Almagestum Novum astronomiam veterem novamque complectens observationibus aliorum et propriis novisque theorematibus, problematibus ac tabulis promotam. Pars posterior tomi primi

New Almagest, embracing both old and new astronomy, advanced by the observations of others and the author's own, and by new theorems, problems, and tables. Volume I, Part 2

[New Almagest, Part 2]

(1651)

by Giovanni Battista Riccioli, S.J. (1598–1671)
Translation by Scott Weisman



Ars Astronomica
2026

"Almagestum Novum astronomiam veterem novamque complectens observationibus aliorum et propriis novisque theorematibus, problematibus ac tabulis promotam. Pars posterior tomi primi (New Almagest, embracing both old and new astronomy, advanced by the observations of others and the author's own, and by new theorems, problems, and tables. Volume I, Part 2)," published in 1651, is in the public domain. All translations, original additions, including illustrations, commentaries, and summaries, are copyright.

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Translator Version 2

Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

The *New Almagest* (Bologna, 1651) is the great Jesuit encyclopedia of seventeenth-century astronomy, by Giovanni Battista Riccioli (1598–1671). This second part, Books VIII–X, turns from the individual heavens to the shape of the whole: comets and new stars, the systems of the world, and a book of practical astronomical problems. Its centre is Book IX, where Riccioli lays out 126 arguments for and against the Earth's motion — the fullest case ever made against Copernicus, set out fairly enough that both sides read it. The first part, with the Sun, Moon, stars and planets and the famous Moon-map, is a separate volume.

A note on the translation process

This translation was produced with the translation-pipeline. See the linked repository for details on the process.

A note on the references. A scriptural or bibliographic reference given in brackets or parentheses is, in most cases, an **editorial identification of an allusion rather than a citation printed in the source**. Books of this kind quote and weave scripture constantly without naming it, and the identification is supplied here as a convenience to the reader. Where the source does print a reference of its own, it is given as printed. **An identified allusion is not evidence that the author cited that passage —** look at the original before resting an argument on one.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

Translated with the assistance of Claude. The translator thanks the Anthropic team for making this work possible.

Notation and abbreviations

A few conventions have been applied uniformly throughout this translation, so that one quantity is always written one way. They change how a value is *written*, never what it is; where the printed source and this translation differ in notation, the source's own form is recorded in the editorial apparatus.

Numerals are Arabic, even where the source sets Roman ones. Roman numerals are kept only in headings, running headers and signatures, which are transcribed as printed.

Dates are given as year, month, day — *1572 November 11*. Dates are Julian, as the source gives them, and have not been converted to the Gregorian calendar.

Times are written with raised unit letters: $7^{h}\ 36^{m}\ 12^{s}$ for seven hours, thirty-six minutes and twelve seconds. An interval longer than a day carries a raised d — $3^{d}\ 7^{h}$. The degree and minute signs below are never used for time.

Angles are written with the conventional signs — 23° 27′ 30″. Early modern astronomy divides the degree sexagesimally and often runs to places for which modern writing has no common symbol; the orders used in this book are:

sign	order	the source's term
°	degrees	*partes*, *gradus*
′	minutes of arc	*scrupula prima*, *minuta*
″	seconds of arc	*scrupula secunda*
‴	thirds	*scrupula tertia*
⁗	fourths	*scrupula quarta*

A **sign** is 30° — one twelfth of the circle — and is written with a raised s: 5^{s} 3° 57′ is five signs and 3° 57′, that is 153° 57′. The source uses the sign both as a place in the zodiac and as a plain unit of 30° for a motion or a distance, and this translation keeps that distinction by writing the count rather than naming a constellation.

The signs of the zodiac. Longitude is reckoned eastward from the first point of Aries.

	Latin	English	longitude
♈	Aries	the Ram	0°–30°
♉	Taurus	the Bull	30°–60°
♊	Gemini	the Twins	60°–90°
♋	Cancer	the Crab	90°–120°
♌	Leo	the Lion	120°–150°
♍	Virgo	the Virgin	150°–180°
♎	Libra	the Scales	180°–210°
♏	Scorpius	the Scorpion	210°–240°
♐	Sagittarius	the Archer	240°–270°
♑	Capricornus	the Goat	270°–300°
♒	Aquarius	the Water-bearer	300°–330°

	Latin	English	longitude
♓	Pisces	the Fishes	330°–360°

The Sun, Moon and planets.

	Latin	English
☉	Sol	the Sun
☽	Luna	the Moon
☿	Mercurius	Mercury
♀	Venus	Venus
♂	Mars	Mars
♃	Iuppiter	Jupiter
♄	Saturnus	Saturn

Abbreviations used in table headings: **Lat.** latitude; **Decl.** declination; **RA** right ascension; **Alt.** altitude.

Where a table in the source spreads one quantity across separate columns for degrees, minutes and seconds, those columns have been joined into a single column and the value written whole. Where a value runs to thirds and beyond, the columns are kept, since there the columns are the notation.

[Front cover of the binding: mottled brown calf, no text.]

[Front pastedown of the binding; no printed matter. A modern pencil note at the head reads "1529 (Rar)", with a pencil stroke running along or through it — a library mark.]

[Front free endleaf of the binding; no printed matter. A modern pencil note at the head, upper right, reads "Rar 947I:1/2 fol." — the library shelfmark.]

Diagram. Engraved frontispiece filling the page within a ruled border; no letterpress. At top centre a hand issues from drapery at the upper right and points to the left; above it stands a Hebrew word, and its three extended fingers carry one word each. Rays fan downward from the hand across the upper half of the plate. In the upper left, putti fly beside a banderole: one carries a globe with the sign ♂ beside it, another a partly shaded sphere with ♀, a youthful face looks out between them, a rayed Sun with a face shines below, and a lower putto holds a shaded disc above the sign ☿. In the upper right, against a dark star-strewn sky, putti carry a body with handles marked ♄, a banded globe in a ring of small stars marked ♃, and a cratered, half-dark Moon; comets streak below them. A second banderole runs across this group. Two inscriptions run along the rays. In the middle ground, on the left, stands a figure whose body is covered with eyes, holding a telescope and pointing upward; on the right a winged woman in a star-strewn robe, with a zodiac belt on which ♏ is legible, holds up a balance by its beam and carries an armillary sphere on her other arm. The beam carries an inscription. The left pan of the balance, riding higher, holds a disc of orbits with the Sun at its centre. The right pan, hanging lower, holds a disc on which a small globe sits at the centre of one circle, with ☾ on that circle, while a Sun with a face stands on a second circle and is itself ringed by smaller circles bearing ♀ and a further sign, with ♂ to the left. Below, an elderly bearded man in a mantle reclines against a cartouche, with an inscription beside his head and his name on the cartouche; the shield in that cartouche shows a star-marked sphere and a globe. At the centre stands a crowned cartouche with a lozengy shield. At lower right a disc of concentric circles lies on the ground, with a small globe at its centre and the signs ♀, ♃ and ♄ near its rim, ☾ on an inner circle and ♂ at the right. The engraver's signature is at lower left inside the border. Below the plate is an oval library stamp.

- Top centre, above the hand: ידך [Hebrew: yadkha, "your hand"] [?]
- On the three fingers: *Numerus*, *Mensura*, *Pondus* — "Number", "Measure", "Weight"
- Banderole, upper left: *Dies diei eructat uerbum* — "Day pours out speech to day"
- Banderole, upper right: *Et Nox nocti indicat scientiam* — "And night shows knowledge to night"
- Ray descending to the left: *Videbo Cælos tuos, opera digitor tuor* — "I shall see your heavens, the works of your fingers"
- Ray descending to the right: *Non Inclinabitur in Sæculum Sæculi* — "It shall not be tilted for ever and ever"

- Along the beam of the balance: *Ponderibus librata suis* — "Balanced by its own weights"
- Beside the reclining man's head: *Erigor dum Corrigor* — "I am raised up while I am corrected"
- On the cartouche beneath his hand: *Cl. Ptolem.* — Claudius Ptolemy
- Lower left, inside the border: *F. Curtus Bon. Incid.* — "F. Curtus [Francesco Curti] of Bologna engraved it"
- Planetary signs on the plate: ♂, ♀, ☿, ♄, ♃, ☾; the sign on each smaller body of the two balance discs and the ground disc is not resolved body by body [?]
- Library stamp below the plate (blue oval): EIDG. POLYTECH[…] […]HULE / […]BIBLIOTHEK [?]; the central device is not read.

[Manuscript, in pencil, upper left: 1529 (Rar)]

AD MAGNANIMVM PRINCIPEM HONORATVM II.

To the Magnanimous Prince Honoré II

Prince of Monaco, Duke of Valentinois, Peer of France, Marquis of Baux, Count of Carladès, Baron of Buis and of Calvinet, Lord of Saint-Rémy, etc.

ALMAGESTI NOVI PARS POSTERIOR TOMI PRIMI.

The Later Part of the First Tome of the New Almagest.

Diagram. An engraved armorial plate, within a plate-mark, beneath the title. A shield of lozenges (fusily) stands at the centre, surmounted by a crown and above it a crowned helm topped with a fleur-de-lis between a plume and a sprig of foliage; heavy acanthus mantling spreads to either side over an ermine-spotted mantle. Two bearded, long-robed figures stand as supporters, one on each side, each raising a sword in his outer hand. A collar of linked chain with scallop shells hangs around the shield, with a small oval medallion and a pendant cross below it at the foot.

- Band of the crown above the shield: a small capital legend, [DEO IVVANTE?] ("with God's help") [?]

Labels read: none securely. Labels not read: the crown-band legend, offered only as a conjecture.

MAGNANIMO PRINCIPI HONORATO II. MONOECI SVPREMO &c.

To the Magnanimous Prince Honoré II, Sovereign of Monaco, etc.

I imagine you were astonished, Magnanimous Prince, that Claudius Ptolemy, risen to life again from a sepulchral pyramid of Egypt, flew from Alexandria into the harbour of Monaco in an unforeseen landing; that soon, having cast the anchors of the deepest trust, he went ashore by stealth; that he eluded the most watchful eyes of your watchtower; and that, not by a creeping climb but by a sudden leap, he made his way up to the height of the Citadel of Monaco, and so into the Court itself. Nay, that he laid hold of the shield of the Grimaldi arms as though it were some Palladium and sacred *ancile*, grasped in a hand so

strong and so tenacious that the hand would not be thought likely to let go of it even if it were cut off!

"What monstrous thing is this?" you say. "From a man unknown, a foreigner, unarmed, half-alive?" Yet this very appearance of grounds for condemning him supplies the argument for pardoning him. He was unknown, unarmed and half-alive: from where could he sooner and better seize a name, arms, a whole life, or even immortality, than from a Prince of royal, and of old even saintly, nobility, of unconquered virtue, and of learning in every field? Horror of the ancient tomb drove him in flight to this refuge. The fear of misfortune, happily turned into boldness, made him seize without hesitation, by usurping it, what he ought to have obtained by entreaty. Having dared things beyond himself, since he had heard the greatest things of You, he took on great courage, for he was about to run to the Magnanimous.

He cast his eyes, moreover, on the hereditary arms proper to your Principality. From their top both the Palm and the Olive put forth leaves — that is, Victory made peaceful, and triumphant Peace — while at the sides, in place of telamons, stand two men of religion, each with his sword drawn: that is, religious Vigour, and armed Religion and Wisdom. Why, then, should our Ptolemy not join his own shield to the shield of so sure and so noble a protection, and entrust to those arms his own — the arms in which the twin globe of Earth and Heaven signifies the wish and the plan to restore Geography and Astronomy together, and to hand them down safely to posterity?

Nor was it without deliberation that I wished this New Almagest, which embraces astronomy old and new — and of it the second volume, but the principal one — to be dedicated and commended above all to You, PRINCE OF MONACO. For turn over for a moment a page or two, and contemplate the figures that stand at the front of the Work. There Argus, with all his lights — nay, all eye — puts the telescope to his knee [genu] rather than to the eye in his cheek [genæ], so that he may not so much see the chief works of God's fingers, namely the Heavens, as venerate them on bended knee. On this side Astraea, herself also jewelled with the most many-eyed stars, weighs the two more recent Systems of the World, balanced by their own weights. In the mobility of the Earth she recognizes an ingenious levity; in its immobility a judgement that outweighs it; and she finds most true what once sounded in her ears from heaven, that saying of David: "It shall not be moved for ever and ever."

Now it was fitting that the system of the world, and the whole weight of this controversy, should be upheld by the authority of that Prince who holds the place of Hercules by hereditary right. For Hercules, not only on account of the Muses, whom the Platonists believed to preside over the nine orbs of the heavens — on account of their company — was called *Musagetes*[1]; he also so restored and advanced astronomy, which was already tottering, that Atlas is

[1] "leader of the Muses"

said to have unloaded the whole sky and the weight of the cosmic sphere onto his shoulders. And yet that same Hercules, as the polymaths report, when he had routed Geryon and withdrawn into this harbour of yours, drove everyone out and lived alone in that citadel, long and much; and so Hercules came to be called Μόνοικος [Greek: Monoikos, "dwelling alone"], that is, the Solitary. What would he do there alone, the leader of the Muses and the nobler heir of Atlas's sphere? Placed nearer the heavens on that summit, would he not refresh himself after the countless labours and wanderings he had endured on land and sea by contemplating the labours of the stars and the wanderings of the planets? From this, then, your harbour here became, and was called, Herculean; from this the citadel of Monaco became, and was called, Herculean — a fact that Strabo, Ptolemy, Pliny the Elder and Tacitus did not pass over in silence, and that the poets, Silius Italicus and Lucan above all, celebrated in song more than once.

Now it is well established that the Grimaldi who is counted fifth in the direct line of descent from Grimoald — brother of Charles Martel and son of Pepin the Fat — was chosen by the Emperor Otto I as one of the chief commanders of his army, and bore himself as so vigorous a hero, first in the storming of Lyon and other cities, then in the siege of Paris, by which he restored Louis IV, King of the Franks, from captivity to liberty, and finally in freeing Italy from the tyranny of Berengar, that he obtained from that Emperor the citadel of Monaco and many neighbouring places in sovereign lordship. He handed down the rights of the dynasty to his descendants, right down to you at the present day; and from then on, in unbroken succession, when one Prince of Monaco was torn away, another has not been lacking.

Among them, your grandfather HONORATVS [Honoré I] and your father HERCVLES [Ercole Grimaldi] so joined martial courage with wisdom and letters that your grandfather seemed to be the arm of the Emperor Charles V in his various battles and his right eye in council, while he sat first among the councillors and envoys of Charles himself and of Philip II; and, cultivated in every kind of learning, he was called the Maecenas of his age. Your father, for his part, when he was already destined for the higher honours of the ecclesiastical order and was making his way towards them through expertise in both laws, was drawn from letters to arms, and was compelled, by a fortunate and noble necessity, to defend the citadel of Monaco from a surprise attack by the enemy, and to weave the siege crown into his scholar's laurels.

You have done nothing, then, foreign to your ancestors if you, HONORÉ, have made your grandfather HONORÉ live again in yourself; if, as son of HERCULES, a most learned Prince, you have held that the ancient Hercules was to be emulated, both in the other disciplines and in the study of mathematics and of the doctrine of the sphere. To these ornaments of intellect were added outstanding piety towards God; towards your subjects, an admirable blend of justice with clemency; an immense greatness of mind alike in adversity and in prosperity; and finally a charm of manner, without loss of majesty, as great

as was once found in Titus Vespasian, who, as we know, was for that reason called the darling of the human race. And it is plainly for these reasons that you have been wished for as a patron not only by other writers but by me as well: that you may show yourself a HERCULES to this work of mine, such as it is, in protecting it, and an HONORÉ[2] in illustrating it and imparting fame to it by your very name. Farewell.

Bologna, on the Day of the Conception of the Blessed Virgin, in the year 1651[3].

Your Excellency's
most devoted and most obedient
Io. Baptista Ricciolus [Giovanni Battista Riccioli], of the Society of Jesus.

EPITOME GENEALOGIÆ GRIMALDÆ GENTIS.

Epitome of the Genealogy of the Grimaldi Family

PROHEMIVM.

Preface

In the dedications of both parts of this volume — namely in the letters to the Most Eminent Cardinal Girolamo Grimaldi and to the magnanimous HONORÉ II, Prince of Monaco — I said that they are of royal stock, and of stock that was once holy. So that no one, then, out of ignorance or an itch to disparage, may put this down to flattery on my part, which has always been abhorrent to my profession and my long-standing habits, or to frivolity in seizing on conjectural arguments of whatever kind, however obscure — something I resolved long ago should be foreign to the good faith, diligence and rigour I have set myself to keep in all my chronological works — I have decided to indicate briefly the foundation of both my assertions.

CAPVT I. De GRIMOALDI, qui Primus Grimaldæ Genti Cognomen dedit, Stirpe, & Propagine.

Chapter I. On the stock and progeny of Grimoald, who first gave the Grimaldi family its surname

All the historians I have been able to see agree that GRIMOALD, surnamed the Just, the legitimate son of Pepin the Fat, sprang from the royal stock of the Franks — indeed, if you consider the female line as well, from imperial stock and from other kings; although a few dissent from most concerning the manner and the series of descent, or concerning the father of Anselbertus, who was the great-great-great-great-grandfather of Pepin the Fat. For according to most of them — especially François de Rosières, Wassebourg, Abbot Bertels and, more or less, Hunibald and Trithemius, in passages to be cited at the end of the chapter — their genealogy ran as follows.

[2]the Latin *Honoratus* also means "honoured"
[3]the feast falls on 8 December

PHARAMVNDVS [Pharamond], who was crowned King of France in the year of the Lord 420, fathered Chlodio by his wife Argotta, daughter of the King of the Cimbri, and died in 430.

[Margin: Chlodio.]

CLODIO, or Clogio, surnamed the Long-haired [Capillatus], King of France — so called because he ordered the Franks to grow their hair as a sign of liberty, the hair that the Romans had wanted shorn as a mark of servitude — married Basina, daughter of Vuidelphus, petty king of Thuringia, by whom he fathered Phriso, Albericus, Raucherius and Reginaldus. But when Phriso died in a certain battle, Chlodio contracted a fatal illness from grief and died in 447, having first declared Meroveus, or Merovingus, Master of the Horse and guardian of his sons.

[Margin: Albericus.]

ALBERICVS, or Albero as others name him, when he had grown older recovered Alsace, Brabant, the Moselle region and almost all of East Francia — partly by defeating Meroveus, his guardian, who had thought that a good part of the kingdom was justly held by himself, and partly by his yielding. He married Argotta, daughter of Theodemir, King of the Goths, and sister of Theodoric, by whom he fathered Vuabertus, or Vuambertus.

[Margin: Vuabertus.]

VVABERTVS, King of East Francia and Prince of the great Ardennes and of Hainaut, enriched with Aquitaine by inheritance from his mother, married Lucilla, daughter of Zeno the Isaurian, Emperor of Constantinople, by whom he had Vuabertus II, who was to die in 528, and Anselbertus, or by contraction Ansbertus.

[Margin: Anselbertus.]

ANSELBERTVS, since he defended the rights of the Empire, was made a Senator by the Emperor, Marquis of the Holy Empire over the Scheldt, and Lord of the Moselle region — provinces that Chlothar I, King of France, had usurped. The marquisate already mentioned comprised Antwerp, Ghent, Tournai, Dendermonde, Mortagne, Condé, Valenciennes, and all the fortresses on both banks of the Scheldt. Nor did Chlothar dare to resist, for fear of the Emperor and of the King of the Goths, through his connection by marriage to whom Anselbertus was powerful; on the contrary, he gave him in marriage his own Blithild, daughter of Clotilde II (for Clotilde I was the saint who married Clovis I and converted him to the faith of Christ).

[Margin: The opinion of more recent writers on the origin of Anselbertus.]

But Scaevola and Aloysius de Sainte-Marthe [Scévole and Louis de Sainte-Marthe], in the *Genealogical History of the House of France*, say that from Ferreolus I, Praetorian Prefect of Gaul under Honorius about the year 400, a native of Auvergne, and from Papianilla, daughter of the consul Syagrius, was born Tonantius Ferreolus II, Senator and Praetorian Prefect of Gaul; that he, by a daughter of the Emperor Avitus, fathered Tonantius Ferreolus III, Senator; that this man in turn had, by Industria, daughter of Clovis I, Anselbertus, Senator and

Duke of Gallia Narbonensis; and that Anselbertus married Blithild, daughter of Chlothar I. It does not seem to me that the authors already mentioned can be reconciled except by saying that Anselbertus was the natural son of Vuabertus but the adopted son of Ferreolus III, who adopted him for lack of natural children, on account of his connection by marriage with Clovis and the proximity or intermingling of rights in Gallia Narbonensis, which was once comprised under Auvergne or Aquitaine.

[Margin: Saints Ferreolus and Condulphus.]

[Margin: Saint Tarsita.]

However that may be, by the consent of all, Anselbertus fathered by Blithild, daughter of Chlothar I, King of the Franks, five children: namely Arnold-Anchises, or Ansegisus; St Ferreolus, or Ferrioldus, Bishop of Traiectum; St Condulphus, pastor of the Tungri; Modericus, Bishop of Auxerre; and St Tarsita, who by others is also called Tarsicia and Ulsida. Anselbertus died in 567, as Vignerius has it, or rather in 570, as most have it.

[Margin: Arnold-Anchises.]

ARNOLD-ANCHISES, or Ansegisus, or, as others call him, Arnold-Buggisus, Marquis of the Holy Empire over the Scheldt, Lord of the Moselle region, Count Palatine and Duke in Austrasia, married Oda, daughter of Cunzo, or Guizo, Duke of Swabia, by whom he fathered St Arnulf; and he was so well received by Gun- tram, King of Orléans, that Guntram wanted to make him heir and successor to the kingdom. But Arnold-Anchises refused, and withdrew from the cares of the world into solitude, having built the monastery of St Martin at Longamuilla, in which he lived devoutly, died and was buried, in 601.

[Margin: St Arnulf.]

ST ARNVLPHVS [Arnulf], Marquis of the Holy Empire over the Scheldt, Lord of the Moselle region, Count [?] of Namur, and then under Chlothar II Count Palatine, Master of the Horse, Mayor of the Palace in Francia and Austrasia, and tutor of King Dagobert, whom Chlothar II had had by Queen Bertrude — urged by the nobles to propagate his line, and dispensed from his vow of chastity, married Doda, daughter of the Duke of Saxony or of Swabia, or, as others say, of the Count of Boulogne, whom Trithemius and Baronius call a saint.

[Margin: St Doda.]

[Margin: St Clodulphus.]

[Margin: St Vuandregisilus.]

By her he fathered St Clodulphus, Ansegisus II, and Valchisius, Count of Verdun and father of St Vuandregisilus, the abbot who was to die on 789 July 22. When Doda died, Arnulf withdrew from court to the monastery of Lérins, where he gave himself to the contemplation of divine things and to theology; but from there, through the eagerness of the people and of Pepin, Mayor of the Palace, and by the authority and almost the force of Chlothar II, he was called back and made Bishop of Metz, and Chlothar was present with his son

Dagobert at his consecration in 620. Moreover, as before, so afterwards the kings of France undertook nothing without consulting him or Pepin, and that kingdom was therefore at its most flourishing then. One Notto, who slandered St Arnulf because the Queen had repeatedly come to his house at night to seek counsel, God punished with a shameful penalty, and the King himself with the loss of his head.

[Margin: St Gauricus.]

[Margin: St Arnulf's generosity repaid by God.]

At length St Arnulf obtained from Dagobert, who was long unwilling, leave to resign his bishopric to his cousin St Guaricus and withdraw again into solitude. Having already sold off even the sacred vessels for the use of the poor, he asked his sons for leave to sell and distribute to the poor the goods that were coming to them by inheritance. Clodulphus, the eldest, refused for his own share; Ansegisus II consented. St Arnulf then prophesied that his house would come to possess far more goods than he himself could leave to his sons, and he blessed his whole posterity. Not long afterwards, in the 18th year of his episcopate, he withdrew into solitude; there, when he was about to die two years later, he summoned Ansegisus and ordered him to marry St Begga, daughter of Pepin the Elder, and then died a most holy death on 640 July 18, or in 641.

[Margin: St Clodulphus.]

His cousin and successor in the bishopric, St Guaricus, died in 654, and was succeeded at last in the same bishopric by St Clodulphus, son of St Arnulf. Though at first Clodulphus had refused his father consent to the distribution of the goods to the poor, he afterwards emulated his father's holiness with such ardour that St Remaclus, Bishop of the Tungri, called him a man full of faith and of the Holy Spirit, and sent St Trudo to him as to an oracle, as Abbot Theodoric relates in the life of St Trudo; and he died a holy death on June 8. But for our purpose it is necessary to insert a little about Pepin, the father of St Begga.

De Pipino Seniore, S. Beggæ Patre.

On Pepin the Elder, father of St Begga

[Margin: Pepin of Landen.]

[Margin: St Allowin Bavo, Pepin's cousin.]

PIPINVS of Landen [à Lauda], or the Elder, Duke of Neustria and Brabant, was the son of Carloman, paternal cousin of Agilulphus, Count of Hesbaye. This Agilulphus, by Adelthrudis, a woman of outstanding uprightness, fathered St Allowin Bavo, famous for his pilgrimages and his hermit's life — even to the point of shutting himself into a hollow tree, and afterwards into the prison of a very narrow cell. In 631, dying a holy death on October 1 in the monastery of St Peter at Ghent, which he himself had built at the urging of St Amand, Bavo appeared to St Gertrude, asking her to send linen cloths to Ghent to bury his naked body.

When St Arnulf first withdrew to the monastery of Lérins, Pepin was Mayor of the Palace of Francia, and in Austrasia, under Chlothar the Great, Dagobert I and Sigebert II. Remarkable are the things proclaimed of his prudence, faith, justice and piety by Abbot Stephen in the life of St Modualdus, and, in Surius, by the ancient author of the life of St Vuiro, the bishop whom Pepin used to summon to the deliberations of the kingdom and to whom he used to go barefoot for holy confession [exhomologeſis].

[Margin: St Itta.]

But this Pepin was most illustrious for his kinship with saints. For his wife was St Yduberga, called by most St Itta, or Ida. Sought by many nobles for a second marriage on account of the singular elegance of her manners, her nobility and her wealth, she preferred, at the urging of St Amand, Bishop of Traiectum, to build the monastery of Nivelles in Brabant under the protection of St Peter, where, having taken the religious habit, she died a holy death on 658 March 8.

[Margin: St Modualdus.]

The brother of St Itta was St Modualdus, made Bishop of Trier at Pepin's urging and regularly summoned, though unwilling, to the business of the kingdom. Having built on the Moselle the convent of St Symphorian the Martyr, he set over it as abbess his own and Itta's sister, St Severa the Virgin; and after she died on July 20, he too died, on May 12, sighing all the more ardently for heaven.

[Margin: St Severa, Virgin.]

[Margin: St Vuandalinus.]

[Margin: St Begga.]

[Margin: St Gertrude.]

[Margin: Blessed Pepin.]

His brother was St Vuandalinus, a monk, or rather a distinguished hermit, after whom a town in Belgium is named, as Trithemius relates. Pepin, further, by St Itta fathered Grimoald; St Begga, of whom more below; and St Gertrude, the most celebrated virgin, whose life is extant in Surius under March 17, the day on which she died a most holy death in the 33rd year of her age. Pepin died in 646 — the author of the life of Blessed Pepin says more of him — and was succeeded in the office of Mayor of the Palace of Austrasia under Sigebert II by Grimoaldus, or, as Trithemius puts it, Grimaldus. On him see Petavius [Pétau] in the *Rationarium temporum*, book 8, chapter 2; for we must return to our former series.

De S. Arnulphi Succeſſoribus vſque ad Theobaldum.

On the successors of St Arnulf down to Theobald

ANSEGISVS II, son of St Arnulf and brother of St Clodulphus, bishops of Metz, by order of his father married St Begga, daughter of Pepin of Landen, by whom he had, in the town or village of Heristellum, or Heristallum, Pepin the Fat, called from it "of Herstal". But he had brought up a little foundling

called Goduinus, found on the road, and enriched him with wealth, and was killed by him while hunting, in 680, in the hope of seizing the principate.

[Margin: St Begga, widow.]

When his funeral was over, St Begga, a woman of lofty spirit and greater than her sex, gave herself wholly to imitating the holiness of her sister Gertrude and of Saints Arnulf and Allowin. She set out for Rome and, furnished by the Supreme Pontiff Adrian with holy relics and his blessing, returned home and built at Adana on the Meuse [Andenne] the Monastery of the Seven Churches, in honour of the same number of churches at Rome; and in it she died a holy death on 698 December 17. Goduinus, however, by no means gained the principate of Ansegisus; Pepin did.

[Margin: Pepin the Fat.]

PIPINVS of Herstal, surnamed the Fat, or the Younger, Mayor of the Palace of France and Duke of Lotharingia, or Austrasia, ruler of the kingdom and in fact, though not in name, its king, had entered on that dignity as early as 687, but did not hold it in peace until 691, after the death of Ebroin and the defeat of Theodoric. This Pepin married Plectrude, daughter of Grimoald, Duke of Bavaria, or Boiaria, by whom he fathered Drogo, surnamed Bosso, Count of Champagne and Duke of Burgundy, and Grimoald, surnamed the Just, as Ado of Vienne says, who was put in his ageing father's place as Mayor of the Palace of France.

[Margin: St Hugh.]

For Drogo, his first-born, after fathering by Adeltrudis, or Austrudis, daughter of Varato, among others Arnold, Duke of Burgundy, and St Hugh, Bishop of Rouen, died either in 699, as Baronius reports from Sigebert under the year 698, or in 708, as Scaevola and Aloysius de Sainte-Marthe say. Of Grimoald's killing we shall speak shortly.

[Margin: Divine vengeance on concubines.]

Pepin, moreover, took Alpais, or Elpais, as his concubine, and put away his lawful wife Plectrude from court and bed. When St Lambert, Bishop of Tongeren, repeatedly rebuked him for this, Alpais had Lambert killed by her brother Dodo at the estate of Liège [villa Legia] as he prayed at the altar of Saints Cosmas and Damian. But Dodo, vomiting up his rotted entrails through his mouth and unbearable to everyone for the stench, was thrown into the Meuse; and while others flocked to the body of St Lambert the Martyr to venerate it, concubines alone were driven back by some hidden force, so that they could not come near to kiss the sacred relics.

Pepin did indeed grieve at St Lambert's death. Yet when he lay mortally ill at Jupille [Iopiliæ] in 714, Plectrude, who was living at Cologne, begged him, through St Suitbert of Vuerda and Agilulphus of Cologne, both bishops, not to deprive the children of his legitimate sons — namely of Drogo and Grimoald — of their inheritance, to the ruin of his soul and the scandal of the kingdom. He nevertheless heard those bishops on the matter with indignation, and chose to leave as his heirs Charles Martel and Childebrand, whom he had had by Alpais.

[Margin: Plectrude's epitaph.]

When he died, however, on 714 December 16, Plectrude, Duchess of Lotharingia, held Charles Martel in custody and fostered her grandson Theobald, Count Palatine and of Gallia Belgica. But when Charles Martel escaped, and defeated and expelled both her and Theobald, she died at Cologne and was buried in the church of the Blessed Virgin on the Capitol, which she had built together with a monastery; and this epitaph was inscribed for her, in perpetual memory of her lawful marriage:

> *Plectrude, joined to Pepin in lawful wedlock, bore Bosso, and the great duke Grimoald.*

[Margin: Why Charles Martel was so called.]

Charles Martel, or Tudes, was so called because, with incredible speed and impetus, he inflicted defeats on the Saracens and other barbarians and beat them down like a hammer falling on them. He fathered Pepin, surnamed the Short, who was long the administrator of the kingdom and at length, because of the inertia of the kings, was elected King of France by the people and confirmed by Pope Zachary. He transferred the kingdom of the Merovingians to the Carolingians, and was the father of Charlemagne [Carolus Magnus], King of the Franks and Emperor of the Romans.

[Margin: Grimoald the Just.]

GRIMOALDVS [Grimoald] the Just, second son of Pepin the Fat by Plectrude, was Count of Gallia Belgica, Mayor of the Palace of France and of Austrasia, Prefect of the Gallic cavalry, and director of the kingdom under Childebert II. He married Theodosinda, daughter of Ratbodus, or Radbodus, petty king of Frisia, and by her fathered Theobald. But whether through the plotting of Theodosinda, burning with jealousy, or of Radbodus, it is certain that Grimoald — having come to Jupille [Iopiliam] to visit his father Pepin, and having turned aside to Liège — was killed while praying in the basilica of St Lambert the Martyr by Rangarius, an assassin of Radbodus, in the month of April, that is, seven months before Pepin's death, in the year 714. Radbodus, for his part, died a most wretched death in 719.

[Margin: Theobald.]

THEOBALDVS [Theobald], son of Grimoald the Just and of Theodosinda, was Count of Gallia Belgica and Prefect of the Gallic cavalry and of the palace under Childebert II and Dagobert II, Kings of France. But Charles Martel feared him as the lawful successor, passed over in Pepin's will; so, with the help of Ramanfredus, who had been put in Theobald's office, and with his own forces, he drove Theobald out of Gaul. Theobald therefore fled with his mother to Pelagius and Alphonsus, Kings of Spain, in 723. He fought vigorously for them against the Moors and Saracens and recovered Salamanca [Salmanticam], and for that reason was given several towns in that province. Among them is a town or village celebrated for the monastery of St John, founded by Theobald and Theodosinda, on whose walls these inscriptions are read even today:

To the once-fortunate Count of Belgica, I. N. Y., and Imp. C. M. F. R. drove him out in the year 722.

That is, Charles Martel, King of France, drove him out. Theobald also built a town there with a citadel, to which he gave a name taken from his father's name, and he wished his descendants to be named from it, that name being turned, as a patronymic, into the surname of his family. When Charles Martel died in 742, Pepin and Carloman succeeded to the principate. When the Franks, weary of the sloth of King Chilpericus, surnamed the Stupid, had elected Pepin king, Pepin soon recalled Theobald to Gaul, about the year 751, and raised him to high offices. Theobald, for his part, having put the Saracens to flight out of Provence [Gallo-Prouincia] and taken Antibes [Antipolim] by storm, obtained the lordship of Antibes. He married Aliarda Clovisia, daughter of the Duke of Orléans, and by her fathered Hugo, of whom we shall speak in chapter 2, and Rainerus Grimaldus, lord of the town of Grimaldi, of Santiuanez and of other places in Spain, a vigorous commander against the Saracens under Alphonsus, who made him perpetual Governor of Salamanca. From him the Spanish Grimaldi were then propagated, who held the fief of Grimaldi until the year 1370. In that year the male line failed; and the female line surviving in Hurraca Bermudez, wife of Petrus Tressius, lord of the citadel of Almofrague, that lordship passed to the Tressius family, from which came Cardinal Tressius.

Auctores ex quibus hæc collecta ſunt.

The authors from whom this has been gathered

What has been said so far we have not gathered wholly from each source; rather, some from some and some from others. Passing over the older chronologers and historians — Fredegar [Fridegario], Flodoard, Aimoin, Sigebert, Ado of Vienne and Otto of Freising — the chief authorities here were: Hunnibaldus, and Trithemius following him, in the *Annals on the Origin of the Franks*; François de Rosières, on the lineage of the Dukes of Lorraine and Bar, whose book is a library in itself; Vignerius; Fauchetus [Claude Fauchet]; and the fragments of the annals of France and of the Mayors of the Palace of France, edited by Pithoeus [Pierre Pithou]. Also: Paulus Vuarnefridus [Paul the Deacon], deacon, chancellor of Desiderius, King of the Lombards, and contemporary of Charlemagne, *On the Deeds of the Lombards*, book 6, chapter 5; the Metz manuscript cited by P. Mauritius in the *History of the Bishops of Metz*; the author of the life of Blessed Pepin; Marcellinus the Priest, a contemporary of Pepin of Herstal, in the life of St Suitbert, in Surius under March 1, from chapter 22; Stephen, Abbot of Liège [Leodiensis], in the life of St Modualdus, in Surius under May 12; Theodoric, Abbot of St Trudo, in the life of St Allowin Bavo, in Surius under October 1; likewise four ancient writers in Surius — of the life of St Gertrude, March 17; of St Vuiro, May 8; of St Vuandregisilus, July 22; and of St Trudo, November 23. Vincentius Lirinensis, in the *Speculum*, book 23, chapter 72; Bede, tome 3, in the acts of St Arnulf; Molanus, in the *Feasts of the Saints of Belgium*; Baronius, in the notes to the Martyrology for the

days March 1 and 17, May 8 and 12, July 20 and 22, October 1, November 23 and December 20, and besides in the *Annals*, tome 8, for the years 614, 615, 626, 631, 644, 698 and 714. Io. Bertelius [Jean Bertels], Abbot of Echternach, in the *History of Luxembourg*; Ambrosius Morales [Ambrosio de Morales], chronicler of Philip II, King of Spain, tome 3, book 13, chapter 16; Wolfgang Lazius [Vuolfangus Lazius], *On the Migrations of Peoples*, book 3; Consaluus Argona, *On the Nobility of Andalusia*, book 11, chapter 20; Salazar Mendoza, *On the Origin of the Dignities* [?] of Castile and León, book 1, chapter 7; Io. Serranus [Jean de Serres], in the *Inventory of French History*; Regino, *Annals*, book 1; Dionysius Petauis [Pétau], *Rationarium temporum*, book 8; Petrus Frizon [Pierre Frizon], in the *Gallia Purpurata*; Scévole and Louis de Sainte-Marthe, twin brothers, counsellors and ordinary historiographers of the Most Christian King, in book 6 of the *Genealogical History of the House of France* and in the tables prefixed to book 1; the manuscript of Cardinal Tressius preserved in the archive of Monaco; and Carolus de Venasque Ferriolus, Doctor of Both Laws, who in 1647 published at Paris the *Genealogical and Historical Tree of the Grimaldi Family*. Among other very substantial witnesses, he cites at the end of the work a testimony that could stand for all of them: that of the Most Christian King, Louis XIV, who expressly acknowledges the Grimaldi Princes of Monaco to be of the stock of the Kings of the Franks, through Pepin, father of Grimoald and of Charles Martel, and attests the same in these very words:

[Margin: Testimony of the Most Christian King for the Grimaldi family.]

[French] *Our very dear and well-beloved cousin the Prince of Monaco, Duke of Valentinois, Peer of France, Knight of our Orders, who is today recognized as the stock of the house of Grimaldi, and who has rendered a signal service to the State by placing his person and that of his children, his fortress, his goods and his subjects under our protection, and by renewing the ancient alliance which his ancestors had made with France — from which even the predecessors of the Princes of Monaco, who formerly held the greatest offices of the Crown, took their origin, through one Grimoaldus, Mayor of the Palace, son of Pepin the Younger and brother of Charles Martel; and in order also to show that we wish the nobility and antiquity of this house to see the light, so as not to deprive the public of so curious a work: etc.*

CAPVT II. De Grimoaldi ac Theobaldi ſucceſſoribus Monœcique Principibus vſque ad Honoratum II.

Chapter II. On the successors of Grimoald and Theobald, and on the Princes of Monaco down to Honoré II

[Margin: Hugo Grimaldus.]

HVGO Grimaldus, lord of Antibes and of other cities and places in Provence, first-born of Duke Theobald, aided the Emperor Charlemagne in command of his army, above all when Charlemagne restored Pope Leo III, oppressed by the Roman nobles, to his see in the year 800. Hugo married the sister of the

Archbishop of Milan, and by her fathered Passanus and Theobald — Thaddus for short — Archbishop of Milan, who died renowned for holiness in 876 and was buried in the church of St Ambrose. So say Nicolaus Grimaldus and Carolus de Venasque in the *Genealogy of the Grimaldi*, and Io. Franciscus Besozzius in the *History of the Pontiffs of Milan*, page 109. Antipolis, moreover, is in French Antibes, between Nice and Forum Iulii, or Fréjus [Freiuls].

[Margin: Passanus.]

PASSANVS, lord of Antibes etc. and Grand Master of France, fathered by his lawful wife Grimaldus and Bombellus Artisius. So Nicolaus and Carolus Venasque in the Genealogy already cited.

[Margin: Grimaldus I.]

GRIMALDVS I, lord of Antibes etc., and afterwards created the first sovereign Prince of Monaco by the Emperor Otto I, because he had borne himself vigorously in command of the army when, after Lyon and other cities had been taken, he freed Louis IV, King of the Franks, merely by the sound of arms in besieging Paris [Lutetiam], and then restored peace to Italy. He obtained the perpetual lordship of Monaco for himself and his successors, as appears from the *Register of Imperial Privileges*. He married Crispina, daughter of Rollo, or Robert, first Duke of the Normans. So Nicolaus and Carolus Venasque in the Genealogy cited, and Fa- nus. Campan.; the manuscript *On the Illustrious Families of Italy*, which is extant in the Ambrosian Library at Milan, chapter 14.

[Margin: Giballinus.]

By Crispina, further, Grimaldus fathered Guido, of whom below, and the hero Giballinus. After the Saracens had been driven out of Provence in the year 980, Giballinus was given by William I, Count of Provence, as the reward of a signal victory, the Sambracitan Gulf [Sinu Sambracitano], or Riuus S. Torpetis, with the places adjoining. Because of that victory and that grant, the gulf is called to this day the Gulf of Grimaldi [Sinus Grimaldus], and it extends in length to about 5 leagues, or 15 Italian miles. The authentic deed of this grant is extant in the acts of the church of Fréjus [Foroiuliensis], and in Carolus de Venasque, page 9. This Forum Iulii, moreover, is in Gallia Narbonensis — in French Friuls — between the Gulf of Grimaldi and Antibes.

[Margin: Crispinus, head of Branch 1.]

Grimaldus also fathered Crispinus, surnamed Ansgothus because on his mother's side he was descended from the Goths, from whom came the first Branch of the Grimaldi family, of which we shall speak in chapter 3. So Nicol. Grim. and Carolus de Venasque in this Genealogy, William of Jumièges [Guillelmus Gemeticensis], Orderic Vitalis [Odericus Vitalis], and others to be cited below.

[Margin: Guido.]

GVIDO Grimaldus, first-born of Grimaldus, sovereign of Monaco and lord of Antibes etc., assisted the Emperor Henry the Bavarian in many battles with a great number of ships and galleys of the Republic of Genoa and some of his

own, and withstood Stephen, King of Hungary. He married Rolanda, daughter of the Count of Alençon [Alanconij], by whom he fathered Prince Hugo.

[Margin: Hugo.]

In the year 1070 Hugo ordered the ears and nose of Antinopus, one of the captains of his galleys, to be cut off, because Antinopus had tried to carry the body of St Devota, Virgin and Martyr, away from Monaco to Florence — though in vain, since (wonderful to say) that ship could not by any force get out of the harbour of Monaco. Guido also fathered Grimaldus II, and Alphanius, Bishop of Apta Iulia [Apt] in Provence, who in 1056 took part in the Council of Toulouse with the nuncios of Pope Victor II, and died in 1065. So Nicol. Grim. and Carol. Venasque in the Genealogy already mentioned, and the catalogue of the bishops of Provence.

[Margin: Grimaldus II.]

GRIMALDVS II, when his brother Hugo died without children, succeeded to the sovereign principate of Monaco and the lordship of the Gulf of Grimaldi, on whose shore he built the tower and town of Grimaldi, in which the arms of the Grimaldi are still to be seen. Having formed an alliance with Robert Guiscard, the Norman, Duke of Apulia, he set free Pope Gregory VII, besieged by Henry III in the Castel Sant'Angelo [arce S. Angeli] at Rome. Afterwards he accompanied Hugh, brother of Philip I, King of the Franks, to Syria, and won famous victories over the Saracens with him. He was the first of the Grimaldi to live in Genoa, and in 1084 married Alexia, niece of the Emperor Alexius of Constantinople. By her he fathered Guido II; Theobald, Cardinal of the Holy Roman Church of Saints John and Paul, of the title of St Pammachius, created by Urban II; and Luca, one of the nobles who went to the war of Jerusalem with Godfrey of Bouillon [Gothofredo Bullionio] and was envoy of the Republic of Genoa to the Supreme Pontiff. From Luca came Luca, Cardinal of the Holy Roman Church of Saints John and Paul, of the title of St Pammachius, created by Innocent II; and Balduinus, shield-bearer of the King of Jerusalem and admiral [Hamiralus] at the capture of Almería and Tortosa in 1147 and 1148. Grimaldus II also fathered Hugo, commander of a great many ships and galleys for the Republic of Genoa in the holy war of Jerusalem, from whom came Guido, Cardinal of the Holy Roman Church of the title of St Balbina, created by Paschal II. So Nicol. Grimald. and Carolus de Venasq. in the Genealogy cited; Platina, under Urban II; Laurentius Pignor. [Lorenzo Pignoria], in the notes to Torquato Tasso; Io. Lup Sard., in the *Epitome of the History of Genoa*; and Platina, under Paschal II and Innocent II.

[Margin: Guido II.]

GVIDO II, Prince of Monaco, lord of the Gulf of Grimaldi etc., admiral of the Emperor Henry IV and Governor of Como [Comi], fought fiercely against the Milanese in 1120. He married Arasima of Carretto [à Carrecto], daughter of Boniface, Marquis of Savona and Finale and Vicar of the Holy Empire; and then a daughter of Fulco de Castro, admiral of the Republic of Genoa. By them, among very many children, he fathered Grimaldus III; Vmbertus, Bishop of

Colonia Iulia Pacensis Augusta and Rector of the church of Fréjus in 1183; Boso, Abbot of Lérins in 1151; and Otholinus, who married Ludovica of Lorraine and was prefect of the crossbowmen of Louis, King of the Franks, and Henry, King of the English, in aid of Pope Alexander III against Frederick I. So Nicol. Grimald. and Carol. de Venasq. in this Genealogy, the catalogue of the bishops of Provence, and the Chronicle of Lérins.

[Margin: Grimaldus III.]

GRIMALDVS III, Prince of Monaco, lord of the Gulf of Grimaldi etc., was one of the army commanders with whom the Emperor Lothar II defeated Roger, Duke of Apulia, an enemy of the Holy See, and drove him into Sicily. As Consul of Genoa in the years 1162, 1170 and 1184 he enclosed part of the city with walls; as admiral of the same Republic he defeated the Saracens and the Pisans with many galleys of his own; and he was twice its envoy to the Emperor Frederick Barbarossa [Ænobarbum]. He married Antonia Pia, daughter of the Count of Provence, by whom he fathered, among others, Obertus; and then Petrus, Bishop of Vintium Nerusiorum [Vence] in Provence, who died in 1210; Fridericus, Prefect-General of the Arms of the Holy See and counsellor, chamberlain and grand master of the crossbowmen of the King of England; likewise Rossus, Consul of Genoa in 1173; Raymundus, Bishop of Antibes in 1190; Guido, Consul of Genoa and envoy to the Republic of Venice, who married Francisca, daughter of the Marquis Malaspina; and Passanus, Seneschal of Normandy. So Nicol. Grim. and Carolus Venasq. in this Genealogy, the catalogue of the bishops of Provence, the archive of Vence [Vintij], Iustinianus [Giustiniani] in the *Annals of Genoa*, and Vbertus Folieta [Uberto Foglietta].

[Margin: Obertus.]

OBERTVS, Prince of Monaco, lord of the Gulf of Grimaldi etc., Baron of San Demetrio in Calabria, and lord of Carrosio and Aymerium in Liguria, was grand master of the palace of the Emperor Frederick I, and was sent by him as envoy to Philip of France and Richard of England, Kings, for the pilgrimage and war of Jerusalem in 1188. In the same year, with his father-in-law Obertus, he built the church of St Luke at Genoa for himself and his successors. Then, in 1197, he was Consul of Genoa, admiral, and again envoy to the Kings of France and England. He married Corradina Spinula [Spinola], daughter of Obertus, lord of Savenio, by whom he had first Grimaldus IV; then Ingo, from whom come the Dukes of Eboli [Euboli], the Grimaldi Cavallerones, and the Grimaldi who live in Bologna;

[Margin: Ingo, head of Branch 2.]

[Margin: And Obertus, of Branch 3.]

and Obertus, from whom come the Grimaldi of Castrum Novum [Castri noui] and of Gattières; and Luchineta, who married Iacobus à Iudicibus, nephew of the Emperor Michael of Constantinople, grand chamberlain and Prefect-General of the army against the Saracens; and Guido, lord of the Gulf of Grimaldi, who came to an agreement with the Genoese on fortifying the citadel of Monaco in 1215 and married Iolanda of Savoy; and Carolus Isnardus,

Marquis of Gavi and admiral of the Republic of Genoa; and Nicolaus, one of the Eight Nobles governing Genoa in 1235 and envoy to the Republics of Florence and Lucca against the Pisans in 1255, who married Argentina, daughter of Nicolosinus ab Auria [Doria], and was head of Branch 4;

[Margin: Nicolaus, head of Branch 4.]

and Renatus, Governor of Provence and master of seven galleys, who married Maria, sister of Ludovicus de Claromonte [Louis de Clermont], a prince of the royal stock; and lastly Adelaide, wife of Pyrrhus Ruffus, Count of Sinopoli and of Santa Cristina. So Nicol. Grim. and Carol. Venasq. in this Genealogy, Iustinianus in the *Annals of Genoa*, the archive of Genoa, and Vbertus Folieta.

[Margin: Grimaldus IV.]

GRIMALDVS IV, Prince of Monaco, lord of the Gulf of Grimaldi etc. and Baron of San Demetrio, was Prefect-General of the Genoese fleet at the capture of Damietta in 1219, and one of the Eight Nobles governing Genoa in 1232 and 1244 — the year in which he received in his own house Pope Innocent IV on his way to France. He married Orieta, daughter of Merlus de Castro, and, when she died, a daughter of the Count of Modigliana [Modilianæ]. By them he fathered Franciscus; then Devotus, Bishop of Grasse in 1263; and Luchettus, lord of Petraelata and Stella and co-lord of Clausana, who, after Ventimiglia [Intemelio] had been wrested from the Ghibellines, governed Genoa as one of the Eight Nobles in 1237, was Praetor of Milan in 1242, and, twice admiral of the Republic of Genoa, was victor over the King of Armenia and the Venetians in 1267. Grimaldus IV also fathered Alexia, who married Philippus Medicaeus, Governor of Florence; and Lanfrancus, one of the Eight Nobles governing Genoa in the years 1240, 1246 and 1249, and Prefect-General of Piacenza. He also fathered Gabriel, a vigorous warrior against the Saracens under the Emperor of Constantinople, lieutenant of Charles I, King of Naples, and Grand Master of the Royal Household, by whom he was honoured with a military belt of 250 pounds of gold in 1269. This man had as sons Iacobus, chamberlain of Charles I, King of Naples, and Guigno, one of the three sent to Rome by the whole family to stipulate peace between the Grimaldi and the Fieschi [Fliscos], Guelfs, and the Doria [Auriacos] and the Spinola, Ghibellines. Present were Bertrand, Cardinal Bishop of Sabina, legate *a latere*, Philip, Emperor of Constantinople, and Charles I, King of Naples, as appears from the instrument executed at the Capitol in the chambers of Charles himself on 1276 June 18, and drawn up by Bartholomaeus de Regio, notary. For the rest: Nicol. Grim. and Carolus Venasque in this Genealogy, the Register of Fréjus and the Register of Charles I in the archive of Naples, Iustinian. in the *Annals of Genoa*, and Vbertus Foliet.

[Margin: Franciscus.]

FRANCISCVS, Prince of Monaco, lord of the Gulf of Grimaldi etc. and Baron of San Demetrio, counsellor and chamberlain of the Emperor of Constantinople, vicar of Charles I, King of Naples, in the County of Provence and of Nice, and his admiral, was delegated by the Emperor of Constantinople to negotiate peace between the Kings Charles II of Naples and James of Aragon

and Sicily. In 1272 he was joined to Aurelia of Carretto, sister of the Marquises of Savona and Finale, by whom, above all others, he fathered Raynerus; then Andaro, Baron of Boleum and lord of Vallis Massae, from whom came Branch 6; and Grimaldus

[Margin: Andaro, head of Branch 6.]

Grand Seneschal of Rhodes, master of many galleys and a warrior against the Turks in 1318; and Antonius, commander of the seven galleys that the Grimaldi family sent against the King of Sicily in support of Charles II, King of Naples, and husband of Antonia Spinella. So Nicol. Grim. and Carolus Venasq. in this Genealogy, and the archives of Naples and of Boleum.

[Margin: Raynerus.]

RAYNERVS, Prince of Monaco and Baron of San Demetrio, Grand Master of the palace [Regiæ] of the Emperor of Constantinople, and envoy of the King of the Franks to the King of Jerusalem and Sicily, aided Charles II, King of Naples, with ten galleys of his own against the Sicilians; for this and for his distinguished exploits he received a military belt weighing 260 pounds of gold. He married Speciosa of Carretto, daughter of Iacobus, Marquis of Finale, by whom he fathered Hugo, sovereign of Monaco, who died without children; Raynerius II; then Guigno, commander of 23 galleys for the Grimaldi family in aid of Charles II, King of Naples, against the Ghibellines who were occupying Monaco, whose son Greppus was guardian of Charles the Great, Prince of Monaco, in 1344; Aymo, commander of a great force of crossbowmen from the Grimaldi and the Fieschi in support of the King of Spain against the Saracens, whom he successfully routed; Rogerius, knight of the chamber [cubicularium Equitem] of Boniface VIII, in 1296;

[Margin: Bartholomaeus, head of Branch 7.]

and Bartholomaeus, or Bertonus, commander of the Grimaldi fleet of galleys in aid of Robert, King of Naples, and his viceroy in Calabria, from whom came the seventh branch. At that time Franciscus, son of Guillelmus, Raynerus's brother — called Massa from the strength of his body — recovered the citadel of Monaco, which the Spinola had occupied, in 1317. So Nicol. Grim. and Carolus Venasq. in this Genealogy, Cor. in the *History of Milan*, and the Register of the archive of Naples.

[Margin: Raynerus II.]

RAYNERVS II, Prince of Monaco, lord of Cagnes [Cagniæ], Villeneuve [Villænouæ] etc., and Baron of San Demetrio, joined fifteen galleys of his own to the fleet of the Republic of Genoa against the Venetians. As admiral of France, with many of his own galleys, he defeated in a famous naval battle Guy, Count of Flanders, who was in arms against Philip the Fair, King of France, and brought him back captive, with the greater part of the nobility of Belgica, in 1304. Afterwards he triumphed over the Pisans and did other distinguished things. Having married Margarita Ruffa, of the Counts of Sinopoli, he fathered Carolus and, among others, Vinciguerra, Baron of San Demetrio and lord of Malapeza in Calabria, viceroy of both Calabrias under Robert; and Lucianus,

lord of Villa Franca, chamberlain, counsellor and Prefect-General of the fleet of Joanna I, Queen of Naples, against the King of Hungary, whom he defeated in 1348, and who, as admiral of Provence and Forcalquier [Folcalcarij] in 1361, brought aid with five galleys fitted out at Monaco to Alphonsus, King of Aragon, against the Sardinians;

[Margin: Antonius, head of Branch 8.]

and Antonius, lord of Pratum, Utellae, Vallis Lantuscae and other places in Provence, proprietor of six galleys, counsellor and Vicar in Provence of Joanna, Queen of Naples, and head of Branch 8. For his son Lucas — who, with the consent of Clement VII, received back into his obedience Antibes, which belonged by most ancient right to the Grimaldi family but had been cut off from it in the course of time — fathered Nicolaus, lord of Antibes, Capria etc., counsellor of Joanna, Queen of Naples, and Governor of Marseille [Massiliæ]. Nicolaus's second son, Lambertus, married Claudia, only daughter of Catalanus, Prince of Monaco, and heiress of the principate, of whom more below in her place. Raynerus II died in 13[?]4. From Nicol. Grim. and Carolus Venasq. in this Genealogy; the registers of the Kings of Naples; Io. Villanus [Giovanni Villani], part 1, book 8; Paulus Aemil.; the brothers de Sainte-Marthe and other French historians; Iustinianus in the *Annals of Genoa*; Nostradamus in the *History of Provence*, part 4; and the archives of Monaco and of Antibes.

[Margin: Charles the Great.]

CAROLVS, surnamed the GREAT [MAGNVS], sovereign Prince of Monaco and of Ventimiglia [Intemelij], lord of Cagnes, Villeneuve etc., and lord of many places in the Balearic Islands, bought back with gold the houses that the Spinola still held in Monaco. Having made a treaty with the King of the French, he recovered the Gulf of Grimaldi by the King's favour; he bought Menton [Mentonum], Roquebrune [Roccabrunam] and Castellionum and joined them to his sovereign dominion. He was counsellor of King Robert and, in 1344, Vicar of Queen Joanna in Provence. As admiral of the Guelfs he took part in the peace between the Grimaldi and the Fieschi and the Doria and the Spinola in 1331; then, as perpetual admiral of Genoa, he won many victories, especially over the Catalans [Catalaunis]. In 1338 he brought twenty galleys fitted out at Monaco by the Guelfs against Edward, King of England, in aid of Philip of Valois, King of France. Later, in 1346, as admiral of France, with 30 galleys of his own and ten thousand legionaries armed at Monaco, he hastened to the aid of the same Philip against the same Edward; in that battle, at Crécy [Cresciacum], he was gravely wounded — but most honourably. He married Luchinetta, daughter of Gerardus Spinula, lord of Lucca and Tortona [Dertonæ], and died in 1363. His sons were many, but the chief were Raynerius III; Lancellotus, chamberlain and counsellor of Queen Joanna I in 1348; Franciscus Baptista, Prefect-General of the Guelf fleet in aid of the King of Naples; and Carolus, co-lord of Menton, Governor of Aquila, counsellor of Queen Joanna, and envoy to the King of the Franks.

[Margin: Carolus, and Henricus, head of Branch 9.]

Carolus's son Henricus, chamberlain and counsellor of Martin, King of Sicily and Aragon — by whom, for his distinguished military exploits, he was created Baron of Buzetta and Scitibillinum and Governor of the city of Castrum Ioannis — was head of Branch 9. So from Nicol. Grimald. and Carol. Venasq. in this Genealogy; the registers of the royal archive of Naples and of Monaco; Zurit. in the *Annals of Aragon*; Iustinianus in the *Annals of Genoa*; Vbertus Folieta; Io. Villanus, part 2, book 12; Matth. Villanus [Matteo Villani], *History of Florence*, book 1; the register of the chancery of Palermo; and Fasel. in the *History of Sicily*.

[Margin: Raynerus III.]

RAYNERIVS III, Prince of Monaco, Menton etc., Baron of the city of Vincium, lord of Baussonis, Boissonis, Turreta, Leventium, Esa, Turbia etc., Baron of Lunellum and Servianum, lord of Vallis Baiae in Piedmont, and lord of the islands of Cephalonia, Leucata, Santa Maura and Zacynthus, of Vallis de Compare and of other cities of Greece, was Prefect-General on land and sea for Louis and Joanna, sovereigns of Naples; Seneschal of Piedmont; chamberlain and counsellor of the King of the Franks; admiral of the Mediterranean Sea; Prefect-General of the land armies in Provence, with supreme right of peace and war; and envoy to the King of England for a treaty. Always most ready to aid the Kings of France and of Naples, he earned eternal praise by signal victories. At Menton in 1378 he received Io. Serra, apostolic nuncio, sent to him as legate by Urban VI with a special charge to defend the Holy See against the schismatics; and he handed over to the nuncio the rod of Moses, with sacred relics and the registers of the Apostolic Camera, which had been carried off by the deserting cardinals but wrested from them. Monaco, seized in his absence in 1395, he recovered in 1402; and there, in 1406, he also received Pope Benedict XIII as his guest. He married Isabella Assenaria, of imperial stock, and died in 1407.

[Margin: Ambrosius, Prince of Monaco.]

His chief children were Ambrosius, Prince of Monaco, who, while busy fishing near Menton, was drowned in the sea in 1422, being unmarried; and Ioannes. From Nicol. Grim. and Carolus Venasq. in this Genealogy; the archive of Monaco; Iustinianus in the *Annals of Genoa*; Vbert. Foliet.; Nostradamus in the *History of Provence*; and the registers of the Kings of Naples.

[Margin: Ioannes.]

IOANNES, sovereign of Monaco etc., after many wars and losses inflicted on the Florentines, made peace and a treaty with that Republic in 1424. Then, as Prefect-General of Philip, Duke of Milan, he routed the Venetians near Cremona on the Po, taking eight thousand of them captive, in 1431. He defeated the Catalans many times and was most skilled in naval warfare. Together with his son Catalanus he approved the aggregation of the Ceba and Oliva families to the Grimaldi in 1448. He married Pomelina, daughter of Thomas Fulgosius, Doge of Genoa, in 1433, and died in 1454. His children were Catalanus and Bartholomaea, wife of Petrus Fulgosius, Doge of Genoa. When Petrus had died

and Francesco Sforza [Franciscus Sfortia] was besieging the citadel of Genoa, Bartholomaea governed Genoa and showed herself a heroine, in 1464. From Nicol. Grim. and Carolus Venasq. in this Genealogy; the archive of Monaco; Iustinian. in the *Annals of Genoa*; Vbert. Foliet.; Sabellicus in the *History of Venice*; Tarcagnota; Blondus; and others.

[Margin: Catalanus.]

CATALANVS, sovereign Prince of Monaco etc., inflicted many defeats on the Catalans with his galleys in defence of the rights of the port of Monaco. He married Blanca of Carretto, of the Marquises of Finale, and died in 1457. His sons Io. and Raynerius died in the cradle; but his daughter Claudia, her father's heir, married Lambertus, of whom presently. From Nicol. Grimald., Carol. Venasq., and the archive of Monaco.

[Margin: Claudia, and Lambertus.]

CLAVDIA, only daughter of Catalanus and heiress of the State, married in 1457 Lambertus Grimaldus, son of Nicolaus I, lord of Antibes, her kinsman in the 4th and 5th degree — namely, the great-great-grandson of RAYNERVS II, Prince of Monaco. By virtue of the marriage Lambertus became sovereign of Monaco etc., and in 1463 he gained possession of the County of Ventimiglia. He was helper, with his galleys and soldiers, counsellor, and envoy extraordinary of René, King of Sicily and Jerusalem, to Louis XI, King of France; and, concluding a treaty with that Most Christian King, he was enriched by him with many privileges. He was, besides, Prefect-General of the fleet of Savoy under Duke Amadeus in 1465, and counsellor and chamberlain of Charles III, King of Sicily and Jerusalem, in 1481, who by codicil gave him two galleys. Some years later, from Charles VIII, King of the Franks, he obtained as duke and royal lieutenant the whole western Riviera of Genoa; and he died in 1493. Of Claudia and Lambertus were born, first of all, Ioannes II, sovereign of Monaco etc., who was killed by his brother and left no children; and Lucianus and Augustinus, of whom below. From the archives of Monaco, of Antibes and of Paris, and Car. Venasq.

[Margin: Ioannes II.]

IOANNES II, sovereign of Monaco and Ventimiglia [Intemelij], counsellor, chamberlain [Cambellanus] and pensioner of Charles VIII and Louis XII, Kings of the Franks, and Knight of the Order of Saint Michael, hastened as commander of his own galleys and troops to the aid of Charles VIII in acquiring the Kingdom of Sicily, and of Louis XII in acquiring the Duchy of Milan and the lordship of Genoa; for that reason he was Prefect-General of the whole western Riviera in 1495 and 1500. He built the citadel of Menton and a bulwark of wonderful architecture. He married Antonia, daughter of Philip, Duke of Savoy, but, being without children, he was killed in a quarrel by his brother Lucianus, who was greedy for power, in 1505. From the archive of Monaco; Carolus Venasq. in this Genealogy; Aret. in the *Compendium of the Princes of Europe*, book 3; and Pignor. in the genealogical tree of the Dukes of Savoy.

[Margin: Lucianus.]

LVCIANVS, then, sovereign of Monaco etc., counsellor, chamberlain and pensioner of Louis XII and Francis I, Kings of the Franks, and enriched by them with many privileges, vigorously defended the citadel of Monaco in 1506, when it was besieged and attacked by the Genoese and the Pisans in a long and stubborn war. In 1506 he recovered Menton and Roquebrune [Roccabrunam], which the same enemies had taken by storm. He married Pontevesia [Pōteueſiam], lady of Cabanae [Cabanarum] in Provence, by whom he fathered Franciscus II, Prince of Monaco, to whom the subjects swore the oath of fealty although he was an infant; but he died at four years old. He also fathered Honoratus, of whom below. At last Lucianus was killed by his nephew Bartholomaeus ab Auria [Doria], lord of Dolceacqua [dulcis aquæ], in 1523. From the archive of Monaco; Iustinian. in the *Annals of Genoa*; Carolus Venasq.; Io. de Auton [Jean d'Auton], *On the Deeds of Louis XII*; and others.

[Margin: Augustinus.]

AVGVSTINVS, Bishop of Grasse, Abbot of Lérins, counsellor and almoner of the Most Christian King, entered the citadel of Monaco when his brother Lucianus was barely dead. Taken on as guardian of his young nephews, he exacted from the subjects an oath of fidelity to himself and to his nephews, and declared himself Prince for his lifetime. By force of arms he stripped his brother's murderer of his state of Dolceacqua, compelling its subjects to swear fealty to himself and his nephews; and, having captured the murderer at Castrum Pennae, he later had him executed at Monaco. Then, in 1524, having made a treaty with the Emperor Charles V and taken his side, he rendered him great services with his galleys, soldiers, port and citadel. For these he was elected Archbishop of Oristano and Bishop of Majorca, and was named Cardinal, although he did not reach the promotion: death forestalled him in 1532, after he had received Charles V as his guest at Monaco in 1529. From the archive of Monaco and of Grasse; the Chronicle of Lérins; Iustinianus in the *Annals of Genoa*; and Carolus de Venasq. in this Genealogy.

[Margin: Honoratus.]

HONORATVS I [Honoré I], son of Lucianus, Prince of Monaco, Marquis of Campagna [Campaniæ], Count of Canosa [Canuſij], lord of Dolceacqua [Dulciſquæ], and counsellor of the Emperor Charles V, from whom he obtained Campagna and Canosa because he had stood ready to serve him with his galleys in various battles — above all at Lepanto [Naupacti] under Io. Austriacus [Don Juan of Austria], at the storming of La Goletta [Goletæ], and at Tunis. This Prince was cultivated in every kind of learning and was the Maecenas of his age; and he had such authority with Charles V, and with Philip II, King of Spain, that those who acted as their ambassadors sat beside him. When his guardian Augustinus died while Honoratus was still under age, he had a second guardian, Stephanus Grimaldus of Genoa, whose niece [neptem] Isabella he married in 1545 in return for the guardianship well discharged; and he died in 1581. His children were Carolus, Prince, of whom presently; Franciscus, the second-born, who, sent by his brother Carolus to the Catholic King on business in 1583, died

at court; Hercules, the third-born; and Horatius, the fourth-born, cupbearer of Philip III, King of Spain, who died at Naples in 1620; besides other sons and daughters. From the archive of Monaco and Carolus de Venasqu. in this Genealogy.

[Margin: Carolus.]

CAROLVS, Prince of Monaco, Marquis of Campagna, Count of Canosa and pensioner of Philip, King of Spain, drove back the French when they made a sudden attack on the citadel of Monaco in 1584; and he died unmarried in 1589.

[Margin: Hercules.]

HERCVLES [Ercole], third-born son of Honoratus, Prince of Monaco, Marquis of Campagna and Count of Canosa, had taken degrees in both laws [I. V.] and was destined for higher rank in the ecclesiastical order. He defended the citadel of Monaco in 1596 against a surprise attack by the French and against a conspiracy within, and so joined arms to letters. In 1595 he married Maria, daughter of Claudius Lando [Claudio Landi], Prince of the Holy Roman Empire and of Val di Taro [Vallis Tari], and of Ioanna of Aragon; she was herself a pensioner [Penſionariam] of the Catholic King. Treacherously killed by some of his Monegasque subjects in 1604, he left Honoratus II as his successor. His daughter Ioanna married Theodorus Triuultius [Teodoro Trivulzio], Prince of Mesocco [Miſochij] and of Val Mesolcina [Vallis Miſolcinæ] in the Holy Roman Empire, Count of Meltium and Gorgonzola, lord of Cothoneum and Venzaghelum, Knight of Santiago [S. Iacobi], pensioner of the Catholic King, commander of two troops of horse for the same King, and Imperial Commissary in Italy. After his wife's death in 1620 he was created Cardinal of the Holy Roman Church by Urban VIII in 1629, and he was afterwards Governor of Milan [Mediola-lani] in 1640, Viceroy of Aragon in 1642, and Viceroy of Sicily in 1646. From the archive of Monaco and Carolus Venasqu. in this Genealogy.

[Margin: Honoratus II.]

HONORATVS II, Prince of Monaco, Marquis of Campagna, Count of Canosa and Knight of the Order of the Golden Fleece, was seven years old when his father died. His guardian was his maternal uncle Fridericus Lando [Federico Landi], Prince of the Holy Roman Empire and of Val di Taro, who, having made agreements with the Spaniards and thinking his ward's safety better provided for, brought a Spanish garrison into the citadel in 1605. When the time of his wardship was over, the Prince returned to the citadel from far away and, not recognizing his liberty as free, he published apologetic writings; then, with an armed band of a few subjects and courtiers, he fearlessly expelled the Spanish garrison by night in 1641, admitted the arms of Louis XIII into the citadel, and sent back the Golden Fleece. In compensation for what the Spaniards had taken from him, King Louis therefore created him Duke of Valentinois [Dux Valentinus], Peer of France, Count of Carladès [Carladeſij], Baron of Buis [Buiſſij] and of Calvinet [Caluineti], lord of Saint-Rémy [S. Remigij], perpetual and hereditary Governor of the royal forces in the citadel

of Monaco, and Prefect-General of its fleet; and before Perpignan, amid the camp drawn up in order, the same King decorated him with his own hand, out of the regular order, with the collar of the Royal Orders. He married Hippolyta [Ippolita Trivulzio], daughter of Theodorus Triuultius, Count of Meltium, and of Catharina Gonzaga, and sister of the Cardinal Prince; she died in 1638. By her he had Hercules, formerly Marquis of Campagna, Knight of the Order of Alcántara, commander of Benfaianen, and Prefect of the cuirassiers [Equitum Cataphractorum] in the Kingdom of Naples, but afterwards created by the Most Christian King Marquis of Baux [Baucij], pensioner, captain in France of a hundred men-at-arms, and knight of the Royal Orders with the collar. In 1641 Hercules married Aurelia, daughter and heiress of Lucas Spinula [Luca Spinola], lord of the city of Molfetta, and of his wife Pelina, likewise a Spinola. By her he had Ludouicus [Louis], Count of Carladès, born 1642 July 25, who in 1643 was held at the sacred font of baptism by the Count and Countess of Alais [Aleſijs] in the name of King Louis XIV and of the August Queen Mother, the Regent. From the archive of Monaco and Carolo de Venasque in this Genealogy.

CAPVT III.

Chapter III.

De Ramis Nobiliſſimis Grimaldæ Arboris, ex Monœci Principibus fœcunditate legitima in Galliam, Hiſpaniam, & Italiam dilatatis.

On the most noble branches of the Grimaldi tree, spread by lawful fruitfulness from the Princes of Monaco into France, Spain and Italy.

Hardly had the name of Grimoald the Just begun to be taken up patronymically as the family surname by his son Theobald, when from that stock [ceſpite] there sprang, as it were, a twin trunk. One of the two spread into France through Hugo Grimaldus, Theobald's first-born; the other into Spain through Ramirus Grimaldus, second son of the same Theobald. Granted that, before the year 1280, the Spanish nobles of this line remained unnamed through the negligence of writers, yet Cardinal Tressius, formerly President of Castile, as is recorded in his manuscript preserved at Monaco, asserts that the Grimaldi family held vassals and fortresses in Spain for about 900 years, and that the Grimaldi surname flourished until the year 1370. In that year the male line failed: the family was indeed continued through a woman, Hurraca [Urraca], but the surname passed with the inheritance to the most noble family of the Tressii.

[Margin: The Spanish Grimaldi.]

For, as the same Cardinal testifies, Petrus Sanz Grimaldus, lord of the castle, or fief, of Grimaldi in Spain, fathered Ruy Perez Grimaldus; and when Petrus died he was buried in 1280 at S. Petrus de masculis near Santiuanez. Ruy Perez Grimaldus, lord of Grimaldi, was one of the judges in the city of Plasencia

[Placentia], recently founded, in 1320; and he fathered Bermudus Guttierez Grimaldus and Ruy Perez Grimaldus, Bishop of Plasencia under the last Alfonso [Alphonſo vltimo]. Bermudus, lord of Grimaldi, fathered Consaluus Bermudez Grimaldus, who in 1340 was in attendance on King Alfonso at the battle of the Salado; and Nico- laus Bermudus Grimaldus, Bishop of Plasencia, who was buried in 1350 in the metropolitan church of Plasencia at the altar of the Blessed Virgin, commonly called *Del perdon*, where his Grimaldi arms are to be seen. Lastly, Consaluus, lord of Grimaldi, fathered Hurraca Bermudez Grimalda, lady of Grimaldi, who married Petrus Tressius, lord of the castle and place of Almolfrague; and so the male line of the Grimaldi in Spain came to an end about the year 1370.

Leaving aside, then, this trunk — a trunk of the stock rather than a branch — from the other trunk, that is, from Hugo Grimaldus, there came forth, besides the straight stem of the Princes of Monaco, Nine Branches, by whose flowers and fruits the nobility of this tree and stock has been confirmed. It is therefore worth pointing them out.

PRIMVS RAMVS

The First Branch

Criſpinorum, nunc Vardij, & de la Boſſe Marchionum.

That of the Crispini, now Marquises of Vardes [Vardij] and of la Bosse.

CRISPINVS, surnamed Ansgothus as being descended from the Goths on his mother's side, was the son of Grimaldus I, Prince of Monaco, and of Crispina, daughter of Robert I, Duke of the Normans. For that reason he lived in Normandy, where he held the barony du Bec in the Pays de Caux [Regione de Caux]. He married Heloysa [Heloyſan], daughter of Rodulphus, Count of Guînes [Guiſnenſium] and of Boulogne [Bononienſium], and of Rosella, of the Counts of Saint-Pol [à S. Paulo]; from them the Norman Crispini sprang. So Carolus de Venasque teaches in this Genealogy, from Nicolaus Grim., William of Jumièges, Orderic Vitalis, a report sent from Normandy, letters of Charles VI, King of France [Galliar.], preserved in the archive of Monaco, and the Norman Chronicle. From these, and from very many other historians and authentic documents, the same Carolus wove together the genealogy of this branch, whose direct line, leaving out the collateral lines, is as follows.

Of Crispinus Ansgothus was born Gilbertus, Baron du Bec and lord of the castle of Thilliers in 1041, Constable of Normandy and commander of its forces, who took the surname Crispinus, in place of Grimaldus, from the proper names of his grandmother and his father, and handed it down to his posterity, though the Grimaldi arms were kept. Of him was born Guillelmus Crispinus, Baron du Bec, a vigorous warrior in England in 1059 and 1066; of him, Guillelmus II Crispinus, surnamed the Great, Baron du Bec, the most celebrated hero of his age, who in the pitched battle of Breniuilla [certamine Breniuillano]

struck Henry, King of the English, two blows on the head in 1119; of him, Goscelinus Crispinus, Baron du Bec and lord of Sterpigneium, in 1153; and of him was born Guillelmus III Crispinus, Baron du Bec, lord of Dangutum and Sterpigneium, in 1196. Then came Gullielmus IV Crispinus, Baron du Bec, lord of Dangutum, Sterpigneium and Lisortium; of him, Gullielmus V, Baron du Bec, lord of Varengibeccum, Dangutum and Sterpigneium, hereditary Constable of Normandy and Marshal of France in 1283; and also Hugo, of whom presently.

Of Guillelmus V were born Guillelmus VI, heir to the same estates, father of Ioanna Crispina and Maria Crispina; and Ioannes Crispinus, lord of Dangutum. Of Ioannes came another Ioannes, likewise lord of Dangutum, and Guillelmus Crispinus, whose son Guillelmus Crispinus, lord of Three Towns [trium vrbium], of S. Dionysius in the forest of Lihuntium, etc., recovered the barony du Bec Crespin, which had been alienated by Ludouicus Chalonius, son of Maria Crispina. He fathered Ioannes Crispinus, Baron du Bec Crespin, lord of Maunium, of Three Towns and of Plana; of him came Ioannes Crispinus du Bec Crespin, Baron of Maunium, lord of Plana, of Three Towns, of Aurichier, etc., hereditary Marshal of Normandy and supreme inquisitor of the waters and forests of all Normandy and Picardy; and also Ioanna Crispina du Bec Crespin, Baroness of Maunium, lady of Plana, Malleporium, etc., wife of Petrus de Brezè [Pierre de Brézé], Grand Seneschal of Normandy, from whom a great many princes sprang. In this way the barony du Bec Crespin, etc., passed out of the Crispin family.

Hugo Crispinus, for his part, fathered Ioannes Crispinus, of whom came Michael du Bec Crespin, Cardinal of the Holy Roman Church with the title of S. Stephanus on the Caelian Hill, created by Clement V in 1312; and Guillelmus du Bec Crespin, father of Iordanus, of whom came Gothofredus du Bec Crespin, lord of Boisdilliers, la Mothe Dusseau [?], etc. He fathered Guillelmus du Bec Crespin, lord of la Mothe Dusseau [?], father of Carolus, Bishop of Bayeux [Baiocenſis] and ecclesiastical senator in the Parlement of Paris, and also of Ioannes du Bec Crespin, Viscount of Canium, lord of Bourium and cupbearer of the King of the Franks. Of Ioannes came Carolus du Bec Crespin, lord of Bourium, Vice-Admiral of France and knight of the King's Order with the collar. Of him were born Philippus du Bec Crespin, Bishop of Nantes [Nannetenſis] and of Saint-Malo [Machlouienſis], afterwards Archbishop and Duke of Reims, first Peer of France and admitted to the Royal Orders, who gave a great proof of learning and integrity at the Council of Trent; and Petrus du Bec Crespin, lord of Vardes etc. Of Petrus came Renatus I du Bec Crespin, Marquis of Vardes and of la Bosse, knight of the Royal Orders of France with the collar, Governor of La Capelle [Capellæ] and of the Thiérache [Regionis Tirraſchanæ] etc.; and he fathered Renatus II du Bec Crespin, Marquis of Vardes, etc. So Carolus Venasq. in this Genealogy, from various historical authors, epitaphs and authentic manuscripts; the Norman Chronicle; William

of Jumièges [Guillelm. Gemeticen.]; Orderic Vitalis [Oder. Vital.]; Matthew Paris [Math. Pariſ.]; etc.

SECVNDVS RAMVS

The Second Branch

In quo Duces Euboli, & Cauallerones, & Bononienſes Grimaldi.

In which are the Dukes of Eboli [Euboli], the Cavallerones, and the Grimaldi of Bologna.

[Margin: Ingo.]

INGO, son of Obertus, the sixth of the Princes of Monaco, was lord of Carrosium, Aymerium, Stella and Campastrum, and one of the Eight Nobles governing Genoa in 1225. As commander of the army of the same Republic against the people of Tortona [Dertonenſes] and Alessandria, he took the fortress of Montanaria by storm. Elected to negotiate peace between the Republic of Genoa and the King of Sicily in 1237, he restored to the Archbishop of Genoa a great part of the Most Holy Cross that had been recovered. He married first Theodora ab Auria [Doria], then Faustina, daughter of the Count of Ventimiglia. From Nicol. Grim. and Carolus Venasq. in this Genealogy; Vbertus Folieta; Iustinianus in the *Annals of Genoa*; and the archive of Genoa.

[Margin: Luchinus.]

[Margin: Boarellus, from whom Part 1 of Branch 2.]

By these wives he fathered Lucas, or Luchinus, lord of Carrosium, Aymerium and Stella, one of the 8 Nobles governing Genoa in 1256 and arbiter of the disputes between the Genoese and the Venetians in 1258. He also fathered Boarellus, of whom below in Part 1 of this Branch; and Petrinus, Prefect-General of the Genoese fleet of 25 galleys against the Venetians in aid of the Emperor of Constantinople in 1263. From Nicolaus Grim. and Carolus Venasq. in this Genealogy; the archive of Genoa; Vbertus Folieta; and Iustinianus in the *Annals of Genoa*.

[Margin: Gabriel.]

[Margin: Perciuallus.]

From Luchinus and Maria ab Auria was born Gabriel, lord of Carrosium, Aymerium and Stella, one of the Eight Nobles governing the Republic of Genoa in 1267. He took part in the agreements between the Republic and Michael, Emperor of Constantinople, in 1261, and the King of Sicily in 1269, and was made Baron and castellan of Mons Sacarius [montis Sacarij] by Charles I, King of Naples. His brother Perciuallus was Prefect-General of the Holy See against Manfred, who was occupying the kingdom of Naples, in 1252, and subdued the cities of Troia and Foggia in 1279. From Nicol. Grim. and Carolus Venasq. in this Genealogy; the archives of Genoa and Naples; Thomas Cost. in the *Life of Innocent IV*; and Matth. Spinellus in the *History of Naples*.

[Margin: Gaspar.]

[Margin: Rabella, from whom Part 2 of Branch 2.]

Of Gabriel were born, among others, Gaspar, lord of Stella, who, after the Doria [Auriacis] and the Spinola had been expelled, was rector and Doge of Genoa together with Carolus Fliscus [Fieschi] in 1317. He defended the city when it was long besieged by the Ghibellines of Lombardy and Tuscany, and in 1318 he handed over the government of the city for 10 years to Robert, King of Naples, who was hastening to its aid. He was pensioner of the said King in 1324, and he married Anna Marina. His brother was Rabella, lord of Carrosium, Aymerium and Stella, head of the second part of Branch 2. So Nicol. Grim. and Carolus Venasq. in this Genealogy; the archive of Genoa; Iustinian. in the *Annals of Genoa*; Villanus in the *Chronicle of Florence*; and the Register of King Robert of Naples.

[Margin: Perciuallus.]

[Margin: Gabriel, from whom Part 3 of Branch 2.]

[Margin: Ioannes.]

[Margin: Antonius.]

Of Gaspar was born Perciuallus, lord of the city of Policastro, counsellor and pensioner of King Robert in 1332 and of Queen Joanna I in 1348; and Gabriel, from whom Part 3 of this Branch 2; and Ioannes, viceroy for King Robert in the district of Otranto [tractu Hidruntino]; and Antonius, lord of Policastro, admiral of King Robert in 1329. Antonius was afterwards admiral of the Republic of Genoa with 45 galleys: he put to flight the fleets of the Kings of Aragon and of Majorca and captured five galleys of the Catalans in 1332; he was admiral again, with 60 galleys, against the Venetians and the Catalans; and by grant of Queen Joanna I in 1375 he was made lord of Baux in Provence for a term. So Nicol. Grim. and Carolus Venasq. in this Genealogy; Iustinian. in the *Annals of Genoa*; Vbertus Folieta; and the Registers of King Robert and of Joanna.

[Margin: Aymo.]

[Margin: Perciuallus II.]

[Margin: Lucianus.]

Of Perciuallus came Aymo, lord of Policastro and counsellor and pensioner of Queen Joanna I; of Aymo, Perciuallus II, who aided the King of Naples with his own galleys, and whose marble inscriptions survive in the church of S. Nicolaus de Busqueto near Genoa; of this Perciuallus, Lucianus, who negotiated peace between the Genoese, the Venetians, the Florentines and the Duke of Milan. Lucianus was envoy of the Republic of Genoa to the Kings of France, Aragon and Naples, and in 1448 procurator and commissary for the aggregation of the Ceba and Oliva families to the Grimaldi. In San Giorgio [In Sancto Georgio] he left, in 1474, most ample legacies for members of the Grimaldi family tossed by adverse fortune; and for that reason he deserved a statue and marble inscriptions in the old hall of San Giorgio.

[Margin: Merualdus.]

[Margin: Augustinus.]
[Margin: Nicolaus the Monarch.]

Of Lucianus was born Merualdus, an Elder [Antianus] of Genoa, whose inscriptions survive in the church of S. Maria Cella, S. Petrus de Arena. Of Merualdus came Augustinus, one of the envoys of the Republic of Genoa sent in 1504 to render obedience to Julius II, Supreme Pontiff, and envoy of the same Republic to receive Charles V. Of Augustinus came Nicolaus, Duke of Eboli, Prince of Salerno, Marquis of Teano [Theani], Count of Pola, and Baron of other places in the Kingdom of Naples, and lord of Tapazium and Rudaranum in Spain, who was called "the Monarch" for the abundance of his riches. He built from its foundations a most splendid palace in the Via Nuova of Genoa [in via noua Genuæ], and in 1570 he married Iulia Cibo, great-granddaughter of Innocent VIII. Of him came Augustinus II, Duke of Eboli etc.; and of Augustinus, Nicolaus II, Duke of Eboli etc., Knight of the Order of Alcántara, who died at Genoa unmarried and without offspring in 1639, and with him this distinguished line became extinct. So Nicol. Grimaldus and Carolus Venasq. in this Genealogy; the archive of Genoa; and the Registers of the Kings of Naples.

SECVNDI RAMI

Of the Second Branch

Pars 1. Nempe Grimaldorum de Caſtro.

Part 1: namely, that of the Grimaldi de Castro.

[Margin: Boarellus.]
[Margin: Conradus.]
[Margin: Boarellus II.]
[Margin: Conradus II.]
[Margin: Boarellus III.]
[Margin: Ingo.]

BOARELLVS, son of Ingo and brother of Luchinus, was one of the Eight Nobles governing the Republic of Genoa in 1262, and envoy of the same Republic to Clement IV, Supreme Pontiff, in 1266, and to Charles I, King of Naples, in 1272. His son Conradus, knight of Charles I, King of Naples, was governor of Barcarium and Camerana in 1272 and Praetor of Bologna in 1306; a prudent and vigorous commander, he took many enemy places by storm. His son Boarellus II aided Robert, King of Naples, with four galleys of his own and was his counsellor; he married a daughter of the Marquis Malaspina. Of them came Conradus II, lord of Stella in 1332; of him, Boarellus III, lord of Stella, a fief that he sold to the Republic of Genoa in 1390. His son Ingo discharged 24 embassies for the Republic of Genoa to various and greater princes of Europe; a Doctor of both laws [I. V. Doctor] and a knight of remarkable eloquence and learning, he earned for these a solemn funeral at public expense.

[Margin: Baptina.]

[Margin: Raphael.]

Ingo's daughter Baptina married Franciscus de Castro, son of Iulianus, who by virtue of the marriage was aggregated to the Grimaldi family. Of them came Raphael, Commissary-General in Corsica for the Republic of Genoa, who took Lecca by storm and laid waste Ginerca, after Paulus à Lecca, governor and rebel, had been driven from the island, in 1487. He was likewise Treasurer of the Holy See, governor of Terni [Interamnij] and Rieti [Reates] in 1496, and Prefect-General of the fleet of the Emperor Maximilian I against the French and the Florentines. He carried the Emperor and the imperial standards in his own ship, and for his distinguished deeds the Emperor created him at Sarzana, before the princes of the court, knight and Count Palatine, with the privilege, extending also to his descendants, of placing the Imperial Eagle on a gilded field in the upper part of the Grimaldi arms, in 1497. He served the King of the Franks at the storming of Mytilene [Mettelini].

[Margin: Franciscus.]

[Margin: Raphael II and Lucas.]

[Margin: Franciscus II.]

His son Franciscus, counsellor of Charles V, was made by him a gilded knight [Eques aureatus] and Count Palatine in 1531, and was envoy of the Republic of Genoa to the same Charles in 1547. He married Caecilia, daughter of Lucas Spinula, by whom were born to him Raphael II, created Count Palatine by Charles V, and Lucas, Doge of Genoa in 1605, who in 1597 married Nicoleta, daughter of Baptista Grimaldus, Duke of Terranova [Terrę nouę]. Of her was born Franciscus II, who married Nicoleta, daughter of Io. Baptista ab Auria, of the lords of Sassello [Saffelli]. So Nicol. Grimald. and Carolus Venasq. in this Genealogy; Iustinian. in the *Annals of Genoa*; the archive of Genoa; the Registers of the Kings of Naples; Marcus Anton. Cecald. [Marcantonio Ceccaldi] in the *History of Corsica*; and Sigonius in the *Life of Andrea Doria* [Andreæ ab Auria].

SECVNDI RAMI

Of the Second Branch

Pars 2: In qua Grimaldi Cauallerones.

Part 2: in which are the Grimaldi Cavallerones.

[Margin: Rabella.]

RABELLA, son of Gabriel and grandson of Ingo, lord of Carrosium, Aymerium and Stella, was the chief author of the expulsion of Opicinus Spinula, who had usurped the lordship of Genoa in 1310. He was Prefect-General of the Guelfs against the Ghibellines in 1317 and Elder of Genoa, 1333; he worked for the solemn peace between the Grimaldi and the Spinola.

[Margin: Rabellinus.]

[Margin: Dorinus.]
[Margin: Rabella II.]

He married Eleonora, sister of Raphael ab Auria. Of her were born Rabellinus, governor [Preſes] of Vallis Grata and Terra Iordana, and viceroy for King Robert in the Principato Citra [Principatu citra]; and Dorinus. Of Dorinus came Rabella II, Elder of Genoa in 1394 and envoy of the Republic of Genoa to the Emperor Sigismund in 1398, who married a daughter of Georgius Adornus [Giorgio Adorno], Doge of Genoa.

[Margin: Dorinus II.]
[Margin: Angelus.]
[Margin: Georgius.]
[Margin: Io. Bapt.]
[Margin: Nicolaus and Stephanus.]

From them was born Dorinus II, Elder of Genoa in 1421, and in 1448 procurator and commissary of the whole family for the aggregation of the Ceba and Oliva families to the Grimaldi. He brought aid to the King of Naples against the King of Aragon in 1443, and was envoy of the Republic of Genoa once to the Pontiff and three times to the Duke of Milan. Of him came Angelus, who flourished in 1448; of Angelus, Georgius, Elder of Genoa in 1471, who dedicated a sumptuous chapel in the church of S. Maria de Consolatione near Genoa in 1497; of Georgius was born Io. Baptista, lord of Montaldeo [Montaldei], created Count Palatine and gilded knight by the Emperor Charles V, together with his brothers Stephanus and Nicolaus, in 1525. The title was transmitted to their descendants, and on account of it they are commonly called the Cavallerones. Nicolaus was most generous to the poor. Stephanus was guardian of Honoratus I, Prince of Monaco, and an excellent administrator of the principality: he restored the church of S. Nicolaus and fortified the citadel with great bulwarks and with artillery. He was a man endowed with every kind of learning, shrewd and prudent, and he was buried in the tomb of the Princes of Monaco in 1561.

[Margin: Augustinus I.]
[Margin: Franciscus.]
[Margin: Georgius.]
[Margin: Thomas.]
[Margin: Dominicus.]

Io. Bapt., moreover, married Magdalena Pallauicina [Pallavicini], and of them were born many illustrious children: namely, Augustinus I, Baron of the city of Mons Pelusius [Montis Peluſij], senator of Genoa in 1581; Franciscus, who served Philip II, King of Spain, as commander of horse in acquiring the kingdom of Portugal, and also with his own galleys; Georgius, who with his own galleys served the same Philip II, King of Spain, in the naval battle of Lepanto [Naupacti] in 1571; and Dominicus, Commissary-General of the galleys of the Holy See under Pope Pius V in the naval battle of Lepanto in 1571. Dominicus was first Bishop of Savona, then of Cavaillon [Cabellicenſis], and finally Archbishop and Vice-legate of Avignon and governor of the armed forces of

the whole State in 1585; together with his brother Thomas, commander of horse, he drove the heretics out of the Comtat Venaissin [Comitatu Venayſſino], took Minerba by storm, and died in 1592. There was also Io. Iacobus, Baron of S. Foelix in the Kingdom of Naples and senator of Genoa in 1606, who married Hieronyma, daughter of Augustinus de Mari, by whom he fathered many sons; among them Hieronymus, Cardinal of the Holy Roman Church, stands out, of whom presently.

[Margin: Nicolaus.]

[Margin: Augustinus II.]

[Margin: Father Iulius.]

Augustinus married Maria, sister of Ambrosius ab Auria, Doge of Genoa, and died in 1592. Of her was born Nicolaus, Baron of Mons Pelusius, who married Paula Maria, daughter of Iulius Spinula, in 1609, and died at Mons Pelusius in 1636. His sons were many, but above all Augustinus II, Baron of Mons Pelusius; Father Iulius Grimaldus of the Society of Jesus; and Fathers Ioseph, Raynerius and Ioannes, Theatines.

[Margin: HIERONYMVS, Cardinal.]

Of Io. Iacobus, brother of Augustinus I, were born Io. Bapt., Baron of S. Foelix, Commissary-General of arms of the Republic of Genoa against the Savoyards in 1625 and Commissary-General of the army of the Holy See under Pope Urban VIII in 1630, who married Magdalena, daughter of Andreas Spinula, in 1613, and died in 1645. Of the same Io. Iacobus was born HIERONYMVS, whom we have tried, after a fashion, to sketch in the dedicatory letter of the first part of this first Tome.

HIERONYMVS, then, was Referendary of both Signatures under Gregory XV, Pontiff, in 1621; Pro-legate of the Province of the Patrimony in 1625; Governor of Rome in 1628; Nuncio extraordinary to the Emperor Ferdinand II in 1632; Governor of Perugia in 1634; Vice-legate of the Duchy of Urbino in 1636; Archbishop of Seleucia and Nuncio to Louis XIII, King of France, in 1641; and finally he was created Cardinal of the Holy Roman Church, with the title of S. Eusebius, by Urban VIII in 1643. The chief management of French affairs was conferred on him for his outstanding skill in civil and ecclesiastical matters and his consummate prudence and sagacity joined to integrity. So Nicol. Grim. and Carolus Venasq. in this Genealogy; Iustinian. in the *Annals*; Vmb. folieta; and the archives of Genoa and Monaco.

RAMI SECVNDI

Of the Second Branch

Pars 3. In Qua Grimaldi Bononiæ Incolæ.

Part 3: in which are the Grimaldi living at Bologna.

[Margin: Gabriel.]

GABRIEL, son of Gaspar, lord of Stella, and great-great-grandson of Ingo (from whom comes the Second Branch), of the lords of Policastro, obtained in 1324, through the liberality of Robert, King of Naples, the freedom of his son Raphael, who had been captured by the enemy. He was Regent of the Vicaria of Naples in 1326, Viceroy of the Abruzzi [Aprutij] in 1328, and envoy of the same King to the Supreme Pontiff; then pensioner of Queen Joanna I in 1343.

[Margin: Raphael.]

[Margin: Gaspar.]

[Margin: Franciscus.]

[Margin: Stephanus.]

His son Raphael, of the lords of Policastro, served Robert, King of Naples, with his own galleys in 1328, and was afterwards pensioner of the sovereigns [Reg.] Louis and Joanna I in 1348. He fathered Gaspar, who flourished in 1381; Gaspar fathered Franciscus, who flourished in 1411; and Franciscus fathered Stephanus, who because of private quarrels, left Genoa and lived at Reggio in Emilia [Regium Æmiliæ] in 1478. He married Catharina Fesseta, a noblewoman of Reggio, by whom was born Grimaldus. In 1525 Grimaldus married Ludovica Anzina, a noblewoman of Reggio, and from them sprang Franciscus II, who in 1556 married Ursina Rampionesia, a noblewoman of Reggio. By her he fathered Paris [Paridem], who lived at Bologna in 1589 and married Anna Catanea, a noblewoman of Bologna; and by her he fathered five children of outstanding character. They are Father Vincentius Maria and Father Franciscus Maria Grimaldi [Francesco Maria Grimaldi] of the Society of Jesus; Iosephus Maria, who married Sulpitia, daughter of Alexius Orsus [Alexij Orſi], Senator of Bologna and Count of San Quintino [S. Quintini], by whom he had Vespasianus; Antonius Maria, who died young; and Paris Maria Grimaldus. So from Nicolaus Grim. and Carolus de Venasque in this Genealogy, the Registers of the Kings of Naples, the archive of Naples, and various instruments of Reggio and Bologna drawn up by public notaries [publica manu rogatis].

[Margin: Grimaldus]

[Margin: Franciscus II.]

[Margin: Paris.]

[Margin: Fathers Vincentius Maria and Franciscus Maria.]

[Margin: Iosephus Maria and Paris Maria.]

De Reliquis Ramis.

On the Remaining Branches.

[Margin: Branch 3.]

The Third Branch [RAMVS Tertius], comprising the lords of Castrum Novum [Caſtri Noui] and of Gattières [de Gattieres] in the County of Nice, originated from Obertus, brother of Ingo and son of Obertus, Prince of Monaco; on it see Nicolaus Grimaldus and Carolus Venasq. in this Genealogy, from p. 124. The male line of this branch died out in Anna Francisca, co-lady of

Turreta [Turretæ Condomina], who married Michael Angelus Lassaris, of the senatorial order of Nice.

[Margin: Branch 4.]

The Fourth Branch originated from Nicolaus, likewise a brother of Ingo and son of Obertus, Prince of Monaco. From him come the Grimaldi living at Carignano [Carignani] in Piedmont, on whom see Nicolaus Grim., Iustinianus in the *Annals of Genoa*, and Carolus de Venasque in this Genealogy, from p. 132.

[Margin: Branch 5.]

The Fifth Branch, in which are the Marquises of Petra [Petræ], the lords of Bellifortis and of Cuneo [Cunei] in Piedmont, and those living at Seville [Hiſpalis] in Spain, originated from Luchetus, lord of Petra Alta [Petræ altæ] and of Stella, and son of Grimaldus IV, Prince of Monaco. On this branch see Iustinianus in the *Annals of Genoa*, Vbertus Folieta, Nicolaus Grimald., and Carolus de Venasque in this Genealogy, from page 140.

[Margin: Branch 6.]

The Sixth Branch, in which are the Counts of Bolci, of the Holy Roman Empire [Sacrocæſarienſes], and of Marantium [Marantij], the Marquises of Bovesium [Boueſij] and Peveranium [Peueranij], and the lords of Leventium [Leuentij]. Their head, Andaro, son of Franciscus, Prince of Monaco, was lord of Bolei and of Vallis Massae [Vallis Maſſæ]. On this branch consult Carolus de Venasque in this Genealogy, from p. 158.

[Margin: Branch 7.]

The Seventh Branch, in which are the Grimaldi living at Seminara [Seminaræ] and lords of Missimerium [Miſſimerij] in the Kingdom of Naples, whose head was Bartholomæus, son of Raynerius, Prince of Monaco. On this branch see Nicolaus Grim. and Carolus Venasq. in this Genealogy, from p. 172.

[Margin: Branch 8.]

The Eighth Branch, in which are the lords of Antibes [Antipolis] and the Marquises of Corbonium [Corbonij], whose head was Antonius, son of Raynerius, Prince of Monaco. On this branch see Nicolaus Grim., the archives of Monaco and Antibes, the Register of the Kings of Naples, and Carolus Venasq. in this Genealogy, from p. 180.

[Margin: Branch 9.]

The Ninth Branch, in which are the Princes of Sancta Catharina [Sanctæ Catharinæ] and the Barons of Busetta, Pasquasia, Canuti, Favara [Fauaræ], Galisium [Galiſij] and Carropipium [Carropipij] in Sicily. Their head was Henricus, on whom see Nicolaus Grim. and Carolus de Venasque in this Genealogy, from p. 188.

Facultas R.P. Vincentij Carrafæ SOC. IESV Præpoſiti Generalis.

Licence of the Reverend Father Vincentius Carrafa [Vincenzo Carafa], Superior General of the Society of Jesus.

Since three theologians of our Society have examined the First Volume of the *Almagestum Novum* of Father Ioannes Baptista Riccioli [Giovanni Battista Riccioli], a priest of our Society, and have approved it as fit to be published, we grant permission for it to be printed, if those whom it concerns see fit. In testimony of this we have given this letter, signed by our hand and secured with our seal, at Rome on 1647 March 12.

Vincentius Carrafa.

Place of the seal.

Facultas R.P. Alexandri Boſelli SOC. IESV Prouincialis in Prouincia Veneta.

Licence of the Reverend Father Alexander Bosellus, Provincial of the Society of Jesus in the Venetian Province.

Since three theologians of the same Society have examined the additions made to the First Volume of the *Almagestum Novum*, divided into two parts, by Father Io. Bapt. Riccioli, a priest of our Society, and have approved them as fit to be published, we too, having received the authority from our Reverend Father Gosuinus Nickel [Goswin Nickel], Vicar General of the Society of Jesus, grant that they be printed, if those whom it concerns see fit. In testimony of this we have given this letter, signed by our hand and secured with our seal, at Bologna on 1651 August 17.

Alexander Bosellus.

Place of the seal.

Examined by Father Bonaventura Cavalerius [Bonaventura Cavalieri], and afterwards by Father Franciscus Maria Grimaldus, professors of mathematics, on behalf of the Most Eminent and Most Reverend Cardinal Archbishop.

Don Andreas Cuttica, Clerk Regular of St Paul and Penitentiary Rector [Pœnitent. Rect.], on behalf of the Most Eminent and Most Reverend Cardinal Archbishop.

IMPRIMATVR

Let it be printed.

Brother Vincentius Pratus à Serravalle, Inquisitor General of the Holy Office at Bologna.

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Book VIII.

DE COMETIS, ET STELLIS NOVIS.

On Comets and New Stars.

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De Cometis.

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SECTIO II.

Section II.

[4]so printed — the third sort is a roman capital S, presumably a wrong sort for 5 (page 125); not checked against the page the entry points to.

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Book IX.

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Hactenus de Argumentis pro Motu Terræ Excogitatis. Sequuntur Argumenta pro Terræ Immobilitate.

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Book X.

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- 38, 39. To deduce the azimuth of a star by observing, in two ways. 554
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Section III.

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- 2. To distinguish the use of the time of the primum mobile and of the solar day. 576
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- 4. To convert equal time into apparent, or the reverse. 576
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- 6. To determine the hours during which the Moon shines, p. 576
- 7 to 10. To find in four ways the true semidiurnal and seminocturnal time of the Sun, or of any star. 576
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- 14. To investigate the horizontal refraction of the Sun required for its parallel not to cut the horizon at a given altitude, p. 578
- 15 to 20. To seek in six ways the moment of time from an observation of the Sun. 578
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- 21 to 24. To deduce in four ways the moment of time from an observation of the fixed stars, without trigonometry. 580
- 25 to 31. To do the same by trigonometry in seven ways. 580
- 32. To trace the time of the true or apparent rising and setting of the Sun. 581
- 33. To define the apparent time of the rising or setting of the limbs of the Sun. 581
- 34. To find the time at which the whole disc of the Sun rises. 581
- 35. To investigate how much earlier a star rises for someone living on the summit than at the foot of a mountain of known height. 582
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- 37 and 38. To find in two ways the time at which the Sun or any star comes to a given vertical. 582
- 39. To find the time at which a star will come to a given altitude on a vertical. 582
- 40 and 41. To seek the time at which the Sun or any star will reach the nonagesimal. 583
- 42 and 43. To find the time at which a given arc of the ecliptic passes through a given vertical. 583
- 44. To determine the time between the transits of two stars, or of two points of the ecliptic, through the midheaven. 583
- 45. To find the moment of time at which a given star or point passes through the midheaven. 583
- 46. To find the day on which a star culminates together with the Sun. 584
- 47 to 49. To seek in three ways the moment of time at which a star or a given point rises or sets absolutely on a given day. 584
- 50. To find the time between the absolute risings or settings of two stars. 584
- 51. To find the day of the rising or setting of some star — respective, but true — whether morning or evening. 584
- 52. To find the time at which some star once truly rose or set. 584
- 53. To determine the arc of vision, or of shining [Fulfionis], and the interval of the ecliptic for the heliacal rising and setting. 584
- 54. To determine the day of the heliacal rising and setting of any star. 584
- 55. To find the time at which a star fitted for it will reach the equator or a tropic, or will cross it. 584
- 56. To find the time at which a star capable of passing through the zenith of a given place will pass through it. 584
- 57. To find the time at which a star that rises and sets will pass into the number of the always visible, or the always hidden, stars, and will remain so, p. 584
- 58. To seek the time at which a star capable of reaching the pole of the world will arrive there, or has already arrived. 584
- 59. To trace the time at which a suitable star will reach the polar circle, whether inside it or outside. 584
- 60. To investigate the time at which the right ascension of a star fitted for it will increase and when it will decrease, or has already increased or decreased, while its longitude still always increases. 585

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Section IV.

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- 1. To find the ratio of a parallel to a great circle of the sphere, and to construct its table. 585
- 2. To investigate the size of the Earth's diameter by the method of Maurolycus [Francesco Maurolico]. 585
- 3. To seek the same by the method of Father Christophorus Griembergerus [Christoph Grienberger]. 586
- 4. To trace the diameter of the Earth by the method of Father Marius Bettinus [Mario Bettini], although it is fallacious. 587
- 5, 6, 7. To investigate the diameter of the Earth by the threefold method of Father Christophorus Clavius [Christoph Clavius]. 587
- 8. To find the diameter of the Earth by the method of Father Paulus Casatus [Paolo Casati]. 587
- 9. To seek the diameter of the Earth by the method of Kepler. 588
- 10. To seek the diameter of the Earth by the method of Father Nicolaus Cabaeus [Niccolò Cabeo] and of Dom Alphonsus Isaeus [?]. 588
- 11. To investigate the circumference of the Earth in the manner of Eratosthenes, and to correct his errors. 588
- 12. To seek the circumference of the Earth in the manner of Posidonius, and to correct his errors. 589
- 13. To measure the circumference of the Earth by the device of Marinus of Tyre and of Ptolemy. 589
- 14. To hunt out the circumference of the Earth in the manner of Almaeon [al-Maʾmūn] and of many Arabs and cosmographers. 589
- 15. To seek the circumference of the Earth by the device of Ioannes Fernelius [Jean Fernel]. 589
- 16. To conclude the circumference of the Earth by the ingenuity of Petrus Nonius [Pedro Nunes] and Iacobus Mazzonius [Jacopo Mazzoni]. 590
- 17. To investigate the circumference of the Earth by the geometry [Geometriſmo] of Villebrordus Snellius [Snel]. 590
- 18. To investigate the circumference of the Earth by our method and that of Father Franciscus Maria Grimaldus [Francesco Maria Grimaldi], p. 591
- 19. To attempt the circumference of the Earth by another method of ours. 591
- 20. To determine the circumference of the Earth from a distance, by another method. 591
- 21 to 26. To measure the height of a tower by altimetry in six principal ways. 592

- 27. To measure the distance of an island, or of any place visible from a shore or a riverbank. 593
- 28, 39 [?], 30. To measure the height of mountains by a threefold method, with an example in the mountains Etna, the Pico and the Caucasus. 593
- 31. To measure the heights of mountains in the manner of the older geometers, which is however doubly fallacious. 594
- 32. To correct the method of the older geometers in measuring the height of mountains. 594
- 33, 34, 35. To measure the height of a mountain by another, threefold, method. 595
  - On that occasion, to define the height of Mount Athos. 596
- 36. To seek the height of a mountain by another method, and on that occasion on the wonderful height of the Caucasus and Mount Casius. 596
- 37. Given the height of a mountain or tower, to investigate the height of another mountain, etc., p. 597
- 38. Given the Earth's semidiameter and the distance between two mountains, the summit of one being visible from the other, to find the height of each, or the least height of the one supposed the greater. 598
- 39. Given the Earth's semidiameter and the height of the eye raised above the convex surface common to land and sea, to find the terrestrial arc visible from the eye, that is, the distance of sight. 598
- 40. Given the height of a mountain, tower, etc., and the Earth's semidiameter, to define the distance from which it can be seen by an eye raised not at all above land or sea. 599
- 41. Given the height both of the object and of the eye, and the Earth's semidiameter, to find the greatest distance from which such an object is visible. 599
- 42. Given the distance of a spring from the sea, to find the height of the level, that is of the spring, required for a river to be able to descend to the sea by the same straight course. 599
- 43, 44. Given the distance between two places and the altitude of the pole, to find the difference of geographical longitude, the angles of position, and the azimuths of the common vertical. 599
- 45, 46. Given the polar altitude of two places and the difference of longitude, to find the distance between them and the azimuths of the common vertical. 599
- 47, 48. Given the altitude of the pole of one place, and its distance from another place, with the difference of longitude, to find the altitude of the pole of the other place, the angles of position, and the azimuths of the common vertical. 599
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- 50 to 54. Either to correct a meridian line already found, or to find one, in five ways, p. 600
- 55 to 59. To find the meridian line in five other ways. 601
- 60 to 66. To attempt the meridian line in seven other ways. 602
- 67 to 70. To seek the altitude of the pole in four ways without trigonometry. 602
- 71 to 74. To investigate the altitude of the pole in four other ways. 603
- 75 to 78. To seek the altitude of the Sun from the shadow of a gnomon, or the shadow from the altitude, and to construct a gnomonic table, and to correct the errors of some geometers. 603
- 79, 80, 81. Given the Sun's shadow, to find the altitude of the pole, and to correct in this the errors of certain cosmographers. 604
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- 86, 87. To find the altitude of the pole in two other ways. 606
- 88. Given the altitude of the pole, to determine the clime and the parallel of the clime. 606
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- 96 to 101. To hunt out the altitude of the pole in six other ways. 607
- 102. Given the altitude of the pole of two places, and either angle of position, to find the difference of longitude and the distance between the places. 608
- 103. Given the distance between two places and the difference of longitude, with either angle of position, to find the altitude of the pole of each. 608
- 104. Given the distance between two places and the altitude of the pole of one, with the angle of position, to find the altitude of the pole of the other and the difference of longitude. 608
- 105. Given the distance between two places and the angle of position of each, to find the altitude of the pole of each and the difference of longitude. 608
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- 106, 107, 108. To investigate in three ways the difference of longitude, that is the distance between the meridians, by eclipses of the luminaries; and which eclipses are better for the purpose. 608
- 109. To investigate the distance [?] between the meridians by clocks inspected at the beginning and the end of a journey. 609
- 110. To investigate the distance between the meridians by eclipses of the satellites of Jupiter. 609
- 111 and 112. To find the distance between the meridians by eclipses or illuminations of the lunar spots. 610
- 113 and 114. To find the distance between the meridians by the place of the Moon, by the methods of Orontius [Oronce Finé], Vernerus [Johannes

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- 115. To attempt the distance between the meridians by the method of Hieronymus Ruscellus [Girolamo Ruscelli]. 610
- 116. To investigate the distance between the meridians by the method of Longomontanus and Kepler, which however needs correcting. 611
- 117. To correct the preceding method of Longomontanus. 611
- 118 and 119. To seek the distance between the meridians by the method of Io. Bapt. Morinus [Jean-Baptiste Morin], although it is fallacious. 612
- 120. To seek the distance between the meridians by the method of Herigonius [Pierre Hérigone], and on that occasion to find the declination of the Moon. 612
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- 121 to 125. Given a moment of time, to define where it is then noon and where midnight; where it is day and where night, at least within the temperate zones and the torrid zone; for which places the Sun is then rising and for which it is setting; and what hour of any kind it is at any given place; and conversely, at what place on Earth there are such and so many hours and minutes of hours. 612

SECTIO V.

Section V.

De Parallaxibus.

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CAPVT II.

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- 7 and 8. Given the horizontal parallax, to find the parallax of any apparent altitude, and the reverse, etc. 614
- 9. Given the parallax of some apparent altitude, to find the parallax corresponding to another given apparent altitude. 614
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Section VI.

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- 7 to 11. To measure in five ways the refractions of rays from air into glass or crystal. 649
- 12 to 19. To estimate, from the inclinations, the simple or compound refractions, and from the refractions the inclinations; and from this, or from their weight, the density of the media. 650

- 20 to 23. To find in four ways the refraction of altitude in a star free of sensible parallax. 652
- 24, 25. To investigate the horizontal refraction of a star whose parallax is smaller than its refraction, and to correct a fourfold fault of Tycho's in the refraction of the Sun. 653
- 26. To find the refraction of a star appearing above the horizon, and the limit of altitude at which sensible refractions cease. 654
- 27. To separate the simple refraction on a circle of altitude from the whole refraction observed. 654
- 28. To find the horizontal refraction of a star whose refraction is smaller than its parallax. 655
- 29, 30. To investigate in two ways the refraction of a star through observation of its altitude and its declination. 655
- 31. From the parallax and the other requisites, to determine whether a star is on the true horizon, below it, or above it. 656
- 32. To deduce the horizontal refraction of the Sun from its apparent rising or setting. 656
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LIBER OCTAVVS DE COMETIS ET STELLIS NOVIS

Book Eight: On Comets and New Stars

SECTIO PRIMA DE COMETIS.

Section One: On Comets.

C A P V T I.

Chapter I.

Prohemiale, In quò & de Auctoribus qui de Cometis ſcripta ediderunt.

Introductory, in which also of the authors who have published writings on comets.

I. Having surveyed, in the first part of this work, the accustomed splendours of the wandering and the non-wandering stars, and having weighed their properties as far as the slenderness of our wit and judgement allowed, we are raised up and roused once more to look up at, and at the same time to discern, the unaccustomed spectacles of the sky. Nor are we merely invited: we are compelled by the laws and command of astronomy, so that it may at last be decided whether, among those eternal, or rather ancient, splendours with which the stage of the heavens has flashed since the beginning of the world, there are to be received, as stars coeval with them or at least temporary, those curly-haired globes of light, or the long-bearded faces of contracted lightnings, or the fires advancing through the tracts of the aether with the long trailing robe of a shining tail, which we call comets; and what place in the system of

this world is to be assigned to them, whether as citizens or as guests. These new portents turn the eyes and minds of mortals to themselves no less than eclipses of the Sun and Moon do. "And so this assembly of stars, by which the beauty of the immense body is set off, does not summon the people. But when something has changed from its custom, everyone's face is turned to the sky. The Sun has no spectator unless it is in eclipse. No one watches the Moon unless she is labouring. Then cities cry out, then each person trembles for himself in vain superstition." So Seneca, who soon adds: "If something is disturbed, or has flashed out contrary to custom, we gaze, we ask, we point it out: so natural is it to wonder at new things rather than at great ones. The same happens with comets. If a fire of rare and unaccustomed shape has appeared, there is no one who does not wish to know what it is; forgetting the rest, he asks about the newcomer, not knowing whether he ought to admire it or to fear it."

[Margin: Seneca's words, bk 7 *Natural Questions*, ch. 1.]

II. Although this controversy about comets holds, for the most part, more uncertainty and trouble than fruit, we must nevertheless proceed by a middle way, so as to handle these matters with a mind neither too timid nor too confident. For well known is that saying of Aristotle in book 2 of *De caelo*, ch. 7, text 34, where, dealing with celestial phenomena, he professes his modesty: "Perhaps," he says, "to attempt to declare something about some things, and about everything, and to pass over nothing, will certainly seem a sign either of great folly or of great readiness. Yet it is not just to rebuke everyone alike; one must see what the reason for speaking is, and moreover how they stand in believing, whether in a human way or more firmly. Therefore when anyone has attained more certain and necessary reasons, then we ought to be grateful to those who find them; but for now, what seems so must be said." And in book 1 of the *Meteorologica*, ch. 4, about to speak of the causes of comets, he says: "Since, concerning things not manifest to sense, we think it sufficient to bring forward reasons that contain nothing impossible."

[Margin: Aristotle's modesty.]

Seneca sprinkles on us a taste of the same sobriety when he says, in book 7 of the *Natural Questions*, ch. 29: "These are the things concerning comets that move others, or me. Whether they are true, the gods know, who have knowledge of the truth. To us it is allowed only to search these things out and to conjecture in the dark, neither with confidence of finding nor without hope." Twin to these is that saying of Cicero in *Tusculans* 1: "Which of these opinions is true, some god will see; which is most probable is a great question." But I cannot yet tear myself away from Seneca, who in that same book 7, ch. 30, pursues the matter in these words: "Aristotle says excellently that we ought never to be more reverent than when the gods are in question. If we enter temples composed, if when about to approach a sacrifice we lower our face, draw in our toga, and put on every sign of modesty, how much more ought we to do this when we dispute about the heavenly bodies, about the stars, about

the nature of the gods, lest we affirm anything rashly, anything impudently, or ignorantly, or lie knowingly? Nor let us wonder that things lying so deep are dug out so slowly." And a little after: "How many things besides these" — comets, that is — "move in secret, never rising to human eyes? For God has not made all things known to human eyes. How small a part of so great a work is entrusted to our eyes?" Yet the same writer is to be rebuked when, in ch. 31, speaking of God, he says: "What this is, without which nothing is, we cannot know. And do we wonder that we know little of some little fires, when the greatest part of the world, God, lies hidden?" For God is not, properly speaking, a part of the world.

[Margin: Seneca's moderation in the inquiry into comets.]

Now therefore, if in any matter at all the opinions of many, or of better, men are to be gathered, and we must learn from many what cannot be sufficiently acquired from one man or from the men of one sect, I judge that this must be done above all in the present controversy, and that we must obey the admonition of Symmachus, who says in book 10, letter 61: "It is fair that whatever all worship should be thought one. We look on the same stars; the sky is common to us; the same world envelops us. What does it matter by what wisdom each seeks the truth? One cannot reach so great a secret by a single road." I shall therefore review the principal authors who have written on comets, and then undertake to hand on to posterity, in order, each of the choicer points drawn from them.

[Margin: Symmachus's excellent maxim.]

Auctores Præcipui, qui de Cometis Scripſerunt.

The Principal Authors Who Have Written on Comets.

III. Those who have written on comets are, first of all, Charimander, as Seneca reports in book 7 of the *Natural Questions*, ch. 3; Aristotle, book 1 of the *Meteorologica*, chs. 6 and 7, and his interpreters; Seneca himself in the whole of book 7 of the *Natural Questions*; Pliny the Elder, book 2, ch. 25, and his interpreters on that place, especially Valla [?] and Milichius [Jacob Milich]; then Plutarch, book 3 of *De placitis philosophorum*, ch. 2; Ptolemy in the *Centiloquium*, aphorism 100, and Pontanus [Giovanni Pontano] on that place; and in the *Quadripartitum*, or *On the Judgements of the Stars*, book 2, and Cardanus [Girolamo Cardano] on that place, commentaries 53 and 54; and again Cardano, book 4 of *De subtilitate*, and against him Iulius Cæsar Scaliger [Julius Caesar Scaliger], exercise 79; Albumasar [Abū Maʿshar], book 8 of *De magnis coniunctionibus*; Haly [ʿAlī ibn Riḍwān] on book 2 of the *Quadripartitum*; Angelus Cato [Angelo Cato], a short work on the comet of the year 1472; Regiomontanus on the comet of the year 1475, and in sixteen problems on comets; Ioannes Vogelinus [Johannes Vögelin] on the occasion of the comets of the years 1527 and 1532; Petrus Apianus [Peter Apian] at the end of the Caesarean work[5], who then was flourishing, and who besides those two described three other comets; Daniel

[5]the *Astronomicum Caesareum*

Santbech, *De observatione phaenomenωn*, proposition 19; Fracastorius [Girolamo Fracastoro] in the *Homocentrica*, section 3, ch. 23; Kechermanus [Bartholomäus Keckermann], book 6 of the *Systema physicum*; Ludouicus Lauaterus [Ludwig Lavater] and Eberus [Paul Eber] in the *Catalogue of Comets*; Antonius Mizaldus [Antoine Mizauld] in the *Cometographia*; Grisaldus [?] in the *Cometologia*; Bodinus [Jean Bodin], book 2 of the *Theatrum*; Leopoldus [Leopold of Austria], tract 5, *De annorum revolutionibus*; Franciscus Iunctinus [Francesco Giuntini] at the end of the *Speculum astrologiae*; Simon Maiolus [Simone Maioli] in the *Dies caniculares*, colloquy 1, from p. 11; Thomas Garzonius [Tommaso Garzoni] in the *Theatro*, pp. 72 and 200; Maginus [Giovanni Antonio Magini] in the *Primum mobile*, book 12 of the problems. To these must be added a great many natural philosophers and writers on meteorology: first of all Nyphus [Agostino Nifo] on book 1 of the *Meteorologica*; Libertus Fromondus [Libert Froidmont], book 3 of the *Meteorologica*; the Coimbra commentators [Conimbricenſes], tract 3 of the *Meteorologica*; Ioannes Cottunius [Giovanni Cottunio] on the *Meteorologica*, from lecture 30 to 33; Bartholomæus Mastrius [Bartolomeo Mastri] and Bonaventura Bellutus [Bonaventura Belluto], disputation 4, *De caelo ac meteoris*, question 2, article 4; Franciscus Resta [?], book 1 of the *Meteorologia*, tract 1; Raphaël Aversa [Raffaele Aversa], tome 2 of his *Philosophia*, questions 32 and 33; Nicolaus Cabæus [Niccolò Cabeo] on book 1 of the *Meteorologica*, text 37, from question 1 to 9; Bartholomæus Amicus [Bartolomeo Amico], *De caelo*, tract 4, question 5, d. 2, article 4, and tract 5, question 1, d. 1, article 7, and question 5, last dubium, a. 5; Libavius [Andreas Libavius] and Telesius [Bernardino Telesio], treatises on comets; Ludouicus Columbus [Lodovico delle Colombe] and Cornelius Frangipanius [?], treatises on comets; Arriaga [Rodrigo de Arriaga], the single disputation *De caelo*, section 3, subsection 2; Ouiedus [Francisco de Oviedo], the single controversy *De caelo*, point 3, paragraph 3; Balthassar Tellez [Baltasar Teles] in the *Physica*, disputation 4, section 1, and disputation 47, section 2; Blancanus [Giuseppe Biancani] in the *Loca mathematica Aristotelis*, marginal number 136, and the *Sphaera*, book 16; Ioannes Camillus Gloriosus [Giovanni Camillo Glorioso] in the astronomico-physical dissertation on comets; Fortunius Licetus [Fortunio Liceti] in the book *De cometis ac novis astris*, and in the disputation on the regular motion and minimal parallax of celestial comets; Adamus Tannerus [Adam Tanner], *De caelo*, question 7, paragraph 4; Descartes, part 3 of the new *Philosophia*, no. 41 and from no. 126 to 139.

[Margin: Writers on the comet of the year 1577.]

Besides these, on the occasion of the comet that appeared in the year 1577, there wrote: Tycho, in the whole of tome 2 of the *Progymnasmata*; Cornelius Gemma; Michaël Mæstlinus [Michael Mästlin]; Gulielmus, Landgrave of Hesse [Wilhelm IV of Hesse-Kassel]; Helisæus Rœslinus [Helisaeus Röslin]; Thaddæus Hagecius [Tadeáš Hájek]; Bartholomæus Scultetus; Andreas Nolthius; Nicolaus Vuincklerus [Nikolaus Winckler]; Io. Prætorius [Johannes Praetorius]; Cunradus Dasypodius [Conrad Dasypodius]; Valentius Steinmetz [?];

Nicolaus Bazelius [?]; Georgius Henischius [Georg Henisch]; Io. Huernius [?]; Theodorus Graminaus [?]; Antonius Paduanus [?]; Antonius Santutius [Antonio Santucci]; in Italian, Marcellus Squarcialupus [Marcello Squarcialupi]; Thomas Erastus; Petrus Surdus [?]; Io. Maria Fiornouellus [Giovanni Maria Fiornovelli]. Again, on the occasion of the comets that shone in the years 1580, 1585 and 1590, Tycho wrote in his letters to the Landgrave of Hesse, or to Rothmann [Christoph Rothmann], which begin at pages 13, 36, 121, 137, 170, 173 and 176. And once more, on the occasion of the comet of the year 1607, Longomontanus and Kepler published writings.

[Margin: Writers on the comet of the year 1618.]

But many more wrote on the comets of the year 1618, namely: the same Longomontanus, in the treatise *De novis phaenomenis*; Kepler, in three books (see him also in the *Optical Part of Astronomy*, pp. 264 and 335, and the *Epitome of Astronomy*, pp. 12, 57, 440 and 442, and in the *Hyperaspistes*, of which more shortly); Ambrosius Rhodius; Io. Remus Quietanus [Johannes Remus Quietanus]; Antonius Santutius; Io. Baptista Cysatus [Johann Baptist Cysat]; Thomas Fienus [Thomas Fyens]; Erycius Puteanus; Villelmus Schicchardus [Wilhelm Schickard]; Antonius Lucianus [?], in Spanish; Villebrordus Snellius [Snel]; Alphonsus Zobolus [?]; Io. Baptista Venantius [?]; Philippus Millerus [?]; Isaac Habrechtus [Isaac Habrecht]; Beniamin Vrsinus [Benjamin Ursinus]; Carolus Piso [?], in the *Speculum* of this comet; the author of the disputation held in the Roman College of the Society of Jesus, on the three comets of the year 1618; and another author, of a little work entitled in Italian *L'Asemblea di Parnaso*; Marius Guiducius [Mario Guiducci], setting out Galileo's opinions; Lotharius Sarsus [Lotario Sarsi] in the *Libra astronomica*, in which he weighed the statements of Mario Guiducci and Galileo, speaking in the person of Father Horatius Grassius [Orazio Grassi] of the Society of Jesus, his master; to which Galileo paid back his own weighing, or consideration, and examination in a little Italian work entitled *Trutinator*, *Il Saggiatore*; Gotardus Arthus Dantiscanus [Gotthard Arthus of Danzig].

Further, after the many listed so far who wrote on the comets of the years 1577, 1580, 1585, 1590, 1607 and 1618, Scipio Claramontius [Scipione Chiaramonti] wrote against many, but above all against Tycho, in his *Antitycho*; and in his defence of himself against the attacks of Liceti; and in the book *De sede sublunari cometarum*. But Kepler, in his *Hyperaspistes*, took up Tycho's defence, and, not content with the shield of protection, attacked Chiaramonti's *Antitycho* in words whether very sharp or very bitter; to which Chiaramonti at once replied with a second *Apologia*. And so these comets, but especially those of the years 1577 and 1618, might seem ominous and threatening on this account alone, that they stirred up so many and such sharp quarrels and contentions: between Kepler and Chiaramonti; between Liceti and the same Chiaramonti; between Galileo and Grassi; between Galileo and Chiaramonti; between Squarcialupi and Erastus; besides those with which the University of Padua resounded in Italian pamphlets.

IV. So much for the astronomers or natural philosophers. But orators too are to be counted in addition: Carolus Malapercius [Charles Malapert], in an oration delivered at the beginning of the school year, and Vincentius Guinisius [Vincenzo Guinigi], *De felicitate Cometis*. And poets too: Pontanus in the *Meteora*, and the interpreters on that passage of Virgil, *Georgics* 2:

> Non aliàs cælo ceciderunt plura ſereno Fulgura, nec diri toties arſere Cometæ.
> *At no other time did more lightnings fall from a clear sky, nor did dire comets so often blaze.*

CAPVT II.

Chapter II.

Nomina, Genera, & Species Cometarum.

The Names, Genera and Species of Comets.

I. *Cometa*, or *Cometes*, in Greek Κομήτης [Greek: Κομήτης (komētēs, "long-haired")], is so called from *coma*, "hair". Although, when it is taken in a narrower and stricter sense, it signifies one particular species of phenomenon — namely a hairy star, or one tressed with a mane of rays, or, to speak with Cicero, a curled star — in a wider sense it is nevertheless the common name. For Seneca rightly says, in book 7 of the *Natural Questions*, ch. 11: "They do not have one form. For although the Greeks have made distinctions among those whose fire is indeed diffused, but tending towards a peak, yet all of these are of the same mark and are rightly called comets." And soon after: "At the very time they appear, the spectators do not agree about their aspect; according as each has sharper or duller sight, so he says that it is brighter, or redder, and that its hair is either drawn forward or divided to the sides. But whether there are some differences among them or not, comets must come about in the same way." In the same way the Latins embraced all forms of such portents under the name of hairy stars. Thus Pliny, book 2, ch. 25, begins: "The Greeks call them comets, we call them *crinitae*, bristling with blood-red hair, and shaggy at the top like a head of hair." In that place he reviews twelve names, or species — granted that Albertus Magnus, tract 3 on book 1 of the *Meteorologica*, ch. 10, makes five differences of comets. Cornelius Gemma, for his part, in book 1 of *De divinis naturae characteribus*, ch. 8, expressed Pliny's forms in pictures, and Leopold confirmed them in tract 5 of *De annorum revolutionibus*. It is worth enumerating them, with the Greek names substituted.

[Margin: Comet, the generic name.]

[Margin: Pliny's division of comets.]

II. Let the first, then, be the κομήτης [Greek: κομήτης (komētēs)], that is, a comet in the strict sense of the name, or a star hairy all round, also called the Rose.

[Margin: 1. The comet.]

The second is the πωγονίας [recte: πωγωνίας] [Greek: πωγονίας (pōgō-nias, "bearded")], that is, a bearded star or bearded comet; for ὁ πὼγον [recte: πώγων] [Greek: ὁ πὼγον (ho pōgōn, "beard")] is "beard", which is also why Constantine, Emperor of the East, was called Pogoniates, that is, Long-beard or Beard-long. It is commonly not distinguished from the tailed comet, because, as Fromondus notes in book 3, ch. 4, only its relation to the Sun makes this difference. For if, by the motion of the primum mobile, it precedes the Sun in the morning, it appears bearded, its hairs running forward towards the west — such was the comet of the year 1618. But if it survives in the evening, after the Sun has set, it is seen as tailed, and marked out behind with a very long trailing robe towards the east — such was the comet of the year 1577.

[Margin: 2. The bearded, or tailed.]

The third is the Ακοντίας [Greek: Ακοντίας (akontias, "javelin-comet")], because it is darted with a long appendage of rays in the manner of a javelin; for in Greek ἀκοντίζω [Greek: ἀκοντίζω (akontizō, "I hurl a javelin")] is *iaculor*, "I hurl a javelin", and τὸ ἀκόντιον [Greek: τὸ ἀκόντιον (to akontion, "the javelin")] is "javelin". Such was the one seen in the year 1533, in the month of July, on the testimony of Milichius; and the one about which Titus Caesar, in his fifth consulship, wrote in a splendid poem, on the testimony of Pliny.

[Margin: 3. The javelin.]

The fourth is the ξιφίας [Greek: ξιφίας (xiphias, "sword-comet")], namely the Sword, or rather the sword-shaped, whose head indeed shows the hilt of a sword, while its mane tapers, shorter, to a point, so that it imitates a dagger rather than a longer sword. Pliny adds that this is the palest of all, with a certain gleam as of a sword, and without any rays — understand: distinct rays. In Greek, τὸ Ξίφος [Greek: τὸ Ξίφος (to xiphos, "the sword")] is "sword",

[Margin: 4. The sword, or dagger.]

and ξιφίδιον [Greek: ξιφίδιον (xiphidion, "little sword")] means a small sword or dagger. Piso, however (chapter 15 of the *Speculum*), thinks that this may be the same as the one the historians called the Goal-post or Pyramid. Such, it is said, was the comet that appeared in the year 1532.

[Margin: 5. The discus.]

The fifth is the Δίσκευς [Greek: Δίσκευς (diskeus, "discus")], or rather δισκοειδὴς [Greek: δισκοειδὴς (diskoeidēs, "disc-shaped")], that is, the disc-shaped, which, shining with a broad roundness, spreads a few rays at its rim and is the colour of electrum, says Pliny.

[Margin: 6. The pithetes, or jar-shaped, and the Tenaculum.]

The sixth is the πιθίτης [Greek: πιθίτης (pithitēs, "jar-like")], because it appears in the shape of a jar, as if hiding a smoky light and dark rays within a hollow in its middle; for in Greek ὁ πίθος [Greek: ὁ πίθος (ho pithos, "the jar")] means a jar. To this is referred the Tenaculum, lurid with a smoke like that of embers under ash.

[Margin: 7. The ceratias, or horn.]

The seventh is the Κερατίας [Greek: Κερατίας (keratias, "horned")], or κερατοειδὴς [Greek: κερατοειδὴς (keratoeidēs, "horn-shaped")], that is, the horned or horn-shaped, whose flame is twisted round in a curve like a horn, an augur's crook, or a Persian scimitar. Such, it is reported, was the comet that sounded from heaven like a war-trumpet as Xerxes was crossing into Greece, and also the last comet of the year 1618. For to the Greeks τὸ κέρας, ατος [Greek: τὸ κέρας, -ατος (to keras, "the horn," with its genitive ending)] is "horn."

[Margin: 8. The lamp.]

The eighth is the Λαμπὰδιας [Greek: Λαμπὰδιας (lampadias, "torch-like")], or Lamp, a comet that imitates a burning torch.

[Margin: 9. The horse star.]

The ninth is the ἱππὲυς [Greek: ἱππὲυς (hippeus, "horseman")], or Horse star, representing a horse's mane, which Piso says more recent writers call the Rose; and it is of very swift motion. In Greek ἵππος [Greek: ἵππος (hippos, "horse")] is "horse."

[Margin: 10. The silver-haired.]

The tenth is the Αργυροκόμης [Greek: Αργυροκόμης (argyrokomēs, "silver-haired")], the Argyrocomus — "silver-maned," as you might say, or "silver-haired" — of which Pliny speaks thus: *A white comet also occurs, gleaming with silver hair so brightly that one can scarcely look at it, and showing within itself the likeness of God in human form.* Fromondus (bk 3, ch. 4) suspects that these words refer to the Star of the Magi, perhaps because of the report, well established, that under Augustus a comet was seen in the East and in Judaea in which the image of a little boy was expressed, signifying God made man. In Greek ὁ ἄργυρος [Greek: ὁ ἄργυρος (ho argyros, "silver")] is "silver."

[Margin: 11. The goat.]

The eleventh is the Goat, a comet surrounded by the look of shaggy hair and a sort of mane, bristling, that is, with very fine fibres of rays; in Greek it would be called τραγοειδὴς [Greek: τραγοειδὴς (tragoeidēs, "goat-like")], since τράγος [Greek: τράγος (tragos, "he-goat")] is "goat."

[Margin: 12. The spear, or spit, or pole.]

The twelfth is the Λογχίτης [Greek: Λογχίτης (lonchitēs, "spear-like")], that is, the one having the form of a spear, which more recent writers have called the Spit or the Pole. Of this one Pliny says: *Once only so far has the likeness of a mane been changed into a spear, in the one hundred and eighth Olympiad, in the three hundred and ninety-eighth year of the City.* In book 2, chapter 25, Pliny adds other species, invented rather than actually seen: as when he says that if they have the likeness of flutes they portend something for musicians, if they are triangular, for men of talent, and so on.

[Margin: Aristotle's division of comets.]

III. If, then, we look to the accidents of colour and shape that Pliny lists, there are twelve species of comets. Aristotle, however (*Meteorologica* bk 1, ch. 8), reduces them to two species, namely the Haired and the Bearded. The first

is of those that spread their hair all round, and to it can be reduced the Disc, the Jar, the Horse, the Silver-haired and the Goat. The second is of those that stretch out a mane, straight or bent, toward one side only, and to it are reduced the Javelin, the Sword, the Horned, the Lamp and the Spear. Albertus Magnus, however, as I said, sets up five classes. Others prefer three kinds, namely the Haired, the Bearded and the Tailed.

[Margin: The astrological division of comets.]

Another division of comets is taken from their matter or place, inasmuch as some are elementary and others celestial; or from the efficient cause, as the astrologers count seven simple species of comet, arising from the seven planets, and assign them different colours and effects. Thus they ascribe leaden comets to Saturn, silver to Jupiter, red to Mars, golden to the Sun, yellow to Venus and blue to Mercury; and the Saracens call the Mercurial comet *Dominus Asconae* [Dominum Aſconæ]. Of these more below, from Iunctinus [Francesco Giuntini], when we deal with the prognostics of comets.

[Margin: The division of Epigenes.]

Lastly, Epigenes establishes two kinds by their motion and substance; of him Seneca writes thus (*Natural Questions* bk 7, ch. 6): *There are, says Epigenes, two kinds of comets: some pour out their fire on every side and do not change their place; others stretch out a wandering fire toward one side in the manner of hair, and pass through the stars.* This does not mean that the first kind lacks the motion of the primum mobile, but that, in the manner of the fixed stars, it does not stray on a roving course and has no proper motion of its own. This division seems to me very apt: since these phenomena, if they are not stars, are nevertheless certain imitations of stars, they may thus be divided into fixed and wandering comets.

[Margin: The waking rod and the seething pot of Jeremiah were not comets.]

But if anyone should try to drag the waking rod and the seething pot divinely shown to Jeremiah into portents of comets, he would surely act incongruously, says Johannes Praetorius at the beginning of his account of comets; and this is confirmed by the common opinion of the sacred interpreters on those passages of Jeremiah.

[Margin: Years before Christ.]

CAPVT III.

Chapter III.

Hiſtoria Cometarum 154. Chronologica & Aſtronomica. Cum euentibus, qui Cometis tanquam cauſis aut ſignis attribui conſueuere ab alijs, potiùs quàm à nobis.

A chronological and astronomical history of 154 comets, with the events that others, rather than we, have usually attributed to comets as their causes or signs.

I. Seneca, before his whole inquiry into comets (*Natural Questions* bk 7, ch. 2), rightly said: *It is necessary, moreover, to have the past appearances of comets*

collected; for because of their rarity their courses cannot yet be grasped, nor can it be explored whether they keep their turns and whether a fixed order brings them forth on their own day.

[Margin: The number of comets observed.]

Lavater (from Keckermann) [Lauatterus ex Kechermanno; Ludwig Lavater, Bartholomäus Keckermann] collects no more than 117 comets from Augustus to the year 1556. Yet Io. Camillus Gloriosus [Giovanni Camillo Glorioso] (bk 1, ch. 1) prudently judges that there were many others, either unobserved in the northern hemisphere and belonging to more southern climes, or inconspicuous for some other cause, or omitted by the historians. Nor indeed can any probable reason be adduced why so many comets should have blazed in the preceding century, and not, in other centuries, if not as many, at least far more than those recorded in the writers.

[Margin: On the ancient observers of comets.]

Nor is it surprising that the ancient astronomers were rather negligent in these matters, since much was still uncertain even about the regular planets. Whether, however, the observation of comets was neglected by the Chaldaeans may be doubted, from Seneca's words (*Natural Questions* bk 7, ch. 3): *Democritus,* he says, *the most subtle of all the ancients, says he suspects that there are more stars that move; but he set down neither their number nor their names, the courses of the five planets not yet being understood. Eudoxus first brought these motions from Egypt into Greece; yet he says nothing about comets. From this it appears that not even among the Egyptians, who took greater care of the heavens, was this part worked out. Conon later, himself a diligent investigator, collected the eclipses of the Sun observed by the Egyptians, but made no mention of comets — and he would not have passed over it had he found anything explored among them.*

[Margin: The contradictory assertions of Epigenes and Apollonius on comets observed in Chaldaea.]

Two men, certainly, who say they studied among the Chaldaeans, Epigenes and Apollonius of Myndus — the latter most skilled in the inspection of natural things — disagree with each other. Apollonius says that comets are placed by the Chaldaeans among the wandering stars and that their courses are known. Epigenes, on the contrary, says that the Chaldaeans have nothing grasped about comets, but that comets seem to be kindled by a certain whirl of air stirred up and twisted. I would reconcile them thus: Epigenes does not deny the Chaldaeans' efforts in observing and expounding comets, but only the certainty and evidence of their causes and matter. Let us come to the history.

[Margin: Year 480.]

[Margin: Ceratias.]

In the first year of the 75th Olympiad, or 480 before Christ, when Xerxes had crossed into Greece, a comet appeared — a Ceratias — shortly before Greece fought it out against him at Salamis, as Pliny reports (bk 2, ch. 25). Xerxes was defeated by Themistocles in a naval battle and returned to Asia, having lost a great part of his land army.

[Margin: The birth of Democritus.]

[Margin: Year 450, approximately.]

In the year 469 before Christ, or the third year of the 77th Olympiad, Democritus was born; he lived 105 years, as Diogenes reports from Thrasyllus. In his time certain comets appeared, and Democritus himself related that on their dissolution certain stars appeared, as Aristotle has it (*Meteorologica* bk 1, ch. 3). In the same century flourished Hippocrates of Chios and Aeschylus, in whose time a comet shone, as Aristotle relates in the same place.

[Margin: Year 431.]

In the year 431 before Christ, or the 2nd year of the 87th Olympiad, a huge comet was seen after sunset for 75 or 60 days, according to Thucydides and to Plutarch in his *Lysander*. With it came a great eclipse of the Sun; the Peloponnesian War began at once; and in the following year a pestilential catarrh ravaged Athens.

[Margin: Year 410.]

In the year 410 there was the comet of which Aristotle writes (*Meteorologica* bk 1, ch. 6): *When Eucles son of Molon was archon at Athens, a comet appeared, the Sun being near the winter tropic, in the month of January.*

[Margin: Years before Christ.]

[Margin: Year 400, approximately.]

In the year 412 before Christ Archelaus, king of the Macedonians, began his reign, and he reigned 14 years; in his time a comet shone, as Praetorius reports from Seneca.

[Margin: The birth and death of Aristotle; the birth [?] and the beginning of the reign of Alexander.]

In the year 384 before Christ, namely the first year of the 99th Olympiad, Aristotle was born, as Diogenes Laërtius reports from Apollodorus; he lived 63 years and died in the 3rd year of the 114th Olympiad, the year in which Diogenes also died, that is, in the year 321 before Christ. Alexander the Great was born in 356 before Christ, or the 106th Olympiad, and began to reign in the 20th year of his age, when Pythodorus was archon, that is, in the year 336 before Christ, the 111th Olympiad, as we have it from Diodorus Siculus, Plutarch, Arrian and others.

[Margin: Year 373 or 372.]

[Margin: A huge comet when Aristotle was a young man.]

Within these years several comets blazed. Notable among them was the one in the 4th year of the 101st Olympiad, that is, 373 or 372 before Christ, in which, as Cottunius [Ioannis Kottounios] says (*Meteorologica* bk 1, lecture 30), Aristotle was completing his eleventh year. Of this comet Aristotle writes thus (*Meteorologica* bk 1, ch. 3): *For that great comet occurred around the time of the earthquake in Achaea and the rising of the waves; it rose from the equinoctial west.* That this is the same comet as the one Seneca speaks of (*Natural Questions* bk 7, ch. 5) is gathered from these words: *Charimander too, in the book he composed on comets, says that Anaxagoras saw a great and unusual light in the sky, the size of a*

broad beam, and that it shone for many days. Callisthenes reports that a fire of a long shape of this kind appeared before the sea covered Buris and Helice. Aristotle says that it was not a beam but a comet; but that because of its excessive heat the scattered fire did not appear, and that as time went on, when it was now blazing less, the comet's own appearance was restored. In this fire there were many things worth noting, but nothing more so than that, as soon as it shone in the sky, the sea was over Buris and Helice.

Ephorus, however, added certain things about the same comet, but of doubtful credit, in Seneca, in the same place, chapter 16. For Seneca says: *But Ephorus, a writer of by no means the most scrupulous good faith, is often deceived and often deceives. So with this comet, which was watched by the eyes of all mortals because it drew after it the outcome of an enormous event, when at its rising it sank Helice and Buris: he says that it divided into two stars, which no one but he reports. For who could observe the moment in which the comet was dissolved and reduced to two parts? And how is it, if there is anyone who saw a comet split into two, that no one has seen one made out of two? And why did he not add into which stars it was divided, since it must have been one of the five stars?* But neither of these conclusions necessarily follows. The beginning of the comet could have gone unobserved, while its end was observed, or its breaking up into two — not into two of the five known planets, but into lesser comets, or unknown planets. Seneca, however, out of Stoic disdain and a swollen wit, is inclined to carp at the Greeks whom the Romans had conquered.

[Margin: A comet occupying a third of the hemisphere.]

Again, in book 1, chapter 7 of the *Meteorologica*, Aristotle adds that a great comet occurred, in these words: *Moreover, all those seen in our time vanished without setting, in a place above the horizon, gradually consumed, so that the body of no single star was left behind, nor of several. And the great star, of which we made mention earlier, appeared in winter, in frost and clear weather, in the evening, when Aristaeus was archon. On the first day it did not appear, as if setting ahead of the Sun, but on the next day it appeared, as far as was possible, for it lagged behind the Sun by the least amount, and it soon set. Its light, however, stretched out as far as a third part of the sky, like a leap; for this reason it was also called the PATH. It climbed as far as the belt of Orion, and there it was dissolved.*

[Margin: Year 373.]

Since, then, he says that this was the great comet of which he had spoken earlier, it certainly seems to be the one that appeared in 373 before Christ, around the time of the earthquake in Achaea. The catalogue of the Athenian archons also supports this, for in that year Astius is listed, perhaps a contracted or corrupted form of the name Aristaeus. That comet was therefore famous on many counts and, as Seneca said, *watched by the eyes of all mortals*. Diodorus Siculus makes this comet a rival of the Moon; and Aristotle must be understood of the same comet in that book 1, chapter 7, when he says: *When the stone fell from the air at the river Aegos, raised up by a wind, it fell by day; and then too a comet had appeared in the evening. At the time of the great comet it was winter, and a*

northerly one, and the flood was produced by the opposition of the winds: for within the gulf Boreas prevailed, while outside a great south wind was blowing. Here he seems to be speaking a third time of that great comet which was followed by the submersion of Buris and Helice: understand this from the words *And at the time of the great*, etc.

[Margin: Years before Christ.]

[Margin: Year 341.]

Again, in the year 341 before Christ, in which, according to the catalogue of archons, Nichomachus presided at Athens, a comet shone out, of which Aristotle writes (*Meteorologica* bk 1, ch. 7): *Further, when Nichomachus was presiding over the Athenians, a comet occurred for a few days near the equinoctial circle, rising in the evening; in its time that wind about Corinth occurred.* It seems probable to me that those three comets were known to Aristotle by sight: one shone out when Aristaeus, another when Eucles son of Molon, and the third when Nichomachus presided at Athens. Nor, perhaps, did the three others escape him of which we shall speak in the following paragraph.

[Margin: Years 356 and 336.]

For both in the year in which Alexander the Great was born, namely 356 before Christ, and in the year in which he began his reign, which was 336, as we said above, comets appeared, two in all, each of which was visible for 70 days, as Justin the historian relates. So, although the first of these comets was unlucky for the Illyrians, since they were defeated by Parmenio, the general of Philip, who stormed Potidaea, it was nevertheless lucky for Philip. The other comet even then began to be unlucky for the Persians: for in that same year 336 Darius Codomannus, who was to be the last king of the Persians, inaugurated his reign, and against him Alexander was elected commander by the common votes of all Greece, if you except the Lacedaemonians. First, however, Alexander took up arms against the Thebans, who had revolted from him, and razed Thebes to its foundations; ninety thousand Thebans were slain and thirty thousand captured — namely in the 2nd year of the 111th Olympiad, that is, the first of his reign.

[Margin: Pindar's house alone was left intact.]

[Margin: Year 348.]

Between the two preceding comets, however, there was the one that appeared in 348 before Christ, of which Pliny says (bk 2, ch. 25): *Once only so far has the likeness of a mane been changed into a spear, in the one hundred and eighth Olympiad, in the three hundred and ninety-eighth year of the City.*

[Margin: Year 146.]

[Margin: A comet no smaller than the Sun.]

After it, at a long interval, came the one Seneca narrates thus (*Natural Questions* bk 7, ch. 15): *After the death of Demetrius, king of Syria, whose children were Demetrius and Antiochus, a little before the Achaean war, a comet shone out no smaller than the Sun. At first it was a fiery and reddish orb, giving out a bright light, enough to overcome the night; then gradually its size contracted and its brightness faded, and at last it vanished altogether. How many stars, then, must come together*

to make so great a body? Though you gather a thousand into one, they will never equal this extent of the Sun. The Achaean war began in the year 146 before Christ; about that year, then, this great comet blazed out, which was unlucky for Corinth and Carthage, both destroyed.

Seneca continues: *In the reign of Attalus a comet appeared, modest at first; then it raised itself and spread out, and came as far as the equinoctial circle, so that, stretched out immensely, it equalled that region of the sky which is called the Milky Way. How many wandering stars, then, must have come together to occupy so long a tract of the sky with continuous fire?* It is credible that he is speaking of the Attalus who flourished after the Achaean war and was so great a friend of the Romans that he made the Romans heirs of his kingdom; he began to reign in the year 137 before Christ and died in 133. Within these years, then, this portent was seen.

[Margin: Year 135, approximately.]

[Margin: 130.]

Not much later Mithridates was born, namely in the year 130 or 129 before Christ. For it is established that he was killed in the consulship of Iulius Silanus and Licinius Murena, that is, in the year 62 before Christ, and that he lived 68 or 69 years and reigned 57, according to Appian.

[Margin: 119.]

A comet seen at each time foretold the beginning both of his life and of his reign, as Justin records (bk 37) in these words: *Celestial portents too had foretold his future greatness. For both in the year in which he was born and in the year in which he first began to reign, a comet shone throughout each period for seventy days so brightly that the whole sky seemed to be on fire. For in size it had filled a fourth part of the sky, and in brilliance it had surpassed the splendour of the Sun; and in rising and setting it took up a space of four hours.* For together with its tail it occupied 45°, which are a quarter of the hemispherical circle; so it is no wonder if, by oblique ascension and descension, it did not wholly rise or set except over a space of 4 hours. Eutropius, however, says that Mithridates reigned 60 years and lived 72.

[Margin: And 60, approximately.]

About the year 60 another comet was observed by Posidonius, as Seneca indicates (bk 7, ch. 20), saying: *We do not see many comets because they are obscured by the rays of the Sun; Posidonius reports that during an eclipse of the Sun a certain comet appeared which the nearby Sun had concealed.* Before

[Margin: Years before Christ.]

[Margin: Year 47, 46 or 45.]

the murder of Julius Caesar, who was killed in the Senate in the year 44 before Christ, not one comet but several appeared, as Virgil sings in *Georgics* 2:

> *Never at other times did more lightning fall from a clear sky, nor did*
> *dread comets blaze so often. So Philippi saw Roman battle lines clash*
> *once more with matching weapons.*

But I believe he is also speaking of the comet or comets that shone out before the civil war when Caesar had crossed the Rubicon, of which Lucan writes in book 1:

> *Dark nights saw unknown stars, the pole ablaze with flames, and torches flying slantwise through the void of heaven, and the hair of the dreaded star, the comet that changes kingdoms on earth.*

[Margin: The comet after Caesar's murder.]

Of these Pliny says (bk 2, ch. 25): *The comet, a star terrifying* magna ex partes [?] *and not lightly expiated, as in the civil disturbance in the consulship of Octavius, and again in the war of Pompey and Caesar.*

[Margin: Year 44.]

After Caesar was killed, a hairy star appeared, of which Suetonius writes thus (*Julius Caesar*, ch. 88): *He died in the fifty-sixth year of his age and was numbered among the gods, not only by the mouths of those who decreed it but also by the conviction of the common people. For at the games which his heir Augustus was giving, the first held in his honour, a hairy star shone for seven successive days, rising about the eleventh hour, and it was believed to be the soul of Caesar received into heaven; and for this reason a star is added to the top of his statue.* Plutarch asserts the same in his *Caesar*, adding that it was a huge hairy star which, after Caesar's death, blazed with remarkable brilliance until the seventh night. But Pliny is fuller on this (bk 2, ch. 25), where he narrates thus: *In only one place in the whole world is a comet worshipped, in a temple at Rome; it was judged very auspicious by the deified Augustus himself, because it appeared at his own beginnings, at the games he was holding for Venus Genetrix not long after the death of his father Caesar, in the college he had founded.*

[Margin: Augustus's words on the good omen of the comet.]

For he expressed his joy in these words: "On the very days of my games a hairy star was seen for seven days in the region of the sky that lies under the Septentriones. It rose about the eleventh hour of the day, and was bright and visible from all lands. By that star the common people believed it was signified that Caesar's soul had been received among the divine powers of the immortal gods; on which account that emblem was added to the bust of his head which we shortly afterwards dedicated in the forum." This he said in public; with a more inward joy he interpreted it as born for himself, and himself as born in it; and, if we confess the truth, it was salutary for the world.

Nor is it unfitting that this comet was seen while the Sun was still above the horizon, since Venus has been seen in the daytime more than once by us and often by others. Some take the following words of Q. Curtius (bk 10) to refer to this very comet: *Rightly and deservedly, therefore, the Roman people professes that it owes its safety to its Prince, who shone out as a new star in the night that we had held to be almost our last. It was his rising, by Hercules, not the Sun's, that gave light back to a darkened world. How many torches did he then put out! How many swords did he sheathe! How great a storm did he scatter with sudden calm!*

[Margin: Year 13.]

[Margin: A comet divided into several.]

In the year 13 before Christ, the year before the death of Agrippa, Dio (bk 54) relates that there was a comet which appeared for many days hanging over the City and was then dissolved into many torches. Perhaps this is the one of which Seneca speaks (bk 7, ch. 17) in the words *the one we saw under Augustus*.

Of the wonderful Star, however, which divinely foretold the birth of CHRIST to the Magi, this is not the place to treat, but below, in Section 2, where we deal with new stars.

[Margin: Years of Christ.]

Hiſtoria Cometarum qui Annis poſt CHRISTI Ortum apparuerunt.

A history of the comets that appeared in the years after the birth of CHRIST.

[Margin: 14.]

[Margin: 54.]

II. The first comet after the birth of CHRIST recorded in history is the one seen in the year 14 of Christ, near the death of Augustus, as Fiornouellus [?] reports; or at any rate the one that appeared a little before the death of the emperor Claudius, in the consulship of Q. Asinius Marcellus and M. Acilius Aviola. Of it Suetonius says (*Claudius*, ch. 46): *The chief omens of his death were the rising of a hairy star, which they call a comet, the striking of his father Drusus's monument by lightning, and the fact that in the same year most of the magistrates of every kind had died.* This comet, according to Seneca (bk 7, ch. 21), rose in the north and went on into the east, ever dimmer. Claudius, moreover, died of poison; and so Pliny (bk 2, ch. 25), having said that a comet is a star terrifying for the most part and therefore not lightly expiated, added: *In our own age, about the time of the poisoning by which Claudius Caesar left the empire to Domitius Nero* — understand: the comet was not lightly expiated.

[Margin: Years of Christ.]

[Margin: Year 60.]

Under Nero, however, who succeeded him, a double comet was seen. One was in the year of Christ 60, in the consulship of Nero for the fourth time and Cossus Cornelius Lentulus, as we have it from Tacitus, *Annals* 14, who says: *Amid these events a star also shone out, of which the common opinion is that it portends a change of kingdoms.*

[Margin: Year 64.]

The other was in the consulship of C. Lecanius Bassus and M. Licinius Crassus, namely the year of Christ 64 and the tenth of Nero's reign; of it the same Tacitus writes in *Annals* 15 in this way: *At the end of the year prodigies were reported: lightning more frequent than ever before, and a comet, a star always expiated by Nero with noble blood.* Pliny (bk 2, ch. 25) says that this star was almost continuous and savage during Nero's principate. For Kepler, in *De Cometis*, p. 111, wrongly makes this one distinct from that one and draws the

earlier back to the year 60 of the common era, not attending to the consuls under whom Tacitus places it.

[Margin: A comet of 6 months.]

Seneca, it is true, says (*Natural Questions* bk 7, ch. 17): *There is no reason for us to think that the comet seen under Claudius was the same one we saw under Augustus; nor that this one, which appeared under Nero Caesar and took away the ill repute of comets, was like the one that emerged after the murder of the deified Julius, at the games of Venus Genetrix, about the eleventh hour of the day.* And in chapter 21: *For six months this comet, which we saw in the most happy principate of Nero, offered itself to view, turning round in the opposite direction to that Claudian one. For that one, rising from the north toward the zenith, made for the east, ever dimmer; this one began from the same quarter, but, tending westward, bent toward the south and there withdrew from sight.* But the grim events that followed, and the deaths of senators and of Seneca himself, showed how happy Nero's principate was.

In the same book, chapter 29, Seneca mentions another comet, saying: *The comet that appeared in the consulship of Paterculus and Vopiscus brought about what Aristotle and Theophrastus had foretold: for there were very great and continuous storms everywhere, and in Achaea and Macedonia cities were overthrown by earthquakes.* I would think, however, that these were suffect or honorary consuls, for I do not find them in the catalogue of the ordinary consuls.

[Margin: Year 70.]

In the year 70 of Christ, in the consulship of Flavius Vespasian (for the 1st time) and Titus his son, Jerusalem was laid waste, a comet — a Xiphias — having preceded it and shone for almost a whole year, hanging over the city of Jerusalem itself, as Josephus reports (*Jewish War* bk 7), and Zonaras (vol. 1). So that it might be plain that those who had not believed the star and the Magi announcing Christ deserved to be deprived of kingdom and priesthood, a comet foretold that disaster.

[Margin: Year 76.]

Later, in the year 76 of Christ, another comet blazed out with a very long mane in the shape of a javelin. On seeing it Vespasian, as Aurelius Victor relates, remarked wittily that it concerned the king of the Persians — or, as Dio has it, of the Parthians — whose hair was more flowing. Of this comet is Pliny's remark (bk 2, ch. 25): *The Acontiae quiver like a javelin, with a very swift significance. This was the one about which Titus Imperator Caesar wrote in a famous poem in his fifth consulship, the last seen to this day.* Dio, however, adds that a memorable drought followed, and fiery prodigies around Vesuvius, and vast images of men which the people called aerial Giants; and that ashes were blown not only as far as Rome, so that the Sun was darkened, but even into Africa and Egypt. But a little before Vespasian's death another comet, pertaining more closely to him, was seen — unless perhaps Dio is to be understood of the earlier one.

[Margin: Year 130.]

After these, in the year 130, a comet was seen that was most auspicious for Viuardus [?], king of England, who, having subdued the tyrant Carduela [?], joined many provinces to himself and to God.

[Margin: 340.]

In the year of Christ 340, in which Constantine, son of Constantine the Great, was killed, a huge comet shone for several months, as Praetorius reports.

[Margin: 383.]

[Margin: 386.]

[Margin: 390.]

Praetorius adds that in the year 383, under Theodosius, a comet like the one over Jerusalem shone; and also that in the year 386 the comet was seen of which Claudian speaks in his Panegyric of Honorius; and, about the year 390, the one that Proclus saw above Jupiter, as Peucerus [Caspar Peucer] has it, cited in Tycho (vol. 1, pp. 619 and 625), and Cysatus [Johann Baptist Cysat] (p. 56).

[Margin: Year 392.]

By far the most celebrated is the one that appeared a little before the strangling of Valentinian, which took place on the Ides of May in the year of Christ 392. For of it Nicephorus (*Ecclesiastical History* bk 12, ch. 37), after recounting the unusual floods of the Nile, says: *Then indeed unusual portents too were seen, which*

[Margin: Years of Christ.]

were to forebode evils for the world. *For first an unexpected and unusual star, shining in the sky about midnight near Lucifer, appeared about the zodiac; because of its flashing rays it was large and bright, and scarcely yielded to Lucifer. Little by little a great multitude of other stars also gathered to it. Had you seen this spectacle, you would have likened it to a swarm of bees flying about their leader. The light flashing out from them by their mutual concourse became a single flame and, shining with a certain terror, bore the appearance of a great and dreadful sword, two-edged throughout. For because the remaining stars fell into the same aspect and seemed to be a single star with the one that had been seen first — which was, as it were, the root or hilt — its flame was carried aloft just like a burning wick from a lamp. Its motion, moreover, differed entirely from the course of all the stars. At first, beginning to move from the place we have mentioned, it rose and set together with Lucifer. Then, gradually withdrawing from it, it made at a slow pace towards the Bears, and moved by an oblique path to the left of those watching. Its circuit, indeed, it completed in common with the other stars it had joined; but by its own motion, which was prolonged to the fortieth day, it at last barely reached the Great Bear, and vanished in the middle part of that constellation, where it had last shone. At that time it appeared sword-shaped.*

[Margin: The age of Nicephorus.]

Where Nicephorus drew this from, I do not know; for he cannot have been an eyewitness, since he is scarcely older than the year of Christ 1300, as is gathered from the dedicatory epistle of his *History*. See what I shall say in ch. 6, Scholium 3.

[Margin: Year 396.]

[Margin: Year 400.]

[Margin: A comet reaching right down to the earth.]

But the fire that appeared, sulphurous, over Constantinople in the year 396 — of which St Augustine speaks in his book *On the Destruction of the City* in these words: *At nightfall, when the world was already growing dark, a fiery cloud was seen from the east, small at first; then gradually, as it approached over the city, it grew, until it hung hugely and terribly over the whole city. A dreadful flame seemed to hang there, nor was the smell of sulphur lacking: all fled to the church,* etc. — that fire, I say, does not seem to have been a comet, although Baronius, at the year of Christ 400, designates another, very similar portent over Constantinople by the name of comet. Gainas [Gaynas] was raging at that time and before [?], and so to this time belongs the comet of which Socrates speaks thus (*Ecclesiastical History* bk 6, ch. 6): *Through the tyranny of Gainas so grave a danger hung over the city of Constantinople that a very great comet, reaching from heaven right down to the earth (such as no one had ever seen before), portended that very thing.* Very similar are Sozomen's words about it (bk 8, ch. 4): *At which time a comet too portended such plots of Gainas; it appeared very great over Constantinople, reaching almost to the earth itself, and is said to have been such as never was before.* And those of Nicephorus Callistus (*Ecclesiastical History* bk 13, ch. 6): *Which plots of Gainas that star portended, bearing the appearance of a sword; it was splendid beyond measure, such as the record of letters relates never to have appeared before: shining above the city itself, and reaching from almost the top of heaven to the earth itself.*

[Margin: Year 405.]

In the year 405, in which Stilicho [?] defeated the Getae, there appeared a comet described by Claudian in his poem *On the Gothic War*, in the verses we shall quote in ch. 4, no. 8.

[Margin: 454.]

[Margin: 538.]

[Margin: 557.]

[Margin: 570.]

[Margin: 587.]

[Margin: 594.]

[Margin: 603.]

[Margin: A comet of 6 months.]

[Margin: 632.]

In the year of Christ 454, the 26th of the Emperor Valentinian, Trithemius reports, besides a comet, other prodigies in the sky, which seemed to foretell the death of Theodosius and the invasions of Attila. In the year 538 a comet was followed by so great a famine that men devoured their own flesh, as Idatius [Hydatius] testifies. Likewise, in the year of the saving Birth 557, a hairy star in the shape of a lance appeared at Constantinople, as Johannes Praetorius relates. And again in the year 570 a comet was seen, and the Lombards, with the eunuch Narses as their leader, occupied Italy; but Alboin is killed in the year 571. And after 17 years, namely in the year 587, which was the fourth of the reign of Maurice, a comet appeared at Byzantium, as Zonaras testifies; and

in his eleventh year, namely the year of Salvation 594, a comet appeared for a whole month, and famine and drought followed it. Then in his last year, which was the year of Christ 603, the comet Xiphias was seen, in the year in which Maurice was killed with his three daughters. Under this emperor, moreover, Mahomet was born, at which time, Johannes Praetorius reports, a hairy star appeared for 6 months — and perhaps it was one of those already mentioned. But before Persia was occupied by the Saracens, another comet was seen in the year 632, bearing the form of a fiery sword, as Petrus Surdus [?] and Spondanus [Henri de Sponde] relate from Theophanes.

[Margin: 676.]

[Margin: Year 684.]

In the year of Christ 676 — or, as others relate, 678 — a terrible comet was watched at Rome for three months. Orosius adds that great rains and frequent thunder followed, but Bede says drought and plague, and that in the following year locusts flew in great swarms through Syria and Mesopotamia. And it is said that after 8 years a great star appeared in the east, though it is uncertain whether it was a comet; after it, in the year 685, Vesuvius vomited out fire, as Sabellicus [Marcantonio Sabellico] relates (*Enneads* 8, bk 6).

[Margin: Year 729.]

[Margin: Two comets.]

[Margin: Or 726.]

[Margin: Or 725.]

In the year of restored Salvation 729, Bede and Palmerius [Matteo Palmieri] in his chronicle affirm that two comets appeared in the month of January for 15 days, one of which went before the Sun like Lucifer, while the other followed it like Hesperus. Yet it could have happened that, because of its great declination towards the pole, the same comet was visible and was seen both morning and evening. To this year Johannes Praetorius and Giovanni Camillo Glorioso refer these comets; although Cardano, in bk 2 of *On the Judgements of the Stars*, text 53, refers them to the year 726, and says they were seen within 14 days in the month of January, with their tails turned towards the north. Vincentius Guinisius of the Society of Jesus, however, in his oration *On the Felicity of a Comet*, sets them back to the year 725, in which there was indeed foul weather throughout the whole world, but Charles Martel triumphed over the Saracens.

[Margin: Year 745.]

[Margin: 761.]

[Margin: 763.]

In the year of Christ 745, after a comet seen in Syria at the end of the year, there followed a plague lasting three years, which ravaged first Sicily, then the islands of the Mediterranean and all Greece, and finally Constantinople, as Johannes Praetorius relates. Then in the year 761 a comet shone in the east for ten days, and another in the west for 21 days — perhaps one and the same, although Functius [Johann Funck] in his *Chronicle* reports them as different. Again, in the year 763 a comet was seen in the east, followed by a great drought

and unbearable cold; the Turks, meanwhile, extended their power with great success.

[Margin: 800.]
[Margin: 814.]
[Margin: 829.]
[Margin: A hailstone from the sky.]
[Margin: 837.]
[Margin: 839.]
[Margin: 842.]
[Margin: 843.]
[Margin: Year 844.]

In the year 800 Johannes Praetorius assigns a comet, the year in which the Roman Empire was transferred to Charlemagne; then in the year 814 a terrible comet appeared a little before the death of that same Charles — from the *Annals of the Franks*. And in the year 829, as Fiornouellus relates, there were seen shooting stars rather than comets, which killed cattle and some men; and in the same year a hailstone four feet long fell from the sky. And in the year 837 Pepin died, a comet having gone before — granted that others carry this forward to Louis the Pious, who died in the year 840, and say that in the year 839 a comet shone out in the sign of Aries, at the sight of which, especially with an eclipse added, he began to prepare himself for death. Then in the year 842, which was the second of Lothar, a comet again shone in the sign of Aquarius; and in the following year several comets are said to have foretold the deadly war that Lothar waged with his brothers. Cardano says there were three comets (on Ptolemy, bk 2 of *On the Judgements of the Stars*, text 54), the first of them in Libra, the second in Aries, the third in Aquarius. In the year 844, too, Abū Maʿshar observed a comet above Venus, as Cardano relates in bk 4 of *De subtilitate*, and from him Tycho (vol. 1, p. 783) and Cysatus (*On Comets*, p. 56).

[Margin: Year 876.]
[Margin: 882.]
[Margin: 906.]
[Margin: 945.]
[Margin: 983.]

In the year 786 [?], on the eighth day before the Ides of October, a comet was seen, and floods and plague followed from it. In the year 882, in which Louis, King of the Bavarians, died, a hairy star appeared on the 15th day before the Kalends of February. In the year 906 great rains followed a comet that was unusually red; and in the year 945 a comet of wonderful size was seen in Italy, followed by famine there and in Gaul. In the year 983, too, in which the Emperor Otto II died of grief, a comet was seen, with famine and plague as its attendants. So Johannes Praetorius tells of these five comets in his *Catalogue of Comets*. Earlier, however, in the year 982, Vesuvius had poured out fire, according to Sigonius [Carlo Sigonio] (*On the Kingdom of Italy* bk 7) — or in this same year 983, as Baronius has it (vol. 10).

[Margin: Year 1005.]
[Margin: 1009.]
[Margin: 1027.]
[Margin: 1066.]

In the year 1005 a comet of horrible aspect was seen for thirteen nights about the Easter feast, as Johannes Praetorius has it. And in the year in which John XVIII died — or, as others say, XIX — namely the year 1009, a terrible comet appeared towards the south, followed by great famine and plague, as Cardano affirms (text 54 on bk 2 of *On the Judgements of the Stars*). And in the year 1027, after the blazing of a comet, there was so dire a plague that the living were not enough to bury the dead, says Petrus Surdus.

In the year 1066 a comet, before the death of St Edward, king

[Margin: Years of Christ.]
[Margin: 1067.]
[Margin: 1071.]
[Margin: 1098.]
[Margin: 1099.]
[Margin: 1106.]
[Margin: 1141.]

of England appeared, as Henricus Spondanus [Henri de Sponde] relates in his *Epitome of the Annals*; and in the year 1067 there was another, according to Fiornouellus, in the year in which the supreme pontiff Alexander was harassed by Cadalus, bishop of Parma. Again, in the year 1071, after a new star seen towards the south and west, a comet with long and flaming hair appeared for 25 days. Like it was the flaming comet of the year 1097 or 1098, which appeared in the west, as Keckermann records; and another in the year 1099, in the east, which changed its place by leaps. A very great comet also appeared in the year 1106, in the season of Lent, in which Henry IV died, according to Johannes Praetorius; and in the year 1141 as well.

[Margin: 1169.]

In the year 1169 a comet appeared in Scotland with two very long rays, burning and huge, and Malcolm IV, King of Scots, surnamed the Maiden, died; so Cardano reports (on the *Quadripartitum*, bk 2, text 54).

[Margin: Year 1200.]

In the year 1200 Haly Ben Rodoan [ʿAlī ibn Riḍwān] the Arab, while applying himself to astronomy as a young man, saw a comet whose solid part appeared three times larger than Venus, and of round shape, and its light, he says, was as great as if a quarter of the Moon were shining. Its place was at 15° Scorpio, and it moved with the motion of the primum mobile from east to west, in such a way that it was carried more swiftly against the order of the signs, until it reached 15° Virgo. So he himself says in bk 2 of the *Quadripartitum*, ch. 9; and Cardano in the same place, text 54, describes many astrological circumstances of this comet from Haly. A rebellion in Africa followed, of which he speaks in the same place. The empire of the Tartars then made its beginnings.

[Margin: Year 1211.]

In the year of restored Salvation 1211, in the month of May, a comet was seen with its tail stretched out over Russia, and it lasted 18 days; from it came the flood of the Tartars into Muscovy and Assyria, as Spondanus relates in his *Epitome of the Annals*, as does Johannes Praetorius.

[Margin: Year 1214.]

[Margin: Twin comets.]

[Margin: Year 1240.]

[Margin: A comet of 6 months.]

But in the year 1214 two comets appeared together in Scotland, one going before the Sun in the morning, the other following it in the evening; in that year King William, a man of outstanding prudence and holiness, died at Stirling aged 74, as Hector Boëtius [Hector Boece] has it (*History of the Scots* bk 13). Again, in the year 1240 a comet appeared in the east, with its hair stretched out right to the middle of the sky, and it reached right to the pole, as Albertus Magnus affirms as an eyewitness (*Meteorologica* bk 1, tr. 3, ch. 5); and it was extinguished scarcely within six months, as Daniel Santbech testifies (book *On Observations*, prop. 19). In that year Tamerlane burst into Asia, as Pontanus [Giovanni Pontano] has it. But Cardano, in bk 2 of *On the Judgements of the Stars*, text 54, puts this comet in the year 1260, saying that Pope Urban died in that year, which is false. One or the other, then, must have erred in the numerals, writing MCCXL for MCCLX, or the reverse.

[Margin: Year 1254.]

[Margin: 1264.]

In the year 1254 a comet was visible in Germany for several months, followed by a violent abundance of winds. And in the year 1264 a huge comet appeared in the east about the beginning of August, and shone for 40 days. At first it rose two hours before sunrise, large and bright, spreading out a long and broad tail, which day by day grew fainter, and it joined the Sun: so the *Annals of Colmar* have it. In this year, moreover, it is established that Urban IV died, about the beginning of October; his death the comet already mentioned seemed to have foretold, and it was seen at Ptolemais, in Italy and in France, as Bzovius [Abraham Bzowski] says in his *Annals*. Pontanus too speaks of the same comet in his commentary on Ptolemy's *Centiloquium*, at the last aphorism, saying: *In the year one thousand two hundred and sixty-four from the day of Christ's birth, in the month of August, a hairy star was seen in the eastern part of the sky, which spread its hair from its very rising to the middle of the sky, and it shone for about three months, nor did it cease to be seen before Pope Urban, Pontifex Maximus, died. Then Charles moved with an army out of France and, having overcome Manfred, occupied the kingdom of Naples.* Since, however, Urban's successor Clement IV was not elected until February of the year 1265, Johannes Praetorius perhaps for that reason draws this comet to the year 1265; but Functius and Camerarius agree with the *Annals of Colmar*. Accordingly Cardano has confused this comet with

the other, and Pontanus, who designated the year in words, not in figures, is rather to be believed.

[Margin: Year 1267.]

[Margin: 1268.]

The *Annals of Colmar* add that three years later, namely in the year 1267, a beautiful and great star rose at sunrise near the Moon, on July 18; advancing with great impetus from the Moon towards the east as far as the middle of the hemisphere, it left behind it a tail, or a white and fiery cloud, and the two ceased to be together. And in the year 1268 a comet of wonderful size was seen for several days in Scotland, towards the south (a great marvel of a portent indeed), as Cardano relates in bk 2 of *On the Judgements of the Stars*, text 54, who attributes to it great disasters in Scotland arising from winds.

[Margin: Year 1283.]

In the year 1283 too, Johannes Praetorius relates, a comet was seen in summer, at the time when the Emperor Adolf was defeated in battle by Albert.

[Margin: 1298.]

[Margin: 1301.]

[Margin: 1303.]

[Margin: 1305.]

[Margin: 1312.]

[Margin: 1313.]

[Margin: 1314.]

In the year 1298, as Spondanus and Bzovius relate in their annals, so great an earthquake suddenly occurred in various regions of the world that it surpassed all memory; it happened on the feast of St Andrew, and at the end of November a comet appeared. Then in the year 1301, before the deaths of Charles Martel and of Andrew, King of Hungary, another comet was watched, according to the same Spondanus. And in the year 1303 there was a comet like a fiery column descending and soon ascending again, as Keckermann relates (*Systema physicum* bk 6, p. 694). And soon after, in the year 1305, at the Easter feast, a comet was the herald of a plague that was to rage through the whole world, says Johannes Praetorius. He adds that in the year 1312 a comet showed itself for 14 days, and another in the following year, which ran from north towards south following the motion of Mars; and finally that in the year 1314 a comet appeared at the end of Virgo for three weeks and directed its flame towards the north; and that there was then a very great famine in Lithuania and the adjacent regions, rains through the whole summer, and from them plague, by which a third part of the living were carried off. Philip the Fair, King of France, dies on November 29, thrown from his horse while hunting.

[Margin: Year 1337.]

[Margin: Twin comets.]

[Margin: Year 1341.]

[Margin: 1347.]

In the year 1337 a dire comet lasted in the sign of Taurus for 4 months and more, and another joined it, which shone together with it in the months of June, July and August. There were many disasters of war, a host of locusts, and plague at Nuremberg. Johannes Praetorius, in his *Catalogue of Comets*, adds that three years later another appeared in the sign of Virgo in the month of March, and that 400 buildings at Nuremberg were consumed by fire; the Emperor Andronicus, moreover, died. And he, together with Fiornouellus, reports that in the year of Christ 1347, in the month of August, there was seen in the sign of Taurus, towards the north, something huge — either a comet or a fiery vapour — as if falling from the sky, to the great terror of those watching; the one, however, was seen for a few days, the other for two months. Throughout Europe, but especially in Italy, a severe famine followed, and in the following year a plague from Asia right to Britain, which lasted three years, carried off the greatest part of mortals, and surpassed all pestilences and diseases within human memory.

[Margin: Year 1352.]

[Margin: 1353.]

[Margin: 1363.]

[Margin: A comet of three months.]

[Margin: 1375.]

Then the years 1352 and 1353 also had their comets; and in the year 1363 a huge comet appeared[6] from the east for three whole months, casting its rays, in a dire burning of the air, towards the west up to the middle of the sky, as Johannes Praetorius says. In the year 1375 the death of Charles IV followed a hairy star; and in the year 1391, too, a comet was seen having the shape of a spit, which astrologers, the same Praetorius says, reckon the worst.

[Margin: 1401.]

[Margin: Year 1402.]

In the year 1400 a huge comet appeared at the eastern cardinal point of the sky, which moved towards the west, and this a few months before Tamerlane, dynast of the Tartars, burst into Asia against Bayezid — who had grown proud with many successes — with very great slaughter of the Turks. The *Turkish Annals* record this slaughter in the year of the Hegira 804, which began in the year of Christ 1401, and they call Tamerlane Temir, as Chalcondylas [Laonikos Chalkokondyles] also does. Pontanus, however, who reports on the comet already mentioned at aphorism 100 of the *Centiloquium*, and Cardano, who does so in bk 2 of *On the Judgements of the Stars*, text 54, call him Tamyris or Tamorlanus. He died in the year 1403, likewise with a comet going before, as Johannes Praetorius relates; but Fiornouellus assigns this comet to the year 1402, saying that it too had the shape of a spit and shone for a month, and that shortly afterwards there died not only Tamerlane but also Galeazzo [Galeatius], Duke of Milan. Perhaps there were two different comets.

[6]printed *appatuit*, for *apparuit*

[Margin: 1407.]
[Margin: 1433.]
[Margin: 1434.]
[Margin: 1439.]
[Margin: 1444.]

Johannes Praetorius assigns, moreover, a comet of its own to the year 1407, in which the Prussians are said to have been defeated by the Poles with great slaughter; a comet of three months to the year 1433; and their comets to the years 1434, in which Trapezuntius [Georgios Trapezountios] flourished, and 1439, in which Turkish power grew and the strength of the Christians was weakened; and besides these, in the year 1444, a huge comet at the time of the solstice, which terrified mortals.

[Margin: Year 1450.]
[Margin: A comet eclipsing the Moon.]

In the year 1450 that comet was seen of which Georgius Phranza [Georgios Sphrantzes] tells thus in bk 5, ch. 21, of his *History*: *In the summer of the year of Christ 1450 a comet above the horizon*

[Margin: Years of Christ.]

began to appear every evening immediately after sunset, in the likeness of a broadsword. Passing beneath the full disc of the Moon, it brought about an eclipse, according to the usual order and motion in the circuit of the heavenly lights. Some, seeing that this comet expressed the appearance of a sword, that it moved from west to east and approached the Moon, and seeing the darkness of the Moon's eclipse, interpreted the matter thus: that the Christian princes, having formed a league, would come against the Turks and would vanquish them. The Turks too fell into no slight fear at that portent. So evident was it that that comet was below the Moon. Our Tannerus [Adam Tanner] also reports this author's words (*De Caelo*, question 7, §4). In that year, moreover, Amurath [Murad II], emperor of the Turks, died of drunkenness.

[Margin: Year 1456.]
[Margin: Year 1457.]
[Margin: A comet occupying 60 degrees and more.]

In the year 1456, as Johannes Praetorius relates, a comet appeared with various prodigies for a whole month in Poland and elsewhere; it was believed to have foretold the death of Ladislaus, King of Bohemia, who was carried off by poison. In the following year, that is 1457, says the same author, there shone the comet of which Pontanus speaks thus in his commentary on Ptolemy's *Centiloquium*, aphorism 100: *When we were young men, a remarkable comet also shone for many days towards the east, in the regions of Cancer and Leo, of such length that with the trail of its hair it occupied more than two signs of the sky. It was followed by the death of King Alfonso, which entangled Emilia, Sabina, Campania and the whole kingdom of Naples in a long and grievous war; a pestilence, somewhat longer-lasting, followed as well.* Alfonso died, moreover, in the year 1458, on the Kalends of July, aged 66.

But Kepler, in his *Physiology of Comets*, p. 130, affirms that Pontanus is to be understood of the comet of the year 1456, when Mehmed [Mahometes] the Turk besieged Belgrade, formerly called Taurunum or Alba, after he had stormed Constantinople in the year 1453, on May 29. But in this year 1456 he was driven from the siege, with great slaughter and disgrace of the Turks, by Johannes Corvinus Huniades [János Hunyadi], guardian of Ladislaus, King of Hungary, then still a boy, on the day sacred to St Magdalene. The chief author of this battle was a most holy man of the Order of Friars Minor, Ioannes Capistranus [Giovanni da Capestrano]: at his urging 40,000 men took up the badge of the cross and arms against the Turks. Johannes Huniades, however, who was voivode, that is, prince of Transylvania, died a happy death in the same year.

[Margin: A comet occupying 50 degrees, of wonderful swiftness and motion.]

[Margin: Year 1471.]

But Pontanus continues, in the place already cited, saying: *In these preceding years another comet was seen, at first with a slender head and very short hair. Soon, grown to wonderful size, it began to turn aside from the east towards the north, now with quickened, now with slackened motion; and because Mars and Saturn were both going back, it was itself carried turned away, with its hair going in front, until it reached the Bears themselves. From there, as soon as Saturn and Mars began to proceed in direct course, it bent its path towards the west with such speed that in one day it traversed about thirty degrees; and when it reached Aries and Taurus, it ceased to be seen. This comet both shone for very many days and, though it looked short at the beginning, grew so much that it occupied fifty degrees and more. Not long afterwards Ussonus Cassanus [Uzun Hasan], king of Parthia and Armenia, having set out for the Euphrates with the design of invading Asia, joined battle with Mahomet, king of the Turks, and fought in such a way that he both inflicted and suffered a huge defeat; for two most powerful armies, two very great commanders, clashed twice within a few days.* And a little after: *We learned that in the same period all Spain was shaken by war, and that King Henry died.*

[Margin: Year 1460.]

[Margin: 1470.]

[Margin: 1471.]

[Margin: or]

[Margin: 1472.]

But this comet was not the one that was seen in the year 1460, from which Ioannes a Capistrano is said to have prophesied the evils of the age, as Johannes Praetorius says; nor the one that ran down with very long hair on the Ides of January in the year 1470, if you believe Bodin (*Theatre*, bk 2). The one Pontanus speaks of is referred, rather, to the year 1471, in which Alfonso, King of Portugal, stormed Tangier and Arzila. But Kepler, in his *Physiology of Comets*, p. 129, says that that defeat of the Turks and Tartars was in the year 1473, and the comet in the year 1472, of which presently.

[Margin: A comet observed by Regiomontanus.]

In the year 1472 two comets appeared, as the Polish history has it. One of them, as Petrus Surdus relates (*On Comets*, part 1), was seen in the month of March having the form of a javelin, and so great a drought followed it that springs were long without water. The other was observed by Regiomontanus on January 20; it lingered in Libra and was followed by a drought of three years, as Johannes Praetorius relates in his *Catalogue of Comets*, and Cardano in bk 2 of the *Quadripartitum*, text 54, where he says that this comet is assigned by Antonius Mizaldus [Antoine Mizauld] now to the year 1472, now to the year 1475. Kepler, in *Tycho's Hyperaspistes*, bk 3, ch. 2, 17 and 18, teaches how this arose: Mizauld admits that he took the account of this observation from Ziegler's *Conceptiones in Genesim* [Jakob Ziegler], and Ziegler easily erred, taking the obsolete figure for two as a five. Fracastoro (*Homocentrica* bk 3, ch. 23) says that it was seen in February 1472, that it was a pogonias [bearded comet], and that from southern it became northern. The words of Regiomontanus, as given in Ziegler and Cardano, are these: *On the Ides of January in the year of the Lord 1475* (read 1472) *a comet was seen by us under Libra, with a star of Virgo. Its head was of slow motion until it was near Spica; then it advanced through the legs of Bootes towards his left, and, departing from there, in one natural day it described a portion of a great circle of 40°. When it was in the middle of Cancer, it stood at its greatest distance from the circle of the signs, 67°, and was then passing between the two poles, of the zodiac and of the equinoctial, as far as the space between the feet of Cepheus; then through the breast of Cassiopeia, over the belly of Andromeda. Afterwards, proceeding along the length of the Northern Fish, where its motion slackened greatly, it approached the zodiac, crossing it near the middle of Aries, until its heliacal setting with the stars of Cetus hid it from us in the last days of February. By this proper motion of its own it described a portion of a great circle, along which it was carried northwards, against the succession of the signs, from Libra into Aries. At the end and at the beginning it moved slowly; in the middle very swiftly, in one day through four signs, from the end of Virgo right to the beginning of Gemini. Its tail, however, being less mobile, by continually facing the stars of Gemini, went round them, never deviating from them through the whole time of the apparition. Therefore at its first emergence it stretched the tail towards the west, because the stars of Gemini were there; but at the end, the comet being placed under Aries, it appeared only in the west because of the nearness of the Sun, with its tail stretched towards the east, because in this situation the stars of Gemini lay there. In the middle of the apparition it turned its tail to the south: the stars of Gemini were then there; and it happened in one and the same night that immediately after sunset the tail faced east; as midnight approached it faced south; after midnight, west; before sunrise it pointed to the region of the north. This variation of the tail in position arose from the diurnal motion, which is always from east to west. By its proper motion, however, the extremity of the tail — though more slowly than the head of the comet — also always moved westward, against the succession of the signs, describing a parallel from the beginning of Libra right to the middle of Taurus: passing beneath the feet of the Howler [Ululans], through the Great Bear,*

approaching Perseus, through whom it went around the Pleiades to the tail of Aries, at about the middle of Taurus. From this it is evident that both the head and the tail of the comet were directed towards the west, and never towards the east, not only by the diurnal motion but also by the proper motion.

[Margin: The size of this comet.]

Kepler, moreover, confirms in the places cited that this is the same comet of which Pontanus spoke above: from the motion Pontanus indicated; from the station and retrogradation of Mars and Saturn, which fall at the end of the year 1471 and the beginning of 1472; from the authority of Eber, who says expressly that it is the one which Pontanus too described, both in the *Centiloquium* and in an elegant poem in his *Meteora*, and which the mathematician of Gurk measured and affirmed to have been more than 4000 German miles long, and 26 miles broad at the head and 81 at the tail; and from Matthias Palmerius [Mattia Palmieri], who called this comet the Pole [Pertica]. We therefore subscribe to these rather than to Fromondus [Libert Froidmont], who (*Meteorologica* bk 3, ch. 4, art. 4) assigns this comet of Regiomontanus to the year 1475. Finally Peter Apian, in part 2 of the *Astronomicum Caesareum*, says that the comet of the year 1472 let its tail down right to the earth, that in his own time old men were living who remembered it, and that it was called the peacock's tail. A great drought followed, and the campaign of Ussumcassanus [Uzun Hasan].

[Margin: Year 1477.]

[Margin: 1491.]

In the year 1477 a blackish comet is noted by Eber in his *Catalogue*, after which followed the slaughter of Charles of Burgundy. Finally, towards the end of that century, that is, in the year 1491, there occurred an eclipse of the Sun and a comet about the beginning of Aries, with southern latitude; Vualtherus [Bernhard Walther] notes nothing else about it in his observations. At the beginning of the following year Ferdinand, having taken Granada, expels 800,000 Moors.

[Margin: 1500.]

In the year 1500 a comet appeared in the north, in the longitude of Capricorn, in the month of April, shortly before the

[Margin: Years of Christ.]

Tartars laid waste Poland and the Muscovites Lithuania. Ludovicus Sfortia [Ludovico Sforza] was captured besides; Vesuvius erupted; and the Christians, alas, were defeated by the Turks — so Io. Praetorius and others.

[Margin: Year 1506.]

And in the year 1506 a hairy star appeared within the Bear [inter Arcton] on April 11 [3. Idus Aprilis]. It lasted 25 days and was seen as foretelling the death of the King of Poland and of Philip, father of Charles V, as Praetorius has it. Cardano, however, in book 2 of the *Quadripartitum*, text 53, reports from Augustinus Niphus [Agostino Nifo] that this comet referred to the great conjunction made in Cancer in the year 1504;

[Margin: A drought of 10 months.]

and he adds that the comet's companion in Insubria was a drought so great that from January to November there was no rain.

[Margin: Year 1512.]

[Margin: 1513.]

The comet seen in the year 1512, in March and April, was blood-coloured, and it was followed by the killing of Bayezid [Baiazetis] and, the following year, by the death of Pope Julius II and a great force of wars, as Petrus Surdus relates. He appends another comet, of the year 1513, seen from the end of December until February 19 of the following year, with various colours and an elongated tail, which ran from the end of Cancer to the end of Virgo; it was visible all night. So King Francis of France came into Italy, and various wars followed.

[Margin: Year 1521.]

[Margin: 1526.]

[Margin: 1527.]

[Margin: 1528.]

In the year 1521 a conspicuous comet appeared in the month of April at the end of Cancer, with short hair, bright, and like the Moon at dichotomy; wars against the Roman Pontiff followed, and the death of Leo X. But the one seen in the year 1526, from August 23 to September 7, was like a fiery sword, on the testimony of Petrus Surdus, part 2. Then in the year 1527 another comet is recorded, observed by Io. Vögelin [Io. Vogelino]; the astronomer Petrus Creuster [?] says that it lasted 1¼ hours. Cornelius Gemma, however, in book 1 of *On the Characters of Nature* [de naturæ charact.], ch. 8, says that it was seen in the Palatinate in the form of a sword. And in the year 1528 another comet appeared in Pisces from January 18, in opposition with Saturn: King Francis came into Italy again, and Rome was taken by Bourbon [à Borbonio].

[Margin: Year 1529.]

[Margin: 4 comets.]

[Margin: Year 1530.]

In the year 1529 four comets are recorded, standing opposite one another and spreading their hair towards the four sides of the world; Vienna was besieged by the Turks; there was a severe famine at Nuremberg. Here Io. Praetorius wishes for a more accurate description of them. In the year 1530 — according to the same Praetorius, to Cardano (text 54 on book 2 of the *Quadripartitum*) and to Petrus Surdus — a comet began to be seen from August 6 and lasted until September 3 or 13. It was seen in the morning before sunrise, and then, after some days, in the evening after sunset; and in that time it passed through Cancer and Leo, Virgo and Libra, in which sign it vanished. At that time many towns in Holland, Zeeland and Brabant were submerged; there were Swiss disturbances and the rustic war of the German peasants; and in the following year an earthquake in February shook Lusitania, so that at Lisbon 1050 houses collapsed, and 600 tottered and came close to ruin. Some, however, ascribe these calamities to the comet of the year 1531.

[Margin: Year 1531.]

In the year 1531 there appeared a comet, a Pogonias or bearded comet, as Milich attests on book 2 of Pliny, ch. 25. Apian described it in part 2 of the *Astronomicum Caesareum*, ch. 1, and observed it at Ingolstadt from August 6 to 23. He affirms that its tail ran out towards the side turned away from the Sun, and therefore, once the Sun had set, rose upwards; that at first it was seen in the morning, but on August 15 set heliacally, and afterwards emerged and was seen in the evening; and that from this it came about that the ignorant common people thought there had been two comets. When it approached the horizon its flame seemed to be extinguished — which is how he understands the words of Pliny, book 2, ch. 25, saying that comets are never in the western part of the sky, that is, that because of their weak light they never appear there; for otherwise, on the 13th, it set at 9^{h} 55^{m} after noon. Further, on August 14 the place of the comet's head was ♌ 23° 39′, with northern latitude 23° 2′; that of the end of the tail ♌ 19° 18′, with latitude 39° 45′. But on the 23rd the place of the head was ♎ 2° 51′, with latitude 14° 31′. The northern node of the comet's orbit with the ecliptic was at Scorpio 14°. The motion of the comet in its own path was gathered from the difference of the longitudes and the complements of the latitudes, as you see in the following small table, from the 13th, the day on which the observations were resumed and treated more accurately: it was, namely, swifter in the middle. This comet was followed by disturbances in Switzerland, in Saxony and on the Baltic Sea; a flooding of the sea into the Low Countries [Belgium], and the towns of Zeeland submerged; the Turkish incursion into Austria; and the imprisonment of King Christian of Denmark [Chriſtierni Regis Daniæ].

Motion of the comet in its own course

From day	To day	Daily motion	Distance of the comet's head from the Sun
13	14	4° 3′	23° 40′
14	15	4° 0′	23° 31′
15	16	6° 0′	22° 15′
16	17	5° 24′	22° 12′
17	18	5° 0′	23° 2′
22	23	3° 40′	28° 16′

[Margin: Years of Christ.]

[Margin: Another comet of the year 1531, and Apian's error.]

It is uncertain, however, whether this comet is the same as the one which Fracastoro, in book 3 of the *Homocentrica*, ch. 23, says he saw in this year 1531 on September 8 and 9, in the morning and much further north than the summer tropic; but that about the 13th it became an evening comet and emerged about the summer tropic; and that on the following days, its longitude little changed, it was driven in latitude beyond the equinoctial and was seen not far from Jupiter, which was then at Scorpio 13°, and about the 18th was gradually consumed. Its motion was therefore from west to east, obliquely — that is, from Magister or

Caurus towards Notapeliotes[7]. But since Apian says that his comet began to disappear on August 23, and did not emerge as an evening comet until after August 15, and always places it on this side of the ecliptic and the equator, who does not see that these were different comets? And the report of two comets was not founded on their having been seen first in the morning and then in the evening, but on the fact that the days and months in which they were seen in the morning and those in which they were seen in the evening were different. But the shorter life of this one, of 10 days, and the slow report of it, or cloudy weather at Ingolstadt, perhaps begrudged Apian the observation.

[Margin: Year 1532.]

In the year 1532 there was the comet called Xiphias by Milich on book 2 of Pliny, ch. 25. It was at first southern, then turned towards the north, and began to be seen by Apian on September 25; it was seen until November 20, while he was staying at Dresden, a town of Meissen, under a pole height of 51°. On October 2 the place of the comet was observed by him in ♏ 8° 24′, with southern latitude 13° 44′; and the comet set before the Sun and was seen only in the morning. But on October 31, at 5^h after midnight, its place was seen in ♎ 21° 30′, with northern latitude 13° 15′; and on the preceding evening it had begun to appear in the evening shortly after sunset, its tail inclining day by day more towards the south, until it became perpendicular with respect to the zenith. Then on November 8, at 5^h 12^m after midnight, the place of the comet was Scorpio 3° 35′, with northern latitude 19° 36′, and the tail bent towards the north; it was also visible in the evening. The northern node of this comet's orbit was at the beginning of Libra. This and more is from Apian. Cardano, however, in book 3 of *De subtilitate*, and Fracastoro, in book 3 of the *Homocentrica*, ch. 23, say that it rose on September 22 and set on December 3, and that in those 71 days it advanced no more than 63°, namely from ♏ 5° to Scorpio 8°; and from this he concludes that it was above the Moon. Fracastoro adds that its head was three times larger than Jupiter and its tail two cubits long, that its motion was from south to north, and that on September 22 it was at Virgo 5° with southern latitude 15°, but on December 3, when it vanished, much further east.

[Margin: Parallax of 6° in this comet.]

Vögelin, a pupil of Regiomontanus, is said to have observed in this comet a parallax of 6°, and to have deduced its diameter as one and a quarter German miles. To this comet is attributed Barbarossa's movement against Io. Auria [Doria].

[Margin: Year 1533.]

In the year 1533 the comet Acontias, or spear-shaped, shone out, on the testimony of Milich on book 2 of Pliny, ch. 25. Fiornouellus says that it was of a blackish colour, turned its tail towards Africa, was seen for 30 days in the northern parts, and shone in advance of the death of Clement VII and of

[7]the north-west and the south-east winds

Alfonso III [?], Duke of Ferrara, who died the following year. Cardano, in book 2 of the *Quadripartitum*, text 54, says that its motion was against the order of the signs, from Gemini through Taurus into Aries. Petrus Surdus, in part 2 of *On Comets*, asserts that it was yellow in colour and was seen from the end of June to August 4; and that there followed the English schism in the year 1534, in which Henry VIII of England declared himself head of the Church; the furies of the Anabaptists; the incursions of Aenobarbus [Barbarossa]; the death of Clement VII; and that of the Duke

[Margin: Years of Christ.]

of Milan. Fracastoro, in book 3 of the *Homocentrica*, ch. 23, says that it was seen by some on July 1 between the Pleiades and the horns of Aries, but by himself only on the 7th, about hour 2 of the night; that it was a little larger than Jupiter, with a tail equal in the sky to the length of a militatis [?] spear; that on the 21st, at hour 8 of the night, it was near the sword of Perseus, and on the 27th near the star of the Chair of Antiope [cathedræ Antiopes]; and that all three comets, of the years 1531, 1532 and 1533, had their tails turned away from the Sun. But it is better to hear Peter Apian, its diligent observer. In part 2 of the *Astronomicum Caesareum* he relates that on June 18 he observed the comet in ♊ 3° 40′, at a distance of 66° from the Sun, with northern latitude 32°, near the star Algenib; that on the 21st the tail was 15° long; and that on the 25th it was in ♉ 15°, with latitude 43°, at a distance of 88° from the Sun. It never set, because of its nearness to the north pole, and after June 24 it was visible from about midnight until sunrise, always between Boreas and Subsolanus, since, when the Sun was about to rise, it had nearly ascended to the meridian.

[Margin: A comet above the Moon.]

He adds that, had it been as near the Earth as the Peripatetics think, it would have been hidden within the Earth's shadow; and that some, from its regular motion against the order of the signs, suspected that it was a star which had come down and afterwards turned back upwards. On this comet, however, Gemma Frisius the elder says in the *Radius Astronomicus*: *We saw a comet in the year 1533, in July, moved from the star of the He-goat [Hirci ſidere] along the course of the Milky Way through the constellation of Cassiopeia — that is, at once against the order of the signs and far into the north.* There followed the devastation of Westphalia by the heretics; the open outbreak of the English schism; the movement of the Turks against the Persian; and that of Barbarossa against Muleasses.

[Margin: Year 1538.]

In the year 1538 a comet shone from January 17 to the 21st, observed by Peter Apian. He says that it had its tail erect towards the zenith, 30° long, and that its place was in ♓ 5°, with northern latitude 17°, on the very neck of Pegasus; on the 17th the comet's head stood 32° 30′ from the Sun. Petrus Surdus says that it was in opposition to Saturn, and that it was followed by the Florentine uprisings against Cosimo the Great [Coſmum Magnum], Duke of Etruria; the arrival of Francis, King of the French, in Italy with a great army; the conference

of the Pontiff with him and with Charles V; and the unhappy league of the Christians against the Turks, when Io. Auria, who was commander of the Spanish, Venetian and papal fleet, was defeated by Barbarossa.

[Margin: Year 1539.]

In the year 1539 the same Apian observed a comet from May 6 to the 17th. He says that on the 17th, at hour 10 of the night, it was in ♌ 17°, with southern latitude 3°, with a short and dim tail, and that it stood 72° from the Sun; and finally that its motion was against the order of the signs, and swifter towards the west than the motion of the primum mobile. To this comet are referred the slaughters and disturbances of Meissen, Thuringia, Brandenburg and Brunswick.

[Margin: Year 1543.]

In the year 1543, as Froidmont relates from Senertus [Daniel Sennert] (*Meteorologica* bk 3, ch. 1, art. 6), the tail of a comet, having fallen down as far as a certain stream, drained it dry. For this reason Scaliger too (*Exercitationes* 79) says that a comet of his own time was seen to fall; and Vicomercatus [Francesco Vimercati] thinks that the fable of Phaethon arose from a similar comet, but Tertullian thinks it arose from the burning above Sodom.

[Margin: Year 1556.]

In the year 1556, on the testimony of Cardano (*De varietate* bk 14, ch. 69), a comet was seen *about the beginning of March, almost equal to half the Moon, with hair not long and not steady, but flickering red as in fires, as in torches while the wind blows; it was dusky, and reddish and turbid.* Cardano adds that in 4 days the comet moved 75° from east to west and 30° from south to north.

[Margin: Homelius's observation.]

Io. Homelius [Johannes Homelius], who also observed it, relates that on March 5 it stood above Spica, beside the left wing of Virgo; on the 8th below the knee of Bootes; on the 9th beside Arcturus — so that it moved, as it were, along the circumference of a great circle straight towards the north pole of the ecliptic, and at that time completed more than 15° of that circle in one day. Then it ascended towards the pole of the equator with great speed, and from there, as if descending from a summit, made for Saturn, which was then in Aries. And just as it had earlier advanced in longitude from Libra against the order of the signs, so afterwards it imitated the motion of Saturn and advanced in the order of the signs through Andromeda to the sign of Pisces, where it was extinguished.

[Margin: Whether the tail is always turned away from the Sun.]

He adds that the rays, which in the evening were turned towards the south, were turned at midnight towards the west; and that its hair was not turned away from the Sun until it had moved less than a quadrant of a circle away from the Sun. Since, moreover, it was far from the Sun and from the ecliptic, it could be seen both morning and evening, and at times did not descend below the horizon.

[Margin: Charles V feared the comet for himself.]

Kepler, in the *Physiologia Cometarum*, p. 113, says that Charles V feared it for himself in vain, since he survived it a long while; that it portended rather the disturbances between the Spanish and French kings, and in Livonia by the Teutons [per Teutonicos]; and on p. 129 he also ascribes to it the hidden quarrels between the brothers Charles and Ferdinand.

[Margin: Year 1557.]

[Margin: 1558.]

In the year 1557 a similar portent was seen in the western region in the month of October, in the sign of Sagittarius; and in the year 1558, Io. Praetorius and Fiornouellus relate, a comet having the form of a spit [Veru] was seen in August for many weeks in Scorpio, with its tail turned towards Rome. Froidmont (*Meteorologica* bk 3, ch. 3) thinks that it foretold the death of Charles V, which occurred this year on September 21. Others say that it had its tail turned towards Spain and was beneath Coma Berenices; and so this is rather the one for which Charles V feared for himself, and from which came that pentameter he composed about it:

By these signs, then, do my fates summon me?

[Margin: Year 1559.]

[Margin: A comet unlucky for many princes.]

Mary, Queen of England, also died on November 17, and on the same day Reginaldus Polus [Reginald Pole]; the queens of Poland and of Hungary died too. Again, in the year 1559 another comet appeared in the east from the end of May until June 22; Henry II, King of France, died, and shortly afterwards Paul IV and 15 cardinals, Hercules [Ercole], Duke of Ferrara, the King of England, the King of Portugal, the King of Denmark, the Doge of the Venetians, and many other princes; and the treachery of the Huguenots broke out more openly.

[Margin: Year 1569.]

In the year 1569 a comet is recorded, shining in the month of November in Ophiuchus [Serpentario] and corresponding in its longitude to Sagittarius and Capricorn; Selim was then intent on seizing Cyprus, which he obtained the following year. The comet advanced from Cancer and, towards the end, abandoning its direct course, turned aside towards Virgo 4°, where it became stationary. So Kepler, in the *Physiologia Cometarum*, pp. 114 and 129. The following year an earthquake shook Ferrara, and the sea flooded the Low Countries [Belgium].

[Margin: Year 1577.]

De Cometa Anni 1577. Compendium ſelectarum Obſeruationum Tychonis.

On the Comet of the Year 1577. A Summary of Selected Observations of Tycho.

[Margin: Tycho's observation.]

In the year of restored salvation 1577, sailors on the Norwegian Ocean, enjoying calmer weather, began to see this comet on November 10, as Tycho

acknowledges in the preface to volume 2 of his works; Tycho himself, however, began to see it on November 13, shortly before sunset, when the tail was not yet visible for the brightness of the Sun. After sunset the tail appeared very long and stretched out towards the east. On that day, about one hour after sunset, the comet was seen just above the head of the constellation Sagittarius [Sagittarij ſtellati], not far from Saturn, which it stood a little higher than, and it threw its tail towards the horns of Capricorn. From there, on other nights, it went with a swifter course towards Antinous, and passing his left hand and going past the tail of Delphinus, it was seen to traverse the nostrils of Equuleus, leaving the head of Pegasus to the south. From there it was carried straight towards Scheat of Pegasus with a slower motion than before, and at last disappeared on January 26 of the year 1578, between Scheat and the two very small stars that are in the breast of Pegasus. It never crossed the meridian by night, but always inclined from it towards the west; and for this reason, and because of the inequality of its proper motion, Tycho acknowledges that the investigation of its parallax set very intricate labyrinths before him.

After these preliminaries Tycho reports, in chapter 1 of that volume 2, the observations of this comet: of the distances of the head and of the end of the tail from the fixed stars, taken with the radius and the sextant; of the azimuths and altitudes, taken with the quadrant, at definite moments of time; and of the apparent size of the head. From these I have chosen those set down below, passing over those which Tycho acknowledges were not exact, and the days on which, for want of clear weather, it was not observed.

[Margin: November observations.]

On November 13 the apparent diameter of the comet's head was seen to be 7′; the length of the tail, though not yet fully visible, 22°. At $5\frac{1}{2}^{h}$ the head was 18½° from the new ☾; but at 6^{h} it was 26° 48′ from the bright star of the Flying Vulture, and 21° 19′ from the lower horn of Capricorn. The tail was

[Margin: Years of Christ.]

curved in an arc, wider at the end than in the middle, and its upper part almost touched the lower of the two stars in the horn of Capricorn.

On the 14th, at 6^{h}, it was 23° 25′ from the bright star of the Vulture, 18° 20′ from the lower horn of ♑, and 10° 12′ from Saturn; the tip of the end of the curved tail was already touching the uppermost star of the horn of ♑. On the 15th, at 6^{h}, it was 20° 25′ from the bright star of the Vulture and 16° 14′ from the lower horn of ♑. The tail, more contracted, was as far from the northern horn of ♑ at its lower end as the two stars of that horn are from each other. The colour of the head was pale and a leaden white; the tail, the farther from the head, the darker and more livid. The clouds and storms that followed interrupted the observation.

On the 20th, at 6^{h} in the evening, it was 11° 7′ from the bright star of the Vulture and 27° 35′ from the mouth of Pegasus. On the 21st, shortly after 6^{h}, it was 10° 40′ from the bright star of the Vulture and 25° 19′ from the mouth

of Pegasus; the tail stretched towards the end of the wing of Pegasus. On the 23rd, on a very clear night, at 6^h, it was 11° 1′ from the bright star of the Vulture and 21° 5′ from the star of the mouth of Pegasus. The colour of the head was not as bright as in the fixed stars; the tail was a dusky red near the head, like a flame struggling out through dense smoke. The tail was seen to end at the upper star in the forehead of Equuleus [Equiculi], and was more curved than usual, raising its convex part towards the zenith.

On the 25th, at $5\frac{3}{4}^h$, it was 17° 21′ from the mouth of Pegasus, and at 6^h 12° 38′ from the bright star of the Vulture; the tail inclined towards the gape [rictum] of Pegasus. On the 29th, at 6^h 15^m, it was 11° 33′ from the mouth of Pegasus and 16° 49′ from the bright star of the Vulture, or Eagle; the end of the curved tail ended exactly at the mouth of Pegasus, and its convexity almost touched the little star which is in the mouth of Equuleus. On the 30th, at 6^h, it was 10° 25′ from the mouth of Pegasus and 17° 45′ from the Eagle; the end of the tail still reached the mouth of Pegasus, while its convexity precisely grazed the little stars of the nostrils of Equuleus.

[Margin: December observations.]

On December 1, at $5\frac{1}{2}^h$, the comet was 18° 47′ from the Eagle and 9° 20′ from the mouth of Pegasus; the tail inclined towards the place between Markab [Marchab] and the pair in the middle of the neck of Pegasus. On the 10th, at $5\frac{1}{2}^h$, it was 4° 43′ from the mouth of Pegasus and 24° 33′ from Scheat of Pegasus; the end of the tail was about 4° from the brighter star of the neck of Pegasus.

On the 12th, at 6^h, it stood 5° 8′ from the mouth of Pegasus and 23° 7′ from Scheat; the tail inclined towards the middle of the quadrilateral of Pegasus. On the 14th, at 6^h, it was 21° 42′ from Scheat and 5° 50′ from the mouth of Pegasus. On the 17th, 19° 35′ from Scheat and 7° 20′ from the mouth of Pegasus. On the 24th the comet was so faint that it could scarcely be observed with instruments; yet with the radius it was seen to be about 16° from Scheat. On the 30th it appeared to be 12° 35′ from Scheat and 11° 56′ from the first star of the neck of Pegasus; the tail inclined towards the more southern of the two stars of Triangulum that adjoin the feet of Pegasus. On the 31st, at 6^h, it was 12° 0′ from Scheat and 14° 0′ from the mouth of Pegasus.

[Margin: Year 1578.]

[Margin: January observations.]

On January 1, at 6^h, the comet was 11° 35′ from Scheat and 14° 25′ from the mouth of Pegasus; it was very faint, and threw its rays towards the lower of the two stars of the neck of Pegasus, where they seemed to end. On the 9th, at $6\frac{1}{5}^h$, it was 8° 15′ from Scheat and 7° 40′ from the brighter star of Triangulum at the feet of Pegasus. On the 13th, at 6^h, the comet was so slight that its distances could not be taken; on the following days it could not be observed, because of either clouds or the presence of the Moon. On the 26th, at $7\frac{1}{2}^h$, scarcely a trace of the comet could be observed, and only by those with sharp sight, between

Scheat of Pegasus and the nearer of the two small stars at the breast of Pegasus; and this was its last appearance.

Concluſiones per Tychonem deductæ ex prædictis obſeruationibus.

Conclusions drawn by Tycho from the observations already mentioned.

First, in chapter 2 of volume 2, Tycho deduces the declination, right ascension, longitude and latitude from the altitudes and azimuths of the fixed stars from which the comet was distant, by the usual problems (which we shall treat in Book 10), and then corrects them by observations of the fixed stars made afterwards with larger and more exact instruments. Second, in chapter 3 he derives the comet's longitude and latitude, and in ch. 4 its right ascension and declination, from its distance from fixed stars already known, likewise by the common problems of the primum mobile and the analysis of triangles. But at the end of chapter 4 he acknowledges that he had used an obliquity of the ecliptic of 23° 27′, which he afterwards found to be 23° 31′ 30″; and he adds:

[Margin: Tycho's candid admission.]

But since the difference from the value first assumed is very small, those few minutes altering the truth of the matter under inquiry insensibly, I was unwilling to repeat the whole calculation, with labour more tedious than useful.

After this, third, he passes to chapter 5, in which he investigates the path of the comet and its situation with respect to the ecliptic and the equator, again by common problems — a path which can also very easily be marked out on a globe once the comet's places are known as to longitude, latitude, right ascension and declination. From these he finally deduces the intersection of the cometary orbit with the equator, or northern node, at 299° 50′, counting from the beginning of ♈ in the order of the signs, and the angle of inclination of the comet's path to the equator, 33° 45′. He appends, moreover, an ephemeris of the places and motion of this comet for each day; but we have put together an epitome of it in the following small table, as a specimen and as sufficient knowledge of what is to be said.

Specimen of the Ephemeris of the Comet of the Year 1577, adapted to hour 6 after noon at Uraniborg [Vraniburgi], from Tycho's volume 2, p. 86. The daily motion is to be understood as running from hour 6 of the preceding day, although that day is not named, to hour 6 of the day given.

Month	Day (Old Style)	Daily proper motion	Longitude of the comet	Northern latitude	Right ascension	Declination
November	9	0° 0′	♐ 20° 55′	0° 0′	260° 8′	23° 9′ A
	10	6° 2′	24° 29′	1° 59′	264° 7′	21° 21′ A
	13	14° 30′	♑ 7° 15′	8° 59′	277° 23′	14° 17′ A
	15	7° 13′	13° 47′	12° 16′	283° 43′	10° 32′ A
	20	13° 56′	26° 59′	18° 15′	295° 34′	2° 50′ A

Month	Day (Old Style)	Daily proper motion	Longitude of the comet	Northern latitude	Right ascension	Declination
	25	10° 31′	♒ 7° 24′	22° 5′	304° 19′	3° 0′ B
	30	7° 25′	15° 3′	24° 26′	310° 32′	7° 8′ B
De- cem- ber	5	5° 40′	♒ 21° 6′	25° 54′	315° 21′	10° 6′ B
	10	4° 17′	25° 47′	26° 52′	319° 4′	12° 23′ B
	15	3° 34′	29° 41′	27° 34′	322° 11′	14° 15′ B
	20	3° 19′	♓ 3° 27′	28° 6′	325° 10′	15° 58′ B
	25	2° 48′	6° 36′	28° 29′	327° 41′	17° 25′ B
	30	2° 23′	9° 15′	28° 44′	329° 52′	18° 30′ B
Jan- uary	5	2° 45′	♓ 12° 23′	28° 59′	332° 26′	19° 48′ B
	10	2° 1′	14° 42′	29° 6′	334° 20′	20° 45′ B
	15	1° 57′	16° 56′	29° 12′	336° 11′	21° 39′ B
	20	1° 44′	18° 54′	29° 14′	337° 52′	22° 25′ B
	25	1° 29′	20° 17′	29° 15′	339° 18′	23° 3′ B
	26	0° 16′	20° 55′	29° 15′	339° 35′	23° 10′ B
	Sum	91° 46′				

8

It began, then, in the ecliptic itself, and on November 22, about midnight, it crossed the equator; and from the beginning to the end of its visibility it completed a quadrant of a circle in its orbit, that is 90°, as Tycho has it. There follows Tycho's chapter 6, in which he contends that this comet was above the Moon and had a parallax smaller than the Moon's. He infers this, first, from its constant progress along the same great circle of the sphere, having completed precisely a quadrant; next, from the comet's distances from the fixed stars, taken on one and the same night, which were the same, or very nearly the same, at different altitudes of the comet; and finally from the comet's distances from the fixed stars as observed by others at other places on the Earth. Then in chapter 7 he shows that the comet's tail was not directed exactly to the part opposite the Sun, but rather to the part opposite Venus, and that its curvature was not real but apparent and optical, because the end of the tail was much nearer to us than the head. In chapters 8 and 9, further, he shows that the comet described a circle about the Sun, larger than the circle of Venus but higher than the Moon. He sets the semidiameter of this epicycle at 8405 parts, of which the radius of the solar orb, or the mean distance of the Sun from the Earth, is 10,000. From this, for November 13, when the apparent diameter of the comet was seen by him as 7′, he infers the comet's distance from the Earth as 210 semidiameters of the Earth; its true diameter as 368 German miles, or in ratio to the Earth's diameter as 3 to 14; and that the body of the comet was smaller than the terrestrial globe 100⅔ times. And finally he infers the length of the tail as 96 semidiameters of the Earth. All of this will have to be examined

[8] A and B after the declinations mark southern (austral) and northern (boreal) declination.

below, together with Chiaramonti's objections against Tycho and Kepler's replies on Tycho's behalf. Meanwhile, reader, enjoy part of another of Tycho's tables, from volume 2, p. 206, in which the comet's motion in its epicycle, or its distance from the Earth, and its horizontal parallax are set out — although afterwards, on p. 268, Tycho seems to retract this motion by an epicycle, as Longomontanus also noted; just as in volume 1 of the *Progymnasmata*, which was published after volume 2, he acknowledged that the tail comes not from the rays of Venus but from those of the Sun.

Table from Tycho's volume 2, p. 206, for the comet of the year 1577, abridged by us to the days written below.

Month	Day of the comet	Motion in the epicycle from the perigee	Distance of the comet from the line of the Sun's mean motion	Distance of the comet from the centre of the Earth (semidiameters of the Earth)	Horizontal parallax of the comet
November	9	6° 2′	25° 4′	173	19′ 52″
	10	6° 45′	28° 31′	179	19′ 12″
	13	9° 32′	39° 22′	211	16′ 18″
	15	11° 23′	44° 38′	235	14′ 39″
	20	16° 9′	53° 10′	308	11′ 10″
	25	21° 49′	57° 51′	423	8′ 7″
	30	28° 31′	59° 44′	515	6′ 40″
December	5	35° 36′	59° 42′	636	5′ 25″
	10	42° 53′	58° 28′	761	4′ 31″
	15	50° 14′	56° 31′	883	3′ 54″
	20	57° 39′	54° 11′	1003	3′ 25″
	25	65° 9′	51° 30′	1144	3′ 4″
	30	72° 39′	48° 37′	1235	2′ 49″
January	5	81° 33′	45° 2′	1362	2′ 32″
	10	88° 55′	41° 58′	1462	2′ 20″
	15	96° 14′	38° 51′	1554	2′ 12″
	20	103° 29′	35° 42′	1640	2′ 6″
	25	110° 37′	32° 33′	1718	2′ 0″
	26	112° 2′	31° 56′	1733	2′ 0″

Ioannis Prætorij Norimbergenſis Mathematici Obſeruatio.

The Observation of Johannes Praetorius, Mathematician of Nuremberg.

Io. Prætorius [Johannes Praetorius], in his account of comets, places this one last among the comets. It was first seen by him on November 12 at Nuremberg, although he admits that it had been seen earlier at Venice on the 8th and at Lyon [Lugduni] on the 9th. "Its colour," he says, "was rather pale; the coma, spreading from a knot of no great size, was diffused widely and lengthwise, and it was always turned away from the Sun, except that in its middle it was seen to be slightly curved, which I think happened from an optical cause. The length

of the coma extended beyond 25 degrees of a circle of the heavens, while in breadth, where it was greatest, it seemed to occupy not fully two degrees."

[Margin: Size of the tail, and its turning away from the Sun.]

That at the beginning it was seen in some places as ruddier and more horrible in appearance, he ascribes to the intervening of vapours or of a thinner cloudlet. Then he goes on thus: "The comet, as to longitude, was seen on November 13 in the middle of Capricorn, with a northern latitude of 15 degrees. Its motion, apart from the diurnal motion it had, was not direct. For although it first appeared below the equinoctial, on the southern side, near the Milky Way, afterwards — not by a straight path — it crossed the equinoctial about November 19 (in the part where 294 of ascension is usually assigned), and, leaving the bright star of Aquila about 4 degrees off, it approached the tail of Delphinus, and about the 29th of the same month it mingled with those last stars of the tail of Delphinus. At that time, too, it seemed swifter in its course, and advanced about one degree, or a little more, in one day. Departing from the said stars, by December 5 it had passed a little beyond the two little stars that are in the mouth of Equuleus [præciſionis Equi], and again it was heading for the star of the third magnitude that is in the gape of the winged horse. But leaving that star too a little below it, it moved between east and north, and on January 1 [Calendis Ianuarij] it formed an equilateral triangle with the two leading bright stars of Pegasus belonging to that quadrangular figure. Afterwards, when its size began to diminish too much, it approached with a slower motion the two little stars that are in the breast of Pegasus, and vanished there, so that after January 10 of the present year 1578 not even a shadow of it remained. From all this its motion can be understood on a celestial globe, and from that it will become plain that it went round with a wandering motion. But if it had proceeded by a straight path to the place where it vanished, then in the said space of time it would have completed about 60 degrees."

[Margin: Apparent motion of the comet.]

Yet he candidly adds: "We could not accurately observe either the true places or the parallax, because we were not furnished with suitable instruments. But at the first time of its appearance it seemed much nearer to the Earth than afterwards, so that I judge that it too was carried upward; about this, however, I have nothing beyond conjecture." He adds that it could be made out through the holes of the sights [dioptrarum foramina] only with the greatest difficulty, because of its dull and weak light, and that it usually disappeared before it reached the horizon. From this he confirms Apian's Plinian interpretation, that comets never stand in the setting part of the sky [in occaſuri cæli parte], and that the comet consisted of terrestrial vapours, not set on fire but illuminated by the Sun, because the tail was turned away from the Sun and its light was more like mere illumination than burning; in this he subscribes to Apian.

[Margin: Praetorius's opinion on the substance of the comet.]

Tycho, however, in volume 2, p. 449, opposes him: because he made this comet elementary; because he gave it a wandering motion, which Tycho asserts

was perpetually constant and regular; because for November 13 he increased its longitude by nearly 8 degrees more than was right and diminished its latitude by 6 degrees; because he assigned the crossing of the equator, which took place between November 22 and 23 at the 300th degree, to November 19 at the 294th degree; and because he set it only 4 degrees from Aquila, when it was never less than 10 degrees from it. So great is the difference between observations made with accurate instruments and observations undertaken roughly with the naked eye or with unsuitable apparatus. That Praetorius did not see this comet beyond January 10, when others saw it until the 26th, must be put down to the keenness of those others' sight and to the weakness of Praetorius's eyes. From this, too — from a false beginning and end — it came about that he gave it a motion of only 60°, which Tycho puts at 91° 46′ from November 9; whereas, since it was seen at Venice on the 8th and on its first day completed not less than 6°, its motion was at least 98 or 100 degrees.

[Margin: Tycho's criticism and disagreement.]

[Margin: Amount of the motion.]

De Obſeruationibus Gulielmi Landtgrauij Haſſiæ & Aliorum circa Cometam Anni 1577.

On the Observations of Wilhelm, Landgrave of Hesse [Wilhelm IV, Landgrave of Hesse-Kassel], and of Others concerning the Comet of the Year 1577.

[Margin: Of the Landgrave of Hesse.]

Tycho reports these in volume 2, from p. 212, and prefers them to the others, as being from larger instruments of solid metal

[Margin: Years of Christ.]

that display azimuths and altitudes. From them he derives the comet's longitudes and latitudes and its apparent diurnal motion; and because he found this retarded by a few minutes, he concludes that the comet had a parallax smaller than the Moon's. In those observations, moreover, the length of the tail on November 16 is noted as 17° and its greatest breadth as 4½°; but on December 30 the length of the tail is given as 4° 10′.

[Margin: Mästlin's observations.]

Michaël Mœſtlinus [Michael Mästlin] covered his observations of this comet in ten chapters. In them he grants that other comets were below the Moon, but says that this one was above the Moon, and that it did not cast its tail exactly towards the part opposite the Sun; he says the tail was at first 30° long. On November 20, he says, the comet was 326 semidiameters of the Earth distant from the Earth, with a horizontal parallax of 10′ 30″. He puts its first appearance on November 12 and its last on January 10. Tycho, however, compares the places of the fixed stars that Mästlin derived from the Prutenic Tables with his own, and shows that they sometimes differ from the truth by a whole degree; having therefore corrected them, he derives truer places of the comet from

Mästlin's observations, and on that occasion he relates that the comet was first seen at Constantinople on November 10.

[Margin: Tail 30°.]

[Margin: Observation of Cornelius Gemma.]

Cornelius Gemma the younger set out his proof [Apodixim] concerning this star in three chapters. In them he asserts that the tail stretched out towards the opposite part of the sky, and that the comet always had a parallax smaller than the Moon's, and therefore shone above the Moon, in the orb of Mercury according to the Ptolemaic system; and that its greatest parallax was 40′ on November 20, but 15′ on the 27th.

[Margin: Parallax 40′.]

[Margin: Roeslin's consideration.]

Heliſæus Rœſlinus [Helisaeus Röslin], using the rather crude observations of Cornelius Gemma and trying to bring the proportions of the comet's motion back to geometric and musical laws, ascribes stations and retrogradations to it, against Tycho's findings [experimenta Tychonica]; although afterwards he thought otherwise and subscribed to Mästlin. But when he affirms that Cornelius's parallax of 40′ on November 20 agrees with Mästlin's, he falls into a serious error: for on that day Mästlin gives the comet a distance from the Earth of 326 semidiameters of the Earth, to which a horizontal parallax of only 10′ 30″ corresponds. He concludes, however, that it was above the Moon — from the regularity of its motion, from the property of its light, from its duration, and above all from its small parallax.

[Margin: Parallax 10′ 30″.]

[Margin: Authors who place this comet below the ☾.]

[Margin: Hagecius.]

The others whom Tycho reports make this comet sublunary. Among them, however, is Thaddæus Hagecius [Tadeáš Hájek], who in his *Dialexis* on this comet gives it a parallax of 5° and some minutes at an altitude of 38°, and sets its tail always turned away from the Sun; afterwards, in his second animadversions on this comet and on the comet of the year 1580, he candidly retracted, and confessed that it was above the Moon.

[Margin: Scultetus.]

Bartholomæus Scultetus [Bartholomäus Scultetus], by contrast, gives this comet a parallax of 5° 22′ at an altitude of 29°. His observations, however, are brought under suspicion of error by many arguments, and this is plain even from the fact that, using the radius, he puts the distance between the seventh little star of Andromeda in the old catalogue and the bright star of the head of Aries at 17° 45′, when it is only 16° 59½′.

[Margin: Nolthius.]

Andreas Nolthius, for his part, attributes to this comet a parallax of 5° 32′ at an altitude of 33°, and a distance from the Earth of 8⅔ semidiameters of the Earth, or 7700 German miles — far above the vapours and exhalations of the atmosphere; all of which Tycho refutes in many ways.

But Nicolaus Vuincklerus [Nicolaus Winckler] infers from his own observations a parallax of this comet of 6° 50′, even when it was 46° high above the horizon, and a distance from the Earth of 8 semidiameters of the Earth. Tycho likewise refutes him, together with others who concluded from their own or others' observations that this comet was sublunary; these I shall review more fittingly below, in their places. Besides these, who were known to Tycho, Petrus Surdus [?] and Ioannes Paduanus [?] and others wrote of it; they say that it began to be seen from November 7 or 9, and ceased about January 10. Kepler, *De Cometis*, p. 129, says that King Sebastian was spurred by this comet to cross into Africa; but Fromondus [Libert Froidmont] laughs at him in book 3 of the *Meteorologica*, chapter 3. In the following year, moreover, Mustapha the Turk was beaten in battle with the loss of 70000 men; and Sebastian, King of Portugal, was slain at Alcazar [Alcazerium] together with the flower of his nobility — though Abd al-Malik [Abdemelecho], his enemy, died of apoplexy; Juan de Austria [Io. Auſtriacus] dies in the camp in the Low Countries; and the English rage against the Catholics.

Sequitur Hiſtoria aliorum Cometarum.

There follows the history of other comets.

[Margin: Years of Christ.]

[Margin: Year 1580. 1582.]

In the year of Christ 1580 a comet appeared, observed by Mästlin, which Fromondus mentions in book 3 of the *Meteorologica*, chapter 1, article 1. And in the year 1582 there was another, on which see Antonius Santutius [Antonio Santucci] and Io. Camillus Gloriosus [Giovanni Camillo Glorioso], book 1, p. 8: from the beginning of March it was a morning star, afterwards an evening star, and it ceased about the middle of April, its tail always turned away from the Sun — so Santucci. But Kepler, in the physiology of comets, p. 120, says that it was retrograde, and that when it became stationary at the beginning of Sagittarius it came nearer to the ecliptic.

[Margin: 1585.]

And in the year 1585 there was a comet observed by Rothmann [Christoph Rothmann] from October 8, Old Style, equal to a star of the fourth magnitude, but, with its coma all round, a little larger than Venus. It was observed by Tycho from October 18, Old Style, to November 15 — on which see Tycho himself in his letters [Epiſtolis] to the Landgrave of Hesse, from p. 4 to 20 and p. 37, where he also indicates that the Landgrave's, or Rothmann's, observations agree with his own. At the beginning this comet was seen nearly equal to Jupiter, but endowed with a thin and nebulous light, near the cord of the more southern Fish, haired all round with rays, without any beard or tail, except on the 20th and 22nd, when it threw out a ray a span long towards the east. It then diminished by degrees, yet remained similar in colour and form to the nebulous ring they call the Manger of Cancer [præſepe Cancri], though smaller than

that in size, whereas at the beginning it had been larger; at last, having become smaller still, it vanished entirely about the middle of November. On the first night on which Tycho observed it, it had passed five degrees or a little more to the east of Saturn, which was retrograde near the cord of the Fishes. On other nights it passed through the whole asterism of Aries, and, leaving the back of Taurus to the south, it turned northwards and was extinguished not far from the asterism of the Pleiades, having attained a greater latitude than they, but a smaller longitude. From selected observations on 9 days (for on other nights either clouds or the glare of the Moon prevented it), he infers that by its own proper path this comet described an arc of a great circle, which nevertheless had a variable angle of inclination to the equator and to the ecliptic; and that it was swifter at first and slower afterwards, yet with a proportional decrease.

[Margin: Parallax of this comet.]

Finally, having examined the comet's apparent places, he affirms that it was subject to scarcely any sensible parallax; and yet, if it had been in the concave of the heaven of the ☾, it would have had to have a large parallax, sensible not only by instruments but even to the naked eye, by mere inspection against the neighbouring fixed stars. And lest anyone should suspect that refraction was the reason this comet kept its motion so conformable to the true and diurnal course, with so great a proportion of advance, Tycho affirms that he chose above all the situation in which the comet was raised so high above the horizon that no sensible refraction could weaken the certainty of the matter under inquiry. For example, on October 18, at $10^h\ 15^m$ after noon, he found the comet's right ascension to be 19° 19′ and its northern declination 4° 25′, the comet's altitude then being 37° 30′. Then at $13^h\ 30^m$ its ascension was found to be 19° 32′ and its northern declination 4° 36½′, and its altitude was then 31° 10′. From these, and from the diurnal motion of the nearest days, Tycho infers that the comet ought to have advanced in its own proper path by 17½′ from $10^h\ 15^m$ to $13^h\ 30^m$, and yet that it had advanced 18⅔′; and therefore that the parallax was no more than 1′ 20″. But if the comet had been in the concave of the Moon, at a distance of 52 17′ semidiameters of the Earth, its parallax at the said altitude ought to have been 27′; and refraction at that altitude is so slight that it could in no way have extinguished so great a parallax, had there been one. In the same way, on November 5 at $7^h\ 50^m$ the comet's altitude was 39° 45′, and at $13^h\ 30^m$ it was 48° 20′; and on the 6th at $6^h\ 5^m$ the comet's altitude was 26°, and at $14^h\ 55^m$ it was 38°. At these altitudes refraction is almost nil. Now a table of the daily motion, drawn up by analogy with the observed motions, is to be taken from Tycho; and it should be known that in the following year Asia Minor, Austria and Hungary were infested by plague, Italy and the Low Countries by famine, and Constantinople by locusts.

[Margin: Years of Christ.]

Daily motion of the comet of the year 1585, for midnight. From Tycho's Letters, p. 19.

Month	Day (Old Style)	Motion in its proper arc	Inclination to the equator	Inclination to the ecliptic	Right ascension	Northern declination	Longitude	Latitude
October	18	6° 24′	45° 28′	23° 17′	19° 25′	4° 33′	♈ 19° 38′	3° 25′ A
	19	8° 42′	45° 26′	23° 7′	21° 3′	6° 11′	21° 45′	2° 30′
	20	10° 53′	45° 23′	22° 57′	22° 36′	7° 43′	23° 46′	1° 39′
	21	12° 58′	45° 21′	22° 54′	24° 7′	9° 11′	♈ 25° 41′	0° 50′
	22	14° 56′	45° 18′	22° 52′	25° 32′	10° 33′	27° 29′	0° 3′ B
	23	16° 49′	45° 16′	22° 50′	26° 55′	11° 56′	29° 13′	0° 59′
	24	18° 33′	45° 14′	22° 47′	28° 13′	13° 2′	♉ 0° 49′	1° 19′
	25	20° 10′	45° 12′	22° 45′	29° 26′	14° 9′	2° 20′	1° 57′
	26	21° 40′	45° 10′	22° 43′	30° 35′	15° 11′	3° 44′	2° 32′
	27	23° 4′	45° 8′	22° 40′	31° 40′	16° 8′	5° 2′	3° 5′
	28	24° 24′	45° 6′	22° 38′	32° 40′	17° 1′	6° 14′	3° 34′
	29	25° 40′	45° 4′	22° 36′	33° 38′	17° 51′	7° 24′	4° 2′
	30	26° 52′	45° 2′	22° 34′	34° 35′	18° 38′	8° 31′	4° 29′
	31	28° 0′	45° 0′	22° 31′	35° 30′	19° 23′	9° 35′	4° 55′
November		29° 5′	44° 58′	22° 29′	36° 23′	20° 5′	10° 35′	5° 18′
	2	30° 6′	44° 55′	22° 27′	37° 14′	20° 44′	11° 34′	5° 40′
	3	31° 4′	44° 53′	22° 25′	38° 2′	21° 21′	12° 28′	6° 1′
	4	31° 59′	44° 50′	22° 21′	38° 50′	21° 56′	13° 21′	6° 20′
	5	32° 52′	44° 46′	22° 16′	39° 35′	22° 29′	14° 11′	6° 38′
	6	33° 42′	44° 41′	22° 8′	40° 17′	22° 59′	14° 58′	6° 55′
	7	34° 28′	44° 36′	22° 0′	40° 58′	23° 26′	15° 43′	7° 10′
	8	35° 12′	44° 31′	21° 53′	41° 37′	23° 50′	16° 24′	7° 23′
	9	35° 52′	44° 26′	21° 45′	42° 13′	24° 13′	17° 3′	7° 34′
	10	36° 30′	44° 22′	21° 41′	42° 48′	24° 35′	17° 40′	7° 46′
	11	37° 6′	44° 19′	21° 37′	43° 21′	24° 56′	18° 15′	7° 57′
	12	37° 42′	44° 16′	21° 34′	43° 53′	25° 16′	18° 49′	8° 7′
	13	38° 16′	44° 13′	21° 31′	44° 24′	25° 36′	19° 22′	8° 18′
	14	38° 49′	44° 12′	21° 28′	44° 54′	25° 55′	19° 53′	8° 28′
	15	39° 21′	44° 10′	21° 25′	45° 23′	26° 13′	20° 24′	8° 38′

9

Rothmann, however, as Tycho testifies in his Letters, p. 118, and in volume 2, p. 442, observed that at its beginning this comet moved three degrees each day by its proper motion; and yet he found so small a parallax in it that he placed it above Jupiter, in the heaven of Saturn.

[Margin: Year 1590.]

In the year 1590 there appeared the comet on which see Tycho in his Letters, from p. 173 to 182. It was seen from February 23, Old Style, to March 6. At the

[9] A and B after the latitudes mark southern (austral) and northern (boreal) latitude; each is printed once, where the latitude changes side.

beginning it appeared larger, though scarcely of 3 minutes, near the head of the northern Fish, in the western quarter of the sky, with a tail about 10 degrees long, directed exactly towards the point opposite the Sun, towards the zenith. Its motion was in the order of the signs, and so rapid that from the first day of observation to the second it completed nearly 8 degrees; afterwards it was slower, and at the end it covered a degree and a half. By Tycho's reasoning it described exactly an arc of a great circle and had no sensible parallax, and was therefore above the Moon — if we believe Tycho. But here, for you, is its daily motion, with the rest deduced by Tycho.

Ephemeris of the comet of the year 1590, adapted to hour 9 after noon at Uraniborg [Vraniburgi], from Tycho's Letters, p. 181.

Month	Day	Motion in its proper path	Daily proper motion	Declination, always northern	Right ascension	Longitude from the beginning of ♈	Latitude, always northern
Febru-ary	23	37° 32′		24° 1′	9° 29′	♈ 18° 27′	18° 14′
	24	45° 6′	7° 34′	28° 16′	16° 26′	♉ 26° 21′	19° 33′
	25	51° 40′	6° 34′	31° 40′	22° 58′	3° 17′	20° 24′
	26	57° 16′	5° 36′	34° 15′	28° 52′	9° 11′	20° 55′
	27	61° 59′	4° 43′	36° 12′	34° 9′	14° 16′	21° 12′
	28	65° 51′	3° 52′	37° 38′	38° 39′	18° 25′	21° 15′
March	1	69° 8′	3° 17′	38° 41′	42° 36′	21° 57′	21° 15′
	2	71° 56′	2° 48′	39° 27′	46° 2′	24° 56′	21° 12′
	3	74° 21′	2° 25′	40° 6′	49° 1′	27° 29′	21° 7′
	4	76° 24′	2° 3′	40° 32′	51° 43′	29° 44′	20° 58′
	5	78° 8′	1° 44′	40° 53′	53° 58′	♊ 1° 34′	20° 52′
	6	79° 42′	1° 34′	41° 10′	56° 0′	3° 15′	20° 46′

The motion in its proper path is counted from its intersection with the equator, or from the node, which would have been at 339° 45′ of the equator; that path, moreover, was inclined to the equator at a constant angle of 42°. The sum of the daily motions makes an arc of 42° 10′.

[Margin: Years of Christ.]

[Margin: 1593.]

The comet of the year 1593 is mentioned by Froidmont [Fromondus] in book 3 of his *Meteora*, chapter 1, article 1; he likewise places it above the Moon on Tycho's authority — but from where he got this I do not know.

[Margin: 1596. Two comets.]

In 1596 Giuseppe Bonfiglio [Ioſephus Bonfilius], in his *Sicilian History*, part 2, book 10, says that two comets were seen beneath Ursa Major, in the sign ♏, and that they were followed by a drought and an earthquake in Asia. Kepler, however, in his *Physiologia Cometarum*, p. 120, says that a comet advanced from Cancer in this year and at the end, from direct, became stationary in longitude, its path bending towards 4° of Virgo. The English fleet burned the

Spanish fleet at Cádiz [Gades]; Matthias of Austria was defeated by Mehmed [Mahomete].

[Margin: Year 1597.]

Finally, in the year 1597 too a comet was seen — on which see Antonio Santucci [Antonius Santutius] — in the month of July, before the 16th; and it lasted until August 9.

[Margin: 1607.]

In 1607 there shone a comet observed by Kepler and Longomontanus, as can be seen from their treatises on it.

[Margin: Kepler's observation.]

Thus Kepler, from p. 25, says that he first saw the comet on September 26 at Prague, at hour 8½ after noon: larger than the fixed stars if viewed through a telescope, and to him without a tail, though to others of sharper sight it had one. The comet stood here near the following knee of Ursa Major, towards the preceding one. The next morning, at hour 3 after midnight, a tail appeared plainly, quite long and turned upwards, while the head, carried by a swift motion in the order of the signs, had moved about 2° 50′ within those 6 hours. But see, in the following table, the core of Kepler's observations — which, however, he admits on p. 30 were not the most precise.

[Margin: The comet here above the ☾.]

Even so, he concludes from them that if the comet had been on the borders of the Moon, the difference of parallax within 6 hours would have been much greater than the proportion of the observed motion allows; and therefore he places the comet above the Moon. He affirms that the tail pointed exactly opposite the Sun, that the comet was always less than a quadrant away from the Sun, that on the 30th it was in conjunction with the Sun in longitude, and that at the end it went from direct to stationary.

[Margin: Distance of this comet from the Earth.]

Assuming the motion of the Earth, moreover, and a rectilinear trajectory of the comet (to be explained below), he gathers that on the last day of observation, which was October 26, the comet's distance from the Earth was 13,187 semidiameters of the Earth — which, on his hypothesis, he says is more than eight times greater than the height of the Sun.

Ephemeris of the comet of the year 1607, from Kepler, with the places of the Earth as seen from the Sun, which are opposite to the places of the Sun as seen from the Earth.

Month	Day	Places of the Earth	Apparent longitude of the comet	Apparent latitude of the comet, always northern
Septem-ber	26	♈ 3° 25′	♌ 18° 30′	35° 30′
	27	4° 25′	♌ 29° 30′	37° 30′
	28	5° 24′	♍ 12° 0′	40° 4′
	29	6° 23′	♍ 24° 30′	
	30	7° 22′	♎ 7° 30′	38° 40′

Month	Day	Places of the Earth	Apparent longitude of the comet	Apparent latitude of the comet, always northern
October	1	♈ 8° 21′	♎ 18° 15′	37° 0′
	2	9° 20′	♎ 26° 0′	35° 0′
	3	10° 20′	♏ 2° 35′	
	4	11° 19′	8° 5′	
	5	12° 18′	12° 30′	27° 20′
	6	♈ 13° 18′	♏ 15° 50′	
	7	14° 17′	18° 40′	23° 30′
	8	15° 17′	21° 10′	21° 50′
	9	16° 16′	23° 0′	
	10	17° 16′	24° 50′	19° 0′
	11	♈ 18° 15′	♏ 25° 50′	17° 35′
	12	19° 15′	27° 0′	16° 40′
	13	20° 15′	27° 57′	
	14	21° 14′	28° 50′	15° 0′
	15	22° 14′	29° 40′	14° 7′
	16	♈ 23° 14′	♐ 0° 24′	13° 36′
	17	24° 14′	1° 4′	
	18	25° 14′	1° 36′	
	19	26° 14′	2° 0′	12° 0′
	20	27° 14′	2° 0′	

Remainder of the preceding table.

Month	Day	Places of the Earth	Apparent longitude of the comet	Apparent latitude of the comet, always northern
October	21	♈ 28° 14′	♐ 1° 58′	
	22	♈ 29° 14′	1° 54′	9° 45′
	23	♉ 0° 14′	1° 50′	
	24	1° 14′	1° 45′	
	25	2° 15′	1° 38′	7° 0′
	26	3° 15′	1° 30′	6° 30′

10

Longomontani Obſeruatio Cometæ Anni 1607.

Longomontanus's observation of the comet of the year 1607.

Christianus Severini Longomontanus [Christen Sørensen Longomontanus], in the Appendix to the *Astronomia Danica*, which is entirely about new phenomena of the heavens, chapter 9, reports his observations of this very comet. Their substance is this. On September 28, New Style, in the evening, a comet appeared in the north, rivalling Jupiter in apparent magnitude but Saturn in

[10] on October 1 the sign of Libra is printed once, with a double base bar.

the leaden hue of its light. It had a very long tail, dense and compact, of the same colour as the head, and pointing throughout its whole duration towards the quarter opposite the Sun.

[Margin: Apparent magnitude of the comet.]

He affirms, moreover, that the comet did not shine out before September 25, since on the 24th he observed Jupiter and saw nothing new in the sky.

[Margin: Motion of the comet.]

And since the mean opposition of the Sun with Jupiter fell on the 25th, he thinks it very fitting that the comet first shone then — the time at which there was a conjunction of Mercury with the Sun and a trine aspect of Saturn with Mars. Its motion, he thinks, was extremely rapid at the beginning, so that in the first days it ran through 9 degrees and more on its own great circle, and afterwards it decreased in proportion with its magnitude.

[Margin: Parallax of the comet 3 minutes.]

Now, on October 22 at hour 6½ he observed the comet at Copenhagen [Hafnia] and found it at 1° 50′ of Sagittarius with a northern latitude of 9° 45′; and Kepler at Prague in Bohemia, on the same day, at the time of evening twilight, observed it at 1° 50′ of Sagittarius with a latitude of 9° 45′. Longomontanus marvels at this remarkable agreement, and at a parallax not so much slight as null. This is confirmed by the agreement of September 28, when at hour 7¼ Longomontanus at Copenhagen and Kepler at Prague observed the comet at a latitude of 40° 30′ and at the end of the 12th degree of Virgo, with some very small difference, namely 3′. One might suspect the cause of this difference from Kepler's words on p. 30, where he says: *"For there is no need of subtlety, since the observation is coarse. For they require the Tychonic instruments and a platform, a free horizon, a skilled observer as companion, and keen eyes — all of which I lacked at the beginning."*

[Margin: The comet above the ☾.]

But Longomontanus assumes the altitude of the pole at Copenhagen to be 55° 43′ and at Prague 50° 6′, the difference in time between the meridians scarcely 6^{m}, the Earth's semidiameter 860 German miles, and the greatest height of the vapours and exhalations above the Earth's surface 13 German miles. By a geometric analysis of triangles he then proves that, had this comet been some meteor in the highest region of the exhalations, there would have had to be a difference of 5 degrees between the place seen by Kepler and the place seen by himself, and the comet would have had to appear lower to Kepler by as many degrees. Since that parallax, however, was at most 3 minutes, he declares that the comet was above the Moon, and that here the Peripatetic strays many parasangs from the truth — even though he does not have enough observations to determine its distance from the Earth precisely. For Longomontanus's words are: *"Although how high it climbed aloft through the aether, the observations of it, made less precisely and less promptly, scarcely allow us to reason; therefore now, the web broken off, etc."*

Now let us have from Longomontanus a table derived from his own observations, which gives the motion of the planet in its proper circle, with its place in longitude and its declination.

[Margin: Years of Christ.]

Ephemeris of the comet of the year 1607, from Longomontanus, for hour 7 after noon at Copenhagen [Hafniæ].

Month	Day	Daily motion in its proper circle	Places in longitude	Declination, first northern, then southern
September	26	9° 40′	♌ 18° 0′	49° 0′ B
	27	9° 30′	♍ 0° 0′	47° 0′
	28	9° 10′	♍ 11° 50′	44° 0′
	29	9° 0′	♍ 23° 30′	39° 15′
	30	8° 40′	♎ 5° 15′	34° 20′
October, of the new calendar	1	7° 50′	♎ 16° 30′	28° 30′ B
	2	7° 0′	♎ 27° 0′	21° 20′
	3	6° 0′	♏ 4° 0′	16° 40′
	4	4° 30′	9° 30′	12° 20′
	5	3° 40′	13° 30′	9° 30′
	6	3° 0′	♏ 17° 0′	6° 20′ B
	7	2° 30′	20° 0′	4° 10′
	8	2° 10′	22° 30′	1° 50′
	9	2° 0′	24° 0′	0° 25′
	10	1° 50′	25° 30′	1° 0′ A
	11	1° 40′	♏ 26° 40′	1° 50′
	12	1° 30′	27° 30′	2° 40′
	13	1° 25′	28° 20′	5° 0′
	14	1° 15′	29° 10′	6° 45′
	15	1° 0′	♐ 0° 0′	8° 15′
	16	0° 45′	♐ 0° 50′	9° 30′
	17	0° 40′	1° 20′	10° 30′
	18	0° 30′	1° 30′	11° 0′
	19	0° 25′	1° 40′	11° 10′
	20	0° 20′	1° 45′	11° 18′
	21	0° 18′	♐ 1° 50′	11° 25′
	22	0° 15′	1° 55′	11° 32′
	23	0° 12′	2° 0′	11° 40′
	24	0° 10′	2° 5′	11° 50′
	25	0° 8′	2° 8′	12° 0′
	26	0° 5′	2° 10′	12° 10′
	Sum	97° 18′		

So between October 9 and 10 the comet crossed the equator, and from then on it became steadily more southern.[11]

[11] B and A in the declination column mark northern (boreal) and southern (austral) declination.

Obſeruationes Primi ac Secundi Cometæ Anni 1618.

Observations of the first and second comet of the year 1618.

[Margin: Year 1618.]

In the year of Christ 1618, besides fiery beams and flying dragons, three or four comets appeared.

[Margin: The first comet.]

The first appeared in Upper Hungary on August 25. As Kepler reports in his work on the comets of this year, p. 47, it came out before sunrise, around the third hour after midnight, casting its tail from east towards west — that is, turned away from the Sun, as comets' tails are. Its tail was short and broader than the head, so that it looked like a clothes-brush. On September 1 it was seen at Linz [Lincij] by Kepler a little below the left forefoot of Ursa Major, sinking towards the head of Leo, so that it seemed to be at 10° of Leo, with a northern latitude of 21½°. On September 2, keeping its latitude, it moved ahead by one degree with a retrograde motion, against the order of the signs; and after some days it became slower in its motion. On the 6th no tail could be observed with the naked eye, but viewed through the telescope it appeared fairly large, like a small cloud, with very short stumps of hair stretched upwards. On the 23rd it was seen to have reached ♋ 29° 42′ with a northern latitude of 23° 41′; seen through the telescope it was shaggy, with a broad and very short head of hair, but nebulous, and with a light that did not twinkle at all. On September 25 it disappeared at about ♋ 28° and a latitude of 23¼°.

[Margin: Kepler's opinion on the motion of this comet.]

Since, however, its parallaxes could not be deduced for lack of observations, nothing certain can be established about its distance from the Earth. Kepler nevertheless thinks, because it lost its tail so quickly, that it was[12] towards the Sun, on a line turned obliquely away from the Earth; and that, assuming the annual motion of the Earth, this comet at the beginning of its trajectory equalled with its daily motion the Earth's daily motion in longitude.

[Margin: The second comet.]

The second comet seems to be confused by writers either with a beam or with a flying dragon, or with the third comet. This is either because its tail appeared long before the rising of its head or its withdrawal from the Sun, or because, as Kepler on p. 50 thinks not improbable, it was afterwards divided into two — as Ephorus once related had happened, Seneca objecting in vain. Wilhelm Schickard [Vuilhelmus Schikardus], in Württemberg [Vuirtembergia], on October 20, New Style, saw a torch or flying dragon passing from the head of Andromeda through the northern Fish to the gaping mouth of Cetus. Perhaps this is the comet that Ursinus [?] reports was seen on October 30 at Cologne [Coloniæ Agrippinæ], with a long train of tail towards the north-east

[12] printed *ruiſſe*; read as *fuiſſe*, "was", the ascender of the f unprinted or a foul sort; *ruiſſe*, from *ruo*, would give "rushed"

[Borrhapeliotem], and which did not last long; but perhaps it was a meteor, not a comet.

Johannes Remus Quietanus [Io. Remus Quietanus], however, in a little book on this comet published at Innsbruck [Oeniponte], writes that at Novesium [Noueſij] in Silesia, on November 10, a fiery phenomenon was seen like an ostrich feather, and curved — the kind the ancients called *ceratias*. He adds that at Rome on November 10, from $16^h\ 20^m$ to $18^h\ 40^m$ after noon, a curved beam was seen, which the Roman observer called a comet with a huge tail at its side. One end of it, towards the horizon, was gathered into the form of a handle, but was no brighter than the rest of the body, nor distinguishable from any star — perhaps because of the nearness of the Sun; its other end lay between the autumnal intersection and 18° of Libra, deflecting from the ecliptic towards the south by about 15 degrees. Therefore, if it was a comet and turned its tail away from the Sun, its head must have been between ♎ 18° and ♏ 19°, near the last star of Hydra, since on that day the Sun stood at 19° of Scorpio.

But Georgius Sualbachus [?] says that he saw the comet at Speyer [Spiræ] on November 17, New Style. And at Linz, farmers outside the town had seen the same brightness about November 12, in the same quarter of the sky and at the same hour of the night as the Romans, on the testimony of Kepler, p. 52. Io. Bapt. Herbenstreitus [?] also testified that the same object was seen at Ulm [Vlmæ] by some travellers some weeks before December 4; and Herlicius [David Herlitz] writes that it was observed in Pomerania on November 19, and earlier in Prussia [Boruſſia].

[Margin: Retrograde motion of this second comet.]

Finally, the same seems to be the one that was seen in the form of a horn among the stars of Crater at Tübingen [Tubingæ] around November 20 by Johannes Straussius [?], and at Linz by Kepler on the same day among the same stars, at hour 5½ after midnight, with a retrograde motion and with its head hidden in the light of dawn. Kepler adds that on November 23, at hour 5 in the morning, it had turned aside from the stars of Corvus towards the south, in the direction of the Heart of Hydra. But he speaks of it in such a way that you cannot tell whether he recognises it as a comet properly so called, or as a meteor that was the forerunner of the third comet, or whether the third was born from this one when it divided into two. The cause of this confusion was that the head was not seen clearly enough by Europeans in the twilight; but in Persia and India the second was plainly distinguished from the third, as I shall shortly relate.

Obſeruationes Tertij Cometæ Anni 1618.

Observations of the third comet of the year 1618.

[Margin: The third comet of the year 1618.]

In the year 1618, about the middle of November, a third comet appeared, distinct from the second (on which see above) and from the fourth (on which

see below), and unknown to Kepler, Longomontanus and most European writers. Their third will be called our fourth; but in Persia the third appeared distinct from the second, and in East India this third appeared distinct from the fourth.

[Margin: Observations of García de Silva in Persia.]

For Longomontanus, in the Appendix to the *Astronomia Danica*, chapter 10, reports an observation of two comets made by García de Silva y Figueroa [Garcia Silua Figueroës], ambassador of Philip III, King of the Spains, in the town of Isfahan [Sphani], not far west of Persepolis, at a polar altitude of 31½°. It was extracted from his letter written to the Marquis of Bedmar [Marchionem Bedmarum] on February 25 [5. calend. Martij], 1619, which was translated into Latin and published at Antwerp in the year 1620. The closing passage of this letter was as follows: *"Account of the twin comets which were a spectacle for us in these days. The one began to shine on November 10, two hours before sunrise; its appearance, between the East and the South was noticed. Its colour was quite like the smoke that rises from very thoroughly refined sulphur powder when it is lit; its head, as it seemed to me, was in Scorpio. Its size was such that it would take up a sixth of the zodiac; its form was like a scimitar [acinacis], as some judged — the kind the Greeks call Xiphias — portending terrible events. To me this comet seemed rather to resemble a year-old palm shoot which, not yet spread out, curves gently at its tip. Its motion, moreover, was towards the south."*

From this, then, it is clear enough that this was the comet — resembling either a scimitar, or a *ceratias*, or an ostrich feather — that we have named, with Kepler, the second comet. García goes on as follows: *"On the twelfth or thirteenth day after the rising of this comet, the other appeared, maned or haired in the usual way, like the star of Venus in colour and equal to it in size, or a little larger. It took its rising straight from the equinoctial east; and although at the beginning it was much smaller than the earlier one, it grew from day to day by much larger increments, so that it nearly equalled the other. It moved with the motion of the primum mobile, and also with a proper motion, not far from the vertical line, towards the north. Then, three or four days after its rising, the other vanished. Both certainly had so short a period that the later one stopped appearing just after the tenth day. It was observed that, as it was ending, it grew redder."*

Now if by "the thirteenth day" you understand November 13, the day on which the third comet (which is García's second) rose, it follows that the second (which is García's first) was extinguished on November 17. But if by "the thirteenth day" you understand 13 days after the rising of the second (which is García's first), then the third comet rose on November 23 and ceased on December 3 — which is closer to the sense of the words. Either way, however, it is plain that this second comet of García's is the third with respect to the first one seen in Europe, and is not to be confused with that third comet which was famous among Europeans, which shone from November 24 until January 20 of the year 1619, and which really should be called the fourth. Longomontanus, it is true, confuses the two; and because of the growth it took on — which can

be understood of its tail — and because of its brevity, he thinks that García, either through carelessness or because of the hindrances of his embassy, did not persist in observing. But it is one thing not to have persisted, another to affirm that the comet ceased — and this García asserted so plainly that in the same letter he said: *"However that may be: if these comets portend evils, at least they will not be long-lasting, like the other things that are seen in great number in the world."*

Rather, it was proper to recognise from this that this comet was quite different from the one that the Europeans saw as the third. This is confirmed by the letters of Father Giacomo Rho [P. Iacobi Rhò] of our Society, written from Goa to Europe. They record observations of the last comet, begun on November 29, which we shall bring forward below in chapter 22, number 23, and compare with the European ones; but they also mention the other comet, although no observation of it is noted — because, that is, it vanished quickly. For so one reads in those letters, as Chiaramonti also has it in the supplement to his *Antitycho*, p. 34: *"One thing alone we could not, and ought not, pass over without serious notice: that when we had scarcely entered port, and were unprepared for that kind of observation, the sky was suddenly seen to shine with two comets in the morning hours. Among the Moors and the heathen, astonished by the novelty of the thing, this raised various rumours."* Afterwards he relates observations made with the astrolabe and the astronomical radius of the College of Goa from November 29 through a great part of December.

[Margin: Four comets in the year 1618.]

It is therefore clear enough that before the comet which appeared from November 24 to January 22, three comets shone in the sky this year. The first lasted about a month, from August 25 to about September 25. The second was seen from November 10 — not only until November 18 in Silesia, or the 19th in Pomerania and Prussia [Boruſſia]; in Persia and in Austria it was seen as well, until November 23. The third was seen in East India and in Persia from November 22 or 23 until December 13. The fourth, from November 24 to January 20, is the one called by antonomasia [Greek: ἀντονομαστικῶς (antonomastikōs, "by antonomasia" — par excellence)] the Comet of the Year 1618, the longest-lasting and most celebrated of all; we must speak of it shortly, and Kepler and the Roman observer call it the third.

[Margin: Two or three beams seen in the same year.]

Fiery beams, moreover, or torches distinct from the comets, were seen to the number of two or three. One was seen by us at Ferrara towards the end of August: as we were walking after supper in the garden of our College, it frightened us with its sudden apparition and very rapid flight. The second was seen on October 20 by Schickard in Württemberg. The third, from November 18 to 29, was seen by our Fathers at Rome and Parma. Remarkably, it moved with the motion of the primum mobile, appeared in the hours before dawn, and occupied nearly 40 degrees of the sky; by its proper motion, however, it advanced, and in those 11 days it advanced from Crater to the Heart of Hydra by 24 degrees, against the order of the signs. Some called it a comet, but

Biancani [Blancanus], in book 16 of the *Sphaera*, chapter 5, called it the tail of the great comet; and from the position of Crater and Hydra it seems to have been Kepler's second comet — although in a disputation held by our Fathers in the Roman College it was designated by the name of a beam. I also remember a fiery globe seen by us, in the month of August if I am not mistaken, rising under the full Moon together with it, and glowing red in a frightening way with a wonderful whirling, while our people ran together to the spectacle; and those nights were hotter and more sultry than any others.

What if so much matter was used up in so many comets and beams that year that, from then until this year 1650, in which we are inserting these words into this account, there was not enough matter left over for comets — and that for this reason, from 1618 to 1650, no comet shone in Europe, that is, for 32 whole years?

Obſeruationes Quarti Cometæ Anni 1618. & 1619.

Observations of the fourth comet of the years 1618 and 1619.

[Margin: Years 1618 and 1619.]

This, of course, is the comet which some writers, aware of only two, call the second, and others, who knew of only three, call the third; we have been compelled to call it the fourth, having weighed the indications in the accounts. This, I say, is the comet watched by so many observers in Europe and in East India, made famous by so many books brought out into the light, and raised above the Moon on so many wings or ladders of arguments — which, however, as I shall show in its place, are not evident. Its distances from the fixed stars, moreover — especially from Arcturus, Spica, the Crown, and the stars of the Bears and of Draco — I shall more conveniently report, and at the same time compare, in chapter 22, number 23. The rest of the more notable points, gathered in summary from various observations, I shall present here, with a threefold ephemeris constructed by our Cysat [Cyſato], by Kepler and by Longomontanus.

[Margin: When this comet first began to be seen.]

Before all else, however, its beginning must be settled. I find that this was on November 24, New Style: for on that day it was first seen, with head and tail very bright, at Budweis [Budouuici] in Bohemia, by the Emperor's soldiers living in winter quarters, as Maximilianus Marsilius [?] of the Society of Jesus, who was staying there at the time, affirmed to Kepler; and he saw it on the same day. Ursinus, however, records that it was seen at Frankfurt on the Oder only on the 26th, as do Millerus [?] at Leipzig and Hebenſtreitus [?] at Ulm. On the 27th, however, it was seen by Ambrosius Rhodius at Wittenberg and by Snel in Holland [Batauia]; and Johannes Remus Quietanus says it was first seen at Rome on that same day. But Vuelperus [?] at Worms, Benjamin Ursinus in the Mark, Gotthard Arthus [Gottardus Arthus] at Danzig [Dantiſci], Gaſpar Herſpachius [?] at Cologne, and finally Gallus Prouenzalis [?] at Paris record

that they saw it only from the 28th. On November 29, however, Biancani at Parma, Gassendi at Aix [Aquis Sextijs], Herlitius in Pomerania, Io. Strauſſiu [?] at Tübingen, and Kepler at Linz in Austria say that they first saw it. Finally, because of rains and fogs during the eight days before December 1 [Kalendis Decembris], it could not be observed either in Denmark or at Ingolstadt — by Longomontanus at Copenhagen, nor by Father Cysat at Ingolstadt — except from December 1.

[Margin: When it ceased to be seen.]

It ceased to be observed by Remus and Vuelperus on December 27, by Longomontanus on December 28, and by Biancani on the 31st, because its tail was small and its body rather dim; but by Ursinus on the Oder [ad Viadrum] and by Kepler at Linz on January 7, because no tail appeared any longer and they were not certain about the head, as Kepler admits on p. 69. Ambrosius Rhodius judged that it disappeared at Wittenberg around January 12, but Schickard on January 18. Father Cysat's last observation was not on the 13th, as Kepler has it from a corrupt sheet, but on January 22, as Cysat himself relates on p. 9 — where, however, he does not say that it was extinguished, but that rains and clouds followed for eight days, after which no further trace of it appeared. So this comet will have lasted from November 24 to December [?] 22, at least 59 days, and perhaps somewhat more, although Kepler sets only 55 days. It will be clear, moreover, from what is to be said, that its rising was 4° below the northern Scale [Lancem Borealem], and its extinction between the head of Ursa Major and the north pole of the world; that its motion was retrograde, obliquely against the order of the signs [in præcedentia], and slower every day, just as the size of the head also appeared smaller every day to the naked eye, though not so through the telescope; and that its latitude, or declination from the ecliptic, increased until January 8 and then decreased, its course bending somewhat towards the south. So it began at 18° of Scorpio with a southern latitude of 3½°, and ended at 21½° of Cancer with a northern latitude of 56⅓°.

[Margin: Year 1618, 1619.]

Obſeruationes Particulares Cometæ Anni 1618. & 1619.

Particular Observations of the Comet of the Year 1618 and 1619

Besides the observed longitudes, which I shall give in a table at the end of this history, and the distances from the fixed stars, which I shall present in chapter 22, number 23, I should like to gather certain other points from various observations.

[Margin: From Maximilianus Marsilius.]

Father Maximilianus Marsilius of the Society of Jesus, at Budweis in Bohemia, was the first of all — of those, at least, who are known — to observe this comet, with a clear light and with its head and tail very bright; for him, however, the second comet had disappeared on the same day, the dawn being already bright,

and yet the fourth comet overcame the dawn's light. So Kepler attests in his work on these comets, pp. 55 and 57.

[Margin: From the Roman observations.]

The Fathers of our Society at the Roman College, and especially Father Orazio Grassi [Horatius Graſſus], began to observe this comet on November 27, between the Scales [inter Lances], that is, about 4° distant from the northern Scale. On the 29th, at hour 12, they saw it 1½° from the same Scale, towards the Notapeliotes [Notapeliotem], commonly called the Sirocco, with a tail of 58° — as long as Kepler, p. 58, reports it was seen at Vienna at that time. On the 30th they saw that the comet had passed the Scale, with a tail at first of 23° and later much longer; through its middle, lengthwise, ran a clear and distinct line like the pith of a tree, while the tail curved somewhat and stretched from the right foot of Virgo towards her knees. On the 9th, with December 10 following, the comet was seen by them between Arcturus and Corona Borealis [Coronam Boream], with a very long tail, curved and swelling towards the south, fainter and more scattered on the left towards the north, but fuller on the right.

[Margin: Froidmont's observations.]

Libert Froidmont marvelled that on December 12, from the 9th and 10th hour of the evening until midnight, not even a hair of the comet's beard was to be seen beneath the wheels of the Wain [Plauſtri], where in the morning he had seen its long beard; but after midnight, as the Sun and the comet — which went a little ahead of the Sun — approached the eastern horizon, the beard returned beside the Wain and gradually grew.

[Margin: From Father Biancani's observations.]

Father Giuseppe Biancani of the Society of Jesus, once my teacher in geometry, saw at Parma on November 29 the fiery beam for the last time and this comet for the first time: ruddy in its centre, but yellowish at the edges. At hour 13¼, Italian civil time, he saw it about one degree below the northern Scale, but 7½° above Mercury, which was at 20° of Scorpio. Besides the distances of the comet from the fixed stars observed on very many nights, on December 14 at hour 8, Italian civil time, he saw it rising from the horizon with a tail of 36°; on the preceding and following days the tail seemed the longer, the darker the air was, or the nearer the comet was to the horizon. On the 20th, having considered the motion of the preceding days, he judged that it had been made along one and the same great circle, and he concluded that this was kept up until the 31st. On the 31st, around civil hour 13½, he saw the comet a little beyond the straight line drawn through the third star of the tail of Ursa Major and likewise through the third of the tail of Serpens, but so diminished in colour, motion and size that it was scarcely visible; so from then on he gave up his observations.

[Margin: Chiaramonti's observations.]

Scipione Chiaramonti, who wrote a little book on this "pogonarian" comet [Pogonari Cometa], gives few observations. In them, however, he affirms that the comet's head diminished daily; that on December 4 he saw the tail 21° long,

when the head was 64° from the zenith [vertice] of Cesena [Cæſenæ], at 12^{h} 40^{m} after sunset; and that on December 18, at the same hour, when the head was 37° from the zenith, he saw the tail 28° long.

[Margin: Gassendi's observations.]

Gassendi [Petrus Gaſſendus] observed this comet at Aix [Aquis Sextijs] from November 29 to January 5, the day on which he says the head was blended with the tail. Its proper motion, he says, increased daily until December 6, when it reached 3° 36′, and then decreased, so that on December 22 it was 2°, on December 31 1° 20′, and on January 5 1°.

[Margin: Snel's observations.]

Willebrord Snel [Villebrordus Snellius] in Holland began to observe the same comet from the 27th, although he admits that it was seen by Antonius Lucianus at Valencia in Spain on the 24th, without a head because of its nearness to the Sun; but that on the 29th the head emerged from the Sun's rays and stretched out a tail of 25° towards the west, turned away from Venus rather than from the Sun, although later it was turned exactly away from the Sun. Its motion began from the northern Scale and ran from there through Boötes and the tail of Ursa Major. From December 17 it no longer set for the Dutch, but moved among the stars that are always visible there. Finally, he says, it was extinguished around mid-January beyond Ursa Major.

[Margin: Longomontanus's observations.]

Longomontanus at Copenhagen [Hafnię] did not catch sight of this comet before November 21, Old Style — December 1, Gregorian Style — because of the storms and rains that raged for the 10 preceding days. Already on December 1 the comet's head was somewhat larger than Spica, its colour midway between Mars and Spica; he saw the tail turned directly away from the Sun, and as long as the stature of a fairly tall man. On December 3 he measured the tail at about 20°, though he could scarcely follow its extreme flickering with his eyes. On the 4th the tail was very long and remarkable for its flickering, and the head still of a larger disc than Spica, and fairly round. On the morning of December 10 the comet, rising almost with the Moon, flickered the rays of its tail beyond its usual extent, up to 104° — that is, as far as the left hand of Auriga — and they were turned away from the Sun; Longomontanus ascribes this to the disposition of the air. But as the comet rose higher, the tail became more contracted. On December 15 the comet's head was rather dim, diminished and ruddy to the naked eye, but livid through the telescope, and the rays of the tail were sparse. On the 24th, with the Moon shining opposite, the comet appeared much attenuated, and scarcely a trace of the tail appeared, as also on the 26th and the 28th, on which day it ceased to be observed at Copenhagen. Further, from the parallaxes he deduces the comet's distance from the Earth as at least 240 semidiameters of the Earth; and because he estimated the comet's head as larger than Spica, and of 3′, he makes its diameter to the Earth's diameter as 377 to 3600, or as 1 to 9½, and the body of the comet to the Earth as 1 to 870, and to the Moon as 1 to 20.

[Margin: Kepler's observations.]

Johannes Kepler [Ioannes Keplerus] at Linz in Austria began his observations of this comet on November 29, by estimating its place against the fixed stars by eye, since he lacked suitable instruments. On that day he saw the comet's head three Moons distant from the northern Scale. The tail flickered, partly yellow and partly ruddy, so that it seemed now long, now short. The head, inspected through the telescope, was round below [infrà] [?]; below, it faded away into hair, and appeared less bright than when viewed with the naked eye. He estimated the length of the tail at 30°, although he says that it was seen at Rome and Vienna at 58°, because of the inconstancy of its length. On the 9th the tail, spreading more and more, stretched as far as the breast of Ursa Major, and indeed seemed to flash out as far as Auriga; but its certain length, conspicuous to everyone's eyes, was 70°, namely up to the breast of the Bear — a tail such as, he denies, has been observed in the past 150 years. Nor was it turned exactly away from the Sun, but rather from Mercury; by its curvature it swelled towards the south, and it was fuller on the right and sparser and fainter on the left. On the 16th it began to be seen in the evening too. On the 23rd it spread its tail widely, as if towards the northernmost of the Wain; and the head, seen through the telescope, looked like a cloud. At last, on January 7, since the tail no longer appeared and he was not certain about the head, he stopped observing. He judges, moreover, that at the beginning this comet was midway between the Earth and the Sun, and thirty times higher than the Moon, that is, 1770 semidiameters of the Earth high, with a parallax of 2′; that it then crossed the orbs of Venus and Mars along a straight line; that when it ceased to be seen it was between Jupiter and Mars; and that on December 9 it had so long a tail that the tail was greater than the comet's distance from the Earth.

[Margin: Rhodius's observations.]

Ambrosius Rhodius, professor of medicine and mathematics at Wittenberg [Vuitebergę], began to observe this comet on November 27; his own observations, however, not as most certain does he put them forward.

But on December 14 he affirms that he saw the tail very long, up to 60°, or at least 50°. On the 17th he saw it contracted to 30°, as also on the 19th, again of 30°, and reaching the lower of the following stars in the quadrilateral of the Great Wain [plauſtri maioris]. Finally he judged that the comet disappeared around January 12 of the year 1619, near the penultimate star of the tail of Draco, at ♌ 4°, with its latitude decreasing.

[Margin: Vuelperus's observations.]

Vuelperus of Alsace [Vuelperus Alſatus] affirms that the comet was first seen at Worms [Vuormatiæ] on November 28; he himself observed it at Loræ, and he says that on December 7 a ray of the comet grazed the reddish star Arcturus and the right leg of Boötes — that same Arcturus which Hebenstreitus at Ulm took for Mars. On the 20th he saw the tail 18° long, and its course through the first Horse [primum Equum]. Beyond December 30 I do not find his observations, and he does not boast that they were the most certain.

[Margin: Habrecht's observations.]

Dr Isaac Habrecht [D.Iſaac Habrechtus] at Strasbourg [Argentorati], or Argentina, on November 30 noted in the comet a kind of triple star, its innermost nucleus shining like gold melted in a crucible [teſta], among the most violent fires of Capella; but this nucleus, he says, was surrounded by a circle of fainter brightness, and this in turn by another circle, inconstant and scintillating. On the same day a star was seen by others in the comet's head, like a young lady seated in a carriage, as was written to Philippus Millerus at Leipzig.

[Margin: Kepler's imperfect eye.]

Kepler, p. 59, attributes this in vain to a defect of the eyes, since he himself was dim-sighted and used to see, in place of one Moon, six or eight interwoven with one another, as he admits in the same place and in the *Astronomia Optica*. But compare these observations with the observations of our Cysat, made with the telescope and shortly to be adduced: you will see that this was no trick of the eyes. On the morning of December 18 Habrecht noticed the comet advancing midway between the shoulder and the wrist of Boötes, and on the 20th it reached the star of the wrist. On January 7 he saw the comet not far from Ursa Minor, and saw that it had already turned aside in its course towards the south, its latitude diminished.

[Margin: Millerus's observations.]

Philippus Millerus, professor of mathematics at Leipzig [Lipſię], began to observe the comet on November 26, and judged that on that day the tail reached the nearest star of the quadrilateral of the Great Wain, that is, was about 90° long. On December 14 he saw the head above the back of Boötes; on the 16th the tail set in between the two front stars of the Wain, and 30° long; and on December 23 he noted the comet between the last star of the tail of Ursa Major and the left star of Boötes. From about that day onwards, he wrote in the little book he published on this phenomenon, the comet bent its path somewhat towards the south; and his observations end at December 27.

[Margin: Remus's observations.]

Johannes Remus Quietanus, physician and mathematician to the Emperor Matthias, published a little book on this comet at Innsbruck [Oeniponte]. From November 27 he brings forward the first observation of others [?]. He set out from Rastadium [Raſtadio] to St Johann [S. Ioannem] in the County of Tyrol, where, lacking instruments on the journey, he observed some distances from the fixed stars. Afterwards he was at Innsbruck, where on December 13 he saw the tail 30° or 36° long, and at hour 3 the comet's head in the same vertical circle as the hip of Boötes; on the 16th he saw the tail touching with each of its ends the two front stars of the Wain; on the 18th the tail reaching up to the penultimate star of the tail of Ursa Major, but bending towards the upper parts of the Wain; and on December 21 he noted the tail between the last and the penultimate stars of the tail of Ursa Major. On the 22nd it was 7° long, and on December 27 he stopped observing.

[Margin: Schickard's observations.]

Wilhelm Schickard, in Württemberg [Vuertembergia] on the banks of the Neckar [Nicri], drew the path of this comet on a chart from his own observations. In this picture you may see, on December 9, the tail turned directly away from Mercury rather than from the Sun, full and gathered on the right but scattered on the left. On December 13 he places the northern edge of the tail faint and touching the lower star of the Great Wain, but the southern edge suffused with a more evident light and running out further; and on the 16th he paints the head beside the shoulder of Boötes. On January 7 he places the comet's head passing the star of Draco, and on January 18 he places the comet at ♋ 23½° with a northern latitude of 58½°. He does not extend his observations any further.

[Margin: Ursinus's observations.]

Benjamin Ursinus, astronomer of the Mark [Aſtronomus Marchicus], caught sight of this comet at Frankfurt on the Oder [Francofurti ad Oderam], or on the Viadrus, on November 28, around the Claws, or Scales, of Libra. On December 16 he saw the comet's head a diameter and a half of the Sun away from the star in the shoulder of Boötes. On January 6 he saw the comet almost touching the penultimate star of Draco, and on the 7th still very near it and verging towards the region of Ursa Minor.

[Margin: Other observers indicated.]

What Hebenstreitus at Ulm, Io. Straussius at Tübingen, Gaspar Herspachius at Cologne, Gallus Provenzalis at Paris, Arthus at Danzig [Dantiſci], Herlicius at Stettin in Pomerania, Nagellius at Magdeburg, Antonius Lucianus at Valencia in Spain, and Carolus Piso in Lorraine observed in this comet is not worth the trouble of begging for. I would rather pass on to the more accurate observations of Father Cysat, and refresh the reader's mind with them.

[Margin: Father Cysat's notable observations.]

Father Johann Baptist Cysat [P. Ioannes Baptiſta Cyſatus] of the Society of Jesus, ordinary professor of mathematics at Ingolstadt, had indeed in good faith shared his observations with others indiscriminately, on sheets he gave out. Some of these came, through Dr Johann Georg Herwart [D. Io. Georgium Heruartum], counsellor of the Duke of Bavaria, into Kepler's hands, as Kepler himself admits on p. 59; but they were mixed and confused with uncertain ones. Afterwards Father Cysat himself, at leisure, selected the more certain ones and described them in chapter 1 of his *Mathemata* on this comet; at its end he complains that those earlier, uncorrected observations were used by Kepler and published under the name of "the Ingolstadt observer". But certainly, as Ovid sings, *Puriùs ex ipſo fonte bibuntur aquæ* — "waters are drunk purer from the spring itself"; so we too must draw from the spring. And just as he sets out separately, in chapter 4, the observations of the tail and of the head, and in chapter 6 the singular phenomena of the head, so we too shall select these separately.

Before the Kalends of December, mists and storms had prevailed at Ingolstadt for 8 days and more; but on December 1 the north wind [Aquilo] drove away

the clouds and disclosed the comet to Cysat a little before hour 5 in the morning. At 5^h 45^m he saw the comet at an altitude of 12° 20′, distant from Arcturus 27° 6′ and from Spica 24° 48′, and therefore at 9° 24′ of Scorpio, with a northern latitude of 11° 37′. On the 4th the comet rose for him from the horizon a little before hour 4 in the morning. On the 7th, between hour 4 and 5 in the morning, the star of the third magnitude in the leg of Boötes was almost in the middle of the rays of the comet's tail, and at hour 7 at their far ends, near the head. On December 16, at hour 9 in the evening, the comet's tail seemed to pass through the middle of the quadrilateral of Ursa Major, while the head still lay hidden, either below the horizon or in the horizon's vapours, from which it emerged around hour 11 of the night. On December 20, at 3^h 54^m in the morning, the comet's head appeared to be very nearly one degree from the star of the fourth magnitude at the left elbow of Boötes, when the comet seemed 48° 30′ high; and it kept the same distance from the same star, without notable difference, almost until sunrise, that is, until the comet's apparent altitude was 76° — which seemed to him an argument for a small parallax.

[Margin: When the comet began to be among the stars that are always visible.]

From this 20th day until January 18 the comet never again set below the horizon, but moved among the stars that are always visible. On September [?] 27, at hour 7 in the evening, the sky was so very clear that stars of the sixth magnitude were distinctly seen, and yet nothing at all of the comet's coma appeared; the head was seen with the greatest difficulty, no larger than stars of the 4th magnitude and much dimmer than the other stars — because, that is, the fuller Moon had almost extinguished the comet, as it had the Milky Way and the nebulous stars. But on the 28th at hour 5 in the morning, with the Moon absent, the comet was more conspicuous, the head much larger, the coma much longer, and the air purer too. At hour 9 in the evening, however, with the Moon shining, nothing of the comet was seen, although stars of the 6th magnitude were visible. On January 7 the comet was in conjunction with the penultimate star of the tail of Draco. Finally, from January 22 onwards, for another 8 days and more (as was said at the beginning), the days were cloudy, after which no further trace of the comet appeared.

The rest, as to the distances from the fixed stars and the altitudes, we shall report in chapter 22, number 23, comparing these distances with the distances taken by others, so that a conjecture may be formed from them about the parallaxes of this comet; but the longitudes and latitudes observed by Cysat over many days we shall present shortly in the first table. There follow the observations of the tail, set most elegantly before the eyes in twelve diagrams of the stars.

[Margin: Position and length of the tail.]

On December 1 the head was a little below the northern foot of Virgo, from which the tail rose, not straight but curved, upwards convex, and somewhat concave downwards, and broader in the middle than the head itself, as happens

in the flames of lamps; at its end, however, it was not scattered like a broom but gathered to a point, which shone with sparser and fainter rays. The length of the tail was then at least 22°. On the 4th the comet's head scintillated seldom and languidly, and the tail undulated as if gently stirred by the wind, leapt with an alternating vibration, and was sometimes sharpened at its end and sometimes spread out into the shape of a broom — which was also noticed on many other nights up to December 28. On the 7th, at hour 7, it stretched out on an almost straight path; its breadth in the middle was ordinarily about 2° 30′, but because of its alternating ejaculation and spreading, sometimes 3½°. Its length appeared 57°, and it extended from the farther knee of Boötes to the stars of the left hind foot of Ursa Major, almost grazing Arcturus (not Mars, as the astrologaster Hebenstreitus thought). On December 8 the tail's length was 58°, its ordinary breadth 3°, its extraordinary breadth 4°. On December 9, at hour 7 in the morning, the tail stretched out up to 75° — namely from about Arcturus to the claws of the right forefoot of Ursa Major — the ordinary breadth of the tail being 2°. On the 17th, at hour 6 in the morning, the tail appeared notably curved, 56° long and a little more than 2° broad. On the 18th it was 54° long and a little less than 2° broad. On the 20th the tail was 34° long and curved, and passed from the wrist of Boötes through the last star of the tail of Ursa Major as far as the left shoulder of the Bear. On the 21st the tail was 20° long. On the 22nd, at hour 8 in the evening, the tail was so short that it seemed about to disappear; but on the 24th it received new increases, and was seen to undulate and to flicker its rays in alternating darts much more often than at other times, so that it reached from the hand of Arcturus as far as the most remote star of the Wain, and at times became 4° broad — which was also seen on the 28th. The tail's length on the 28th and 29th was 35°; it then decreased so much that on January 7 it scarcely seemed 8°. Finally, he notes that the fineness of the extreme rays and the length of the tail were seen differently by different observers, because of the variety of eyes, air and twilight; but that the tail always stretched directly towards the part opposite the Sun;

[Margin: Digression from the Sun.]

and that the comet's head had digressed from the Sun by 31° 27′ on December 1, by 90° 34′ on December 21, and by 123° 3′ on January 18, but on the 22nd by only 122° 2′.

[Margin: Observations of the head.]

Further, he found the comet's head, to the naked eye, on the first day of December far larger than any star of the first magnitude, its light dull and pale, not scintillating, but very like Saturn. Measuring it with an astronomical radius, he found it, until December 10, to be of 7′ or 6′; but on January 3 so diminished [?] that it scarcely equalled stars of the 4th magnitude. Using two telescopes [tubo∫picillis], however, one of 6 feet and the other of 10, he saw in the middle of the head a round nucleus of more condensed light, of greater diameter than any star of the first magnitude likewise viewed with the telescope, yet smaller than Jupiter's diameter as seen through the tube in those days, so that at most

it contained two-thirds of Jupiter's diameter. Around that nucleus was spread another, paler light, with a rim almost twice as broad as the nucleus's diameter, like a mane or halo. The nucleus's diameter was 2′, the rim's breadth 3′, and the whole diameter about 8′. On the 8th the nucleus taken by itself appeared twice as large as Arcturus, and of 3′ or 4′, whereas on the first day it was seen much smaller; nor was it round any longer, but split into three or four globules of irregular shape, cohering with one another. On the 17th and 18th of December, in place of that nucleus there were now several very small stars, with a very dull light spread between and around them; and on the 20th it appeared more plainly dissolved into many little stars, of which three were seen more distinctly, the largest of them like stars of the 5th magnitude. The diameter of this nucleus, or cluster of little stars, was 5′ or 6′, notably larger than on December 1. But on December 24 the little stars of the nucleus appeared more dissipated and with a much fainter light, and of those three conspicuous ones only one was seen; the others, all of similar magnitude, could not be counted because of their frequent scintillation and inconstancy. The nucleus's diameter, however, was seen to be 6′, the breadth of the surrounding rim 5′, and the whole diameter of the head 16′. Afterwards it could not be viewed with the tube, since those little stars were by then dissipated.

[Margin: The comet a cluster of little stars.]

It was remarkable, therefore, that quite the opposite happened for those viewing it with the naked eye: at the beginning it seemed larger, because of its condensed and dense body and its more intense light, and at the end it could scarcely be discerned and seemed of the smallest magnitude; whereas, on the contrary, through the telescope the head was seen as 8′ on December 1 and as 16′ on December 24. Chrysostomus Gall., afterwards public professor of mathematics in Portugal [Lusitania], was present; and the observation, repeated with different tubes and often on the same nights, frees the mind of the suspicion of a trick of the eyes. See also what Dr Habrecht saw at Cologne [Coloniæ], above. And here, at the end of this observation, is the figure in which the comet's head appeared from December 17 to the 20th. A similar cluster is seen with the telescope in the nebula of Cancer and at the end of Orion's sword. Finally, Cysat thinks that the distances from the Earth were such as we shall set out at the end of table 4, and that the comet moved on an epicycle around the Sun, although he does not deny that it could have moved along a straight line; and that the head was 175 German miles in diameter, its body 950 times smaller than the Earth but 22 times smaller than the Moon. Next I shall present the tables: first those of the observed longitudes and latitudes, then ephemerides put together from a few observations, but arranged by a probable conjecture of the motion and extended to many other days on which the comet was not observed.

[Margin: Effects of the comet, from Kepler.]

As to its effects, says Kepler, the winter of the year 1619 was dry and squalid, so much so that, although the south wind blew, the Moselle never overflowed as

it usually does. About this time the Empress Anna died, and then the Emperor Matthias, who had stripped Mansfeld of the Palatinate; from this came huge disturbances and disasters, the Bohemian war, the incursions of Gábor Bethlen [Bethlemi Gaboris] into Hungary, and many other calamities, which afflicted Germany and Italy in particular.

Diagram. An uncaptioned woodcut set in the right column beside the account of the comet's effects: the comet's head drawn as a ring of fine lines radiating outwards around a clear central disc, which holds a cluster of small six-pointed stars — the figure, announced in the text, of the head as it appeared from December 17 to 20. The block carries no letters, figures or labels.

TABVLAE Ex Obſeruationibus Cometæ Anni 1618. & 1619. Conſtructæ.

Tables Constructed from the Observations of the Comet of the Year 1618 and 1619

The places at which the authors listed below observed are these: Longomontanus at Copenhagen; Cysat at Ingolstadt; Snel in Holland; Rhodius at Wittenberg; Habrecht at Strasbourg [Argentinæ]; Remus at Innsbruck; Vuelperus at Loræ [Lorę] in Alsace; Schickard in Württemberg; Millerus at Leipzig; Ursinus at Frankfurt on the Oder; Kepler at Linz; Chiaramonti at Cesena; the Dillingen observer [Diligenſis] at Dillingen; the Roman Fathers at Rome.

Table I. Longitude and latitude of the last comet of the year 1618 and 1619, as observed. Latitude always north.

Day (New Style)	Observers	Hour after noon	Longitude	Latitude
November 28	Kepler	$18^h\ 30^m$	♏ 12° 43′	7° 30′
29	Fathers of the Roman College	✠	11° 30′	10° 0′
30	Longomontanus	$17^h\ 45^m$	9° 26′	11° 47′
	Father Cysat	$17^h\ 45^m$	9° 22′	11° 50′
December 1	Father Cysat	$16^h\ 48^m$	7° 56′	14° 31′
	Snel	✠	8° 24′	14° 12′
2	Longomontanus	$18^h\ 30^m$	6° 24′	16° 56′
	Rhodius	✠	3° 30′	16° 0′
3	Longomontanus	$18^h\ 30^m$	4° 53′	19° 33′
	Father Cysat	$18^h\ 52^m$	4° 56′	19° 18′
	Chiaramonti	$17^h\ 46^m$	3° 35′	20° 43′
	Habrecht	✠	3° 0′	19° 40′
	Rhodius	✠	1° 24′	20° 0′

The Remainder of the Preceding Table.

Day (New Style)	Observers	Hour after noon	Longitude	Latitude
December 4	Remus	$16^h\ 0^m$	♏ 2° 0′	22° 0′
5	Remus	✠ ✠	0° 45′	24° 30′
6	Father Cysat	$18^h\ 51^m$	♎ 29° 23′	27° 37′

Day (New Style)	Observers	Hour after noon	Longitude	Latitude
	Habrecht	✠	28° 30′	27° 30′
	Vuelperus	✠	29° 30′	26° 20′
7	Longomontanus	$18^h\ 0^m$	27° 45′	30° 0′
	Father Cysat	$15^h\ 0^m$	28° 21′	29° 47′
8	Longomontanus	$18^h\ 30^m$	26° 10′	33° 0′
	Father Cysat	$18^h\ 22^m$	26° 18′	32° 29′
	Kepler	$18^h\ 0^m$	26° 40′	32° 0′
	Remus	$17^h\ 0^m$	27° 0′	34° 0′
	the Dillingen observer	✠	24° 0′	31° 48′
9	Longomontanus	$17^h\ 30^m$	24° 0′	35° 0′
	Father Cysat	$18^h\ 25^m$	23° 37′	35° 4′
10	Snel	✠	23° 21′	34° 4′
11	Remus	$17^h\ 30^m$	22° 0′	40° 20′
12	Remus	$16^h\ 15^m$	18° 20′	41° 45′
	Schickard	$17^h\ 30^m$	17° 49′	42° 1′
	Rhodius	$18^h\ 0^m$	16° 33′	41° 48′
13	Longomontanus	$17^h\ 30^m$	16° 20′	43° 15′
	Remus	$16^h\ 30^m$	17° 0′	44° 0′
	Schickard	$17^h\ 52^m$	15° 42′	44° 4′
	Snel	✠	15° 1′	46° 4′
	Rhodius	✠	14° 30′	43° 53′
14	Longomontanus	$17^h\ 0^m$	14° 40′	51° 36′
	Rhodius	✠	12° 31′	45° 58′
	Habrecht	✠	13° 30′	46° 50′
15	Rhodius	✠	10° 21′	47° 42′
	Remus	✠	11° 27′	49° 33′
16	Father Cysat	$16^h\ 7^m$	8° 49′	49° 20′
	Habrecht	✠	11° 0′	48° 0′
	Vuelperus	$12^h\ 0^m$	9° 0′	50° 30′
	Rhodius	✠	8° 0′	49° 40′
17	Father Cysat	$16^h\ 46^m$	6° 20′	51° 2′
	Habrecht	✠	8° 0′	51° 0′
	Remus	✠	8° 0′	50° 30′
18	Rhodius	✠	3° 0′	53° 0′
19	Father Cysat	$15^h\ 54^m$	♎ 1° 21′	54° 11′
	Kepler	$17^h\ 0^m$	1° 30′	54° 30′
	Rhodius	✠	✠ ✠	54° 42′
	Vuelperus	✠	3° 0′	55° 30′
20	Father Cysat	$17^h\ 44^m$	♍ 28° 23′	55° 20′
	Remus	$16^h\ 30^m$	26° 30′	56° 30′
21	Remus	$14^h\ 30^m$	25° 0′	57° 0′
	Rhodius	✠	25° 25′	57° 0′
	Habrecht	✠	22° 0′	56° 0′
22	Remus	$12^h\ 0^m$	23° 0′	57° 30′
	Vuelperus	$14^h\ 30^m$	24° 0′	59° 0′
	Kepler	$17^h\ 30^m$	23° 0′	57° 40′
	Schickard	$18^h\ 0^m$	19° 40′	59° 0′
	Rhodius	$18^h\ 0^m$	20° 30′	59° 0′
23	Father Cysat	$18^h\ 19^m$	21° 3′	58° 44′

Day (New Style)	Observers	Hour after noon	Longitude	Latitude
	Snel	✠	20° 44′	58° 31′
24	Longomontanus	10^h 0^m	♍ 21° 0′	58° 20′
	Kepler	18^h 0^m	18° 0′	59° 40′
26	Longomontanus	7^h 20^m	12° 58′	60° 19′
	Rhodius	✠	14° 49′	60° 45′
27	Longomontanus	5^h 30^m	8° 45′	60° 40′
	the same, again	18^h 45^m	7° 30′	61° 0′
	Vuelperus	✠	15° 0′	62° 30′
	Father Cysat	17^h 4^m	8° 31′	60° 57′
	Habrecht	18^h 30^m	8° 40′	60° 30′
28	Father Cysat	16^h 28^m	6° 16′	61° 27′
	Kepler	16^h 30^m	6° 20′	62° 0′
In the year 1619				
January 3	Father Cysat	7^h 22^m	♌ 20° 38′	62° 36′
4	Schickard	✠	20° 0′	62° 0′
6	Ursinus	✠	21° 45′	62° 0′
7	Father Cysat	8^h 18^m	♌ 11° 38′	62° 28′
12	Cysat	8^h 3^m	1° 7′	59° 47′
13	Cysat	7^h 51^m	0° 6′	59° 28′
14	Cysat	9^h 15^m	♋ 28° 31′	58° 57′
15	Cysat	8^h 3^m	27° 47′	58° 44′
16	Cysat	8^h 30^m	26° 46′	57° 51′
17	Cysat	8^h 30^m	25° 39′	57° 16′
18	Cysat	6^h 44^m	24° 13′	56° 51′
	Schickard	✠	23° 30′	58° 30′
21	Father Cysat	16^h 52^m	♋ 21° 20′	56° 22′

Table II. Ephemeris of the comet of the year 1618 and 1619, constructed by Johannes Kepler for the 6th hour after midnight at Linz [Lincij].

Day of the comet	Daily motion	Place in longitude	Latitude
November 24	0° 0′	♏ 18° 2′	north
25	2° 12′	17° 2′	
26	2° 16′	16° 1′	
27	2° 21′	14° 58′	
28	2° 26′	13° 53′	4° 40′
29	2° 32′	12° 43′	7° 30′
30	2° 38′	11° 31′	10° 0′
December 1	2° 45′	♏ 10° 14′	10° 36′
2	2° 54′	8° 50′	13° 40′
3	3° 4′	7° 23′	16° 0′
4	3° 20′	5° 43′	19° 40′
5	3° 21′	3° 58′	22° 0′
6	3° 17′	2° 11′	24° 30′
7	3° 11′	0° 22′	28° 0′
8	3° 5′	♎ 28° 33′	30° 13′
9	2° 58′	26° 40′	32° 0′

Day of the comet	Daily motion	Place in longitude	Latitude
10	2° 52′	24° 45′	35° 25′

The Remainder of the Preceding Table [Table II, continued].

Day of the comet	Daily motion	Place in longitude	Latitude
December 11	2° 47′	♎ 22° 46′	38° 0′
12	2° 42′	20° 43′	40° 20′
13	2° 37′	18° 37′	42° 1′
14	2° 32′	16° 26′	44° 4′
15	2° 23′	14° 9′	45° 58′
16	2° 24′	11° 48′	47° 42′
17	2° 19′	9° 22′	50° 15′
18	2° 15′	6° 50′	51° 0′
19	2° 10′	4° 13′	53° 0′
20	2° 6′	1° 29′	54° 42′
21	2° 1′	♍ 28° 43′	55° 30′
22	1° 56′	25° 53′	57° 0′
December 23	1° 52′	♍ 23° 0′	57° 40′
24	1° 48′	20° 40′	59° 0′
25	1° 43′	17° 7′	59° 40′
26	1° 38′	14° 10′	60° 45′
27	1° 33′	11° 16′	62° 30′
28	1° 29′	8° 23′	61° 20′
29	1° 25′	5° 32′	62° 0′
30	1° 21′	2° 45′	62° 20′
31	1° 18′	0° 1′	✠ ✠
January 1619, 1	1° 15′	♏ 27° 20′	
2	1° 13′	24° 41′	
3	1° 10′	22° 5′	63° 15′
4	1° 8′	19° 36′	62° 0′
5	1° 6′	17° 9′	
6	1° 4′	14° 47′	62° 0′
7	1° 2′	12° 29′	62° 0′
8	0° 59′	10° 18′	
9	0° 57′	8° 13′	
10	0° 55′	6° 13′	
11	0° 53′	♏ 4° 18′	
12	0° 51′	2° 29′	61° 15′
13	0° 49′	0° 45′	
14	0° 48′	♋ 29° 5′	
15	0° 46′	27° 30′	
16	0° 44′	26° 1′	
17	0° 42′	24° 37′	
18	0° 41′	23° 16′	58° 30′
19	0° 39′	22° 0′	
20	0° 37′	20° 50′	

Table III. Ephemeris of the last comet of the year 1618, from Longomontanus, at Copenhagen [Haſniæ], at the 9th hour of the morning.

Day of the comet	Daily motion on its own circle	Longitude	Declination, north from the 23rd
November 21	3° 20′	♏ 9° 25′	3° 20′ A
22	3° 17′	7° 55′	0° 35′ A
23	3° 14′	6° 25′	2° 20′ B
24	3° 11′	4° 55′	5° 30′ B
25	3° 9′	3° 7′	9° 0′ B
26	3° 6′	♏ 1° 20′	12° 0′ B
27	3° 3′	♎ 29° 30′	15° 20′
28	3° 1′	27° 45′	18° 10′
29	3° 0′	26° 10′	20° 40′
30	2° 57′	24° 10′	23° 15′
December 1	2° 47′	♎ 22° 0′	26° 10′
2	2° 38′	20° 0′	29° 10′
3	2° 24′	18° 0′	32° 0′
4	2° 19′	16° 20′	34° 20′
5	2° 16′	14° 40′	36° 30′
6	2° 14′	♎ 12° 30′	38° 40′
7	2° 12′	10° 20′	40° 50′
8	2° 10′	7° 40′	43° 0′
9	2° 8′	5° 0′	45° 0′
10	2° 6′	2° 20′	47° 0′
11	2° 3′	♍ 29° 30′	49° 0′
12	1° 59′	26° 40′	51° 0′
13	1° 55′	23° 40′	53° 0′
14	1° 50′	20° 50′	55° 0′
15	1° 43′	18° 0′	56° 40′
16	1° 33′	♍ 14° 40′	58° 10′
17	1° 18′	10° 11′	59° 35′
18	1° 8′	7° 45′	61° 0′

Table IV. Ephemeris of the comet of the year 1618 and 1619, at Ingolstadt [Ingolſtadij], from Father Cysat. In morning hours, i.e. after midnight (M), or evening hours, i.e. after noon (V).

1618: day, hour, minutes	Longitude	Latitude, north	Digression from the Sun	Daily motion
December 1, 5^h 45^m M	♏ 9° 24′	11° 37′	31° 27′	
1, 6^h 56^m M	9° 22′	11° 50′	35° 0′	3° 14′
2, 4^h 48^m M	♏ 7° 56′	14° 31′	35° 0′	
4, 6^h 52^m M	4° 56′	19° 18′	41° 9′	
7, 6^h 51^m M	♎ 29° 23′	27° 37′	✠ ✠	
8, 3^h 0^m M	28° 21′	29° 47′	51° 42′	
9, 2^h 46^m M	♎ 26° 42′	31° 52′	✠ ✠	
9, 6^h 22^m M	26° 18′	32° 29′	57° 48′	3° 0′

1618: day, hour, minutes	Longitude	Latitude, north	Digression from the Sun	Daily motion
10, 6^h 25^m M	♎ 23° 37′	35° 4′	61° 40′	
17, 4^h 7^m M	8° 49′	49° 20′	81° 13′	
18, 4^h 46^m M	6° 20′	51° 2′	✠ ✠	
20, 3^h 54^m M	♎ 1° 21′	54° 11′	89° 53′	2° 20′
20, 6^h 22^m V	✠ ✠	✠ ✠	✠ ✠	2° 10′
21, 5^h 44^m M	♍ 28° 23′	55° 20′	90° 34′	
24, 6^h 19^m M	21° 3′	58° 44′	✠ ✠	
28, 5^h 4^m M	8° 31′	60° 57′	103° 10′	
29, 4^h 28^m M	♍ 6° 16′	61° 27′	✠ ✠	
29, 6^h 34^m M	♌ 6° 6′	61° 28′	104° 12′	1° 34′
1619: day, hour, minutes				
January 3, 7^h 22^m V	♌ 20° 38′	62° 36′	111° 25′	
7, 8^h 18^m V	11° 38′	62° 28′	115° 49′	
12, 8^h 3^m V	1° 7′	59° 47′	119° 51′	
13, 7^h 51^m V	0° 6′	59° 28′	✠ ✠	
14, 5^h 56^m V	♋ 28° 38′	59° 1′	✠ ✠	1° 3′
14, 9^h 15^m V	28° 31′	58° 57′	120° 54′	
15, 8^h 3^m V	27° 47′	58° 44′	121° 14′	
16, 6^h 15^m V	♋ 26° 49′	57° 54′	✠ ✠	
16, 8^h 30^m V	26° 46′	57° 51′	122° 8′	
17, 4^h 30^m M	25° 57′	57° 21′	✠ ✠	
17, 8^h 30^m V	25° 39′	57° 16′	122° 43′	0° 54′
18, 6^h 44^m V	24° 13′	56° 51′	123° 3′	
22, 4^h 52^m M	21° 20′	56° 22′	122° 2′	

From the same Father Cysat.

Day of December	Distance of the comet from the centre of the Earth, in Earth semidiameters	Distance of the comet from the centre of the Earth, in German miles	Semidiameter of the comet's epicycle, in Earth semidiameters	Length of the comet's tail, in Earth semidiameters
1	72	62139	1041	45
9	96	83172	1157	137
17	132	113456	1129	185
20	154	132921	1111	✠
29	252	216625	1189	445

CAPVT IV. De Proprietatibus ſeu Accidentibus Cometarum ex Præcedenti Hiſtoria collectis in vnum.

Chapter IV. On the Properties or Accidents of Comets, Gathered Together from the Preceding History

I. To satisfy those eager for learning, and so that many of the things usually needed for solving questions about comets may be ready to hand, I have decided to select from chapter 3 the properties, or conditions, of comets set out below.

Numerus Cometarum.

The Number of Comets

[Margin: 154 comets known for certain.]

Lavater [Ludwig Lavater] and Keckermann [Bartholomäus Keckermann] count 117 comets from Augustus to the year of Christ 1556. We, however, have counted 154 from the year 480 before Christ to the year of Christ 1618. About those that were not observed or recorded in history, or were not seen on the horizons of Europe, we can establish nothing certain. It is nevertheless very likely that many more have shone upon the whole world from the beginning of the world down to this day. Add that, as Seneca says in Book 7 of the *Natural Questions*, chapter 20: "We do not see many comets, because they are obscured by the rays of the Sun; Posidonius writes that during an eclipse of the Sun [quo deficiente] a certain comet appeared which the neighbouring Sun had hidden."

Numerus Cometarum eodem tempore.

The Number of Comets at the Same Time

II. It is a pronouncement of Aristotle (*Meteorologica* I.6) that "more comets than one have appeared at the same time"; on which Pliny (II.25) says: "Aristotle reports that several are seen at once — a thing found out by no one else, as far as I know." Yet it was found out by Bede in the year of Christ 729, in which, as Palmerius also reports in his *Chronicle*, two comets were seen for 15 days in the month of January, one in the morning, the other in the evening. The same is recorded to have happened in the year 761, so that one appeared in the east for 10 days and the other in the west for 21 days. In the year 1214, the *History of the Scots* reports two seen at once, one in the morning going before the Sun, the other in the evening following the Sun. Vuenceslaus Pantaleon affirms that while he was living among the Indians [apud Indos] two comets were seen at once; likewise in the year 1529, 4 comets are recorded to have been seen standing opposite one another; and Franciscus Fernandes says that five can be seen at once. In the year 1337, too, a solitary comet appeared in May, and another later joined it, visible together with the first in June, July and August. Julius Caesar Scaliger, too, in exercise 79 against Cardano, says that at the time when Charles, King of the French, invaded Italy, a comet was seen at sunset which a few days later went ahead of the Sun; and then he adds: "We saw two at once, one in the morning, the other in the evening, for many days, at different latitudes, not so many years ago." Sometimes, however, a comet that was really one has been believed to be twin, because from a morning comet it became an evening one, its declination and right ascension having changed — as Apian and Fracastoro noted happened in their own time with the comets of the year 1531 and 1532. But in the year 1618 two were plainly seen together in the morning by Franciscus Garcia in Persia and by Father Iacobus Rhò [Giacomo Rho] of the Society of Jesus at Goa in East India, as we related in the history of that year, chapter 3.

Cometæ in plures diuiſi aut reſoluti, aut ex pluribus Coaleſcentes.

Comets Divided or Resolved into Several, or Coalescing from Several

[Margin: One comet going off into several.]

III. In the year 450 before the Christian era, or thereabouts, Democritus saw a comet dissolved into several stars, as is found in Aristotle, *Meteorologica* I, chapter 6. Then, in the year 372 before Christ, the historian Ephorus affirms that the great comet of which Aristotle speaks in the same chapter, and which preceded the submersion of Helice and Bura, cities of Achaea, broke up into two stars. Seneca (*Natural Questions* VII.16), however, scorns this with arguments half drawn from rhetorical commonplaces [argumentis ſemitopicis], as we reported at that year. On Ephorus's side, by contrast, Kepler, in his book *On Comets*, p. 49, said: "No doubt the Romans, lords of the world and haughty, for whom contempt for a conquered nation counts as part of their wealth. And Seneca, in a declamatory style, rattling the boxes of commonplaces, plays his own part." Seneca, that is, was wholly bent on persuading, against Artemidorus, that comets are not a coupling of two known stars; but Ephorus had not said that the comet was resolved into two known stars, but into two unknown ones.

[Margin: Several comets coming together into one.]

So too in the year 13 before Christ, according to Dio, a comet that had long hung over the City was afterwards dissolved into many torches. Kepler, in the passage cited, thinks that the very same thing happened with the two comets seen in the year 1618. Nicephorus, on the contrary (*Ecclesiastical History* XII.37), reports of a new star, or comet, of his own time: "Little by little a huge mass of other stars gathered to it, like bees to their leader."

Tempus Apparitionis Cometarum.

The Time of the Appearance of Comets

IV. Simon Maiolus, in colloquy 1, and Junctinus [Francesco Giuntini], in his annotation, said that comets are generated more often in autumn; Hippocrates, in Aristotle (*Meteorologica* I, chapter 4), around the summer solstice; Suessanus [Agostino Nifo], in the same place, that the most suitable time for it is spring and autumn; Franciscus Resta, *Meteorology* I, tract 1, chapter 7, said that it happens more often in autumn, but that the other seasons of the year also have their comets.

Giovanni Camillo Glorioso, *On Comets* I, p. 12, says that it happens rarely in springtime, more rarely in summer, most rarely in winter, but most often in autumn, around the equinox or a little after. Froidmont, *Meteorology* III, chapter 4, article 6, declares that comets sometimes arise in every month of the year, but most of all in autumn and winter. What should really be said will at once become clear from the history, which you may see in the following synopsis, in the years of Christ — for before that it can scarcely be established in which particular month they appeared; as when Aristotle says that the great comet appeared in winter, when there was frost.

Years of Christ in which comets are established to have arisen in definite months.

Winter	Spring	Summer	Autumn
December	*March*	*June*	*September*
745	1341	1283	1531
January	1472	1337	1532
726	1512	1444	1607
882	*April*	1533	*October*
1470	1500	*July*	876
1472	1506	1268	1557
1513	1521	1533	1585
1528	*May*	1597	*November*
1538	1211	*August*	1298
February	1539	1264	1559
1590	1559	1347	1577
		1526	1618
		1530	
		1531	
		1558	
Total 9	9	13	10

Summer therefore wins, and among the summer months August, among the winter months January, as far as is established from history. But if the definite times of the other comets had been recorded, and their true efficient cause and matter were evidently known, perhaps another season of the year and other months would carry off the palm — and perhaps not. We have already said above that some appeared as morning comets, others as evening comets, and others first as morning and then as evening comets: see the years 1530, 1531 and 1532.

[Margin: A comet staying up all night, or not setting.]

Some, moreover, either stayed visible all night, as in the year 1513, or never set below the horizon, as in the years 1513, 1533 and 1556; and the last comet itself, of the year 1618, began from December 20 to January 18 and was seen among the stars that are always visible.

Duratio Cometarum.

The Duration of Comets

V. Pliny (Book 2, chapter 25) certainly spoke far too narrowly about these when he said: "The shortest period during which they have been observed is recorded as seven days, the longest as eighty." But since Seneca (*Natural Questions* Book 7, chapter 21) affirms that a comet appeared continuously under Nero for 6 months, and Pliny cannot have been unaware of this, Muretus [Marc-Antoine Muret], in his notes on that chapter 21 of Seneca, judges that the letter C should be added to Pliny's number, so that it reads "C. Octoginta" [a hundred and eighty], because six months consist of that many days.

[Margin: The shortest duration of comets.]

The shortest duration, however, was that of the comet which the Coimbra commentators [Conimbricenſes] report (tract 3 on the *Meteorologica*, chapter 3): when it had risen a little above the horizon, it was suddenly dispersed and vanished from sight. Not much longer-lasting was the one which the astronomer Petrus Creusser saw in the year 1527 — terrible indeed, but lasting no more than five quarters of an hour, as Keckermann reports (*Systema Physicum* Book 6, p. 680).

[Margin: The longest duration of comets: a year.]

On the other hand, the longest duration of comets can be seen to be a year, if indeed, on the testimony of Josephus (*Jewish War* Book 7, chapter 17), a comet, the Xiphias, was seen for a whole year hanging over that city shortly before the destruction of Jerusalem. But Tycho (vol. 1 of the *Progymnasmata*, pp. 330 and 626), because he holds that all comets are supralunar, considers that this comet was not natural but extraordinary — as it were a supernatural portent kindled by God to terrify the Jews. For in this way it could hang over the city and not be carried round by the primum mobile, as the Peripatetics maintain of the others, and could terrify the Jews rather than other nations, and finally persist for a whole year. Froidmont (*Meteorology* Book 3, chapter 4, article 5), Glorioso (*On Comets* Book 1, p. 9) and Cottunius [Giovanni Cottunio] (Book 33 on *Meteorologica* 1, p. 312) subscribe to this view.

[Margin: Three comets lasting six months.]

With this one excepted, then, no other is reported to have lasted beyond 6 months, and I recall that only three of that kind have occurred. The first was under Nero, in the year of Christ 64. The second shone out around the year 603, at the rise of the most impious Mahomet. The third, in the year 1240, at the outbreak of the great Tamerlane. About the rest, different authors say different things; but here, drawn from the history in chapter 3, is a second synopsis for you, in which we take the months that comets are reported to have lasted as 30 days.

Synopsis of the Duration of Comets, in the years after Christ — but before Christ where A is placed.

Days	Year
0 5/96	1527
7	44 A
10	761 and 1531
11	1590
12	983
14	1312
15	1526
17	1531
18	1211
21	1314
25	1071 and 1506 and 1559, 1597

Days	Year
28	1618, 1585
30 or 31	676 and 1456 and 1533 and 1607
40	392, 1264 and 1530
55	1513
56	1532
57	1618
60	1347
79	1577
70	356 A, 336 A, 130 A, 119 A
71	1532
75	431 A
90	676 and 1264, 1363 and 1433
120	1337
180	64 and 603 and 1240
365	70

13

Magnitudo Capitis Cometæ Apparens.

The Apparent Size of the Head of a Comet

VI. The apparent size of the head of comets is usually very inconstant and varied. Few have written about it, and they have written little: Biancani (*Sphaera* Book 17, chapter 1), Piso (chapter 18 of the *Speculum*), Glorioso (*On Comets* Book 1, p. 4), Froidmont (*Meteorology* Book 3, chapter 4, article 5), Cottunius (lecture 32 on the *Meteorologica*, p. 283) and Resta (*Meteorology* Book 1, tract 1, p. 42); to these we shall add more. Many comets, viewed with the naked eye, were large-headed at first and afterwards gradually decreased; sometimes, however, they were seen small at first and larger afterwards, either with the naked eye, as Pontano observed in the comet of the year 1460, or at any rate when inspected with the telescope, as we shall shortly report from Cysat about the comet of the year 1618.

[Margin: A comet equal to the Sun.]

The greatest of all seems to have been the one that shone out in the year 146 before Christ, of which Seneca says (*Natural Questions* Book 7, chapter 15): "Shortly before the Achaean war a comet blazed forth no smaller than the Sun."

[Margin: Comets equal to ☾ □, and one three times greater than Venus.]

Large indeed, too, were those seen equal to the Moon at dichotomy — that is, shining in quadrature with the Sun — in the year of Christ 1200, on the testimony of Haly [ʿAlī ibn Riḍwān], and in 1521 and 1556, on the testimony of Cardano (*De Varietate* Book 14, chapter 69). Haly adds (on the *Quadripartitum*

[13] the first entry, given here as 0 5/96, is printed as a small round sort, then a 9 standing alone at mid-height, then a small 5 over 6 whose fraction bar spans the 6 only. The print can be read as 5/96 or as 9 5/6; 5/96 of a day, the five quarter-hours of Creusser's comet of 1527 in the text above, is a construal, not a reading of the bar.

Book 2, chapter 9) that the comet of the year 1200 had a head three times greater than Venus.

[Margin: A comet three times greater than Jupiter.]

Fracastoro, in the year 1532, as he relates in the *Homocentrica* (Book 3, chapter 23), saw a comet three times greater than Jupiter, with a tail two cubits long. And the comet of the year 1577, which Tycho brought under measurement with the astronomical radius and estimated at the beginning at 7′, as he has it in vol. 2, pp. 2 and 202, will have been twice as great as Venus at perigee, whose apparent diameter for the same Tycho (vol. 1 of the *Progymnasmata*, p. 475) is 3¼′.

[Margin: A comet twice as great as Venus.]

[Margin: Comets equal to Jupiter.]

The comet of the year 1585, at its first rising, appeared to Tycho almost equal to Jupiter, namely 3′; so too did the comet of the year 1590, as he himself relates in the *Epistolae*. The comet of the year 1618 also appeared as large to Longomontanus, since in the Appendix to the *Astronomia* (chapter 10) he says that it was seen a little larger than Spica, and of 3′; Tycho, however (vol. 1 of the *Progymnasmata*, p. 481), assigns Spica 2′ and Jupiter 3′. But Cysat (*On the Comet of the Year 1618*, chapters 1 and 4) speaks very differently. He says that the comet, observed with the naked eye by means of the radius, was seen at the beginning at 7′ and greater than any fixed star, and afterwards decreased down to a single minute; but that, viewed with the telescope on the contrary, on December 1 and 4 it had in the middle of its head a nucleus of denser light about 2′ in diameter, while the whole head, with its fringe of rarer light, was seen at 8′. On December 8 that nucleus was seen twice as great as Arcturus, if Arcturus too were viewed with the telescope; and on December 24 he saw with the telescope a nucleus of 6′, but the whole head of 16′, scattered into several stars, as he had seen it before as well — though of these only one, or a few lying nearer to one another, could be seen with the naked eye. And so on January 3, viewed with the naked eye, it appeared scarcely equal to stars of the 4th magnitude.

In view of these observations, it seems we should subscribe to our Biancani, who asserts that no comet has been seen at its beginning either greater than the Sun or smaller than the stars of the first magnitude. For Scultetus's statement that the apparent diameter of the comet of the year 1577 was 54′, and so almost twice as great as the diameter of the Sun, is refuted as false by Tycho, an eyewitness (vol. 2, p. 415), and must be attributed either to weakness of the eyes or to a halo which, perhaps poured round the comet, enlarged it optically. Here, however, one must remember that the radius is very deceptive in taking distances and diameters, as Tycho himself admits (vol. 1, pp. 247, 335, 343, 559 and 671; vol. 2, p. 31; and in the *Mechanica*). By what means the diameters of the fixed stars, of the lesser planets, and so also of the heads of comets can and should be observed more certainly and accurately, we have taught sufficiently in Book 7, Section 6, chapters 9, 10 and 11.

Magnitudo Apparens Caudæ vel Barbæ Cometarum.

The Apparent Size of the Tail or Beard of Comets

VII. The tail and beard of comets generally grow for a few days after the first appearance, and decrease at the end. It is already established from the preceding history that the comet seen by Aristotle in the year 341 before Christ

[Margin: Tail 60°.]

occupied a third part of the hemisphere, namely 60°. Equal to it were the comets of the years 1456 and 1460, of which one had a tail occupying two signs, the other more than 50°, on the testimony of Pontano. The tail of the comet of the year 135 before Christ must certainly have been very great; of it Seneca says (*Natural Questions* Book 7, chapter 15): "In the reign of Attalus a comet appeared, modest at first; then it raised itself and spread out and came as far as the equinoctial circle, so that, stretched out immensely, it equalled that region of the sky which is called the Milky Way." Apian found in the comet of the year 1533 a tail of 15°, but in the comet of the year 1538 one of 30° — as great as Mästlin too noted in the comet of the year 1577; granted that Johannes Praetorius made it only 25° long and 2° wide, and Tycho only 22°. But in the comet of the year 1585 Tycho could scarcely make out a tail a span long, and in the comet of the year 1590 one of 10°. No comet, however, was more remarkable for its size and for the variation of its size than that of the year 1618: on the same nights it was seen of different lengths not only by different observers but by the same observer, now short, now longer through the vibration and darting of its rays. But the variety of this and of the preceding comets will be more pleasing if it is brought into the following little table, in which the days and months are set out separately only where the diversity of the observations required it.

The Apparent Length of the Comet's Tail

Years	Observers of the tail	Length of the tail, in degrees
Years before Christ		
341	Aristotle	60
135	From Seneca, *Natural Questions* Book 7, chapter 15	Equal to the Milky Way, and immense
Years of Christ		
1456	Pontano	60
1460	Pontano	50
1533	Peter Apian	15
1538	Peter Apian	30
1539	Peter Apian	very thin
1577	Tycho	22
	Johannes Praetorius	25
	Mästlin	30
1585	Tycho	a span long
1590	Tycho	10

Years	Observers of the tail	Length of the tail, in degrees
1618		
November 26	Millerus	90
29	the Roman Fathers of the Society of Jesus	58; so too at Vienna
29	but Kepler	30
December 1	Father Cysat	22 at least
4	Chiaramonti	21
7	Cysat	57
9	Cysat	75
9	Kepler	70 at least
10	Longomontanus	104, with an enormous vibration
12	the Roman Fathers	60
13	Remus	36
14	Rhodius	50 or 60
14	Biancani	36, but at times greater, in greater darkness of the air
17	Rhodius	30
17	but Cysat	56
18	Chiaramonti	28
19	Rhodius	30
20	Cysat	34
20	but Rhodius	18
22	Remus	7
28	Cysat	35
29	Cysat	35
January 7	Cysat	8
Foot: The tail of the comet of the years 70, 400, 1472 and 1543 was seen to reach down to the Earth.		

Magnitudo Vera Capitis & Caudæ Cometarum.

The True Size of the Head and Tail of Comets

VIII. This size depends on the distance from the Earth both of the comet's head and of the extremity of its tail, and on the apparent size. Moreover, most authors, because of the resemblance of comets to stars, suppose a comet's head to be round, and usually measure only the length of the tail; a few, however, also measure its thickness or solidity, just as if it were a kind of cylinder. How each is done geometrically I shall teach below, in chapters 23 and 24. Meanwhile, for those who pursue learning pure and simple and relish only the marrow, let the various opinions on the size of the head and tail, with their grounds, be set out in the following synopsis.

[Margin: Year of Christ 1472.]

Regiomontanus estimated the comet's diameter at 16 German miles, that is, 64 Italian miles. But Gurcensis estimated it as longer: the head together with the tail at 4000 German miles, the breadth of the head at 26 miles and that of

the tail at 81 German miles — each of which is usually equated with four Italian miles. From this, in chapter 11, number 15, we shall work out the solidity of the whole comet with its tail, relative to the Earth, as 32 to 296,000.

[Margin: Year 1532.]

Johannes Vögelin, setting the distance from the Earth at 8 Earth semidiameters, judged the diameter of the head to be 1¼ German miles, or 5 Italian miles, and the tail 182 German miles long. Hence Carolus Piso said that it is not impossible for comets to equal a mountain of one or two German miles.

[Margin: Year 1577.]

Tycho (vol. 2, pp. 202 and 205) judged that the comet of the year 1577, on November 13, was distant from the centre of the Earth 210 Earth semidiameters as to its head, but 157 Earth semidiameters as to the extremity of its tail; that the apparent diameter of the head was 7′, and the true diameter 368 German miles, in ratio to the Earth's diameter as 3 to 14 but to the Moon's diameter as 1 to 4; that the body, or solidity, of the head was to the solidity of the Earth as 1 to 101⅔, but to the solidity of the Moon as 1 to 2⅓; and finally that the apparent length of the tail was 25°, but its true length 95 or 96 Earth semidiameters, or 81,700 or 82,560 German miles. From this, in chapter 11, number 15, we shall work out the solidity of the whole comet with its tail to the solidity of the Earth as 14 or 33 to 296.

[Margin: Year 1577.]

Scultetus, however, as reported by Tycho (vol. 2, pp. 409 and 415), made the diameter of the head of the comet of the year 1577 124 miles, of which he gives the diameter of the Earth 1718 and that of the Moon 505; and the length of the tail 2623 of the same miles. But he supposed the apparent diameter of the comet and of the Moon to be 54′ — a manifest error indeed — and its distance from the Earth 8047 German miles, and the parallax 5° 22′.

[Margin: Year 1577.]

Snel (*On Comets*, chapter 6) supposed the same comet to be distant from the centre of the Earth 300 Earth semidiameters, and of apparent diameter 7′; and hence its solidity relative to the Earth as 8 to 27, but relative to the air compressed into a single globe as 8 to 1. We shall present his method in chapter 11, number 13.

[Margin: Year 1618.]

Longomontanus, in the Appendix to the *Astronomia* (chapter 10), estimated the head of the comet of the year 1618, on December 27, at 3′ in apparent diameter. From this, and from its distance from the Earth of 240 Earth semidiameters, he works out its true diameter as 175 German miles, in ratio to the Earth's diameter as 377 to 3600, and the solidity of the head to the Earth's globe as 1 to 870, but to the Moon's globe as 1 to 20.

[Margin: Year 1618.]

Father Johann Baptist Cysat, on this same comet (chapter 4), says that on December 9 the head was distant from the centre of the Earth 100 Earth semidiameters, and its apparent diameter 7′, but its true diameter 175 German

miles, its circumference 550 German miles, and its solidity to the Earth's globe as 1 to 950, to the Moon's globe as 1 to 22; and that the length of the tail varied on various days, as in this little table.

Day of December	Earth semidiameters	German miles
1	45	38700
9	137	117820
17	185	159100
29	445	382700

[Margin: Year of Christ 1618.]

But our Fathers at the Roman College, on the hypothesis that the distance of the comet of the year 1618 was at least as great as the least distance of the Moon in Ptolemy, namely 34 Earth semidiameters, and taking the apparent length of the tail as 60° and its breadth as 2′ — as they say it appeared on December 12 — assign to the tail 127499 Italian miles in length and 70 5/7 miles in breadth, and to its cylindrical solidity 490871150 cubic miles. So Chiaramonti also reports (*Anti-Tycho* Book 2, chapter 30), and Cottunius (lecture 33 on *Meteorologica* 1), who adds, however, that by others, who place the comet far below the Moon, it was estimated at only 1300 cubic miles.

Chiaramonti, on the other hand (*Anti-Tycho* Book 2, chapter 30), supposing that on December 4 the tail appeared to him as 21°, the comet's distance from the Earth as 5000 Italian miles, and the breadth of the head as 2′, works out the breadth as 2 9/10 miles, the length as 1834 miles, and the cylindrical solidity of the tail together with the head as 12,838 cubic miles.

Situs Caudæ reſpectu Solis, Veneris, aut Mercurij; & Plaga Cæli, in quam vergere viſa eſt.

The Position of the Tail with Respect to the Sun, Venus or Mercury; and the Region of the Sky toward Which It Has Been Seen to Point

IX. The position of a comet's tail could have been conjectured long ago from the words of Dio about the comet conspicuous during the reign of Macrinus, where he says: "A star stretching its tail from west to east for many nights brought us grave disturbance"; and more clearly from the account of the comet of the year 1472 seen by Regiomontanus, for it is said: "It happened that immediately after sunset the tail faced the east; in the middle of the night, the south; after the middle of the night it faced the west." Even so, the first — as Tycho and Gemma Frisius noted — was in fact Peter Apian, who observed that comets always cast their tail toward the part turned away from the Sun. Consequently, comets that run ahead of the rising Sun in the morning are seen bearded, and their beard, turned toward the west, rises before the head; whereas evening comets, which follow the Sun after it has set, go down head first, before the tail. This, I say, he observed in five comets of his own time, namely those of the years 1531, 1532, 1533, 1538 and 1539, by observing the moments of time

and the place of the head and of the end of the tail in longitude and latitude, the Sun's place at that moment being already known from the Alphonsine Tables. Fracastoro observed the very same thing at the same time in three comets, of the years 1531, 1532 and 1533, and Gemma Frisius in 8 comets from the year 1532 on, which he records in his treatise on the astronomical radius, chapter 19, and in the *Astrolabe*, chapter 82; and his son Cornelius Gemma subscribed to him (*De divinis naturae characterismis* Book 1, chapter 6). Of the same opinion — that the tail is turned away from the Sun — were, later, Cardano (*De subtilitate* Book 4), Scaliger (exercise 79 against Cardano), Mästlin on the comet of the year 1580, Cysat on the comet of the year 1618 (chapter 4, proposition 2), Biancani (*Sphaera* Book 16, chapter 12), Cottunius (lecture 33 on *Meteorologica* 1, p. 307), Froidmont (*Meteorology* Book 3, chapter 4, article 1), Cabeo [Niccolò Cabeo] (*Meteorologica* Book 1, text 37, question 7), Longomontanus (Appendix to the *Astronomia Danica*, chapter 6), and Tycho in five comets that he observed.

Thus in the *Epistolae*, p. 173, on the occasion of the comet of the year 1590, Tycho says: "It directs its tail, thin but about 10° long, exactly opposite the Sun and toward the zenith"; and on p. 177 he says of the same comet: "It also directed its tail, very thin and rather sparse, exactly toward the parts opposite the Sun, for as long as it could be made out";

[Margin: Tycho's genuine opinion on the position of the tail.]

and a little below: "From the extensions of the tail which we have indicated, it is evidently gathered that it was everywhere turned exactly away from the Sun, provided only that the positions of the Sun are compared with the places of the comet's head which they occupied at the same times." He teaches the very same expressly in vol. 1 of the *Progymnasmata*, p. 651. And although in vol. 2, chapter 7, he judged that the tail of the comet of the year 1577 was directed not exactly toward the part opposite the Sun but toward the part opposite Venus, in chapter 9, p. 204, he does not rest content with this because of the weak light of Venus. For that reason, in a letter of the year 1589 to Rothmann [Christoph Rothmann], p. 142, he says: "As regards the drawing out of the tail, I showed that in the comet of the year 1577 it constantly faced the star of Venus — although afterwards I would rather incline toward the side of the Sun, because of some hidden optical reason, since it can scarcely be believed that the star of Venus emits such strong rays."

[Margin: Venus powerless to produce a comet's tail.]

And although in the same place he grants to Giovanni Battista Benedetti that the part of the Moon that is dark in itself is nonetheless lit somewhat by Venus, he adds: "Yet to tell the truth, this reasoning does not satisfy me so far that I would dare attribute to Venus so great a power of its rays as could have produced the tail of a comet." Resta (*Meteorology* Book 1, tract 1, chapter 1) therefore wrongly attributes the opposite view to Tycho.

So, whether because of a hidden optical cause, as Tycho suspects, or because the comet's head is not exactly spherical, as Cabeo thinks in this controversy, it happens that the tail sometimes seems to recede a little from the point opposite

the Sun — so that Kepler (*On Comets*, p. 54) said: "It is customary for comets that their tails deflect somewhat from the point opposite the Sun." The same must be said of the comet of the year 1618, whose tail Kepler and Schickard saw on December 9 stretched out opposite Mercury rather than opposite the Sun, and Snel saw it stretched away from Venus, although they admit that it afterwards directed itself opposite the Sun. So too with Mästlin: the tail of the comets of the years 1577 and 1580 at first deflected from opposition with the Sun, but afterwards was ranged in the same great circle with the Sun and the comet's head, and turned directly away from the Sun. Johannes Hommel too observed that the tail of the comet of the year 1556 did not appear turned exactly away from the Sun for as long as the comet was less than a quadrant of the circle away from the Sun, but afterwards stretched directly toward the region opposite the Sun. As for the Landgrave of Hesse appearing, in the year 1558, to see the tail deflect 10° from the point opposite the Sun, Tycho attributes that to the deceptiveness of the torquetum, since Gemma's son observed the same tail in the same year stretching toward the part turned away from the Sun. From all this it is gathered that the deflection from that direction arises accidentally from the position of the Sun, the comet and the observer — other causes perhaps contributing that lie hidden in the matter and figure of the head, or in the distance and transverse motion of the comet — but that of itself the tail is always carried toward the region opposite the Sun.

[Margin: The turning of comets' tails away from the Sun confirmed.]

Finally, many phenomena indicated by Tycho (*Epistolae*, p. 143) confirm this very point. For comets that were seen to stretch toward the point opposite the Sun, such as the comet of the year 1585, showed no tail or a very small one, since the tail lay hidden behind them and was concealed from us, who stood between the Sun and the comet's head; whereas those that were nearer the Sun, such as the comets of the years 1580 and 1582, were seen to put out their beard more and more. Hence it is that morning and eastern comets are bearded, with their rays extended toward the west, as in the year 1363, while evening and western comets are tailed, as in the year 1577, with their rays turned away from the west toward the east; and that, when the Sun is very far depressed below the horizon, the tail of comets seems raised up toward the zenith, and the beard of a morning comet rises before its head, whereas the head of an evening comet sets first.

[Margin: The regions of the world toward which a comet's tail points.]

Granted, by reason of the declination of both the comet and the Sun the tail has commonly been seen to point toward other regions and to portend something to them. Thus the comet of the years 726, 1314 and 1347 is said to have stretched its tail toward the north; that of the year 1211 over Russia; that of the year 1533 toward Africa; that of the year 1558 toward Rome and Spain; and that of the year 1532 more toward the south each day, and then to have bent toward the north. But the tails of those comets that were seen to slide down to

the earth so as to lick up and swallow streams were meteorological kindlings similar to comets, but not properly comets.

Plaga Cœli in qua Orti, & in quam delati, & in qua extincti ſunt Cometæ.

The Region of the Sky in Which Comets Arose, toward Which They Were Carried, and in Which They Were Extinguished

[Margin: The prerogative of the northern hemisphere over the southern.]

X. I have said elsewhere that the northern hemisphere is furnished with more fixed stars than the southern; that the Sun spends about 8 days longer in it than in the southern; and that it holds more habitable land, and the greater and better part of the human race. What wonder, then, if the greater — indeed the greatest — part of the comets observed was on this side of the equator, for either the whole or the greater part of its duration? For it was fitting that more things should be signified and portended from heaven to these regions — granted that Aristotle (*Meteorologica* Book 1, chapter 3), though citing no history, says: "And toward the south many have arisen"; for him, however, it is enough that they arose beyond the equator. But Seneca (Book 7, chapter 11) says that they arise

[Margin: Years of Christ.]

most frequently around the north.

As for the points from which and to which comets have made their passage, there is scarcely one, so far as is yet established, that has crossed directly from one of the four cardinal points of the sky to the opposite cardinal point; all of them have taken an oblique path, with an unexpected variety, as a few examples will make clear. I said "scarcely", because perhaps the comet of the year 1200, observed by Haly, travelled from east to west by a straighter route, since he says that it was carried by the motion of the primum mobile from east toward west, from the 15th degree of Scorpio to the 15th degree of Virgo.

[Margin: Comets from east toward west or north.]

From east toward west, but obliquely, the comets of the years 392, 405, 1471, 1475, 1532, 1533, 1539, 1556 and 1618 swam across the sky. For the comet of the year 392, see what I reported under that year from Nicephorus; and of the comet of the year 405 Claudian sang thus in *De bello Getico*:

[Margin: Path of the comet of the year 405.]

> And the comet, never seen in heaven with impunity, which, first brought forth from the rosy rising of Phoebus, where old Cepheus glitters with his star-bearing wife, then, driven little by little from the Lycaonian Bear, defiled with its wandering hair the stars of the Getic Wain, until, dying, it faded into a meagre fire.

So it was heading from the summer rising point, or short of it, through the Bears toward the west. Of the comet of the year 1471, Pontano says at the end of the *Centiloquium*: "In these recent years another comet was seen, at first with a slender head and a very short coma; soon, having grown to a marvellous size, it began to turn aside from the east toward the north, etc., until it reached the

Bears themselves, from where it bent its path toward the west." I have reported the rest of his words in chapter 3, under the year 1471.

[Margin: Path of the comet of the year 1471.]

[Margin: Path of the comet of the year 1472.]

Next, the comet of the year 1475, or rather 1472, observed by Regiomontanus, began its path from nearly the equinoctial rising point, near Spica, and went on through the legs of Boötes, between the two poles of the zodiac and of the world, through the feet of Cepheus, the breast of Cassiopeia and the belly of Andromeda, as far as the northern Fish; and a little beyond the equinoctial setting point it set heliacally with the stars of Cetus. The comets of the years 1532 and 1539 too, if you weigh Apian's words properly, crept out from the east with southern latitude and, having then acquired northern latitude, went in an oblique course roughly toward the summer setting point. But the comet of the year 1533 went from the Goat [Hircus] through Cassiopeia to the north and against the order of the signs [in praecedentia], as Gemma Frisius testifies.

[Margin: Path of the comet of the year 1533.]

[Margin: Path of the comet of the year 1556.]

Again, the comet of the year 1556, very like that of Regiomontanus, rising nearly at Spica, or a little beyond the equinoctial rising point, ascended through the knee of Boötes and Arcturus toward the pole of the zodiac, then toward the pole of the world; and descending from there through Andromeda, it turned to the sign of Pisces, where it was extinguished, as Johannes Hommel saw. To this belongs the comet which, as Vicomercatus relates without indicating the year, rose from Vindemiatrix, was carried to the north and expired among the coils of Draco. Finally, the comet of the year 1618, seen in the morning a little beyond the equinoctial rising point, set out from Libra through Boötes to the tail of Ursa Minor — in such a way, however, that, as Cysat, Snel and others observed, it described at the end something like a spiral around the pole of the world, or a circle, remaining within the circle of perpetual apparition from December 20 to January 23 and never descending below the horizon of Ingolstadt. The path of these comets, then, was from east toward west and north.

[Margin: Path of the comet of the year 1618.]

[Margin: Comets from the west toward the east.]

[Margin: Path of the comets of the year 373 before Christ and 1577 after Christ.]

Others, by contrast, held their course from the west toward the east, some turning toward the south, others toward the north. For the great comet seen and described by Aristotle in the year 373 before Christ began from the equinoctial setting point and went from there as far as the belt of Orion, where it was also extinguished. And in the year 1577 the evening comet, first seen by Tycho a little above the head of the starry Sagittarius, beginning on November 9 or 8 beyond the equinoctial setting point, crossed the equator on the 22nd by the left hand of Antinous; passing the tail of Delphinus, it went through the nostrils

of Equuleus and, leaving the head of Pegasus to the south, made straight for Scheat of Pegasus, until it faded away between Scheat and two small stars that glitter in the breast of Pegasus; nor did it ever reach the meridian.

[Margin: Path of the comet of the year 1585.]

Again, the comet of the year 1585, observed by the Landgrave of Hesse and by Tycho, rising between the equinoctial and the summer setting points, stood about 5° from the cord of Pisces toward the east; then, passing through the whole constellation of Aries, it emerged by the back of Taurus toward the north and ceased to be seen not far from the Pleiades, which it surpassed in northern latitude but did not reach in longitude.

[Margin: Years of Christ.]

[Margin: Path of the comet of the year 1590.]

Not much later, namely in the year 1590, a comet began to be seen in the evening in the western region of the sky, at the head of the northern Fish, near the summer setting point; it vanished toward the feet of Andromeda, at the feet of Cassiopeia, having acquired a greater northern latitude and declination day by day.

[Margin: Path of the comet of the year 1607.]

Lastly, the comet of the year 1607 began to be seen on September 26 between the north and the summer setting point, with a northern declination of nearly 50°; then, continuing its motion in the order of the signs [in consequentia], it crossed the equator on October 10 and vanished at the leading shin of Serpentarius, having attained a southern declination of 12° and more.

[Margin: Path of the comets of the years 1513 and 1450.]

The comet of the year 1513, which flew from the end of Cancer to the end of Virgo, also seems to belong with these; but the comet of the year 1450 plainly belongs here.

[Margin: Comets that arose from the north.]

From the north itself, I note, one was carried toward the east, namely in the year 54 of Christ, under Claudius; another toward the west, in the year 64, under Nero; a third, in the year 1313, as also in the year 1531, toward the south. On the first two the evidence is Seneca (*Naturales quaestiones* Book 7, chapter 21), who says: "This one, which we saw in the most happy principate of Nero, went round in the opposite direction to that of Claudius: for that one, rising from the north toward the zenith, made for the east, growing ever dimmer; this one began from the same part but, heading toward the west, bent toward the south and there withdrew itself from our eyes."

[Margin: Path of the comets of the years 54, 64, 1313 and 1531.]

Of the third, Johannes Praetorius testifies, saying that it ran forward from north toward south and followed the motion of Mars; for that reason I would not believe that it went quite straight to the south, but that it deflected somewhat to the sides. Finally, many others are said to have risen from the north: above all the one that appeared in the year 44 before Christ, after Julius Caesar was killed, and that of the years 1500, 1506 and of the year 1533. But since the carelessness

of the old observers forbids us to say more about the path of comets, let us pass to other conditions. From what has been said, however, it is clear that Pliny the Elder (Book 2, chapter 25) spoke too confidently: "almost all under the north itself, in no fixed part of it, but most of all in the white region that has received the name of the Milky circle." This notable frequency of comets in the Milky Way has the support of Tycho (vol. 1 of the *Progymnasmata*) and of Aristotle (*Meteorologica* Book 1, chapter 7), who says: "Most of all it gathers in the region of the Milk."

De Motu Cometarum in Conſequentia vel in Præcedentia, & de Motu Primi Mobilis.

On the Motion of Comets in the Order of the Signs or against It, and on the Motion of the Primum Mobile

XI. It is clear from what was said in the preceding paragraph that some comets moved in the order of the signs, because day by day they appeared more easterly than the fixed stars, and others against the order of the signs, because they ran ahead toward the west with a swifter motion than the motion of the primum mobile carries. To have them gathered together: the comets of the year 373 before Christ, and after Christ of the years 54, 1450, 1513, 1532, 1533, 1577, 1585, 1590 and 1607, advanced in the order of the signs; the comets of the years 64, 405, 1200, 1471, 1472, 1539, 1556 and 1618 moved against it. Even so, it must be added that some comets, from being retrograde and moving against the order of the signs, became direct and began to go in the order of the signs, as the comet of the year 1556 did; and that some, from being direct, became retrograde or stationary, as the comets of the years 1569 and 1582 did.

[Margin: The daily motion of comets, or that of the primum mobile.]

The daily motion, which they call that of the primum mobile, was shown to be included in the motions already mentioned by the fact that most of the morning comets returned at dawn or before dawn, and were not seen at sunset or as the night came on in the place where they had been seen in the morning — as they would have had to be if they had stayed in the same place, or had moved only by their own motion — but only as daybreak approached. Just so, most of the evening comets appeared after sunset near the western horizon in such a way that, notwithstanding their effort in the order of the signs, some nevertheless went down below the western horizon before midnight, others at least before sunrise. Among them, however, some rose with a true evening rising and set with a morning setting, as the comet of the year 119; some were seen all night, or even never set at all, being enclosed within the circle of perpetual apparition, as were the comets of the years 1513, 1533 and 1556, and that of the year 1618, but only from December 20 on.

[Margin: Comets seen all night or never setting.]

But whether the comet that was seen hanging above Rome in the year 13 before Christ, and the one that in the year

[Margin: Whether comets are moved or motionless.]

of Christ 70 hung over Jerusalem for a whole year, were driven by any motion, common or proper, is unknown to us; nor is that saying of Pliny the Elder (Book 2, chapter 25), "Some move in the manner of the wandering stars, others stay motionless", necessarily to be understood of both motions.

De Arcu ſeu Via Cometarum, eiuſq. quantitate, & de Diſtantia à Sole apparenti.

On the Arc or Path of Comets and Its Size, and on the Apparent Distance from the Sun

XII. There is a great controversy, to be argued further on, in the last chapter, whether the path of all comets, or of many, runs along a straight line, as Kepler, Guiduccius [Mario Guiducci], Galileo, Cysat, Glorioso and Froidmont hold, or along a circular one, as most do; and, if along a curved one, whether in the plane of a great or greatest circle in the sphere, as Regiomontanus, Tycho, Longomontanus and many of the more recent writers contend. This cannot be adequately explained from the simple history of observations that we follow here, without trigonometry and diagrams. Yet even if we were to grant that the path runs along a straight line projected upward or downward, that apparent motion, at so great a distance from the Earth, is still similar to the motion of the planets, which we judge to take place along an arc. This apparent arc, then, was shorter in some comets and longer in others. Its size is gathered from the daily proper motions in the comet's own course, added together into one; or from the end-point at which the motion began to the end-point at which it ceased, the comet's path being marked on the surface of a globe and taken off with compasses; or else by the observer's bare estimate by eye — in the way that Seneca (*Naturales quaestiones* Book 7, chapter 29) says that the comet seen under Nero ran through half the sky in six months. For the size, then, consult the following small table, for a few comets; for the rest it has not yet been possible to determine it.

Years of Christ	Arc	Days
1472	about 170°	✠
1533	about 50°	30
1556	about 180°	✠
1577	91° 46′	79
1585	39° 21′	28
1590	42° 10′	11
1607	97° 0′	31
1618	107° 30′	28

[Margin: Distance from the Sun.]

Add, however, from Wendelinus [Govaert Wendelen] and Froidmont, that comets sometimes twist themselves like a spiral and deflect from the segment of a great circle, as did those seen toward the end in the year 1618. Apian observed the apparent distance of the head from the Sun in the comet of the year 1533 as

88°; it is false, therefore, that comets do not stand more than 30° or 60° from the Sun.

De Velocitate Cometarum.

On the Speed of Comets

XIII. Three questions can be asked about this speed: whether it was uniform or non-uniform, that is, unequal; and, if unequal, whether it was greater at the beginning, in the middle or at the end; and finally how great it was. To the first the answer is that, if one may judge of the rest from the few that are established, it was unequal — and rightly so, since not even to the planets is equality of apparent motion granted.

[Margin: Comets faster at the beginning.]

To the second I reply that some comets were fast from the start and gradually became slower: such were the comets of the years 1577, 1585, 1590, 1607 and 1618, although Kepler thought this last slower at the beginning.

[Margin: In the middle.]

Others, again, were faster in the middle than at the extremes, such as the comet observed in the year 1472 by Regiomontanus, the comet of the year 1531 observed by Apian, and the comet of the year 1556 seen by Hommel.

[Margin: At the end.]

Finally, some were faster at the end, an example of which we have in the comet of the year 1471 seen by Pontano, who, besides the words from the *Centiloquium* reported by us above under that year, sings of it thus in the *Meteora*:

> For I remember how once, fallen from the Icarian star, it bore its squalid hair before it, and with slow course bent beneath the cold orb of piercing Boreas; how from there it bore its head before it again, and with a favouring course turned to the west and pressed on with slackened reins, until it felt the fierce horns of Agenor's Bull.

The measure of the greatest speed in the comets already mentioned (for of the others it is not clear) is set out in the following small table.

Year	Greatest daily motion
1471	30° 0′
1472	40° 0′
1531	6° 0′
1556	15° 0′
1577	6° 2′
1585	2° 18′
1590	7° 34′
1607	9° 40′
1618	3° 20′

De Loco Cometarum, ſeu Diſtantia à Terra.

On the Place of Comets, or Their Distance from the Earth

XIV. We shall have to discuss this matter at greater length below, from chapter 15 to chapter 23, in paragraph 5 of which I shall present a table of the altitudes and parallaxes that some have assigned to comets. For now I shall list only the bare names of the authors, divided into three classes, as can be gathered from the passages of chapters 6 and 23 to be reviewed later.

[Margin: First class: placing all comets below the Moon.]

The first class is of those who asserted that all comets, or comets without qualification, were below the Moon. Such were Aristotle with the Peripatetics, Xenophanes, Metrodorus, Strato, Heraclides, Epigenes, Bohetus, and many of the Stoics; Regiomontanus, Pontano, Johannes Vögelin; Fracastoro; Peter Apian, Iacobus Milichius [Jakob Milich], Franciscus Iunctinus [Francesco Giuntini], Julius Caesar Scaliger; Bernardinus Telesius [Bernardino Telesio], Io. Paduanus; the Coimbra commentators [Conimbricenses]; Petrus Surdus; Chiaramonti; Cottunius, Piso; Laurentinus; Raphaël Auersa [Raffaele Aversa]; Bartholomaeus Mastrius [Bartolomeo Mastri]; Bonauentura Bellutus [Bonaventura Belluto]; Andreas Nolthius; Georgius Buschius, Theodorus Graminaeus, Nicolaus Vuinchlerus, Conradus Dasypodius [Conrad Dasypodius], Valentin Steinmetz, Georgius Henischius [Georg Henisch], Nicolaus Bazelius, Ioannes Huuernius, Thomas Erastus. But that Hájek is wrongly counted among these by Aversa and Mastri I shall show in chapter 6, scholium 5.

[Margin: Second class: placing all comets above the Moon.]

The second class places all comets above the Moon. In this class were the Chaldeans, the Pythagoreans in Italy, Democritus, Anaxagoras, Zeno, Artemidorus, Apollonius of Myndus; Seneca, Cardano, Tycho, Longomontanus, Galileo, Mario Guiducci, Io. Echius, Biancani, Santucci, Isaach Habrechtus [Isaac Habrecht]; Libavius, Iordanus Brunus [Giordano Bruno]; Oddus Patauinus; Cornelius Frangipanius, Marcellus Squarcialupus [Marcello Squarcialupi]; Ambrosius Rhodius, Thomas Fienus [Thomas Fyens]; Christoph Rothmann, once he yielded to Tycho; Erycius Puteanus; Snel, Ludouicus Columbus [Ludovico delle Colombe]; Aresius [Paolo Aresi], and Renatus de Cartes [Descartes]. That Fracastoro, Abū Maʿshar, Nicephorus, Proclus and Piso are wrongly listed among these I shall show in chapter 6, scholia 3, 4 and 5.

The third class places either some comets above the Moon and some below it, or the same ones now below and now above the Moon. In it were Mästlin, Cornelius Gemma; Hájek, Elisaeus Roeslinus [Helisaeus Röslin]; Bartholomaeus Scultetus [Bartholomäus Scultetus]; Johannes Praetorius, Philippus Apianus [Philipp Apian]; Elias Camerarius, Gulielmus Hirsaugensis [William of Hirsau], Glorioso; Clavius; Franciscus Maurolycus [Francesco Maurolico]; Adamus Tannerus [Adam Tanner]; Froidmont; Fortunius Licetus [Fortunio Liceti]; Cabeo; Rod. Arriaga [Rodrigo de Arriaga]; Franciscus Ouiedus [Francisco de Oviedo]; Resta, Balthasar Tellez [Baltasar Téllez]. But in Hájek and Scultetus it is the later, not the earlier, opinion that must be considered. That one and

the same comet was below the Moon on one day and above it on another is admitted by Kepler and Ioannes Baptista Cysatus [Johann Baptist Cysat] — though mostly above the Moon.

Figura, Color, Odor, Diaphaneitas Cometarum.

The Shape, Colour, Odour and Transparency of Comets

[Margin: Comets with short coma, and round.]

XV. The shape of comets is varied. From our history, however, some were seen round, or hairy or comate all round, or with short hair — those that some call *circaei* — such as the comet of the year 44 before Christ, and those of the years 1200, 1521, 1556 and 1585 after Christ, although this last had for two days a ray a span long.

[Margin: Acontias.]

[Margin: Lance.]

[Margin: Spit.]

[Margin: Rhomphaea.]

Some had the shape of a javelin or spear and were called *acontiae*, as in the years 348 and 76 before Christ and 1533 after Christ. Little unlike these are those that imitated either a lance, as in the year 557, or a spit, as in the years 1391 and 1558. Others had the form of a rhomphaea, as in the year 1450; others the image of a sword, a dagger or a blade, as in the years 392, 603, 662, 1526 and 1532 — those which we said are called *xiphiae*.

[Margin: Xiphias.]

[Margin: Pogonias.]

[Margin: Tailed.]

Many, as long as they preceded the Sun as morning comets, were seen bearded, or *pogoniae*, as in the years 1531 and 1618. Many evening comets that followed the Sun were tailed, as in the years 1211, 1539, 1577 and 1607, and that very comet of the year 1531 which, from morning and bearded, turned out evening and tailed. But reread what was said in chapter 2.

[Margin: Bright and shining.]

[Margin: Fiery or red.]

[Margin: Yellow.]

[Margin: Silver.]

[Margin: Leaden.]

[Margin: Black or dusky.]

The colour mixed with the light, as observed in comets, was sometimes bright and shining like the Sun, as in the year 119 before Christ and in 1264 and 1521 after Christ; sometimes fiery, ruddy and glowing red, as in the year 146 before Christ and in 662, 1526 and 1556 after Christ. In others it was yellow or golden — those they call *chryseoi* — as in the year 1533, which, however, afterwards degenerated into a blackish colour; in others silver — those they call *argenticomi* — as in the year 1200; in some it was leaden and livid, as in the year 1607. Some turned blackish, or were seen dusky with a cloudy light rather than

shining, as in the years 1477 and 1585; and finally some became of every hue and variegated in succession, as in the year 1513.

[Margin: Odour.]

A sulphurous odour could be perceived in a single comet only, that of the year 396.

[Margin: Transparency.]

Transparency seems to be attributed to all comets by Seneca, who says in chapter 26: "We send our sight through comets." Piso subscribes to this (*Speculum*, chapter 18), adducing the experiments of Father Philippus Plumeiesi [P. Philippi Plumeieſi]; and Tycho experienced the same in the comet of the year 1577, for he saw fixed stars through its tail, as he testifies in the *Epistolae*, p. 143. But transparency fits at least all those comets whose tail or beard is nothing other than a propagation of the Sun's rays through the head, shining through it as through a glass sphere, in line with what is to be said below. Seneca, however, speaks from the opinion of others, for he adds: "First, if this happens, it happens not in that part where the star itself is of dense and solid fire, but where a thin splendour runs out and is scattered into hair: you see through the intervals between the fires, not through the fires themselves."

So much for the accidents of comets that are manifest to sense and experience, except their altitude and the shape of their path, from which their place, matter and efficient cause will have to be gathered hereafter.

SCHOLIA.

Scholia

[Margin: Pliny's error over the text of Aristotle.]

I. That saying of Aristotle (*Meteorologica* Book 1, chapter 3): "Moreover, all the comets that have been seen in our times disappeared without setting, consumed little by little in their place above the horizon, so that the body of no single star was left behind them, nor of several." Pliny the Elder (Book 2, chapter 25) rendered this passage very badly, as Milichius pointed out against him; for Pliny's words are: "But a comet is never in the part of the sky that is about to set."

[Margin: Comets extinguished before setting in the time of Aristotle.]

For Aristotle did not deny that comets, even of his own age, descended below the horizon or appeared in the west. Rather, he argues against Democritus, who said that comets are an aggregate of stars and are dissolved into them, and he destroys that universal claim by the example of the comets of his own time, which, by night, before they reached the horizon, were extinguished, leaving behind them no trace of one star or of several. So Glorioso (*De cometis* Book 1, p. 10) and Cottunius (lecture 1 on *Meteorologica* 1) rightly understand and explain this text — although he also embraces the other exposition, concerning the heliacal setting, which was the interpretation of Kepler (*De cometis*, p. 96), who thought that Aristotle was arguing against the Pythagoreans, denying that

comets usually go out, and holding instead that on approaching the Sun they are hidden under its rays.

[Margin: The interpretation of Kepler, Apian, Praetorius and Tycho is foreign to the text.]

But this interpretation, though it does no harm to Aristotle's argument, is nevertheless foreign to the text. For it is clear from the text that he is arguing against Democritus, and the Greek expression signifies not an occultation or heliacal setting, which is usually called [Greek: κρύψις (krypsis, "concealment")], but a descent or sinking below the horizon; for the words of the text are [Greek: ἄνε[?] δύσεως ἠφανίσθησαν (ane[?] dyseōs ēphanisthēsan, "they disappeared without setting")], that is, "they disappeared without a descent". Apian, moreover (*Astronomicum Caesareum* part 2, chapter 1), says of the comet of the year 1531 that when it was nearer than 4° to the western horizon its flame seemed to go out. In this way he himself and Johannes Praetorius understand Pliny's words: not that comets really go out and are afterwards reborn, but that, because their light is less compact and less vivid than in other stars, they cease to appear before they set. In this sense Tycho too understands Pliny (*Progymnasmata* vol. 1, p. 400, and vol. 2, p. 450). But this happens, other things being equal, in any position of the horizon, if comets and the smaller stars are only a little way from it. Let us admit, therefore, that Pliny erred through misunderstanding Aristotle — no differently from the man who, as reported by Glorioso, said that Aristotle spoke of comets enclosed within the circle of perpetual apparition and therefore never setting: certainly a careless and crass opinion. For Aristotle relates that a certain comet of his time, setting out from the equinoctial setting point, went as far as the belt of Orion and was extinguished there; yet the whole of Orion, let alone its belt, was, with respect to Greece, outside the circle of perpetual apparition.

II. It is certain that the star of Venus near the horizon has quite often appeared among vapours so enlarged by rays, all round or toward one side, that many have believed it a hairy star, or comet. And — not to stray too far from my own memory — Claudius Achillinus [Claudio Achillini], formerly public professor of jurisprudence at Parma, came running to me at the prompting of some astrologer or other, announcing that a comet was to be seen in the evening from the tower of the Franciscan Fathers.

[Margin: Venus taken for a comet.]

I replied that it was the evening Venus, and so it was. The very same thing happened here at Bologna, so that three times there was doubt about a comet, when I asserted — and truly — that it was Venus. But whether that star of which Claudian sang in *On the Fourth Consulship of Honorius* was Venus, or a comet, or some new star —

[Margin: A star or a comet in the time of Honorius.]

> A star was seen even at midday, while the peoples marvelled, a bold star, in no way to be doubted, nor faint with blunted hair, but as great as Boötes is reckoned by night —

is quite uncertain; for we have more than once seen Venus at midday here at Bologna, and Father Grimaldi has shown it to many. Tycho, however (*Progymnasmata* vol. 1, p. 543), suspects that Claudian's portent is to be referred to the year and birthday of Honorius, which was the year 388 or 389, in which, as Cuspinianus [Johannes Cuspinian] reports, a certain star was seen to rise from the north around cockcrow, to shine like Lucifer, and to disappear within three weeks. But how does Tycho reconcile cockcrow with midday?

CAPVT V.

Chapter 5

De Prognoſticis, & Cauſa Finali Cometarum.

On the Prognostics and the Final Cause of Comets

I. A most widely received opinion has sat in the minds of mortals from every age: that comets are certain funeral torches, certain secret thunderbolts, and lamps of death rather than of life; and that they never appear without portending, with most certain terror — like the threatening eyes of divine vengeance and the tongues of an angry deity — the death of princes, plague among peoples, famine, earthquakes or dreadful storms. Very few are excepted, and even these, though they seemed favourable to someone, were equally unfavourable and hostile to others. To begin, then, with the prince of the Peripatetics: he laid it down thus in *Meteorologica* Book 1, chapter 7: "That a comet is of fiery consistence is shown by the fact that many such comets, when they come about, signify winds and droughts."

[Margin: The opinion of Aristotle and of Seneca.]

And Seneca (*Naturales quaestiones* Book 7, chapter 28) thinks that comets are at once natural signs and causes of storms, even though he thinks them stars; for he says: "Ari- stotle says that comets signify a storm, and an excess of winds and rains. Why, then, do you not judge it to be a star, since it announces the future? For it is not a sign of storm in the way that these are signs of coming rain:

Oil sparks, and rotten fungi thicken,

or the way it is a sign of a raging sea if

Sea-coots play on dry land, and the heron leaves The familiar marshes and flies above the high cloud.

It is a sign rather in the way the equinox is a sign of the year turning to heat and cold; or in the way of what the Chaldaeans chant — what a star sets down as sad or happy for those being born. To show you that this is so: a comet, once risen, does not threaten winds and rains at once, as Aristotle says, but makes the whole year suspect. From this it is plain that it did not draw from close at hand the signs it would give for what is close at hand, but holds them stored up and compressed by the laws of the universe."

In chapter 17 Seneca had assumed that the greatest part of comets labour under the infamy of misfortunes, saying: "Nor this one, which appeared under

Nero Caesar and took the infamy away from comets." Why? Because in chapter 21 he adds: "For this one, which we saw in the most joyful principate of Nero, showed itself to be watched for six months." Most joyful, really? Was this flattery not unworthy of Stoic gravity? Or had Nero not yet begun to spew the poison of his cruelty on others, so that Seneca might fear for himself? So not even this comet should have been made an exception. Cicero, *On the Nature of the Gods* 2: "Both by torches seen in the sky and by those stars which the Greeks call comets and our people *cincinnatae* [curly-haired], which recently, in the Octavian war, were the forewarners of great calamities."

[Margin: Pliny's opinion.]

Let us now hear Pliny the Elder, Book 2, chapter 25. He first says: "But the comet is for the most part a terrifying star, and not lightly expiated, as in the civil disturbance when Octavius was consul, and again in the war of Pompey and Caesar; in our own age, about the time of the poisoning by which Claudius Caesar left the empire to Domitius Nero, and then during Nero's principate, almost continuous and savage." He calls the comet continuous because it shone for six months, and savage from its effects, from Nero's savagery. It was therefore not most joyful, nor did it take the infamy away from comets — if it is an infamy to serve divine Providence. Pliny goes on to say that for the kind of portent it matters in which direction the comet darts its rays, or from which star it receives its powers, or what likeness it displays: if it has the appearance of flutes, it portends something for the art of music; if it is triangular, for talents and learning. He adds, however, that in one place in the whole world a comet is worshipped as a favourable star, because it signified both the soul of Caesar received into heaven and the monarchy of Augustus, as I reported in chapter 3 at the year 44 before Christ; and so he concludes: "And if we confess the truth, it was salutary to the earth."

[Margin: Nero's terror at comets.]

But I return to Nero, who, by the very fact that he feared the comet, made it something for others to fear. For Tacitus, in *Annals* 14, after recording certain prodigies seen in the consulship of Nero (for the fourth time) and Cornelius Cossus, adds: "Among them a star also shone out, of which the common opinion is that it portends a change for kingdoms"; and a little below: "the common people everywhere, as if he were already driven out" — namely Nero — "were asking who would be chosen." But since many comets arose throughout his principate, he used to expiate them with illustrious blood, to turn the disaster away from his own head, taught by the astrologer Babilus, as Suetonius relates in his *Nero*, chapter 36. And Tacitus again, in *Annals* 15, says: "At the end of the year prodigies are spread abroad: a force of thunderbolts more frequent than ever before, and a comet star, always expiated by Nero with illustrious blood."

[Margin: The opinion of Tacitus and Suetonius.]

Tacitus, it is true, attributes the fear of the divine power to common opinion, and so does Suetonius in his *Nero*, saying: "A hairy star, which is commonly thought to portend destruction to the highest powers." Yet in his *Claudius*,

chapter 46, Suetonius speaks outright, as if giving his own opinion: "The chief presages of his death were the rising of a hairy star, which they call a comet."

[Margin: Vespasian mocks the terror of a comet.]

Vespasian, milder and wittier, replied when a comet was pointed out to him that it concerned the king of the Persians, whose hair was more flowing, as Aurelius Victor relates; or the king of the Parthians, because "he," he said, "is long-haired, and I am bald," as Dio relates.

II. We have heard some philosophers and historians; now let us lend an ear to the poets.

[Margin: Aratus.]

Of these the most ancient, Aratus, rendered into Latin, sang thus:

A dry season often brings dreadful comets.

And elsewhere:

That a happy and golden year may fall to one who prays for it, Let no hairy star ever be seen in the air; Let not one comet shine, nor two, nor more; For a dry season breeds many comets.

[Margin: Claudian.]

For there was a proverb among the Greeks, Οὐδεῖς κομήτης ὅστις οὐ κακὸν φέρῃ [Greek: "no comet that does not bring evil"]: "No comet was ever seen that did not bring evil." With it agrees that line of Claudian:

And never was a comet seen on earth with impunity.

Manilius alludes to this in the verse:

Never has the aether blazed with idle fires.

[Margin: Virgil.]

But the prince of poets had gone before, in *Georgics* I. Among the other portents that followed the murder of Caesar, when civil war was about to rage again on the fields of Philippi, he counts this one too:

Never did more lightning fall from a clear sky, Nor did dire comets so often blaze: So Philippi saw Roman ranks clash again With equal standards, and the gods did not think it unworthy That Aemathia and the broad plains of Hemus should twice grow fat with our blood.

[Margin: Lucan.]

Lucan too, in Book I, sang this of the same portents before the civil war:

Dark nights saw unknown stars, The pole burning with flames, and torches flying Slantwise through the void of the sky, and the hair of the dreaded Star, the comet that changes kingdoms on earth.

[Margin: Silius.]

No differently Silius Italicus, first in Book I of the *Punica*:

As when a comet with flame-bearing hair terrifies fierce kingdoms, Scattering blood-red fire, the torch vomits dark, reddening Rays from the sky, and the star, flashing with savage light, Sparkles and threatens the earth with the worst.

Then in Book 8:

..... Not one comet with flashing hair, Overthrower of kingdoms, has glowed red with death.

[Margin: Tibullus.]

And before him Tibullus, Book 2, elegy 5, had called comets

..... evil signs of war.

Valerius Flaccus, *Argonautica* 6:

..... and comets, summoned by angry Jove, fatal to unjust kingdoms.

[Margin: Pontano.]

Nor is Pontano unworthy of their company: in his *Meteora* he gathers many arguments on this subject in this elegant poem:

Comets will also give you sure signs of winds. They threaten, too, the commotions of war and fierce arms, And the slaughter of great peoples, and the funerals of kings. As when, whether toward the setting of the sky or toward the rising of the Sun, They have lifted their unlucky head, and with threatening hair Have filled cities far and wide with horrible terrors: If they stand still and fix their footsteps in a fixed place, Neighbours will invade in war, and civil arms Will defile allied hands with kindred blood. But if they make toward the East, know that foreign wars Are being stirred, and that an enemy will come from foreign shores.

[Margin: The opinion of St John of Damascus.]

III. But it is better to appeal to the Fathers and theologians. Let St John of Damascus stand for them all, in Book 2 of *The Orthodox Faith*, chapter 7, where he speaks expressly of comets thus: "Comets also frequently come into being, certain signs of the death of kings. They are not among the stars that were made at the beginning of things, but by divine command are formed at certain times and dissolved again."

[Margin: Tertullian's opinion.]

Speaking generally of these and similar portents, Tertullian, *To Scapula*, chapter 3, concludes after a long discussion of the matter: "All these are signs of the imminent wrath of God, which we must, in whatever way we can, both announce and proclaim, and meanwhile pray that it be local. For the universal and supreme wrath will be felt in its own time by those who interpret its examples otherwise."

[Margin: St Augustine's opinion.]

Yet one could say of these what St Augustine said, in the book *On the Destruction of the City* (tome 9), of a certain comet or chaumate [?]. When it threatened the city of Constantinople, around the year 496, a revelation was made first to a certain soldier and through him to the bishop, and the bishop called the people together and drove them to penance and to seek the sacraments. "For God," says St Augustine, "wishing to terrify the city and by terrifying to amend it, by terrifying to convert it, by terrifying to cleanse it, by terrifying to change it, to his faithful servant, a soldier, as they say, etc."; and after a little: "At the beginning of the night, as the world was already darkening, a fiery cloud was seen from the East, at first small; then gradually, as it came over the city, it grew, until it hung hugely and terribly over the whole city. A horrible flame seemed to hang there, nor was the smell of sulphur lacking. Everyone fled to the church; it could not hold the multi- tude. Everyone

wrested baptism from whomever he could. Not only in the church but through houses, streets and squares the saving help of the sacrament was demanded, so that the wrath might be put to flight — not, to be sure, a present wrath, but one to come. The people, moreover, fled with Arcadius outside the city and flocked together in one place to pray to God, and they did not return into the city until that cloud vanished into smoke and they were rid of the danger." St Augustine concludes: "What shall we say? Was this the wrath of God, or rather his mercy?" and after a little: "Exactly so are hands usually raised to strike, and, once the one who was to be struck is terrified, called back by pity: so it happened to that city." Read the same author, *City of God* Book 21, chapter 8, Book 2, chapter 23, and Book 3, chapter 1, together with Nicephorus, Book 3, chapter 4.

Nor does the oracle of the Erythraean Sibyl, who is also called the Babylonian, seem contemptible. It is read in Book 4 of the *Sibylline Oracles*; to whatever time it finally refers, granted that it seems to look to the last times of the world:

Then a huge Sign will be given from heaven, For a most brilliant star will shine out, Very like the bright one of the Crown, flashing in the shining sky, And for many days will be seen by the world, a thing to watch: But under the setting Sun a star called a comet Will shine, a herald to mortals of the sword, And of famine and death, and of the destruction of illustrious men And of great leaders and nobles.

And elsewhere:

But from the fourth year after a huge star has shone, A Star will come from the heavens which, hurling flame, Will flash through the great sea and the deep Pontus.

Granted that Tycho, *Progymnasmata* tome 1, page 815, draws this last oracle to the comet of the year 1577, and the first to the new star seen in Cassiopeia in the year 1572, because four years passed between the one and the other. Yet the same Sibyl sings of happier things after that time:

..... Again the God who dwells in great heaven Will preserve the remnants of men utterly, everywhere. Then peace and the highest wisdom of truth will flourish, And the fruit-bearing earth will bear more abundant crops, No longer divided, nor to be enslaved from then on.

For the other Fathers, who refer such portents to divine Providence threatening mortals for their good, see our Father Adam Tanner in his *Dissertation on the Heavens*, question 7, paragraph 4; he too admits that this is a way of philosophizing worthy of a Christian.

Astrologorum Placita.

The Tenets of the Astrologers

IV. It neither befits me, nor have I the leisure, to follow up or pursue at length the opinion of the astrologers on the presages of comets; yet it should not be passed over entirely.

[Margin: Ptolemy's opinion.]

Their leader, Claudius Ptolemy, in the *Centiloquium*, aphorism 98, decrees thus: "Shooting stars [traiectiones] and comets [crinitae]," in Greek διάττοντες

καὶ κομήται [Greek: diattontes kai komētai, "shooting stars and comets"], "hold the second place in judgements" — for in aphorism 96 he had given the first and chief place to eclipses.

[Margin: The prognostics of shooting stars.]

Then in aphorism 99 he adds: "Shooting stars indicate dryness of the air. If they are carried toward one part, they indicate wind from that quarter; but if in different directions, they indicate the diminishing of waters, disturbances of the air, and the incursions of armies." These shooting stars are the ones Virgil described in the *Georgics*:

And long trails of flame grow white behind them.

And Pontano, in his *Meteora*, following Ptolemy:

Moreover, where the glowing fire has lifted itself at night, Marking the clear air with a bright trail, From there the wind will rush as soon as its gates are open.

Finally Ptolemy, in the last, or 100th, aphorism: "Comets whose interval from the Sun is eleven signs: if they appear in the angles [cardines], the king of some kingdom, or one of the princes of the kingdom, will die. But if in a succedent place, the things that pertain to his treasuries will go well, yet he will change his government. And if in a cadent place, there will be diseases and sudden deaths. But if they move from the west to the east, a foreign enemy will overrun the regions; if they do not move, the enemy will be a provincial one." We have already seen this opinion expressed by Pontano in the verses quoted at no. 2.

[Margin: Ptolemy on the distance of comets from the Sun, explained and defended.]

At this place many attribute to Ptolemy so great a distance as to have him declare that all comets are eleven signs distant from the Sun. But he meant nothing of the kind. Rather, he attributed visibility to those alone that are so distant, and — compared with those that are less distant — the power of signifying the death of kings, because of their approach to the mean conjunction with the Sun, which they approach by their proper motion in the order of the signs when it is a single sign away. For if they recede from its wide [platica] conjunction, they do not have that power, granted that they may be visible; and if they are less distant from the Sun than one sign, or thirty degrees, they do have a great power of themselves, but accidentally they cannot signify it, because, overwhelmed by the light of twilight and the nearness of the Sun, they do not appear. For them to have the power of signifying the death of kings and princes, then, two things are required: that they approach conjunction with the Sun, and yet be so far from it that they can appear on the eastern horizon. I later found nearly this interpretation in Pontano on that aphorism.

[Margin: Ptolemy's second opinion.]

Again Ptolemy, in the *Quadripartite Construction*, or *On the Judgements of the Stars*, Book 2, chapter 9, text 53, declared: "Comets too must be observed, whether they shine at eclipses or at any other time, in universal considerations — those of the kind called beams, trumpets, jars, and the like," as Cardano's

version has it. But my Greek codex, with the notes of Porphyry, has ξιφοειδέσι, καὶ εἰδολομόρφοις, καὶ τὰ τοιαῦτα [Greek: xiphoeidesi, kai eidolomorphois, kai ta toiauta, "sword-shaped, and image-shaped, and the like"], that is, "sword-shaped, or image-shaped, etc. For the working of their nature is of the property of Mars and Mercury: of wars, heat, turbulence, and what usually follows these." And text 54: "The places in which these things will happen are designated both by the parts of the zodiac in which such stars have gathered and by the inclination of the comet and by its form. If they have a human form, they portend for humans; if a beast's, for beasts; if a beam's, for stakes or plants. The length of the duration you will know from the duration of the appearance: a short one indicates a short duration of the effects, a long one a long. From the schematism of the aspect, or the configuration to the Sun, we shall know whether the effects and the beginning of their working will follow at once or somewhat later. For when these portents are oriental, and precede the Sun and appear in the East, they signify the speed of the coming event; occidental and vespertine ones, its slowness." This, namely, is how I think the text should be translated from the Greek, since Cardano's version is both defective and corrupt. Yet his notes and Porphyry's on these two texts may be consulted, and Pontano's on the aphorisms cited.

V. We have heard the master; are the disciples not to be heard as well? But the matter is long-winded, and stuffed and infected with a great deal of vanity. Anyone who wishes, and is permitted, may consult Leopold of Austria, treatise 5, *On the Revolutions of Years*, and, standing for them all, Francesco Giuntini, after his *Astrological Calendar*, in the annotation on comets. There he sets out distinctly and at length the properties and prognostics of comets, running through each of the planets whose nature they imitate, and each of the signs of the zodiac in which they appear. So, to take at least a taste of it:

[Margin: The prognostics of a Saturnine comet.]

A Saturnine comet, that is, a leaden and livid one, appearing in the horoscope of someone's nativity, signifies melancholic and chronic diseases, that is, slow and long ones; in the horoscope of the beginning of the year, plagues, want, rainstorms, fogs, snows and lasting ice, locusts, etc.; and for men subject to the rule of Saturn, an incitement to and increase of spite, voracity, solitude, austerity, envy, avarice, sadness, obstinacy and very great malice.

[Margin: The prognostics of a Jovial comet.]

A Jovial comet, that is, a silver one, portends fertility and healthy rains, but also pleurisies, ophthalmias, measles, pneumonias, stones, colic pains, lethargies and lientery, especially for men of a Jovial nature, that is, beneficent, magnanimous, truthful, clement, etc.

[Margin: What a Martial comet portends.]

A Martial comet, which he says is ruddy and has the form of a pole or a spit, signifies the effects that usually proceed from hot dryness: winds, earthquakes, storms, thunder, lightning, dysenteries, erysipelas, very acute fevers, choleric phrenitis, and then incitement to brawls, seditions, wars, etc., especially for men

of a Martial nature, namely the red-haired, slender, talkative, restless, arrogant, crafty, fiery, untamed, eager for revenge, etc.

[Margin: What a Solar comet portends.]

A Solar comet, that is, a golden one rivalling the Sun, portends the death of princes, or at least tumults and a change of state, or the danger of a change, and also heats and droughts.

[Margin: A Venereal comet.]

A Venereal comet, that is, a yellow and reddish one, threatens the stomach, the womb and the kidneys, and threatens dropsy, paralysis, apoplexy, a change of laws, and of commonwealths. A Mercurial comet, or blue one, called *Dominus Asconae*, threatens phrenitis, lethargy, epilepsy, famine and plague, especially for the ingenious, the eloquent, the witty, the shrewd, the suspicious, etc. Finally, a Lunar comet portends some evil for the female sex and for the common people: sterility, light wars, dropsy, and diseases arising from excessive moisture. Comets of a mixed nature, which arise from the conjunction of two or more planets, are thought to presage mixed events as well. I forbear, moreover, from the examples of Alliacensis [Pierre d'Ailly], Nyphus, Giuntini, Surdus and Praetorius, by which they try to establish the power of comets to portend evils; for far more are available to us from the history set out in chapter 3. Nor do I heap up here the bundle of prognostics with which, on the occasion of the comets of the years 1577, 1607 and 1618, they filled Europe with innumerable pamphlets and with terrors that were for the most part false.

Meteorologorum Recentiorum Opinio.

The Opinion of the More Recent Meteorologists

[Margin: The opinion of Julius Caesar Scaliger.]

VI. As one turns from the astrologers to the astronomers and natural philosophers, the first to come to meet him is Julius Caesar Scaliger, in Exercise 79 against Cardano. He holds that comets are neither signs nor causes of calamities, and that it is ridiculous madness to think that a king is killed by a comet, let alone that a province is overthrown. For he says that a comet is not a sign of any evil in the way smoke is a sign of fire, but only through a certain coherence from a common cause — the way a hen's cackling is a sign of the egg she has laid, because the hen is the cause both of the laying and of the cackling, while neither depends on the other. And he adds: "Many comets, then, have been seen by us that were followed by no destruction of mortals anywhere in all Europe; and many very famous men have met their fate, many principalities have been overthrown, most illustrious families ruined, without any indication from a comet." He therefore thinks Cardano is joking when he writes that princes die more easily than other people in dry seasons, because of foods that are fragrant and seasoned with spices, or because they are worn out by cares and vigils: as if comets did not arise in winter too, or as if many of the common people or of the country folk who eat garlic, onions, bulbs, leeks, salt and

pepper — or smiths, rowers, glassmakers and reapers — were not more disposed to the diseases that spring up from an excess of heat and dryness.

[Margin: The opinion of Carolus Piso.]

A little more distinctly, Carolus Piso, in his *Mirror of the Comet*, from chapter 28 to the end, teaches that the troubles comets threaten are neither so certain nor so common to all regions of the earth, and that the comet is not the cause of evils but a sign of a concomitant or future dryness. For comets arise only from a great quantity of exhalations boiling upward, chiefly from the sea, and from lands heavy with sulphur and hot fat, and so they are generated by a concurrence of causes that brings dryness to the lands. The comets themselves, however, do not dry out the regions over which they pass, nor bring drought upon them: first, because they are very high and stand at an immense distance; second, because they change course not only from day to day but from hour to hour, and do not stand long directly over one region; and finally, because many comets have rather had very cold winters. "But neither," he says, "are they necessarily signs of famine. For dryness does not always bring famine, even if we granted that comets, dissolved into embers and ashes, fall back to the earth — just as the embers neither of Etna nor of Vesuvius harm the fertility of the crops and the greenery of the Sicilian or Neapolitan countryside." From this he infers that they are not necessary indicators of plague either, especially of the plague that usually arises from famine, not among the wealthy but among the common people and the poor. Plague that comes from an infection of the air arises from an extraordinary putrefaction owing to heat in moisture, as Hippocrates teaches in *Epidemics* 2, and Galen throughout his works; and it occurs when the south winds prevail. It therefore has nothing in common with comets, which signify an excess of heat and dryness, unless south winds prevail from some other cause. He grants, indeed, that winds are signified and even produced by comets, since their fumes, dispersed after the burning, are spread into the air; but he holds that for the most part the boilings of the sea — from which come floods and earthquakes about maritime places — are indicated by comets, not produced by them, because comets indicate winds shut up in the bowels of the earth and sea; for he denies that comets arise without a huge quantity of winds.

In his last chapter, moreover, he contends that the comet is unjustly ill spoken of as a torch of wars or a regicide, on the ground that a change of state and of laws is usually feared under a new king. For granted that princes are more delicate and more elated, and have humours more apt to inflammation, yet the headlong anger of the kings of Europe, especially the Christian ones, is rarely the cause of war; and if it were, they themselves are mostly out of the weapons' range; and most of them live soberly, or at their physician's nod. So he bids princes be free of fear from the comet of the year 1618.

[Margin: Piso's prudent [?] conclusion.]

At last, however, he concludes the chapter thus: "I think it must be held that by divine Providence various disasters usually fall upon the human race, so that

the fear may indeed reach many, but the punishment few. For it is not my mind to regard such portents as mere empty bugbears; and for my part, let this opinion sit more deeply in everyone's minds: that grave evils and calamities are portended by them, by which GOD — who both hears and sees what we do, and is also judge and avenger — usually avenges the injuries done to his name. Meanwhile, let fathers of families and stewards, warned by the comet, use their stored crops and fodder prudently and more sparingly, as befits thrifty people; and let magistrates and princes take timely care for the serenity and healthiness of the air, and strive with all care and zeal to turn future diseases and wars away from their own heads. Finally, let the preachers of the divine word be permitted seriously to rebuke and admonish the profligate, the gluttons and the impious: that the comet, from that most lofty station, announces and proclaims far and wide that rivers of eternal fire run hidden in the abyss of the earth, in whose never-dying flames wicked heads pay to divine justice the deserved penalties of their wickedness and impiety — penalties most bitter and immortal."

[Margin: Froidmont's opinion.]

VII. Froidmont, in *Meteorologica* Book 1, chapter 3, after much discussion with Thomas Fienus [Thomas Fyens], concludes that a comet is not a physical cause of calamities but rather a sign at the pleasure of God, who uses the comet, although it is a natural effect, to terrify and amend mortals in some place, as he used the rainbow to signify his covenant with our race. And although certain comets have been favourable to some people, or seemed so, they are yet not sent for this end, but to announce sadder things; and it would be an endless task to patch together the deaths of princes and the calamities that have happened with comets announcing them beforehand. Celestial comets, moreover, are more apt to portend these things than elementary ones, because the former, being higher and longer-lasting, are seen by more people and spread the fear of the divine power more widely; whereas elementary comets, according to Aristotle's opinion, indicate, but do not produce, the exhalations from which they arise and, from these, other related effects.

[Margin: The opinion of Cottunius.]

Cottunius, however, in lecture 33 on *Meteorologica* Book 1, toward the end, denies that disasters or the death of princes are presaged by a comet — granted that the death of princes is usually more noticed than that of common people — because those disasters did not happen when comets arose, or did not happen more than at other times; and conversely they have happened with no comet preceding. He grants, however, that comets are signs of blasts and winds, and of what is connected with them; and that if they are northern, they signify sterility arising from dryness, the stepmother of crops, and earthquakes, storms and fits of anger; but if southern, pestilences and floods.

[Margin: The opinion of Franciscus Resta.]

Franciscus Resta, *Meteorologia* Book 1, tract 1, chapter 8, says at the beginning that, of many effects attributed to comets, comets are nothing but signs, and perhaps lying ones. Yet he is of the same opinion as Froidmont: namely, that

God uses these natural effects as signs, by divine institution, to strike a salutary fear into us, as he once used the rainbow to relieve the fear of the flood; and yet that we do not stray from truthful philosophy if we say that celestial comets are formed by the ministry of angels for this very end.

[Margin: The opinion of Father Cabeo.]

Father Niccolò Cabeo, on *Meteorologica* Book 1, text 37, question 9, admits that some comets are unfavourable and some favourable, both of themselves and for everyone, and accidentally, because nothing is ever so harmful to one that it does not benefit another, and if one man loses an empire, another gains it. He then subscribes to Cardano: elementary comets, from the great quantity of sulphurous spirits that they summon, produce or signify droughts and an intemperate air, by which weaker bodies are more easily harmed, as well as those more delicately nourished, or subject to cares and vigils — of which kind he thinks princes are, and many children and delicate women; but the deaths of these are not noticed in the same way as those of princes. To celestial comets he attributes, in due proportion, as much power as the astrologers attribute to the planets themselves — on the hypothesis, however, that the astrologers have a sufficient induction for establishing general rules from experience. But at least they are signs that God displays, to give to those who fear him

a signification, that they may flee from before the bow; so that the comet is, as it were, a tongue of heaven, by which we are warned to flee from the wrath to come. So much for the principal meteorologists of our century.

Aſtronomorum Recentiorum Opinio.

The Opinion of the More Recent Astronomers

[Margin: Tycho's opinion.]

VIII. What the Phoenix of astronomers, as some call Tycho, has to sing on the prognosis of comets must be sought in volume 1 of the *Progymnasmata*, from page 800. There, treating of these and of the new stars alike, he condemns those who, because of the errors of the astrologers and the weakness of the human mind in attaining certain presages of these portents, take from them all energy and power of signifying — as if God and Nature were playing idly with these new spectacles and wished nothing to be foretold to the world. It is no wonder, however, he says, if the foreknowledge of them is conjectural [ſtochaſtica] and probable only by mere conjectures, since not even the motions of the stars that have shone perpetually in heaven from the origin of the world — let alone their particular faculty of influence — are sufficiently understood. The rest that he appends there pertains not to comets but to the prognostics drawn from the new star of the year 1572 seen in Cassiopeia, which is to be discussed in the following section.

[Margin: The opinion of Longomontanus.]

Tycho's pupil and assistant Longomontanus, in the Appendix to the *Astronomia Danica*, chapter 7, rebukes Thomas Erastus and others, who allow the ordinary stars a slight power of producing or signifying effects, but comets

none. For God and nature, he says, do nothing in vain, and God orders to a good end all the evils that comets signify: namely dearth of provisions and sterility, so that the earth, as though keeping a holiday, may meanwhile be prepared for an abundance of crops; storms of the air, so that the air, stirred up in this way, may come out purer; and diseases and wars, so that, with wicked men exterminated, the world may be renewed — according to those verses of Palingenius [Marcellus Palingenius Stellatus]:

[Margin: Palingenius's fine reflection.]

> *Such men, then — nay, shadows — at a fixed time God sends to arms, to be slain by a cruel death. So he purges the human race, and for many years those who remain live cheerfully, this part removed, until evil weeds grow up again, once more to be plucked out by bitter war and cut down by the sword. Then once again Jupiter skims the unclean pots; then once again, with furies let loose, he sweeps away the refuse.*

[Margin: An argument for divine Providence in the very calamities.]

Meanwhile, the innocent are harmed by these disasters only through a wrong opinion, because that golden saying of Chrysostom stands: *"No one is harmed except by himself."* And although these new phenomena are contained within the bounds of nature as to their matter and efficient cause, yet as to their end they are chiefly destined, by divine foresight, to signify the overthrow and changes of commonwealths and to prick the excessive security of men; and comets are prophets like Jonah, but mute, who announce the divine wrath and are signs of a world that is to perish. He then passes to chapter 8. He refers the variety of effects, whether from the stars or from comets, to the varying disposition of regions and of living things, or to the power of free will, which cannot be compelled by them but only inclined; and he teaches much toward the general use of astrology, which, however, he says must keep within the bounds of general judgement, on account of that first pronouncement of Ptolemy in the *Centiloquium*: *"Only those inspired by the divine power foretell particulars."* After this, in chapter 11, he erects a celestial horoscope for the rising of the comet of the year 1607, for September 15 at $15^{h}\ 24^{m}$ and a pole height of 56°. He considers that it arose at the mean opposition of Jupiter with the Sun, and nearly in the quartile of Jupiter with Saturn, and in a quartile aspect of the Sun with Mercury; that Mercury was lord of the ascendant, and Mars not far from the ascendant; the Saturnine colour of the comet; and finally the full moon that followed next, on the 25th, and its horoscope. He thinks it signified the winter of the year 1608, so savage in the Low Countries, Germany, Switzerland, France, Spain and Italy that many animals and men perished of cold, although it afflicted Iceland and Norway more mildly; and from it the dearth of provisions, and the chronic diseases, especially the quartan fevers so frequent in Germany from that time, and the wars of the year 1610 in Tartary, and things like these — extending the significations of this comet up to the year 1612. Erecting likewise a horoscope of the comet of the year 1618, both at its rising, for November 11 at 11^{h} before noon, and at the full moon that followed

next, on November 21, he contemplates its rising in the quartile aspect of the Sun and Jupiter, with the malefics Saturn in the 5th house, Mars in the 8th and Mercury in the 9th. He says that many ominous things were foretold by it, and that it was fatal to Archduke Maximilian of Austria (who had, however, already died on September 5), to the Empress, to Anne, Queen of Britain, and to the Emperor Matthias, who died in the year 1619 on March 10, Old Style. From here he does not hesitate to range out from this portent to other evils to be announced, especially in the year 1624; for although, according to the tenets of Ptolemy, this comet, having risen in the morning, would have had swift significations, yet from the size of its tail and body, and from the constancy and duration of its motion, they could extend up to that year. But it is wearisome to reduce to a summary the long heap of divination affected by this astronomer — a heap of words rather than of things; and this itch for divining you may also see, and lament, in Kepler.

[Margin: Kepler's opinion.]

IX. Kepler, however, treating the physiology of comets, puts forward many shrewdly devised ideas. For he says that comets were made for this end: that the ethereal region should not be empty, for so long a time, of its own monsters, so to speak, just as the Ocean is not empty of larger fish that go raiding on rare excursions; and that, the thick fat of the aether being gathered into one abscess as a kind of excrement, the celestial air may be purged — lest there should often happen what the histories testify once happened, namely that the Sun, for some days, as in the year 1547 from April 24 to 28, or for a whole year, as in the year of the murder of Julius Caesar, shone malignly, dulled to a rusty or blood-red colour. He then grants that comets, though rarely, reach down to the earth with their tail and bring in pestilence; but that comets for the most part move above the Moon, and that therefore the things attributed to them concerning the signification of winds and earthquakes are futile, since they rest on two false opinions, namely that comets burn and that they are below the Moon. The genuine cause, therefore, why a comet, though absent, changes the state of sublunary things he holds to be that animal faculty of the sublunary world by which, through its sympathy with heaven, he thinks minerals, rivers and rains are also produced; for this faculty is moved objectively by comets and stirred up to generate those things that bear an analogy to comets, and to rouse men's passions toward war. Yet for a comet to appear and signify in definite places, its trajectory must be directed and destined by some mind supremely intelligent in geometry and in the whole world — whether it be God, or another spirit, good or evil — which produces comets in the aether, like whales in the sea, as monsters and portents of heaven, or leads them, once produced, to definite limits. With this set out first, in Book 3 he passes to the significations of the comet of the year 1607, to which he attributes drought and dearth of provisions and many other evils to follow from it. But those words on page 111 are worth noting: *"For why should I alone speak against the universal consent of mankind, which each person confirms according to his capacity with very*

frequent histories? Therefore I affirm that this comet was displayed by God among the stars to be a testimony to each and every one of their mortality, and that they might be admonished that God has decreed shortly to remove from this world a good part of the human race, of every condition alike, by whatever law of fate: this being certain in general, but most uncertain as regards individuals. For the strong have no privilege over the weak, nor the young over the old, nor the seer over those who consult him. The celestial herald therefore warns each person to be reconciled to God for his own sake, to prepare himself for his departure, and so to settle these earthly affairs as he wishes to see them kept, once settled, after his decease." No less prudently he adds that, if these warnings are obeyed, it will come about that whoever death snatches away will depart happily, and that whoever survives will not take it ill to have been so happily deceived by the astrologer. He explains himself further, and denies that a comet has the power of killing men: rather, by the comet's indication, the universe and heaven itself are corruptible, and so a man must bear the condition of his mortality with equanimity when he sees a comet, a blemish of heaven, and must prepare himself for a departure that will not be so far off. And, he says, the histories, and the wisest historians and philosophers, testify that hardships follow shining comets — not only through the death of princes and the occasion it gives for revolution, but in a thousand other ways. Further on, after much else, he adduces the troubles and disasters after the comets that arose in the year 1531 in Switzerland, Saxony and the Baltic Sea; in the year 1532 in Westphalia and the Low Countries; and in the year 1533 in

[Margin: Years of Christ.]

Wittenberg and elsewhere; in the year 1538 in England; in the year 1539 in Meissen, Thuringia, the Mark of Brandenburg and Brunswick; in the year 1558 again in England, France and the Low Countries; in the year 1569 in Saxony; in the years 1577 and 1580 in very many places in Germany; in 1582 and 1585 at Cologne, at Aachen and in France; in 1590 and 1593 in Styria on the Rhine, and here and there in Germany; and in 1596 in Poland, Sweden and Styria. There are countless things, moreover, that he tries to predict by conjecture about the regions of Europe and about persons in general; but at last, recognising that his effort is vain, he concludes Book 3 in these words: *"And so I stop soothsaying, since even this may be too much."* He then appends the events after this comet of the year 1607, and relates that a very harsh and long winter followed, but that the crops came up excellently in Germany; that deaths were not heard of so frequently; and that of the monarchs of Europe none died except Mahomet. The other things he relates are separated from this comet by too great an interval of years, or are not so remarkable that the comet should seem to have been destined for them.

[Margin: Kepler's prognostic on the comet of the year 1618.]

X. There follows Kepler's judgement on the comet of the years 1618 and 1619. From its size and its tail — the longest of all that have shone within these 150 years — he says it is likely that sublunary Nature was violently struck, together with that faculty which resides in the globe of the earth and which, through

sympathy, rests on imagination; that from this a great part of the Bohemian and the imperial army perished of an unusual disease, and that a very snowy winter of two months followed. He then repeats from history the disasters of the comets of old, which we indicated in chapter 3.

[Margin: The opinion of Father Cysatus.]

XI. Our own Cysatus, in the *Mathemata* on comets, page 79, thinks it probable that comets portend great evils to the world and to mortals, but uncertain ones; that it is rash to assert what they portend, or to whom; and that, although they are natural, by divine providence they are nevertheless an omen of great calamities and of God's anger against the guilty — granted that the calamity does not reach everyone, but perhaps only those over whose heads the comet hangs perpendicularly, or above whose horizon it remains for many hours.

[Margin: The opinion of Father Guinisius.]

By contrast, our own Vincentius Guinisius, likewise, in his oration on the felicity of the comet, contends that the comet of the year 1618 promises no more unhappiness than happiness, because many comets have been favourable to many princes — for example, to Alexander the Great, Mithridates, Augustus and others — and with other arguments that we have already indicated above.

[Margin: Our opinion.]

Our opinion, finally, is this. If comets are elementary, they certainly signify naturally some notable change brought about in sublunary nature, whose effects have not yet entirely ceased but will come forth not much later, according to the disposition of the matter and the concurrence of other causes. But if they are celestial, they are certainly ordained by divine providence: first, so that mortals may raise their eyes to heaven and acknowledge that there are many things whose cause and nature they are forced not to know, and so lower their pride and their fasces before the divine wisdom; and then, so that they may expect from this something great, either in themselves and their own regions or in others — for the most part an evil, which will either actually come about if the Deity is not appeased, or is only threatened, so that through fear of the Deity they may thus be turned to sounder counsels; and so that from this, at least, they may conceive beforehand some sample of those prodigies of which our Saviour speaks in the Gospel: *"There shall be signs in the Sun and the Moon and the stars, etc."* and *"the stars shall fall from heaven, etc."* Prudently, therefore, on the occasion of the comet of the year 1460, that most holy man Ioannes de Capistrano [Giovanni da Capestrano], preaching, announced calamities to come unless mortals seriously amended their life and morals. For this agrees both with the history already set out in chapter 3 and with the wisest men, not only pagan but Catholic too. But to wish — I do not say to divine, but to attempt it, and to publish prognostics of comets to the common people from the rules of Ptolemy or of other astrologers — I judge utterly empty, and rather a provocation of a Deity already angered than the obedience or expiation owed to him. For a prognosis of this kind would require that several comets of the

same condition in every respect had appeared, whose ensuing effects or events had for the most part been very similar. But if you read the history of comets attentively, no comet can yet be shown to have shone similar to another in figure, place, motion, duration, season of the year, etc.; nor any comets, similar among themselves in a number of conditions, whose effects, or the events they signified, were also similar. Yet for the benefit of those who wish the comets favourable to some to be distinguished from the unfavourable ones, or who wish to see those that preceded the death of princes, it has pleased us to set out under one view the comets listed below, from our catalogue in chapter 3; a fuller knowledge of them must be sought from that same chapter 3.

Cometæ Faufti aliquibus. — *Comets favourable to some*

Years before Christ	**Persons to whom they were favourable**
356	Alexander the Great, then born
336	Alexander the Great, entering on his reign and on victory over the Persians
130	Mithridates, then born
119	Mithridates, entering on his rule and on victories from it
44	Augustus, taking from it an omen of empire
Years of Christ	
130	Usuardus, King of England, victorious
405	Stilicho, conquering the Getae
725	Charles Martel
800	Charlemagne, then created emperor
1200	The empire of the Tartars, beginning
1283	Adalbert, defeating Adolf
1456	Tamerlane, triumphing over Bayezid; and Io. Corvinus, repelling him
Years of the comets before Christ	**Leading men who died in the same or the following year**
336	Philip, father of Alexander the Great
336	Darius Codomannus, King of the Persians
44	Gaius Julius Caesar
Years of Christ	**Princes who died in this or the following year**
13	Agrippa
14	Emperor Augustus
54	Emperor Claudius
70	Emperor Vitellius
80	Emperor Vespasian
213	Emperor Severus
340	Emperor Constantine the Younger
363	Julian the Apostate
392	Emperor Valentinian
454	Emperor Theodosius
571	Alboin, King of the Lombards
603	Emperor Maurice
814	Emperor Charlemagne
837	Pippin, King of the Franks
839	Emperor Louis the Pious

Cometæ Fauſti aliquibus. — *Comets favourable to some*	
882	Emperor Louis the Bavarian
983	Emperor Otto II
1009	Pope John XVIII
1066	St Edward, King of the English
1106	Emperor Henry IV
1169	Malcolm, King of Scots
1214	William, King of Scots
1264	Pope Urban IV
1301	Andrew, King of Hungary, dies without children
1314	Philip the Fair
1341	Andronicus, Emperor
1375	Charles IV
1402	Tamerlane, and Galeazzo Visconti [Galeat.Vicecomes]
1450	Amurath, Emperor of the Turks
1456	Ladislaus, King of Poland
1457	Alfonso, King of Naples
1477	Charles of Burgundy
1505	King Philip
1506	Alexander, King of Poland
1512	Pope Julius II, and Bayezid, Emperor of the Turks
1521	Pope Leo X

Reſiduum Tabulæ præcedentis. — *Remainder of the preceding table*	
Years of Christ	**Princes who died in this or the following year**
1533	Clement VII; Alfonso, Duke of Ferrara; and the Duke of Milan
1558	Emperor Charles V; the Queens of Poland, England and Hungary
1559	Pope Paul IV; Henry, King of France; the King of England; the King of Portugal; the King of Denmark; the Doge of Venice; Ercole, Duke of Ferrara; and 15 cardinals
1577	Sebastian, King of Portugal
1585	Osman, Emperor of the Turks, and Stephen, King of Poland
1590	Pope Urban VII, and Archduke Charles
1607	Charles, Duke of Lorraine
1618	Emperor Matthias

CAPVT VI.

Chapter VI

Proponuntur ſummatim Opiniones de Materia Loco & Cauſa Efficiente Cometarum, ſumma fide ac diligentia conquiſitæ.

The opinions on the matter, place and efficient cause of comets are set out in summary, gathered with the greatest fidelity and diligence.

CAPVT VI.

I. Many have reduced all the opinions on the matter, place and effecting cause of comets to three main heads, but with differing distributions.

[Margin: Seneca's distribution of the opinions on comets.]

Seneca, indeed, in *Natural Questions* Book 7, chapter 19, does so thus: *"Some, then, think that there are no comets, but that appearances of them are produced by the reflection of neighbouring stars, or by the conjunction of stars that cohere. Some say that they exist, but have their own courses, and come out into the sight of mortals after fixed periods. Some say that they exist, but are not such that you could give them the name of stars, because they slip away, do not last long, and are dissipated in a short space of time."* As if he were saying: some say that a comet is a mere appearance, and that nothing new is produced or distinct from the known bodies; some, that comets are perpetual stars, but rarely appearing; some, that they are produced anew and do not deserve the name of star, because of their swift destruction.

[Margin: Piso's distribution.]

Carolus Piso, on the other hand, in the *Mirror of the Comet*, chapter 1, recognises only three probable opinions: namely those that affirm the comet to be *either some particle of heaven, whether a fixed or a wandering star; or an imperfect mixed body, and that truly on fire; or merely shining and flashing with another's light, namely the Sun's, which, having passed through it, then pours itself out further and farther beyond.*

[Margin: And Glorioso's.]

Giovanni Camillo Glorioso, in Book 1 of *De Cometis*, chapter 2, distributes the primary opinions into three classes in a slightly different way. The first, he says, is that of the Chaldaeans, who assert that comets are wandering stars. The second is that of the Peripatetics, who hold that comets are kindled terrestrial exhalations. The third and newest is that of certain more recent astronomers, who think that comets consist of ethereal matter, but illuminated by the Sun. For a comet is either an emphasis [emphaſis] — the appearance of a mere image; or a real hypostasis, but of an eternal body, that is, a star, or a symphasis [ſymphaſis] of stars; or finally a real one, but of a temporary body, either kindled or only illuminated. So he.

[Margin: Cottunius's distribution.]

After these, Io. Cottunius, in Lecture 32 on Book 1 of the *Meteorologica*, reduces to three heads the 25 opinions on comets collected by Fortunio Liceti in Book 2 of *De novis astris et cometis*, according to three general controversies, namely on the place, on the matter and on the tail of comets. As to place, there are three opinions: one of those who put all comets below the Moon, another of those who put them all in the heavens, and a third of those who put some above the Moon and some below. Again, as to matter, a comet is either something eternal or something temporary: if eternal, either one star or several stars together; if newly generated, it is either kindled in sublunary matter, or only illuminated, whether in elementary matter carried up into the heaven, or from celestial matter.

[Margin: Longomontanus's distribution.]

Longomontanus, in the Appendix to the *Astronomia Danica*, chapter 2, does it otherwise: he reduces the opinions to three kinds, but subdivides each in two. The first kind is taken from the matter: for a comet arises either from celestial or from terrestrial matter. The second is from the form of its appearance: for a comet is produced either by mere inflammation, like a kindled fire, or by mere illumination from the Sun or the other stars. The third is from place: for a comet is either above the Moon or below it, or sometimes above and sometimes below.

[Margin: Our division and subdivision.]

To me, however, setting aside the fables, of which I shall speak in the Scholia, all the opinions seem reducible to three classes, though these are to be subdivided. For a comet is either a mere appearance, or something real and eternal, or something real and newly made — which was more or less Seneca's partition. If it is something merely apparent, it is either an emphasis arising from the reflection, or also the refraction, of light in celestial or elementary matter, or a symphasis of several known stars. If it is something real and eternal, it is either some perpetual planet, unknown however and rarely appearing to us, having such a figure of itself, or illuminated by the Sun and from there taking on various forms; or it is a heap of several old small stars and unknown little planets coming together into one extraordinary planet. If, however, it is something real but newly made, it is either produced from elementary matter, by the generation of fire or by mere illumination, whether it remains below the Moon or ascends above the Moon; or it is produced from celestial matter, either by substantial generation or by mere alteration — namely of density, opacity and light — and it remains above the Moon or is carried below it; or it is produced from nothing by mere creation. Or, finally, some comets are celestial in place and origin, and some elementary. Thus 12 opinions will arise, to be set out distinctly in a moment.

[Margin: 1. The opinion of mere appearance through the reflection of light.]

II. The *first* opinion was that a comet is not something real and distinct from bodies already existing, but a mere ἔμφασιν [Greek: ἔμφασιν (emphasin, "appearance, reflected image")], that is, an appearance, arising from the reflection, or also the refraction, of the rays of the Sun or of the Moon, in much the same way as the rainbow, the halo and parhelia. So Panaetius held, of whom Seneca says in *Natural Questions* Book 7, chapter 30: *"With Panaetius, and with those who wish it to appear that a comet is not an ordinary star but the false appearance of a star, we must diligently discuss whether every part of the year is equally suited to producing comets."* He had indicated the other authors of this stamp, without naming them, in chapter 19, saying: *"Some, then, think that there are no comets, but that appearances of them are produced by the reflection of neighbouring stars."* In the same way Plutarch too, in Book 3 of *De placitis philosophorum*, chapter 2, after certain unnamed persons who thought that a comet is *"a reflection of*

our sight toward the Sun, very like the images in mirrors", a little later nevertheless names others akin to this opinion, but belonging rather to opinion 6.

[Margin: 2. The opinion of the co-appearance of several known stars.]

III. The *second* opinion likewise denies that comets are something newly existing on high; for it says that a comet is a mere σύμφασιν [Greek: σύμφασιν (symphasin, "co-appearance")], or co-appearance, of several stars already known, joined in a quasi-corporeal conjunction, as the astrologers say, and so presenting the appearance of a longer star. Of this opinion I have so far found no certain author. For Seneca, *Natural Questions* Book 7, chapter 19, says only: *"Some think that there are no comets, but appearances of them; or that cohering bodies are produced by conjunction."* And when he had said in chapter 3, *"Democritus too, the subtlest of all the ancients, says he suspects that there are many stars that move; but he gave neither their number nor their names, the courses of the five stars not yet being understood,"* he says later, in chapter 11: *"Let us leave Epigenes and pursue the opinions of others"*; and a little after, naming no one, he adds: *"This one thing must be agreed: that a new appearance of a star is seen beyond the usual, drawing round it a scattered fire. Some of the ancients favour this explanation: when one of the wandering stars has applied itself to another, their two lights being merged into one, the appearance of a longer star is produced. Nor does this happen only when a star touches a star, but also when it has approached; for the interval between the two is lit up by both and set on fire, and makes a long fire."* This opinion he at once attacks in chapter 12 with arguments that are truly very easy and obvious; for that notion is so crass as to be unworthy of Democritus, whom he had called the subtlest of the ancients, and who, as Cicero asserts in *Tusculan Disputations* Book 5, *"made such progress in every kind of philosophy that a Pentathlos, or Quin-*

quertio, he could be called." So Democritus spoke of planets, or of unknown stars, since, on the testimony of Diogenes Laërtius in his *Life*, he had been instructed by the Chaldaeans, from whom he learned theology and astrology; and the Chaldaeans held that comets were wandering stars, distinct from the seven planets commonly known. Accordingly, although Aristotle (*Meteorologica* Book 1, chapter 4) ascribes to Anaxagoras and Democritus this opinion of the co-appearance of planets already known, Cottunius — most devoted to Aristotle as he is — holds in Lecture 30 on *Meteorologica* Book 1, and Mastrius [Bartolomeo Mastri] with him (disputation 4 on the heavens, question 2, article 4), that Democritus and Anaxagoras were not of this opinion but of the fourth of those to be reported below. And rightly, granted that the Coimbra commentators (tract 3 on the *Meteorologica*, chapter 1), Resta (tract 1 on the meteors, chapter 3) and Glorioso (Book 1, chapter 9) held the opposite; as did Alexander, Philoponus, Olympiodorus and St Thomas, who take Democritus and Anaxagoras to mean the five lesser planets, namely Saturn, Jupiter, etc. Nor did Ptolemy, *On the Judgements of the Stars* Book 2, text 53, if he said that comets arise from Mars and Mercury, mean that they arise from the conjunction of those planets as from elements intrinsic to the comet itself, but as from concurrent efficient causes.

[Margin: 3. The opinion that a comet is an extraordinary planet.]

IV. The *third* opinion was that a comet is a wandering star, or some planet — not one of the seven already commonly known, but other, unknown ones, which appear rarely: either because of their small distance from the Sun; or because of their very great distance from the Earth, which they approach after long intervals and so become visible; or because parts of the heaven or of the air that are more transparent run between; or for some other cause. So held the Italic sect of the Pythagoreans, of whom Aristotle says in *Meteorologica* Book 1, chapter 6: *"Some of the Italians, those called Pythagoreans, say that the comet is one of the wandering stars, but that it does not appear except after a long time, and stands out only for a short time, which also happens to Mercury; for because it rises only a little above the horizon, it often does not appear, except after a long time."*

[Margin: The opinion of the Pythagoreans.]

[Margin: And of Hippocrates of Chios and Aeschylus.]

The same author adds that Hippocrates of Chios and his disciples, Aeschylus first among them, held this, although they said that this planet which is the comet does not have its hair [coma] of itself, but from moisture tinged by the force of the Sun's colour and refracting the light; and that for this reason it appears neither at the summer solstice nor within the tropics, because there the moisture is burned up by the Sun's passage, but outside the tropics; and rarely towards the south as well, because the greater part of the circle on which that planet is carried is below the horizon. And Plutarch, *On the Opinions of the Philosophers* [*De placitis*] Book 3, chapter 2: *"Some of the Pythagoreans said that the hairy star is one of those which do not appear continually in the heaven, but which, at fixed times, having completed a certain circuit, are accustomed to rise again."* And a little after: *"Diogenes: that hairy stars are truly stars."*

[Margin: Diogenes.]

[Margin: Favorinus.]

[Margin: The opinion of the Chaldaeans and of Apollonius of Myndus.]

For Favorinus too, in Aulus Gellius Book 14, chapter 1, argues sharply that there are many wandering stars still unknown; and the Chaldaeans, Apollonius of Myndus and Artemidorus (in Seneca) thought the same, and finally Seneca himself. For in *Natural Questions* Book 7, chapter 3, he has this: *"Two who say that they studied among the Chaldaeans, Epigenes and Apollonius of Myndus — the latter most skilled in examining natural things — disagree with each other. For Apollonius says that comets are placed by the Chaldaeans among the wandering stars and that their courses are known; Epigenes, on the contrary, says that the Chaldaeans have grasped nothing about comets."* And chapter 17: *"Apollonius of Myndus is of a different opinion. For he says that a comet is not made out of many wandering stars, but that many comets are wanderers. 'It is not,' he says, 'a false appearance, nor fire spread out where two stars meet; a comet is a star of its own, like the Sun or the Moon. Its form is this: not confined to a round, but taller and drawn out lengthwise. Its course, moreover, is not open to view: it cuts through the higher regions of the world, and appears only when it has come to the lowest point of its course. Nor should we think that the one seen under Claudius was the same as the one we saw under*

Augustus; nor that the one which appeared under Nero Caesar, and took away the ill repute of comets, was like the one which, after the murder of the deified Julius, at the games of Venus Genetrix, emerged about the eleventh hour of the day. They are many and various, unequal in size, unlike in colour,'" etc. *"'They diminish and increase their light, as other stars do, which are brighter when they have descended; and they are seen larger from a nearer place, smaller and dimmer when they recede, because they withdraw further.'"* Thus far Seneca on the opinion of Apollonius.

[Margin: Posidonius favouring the Pythagoreans.]

Posidonius too seems to favour this opinion, or that of the Pythagoreans, who held that comets, like Mercury, do not appear unless they pass out from the Sun's rays on a wider circuit; for Seneca says in the same Book 7, chapter 20: *"We do not see many comets, because they are obscured by the rays of the Sun; Posidonius reports that, when the Sun was eclipsed, a certain comet appeared which the nearby Sun had covered."*

[Margin: The opinion of Artemidorus.]

But Artemidorus was plainly of that opinion. Of him Seneca, in the same Book 7, having argued in chapter 12 against those who say that a comet is a conjunction of several of the known planets, at once adds in chapter 13: *"Against this, Artemidorus says the following: that not only these five stars run their courses, but that they alone have been observed. Innumerable others, however, are carried along out of sight, unknown to us either because of the dimness of their light or because their circles are so placed that they are seen only when they have come to the extremity of them. Therefore certain stars,"* he says, *"new to us, run in between, which mingle their light with the stationary ones and put forth a greater fire than stars have."* For this reason Artemidorus held that these unusual planets draw their hair from their own light joined with the light of the fixed stars.

[Margin: Seneca's view.]

Finally, Seneca himself adhered to the opinion of the Chaldaeans and the Pythagoreans, leaving Zeno aside for the moment; for he says in Book 7, chapter 22: *"I do not agree with our people; for I do not think that a comet is a sudden fire, but that it is among the eternal works of nature."* And chapter 23: *"No fires endure except in their own: I mean those divine fires which the world holds as eternal, because they are its parts and its works."* And a little below: *"A comet has its own seat, and therefore is not quickly driven out, but measures out its space; nor is it extinguished, but passes beyond."* Then, meeting an objection — *"If it were a wandering star,"* he says, *"it would be in the zodiac [Signifer]"* — he replies: *"Who sets one limit for the stars? Who drives divine things into a narrow space? These very stars which alone you believe to move have circles that differ one from another. Why then should there not be some which have withdrawn onto a path of their own, far from these? What reason is there that the heaven should be impassable in some part? But if you judge that no star can touch anything but the zodiac: a comet can have so broad a circle that it still falls into the zodiac with some part of itself — which is not necessary, but possible."* Hence in chapter 24 he rises higher and continues: *"Consider whether this does not better befit the greatness of the world, that it be divided into many paths,*

and not wear down this one track while lying idle in its other parts. Do you believe that in this greatest and most beautiful body, among innumerable stars which adorn the night with varied beauty and do not allow the air to be empty and inert, only five are permitted to exert themselves, while the rest stand still, a fixed and motionless people? If anyone at this point asks me, 'Why then has their course not been observed, as that of the five stars has?', I shall answer him: there are many things which we believe to exist, but what they are like we do not know"; and he brings the example of our soul and of God. And chapter 25: *"Why then do we wonder that comets, so rare a spectacle of the world, are not yet held by fixed laws, and that their beginnings and ends are not known, since their return is from vast intervals?"* etc. *"The time will come when daylight and the diligence of a longer age will draw into the light the things that now lie hidden. A single age is not enough for the investigation of things so great, even were it wholly free for the heavens."* And finally chapter 26: *"Someone will one day show in which parts comets wander, why they go so far apart from the others, how large they are and of what kind. Let us be content with what has been found; let posterity too contribute something to the truth."* The tail of comets he ascribes, in the same chapter, to the splendour of the comet: *"For who will grant you that comets are long? Their nature, like that of the other stars, is a globe; but their brightness is extended. Just as the Sun sends its rays far and wide, yet the Sun itself has one form and the light that flows from it another, so the body of comets is itself rounded, but its splendour appears longer than that of the other stars."*

[Margin: Summary of these opinions.]

Accordingly the Italic Pythagoreans, the Chaldaeans, Apollonius of Myndus, Artemidorus, Seneca and perhaps Posidonius held that a comet is an extraordinary planet, and that comets do not appear for a long time either because they lie hidden within the brightness of the nearby Sun, as the Pythagoreans and Posidonius held, or because of the size and slowness of their circle, near whose perigee alone they can be seen, as the rest held; and that their tail is either the very form of the planet itself, as Apollonius held, or comes from moisture lit by the Sun's rays, as Artemidorus held, or from the splendour flung out by the comet, as Seneca would have it.

[Margin: Cardano.]

Among the more recent authors, this opinion was approved by Cardano, *De subtilitate* Book 4, who says, on the supposition that no generation takes place in the heavens, that comets are stars of a rarer substance, which

[Margin: Oddus.] [Margin: Renatus.] [Margin: Tannerus.] [Margin: Frangipanius.] [Margin: Columbus.] [Margin: Raymundus.] [Margin: Vallesius.] [Margin: Echius.] [Margin: Liceti.] [Margin: Fienus.]

cannot be seen except in very dry and clear weather; Oddus of Padua, *On the Plague* Book 2, chapter 4; Descartes, *New Philosophy* Book 3, number 132; and our Tannerus [Adam Tanner], *On the Heavens* p. 148, who attributes to the size of the epicycles the fact that comets come near us so rarely and become visible; Frangipanius, when he says of comets that they are tiny stars which are not seen except when their light is increased; Ludovicus Columbus

[Lodovico delle Colombe], who says that they are above the ninth heaven and are not seen except when a denser part of the heaven glides beneath them; Annibal Raymundus [Annibale Raimondo], in Resta, tract 1 on the *Meteorologica*, chapter 1: that they become visible because of fumes drawn up to them; Vallesius [Francisco Vallés], because of a denser part running beneath; Liceti, when they have fallen into the denser lower parts of very many spheres, along the perpendicular; Io. Ekius, that they are seen at the time when a rarer part of the heaven glides beneath. For to all of these, among the other opinions, this one seemed probable, as it did also to Thomas Fienus [Thomas Fyens], in Froidmont, Book 3 of the *Meteorologica*, chapter 2, article 2.

[Margin: 4. The opinion of a comet composed of many stars.]

[Margin: Anaxagoras, Democritus and Zeno.]

V. The *fourth* opinion was that comets are an aggregate of many small stars — eternal ones, or ancient ones — coalescing into one, and afterwards vanishing when they separate. For it is of these, and not of the commonly known planets, that Anaxagoras and Democritus are to be understood, as we showed in number 3; of them Aristotle says, *Meteorologica* Book 1, chapter 6: *"Anaxagoras, then, and Democritus say that comets are a co-appearance of wandering stars, when, because they come close, they seem to touch one another."* And a little below: *"Democritus contended for his opinion; for he says that, when comets dissolved, certain stars appeared."* Plutarch, too, is to be taken in the same way when he says of comets in *De placitis* Book 3, chapter 2: *"Anaxagoras and Democritus: that it is a synod of two or more stars illuminating one another."* Indeed Seneca, Book 7, chapter 3, testifies that Democritus spoke of stars unknown in number and in name, in these words: *"Democritus says that he suspects there are more stars which run their courses, but he set down neither their number nor their names."* Zeno held the same, as Seneca relates in Book 7, chapter 19, saying: *"Our Zeno is of this opinion: he judges that stars come together and join their rays with one another; that from this fellowship of light there arises the image of a longer star."*

[Margin: Probable to Tanner, Cysat, Liceti, Resta, Cardano, Bruno.]

This opinion Tanner, *On the Heavens* p. 150, and Cysat, *On Comets* chapter 7, thought very probable; and Liceti, Book 5, chapters 52 and 55, and Resta, *Meteorologica* Book 1, tract 1, chapter 3, adopted it as regards celestial comets; and before them Cardano, *De varietate rerum* Book 1, chapter 1, when he admitted that comets are knots formed from many small stars intersecting one another, but lit by the rays of the Sun and drawing a tail; and Iordanus Brunus [Giordano Bruno], in the book *De innumerabilibus*. Liceti, however, held this of the celestial comets, for he does not deny that some are below the Moon.

[Margin: 5. The opinion that a comet is a sublunary meteor and an exhalation catching fire.]

VI. The *fifth* opinion is that of Aristotle, *Meteorologica* Book 1, chapters 7 and 10, and of very many interpreters there, and the common opinion of the Peripatetics; but above all of the Coimbra commentators (tract 3 on the *Meteorologica*, chapter 3), of Cottunius (Lecture 31 on *Meteorologica* 1), and of

Chiaramonti in the *Antitycho*, Book 3, last chapter, and in all the other booklets he published on the sublunary seat of comets or in defence of the *Antitycho*.

[Margin: Aristotle.] [Margin: Coimbra commentators.] [Margin: Cottunius.] [Margin: Chiaramonti.] [Margin: Giuntini.] [Margin: Pontano.] [Margin: Regiomontanus.] [Margin: Vögelin.] [Margin: Milich.] [Margin: Nolthius.] [Margin: Vuinchlerus.] [Margin: Dasypodius.] [Margin: Steinmetz.] [Margin: Bazelius.] [Margin: Henisch.] [Margin: Huuernius.] [Margin: Graminaeus.] [Margin: Erastus.] [Margin: Surdus.] [Margin: Paduanus.] [Margin: Piso.] [Margin: Aversa.] [Margin: Mastri.] [Margin: Belluto.]

In this the Peripatetics were followed, among the astronomers or astrologers, by Giuntini in his *Adnotatio de Cometis*; by Pontano in the last aphorism on Ptolemy's *Centiloquium*; by Regiomontanus and Io. Vogelinus [Johannes Vögelin] in their observations of comets, the former of the comet of the year 1572, the latter of that of 1532; by Iacobus Milichius [Jacob Milich] on Pliny Book 2, chapter 25; and (if we believe Angestus, *On the Properties of Forms* Book 2, p. 34) by all the ancient astronomers. To these must be added the more recent: Andreas Nolthius, Nicolaus Vuinchlerus [Nikolaus Winckler], Cunradus Dasypodius [Conrad Dasypodius], Valentinus Steinmetz [Valentin Steinmetz], Nicolaus Bazelius, Georgius Henischius [Georg Henisch], Io. Huuernius, Theodorus Graminaeus, Thomas Erastus, Petrus Surdus, Antonius Paduanus and Carolus Piso, in their tracts on comets; Raphaël Aversa [Raffaele Aversa], *Philosophia* tome 2, from question 32; and Bartholomaeus Mastrius and Bonaventura Bellutus [Bartolomeo Mastri and Bonaventura Belluto], disputation 4 on the heavens, question 4, article 2, number 36.

The opinion of Aristotle and of these authors, then, is that a comet is a fiery meteor [meteoron], newly generated from a great quantity of exhalations raised up from the earth and the sea as far as the highest region of the air. This quantity, when that air is dispersed, is separated off and, well compacted, acquires consistency; and, carried round by the sweep of the primum mobile by which the highest air is revolved, it catches fire and burns for a long time, as long as that sulphurous, unctuous, fatty and oily or nitrous quantity of exhalations (these epithets are used not by Aristotle but by his followers), gathered either from the beginning or successively, supplies tinder or food for the flame; and when this fails, comets gradually diminish and at last are extinguished. Granted, Aristotle admits that some comets, which seem to move with the motion of some star, acquire hair or tresses from rays refracted in vapours drawn upward, and that exhalations suitable for this are separated off by the force of that star. By the same reasoning he thinks that, by the force of the very many and great stars which are above the Milky Way [lacteus circulus], those exhalations are separated off and intensified, so as to produce there a kind of perpetual comet — for that is what he calls the Galaxy in that chapter 10. In the same way he also affirms that one of the stars in the hip of the Dog was observed with some sort of hair, and that a comet followed its motion.

Pontano, however, Milich, Mastri and Piso hold that comets are not swept along by the daily motion of the air, but are kindled, or set aflame, by the stars and above all by the Sun; others, by the sphere of fire; others, by antiperistasis, as happens with lightning and shooting stars. Piso, moreover, places hairy comets in roughly the middle region of the air, but tailed ones in the highest, or even a little below the Moon.

[Margin: Strato.] [Margin: Bobetus.] [Margin: Xenophanes.] [Margin: Epigenes.]

Among the more ancient authors, certain others seem to be brought under this head, of whom Plutarch, *De placitis* Book 3, chapter 2, reporting the opinions on hairy stars, says: *"Strato: that it is the light of a star enclosed in a dense cloud, as happens also with torches; Epigenes: that it is a spirit having something earthy in it, carried aloft and catching fire there; Bobetus: that liquefied air presents an appearance of this kind; Xenophanes: that they are clusters or flashings of kindled clouds."* But on Epigenes Seneca is more explicit, Book 7, chapter 3: *"Epigenes, on comets: that they seem to be kindled by a certain whirlwind of agitated air"*; and chapter 6: *"There are,"* says Epigenes, *"two kinds of comets. Some pour out their heat on all sides and do not change place; others stretch out a wandering fire in one direction like hair, and pass by the stars, such as the two seen in our own age. The former, hairy on all sides and motionless as well, are generally low, and are formed from the same causes as beams and torches, and from the disorder of turbulent air, which turns over with it many dry and moist things exhaled from the earth. For spirit forced through narrow places can kindle the air placed above it, full of nourishment suited to fire,"* etc.

[Margin: Perhaps the Stoics too.]

Finally, there are also referred to this head those — whether they were Stoics or Latins — of whom Seneca wrote in the same Book 7, chapter 21: *"Our people therefore hold that comets, like trumpets and beams and the other portents of the sky, are created from dense air; and that for this reason they appear most often around the north, because there is a very great deal of sluggish air there."* And, giving their mind, he adds that a comet creeps to where the vein of its fuel leads, and does not advance like a star but feeds like a fire, etc.

[Margin: 6. The opinion that a comet is a sublunary meteor or exhalation, but illuminated by the Sun.]

VII. The *sixth* opinion is that a comet is indeed a sublunary meteor made of subtler exhalations [halitus] or vapours, but not so much inflamed as illuminated by the Sun because of a certain transparency, like glass phials tinted with a yellow or red colour and full of water; or, as Scaliger says in *Exercitation* 79 against Cardano (Telesius [Bernardino Telesio], *On Comets* chapters 2 and 3, favours him), and Apian in *On the Comet of the Year 1531*: that it is a subtle vapour carried up by its own lightness, or drawn by the stars into the highest region of the air, like a smoke or soot resembling twilight, but so thinned that it receives and then transmits the rays of the Sun. Thus, on the testimony of Plutarch, *De placitis* Book 3, chapter 2, Heraclides Ponticus thought that a

comet was a very high cloud lit by light; and Metrodorus, that it was the Sun striking more violently into the clouds and sparkling.

[Margin: Scaliger.] [Margin: Apian.] [Margin: Telesius.] [Margin: Heraclides.] [Margin: Metrodorus.]

[Margin: 7. The opinion that a comet is an earthly exhalation, but raised above the Moon and illuminated.]

VIII. The *seventh* opinion is that a comet is indeed made of elementary matter, that is, of subtler exhalations, which strive upward by their lightness so far that they climb above the Moon, and by their transparency drink in the rays of the planets, especially of the Sun, and transmit them behind them into a tail and beard. So Marius Guiducius [Mario Guiducci], in his treatise *On Comets*; Galileo, in Dialogue 1 *On the System of the World* and in his *Trutinator* [*The Assayer*]; Aresius, Johannes Praetorius and Christoph Rothmann, in his treatise on comets and in the letters to Tycho, pp. 91 and 161 — although Rothmann thinks that these exhalations are raised by angelic power right up into the heaven, to terrify mortals. Tycho, however, in the letters, p. 161, indicates that Rothmann retracted and conceded that comets not only are, but are also generated, above the Moon.

[Margin: Galileo.] [Margin: Rothmann.] [Margin: Aresius.] [Margin: Praetorius.] [Margin: Guiducci.]

[Margin: 8. The opinion that comets are made from celestial matter, con-]

IX. The *eighth* opinion is that comets are made anew from celestial matter, not through the generation of a new substantial form, but through condensation with a mixture of opacity and

[Margin: densed and illuminated.]

transparency, so that they receive and transmit the light of the Sun; or through an addition of light made in parts of the heaven.

[Margin: Libavius.] [Margin: Liceti.] [Margin: Tanner.] [Margin: Glorioso.] [Margin: Resta.]

So holds Libavius in a separate treatise, part 2, in the chapter on comets; and Liceti accepts this opinion above all others, *On Comets* Book 5, chapters 52 and 55; so too Tanner, *On the Heavens* p. 148, and Camillus Gloriosus [Glorioso], *On Comets* Book 5, chapter 1, as regards motionless comets and new stars; and Franciscus Resta, *Meteorologica* Book 1, tract 1, chapter 4. Froidmont had once embraced the same opinion, but afterwards rejected it, Book 3, chapter 2, article 5. If, however, this condensing [addensatio] is made from the small stars of the Milky Way, Liceti denies that it should be called condensation in the strict and proper sense. Froidmont, for his part, thinks that Tycho held this opinion, that comets were made through the condensation of the Milky Way.

[Margin: 9. The opinion that comets are generated anew from celestial matter.]

X. The *ninth* opinion holds that comets are generated anew in the heaven from celestial matter, and afterwards corrupted. Some hold this of all comets:

so Tycho indicates here and there in the letters and in tome 2 of the *Progymnasmata*, chapter 6 and the last chapter (for he uses the word "generation"), and Longomontanus in the Appendix to his *Astronomia*, chapter 3. Others hold it of very many, or without specifying: so Kepler, *Epitome of Astronomy* Book 1, pp. 55 and 57, and in the *Harmonice*, p. 242, but more fully in *On Comets* Book 2; Io. Bapt. Cysatus [Johann Baptist Cysat], *Mathemata* on comets, chapter 7; Thomas Fienus, Marcellus Squarcialupus [Marcello Squarcialupi], D. Habrechtus, Erycius Puteanus, Ambrosius Rhodius and Villebrordus Snellius [Snel], in their treatises or booklets on comets.

[Margin: Tycho.] [Margin: Longomontanus.] [Margin: Kepler.] [Margin: Cysat.] [Margin: Fienus.] [Margin: Puteanus.] [Margin: Squarcialupus.] [Margin: Habrechtus.] [Margin: Rhodius.] [Margin: Snel.] [Margin: Mästlin.] [Margin: Gemma.] [Margin: Röslin.] [Margin: Hájek.] [Margin: Scultetus.] [Margin: Glorioso.] [Margin: Froidmont.] [Margin: Cabeo.] [Margin: Teles.]

The same was held — not of all comets, but of some — by Mästlin, *On the Comet of the Year 1577*; by Cornelius Gemma in his *Physical and Mathematical Proof* [*Apodixis*] on the comet of the year 1577; by Helisaeus Röslin in his writing on the same comet; by Hájek, *On the Comets of the Years 1577 and 1580*, having retracted his earlier view, in which he had maintained that all comets are made from sublunary matter and are below the Moon, as Tycho relates, tome 2, p. 367; and by Bartholomaeus Scultetus, who himself also abandoned his earlier opinion on comets. Finally the same was judged — not of all comets, but only of the celestial ones (for they also admit elementary or sublunary comets) — by Io. Camillus Gloriosus [Glorioso], *On Comets* Book 5, chapter 1, as regards moving and wandering comets (for he holds that the motionless ones are made by condensation); by Froidmont, *Meteorologica* Book 3, chapter 2, articles 6 and 7; by Cabeo on *Meteorologica* Book 1, text 37, question 6; and by Baltazar Tellez [Baltasar Teles], physics disputation 40, section 1, and disputation 47, section 2.

[Margin: Various modes of this generation.]

Not all, however, explain this generation in the same way. For Tycho and Longomontanus think that comets are made from that matter of the heaven which is in the Milky Way, as being more nearly prepared and disposed for it, and that they are bodies not blazing, but transparent and translucent. Kepler, however, thinks that the ethereal air is crammed with thicker fumes and soots exhaled by the planets. Like excrements, these are gathered into one, like an abscess [apostema], by a certain animal faculty of the heaven, so that they may not cloud over the Sun and stars any longer — as he teaches they have done more than once; and at last, when by their transparency they have transmitted the rays of the Sun (from which they get their tail), they are dispersed. So the heaven is from time to time purged of these impurities; and as whales and monsters are begotten in the sea, so comets are begotten in the heaven in that extraordinary way.

[Margin: What the solar spots are.]

But Fienus, Snel, Puteanus, Cysat, Froidmont and Glorioso hold that comets, as to origin and matter, are made in the same way as the solar spots or faculae. Thus Snel calls them κατακαύματα [Greek: katakaumata, "scorchings"; glossed on the page by the Latin *exustulationes*], or scorchings emitted by the Sun, or rather breathed out from the surrounding [peristatic] matter of the Sun and blown far off elsewhere. Cysat holds that many of those faculae are gathered into one, and that for this reason a compact nucleus made of small stars is seen with the telescope, and these are afterwards dispersed. Puteanus, most elegantly, says that a comet is *"a citizen of the heaven, an offspring of the Sun, a part and a birth of the aether; a torch in its light, a wanderer in its motion; with a body of its own, a light not its own, a motion its own and not its own"*; and that it is generated from the corpuscles which spot the Sun, but purged and illuminated by the Sun. Froidmont holds that they are not breathed out by the Sun, lest this universal lamp of the world be exhausted, but drawn out from the other planets. Our Cabeo, by contrast, though he had said that the faculae of the Sun are spots, but now purged and illuminated by the Sun, and that comets are sometimes made from them, nevertheless denies that these effluvia or volatile soots are exhaled from the Sun alone, or chiefly from the Sun; rather, they are gathered from all the other stars, at least the wandering ones, towards the Sun, and are from there purged again into faculae and become comets. Finally, our Teles too recognizes that comets are generated from the exhalations of all seven planets. As to the remaining authors, it is not clear enough to me in what way they understood this generation of comets.

[Margin: 10. The opinion that comets are of celestial matter, but have come down below the Moon.]

XI. The *tenth* opinion — held so far, as far as I know, by no one except Kepler, as I shall say in the last chapter, but conceivable nonetheless — is that comets are made anew from celestial matter, but that some are sent down below the Moon, just as Opinion 7 said that they are made from elementary matter but ascend above the Moon. And certainly, if they were effluvia of the planets, there would be no necessary cause why those exhaled from the Sun, Venus, Mercury and the Moon at their perigees could not sometimes be carried below the Moon.

[Margin: 11. The opinion that comets are made by a supernatural creation or generation by God, directly or through angels, and are moved by the angels.]

XII. The *eleventh* opinion is that, if not all comets, then at least many, are either created by God from nothing, as Mästlin suspected (in Liceti), or else are made, and afterwards moved wherever it may please, through angels, out of whatever matter may please, celestial or elementary, with any form and figure whatever imposed on them, to terrify mortals and to signify the calamities discussed in chapter 5. And in this way all questions about comets are easily solved.

[Margin: St Damascene.]

St Damascene [John of Damascus] seems to approve this opinion, *On the Faith* Book 2, chapter 7, when he says: *"Comets too are frequently produced, certain signs of the deaths of kings; they are not of those stars which were made at the beginning of things, but are established by divine command at a fitting time and dissolved again"*; and he compares them with the star of the Magi as regards their supernatural generation.

[Margin: Tanner.] [Margin: Arriaga.] [Margin: Oviedo.]

The same opinion Tanner defends as highly probable and worthy of a Christian philosopher, *On the Heavens* from p. 141 to p. 147. Ambrosius Rhodius goes over to it absolutely, as does Arriaga [Rodrigo de Arriaga] in his single disputation on the heavens, section 2, subsection 3, to whom Oviedus [Francisco de Oviedo] subscribes (single controversy on the heavens, point 3, paragraph 3); and so does Gulielmus Hirsaugiensis [Wilhelm of Hirsau] in his *Institutes of Philosophy and Astronomy*, whose definition is: *"A comet is a fire kindled according to the will of the Creator to signify something."*

[Margin: Rothmann.] [Margin: Froidmont.] [Margin: Resta.]

As to the motion of comets, and their direction by angels towards the region that Divine Providence has appointed, this was the view of Christoph Rothmann; nor did it wholly displease Froidmont, *Meteorologica* Book 3, chapter 4, article 4; and it did please Franciscus Resta, tract 1 on the *Meteorologica*, chapter 6, where he teaches *a fortiori* that these temporary planets need the help of ruling angels, since not even the others are moved from within, and there is nothing that is, or is moved, in the heaven which is not subject to the administration of angels.

[Margin: Cabeo.]

And Cabeo, *Meteorologica* Book 1, text 37, question 8, having considered the conditions of the motion of comets, says that no sufficient cause can be assigned why they move as they do, unless recourse is had to some Intelligence moving freely towards a predetermined end.

[Margin: The view of Tycho and of Regiomontanus.]

Whether we speak of the making or of the motion of comets: although Longomontanus says in the Appendix, chapter 4, that our God does whatever he wills in the heavens, but mostly uses secondary causes, and in producing comets uses chiefly the conjunction, opposition or quadrature of Mars and Mercury, he nevertheless quotes words of Tycho commending Thaddaeus Hagecius [Hájek], which go far to support this 10th opinion. For Tycho, *Progymnasmata* tome 1, p. 513, speaks thus of Hájek: *"At the end of the same chapter he touches on some points concerning the celestial generation of comets. Although he frankly confesses that he does not know it, he does not on that account deny that it can happen, but ascribes how it comes about to the wonderful works of GOD, and asserts that it is far removed from our full knowledge — which I too do not deny,"* etc. *"Meanwhile I willingly agree with those who place comets among the secrets of nature not yet explained, and who assert that they arise from hidden causes, as Regiomontanus proves from Māshāʾallāh [Meſhala]."* Further, Regiomontanus says in his little work on

the comet of the year 1472: *"Comets come from hidden causes of nature; of which opinion is Meshala the Arab."* Here Seneca would exclaim, Book 7, chapter 30: *"How many comets besides these go about in secret, never rising for human eyes! For God has not made all things known to human eyes."*

[Margin: 12. The opinion admitting some comets above the Moon, others below the Moon.]

XIII. The *twelfth* opinion is not distinct from all the preceding ones, but rather distinguishes celestial comets from elementary, or sublunary, ones as to place and matter. For it admits that some comets are made below the Moon from exhalations of the terrestrial globe, whether inflamed or illuminated, but that others are above the Moon and made of celestial matter, in the ways reported in number 10. In this class are Mästlin, in Tycho, tome 2, p. 254, and, in the same author, Gemma, Röslin, Scultetus, Hájek, likewise Kepler in the *Hyperaspistes*, p. 82, and in *On Comets*, p. 99; Tanner, *On the Heavens*, question 7; Liceti, *On New Stars and Comets*, Book 5; Froidmont, *Meteorologica* Book 3, chapter 1, articles 1, 3, 4, 6; Cabeo on *Meteorologica* Book 1, text 37, questions 2, 4, 5, 6; Habrectus, in Longomontanus, in the Appendix, chapter 2; Resta, Book 1, treatise 1 of the *Meteorology*, chapters 3 and 4; and Rothmann, in Froidmont. Whether Aristotle should also be counted among these authors, as Liceti would have it, I shall discuss in Scholium 2; and whether Abū Maʿshar, Proclus, Haly and Nicephorus should be, as some think, in Scholia 3 and 4.

[Margin: The method to be followed from here on in this controversy.]

We shall therefore set out the foundations of these opinions, together with the arguments for and against them and the solutions of those arguments, so that this synopsis of the reasons may make it easier to pass judgement on them. Then we shall weigh separately those arguments that need deeper examination and that bear on several opinions at once: namely, on the motion of comets, whether curvilinear along a great circle or rectilinear; on the altitude of the regions of the air; and on the parallax and altitude of comets. The problems, however, and most of the geometrical figures that belong to this controversy and to a more refined examination of parallaxes, we shall set out from chapter 16 to chapter 22.

SCHOLIA.

Scholia.

[Margin: Bodin's ravings about comets.]

I. It was shameful indeed to count, among opinions in any way probable, the ravings of Bodin, who says in Book 2 of the *Theatre*: "The opinion of Democritus came to my mind, so that I think that comets are the minds of illustrious men, which, after flourishing on earth through countless ages, at last, when about to die, celebrate their final triumphs, or are called back into the starry heaven like splendid stars. And for that reason famines, epidemic diseases

and civil wars follow, as though cities and peoples were deserted[14] by those excellent leaders and governors who used to calm divided furies." He does add that he does not wish to assent to anything on such difficult matters; but by the very fact that he puts these things forward and does not condemn them, he fails in the office of a good statesman. He has therefore deservedly been condemned himself by Franciscus Resta, *Meteorology* Book 1, treatise 1, chapter 1, and by Froidmont, Book 3, chapter 2, article 2, who calls him a political fool, a rabbinical philosopher, and a scourge of the Peripatetics [Peripateticomastix].

[Margin: Comets are not the souls of illustrious men.]

And indeed, on what ground does he impose this foolishness on Democritus? Is it because Democritus thought that comets were celestial bodies and stars unknown to us? And what are those countless ages in which the heroes lived? Are these things not drawn from the Pythagorean transmigration, that μετεμψυχῶσι [Greek: metempsychōsis, "transmigration of souls"] so often condemned by Catholics? Next, how are comets the souls of illustrious men already dead, if the more common consensus of the wise is rather that they shine out for some days or months before the death of illustrious men? Do they die before themselves, and become comets twice, so as to be both the Lucifer and the Hesperus of their own deaths? Yet on the testimony of Suetonius and Pliny the Elder, Augustus wished it to be believed that the comet which arose in his time was Julius Caesar received into heaven, and he set a long-haired star on the crown of Caesar's statue — as Pliny and Plutarch also relate. Again Lucan, the Pompeian poet, so as not to make Pompey lesser than Caesar, sang that Pompey's soul was gathered into the eternal orbs; indeed, as the shamelessness of flattery grew, he boasted that Phoebus would yield his own star to Nero. And many others among the pagans, whether philosophasters or poets, have declared that the souls of heroes descend from the stars and return into them. Very well; but all these things come from the stench of paganism and of a foolish mythology, which, so far as lay in them, befouled or offended heaven itself with a most foul stink.

[Margin: Liceti's false opinion about supralunar comets recognised by Aristotle.]

II. It will be better worth the effort to see whether Aristotle recognised some comets as celestial, as Fortunio Liceti contends in *On New Stars*, Book 5, chapter 41. He argues from *On the Heavens* Book 2, chapter 7, where the Philosopher says that the stars are made of the body in which they have their motion [latio], and from chapter 8, that the stars are spherical and that this befits what generates them out of a spherical body, which is the heaven. Yet Aristotle does not seem to be speaking of the ancient stars, which he posits from eternity; therefore he speaks of stars newly generated from celestial matter. Moreover, in *Meteorology* Book 1, chapter 1, he affirms that comets are born in the place nearest to the motion of the stars; and in chapter 7 he said that

[14]printed *defererentur*, with f; construed as *defererentur*

when a condensation of exhalation occurs beneath some star, wandering or not wandering, then the comet becomes one of them, and when such a concretion occurs next to a star, the comet is necessarily carried with the same motion as that star, and so without parallax.

But Liceti is opposed by Froidmont, *Meteorology* Book 3, chapter 1, article 5; Cottunius, lecture 31 on *Meteorology* Book 1; Franciscus Resta, *Meteorology* Book 1, treatise 1, chapter 2; and Mastri, disputation 4 *On the Heavens*, question 2, article 4, number 43 — and rightly. For it is manifest that Aristotle, in *On the Heavens* 2, chapters 7 and 8, speaks not of some stars newly generated, but of all that are in heaven — those to which, as he confirms from their motion and roundness, celestial matter belongs, and which are generated (that is, from eternity, as Aristotle holds) by an eductive action out of matter, yet without that matter pre-existing by priority of time. And since in so many other places he asserts that the heaven is ingenerable and incorruptible, there was no call to beg little arguments for a new generation of comets in heaven from those two places on account of the particles "ex" and "genitum". Nor is it less manifest from Aristotle that the place of comets is the highest region of the air, in which ἐκπυρομένα φάσματα [Greek: ekpyromena phasmata, "ignited apparitions"] arise, that is, fiery appearances — granted that he says this region, compared with the lower regions of the air, is very near to the motion of the stars.

As for the second kind of comet, which he recognises in *Meteorology* Book 1, chapter 7, what kind it is is plain to anyone who looks at the text. For he meets an objection that someone could raise: that comets are sometimes seen nearest beneath the stars themselves and moving with the same motion without any parallax, and are therefore in heaven. And he says: "When, however, an exhalation has settled through the motion beneath one of the stars, wandering or not wandering, then one of these becomes a comet. For the coma does not form at the stars themselves; but just as halos [areae] appear around the Sun and the Moon and follow their motion, even though the stars are carried along, when the air has been so condensed that this affection occurs beneath the Sun's path, so also the coma is like a halo to the stars. Yet a halo of that kind arises through refraction, according to the apparent colour, whereas there the colour arises in the exhalations themselves." Just as the area, or halo, of the Sun or Moon therefore appears without parallax, as if it were a corona surrounding the Sun or Moon, and yet is really in the elementary region — because it forms in one part after another of a little cloud or of vapour spread beneath them, or at any rate because that vapour itself follows the motion of the luminaries by a motion either of impulsion or of attraction — so, when an exhalation condensed and mixed with vapour has moved with the motion of some star, it will seem to be immediately beneath that star without any variation of view, and the coma of comets will seem to be the coma of the star, or to be one thing with it, although in reality it is in an exhalation and in an elementary place.

[Margin: The Dog, Venus and Arcturus seen with their halo.]

Aristotle further adds the example of a certain star in the coma of the Dog, which he saw crowned as if with a kind of halo; just as we have often seen Venus wrapped about by a halo among vapours, so far as appearance goes. Froidmont, in the place cited, relates that he saw this very thing in Venus in the year 1618 and in Arcturus in 1624; indeed, we too have observed it in Jupiter more than once.

[Margin: Nicephorus wrongly adduced for a comet seen above Venus.]

III. I am no less doubtful whether, as our Cysat affirms in his *Mathemata* on comets, chapter 3, proposition 7, Abū Maʿshar observed a certain comet above Venus in the year of Christ 844, and Proclus another comet above Jupiter around the year 390. This very thing Giovanni Camillo Glorioso, *On Comets* Book 4, chapter 8, attributes to Proclus on Tycho's authority and to Abū Maʿshar on Cardano's, adding, about the comet seen above the sphere of Venus, that Nicephorus and Haly Ben Rodoan report that the same thing happened in their age; he cites Nicephorus, Book 12, chapter 37, and Haly on Book 2 of the *Quadripartitum*, chapter 9. But to begin with Nicephorus: already in chapter 3, at the year of the Lord 392, we reported his words, from which nothing is gathered about a place of the comet above Venus, but only that it appeared next to Venus and in light and apparent size was not much inferior to Venus — that is, a little less bright and large. Besides, as I said in the same place, and as Froidmont points out in *Meteorology* Book 3, chapter 2, article 3, Nicephorus lived around the year of Christ 1300, whereas the comet he describes shone out a little before the emperor Valentinian was found strangled in his bed, which happened in the year 392.

[Margin: The pseudochronism of Giovanni Camillo Glorioso.]

So Glorioso, inglorious in my view in this place, falls into a pseudochronism when he refers the observation of that comet to the age of Nicephorus.

[Margin: The doubtful testimony of Proclus for a comet seen above Jupiter.]

Nor does he skilfully lay at Tycho's door what is said of Abū Maʿshar and Proclus; for in tome 1 of Tycho's *Progymnasmata*, pp. 619 and 625, which Glorioso adduces, nothing of the kind is asserted absolutely by Tycho; only the words of Caspar Peucer are reported, who says: "Many have so far laughed, as at something incredible, at what Proclus noted somewhere — that a certain comet was seen above Jupiter, I know not when. In this indeed he could have been free of error if that comet was in conjunction with Jupiter: for the eyes could then have shown this; but if there was no conjunction, that philosopher ought to have shared with us how he had detected that it was higher than Jupiter" — as if he were saying that the comet could have been asserted to be higher than Jupiter if it had been seen so conjoined with Jupiter that Jupiter hid it by a kind of eclipse. Likewise, on page 625, the words of Wolfgang Schuler are adduced, replying to Peucer himself in this way: "To this kind, I judge, the comet too should be referred which Proclus mentions in the commentaries he wrote on Plato's *Timaeus*." But Wolfgang adds that he would have wished Proclus, or the anonymous author whom Proclus cites, to have added the time,

the place, the author and the manner of that observation, indicating that it was suspect to him; if, however, it is true, the comet should be placed among the extraordinary τέρατα [Greek: terata, "portents"], or signs sent by God beyond the customary course of secondary causes. Tycho himself, moreover, in the same place, p. 626, adds — not absolutely but hypothetically: "If Proclus asserts that some comet was observed above the revolutions of Jupiter, he has said nothing absurd." And a little later: "For it is not likely that he pronounced this without evident reasons, since he was without doubt not ignorant of the Aristotelian doctrines." But what if that observer said that the comet was seen higher than Jupiter, or above Jupiter, in respect of altitude from the horizon on a vertical circle? So that, if Jupiter appeared 40° high, the comet appeared 41° high or thereabouts. For Nicephorus indeed holds, in Book 1, chapter 13, that all comets are made in the elementary region.

[Margin: Was the comet seen by Abū Maʿshar and Haly higher than Venus?]

IV. Let us come to Abū Maʿshar. Tycho, in tome 1 of the *Progymnasmata*, p. 783, reports the words of Adam Ursinus of Nuremberg, who adduces from an aphorism of Cardano these words: "Abū Maʿshar said that a comet was seen above Venus; therefore it is not in the elementary region, contrary to the Philosopher." Tycho adds that Cardano said this rightly, because comets properly so called are ethereal. I, however, find in Cardano, *On Subtlety* Book 4, p. 155, these words, in which he attacks Aristotle's opinion that a comet is made in the elementary region as absurd: "There are many things besides this that show the absurdity of this opinion, from which I refrain: such as that saying of Abū Maʿshar, that a comet was above Venus; and Venus is much higher than the Moon." But perhaps Abū Maʿshar meant that Venus was lower than the comet in respect of altitude from the horizon. Further, the words of Haly, on Book 2 of the *Quadripartitum*, chapter 9, are these: "I wish to relate a portent which I saw when I was young and studying: a comet appeared whose solid part was of round shape and three times larger than Venus." From this, I think, arose the occasion for someone to read that it was three times higher than Venus. Nothing certain, therefore, follows from this.

[Margin: The error of Froidmont, Cottunius, Tycho, Mastri and Aversa in reporting authors.]

V. From the passages we adduced in the preceding chapter you will gather that certain opinions are falsely ascribed to certain authors, either through lapse of memory or through carelessness. For Froidmont, *Meteorology* Book 3, chapter 1, article 1, counts Piso among the champions of superlunary comets, though Piso plainly stands for sublunary ones. The same Froidmont, chapter 2, article 2, and Tycho, tome 2, p. 161, list Fracastoro among the defenders of celestial comets; yet he does not teach this, but only posits a circle passing through the heaven of the Moon and moving the Moon and the air beneath it, and by its means moving comets in latitude, in the way that he teaches they are carried in longitude by the motion of the lunar heaven. Indeed he plainly places them in the air, as will be clear to anyone who reads section

3 of the *Homocentrica*, chapter 23. Cottunius too, at the end of lecture 32, places Epigenes among those who asserted that comets are celestial and eternal, although it is evident from Seneca and Plutarch that Epigenes thought comets were a sublunary meteor kindled by the force of a whirlwind and of stirred-up air. Finally, Mastri, disputation 4 *On the Heavens*, question 2, article 4, number 39, following Aversa, lists Hájek among those who gave all comets a sublunary origin and seat.

[Margin: Hájek's palinode on the seat of comets.]

But he ought to have read Tycho, tome 2 of the *Progymnasmata*, p. 366, where Hájek, retracting his earlier opinion, says: "In the most recent comet, of the year 1577, for the reasons given above, I could not apply equal diligence. So it could very easily have happened that some error crept into my givens [didomena] too. For I myself have only now noticed, in these second thoughts of mine, that the parallax I assigned cannot correspond to the distance of the comet taken from the neighbouring stars at different hours and varying by scarcely 4′ or 5′. So I would judge that that comet too should be placed above the Moon." For he also set the comet of the year 1580 above the Moon, as Tycho relates in the same place.

CAPVT VII.

Chapter VII

Cometam non eſſe meram Apparentiam Luminis reflexi aut refracti, aut Coapparentiam plurium notarum ſtellarum.

That a comet is not a mere appearance of reflected or refracted light, nor a joint appearance of several known stars.

I. Against Panaetius, who said that a comet is a false appearance of a star, or a mere φὰσμα [Greek: phasma, "apparition"] and image, or against others who, as I said in chapter 6, number 2, held that it is a reflection of the visual rays made towards the Sun in a mirror of air, according to the first opinion on vision, that it takes place by extramission — against these argue Seneca, *Natural Questions* Book 7, chapter 30; Piso, in *The Mirror of the Comet*, chapters 13 and 14; Glorioso, *On Comets* Book 1, chapter 3; Franciscus Resta, treatise 1 of the *Meteorology*; Cabeo, *Meteorologica* Book 1, text 37, question 3; and Lotharius Sarsius [Orazio Grassi], *The Astronomical Balance*, p. 28, with Galileo's comments in *The Assayer* [Trutinator] from p. 113 to p. 121 — with roughly the following arguments.

[Margin: 1. Argument from long duration.]

First: phenomena that are mere emphases and consist in a kind of appearance — such as rainbows, rods [virgae], areae or halos, and parhelia — are not long-lasting, but quickly dissolve, and in that brief time, which seldom fills an hour, they take on various forms or positions. But comets persist through many days and months under the same principal appearance in which they first appeared, as is clear from what was said in chapter 4, number 5. Therefore comets are not mere phantoms [idola] of stars.

[Margin: 2. Argument from matter and place.]

Second: phenomena that have no real substance [consistentia], such as rainbows, coronas and the like, require a matter suited to marked refraction, or reflection, or both — namely a dewy cloud, or denser vapours, or a mirror, so to speak, of vapour, which is therefore raised little from the earth by its own weight and is not visible at the same time in widely separated parts of the earth. But the matter of a comet is seen raised far above clouds and vapours of this kind, as will be plain from what is to be said about the parallax of comets. Therefore comets are not mere images like a rainbow, a halo and the like.

[Margin: 3. Argument from the freedom [?] of motion.]

Third: phasmata, which are images of the Sun, the Moon or other stars, have such a connection with their prototype that they follow its motion — either towards the same side, if they are made by mere refraction, as halos are; or towards the opposite side, if by reflection with refraction, as rainbows are, which rise as the Sun descends and descend as it rises, since through the circular motion of the stars the angles of incidence on the reflecting matter vary — or else they are bound by a similar optical law to the motion of their prototype, as will be said in the scholia. But the motion of comets is its own master, and has no such connection with the motion of the Sun or of the other stars, as will be quite clear to anyone who rereads what we related in chapter 4, number 10.

[Margin: 4. Argument from an appearance [?] not bound by optical laws.]

Fourth: whatever things are mere emphasis, seen by reflection or refraction, are seen only by an eye placed in a determinate position, not by all, nor equally; and meanwhile some people in the same province or kingdom do not see them, even if nothing else prevents it. For when I see a rainbow or a halo at Bologna, someone at Ferrara or Florence does not see the same one, let alone someone at Rome. And if it is a halo or a corona, these must lie between the Sun or Moon and our eye, or be conjoined with their star; but if they are like rainbows, the eye must be on the same straight line between the Sun and the rainbow, which appears only in opposition to the Sun. Or at least, to see images of this kind the eye must be on the line of reflection, the line that makes angles in the reflecting matter equal to the angles of incidence; for to eyes moved outside that position they do not appear. But comets, with no necessity from these laws, appear at the same time, in the same form and shape, to the eyes of all, in diverse and widely separated regions; and scarcely a comet will be found that has appeared conjoined with the Sun or the Moon or another planet, or opposite to it.

[Margin: 5. Argument from the presumption of real being in everything that appears, unless the opposite is proved.]

Fifth: whatever appears to the eyes must be called real objects and true physical beings, unless we are convinced on other grounds, by strong arguments, that they are mere phenomena, or fleeting and empty images of bodies (as we are convinced by optical and geometrical arguments of the mere appearance of rainbows, parhelia and halos). This holds, first, because one must not presume that what appears to be is not, since possession stands for being rather than

for not being, and the burden of proof falls on whoever says that these things have no fixed and firm being; and secondly, because otherwise many other phenomena of the heaven and of the air could be called into doubt — and from the fact that images of the stars appear in waters and in other reflecting bodies, someone could assert that the planets are images of the Sun shining back from transparent little clouds, and so on for similar cases. Let it stand, therefore, by the common consent of the observers who have ever seen a comet, that it is not an empty spectre, or an illusory image appearing to us through the ἀνάκαμψιν [Greek: anakampsin, "reflection"] or ἀνάκλασιν [Greek: anaklasin, "refraction"] of the solar rays — that is, their rebound or refraction — but some true body, shining either because it is kindled or by a light it has received and retains.

Cometam non eſſe σύμφασιν, ſeu Coapparentiam duorum vel plurium ex iam notis ſideribus.

That a comet is not a σύμφασιν *[Greek: symphasin, "joint appearance"], or co-appearance, of two or more of the stars already known.*

II. We pass without any trouble from emphasis to the demolition of symphasis; for its overthrow is so easy, and this opinion so weak, that no named author, so far as I know, has taken up its defence. It was sufficiently shown in chapter 6, number 3, that it was not the opinion of Anaxagoras, Democritus or Zeno. But from whatever ancients it may have trickled down — in a timid whisper, I believe, and with the person concealed — the view that the oblong figure of a comet arises from the synod or conjunction of two or more of the stars we already know, and whose course we grasp after a fashion, such as Saturn, Jupiter and Mars, which seem as it were to touch one another and to light up the space between with a longer trail, is rejected by Aristotle, *Meteorology* Book 1, chapter 6; Seneca, *Natural Questions* Book 7, chapter 12; Glorioso, Book 1, chapter 9; Cottunius, lecture 30 on the *Meteorology*; and the Coimbra commentators, treatise 3 on the *Meteorology*, chapter 2. Granted, Guiducci, *On Comets*, p. 4, tries to weaken the last three of Aristotle's reasons; but they are firm, if they are taken in a sound way and without cavilling. I shall nevertheless add other arguments to them.

[Margin: 1. Argument from place.]

First, then: a conjunction of the common planets takes place only within the zodiac, whereas comets often appear with their whole body outside the zodiac and far from it, as is clear from what was said in chapter 4, number 10.

[Margin: 2. Argument from rarity.]

Second: conjunctions of the lesser planets with one another, as it were bodily, have often taken place, as is clear from the ephemerides — and indeed at night, when they could be observed — and yet nobody has said that a comet was made on that account. Conversely, when comets were made, never, so far as is known, was there at the same time an as-it-were bodily conjunction of these planets. (A platic conjunction does not suffice; they must as it were

touch one another in appearance, or cohere.) Aristotle gives the first part of the argument in these words: "The Egyptians say that conjunctions of the wandering stars take place with one another and with the non-wandering stars; and we ourselves have already twice seen the star of Jupiter pass beneath a certain star in Gemini and hide it, but no comet was made." A visible contact had, at any rate, preceded the occultation — lest you say with Resta that the occultation of a higher star by a lower one is unfit to generate the symphasis of comets. And if it were established that all the lesser planets had been observed separately on some night in different parts of the zodiac, and that at the same time a comet was seen distinct from each and all of them, we should have a new and most evident argument against this opinion. But we do not need it, nor is it worth the effort, in order to fish it out, to unroll the ephemerides and examine the places of the planets at the time of every comet seen. If, however, a comet were a conjunction of planets, it could be foreseen with certainty from the astronomical tables; and it is plain that this can by no means be done.

[Margin: 3. Argument from duration.]

Third: the conjunction and visible contact of the common planets, however slow, does not last beyond one day, and the conjunction of the luminaries itself does not exceed five hours; but most comets have appeared for whole months, and some for 6 months, as is clear from what was said in chapter 4, number 5.

[Margin: 4. Argument from size.]

Fourth: it is clear that a comet is not formed from a conjunction of the luminaries, since the Sun's apparent size is rather diminished when the Moon passes beneath it. And from a conjunction of all five planets, even if they were then also conjoined with one of the fixed stars of the first magnitude, no phenomenon so great can be composed as to equal many comets in apparent size, whether in the head or in the tail. For as to the head, some have been seen no smaller than the Sun, some nearly equal to the Moon; as to the tail, some occupied 22°, some 30°, some 45°, some 60°, some 75° in the sky, as may be seen from chapter 4, numbers 6 and 8. Moreover, some were smaller at the start and later became larger, from what was said in the same place, number 6, whereas the case ought to have been otherwise if a comet were a mere coalescence of two or more stars.

[Margin: 5. Argument from motion.]

Fifth: the falsity of this opinion is proved from the motion of comets. For many comets were carried obliquely against the order of the signs [in praecedentia] towards the north, and some towards the south, to a greater latitude, or departure from the ecliptic — 8° or 10° — than is granted to any of the common planets, as is clear from the comets of the years 1472, 1577, 1580, 1585, 1590, 1607 and 1618, and others. And some within one month completed an arc of a great circle, or a path, of 39°, as the comet of the year 1585 did; or of 50°, as in the year 1533; or of 97°, as in the year 1607. Some completed 42° in 11 days, as in the year 1590; some 107° in 57 days, as in 1618; some 180° within the space of three months. No planet, alone or conjoined with others, can complete so

great an arc in so short a time by its natural course. Finally, some were so swift in their own proper motion that in one natural day they got through 10°, 15°, 30° or 40°, as is clear from chapter 4, number 13, whereas none of the lesser planets, even when conjoined with another, can complete two degrees in the sky in one day.

[Margin: 6. Argument from setting or extinction.]

Sixth and last: some comets were seen all night in such a way that they did not go down below the horizon, remaining within the circle of perpetual apparition, which is granted to none of the common planets in our climes. And many were extinguished while still standing above the horizon in such a way that they were not dissolved into two planets or into two stars, as they would have had to be if the comet had been an aggregate of them — unless you say, raving still more, that they were extinguished because both ascended into the heaven towards their apogee and at the same time were diminished to the senses. For when does even a single one of the common planets, Mars for example, ascend so far that it ceases to be visible at night, when the Sun is absent?

[Margin: Seneca's arguments.]

III. But I wish to gather some of the arguments already mentioned in another way, and succinctly, in the words of Seneca himself, Book 7, chapter 12, who says: "To this we shall reply that the number of the moving stars is fixed, and that these and the comet usually appear at the same time; from which it is manifest that a comet is not made by their meeting, but is something proper and its own master. Moreover, a star frequently comes beneath the track of a higher star, and Saturn is sometimes above Jupiter, and Mars looks down on Venus or Mercury in a straight line; and yet a comet is not made on account of this course, when one passes beneath the other — otherwise one would be made every year. For in every year some stars are together in the same sign. If a star coming upon a star made a comet, it would cease to be in a moment; for the speed of those passing is very great. That is why every eclipse of the stars is brief: because the same course that brought them together quickly draws them apart. We see the Sun and the Moon freed within a short time once they have begun to be darkened. How much faster must the meeting be in stars so much smaller? Yet comets remain for six months, which would not happen if they were generated by the meeting of two stars; for those cannot cohere for long, and the law of their speed must separate them." And at the end of the chapter: "The stars have their course within the zodiac; they press upon this circuit. Comets are seen everywhere. There is no more a fixed time for them to appear than any place beyond which they do not go." In these words, however, I should think that Seneca said that all the planets appear at the same time separately from the comet out of confidence in a possible case rather than from any astronomical history or ephemeris; much less is it true that it is usual, or that every year there occur conjunctions of the lesser planets that are fitted to represent a comet by their coherence. For although they are conjoined as

to longitude — that is, the arc drawn and produced through them from the pole of the zodiac falls on the same place of the zodiac — yet the latitude, being greater in one planet than in the other, usually prevents their conjunction from being bodily, or as it were bodily. Rather, it most often takes place πλατικῶς [Greek: platikōs, "broadly"] — platically, I say, that is, in a broad manner, and with a large gap between them as to latitude, that is, as to departure from the ecliptic towards the north or towards the south.

[Margin: Philosophers without astronomy are prone to erroneous propositions in astronomical matters.]

It is no wonder, however, that such things as these slip out of philosophers only very lightly versed in astronomy. It would have been wiser for them either to consult more skilled astronomers or to refrain from pronouncing an opinion on matters they little understand. But the security of knowing that few are able to refute them emboldens them, so that at times they pronounce even more intolerable things — as I myself have heard more than once. Indeed, if even some of us who are commonly called mathematicians are liable to errors unless we examine these matters carefully and attentively, how much more so are others, who have despised mathematics, or have scarcely brought themselves to greet it from the threshold? See, then — not to go further afield — the following Scholia.

SCHOLIA.

Scholia.

I. Lotharius Sarsius — that is, Father Orazio Grassi, hiding under his persona — in the *Astronomical Balance*, p. 29, among the arguments by which he labours to show that a comet is not a mere phenomenon arising from reflection or refraction of the solar rays, brings forward this one: that images of the Sun of this kind follow the Sun with a motion that attends and obeys it, and are carried in the same direction as the Sun; whereas the comet of the year 1618 went in the contrary direction, that is, northward, from 29 November, although from that day the Sun, moving from the 7th degree of Sagittarius towards Capricorn, became daily more southern. Among other diagrams he uses the following one. In it let BO be the surface of the sea, the eye at A, and let the Sun ascend from the point O of the horizon to H. When the ray of incidence HI is drawn, the image of the Sun will be seen at I from the eye A, along the reflected ray AI, provided the angle of incidence HIO and the angle of reflection AIB are equal. Similarly, when the Sun ascends to G and radiates along the line GE to E, it will be seen at E along the reflected ray AE; and when it has ascended to F, it will be seen at D along the reflected ray AD. Conversely, as the Sun descends towards its setting from F, where its image was seen at D, at G it will be seen at E, and when the Sun is at H its image will be seen at I. From this he deduces the following proposition: "From these things anyone may understand that these appearances always move in the direction

in which the luminous bodies themselves, by which they are produced, are carried." Afterwards, however, he adds that if anyone imagines them to be carried the contrary way, they must still keep the same law: namely, that if the Sun's image is carried northward while the Sun recedes towards the south, then when the Sun afterwards returns northward the image must be carried southward; but that the contrary happened with the comet of the year 1618, which went on heading northward even after the Sun, from 22 December, had begun to return towards the north, the winter solstice being now past. This second assertion is sound and true.

Diagram. A woodcut set in the left column beside Sarsius's construction: a semicircle with its diameter upright on the left, a horizontal line running right from B (the surface of the sea), the eye A inside the semicircle on the left, and straight lines from A and from points on the arc meeting the horizontal line. The labels on the right-hand arc run from top to bottom.

- A (left, inside the semicircle): the eye
- B (left end of the horizontal line): one end of the sea's surface BO
- D, E, I (along the horizontal line): points of reflection
- F, G, H (upper right arc): successive positions of the Sun
- R (top right, head of a vertical line): end of the straight line OR
- Q (below R): not cited in the text
- P (right): not cited in the text
- O (right, on the horizontal line's end): the point of the horizon
- L, M, N (lower right arc): the Sun's image below the horizon, per §II

[Margin: Galileo's criticism of Sarsius.]

II. But Galileo, passing over in silence the second part of the doctrine just mentioned, and weighing the first proposition more subtly in his *Assayer* [Trutinatore], p. 118, teaches that it is not true that the Sun moves in the same direction as its image is carried, and he shows this in two ways. First, if the arc FO is continued below the horizon through ON, and AI is produced to L, AE to M and AD to N, the image of the Sun will appear, in the arc of the sky beneath the waters, to be carried in the contrary direction — namely from L to M and from M to N — while the Sun ascends the other way, from H through G to F; and the image will be seen to recede from the eye A while the Sun is seen to approach it. Again, if the luminous body were imagined to ascend along the straight line OR and to move further and further from the eye A, its image would nevertheless move from I through E to D, and approach the eye. It is false, therefore, that the image of a luminous body always moves in the direction in which the luminous body that produces it is carried. But let us leave these subtleties aside for now.

CAPVT VIII.

Chapter VIII.

CAPVT VIII.

An Cometæ ſint Planetæ Extraordinarij, ſed perpetui, poſt longiora temporum interualla nobis apparentes.

Whether comets are extraordinary planets — but perpetual ones — that appear to us after longer intervals of time.

[Margin: Arguments for this opinion.]

I. We showed in chapter 6, number 4, that this third opinion was held by the Chaldaeans, and in Italy by the Pythagoreans, as well as by Hippocrates of Chios, Aeschylus, Diogenes, Apollonius of Myndus, Artemidorus, Seneca and Favorinus; and that among more recent writers it found favour, or at least no disfavour, with Oddo, Renatus, Frangipanius, Tannerus, Columbus, Raymundus, Vallesius, Liceti, Echius and Fienus. And indeed at first sight it seems not only not unfitting but rather fitting to the vastness of the heavens and the majesty of their Maker that, besides the five planets already commonly observed, there should be some others, of a different shape and colour — such as comets are — which show themselves more rarely and at longer intervals, but for the most part lie hidden: either under the rays of the Sun; or because they have not yet descended towards their lowest apsis, around which they come nearer to our eyes; or because they are not in a position fit for reflecting the Sun's light to us; or because they are not yet purified enough to be illuminated and to pour their light back to us; or because parts of the heaven or of the air transparent enough to allow us to see them have not yet run beneath them.

[Margin: 1. Argument from the possible and from the vastness of the heaven.]

The first argument, then, is from the possible, and from the congruence drawn from the vastness of the heavens, or of the planetary system. To this is added the comparison with the many standing, or fixed, stars. For the number of seven planets seems too small when compared with the innumerable fixed stars, and the vast expanse of the planetary heavens would seem too empty and idle if only seven wandering stars ran within the zodiac, and there were not many other planets ranging through every region of the sky. Seneca touches on this argument in Book 7, chapters 23 and 24, in these words: "'If it were a wandering star,' you say, 'it would be in the zodiac.' Who sets a single limit to the stars? Who confines the divine within a narrow space?" etc. "Consider whether it does not better befit the greatness of the universe that it be divided into many paths, rather than wear down this one track and lie torpid in its other parts. Do you believe that in this greatest and most beautiful body, among the innumerable stars which adorn the night with varied beauty and do not allow the air to be empty and inert, there are only five that are permitted to exercise themselves, while the rest stand still, a fixed and motionless people?"

[Margin: 2. Argument from duration.]

Second: that a comet is among the eternal works of nature, as Seneca puts it, seems to be proved from its duration over some months, of which I have already spoken in chapter 4, number 5. For it is from this that Seneca argues, for his own view and against the Peripatetics, in Book 7, chapter 22: "I do not agree with our people. For I do not consider a comet a sudden fire, but one of the

eternal works of nature. First, whatever the air creates is short-lived, for such things are born in a fleeting and changeable element." And a little further on: "It cannot be that a definite fire should sit in a wandering body and cling to it so persistently." And in chapter 23: "No fires have any stay except in what is their own — I mean those divine ones which the universe holds as eternal, because they are its parts and its works." And shortly after: "I was saying just now that nothing kindled by a fault of the air is lasting; now I add further that it can in no way linger and stand still. For the torch, the thunderbolt, the shooting star and whatever other fire is squeezed out of the air is in flight, and does not appear except while it is falling. A comet has its own seat, and therefore is not quickly driven out, but measures out its course; nor is it extinguished, but departs."

[Margin: 3. Argument from the pursuit of fuel.]

Third: if a comet were not an eternal planet but something kindled in matter which it needed as fuel, it would follow its fodder in a wandering course and would sometimes descend towards the earth. For so the same Seneca argues in Book 7, chapter 12: "Further, if it clung to its nourishment, it would always descend; for it is thicker the nearer it is to the earth. A comet is never let down to the lowest region, nor does it approach the ground. Moreover, fire goes where its nature leads it, that is, upward, or where the matter to which it has clung, and on which it feeds, draws it."

[Margin: 4. Argument from size and shape.]

Fourth: if a comet were not something perpetual but a temporary fire kindled in sublunary matter, it would change its size and shape very quickly — the contrary of what is found to happen in comets. Hear Seneca again, Book 7, chapter 23: "For they would become larger or smaller on alternate days, if the fire were a gathered one, and sudden from some cause. For it would be smaller or larger according as it was fed more fully or more meagrely."

[Margin: 5. Argument from circular motion.]

Fifth and last: the circular motion itself along the same orb, or great circle — such as we indicated appears in comets in chapter 4, numbers 11 and 12 — is a sign that a comet is a celestial star rather than a sudden fire. This very point is pressed by Seneca, Book 7, chapter 23, where he says: "No fires, ordinary and celestial, have a curved path. It is proper to a star to trace an orbit. Whether other comets have done this I do not know; two have done it in our age." Add the constancy and the regular anomaly of the comets' motion, of which enough has been said in chapter 4, numbers 11 and 13, and of which more below, when this argument is explained at greater length.

[Margin: Solution of the arguments just mentioned.]

II. But in truth the four latter arguments do not show that a comet is a planet that has been in the sky since the beginning of the world; at most they show that it is not a sudden flame, kindled in rare matter, or in matter that is quickly combustible and variable. But, say the Peripatetics, if it is kindled in matter that is dense indeed and well compacted, but also viscous and smeared with much

grease, and at the same time lighter than the thicker air around the earth, it can last a long time under the same apparent shape and size, and can move not downward but in another direction, following the motion of the air or by the force of some other cause — since motion in a circle belongs to the sea, to rivers, to clouds and to elemental fire alike. To the first argument, moreover, one could reply that planets are multiplied in vain, and without evident or strongly probable cause, especially since there are equally probable arguments to the contrary — namely, that planets of this kind do not pre-exist before comets appear, as I shall say presently.

III. Accordingly, against this opinion rise the Coimbra commentators, treatise 3 on the *Meteorology*, chapter 2; Froidmont, Book 3 of the *Meteorology*, chapter 2, article 2; Glorioso, Book 1 *On Comets*, chapters 6, 7 and 8; Resta, treatise 1 on the *Meteorology*, chapter 1; and Cabeo, Book 1 on the *Meteorology*, text 37, question 6. Take their arguments, or ours.

[Margin: 1. Argument from size at the first phase.]

First: if there were very many planets in the sky which were not visible to us except because of their nearness — that is, after making a descent towards us by means of epicycles, or of orbs equivalent to epicycles — they would at first appear very small, like a star of the sixth or seventh magnitude, and then little by little, by a proportional increase, would be seen larger and larger. Having reached their greatest apparent size in the perigee of the epicycle, they would, as they re-ascended, decrease little by little through almost every degree as far as sight goes, until they became equal to a little star of the lowest magnitude among the visible ones; nor would they vanish at night above the horizon before such a decrease. Yet history shows that it happened otherwise in most comets, on which see also chapter 4, number 6. For several appeared very large from the very beginning, or a little after it, although they had never been seen anywhere before. Among these one was memorable — indeed, worth recording for Seneca — the one which, as he himself relates in Book 7 of the *Natural Questions*, chapter 15: "A little before the Achaean War a comet shone out, no smaller than the Sun. At first it was a fiery and ruddy orb, giving off a bright light, enough to overcome the night; then little by little its size was contracted and its brightness vanished; finally it disappeared entirely." That is, it disappeared whole, as great as it still was — smaller, granted, than at the beginning, but not yet become perceptibly very small. How then, good Seneca, did you deny that comets are extinguished, and prefer that they depart and vanish little by little as they ascend — in chapter 23, that is?

[Margin: 2. Argument from the vastness, number and variety of the epicycles.]

Second: if so many and such varied planets pre-existed, and appeared only for a few days or months while lying hidden for many years because of their upward motion, the portion of the epicycle that carries the comet and grants us sight of it would have to be incomparably smaller than the portion of that circle which would render the comet invisible through its distance from our

sight. For an epicycle of this kind would have to be larger than the whole sphere of the fixed stars, since in it a comet so great that even without a tail it was seen larger than Venus and Jupiter, and sometimes equal to the Sun, would certainly still be visible, seeing that stars of the sixth magnitude are seen in that sphere. Next: either the same comets frequently recur, and so their return would be observable; or they do not recur — which agrees better with history, because of the incredible variety of past comets — and so the epicycles would have to be immense in vastness, innumerable in number, and beyond conception in situation and position, because of the windings and labyrinths, and the mutually intersecting paths, which comets, compared with one another, have so far described by their motion — as will be plain to anyone who considers what we said in chapter 4, numbers 10 and 11.

[Margin: 3. Argument from rising and setting other than heliacal.]

Third: suppose that so many planets, of such varied form and of such size, had been moving in the sky ever since the beginning of the world, and were hidden not by reason of their ascent and their distance from our sight but because of their nearness to the Sun, as Mercury is often hidden — which seems to have been the view either of the Pythagoreans or of Posidonius. Then they would never rise for us except heliacally; and yet they would rise as soon as they stood from the Sun by an arc of vision of 10° or 12°.

[Margin: See Book 1, chapter 32, on this arc.]

For no greater distance than this is required for heliacal emergence in any of the planets, or in stars of the first magnitude — and many comets are known to have been larger than these. Likewise they would not vanish except by a heliacal setting, or occultation, as they returned to the Sun. But it is clear from the history of comets that many of them began to appear quite large when they stood from the Sun by much more than 12°, and departed from the Sun towards the north, directly or obliquely, and were extinguished there very far from the Sun.

[Margin: 4. Argument from the rarity of their sighting without a reasonable impediment.]

Fourth: if so many planets of such changing forms wandered roaming through the fields of the sky, and yet were hidden neither by their ascent in the epicycle nor by the glare of the neighbouring Sun, then they would be hidden either for want of rarity and of a pure and purified transparency, or of a notable serenity, as Cardano and Echius would have it — and this does not square with our sight of the fixed stars, which we see so frequently, though they are much smaller than comets, in every region of the sky on moderately clear nights; or for want of a denser part of the heaven, as Columbus, Vallesius and Liceti would have it — but since the regions of the heaven in which comets have appeared, and through which they have led their courses, have extended through almost every tract of the aether, that density would have to be lacking for the greatest part of the time in almost the whole heaven, and its absence would therefore render most of the fixed stars inconspicuous. Besides, so great

a variety of parts, infesting the whole heaven with so tangled an intersection, is incredible in itself. Or, certainly, they would fail to appear for this reason: that they did not have beneath them attracted humours or fumes in which the light propagated from them might shine in the manner of a tail, as Hippocrates, Aeschylus and Raymundus held. In that case they would at least appear without a tail; or if they did not appear even so, then they themselves, no less than the tail, are something new rather than ancient planets. And I say the same against Frangipanius, who said that these planets are not seen except when their light has been increased. For either this increase takes place all at once from the beginning — and then by the same argument by which the newness of the light is asserted, the newness of the whole phenomenon must also be asserted, unless the opposite is established; otherwise one could say that the flame of a furnace and live coals are perpetual, or that the Sun is perpetually above the horizon but is not seen except when its light is increased — or that increase takes place gradually and successively, and this is against the history of the origin of several comets.

[Margin: 5. Argument from the signs of their newness of being.]

Fifth: the tail, for the most part turned away from the Sun; the duration, so short in comparison with the permanence of the other stars; and the motion of comets, so varied, as may be seen in chapter 4, numbers 10 and 11 — especially their motion by a leap, as in the comet of the year 373 before Christ, which for that reason, on Aristotle's testimony, was called the Path [Semita], and in the year 1099, when another comet too ran about by certain leaps — are evidence of a new phenomenon, and by no means of an eternal one. And universally, whenever a multitude of new sensible accidents appears at once that did not appear before, it is physically evident that something has been made anew, at least by alteration, as clouds, rain and the like are made anew — unless the opposite is evident from elsewhere; and the burden of proving pre-existence lies on whoever asserts that the thing existed before.

Let this, then, be the conclusion:

It is hardly probable that comets are ancient planets, much less eternal ones; and it is more probable that they are made anew.

CAPVT IX.

Chapter IX.

An Cometæ ſint Aggregatum ex pluribus Stellis paruis, ab initio Mundi præxiſtentibus, ſed ſecundùm ſe incognitis, ſeu ſeorſim inconſpicuis.

Whether comets are an aggregate of many small stars that have pre-existed from the beginning of the world, but are unknown in themselves, that is, inconspicuous taken separately.

I. We reported in chapter 6, number 5, that Anaxagoras, Democritus and Zeno went over to this opinion, and we showed in the same place that it seemed not improbable to Cardano, Giordano Bruno, Tannerus, Cysatus, Liceti and

Resta — but above all to Francesco Resta, who, in treatise 1 of his *Meteorology*, chapter 3, distinguishing ethereal comets from elementary ones, contends that the ethereal ones are a concourse of many stars. And it seems that it can be urged by the following arguments.

[Margin: 1. Argument from observation.]

First: Democritus, as Aristotle reports in *Meteorology* 1, chapter 6, used to relate that when comets dissolved, certain stars appeared; and Ephorus the historian, in Seneca, *Natural Questions* Book 7, chapter 16, said that the comet which by its rising sank Helice and Buris split into two stars. Granted, Seneca there detracts from Ephorus's credibility, saying, with arguments that are by no means conclusive, that nobody besides Ephorus observed this in that comet, "which was nevertheless watched by the eyes of all mortals, because it drew after it the outcome of an immense event. For who could have observed the moment at which the comet, dissolved, was reduced into two parts? And how is it, if there is someone who saw a comet split into two, that nobody has seen one made from two? And why did he not add into which stars it was divided, since it must have been one of the five stars?" But these objections are empty. For the end of this comet could have been observed while its beginning was not observed, or was not recorded; and we, with Democritus, are speaking not of the five usual planets but of extraordinary ones. Moreover, Dio, Book 54, relates that the comet which appeared under Augustus at the death of Agrippa went off dissolved into torches. Conversely, Nicephorus, Book 2, chapter 31 — as I reported in chapter 3 at the year of Christ 392 — tells of a comet that was seen to which a huge throng of other stars gathered, like bees to their king. Finally, Father Cysatus, in the comet of the year 1618, observed with the help of the telescope a nucleus compacted of many little stars, which afterwards separated; we have already given his diagram of it in chapter 4, in the history of that observation.

[Margin: 2. Argument from the Milky Way.]

Second: most comets, as Aristotle admits in Book 1 of the *Meteorology* at the end of chapter 7, gather into the region of milk, that is, into the Milky Way; and Pliny asserts the same, Book 2, chapter 25, in these words: "Almost all of them under the north itself, in no fixed part of it, but chiefly in the white part, which has received the name of the milky circle." Tycho too noted this in tome 1 of the *Progymnasmata*, and from him Liceti, Book 2 *On New Stars and Comets*, chapter 40, and Froidmont, Book 3 of the *Meteorology*, chapter 2. But the Galaxy is nothing other than a heap of very minute stars, as the telescope has revealed. It is therefore highly probable that very many comets are made in it or along it for this reason: that they are made from those little stars when they are gathered closer together into one. This holds especially if it is necessary to admit some celestial comets; for it is less absurd to admit this mode than generation and corruption in the heavens, since that packing and condensing can be accomplished by the force of local motion alone.

II. Nevertheless this opinion does not please at all. It is rejected by Aristotle, *Meteorology* Book 1, chapter 6; Seneca, Book 7, chapters 15 and 16; Glorioso, Book 1 *On Comets*, chapter 9; Froidmont, *Meteorology* Book 3, chapter 2, tr. 3; Mastri, Disputation 4 *On the Heavens*, question 2, article 4, number 36; and Cabeo, *Meteorology* Book 1, text 36, question 6; and Cysatus himself, *On Comets*, p. 76, abandoned it and chose little planets formed anew out of the same matter from which the solar spots and faculae are made.

[Margin: 1. Argument against this opinion.]

First: suppose comets were made by an aggregation of little stars and dissolved into them again, or that those little stars were visible separately without the telescope, as is supposed in the first three cases of the first argument, those adduced from Democritus, Ephorus, Dio and Nicephorus. Then in all comets, when they first appear, those stars that were gradually being aggregated into one would have to appear beforehand, as in Nicephorus's case; or at least, at the dissolution of every comet, those two or more little stars, or little torches, would have to appear — in accordance with Aristotle's remark against Democritus, Book 1 of the *Meteorology*, chapter 6: "But this ought not to happen sometimes and sometimes not, but always." Yet apart from those three cases of a comet dissolved into several stars or torches, this is established of no other comet. On the contrary, it is established that many were extinguished before they went down below the horizon, without any such breaking up into several little stars or little torches — as Aristotle, Seneca, Pontano, Regiomontanus, Apian, Fracastoro, Tycho, Hommel and others who were able to observe the end of comets relate of the comets they saw, and as we related in chapter 3 at the years before Christ 373 and 348, and of Christ 54, 64, 392, 1264, 1267, 1471, 1472, 1532, 1539, 1556, 1577 and 1585. Much less is there any evidence of a composition such as Nicephorus reports at the origin of his comet. Granted, the first rising of several comets is scarcely observable, and only after some days does report of it, or a sure distinction from the other stars, prevail. The point is therefore concluded more evidently from the death of comets, especially of those which disappeared whole before they were reduced to a very small and imperceptible little star.

[Margin: 2. Argument against the perpetuity of the comet's parts.]

Second: even if it were established that all comets were made by an aggregation of semi-comets, or of small little stars, and that all were dissolved into little stars, it would still not follow from this that these had always pre-existed and would continue to endure — unless those very stars had always appeared, before and after, in that small quantity of theirs. Otherwise it would rather have to be said that they too had been made anew and had afterwards perished, than that they had been and would be perpetual: in the way that a great cloud is made from many little clouds, or from flakes of snow, and is afterwards dissolved. And this, it is prudently believed, is what happened in Nicephorus's comet.

[Margin: 3. Argument against [?]position from stars of the Milky Way.]

Third: if a comet is supposed to be composed of ancient and perpetual little stars of the Milky Way, many things stand in the way. For their colour is whitish, whereas that of comets is often fiery and ruddy, or even dusky. Next, the distance of the fixed stars from one another is always the same, and therefore so is the distance of these; and it must not be said that they change the distance they have had from the beginning unless this is proved by strong arguments. For possession stands in favour of the fixed stars, which are distinguished from the wandering ones chiefly in this, that they always keep the same distance from one another; nor has the telescope so far revealed otherwise. Besides, unless one has recourse to God, or to angels who would conglomerate those stars and drive them together into one comet, no other reasonable cause that gathers them together can be devised, except gratuitously and by mere fancy. Yet even with such causes posited, it is no easier for God or the angels, or for any other feigned cause, to gather the stars of the Milky Way than any others — especially since not all comets arose in the Milky Way, but far from it, nor did they return into it when dissolved. For it is one thing that many travelled along the Milky Way for some time, another that they arose in it and out of it. But if it is said that they are composed of wandering little stars which do not appear, because of their smallness or for some other cause, but which at certain times come together into one visible star — why are so many little planets not seen at some time through the telescope, as the satellites of Jupiter are seen? Why do they not more often obscure the Sun or the other planets? Or why, at least, do they not more frequently come together and conglomerate? And on what foundation, finally, except a gratuitous and purely arbitrary one, are innumerable little bodies, invisible in themselves, feigned to exist from the beginning of the world? For they must be innumerable, and almost infinite, if singly they are so small as to be invisible, and yet by their confluence they must make up a comet sometimes no smaller than the Sun, together with its tail, which sometimes occupies 60°, 70° or 180° in length. This is Seneca's argument, Book 7, chapter 15, about the comet which "shone out before the Achaean War, no smaller than the Sun", when he says: "How many stars, then, must come together to make so great a body? Gather a thousand into one, and they will never equal this circuit of the Sun." And soon after: "In the reign of Attalus a comet appeared, moderate at first. Then it raised itself and spread out, and came as far as the equinoctial circle, so that, stretched out immensely, it equalled that region of the sky which has the name of milky." How many [?], then, to have come together must the wandering bodies be, to occupy so long a tract of the sky with continuous fire?

[Margin: 4. Argument from duration.]

Fourth: if those innumerable little planets, or those little fixed stars of the Galaxy [Fixulæ illæ galacticæ], are gathered together to compose a single comet, then either they are gathered by a chance and tumultuous motion and concourse — and so would at once be dispersed again [digregarẽtur], as happens with straws and little balls that come together in some eddy of a river or of the sea and

are scattered shortly afterwards; whereas many comets persist for whole months, and for several of them. Or else there is some cause that of itself aims at their coming together, on account of which, once coupled, they dwell together for a long time, or migrate elsewhere in a close-packed and compact column, like cranes. In that case, since this opinion does not admit generation, corruption or corruptive alteration in the heavens, what is the reason why they do not keep that union always, or at least for far longer? For no contrariety stirs up that sedition and secession. You will say that the cause is indeed unknown, but that there are many things which we know to exist and yet do not know how they come about or exist. True enough — but that holds of things we know for certain to exist; and we are not yet certain that comets are an aggregate of many little stars produced from the beginning. For this is the very point under dispute.

[Margin: 5. Argument from the tail.]

Fifth: one may argue from the tail of comets, which, it is established, was for the most part — indeed in all those observed somewhat more accurately and by the more expert — turned away from the Sun. For either the tail too consists of a concourse of little stars, and then no reason can be given why that long train [ſyrma] inclines towards the region turned away from the Sun rather than elsewhere. Or nothing but the comet's head consists of those little stars, which, being semi-transparent, transmit the Sun's rays received within them out behind themselves into a longer tract. Then some semi-opaque matter must be brought in, fit to stop those rays and reflect them to our eye. Either, then, that very matter or subject coalesces out of semi-opaque corpuscles, and the previous argument returns, since not even for these can a reason be assigned why they should be arranged in this way only behind the back of the comet's head, and not surround it on every side, and so stop the Sun's rays and send them back to us from whatever region of the sky. Or that subject is some smoky matter carried there, or exhaling from the comet's head; and so in comets one must recognise newness — some new generation or alteration — and not merely a motion by which little stars congregate.

[Margin: Reply to the principal argument for the affirmative side.]

III. Now to the last part of the second argument for the affirmative side, where it was said that, if it is necessary to admit some celestial comets, the mode by which comets are composed by the force of local motion alone is to be chosen rather than generation and corruption in the heavens. The reply is to concede this if the comparison of local motion with generation and corruption is made strictly; but to deny the assumption if this local motion brings with it inconveniences no smaller. For once it is posited, one must admit either that the fixed stars of the Milky Way are not fixed, but, against the common lot of the other non-wandering stars, are torn from their place and migrate into another — the space they occupied being meanwhile left empty, or filled by rarefaction or some other motion — and that they change colour. Or, whether they are fixed or wandering little stars, one must admit a certain tumultuous and

chance motion, through a heaven that is now liquid, by which they are gathered without any rule, and a composition of the tail made almost at random. Taken together, these things are no less unsuited to a celestial body than generation and corruption. Therefore, once that dislocation of the fixed stars, the fluidity of the heaven and irregular motions have been admitted, generation and corruption in a celestial body can be swallowed and digested with the same stomach.

CAPVT X.

Chapter X.

AN COMETÆ fiant ex elementarium EXHALATIONUM COPIA conſtipata ac incenſa, & in ſuprema regione aeris vel infra LUNAM permanente.

Whether comets are made from an abundance of elementary exhalations, packed together and kindled, and remaining in the highest region of the air or below the Moon.

I. This opinion of the Peripatetics and of the older astronomers — very widely held, and once received in most of the academies, as we said in chapter 6, number 6 — seems capable of being established by the following arguments. To these, however, we shall subjoin one by one their solution and the arguments diametrically opposed to them, so that, in Aristotle's dictum, opposites set beside one another may shine out the more, and may be weighed and assessed on either side as in a double scale. The remaining arguments that tell against this opinion, and do not bear on the immediate solution of the reasons adduced for the Peripatetics, we shall bring forward separately in chapter 11, together with their solutions too.

[Margin: 1. Argument from the dryness and winds that accompany a comet.]

II. First: when a comet is made, it is preceded or accompanied by a great dryness with winds. These either shake the very ground of the lands with earthquakes, or burst out through Vesuviuses, or swell the seas into a surge — hence the submersion of islands or of maritime cities — or rage in long-lasting storms and whirlwinds, and bring upon crops and animals other calamities akin to violent, or even virulent, winds. Not, indeed, always in every region, but in some regions more disposed to it, or over which the comet more predominates, as may be learned here and there from the history of comets set out in chapter 3 — especially of those which Aristotle reported, there at the years before Christ 373 and 341; read Aristotle's words there again. It is therefore a sign that a comet is made from an abundance of hot and dry sublunary exhalations, and that through it the comet does harm to the lands from nearby rather than from a distant and supralunar place. And this is the Philosopher's first argument, *Meteorology* Book 1, chapter 7.

[Margin: Solution of the first argument.]

The reply, however, is to deny the universal inference. For it is established that at times, when a comet was made, no such abundance of dry winds existed, nor any disaster arising from it — unless, out of the Peripatetic opinion already presumed, one were to assert, fictitiously and too confidently, that

it existed somewhere, and in places about which all history is silent. Indeed, it is established that a comet has at times had heavy rains and clouds near it, which hindered the beginnings and ends of the observation or disturbed[15] an observation already begun; or that health among the animals, abundance of crops and calm weather followed. Conversely, the evils and misfortunes enumerated in the argument have far more often occurred without any comet going before or following close upon them. For example, Vesuvius, notorious for twelve eruptions since the birth of Christ, scarcely once had a comet as a torch announcing it or as a hired mourner — namely under Titus, or again in the year 983. And who could enumerate the earthquakes, shipwrecks, swellings of the sea, famines, plagues and other calamities akin to these, by which mortals have perished with no comet shining before them or shining upon them? This sign, therefore, is not convertible with a comet, either simply or in a certain respect, and is universal from neither side. Let Aristotle, then, suffer to be turned back against himself what he himself hurled at Democritus in *Meteorology* Book 1, chapter 6: "But this ought not to happen sometimes and sometimes not, but always."

[Margin: 2. Argument from likeness with other fiery meteors.]

III. Second: all the other fiery impressions similar to comets — namely burning beams, flying dragons, torches, falling or running stars, will-o'-the-wisps [ignes fatui], leaping goats, the fire of the Tyndarids about the yards of ships, and so on — are made from a hot, dry, sulphurous or nitrous breath that has been kindled. They do not differ from a comet except in shape and duration; nor would they differ if that exhalation were denser, better compacted and fatter, or if fuel were supplied to it for longer — for running stars, which are extinguished for lack of fuel before completing their passage, would, if they had long-lasting tinder, fulfil their course over a longer time and become comets. This argument from likeness, with the dissimilarity already mentioned, is Aristotle's, *Meteorology* Book 1, chapter 7, where he reasons thus: "When, therefore, on account of the upper motion, a fiery principle has fallen into such a condensation — neither so strong as to burn out quickly and over much space, nor so weak as to be quickly extinguished, but great, and fit to last a long time — and at the same time a well-tempered exhalation happens to rise from the lower part, a comet is made, in whatever way the exhalation has been shaped, etc." And a little below: "For just as, if someone thrusts a firebrand into a great heap of chaff, or throws in a small beginning of fire, the running of the stars will be like this; for quickly, because of the suitable disposition of the kindled tinder, a conflagration follows along its length. If, then, this lasts, and is not consumed in a swift passage where the kindled tinder has been most densely packed, the motion, once begun, will certainly complete its course. Such too is the comet, a hairy star [Comata ſtella], as a running star is, but having the beginning and end of its motion complete."

[15]printed *intertubarunt*, for *interturbarunt*

[Margin: Solution of the 2nd argument.]

The reply, however, is that a comet is far more unlike the other meteorological inflammations, and in many other respects more like the celestial stars. It is so chiefly in splendour, in colour, and in its motion, which is for the most part uniformly difform, regularly unequal and yet constant, along a circle that is sensibly a great circle, or along a path of its own like the path of the planets. It is so, next, because it does not follow its fuel, nor leap across to lower places, plucking at tinder supplied to it, but rather, when about to die, is carried up on high. And it is so, finally, in its lasting for several months. Although such duration is no less in snows, and sometimes in clouds, because these have no destructive and resolving principle within them, in a body inflamed and kindled, as is supposed, it does not seem probable — especially since even the fire-vomiting chasms, full of sulphur and of bitumen far more viscid than the matter attributed to comets, scarcely anywhere spew out a continuous flame for 6 months, as comets have done, but now and then languish, or even go out, and are rekindled later. But since Aristotle compared comets with falling or flying stars, let us too compare their duration on the one hand with momentary and falling stars, and on the other with perpetual or ancient ones. One of the falling stars is lit almost in the twinkling of an eye and is extinguished in about two or three beats of the human pulse; that is, in its run it scarcely equals two hourly seconds. Now there are 3600 seconds in one hour, that is, 1800 lifetimes of running stars; but let us suppose there are only 1000, so that the life of a falling star is about four seconds. Then in one day there will be twenty-four thousand times, each equal to the time belonging to one glowing star, and in one month of 30 days there will be 720,000 such times, and in six months 4,320,000. But from the beginning of the world down to us not yet 6000 years have elapsed, that is, not yet 72000 months. Therefore the excess of the duration of the stars so far elapsed (for how great it will be in future is unknown, and it is probably believed that it will not be much greater) over comets of a month's duration, of which you may find many in chapter 4, number 5, is smaller than the excess of the duration of a single month-long comet over the duration of a single running star. For the former is as 72,000 to 1, but the latter as 720,000 to 1 — that is, ten times greater in the latter case than in the former. And if a falling star is compared with a comet of six months — of which kind it is established from history, and from chapter 4, number 5, that there were three, and it is probable that there were several others from the beginning of the world down to Christ — this excess would be sixty times greater. A greater likeness in duration therefore holds between comets and the celestial, permanent stars than between comets and the falling or running stars. It is therefore not expedient to uphold this Peripatetic doctrine so stubbornly with these topical arguments; for many others will be set against Aristotle when he uses this likeness, on which see chapter 11, argument 7.

[Margin: 3 [?]. Argument from the many and great sources of exhalations.]

IV. Third: if a comet is not a hot and dry exhalation kindled by some motion or from elsewhere, this is certainly because of its long-lasting and persistent flame, which sometimes perseveres for as long as six months. Yet this is no obstacle at all, if you look about at how many exhalations, imbued with a sulphurous, nitrous or vitriolic unguent, are perpetually drawn up on high — from the Ocean on the one side, and from the furnaces and seething chasms of the Earth on the other. For that the sea is oily, and in warmer weather sweats out a kind of oil, not only did Aristotle assert, when he said [Greek: ἀπὸ γ[[α/ὰ]]ρ τῶν ἁλῶν ἔλαιον ἀφαιρεῖται (apo gar tōn halōn elaion aphaireitai, "for oil is drawn off from the salts")], but it is also gathered from many experiments: its water stains clothes rather than washing them, by its fatness bears up swimmers more easily, and, thrown upon flames, rekindles them rather than putting them out, and for that reason it is congealed into ice only with great difficulty. The Earth, moreover, teems in innumerable places with sulphurous fires and has fire-vomiting mountains or flame-flowing chasms, from which hot, dry spirits and volatile salts endowed with some oil are perpetually exhaled. If these are collected into one mass, packed together, and brought to a certain temper which is apt to conceive a flame and, by its viscosity, to nourish it for longer, there seems to be no inconvenience in life and strength being supplied from that source even to comets lasting six months. That this happens rarely, however, is because those two conditions rarely concur: namely, that an abundance of such breaths is gathered into one place, and that it attains the sufficient packing and temper required for a long-lasting flame — its dryness preserved, in the sense in which dryness is opposed to watery humidity, and the fat, airy and unctuous moist not excluded.

[Margin: Reply to the 3rd argument.]

The reply, nevertheless, is that what has been said so far is indeed not improbable, so long as the opposite has not been made evident; but in this controversy the question is what is evident, or what is more probable. Therefore, if it is evidently established from the parallax of some comets that they are above the Moon, and in the highest aether, the probability just mentioned will collapse and will not be able to stand for all comets. For then the force of terrestrial and maritime breaths, especially of viscid ones, will be able to penetrate as far as the heaven far less, or perhaps not at all — and yet that force would have to be greater than if the comet were made in the highest region of our atmosphere, by as much as the same comet, seen under the same angle, that is, with the same apparent magnitude, is really of greater bulk if it is in the heaven than if it is near the earth. But let us grant for the moment that this is not yet evident from parallax. There are nevertheless many other arguments by which it is concluded, with equal or perhaps greater probability, that not all comets are generated from that matter and in that manner which the Peripatus suggests to us — as is clear from what was said in the replies to the 1st and 2nd argument, and as will further be clear from what is to be said in the following chapter.

[Margin: 4. Argument from manifest indications.]

V. Fourth: the sudden kindling and very wide diffusion of comets, made either by leaps, or their dispersal into torches, and their falling towards the earth — so that at times a sulphurous smell has not been lacking — and finally a partial eclipse of the Moon made by a comet gliding beneath it, are manifest indications of a sublunary comet and of a meteorological inflammation in elementary matter. On the sudden diffusion, see chapter 3 at the year before Christ 146; on the leap, see the same place at the year 373 before Christ and 1099 after; on a tail let down almost to the earth, sometimes with a sulphurous smell, sometimes even to the point of drying up a stream, see in the same chapter the years after Christ 70, 400 and 1543. On a comet inducing an eclipse on the Moon itself, see the same place at the year 1450. Finally, on dispersal into torches, see the comet of the year 13 before Christ, about which Dio writes, Book 54.

[Margin: Reply to argument 4.]

The reply is that this argument holds not for all comets but only for some, and that therefore some sublunary comets must certainly be admitted.

[Margin: 5. Argument from disordered motion.]

VI. Fifth: Chiaramonti, in the *Antitycho*, Book 3, chapter 1, argues from the irregularity of cometary motion, which he tries to prove from Tycho's own observations; and Carolus Piso does the same in the "Mirror of the Comet" [ſpeculo Cometæ], chapter 27, on the comet of the year 1618, although he also mixes in other accidents and indications of a sublunary inflammation. For he says: "For if, even before the comet's appearance, a certain black and murky cloud, like a huge beam, was seen hanging for a long time in the highest air towards the south, by the report of the Romans; and the brightness of the comet, once kindled, was by no means pure and clear, but rolled much smoky burning about itself, and differed from the purer light of the stars; and its motion, moreover, was singular, disordered and wandering, and wholly most unlike the constant and ordered motion of the stars; and finally its magnitude was such that it was gradually drawn in, and its brightness faded, and at the very last the whole comet perished — why do we doubt to declare the recent comet a tumultuous and improvised fire residing in the fluid elementary region, rather than to name it an eternal work of nature and assign it to the eternal fires?" This argument, put into syllogistic form, we shall bring forward more opportunely in the following chapter, number 10.

[Margin: Reply to the 5th argument.]

The reply, however, is that the comets' sublunary origin and nature are so far from being proved by their motion that, on the contrary, most people probably affirm that the comet should be counted among the celestial rather than the elementary bodies. They do so in view of its uniform difformity, and of an inequality which is real but regular, increasing or decreasing in a certain proportion and like the anomaly of the planets; and further from its constant motion along one arc of a great circle, or from the comet's proper motion, much slower than the motion of the Moon. But we shall have to discuss this

motion separately in the following chapter, arguments 4 and 5. Meanwhile, if some comet is found to wander with a disordered and irregular motion — like the one of the year 1472, slower at first, then suddenly so headlong that it flew across 40° in a single day, and afterwards slackened — the argument is not on that account valid when extended to all comets, and much less to those which are established to have completed their course with an ordered motion.

[Margin: 6. Argument from the ease of explaining the accidents of comets.]

VII. Sixth: if it is asserted that a comet is generated from a hot, dry and fatty mass of exhalations, well compacted and tempered, and kindled either by the motion of the air or of fire under the force of the primum mobile, or by antiperistasis, or by the very temper closely disposed to receive fire and to produce it in the same subject, then many things that appear in comets are easily explained, and a sufficient reason is given for them. For example, why comets are not made within the tropics except very rarely: namely because exhalations raised upward in that tract, by the excessive heat of the Sun and the swifter motion of the primum mobile there, are either cooked away or dissipated, so that they cannot be packed together into one; whereas outside the tropics, especially towards the poles of the world, where the air is thicker and fatter, they are condensed more easily. Likewise why comets occur more frequently in autumn: namely because many exhalations, already raised in the preceding summer and after that equinox made fatter but not yet dissipated, are able to hold together. And so with similar cases.

[Margin: Reply to argument 6.]

The reply given by the opponents is to deny the antecedent. For there are many other properties of comets which are explained only by force, or not very probably, in this Peripatetic hypothesis: such as their enormous duration; the constancy and order in the motion of many comets; a splendour and scintillation much like that of the celestial stars; the diaphaneity of the tail, and its turning away almost always from the Sun into the opposite part; the same apparent magnitude whether the comet is viewed through the telescope or with the naked eye; the monstrous bulk of the comet's body at so great a distance from the earth as many have measured, whether they place comets below or above the Moon; their motion, slower than the Moon's; and their long stay above the horizon. As for what is said about comets being more frequent in autumn, it is established to be false from our Epilogue to the history of comets, on which consult chapter 4, number 4: for more appeared in summer — a time at which fewer ought to have appeared, because the exhalations in our climes are consumed by the heat of the Sun, or too much attenuated and dissipated, if it is for a similar cause that comets are not so frequent within the tropics.

[Margin: 7. Argument from parallax.]

VIII. Seventh: Chiaramonti, in the *Antitycho*, argues from parallax, which he contends was greater than the Moon's parallax in all comets observed with mathematical instruments. Now it is certain that a phenomenon is nearer the earth whose variation of view, or parallax, at an equal altitude above the horizon

is greater — that is, whose difference between the place seen from the surface of the earth and the true place reckoned from the centre of the earth is greater — as we taught in Book 1, chapter 35.

[Margin: Reply to the 7th argument.]

The reply: "This is the task, this the toil." On this the reader must suspend judgement until the examination of these parallaxes, to be carried out below; for geometers and astronomers in no way inferior to Chiaramonti — Tycho, Longomontanus, Kepler, Mästlin and many others — affirm that they found parallaxes smaller than the lunar ones in the comets they observed, and on their authority very many more recent writers have gone over to that opinion with their feet rather than with their heart.

CAPVT XI.

Chapter XI.

Argumenta Contra Opinionem Ariſtotelicam de Cometis, cum ſolutione qualicumque ipſorum.

Arguments against Aristotle's opinion on comets, with some solution of them, such as it is.

Primum Argumentum ab auerſione Caudæ à Sole.

First argument: from the tail's turning away from the Sun.

I. FIRST: the tail of all the comets observed in these last three centuries by astronomers — not by the common people or by those unskilled in astronomy — has been found by them to be turned away from the Sun and to stretch towards the part opposite the Sun, as is clearly established from chapter 4, number 9, and from the history set out in chapter 3. Hence it has come about that the tail emerged before the head, and from the same point of the horizon. But the Peripatetics can by no means adduce a reason for this phenomenon. For if a comet is an inflamed elementary exhalation, taking on whatever shape that exhalation, closely disposed for kindling, has acquired — as the Philosopher expressly teaches, *Meteorology* Book 1, chapter 7 — and is kindled because of the air stirred up by the force of the upper motion and carried round by the primum mobile, then it can have no connection with an opposition of position towards the Sun. The other fiery meteorological impressions have none either: by their own burning they stretch themselves out in whatever direction the matter and fuel have enticed them, without any relation to the Sun. Accordingly, the tail of such comets, if it is a flame rarer and lighter than the comet's head, would have to be either raised upward in the shape of a pyramid, as happens in the other flames around us; or always incline towards the east, because of the motion of the primum mobile snatching the comet's head along against the order of the signs [in antecedentia] — the head being denser and more apt to break through the air, while the hair meanwhile yields and streams out behind comets. Or at least the tail would have to stretch out in whatever direction

the tinder and the exhalation tempered for burning invite it — and you could only gratuitously say that this is always found in the part that lies behind the comet's trunk, in the region opposite the Sun. Finally, a comet that rises in the east in the morning and appears as a pogonias, or bearded comet, its streamers going before it towards the west, would not turn into the same comet tailed, that is, with its hair turned towards the east, when in the evening it follows the Sun after the Sun has already sunk below the horizon — as is established to have happened more than once, especially in some comets observed by Peter Apian, as we related in the history of comets. Therefore not the whole comet is a flame kindled in an elementary breath: its greatest part, taken adequately — namely the tail — is not.

[Margin: Reply to the 1st argument.]

II. The more recent Peripatetics reply that the outflow of the comet which we call its beard or tail is a thinner and spirituous matter exhaling from the comet's body — just as we see little rods streaming from some fires, like a dart or a javelin — or else that it is adventitious matter, newly joined to the comet, and rarer, yet becoming fiery around it. The part that lies between the comet and the Sun, they say, is not seen, because it is overwhelmed by the Sun's rays or consumed by them, and is effaced in front; whereas the part behind the comet appears, because it is neither consumed by the Sun's rays nor overwhelmed by a greater splendour, the density and opacity of the comet's head standing in the way. As for comets that are hairy all round, and whose locks stretched out towards the Sun the Sun does not shear off: either they are like haloes, or they resist the consuming or illuminating action of the Sun for longer because of their denser matter; or at any rate no argument founded on the turning-away of the tail, which is the present difficulty, is drawn from them.

[Margin: Objection against the reply.]

But if that exhalation floating about the comet appears only behind the comet's head for this reason — that it shines in the comet's shadow, while the rest lies hidden because of the presence of the Sun — then the comet's tail or beard would have to either taper into a turning-post, that is, into a cone, and be of the kind which the Greeks call Lampadias, or the figure of a two-edged sword display — such as was the comet we spoke of in chapter 3, from Nicephorus, under the year 392. For the Sun is larger than the comet's globe, and the shadow of the comet's head would be conical, as the optical law on the differences of shadows given in Book 5, chapter 3 requires. Or at any rate the tail or beard could not go out beyond the limits of the conical shadow, nor be columnar, or calathoid [basket-shaped], or widened and spread out like a broom; and much less could it escape from the boundary of the shadow so as to curve in the manner of a sickle, or a Persian scimitar, or a mane bent downward — and yet it has been observed more than once that the tails or beards of comets had these very figures.

[Margin: Carolus Piso's weak reply.]

Therefore Piso was wrong to assert, in the *Speculum Cometae*, chapter 14, that the tails of comets do take on various forms according to the comet's greater or lesser distance from the Sun, but stay within the cone of the solar rays; and that a comet which is nearer the concave of the Moon but farther from the Sun — such as he thinks the comet of 1618 was — can have a coma equal in breadth to the head, that is, cylindrical, because the Sun's rays then cut nothing away from it. For although a globe smaller than the Sun casts a wider and longer shadow the farther it recedes from the Sun, the shadow is always conical, and the more pointed the longer it is. For it to become, to the senses, as it were cylindrical, an almost immense distance from the Sun would be required, or at least a difference of distance much greater than the interval between the Moon and the Earth. Finally, what would Piso do with comets that have a calathoid and sickle-shaped tail, as regards the part that is not covered by the comet's head but lies plainly exposed to the Sun's rays?

[Margin: Chiaramonti's slight reply.]

Chiaramonti, however, replied differently in the *Antitycho*, Book 2, chapter 31. There he admits this turning-away of the comet's tail from the Sun — indeed he affirms that he demonstrated it himself in his little book on the bearded comet of 1618 — but denies that it follows that comets shine not by their own light but by the Sun's: for the tail can lean toward the part opposite the Sun "*even if that impression glows with its own light, or because the head is turned toward the Sun with its right side, as we judged probable in the book on that same bearded comet, or for some other reason.*" So he scarcely deigned to answer an argument of such weight, as Glorioso taunts him in Book 3, chapter 8. Yet Apian, Cardano, Scaliger, Telesio, Guiducci, Galileo, Rothmann, Tycho, Kepler, Longomontanus, Cysatus and Aresius, besides others listed in chapter 6, number 10, made so much of this argument that on its account they held that the tail of these comets is not flame, but the Sun's rays transmitted through the comet's transparent head and refracted in various ways according to the disposition of the matter surrounding the comet and the varied figure of the head. But since this question about the tail is entangled with geometrical difficulties and optical intricacies, we must weigh it separately in chapter 25.

2. Argumentum à Motu Cometæ Diurno imitante motum Primi Mobilis.

Second argument: from the comet's diurnal motion, which imitates the motion of the primum mobile.

III. SECOND. Aristotle supposes that fire, and nearly all the air, lying between the lunar heaven and the Earth, are moved circularly, and that by this agitation a dry and hot exhalation is kindled, when it has been well tempered, and blazes up into comets. For he says, in *Meteorology* Book 1, chapter 7: "*For we suppose that the first part of the world, the part around the Earth, the whole of which lies under the circular revolution, is a hot and dry exhalation. This, and a great part of the air still below it, is continually carried round the Earth by the revolution and circular motion*, etc."

[Margin: Whether comets are carried round together with the air by the celestial primum mobile.]

But all the arguments by which the solidity of the heaven is attacked — and how valid and solid they are will appear in Book 9 — also attack the motion of elementary fire and of the air around the Earth, if that carrying-round is supposed to be effected by the concave of a solid lunar heaven, in the way that, when a basin is turned round, the water enclosed in it is whirled round with it. But suppose instead that elementary fire and the greatest part of the air are set moving by a heaven, however fluid, in the way that, when water enclosed in a basin is turned round, the air nearer to it is also rolled round, though somewhat more slackly and sluggishly. (So Sarsius teaches, against either Galileo or Guiducci, in the *Libra Astronomica* from p. 43 on, and indeed by an experiment of Father Orazio Grassi: namely, of a small candle flame, or of a leaf of paper hung from a thread and let down near water. The leaf imitates the eddies of the water in a following circuit, whereas the flame leans to the side opposite the air set in motion.) If this, I say, is supposed, there is no reason why the whole of that small body of air which remains above the mountains and below them, right down to us, should not be moved by the same motion; or why clouds, when no wind is blowing toward the east, should not be carried perpetually westward by that motion — let alone papers or straws thrown upward. Since, then, neither the flames of a pyre, nor hairs or very light reeds set upright, nor the shavings or motes that we see floating in the Sun's rays let into a dark room, nor straws or paper balls thrown upward, or dropped down by us from a tower, nor threads let down perpendicularly from towers, nor finally clouds, however calm the weather and with no wind blowing against them, are carried toward the west, we seem to have much more probable indications that not even the upper part of the air is moved with the motion of the heaven, whether solid or liquid — and much less moved so rapidly that an exhalation catches fire from it — whatever effort Chiaramonti makes, in his little work on the sublunary seat of comets, part 2, chapter 12, to establish that motion.

[Margin: Reply to the 2nd argument.]

One can reply nonetheless that the motion of the primum mobile is not felt by us, both because of the enormous distance from the heaven and because of the sluggishness and thickness of the air in our parts. For if someone on a lake or a vast sea were to describe the circumference of a circle at a distance of 100 miles or more, the whole of the water would not on that account be stirred by a sensible whirling from that circumference right to the centre; rather, the parts nearer the centre would stay at rest just as before. And if it does not seem likely that the air nearest us is free from the motion of carrying-round, while the uppermost region of the air is snatched along by it at such a speed that an exhalation is kindled there into a comet, the Peripatetics will reply that the comet's matter does indeed arise from our exhalations, but afterwards rises higher, through the region of elementary fire, until it experiences the most

violent motion of the primum mobile. And indeed it is surely more likely that such a motion of the primum mobile is communicated to the already-mentioned region of fire — and even, at least more slackly, to an exhalation already raised to that place.

3. Argumentum à mora Cometæ ſupra Horizontem.

Third argument: from the comet's stay above the horizon.

IV. THIRD. Let it indeed be granted to Aristotle that the uppermost region of the air, into which hot and dry exhalations gather, as he himself holds, is carried round very violently by the motion of the heaven, and that comets therefore catch fire. Granted this, as soon as they had kindled they would be carried with the greatest speed toward the west and would be seen to fly there like flashes of lightning; and in this diurnal motion they would appear to us faster than the Moon and the other stars by as much as they are nearer the Earth than the Moon is; and the comets that were near the Equator would pass across from east to west not in 12 hours but in far fewer — which, however, does not agree with experience. This argument will be better understood from the diagram of chapter 9 in the section on parallaxes, Book 10. Nor could that speed be retarded by the comet's own motion striving eastward, in those comets which are established to have moved against the order of the signs, as we showed in chapter 4, number 11.

[Margin: Reply to the 3rd argument.]

The reply is that the comet's proper motion is so varied — by reason of the matter it follows as its tinder, and of the impelling principle, whether that is like the principle that drives the winds or thunderbolts, or something else — that it is not impossible, nor even very surprising, that a comet should be carried round in this way by the motion of the primum mobile and return to the horizon after the manner of the stars, and yet be borne, either obliquely toward the poles of the world or straight toward the east, in such a way that it retards that speed, so that in the space of one hour it is not advanced fifteen degrees westward by the motion of the primum mobile alone, but much less. Hence, then, it can happen that comets remain above the horizon for many hours — especially if the place of comets is not at the boundary of fire and air, as is attributed to Aristotle, but next to the concave of the Moon, which the Philosopher seems to hint at in *Meteorology* Book 1, chapter 1, when he says that the place of comets is very close to the revolution of the stars. But this argument will be discussed again below, in chapter 15, from Problem 7.

4. Argumentum à motu proprio tardiore Cometarum quàm Luna.

Fourth argument: from the proper motion of comets, slower than the Moon's.

V. FOURTH. If a comet were a meteoric inflammation below the Moon, the comet's proper motion toward the east would have to be faster than the Moon's motion. For the farther each star is from the Earth, the slower it is in its proper motion, as Aristotle himself admits in *De caelo* Book 2, text 58,

and as is evident to anyone who compares the Moon's motion with the Sun's, and the Sun's with that of Mars, etc. But many comets have been slower in proper motion than the Moon, as is evident from the history of chapter 3 taken together with chapter 4, numbers 10, 11 and 13. Therefore those comets at least are above the Moon and different from meteoric kindlings — granted that this argument does not hold for the comets, which I indicated in the same chapter 4, that were faster in proper motion than the Moon. Tycho uses this argument in volume 2 of the *Progymnasmata*, p. 104. There, having related that he had observed in the comet of 1577 a motion always slower than the Moon's from beginning to end, he had inferred: "*Hence it always kept a proper progress slower than the Moon's; and for that reason the harmony of the celestial circles and motions demands that it was far more remote from us than the Moon's orb is.*" Indeed, earlier, on p. 94, he had asserted of the same comet: "*It is a proof that this comet was by no means sublunary that its proper diurnal motion was never so great as to equal the Moon's diurnal course even at its slowest. For the Moon, when it appears slowest, completes more than ten degrees in one day; whereas this comet, at the beginning, when it was fastest, was found by us to advance not much beyond five degrees within a single revolution of the primum mobile. And that it therefore completed its course far above the Moon's sphere can at once be proved quite plainly from this: the farther the stars are from the Earth and the nearer to the eighth sphere, the slower the proper motions they appear to us to have; and conversely, the nearer they are, the faster they are seen to be moved. Therefore the comet is proved not only to have stood just above the lunar sphere, but to have completed its path not far from the orbs that* ♀ *and* ☿ *describe around the Sun.*" Finally, the same argument is put forward, and not resolved, by Biancani in *Loca Mathematica Aristotelis*, p. 93.

[Margin: Reply to Argument 4.]

VI. I REPLY that the argument already mentioned holds for stars that complete their revolutions and that are established on other grounds to be ordinary stars, or higher than the Moon; and that it does not hold unless that greater height is certain on other grounds — otherwise one would be allowed to put tortoises above the heaven. Indeed, it does not hold for every planet in every position and at every time. For in the Ptolemaic hypothesis Mercury and Venus are below the Sun, and yet in their mean motion in longitude they are as fast as the Sun, while in their apparent motion they are sometimes slower. In the Tychonic and the Copernican hypothesis, by contrast — indeed in the hypothesis of all the distinguished more recent astronomers — the same Mercury and Venus are sometimes above the Sun and sometimes below it; and Mars, which is slower than the Sun, is nevertheless much nearer the Earth than the Sun is when it has become acronychal, that is, when it is opposite the Sun, especially at the perigee of its eccentric — as Copernicus, Tycho, Kepler, Longomontanus, Lansbergen and Ismaël Boulliau concede.

As for Tycho, I say that when he wrote those things about the comet of 1577 he was younger, and that he did not himself bring out the whole of that volume 2; rather, he first published volume 1 of the *Progymnasmata*, while volume 2 as a

whole was published by his heirs after his death, as is clear from the letter to the reader prefixed to volume 2. So it is no wonder if Tycho did not correct those propositions of volume 2. For when he had become more learned than his former self, he recognized that this argument is only probable and establishes nothing certain for the greater or lesser distance of a phenomenon from the Earth.

[Margin: Does Tycho contradict himself, or correct himself?]

For in a letter to Rothmann of 1595 January 14, indignant that a certain Scot had thrown up to him the futility of this argument, he says: "*And because I would say that it is not without likelihood that that comet was slower in its proper motion than the Moon, and therefore higher, yet I would not have tied any necessity to these points — he has scarcely found sneers enough to attack me with.*" Yet, as we saw, he had once called this argument not merely probable but *plainly convincing*. But a man who corrects himself must be spared. For in a letter of 1588 August 17 to the same Rothmann, p. 118, he has these words: "*Although, to tell the thing as it is, speed and slowness of motion do not sufficiently distinguish the remoteness or the nearness of comets, so that what you seem to raise against us is no obstacle — namely, that the speed of motion in Regiomontanus's comet, which is put at 40° of a great circle in one day, proves that it was sublunary and had as great a parallax as he attributes to it. For if the comet of 1585 could lie in the highest orb of Saturn, even by your own judgement, and nonetheless complete a diurnal motion of three degrees at the beginning, while the star of Saturn itself, even at its fastest, completes only 8′ in one day — so that the comet was then more than twenty times faster than Saturn even at its swiftest — how, I ask, can it not also be granted that the comet described by Regiomontanus was above the Moon, even though it was about three times more rapid than the Moon's diurnal motion?*"

Rightly indeed. And consistent with this is what he says against Thaddaeus Hagecius [Tadeáš Hájek] in volume 1 of the *Progymnasmata*, p. 512, where he writes: "*That argumentation which Thaddaeus puts forward in the same chapter, from the proper motion of comets — that comets which resist the first motion faster than the Moon are below it, and those that resist it more slowly are above it — does not seem to me sufficiently apodictic. For comets can be moved by their proper motion in a swifter course than the Moon itself, whether forward, or in the order of the signs, or even sideways toward the poles, and yet by no means lie below the Moon. For comets have their own laws of motion, exempt from what is prescribed for the planets.*" But why do we fish for Tycho's mind in his other volumes? Did he not, in that very volume 2 in which he had called the argument already mentioned *plainly convincing*, later, on p. 440, call it merely *tolerable*? And on p. 441 he added: "*Thus the last comet, that of 1585, which beyond all doubt was carried far above the Sun — so much so that that most learned man Christoph Rothmann, the Landgrave's mathematician (who agreed with me closely in unravelling its parallaxes), did not hesitate to place it in the highest sphere of Saturn at the beginnings of its appearance — resisted the revolution of the primum mobile in its proper diurnal motion by a full three degrees, whereas the star of Saturn opposes it by at least 2′, Jupiter by 5′, Mars*

by 32½. It was therefore in no way subject to the same laws of motion as any of these three planets that are called superior, although in truth it was as far from the Earth as one of them; but it was carried from west to east with a much swifter impulse."

[Margin: Rothmann's admission of the weakness of this argument.]

Finally Rothmann himself, on this comet of 1585, p. 101, speaks thus: "*If motion in comets were an argument of height, the comet would have to be placed not only below Saturn, Jupiter and Mars, but also below the Sun, Mercury, and below Venus herself — which, however, the absence of parallax entirely refutes. And therefore, given and granted that the comets observed by Pontano, Regiomontanus and others were faster than the Moon, they are under no necessity of being placed below the Moon.*" To gather, then, what these men have said into a few words: with Tycho and Rothmann assenting, the comet of 1471 observed by Pontano could cover 30° in one day, and that of 1472 observed by Regiomontanus 40°, and so be faster than the Moon and yet be above the Moon; and the comet of 1585 could cover 3° a day at its beginnings, by Rothmann's observation, and so be faster than the Sun, Mars, Jupiter and Saturn and yet lie in Saturn's sphere. Just so, the comet of 1577 could cover only 5° and be slower than the Moon, and yet be below the Moon. And so from this slowness or speed of the comets' proper motion nothing certain is gathered about their distance from the Earth; and therefore this argument is rightly belittled and rejected by Chiaramonti in the *Antitycho*, Book 2, chapters 7 and 9; by Glorioso, *De Cometis* Book 3, chapter 6; by Cottunius on *Meteorology* Book 1, lecture 33; and by Mastri, disputation 4 *De caelo*, question 2, article 4, number 46. Granted that Kepler in the *Hyperaspistes*, p. 30, would have this argument be fairly probable in itself, or a matter of opinion — that is, to one who considers it rather crudely; for examined inwardly it is worth little or nothing. Meanwhile, partly from our limitation and partly with Chiaramonti in the *Apologia pro Antitychone*, Book 1, he lays down that the syllogism must be arranged thus: "*Every phenomenon that is a star and has a proper revolution slower than the Moon's is above the Moon; but a comet is of this kind;*

therefore, etc." In the minor premise, however, it must be said that a comet is a star — which the Peripatetics will deny.

5. Argumentum à Motu Cometæ per eumdem Circulum Maximum, & quidem Ordinato ac Regulari.

Fifth argument: from the comet's motion along the same great circle, and a motion, moreover, that is ordered and regular.

VII. FIFTH. If a comet were a meteorological impression and sublunary, its motion would not be along the same great circle, and it would be disordered and irregular in its own path — that is, not only unequal, but inconstant in its very inequality, and so not even uniformly non-uniform. For it would not go by a fixed law, except perhaps for a few hours; it would be snatched along in whatever direction it was called away by the tinder of the matter to be kindled in succession, or by air stirred up by blasts of wind. But it is established that

many comets both described one and the same great circle in the sphere and were uniformly non-uniform in their proper motion for many days and months — that is, kept a certain proportion in the increase or decrease of their motion — as anyone finds who rereads what we said in chapter 4, numbers 12 and 13, and the history of the comets of 1577, 1585, 1590, 1607 and 1618. These comets at least, then, were rather celestial, and generated or formed otherwise than Aristotle taught. Seneca presses this argument home elegantly and acutely, in his usual way, although he wrongly supposes that it is not known how far a comet has advanced in one day. For in *Natural Questions* Book 7, chapter 8, against Epigenes, who had asserted that comets are kindled in the air by the force of a whirlwind stirred up and twisted, he says:

[Margin: Seneca's opinion, but sprinkled with falsehood.]

"*Whoever said this does not note what the course of whirlwinds is like, and what that of comets is like. The course of whirlwinds is rapid and violent, and more agitated than the winds themselves; that of comets is gentle, and conceals how much ground it has covered in a day and a night. Again, the motion of whirlwinds is wandering and scattered and (to use Sallust's word) eddying; that of comets is composed and keeps to a set path. Would any of us believe that either the Moon or the five stars are snatched along by the wind or spun round by a whirlwind? I think not. Why? Because their course is not disturbed and uncontrolled. Let us transfer the same to comets. They do not move confusedly or tumultuously, so that anyone should believe they are driven by turbulent and inconstant causes.*" And in chapter 32, pointing to motion along a great circle, he says: "*No ordinary and celestial fires have a bent path. It is proper to a star to trace out an orbit; whether other comets have done this, I do not know: two have done it in our own age.*"

[Margin: Tycho's epicheireme for the celestial place of the comet of 1577.]

Tycho too relies on the same argument in volume 2, where he discusses the comet of 1577 from p. 92 to p. 105, and argues thus: "*Add that in this way, by reason both of the ecliptic and of its own path, it appears to have completed a quadrant of a great circle in the sphere; and this too does much to persuade us that a celestial, not an elementary, nature belonged to this comet. Therefore, since it has been sufficiently declared and demonstrated that this comet by its proper motion described a circle in the sphere that was exactly a great circle, contained midway between its poles, and never deviated from it in this direction or that during the whole time of its duration; moreover, that under this circle it kept an ordered and not an unstable motion, slackening gradually and step by step, as usually happens in the wandering stars; that it never displayed this motion faster than the Moon's slowest diurnal motion, but on the contrary much slower; and finally that it ended the places of the beginning and of the end of its motion within each of the tropics, near the limits of the Sun's digression, and in that way completed a quadrant of the sphere — therefore, since so many reasons and indications concur, drawn from the comet's proper path, which he observed through the whole time of its duration, it is proved clearly enough that it had a motion conforming to the ethereal region: it becomes manifest that its place and course lay in the immense capacity of the celestial world, and by no means in the*

sublunary and elementary orb. This we proposed to demonstrate first, by this more general way." You see, reader, from those marks — "*becomes manifest*" and "*we proposed to demonstrate*" — that Tycho judged this argument, if not in each of its parts, yet gathered from all of them together, to be not only persuasive and probable but also demonstrative and evident.

[Margin: How Kepler defends this argument.]

VIII. Now Kepler in the *Hyperaspistes*, upholding this argument not from his own opinion but in the person of a champion of Tycho, says on p. 19 that, since Tycho showed that the comet of 1577 described a great circle by its proper motion — if not most exactly, yet more exactly than the Moon describes one in its progress — he rightly inferred that that comet was rather celestial, and not lower than the Moon. And from p. 19 to 26 he teaches that the motion of the same comet, by Tycho's observation at least, was for three months so ordered and so uniformly non-uniform that in this it was far more like the celestial motions of the planets (in which all those skilled in astronomy know well enough that certain small irregularities are to be disregarded and do not impair the proportion of the whole motion, unless one goes looking, out of season, for a knot in a bulrush) than any elementary motions: for none of these keeps an ordered increase or decrease over several days, let alone months; and that Tycho therefore rightly argued from this that the comet was celestial rather than elementary. Yet Kepler admits in the same place, pp. 5, 18 and 19, that this argument is only probable, as will be clear from his words, to be adduced in chapter 18, number 3.

[Margin: Father Cysatus's similar argumentation.]

Cysatus uses the same argument — namely, that of motion ordered and along a great circle — to establish the comet's celestial region and origin, in *De Cometa anni 1618*, chapter 3, proposition 4, proof 1, in these words: "*For the comet, as is clear from chapter 2, from the day it first shone, advancing thereafter with a regular and proportioned motion, described a great circle beneath the firmament, midway between its two opposite poles, and did not sensibly deviate from that orbit, which divided the sphere into two equal hemispheres, any otherwise than the Sun proceeds along the ecliptic, or any other planet along its own path. Therefore this comet, no less than any other planet, just as it had celestial motions and laws, so too was endowed with an ethereal nature and place.*" And a little later: "*Finally, the comet's motion in this great circle was indeed unequal, but it consisted in a lawful and most ordered inequality: at the beginning, namely of December, it was swiftest, as exceeding 3°; afterwards it decreased gradually day by day, not by a random leap but by a reasonable falling-off, exactly as is the motion of the Sun and of the other planets too. Therefore the comet too is to be numbered among the planets.*" He strengthens this argument by a comparison with other fiery meteors: "*For an elementary fiery meteor,*" he says, "*must wander off in the air this way and that without law and without proportion of motion, being driven at random and violently by the impetus either of an agitating wind or of a seething fire — as is evident in falling stars, lightning, leaping goats, the flying dragon and other genuine aerial meteors.*" Understand this

not of a small portion of motion, or of a whole motion of brief duration, but of a whole and long-lasting motion. Meeting the objection that Chiaramonti later made much of, he says: "*Nor can it be said that the comet was snatched along by the heaven or by some star in so ordered a way: for no motion of the heaven or of any star tends from south to north, as this comet's did.*" Yet he at once admits that a more effective argument for this is taken from parallax.

[Margin: Lotharius Sarsius used the same argument.]

Guiducci, following Galileo's mind, had objected to the disputation held in our Roman College on the three comets of 1618, which had also used this same argument, that the motion of comets can be defended without any circle, let alone a great one, and can take place by a rectilinear passage. To this Lotharius Sarsius replies in the *Libra Astronomica*, p. 9, from the teaching of Father Orazio Grassi, that in this controversy it was supposed that the comet is carried with a circular motion, and that the disputation was with those who already supposed this; and on that supposition, that the comet's path was ordered along a great circle, it is rightly inferred from this that its nature is celestial rather than elementary — although he says that that rectilinear motion is entangled in great difficulties, of which more elsewhere, below.

[Margin: And Biancani.]

Biancani too, in *Loca Mathematica Aristotelis*, p. 93, puts forward both parts of this argument — from the great circle and from motion regularly unequal — yet he ventured to define nothing in this controversy.

[Margin: And Froidmont.]

But Froidmont, *Meteorology* Book 3, chapter 1, article 3, affirms from his own observation at Louvain as well that the comet of 1618 crept through Libra, Boötes and the Bear to the Arctic circle, and only toward the end strayed a little from the segment of a great circle; and that the same was observed by Kepler at Linz, by Cysatus at Ingolstadt, and, in the comet of 1577, by Tycho. From this, and from the regularity of the motion, he concludes thus: "*I conclude, therefore, and lay down as probable, that those comets, so composed and unwavering under the same cir-* cle — a great circle — gliding by their proper motion, are carried round in an ethereal region that is not so turbulent and unsteady as this sublunary one is.*"

[Margin: And Resta.]

[Margin: And Cabeo.]

Franciscus Resta [Francesco Resta] likewise uses this argument, in his *Meteorology*, tract 1, chapter 2, and so does Cabeo on Book 1 of the *Meteorology*, text 37, question 1. Gathering the argumentations of those already mentioned into syllogistic form, then, take it thus:

[Margin: Major premise.]

1 *No elementary and sublunary mobile, especially a fiery one, moves by its own proper motion for many days, let alone months, sensibly along the same great circle of the sphere, with a motion either equal, or unequal indeed but sensibly regular and ordered — or at least not as exactly as the Moon.*

[Margin: Minor premise.]

2. *But some comets have moved by their proper motion for many days, or even months, sensibly along the same great circle of the sphere, with a motion unequal but sensibly regular and ordered; and at least more exactly than the Moon.*

[Margin: Conclusion.]

Therefore *some comets are not elementary and sublunary mobiles.*

The *major* is evident by induction through all elementary mobiles, in none of which do all the conditions enumerated in the major of the syllogism concur; and it will become still clearer in the solutions of the replies. The *minor* is proved from the history of the observations set out in chapter 3, on the comets observed from 1577 to 1618. If you yourself mark on the surface of a celestial globe the places those comets passed through, point by point, you will see that many of them moved along the arc of a circle whose plane passes through the centre of that globe and cuts it into two hemispheres — which is the property of a great circle in the sphere, from Theodosius, *Sphaerica* Book 1, proposition 6. And if you compare the motion they made from day to day in the tables given in that chapter 3, you will see that several of them diminished their speed by a proportional decrement, with a difference either nil or smaller than that found in the motion of the ☾ or of the other planets.

[Margin: First reply to Argument 5.]

IX. It can be replied, first, as some Peripatetics reply on their side — namely Chiaramonti in the *Antitycho*, from p. 110 to p. 151, and in the *Apologia* for the *Antitycho*, from p. 15; and, following him, Cottunius, *Meteorology* Book 1, lecture 33, and Glorioso, *De Cometis* Book 3, chapter 5 — by denying the major premise. They add that even if it were granted that some comets moved with an ordered and regular motion, it by no means follows from this that they were celestial rather than sublunary or elementary. First, because the motion of comets, which these authors posit as made along a great circle, can be saved by a rectilinear trajectory without any circle at all, let alone a great one. So Kepler did in the *Optics*, p. 335, and in his little book on comets, where for that reason he said: "*Those who demonstrated the motion of the comet of 1577 by circles took on a most difficult task, and yet it did not wholly succeed.*" And in the *Hyperaspistes*, pp. 6 and 10, he granted Chiaramonti that this proposition is not at all necessary if it is taken apart from parallax — indeed, that the argument drawn from it is faulty and weak. On rectilinear motion Cysatus, *De Cometis* Book 2, proposition 8, and Froidmont, *Meteorology* Book 3, chapter 4, article 4, subscribe to Kepler; and, with Galileo, so does Mario Guiducci in his opuscule on comets. Next, because for a motion along a great circle in the sphere of the world it suffices that the circle of the motion be described from the centre of the world, and this can happen just as well with an elementary mobile, however near the Earth, as with a celestial one. Hence it does not follow from the nature of a great circle that the mobile is celestial rather than elementary or sublunary.

[Margin: Many elementary things travel in the plane of a great circle.]

Moreover, along the same plane of a great circle — and indeed keeping the same tenor, whether of equality or of sensibly uniform non-uniformity — there move many missiles, whether propelled by hand or by the bowstring through ballistas or other machines, or by sulphurous and nitrous powder; and thunderbolts, before they strike a resisting body and are twisted into spirals and vortices by some other motion *per accidens*; and certain perennial winds, driving sails and ships along the same great circle for many days; and certain canals led in a straight course over a long tract of land. For the example that Chiaramonti adduces of the sea, and of the ebb and flow of the sea keeping a proportional increase or decrease, is rightly rejected by Kepler in the *Hyperaspistes*, p. 21: the sea is not poured onto the land and drawn back with the tranquil tenor of a balance, but with waves driven back and forth in different directions, and with disordered motions, greater in one place and less in another. More to the point is Chiaramonti's instance on p. 117, about the motion of heavy and light bodies thrown in a circle by an external cause: for while these try to descend to the middle, or to ascend from the middle, they tend toward the centre and proceed along a plane cutting the sphere at the centre of the world. But what Io. Bapt. Benedictus [Giovanni Battista Benedetti] says in his letters — that ships always incline toward the equator, and, if land did not stand in the way, would rush to the equator so as to describe a great circle there — lacks solid foundation. For the ocean is not lower under the equator, however much vapour and exhalation the Sun draws from it there, such that the waters would have to flow down there because of the slope; and ships can describe a great circle by their own motion even when they are not at the equator, by proceeding along vertical circles, as we showed in the Geographical Book, where we treat of hydrography. Moreover, the motion of a pendulum held in the same plane, swinging up and down to and fro, takes place along the plane of a great circle.

As for the fact that no other fiery meteor persists for so long a time in the same great circle, and in a motion unequal in an ordered way, as some comets do: this happens (say the Peripatetics) because the other meteors do not have so much matter, or matter so densely packed and tempered, which, kindled all at once or successively, could feed a flame for many months or days, as comets have. In this respect comets are rather to be compared with certain vast lamps which, owing to the quantity of oil and a wick that is not quickly consumed, burn for many days, and for still more if crushed salt is mixed into the oil — unless you prefer to compare them with the lamps found in the tombs of the ancient Romans and shining there for whole years, about which Liceti wrote learnedly. That major premise, therefore, is not universally true, even if comets have some properties that are not found in other fiery meteors, and on account of which they have a somewhat greater agreement with the stars and planets. In the same way light has some properties in which it differs from all other corporeal qualities, and a certain greater likeness to spiritual things, and yet we do not conclude from this that it is a spiritual quality. Or again, the more

recent astronomers admit that the spots and faculae of the Sun are now and then generated and destroyed — in which respect they are more like sublunary fiery meteors than stars — and yet they will not forthwith concede to the Peripatetics that these are elementary or sublunary bodies.

[Margin: Glorioso recognised nothing but verisimilitude in this argument.]

Therefore, just as we saw in the reply to argument 4 that it does not necessarily follow from a motion slower than the Moon's — as Tycho himself at length acknowledged — that comets are above the Moon, however great their likeness in this respect to the planets slower than the Moon, so neither does it follow from motion along a great circle, and ordered motion, that comets are above the Moon. Glorioso frankly admits as much in Book 3, chapter 6, p. 141, saying: "*And so, although these reasons drawn from the speed and slowness of the proper motion, and also from the great circle, may seem in some way weak if they are put forward in this way and simply, nevertheless they are not at all to be neglected if they correspond with the other circumstances of the distances and of the motion. Since, therefore, the astronomers noticed that comets do not keep a wholly irregular inconstancy in the speed and slowness of their motion, nor in the straightness of their path and route, and since they saw that comets agree with the stars in very many things, they believed they could at least establish this quite probably: namely that comets are very far distant from the Earth and do not consist of flaming matter, which by its nature can keep no proportion and constancy either in motion or in path.*" Neither Kepler nor Froidmont recognised anything in this argumentation but a certain probability or verisimilitude; and to it the Peripatetics could nevertheless oppose a conclusion no less verisimilar, by replying through an instance, as follows.

[Margin: Reply 2, by instance.]

X. *It can be replied*, therefore, secondly, by turning a similar argument back upon the adversaries.

[Margin: Major premise.]

1. *No celestial body observable with the naked eye moves either outside the zodiac, or in such a way that it does not describe a circle parallel to the zodiac; nor is any sensibly generated anew and extinguished before completing one revolution.*

[Margin: Minor premise.]

2. *But very many comets move outside the zodiac, in circles not parallel to the zodiac; and they are sensibly generated anew and extinguished without completing even one revolution.*

[Margin: Conclusion.]

Therefore *very many comets are not a celestial body, etc.*

But who, unless his mind were already preoccupied by prejudice, or by a prior inclination toward the Tychonic opinion, would deny that, in this deviation of their motion from the zodiac and its poles — a deviation, indeed, as great as is gathered from what was said in chapter 4, numbers 10 and 11 — and in their rising and perishing, comets bear a greater likeness to sublunary things than to celestial bodies? With these topical arguments weighed on either side,

then, we are compelled to remain almost in equilibrium, until the weightier moments of other reasons incline us to one side.

[Margin: 3rd reply to Argument 5.]

XI. *The Peripatetics already named reply*, further, thirdly: granting or passing over the major of Argument 5, they deny the minor proportion [?]. They think that Tycho was refuted by Chiaramonti — as Chiaramonti himself also asserts — and that it was shown from data given by Tycho himself that the comets neither described the same great circle geometrically nor kept an exact proportion in the increase or decrease of their speed; especially since neither Tycho nor others observed the comets daily from the beginning of their first appearance until their extinction, but gathered the unobserved motion of the other days by probable conjecture from the observations of a few days.

[Margin: Chiaramonti's reasons against Tycho, and their solutions, as regards the progress along a great circle.]

That it did not describe a great circle, Chiaramonti proves in the *Antitycho. First*, on p. 119, because he had already shown in Book 1, chapter 11, that if a comet's true motion is along a great circle, its apparent motion cannot be along a great circle if the parallax is sensible. But from what will be said below in chapter 17, theorems 3 and 4 and the following, it will be clear that this holds only in proportion to the size of the comet's parallax. Hence if the comet's parallax is less than the lunar, as Tycho already presupposed at the time, the true motion along sensibly the same great circle can be deduced from the apparent one. He proves it *secondly*, in the same work, Book 2, chapter 6: when the comet's places in longitude and latitude given by Tycho are examined, it is gathered that the comet of 1577 did not keep the same inclination to the ecliptic in its course, nor describe the same great circle most exactly, as Tycho said, but followed a tortuous path, like the alternating windings of serpents. Indeed, at the beginning that arc cut the ecliptic at 20° 51′ of ♐, but later at 20° 20′ of ♐ — a difference not of only 5′, which Tycho acknowledged and judged negligible, but a difference of 31′, which does not seem negligible, and at times a difference of 55′. But this does no harm to Tycho, or to Kepler, his Hyperaspistes, who on p. 25 grants this difference, though always less than one degree, as Chiaramonti admits in those places. And this does not prevent its being said that the comet described the same great circle more exactly than the Moon, since Chiaramonti admits in the same chapter that none of those arcs deflects from its great circle by more than a sixtieth part of the lunar flexure, that is, of the amount by which the Moon deflects from its great circle. Would that astronomers could always keep within a certainty of one degree for the places of the nodes, or sections, and of the apsides of any planet! Astronomy would be happier for it, as Kepler admits in the same place. Tycho would nevertheless have spoken more cautiously if in volume 2, pp. 92, 93, 103 and 105, he had not called that circle of the comet "*a circle exactly, indeed most perfectly and most exactly, great*", and like the ecliptic, indeed more perfect than the Sun's course, and so given

Chiaramonti occasion to refute him on this point. But this argument will be discussed again more closely in chapter 18, from number 2 to number 6.

[Margin: Chiaramonti's other attempt against Tycho, repelled by Kepler.]

XII. The other part — namely that the comet's motion was not ordered — *Chiaramonti* tries to show in the same Book 2 of the *Antitycho*, chapter 6. He examines the observations of seven days and reduces Tycho's given longitude and latitude of the comet to geometrical calculation; from this he establishes that the proportion of decrement by a sixth part at each step, which he says Tycho asserted, was not kept; that the motion even increased at times by a sixth part; and that not only was the sesquialteral proportion not kept in the decrement, but no other kind of ordered and continuous proportion either. But *Kepler* in the *Hyperaspistes* denies that Tycho asserted a continuous sesquialteral proportion in the decrement of the comet's motion, or one made continuously by a sixth part of one degree. Granted, Tycho said in volume 2, chapter 6, p. 98, that from November 15 to November 20 the comet advanced so that in five days it completed 14° 3′, "*so that, if it had advanced equally each day, it was found to complete now not fully three degrees, falling short by about a sixth part*" — that is, falling short by about 10 minutes, so that the motion, if it is supposed equal, would have been 2° 50′ daily; whereas he says that from the 14th to the 15th it completed 3° 48′, and from this he gathers that it was swifter at the beginning and then decreased successively. But nowhere in that whole chapter 6 does Tycho mention a continuous decrement by a sixth part of a degree, or that sesquialteral proportion which Chiaramonti attributes to Tycho. Kepler read that chapter, I have reread it, and let anyone who wishes read it again: he will not find what neither Kepler nor I found. So far, indeed, was Tycho from meaning this that he recognised in that cometary motion a retrogradation and station similar to the motions of the planets, to which a continuous proportion in decrement certainly does not belong.

[Margin: Tycho's legitimate meaning on the proportional decrement of the comet's motion.]

For hear what Tycho says in that chapter 6: "*Add that in this very circle, although the comet was seen to move unequally, as the planets themselves also do in their orbs, it nevertheless by no means admitted a disordered inequality — one, that is, that would leap suddenly from slower to faster, and back again from this to that, with a wandering course — in the way that meteors generated in the elementary region are observed to have such an uneven and inconstant motion. Rather, this comet, under that portion of a great circle, was carried from a faster apparent motion into a slower one successively and proportionally, keeping a like course of inhibition, so that it was found to have, no less than the planets, an alteration of the restraint of their ordinary motion when from a faster course they end in stations. For on no following day did it become faster; it always slowed its motion by a like course, so to speak, until in the last days it scarcely completed a third part of one degree each day, whereas in the earlier days it seemed to complete five whole degrees, keeping meanwhile, as I said, the proportion of the ordinary falling-off.*" Nor does he say otherwise on pp. 102, 104

and 105, where he inculcates the same doctrine. It is true, though, that Tycho supplied the motion of some unobserved days by a verisimilar analogy, as is also done in the tables of the celestial motions and their equations, although immediate observations are not had for each and every day of the period.

[Margin: Cottunius and Glorioso subscribing to Chiaramonti with too little circumspection.]

Chiaramonti therefore interpreted Tycho with excessive rigour; and so did Cottunius, who copied certain numbers from Chiaramonti on p. 295 without examining Tycho closely, or Tycho's Hyperaspistes, which had come out long before. *Glorioso* too, *De Cometis* Book 3, p. 132, gave his vote to Chiaramonti against Tycho on this point far too hastily. Let it stand, then, that if the major proposition of the fifth argument were true, nothing in the minor would prevent some comets from being called celestial rather than elementary and sublunary. But because the major is not certain, only probable, it likewise becomes probable from this, though not evident, that the comet was above the Moon — saving, however, the probability of the opposite, until it is evidently refuted from another quarter.

6. Argumentum à Magnitudine Cometarum.

Sixth argument: from the size of comets.

XIII. SIXTH. It is scarcely likely that so great a quantity of exhalations is raised from the earth and sea up to the highest region of the air — let alone to a higher region — as is required to compose the bulk of comets, which sometimes rival the size of the Sun and with their tail occupy 60 or 70 degrees in length and more, as is certain from the history summarized in chapter 4, numbers 6 and 8. This is especially so when they last for several months, or when several comets arise at the same time in the same region, or in immediately following years, and not enough time passes for a sufficient heap of exhalations to gather little by little. For this is why earthquakes occur in the same region only very rarely; and so too the blasts by which the swelling ocean overwhelms islands or cities; and the eruptions of Vesuvius, of which only thirteen are counted from the birth of Christ to our own times. And indeed, if the conditions that the Peripatetics require for forming and kindling a comet are considered, it will be clear how little likely that reasoning is. For grant that exhalations of this kind are drawn not only from the sea but also from the earth (which, however, Plato denied, according to *Olympiodorus*, *Meteorology* Book 1, section 5, and so did *Io. Baptista Porta* [Giambattista della Porta], *De aeris transmutationibus* Book 1, chapter 8), and that by the force of their lightness they make for the upper region;

they still ascend only so far as their acquired heat lasts. But when they reach the colder region of the air — in which, even in summer, rain and hail form, and in autumn and spring hoar-frost, and in winter snow — a great part of them is chilled and their lightness is blunted, so that they penetrate no further. Mixed with vapours, and enclosed or hemmed in by clouds, this

matter either escapes, or rages in an oblique uprising into winds and storms, or rumbles and roars in thunder, or threatens an outlet in lightning-flashes and coruscations, or at last breaks out in thunderbolts once the prison of the clouds is burst. Indeed, if this matter had to be unctuous, that is fatty and oily, as the more recent Peripatetics prefer — so as to be better suited to cohesion and to feeding for a long time the flame it has conceived — it retains by that very grease a certain viscosity, in which the wings of its lightness, so to speak, are entangled as if in birdlime, so that it cannot fly up to the highest region. This is why odours from ointments and oils, however very sharp, spread only to a small distance unless they are carried further on the wings of the winds, while the other hot and dry exhalations, lacking oiliness, ascend much higher. And those small remaining quantities of such exhalations which, having broken through the battle-line and rampart of the second region, managed to struggle into the third or highest region, are either at once kindled by the heat of fire, if there is fire there, and, burnt up, fall down as ashes or very fine soot; or else — if one may use Aristotle's opinion against Aristotle — they are carried off by the motion of the primum mobile and dispersed rather than gathered, for on account of this carrying-off too the Philosopher denied, *Meteorology* Book 1, chapter 4, that there can be clouds in that highest region. Nor can any sufficiently probable cause be adduced that would gather those exhalations, since that motion of carrying-off is of itself uniform, and toward the west, and there are no winds there, or whirlwinds, or eddies of water, by which we see sand, chaff, straws, shavings, foam, filth and other such things otherwise scattered sometimes gathered at random into one heap or one mass. Therefore either those exhalations are purely hot and dry, and these are not fit for packing together, for mutual adhesion, and for a long-lasting tinder of flame; or they are viscous and steeped in a kind of oil, and then only a very small part of them can penetrate beyond the second region of the air and be gathered into as great a quantity as large, long-lasting and frequent comets demand.

[Margin: Seneca's sharp dilemma.]

Seneca indicated this in a few words in *Natural Questions* Book 7, chapter 9, against Epigenes: "*Let him choose, then, whichever he likes: either a light force will not be able to reach so high, or a great and agitated one will sooner break itself. Moreover, those lower comets (as they think) do not go out higher for this reason, that they have more of the earthy: their own weight holds them close by. And yet these longer-lasting and loftier comets must have fuller matter. For they would not appear for longer unless they were sustained by greater nourishment.*" Regiomontanus too, on the comet of 1472, says: "*No aerial impression can, from the natural causes of flame-bearing exhalations, supply the matter of a comet for the space of one year, such as was the one which Josephus testifies foretold the destruction of Jerusalem; but comets come from hidden causes of nature — of which opinion is Messhala* [Māshāʾallāh] *the Arab.*"

[Margin: How great the size of comets' heads is reckoned geometrically.]

XIV. But how vast a quantity of the exhalations already mentioned the size of comets demands cannot be estimated well enough by the physicists from their apparent size together with their altitude known by parallax, unless they take refuge with the geometers. You must recall their opinions at this point from chapter 4, numbers 6, 7 and 8.

[Margin: Snel's calculation of the size of a comet compared with the whole air.]

Here I wish only to bring forward what Villebrordus Snellius [Snel], in his book on the comet of 1618, chapter 6, concludes: that the head of the comet of 1577 was larger than the whole air, if the air were contracted into a single globe. For this calculation he supposes the height of the atmosphere to be 13 German miles, or 52 Italian, and the semidiameter of the Earth 1089 German miles, and therefore the whole semidiameter from the centre of the Earth to the top of the atmosphere 1102. Then, cubing 1089 and 1102 separately, the cube of the number 1089 comes to 1,291,467,969 cubic miles, while the cube of the number 1102 is 1,338,273,208 cubic miles. Now spheres are to one another as their cubes; hence from the difference between those cubes one obtains the proportion between the solidity of the air alone, gathered into one globe, and the solidity of the Earth, and it stands to the latter as 46,805,239 to 1,291,467,969, or as 1 to 27½. Now if the Earth were placed at a distance of 300 terrestrial semidiameters from the centre of the Earth, it would appear, he says, under an angle of 11′ 30″, and at the same distance the whole air, reduced into one sphere, would appear under an angle of 3′ 50″. But the comet of 1577, according to Tycho, subtended 7′ in diameter and was about 300 semidiameters of the Earth distant from the Earth. Therefore that comet was nearly twice as large in diameter as the whole solidity of the air reduced into one globe, and in solidity nearly eight times larger; and if it had had only 4′ in visual diameter, it would still have been larger than the whole sphere of the atmosphere. "Let this deep lethargy, then," says Snel, "be shaken off by men all too credulous, and let them recognise that not only all the exhalations of earth and sea together, but not even the whole air, suffices for the matter of the head of one comet."

[Margin: Size of the comet's tail.]

XV. The size of the tail, moreover, has varied, even in the same comet observed on different days, as is clear from what was said in chapter 4, number 7; but in many it was so great that with its length it occupied 70, 90 or 104 degrees, or even the whole tract of a hemisphere. But to propose two examples of true size, one for celestial comets and the other for those below the Moon, we shall use problem 3 of chapter 24, to be given below. From chapter 4, number 8, we shall take it, from Regiomontanus's measurements, that the comet of 1472 had a true diameter of head of 16 German miles, and a tail 4000 German miles long, as Gurcensis estimated. Multiplying 16 by 16, then, gives the square that can be circumscribed about the disc of the head, of 256 square miles; multiplying these by 4000 gives a prism of 10,240,000 cubic German miles, the third part of which is 3,413,333 cubic miles, if we wish the tail to have

been conical. But if we make, as 14 to 11, so 10,240,000 to another number, the cylindrical solidity of the tail will be 8,045,714 cubic German miles, which are 32,182,858 cubic Italian miles. In the terraqueous globe, however, there are 296,457,106,140 cubic Italian miles, as we showed in Book 2, chapter 7; hence the solidity of the comet was to the solidity of the Earth very nearly as 32 to 296,000. And let this be the example for a sublunary comet. For a celestial one: if, as I said in chapter 4, number 8, the true diameter of the head of the comet of 1577 was, according to Tycho, 368 German miles, and the length of the tail at least 81,700 of the same miles, the bulk of the whole comet is gathered by the same method in cubic German miles — cylindrical, 8,307,382,058; conical, 3,524,646,933. That is, compared with the Earth's solidity, the cylindrical is as 33 to 296, and the conical as 14 to 296.

[Margin: Reply to Argument 6.]

XVI. *It can be replied*, however — as Chiaramonti more or less replies in the *Antitycho*, Book 2, chapter 30, and in the Appendix to the three opuscula on the sublunary seat of comets, pp. 92 and 179; Cottunius, *Meteorology* Book 1, lecture 33; and, for sublunary comets, Froidmont, *Meteorology* Book 3, chapter 2, article 4, Cabeo, *Meteorology* Book 1, text 37, question 5, and Resta, tract 1, Book 1 of the *Meteorology*, chapter 4 — by denying that what is said is scarcely likely. For the arguments brought against the ascent and gathering of suitable exhalations prove only that it is difficult for so great a quantity of such exhalations to arise as the kindling, quantity and duration of a comet require, and that comets therefore arise rarely. Nor, however, is matter required that is actually unctuous: volatile salts suffice, which are such in virtue, that is, extracted from an oily substance. Nor is so great a density required as is supposed in that argument, but only that the matter be well compacted and concocted, so as to acquire opacity (which is no doubt different from density, as is clear in crystal, which is denser than smoke and yet less opaque), and at the same time a temper apt for conceiving and feeding fire — for which density is not required. For we see that the fresh smoke of an extinguished candle, if it is brought up to a flame from the side or even from below, is rekindled even though it does not touch the flame. Further, does not so much smoke rise from a single log — nay, from a piece of wood — that we see a whole room filled, even to the discomfort of our eyes? And this whatever may be said of that experiment of Snel's by which, as Froidmont relates in *Meteorology* Book 2, chapter 3, article 14 — and Cabeo and Resta after him, in the places just cited — he found that a grain of gunpowder expands into a flame of a bulk 125,000 times greater than that grain.

[Margin: Size of the flame of gunpowder.]

Besides, do we not see clouds occupy the whole vault of the sky, which, if they were raised much higher, would produce a body apparently equal to any comet. Or, to use a more apt example: do not those exhalations which become the twilight lamps as the Sun approaches the horizon encircle a very broad and long tract, so that with their whiteness they seem to occupy half the hemisphere

all round? What wonder, then, that over many years so great a quantity of exhalations should be gathered from earth and sea, which at last coalesce into one mass and blaze up into one comet or several — whether through the motion by which the highest region of the air is necessarily moved along with the motion of the lower air driven by the force of the winds, so that no vacuum may arise, or through their own wandering lightness?

As for what was said about the bulk of comets: first, it is replied that it is not necessary for their heads to be perfectly spherical globes, and their tails of cylindrical or pyramidal solidity; for they can be flattened like certain clouds, and so of smaller bulk. Next, if comets are in the highest region of the air, as Aristotle supposes, their bulk does not turn out so vast as to exceed the powers and the quantity of the exhalations required for a comet. This is clear a fortiori from Regiomontanus's comet. For if that comet, though several semidiameters of the Earth high above the Earth, was nevertheless to the bulk of the Earth only as 32 to 296,000, as we said in number 15, it would have been far smaller had it stood no further from the Earth than the clouds do, and would have been smaller in bulk than many clouds, which sometimes occupy the whole visible hemisphere in length. But if comets ascend as far as the Moon or beyond, their bulk does indeed turn out excessive when compared with the quantity of elementary exhalations, as we showed above in number 15 from a calculation resting on Tycho's foundations. That altitude, however, is not conceded by the Peripatetics unless they are forced by the force of parallax to concede it; and once that has happened, they will likewise concede that some comets are made from matter other than elementary. Parallax set aside, however, that altitude is not to be supposed against the Peripatetics; against them one must rather use the mere altitude of the highest region of the air.

7. Argumentum à Duratione Cometarum.

Seventh argument: from the duration of comets.

[Margin: Fracastoro's argument.]

XVII. SEVENTH. Granted that a quantity of exhalations sufficient and suitable for forming a comet could be heaped up, it is still not likely that a quantity so great could be either gathered from the beginning or substituted in succession that it should burn for many months with the same tenor of shape, and the same temper of colour and motion, without leap or rush of flame. That comets lasted several months, and more than once six months, we showed in chapter 4, number 5; and that many of them kept their former colour, shape and steadiness of calm motion is attested, among others, by Fracastoro in the *Homocentrica*, section 3, chapter 23, where he argues from it as follows: "*For that they follow their fuel does not seem likely enough. First, because it would be very wonderful that all comets should have that fuel in breadth so portentous* [?]; *and then it would seem still more wonderful that it should be set out through so great a space with such order and evenness, when in other kindlings of this kind that follow their fuel we see them borne without order* [?] *and as if by certain leaps, and see their shapes change*

according to the diversity of the fuel, and their colours, and the rest. But in the three comets already mentioned the same order, the same shape and colour, all remained unchanged." But long before Fracastoro, Seneca brought more to this argument in *Natural Questions* Book 7. For in chapter 22 he says: "*Whatever the air creates is short-lived, for they are born in a fleeting and changeable thing*"; and again: "*It cannot be that a fixed fire should settle in a wandering body and cling so stubbornly. Then, if it clung to its fuel, it would always descend; for it is the thicker, the nearer it is to the earth. A comet is never let down to the lowest region, nor does it approach the ground. Further, fire either goes where its nature leads it, that is, upward, or where the matter to which it has adhered, and on which it feeds, draws it*"; and in chapter 23: "*Everything that a temporary cause kindles quickly perishes: so torches burn while they pass; so thunderbolts avail for a single stroke; so the stars called cross-flying and falling fly past and cut the air. No fires have any stay except in their own — I mean those divine ones, which the world holds eternal.*" Add to this that not even the chasms or the fire-vomiting mountains — Vesuvius, Etna, Hekla and the like — belch out flames perpetually, or for several months; and yet the matter of that flame is fattier than that of comets, and more tenacious, as is recognised from the naphtha and bitumen they sometimes vomit forth; they rekindle the slumbering flame after intervals. The duration of snows and clouds, moreover, is in no way comparable with the duration of comets, because those do not have in themselves a principle that dissolves or consumes their matter as quickly as all burnt and inflamed things have. On the contrary, we showed in chapter 10, in the solution of Argument 2, that the duration of comets, even of one month, let alone of 6 months, is more like the definite and manifest duration of stars and planets than the duration of falling stars.

[Margin: Cardano's opinion.]

The force of this argument grows, moreover, if the duration of comets is combined with their bulk, especially that of the higher ones. This is what Cardano did in Book 4 of *De subtilitate*, when he said: "*Most comets exceed a second month, and some do not end even in the third; for this conflagration the whole fabric of the earth would not suffice. For it was shown above that fire is never in the same matter, but perpetually requires new. This star with a tail, however, is* [?] *for the most part in the aether as well: so it comes about that it is not smaller than the Moon. And that so much matter should burn, and last for three months, is altogether impossible.*" How much less for six? This argument taken from duration is used besides by many: above all by Biancani, on the mathematical passages of Aristotle, p. 93; Tycho, volume 2, chapter 6; Santucci, p. 23; Lotharius Sarsius in the *Libra Astronomica*, p. 33; and Thomas Fienus, *De Cometis*, p. 29.

[Margin: Reply to Argument 7.]

XVIII. A reply is made, however, partly by Chiaramonti in the *Antitycho*, Book 2, chapter 30, and in the opuscula on the sublunary seat of comets, part 2, chapter 10, and partly by Galileo in the *Trutinator*, p. 125: that the argument made is valid at most against Aristotle, who asserts that a comet is a kindled exhalation that needs fuel, but not against all who assert that it is sublunary.

For someone could say that its matter is purged and cleansed of dregs, and afterwards illuminated by the Sun; or that fire needs fuel in a foreign matter and region, but in its own no longer needs it; and that the comet, condensed or having acquired opacity, can shine and display a gleaming whiteness — in the way that the exhalations which reflect the light of twilight to us, and perpetually spread the light of the whitening dawn in some hemisphere from whose horizon [?] the Sun, placed below it, is about 18 degrees distant. By this example, it seems, Aristotle's opinion too can be defended; for it does not seem impossible that so great a quantity of exhalations could come together as to rival in duration the twilight, which is of itself perpetual if the Sun approaches. Others, indeed, use the example of sepulchral lamps, kindled in such matter that they can burn for many years. And certainly, if a cone with a wick is made from ground salt mixed with oil, the flame lasts much longer in a little oil than without the admixture of salt. What then forbids that the volatile salts of which a comet is made should come not from the spirit of nitre alone (from which falling stars perhaps come), which is very easily kindled but very quickly extinguished, but from another matter fit to preserve a flame for several months, until, that matter consumed, the whole comet is extinguished? For the example of twilight is indeed valid for mere brightness; but through it no account is given that fits a kindling, nor why the comet does not always last, or return at the Sun's approach, as twilight returns every day, but is extinguished and rarely returns — so much so that from the year 1618 up to this year 1650 no comet has been observable, or observed, in our regions.

[Margin: Second reply, according to the mind of Aristotle.]

Aristotle himself, however, would rather reply with the example of the Galaxy, since in *Meteorology* Book 1, chapter 10, he says that the milky circle is as it were a perpetual comet, or the stable hair of a great circle. For he holds that the Milky Way is exhalations in the highest region of the air which are in potency fire, so that, when the air is dispersed by the motion, such a consistency is separated out as there is in those comets that are kindled not by the mere carrying motion of the heaven of itself, but also by the motion and action of some star.

[Margin: How the milky circle is, according to Aristotle, a perpetual comet.]

For just as such comets are an exhalation kindled by the motion of the heaven and by the motion and action of that star, whose motion they therefore follow and from which they draw their hair, so by the motion of the heaven and the force of the very many great fixed stars that lie in the tract of the Galaxy, is separated out and kindled the quantity of exhalations that imitates the milky whiteness; and the milky circle is the perennial hair of that whole starry circle, and so a perpetual comet. For this is the legitimate sense of Aristotle in that passage. But Aristotle's reply is now refuted by the telescope, which shows us that the whiteness is an aggregate, and as it were a hedge, of very tiny little stars. Next, if the Galaxy is not only a brightness but a kindling, which nonetheless persists, why do the other comets too — by the force of the other stars, many

and great, that are outside the Galaxy — not either persist longer, small as they are, or return more frequently? Lastly, if the multitude of stars of the milky circle has so much force that it perpetually kindles that tract, or keeps it kindled, why does it not at last produce as great a splendour as has been seen in many comets that rivalled the Moon and the Sun? A comet, therefore, is something other than a mere kindling, and has another cause of its rising and perishing than the one that is chanted in the Peripatos.

8. Argumentum à Caudæ Diaphaneitate.

Eighth argument: from the transparency of the tail.

XIX. EIGHTH. No flame in packed and consistent matter, such as Aristotle requires for comets, is so transparent that through it — especially when it is placed far off — the remotest stars can be seen by us. But through comets, or at least through their tail, stars can be seen and have in fact been seen, as Tycho observed in the comet of the year 1577, from p. 143 of the *Epistolae*; and Carolus Piso confirms it, in the *Speculum Cometae*, chapter 19, by an experiment of Father Philippus Plumeretus on the comet of the year 1618.

[Margin: Seneca's opinion on the transparency of the tail.]

And long before, Seneca set this down as asserted by others, *Natural Questions* Book 7, chapter 18, raising it as an objection to himself and to Apollonius of Myndus, and saying: "*A star never appears through a star: our sight cannot pass out through the middle of a star so as to see the higher things through it. But through a comet the further things are seen no otherwise than through a cloud: from which it appears that it is not a star, but a light and haphazard fire.*" To this objection he replies in chapter 26 by granting the transparency of the tail, thus: "*We do not see further things through stars, you say; through comets we send our sight. First, if this happens, it happens not in the part where the star itself is of dense and solid fire, but where a thin splendour runs out and is scattered into hair. You see through the intervals of the fires, not through the fires themselves.*" The tail, therefore, is a ray emitted by the comet and not of the comet's substance, in Seneca's judgement.

The Peripatetics, however, *will reply* by denying that the flame of a comet cannot be like our flames in transparency — the transparency by which a candle placed and lit beyond a flame is seen through it, just as the image of that little flame is also seen in a mirror between which and the candle there is a larger flame, for instance that of a torch or of a lit pyre.

9. Argumentum à Magnitudine apparenti Cometæ per Telescopium spectati.

Ninth argument: from the apparent size of a comet viewed through the telescope.

XX. NINTH. A telescope usually represents distant objects as far larger than when they are viewed with the naked eye, but on this condition: that it shows the Moon, the Sun and the nearer planets larger by a notable increment, and increases their visible diameter twentyfold, thirtyfold and much more, if it is one of the more excellent telescopes; whereas the fixed stars, because they stand at a huge distance, it increases either not at all or very slightly. But the comets

of the year 1618, viewed at Rome and elsewhere with a very large telescope, received no increment; and people of earlier times would probably have found the same in several comets, had they known the use of the Netherlandish tube. It is therefore a sign that that comet and similar ones were above the Moon and stood at a very great distance from us. This argument, indeed, was used by the authors of the disputation held in the Collegio Romano, and by Lotharius Sarsius in the *Libra Astronomica*, p. 15.

[Margin: First reply, Chiaramonti's — but trivial, and in part false.]

XXI. Chiaramonti replies, *Antitycho* Book 2, chapter 32: first, that he had used a tube not wholly to be despised, by whose force, however, the Moon grew by no conspicuous increment; next, that comets therefore do not acquire from the telescope an increment such as the Moon does, because they are not such great and bright objects but, compared with the Sun and Moon, much smaller and dimmer, and do not have so great a sphere of activity. But Kepler, in the *Hyperaspistes*, p. 124, rightly censures him for seeming to deny that the Moon is enlarged by the telescope, since a single one of its spots — of those which scarcely equal a fortieth part of the Moon — is seen distinct in its parts and nearly equal to Jupiter seen with the naked eye. And indeed, from where does it come that with more perfect tubes not the whole Moon but only a part of it can be taken in by the telescope at a single glance, if not from its notable enlargement? Chiaramonti's second reply, moreover, cannot be fitted to the fixed stars, which in Chiaramonti's own opinion are larger and brighter than comets, and yet receive no notable increment from the telescope.

[Margin: Second reply, by others.]

XXII. A reply is therefore made, and a better one, by Mario Guiducci, *De Cometis*; Galileo in the *Trutinator*, from p. 65 and p. 217; Tannerus, *De caelo*, question 7, p. 138; and Mastri, disputation 4 *De caelo*, question 2, article 4, number 46. First, by turning the argument back against its own authors, who did not place the comet of the year 1618 above the Sun, as the force of the argument required, since by their own admission the Sun does in fact receive a notable increment from the tube. Second, by denying the causal proposition placed in the major premise of the syllogism — that is, that the cause why the fixed stars receive from the telescope no sensible increment, and none proportionally as great as the Moon and the Sun, is their enormous distance.

[Margin: The slip of Mastri and Chiaramonti.]

The cause of this difference that Mastri offers, taken from Chiaramonti — namely that a comet does not have a lively splendour and glitters less, whereas splendour and greater brightness contribute to acquiring an increment from the telescope — is not an adequate cause; nor can it be applied to the fixed stars, which glitter most brilliantly and twinkle, especially the Dog, and yet receive no increment from the telescope.

[Margin: The true cause of the differing increment produced by the telescope.]

The truer cause, therefore, which Tannerus gave in a few words and Guiducci and Galileo in more, is that stars, as also comets and flames, viewed from afar with the naked eye, appear to us crowned and girdled with certain adscititious rays, which are not, however, in the object itself or near it, but are an affection of our dilated pupil and a deception of our sight. That mane of spurious or adventitious rays is very small in the Sun, and almost nothing in the ☾, compared with the true size of the solar and lunar disc, so that it is a sixtieth or a fortieth part of their apparent diameter. But the corona that appears around Saturn and Jupiter is somewhat larger, and larger still around Mars; larger again around Venus and Mercury — larger, I say, compared with the true size of the apparent disc — and by far the largest around comets, flames of lamps viewed at a distance, and the fixed stars. Now the telescope so fortifies and protects the eye that it does not allow any prevailing in the pupil by its glare, and so takes away from the objects already mentioned those adscititious rays; on the other hand, by the force of the refraction produced by the crystal lenses, it widens the visual angle of any visible point, and so visibly enlarges the object, without however enlarging the adventitious rays, since it rather strips the object of them. The greater, therefore, the corona of adventitious rays compared with the object taken without them, the less the telescope will seem to enlarge that object which you previously viewed with the naked eye. For suppose that the corona around Venus, or the Dog Star Sirius, is of 10 parts, of which their true diameter is one, and that the Moon's corona is of one part, of which the Moon's diameter is 10 (which I feign for the sake of an easier example). Suppose next that the telescope has the power of multiplying or enlarging the true diameter of the stars tenfold. Then both Venus and the Moon, viewed through the telescope, will appear tenfold larger; but not tenfold larger than when viewed with the naked eye together with that corona. For Venus, stripped now of nine parts and keeping one, but that one increased tenfold, will appear through the telescope as large as she and her mane together seemed to the naked eye. But the Moon, since she is feigned to be stripped of a tenth, adscititious, part of herself (if indeed of herself), and keeps nine parts, which are made tenfold by the telescope, will appear nine times larger than when viewed with the naked eye. The true cause, therefore, why the telescope gives a notable increment to the Moon and the Sun and so on, but not to the fixed stars, is not the enormous excess of the distance of the fixed stars over the distance of the Moon and the Sun, since the telescope does not give, even to lamps viewed from a distance within one mile, that increment proportional to the increment of the Moon. It is rather that it strips them of a much greater corona of adventitious rays, without which the true object is much smaller than it was estimated with the naked eyes; and so it happens with comets. Moreover, the same telescope, with the same or a slightly different draw of the tube kept, enlarges comets and fixed stars in the same proportion as the Moon, if they are compared with one another without the rays that arose from the deception of our sight before the use of the telescope. Yet it does not make the heads of

comets appear as vast as the Moon, because they are not of so great a size in themselves.

[Margin: The telescope's differing effects from its differing extension.]

XXIII. But, you will say, why does the telescope so notably enlarge objects viewed within a single room, or a church or hall, and much more than proportionally to what it does with the Moon? Is this not a sign that greater distance is the cause of the smaller increment? I reply that this by no means comes from distance as such. Otherwise, since the distance of the Sun from the Moon is much greater than that of the Moon from the Earth, the Sun would have to acquire from the telescope an increment smaller than the Moon's by as much — and by much more — as the increment the Moon receives from it is smaller than the increment received by an object placed within a room. The cause of this difference, rather, is that an object too near cannot be seen with a telescope without confusion of many visual rays, nor well bounded, unless the tube is drawn out a great deal; and when it is drawn out, the object appears larger than when it is not. But when the object is remote — say beyond one, or at least ten, miles — the rays falling from from [recte: from] one point of the object on the lens of the telescope begin to approach parallelism. So it comes about that to view a remote object properly no notable difference in the drawing-out of the tube is required, but almost the same draw serves the same eye, whether it views the Moon or the fixed stars. These things, said perhaps at too great length, had nevertheless to be said against those who think that the difference in the increment produced by the telescope, or its differing draw, is an evident argument of a greater or lesser distance among great ones. Even if we were to grant that, to see the sunspots or represent them perfectly on paper, a longer-drawn tube is required when the Sun is at perigee and nearer than when it is at apogee and more remote, it could still be said that this happens not by reason of distance but by reason of the vapours that intervene in the winter season, when the Sun is at perigee. Or certainly, in our case of comets, it would have to be established by experiments that are not doubtful that a more contracted tube must be used to see the Moon clearly and distinctly, but a more drawn-out one for a comet — which is not yet established.

10. Argumentum ab Altitudine Cometarum requisita, vt à pluribus locis distantibus spectari queant.

Tenth argument: from the altitude of comets required for them to be seen from many distant places.

XXIV. TENTH. There is still in force the argument which, as Biancani says on the mathematical passages of Aristotle, p. 92, mathematicians used to employ before the new observations to show that comets cannot be in that highest region of the air which lies immediately above the clouds and does not exceed the altitude of the exhalations that make the twilight, that is, an altitude of about 50 or 60 Italian miles; and which was then used by Cysat, chapter 3; Froidmont, *Meteorology* Book 3, chapter 1, article 4; Glorioso, Book 3, chapter

3, number 24 — although in chapter 5, from number 22, he adds its solutions — and Kepler in the *Hyperaspistes*, p. 169. For on the one hand it is certain that the altitude of the exhalations that produce the twilight is less than 80 Italian miles, as will be shown in chapter 14; on the other, that the same comet — especially that of the year 1618 — was seen at the same time, and indeed for several hours above the horizon, in very distant places: namely at Rome, Florence, Goa, Parma, Milan, Antwerp, Innsbruck, Ingolstadt, Wittenberg, Tübingen in Batavia, Paris, Aix-en-Provence &c., that is, in almost the whole of Europe and in the East Indies. But for that comet to have been visible in such places at the same time for several hours above the horizon, an altitude far greater than 80 Italian miles is required. For if it had been only 80 Italian miles high, or 20 German, and had had no motion other than that of the primum mobile, it would have set within two hours, seen to fly from east to west, as will be shown below; and much less could it have described, over several nights, a circle around the pole of the world within the circle of perpetual apparition, as it did from December 20 to January 23, as Cysat observed. But this argument cannot be fully treated without many problems to be given below in chapter 15.

[Margin: Reply to Argument 10.]

Some Peripatetics, however, *will reply* that, to save phenomena of this kind, it suffices for the comet to be in the region of fire, provided it remains below the Moon; and that Aristotle is not against this, since in *Meteorology* Book 1, chapter 1, he said that comets are in the place nearest to the carrying motion of the stars, that is, the concave of the Moon. With these things laid down, its elementary origin from a quantity of kindled exhalations still stands; for by that name he comprehends the whole place of elementary fire, in which he granted that fiery meteors are made, when he says: "*These are whatever things happen according to nature indeed, yet with the disorder of the first body among the elements, about the place nearest to the carrying motion of the stars — such as concern the Galaxy, and comets, and fiery and moving images.*"

11. Argumentum ab Altitudine requisita, vt Cometa sit conspicuus extra Vmbram terræ.

Eleventh argument: from the altitude required for a comet to be visible outside the shadow of the Earth.

XXV. Apian uses this argument, supposing that comets, because of the perpetual turning-away of the tail from the Sun, are illuminated by the Sun, and so cannot be seen at night if they lie hidden within the shadow of the Earth; and that they are sometimes in such a position that, if they were not higher than the highest region of the air, they would lie hidden in the shadow of the Earth. But this argument cannot be fully understood except from problem 9, to be given in chapter 15. Meanwhile, however, it is not proved from this that these comets are above the Moon, because it suffices that they avoid that shadow at its sides.

12. Argumentum est à Parallaxi.

Twelfth argument: from parallax.

XXVI. This is the Achilles, this the Atlas of the astronomers; and therefore it is to be treated more fully below, and not without geometrical diagrams, from chapter 16 to 21.

SCHOLIVM.

Scholium.

[Margin: Was parallax known to Aristotle?]

Some have doubted whether parallax was known in Aristotle's age. For in Meteorology *Book 1, chapter 7, he recognised that certain comets follow the star from which the exhalation was drawn out and separated, and are carried along beneath it by the same carrying motion, as though affected by no difference of appearance as regards place; and in chapter 8 he taught that the milky circle is not a phenomenon arising from the reflection of our sight from that place of the heaven to the Sun, as from a mirror, because the Galaxy always appears in the same place of the heaven — which is the same as saying that the Galaxy suffers no variation of view, or parallax. But* Glorioso, *in* De Cometis *Book 4, chapter 6, rightly drew a distinction, and taught that the doctrine already mentioned is indeed connected with parallax, but that the parallax which astronomers after Ptolemy discuss — namely the one by whose force we investigate the distance of the stars, or of any phenomenon, from the Earth — was not known to Aristotle; for the controversy between him and the ancients was not about the distance of the milky circle from the Earth, but about its substance. Certainly the first to mention parallax was Hipparchus, in Ptolemy; but he himself treated it more clearly in the Moon, as I taught in Book 4, chapter 13; and the first who carried its use over to comets was Johannes Regiomontanus.*

CAPVT XII.

Chapter XII.

An Cometæ ſint ex materia Elementari, non accenſa ſed illuſtrata à Sole, & eleuata priùs aut ad ſupremam aeris regionem, aut ſupra Lunam.

Whether comets are made of elementary matter, not kindled but illuminated by the Sun, having first been raised either to the highest region of the air or above the Moon.

[Margin: On comets made of elementary matter below the Moon, not burning but shining.]

I. The first part of the opinion indicated in the title of this chapter, and in chapter 6, number 7, was taught by Scaliger and Telesius; nor does it seem very different from the view of Heraclides and Metrodorus, as we showed in the same place. The second part was handed down by Apian, Guiducci, Galileo, Aresius, Praetorius and Rothmann, as I said in the same place, number 8. That a comet is something separated out of elementary matter, but so compacted and tempered that it can receive the Sun's rays into itself — partly stopping them, so

that it shines for us, and partly transmitting them through its translucent body, so that it draws a tail from them — is urged by the turning of the cometary tail away from the Sun, about which much was said in chapter 11, in the first argument, and more must be said again in chapter 24. Against the first part stand all the arguments made against the Peripatetics in chapter 11, except the first; but in its favour stand the replies to those arguments, such as they may be.

[Margin: On the elementary matter of comets raised above the Moon.]

II. Against the second part — elementary matter ascending, or drawn up, above the Moon and shining there under the Sun — many points are gathered by Lotharius Sarsius in the *Libra Astronomica* from p. 27; by Tycho in his letter of 1589 November 24, p. 162; by Piso in the *Speculum*, chapter 2; by Cottunius, lecture 32 on Book 1 of the *Meteorology*; by Froidmont, *Meteorology* Book 3, chapter 2, article 4; and by Cabeo on Book 1 of the *Meteorology*, text 37, question 6.

[Margin: 1. Argument against the second opinion.]

The first argument is drawn from the solidity of all the heavens, in the opinion of Cottunius and Piso, or of the Moon's heaven, in the opinion of Cabeo. Cabeo holds that the heaven of the fixed stars and that of the Moon are solid: the Moon's, because of its kinship with the Earth, which it circles like an attendant and companion, and so that its solidity may prevent volatile salts from being raised to an infinite distance, lest sublunary nature be exhausted — indeed, so that they may be forced to return downward; the intermediate heavens, however, he concedes to be fluid. Now if at least the heaven of the Moon is solid, elementary exhalations certainly cannot penetrate it, however keen the subtlety with which they are endowed.

[Margin: Reply to Argument 1.]

The authors of the opposite opinion, however, will reply by denying that solidity, as will be said in Book 9.

[Margin: 2. Argument.]

III. Cottunius draws the second argument from the motion of comets toward the north and the south. Exhalations, by their innate lightness, are carried straight upward; or if, as Lotharius Sarsius contends, they are carried obliquely in a wandering and restless dispersal, and carried upward crosswise, it will never come about that they can be gathered at the heaven of the Moon into one comet, still less above that heaven.

[Margin: Reply to Argument 2.]

Those authors, however, will reply either, with Rothmann, that the exhalations are carried up by Intelligences to form a portent of this kind; or that, whether they are carried straight or obliquely, many of them, warmed by the Sun and made lighter, are nonetheless so tempered once they have ascended above the Moon that they can acquire a particular motion arising from some source other than lightness. How arbitrary an invention this is, there is no one who does not see. Galileo, however, is under a particular necessity to explain this motion, since he follows Copernicus, who explains the motion

of elementary bodies away from the centre or toward it otherwise than the Peripatetics do.

[Margin: 3. Argument.]

IV. The third argument, which seems to have greater force, and which Tycho, Froidmont and Cottunius point to, is drawn from the size of comets together with their duration. For if exhalations must ascend from land and sea above the Moon to form a comet, then either they must be thinned out and rarefied almost to infinity, so as to occupy as much space as a comet with its tail — and so they will not be fit to reflect so strong and vigorous a light from the comet's head — or else that matter must be packed close, so that it lasts so long and keeps its original shape, and at the same time, on account of its opacity, reflects so great a force of light to us; and then it is not probable that the whole earth and water would suffice with their exhalations to form a comet. At this point what was said about the size of comets in chapter 11, Argument 6, and about their duration in Argument 7, should be read again; from it, nonetheless, some replies may be gathered. As for those *hypecaumata* and glows which we believe to lie outside the Earth's shadow on summer nights: they are not in truth beyond the apex of the Earth's shadow, and so lit by the Sun; they are exhalations catching fire, in the way that shooting stars do, or, if they are above the Earth's shadow, they are rather effluvia of the planets.

CAPVT XIII.

Chapter XIII.

An Cometæ, ſi cæleſtes ſint & origine & loco, fiant per condenſationem vel ſolam alterationem partium Cæli, An per generationem ex materia effluente à Sole, cæteriſque Planetis.

Whether comets, if they are celestial both in origin and in place, come about through condensation or mere alteration of parts of the heaven, or through generation from matter flowing out from the Sun and the other planets.

I. It is assumed here that it is established — with certainty from the parallax of comets, and very probably from the constancy of their regular motion and the similar arguments adduced in chapter 11 — that some comets are above the Moon; and next, from what was said in chapters 7, 8 and 9, that a comet is neither a symphasis of two or more of the five commonly known planets, nor one or more of some perpetual but unknown bodies, whether fixed stars or planets, but comes into being anew and passes away. This granted, the question is whether comets come about through a merely accidental change, by which density, or opacity, or light, or all of these, are added to parts of the heaven, according to the opinion reported in chapter 6, number 9; or rather through generation, according to the opinion given in the same place, number 10. For once it is supposed that some comets are born and die in the heaven, one of these two must be said — unless someone wishes these comets to be made by God out of nothing, or out of some matter or other, by Him either directly or

through angels, by an extraordinary concurrence. Nor are the arguments of the Peripatetics, or of others, relevant here — those who contend that the heaven is both solid and incapable of generation or corruption, or of such alteration as is required to form comets, and who therefore assert that comets are all sublunary. For here, as I said, it is assumed as certain from parallax that some comets are celestial; and from this many of the moderns try to show that the heaven is fluid, and pervious to the passages and the truly very diverse motions of comets, and capable of generation and corruption, or of an alteration very like it — especially since they have very probable arguments for this both from the new stars and from the spots and faculae near the Sun. And as regards the authority of the Fathers and the Wise, so many are for the fluidity of some heavens, and the generability and corruptibility of some parts of the heaven, that they need not fear the authority of the other Fathers or Doctors who held the opposite.

[Margin: Arguments for mere alteration.]

II. In favour of a merely alterative change stands that well-worn but not contemptible argument by which philosophers contend that God and Nature do not do through more what can be done through fewer — that is, do not multiply entities without necessity. Therefore, if the phenomena of comets can be saved by the mere condensation of celestial matter, or at most by opacity mixed with transparency — so that they can reflect the Sun's light to us insofar as they are opaque, yet transmit it insofar as they are transparent, so that the tail is turned away from the Sun — no other, greater change seems to need introducing. Add that clouds too are sometimes so tempered that, partly by density and partly by transparency, they become fit to present shining globes to us.

III. Against mere alteration, however — especially condensing alteration — no few points are brought. First, if its efficient cause is natural, and operating in a natural way, [Margin: Argument against mere alteration, especially through mere condensation.]

it can be identified only with the greatest difficulty. For cold in the heaven cannot be imagined except gratuitously; nor would the motion of the planets suffice, driving the heaven and producing a moderate density in various parts of it so that no vacuum arises, since that density is dispersed, as it is in water or air when they are pushed. To attribute this to the substantial form of the heaven, as Liceti does — a form that would try to repair the parts of the heaven rarefied by the Sun's heat and restore them to their original density — is to weave and unweave Penelope's web: for as much density as is repaired while the Sun is absent would be removed on its return, and from these alternating vicissitudes a comet could never be formed. But if this cause is supernatural, or operates in a supernatural way, then it is all one to it whether it uses an alterative or a generative action. Moreover, condensation alone does not make an object visible and able to stop the Sun's rays unless some opacity is present. Besides, so that no vacuum arises, by as much as the heaven was condensed in one part, by so much would it have to be rarefied in another. Finally, if density and opacity

alone were added to one part of the heaven, that would suit motionless comets, as Camillo Glorioso holds them to be, better than moving ones. No evident arguments, however, are to hand either for this opinion or against it; and so I am loath to be more long-winded over these conjectures.

[Margin: Arguments for a substantial change in the heaven.]

IV. In favour of substantial generation, on the other hand, stand almost all the reasons that our Scheiner adduced in Book 4 of the *Rosa Ursina* for spots and faculae newly generated around the Sun, and that others adduced too, some of whom we indicated in Book 3, chapter 3, number 10. To these must be added the many named here in chapter 6, number 10: namely Cysat, Cabeo, and Tellez of our Society; then Kepler, Fienus, Snel, Glorioso, Puteanus, Froidmont, and most recently Johannes Hevelius in chapter 5 of the *Selenographia* and in its Appendix. The chief reason is that many of those spots and faculae perish and vanish in the middle of the Sun's disc before they reach the edges, while some appear there anew which had not been seen near the edges; and those that seem to return after about 26 days rarely keep the same shape. From this, then, it is inferred by a very probable argument that they flow out from the Sun, and are as it were soot, or corpuscles boiling up out of that immense furnace or fountain of the universe's heat. It is therefore not improbable that comets are made out of the same matter, or also out of effluvia of the other planets collected over intervals of time, thoroughly purged, and illuminated by the Sun; and that they are nothing other than faculae that are larger and further from the Sun.

[Margin: Two confirmations of this change.]

This is confirmed, first, because in the year 1618, in which three comets appeared, no spots appeared around the Sun, as I reported in Book 3, chapter 3, number 5 — although none appeared in 1632 either, for many months, during which there was a remarkable drought; but nothing is established about comets having arisen in that year. Second, because those obscurations of the Sun which were observed far from new moons (so that you could not say they were eclipses), and recorded in history, seem to have arisen from effluvia of this kind, as it were mists exhaled from the Sun or the other planets. On these see our Book 3, chapter 3, scholium 1, and Book 5, chapter 1, scholium 3.

[Margin: Obscurations of the Sun arising otherwise than from eclipses.]

To these add the darkness around the Sun lasting 17 days in the year 790, on the testimony of Paulus Diaconus as cited by Patrizi in Book 19 of the *Pancosmia*, p. 111. So too in the year 1547, from April 24 to 28, the Sun shone everywhere with a sad and rust-coloured appearance, as Froidmont reports in *Meteorology* Book 3, chapter 2, article 6; in that year, on March 31 [pridie Calendas Aprilis], Francis I, King of France, had died.

[Margin: Arguments against a substantial change of this kind.]

V. Against this mode, however, besides the corruptibility and fluidity that many think unworthy of the heaven, there are these arguments. First, if so many exhalations flowed out perpetually from the planets, and as large as certain

great spots that appear around the Sun when viewed through the telescope, their substance would seem bound to be exhausted. For that reason Froidmont preferred that the Sun — *an admirable vessel and the work of the Most High*, as it is called in Ecclesiasticus 43 — should, as the lamp of the universe, be immune from these evaporations, and he said that they are breathed out from the other planets: as though there were not more to fear on this account for the other planets, which even taken together are of smaller bulk than the Sun. Next, why do comets not appear more frequently, and around the Moon and the Sun? Finally, what drives those exhalations, with so great a variety of motions, out of the zodiac as far as the north or the south? Fienus, Puteanus and Froidmont reply, however, that far more frequent generations and perishings may take place in the heaven which we nevertheless cannot see, since they are either drowned by the Sun's splendour or hidden by their distance and smallness; and that there is no danger to such vast bodies if particles of them, or particles arising from them, perish. For if we were on the Moon or on the Sun, we could by no means discern the births and deaths of animals and plants, or even of cities or mountains when they are overthrown: for how tiny a part are these with respect to the whole globe? Therefore, just as the Earth remains, and the seas and the kingdoms of the lands remain, and in spite of those changes the Earth is still the Earth and the ocean does not cease to be the ocean, so the changes already mentioned in the heaven will not make it cease to be the heaven; indeed, not even any planet will cease to be of such a kind, or as large, as it is seen by us from the Earth.

[Margin: Our opinion is indicated.]

VI. These points, indicated within the bounds of probability rather than argued out, keep me greatly in suspense and wavering this way and that. The suspense is increased both by the so varied motion of comets and by their perishing, if they are made not of burning matter, so as to fail when their fuel fails, but of matter illuminated by the Sun or by the other planets. Nor do I care to sharpen my wits devising efficient causes that would keep both the substance and the motion of this celestial phenomenon in being for a definite time. I therefore lean strongly toward the opinion that attributes the contriving and governance of these portents to God or to Intelligences, according to what was said in chapter 6, number 12; or at any rate toward the one that counts comets among the secrets of the world and the most hidden miracles of Nature. This was done by Māshāʾallāh, Regiomontanus, Tycho, Hájek, Tanner, Ambrosius Rhodius, Arriaga, Oviedo and William of Hirsau; and we showed in the same chapter 6, number 12, that Froidmont, Cabeo and Resta did not shrink from it either. For why should God not display to the world these new and portentous spectacles, recurring in no order of times and at no intervals — both to raise them up toward heaven and prepare them for that signal catastrophe of the world which is to come at the end of the ages, and so that it may be manifest that He is held by no necessity or laws of nature from doing whatever He wills below and above the Moon; and so as to set a crux not only for the Peripatetics,

who have deserved ill of the Creator's freedom, but for every mind, or to propose a splendid enigma never to be solved?

The Philosopher's excuses are well known — noble tokens of a most moderate mind. In Book 2 of *On the Heavens*, chapters 3 and 5, he says that he speaks of celestial things *from afar, not as if in the place itself: and all the more so because we have perception of very few of the things that happen in the heavens*. Then he adds: *if anyone shall have attained more certain and more necessary reasons, thanks must be given to the discoverers*. And in Book 1 of the *Meteorology*, chapter 7, about to speak of comets, he declared: *Since, however, concerning things not manifest to sense, we think we have dealt with them sufficiently if we have brought our reasons to the point of saying nothing impossible*. On this, then, or on others of his sayings, Seneca, *Natural Questions* Book 7, chapter 30: *Aristotle says admirably that we ought never to be more reverent than when the gods are in question. If we enter temples composed, if, about to approach a sacrifice, we lower our face, draw our toga close, if we put on every mark of modesty, how much more ought we to do this when we discuss the constellations, the stars and the nature of the gods, lest we affirm anything rashly or impudently, either affirming in ignorance or lying knowingly?* These things the author brings in on the occasion of comets, as also the rest that we related in chapter 1, number 6. [Margin: The modesty of Aristotle.] [Margin: Seneca's opinion of the modesty of Aristotle's mind.]

CAPVT XIV.

Chapter XIV.

De Altitudine Exhalationum & ſupremæ regionis Aeris, infra quam Cometæ ſupponuntur eſſe non poſſe. Qua occaſione traduntur Problemata ad illorum Altitudinem ſpectantia.

On the altitude of the exhalations and of the highest region of the air, below which comets are supposed unable to be; on which occasion problems bearing on their altitude are set out.

I. Many things about the variety of the twilight, and about the altitude of the exhalations which, reflecting to us the light of the Sun as it approaches the horizon, or after it has sunk a little before below the horizon, produce morning or evening twilight for us, were said by us in Book 1, chapter 31, and it will be worth rereading them here. First of all it must be recalled that the varying altitude of these exhalations, which are broadly called vapours, depends both on the varying season of the year and the constitution of the air, and on the obliquity of the sphere, or of the horizon; and that its measure depends both on the depth of the Sun below the horizon at the beginning of morning twilight or the end of evening twilight, and on the size of the Earth's semidiameter — on which, if the authors adduced in Book 2, chapter 7, agreed about the size of the Earth, there would be less disagreement among them on this head about this altitude of the exhalations. Moreover, in the opinion of Kepler, Vendelinus [Govaert Wendelen], Glorioso, Gassendi and ourselves, it

depends on the refraction of the Sun near the horizon, which certainly makes the altitude much smaller. But since in surveying the authors on comets I have come upon certain other opinions about this altitude, it seemed good to present them in a single view together with those I reported in Book 1, chapter 31 — but first to set out here the problems bearing on this, which I had deferred to Book 10 of this volume or to the book on optics, adding the depth of the Sun below the horizon, which, however, not all of them state. If anyone wants the opinion of the authors listed below on the size of the Earth's semidiameter, let him consult either Book 1, chapter 31, in the last column of the second table, or Book 2, chapter 7.

[Margin: The altitude of the air, according to Tanner, Mastri and Chiaramonti.]

Speaking, however, of the subtler exhalations, which are not so opaque that they can make twilight, Tanner, *On the Heavens*, question 7, p. 140, thinks that they can ascend almost as far as the Moon. Mastri, on the other hand, *On the Heavens*, disputation 4, question 2, article 4, thinks it better to raise their altitude and the bounds of the air than to carry comets up into the heaven. Chiaramonti too, in Book 3 of *On the New Stars*, chapter 26, and in the opuscula on the sublunary seat of comets, p. 179, says that the altitude of the air is almost 26 semidiameters of the Earth, that is, about 90,000 Italian miles.

II. The authors I have seen on this matter are: Posidonius, as reported by Pliny the Elder, Book 2, chapter 32; Ibn al-Haytham, *On Twilight*, last proposition; Vitellio [Witelo], Book 10, proposition 60; Nonius [Pedro Nunes], Book 2, *On Observation*, from p. 82, and *On Twilight*, part 2, proposition 18; Clavius, *On the Sphere*, p. 131 in my copy, and in chapter 3 of the *Sphere*, in the digression on twilight, proposition 6; Cardano, *De subtilitate* Book 4; Abrahamus Roggembach, as cited by Tanner, *On the Heavens*, question 7; Barocius [Francesco Barozzi], *Cosmographia*, p. 198; the Coimbra commentators [Conimbricenſes] on *On the Heavens* 3, chapter 5, question 2; Maginus [Giovanni Antonio Magini], *On the Primum Mobile*, Book 11, problem 30; Gemma Frisius in the supplement to the *Astrolabe*; Tycho, *Progymnasmata* volume 1, pp. 95 and 733, and volume 2, p. 410; Scultetus, as cited by Tycho in the same place; Rothmann, in his letter to Tycho of 1588, of February 21 and 24; Biancani, *On the Sphere* Book 6, chapter 5, and Book 10, chapter 15; Simon Stevin in the *Cosmography of the Atmosphere*, that is, in the *Mathematical Memoirs*, Book 3 of the *Geography*, proposition 2; Longomontanus, Book 2 of the *Sphaerica*, chapter 11; Kepler, *Optics*, p. 288, and *Epitome of Astronomy*, pp. 74, 309, 369; Snel, *On Comets*; Froidmont, *Meteorology* Book 1, chapter 1, article 2, and Book 6, chapter 4, article 2; Cabeo, *Meteorology* Book 1, text 15, question 2; Gassendi, *Institutio Astronomica* Book 1, chapter 18; Glorioso, *On Comets* Book 2, chapter 2; Resta, *Meteorology* Book 2, tract 1, chapter 2; Cottunius, *Meteorology* Book 1, lecture 33; Wendelen, preface to the *Eclipses*, p. 5. Of these, Froidmont and Cottunius follow the ancients on the 52 miles, yet in such a way as not to reject the opinion of Nonius or of Stevin.

Problema 1. Data Solis Profunditate ſub Horizonte ad initium Crepuſculi Matutini, & Semidiametro Terræ, Inueſtigare Altitudinem Vaporum Crepuſculinorum Methodo Cardani.

Problem 1. Given the depth of the Sun below the horizon at the beginning of morning twilight, and the Earth's semidiameter, to find the altitude of the twilight vapours by Cardano's method.

III. Following Cardano, *De subtilitate* Book 4, let there be the globe of the Earth ADG, whose semidiameter AC is known in leagues, or miles, or stades; Cardano sets it at five thousand times a thousand paces, that is, 5000 Italian miles. Let there be given, at the beginning of morning twilight, the depth of the Sun below the horizon, which he sets at 19°, so that under the equinoctial circle there is about an hour and a quarter from twilight to sunrise. At the beginning of twilight let the Sun's ray be DB, touching the Earth at D and continued in a straight line as far as B, the vertical point, or aerial zenith, overhanging the head of the observer at A. For the altitude of the vapours at B will be AB itself, and the Sun's ray reflected from the vapours at B to the eye at A will be the same BA. Let AFE be the physical horizon, meeting BAC at right angles, and let the Sun's ray DB cut it at F. When the radius CD is drawn to the point of contact D, angle CDB will be a right angle by proposition 18 of Book 3 of Euclid, as is BAF; and angle B is common to, and the same in, the two triangles ABF and CBD. The triangles are therefore similar, and so angle BFA will be equal to the remaining angle BCD, that is, 19°, because BFA is equal to angle DFE; "for on account of the Sun's very great distance from the Earth, angle DFE is just as if it were at the centre of the Earth," says Cardano. From this, then, by analysis of the triangles he infers AB to be 288 miles, of which AC, or CD, is 5000. But if the twilight lasts two hours before sunrise, the depth of the Sun with respect to the equator — that is, angle BCD, that is, AFB — will be 60°, as Cardano estimates; from which he infers AB to be 772 miles, beyond which altitude he denies that rainbows, burning rods, parhelia and so on can be produced.

Diagram. Cardano's construction for the altitude of the twilight vapours, set in the right column. A circle (the Earth) with centre C; a vertical line runs from G at the foot of the circle up through C and through A on the upper rim, and continues above the circle to B. A short line through A, the physical horizon, carries F and ends at E on the right. A line from B descends through F to D on the right-hand rim (the Sun's ray touching the Earth at D). A dotted line joins C to D.

- B (top, above the circle): the vertical point, or aerial zenith, over the observer
- A (upper rim, on the vertical): the observer
- F (on the horizon line, right of A): where the ray DB cuts the physical horizon
- E (right end of the horizon line): end of the physical horizon AFE
- D (right-hand rim): the point where the Sun's ray touches the Earth

- C (centre): the Earth's centre
- G (foot of the circle): end of the Earth's diameter through A

[Margin: The altitude of a comet according to Cardano.]

Since he says that some comets were seen at Milan under the winter circle, he adds the following words to show that they are much higher than the altitude of the vapours: *Let us suppose, then, since we are distant from the circle of Capricorn by about 68°, that the eye is at D, at Milan, and that the point seen under the circle of Capricorn is B. It is established, then, that the ratio of CB to CD is about as 8 to 3. Since, then, CA is equal to CD, the ratio of CB to CA will also be as 8 to 3; therefore AC to AB as 5 to 3; therefore AB is 8000 thousand paces and 333 besides, which I do not trouble about. It is clear enough that the place which an inhabitant of Milan must see under the winter circle is more than ten times as far from the Earth as the altitude of the vapours reaches. But comets are seen by us there; therefore they are not made of vapours.* So he.

Reprobatur Cardani Methodus.

Cardano's method is rejected.

IV. But is this, Cardano, that subtlety of yours? On the contrary, there is great obesity and fatness in this false pronouncement of yours. First, who ever saw the air that stands perpendicularly over our heads whiten from the Sun's rays at the beginning of twilight, and reflect them from there to the eye? Do not any country people and rustics know that that whiteness rises from the eastern horizon and girds the horizon like a belt? Rightly, then, did Tycho, *Progymnasmata* volume 2, p. 411, say *that Cardano went enormously astray when he too inconsiderately laid down that the highest vapours producing twilight are to be observed near the vertex, since the first reflection of the ray that begins the twilight rather takes place at the horizon, and has never been seen by anyone near the zenith overhead*; and in the same place he rebukes Scultetus, who follows Cardano in this. [Margin: Tycho's censure of Cardano.] Second, angle DFE cannot be equal to the angle that the Sun's ray, drawn from the Sun's centre through the centre of the Earth, makes with the astronomical horizon, since that ray is not parallel to the ray drawn from the Sun's limb and touching the Earth, however great the Sun's distance from the Earth may be; angle DFE is smaller by as many minutes as the Sun's apparent semidiameter subtends, as we shall show in Problem 3. In the last reasoning, moreover, if CA is 5000 miles and is to AB as 5 to 3, then AB will certainly be 3000 miles. But perhaps there is a fault in Cardano's text.

Problema 2. Data Solis Profunditate infra Horizontem ad initium matutini, vel finem veſpertini Crepuſculi facti aut ficti à radijs Solis reflexis, ſed irrefractis; & Portione terræ à Sole illuſtrata: Inueſtigare altitudinem Vaporum Crepuſculinorum in partibus, qualiũ Terræ ſemidiameter nota fuerit.

Problem 2. Given the depth of the Sun below the horizon at the beginning of morning twilight, or the end of evening twilight, produced — or supposed to be produced — by

rays of the Sun that are reflected but not refracted, and given the portion of the Earth illuminated by the Sun: to find the altitude of the twilight vapours in parts of which the Earth's semidiameter is known.

V. This problem is that of Ibn al-Haytham and Witelo, and, with a few changes, of Magini and Longomontanus, at the places already adduced in number 2. Following them, in the diagram below, from the Earth's centre A describe the Earth's globe BCD; and the great circle EHKG of the solar heaven. Let GAH be the astronomical horizon, and LBM the physical horizon, touching the Earth at B and making right angles at B with the Earth's semidiameter AB, by proposition 18 of Book 3 of Euclid; and let AB be extended in both directions, to K, the zenith of the observer at B, and to T, the nadir. For KAT will be the vertical line, cut at right angles at A by the horizon GAH, which may also represent the plane of the meridian.

Diagram. The construction for Problem 2, used again for Problem 3, set in the left column. A large circle (the great circle of the solar heaven) with the Earth's centre A at its middle; about A, two close concentric small circles (the Earth and the sphere of the twilight vapours). A horizontal diameter G–A–H (the astronomical horizon); above it a horizontal chord L–B–V–M (the physical horizon), touching the inner small circle at B; a vertical line K–B–A–T. A line runs from F at the top of the circle down through V to N on the lower right, and F is joined to a point on the lower left near O; lines run from A to C, to V through I, to Z, to N, to E, and down through R. At the foot an arc through N, P and O (the Sun's disc), with its centre E on the great circle; a short mark Q on the line near N.

- F (top of the great circle, left of K): vertex of the shadow cone DFC
- K (top): the zenith of the observer at B
- L (left end of the upper chord), M (right end): ends of the physical horizon LBM
- B (on the upper chord, at the small circle): the observer, where the physical horizon touches the Earth
- V (upper chord, right of B): where the Sun's ray NF cuts the physical horizon
- I (on AV, at the small circle): foot of the altitude IV
- C (on the small circle, lower right of I): point of contact of the ray NC
- G (left end of the diameter), H (right end): the astronomical horizon GAH
- D (left of A, on the small circle): point of contact of the ray OF
- A (centre): the Earth's centre
- Z (on GH, right of A): where the ray NF cuts the astronomical horizon
- S (lower left of A, on the outer small circle): on the sphere of vapours VS
- R (below A, on the small circle): mid-point of the illuminated arc DRC
- P (in the lower arc): on the Sun's disc NPO
- N (lower right of the great circle): the Sun's limb

- Q (on the line near N): end of the portion QN, set in a script-like italic form
- O (lower left of the arc): the Sun's other limb
- T (bottom): the nadir, the point of midnight
- E (bottom, right of T): the Sun's centre

Now let the Sun's centre have passed T, the point of midnight, and be at E; from this point describe the Sun NPO, and so on. Let its central ray, drawn to the Earth's centre, be EA, and let the extreme rays, drawn from the Sun's limb on either side, be NF and OF, touching the Earth at C and D and enclosing the cone DFC of the Earth's shadow. To the point of contact C draw the Earth's semidiameter AC, which makes the right angle ACV by the same proposition 18 of Book 3. Further, draw the straight line AIV as far as the point V, where the Sun's ray NF cuts the physical horizon LBM. For VB will be the first ray reflected from the vapours at V to the eye at B, the one that makes the beginning of morning twilight; the sphere of the twilight vapours, described from the centre A through V, will be VS; and IV will be the altitude of the vapours that is sought. Finally, since EN, the Sun's semidiameter, is greater than the Earth's semidiameter AC, cut off from EN a portion QN equal to AC, and draw from A to Q the straight line AQ, parallel to CN; for by propositions 29, 33 and 34 of Book 1 of the *Elements* every angle of the parallelogram ACNQ is a right angle.

[Margin: First step.]

With these laid down: since the portion DRC of the arc illuminated by the Sun is known, half of which is the arc RC, the angle EAC, or RAC, will also be known. And since the Sun's depth below the astronomical horizon is given, namely EH, which is measured by the angle EAH, if this angle is taken from the angle EAC, the angle CAH will remain known; and this, taken from the right angle KAH, or BAH, will leave the angle BAC known. Now the triangles ACV and ABV are right-angled at the points of contact B and C; they have the common base AV and the equal sides AB and AC; and since the tangents VB and VC are drawn from the same point V, the sides BV and CV are also equal to each other, by theorem 58 of Book 1 of Witelo. Therefore these triangles are equal to each other, and the angle BAV is equal to the angle CAV, by proposition 8 of Book 1 of the *Elements*. But the whole BAC is known; therefore its half, that is, either of the equal angles BAV and CAV, is known as well.

[Margin: Second step.]

Secondly, then, in either of these triangles — say in the triangle ABV, right-angled at B — since besides the right angle the angle BAV is given, and the side AB, that is, the Earth's semidiameter, is known in determinate parts, say in stades or miles, you will easily find the base AV by the analysis of plane right-angled triangles. Subtracting from it the Earth's semidiameter AI, the altitude of the vapours IV will remain known, which was to be done. Briefly, then: to the logarithm of the semidiameter AB add the residue of the second

logarithm of the angle BAV, and the sum will be the logarithm of the base AV. Or otherwise: look up in the tables of sines the secant AV of the angle BAV, and make: as the radius, or whole sine, AB is to the secant, so are the stades or miles contained in the semidiameter AB to the stades or miles contained in AV; and when the stades or miles of AB, that is of AI, are taken from AV, there remain the stades or miles of IV itself, that is, of the altitude of the twilight vapours — if, that is, twilight were produced, as the ancients thought, by rays of the Sun that are reflected and not refracted at all in the denser air around the Earth. [Margin: A new short form of the rule.] You will work more briefly still if you subtract from the Sun's depth the excess of the semiarc of the Earth illuminated by the Sun over a quadrant, and look up the secant of half the remainder, and so on.

EXEMPLVM.

Example.

VI. According to Ibn al-Haytham at the end of his little work *On Twilight*, and Witelo, Book 6, proposition 33, the angle EAH of the Sun's depth below the astronomical horizon at the beginning of morning twilight is 19°, and the semiarc of the Earth illuminated by the Sun, that is, the angle EAC, is 90° 13′ 56″, by proposition 59 of Witelo's tenth book and Ibn al-Haytham's penultimate proposition. If the former is taken from the latter, CAH will be 71° 13′ 56″; and when this is taken from the right angle BAH, that is, from 90°, there remains BAC, 18° 46′ 4″, and its half, the angle BAV, 9° 23′ 2″, whose secant AV is 10,135,635 of the parts of which the whole sine AB is 10,000,000. But the circumference of the Earth according to Ibn al-Haytham, Abbomadus and Witelo is 24,000 miles. Therefore, by Archimedes, from what was said in Book 1, chapter 4, BR will be 7643 and AB 3821½. Let it be, then: as the radius 10,000,000 is to the secant 10,135,635, so is 3821½ to 3873 3,829,152/10000000. If you subtract 3821½ from these, there remains IV, 51 88/100 miles, or approximately 52 — although Ibn al-Haytham and Witelo, by the longer detours of chords, arrived at 51 miles 47′ 34″.

Problema 3. Altitudinem eamdem Vaporum Methodo Steuini & Glorioſi inquirere.

Problem 3. To investigate the same altitude of the vapours by the method of Stevin and Glorioso.

VII. Simon Stevin, in his *Mathematical Memoirs*, Book 3 of the *Geography*, proposition 1, and with him Giovanni Camillo Glorioso, *On Comets* Book 2, chapter 2, use the Sun's apparent semidiameter in place of the semiarc of the Earth illuminated by the Sun, because the arc that the Sun illuminates beyond the Earth's hemisphere differs by very few minutes from the Sun's apparent diameter — as anyone may see who compares this arc, which we treat in Book 3, chapter 11, problem 4, with the Sun's apparent diameter, treated in the same place, chapter 10. In our preceding diagram, therefore, since the Sun's limb N is less distant from the astronomical horizon HA than the Sun's centre E by

the arc EN — that is, less by as much as the Sun's apparent semidiameter — they accordingly subtract the Sun's semidiameter from the angle EAH of the Sun's depth; and when AQ is drawn parallel to the tangent ray NC, they are left with a known angle QAH, which is equal to the exterior angle NZH by proposition 29 of Book 1 of the *Elements*. For on account of the small bulk of the Earth compared with the Sun's distance, they think it is just as if the angle Z were made at the Earth's centre A and there measured the distance of the Sun's nearer limb from the horizon. In the same way Cardano too used the angle BVF, equal to the angle NVM, because this, owing to the parallelism of the horizons BM and AH, is equal to the angle NZH, which he assumed without any correction as 19°. Knowing QAH, then, and subtracting it from the right angle QAC, ZAC remains known; and when this is subtracted from the right angle BAZ, BAC remains known, half of which is BAV. With this and the semidiameter AB, the base AV is found in the triangle BAV, right-angled at B; and when AI is taken from it, the altitude IV becomes known. But the method of Ibn al-Haytham, Witelo, Magini and Longomontanus is certainly somewhat more accurate.

Probl. 4. Deſcribere idoneam figuram ad diſcernendum, An & quantùm altitudo Halituum Crepuſculinorum per radios Solis irrefractos inuenta, ſit ob radios Refractos diminuenda: Et Datis quæ in Probl. 2. ac præterea Refractione Solis horizontali, atq. altitudine Aeris refractiui; Inuenire punctum in ſuperficie Aeris, per quod ingreditur & egreditur radius Solis gemina refractione refractus.

Problem 4. To describe a suitable figure for discerning whether, and by how much, the altitude of the twilight exhalations found by means of the Sun's unrefracted rays must be diminished on account of refracted rays; and, given the quantities in Problem 2, together with the Sun's horizontal refraction and the altitude of the refracting air, to find the point on the surface of the air through which the Sun's ray, refracted by a double refraction, enters and leaves.

VIII. I suppose as certain, with Kepler in the *Astronomiae Pars Optica* [Aſtronomia Optica], pp. 79, 129 and 135, that on account of the greater lightness of the twilight exhalations and fumes than of the thick air, which watery vapours make denser for refracting the rays of the stars, the twilight exhalations are higher than the refracting air — granted that Tycho, *Progymnasmata* volume 1, p. 95, and Longomontanus, Book 1 of the *Sphaerica*, chapter 11, problem 2, take the altitude of the vapours that produce twilight and that of the refracting air for the same thing. We shall certainly show in Book 10, section 6, problem 51, that the altitude of the refracting air extends scarcely beyond 20 Bolognese miles; and yet the altitude of the exhalations, whether sought in the ancient way or by means of refracted rays, as below, is always greater, as we shall show below. With this laid down, the horizontal ray by which we first see twilight must be a refracted one; and the ray from which it comes must come from an exhalation that bounds the Sun's direct ray, as a little cloud would, and reflects it into the refracting air — but below the line of the physical horizon. Now,

so that the position of these rays may be understood in some fashion, let A be the Earth's centre, from which are described the terrestrial quadrant BD and the quadrant EF of the refracting air, enclosed by the true horizon AF and the vertical and perpendicular line AE.

Diagram. The figure for Problems 4 and 5, set in the left column: a quadrant construction for twilight with refraction. A vertical line from A up through B to E at the left, and a horizontal line from A along the foot through D to F; a quadrant arc from B down to D (the Earth) and a larger arc from E down to F (the surface of the refracting air); a horizontal line from B through S and H to T across the top. Straight lines fan out from A towards N, S, X, a, L, V, I, R and Q, Z, P and M. A line from G at the foot rises through Z to H; a line from K at the foot rises through M; short dotted lines run near H–V and P–M.

- E (top of the vertical): the top of the quadrant of the refracting air, EF
- B (on the vertical, below E): the observer's eye on the Earth's surface
- S (on the line BT): the point of the ray reflected to the eye
- N (on the outer arc, below S): end of the altitude NS of the refracting air
- a (small italic, below the line BT): the exhalation
- H (on the line BT): where the unrefracted ray meets the tangent TB
- o (small, just below H): the point O on the line AH
- T (right end of the top line): end of the physical horizon TB
- X (between N and the arcs): a point on the figure, not named in the text on this page
- I (on AH, below o): where the refracted ray Va cuts AH
- c (small, on the arc near I): the point C on the arc EP
- L (on the inner arc, on AH): mid-point of the arc BZ
- V (on the inner arc, right of L): the point of exit V
- R (on the Earth's arc, lower right): the point of contact R
- Q (on the outer arc, beside R): end of the line ARQ
- P (on the outer arc, below Q): the point P
- Z (below P): the point where the unrefracted ray touches the Earth
- M (on the outer arc, lower right): the point of entry M
- A (lower left corner): the Earth's centre
- D (on the foot line, at the inner arc), F (at the outer arc): ends of the quadrants BD and EF
- G (at the foot, below D and F): the unrefracted ray passing through G
- K (at the foot, beside G): the ray KM

Let the eye of the observer of the twilight be at B, on the surface of the Earth, which the line TB of the physical horizon touches at B. [Margin: First part of the problem.] Now let a ray issue from the Sun's right-hand limb, passing through G and through the air unrefracted, as the ancients thought, and touching the Earth at Z, until it meets the tangent TB at H. Draw the straight line AH to this point; for by it the arc BZ will be bisected at L, the arc EP at C, and the angle BAZ into the two equal angles BAL and LAZ, by proposition 1 of Problem 2, set out above; and by the same problem AH will

become known, and therefore also the altitude LH that the exhalations at H would have if the Sun's rays were not refracted. Now let there be another ray of the Sun from the same right-hand edge, namely KM, sensibly parallel to the former, GZ, because of the Sun's very great distance and the very small distance between the surface of the Earth and that of the air. Falling on M, this ray does not travel straight to P but is refracted towards the perpendicular and touches the Earth at R; proceeding from there, it issues through V, and would go straight on to O were it not refracted away from the perpendicular towards I. So the horizontal refraction of the solar ray is, at entry, the angle PMZ, and at exit, OVa. The ray, then, refracted a second time and proceeding along the line Va, before it reaches the horizontal line BT strikes exhalations — the atoms, as it were, of little clouds — which their opacity makes fit to reflect the Sun's ray to the point S, so that from there it is refracted to the eye at B. For if these exhalations were above the line BT, the ray reflected from them and falling into the air could not be refracted along the horizontal line to the eye at B; and so twilight would not be seen first by means of it, but some time after its beginning. Let us now suppose, for the sake of explanation, that the exhalation is at a, and that the ray reflected from there is aS. And although the lines AZ and AH are in reality much nearer to the line AB than you see in this figure, they are nevertheless placed in this position so that the narrowness of the space may not breed confusion among the lines and angles to be used below. Finally, from the centre A draw the straight lines ARQ through the point of contact R, AV to the point of exit V, Aa to the exhalation, and AS to the point of incidence of the last, the third, refraction. Now, so that it may appear that the point V of the exiting ray always lies between the lines AH and AQ, and so that its position — and indeed that of the point M of the entering ray — may be known, let us now complete the second part of the problem.

[Margin: Second part of the problem.]

IX. The ray MRV, then, touching the Earth at R, is the chord of the arc VQM, and its half VR is the sine of the arc VQ, as MR is of the arc MQ. Now the Earth's semidiameter AR is given, and when the altitude RQ of the refracting air is added to it, AQ is given in miles. Make, then: as AQ is to AR in miles, so is the whole sine AQ, of 10 000 000 parts, to AR, the second sine [Sinum Secundum] of the arc VQ, or QM; and so both arcs will be known. Now by Problem 2 the arc ECP is given, that is, the angle EAP, which is as great as the given depth of the Sun once you subtract from it the semiarc of the Earth illuminated beyond the hemisphere by the Sun's unrefracted rays; and its half is the arc CP, or LZ. But the ray refracted at M touches the Earth at R, beyond Z, with the arc RZ lying between, which is as great as the Sun's horizontal refraction, as is clear from what was said with Kepler, Book 5, chapter 5, problem 3. Subtracting RZ from LZ, therefore, LR, or CQ, will be known; if VQ is subtracted from this, the point V through which the Sun's ray exits will be known; and if to the point Q, now known, you add QM, equal to VQ, the position of the point M through which the Sun's ray enters will be known.

Now from the four extreme cases to be set out below in Problem 6 it will be clear that the point V always lies on this side of the line AH, towards AP; and it is demonstrated in general as follows. The semiarc of the Earth beyond the hemisphere illuminated by the Sun's unrefracted rays is not greater than 16′ 8″, from what was shown in Book 3, chapter 11, problems 4 and 5. The Sun's depth below the horizon at the beginning of morning twilight is not less than 16°, and at the end of evening twilight it is always greater, as is gathered from the observations to be reported in Problem 7. Therefore the angle BAZ, or the arc BLZ, is not less than 15° 43′ 52″, nor LZ less than 7° 51′ 56″. If you subtract from this RZ, the ordinary horizontal refraction of the Sun, which is about 34′, there remains LR, or CQ, 7° 17′ 56″. Now the altitude RQ of the refracting air, from what will be said in Book 10, section 6, problem 51, is scarcely greater than 20 miles; but let us grant it to be 30, of the parts of which AR, from what was said in Book 2, chapter 7, is 4139, so that the whole AQ is 4169. If one makes: as 4169 is to 4139, so is 10 000 000 to another number, AR will come out as 9 928 040, the second sine of the arc VQ of 6° 51′ 40″, which is less than the arc CQ of 7° 17′ 56″. But if you take RQ as 20 miles, AR will be 9 951 912 parts, the second sine of the arc VQ of 5° 37′ 16″. And so, the smaller the altitude RQ, the farther the point V will lie from the line AH, towards the line AP.

Probl. 5. Datis & inuentis quæ in Problem. 2. & 4. & Refractione triplici radij Solis horizontali Inuenire in linea altitudinis halituũ per irrefractos inuentæ, Punctum, per quod tranſeunt ſi producantur ad eam, radius Solis in egreſſu refractus & idem reflexus ad pũctum horizontis phyſici.

Problem 5. Given and found the quantities in Problems 2 and 4, and given the triple horizontal refraction of the Sun's ray: to find, on the line of the altitude of the exhalations found by means of unrefracted rays, the point through which pass, if they are produced to it, the Sun's ray refracted at its exit and the same ray reflected to the point of the physical horizon.

[Margin: First part, for the reflected ray.]

X. We begin with the reflected ray aS, but extended to O, a point of the line AH; and we seek the distance of the point O from H, which we shall find as follows. First, in the triangle BAH, right-angled at B, there are given, by Problem 2, AB, the Earth's semidiameter; AH, made up of the Earth's semidiameter AL and the altitude LH of the exhalations found in the ancient way; and the angle AHB, the complement to a right angle of the angle BAH. Therefore, by the laws of right-angled triangles, the side BH will become known. Secondly, in the triangle BAS there are given AB, and AS, made up of AH and the altitude NS of the refracting air; therefore, by the same laws, the side BS will become manifest, and this, subtracted from the side BH, will leave the side SH known. Thirdly, in the triangle SOH, the angle SHO is given, since it is the same as AHB, and the refraction of the entering ray, which is OSH, and therefore also the third angle SOH (the supplement of two right

angles). [Margin: By proposition 32 of Book 1 of Euclid.] Therefore, since SH is given, the side OH will not be unknown, by the laws of oblique-angled triangles.

[Margin: Second part, for the refracted ray.]

For the position of the refracted ray Va, cutting the straight line AH at I — that is, for the distance IH — draw the chord CV, so that the triangle COV arises. Now in the triangle AVR, right-angled at the contact R, the arc VQ was found by Problem 4, that is, the angle VAQ, whose complement to a right angle is the other angle, AVR; and the supplement of this to two right angles is AVO. [Margin: By propositions 32 and 13 of Book 1 of Euclid.] Next, by the same Problem 4 the arc CV was found, that is, the angle CAV in the isosceles triangle CAV. When this is subtracted from two right angles, therefore, the sum of the angles at the base CV is given, half of which is the angle ACV, and AVC. When this is subtracted from AVO, CVO remains; and when the refraction OVI is subtracted from this, IVC remains, known in the small triangle, in which ICV is also known, the supplement to two right angles of the angle ACV, and therefore also the third angle, CIV. [Margin: By proposition 5 of Book 1 of Euclid.] [Margin: By propositions 13 and 32 of Book 1 of Euclid.] Let there be found, then, in the isosceles triangle CAV, from the given angles and the sides AC and AV, the base CV; for the sides are made up of the Earth's semidiameter AL and the altitude LC of the air. Then with CV and the angles CIV and CVI you will find CI. Add this to the altitude LC, subtract the sum LI from the altitude LH found by Problem 2, and you will have the distance sought of the point I from H; the difference between this and the distance OH will be OI, which you should keep. And do not think that the rays of the Sun we use here hold some other position that conflicts with this demonstration; for from what will be said in Problem 6 it will be established that no false drawing [pſeudographiam] is contained in this diagram, that the twilight exhalation is at a, between AH and AS, and that the refracted ray Va falls below O — unless it meets the line AH at the same point at which the reflected ray Sa falls on it.

Probl. 6. Datis ac Inuentis, quæ in Probl 2.4. & 5. Inuenire Altitud. Halituum Crepuſcul. refractione radiorum Solarium Correctam.

Problem 6. Given and found the quantities in Problems 2, 4 and 5: to find the altitude of the twilight exhalations corrected for the refraction of the solar rays.

XI. If the difference IO found by Problem 5 were nil, the refracted ray VI and the reflected ray Sa would meet at the same point of the line AH; and so the portion OH, found by the first part of Problem 5 and taken from the altitude LH, would give the corrected altitude LO. But if they do not meet (as usually happens), use the triangle aIO, in which the difference, or side, IO has been found and all the angles are given. For aOI is the supplement to two right angles of the angle SOH, known by the first part of Problem 5; and aIO is vertically opposite, and equal, to the angle CIV, known by the second part of Problem 5; therefore the third angle, IaO, is also known. [Margin: By

propositions 13, 15 and 32 of Book 1 of Euclid.] Find, then, the side aI. Next use the triangle AIa, in which, besides the side aI just found, there is given the side AI, made up of the Earth's semidiameter AL, the altitude of the air LC, and the portion CI found by the second part of Problem 5; and there is also given the angle AIa, since it is the supplement to two right angles of the angle CIV, already known by the second part of Problem 5. The side Aa will therefore not be hidden; subtracting from it the Earth's semidiameter AX, the remainder will be the altitude Xa of the twilight exhalations, corrected for the refraction of the solar rays.

EXEMPLORVM 4. Epitome.

A summary of four examples.

XII. I have taken the Earth's semidiameter from Book 2, chapter 7; the altitude of the refracting air as at least 2 Bolognese miles and at most 20, from what will be said in Book 10, section 6, problem 51; and the Sun's depth as at least 16° and at most 26°, for in the temperate zones it is never usually outside these limits when twilight begins in the morning or ends in the evening. The ordinary horizontal refraction of the Sun's ray in the same zones I have taken from Tycho as 34′, and I have given no greater one to its ray reflected from the exhalations into the refracting air and refracted a third time to the eye. The semiarc of the Earth illuminated by the Sun beyond the hemisphere, by rays supposed unrefracted, I have taken as 15′, from Book 3, chapter 11, problems 4 and 5. The remaining angles and lines I have worked out, as in the following table, by applying Problems 2, 4, 5 and 6 in order.

Angles or arcs (altitude of the air →)	Sun's depth 16°, air 2 miles	Sun's depth 16°, air 20 miles	Sun's depth 26°, air 2 miles	Sun's depth 26°, air 20 miles
Use of Problems 2 and 4				
BAZ	15° 45′	15° 45′	25° 45′	25° 45′
BAL; LAZ	7° 52½′	7° 52′	12° 52½′	12° 52½′
BHA, SHO	82° 7½′	82° 7′	77° 7½′	77° 7½′
HSO	0° 34′	0° 34′	0° 34′	0° 34′
SOH	97° 18½′	97° 18′	102° 18½′	102° 18½′
ASB	88° 16′	84° 23′	88° 16′	84° 23′
Problem 4, second part				
LAR, or CQ	7° 18½′	7° 18′	12° 18½′	12° 18½′
RAZ, or RZ	0° 34′	0° 34′	0° 34′	0° 34′
VAQ, or VQ	2° 9½′	5° 37¼′	2° 9½′	5° 37¼′
AVR	87° 50½′	84° 22¾′	87° 50½′	84° 22¾′
AVO	92° 9½′	95° 37¼′	92° 9½′	95° 37¼′
CAV	5° 9′	1° 41¼′	10° 9′	6° 41¼′
ACV, and AVC	87° 25½′	89° 9⅓′	84° 55½′	86° 29⅓′
OVI	0° 34′	0° 34′	0° 34′	0° 34′
AVI	91° 35½′	95° 3¼′	91° 35½′	95° 3¼′
CVI	4° 10′	5° 53⅚′	6° 40′	8° 33⅚′
ICV	92° 34½′	90° 50⅗′	95° 4½′	93° 30⅗′

Angles or arcs (altitude of the air →)	Sun's depth 16°, air 2 miles	Sun's depth 16°, air 20 miles	Sun's depth 26°, air 2 miles	Sun's depth 26°, air 20 miles
CIV, and aIO	83° 15½′	83° 15½′	78° 15½′	77° 55½′
aOI	82° 41½′	82° 41½′	77° 41½′	77° 41½′
IaO	14° 3′	14° 3′	24° 3′	24° 3′
AIa	96° 44½′	96° 44′ + fraction [?]	101° 44½′	102° 4½′

Lines, in Bolognese miles (altitude of the air →)	Sun's depth 16°, air 2 miles	Sun's depth 16°, air 20 miles	Sun's depth 26°, air 2 miles	Sun's depth 26°, air 20 miles
Problem 5, first part				
AB	4139	4139	4139	4139
AS	4141	4159	4141	4159
LH	40	40	107	107
BH	572	572	946	946
BS	125	407	125	407
SH	447	165	821	539
HO	4½	1 7/10	8 3/10	5½
Problem 5, second part				
CV	372	128	733	486
CI	27¼	13	87	77 7/10
CH	38	20	105	87
IH	11	7	18	9 3/10
IO	7	5 3/10	9 7/10	3 9/10
aO	28⅕	21⅗	23⅓	9⅖
aI	28⅐	21⅗	23 2/9	9⅖
Aa	4172	4174 6/10	4232⅘	4238⅗
Conclusion				
Therefore the corrected altitude Xa of the exhalations	32 7/10	35⅗	93⅘	99⅗
The uncorrected altitude LH was	40	40	107	107

Although, then, Kepler in the *Epitome of Astronomy*, p. 75, and in the *Astronomiae Pars Optica*, p. 288, Glorioso, *On Comets* Book 2, chapter 2, Wendelen in the *Eclipses*, p. 5, and Gassendi, *Institutio Astronomica* Book 1, chapter 18, rightly advise that the altitude of the twilight exhalations asserted by the older writers must be diminished, chiefly on account of refractions (for Kepler counts as part of the cause the inflamed matter around the Sun, which enlarges its diameter and apparently hastens its arrival at the horizon), Wendelen nevertheless did not speak rightly when he said: *The altitude of the twilights would come out at about 50 Italian miles, except that through refractions it seems to be contracted to only 24, which I hold to be truer.* [Margin: Wendelen's opinion on the altitude of the twilight exhalations is false.] But I, who hold that the least altitude of the refracting air is greater than two miles, and do not think that the Sun's depth below the horizon at the beginning or end of twilight reaches 26°, determine

from my own observations their altitude, which you see at the end of the table of Problem 7.

Probl. 7. Profunditatem Solis ſub Horiz. ad initium matutini, vel finem veſpert. Crepuſculi obſeruando Inuenire.

Problem 7. To find by observation the depth of the Sun below the horizon at the beginning of morning or the end of evening twilight.

XIII. I gave this problem in Book 1, chapter 31, scholium 2, but there I supposed that the time was to be gathered from the altitude of the fixed stars by Book 10, section 3, problem 25. Using this, then, we observed the Sun's depth at Bologna with the greatest diligence at the solstices and equinoxes. Tycho's disputes with Rothmann on this matter may be seen in Tycho's *Letters*, pp. 112, 124, 140, 152 and 158 — and no wonder, since the two did not observe at the same season of the year. The opinions of the rest, from the authors adduced in Book 1, chapter 31, I repeat here, so that they may be at hand for this controversy.

Altitude of the vapours or exhalations that produce twilight, in Italian miles, 4 of which make one German mile.

Depth assumed in deriving the altitude of the vapours — authors, with their passages reviewed at No. 2	Depth of the Sun below the horizon	Altitude of the vapours (Italian miles)
Posidonius, 400 stades	19°	50
Ibn al-Haytham, Witelo, Clavius, Barozzi	19°	52
The Coimbra commentators, Cottunius, Snel, Resta, Surdus	19°	52
Petrus Surdus, from another hypothesis	19°	73
Cardano, from a twilight of 1¼ hours	19°	288
Cardano, from a twilight of 2 hours	60°	772
Scultetus and Abram Roggembach	60°	772
Gemma Frisius, Magini, Biancani	18°	43
Petrus Nonius, from someone else's hypothesis	19°	68
Petrus Nonius, from his own observation	16° 30′	48
Tycho Brahe	16°	48
or	17°	52
Christoph Rothmann	24°	60
Christianus Severini Longomontanus	20°	52
Kepler, from various hypotheses of others and with refraction excluded; for he thinks the altitude of the vapours must be diminished on account of refraction	16°	34
	16°	48
	18°	40
	19°	44
Simon Stevin and Glorioso, neglecting refraction	18°	41
Gassendi, if refraction is neglected	18°	40
Govaert Wendelen, if refraction is neglected	19°	50
But with refraction taken into account	19°	24

We, from our own observation		Depth of the Sun	Altitude of the vapours, Italian miles: least	mean	greatest
At the equinoxes	morning	16° 0′	38	50	63
	evening	20° 30′	63	75	88
At the solstices	summer	21° 25′	70	82	95
	winter	17° 25′	46	58	71

Scaliger, Piso, Chiaramonti, Tannerus and Mastrius held, however, that lighter exhalations, which do not produce twilight, rise much higher, towards the Moon. Remember that 4 Bolognese miles, which we use, equal 5 ancient Roman miles.

[Margin: Kepler, Glorioso, Gassendi and Wendelen think that the altitude of the vapours is diminished by the force of refraction.]

CAPVT XV.

Chapter XV.

De Altitudine Cometarum ex vno, vel diuersis Terræ locis spectabilium, inuestiganda, independenter à Parallaxi.

The altitude of comets visible from one place, or from different places, on the Earth, to be investigated independently of parallax.

I. To understand the various problems that bear on this, the following figure must be set out. In it, from the Earth's centre A, describe the Earth's circle BCOD; the circle LNMP of the lower comet; and the circle EFGH of the higher comet. For convenience these are now to be understood as lying sometimes under the equator, sometimes under the meridian, as we shall need below. Let EAG be the vertical line passing through the zenith E and the nadir G of the inhabitant at B, cut in A πρὸς ὀρθὰς [Greek: pros orthas, "at right angles"], that is, at right angles, by the rational or astronomical horizon HAF; and at B let IBK be the physical or sensible horizon. Supposing EHGF to be the meridian, let QR be the equator and ST its axis, so that HT is the altitude of the pole and QF the altitude of the equator with respect to the inhabitant at B. Now let the two points I and K of the physical horizon be joined to A by the straight lines AI and AK, just as its points L and M have been deliberately joined to A by the same lines that represent the axis of the equator and the diameter of the equator. In the same way, deliberately, so as not to multiply lines, two straight lines drawn from E touch the Earth's circle at the points C and D, through which pass AI and AK, terminated at the extremities of the physical horizon — granted that none of this was at all necessary. Just so, the drawing of this whole figure is not needed for each of the following problems: one part serves for one, another part for another.

Diagram. The figure for Chapter XV, set in the right column. A large circle (the circle of the higher comet) with the Earth's centre A at its middle; about A, two concentric small circles (the Earth and the circle of the lower comet). A vertical diameter running from E at the top through N, B, A and P to G at the foot; a horizontal diameter H–C–A–D–F; above the centre a horizontal chord I–L–B–M–K (the physical horizon). Lines run from A to I and to K, and from E down to the small circle on either side. A line runs from T at the upper left through A to S at the lower right, and another from Q at the upper right through A to R at the lower left.

- E (top): zenith
- G (foot): nadir
- H, F (left and right ends of the horizontal diameter): the rational horizon HAF
- I, K (ends of the upper chord): the physical horizon IBK
- L, N, M (on the inner circle): the lower comet's circle
- B (above A): the inhabitant
- V (small, between L and B): label not named in the text on this page
- C, D (on the small circle, either side of A)
- A (centre): the Earth's centre
- O (small, below A)
- P (on the vertical, below the small circles)
- T (upper left), S (lower right): the axis ST
- Q (upper right), R (lower left): the equator QR

Problema 1. Quanta debeat esse Altitudo Cometæ sub Aequatore moti ad motum Primi Mobilis, vt possit apparere supra Horizontem Physicum tanto tempore dato, si Semidiameter terræ nota sit in suis partibus.

Problem 1. How great the altitude of a comet moving under the equator with the motion of the primum mobile must be for it to be able to appear above the physical horizon for a given length of time, if the Earth's semidiameter is known in its own parts.

II. In the preceding figure it is already plain that if one comet moves under the equator along the smaller circle LNMP, and another along the larger HEFG, with no motion other than that of the primum mobile, then — since the whole equator revolves with uniform motion in the space of 24 hours, and 15 of the 360 degrees of it, and of the circles similar to it, rise in one hour — the higher comet, rising physically at I, will remain above the physical horizon IBK for a longer time, before it reaches its setting at K by way of E, than the lower comet, rising at L and passing through N to its setting at M. For the arc IEK is nearly 180°, or comes closer to the semicircle HEF, than the arc LNM, which in this diagram is not a whole quadrant of its circle. [Margin: Cases 1 and 2.] Now let the time be given during which the lower comet has appeared above the horizon from its sensible rising in the east at L as far as the meridian N, or even as far as its setting at M. For when this is given and converted into

parts of the equator, the arc LN is given, or the whole LNM, whose half is LN, measured by the angle LAN. Therefore in the triangle LAB, right-angled at B by construction (as the intersection of the horizon with the vertical line requires), two angles are given, namely the right angle B and the acute angle LAN; given too are the stades or miles contained in the Earth's semidiameter AB; and so by the rules of triangles the base AL will become known. And since AN is equal to AL, as they are radii of the same circle, if from the whole AL, that is, from the whole AN, the Earth's semidiameter AB is subtracted, BN will remain known, that is, the altitude of the comet that was sought. In the same way, if the time of the higher comet from its rising at I to the meridian E, or to its setting at K, is given, the arc IE, half of the whole IEK, will also be given; it is measured by the angle IAE, that is IAB, in the same triangle right-angled at B. Therefore from the semidiameter AB the secant or base AI will become known, to which AE is equal; and so, when AB is taken away, the comet's altitude BE will be known, if its distance from the Earth's surface is sought; for if its distance from the centre is sought, it will be AE itself, or AI. And these are the first two cases.

III. [Margin: Case 3.] The third case is if the comet has been observed from its rising, say at I, up to its elevation at T, at which time it vanished because of the Sun's rising, and its distance from the meridian was then observed by azimuth (which will nevertheless be the true distance, notwithstanding that the parallax is unknown). For from the time from the comet's rising to its elevation at T the angle IAT will be given; and from the remaining part of its ascent to the meridian, derived by the azimuth, the angle TAE will be given, and so the whole angle IAE; with this, and with AB, as above, the distance AI will be known. [Margin: Case 4.] The fourth is if it happens that the comet appears at Q as the Sun rises, and the angle EAQ, or the arc EQ, is known from the azimuth; for this, subtracted from the arc IEQ, which answers to the time from the comet's rising to its position at Q, will leave the angle IAE known, as above. [Margin: Case 5.] The fifth case is if the comet appears at T as the Sun sets, and when its azimuth is taken the angle TAE becomes known; and when the time from T up to the comet's setting at K is counted, the arc TEK is known. For when the arc TE is subtracted, the arc EK will be known, that is, the angle EAK or BAK, with which, and with AB, the secant AK will become known. [Margin: Case 6.] The sixth case is if the comet appears at Q at sunset or a little after, and when its azimuth is taken the arc EQ, or the angle EAQ, becomes known; for this, added to the angle QAK, which is known from the time in which the comet came from Q to K, will make up the angle BAK, and so on. [Margin: The case is extended to the parallels of the equator.] From these, then, it will not be difficult to solve the other cases, whether the comet is under the equator or in a parallel of known declination, as will be said more fully in the problems of the primum mobile.

Meanwhile let it be supposed that it can be known that some comet moves under the equator, setting parallax aside — as if, for example, its motion had

been observed by another inhabitant in the right sphere — granted that this has perhaps never happened.

Problema 2. Data Cometæ altitudine seu distantia, sub Aequatore moti ad motum primi Mobilis, Inuestigare tempus, quo apparere poterit supra Horizontem.

Problem 2. Given the altitude or distance of a comet moving under the equator with the motion of the primum mobile, to find the time for which it will be able to appear above the horizon.

IV. This problem also includes six principal cases. For what is sought is either the whole time from its apparent rising to its setting, in which the comet runs through the arc IEK; or the time that passes between its first or last appearance at the point E of the meridian and its setting at K, or from its rising at I; or the time from its rising at I until its vanishing at T, the Sun now rising; or the time from its rising at I until its vanishing at Q, the Sun now rising; or the time from its appearance at T, or at Q, a little after sunset, until the comet's setting at K. Now in the first two cases we must use the triangle IAB or KAB, right-angled at B, in which are given the Earth's semidiameter AB and the comet's altitude, or distance from the Earth's centre, in the same kind of parts — that is, the secant or base AI or AK. From these three data, therefore, the angle IAB or KAB, which measures the arc IE or KE, will be found by the rules of triangles; and this arc, converted into times of the equator, will show the time from the comet's rising at I to its ascent to the meridian at E, or from there to its setting at K, and the sum of the two will be the time from the comet's rising to its setting. In the other cases, by contrast, the azimuths will show the differences of the angles that exceed or fall short of the angle IAB or KAB; and these excesses or defects, converted into times of the equator, will make plain the time desired.

I should like now to put into a small table the altitude of a comet, or its distance from the Earth's surface, corresponding to the whole time for which it can appear above the physical horizon under the equator or very near it, so that conjectures about the rest can be made from it — although it serves also for places situated under the same meridian, or under any one vertical, from which the phenomenon can be seen in the physical horizon of each.

Altitude above the Earth's surface necessary for a comet moving under the equator with the motion of the primum mobile to be able to appear above the horizon for the number of hours written below, from rising to setting.

Altitude of the comet: or semidiameters of the Earth	and miles	Italian miles	Time of stay	Arc of the equator
		10	$0^h\ 30^m$	7° 30′
		36	$1^h\ 0^m$	15° 0′
		146	$2^h\ 0^m$	30° 0′
		341	$3^h\ 0^m$	45° 0′
		641	$4^h\ 0^m$	60° 0′

Altitude of the comet: or semidiameters of the Earth	and miles	Italian miles	Time of stay	Arc of the equator
		1078	$5^h\ 0^m$	75° 0′
		1715	$6^h\ 0^m$	90° 0′
		2660	$7^h\ 0^m$	105° 0′
1	0	4139	$8^h\ 0^m$	120° 0′
1	2538	6677	$9^h\ 0^m$	135° 0′
2	3575	11853	$10^h\ 0^m$	150° 0′
6	2737	27571	$11^h\ 0^m$	165° 0′
14	1200	59146	$11^h\ 30^m$	172° 30′
29	2329	122360	$11^h\ 45^m$	176° 15′
47	1067	185600	$11^h\ 50^m$	177° 30′
90	1960	374470	$11^h\ 55^m$	178° 45′

Corollaria ex Præcedenti Tabella deducta pro Altitudine Cometarum.

Corollaries deduced from the preceding table for the altitude of comets.

V. First. If a comet were in the highest region of the air — taken as the region bounded at the altitude of the twilight vapours — it could not appear above the horizon for two whole hours, either with the motion of the primum mobile alone or with a motion against the order of the signs, towards the west, added to it; rather, it would plunge to its setting before two hours from its rising had passed. [Margin: Corollary 1, for comets not [?] exceeding the highest region of the air [?].] This proposition, understood with this limitation, is Cysat's, in the *Opusculum de Cometis*, chapter 3, proposition 1, and, from him, Froidmont's, in Book 3 of the *Meteorologica*, chapter 1, article 4. It is evident because, according to the preceding table, a comet that appears above the horizon for a stay of two hours requires an altitude above the Earth's surface of 146 Italian miles. But according to the last table of chapter 14, the altitude of the twilight vapours does not exceed 100 Italian miles, let alone 146, in the opinion of any author that can be tolerated; for the opinion of Cardano, Roggembach and Scultetus is intolerable and manifestly false, from what was said there at No. 4.

Secondly. [Margin: Corollary 2. An argument that the comet is above the Moon from its diurnal motion alone [?].] If a comet's diurnal motion under the equator by the force of the primum mobile takes very nearly 12 hours, so that it falls short of this by no more than 8^m, with refraction and parallax removed — or if it takes at least $11^h\ 52^m$ — the comet will be higher than the Moon. And this is what Froidmont meant to suggest in Book 3 of the *Meteorologica*, chapter 1, article 4, in these words: *Because many comets have drawn out a stay above the horizon equal to that of the stars, they are proved also to have had an altitude no lower.* Yet no comet is known to have moved under the equator. This is evident from the preceding table, in which a stay above the horizon of $11^h\ 50^m$ requires the comet to have an altitude above the Earth's surface of 47 terrestrial semidiameters and 1067 miles besides, while a stay of $11^h\ 55^m$ requires an altitude of 90 semidiameters and 1960 miles; therefore a stay of $11^h\ 52^m$ will require

an altitude of 64 terrestrial semidiameters. But the Moon's distance from the Earth's centre is about 64, and from its surface 63, as is clear from what was said in Book 4, chapter 14. The whole motion of the comet just described, from east to west, can, however, be known even if it has been observed for a few hours from its rising or before its setting, according to the methods indicated in Problem 1. Thus Cysat, as he has it in chapter 3 of *De Cometis*, proposition 1, observed on 1618 December 1 a comet that had risen on the eastern horizon at the 5th hour of the morning, that is, after midnight, at Ingolstadt, and had been elevated like the other stars, by the motion of the primum mobile, for three hours; when these had passed, at the approach of sunrise, it ceased to be seen, being distant from the meridian by an interval of more than two hours, indeed of nearly three. From this he rightly concludes that its semidiurnal arc was more than 5 hours, and its diurnal arc more than 10. On the following day it emerged from the horizon at the 4th hour of the morning, that is, 4 hours after midnight. That comet was therefore higher than three semidiameters of the Earth, as is evident from our table, and as Cysat shows in another way about that comet, in chapter 3, proposition 2; otherwise it would have set much sooner and risen much later, or (which comes to the same thing) it would have been above the horizon for fewer than 10 hours and below the horizon for more than 14. These things are truer of that comet at the time when it had a southern declination, since they are true of a comet under the equator itself.

[Margin: Corollary 3, for comets moved by a motion other than that of the primum mobile.] Thirdly. If a comet under the equator moves not only with the motion of the primum mobile but also with a proper motion against the order of the signs, what we said in the first and second corollaries is much more true. For motion against the order of the signs, that is, towards the west, goes along with the rapt motion [motus raptus], that is, the motion of the primum mobile; and the more it adds to that motion, the longer is the arc of the equator that would be owed to the comet from east to west if it were carried precisely by the motion of the primum mobile. So if this comet in one day completed 15 degrees by its proper motion against the order of the signs, and therefore rose one hour earlier than on the preceding day, then if it required two hours from its rising to its setting, that would be a sign that the diurnal arc of the equator was of three hours, or 45 degrees — one of which hours, however, it would have gained by its own speed towards the west. It is otherwise, by contrast, if the comet moves with a proper motion towards the east. For then, from the diurnal motion towards the west, unless this is corrected by subtracting as much as the eastward motion demands, so great an altitude of the comet cannot be inferred as from the motion of the primum mobile alone.

[Margin: Corollary 4, for comets declining from the equator.] Fourthly. If the comet declines towards the region opposite the observer, and moves only with the motion of the primum mobile, or only towards the west, what we said in Corollaries 1 and 2 is much more true. For the diurnal arc of the parallel,

because of the obliquity of the horizon, would be of fewer degrees than the diurnal arc of the equator, as will easily become clear.

Diagram. The figure of p. 91 set again in the left column (the text below calls it the figure set out at No. I). A large circle with the Earth's centre A at its middle; about A, two concentric small circles. A vertical diameter running from E at the top through N, B, A and P to G at the foot; a horizontal diameter H–C–A–D–F; above the centre a horizontal chord I–L–B–M–K. Lines run from A to I and to K, and from E down to the small circle on either side. A line runs from T at the upper left through A to S at the lower right, and another from Q at the upper right through A to R at the lower left.

- E (top), G (foot): zenith and nadir
- H, F: the rational horizon
- I, K: the physical horizon
- L, N, M (on the outer small circle)
- B (above A): the observer
- V (small, between L and B)
- C, D (on the small circle, either side of A)
- A (centre)
- O (small, below A)
- P (on the vertical, below the small circles)
- T (upper left), S (lower right)
- Q (upper right), R (lower left)

Problema 3. Data Cometæ ad oppositam spectatoris Plagam declinantis, Declinatione, sed vera, & Altitudine Poli, Inquirere minimam Cometæ à centro terræ distantiam.

Problem 3. Given the declination — but the true one — of a comet declining towards the region opposite the observer, and given the altitude of the pole, to find the least distance of the comet from the Earth's centre.

VI. Let the comet have been observed at the zenith by someone whose zenith's distance from the equator is known, and let the comet's true declination have been derived from this, independently of parallax. Or let the comet's true declination be supposed from some other source, feigned or given — the declination measured by the arc of the meridian, or of any great circle drawn through the poles of the equator, intercepted between the comet and the equator. On these suppositions, let the figure set out at No. I be repeated here, and let the observer be a northern one at B, whose altitude of the pole is HT, and altitude of the equator FQ, from which the comet declines to the south by the arc QK (the comet being feigned for the moment at K, a point of the physical horizon, when it appears on the meridian). For when EQ, the distance of the equator from the zenith (which is always as great as the altitude of the pole), is added to the declination QK, the arc EQK becomes known; and from it the angle EAK, which that arc measures, and hence the angle BAK in the triangle ABK, right-angled at B. In this triangle, therefore, since the Earth's semidiameter AB is supposed known — whether it is taken as 1 or as so

many miles — the base or secant AK can be found by the rules of triangles. So if you make: as the whole sine, or whole radius, that is, the radius AB, is to the secant AK of the angle BAK, so is the Earth's semidiameter AB, 1 or so many miles, to a fourth term, then AK will be known in semidiameters of the Earth or in miles. Or add to the logarithm of the side AB the residue of the second logarithm of the angle BAK, and the result will be the logarithm indicating the base AK. Now AK is the least distance of this comet from the Earth's centre; for if, as is supposed, it appears above the horizon outside the meridian, it will appear much higher above the horizon on the meridian. Therefore from its declination towards the opposite region a much greater distance of it from the Earth's centre than AK is inferred — which will also be clear to anyone who produces AK in a straight line. But if the comet is on the equator, its least distance is AM, which will be greater if it appears higher on the meridian, say at Q. By this device the following small table was constructed, or rather transferred from the preceding one to this use. It gives the least distance from the Earth's centre that a comet must have when placed on the equator, or declining towards the opposite region, corresponding to the comet's meridian distance from the zenith — a distance made up, namely, of the equator's distance from the zenith and the comet's declination just mentioned.

Least distance of a comet from the Earth's centre, corresponding to the altitude of the pole if the comet is on the equator, or to the sum of the altitude of the pole and the comet's declination towards the other pole, if it so declines.

Altitude of the pole, or its sum with the comet's declination	Least distance of the comet: semidiameters of the Earth	Italian miles
3° 45′	1 and	10
7° 30′	1	36
15° 0′	1	146
22° 30′	1	341
30° 0′	1	641
37° 30′	1	1078
45° 0′	1	1715
52° 30′	1	2660
60° 0′	2	0
67° 30′	2	2538
75° 0′	3	3575
82° 30′	7	2737
86° 15′	15	1200
88° 7½′	30	2329
88° 45′	48	1067
89° 22½′	91	1960

Corollaria ex præcedenti Tabella deducta pro Altitudine Cometarum.
Corollaries deduced from the preceding table for the altitude of comets.

VII. First. If the comet is under the equator, and the altitude of the observer's pole is not less than 15°, that comet will be higher than the twilight vapours. [Margin: Corollary 1, for comets moving under the equator.] For, according to the preceding table, it will be distant from the Earth's centre by one semidiameter of the Earth and 146 Italian miles besides, which far exceeds every probable distance of the said vapours from the Earth's surface, as is evident from the last table of chapter 14 and from No. 5 of the same chapter.

Secondly. [Margin: Corollary 2, for comets appearing in either of the tropics.] If the comet is on the tropic of Capricorn, or beyond it, and the altitude of the north pole is 65° 30′ or more, that comet will be above the Moon. For from the altitude of the pole, 65° 30′, and the declination of the tropic of Capricorn, which for me is always 23° 30′, there is made up a sum of

89 degrees. To this sum the preceding table assigns a least altitude roughly midway between 48 and 91 semidiameters of the Earth, that is, about 70 semidiameters, which exceed the Moon's distance from the Earth, from what was said in Book 4, chapter 14. How much more so if the comet, when it is on the meridian, is raised above the horizon, or is beyond the tropic of Capricorn. These things hold equally if the comet is on the tropic of Cancer or beyond, and the altitude of the south pole is not less than 65° 30′.

Thirdly. [Margin: Corollary 3, for comets seen at Milan on the tropic of Capricorn.] Some comets seen at Milan by Cardano or by others would have been distant from the Earth's centre by at least three semidiameters of the Earth, if some of them — seen at Milan on the tropic of Capricorn, as Cardano testifies in Book 4 of *De subtilitate*, and so declining to the south by 23° 30′ — had really been there. For at Milan the distance of the equator from the zenith, or the altitude of the pole, is 45° 28′; the sum is therefore 68° 58′, or very nearly 69°. To this sum the preceding table assigns at least 3 semidiameters of the Earth, since to 67 degrees are assigned 2 and 2538 miles, and to 75 degrees 3 semidiameters and 3575 miles — especially since this is the least distance, and the distance of those comets must necessarily have been greater, seeing that they were seen above the horizon on the tropic of Capricorn. Cardano, however, supposing there a distance of Milan from the circle of Capricorn of 68°, infers a smaller distance. I said "if they had been", because it does not follow from their having been seen on that tropic that they really were on it. For they could have had a much smaller true declination from the equator, and yet, because a large parallax depressed them towards the horizon, have seemed to decline from the equator as far as the tropic of Capricorn declines. [Margin: Cardano's slip concerning the altitude of comets.] Cardano's inference, therefore, which rests on the apparent declination, is false or uncertain. Similar to it seems the inference of our Father Cysat, who, on the comet of the year 1618, chapter 3, proposition 2, affirms that its least distance from the Earth's centre was 3322 German miles, because that comet was seen on 19 November at Frankfurt an der Oder when it had a southern declination of at least 23°, and so described the tropic of Capricorn by the motion of the primum mobile. He supposes, moreover, the

altitude of the pole at Frankfurt to be 52°, and the Earth's semidiameter to be 860 German miles. Indeed, if the same comet was seen on the same day in Prussia under a pole altitude of 54° (at Danzig, I believe, where for me it is 54° 23′), he infers that the comet must have been at least 3823 German miles distant from the Earth. But Cysat, aware that the parallax of this comet was smaller than the parallax of the Moon — although that had not yet been demonstrated at the time — seems to have taken as certain a true declination of the comet not less than 23°, especially since the comet was seen beyond the tropic of Capricorn; and yet he assumed it to be on the tropic itself.

Problema 4. Data diſtantia in circulo maximo inter duo loca Terræ, Cometam eumdem eodem tempore ſpectantia, & ſemidiametro Terræ, Inquirere minimam Cometæ à centro terræ diſtantiam, ſeu quanta ſaltem eſſe debeat.

Problem 4. Given the distance on a great circle between two places on the Earth that observe the same comet at the same time, and given the Earth's semidiameter, to find the least distance of the comet from the Earth's centre, or how great it must at least be.

VIII. With the figure set out at No. 1 repeated here, let there be two places on the Earth's surface, C and D — for convenience, on the equator itself, or under the same meridian, or under the same vertical circle — and let two straight lines EC and ED, meeting at E, touch those places. For when radii are drawn to the points of contact C and D, the angles at C and D will be right, by proposition 18 of the third book of the *Elements*. Let the straight line AE be drawn from the Earth's centre to the comet; for the least distance at which the comet can be seen will be AE — namely, if the comet is on the physical horizon of both places, CE and DE. But if it is seen above the horizon, it will of course be seen beyond E, and the comet's distance will be greater than AE; whereas we are seeking the least distance, below which it can in no case be seen except by the help of refraction. Therefore the distance of the two places under the same great circle, which is the arc CBD, when bisected, will give the arc CB, which measures the angle CAE. With this angle and the Earth's semidiameter AC, in the triangle CAE, right-angled at C, the least distance — that is, the base or secant AE — will easily be found. You see, moreover, that the problem is solved in the same way as Problem 3, for the triangles BAK and AED, or AEC, are equal and equiangular. Therefore the table placed at the end of No. 6 serves this problem too, if half the distance between the two places on the Earth is looked up in it.

Diagram. The figure of No. 1 set again at the head of the right column: a large circle; inside it, about the centre A, two concentric small circles; a vertical diameter through E, N, B, A, P, G; a horizontal diameter through H, C, A, D, F; a horizontal chord through I, L, B, M, K above the centre; lines from E down to the small circle; and two lines through the centre, from T to S and from Q to R. For this problem, C and D are the two places on the Earth's surface, E the comet, and EC, ED the tangents meeting at E.

- E (top): the comet, where the tangents EC and ED meet

- G (foot)
- H, F: ends of the horizontal diameter
- I, K: ends of the horizontal chord
- L, N, M (on the outer small circle)
- B (above A)
- V (small, between L and B)
- C, D (on the small circle, either side of A): the two places on the Earth
- A (centre): the Earth's centre
- O (small, below A)
- P (on the vertical, below the small circles)
- T (upper left), S (lower right)
- Q (upper right), R (lower left)

For example, if the places are 60° apart, look up 30° in the first column of the table; for to these corresponds a least distance of the comet of one semidiameter of the Earth and 641 Italian miles besides. Other places on the Earth, included within C and D towards B, will be able to see the same comet much more easily.

Corollaria ex hoc Problemate.

Corollaries from this problem.

IX. First. [Margin: Corollary 1, for a comet seen from two widely separated places on the Earth.] If two places on the Earth under the same great circle are not more than 176 degrees apart, it cannot be inferred from their seeing the comet at the same time that the comet is above the Moon, unless other conditions of the phenomenon are added. For from the table of No. 6 and the preceding problem, if those places are 176 degrees apart, half the distance is 88°, to which corresponds a least distance of the comet smaller than 30 semidiameters of the Earth — that is, smaller than the lunar distance, from what was said in Book 4, chapter 14. And unless something else is added, the comet can be said to have been seen on the physical horizon of both places. On this occasion, note that Giovanni Camillo Glorioso, in Book 2 of *De Cometis*, chapter 6, and Book 3, chapter 5, using this problem, rightly said that it cannot be argued from the comet of the year 1618 having been seen at the same time at Goa and in the Fortunate Isles that it was therefore above the Moon. For he assumes a difference of longitude between those places of 115° 10′ and an Earth's semidiameter of 3536 Italian miles, from which he deduces the secant AE, or least distance, as 5410 miles — that is, far smaller than the least distance of the Moon according to Ptolemy, which Glorioso puts at 178,672 miles. And Chiaramonti, in his opuscule *De sede sublunari Cometarum*, chapter 14, shows that the comet of the year 1618 could have been seen at the same time at Cesena and at Goa even if it had been no higher above the Earth's surface than 95 Italian miles — seen, I mean, from each place on its own physical horizon.

Secondly. [Margin: Corollary 2, for the same.] If two places on the Earth's surface that see the same comet at the same time are a whole semicircle of a

great circle apart, or at least 178 degrees, that comet will be above the Moon, unless an enormous refraction is feigned to stand in the way of this true altitude. For if they are 178 degrees apart, half the distance is 89°, to which the preceding problem, or the table of No. 6, assigns as the comet's least distance at least 68 or 70 semidiameters of the Earth, which certainly exceed the Moon's distance, from what was said in Book 4, chapter 14. And the comet will be all the higher, the higher it is seen above the horizon from both places. Would that we had a comet observed at the same time in two such places! But from what is to be said below, especially in chapter 22, No. 23, it will be established that such good fortune has not yet fallen to astronomers: for the comet of the year 1618, which of all comets was observed from the most numerous and most widely separated places on the Earth, was not seen over a greater interval than that between Goa and Antwerp — and not even at the same moment.

Problema 5. Data Cometæ diſtantia à centro Terræ, Inquirere ſpatium terræ, intra quod & non extra videri poterit eodem tempore.

Problem 5. Given the distance of a comet from the Earth's centre, to find the space on the Earth within which, and not outside it, the comet can be seen at the same time.

X. Giovanni Camillo Glorioso proposes this problem in other words in Book 2, chapter 5, of *De Cometis*. But it is easily solved by the preceding figure. For with the comet placed at E, and with its distance AE given — the secant or base in the triangle ACE, right-angled at C — and the Earth's semidiameter AC given as the radius or side, the angle CAE is easily found. For make: as AC, in the miles contained in the Earth's semidiameter, is to AE, the comet's distance, so is the radius or whole sine AC to AE, the secant of the angle CAE, which measures the arc CB. This arc, doubled, gives the arc of the Earth CBD, within which alone such a comet can be seen at the same time.

Problema 6. Data Cometæ Altitudine ab Horizonte eodem momento viſa ex duobus terræ locis, ſub eodem verticali cum Cometa poſitis, & Diſtantia locorum terræ in gradibus ac minutijs circuli maximi, nec non Semidiametro Terræ in Milliaribus; Inueſtigare diſtantiam Cometæ à Terra.

Problem 6. Given the altitude of a comet above the horizon seen at the same moment from two places on the Earth placed under the same vertical circle as the comet, and given the distance of the places on the Earth in degrees and minutes of a great circle, and also the Earth's semidiameter in miles, to find the comet's distance from the Earth.

XI. From the centre A describe the Earth's semicircle CBE, and the quadrants DS and VT, of which let VT be the one in which the comet moves. Let there be two places on the Earth, B and G, through which let the vertical lines ABD and AGL pass from A up to the zeniths, and let the places B and G be joined by the chord BG. An isosceles triangle ABG will thus arise, in which, from the arc BG of the distance of the places, the angle BAG that measures it is known; its supplement to two right angles will be the sum of the angles ABG and AGB, by proposition 32 of the first book of Euclid. Hence those angles at the base

of the isosceles triangle will be known separately, since they are equal to each other, by proposition 5 of the first book of Euclid. All the sides will also be known: for both AB and AG are the given semidiameter of the Earth, and BG is the chord of the arc BG, known from the tables of sines in parts of which AB is the whole sine. So make: as AB, the whole sine 100,000, is to the miles contained in AB, so is the chord BG to the miles contained in BG.

Diagram. A figure in the left column for Problem 6: an outer quadrant arc running from D at the top right down to S at the bottom left, and an inner concentric arc from V down to T; a vertical line through D, V, B, A on the right; a base line through S, T, C, A, E; a small semicircle of the Earth on the base, passing through C, G and B and down to E, with a short upright at m; and many straight lines drawn from the places B and G out to the two arcs. Star-marks for comet positions stand beside Y, beside X, and at the top of the outer arc.

- A (bottom, on the base): the Earth's centre
- C, G, B, E (on the small semicircle): C and E at its ends on the base; B and G the two places on the Earth
- D (top right): the zenith of B; V (on the vertical below D, on the inner arc): a comet position
- L, M, N, O, P, H, F, I (on the outer arc); H, F, I set close together at its top
- K, Q, R (inside, on or near the inner arc): comet positions named in the cases
- X, Y (left, beside the arcs, each with a star-mark)
- S (bottom left), T (on the base, below the inner arc)
- m (right, at the end of the short upright beyond B): end of the physical horizon Em
- one small letter-form below the star-mark at the top of the outer arc: [?]

On these suppositions, there are four cases of the problem.

[Margin: Case 1 of the problem.] The first case is if the comet is seen at the zenith of one of the places — for example, if it is at Q and is seen from G under the zenith L, or if it is at V, seen from the place B under the zenith D — but in the other place is seen to be distant from the zenith. Let it be, then, at Q. When the line BQM is drawn from B, the apparent place will be under M, but the true place under L; therefore BQA will be the parallax of altitude itself, or its measure, and it will be known. For in the triangle BQG, when the angle AGB is subtracted from two right angles, the angle BGQ will be known, by proposition 13 of the first book of Euclid; and by the same proposition, by subtracting from two right angles the sum of the angle ABG and the angle DBQ — which is known from the comet's apparent distance from the zenith D — the angle GBQ will be left known. Therefore, by adding this to the angle BGQ and subtracting the sum from two right angles, by proposition 32 of the first book of Euclid, the parallactic angle BQG, or BQA, will be known. Now, then, in the triangle AQB there are given the angle Q, the angle BAQ, known

from the arc BG, and the side AB, that is, the Earth's semidiameter. Therefore geometers cannot fail to know the side AQ, which is the comet's distance from the Earth's centre.

[Margin: Case 2.] The second case is when the comet is between the zeniths of both places, for example at K. The sign of this case will be if the comet is towards one region of the world for one of the places, but towards the opposite region for the other. So, drawing the lines BKH and GKI from B and G through K, and the line of the true place AKF, consider the triangle BKG. In it the angle GBK is known if you subtract from two right angles the sum made of the angle ABG and the angle DBK, known from the comet's apparent distance from the zenith D; the angle BGK is also known, if you subtract from two right angles the sum made of the angle AGB and QGK, known from the apparent distance from the zenith L. Therefore the third angle GKB too (which in this case is the sum of the parallaxes) will be known. But the base BG in miles is also known; so neither the side BK nor GK will remain hidden. Next, in the triangle ABK there are now given the two sides BK and AB and the angle ABK they contain, if DBK, that is, the comet's apparent distance from the zenith D, is subtracted from two right angles. Therefore the base AK, which is the comet's distance from the Earth's centre, will not remain hidden.

[Margin: Case 3.] The third case is when the comet is outside the arc comprehended by the zeniths of both places, as if it is at R. The sign of this will be if it is so distant from each zenith that for both observers it inclines towards the same region of the world. So draw through R the line of the true place ARN, and from the places B and G through R the lines of the apparent places BRP and GRO. First consider the triangle BGR, in which there are given the side BG and the angle GBR, if you subtract from two right angles the sum made of the angle ABG and DBR, which is known from the comet's apparent distance from the zenith D. The angle BGR is also known, if from the sum made of two right angles and the comet's apparent distance from the zenith L, which is the angle RGL, you subtract the angle AGB. Therefore, by proposition 32 of the first book of Euclid, the third angle BRG will be known, as being the remainder to two right angles (in this third case it is, and is called, the difference of the parallaxes). Find, then, the side GR. Next, in the triangle AGR, find the distance AR of the comet R from the Earth's centre; for in it there are now given the sides AB and GR and the contained angle AGR — namely, if you subtract the angle RGL from two right angles.

[Margin: Case 4.] The fourth case is when the comet appears to both observers on the physical horizon, or apparently distant from each zenith by 90 degrees; enough has been said of this in Problems 1, 2 and 3.

But if two places on the Earth are a whole quadrant apart, or more, only the second or the fourth case can occur. For let the two places be B and E, and let the comet be seen from B under the zenith D. It cannot be seen from E, because an observer can see nothing below the physical horizon Em, except accidentally by the help of refraction; much less will he be able to see a comet

placed beyond the straight line AD, towards K. Neither the first nor the third case, therefore, will occur.

Problema 7. Data Diſtantia Phænomeni à centro terræ, datoque vero Parallelo, in quo motu diurno vertitur, & data cuiuſuis loci terræ propoſiti Latitudine ac Semidiametro terræ; Inueſtigare Differentiam inter moram Phænomeni ſupra Horizontem Phyſicum, & Aſtronomicum in motu diurno.

Problem 7. Given the distance of a phenomenon from the Earth's centre, and given the true parallel on which it turns in the diurnal motion, and given the latitude of any proposed place on the Earth and the Earth's semidiameter, to find the difference between the stay of the phenomenon above the physical horizon and above the astronomical horizon in the diurnal motion.

XII. In the following figure, from the Earth's centre A describe the celestial meridian BOD, and under it the terrestrial meridian ECF; and let there be on it an inhabitant at C, whose vertical line is ACO. Let the natural or physical horizon GCK cut this line at right angles at C, and parallel to it let the rational or astronomical horizon BAD be drawn. Further, let the axis of the world be TAV, and the plane of the equator AL standing perpendicular to it, so that the altitude of the north pole is the arc BT, which is measured by the angle BAT, equal to the angle OAL, that is, to the distance of the equator L from the zenith O. Now, if the phenomenon is on the equator, draw from the point I, the common section of the equator with the physical horizon, the perpendicular IP to the astronomical horizon. But if it is on a southern parallel, describe that parallel to the equator, for example HF, and from the point of its common section with the physical horizon draw the perpendicular MN to the astronomical horizon; join H also to C by the straight line CH. If, however, the phenomenon is on this side of the equator, describe its parallel, for example QE, and from its common section with the physical horizon draw the perpendicular RS to the astronomical horizon — just as, if it passed through the zenith, the parallel to the equator OX would have to be drawn. On these suppositions we shall proceed as follows, with Chiaramonti, Book 3 of *De stellis novis*, chapter 30.

Diagram. A figure in the left column for Problem 7: a large circle, the celestial meridian, with O at the top; inside it, about the centre A, a small circle, the terrestrial meridian, with C at its top and F below; the horizontal diameter, the astronomical horizon, running from B through X, E, S, A, P, Z, N to D; the chord GK, the physical horizon, through C; the axis of the world from T at the upper left through A to V at the lower right; and many lines from A and from C out to the circle, with short perpendiculars dropped to the diameter.

- O (top): the zenith
- Q, L, H (on the circle, right of O, in that order downwards)
- K (right end of the chord), G (left end of the chord): the physical horizon GCK

- B (left end), D (right end of the diameter): the astronomical horizon BAD
- T (upper left), V (lower right): the axis of the world TAV
- A (centre): the Earth's centre
- C (top of the small circle): the inhabitant; F (on the small circle, below the diameter)
- X, E, S (on the diameter, left of A); P, Z, N (right of A)
- R, I, M (on the chord, from left to right)
- β, ε (small Greek letters, at the left, inside the circle)
- a (small italic letter, left of A, above the diameter)

[Margin: Case 1, if the phenomenon is on the equator.] The first case is if the phenomenon moves under the equator AL. Then use the triangle AIP, right-angled at P, in which there are given the side IP, as being equal to the terrestrial semidiameter AC, and the angle IAP, which measures the arc LD, that is, the altitude of the equator, or the complement of the altitude of the pole. From these, by the rules of triangles, find the base AI; subtract it from the whole AL, namely the given distance of the phenomenon from the Earth's centre, and IL will be left, known in miles. Reduce it by the rule of proportion to parts of which the whole AL is the radius 100,000: for IL will be the versed sine of an arc which, doubled, will give the arc of the phenomenon's diurnal revolution above the physical horizon. Convert its degrees, then, into times, which will give the phenomenon's stay above the horizon; and this, subtracted from 12^{h}, will leave the difference between it and the stay above the astronomical horizon. But if, in the triangle AGC, the horizontal parallax AGC has been found from the phenomenon's distance AG, its double, converted into times of the equator, will give the difference just mentioned very readily. These things, and what follows, are to be understood with refraction left out of account, which usually increases a stay of this kind.

[Margin: Case 2, of a phenomenon placed on a southern parallel.] The second case is when the phenomenon is on a southern parallel, for example under the tropic of Capricorn FH. Then first use the triangle ZMN, right-angled at N, in which there are given the side MN, as being equal to the terrestrial semidiameter AC, and the angle MZN, by proposition 29 of the first book of Euclid — for it is the exterior angle to the two parallels AL and ZH, equal to the interior LAD, which is known from the altitude of the equator, or the complement of the altitude of the pole. Find, then, the base ZM. Next use the triangle AHF, right-angled at F (since all the parallels of the equator cut the axis of the equator at right angles); for in it are given the base AH, namely the distance of the phenomenon from the Earth's centre, and the angle AHF, by proposition 29 of the first book, for it is equal to the alternate angle LAH, which is known from the declination HL of the given parallel. Hence you will be able to find the sides FH and AF. Afterwards, in the triangle AFZ, right-angled at F, since there are given the side AF, just found, and the angle AZF, equal to the angle HZD already known above, find the side FZ; add it to

the base ZM already found, and subtract the sum from the side FH found; for MH will be left. If you reduce it by the rule of proportion to parts of which the straight line FH is the radius 100,000, you will have the versed sine of the arc to be doubled, so as to have the arc of the whole stay of the phenomenon above the physical horizon. In the same way you will find ZH, that is, the versed sine of the arc to be doubled so that the arc of the stay above the astronomical horizon may be known; and these stays will be known when these arcs are converted into times of the equator, etc.

[Margin: Case 3, of a phenomenon placed on a northern parallel.] The third case is if the phenomenon is on this side of the equator, for example on the parallel aQ. In that case, in the triangle ERS, right-angled at S, find the base ER; for in it are given RS, equal to the terrestrial semidiameter AC, and the angle RES, equal to the angle LAD, exterior to the two parallels EQ and AL, that is, to the altitude of the equator. Next, in the triangle EAQ, find the base EQ and the side AE; for in it are given AQ, the distance of the phenomenon; the angle AQE, for it is equal to the co-alternate angle QAL, known from the declination QL of the given parallel; and the angle EAQ, composed of the right angle EAO and the angle OAQ, known from the distance of the parallel AQ from the zenith. Afterwards, in the triangle AaE, right-angled at a, there are given AE, just found, and the angle AEa, which equals the altitude of the equator: find, then, the side Ea. Subtract it from the base ER found above, and you will have aR. Subtract ER, moreover, from the whole EQ, and you will have RQ; so, by adding aR to this, you will have aQ. Now reduce these lines EQ, or also Ea, and RQ to parts of which aQ is the radius 100,000. Then RQ will be the versed sine of half the stay of the phenomenon above the physical horizon, and EQ the versed sine of half the stay above the astronomical horizon; or Ea will be the sine of the arc to be doubled and added to a semicircle, so that the stay above the astronomical horizon is made up.

[Margin: Case 4, of a phenomenon passing through the zenith.] But if the phenomenon passes through the zenith O: first, in the triangle AOX, right-angled at A, there are given AO, the distance of the phenomenon, and the angle O, which is equal to the alternate angle OAL, that is, to the distance of the equator from the zenith, which is as great as the altitude of the pole. Find, then, the base OX and the side AX. Next, in the triangle COε, right-angled at C, find the base εO; for the angle O is given, and the side CO, if you subtract AC, the Earth's semidiameter, from AO. Afterwards, in the triangle AβX, right-angled at β, in which the side or base AX has already been found, and the angle XAβ, that is, the altitude of the pole, is given, find the side Xβ. Then, when εO is subtracted from XO, Xε becomes known; and when this is taken from Xβ, εβ remains known; while Xβ, taken from the whole XO, will give βO. Reduce, then, both εβ and Xβ to parts of which βO is the whole sine 100,000, and you will have the sine εβ, whose arc, doubled and added to 180 degrees, will make the arc of the stay above the physical horizon; while the sine Xβ will be the

sine of the arc to be doubled, so that from it and 180 degrees the arc of the stay above the astronomical horizon is made.

From these you will work out any other cases without difficulty; but for the stay above the astronomical horizon you will be able, from the given declination of the parallel, to use the customary tables of diurnal arcs.

Problema 8. Data mora Phænomeni ſupra Horizontem vtrumq. loci, cuius data ſit altitudo poli; & data Paralleli, in quo Phænomenon verſatur, declinatione vera, vnà cum ſemidiametro terræ, Inueſtigare diſtantiam Phænomeni à centro terræ.

Problem 8. Given the stay of a phenomenon above both horizons of a place whose altitude of the pole is given, and given the true declination of the parallel on which the phenomenon moves, together with the Earth's semidiameter, to find the distance of the phenomenon from the Earth's centre.

XIII. You see that this problem is the converse of the preceding one, so it will be enough to indicate the method in a single case. Let us suppose, then, the phenomenon at H, turning on the southern parallel FH. First, then, convert the half-stay above the physical horizon into degrees of the equator, and look up their versed sine in the tables of sines: for you will have the side MH in parts of which the whole sine FH is 100,000 parts. In the same way, from the half-stay above the astronomical horizon converted into parts of the equator, and from their versed sine, you will have the side ZH, likewise in parts of which FH is 100,000; and by subtracting MH from this, the base ZM of the right triangle ZMN will remain known. In this triangle the angle MZN, that is HZD, is given, equal to the angle LAD, that is, to the altitude of the equator; therefore the side MN will not remain hidden, in parts of which FH is 100,000. Therefore, since MN is given in miles — inasmuch as it is equal to the Earth's semidiameter AC — the whole of FH will become known in semidiameters of the Earth by the golden rule [auream regulam]. Finally, therefore, in the triangle AHF, right-angled at F, with FH now given in semidiameters of the Earth, and with the angle AHF, which equals its alternate angle LAH, that is, the true declination of the parallel from the equator, you will find AH, the distance of the phenomenon from the Earth's centre. And by converting in the same way you will work out the remaining cases proposed above.

Corollarium duplex ex Problemate 7. & 8.

A twofold corollary from Problems 7 and 8.

XIV. [Margin: Corollary 1.] Infer from what has been said that, as long as we are uncertain whether a phenomenon undergoes parallax at its meridian altitude, and how much, we cannot use Problem 7 or 8, because these require the true parallel, or the true declination of the phenomenon. If it has a sensible parallax, Problem 7 will indeed yield the true stay above the horizon, but not the apparent one; and Problem 8 will not yield the correct distance from the apparent stay unless the apparent parallel is first converted into the true one, or the apparent declination of the phenomenon into the true one. For this conversion

the parallax must be known beforehand, since it always visibly depresses the phenomenon from the true parallel to the apparent one. Consequently, from a comet's stay above the horizon, if the parallax is not known, its distance from the Earth's centre cannot be determined with certainty. Scipione Chiaramonti noted this shrewdly in Book 3 of *De tribus novis stellis*, chapters 31 and 32, and in the supplement to the *Antitycho*, page 106, against Antonio Santucci and others, who think that from the mere stay of a comet or a new star above the horizon its distance from the Earth can be inferred, and from that the parallax, which was not known before.

[Margin: Corollary 2.] Infer, secondly, that since Problems 7 and 8 deal with the mere common diurnal motion of the primum mobile, if a proper motion of the phenomenon towards the east intervenes, the motion of the primum mobile must be increased, and if towards the west, diminished — as is done with the planets, whose diurnal arc is inferred from their proper motion.

Problema 9. Quanta ſit altitudo ſufficiens Cometæ à Sole lucenti, vt conſpici poſſit tota nocte, nec in vmbram terræ incidat: Datis Cometæ Declinatione, & loco Solis in Ecliptica, & portione terræ à Sole illuminata, ac Semidiametro terræ.

Problem 9. How great an altitude suffices for a comet lit by the Sun to be visible all night and not to fall into the Earth's shadow, given the comet's declination, the Sun's place on the ecliptic, the portion of the Earth illuminated by the Sun, and the Earth's semidiameter.

XV. From the Earth's centre A let the Earth's globe BDC be described, and another circle under the meridian, VGSL. Let an inhabitant be at B, from whose astronomical horizon GAL let the perpendicular line AV be erected towards the zenith. Let the Sun be imagined at midnight at S, whose extreme rays HF and KF, touching the Earth at C and D, form DFC, the cone of the Earth's shadow; and let the rays be supposed, for the present, unrefracted. Let AF be the axis of the shadow. Then let the Earth's semidiameter AC be drawn to the point of contact C, so that the angle ACF is right [Margin: By proposition 18 of the third book of Euclid.], and let the angle FAC be known from the arc CBO; for it is half the arc COD, which is not illuminated by the Sun.

Diagram. Set in the left column beside the construction of No. XV. A large circle, the meridian circle VGSL, with the horizontal diameter G–A–L (the astronomical horizon) through the centre A; about A a small circle, the Earth, with B above A and D and C on either side; above the large circle at upper left the point F, the apex of the shadow, from which two long lines run down to H and K, touching the small circle; the vertical AV from the centre to the zenith; a line from A through R to M; points I and E near the top of the circle; a small tailed comet drawn at T, with a dotted line running from it down to S on the lower part of the circle; an arc in the lower half of the circle passing through H and K. Star marks stand at E, P and R.

- F (above the circle, upper left): apex of the shadow-cone; the axis of the shadow is AF

- I, E (top of the circle, left of V): IV is the distance of the shadow's axis from the zenith; E is the comet at the border of the shadow in Case 3
- V (top): the zenith
- M (upper right, on the circle): end of the line ARM
- P (on the vertical below V): the comet at the border of the shadow in Case 2
- O, B (above A): B the inhabitant
- R (right of B): the comet at the border of the shadow in Case 1
- T (right, by the drawn comet): not named in the text of this page
- C, D (on the small circle): the points of contact of the extreme rays
- G, L (ends of the horizontal diameter): the astronomical horizon GAL
- A (centre): the Earth's centre
- H (bottom), K (lower right): ends of the extreme rays HF and KF
- S (lower part of the circle): the Sun at midnight

With these premises, let the comet be on the meridian, where there are two dangerous cases in which the comet's altitude may not be sufficient for it to avoid the Earth's shadow. The first is if the comet is diametrically opposite the Sun without any latitude; in that case it would have to be higher than the apex of the Earth's shadow. The other is if it declines, but only a little, from the axis of the shadow. [Margin: Case 1.] Let it be placed, then, at the border of the shadow at R — that is, if by reason of its declination it lies beyond the vertical line, towards the side opposite the shadow. For when the straight line ARM is drawn through it, the triangle ACR results, right-angled at C, in which the side AC, the Earth's semidiameter, is given, and the angle RAC. For from the place of the Sun, opposite and culminating, the declination of that place is known from the tables of declinations of the ecliptic; and accordingly, from the distance of the equator from the zenith, or the altitude of the pole, the distance of the shadow's axis from the zenith on the meridian will be given, that is, the arc IV, which the angle IAV, that is, OAB, measures. When the angle BAR, known from the comet's distance from the zenith on the meridian, is added to it, the angle OAR becomes known; this, subtracted from the arc OBC, which is known as we said above, leaves the angle RAC. With this angle and the side AC you will find the base AR, that is, the altitude of the comet sufficient for it to escape the Earth's shadow. For if there is a danger of its falling into the shadow at such a declination, the danger is greatest on the meridian, when the axis of the shadow also passes through the meridian, since the Earth's shadow is then raised highest. Yet it will not fall into it, because in reality the Sun's rays are refracted and the Earth's shadow is everywhere more contracted, as we showed in Book 5, chapter 5.

[Margin: Case 2.] But if the comet should appear on the vertical line, let it be at P, at the border; then use the triangle PAC, right-angled at C, in which AC is given, and the angle PAC, the remainder after the arc OB — known, as above, from the distance of the shadow's axis from the zenith — has been

subtracted from the arc OBC; and find the base AP. For that will be the comet's most amply sufficient altitude for avoiding the Earth's shadow all night.

[Margin: Case 3.] Finally, if the comet is on the same side of the zenith as that towards which the Earth's shadow inclines, let it be placed at E, the border of the shadow. When its distance from the zenith is subtracted from the distance of the shadow's axis from the zenith (for both will be known from the declination and the distance of the equator from the zenith), the angle IAE will be known; subtracted from the angle OAC, it leaves the angle EAC in the triangle EAC, right-angled at C, in which AC is given. The base AE, therefore, will not be unknown, that is, the comet's distance from the Earth's centre that is most amply sufficient for a comet holding the meridian to avoid the Earth's shadow even at the very middle of the night. For if it is placed off the meridian, or on the meridian indeed but with the shadow already inclined away from the meridian, its danger of eclipse will be much smaller.

E X E M P L V M.

Example.

XVI. Peter Apian, in Part 2 of the *Astronomicum Caesareum*, says that the comet he saw on 1533 June 25, declining from the pole of the world by 33° 20′, was visible around midnight and continuously until sunrise. He does not say, however, that it was on the meridian at midnight; on the contrary, he says it had climbed to the meridian when the Sun was about to rise. He puts the Sun at 11° 48′ of Leo, and the comet at 15° of Taurus with a northern latitude of 43°. He then says that if it had been near the Earth, that is, in the highest region of the air, as the older Peripatetics hold, it would not have appeared on those nights but would have lain hidden in the Earth's shadow; whereas in fact it was visible, even if it was wholly lit by the Sun, as he had shown from the projection of its tail towards the part turned away from the Sun.

But let us suppose, to make the danger greater, that it was on the meridian at midnight; that in longitude it was at 11° 48′ of Aquarius, where the axis of the shadow was; and that it declined from the pole by 33° 20′. The altitude of the pole at Ingolstadt, where Apian was then observing, is 48° 40′, and the distance of the pole from the zenith 41° 20′. Subtract 33° 20′ from this, and the arc VM remains, 8° — the comet, that is, being placed at R. The declination of 11° 48′ of Aquarius, under which the axis of the Earth's shadow is placed, is 17° 17′ to the south; adding to it the distance of the equator from the zenith, which at Ingolstadt is 48° 40′, gives the distance of the axis from the zenith, 65° 57′, namely the arc IV; joined to the arc VM of 8°, this gives the whole IVM, 73° 57′. Now, from what we said in Book 3, chapter 11, the arc OC is 89° 30′ 20″; therefore, when 73° 57′ is subtracted, the angle RAC remains, 15° 33′ 20″. And AC is, for us, 4139 Bologna miles. Therefore in the triangle RAC, right-angled at C, the base AR is found to be

4297 miles. Subtract 4139, and there remains the comet's distance from the Earth's surface, 158 miles — most amply sufficient for it to have avoided the

Earth's shadow in the greatest danger. How much more sufficient, then, was it, given that the shadow is narrower because the Sun's rays are refracted, and that the shadow declined from the meridian? For the comet appeared between the north and the east [inter Boream & Solanum] from midnight until sunrise, during which time the Earth's shadow was inclining more and more towards the west, to the quarter lying between the south and the west [inter Auſtrum & Fauonium].

CAPVT XVI.

Chapter XVI.

An & quantam vim habeat Argumentum à Parallaxi deſumptum ad determinandam Cometarum & cuiuſuis Phænomeni Diſtantiam à terra: & de vſu vel abuſu illius in communi Diuerſæ Opiniones præſertim Tychonis & Claramontij ſectatorum: Qua occaſione de vtriuſque Auctoritate in hoc genere.

Whether, and how much, force the argument drawn from parallax has for determining the distance of comets, and of any phenomenon, from the Earth; and the differing opinions on its use or abuse in general, especially those of the followers of Tycho and of Chiaramonti; and, on this occasion, on the authority of each in this field.

I. The chief argument for showing the distance of comets from the Earth is the one taken from the variation of view [diuerſitate aſpectûs], which the Greeks call parallax [Parallaxin]. It arises from the fact that we observe comets not from the Earth's centre but from its surface. For this reason their true place, marked by the line of the true place — that is, the line drawn from the Earth's centre through the comet's centre to the sphere of the fixed stars — differs from the apparent place, marked by the line drawn from the eye, stationed on the Earth's surface, through the comet's centre to the sphere of the fixed stars. This is called the line of the apparent place, and it never coincides with the line of the true place except when the comet is at the zenith, that is, at the vertex itself. Hence some of the older physico-mathematicians, and some of the more recent, constructed an argument of this sort: *Every phenomenon that, at the same apparent altitude above the horizon, has a greater parallax than the Moon is below the Moon; but comets are of this sort; therefore comets are below the Moon.* [Margin: Syllogism and counter-syllogism from parallax.] Not a few of the more recent, however, especially the Tychonics, argue in an entirely contrary way, thus: *Every phenomenon that, at the same apparent altitude above the horizon, has a smaller parallax than the Moon is above the Moon; but comets are phenomena that have a smaller parallax than the Moon; therefore comets are above the Moon.*

II. Faced with an argument of this kind, some have either dared to deny the major premise, or have admitted it in a wholly alien sense, because they did not understand the term parallax. [Margin: The term "parallax" unknown to some philosophers or astrologers.] Among them, a certain public professor of philosophy at Padua approached Galileo and asked him to explain what was meant by the term parallax; for he had already resolved in his mind to attack it,

having heard that some were using it against Aristotle's opinion on the place of comets. Galileo, smiling gently, said to him: "So, my lord, have you set your mind on disputing against a term whose meaning you do not know?" Another man in the same place, famous for his medical treatises, as Kepler relates in his book *De stella nova*, chapter 15, *dared to assert that astronomers are deceived in the doctrine of "paralapses"* [Paralapſeon] — meaning, of course, to say "parallaxes" [Parallaxeon]. Kepler mocked him splendidly in that place, and in the *Hyperaspistes*, page 2, says that he was Antonius Fabricius of Montepulciano. Petrus Surdus, too, in his book on comets, page 11, took the variation of view of the Moon for its different phases, in which it appears to us now full of light, now bent into a sickle, now swelling with gibbosity. Among the mockers and despisers of parallax, Longomontanus in the Appendix to the *Astronomia*, chapter 3, Glorioso in Book 3 of *De Cometis*, chapter 6, number 4, and Froidmont in Book 3 of the *Meteorologica*, chapter 1, article 2, list Bodin [Jean Bodin], John Craig the Scot, and finally Julius Caesar Scaliger.

For Bodin, in Book 2 of the *Theatrum*, calls the doctrine of parallaxes fallacious, and most ignorantly cites Ptolemy on his side in this — Ptolemy, who nevertheless gave a very great deal of weight to parallax in Book 5 of the *Almagest*. [Margin: Scaliger's ignorance about parallaxes is rebuked.] And Scaliger certainly seems not to have understood what a parallax sensible in itself is. For he says, in Exercise 79 against Cardano, that comets have a smaller parallax than the Moon because they follow a star as their author and are moved at an equal pace with its motion. Yet although this can happen with respect to some one observer, for others at widely separated places on the Earth's surface, if the comet is below the Moon, it will appear at the same moment to be far distant from that star — if indeed the comet is something real, and not a mere apparition [emphaſis] like a halo; and so it will have a parallax sensible in itself. Accordingly, Scaliger was given a thrashing by Christoph Rothmann in his little work on the comet of the year 1585, who says: *"But truly, my Scaliger, this subtlety of yours deserves the whip. You insultingly call the most excellent Homer, approved by all, a Greekling; what name, then, shall I call you here? But since one must not dispute with a man ignorant of the principles, I shall first hand you over to the mathematicians to be chastised and instructed, so that you may learn from them what parallax is, and where it comes from. Once you have learned these things, my contest with you will be easy; indeed you yourself will acknowledge your error, to the point of shame, and you will not deny that comets are lodged above the Moon in the spheres of the planets."*

[Margin: What those unskilled in mathematics should do in this controversy.] To these, then, and to men of this kind, our advice is this: either let them become versed in the doctrine of parallaxes — whether from elsewhere or from our Book 1, chapter 31, but more fully from Book 10, the penultimate section, which is on parallaxes — and only then approach this judgement; or, if they are unwilling or unable to learn this doctrine thoroughly, let them refrain from pronouncing an absolute verdict on matters they do not understand, so

as not to make themselves a laughing-stock to those who do. At most let them dispute on the nature and origin of comets on the hypothesis that the mathematicians have demonstrated that all of them, or some, are above, or below, the Moon; but let them leave the absolute judgement to the astronomers, and defer it to them as experts in their own art — just as Aristotle judged that, in matters pertaining to astronomy, one should defer to Eudoxus and Callippus, and appeal to them in doubtful cases.

III. [Margin: Authors doubting the use of parallax, or contending about its abuse.] There are others, however, who, understanding more or less what parallax is, grant indeed that it holds good for the fixed stars and for the planets, which are by now commonly known, but say that for comets it does not hold good, incidentally, or that only uncertain arguments are drawn from it. They say this either because the observations suited to determining a certain parallax are required to be so subtle and exact that they can easily escape anyone's diligence, as our Tannerus says, question 7 *de caelo*, page 37; or because refraction or the irregularity of the comet's motion renders this treatment [pragmatiam] uncertain, as Cottunius, together with Aversa, affirms in Book 1 of the *Meteorologica*, lectures 32 and 33, and Mastrius in disputation 4 *de caelo*, question 2, article 4, number 47. They confirm this from the disagreements of the astronomers: for some, while using parallax, have nevertheless placed comets far below the Moon, such as Regiomontanus, Vögelin, Nolthius, Sultetus [?], Vuinchlerus and Chiaramonti; others above the Moon, such as Apian, Gemma, Tycho, Longomontanus, Kepler, Rothmann, Cysat, Snel, Santucci and many others. Accordingly, they either deny the minor premise of the syllogism proposed in No. 1, or they evade it and twist it towards the uncertainty of parallax. Among these, however, Scipione Chiaramonti alone, so far as I know, appears to have undertaken this business by geometrical inquiry. [Margin: Chiaramonti's opinion on the place of comets from parallax.] In his *Antitycho*, after a whole first book on the kinds of parallax, he brings forward in Book 2 the reasons adduced by Tycho and others for the celestial place of comets, and contends that they are either false, or neither necessary nor evident; and in Book 3 he contends that from the observations of Tycho himself, and of others reported by Tycho himself, if they are accepted, it is demonstrated that those comets were sublunary. And yet in his *Apology* for his *Antitycho*, page 9, he later said: *If the observations of Tycho and of the Landgrave of Hesse, approved by Tycho, are accepted, the comets of the years '77 and '80 are concluded demonstratively to be celestial; likewise, if the observations of others too are accepted, as corrected by Tycho as well, it is demonstrated from their comparison with Tycho's that the comet of the year '77 was sublunary.* Indeed, a little before he had said: *If the observations of Tycho are granted, and the others of those who at the same time as Tycho observed, and of these the ones that Tycho approved and also corrected — then the comets of the years '77 and '80 are demonstrated most evidently and necessarily to be sublunary.* Yet he asserts the very same of the comets of the years 1475, 1532, 1556, 1582, 1590, 1593, 1597, 1607 and 1618: partly in Book 3 of the *Antitycho*, chapters 17 and 21;

partly in the *Apology* for the *Antitycho*, Book 2, from chapter 7 to 12; and partly in two little works entitled *De sede sublunari Cometarum*. Chiaramonti shows himself, moreover, highly skilled in geometry and in astronomical matters, to say nothing of Peripatetic philosophy; and he has candidly set out his theorems and problems, together with calculations arranged in order, for anyone to examine.

IV. [Margin: Opinions about Chiaramonti and the Tychonics.] Hence, again, disagreements have arisen among the Peripatetics of our time, and among the Tychonics and other mathematicians, and differing judgements on Chiaramonti's doctrine in this field, set against the assertions of Tycho, Kepler and other Tychonics, as to which of the two is rather to be believed. Kepler himself, in his *Hyperaspistes* of Tycho, tried to defend Tycho — and indeed himself too, since he knew that he could not stand if Tycho were overthrown. For so he himself speaks in the preface to the *Hyperaspistes*: *If I were to take up the defence of the part of Tycho Brahe, once my master, even in a lost cause, I believe that not only pardon for the defence but praise besides for the dutiful service would not have been wanting. Now, after I have for thirty whole years continued, secure and undisturbed, in the possession of this knowledge — that comets are seen above the Moon — and have already resolved to enter this possession, as part of my estate, to posterity, if posterity will have any care for my works:* [Margin: Praises of Chiaramonti from Kepler himself.] *behold, out of the blue there arises a learned man, skilled in geometry, accomplished in the doctrine of triangles and in the doctrine of parallaxes, which contains the foundations of my claim — not one who, like Antonius Fabricius of Montepulciano once, hallucinated childishly (an author I dealt with as his merits deserved eighteen years ago, in my book De Stella Nova Serpentarii, folios 79 and 80), but one who treats that whole part of sidereal doctrine most carefully in the entire first book of his work. This man, I say, armed with so many prejudgements and strong in so many clients, brings a suit against me, intending to eject me from this possession of mine. Truly, I think it concerns both the profit of all who have read me and my own private reputation that I be watchful, and that I enquire most diligently whether, now that my age is declining, I must unlearn anything of what I learned as a youth and taught as a man, and unteach it to others. This most illustrious man, who summons us to the pursuit of truth, must be humoured; and I must take care not so much lest, as author, I wrong my master Tycho by heaping false charges on him, as lest I wrong myself by cherishing errors once they are exposed.*

A little later, it is true, he adds: *For I have never thought that teachers should be praised in such a way that error is entrenched, nor that they should be spared in such a way that the truth lies hidden.* Yet after saying this, Kepler later, his bile rekindled, tried to dishonour Chiaramonti with many insults — whether deservedly or undeservedly, let others judge. [Margin: Kepler's inconsistency or forgetfulness.] And finally, on page 183 of the *Hyperaspistes*, as if he had forgotten what he said in the preface, he says of Chiaramonti's book: *The book is indeed of such futility that I would have scorned it; and so I declare even now that, unless Chiaramonti comes forward hereafter better instructed in our art — whether he*

chooses to harass Tycho or myself and the works I have published in the mathematical part of philosophy — I shall have no reason to undertake a defence and write a book publicly. Unless you say that Kepler praised Chiaramonti's first book and left the rest unpraised.

[Margin: Froidmont subscribes with Kepler against Chiaramonti.] On Kepler's side against Chiaramonti, however, stood Froidmont, in Book 3 of the *Meteorologica*, chapter 1, article 2, already prepossessed by the authority of Tycho; for he says: *But Scipione Chiaramonti makes greater trouble. In two books of his Antitycho he draws back into the air the comet of the year 1577, which Tycho had established in the ethereal region above the Moon by so many reasons. For he tries to show that it had a parallax larger than the Moon's, and that Tycho now set down a calculation badly, now did not sufficiently consider the refraction that was disturbing the parallax, and other things of that kind — which not a few would perhaps have believed, had not Kepler's Hyperaspistes promptly withdrawn credence from Chiaramonti. To try to refute his innumerable numbers and computations would be an endless task, and not in keeping with the scope of this book. Kepler has done it better than I could, and splendidly; consult him, if you like, in the Hyperaspistes. I add only this: unless we trust Tycho's instruments and that most skilfully made sextant of his, the credibility of all will have to be thrown away. For since the times of Ptolemy there has been no one — or perhaps never anyone — who has attended with greater care to this matter of instruments.* And after a few words on Tycho's instruments and his primacy in astronomy, he adds: *Nor was Tycho such an infant as not to know that refractions raise the stars which parallax depresses, or as not to attend to them in his observations where they might perhaps disturb the parallax somewhat.*

But you do not distinguish the times, my Froidmont. For there was a time when Tycho either did not know of refractions or did not use them — namely, when he wrote the volume on the comet of the year 1577, as I shall show below — granted that afterwards, grown more learned than his former self, he treated them splendidly. It is also false that Kepler examined Chiaramonti's calculations one by one, since he left almost all of them untouched. Further, although Tycho's observations of the comet of the year 1577 had been made with most exquisite instruments — some of them, however, were made by Tycho with the astronomical radius alone and with the azimuthal quadrant, instruments afterwards disapproved by Tycho himself and brought under suspicion of being deceptive — it still does not follow that the conclusions were evidently deduced from those observations. Nor should the mere mass of the Tychonics' instruments be thrust upon Chiaramonti, who denies this and resists with engines of argument to the contrary; for that is to resort to the machine rather than to untie the knot. I would say this, since the occasion has offered, not in order to bring out my own opinion here for either side, but to prepare the reader's mind for myself, free from all prejudice, for a true judgement on this matter to be passed later.

[Margin: Franciscus Resta casts his vote with Kepler and Froidmont against Chiaramonti.] Likewise, as a back-bench senator [Pedarius Senator], Franciscus

Resta subscribes to Froidmont, for Kepler against Chiaramonti, in Book 1 of his *Meteorologia*, tractate 1, chapter 2, saying of Chiaramonti: *He heaps up a good many things at some length, and sets them out so ingeniously that he could draw many over to his opinion, had not Johannes Kepler set his Hyperaspistes against him, exactly refuting Chiaramonti's arguments; which need not be done again here. Let it suffice to note what Froidmont adds, who considers that Tycho was the more celebrated in astronomical matters in his own age.* And a little below, of Tycho: *It is altogether unworthy to suspect of him that he treated these matters so childishly as not to notice at all the elevation that could arise from the refraction of vaporous air. And if we give room to these doubts of Chiaramonti, the things deduced from similar observations concerning the order of the planets, and concerning other astronomical observations, could at once be called into doubt.*

By no means, good Resta. For at another time, with better instruments, more experienced through practice and by then aware of refractions, Tycho left us many things about the planets that are more certain than those about the comet of the year 1577 — although even in the very things he discussed about the Sun and the Moon, because of an excessive solar parallax, assumed rather than observed, one may depart from Tycho without injury to Tycho, as is plain from what we demonstrated in Book 3, chapter 7, and Book 4, chapter 13.

[Margin: Father Cabeo defers in part to Chiaramonti, but more to Tycho.] Father Cabeo also favoured Tycho in this, on Book 1 of the *Meteorologica*, text 37, question 1, where he briefly weighs the tenets of Tycho and of Chiaramonti, though rather moderately. For he says, among other things: *I do not claim so much for myself that I can sit as judge in this suit*; and again: *I do not intend, as I also said above, to set up an examination of these difficulties, nor to call them to judgement; I do not wish to weigh the calculations or the demonstrations of either side.* Yet neither does he think Tycho's lie should be defended — about a certain observation fabricated on days on which, as he had confessed elsewhere, no observations could be made because of cloudy weather. After many solutions of Chiaramonti's reasons, however (always leaving the calculations untouched), he concludes that if one must deal with mere philosophers, or if judgement must be made from presumptions, the suit is rather to be awarded to Tycho, because of his experience of many years in observing and his diligence and good faith, long proven, in determining the motions of the planets exactly; nor should the omission of refraction be charged against one who even set up for us special tables of the refractions of the Sun, the Moon and the fixed stars. On this, I know well, Chiaramonti complained rather bitterly in a little book recently published: that in a mathematical controversy, to be argued out by geometrical reasons, his reputation had been diminished by a presumption in favour of Tycho's authority, and that he recognised a sentence pronounced against him, though under the appearance of suspending judgement.

[Margin: Cysat a supporter of Tycho.] Before all these, our Cysat too, in *De Cometis*, page 33, said that Tycho's observations ought not to be suspected by

even a single minute — which should have been understood of Tycho when he was already better instructed, but not universally.

V. There were nevertheless some who would rather give their vote to Chiaramonti — not only among those Peripatetics who made no great profession of the mathematical sciences, but also among those who seemed more than lightly versed in them. Thus Giovanni Camillo Glorioso, in Book 3 of *De Cometis*, chapter 5, number 21, and chapter 7, calls Chiaramonti's arguments against Tycho "sufficiently evident, and clearly demonstrated." Nor does he contradict himself in chapter 8, when he says: "If we begin to cavil at other people's observations on the strength of conjectures, the business of astronomy will collapse; one may depart from other people's observations only on the strength of evidence" — although in Book 5, without any mention of Chiaramonti, he sided with Kepler and Tycho in placing many comets above the Moon. [Margin: Glorioso's vote for Chiaramonti.]

Galileo likewise, in his *Trutinator*, page 24, has these words, rendered into Latin: "I say again that it is utterly false that I criticized the followers of Tycho, though I could rightly have done so, as his followers will in the end be able to see from the *Antitycho* of the knight Chiaramonti." Granted, once he learned that Chiaramonti rejected the Copernican hypothesis, which Galileo defends tooth and nail in his dialogues on the system of the world, he now thinks Chiaramonti's prolixity against Tycho's loquacity (as he himself puts it) superfluous, and now, in the person of Salviati, says: "The inconstancy of the observations on which the computation of parallaxes rests makes both these opinions and those equally suspect to me, especially since the *Antitycho*, as it seems to me, sometimes either accommodates observations to its own arguments, or accuses of falsity those that conflict with its purpose." [Margin: And Galileo's, formerly.]

But Cottunius, a Peripatetic, awards the palm outright to Chiaramonti, a Peripatetic, on Book 1 of the *Meteorologica*, lecture 32, where he says: "Scipione Chiaramonti, a man of solid learning, contended outstandingly on this subject in a twofold work: to one part he gave the name *Antitycho*; the other is *De tribus novis stellis*, on the three new stars that appeared in the years 1572, 1600 and 1604. In both he demonstrates that all comets, and also the said new stars, existed below the Moon, in the elementary region." And somewhat later in the same lecture: "Knowledge of this distance, which is derived from parallax, is so difficult that it does not afford a certainty in which the intellect can wholly rest. This is not because parallax, truly taken, cannot show the true distance of some phenomenon — to deny that would be foolish — but because, as Giovanni Camillo Glorioso candidly reports in Book 4, chapter 8, in all observations there is always some scruple and ambiguity; Chiaramonti says much on this matter." [Margin: Cottunius's testimony for Chiaramonti.]

Later, Bartolomeo Mastri and Bonaventura Bellutus [Bonaventura Belluto], in their disputations on the heavens and on meteors, disputation 4, question 2, article 4, number 47, conclude after much discussion: "From this it is plain that

many of Tycho's arguments, which he draws from parallax, do not conclude much; but whoever wants an exact solution and a subtle examination should consult Chiaramonti, Book 2, chapter 15." [Margin: Mastri's and Bellutus's vote for Chiaramonti.]

VI. The testimonies, then, sway this way and that — both for the use of parallax and against its abuse, on one side for the Tychonics and on the other for Chiaramonti and his followers. I myself have more than once been asked by friends what I thought of each side, and never had the leisure or sufficient opportunity — which I now enjoy by the kindness of GOD and of my superiors — to look thoroughly into a case of this kind. It is at last time, therefore, that with all the good faith, diligence and depth of inquiry I can command, I should separate the certain from the uncertain in so tangled a controversy, and weigh separately the arguments of each party drawn from parallax; for the remaining arguments have already been treated in chapters 10 and 11. So that I may do this with greater profit and happier success, I have resolved to ask four Postulates of the reader.

VII. *First*, I postulate that for the present we be allowed — not indeed to despise probability and mere likelihood, for later we shall draw our own assertions from it, but to pass it by as though with a deaf ear and leave it aside; and to direct the eye of the mind to the single goal of certainty and evidence — not, indeed, purely metaphysical certainty, but Phyco-Mathematical [?]. For this is what is required first and foremost, and Chiaramonti has very often professed expressly that this is what he is contending for. In this he cannot fail to be commended by mathematicians and astronomers, provided only that he is content with the kind of certainty on which most of the astronomers' doctrines rest; and to be commended more than Tycho and Kepler, who more than once confuse likely arguments with evident and necessary ones, as we showed of Tycho in chapter 11, number 6, and as will appear of Kepler below. [Margin: Postulate 1: probability is to be disregarded for the present.]

VIII. *Second*, I postulate that it not be put down to cavilling or sophistical inquiry — as though we were looking for knots in a bulrush — if in determining the parallax we require the subtlety that the evidence already mentioned demands. For although in the parallax itself, once it is established that it is many minutes greater or less than the lunar parallax, a difference of one minute or another harms neither side, yet in the very elements or data from which it is derived, the neglected error of a single minute can not infrequently cause us to conclude that the parallax is greater than the Moon's when in fact it is less, or less when it is greater. Thus we in the penultimate section, Book 10, chapter 12, numbers 11 and 12 — and Chiaramonti before us — have demonstrated that it can happen that the difference between the parallaxes of the same phenomenon, observed from different places on the Earth by different observers, is of only one minute, and yet a parallax of the phenomenon much greater than the Moon's follows from it. [Margin: Postulate 2: on subtlety in the inquiry into parallaxes.]

Let the Tychonics indeed remember how great a subtlety Tycho required in investigating the place of the New Star of the year 1572. For in volume 1, near the beginning of chapter 6, page 396, he said: "What we are now undertaking, however — namely, to know with certainty and to demonstrate evidently the star's remoteness from the Earth, and in which part of the world it shone — is a matter of far greater industry, labour and SUBTLETY, more arduous, and exposed to more windings of difficulty." How much more is this so in comets, whose place and motion are wandering and inconstant? And in the same place, page 413: "To measure the remoteness of a phenomenon from the Earth, and to demonstrate it, requires great subtlety." In volume 2 as well, page 145, censuring Regiomontanus's method of finding parallaxes as insufficiently scrupulous, he says: "For he builds the greatest things out of the least, so that an error of one minute or another in the time grows into an excessive deviation. Moreover, unless the azimuths and altitudes are obtained so scrupulously that nothing is lacking, you will have wasted your oil and your labour; for a slip of a few scarcely perceptible minutes in the time and the other data can easily produce a departure of several degrees from the mark of truth." He repeats things quite like these on page 210.

This was surely the reason why he thought little weight should be given to observations of azimuths and altitudes taken together with the same quadrant: a slight error in the time and in the azimuth, owing to the sudden great variation in altitude, passes on a great error to the conclusions derived from them. For in the *Epistolae Astronomicae*, page 295, against someone who preferred that a comet's parallaxes be deduced from altitudes and azimuths rather than from distances from the fixed stars, he says: "The method by distances, applied in due fashion, is more certain than the one taken from altitudes and azimuths, in which a scarcely perceptible error becomes extremely fertile." And in volume 1 of the *Progymnasmata*, speaking of the Landgrave's observations, page 604, he adds: "In the very times of the azimuths and altitudes taken (for this path of investigation was then proceeding too slipperily), a small discrepancy was able to vary the star's place perceptibly." The scrupulousness, then, that Tycho requires in others, it should be judged not unfair, but fair and equitable, to require equally of him.

IX. *Third*, we postulate that the authority Tycho deservedly gained from those observations which he carried out with larger instruments, and when he had himself become more learned, should bring no prejudice to this case, and should not be transferred to those observations of comets which he attempted with less suitable instruments, when he was younger and had not yet gained expertise in refraction and other matters. For the times must be distinguished here. In the years 1572 and 1577, when he observed the New Star and then the first comet, and wrote about it, he had only the astronomical radius, the first sextant (which served a single observer), and an azimuthal quadrant not quite two cubits in size, as he himself relates in volume 2, pages 1 and 31. And we saw just now, at the end of number 8, that azimuthal observations, especially

those made with so small an instrument, were later less approved by him. That the radius as well, and indeed that earlier sextant, were deceptive, he himself teaches in volume 1, pages 247, 335 and 343. Moreover, neither in 1572, nor in 1577, nor perhaps before the year 1582 or 1584, had he caught the scent of the power of refraction to raise the stars; for of those earlier years he himself speaks thus in volume 1 of the *Progymnasmata*, page 91: "Long before greater [Margin: Tycho's observations from different periods are not all equally accurate.]

and more exact instruments were at hand, I had a less well-ascertained understanding of refractions." And in volume 2, page 31, speaking of the year 1577: "For at that time I had not yet explored the impediments caused by this refraction." He repeats this in the addition on page 74. He came to suspect it, however, when he saw at the winter solstice that the Sun stood four minutes and more higher than the altitude of the pole of Uraniborg required — an altitude taken later, with larger quadrants, from the altitudes of a circumpolar star. Having had one larger instrument after another made, and always meeting with the same discrepancy, he at last began to suspect that it arose from refraction, as is found in volume 1 of the *Progymnasmata*, pages 15 and 91. This happened around the year 1584 — the year from which, according to volume 1, page 13, we have the first more exact observations of the equinoxes and solstices made by him — or at the most in the year 1582, in which, as he himself testifies in the same place, page 152, and in volume 2, page 32, he began to observe the fixed stars more accurately, and with larger instruments than before, by way of the distance of Venus between them and the Sun. Granted, not even these are entirely exact, because a parallax of Venus and the Sun larger than is right was used, as Kepler notes on page 15 of his *Ephemerides*. [Margin: Tycho: for how long was he ignorant of refractions?]

From this difference of periods and of instruments, and from his ignorance of refractions at first and his knowledge of them afterwards, it came about that in volume 1 of the *Progymnasmata*, part 1 — which is contained in chapters 1 and 2 and treats of the Sun, the Moon and the fixed stars from more exact observations — Tycho used a polar altitude for Uraniborg of 55° 54′ 40″ or 45″, an altitude of the equator of 34° 5′ 20″, and an obliquity of the ecliptic of 23° 31′ 30″, as is plain from volume 1, pages 17, 28, 86 and 218 taken together. But in volume 1, part 2, and in volume 2 he consistently used an altitude of the equator of 34° 7′ and an obliquity of the ecliptic of 23° 27′, as can be seen from volume 2, pages 31, 37, 38, 66, 109, 123, 148 and 152 taken together.

For he began to write volume 2, on comets, immediately after the appearance of the first comet in 1577, and he did begin to commit it to print, but did not publish it. For one thing, for twenty years and more thereafter he was wholly taken up with observations of the fixed stars and planets, the province of comets being set aside in the meantime — a task he intended either to entrust to Longomontanus or to complete himself later, as is related partly in the preface prefixed to volume 2 by his heirs, and partly at the end of the *Mechanica Astronomica*, page 44. And he dealt with this matter only occasionally and in

passing, either on the occasion of other comets or by a digression made to sheets already printed.

So in the year 1586 he noticed that the places of the fixed stars he had used in determining the place of the comet of 1577 differed by two, three or even six minutes from the places found with more accurate instruments; that refraction had been omitted; and that the altitude of the pole he had used was about three minutes smaller than was right, and the obliquity of the ecliptic by four. Yet since all the chapters of volume 2 had by then been printed — indeed the first 9 chapters had gone to press in the year 1578 — he was unwilling to correct them or to undergo the tedium of repeating so many calculations. Instead he was content for the present to warn the reader of this disagreement, by resetting folio 31 of volume 2 and the two following, and to indicate more correct longitudes of the fixed stars. Yet he applied no correction to the comet's distances themselves, none to the altitude of the pole, none to the obliquity of the ecliptic, nor did he take the benefit of using refraction, although he now recognized it — evidently from an assurance he had formed that no perceptible error could overflow from these into the parallaxes of this comet. And he scorned that scrupulousness which, as we showed in number 8, he required of others. You will learn of this assurance of Tycho's, reader, and of his acknowledgment of the rather crude observations of this comet, if you read through volume 2 from page 31 to page 34, and page 74. [Margin: Tycho acknowledges errors and does not correct them; and why?]

Whether, however, from all these small minutes running together into one, so great an error arises in Tycho's calculations as to make this comet sublunar, or uncertain of its seat, I would not yet affirm. The one thing I argue here is this: the claims of those (Froidmont, Resta and Cabeo, for instance) do not hold who apply Tycho's authority and his diligence in observing equally to all periods, or to the time of this comet; nor the claims of those who say that, because he was at some time acquainted with refractions, it is ridiculous or unworthy of him to have left refraction untouched in observing this comet. Granted, I would not compare Chiaramonti's observations with almost any of Tycho's, even those begun from the year 1572; for he himself says in the *Antitycho*, Book 3, chapter 20: "There remains the bearded comet [Pogonaris] of the year 1618, on which I published a little book poorly furnished with observations, but well enough supported by theorems and problems." And I have learned from elsewhere that he used to use few and small instruments.

X. For indeed Chiaramonti's praise is to be sought not from his observations but in geometry, and in calculations derived from trigonometry, in which he is almost always more exact than Tycho is in many calculations of volume 2. I said "almost always", for he himself, in the *Apology* for the *Antitycho*, Book 2, chapter 9, candidly admits that he erred in a calculation in Book 3 of the *Antitycho*, chapter 18, by using the comet's apparent distance as observed by the Landgrave in place of its apparent distance as observed by Tycho. And he greatly marvels at the sluggishness of Kepler, who, though his fiercest enemy,

did not throw this error at him, because he did not recognize it — Kepler, who would doubtless have trumpeted a triumph over the remaining calculations once this one error was detected, as though it were right to suspect crimes, or rather slips, in all the calculations from a crime in one. [Margin: Chiaramonti's praise comes not from his observations but from geometry, in which he was more skilled than Tycho once was.]

Grant that Chiaramonti erred once or twice; Tycho was no more fortunate in this, for he fell into a pseudographic and most fallacious semblance of a demonstration, one that conflicts with his own positions, as we shall show in Book 10, in the section on parallaxes, chapter 12, number 5. So much so that Kepler, in the *Hyperaspistes*, Book 2, chapter 13, after many evasions, conceded that Tycho's demonstration was pseudographic and erroneous, and said: "But by what Tycho was deceived, so as not to see that he was at this point acting wrongly, or to what errors he had wholly bound himself, from which he in turn is immune — on this, I think, something will become apparent. I will say nothing of the time at which this demonstration was conceived or worked out, nor of how we do not always progress with age, but sometimes even go backwards and forget these subtleties — as often happens to me too — distracted either by a multitude of studies or by domestic business, as Tycho indeed was." And in chapter 20 he says of the same thing: "I have conceded that that figure is pseudographic." [Margin: Tycho's pseudographic figure conceded by Kepler.]

But what displeases more in Tycho is that he invented, on doubtful faith, certain observations as made by himself. For in volume 2, in the history of the observations of the comet of 1577, November, page 3, after saying: "On the 15th, about 6^h, the comet was distant from the bright star of the Vulture 20° 25′, from the lower one in the horn of ♍ 16° 14′," he gives no observation for the following four days, and states the reason, saying: "On the following four days the air with us was filled with dense obscurities, which shut off the view of the comet, the 18th being not only dark but also very foggy, and the 19th raging fiercely with storms of wind and rain; so the observation of the comet was deferred until the 20th." After this he adds the observations of the 20th, 21st and 23rd, and once these are finished he immediately subjoins: "On the following day, the 24th, no clear sky shone on our horizon." [Margin: Tycho's good faith doubtful in places.]

Further on, however, wishing to compare some observation of his own with an observation of Tadeáš Hájek, so as to confirm his opinion on the place of comets from observations made in different places on the Earth, he forgets or conceals what he had related in the history already mentioned, and invents the same comet as observed by himself on the 16th and the 24th — that is, on the same days and at almost the same hour at which Hájek observed it at Prague. And on page 124 he says boldly: "Therefore we shall take the first observation of Tadeáš Hájek, made at Prague in Bohemia on November 16, from the bright star of the Vulture, where he measured the distance from the said fixed star with the radius as 17° 52′. We, however, here, with a similar instrument on the same

day, around six in the evening, found it 17° 50½′." Then on page 127: "Taking in the same way another distance, made by the same Tadeáš Hájek to the same fixed star on November 24, and comparing it with ours taken on the same day, we shall confirm the same thing. For on that day Hájek observed the comet's distance from the Vulture as 11° 43′, which we here found to be 11° 45′."

Is that so, Tycho? But how did you observe the comet on November 16 and 24, if from the 15th to the 20th its observation was deferred because of dense obscurities, fogs, storms and rains, and on the 24th no clear sky shone on your horizon? Or was it because, from the preceding and following observations and from an opinion already formed about the comet's motion, you had no doubt that its distance from the Vulture on those days would have been such, that you were therefore permitted to invent, in chapter 6, as observed, what in chapter 1 you had related in such plain words as not observed?

Rightly, then, are you compelled to bear the contradiction and inconsistency charged against you by Chiaramonti in Book 2 of the *Antitycho*, chapter 13, page 184. Nor could your *Hyperaspistes*, Kepler, in Book 2, chapter 13, cover your blushes with any other shield than by himself inventing observations made by Tycho on the sly on those days. For this is the reason he devises why Tycho did not enter the observations of the 16th and 24th in the catalogue: "For straight away in the catalogue of chapters, with its first word, he promises only the more certain observations, and on folio 1 only those which he obtained with the radius and the sextant together. So he scorned those stolen glimpses of the comet as less certain, content with the most certain ones of the days around them." [Margin: Kepler protects Tycho's fiction with a new fiction.]

And yet it is false that in that first chapter Tycho reports only the most certain or the more certain observations, as will be plain to any reader who looks through it. For he omits only the altitudes and azimuths, deferring mention of them to another place; but the distances of the comet taken from the fixed stars he promises to record, all of them, as many as appeared, since he says clearly: "But what I noticed in these very observations of the comet, on the days on which it appeared to us, I shall now record in order — at least as far as concerns the distances from certain fixed stars, which I diligently obtained with both the radius and the astronomical sextant. The altitudes and azimuths, which were found with the quadrant at certain times, I shall review in those places where their use requires; and I shall make no mention of any of them here, because they were much fewer than the observed distances." This is the sum of Tycho's promise. Let us now see what he actually delivered.

Thus in the history of the 13th he says: "Then, when the middle parts of the asterism of Orion were rising, the comet seemed to be heading for its setting — though clouds intervened and prevented exact certainty in this matter." And after relating the interruption of the observations from the 15th up to the 20th exclusive, he resumes the history and says: "On the 20th, although it was not altogether clear, it was nevertheless possible to observe diligently enough"; and after giving some observations of that night, he concludes: "This consideration,

however, was not exquisite enough, on account of clouds intervening from time to time." Of the 21st he adds: "That evening the head of the comet was also seen at an equal distance from the first star in the wing of Pegasus and from the tail of Cygnus, distant from each by more or less 46°, and this by the radius, at the seventh hour; but this observation has less exact certainty." Of the observations of the 23rd, on the other hand, he says: "And all these observations were exact, because the sky was perfectly clear on all sides for this whole time." By contrast, of the observations of the 25th he has this: "At $6\frac{1}{2}^{h}$ it was distant from Scheat of Pegasus by approximately 42°, which, however, could not be concluded exquisitely enough, because of intervening clouds"; and of the 28th: "But these observations were not exquisite enough, owing to a clear sky that did not last, and thin clouds, etc." Of the observations made in December, on the 2nd, he likewise testifies thus: "But in all these there is no exquisite certainty, owing to clouds intervening from time to time; for this reason too I made no observation worth recording with the instruments this evening." And of the 9th: "It was not yet possible to take any distances exquisitely, because of rather thin clouds; I noticed, however, etc." And on the 18th: "On the 18th, although because of intervening clouds nothing certain could be observed as far as the distances are concerned, I noticed the tail, however, etc." And of the 24th: "By the radius, however, it was seen to be about 16° from Scheat of Pegasus, and this only roughly, because of the faintness of the comet."

I have not minded gathering these testimonies of Tycho himself into one place, so that it might be plainly established that Tycho entered in the catalogue not only certain but doubtful observations, and not only those obtained with the radius and sextant together, but also those obtained somehow with the radius alone, and indeed by mere estimation by eye. It follows that Kepler — with a new lie, if he noticed this, or with a careless fiction rather than a contrivance — tried in vain to defend Tycho's lie or fiction; and that while we were chasing a single little fox, two were caught in almost the same snare. But see how quickly Tycho incurred the penalty for violating astronomical good faith (for there is no greater punishment for a sin than the sin that follows it). For between those very pages 124 and 127, on which he invented the comet as observed by himself on November 16 and 24, he fell into that pseudographic figure we mentioned above, whose diagram you will see in Book 10, in the section on parallaxes, chapter 12, number 3, and its fallacy shown in the same place, number 5. In it he supposes two parallel straight lines falling perpendicularly on the convexity of the Earth, and invents angles such that, if they were true, the distance of the bright star of the Vulture from the zenith of Uraniborg would be many degrees smaller than its declination, as asserted by Tycho himself, requires. For this he was rightly thrashed by Galileo on page 22 of the *Trutinator*, and by Chiaramonti in the *Antitycho*, Book 2, chapters 12 and 13. Thus, from a disturbance either of conscience or of memory and imagination, he slid at once from pseudology into pseudography. [Margin: Tycho, from one falsehood, at once slid into another.]

XI. Let what has been said so far serve to prepare the reader's mind, free of all prejudice, for the naked truth, all authority, whether of the Peripatetic school or of the Tychonic sect, being set aside for the present. I should not at all wish anyone to suspect that I meant to detract from Tycho's authority and to vote for Chiaramonti's name because I wished to favour the one rather than the other, or that I was led to it by some greater affection toward Don Scipione Chiaramonti — as being an Italian and a Catholic, afterwards a priest of wonderful integrity, a very great friend of our Society, and finally the uncle of Father Girolamo Chiaramonti of the same Society, who was once most closely joined to me in literary fellowship and in the study of mathematics. All these reasons, which are not otherwise to be disapproved when other things are equal, I passed by as though with a deaf ear of the heart; and having entered this controversy out of the simple love of truth, I have weighed the arguments of both sides to the end in an even balance.

I confess, however, that out of zeal for my Institute and for the religious life I profess, I have more than once grieved deeply that Tycho, in volume 1, page 712, poured himself out so lavishly in praise of Philipp Melanchthon and David Chytraeus, men of most corrupt faith and most bitter enemies of the Catholic Church. But I grieve still more deeply that the same man, in the same volume 1 of the *Progymnasmata*, pages 776 and 777, follows Luther with such devotion as to brand Theodorus Graminaeus [?] with the mark of iniquity and malice because he thought ill of Luther's heresy. And lest he seem to want to pronounce on religion, Tycho says that he appeals to the judgment of God, and that God alone can judge and decide infallibly on these things — a saying by which, under the appearance of piety, he shows clearly enough how little piously, and how much less catholicly, he thinks of the authority of the Roman Pontiff in settling controversies of religion. [Margin: Tycho suspected of heresy.]

XII. *Fourth* and lastly, I wish the readers to be asked not to require of me in this place all the theorems and problems of parallaxes, but to consult my Book 10, in its penultimate section, which is entirely on parallaxes; for I hope they will be abundantly satisfied there. Since that doctrine is common not only to comets but also to new stars, and indeed to all phenomena old and new, I had to reserve it for that book which treats of, and is entitled, the Common Problems. Besides, that treatise would have been tiresome to some, out of place here, and would have interrupted the course of this dissertation too much. To that section, therefore, I shall myself refer where there is need, and I shall send the readers there — not unwillingly, I believe. Some things from it, however, that are most necessary for parallactic subtlety and for this controversy, I shall select and give a foretaste of in the following chapter, gathering into a summary, as it were, what is set out more fully there. [Margin: Postulate 4.]

CAPVT XVII.

Chapter XVII.

In quo Definitiones, & Axiomata ad Parallaxes earumque proprietates magis neceſſaria.

In which: the definitions and axioms more necessary for parallaxes and their properties.

I. In my Book 10, in its penultimate section, I treat parallaxes at length, and give selected and necessary — or more useful — diagrams suited to illustrating the parallactic theorems and problems; to these I shall refer the reader from time to time. But I cannot pass over the first diagram here, in which, from the Earth's cen- tre A, let there be described a semicircle ZBY of the terrestrial hemisphere, beneath a semicircle HVR of the supreme heaven of visible things described from the same centre A. Let this semicircle be part of a circle of altitude, or vertical circle. (Among the vertical circles — that is, the great circles of the sphere drawn through the zenith or vertex of the inhabitant — any meridian is also counted; and in the right sphere the equator, and in the Torrid Zone the ecliptic, coincides for some people with a vertical circle.) Let the semicircles already mentioned be bounded by the straight line HAR drawn through the centre A, which is called the true, or astronomical, or rational horizon.

[Margin: Delineation of the primary diagram for parallaxes.]

Diagram. A semicircle on the diameter H–A–R, set in the left column, with three smaller concentric arcs about the centre A. The vertical A–B–V rises from the centre to the apex; a horizontal O–B–N crosses it at B; straight lines radiate from A and from B to the outer arc. A small semicircle about A bears Z and Y.

- V (apex of the outer semicircle): the vertex or zenith
- ε, P (on the vertical, below V): the Sun and the Moon at the zenith
- D, E, F (outer arc, upper right): ends of the lines ACD, BGE, BCF
- G (inner arc, right of the vertical): the Sun on the line ACD
- C (inner arc, right of the vertical): the Moon on the line ACD
- Q, T, X (outer arc, upper left): ends of the lines ADQ, AIT, BDX
- I (inner arc, left): the Sun on the line BDX
- D (second D, inner arc, left): the Moon on the line BDX
- B (on the vertical): the observer's eye on the Earth's surface
- K, M, N (outer arc, right, above the diameter): ends of the lines ALK, ASM and the horizontal OBN
- L, S (inner arcs, right): the Moon and the Sun on the physical horizon
- O (left end of the horizontal through B)
- H, R (ends of the diameter)
- Z, Y (small terrestrial semicircle about A)
- A (centre, foot of the vertical)

Upon this line, from A, let the perpendicular ABV be erected. It will be the vertical line with respect to an eye placed at B, a point on the Earth's surface,

and V will be its vertex or zenith, which Regiomontanus, with the more ancient writers, calls "the summit of the heads" [ſummitatem capitum]. The straight line OBN drawn through B parallel to HAR, or orthogonal to AV, is called the apparent, or physical, or sensible horizon with respect to the eye B. Now from the centre A, beneath the vertical HVR, let there be described a semicircle SGεI etc. for the higher phenomenon and LCPD etc. for the lower; for clarity let the former be the Sun and the latter the Moon. First, let both appear together on the physical horizon, the Sun at S and the Moon at L, and through the centres of each let the straight lines ASM and ALK be drawn from the centre of the Earth up to the supreme sphere. Next, let both appear above the physical horizon, but lie on the same line drawn from the centre of the Earth through their centres up to the supreme heaven — such as the line ACD — so that the Sun is at G and the Moon at C; and through their centres let different lines, BGE and BCF, pass from the eye B as far as the fixed stars. Finally, let both appear above the physical horizon, but on the same line BDX, from the eye B through their centres to the supreme heaven — namely the Sun at I and the Moon at D; and through their centres let different lines pass from the centre of the Earth to the supreme heaven, AIT through the Sun and ADQ through the Moon. With this laid out, you will understand the following Definitions and Propositions very easily; in them, what I say of a heavenly body holds for any lofty phenomenon.

DEFINITIONES.

Definitions.

II. The true place of a heavenly body, taken absolutely, is the point in which the centre of its body actually is; but relative to the supreme heaven of visible things, beneath which it is considered, it is the point of the supreme heaven at which the straight line drawn from the centre of the Earth or universe through the centre of the body terminates. And so the *line of the true place* is the straight line drawn out from the centre of the Earth or universe, through the centre of the body, up to the supreme heaven. Thus if the Moon is at L, its true place is absolutely the point L, but relatively the point K, and the line of the true place is ALK. Hence the *true distance from the vertex* is the arc of the supreme heaven intercepted between the vertex and the true place, and the *true altitude* is the arc intercepted between the true horizon and the true place.

[Margin: True place, and line of the true place.]

[Margin: True distance from the vertex, and true altitude.]

The *seen* or apparent place [Viſus locus ſeu apparens] is the point of the supreme heaven beneath which the centre of the body is seen to be, or at which the straight line drawn out from the eye through the centre of the body terminates; and so the *line of the apparent place* is the straight line drawn from the eye through the centre of the body to the supreme heaven. If, for example, the Moon is at L, beneath K, but, seen from B along the line BLN, appears

beneath the point N, its apparent place will be the point N, and the line of the apparent place will be BLN. If, however, it is at C, beneath D, but appears beneath F, the Moon's apparent place will be the point F, and the line of the apparent place BCF. Hence the *apparent distance from the vertex*, or the *altitude from the horizon*, is the arc of the supreme heaven between the apparent place of the body and the vertex, or the sensible horizon.

[Margin: Apparent place, and line of the apparent place.]

[Margin: Apparent distance from the vertex, and apparent altitude.]

The reason why the one is called apparent and the other true is that the true place of a single, unreplicated body is single, from whatever place on Earth it is viewed, and common to all who view it at the same moment; and it cannot be such unless it is considered from the common centre of the Earth or universe. The apparent place, by contrast, is as varied and manifold as the position of the eye from which the body is seen. The reason why the lines of the true and apparent place are drawn out all the way to the supreme heaven — when otherwise, for any body taken separately, any heaven above it would suffice — is that this heaven is the universal place of all visible things, and suited to setting out the parallaxes not only of all the planets but even of the fixed stars themselves, however slight those parallaxes may be.

[Margin: Why is the one called the true place, the other the apparent?]

[Margin: Why is the supreme heaven used for both places?]

Identity of aspect [Identitas aſpectûs] is when the true and the apparent place are the same, and the lines of the true and apparent place consequently coincide in one line. This happens either because the eye (if this were possible) is transferred to, or is at, the centre of the Earth or universe; or because (as actually happens) it is indeed on the Earth's surface, but the body is beneath the vertical point. So if the Moon is at P and the Sun at ε, they will be, and will appear, together beneath the zenith V, and the line AV of the true place will coincide with the line BPεV of the apparent place, granted that one is shorter than the other.

[Margin: Identity of aspect.]

Variation of view [Diuerſitas aſpectûs] is the difference between the true and the apparent place of a body. This aberration, or hallucination, or deviation of the apparent place from the true, the Greeks name *parallaxis* [Greek: παράλλαξιν (parallaxin, "parallax")], from the verb [Greek: παραλλάσσω (parallassō, "to deviate")], which means to deviate, or to go off in different directions. This difference, taken formally, is the arc of the supreme heaven intercepted between the true and apparent place of the body. Thus, with the Moon at L, the true place is K but the apparent N, and its parallax is the arc KN; with respect to the Sun, placed at S, the parallax is the arc MN. But if the Moon is at C and the Sun at G, the Moon's parallax is the arc DF and the Sun's DE; and if, finally, the Moon (on the left side of the diagram) is at D and the Sun at I, the Moon's parallax is the arc QX and the Sun's TX.

[Margin: What variation of view, or parallax, is.]

[Margin: Parallax taken formally.]

Taken mensuratively, however, or equivalently, parallax is the angle formed at the centre of the body by the lines of the true place on the one side and of the apparent place on the other: if the Moon is at L, the angle ALB; if the Sun is at S, the angle ASB; if the Moon is at C, the angle ACB; and so for the rest. This is not because the angle vertical to it — say the angle KLN — is the measure of the parallactic arc KN; for an arc of this kind is not described from the angle's point L as centre, since it is described from A. It is because, by Euclid I.32, ALB is the interior angle by which the other interior angle BAL — that is, VAK, which measures the arc VK of the true distance from the vertex — is exceeded by, and differs from, the exterior angle VBN, which measures at B, the centre of the instrument (the natural one, the eye, or an artificial one), the apparent distance VN of the body from the vertex. Accordingly it shows indirectly how great the parallactic arc KN is, by which the true distance from the vertex VK differs from the apparent distance VN.

[Margin: Parallax taken equivalently in two ways.]

I know, indeed, that Regiomontanus and some others draw from the centre of the Earth a line parallel to the line of the apparent place — so that, with the Moon at C, a line AK is drawn parallel to the line BCF — in order to measure the parallactic arc from the centre of the universe, from which the supreme heaven is described, and not from a foreign centre. For in this way the angle VAK will be as great as VBF, by Euclid I.29, and the angle DAK is that by which VAK exceeds the angle VAD of the true distance. But these do not take the parallactic arc formally either: the arc DK, enclosed between those two parallels, is not the actual difference between the true place D and the apparent place F, since we suppose the Moon at C to be seen beneath F, not beneath K; it is that difference only equivalently. As to the measuring angle, however, it comes to the same thing: because the lines BCF and AK are parallel, the angle BCA is equal to the alternate angle DAK, by Euclid I.29, and each is the measure of the parallax. That parallel line is therefore superfluous; indeed, it seems contrary to the nature of parallax — which arises from the difference of the centres, of the world and of the eye — to try to measure it from the same centre from which the true place is measured.

Parallax of altitude [Parallaxis Altitudinis] is the difference between the apparent and the true altitude; or it is the arc of the circle of altitude intercepted between the true and the apparent place of the body. In substance and in magnitude it is the same as the parallax of distance from the vertex, which some call the vertical parallax: for that is the difference between the true and the apparent distance from the vertex, or the arc of the vertical circle intercepted between the true and the apparent place. The vertical circle and the circle of altitude are the same, and do not differ except in name, or formally by relation to different terms: the vertical circle is referred to the vertex, whereas the circle, or the parallax, of altitude is referred to the horizon and reckoned from it. So if the Sun is at G, the same arc DE is the parallax of altitude with respect to

the horizon, since it is the amount by which the true altitude RD differs from the apparent altitude NE; and it is the vertical parallax, since it is the amount by which the true distance VD from the vertex V differs from the apparent distance VE. The latter, however, is simpler than the former, because the term V from which they are reckoned is the same, whereas the horizons from which the true and apparent altitudes are counted are different.

[Margin: Parallax of altitude, and of distance from the vertex, or vertical parallax: what they are, and whether they are the same.]

[Margin: Vertical parallax is simpler than parallax of altitude.]

Horizontal parallax [Parallaxis Horizontalis] is the parallax of a body appearing on the physical horizon — that is, when it appears in such a way that it seems to be no distance at all from the sensible horizon, but at the greatest distance, a full quadrant of a circle, from the vertex. It therefore is, and is called, the greatest parallax of all for a body that keeps the same distance from the centre of the Earth. So if the Moon appears at L, the arc KN, or the equivalent angle ALB, is the horizontal parallax, and the greatest of all that the Moon can undergo so long as it keeps the same distance AL from the Earth's centre. If, for instance, it rises from L to C, but AC is equal to AL, ALB will be greater than ACB; and so for the rest.

[Margin: Horizontal and greatest parallax.]

SPECIES PARALLAXIVM.

Kinds of parallax.

III. Of parallaxes, one is true, and another false, or merely apparent. The true is the difference between the true and the apparent place, arising from the distance both of the centre of the eye from the centre of the Earth or universe and of the body from each of these centres; it therefore varies when either of them varies, whether the variation is due to the motion of the body or to the motion of the eye. This parallax, considered causally or radically, we may call *parallax from the interval* [?]. False or apparent parallax is the difference not between the true and the apparent place, but one that is thought to lie between the true and the apparent place of the body: either because it is the difference between two places seen by the same eye at different times, or by different eyes at the same time; or because it is the difference between two true places arising from the true motion of the body, and gives the inexperienced occasion to think that it arises from the interval. This could be called parallax from mere motion. If, however, the place seen by one of the eyes were beneath the vertex, and so the same as the true place, while the place seen by the other observer is far from the vertex, the difference between the seen places will be a true parallax.

[Margin: True parallax, or parallax from the interval.]

Of true parallaxes, one is sensible in itself, another insensible in itself. Sensible in itself is the one that can be discerned with some instruments by some observer

on the Earth's surface, even if, by accident, some observer could discern it with none of his instruments. Insensible, by contrast, is the one that can be perceived by no observer on the Earth's surface, however great the instruments and however fine the sights [dioptrarum]. As the smallest of the sensible parallaxes some set 10′ or 12″[16], others a greater or a smaller value; we set 5″ or 6″, in seconds. In that case the lines of true and apparent motion are judged to be not so much parallel as coincident to sense, and to make no angle at the centre of the body; and the distance of the body from the Earth is judged to have no sensible proportion to the Earth's semidiameter, or to the distance between the Earth and the eye; and so the body is seen just as if the eye were at the centre of the Earth — or, in other words, the globe of the Earth is judged to be like a point with respect to the heaven in which that body is.

[Margin: Parallax sensible or insensible in itself.]

Of true and in-itself sensible parallaxes there are, moreover, many kinds. The first, and the root of all the others, is the parallax of altitude, or vertical parallax, which was discussed in number 2. For as many as are the great circles of the sphere in which the body is, or to which it is referred by reason of its place or motion, so many kinds of parallax sprout from the one and same parallax of altitude; but the chief ones are those written below.

Parallax of depth [Parallaxis Profunditatis] is the difference between the true and the apparent depth. For the eye can be raised above the convexity of the land and the sea, so that a body which is truly either above the true horizon, or on the horizon, or a little below it, appears to the eye below the horizon already mentioned, or lower than it is; on this see more in Book 10, penultimate section, chapter 3. This parallax is either the difference between the true and the apparent depth, or the aggregate of the true altitude and the apparent depth of the body; and it is considered in the vertical circle continued below the horizon.

[Margin: Parallax of depth.]

Parallax of declination is the difference between the true and the apparent declination; or it is the arc by which the declination in the circle drawn through the true place differs from the declination taken in the circle drawn through the apparent place of the body; and this is its measure.

[Margin: Parallax of declination.]

Parallax of right ascension is the difference between the true and the apparent right ascension; or it is the arc of the equator, less than a quadrant, intercepted by two circles of declination, one of which is drawn through the true place and the other through the apparent place of the body. The angle made at the pole of the world by these circles is the measure of this parallax. Consider the figures of both kinds in Book 10, penultimate section, chapter 6.

[Margin: Parallax of right ascension.]

[16]the first value is printed with a single prime, the others with double primes; kept as printed

Parallax of latitude, or in latitude, is the difference between the true and the apparent latitude of the body; or it is the arc by which the circle of latitude drawn through the true place differs from the circle of latitude; and the same arc is its measure.

[Margin: Parallax of latitude.]

Parallax of longitude, or in longitude, is the difference between the true and the apparent longitude of the body; or it is the arc of the ecliptic, less than a quadrant, between two circles of latitude, one of which is drawn through the true place and the other through the apparent place of the body. The measure of this parallax is the angle made at the pole of the ecliptic by the two circles already mentioned. For diagrams of these two kinds see Book 10, penultimate section, chapter 7.

[Margin: Parallax of longitude.]

Parallax of distance — not from the vertex, but from another body — is the difference between the true and the apparent distance of two bodies, a distance measured in a great circle of the sphere. The parallax of distance is therefore the arc of a great circle by which the true distance differs from the apparent either in quantity, or at least in position; for it can differ in position alone while being equal in quantity. On these, more in Book 10, penultimate section, chapter 5.

[Margin: Parallax of distance.]

Parallax of the annual orb [Parallaxis Orbis Annui], taken inadequately, is the difference between the true place of a star, considered from the centre of the universe as distinct from the centre of the Earth, and its place seen from the Earth; it arises from the annual motion of the Earth about the Sun, according to the Samian and Copernican school, for which the centre of the universe is either the Sun or a point as far from the Sun as the Sun's eccentricity is for other authors. Taken adequately, however, it is the difference between the place of a star seen from the Earth occupying one point of the annual orb, and another place of the same star — motionless, or taken as motionless — seen from the Earth occupying the point diametrically opposite the first: as if an inhabitant of the Earth, when it stands beneath Cancer, sees the Pole Star, and the same person sees it again after the Earth has been carried beneath Capricorn. A parallax of this kind does not arise from the parallax of altitude, granted that it can be reduced to it; nor is it sensible unless the diameter of the annual orb is supposed to have a sensible ratio to the distance of the star from the centre of the Earth or universe; otherwise it is not sensible. But on this, more in Book 10, penultimate section, chapter 4.

[Margin: Parallax of the annual orb.]

Parallax of motion is the difference between the true and the apparent motion of a body; or it is the arc, or time, by which the true motion of the body differs from its apparent motion; and it is of as many sorts as there are kinds of motion of the body. See the figure and examples in Book 10, penultimate section, chapter 9.

Difference of parallaxes is the difference between two or more parallaxes of the same kind. It is neither parallax itself, nor the difference between two seen places, as some who otherwise think rightly speak wrongly — except in the case of the parallax of the annual orb, or in the case in which the place seen by one observer is also the true place. See more on this in Book 10, penultimate section, chapter 9.

AXIOMATA PARALLAXIVM.

Axioms of parallaxes.

IV. *A body beneath the vertex, or seen from the centre of the Earth, has no parallax*; its apparent and true place are the same — at least outside the case of the parallax of the annual orb.

[Margin: Axiom 1.]

A body appearing on the sensible horizon has the greatest parallax of all: by the definition of horizontal parallax. Understand this other things being equal, and with the same distance of the body and of the eye from the centre of the Earth preserved.

[Margin: Axiom 2.]

For the same body at the same distance from the centre of the Earth, the greater the distance from the vertex, the greater the parallax; and the smaller that distance, the smaller the parallax too: as in the figure of number 1,

[Margin: Axiom 3.]

the parallax KN of the Moon at L is greater than the parallax DF of the Moon at C.

Of any two bodies having an equal apparent, or an equal true, distance from the vertex, the parallax of the one that is less distant from the centre of the Earth is greater, and that of the one more distant is smaller. As in the figure of number 1: the parallax DF of the Moon at C is greater than the parallax DE of the Sun at G; and, on the left, the parallax QX of the Moon at D is greater than the parallax TX of the Sun at D.

[Margin: Axiom 4.]

As great as the horizontal parallax doubled, so great is the portion of the semicircle of that hemisphere, in whose heaven the body moves, that lies hidden beneath the physical horizon. If, for example, the Sun's horizontal parallax were 3′, then of the semicircle of the solar heaven standing above the true horizon, three minutes on this side and three on that would lie hidden beneath the apparent horizon — 6 minutes in all; and the apparent semicircle is only 179° 54′.

[Margin: Axiom 5.]

As great as the horizontal parallax of a body, so great is the apparent semidiameter of the Earth seen from the centre of that body. For the angle that measures the one and the other is the same, or equal to it.

[Margin: Axiom 6.]

As great as the greatest parallax of the annual orb, taken inadequately, with respect to some star, so great is the apparent semidiameter of the Earth's annual orb seen from that star. For in this hypothesis it stands exactly as the Earth's semidiameter stands to the horizontal parallax.

[Margin: Axiom 7.]

The parallax of altitude and of depth always, in itself, depresses the body apparently; and so it diminishes the true altitude from the horizon but increases the distance from the vertex — not truly, but apparently. Hence, if a true altitude is to be converted into an apparent one, the parallax belonging to the given true altitude must be subtracted from the true altitude; if an apparent altitude is to be converted into a true one, the parallax belonging to the given apparent altitude must be added to the apparent altitude. The contrary must be done in each case if a distance from the vertex is to be converted, true into apparent or apparent into true.

[Margin: Axiom 8.]

The true and the apparent altitude of a body undergoing parallax cannot be the same at the same moment of time: otherwise, if they were the same, there would be no difference between them, and consequently no parallax. Hence it follows that there cannot be the same, or an equal, parallax fitting two equal altitudes, one apparent and the other true. Those who use the same table for both therefore err. But see what I shall say in Book 10, penultimate section, chapter 2, number 18.

[Margin: Axiom 9.]

Refraction raises the body apparently. It is therefore opposed to parallax, and the refraction can be so great as only to diminish the parallax; or so great as to equal it, and so cancel it; or so great as to exceed it. If it equals it, the true and the apparent place of the body are the same. Hence, likewise, for converting a true altitude into an apparent one, or the latter into the former, rules opposite to those indicated in Axiom 8 must be observed as regards the correction for refraction.

[Margin: Axiom 10.]

A body has no parallax, or refraction, in azimuth: for each produces its effect in the same circle of altitude, and azimuth is considered by the arc of the horizon between the vertical in which the body is and the meridian — not in the arc of some determinate circle parallel to the horizon, drawn through the body's true or apparent place.

[Margin: Axiom 11.]

A body appearing, or truly standing, in the meridian has no parallax of right ascension: for the meridian is then a circle both of altitude and of declination. A body therefore cannot appear in the meridian and be outside the meridian, or be in the meridian and appear outside it.

[Margin: Axiom 12.]

A body standing, or appearing, in the ninetieth degree of the ecliptic has no parallax of longitude: for in that case the circle of latitude is the same as the circle of

altitude, and determines the place in the ecliptic through which it is drawn; the true and the apparent ecliptic place are therefore the same. The ninetieth degree of the ecliptic is the one that stands ninety degrees from the horizon on either side.

[Margin: Axiom 13.]

A body standing, or appearing, in the ninetieth degree of the equator has no parallax of right ascension; for it cannot be in the ninetieth degree of the equator reckoned from the horizon without being in the meridian. Axiom 12 therefore holds here.

[Margin: Axiom 14.]

The parallaxes of right ascension and of longitude in themselves increase, in the eastern quadrant of the equator or of the ecliptic, and in the western, in themselves or for the most part, diminish — the former the right ascension, the latter the longitude of the body: for there parallax depresses the body apparently in the order of the signs, here against the order of the signs. Rarely, however, or by accident, it can happen that in the eastern quadrant they place the body visibly at the beginning of Aries, or beyond, and so either cancel the whole longitude reckonable from the beginning of Aries, or show a smaller one from a new beginning of reckoning: as if a body is truly at the end of Pisces and appears in the first degree of Aries. And conversely, in the western quadrant parallax can by accident increase it: as if a body is at the beginning of Aries but is depressed so that it appears at the end of Pisces.

[Margin: Axiom 15.]

The parallaxes of declination and of latitude — the former the declination, the latter the latitude — apparently either increase it, or diminish it, or generate the whole of it, or cancel the whole of it, or change it from one region, or kind, into the other.

[Margin: Axiom 16.]

For this, see the figures and expositions in Book 10, penultimate section, chapter 6, number 2, and chapter 7, number 2. There are, indeed, many theorems of parallaxes that bear on this subject, and ones unknown to the more ancient astronomers; but so as not to be too long, I ask the reader at least to run through those I shall give in Book 10, penultimate section, chapter 9.

CAPVT XVIII.

Chapter XVIII.

An Cometa Anni 1577. Demonſtratus ſit ſupra Lunam an infra : ſeu Vtrùm oſtenſus ſit minorem, an maiorem habere Parallaxin Lunari?

Whether the comet of the year 1577 was demonstrated to be above the Moon or below it; that is, whether it was shown to have a smaller or a greater parallax than the Moon's?

I. Tycho Brahe, in volume 2, chapter 7, affirms that the comet of the year 1577 was in the ethereal region of the world and not in the elementary region — that is, that it was higher than the Moon — and that he demonstrated this

by five arguments. Although we have already adduced some of these, as general arguments, in chapter 11, they must nevertheless be repeated briefly here, because they bear indirectly on the comet's parallax: for if they establish that it was above the Moon, it follows also that it had a smaller parallax than the Moon. We shall reduce his arguments to syllogistic form, but with the conditions added which Tycho has scattered about.

PRIMUM ARGUMENTUM TYCHONIS À MOTU PER CIRCULUM MAXIMUM.

Tycho's first argument, from motion along a great circle.

II. The first argument is of this kind: *The comet of the year 1577, by its motion, marked out with the utmost exactness, to sense, a portion of a great circle in the sphere, no less than the Sun, the Moon and the other wandering stars. But if it had been a fiery meteor blazing in the elementary region, as the Peripatetics hold, it could not by its motion have marked out with the utmost exactness, to sense, a portion of a great circle in the sphere. Therefore it was necessarily in the highest aether, and completed its course there.* The major premise is proved because, from the comet's longitude and latitude, observed by distances from the fixed stars, it is established that from the beginning to the end of its motion it always kept the same angle of inclination to the ecliptic. The minor is shown from this: if it had been a fiery meteor, it would have been carried with a wandering motion wherever the matter it needed for fuel had drawn it, or wherever the force of the winds had violently driven it. And even if it were said to be swept along by the force of some wandering star, still, since the planets themselves do not by their sensible motion describe an exactly great circle, much less could they have imparted one to a comet compacted of such fluid matter. And the non-wandering stars, since they have no sensible proper motion within a few days, could much less have communicated such a motion to a body so foreign to them.

III. I reply, first, by granting for the moment the major and minor premises and denying the consequence, for many reasons. For first, it is established from Tycho himself that this conclu-

[Margin: First reply, denying the consequence.]

sion is not necessary, or demonstrative. For although on pages 91 and 105 he uses the word *demonstrandi* ("of demonstrating"), yet on pages 92 and 93, speaking of this very argument, he says: *It is proved with sufficient probability*; and a little later: *It remains, therefore, that we conclude reasonably.* And Kepler himself, in the *Hyperaspistes*, admits that this argument does not exceed the limits of probability and likelihood. For he says on page 5: *But it is likely, if indeed these endowments are taken to have been given to the comet's body itself, that this body is in a place higher, etc., than this elementary region of ours. The authority, etc., increases the likelihood of the opinion.* And on page 18, again speaking of this argument: *I reply that the probability of Tycho's argument rests on its own degrees: so that it holds for the Moon nonetheless, even though not even this celestial planet, as Tycho has expressly admitted, appears to proceed exactly under the same great circle — as much because of its rather large parallax as for other reasons. The closer,*

therefore, the comet keeps to the path of a great circle, the greater is the probability of its celestial nature; and if only a twentieth part of the Moon's deviation is found to exceed the bounds of the great circle, it is believed twenty times more probably to be in the heavens than the Moon itself — by this argument, at least. And finally on page 19: *Let the assertion stand unshaken, that the comet proceeded under a great circle, and on that account appears to have been above the Moon*; where the word *videri* ("to appear") sounds of mere probability. Tycho, however, wrongly confuses probability with demonstration, or opinion with knowledge; we, for our part, are meanwhile disputing about demonstration and the necessary condition. What is to be said about probability we shall settle elsewhere, below.

[Margin: For the consequence is only probable, not necessary, by the admission of Tycho and Kepler.]

Secondly, because the syllogism has four terms, and more is inferred in the conclusion than is contained in the premises. For it infers not only that the comet was not a fiery meteor blazing in the elementary region — which alone ought to have been inferred by force of the syllogistic form — but also that it was in the highest aether. Yet there are not a few who think that comets shine by the Sun rather than blaze and feed on elementary matter as fuel, and who nevertheless place them below the ☾.

[Margin: 2. Reason for denying the consequence: because Tycho's syllogism is faulty and formless.]

Thirdly, because to conclude that this comet was in the aether, the comet's true motion — that is, the motion distinguished from the seen motion — would have had to be along a great circle. For the planets are not recognized as celestial from the fact that by their seen motion they describe the same circle, great to sense, since all those that have a sensible parallax, or a change of latitude, or both — the Moon especially — are carried by their seen or apparent motion along the same great circle; but because by their true, or rather mean, motion they are carried along the same orbit of a great circle. Therefore Tycho ought to have deduced from the seen motion along a great circle the true motion along a great circle, as Chiaramonti rightly observes in the *Antitycho*, Book 1, chapter 11, in the corollary to proposition 2, and in Book 2, page 119. But from seen motion along a great circle, true motion along a great circle is not necessarily inferred unless the moving body lacks sensible parallax. For if it has a sensible parallax, then if one of these motions is along a great circle, the other is not, from what will be said in Book 10, penultimate section, chapter 9 — and Chiaramonti and Cabeo, adduced there, agree. Therefore, before it is known whether there is any parallax at all, and whether it is sensible or insensible, the kind of circle of the one motion cannot be known from the circle of the other motion, as we shall say there with Chiaramonti, in the *Apologia* for the *Antitycho*, page 53, and the *Antitycho*, Book 1, chapter 11, proposition 10.

From this it is clear that Kepler erred in three ways in the *Hyperaspistes*, page 12. For he thought, first, that by the very fact that a moving body is carried by its seen motion along the same great circle, it lacks sensible parallax; second,

that it therefore necessarily follows that it also moves by its true motion along a great circle; and finally, that the kind of circle marked out by the seen motion must be investigated first, so that the parallax may be determined from it, rather than that the parallax must be known beforehand, so that the circle of each motion may be determined from it. For he says in the *Hyperaspistes*, page 12, against Chiaramonti: *Hear me, Peripatetic: we attend to whether the comet proceeds on a great circle precisely so that we may determine the parallax; we seek the parallax so that from it we may also learn its place along the diameter of the world. In this inquiry the kind of circle comes first; it does not follow. The closer one keeps to a great circle, the less the comet is understood to change its place through the different positions in the first motion.* Then, on page 18, he says: *For it is true that if the co-*

[Margin: Kepler's threefold error in the method of parallax.]

met by its true motion held the path of a great circle, it did not hold it by its seen motion, since it produced a parallax. But, with less circumlocution, this too is true — which Scipio [Scipione Chiaramonti] has not yet refuted — that if by its seen motion it marked out an arc of a great circle, it also marked one out by its true motion, because in that way sensible parallax is taken away from it. But we, in Book 10 of this work, penultimate section, chapter 9, have demonstrated much more clearly than Chiaramonti that a phenomenon can have a very large and sensible parallax and yet be carried by its seen motion along the same great circle, and consequently not describe a great circle by its true motion. This confirms what Chiaramonti rightly taught in Book 1 of the *Antitycho*, chapter 11, proposition 10: that before deciding the kind of circle along which the comet proceeds by its true motion, one must decide about the parallax — whether it is sensible or not. And indeed, even without drawing any figure, who would not understand that, if parallax always depresses a phenomenon below its true place, it cannot happen — if the circle of the seen motion is distinct from the vertical circle — that the circle of the true motion is not always above the circle of the seen motion, and that the one never cuts the other? And if it does not cut it, it is certain that both cannot be great circles, by proposition 11 of Book 1 of Theodosius's *Spherics*. Therefore, if the parallax is sensible, who would not understand that the circle of the seen motion deviates sensibly from the circle of the true motion?

Finally, Tycho, in volume 2, page 207, grants this comet a parallax of 19′ in its first days, and for almost the whole time of its duration one greater than 3′ 7″; but in volume 1, page 80, he does not grant the Sun a parallax greater than 3′ 7″. Therefore this comet's parallax was almost always greater than the Sun's parallax, let alone than Jupiter's and Saturn's; and yet, on Tycho's assertion, it described a portion of a great circle by its seen motion. And so Kepler, while defending Tycho, contradicts Tycho: for he affirms that by the very fact that the comet describes a great circle by its seen motion, it lacks sensible parallax.

[Margin: Kepler contrary to Tycho in his very defence of Tycho.]

IV. Chiaramonti, however, replies secondly, in Book 2 of the *Antitycho*, chapter 4, by denying the major premise and its proof. First, because if we wish to follow full rigour, it does not follow that, if the comet kept a constant angle

to the ecliptic, it described a great circle: for all the parallels along which the Sun proceeds are circles that are not great, and yet they keep the same constant angle to the ecliptic. Next, because in chapter 6 he shows from the data given by Tycho that the comet's seen path departed from the same point of intersection with the ecliptic not only by 5′ at a time — which Tycho acknowledged and disregarded — but also by 19′, and sometimes by 55′, as I shall say below at number 17. Finally, on the ground of parallax: because he thinks that Tycho spoke, or ought to have spoken, of the true motion; but if by its true motion the comet described a great circle, and had a parallax not only of 3 minutes but sometimes greater, up to 19′, it could not have described a great circle by its seen motion.

[Margin: Chiaramonti's objection against the major premise of the syllogism set out in number 2.]

But this last reason is invalid, because Tycho spoke of the motion observed by himself, the seen motion. Granted that, for the conclusion lifting the comet into the heavens to be valid from a seen motion made along a great circle, he ought to have deduced from it a true motion along a great circle — which he could not do, given a parallax so sensible. The second reason, however, does hold against Tycho, insofar as, with excessive exaggeration [?], he will have it that the comet described a great circle with the utmost exactness, and no less than the Sun. For the Sun never deviates by its seen motion, by reason of parallax alone, from the track of a great circle by 5 minutes, let alone by 19, as much as this comet deviated (for we are not considering here the refractions near the horizon common to the Sun and the comet). But it does not hold for the comet compared with the Moon, which, because of a parallax sometimes of 60′ and more, and because of the variation of its latitude, departs from a great circle more than this comet does. And it is enough for Tycho, and for Kepler — as is clear from his words adduced in number 3 — if the comet described a portion of a great circle more exactly than the Moon. Nor does Chiaramonti win on this part, but on the other, just mentioned. The fourth reason, on the rectilinear motion which Kepler attributes to this comet, does no harm to Tycho, because it is enough for Tycho that the comet was in the same plane of a great circle, even if it did not describe the circumference of the circle.

[Margin: Chiaramonti's reasoning against Tycho, partly valid, partly invalid.]

V. The third reply is to deny the minor premise. For nothing prevents some elementary body from being carried along a great circle, even if it is fiery or blazing, since

the very matter of its fuel can be so disposed along that circle. And that there are elementary bodies which move naturally along the plane of a great circle, I have already taught in chapter 11, number 9.

Secundum, Tertium, & Quartum Argumentum Tychonis.

Tycho's second, third and fourth arguments.

VI. Tycho's three following proofs, in volume 2 from page 103 to 105, reduced to the form of a single syllogism, run as follows: *Every body that moves with a regular and ordered motion — that is, either uniform or uniformly difform — and that is slower than the Moon in its own proper motion from west to east, is celestial and above the Moon, especially if, beginning at one tropic, it ends at the other. But the proper motion of the comet of the year 1577 was regular, etc.* For at the beginning it was seen to complete about 4 degrees daily, then 3½, afterwards 3, next 2, afterwards 1, and finally, towards the end, not even one. From this it is also clear that it was slower than the Moon's motion, which is never less than 10 degrees. Finally, the beginning of this motion was from the tropic of Capricorn above the head of Sagittarius, and its end at the tropic of Cancer near Scheat of Pegasus, a quadrant of a circle having been completed both with respect to the ecliptic and with respect to its own track — which savours of the nature of a celestial body. Therefore that comet was above the Moon.

But this argument was sufficiently answered in chapter 11, from number 9 to 12, and there is no need to repeat the reply, since Tycho and Kepler at length acknowledged that, even if the minor premise were granted, the major is nevertheless not necessarily but only probably true. Indeed, from what was said in chapter 11, number 10, it is probable in such a way that its opposite is nevertheless equally probable, because of the many other disagreements which the motion of this comet had with the motions of the celestial bodies.

Quintum Argumentum Tychonis ex differentia Diſtantiarum Cometæ ab eadem ſtella Fixa diuerſis locis obſeruatarum, orta ex Parallaxi.

Tycho's fifth argument, from the difference, arising from parallax, between the comet's distances from the same fixed star observed in different positions.

VII. For the force of the argument that Tycho uses in volume 2, chapter 6, from page 106 to 123, to be understood, certain things must be presupposed which he himself either demonstrated from his observations or seemed, to himself and to others, to have demonstrated. Thus in chapter 1 he reports all the observations of the comet, made on clear nights as well as on nights that were not clear — that is, the certain as well as the doubtful — namely the distances from fixed stars obtained with the radius and the old sextant. Then, in chapter 2, he corrects the places of the fixed stars that were corrupt in Alfonso X and Copernicus, from their declinations and right ascensions, observed by himself with the help of the azimuthal quadrant, through altitudes, azimuths and definite moments of time; for he admits that he did not then have the armillae. Afterwards, on page 31, he admits that not only the radius but that sextant too was liable to some error, and that later, equipped with larger and more accurate

instruments and with a knowledge of refractions, he obtained a more exact obliquity of the ecliptic and altitude of the pole, and more correct places of the fixed stars; and therefore he corrects, for the time of the comet, the places of the stars he had used in observing the comet. Yet because their places did not differ from the earlier ones by more than six minutes, and often by less, he chose to correct neither the distances from the fixed stars taken with the radius and the old sextant nor the innumerable calculations built upon them, judging that no evident difference could arise from this. For when he recognized the fault of the earlier instruments and obtained more correct places of the fixed stars, he had already finished almost the whole second volume — granted that, by remaking or reprinting folios 31 to 34, he inserted this acknowledgement and addition. Moreover, since he always used the same places of the stars, uncorrected though they were — not sometimes corrected and sometimes uncorrected — he judged that, for the sake of coherence, the differences of the parallaxes would come out the same from the corrected places as those he had gathered from the uncorrected ones. And accordingly, on page 34, he says: *Therefore, since there is no difference of any evident moment between the places of the stars which I used in this book from the earlier observations and those which we afterwards noticed, more exactly, that they actually hold in the heavens, I did not wish, on account of*

so few minutes as are lacking, to call back to the anvil the entire calculation founded on their places throughout this book, with a labour tedious and troublesome rather than useful or necessary, etc. Thus Tycho in chapter 3, from the uncorrected places of the fixed stars and the uncorrected distances of this comet from the fixed stars, gathers by the analysis of triangles the comet's seen longitude and latitude for each day of observation; in chapter 4, the comet's right ascensions and declinations; and in chapter 5, the portion of the circle which the comet described by its motion, with its inclination to the ecliptic and the equator, and the point at which the comet's path cut both the ecliptic and the equator.

VIII. After this, in chapter 6, having first set out the four arguments discussed above, he proceeds to the 5th argument, which, reduced to form by me, runs as follows: *If the comet of the year 1577 had been below the Moon, even if it had been in the very concave of the lunar heaven, it would have had to undergo so great a parallax, and no smaller, that on November 23 its own proper motion eastward over three hours would be retarded by 15 minutes and more, and that after the three hours it would appear as far from the mouth of Pegasus as at their beginning, or farther. But it did not undergo so great a parallax* [?]*, nor was its motion retarded in those three hours by 15′, but only by three. Therefore the comet of the year 1577 was not below the Moon.*

[Margin: Tycho's 5th argument, gathered into a few words.]

He attempts to show the minor premise thus. On November 23, at $5^{h}\ 30^{m}$ after noon, at Uraniborg, the comet's seen distance from the mouth of Pegasus was 21° 8′; then, at $8^{h}\ 35^{m}$, the comet's seen distance from the same mouth was 20° 56′. Therefore in $3^{h}\ 5^{m}$ the comet approached the mouth of Pegasus (for that mouth was more easterly than the comet) by 12′. But from the comet's

proper motion eastward, taken from the 21st and the 25th, from which the 23rd is equally distant on either side, the comet ought to have approached the mouth of Pegasus by 15′: for its daily motion on those days was exactly 2 degrees, or 120′, which, divided by 24 hours, gives 5′ to each hour and 15′ to every three hours. Hence, since at the second observation it was nearer the horizon, and parallax depressed it towards the west, retarding its visible motion eastward, it is concluded that the three minutes by which the comet's approach to the mouth of Pegasus was retarded were due to parallax.

[Margin: Proof of the minor premise.]

Moreover, on the same night, at $5^h\ 45^m$, the comet was seen to be 4° 38′ from the star in the left hand of Antinous, and at $6½^h$ to be 4° 40′ from it, this star being more westerly than [?] the comet. But from the hourly motion of 5 minutes, 3′ 45″ were owed to three quarters of an hour; therefore, had the comet had no parallax, it ought to have receded from that star by 3′ 45″ at the second observation; but it receded only 2′. The remainder, therefore, 1′ 45″, was due to parallax depressing the comet towards the west. Now let us come to the proof of the major premise, which he reaches gradually, through many steps, up to number 15.

[Margin: 1st step, for the difference of the distances observed from the same fixed star.]

[Margin: The proof of the major premise begins.]

IX. *Secondly*, since he did not observe the comet's altitudes at the hours mentioned, he investigates them on page 109 by spherical triangles. In this analysis he assumes the distance of the pole of the world from the vertex, that is AB, to be 34° 7′; the place of the Sun at $5^h\ 30^m$ in the afternoon of November 23 to be ♐ 11° 8′; and its right ascension 249° 55′. If to this are added 82° 30′ of the equator, which correspond to $5^h\ 30^m$, the right ascension of mid-heaven becomes 332° 25′. If from this is subtracted the comet's right ascension already found by him on page 70, which was 301° 7′ — though here, on page 109, it is set at 301° 5′ — there remain 31° 20′: that is, ME, or the angle ABD which the meridian HAC makes at the pole of the world B with the quadrant BE of the circle of declination drawn through the comet D as far as the equator ML. And the declination DE of the comet D, found on page 70, was 51′ to the north, and its complement BD 89° 9′, although Tycho here sets 89° 10′. Therefore, by the analysis of triangle ABD, AD comes out at 60° 35½′, and so the altitude ID at 29° 24½′. Similarly, at the second observation, with the comet at F, at $8^h\ 35^m$, the Sun's place for Tycho was ♐ 11°

[Margin: 2nd step, for each seen altitude of the comet.]

Diagram. A semicircle standing on its diameter HC, set at the right of the right column beside the computation of §IX. From A at the top of the arc and from B on the upper right of the arc, straight and curved lines descend to the diameter; arcs cross the interior through D, F, E and G; M stands on the left of the arc; I, K and L stand on the diameter. The text makes HAC the meridian, B

the pole of the world, D the comet at the first observation and F at the second, BE a quadrant of a circle of declination, ML the equator, ME the hour angle, and ID the comet's altitude; these roles are taken from the text, not printed on the block.

- A (top of the arc)
- B (upper right on the arc)
- M (left on the arc)
- D (interior, below and right of A)
- F (interior, below D)
- E (interior, left, below M)
- G (interior, below E)
- H (left end of the diameter)
- I (on the diameter, left of centre)
- K (on the diameter, beside I)
- L (on the diameter, right of K)
- C (right end of the diameter)

36′, and its right ascension 250° 4′. To this he adds 128° 45′, corresponding to 8^{h} 35^{m}, and the right ascension of mid-heaven (together with a whole circle) becomes 378° 49′. And since the comet's right ascension for that time, verified proportionally, was 301° 18′, when this is taken from 378° 49′ there remains MG, or the angle ABF, 77° 31′. The declination FG, verified for that time, was 1°, and its complement BF 89°. Therefore in triangle ABF, from AB 34° 7′, ABF 77° 31′ and BF 89°, AF comes out at 82° 12′, and the altitude KF at 7° 48′ — the altitudes that had to be found.

X. *Thirdly*, from page 110 he investigates the lunar parallaxes that the Moon at perigee would undergo at the two altitudes of the comet found just before, namely at the altitude ID, 29° 24½′, and KF, 7° 48′. He assumes, from Copernicus, the distance of the Moon at perigee as 52 semidiameters of the Earth; but, to work more finely, he converts it to 5,200,000 parts, of which the Earth's semidiameter is 100,000. With these assumed, in the following figure let there be described from the Earth's centre A the Earth's quadrant BH, and that of the concave of the lunar heaven, KG, enclosed by the vertical line AK and the horizontal AG. And let the Moon first be at D, from which let DBF be drawn through the eye B until the perpendicular AF falls on it at F; then, with the Moon placed at C, let CBE be drawn, and the perpendicular AE to it at E; and let D and C be joined to the Earth's centre by the straight lines AD and AC.

[Margin: 3rd step, for the parallaxes corresponding to each altitude, if the comet had been in the concave of the Moon [?].]

Diagram. A quadrant of a circle set at the right of the left column, its vertical side AK and horizontal side AG meeting at A at the foot. An inner arc runs through B on the vertical; straight lines run from A and B out to the outer arc and beyond it to the right, where E and F stand outside the quadrant and I between them and A. In the text A is the Earth's centre, BH the Earth's quadrant, KG the concave of the lunar heaven, B the eye, D and C the Moon's

two positions, and F and E the feet of the perpendiculars from A; these roles are taken from the text, not printed on the block.

- D (top of the outer arc, left of K)
- K (top of the vertical side)
- C (on the outer arc, left of D)
- L (on the outer arc, left)
- B (on the vertical side, at the inner arc)
- E (outside the quadrant, right of B)
- I (between A and E)
- F (outside the quadrant, right, below E)
- G (left end of the horizontal side)
- H (on the horizontal side, between G and A)
- A (corner at the foot of the vertical)

First, in triangle ABF, right-angled at F, the angle ABF is given, 60° 35½′, since it is equal to the angle KBD, that is, to the complement of the comet's first altitude; the base AB is also given, of 100,000 parts; therefore, by the rules of triangles, the side AF is found, of 87,114 parts. *Secondly*, in triangle ADF, right-angled at F, AF 87,114 and AD 5,200,000 are given; therefore the parallactic angle ADF is 57′ 36″. *Thirdly*, in triangle ABE the angle ABE is given, 82° 12′, since it is equal to the angle KBC, which is the complement of the comet's second altitude, and the base AB, 100,000; therefore AE is found, of 99,075 parts. *Fourthly* and lastly, in triangle ACE, right-angled at E, AE 99,075 and AC 5,200,000 are given; therefore the parallactic angle ACE is 1° 5½′. So the parallax of altitude of the Moon at perigee would have been 57′ 36″ at the comet's first altitude and 65′ 30″ at the second, and the difference of the parallaxes 7′ 54″.

[Margin: By Euclid I.15.]

XI. *Fourthly*, from page 112 he investigates the angle which the vertical circle makes with the comet's path at each time of observation. For this, in the semicircle of the meridian HAR, let A be the vertex and B the pole of the equator EQ; let the comet be at C, and its path FCL; and through the comet let there be drawn the quadrants ACK, of a circle of altitude, and BCG, of a circle of declination. With these described, the angle ACF is sought.

[Margin: 4th step, for the angles of the comet's path with the vertical.]

Diagram. A semicircle on the diameter HR, set across the left column below the statement of §XI. Arcs from A at the top and from B on the upper right of the arc pass through C in the interior; a dotted line runs from A to O; below C stand G, M, D and V, and on the diameter K, L and Q. In the text HAR is the meridian, A the vertex, B the pole of the equator EQ, C the comet, FCL its path, ACK a circle of altitude and BCG a circle of declination; these roles are taken from the text, not printed on the block.

- F (top of the arc, left of A)
- A (top of the arc)
- O (interior, at the end of the dotted line from A)

- B (upper right on the arc)
- E (left on the arc)
- C (interior, centre)
- G (interior, below and left of C)
- M (interior, below C)
- D (interior, below G)
- V (interior, below M)
- H (left end of the diameter)
- K (on the diameter, left of centre)
- L (on the diameter, centre)
- Q (on the diameter, right of L)
- R (right end of the diameter)

First, then, in triangle ACB, AB is given, the complement of the altitude of the pole, 34° 7′; BC, the complement of the comet's declination at the first observation, 89° 10′; and AC, the complement of the altitude then found, 60° 35½′. From these the angle ACB is gathered, 19° 34′. *Secondly*, in triangle CGV, right-angled at G,

the angle CVG is given, found by Tycho on page 85 — that is, the inclination of the comet's path to the equator, 33° 45′; and the side GV is known from the difference between the comet's right ascension and the point of the equator at which the comet's path cut the ecliptic, which point Tycho found on page 81 at 299° 50′. Hence from these data he finds the angle GCV — that is, FCB, vertical to it — 56° 15½′; and the angle ACB, 19° 34′, subtracted from this, leaves the angle sought, ACF, about 36° 42′. By a wholly similar method he finds at the second observation, that is for 8^{h} 35^{m}, the angle ACF, 22° 42½′.

XII. *Fifthly*, on page 114 he investigates the comet's parallax of longitude and latitude, not with respect to the ecliptic but with respect to the arc which the comet described by its own proper motion — in the preceding figure the arc FL — which he conceives as described from its own proper pole, for which B may now serve, although in reality that pole was off the meridian. And so from the comet's pole B let the arc BC be drawn through C, the comet's true place, and another arc BD to the seen place D. For since the arc FL is described from the pole B, it follows that the arcs BC and BM fall on the arc FL at right angles. Hence the triangle DCM is right-angled at M, and in it the base DC is given, namely the parallax of altitude from the 3rd step: 57′ 36″ at the first observation and 65′ 30″ at the second. And by the 4th step the angle ACF is given, that is, its vertical angle DCM: 36° 42′ at the first observation and 22° 42½′ at the second. He therefore finds the side DM, that is, the parallax of latitude, 34′ 25″, and CM, that is, the parallax of longitude, 46′ 42″, at the first observation; and at the second, DM 25′ 17″ and CM 60′ 14″.

[Margin: 5th step, for the parallax in longitude and latitude with respect to the comet's path.]

XIII. *Sixthly*, on page 115 he assumes, from what he set out in the table of the comet's motion found on page 86, that on the 23rd at $5\frac{1}{2}^{h}$ the comet stood

distant from the point V by the arc CV, 46° 30′. Subtracting, then, the first parallax of longitude, 46′ 42″, there remains the comet's seen longitude MV from the point of intersection V, 45° 43′ 18. But since from 5^h 30^m to 8^h 35^m the comet completed by its daily motion, as Tycho says, exactly 15′ (from what was said in the 1st step), and was moving eastward, that is from C towards F (for FL is the western arc), the comet's true elongation CV at 8^h 35^m was therefore 46° 45′; and when the parallax of longitude, 60′ 14″, is subtracted from it, the true elongation 45° 44′ 46″ is left.

[Margin: 6th step, for the comet's seen longitude from the intersection V.]

XIV. *Seventhly*, again by spherical triangles, on page 116, he determines the position of the star in the mouth of Pegasus with respect to the comet's path, from the longitude and latitude of the mouth of Pegasus, from the comet's parallax of latitude, and from the comet's longitude from its intersection with the ecliptic, already found in chapter 5 together with the angle of inclination of the comet's path to the ecliptic. From these, I say, he finds the seen longitude of the mouth of Pegasus with respect to the comet's path, 67° 4′, with a seen southern latitude, with respect to the comet's track, of 4° 44′.

[Margin: 7th step, for the position of the mouth of Pegasus referred to the comet's path.]

XV. *Eighthly*, and finally, on page 117, through many windings and the analysis of other triangles — whose diagrams are not at all necessary to us; see them in his book if you wish — he investigates, from what was found above, the seen distance which the comet ought to have had from the mouth of Pegasus at each observation if it had been in the concave of the Moon. At the earlier he finds a distance of 21° 44′ 16″, at the later 21° 44′ 44″, and so the later greater than the earlier by 29″, or almost half a minute. Therefore the major premise of the argument set out in number 8 seems proved — which, however, we shall presently repeat and explain.

[Margin: 8th step, for the distances at which the comet should be seen from the mouth of Pegasus.]

[Margin: Proof of the major premise.]

For with these things laid down, Tycho, on page 118 — rather obscurely and with a very involved line of reasoning, but here reduced by us to a better form — argues thus. The comet by its proper motion moved in three hours eastward, and at the same time towards the mouth of Pegasus, by 15′ in the space of 3 hours. Therefore, had no parallax intervened, after 3 hours it ought to have approached the mouth of Pegasus by 15 minutes, that is, to be less distant from it than before by a quarter of a degree. But if a parallax intervened, depressing the comet towards the west and apparently increasing its distance from that more easterly star, its motion at the later observation ought to have been visibly retarded, so that it would seem to have approached the mouth of Pegasus by less than 15 minutes — and by so much the less, the greater the parallax, or the greater its nearness to the Earth. So that, if we suppose the comet to have been in the concave of the lunar heaven, it follows from the other circumstances of

the comet, by geometrical calculation, that its motion toward the east — its approach to the mouth of Pegasus — ought to have been retarded not merely by 15 minutes, so that its whole proper motion would apparently have been consumed, but that it ought even to have been carried back farther toward the west. For in the earlier observation it ought to have stood 21° 44′ 16″ from the mouth of Pegasus, and in the later one, when it was nearer the horizon, 21° 44′ 44″ — that is, almost half a minute farther — as is clear from the 8th step. But according to the observed distances it approached the mouth of Pegasus by 11½ minutes; that is, its motion, which in itself was of 15 minutes, was retarded by only 3′ 12″. Therefore that supposed seat of the comet in the concave of the lunar heaven is false, and the comet's distance from the Earth must have been much greater than the distance of the Moon at perigee, that is, of the Moon placed in the concave of its own heaven.

[Margin: Tycho's argument.]

[Margin: The argument of number 8 is repeated.]

XVI. Let the argument set out in number 8, then, be repeated, but gathered into fewer points and somewhat differently, as follows. *If the comet of the year 1577 had not been above the Moon but below it — however close to it, and in the concave of the lunar heaven — then on the 23rd, at 8^h 35^m after noon, it would have had to stand, by force of parallax, as far from the mouth of Pegasus as it had stood at 5^h 30^m, or indeed farther, as regards the seen distance. But at 8^h 35^m it did not, by force of parallax, stand as far from the mouth of Pegasus as it had stood at 5^h 30^m, or farther, as regards the seen distance; for it stood less far, and indeed less by 12 minutes. THEREFORE the comet of the year 1577 was above the Moon.*

[Margin: Understand the hours as reckoned for Uraniborg.]

And lest you say that this proves at most the comet's position on November 23, he confirms the same thing by a similar method for November 29 and 30, and for December 1, 13 and 31. For on those days, although it had come nearer the horizon in the hours that elapsed between the paired observations, it nevertheless underwent so small a parallax that its motion toward the east was hardly retarded at all, but it approached the same fixed star very nearly as much as its proper motion, corresponding to those hours, required. He confirms these things from the distances taken with the radius, pages 120 to 123. See the summary of these confirmations in the following table.

Day of the month	Hour and minute after noon, at Uraniborg	Seen distance of the comet from the fixed star, by the radius	Fixed star from which it stood distant	Proper motion of the comet due in the interval of hours	Seen motion by which the comet approached the fixed star	Difference
(days)	(hours, minutes)	(degrees, minutes)	(the more easterly)	(minutes, seconds)	(minutes, seconds)	(minutes, seconds)
Novem~~6~~ber 29	6^h 40^m; 9^h 10^m	35° 45′; 35° 36′	Scheat of Pegasus	8′ 20″	9′ 0″	0′ 40″

Day of the month	Hour and minute after noon, at Uraniborg	Seen distance of the comet from the fixed star, by the radius	Fixed star from which it stood distant	Proper motion of the comet due in the interval of hours	Seen motion by which the comet approached the fixed star	Difference
November 30	$6^h\ 0^m$; $9^h\ 15^m$	10° 25′; 10° 14′	Mouth of Pegasus	9′ 30″	11′ 0″	1′ 30″
December 1	$7^h\ 10^m$; $9^h\ 30^m$	9° 17′; 9° 11′	Mouth of Pegasus	6′ 0″	6′ 0″	0′ 0″
December 13	$7^h\ 40^m$; $9^h\ 30^m$	22° 18′; 22° 14′	Scheat of Pegasus	3′ 30″	4′ 0″	0′ 30″
December 31	$6^h\ 0^m$; $9^h\ 0^m$	12° 0′; 11° 56′	Scheat of Pegasus	3′ 45″	4′ 0″	0′ 15″

From this it is clear, if Tycho's observations are to be relied on, that the comet — when it was in a position lower down on account of the daily motion of the primum mobile — should, if it is supposed to have had a sensible parallax, have approached that same fixed star visibly less than its own proper motion of approach toward that more easterly star required; and yet it approached it as much as that motion required, or indeed somewhat more. And so its approach was not retarded at all by parallax. From this he concludes that it underwent scarcely any sensible parallax, and was therefore far above the Moon.

Deſtructio Argumenti Quinti Tychonici per Inſtantias Scipionis Claramontij.

The demolition of Tycho's fifth argument by the objections of Scipione Chiaramonti.

[Margin: 1st objection of Chiaramonti against Tycho.]

XVII. But Chiaramonti, not at all moved by the machinery already mentioned, tries in Book 2 of the *Antitycho*, chapter 11, to destroy the force of Tycho's argument just set out by five objections [instantiae], to use his own word. The first of them is this. In the steps already mentioned Tycho supposes that the comet moved along the same great circle, and with a motion so regular that its hourly motion could be determined from its daily motion — for example, on November 23 an hourly motion of 5 minutes and a three-hour motion of 15 minutes — and that the comet's path kept the same constant angle to the ecliptic and the equator, and cut those circles at the same point. But Chiaramonti, in Book 2 of the *Antitycho*, chapter 6, demonstrated from the data supplied by Tycho that the comet did not move along the same circle but described a winding path, with a varying angle to the ecliptic and the equator, intersecting those circles at one point and then another; so much so that the point of the ecliptic at which it cut the comet's path differed not only by 5 minutes, which Tycho acknowledged and disregarded, but by 19′ and

sometimes by 55′. But whether the comet's motion in the intervening hours was uniform and proportional to its daily motion, or grossly non-uniform, cannot be established with certainty — and much less so if the kind of circle along which the comet moved is not established.

I said that Chiaramonti demonstrated this because, having examined his calculations as he sets them out in Book 2, chapter 6, I found in them no discrepancy except in a few seconds, which is entirely negligible.

[Margin: Kepler approves Chiaramonti's demonstration.]

Nor does Kepler object, in the *Hyperaspistes*, page 24; on the contrary, he reduces to a summary the calculations given by Chiaramonti in Book 2, chapter 6, and approves them in these words: *Chiaramonti is a good calculator; but the use of his calculation he either, through want of skill, does not know, or, ill-disposed toward his adversary, maliciously conceals.*

[Margin: But rejects the use made of it.]

Let us look at the sum of his seven operations: this is what he finds, this is what he concludes.

Degree and minute of ♐ toward which the comet's arc tends	From November day	to day
20° 19′	13	14
21° 14′	14	15
21° 0′	15	20
20° 58′	20	21
20° 34′	21	23
20° 51′	23	25
20° 20′	25	29

Therefore they do not tend toward the same point, but toward wandering points between ♐ 20° 19′ and ♐ 21° 14′, which are separated from one another by a space of 55 minutes, and are not one point. I concede the whole argument, even without examining this calculation. Tycho himself grants this very thing when, taking the point ♐ 20° 55′ — which came out for him more often, or lay midway among the principal results — he computes how far the places differ from those he had previously computed from the observations. And a little later he says that if the points of intersection of the comet's path and the ecliptic differ by no more than one degree or 55′, the comet's path from a great circle by no more than one minute — which is negligible in this astronomical business, and in a hurried observation of a comet.

[Margin: Kepler thinks Tycho's argument only probable.]

And he immediately adds: *Nor is the probability of his assertion, that comets are above the Moon, overturned by these minutiae.* Afterwards he says that Chiaramonti is looking for a knot in a bulrush, *and that the imperfections of the motions in figure and time are so slight that we are compelled to tolerate greater ones in astronomy itself, that most divine science.*

[Margin: Chiaramonti's reply to Kepler.]

To this Chiaramonti replies, in the *Apologia* for the *Antitycho*, pages 6 and 75, that in the matter of parallax — especially a small one, on which hangs the

decision whether a comet is above the Moon, or the Sun, or Saturn, as some contend — the geometrical calculation must be more accurate than in some part of astronomy, though not in every part. And he shows, from Tycho himself, that disregarding a few minutiae can produce a great error in this matter; he quotes Tycho's words from volume 2 of the *Progymnasmata*, page 220, which are these: *In which, unless the most exact precision is applied, an easy slip creeps in over certain very small things, which afterwards, when one comes to geometrical practice, grows into an intolerable deviation.*

[Margin: Chiaramonti undeservedly censured here by Kepler.]

For although this comet described a less winding path by its motion than the Moon does, and, considered strictly in this respect, it would seem more probable that the comet was more celestial than the Moon, as we said in chapter 11, number 9, and chapter 18, number 2, nevertheless, if in Tycho's steps 4, 5, 6 and 7 there was an error of some minutes in the angle of the comet's path with the ecliptic and the equator, there could have been an error of as many minutes and more in deriving the comet's distance from the mouth of Pegasus. And these minutes are not negligible, since the dispute was whether the comet's motion on November 23 was retarded by 3′ or by 11′ ½. Therefore Chiaramonti's diligence in this matter deserved praise rather than Kepler's carelessness and complacency; nor ought he on that account to have been branded with lack of skill in the use of calculation, or with malicious ill will against Tycho.

[Margin: 2nd objection of Chiaramonti.]

XVIII. Chiaramonti's *second* objection, on page 166 of the *Antitycho*, is this. Tycho, he says, could not know the parallax before he distinguished the true place and motion from the seen place and motion; but he either confused these or tried unsuccessfully to distinguish them, and he supposed as true the comet's distance from the point where the ecliptic intersects the comet's path, which was nevertheless a seen distance, since it was derived from the seen distances of the comet from the fixed stars. Recall the figure set out at number 11, and then step 6: there Tycho says that the distance of the comet C from the intersection V had previously been gathered by him as 46° 30′, and, subtracting the parallax CM, he converts it to the seen distance MV, 45° 43′ 18″. And indeed Tycho's words in volume 2, page 115, are these: *Therefore, since the comet's true longitude from the point of intersection is presupposed from the foregoing to have been 46° 30′ at the first observation, made at 5½^{h}, then, subtracting the parallax of longitude just found for this time, the seen longitude comes out as 45° 43′ 18″.*

But Tycho called the former distance true not absolutely — for he knew he had derived it from seen distances — but comparatively, with respect to the comet's other, visible elongation: the one that would have been seen if the comet had been in the concave of the Moon and had undergone, at each seen altitude, the parallaxes we recorded in step 3, that is, in number 10. Nor did he investigate the parallax of longitude and latitude with respect to the comet's path absolutely, but from a feigned hypothesis of the comet's distance from the

Earth, and from the parallax of altitude derived from it. Tycho therefore uses two kinds of seen places, which Chiaramonti did not distinguish, attending here to Tycho's words rather than to his meaning: one, which he called true — understand, comparatively, with respect to the other, which is more "seen", that is, would have had a greater difference of appearance if the comet had been below the Moon.

[Margin: Chiaramonti's slip in attacking Tycho.]

Therefore, when Chiaramonti, on page 168 of the *Antitycho*, reproaches Tycho for not having used the earlier places seen through the observations in order to investigate the comet's seen distances from the mouth of Pegasus, he is mistaken. For in step 8 Tycho was not seeking these distances — he already had them, known from the observation cited in number 8 — but the visible distances there would have been if the comet had been below the concave of the Moon; and for these he had to use other seen places, to which the comet would have been carried from its earlier seen path *by force of the parallax* of longitude and latitude. So Tycho supposed the comet's earlier path, which he knew to be apparent, as if it were true, in order to see what kind of seen path, and how great, there would have been if the comet had been below the Moon — as Kepler points out, in rather obscure words, in the *Hyperaspistes*, page 37.

[Margin: 2nd part of the 2nd objection, or 3rd objection of Chiaramonti.]

XIX. From the preceding objection arose the *third*, which Chiaramonti accordingly merges with it into one, on page 168 of the *Antitycho*; and it runs as follows. In the step we numbered 8 above, Tycho, from places on the comet's path which he thought true, deduced that the comet's earlier distance from the mouth of Pegasus was smaller than the later one — namely, the former 21° 44′ 16″, the latter 21° 44′ 44″. But Chiaramonti, from the same data given by Tycho, gathers the earlier distance as 21° 5′ and the later as 20° 50′, and so the former greater than the latter by 15′, contrary to what Tycho deduced. From this he gathers, on page 171, that the comet had as great a parallax as would suffice for Tycho to place it below the Moon; for a parallax that great would retard the comet's proper motion by the full 15′, as is clear from what was said at number 15.

[Margin: A second slip of Chiaramonti, rightly reproved by Kepler.]

But Chiaramonti does not prevail in this either. For he investigates the seen distances of the comet from the mouth of Pegasus which it actually had, without supposing the parallaxes that Tycho deduced from the hypothesis of a comet feigned below the concave of the Moon; whereas Tycho, in that step 8, inquires into the seen distances of the comet from the mouth of Pegasus — or rather, those it would have been seen at, had the comet been below the concave of the Moon — applying the parallaxes of longitude and latitude with respect to the comet's earlier path. It is no wonder, then, that Chiaramonti finds distances different from Tycho's; and so Kepler rightly reproves him on this point in the *Hyperaspistes*, page 40, nor did Scipione clear himself sufficiently of these charges in the *Apologia*.

[Margin: 4th objection of Chiaramonti.]

XX. The *fourth* objection, which Chiaramonti counts as the third, in Book 2 of the *Antitycho*, page 172, arises from what Tycho said and we reported at number 8. For Tycho said that in the later observation the comet, on account of its proper motion toward the east of 15 minutes, which corresponded to three hours, ought to have been nearer the mouth of Pegasus — that is, the more easterly star — nearer, I say, than before by the full 15′; but that, since the parallax depressed it visibly toward the west more than in the first observation (for in the second it was far less high above the horizon than in the first), it approached the mouth of Pegasus by only 12′; and so he attributed this difference of three minutes to parallax. Chiaramonti, however, denies that this difference was due to parallax, and says it was due rather to refraction raising the comet.

[Margin: Chiaramonti's blunder.]

But Chiaramonti is deceived on this point, because he thinks Tycho said that the seen distance of the comet from the mouth of Pegasus — the more easterly star, and the one nearer the vertex — was diminished by parallax. Had Tycho said that, this effect would rightly be attributed to refraction rather than to parallax, since it belongs to refraction to diminish the distances of a phenomenon from the vertex, or from a point nearer the vertex, and to parallax to increase them. But Tycho did not say this. He said that the comet's motion, which ought to have been seen as 15 minutes, and by which the comet should have approached the mouth of Pegasus, was retarded by parallax, so that it was seen to move only 12 minutes. Therefore, since the comet was seen farther west and nearer the horizon in the second observation than, considering its motion, it otherwise should have been, that effect could rightly be attributed to parallax rather than to refraction.

I am truly astonished that Chiaramonti, a man otherwise of the sharpest intellect, was so plainly deceived, and gave Kepler an occasion to rebuke him in the *Hyperaspistes*, pages 41 to 44. There, however, Kepler acknowledges that the comet at an altitude of 7° 48′ — that is, in the 2nd observation — was subject to a refraction of no more than 7′ in altitude, and of at most 4′ in its distance from the mouth of Pegasus; with these removed, the later distance would turn out smaller than the earlier by only 7′ or 8′. This does Tycho no harm, since for the comet to be sublunary he required it to be 15 minutes less distant from the mouth of Pegasus in the later observation than in the earlier. But Chiaramonti says, in the *Apologia*, page 114, that he had understood from Tycho that the refractions of phenomena lower than the Sun are greater than the solar ones, or not less, and hence 11′ 30″ at that altitude of 7° 48′. But even this does not suffice to make the comet sublunary: for that refraction would lie in the circle of altitude, but in the arc of distance from the mouth of Pegasus it would come out smaller; so the retardation of the motion of approach toward the star would not reach 15 minutes in the arc of distance itself.

Noſtra Reſponſio ad Quintum Argumentum Tychonis.

Our reply to Tycho's fifth argument.

XXI. Tycho's argument at numbers 8 and 16 was this: *If the comet of the year 1577 had not been above the Moon but below it — however close to it — and in the concave of the lunar heaven, then on November 23 at 8^h 35^m after noon, at Uraniborg, it would have had to stand, by force of parallax, as far from the mouth of Pegasus as it had stood at 5^h 30^m, or indeed farther, as regards the seen distance. But at 8^h 35^m it did not, by force of parallax, stand as far from the mouth of Pegasus as it had stood at 5^h 30^m, or farther, as regards the seen distance; for it stood less far, and indeed less by 12 minutes. Therefore this comet was not below the Moon, but above it.*

[Margin: 1st reason for denying the certainty of the major premise.]

I reply: if the major premise is asserted as probable, it can be admitted; but if as certain and evident, it can be denied, for two reasons. *First*, because it supposes that the comet's proper motion toward the east, corresponding to three hours of the 23rd, was of 15 minutes, which is very uncertain. For this motion had to be extracted from the daily motion of November 23 compared with the motion of the 22nd or the 24th; but on the 22nd and the 24th the comet was not observed by Tycho, as he himself admits in the catalogue of observation, pages 3 and 5 — and perhaps on those days it had some irregularity unknown to us. And although from 6^h on the 21st to 6^h on the 25th the comet completed 8° 12′ on its own course, it does not follow that in one day, and that the 23rd, it completed exactly two degrees, as Tycho puts it on pages 86 and 107. For on that day it could have been much faster, and so in those three hours could have completed more than 15 minutes — so many more that by its speed toward the east it would have absorbed all that difference of distance from the mouth of Pegasus which would otherwise have appeared on account of a large parallax, greater than the Moon's.

[Margin: 2nd reason for denying the certainty of the major premise.]

Second, I say that this major assertion is not certain and evident, because Tycho's proofs of it, reported above from number 9 to number 15, rest on suppositions that are either erroneous or uncertain. They do so on account of many errors, each small taken singly, but which, heaped together by their number, could either have grown into a large error or at least have left uncertain and doubtful the conclusion derived from them.

[Margin: Tycho's errors in the foundation of his calculations.]

To make this plain, the certain errors, or strong suspicions of errors, must be reviewed one by one.

[Margin: 1st error.]

The *first* error was in the altitude of the equator at Uraniborg. Throughout volume 2, dealing with this and other comets, he uses 34° 7′ — especially on pages 109, 113, 135, 148, 152, 153, 154, and here and there elsewhere — but later, with larger instruments and more accurate observation, he found it to be 34° 5′

30″, or 5′ 20″, or 5′ 15″, as is clear from his volume 1, pages 28 and 218. Hence an error of a minute and a half.

[Margin: 2nd error.]

The *second* error was in the obliquity of the ecliptic, which in volume 2 he takes as 23° 27′, as he himself admits on page 74, when he investigates both the places of the fixed stars he used for investigating the comet's places, and the places of the comet. But later he arrived at 23° 31′ ½, as is established from his volume 1, pages 66, 70 and 90. Hence follows an error of 4′ ½ or 3′ ½, which in some places alters the longitude by nearly 10′.

[Margin: 3rd error.]

The *third* error was in the comet's distances taken with the radius and the old sextant. It is manifest from Tycho's own admission, in volume 2, pages 31 to 34, that their unreliability was later recognized, and yet that he did not correct these intervals, as we said in chapter 16, number 9.

[Margin: 4th error.]

The *fourth* error is in the angle of the ecliptic with the comet's path, and in the point of the ecliptic at which it cut the comet's path. He treated each as always the same and unvarying, although there was variation in both, as Chiaramonti demonstrated and Kepler conceded — variation, I say, of up to 55′ in the place of the point already mentioned, as we said at number 17.

[Margin: 5th error.]

The *fifth* error is in the comet's right ascension. On page 86 he puts it at 301° 3′ for 6^h on November 23, but on page 109 at 301° 5′ for $5½^h$, whereas the latter ought rather to have been smaller.

[Margin: 6th error.]

The *sixth* error is in the comet's distance from the mouth of Pegasus on November 23 at $5½^h$: on page 4 he puts it at 21° 8′, but on page 51 at 21° 5′.

[Margin: 7th error.]

The *seventh* error was the omission of refraction, whose use, he himself confesses in volume 2, on the reprinted leaves, page 31, was unknown to him when he was writing about this comet. Now to the seen altitude of the comet, which on the 23rd at $8^h\ 35^m$ was for Tycho 7° 48′, as he himself has it in volume 2, page 110, there corresponds a refraction of at least 11′ 33″ — as much as corresponds to the Sun at such an altitude, from volume 1 of Tycho, page 79, where the table of solar refractions is given; since, according to page 280 of the same volume, things nearer the Earth have a greater refraction, and the comet was then nearer the Earth than the Sun.

With all the errors already mentioned taken together, then, it is readily clear that in the end an error of 15 minutes and more could have been committed in the comet's distance from [?] the mouth of Pegasus. And since, as Tycho himself demands (as I said at number 17), the most exact precision is required in this business, and the dispute is over a retarded motion of 15′ — for if it had been retarded by 15′ or 16′, Tycho concedes that the comet would have been sublunary — it follows that the major premise built on this foundation is either

erroneous or uncertain. As for correcting the errors already mentioned: neither Tycho himself, out of weariness and because of the multitude of triangles to be solved, nor Kepler was willing to do it, although it was very much in their interest to do so; nor have we cared to, since we do not know how great an error was admitted in using the radius and the old sextant, and since we are always pressed by that uncertainty of the proper motion, which could have been swift enough to make good a very large parallax of the comet, or the effect of parallax.

Sextum Argumentum Tychonis, ex Diſtantijs Cometæ à Fixis in diuerſis Orbis terreni locis habitis, & Solutio eius expeditiſſima.

Tycho's sixth argument, from the distances of the comet from the fixed stars taken in different places of the terrestrial globe, and its most ready solution.

XXII. There follows Tycho's third proof (for he includes the first three arguments under the heading of one proof, the first), in volume 2, from page 123. There he takes the comet's distance from the bright star of the Vulture, observed on November 16 at 6^h after noon at Prague by Tadeáš Hájek with the aid of an astronomical radius, which was seen as 17° 52′; and to this he adds: *But we here, with a similar instrument, on the same day, about six in the evening, found 17° 50′ ½.* And he says that the comet, together with the bright star of the Vulture, was virtually under the same vertical circle common to Prague and Uraniborg, and that Prague is east of Uraniborg by a quarter of an hour; from which he tries to show that the comet was above the Moon — but in vain.

[Margin: Tycho guilty of violating astronomical good faith.]

For, *first*, Tycho invented a spurious observation of his own, so that he could compare it with Hájek's. For from the history of the observations of this comet, and from the complete catalogue of them that Tycho gives in volume 2, pages 2 to 13, it is plainly evident that he reports not only the more certain observations of distances, or those taken with the radius jointly with the sextant — even granting that in the preface he had promised only these, and that Kepler, in the *Hyperaspistes*, page 47, thinks that only these are recounted — but absolutely all of them, distinguishing the less exact ones, owing to passing clouds or impurity of the air, from the exact ones. You will see this done on pages 2 and 3 twice, on page 4 once, on page 5 three times, on page 7 twice, on page 9 twice, and on page 12 once; and I have already quoted his words in chapter 16, number 10. So too, on pages 4 and 9 he reports two observations taken with the radius alone. Then, in the same catalogue, page 3, after reporting the observation of November 15 at about 6^h, he adds: *On the following four days the air with us was filled with dense obscurity, which shut off the sight of the comet: the 18th being not only dark but also very foggy, and the 19th raging fiercely with storms of wind and rain; so the observation of the comet was put off until the 20th.* It is false, therefore, that the comet was observed by Tycho on the night following the 16th; and it is no less false that he observed it on November 24, the day on which it was observed by Hájek — even though Tycho, of doubtful good faith, shamefully

invented that observation in volume 1, page 127, in order to have a second little observation to compare with Hájek's, when he had nevertheless formerly said in the catalogue, page 5: *On the following day, the 24th, no clear sky shone on our horizon.*

[Margin: Tycho rightly condemned by Chiaramonti for a feigned observation.]

So Chiaramonti, in the *Antitycho*, Book 2, chapter 14, was entirely right to call these two observations into question and to hold them fictitious; and Kepler, in the *Hyperaspistes*, pages 45 to 47, could not cover up their falsity by any evasion whatever. Indeed, striving to protect one fiction with a new fiction — pretending that Tycho entered in the catalogue only the more certain and select observations — he deserved to be turned from Tycho's advocate into a defendant charged with violating astronomical good faith.

[Margin: Kepler protects Tycho's fiction with a new fiction.]

But let us grant that this observation was left out of the catalogue because it was even more uncertain than some uncertain ones: even from this it follows that whatever is deduced from it is uncertain, and by that very fiction the sinews of Tycho's argument are cut.

Secondly, even if we granted that Tycho observed the comet on November 16 and 24 (which we nonetheless flatly deny), the scheme he uses to demonstrate the comet's parallax is thoroughly faulty, as will be clear from what is to be said in Book 10, the penultimate section, chapter 11, number 5. There we shall show its faults, detected by Galileo and Chiaramonti, and confirmed not only by us but by Kepler's own admission. Here it is enough to say that Tycho, in a great circle of the Earth drawn through Uraniborg and Prague, draws the chord from Uraniborg to Prague, requires at its ends two parallel lines, each making a right angle with that chord, and then supposes that they fall perpendicularly on the convex surface of the Earth. This is both false in itself and, in the case proposed, at odds — by many degrees, not minutes — with the declination of the bright star of the Vulture that Tycho himself gives in his tables for that time.

[Margin: Tycho pays the penalty for violating astronomical good faith, and falls into a false figure.]

So, no doubt by the just judgement of God, he at once incurred this penalty of his lie as well: from a falsely feigned observation he slid into a false and distorted form of demonstration.

Septimum Argumentum Tychonis ex Diſtantijs Cometæ à Stellis Fixis, quas obſeruauit Cornelius Gemma, collatis cum diſtantijs ab ipſo Tychone obſeruatis.

Tycho's seventh argument, from the distances of the comet from the fixed stars observed by Cornelius Gemma, compared with the distances observed by Tycho himself.

XXIII. From Hájek's observations Tycho passes, in volume 2, page 131, to the observations of Cornelius Gemma made with the astronomical radius at

Louvain, whose polar altitude he takes as 50° 50′ from the universal astrolabe [Astrolabium catholicum] of Gemma Frisius, Cornelius's father. The comet's distances from the fixed stars observed by Cornelius on the same days as by Tycho are set out below.

Distances of the comet observed from the bright star of Aquila, or from the mouth of Pegasus, in the year 1577.

No.	November	Star	Tycho	Cornelius Gemma
1	16	Aquila	18° 0′	17° 50½′
2	21	Aquila	10° 34′	10° 40′
3	28	Pegasus	12° 40′	12° 45′
4	30	Pegasus	10° 20′	10° 25′
	December			
5	1	Pegasus	9° 14′	9° 15′
6	31	Pegasus	13° 48′	14° 0′

From their difference he tries to persuade us (for he offers no demonstration) that the comet was above the Moon: otherwise, he says, a greater difference would have appeared between his own distances and Cornelius's. But, *first*, it is false that the comet was observed by Tycho on November 16, as I have already shown at number 22 and in chapter 16, number 10. *Secondly*, Cornelius's distances were taken with the astronomical radius, that is, with a very deceptive instrument — so much so that Tycho says of the first observation: *He seems not to have taken account of the utmost exactness*; of the 2nd: *Something is lacking in the observation*; and of the 4th: *I judge that some error crept into this observation of Gemma's.* And of others, not reported here but found in Gemma, Tycho says on page 133: *For the intermediate distances in Gemma are less certain and do not agree with one another*; and on 134: *There were also some of Cornelius's observations that were not exact enough.* To what end, then, does he defend his opinion with such deceptive observations in a matter so exacting? *Thirdly*, Cornelius Gemma did not record the hour of observation: *which I greatly miss both in him and in Tadeáš Hájek, especially when the comet was swifter in its daily motion*, says Tycho on page 133. Yet if anything certain were to be gathered from this, the observations in both places ought to have been made at the same moment of time, or at least at similar hours, or else these should be derivable from a recorded hour of observation. Wrongly, therefore, did Tycho say on page 134, from these and from Hájek's observations: *It can be clearly established that this comet was far above the Moon and completed its course in the aether itself; which we proposed to demonstrate from distances taken in remote parts of the globe by different observers.*

Octauum Argumentum Tychonis, ex Azimuthis & Altitudinibus Cometæ Vraniburgi diuerſis horis obſeruatis.

Tycho's eighth argument, from the azimuths and altitudes of the comet observed at Uraniborg at different hours.

XXIV. Let the figure set out at number 9 be placed here again, for Tycho uses a similar one in volume 2, page 136.

[Margin: 1st step.]

First, he brings in the azimuths and altitudes of the comet that he observed, for a definite moment of the day, though at different hours of the same night.

[Margin: 2nd step.]

Secondly, from these azimuths, from the declinations of the comet that he found in chapter 4, and from the complement of the polar altitude, he seeks each altitude as seen, or visible. For in the first observation, when the comet appeared higher above the horizon, he uses the triangle ADB, in which are given AB, the complement of the polar altitude; BD, the complement of the declination; and the angle BAD, the complement of the observed azimuth. So by the laws of triangles he gathers AD, the complement of the altitude, whose complement to 90° is the altitude ID. In the same way, in the second observation, using the triangle AFB — with the arc AB given as above, BF the complement of the declination, and the angle FAB the complement of the observed azimuth — he seeks AF, and from it the comet's altitude KF; and finally the differences of the altitudes.

Diagram. Set into the right column beside the text of number XXIV: a semicircle standing on its diameter, with several arcs drawn from the top of the semicircle down to the base and two arcs crossing from the right-hand point on the circumference towards the left. Labels:

- A (top of the semicircle)
- B (on the circumference, upper right)
- M (on the circumference, upper left)
- D (inside the figure, below and left of A, where arcs cross)
- F (inside the figure, below D)
- E (inside the figure, left, below M)
- G (inside the figure, below E)
- H (left end of the diameter)
- I (on the diameter, left of centre)
- K (on the diameter, beside I)
- L (on the diameter, right of K)
- C (right end of the diameter)

EXEMPLVM TYCHONICVM.

Tycho's example.

On November 30 at $5^{h}\ 26^{m}$, the complement of the azimuth, DAB, observed by Tycho, was 143° 40′, and the complement of the declination DE, namely DB, was 82° 52′. From this, and from AB, 34° 7′, as the complement of the polar altitude, Tycho gathers the arc AD, 53° 49′, and the altitude ID, 36° 11′ — whereas he had observed that altitude at the time as 36° 10′. Again at $7^{h}\ 54^{m}$, when the comet was at F, the complement of the azimuth, that is the angle FAB, observed by Tycho, was 105° 50′, and the complement of the declination

GF, namely FB, was 82° 48′. From this, with the arc AB, 34° 7′, he gathers AF, 70° 58′, and the altitude KF, 19° 2′, which nevertheless he says was observed as 19° 4′. Therefore the difference between the observed altitudes was 17° 6′, but between the altitudes derived trigonometrically 17° 9′.

XXV. With these premises, Tycho argues on page 137 roughly as follows. At different hours, in different azimuths, the comet showed almost the same altitude as it would have shown if it had been at an immense distance from the Earth, or at one so great that the Earth's semidiameter compared with it was scarcely sensible; for the difference between the observed altitudes is 17° 6′, and between those derived by calculation 17° 9′, that is, differing by only three minutes. Therefore this comet was far above the Moon.

[Margin: 3rd step.]

This he tries to confirm by another step. For, *thirdly*, taking the least distance of the Moon as 52 semidiameters of the Earth, by exactly the same method we set out at number 10, he gathers from two seen altitudes of the comet the comet's parallaxes for those altitudes: from the earlier seen altitude, 36° 10′, a parallax of 53′ 22″, and from the later altitude, 19° 4′, a parallax of 62′ 30″; and so the true altitudes, as seen from the centre of the Earth, would have been 37° 3′ 20″ in the first case and 20° 6′ 30″ in the second. Hence the difference of the visible altitudes, had the comet been in the concave of the lunar heaven, would have been 17° 6′, but that of the true altitudes 16° 57′; and so the difference of the parallaxes of altitude would have been 9′ 8″, whereas from observation it was no more than 3′ — which, on page 139, he thinks almost negligible, or else to be attributed to refraction. By a similar method he tries to confirm the same thing from the observations of azimuths and altitudes of December 13 and of January 11.

Diſſoluitur Argumentum Octauum Tychonis.

Tycho's eighth argument refuted.

XXVI. Chiaramonti, in Book 2 of the *Antitycho*, chapter 14, set out at greater length and in another form Tycho's reasons, which we have reported under the heading of the eighth argument, and then refuted them in chapter 15 — and soundly. For (as I noticed at once on reading Tycho, and as anyone even moderately versed in optics and geometry will be able to notice) Tycho supposes that the altitudes derived trigonometrically from the azimuths, the complement of the polar altitude and the declinations are true, and correspond to the comet's true place. But this is plainly false. The declinations Tycho uses here are those he had derived in volume 2, chapter 4, from the comet's distances from the fixed stars, observed and seen with the radius and the sextant; and so the declinations derived from them are likewise seen, not true, and correspond to the comet's seen place; nor could he distinguish them from the true ones while the parallax was unknown. It is no wonder, then, if he found the trigonometrically derived altitudes to differ from the observed ones by only one minute or two; indeed, they ought to have differed from each other by

no minute at all; and much less ought the true altitude, as he supposed, to be exceeded by the seen one by 2′ — unless we have recourse to refraction, which at an altitude of 19° is about 6′. But I do not dwell on correcting these things and examining them minutely, because Tycho's calculations are built on an erroneous foundation: namely, an erroneous polar altitude, an erroneous obliquity of the ecliptic, distances taken with a deceptive instrument, and the other things discussed at number 21 — besides the fact that he admitted that the azimuths and altitudes he uses here are subject to some deception, as we shall show presently, towards the end of number 27.

[Margin: Kepler's fictions in protecting Tycho.]

It is wearisome now — indeed shameful — to report here the fictions and evasions with which Kepler, in the *Hyperaspistes*, page 55, labours to protect Tycho; nor can I wonder enough at how much he falsely attributes not only to Chiaramonti but to Tycho as well. One thing, however, I cannot pass over in silence, which he has on page 59. There he says that the comet's altitudes and declinations were not observed by Tycho at the same moment, for the altitudes were observed, one at $5^{h}\ 26^{m}$, the other at $7^{h}\ 54^{m}$: *Since the declination was observed at the 6th hour*, he says — adding that the declination was adjusted to those two moments of the altitudes by keeping to the proportion of the comet's motion. But it is false that the comet's declination was observed by Tycho. He did not yet have the equatorial armillae when he was observing the comet, since he says in volume 2, page 31, that at that time he had at Uraniborg nothing besides the radius and the sextant except the azimuthal quadrant; and in chapter 4 he derives the declinations from the comet's longitude and latitude, which in chapter 3 he had derived trigonometrically from the observed distances between the comet and the fixed stars. Nor does Tycho anywhere mention declinations of this comet observed by himself with any instrument. And those that he entered in the table on page 86, adjusted to the 6th hour from noon, are the very ones he had derived geometrically from the comet's observed distances from the fixed stars, as is clear from the preface before that table, which stands in volume 2, page 85. But let us grant that the declinations were observed: it follows, then, that they were seen, not true, and so the comet's altitudes gathered from them were seen, not true — which is what I contend, and Chiaramonti before me, rightly blunting Tycho's arguments on this point.

[Margin: Chiaramonti refutes Tycho partly rightly, partly not rightly.]

Yet Chiaramonti wrongly condemns Tycho, on page 201 of the *Antitycho*, for having extended the calculation beyond the parallaxes to the difference of the parallaxes and of the altitudes. For if Tycho had not erred in other respects, he would have skilfully compared with each other the differences of the parallaxes corresponding to the two altitudes — namely, the one that had resulted from the observations, and the one that would have resulted from a sublunary position of the comet — showing that the latter ought to have been greater; so that, since the observations exhibited the former as smaller, he would rightly have concluded, as far as this goes, that the comet was not sublunary.

For a smaller difference of parallaxes, derived from the same altitudes above the horizon, also shows smaller parallaxes and a greater distance from the Earth. It should be noted, however, from what will be said in Book 10, the penultimate section, chapter 8, proposition 7, that if one takes on the one hand the difference between two seen altitudes, and on the other the difference between the two true altitudes corresponding to those seen ones, or differing from them only by parallax, and then derives the difference of those two differences, this is equal to the difference of the two parallaxes by which, of course, the true altitudes differ from the seen.

Nonum Argumentum Tychonis ex Methodo Regiomontani ſuis obſervationibus accommodata, exponitur & expungitur.

Tycho's ninth argument, from the method of Regiomontanus adapted to his own observations, set out and struck down.

XXVII. Johannes Regiomontanus, in his little work on comets, had devised several problems for investigating the parallaxes of comets. All of them suppose that the comet either moves with no motion other than that of the primum mobile, or with a proper motion indeed, but one so regular that its hourly motion can be known from its daily motion by the golden rule of proportions, or can be neglected without notable error because of the shortness of the time, as is clear from what he says in Problems 2, 3 and 4. Tycho therefore chose Problem 2, and so, in volume 2, from page 145 to 158, he tries to raise this comet above the Moon on Regiomontanus's ladders. That problem rests on two observed altitudes with their azimuths, both before or both after noon, with the moment of each observation known, together with the declination and, finally, the polar altitude — a problem we shall set out more fully in Book 10, the penultimate section, chapter 10, problems 8 and 9, with the aid by which Tycho taught that it could be used as if the comet were carried only by the motion of the primum mobile. And since by this method he found the parallax of this comet to be either nil or insensible, he affirmed that it was carried beyond the Moon.

[Margin: 1st objection against Tycho.]

But Tycho's argument totters on two counts. *First*, it rests on the comet's proper motion, whose measure was known only by probable conjecture; on an erroneous polar altitude; on deceptive distances of the comet from the fixed stars, with a very sensible refraction neglected; and on seen declinations taken for true ones.

[Margin: 2nd objection.]

Secondly, it rests on altitudes, azimuths and moments of time which, unless they are most accurate, are enough to sniff out parallaxes in a rough way, but not those parallaxes whose slight difference can call the comet down from the heavens into the air, or call it up from the air into the heavens. Yet Tycho observed neither the altitudes of this comet, nor its azimuths, nor the moments of time as accurately as was needed. I shall show both points from Tycho

himself. For in volume 2, page 145, speaking of Regiomontanus's problem, he says: *By two altitudes taken before or after the meridian in different azimuths, with the time elapsed between those two observations also known, it makes each parallax in the circle of altitude known — a subtle and quite ingenious speculation indeed, but one that by no means deserves a place in those smaller parallaxes such as occur in the aether. For it builds the greatest things out of the smallest, so that an error of one or two minutes in the time, which can easily creep in, grows into an excessive deviation; and then too, unless the azimuths and altitudes are obtained so scrupulously that nothing is lacking, you will have wasted, as the saying goes, your oil and your labour.*

[Margin: The reckoning by azimuths with the moments of time is slippery.]

For a slip of a few barely sensible minutes in the time and in the other data easily brings about a departure of several degrees from the mark of truth. But since Regiomontanus, led by the arguments and authority of the Peripatetics, believed rather that comets were sublunary and were generated in the upper region of the air, where they would produce a parallax of many degrees, he did not hesitate to put this method forward, however little suited to practice in a subtler business, and useful at least for these larger parallaxes. And on pages 151 and 157 he twice admits, from experience of the calculation, which led him to it, that this way of Regiomontanus is *less suited to this business*; and so he halts and breaks off the calculation he had begun, seeing that what he had hoped for did not follow from it. And in volume 1, the *Progymnasmata*, page 604, hear what he wrote about this way of observing: *But since he examined these things mechanically on a certain bronze globe three feet in diameter, it was not possible to reach exact precision. In the very times, too, of the azimuths and altitudes taken (for at that time he proceeded by this all too slippery way of investigation), some slight discrepancy could sensibly alter the place of the star*; he is speaking there of the Landgrave's observations. Again, in the *Epistolae Astronomicae*, page 295, refuting the objections of a certain physician, he writes as follows: *He further infers that the comet's parallax should have been sought not from distances but only from altitudes and azimuths — both of which, however, we carried out. But even if he does not know it and is purblind to it, the method by distances, duly applied, is more certain than the one taken from altitudes and azimuths, in which a barely sensible error turns out to be very prolific: as these things are well known to you, who have handled them; and you know for certain how easily one errs in altitudes and azimuths, but not likewise in distances.*

Let this suffice for the first part of our assertion. For the second part, take an open admission from Tycho himself, who, after writing almost the whole of volume 2 on this and other comets, nevertheless inserted the following on folio 31 and the two following, which were reprinted: *When this comet shone, I had, besides the astronomical radius and the sextant — more convenient than the radius itself for measuring the intervals between stars — only a single quadrant, made skilfully of solid brass and finely subdivided by transversal points in our customary way; but it was not fully two cubits, and it turned standing on a steel azimuthal horizon no more than three cubits in diameter, and so it did not give a large enough size to the single minutes. Since, then, in the following years I had much larger and*

more precious astronomical instruments made, I did not trust that quadrant I was then using safely enough where a matter had to be handled very scrupulously; and so it has long since been retired by us in those observations that require the utmost precision. He repeats this on page 465; and on page 32 he admits that at that time he could scarcely obtain the moments of time by the clock precisely enough.

From this we may now turn the following syllogism back against Tycho himself. In investigating those parallaxes on which the solution of the question depends — whether the comet is below or above the Moon — the comet's altitudes, azimuths and moments of time must be observed so scrupulously that nothing is lacking in them, and exact precision must be applied; otherwise an error of a few minutes grows into a large error. But the altitudes, azimuths and moments of time that Tycho uses in applying Regiomontanus's problem were not observed in this way, but with instruments that did not attain the utmost precision, and of such a kind that it had to be retired because of its uncertainty. Therefore they are not so certain that a certain and evident conclusion about this comet's parallax can be drawn from them. Tycho's argument, then, stands sufficiently refuted.

[Margin: Another way in which Chiaramonti breaks Tycho's argument.]

XXVIII. Chiaramonti, however, in Book 2 of the *Antitycho*, chapter 17, met Tycho's argument in another way. By the force of refraction, which Tycho omitted in correcting the altitudes, he tries to break Tycho's argument and calculations — and certainly not without effect. For I note that Tycho, in reducing Regiomontanus's problem to practice, used on page 150 a seen altitude of 28° 56′ in the first observation and 12° 12′ in the second; that then, on page 151, having used the altitudes 33° 7′ and 19° 19′, he found from them and from the azimuths and moments of time no parallax, but rather a greater altitude of the comet in the second observation, when he ought to have found a smaller one; and that on page 155 he attributed this raising to refraction, whose effect he then knew after a fashion, but whose measure he did not yet know. Now, from Tycho himself, about 10′ of refraction are due to an altitude of 12°. Their omission could have introduced no small error into Tycho's computation.

But what does Kepler, Tycho's *Hyperaspistes*, say to this? From page 69 to 81, of course, he entangles himself and Tycho in many evasions, and concedes more to Chiaramonti and to us than one could have hoped.

[Margin: Kepler, by his very advocacy, harms Tycho's cause.]

For on page 72 he says that the ordinary refraction belonging, from Tycho's table, to an altitude of 12⅕° is 4⅖′. From this, and from the other things Tycho laid down, he continues the procedure in Regiomontanus's problem that Tycho had broken off, and by trigonometric calculation gathers, for the first altitude of the comet, 28° 56′, a parallax of 3° 50′, and for the second, which was 12⅕°, a parallax of 3° 54′. Then he adds: *I continued this procedure so that it might appear that what Tycho wrote of it is true, that the greatest things are built out of the smallest*; and a little later: *So not only does this demonstration have no place in the smallest parallaxes, as Tycho said; not even in the large ones, which Tycho*

was ready to concede to Regiomontanus, can any security be hoped for. In short, so great an agreement of the time interval with the azimuths cannot be hoped for from an observer's diligence that the two can be combined in the same demonstration, the one being derived from the other by reasoning, as the method of the demonstration requires. He therefore grants what we want: namely, that whatever Tycho deduced from that method of Regiomontanus, and from moments of time and azimuths that were not of the utmost exactness, is uncertain; or else that this comet's parallaxes, had Tycho's assumptions been sound, were greater than the Moon's, amounting as they do to several degrees.

As for the claim of the *Hyperaspistes*, on pages 72, 75 and 76, that from Tycho's table no more than 4⅖′ of refraction corresponds to 12⅕°: this would indeed be true if the refraction to be ascribed to the comet were the one appropriate to the fixed stars, as is clear from Tycho's table in volume 1 of the *Progymnasmata*, page 280; but it is false if the refraction to be attributed to it is the one appropriate to the Moon and the Sun. For from Tycho's tables in volume 1, pages 73 and 124, a refraction of about 12′ is due to 12⅕°. Moreover, to lower phenomena, other things being equal, a greater refraction is due, in Tycho's opinion (volume 1 of the *Progymnasmata*, page 280); and this comet, according to Tycho, volume 2, page 197, was lower than the Sun for many days; so a refraction no smaller than the Sun's must be granted to it. Now, if a refraction of 4⅕′ is granted, Kepler by his calculations from the rest of Tycho's data gathers a parallax of the comet of almost 4°; much more, then, would he have gathered had he applied a refraction of 12′. Hence in this part of the defence he harms Tycho's cause by his advocacy more than Chiaramonti would have wished. For one of these two follows: either the method of Regiomontanus that Tycho used to establish this comet's position above the Moon is most uncertain; or, if it is applied and Tycho's data are accepted, but with a correction for refraction of at least 4′ applied, the comet had a parallax of nearly 4° and was far below the Moon. Either of these overturns the foundations of Tycho's argument set out last.

Decimum Argumentum Tychonis ex Aliorum obſeruationibus ſumptum, eiuſque Solutio.

Tycho's tenth argument, taken from the observations of others, and its solution.

XXIX. Tycho had finished nine chapters of volume 2 in the year 1578, towards the end of whose January the comet ceased to be seen. Then, nine years later, when the leaves had already been printed, in reworking, that is, inserting a new leaf — the one we have from page 31 to 34 of volume 2 — he himself relates this: *What is contained in this second book I had written immediately after this comet's disappearance, nine years ago, except only the last chapter, in which, after more writings published by others on this very matter had come in, I made most things fuller. But what is contained both in the earlier book and in the following one was written by us not long ago, after the places of the stars had been restored more precisely with new instruments, and the polar altitude and the greatest declination had been more thoroughly explored.* And in chapter 10, page 268, he says: *As also the*

comet seen in the year 85 just past, etc. From this it appears that the last chapter, that is the 10th of volume 2, was written by Tycho long after the first 9, when he had become more learned — namely in the year 1586. So it is no wonder if he often mentions refraction in it, and seems to have known its determinate quantity. In chapter 10, moreover, he reports the observations and opinions of others about this comet that we are now discussing; yet nowhere in this chapter does he deduce the parallax of this comet from two observations made at different places on the Earth, by the means we shall teach in Book 10, the penultimate section, chapter 11, or by similar problems. Instead he uses the observations of others, made at a single place on the Earth, each set separately, in the same way as he had used his own observations made at Uraniborg to investigate the comet's parallax — because, I believe, he did not find two places on the Earth that had been under the same vertical circle as the comet and had observed it at the same moment of time. He begins, moreover, with the observations of Wilhelm, Landgrave of Hesse, which he considers the more exquisite.

Parallaxes Cometæ ex Lantgrauij obſeruationibus à Tychone inueſtigatæ.

The parallaxes of the comet investigated by Tycho from the Landgrave's observations.

XXX. Tycho's method is as follows. First, from page 217 on, he takes the moments of time, the altitudes and the azimuths of the comet observed by the Landgrave of Hesse; assuming the polar altitude at Kassel to be 51° 18′, the obliquity of the ecliptic 23° 31′, and a place of the Sun more correct than before, he gathers by the analysis of triangles the comet's declination, right ascension, latitude and longitude. Then he selects two seen longitudes of the comet on the same night at different hours and, from the analogy of the comet's proper motion, considers how much that motion was retarded by the force of parallax. This will be better understood by an example.

[Margin: An example of the parallax investigated.]

On November 16, at 4^h $59\frac{1}{2}^m$, the comet's seen altitude was 24°, and its seen place in ♑ 16° 20′; but at 7^h 16^m it was in 16° 35′ — that is, after 2¼ hours. On the 17th, at 24^h 36^m, its seen altitude was 25°, and its seen place ♑ 19° 21′. From this Tycho gathers a daily motion in longitude of 3° 1′. So in those 2¼ hours of the 16th that elapsed between the first and the last observation a motion of 17′ was due, yet the comet was not seen to advance more than 15′, namely from 16° 20′ to 16° 35′ of Capricorn. Therefore, says Tycho, the parallax that retarded the comet's proper motion was only 2′, whereas it would have been much greater if the comet had been in the concave of the lunar heaven. In this way, from four similar pairs of longitudes, he contends that the comet was above the Moon. Lastly, from azimuths, altitudes and moments of time observed by the Landgrave, he hunts for the parallaxes by Regiomontanus's problem, discussed in number 27, and finds them either insensible, or else some arcs that came out larger, although as parts they ought to have been smaller than their whole.

From this he confirms that neither were all the observations so accurate, nor is Regiomontanus's method useful for discerning very small parallaxes. Yet since he gathers from it insensible parallaxes, or an effect contrary to parallax — namely, the comet advanced at the lower altitude towards the vertex, when it ought to have been moved away by the parallax — he at least persists, on this ground, in his opinion that the seat of this comet was above the Moon.

[Margin: The non-evidence of Tycho's argument.]

XXXI. But the method just described, although it presents no small probability, is nevertheless not evident. For whatever was said against Regiomontanus's method in number 27 holds equally here, since it requires the utmost precision, so that nothing is lacking in the azimuths, altitudes and moments of time, as we made plain from Tycho himself in the same number 27. But such precision was not present in the Landgrave's observations, of which the same Tycho says, in volume 2, page 212: *Yet in these very observations a deviation of a few minutes could easily have occurred, especially since His Highness at that time was using instruments that were not so exact and corrected*; and on page 226: *It may indeed seem strange that the longitude from these observations, on some preceding days, came out everywhere somewhat greater than our findings, but is now, from this record, found to be smaller — which is an indication that these determinations of the azimuths and altitudes, or even of the time taken together with them, are not exact in every respect*; and on page 227: *It seems strange that, whereas the longitude from these observations, found earlier after the middle of our sixth hour, exceeded by only 10′, now, near the ninth hour, it comes out more than half a degree further off. This must be imputed to these less exquisite observations.* Whether, then, this is to be attributed to the fallacy of the observations or to the irregularity of the comet's motion, it is clear that the conclusions drawn from them labour under some uncertainty.

The earlier method too — by which Tycho, comparing longitudes of the same night with one another, hunts down how much the parallax retarded the comet's proper motion — besides resting on the observations already mentioned, which are not entirely exact, or at least not free of suspicion, also supposes that the comet's daily motion was not only regular but uniform, so that if it advanced 3° 1′ in one day, it follows that in 2¼ hours it advanced 17′. But if that motion, gathered from various observations, is weighed, either it is shown to be very inconstant and unequal, or the observations are found not to agree with one another. For, as Tycho admits on pages 219 and 220, the motion gathered from the altitudes and azimuths on November 27 from 4^h $59½^m$ to 7^h 16^m was 15′, and yet from 4^h 56^m to 7^h 16^m it was only 10′, when it ought to have been greater than 15′. And since in this second case Tycho admits that the comet's motion seems to have been retarded by 7′ because of parallax, if you add to these the 11′ of refraction that are due to an altitude of 8°, the whole parallax will come to 18′. But all these things are uncertain.

Mœſtlini, Gemmæ, & Rœslini Opiniones. de loco & Parallaxi Cometæ huius.

The opinions of Mästlin, Gemma and Röslin on the place and parallax of this comet.

[Margin: Mästlin's opinion.]

XXXII. What Michael Mästlin wrote about this comet in ten chapters, Tycho condensed into fewer, in volume 2, from page 245 to 288. Mästlin had pronounced, in chapter 3, that this comet was above the Moon, because its difference of parallax at different altitudes was barely sensible, especially on December 11. But that this was not demonstrated by Mästlin you will gather from the following: first, that he used places of the stars taken from the Prutenic table, which sometimes differ by a whole degree from the places corrected by Tycho, as Tycho himself testifies on page 261; second, that he considered the comet's motion in the circle of altitude, when it should have been considered in its own track, which was different from the vertical circle, as Tycho points out against him on page 251; and finally, because of the various disagreements between his observations and those of Tycho and of the Landgrave, which Tycho detects again and again. For, given these, his demonstrations remain uncertain.

[Margin: Cornelius Gemma's opinion.]

Tycho presents Cornelius Gemma's observations in volume 2, from page 288 to 305. Among them the most notable for our purpose is the comet's distance, taken with the radius, from the star in the hand of Antinous when it stood 22° 4′ high on November 20; this distance was seen almost unchanged when the comet was near its setting, and from this Gemma gathers that the parallax was small, and smaller than the Moon's. However, as Tycho says on page 291: *This is merely asserted, baldly, by Gemma himself, without any demonstration; nor is it expressly indicated how great a distance the comet had from that star near its setting, and the assignment of both the time and the altitude — the two things most required — is neglected as well.* Gemma, moreover, held that the comet moved in the orb of Mercury.

[Margin: Röslin's opinion.]

Lastly, among others who placed this comet above the Moon was Helisaeus Röslin, about whom Tycho has no little to say in volume 2, from page 306 to 316; for he too thought that this comet had its seat or course in the orb of Mercury. But anyone who reads through what Tycho reports about him will see that this was not shown by Röslin by a conclusive demonstration. For among other things Tycho says of Röslin on page 314: *But his placing the comet in the orb of ☿, acquiescing in the views of Cornelius Gemma, while he tries to prove from the analogy of its motion that this is consistent, lacks foundation: for Gemma's parallaxes are not exact, nor did he demonstrate them mathematically from certain observations, as he ought to have done, but simply asserted that they were so. And Röslin's own opinion, that this comet imitated the motion of ☿ as closely as possible in speed, slowness, stations and retrogradations, is utterly wide of the mark.* Finally, Tycho, about to report the opinion of both men, had said on page 288: *Since*

they do not treat the matter itself mathematically enough, I will be briefer and dwell on them less.

[Margin: Squarcialupi's opinion.]

Nor did Marcello Squarcialupi, as Tycho testifies on page 450, show by any demonstrations what he asserted: that comets move in the aether above the Moon.

Antonij Santutij Argumentum pro Cometæ huius loco ſupra Lunam.

The argument of Antonio Santucci for this comet's place above the Moon.

XXXIII. This comet, says Santucci in chapter 10 of *De Cometis*, was on November 11 in a southern parallel declining from the equator by 22° 20′, and on that day it stayed above the physical horizon for $8^h\ 56^m$; but to that parallel is due a stay of $8^h\ 56^m$ above the astronomical horizon; therefore it had no parallax, or else it was above Venus and about as far from the Earth as the Sun, since otherwise it could not have been seen for so many hours. The reply is, first, to deny that it was in such a parallel on that day, since Tycho says that its declination then was 18° 50′. Secondly, the argument from the visible stay to the distance from the Earth is not valid so long as the parallax is unknown, as we said in chapter 15, in Corollary 1 of Problems 7 and 8.

An Claramontius ex obſeruationibus Tychonis, Lantgrauij, Mœſtlini, Cornelij Gemmæ & Rœslini demonſtrarit Cometam Anni 1577. fuiſſe Sublunarem.

Whether Chiaramonti demonstrated, from the observations of Tycho, the Landgrave, Mästlin, Cornelius Gemma and Röslin, that the comet of the year 1577 was sublunar.

XXXIV. Chiaramonti certainly promised this in Book 3 of the *Antitycho*, pages 261 and 269, and affirms that he demonstrated it on pages 289, 319 and 321, and in the *Apologia* for the *Antitycho* from page 255. To this end, *first*, he himself candidly admits, in Book 3 of the *Antitycho*, chapter 3, that among the other problems he had proposed in Book 1 he chose Problem 1 of chapter 7, because he had newly devised it himself. The problem runs as follows: *Given the seen latitudes and longitudes of some phenomenon at the same time, and hence given the difference of the parallaxes relative to the ecliptic with respect to two places on the Earth differing in latitude alone, or in longitude alone, or in both, provided their difference is given, and given the point of the ecliptic then standing on the meridian: to investigate the parallax of the phenomenon in the vertical circle, or circle of altitude, appropriate to each place on the Earth.* This problem is solved by solving no fewer than 16 spherical triangles, which Scipione laboured through, most patiently, more than once; and we shall give a specimen of this most ingenious problem below, in chapter 19, numbers 11 and 12. *Secondly*, from the record of the Landgrave's observations, which Tycho reports in volume 2 from page 211, and from the observations of Tycho himself, he assumes: that in the year 1577, on the first day of December, at $6^h\ 45^m$ after noon at Kassel, the comet's azimuth was observed to be 59° 25′ and its seen altitude 31° 0′; that it was then $7^h\ 10^m$ at Uraniborg,

this place being more easterly than Kassel by 25^m of time; that therefore the two observations fell at the same moment of time; that the difference of the observed longitudes was 10′ 15″, and the difference of the seen latitudes 24′; and finally that the complement of the polar altitude is 34° 7′ at Uraniborg, as Tycho puts it throughout volume 2, and 38° 41′ at Kassel.

[Margin: The sum of Chiaramonti's calculation.]

Thirdly, from the data or assumptions just mentioned, after the analysis of 16 triangles, he concludes on page 289 that the comet, seen from Kassel, had a parallax of 12° 27′ 47″, and from Uraniborg one of 12° 29′ 38″. From this, and from the Earth's semidiameter of 3035 miles, he gathers on the following page the comet's distance from the centre of the Earth as 12,657 miles at Uraniborg, but 12,059 at Kassel.

XXXV. However, in the assumptions just mentioned, not everything seems to have been taken precisely from Tycho himself. For Tycho, in volume 2, page 87, gives for the first day of December the comet's seen longitude in Aquarius 16° 21′ and its northern latitude 24° 47′, and for December 2 longitude ♒ 17° 38′ and latitude 25° 6′, both for 6^h after noon. Hence, keeping the proportion, the longitude for $7^h\ 10^m$ on the 1st would have been 16° 24′ of Aquarius, and the latitude 24° 48′. Then Tycho, on page 224, from the Landgrave's observations of December 1, deduces for $6^h\ 34^m$ at Kassel a longitude of the comet in Aquarius of 16° 38′ and a latitude of 24° 20′, and for $7^h\ 49^m$ longitude ♒ 16° 49′ and latitude 24° 33′; hence at $6^h\ 45^m$ the longitude would have been ♒ 16° 40′ and the latitude 24° 22′. So, for Tycho, the difference of the longitudes was 16′ and that of the latitude 26′. Yet from these very figures Scipione could have gathered a very large parallax of altitude. Nor was the difference of the meridians, which Chiaramonti puts at 25^m in time, taken from Tycho: for Tycho, in the *Epistolae Astronomicae*, pages 58 and 72, puts the difference between Uraniborg and Kassel at 12^m or 13^m in time, or a fifth part of an hour — as much as both Longomontanus and Kepler in their catalogues, and we from eclipses, have gathered, although Wendelen thinks it is only 5^m. So, on Tycho's data, that observation was not made at the same moment in both places; it was adapted by Chiaramonti, from the proportion of the motions, to suit his calculation and the example of the problem he had devised. Moreover, if they had observed at the same moment, and the parallax had been so great, then at Kassel, the less northerly place, the comet would certainly have been seen more northerly than at Uraniborg, the more northerly place, since the parallax depresses the comet towards the horizon more with respect to Uraniborg than with respect to Kassel. But the opposite is found from the observations: the seen latitude was 24° 22′ at Kassel and 24° 48′ at Uraniborg. I pass over other points that Kepler, in the *Hyperaspistes* from page 129 to 136, objects against Chiaramonti — above all this one: that from the Landgrave's azimuths, altitudes and times neither correct places of the comet nor correct parallaxes are elicited.

[Margin: Chiaramonti's second example.]

XXXVI. But Chiaramonti, in Book 3 of the *Antitycho*, chapter 5, saw most shrewdly that the former example was not adequate for his purpose, because in it the comet's northern latitude was seen smaller at Kassel than at Uraniborg, when the opposite ought to have been seen. He therefore chose two other observations, of November 16: on that day at Kassel, at $7^h\ 16^m$, the seen longitude was ♍ 16° 35′ and the latitude 13° 59′; at Uraniborg it was then $7^h\ 41^m$, and the comet's seen longitude was then ♍ 16° 50′ and its latitude 13° 45′, as is gathered from Tycho's ephemeris and from the comet's daily motion in longitude laid down by Tycho himself. Accordingly, although Tycho did not observe at the same moment, his observation is nevertheless reduced to the same moment by keeping the proportion of the motion, in the way Tycho himself often uses. The difference was therefore 15′ in longitude and 14′ in latitude; and indeed the seen latitude was greater at Kassel than at Uraniborg, as the parallax in latitude required. From this difference of 15′ and 14′, then, repeating the calculation by the problem already mentioned, he gathers, correctly and without any error of calculation, a parallax of the comet of 7° 29′ in the circle of altitude with respect to Uraniborg, and a distance from the Earth of 24,276 miles; with respect to Kassel, the parallax is 7° 28′ 24″ and the distance 24,113 miles. But here too, in reducing the seen longitudes and latitudes to the same moment, Chiaramonti supposes a difference of time of 25^m between Kassel and Uraniborg; yet it is not greater than 13^m, from which there would follow, between the Landgrave and Tycho, a difference of longitude of only 8′ and of latitude of 7′. And yet even from this a parallax of altitude greater than the Moon's is gathered. But if the difference of the longitudes had been 15′ and of the latitude 14′, Chiaramonti's demonstration would hold.

[Margin: Mästlin's observation compared with Tycho's.]

XXXVII. Using, further, the same method and the same problem, Chiaramonti, in Book 3, chapter 8, gathers the difference of longitude and latitude from certain of Tycho's observations compared with Mästlin's and reduced to the same moment by keeping the analogy of the motion; and whenever he finds it equal to 15′ in longitude and 14′ in latitude, or greater, he at once asserts a parallax greater than 7° at the least, and a distance of the comet from the Earth not greater than 24,278 miles. For example, in chapter 8 Chiaramonti considers the observations of November 12, on which day Tycho, from Mästlin's data, in volume 2, page 261, though using corrected places of the fixed stars, deduces for 6^h the comet's place at 4° 2′ of Capricorn and its northern latitude at 6° 34′. But from Tycho's ephemeris, in volume 2, page 86, the comet's longitude for the 6th hour of that day is given in Capricorn 3° 27′ 20″ and its northern latitude 6° 56′, and so a difference of longitude of 35′ and of latitude of 22′. Tycho, however, reckons that the place where Mästlin observed lies further west than Uraniborg by the space of half an hour, a time to which, keeping the analogy of the motion as Tycho usually does, about 5′ correspond. So when it was $6\frac{1}{2}^h$ at Uraniborg, it was 6^h at Mästlin's place; or, when it was 6^h at Uraniborg,

it was 5½^{h} at Mästlin's place. In either case, it follows that the difference of longitude cannot be cut down by more than 5′, nor that of latitude by more than 2′. From these data, therefore, the difference of longitude was 30′ and that of latitude 20′. Therefore, says Chiaramonti, if a difference of only 15′ in longitude and 14′ in latitude between the Landgrave and Tycho yielded a parallax of either 12°, or at least 7°, how much more will a difference of 30′ in longitude and 20′ in latitude yield it — and so a distance from the Earth not greater than 24,278 miles? And how much more does this very thing follow if at some time the difference of the longitudes grew to as much as a degree and a half, as Tycho admits that it did? Moreover, to say that Mästlin's observations are imperfect because they disagree with Tycho's is not to use the observations as they lie, nor to keep to Chiaramonti's hypothesis: for he does not affirm that so great a nearness of the comet is demonstrated absolutely from them, but only if some observations of both men are accepted, however much they were corrected in some respect by Tycho.

[Margin: From this comparison a parallax of the comet greater than the Moon's is deduced.]

XXXVIII. If you have understood Chiaramonti's reasoning set out in the preceding number, you will easily understand also the reasoning by which, in Book 3 of the *Antitycho*, he gathers the difference of the longitudes and latitudes — in chapter 9 from the observations of Tycho and Cornelius Gemma, and in chapter 10 from those of Tycho and Röslin — and, wherever it is greater than 15′, confirms from this, not absolutely but on a hypothesis (that is, if those observations are accepted as they lie), that a parallax of the comet of more than 7° follows. Here, then, are the differences of longitude and latitude, from Tycho himself, volume 2, from page 290:

On November 16, the comet's longitude greater for Gemma than for Tycho by 40′, and the latitude by 20′.

On the 19th, the longitude smaller for Gemma than for Tycho by 1½°; the latitude greater by 15′.

On the 25th, the longitude smaller for Gemma than for Tycho by 2½°; the latitude likewise greater by 15′.

On the 26th and 27th, Gemma's longitude smaller than Tycho's by more than a degree, and the latitude greater by a whole degree.

On the 29th, the longitude smaller for Gemma than for Tycho by 1° 45′.

On the 30th, the longitude smaller for Gemma by 1° 33′, and the latitude greater by 1° 34′.

On the 30th, the longitude smaller for Röslin than for Tycho by 2°, and the latitude greater by 1° 30′.

On January 12, the comet's longitude smaller for Röslin than for Tycho by 4½°, and the latitude greater by almost as much, again for Röslin.

Nor is the difference between the meridians of Uraniborg, where Tycho observed, of Louvain, where Gemma observed, and of Zabern in Alsace, where Röslin observed, so great that, by reducing the observed longitudes and latitudes

to the same moment of time by the proportional part of the motion, a difference of longitude and latitude smaller than those 15′ could be obtained. Nor, again, is the difference in the polar altitudes so great that the demonstration which held for the Landgrave's observations compared with Tycho's would not hold for these as well. Nor can Tycho reject the analogy of the motion in longitude, since he himself uses it very often, and gathers by the proportional part, from the comet's daily motion, how much motion belongs to each hour.

[Margin: Kepler's reply insufficient.]

Nor, finally, does Kepler answer this sufficiently in the *Hyperaspistes*, page 138, by reproaching Chiaramonti for having fished out from the more imperfect observations of Mästlin and Gemma parallaxes of the comet greater than the Moon's, while leaving aside and passing over in silence the more perfect ones, from which Tycho elicited parallaxes smaller than the Moon's. For Chiaramonti does not proceed absolutely, but on a hypothesis — namely, if certain of their observations are accepted; just as he says in the *Apologia* for the *Antitycho*, in part 5 of the preface, that there are certain observations of Tycho and the Landgrave which, if they are accepted, show the comets of the years 1577 and 1580 to be sublunar, and certain others which show them to be above the Moon. Further, if it is not established on other grounds, rather than from an opinion already prejudged about the place of comets, whether observations are imperfect, they are not to be condemned merely because they disagree with Tycho's opinion and parallaxes. The rest that Chiaramonti says in Book 3, chapters 12 and 13, about the refraction of this comet, so as to increase the comet's total parallax, we pass over as less necessary for us and less certain. Let those who care to pursue these matters at greater length look into them.

Reliquæ Opiniones de Parallaxi & Loco huius Cometæ infra Lunam.

The remaining opinions on the parallax and place of this comet below the Moon.

XXXIX. Among the many who, from their own observations, hold that the comet of the year 1577 was sublunar, Tycho counts, in volume 2 of the *Progymnasmata*, page 318, Tadeáš Hájek ab Haych, who observed it at Prague and, at an altitude of 38°, assigned the comet a parallax of 5° and some minutes; but after seeing Mästlin's writings he changed his opinion and conceded that this comet was above the Moon, as Tycho relates on page 366.

[Margin: The parallaxes of this comet according to some.]

Bartholomäus Scultetus of Görlitz, by contrast, gave this comet a parallax of 5° 22′ at an altitude of 29°; Andreas Nolthius of Einbeck a parallax of 5° 32′ at an altitude of 33°; and Nikolaus Winckler, who observed at Schwäbisch Hall, assigned the same comet a parallax of 6° 50′ at an altitude of 46° — on the testimony of the same Tycho, page 318, who afterwards disapproved of them because of these authors' imperfect instruments, or other signs of imperfect observations. Finally, in volume 2, from page 449, he lists several others who placed this comet below the Moon without mathematical proof. Such were Johannes Praetorius, the astronomer of Nuremberg; Thomas Erastus; Simon

Grinaeus; Conrad Dasypodius of Strasbourg; Georg Henisch of Augsburg; Nicolaus Bazelius, a Batavian; Valentin Steinmetz of Gersbach, professor at Leipzig; Ioannes Huernius of Utrecht; Theodor Graminaeus of Cologne; and Georg Busch of Erfurt, a painter rather than a mathematician, who attributed to this comet a parallax of 2° 21′ and, from inexperience, thought that the seen azimuth differed from the true by as many degrees and minutes — whereas, however great the parallax of altitude may be, the true and the seen place of any phenomenon lie in the same vertical circle, and so in the same azimuth, as Tycho rightly points out on this occasion in volume 2, page 456.

Concluſiones ex dictis, ac Noſtra Opinio.

Conclusions from what has been said, and our own opinion.

Conclusion I. *Neither by Tycho nor by anyone else, so far as I know, has it so far been demonstrated absolutely that the comet of the year 1577 was above the Moon.* This is clear from what was said from number 1 to 34 — even though Magini, in his letter to Tycho, Froidmont, in Book 3 of the *Meteorologica*, chapter 1, article 1, and very many others believed that Tycho had demonstrated it, as did Cysat too, on page 33.

Conclusion II. *Yet, if some hypothesis is made and some observations are accepted, it is demonstrated that the comet of the year 1577 was above the Moon; while, if certain other observations are accepted, it is demonstrated that it was below the Moon.* The first part is established by the many arguments of Tycho already adduced above — namely, if it is admitted that the comet travelled along a great circle and, within a few hours of the same night, advanced with a proper motion uniform and proportional to the motion of one whole day, and that the observations of Tycho and the Landgrave, though made with smaller instruments and not emended by the correction for refraction, could not have produced a very sensible difference; which the more skilled astronomers will scarcely concede. The second part is certain from what was said from number 34 to 39 — namely, if certain other observations are accepted as they lie, or even as corrected by Tycho, and the comet's proper motion within a few hours is likewise accepted as uniform.

Conclusion III. *Although it is not improbable that the comet of the year 1577 was below the Moon, and indeed very near the Earth, though higher than the twilight vapours and exhalations, it is nevertheless more probable that it was above the Moon.*

That this comet was not lower but higher than the vapours and exhalations that make the twilight appears clearly enough, first from their altitude, which we showed in chapter 14 does not exceed 100 Italian miles and is indeed much less; and then from the altitude assigned to the comet even by those who pressed it down furthest below the Moon. For Chiaramonti, who of all of them strove most to bring it near the Earth, taking account of parallax and refraction, in Book 3 of the *Antitycho*, chapter 13, assigns to this comet, at an altitude of 4°, a parallax of 19° 35′ and a distance of 8333 or 9025 Italian miles, of which the Earth's semidiameter has 3035. Scultetus, on the other hand, as Tycho reports in

volume 2, page 409, says that it was 8047 miles and 1754 paces from the centre of the Earth; but he is speaking of German miles.

That it is not improbable that this comet was below the Moon is clear both from the hypothetical demonstrations discussed in Conclusion 2, and from the opinions of many authors discussed in number 39; these will keep their probability for as long as the opposite has not been absolutely demonstrated.

But that it is more probable that it was above the Moon follows not only from the authority of the most eminent mathematicians — such as Tycho, Mästlin, the Landgrave of Hesse with his men, Hájek, Cornelius Gemma and Röslin, besides those who held that all comets, or many, and this one among them, were celestial, whom I have already listed in chapter 6 — but above all from Tycho's arguments. These are so many, and established by so great a variety of confirmations, that they fall little short of exact mathematical demonstrations. For many of those arguments rest on the measure of the comet's proper motion decreasing in an orderly way, which is more probably granted than denied, since the way of gathering the quantity of that motion is very like the way in which, in the planets, the motion of the intermediate days is gathered from the motion of a few days.

CAPVTXIX.

Chapter 19.

De Cometarum reliquorum ab Anno 1577. ad Annum 1590. Parallaxi & Altitudine.

On the parallax and altitude of the remaining comets from the year 1577 to the year 1590.

I. In the year 1580, from the beginning of October to the middle of December, a comet was seen, as Tycho says in volume 2, page 267. Mästlin, Rothmann, Hájek and Röslin wrote about it, affirming that it was above the Moon, which Tycho, on that page 267, promises to demonstrate in volume 3 of the *Progymnasmata* [?]. Mästlin, indeed, did not hesitate to raise it as far as the sphere of Saturn, and Tycho, on page 442, even beyond the eighth sphere. Froidmont subscribed to these views in book 3 of the *Meteorologica*, chapter 1, article 1. But Chiaramonti opposes them in book 3 of the *Antitycho*, chapter 18, on account of the difference of 18′. About these minutes Tycho himself, in a letter to the Landgrave of Hesse dated 1592 February 20, old style — in which, on page 267 of my copy, these words of Tycho's stand — wrote: *I have written, moreover, to Your Highness's mathematician about those eighteen minutes that are everywhere lacking in the observations of the comet of 1580 communicated to me. For whenever this comet crossed the meridian, I found that exactly 18 minutes less than was right had been observed at Kassel, whereas my observations made at the same time always have just as many minutes more. From this I conjecture that such a defect was committed because the quadrant, through a fault of the plumb-line, was not correctly*

set by so many minutes. I also conferred about this matter with Rothmann while he was here, and demonstrated it by a comparison of your observations and mine.

Chiaramonti, however, both above and in the *Apologia* for the *Antitycho*, pages 279 and 289, thinks it improbable that an error of so many minutes was committed by the Landgrave's mathematician, since elsewhere, in the *Epistolae Astronomicae*, page 39, Tycho wrote that the error due to the plumb-line was one or two minutes in the Landgrave's observations. Instead he judges that the discrepancy came from parallax, because of which the comet was seen to stand farther from the vertex of Kassel than Tycho would otherwise have it; and from this he draws out a parallax much greater than the Moon's. To do so he takes from Tycho that this comet was hidden from a great part of Germany in the lower part of the meridian, and from this he gathers that it could not have been less than 81° from the vertex of Uraniborg. Otherwise — if less, or indeed if it had been only 81° away — then, subtracting 34° 7′, that is, the complement of the pole's altitude at Uraniborg, from 81°, there remain 46° 53′, that is,

the comet's distance from the pole, which is the geographical latitude at which Germany begins. But if the comet had been 86° from the vertex of Uraniborg, it would not have been visible at Kassel, which lies 4° 34′ farther south than Uraniborg; for with these added to the 86°, the comet's meridian distance from the vertex of Kassel would come to 90° 34′, and so, unless the refraction had been enormous, it would then have lain hidden below the horizon. He therefore assumes what is, as it were, an intermediate distance between 81 and 86, namely 84, so that from this it may appear what is to be judged of the intermediate distances.

Problema Claramontij.

Chiaramonti's problem.

[Margin: Step 1.]

II. With these assumptions: first, in the following figure, let the place of Kassel be at A and that of Uraniborg at I, through which let the vertical lines TAR and TIQ be drawn from the centre of the Earth, and let those places be joined by the chord AI. Let the comet be at L, and through it let the lines of the seen place, ALP and ILO, be drawn, and also the line of the true place, TLM. For the seen place of the comet at Kassel will be P, and at Uraniborg O, but its true place M; hence the parallax at Kassel will be the arc PM, which the angle ALT measures, and at Uraniborg the arc OM, which the angle TLI measures; hence the difference of these angles, that is, the angle ALI, will be the difference of the parallaxes. Now let the Earth's semidiameter AT be given, which Chiaramonti assumes

Diagram. A woodcut set in the right column: a large semicircle on the base S–T–Z, a concentric middle arc from V to X, and a full small circle about T (the Earth) reaching down to u; a horizontal H–T–N across the small circle; a vertical from T up to R; straight lines from T and from points on the small circle out to the arcs.

- R (top of the outer semicircle, on the vertical from T)
- D. (on the outer arc, just right of R; printed with a following dot)
- D (on the outer arc, farther right)
- Q (upper left of the outer arc)
- M, O, P (left of the outer arc, from top to bottom)
- L (on the middle arc, left)
- A (above T, near the centre)
- G, C (right of A, on the inner lines and middle arc)
- E, F (right of the outer arc)
- m (small letter at the right end of a horizontal line)
- I, B (on the small circle, left and right of the vertical)
- H, N (ends of the horizontal across the small circle)
- S, V, T, Z, X (along the base, from left to right; Z is a Z-shaped sort with a descending tail)
- u (small letter at the foot of the small circle)

to be 3035 Italian miles. Let this be, under another ratio of parts, the whole sine, or radius, of 100,000 parts. Let the arc AI be 4° 34′, the amount of the difference in the pole's altitude between Uraniborg and Kassel, and its chord 7968 of the parts of which AT is 100,000; therefore, of the parts of which AT is 3035, AI is 241 miles — a figure to be kept in reserve.

[Margin: Step 2.]

Second, let the comet's seen distance from the vertex of Uraniborg, that is, the angle LIQ, be 84°. The angle ATI, moreover, is 4° 34′ in the isosceles triangle ATI; therefore the sum of the angles at the base, by I.32 of Euclid, is 175° 26′, and each angle separately, namely TAI as well as TIA, is 87° 43′, by I.5. Hence the adjacent angle, by I.13 — namely IAR, as also QIA — will be 92° 17′. Adding, then, QIA and LIQ, the whole angle LIA comes to 176° 17′. Now let the difference of the parallaxes, that is, the angle ALI, be given as 17′. For in the triangle AIL, from the two angles AIL and ALI and the side AI of 241 miles, the comet's distance from the Earth's surface, that is, IL, is gathered by the rules of triangles to be 2921 Italian miles.

[Margin: Step 3.]

Third, in the triangle TIL there are given TI, 3035 miles, IL, 2921, and the angle TIL, 96°; for it is the supplement of the angle LIQ to two right angles. Therefore, by solving this triangle, the distance TL is found to be 4452 miles, and the parallax with respect to Uraniborg, namely the angle TLI, 42° 59′; adding to this ILA, 17′, the parallax TLA with respect to Kassel comes to 43° 16′.

[Margin: Chiaramonti's ingenuity in hunting down parallaxes.]

III. But since that difference in the comet's distance from the vertex of Kassel could have appeared greater than it should because of some larger error in the observation, and the difference of the parallaxes might not reach 18′ or 17′, and the comet's seen distance from the vertex might be some other value between 81° and 86°, what did the most ingenious Chiaramonti do? He assumes a

difference of the parallaxes, that is, an angle ALI, of at least one minute, or even half a minute, and a seen distance from the vertex now of 84°, now of 86°, and by the same method described in number 2, with calculations that are truly exact and checked by me, he gathers the distance of the comet from the Earth, and the parallaxes belonging to Uraniborg and to Kassel. I have decided to bring all of these together into a single synopsis; you should not be surprised if it lacks things that are not found worked out in the author himself.

Given: difference of the parallaxes	Given: seen distance of the comet from the vertex of Uraniborg	Resulting distance of the comet from the centre of the Earth, less, in Italian miles	Parallax of altitude: at Uraniborg	Parallax of altitude: at Kassel
17′	84°	4452	42° 59′	43° 16′
17′	86°		64° 43′	65° 0′
1′	84°	56696		
1′	86°	27691		
½′	84°	89554		
½′	86°	54368		

But who would not grant that the difference of the parallaxes, free of refraction, was at least half a minute? Therefore, from the observations of the Landgrave and of Tycho, if they are taken as they stand in the authors and are not corrected on the presumed opinion that comets are to be carried up into the heaven, it follows that this comet was below the Moon — not in the sphere of Saturn or of the fixed stars — and was not more than three semidiameters of the Earth away from the Earth. Tycho, moreover, could not spurn the Landgrave's observations made in 1580, since he had already accepted the observations of 1577 and had tried to confirm his opinion from them, as we said in the preceding chapter.

[Margin: Kepler substitutes insults for replies.]

But Kepler, in the *Hyperaspistes*, from page 165 to 171, leaves this problem untouched and unexamined; he meets Chiaramonti with nothing but insults and his usual subterfuges, and looks down on him as "trampled underfoot", as he himself puts it. This is quite undeserved: such a thing was not to be expected from an astronomer, let alone from a Christian man — a thing that Ptolemy, pagan though he was, would certainly not have dared. Kepler erred, moreover, in holding that the comet was seen towards the south; for if that were so, it would have been seen in Italy, where it was nevertheless not seen, as is gathered from Santucci, who by order of Cosimo, Grand Duke of Tuscany — as is clear from his dedicatory letter to Cosimo II — observed and described the comets of these years, and makes no mention of this one.

De Cometa Anni 1582.

On the comet of the year 1582.

IV. Kepler mentions this comet in the *Phisiologia Cometarum*, saying that it turned retrograde at the beginning of Sagittarius and nearer the ecliptic; but

he has nothing about its seat or its parallax. Antonio Santucci, in chapter 13 of his work on comets, says that it rose about the beginning of March and ceased to be seen around the middle of April. Now on March 10 the comet's stay above the natural horizon of Rome was observed to be $14^h\ 10^m$, when it was in a northern parallel declining 17° from the equinoctial. For the Sun, which was in ♓ 20°, set at $5^h\ 48^m$ after noon, while the comet was on the meridian at $2^h\ 50^m$ after noon and set at $9^h\ 55^m$; its semidiurnal arc was therefore $7^h\ 5^m$, and its diurnal arc $14^h\ 10^m$. In that parallel, moreover, the Sun's true diurnal arc is $14^h\ 10^m$, so that there was no sensible difference between the true and the apparent diurnal arc. Since this is so, Santucci concludes that the comet was above the Moon, and at least in the sphere of the Sun.

But Chiaramonti, in the supplement to the *Antitycho*, part 2, chapter 16, says that Santucci could not have observed the comet's arrival at the meridian, because the Sun was then above the horizon, and that this was invented from Santucci's own hypothesis and imagination. Next, he says that with the parallax unknown, the true parallel in which the comet stood cannot be known from its seen stay above the horizon; and with that parallel unknown, the comet's distance from the Earth cannot be demonstrated from that stay. That this is true you will learn from what was said in chapter 15, problems 7 and 8, corollary 1. So it has not yet been demonstrated to me that this comet was above the Moon, although that is not without probability, if what Santucci relates is true.

De Cometa Anni 1585.

On the comet of the year 1585.

[Margin: The opinion of Tycho and of Rothmann on the parallax of this comet.]

V. Tycho treats this comet in volume 2, chapter 10, where on page 268 he has this: *Just as the comet seen in the year '85, just past, clearly testified the same thing; for by my most accurate observations it was found to admit a parallax of scarcely one minute; as also the mathematician of the most illustrious Prince Wilhelm, Landgrave of Hesse, Christoph Rothmann, a most learned man and excellently skilled in astronomical matters, demonstrated from its apparent motion that this comet had no parallax at all.* But he treats this very matter at greater length in the *Epistolae Astronomicae*, from which we shall select what bears on our subject. Thus on page 4 he reports these words from a letter of the Landgrave, written at Rotenburg to Tycho, 1585 October 20, old style: *Our mathematician Christoph Rothmann has recently written to us that on the 8th of this month, at 9^h in the evening, he observed the comet in ♓ 23° 15′ with a southern latitude of 14° 0′, and on the 9th of the same month in ♓ 25° 50′ with a southern latitude of 13° 6′. We ourselves noted it yesterday, the 19th, in the evening — observed not with any instrument but only by straight lines on our globe — and found it in ♈ 21° with a latitude of 4° south. It is small, and hairy on every side, so that we judge it to be of the kind of comets they call circaean [circæos], which usually appear at the time of the greatest conjunctions. What it portends, especially since it is Saturnine and conjoined with Saturn, time will tell.*

[Margin: A circaean comet, and Saturn conjoined.]

To this Tycho replies, on page 6, that because of clouds and storms he could observe this comet only ten days after Rothmann, that is, from October 18. He narrates his observations at length from page 13 to page 20; in them he records the comet as observed on October 18, 20 and 22, and on November 4, 5, 6, 7, 9 and 12. Of the remaining days he says: *The sky was so covered with clouds that the comet could not be observed with any instruments during all that time* — granted that he wove the continuous ephemeris I mentioned in chapter 3 out of conjecture from the other days.

He affirms, moreover, that its motion was along a great circle, though with its inclination to the ecliptic varying by 45′ from beginning to end, and that it decreased regularly; and he adds: *So that from the 18th to the 19th, about the time of the first observation we made, it was 2° 18′; and on the following days it became successively slower, so that from the 4th to the 5th of November it advanced only 53½′, and from the 12th to the 13th of the same month only 34′.* From this he gathers that it was not a fiery meteor generated in the elementary region, but in the heaven. This, however, he tries to confirm with stronger arguments taken from parallax, which are as follows.

[Margin: Tycho's first argument.]

VI. From October 18 at $10^h\ 15^m$ to $13^h\ 30^m$, that is, over 3¼ hours, the comet's apparent motion along its own path is gathered, from its observed declinations and right ascensions, to have been 17½′. *If with this*, says Tycho, *one compares the daily course — found from some definite position until, after one revolution of the primum mobile, it returns to a like one (for parallax, if there were any, introduces no difference from the true motion in this way) — that course is found, taking account of the observations made on the two following days, to be 2° 18′. Therefore, according to this daily motion, the comet ought truly to have advanced 18⅔′ in the stated interval of time, the motion by the apparent observation falling short by not much more than a single minute, by which, at least, the parallax anticipated the true course.* But if the comet had been in the concave of the lunar heaven, Tycho says likewise, at a distance from the Earth of only 52 semidiameters of the Earth and 17′, it would have induced a parallax of 27′ in its proper motion, and between the first observation and the second there would have been a difference of almost 26′, which could have been perceived even with a very small instrument.

[Margin: Tycho's second argument.]

In the same way, on October 22, from $13^h\ 15^m$ to $16^h\ 10^m$, that is, over $2^h\ 55^m$, the comet advanced along its [?] arc by 12¾′, from the declinations and right ascensions observed; whereas, at the rate of its daily motion, which was 1° 53′, it ought in the meantime to have completed 13⅚′. And so here too the apparent motion differs from the true motion by not much more than a single minute; but if the comet had been near the confines of the Moon, it would have produced a far greater discrepancy in its motion.

[Margin: Tycho's third argument.]

Again, on November 5, observing in the same way, from $7^h\ 50^m$ to 13^h

30^m, the comet was found to have covered 11⅓′ along its own path in the space of 5^h 40^m, whereas with respect to its daily motion (which, by comparison with the following day, was 50′) it ought in the meantime to have covered 11⅚′. The observed motion thus fell short of the true motion by only half a minute, and so the parallax turns out almost insensible. But if the comet had been in the concave of the Moon, it would have had a parallax in its proper motion of 47⅔′, as is clear from the computation of triangles.

[Margin: Tycho's fourth argument.]

But also, on the 6th, from 6^h 5^m to 14^h 55^m, that is, over 8^h 50^m, the comet's apparent motion along its path was 16½′; but its daily motion was 46′, so that in 8^h 50^m it ought to have been 16′ 56″. Hence, on account of parallax, the apparent motion fell short of the true by not even half a minute; and yet over so long an interval of time, if the comet had been in the concave of the Moon, it would have had a parallax of 1° 1′ — a discrepancy that could have been detected without instruments, by sight alone, against the neighbouring fixed stars.

[Margin: The suspicion of refraction is removed.]

And lest anyone say that the parallax was diminished by refraction, he says that on October 18 the comet was 37½° high at the earlier observation and 31⅙° at the later; on November 5, 39¾° high at the earlier observation and 48⅓° at the later; and on the 6th, 26° high at the earlier observation and 38° at the later — altitudes at which there is almost no refraction.

[Margin: Tycho's fifth argument.]

Finally, he says that this comet, observed at different hours in a higher and a lower position, kept the same distance from Capella, towards which it was directing its course, with no other difference besides the one its daily motion required. From this, on page 18, he concludes that this comet was in the highest aether, above the Sun, or not much below it. He was greatly confirmed in this opinion by the agreement of his own observations with the Hessian ones obtained by Rothmann, and by the opinion of Rothmann himself and of Prince Wilhelm, who said that no parallax, or an insensible one, had been detected in this comet, as Tycho testifies in the *Epistolae Astronomicae*, pages 37, 57 and 58. There, however, he admits that in the longitude of this comet there was at times a discrepancy of 6′ between himself and Rothmann, and of 2′ in latitude, which he thinks arose from uncorrected places of the fixed stars. It is no wonder, then, that Froidmont, subscribing to Tycho and Rothmann in book 3 of the *Meteorologica*, chapter 1, articles 1 and 2, taught that the comet of 1585 was above the Moon.

Solutio Argumentorum Tychonicorum ex Claramontio, & Kepleri Reſponſio.

The solution of Tycho's arguments, from Chiaramonti, and Kepler's reply.

VII. I have reported the arguments and their foundations from Tycho himself; you will find them set out afterwards, in a somewhat different form and more briefly, by Chiaramonti in book 2 of the *Antitycho*, chapter 21, and their solution

in chapter 22. There, first, he denies that the comet's motion was along a great circle, since, as Tycho himself grants, it changed its inclination to the ecliptic by almost a whole degree. Second, he denies that the motion was exquisitely regular. Third, he points out that Tycho calls the daily motion true, although it was only the seen motion, deduced from observations, and with the parallax unknown could not be distinguished from the true motion; and since the comet's motion was not along a vertical circle, it is wrongly denied that any sensible difference between the true and the seen daily motion arises from parallax. Fourth, he affirms that a great refraction could have occurred even at an altitude of 31°; for in book 3, chapter 13, he promises to demonstrate that the comet of 1577 suffered at Kassel a refraction of 11° 7′ at an altitude of 31° — which, however, he shows not absolutely but on a hypothesis. Fifth, he denies that from the persisting equality of the comet's distance from Capella, observed at different hours of the same night, an evident argument is gathered for no parallax, or an insensible one; for in book 1 of the *Antitycho*, chapter 10, he says he has demonstrated that a case occurs in which a phenomenon has a large parallax and yet this is tempered from another source, by reason of the angles; and he notes there, on pages 93 and 233, that this very case occurred in this comet. Sixth, he denies that from the agreement between Tycho and Rothmann, with no difference exceeding one minute, it follows that the comet lacked parallax; for he had demonstrated, in book 2, chapter 20, pages 223 and 227, that there are many cases in which two observers differ by a single minute in the seen place of a phenomenon even though it has a large parallax — which we too shall show in Book 10, the penultimate section, chapter 11, numbers 11 and 12.

Kepler, however, in the *Hyperaspistes*, from page 106 to 112, replies that in a comet a motion along a great circle is not to be required more exquisitely than in the Moon, nor a motion regular according to a continuous proportion in its increase or decrease, since not even the motion of the planets is regular in this way, and yet it is considered ordered and worthy of the heaven. Next, he replies that although the termini of the seen and the true place differ, the spaces lying between the true places are equal to the spaces lying between the seen places, because after exactly 24 hours the same position and the same parallax return; and so the true daily motion is rightly gathered from the seen one, from the comet's return to the same position after 24 hours. Afterwards he says that Chiaramonti's propositions — by which he tries to demonstrate that a phenomenon, notwithstanding a large parallax, can keep the same visible distance from the same fixed star at different hours — are wrapped in darkness, partly through the fault of the printer, partly of the engraver, partly of the author. He does not, however, deny Chiaramonti's demonstration, but says that it is not rightly applied to this comet, because from the ephemeris of its motions and from inspection of the globe alone it is clear enough that Capella was always nearer the vertex than the comet, especially in the western quadrant

of the world; and so, if the comet had had a quite sensible parallax, its seen distance from Capella would have become ever greater and greater.

[Margin: Kepler convicted of falsity by Chiaramonti.]

Yet Kepler demonstrates nothing but is content with bare assertions, and for that reason he is deservedly reproached by Chiaramonti in the *Apologia* for the *Antitycho*, from page 204 to 212, where Chiaramonti convicts certain propositions of Kepler's of falsity — and does so by bringing geometrical demonstrations, as any geometer reading Chiaramonti will easily see.

Noſtra Opinio de Argumentis Tychonicis.

Our opinion on Tycho's arguments.

VIII. For my part, I see that not a few objections can be made to Tycho which render the conclusion about the comet's place uncertain. First, in the first and second arguments, the comet's proper and daily motion — from the 18th to the 19th 2° 18′, and from the 22nd to the 23rd 1° 53′ — is posited not from observation on the 19th and the 23rd, but only from observation on the 18th, 20th and 22nd. For Tycho affirms in the *Epistolae Astronomicae*, page 14, that he observed the comet on these days only (speaking of October), and that on the others it went unobserved because of clouds. Hence, when he says on page 18 that the comet's motion was gathered *taking account of the observations made on the two following days*, this must be understood of the 20th and the 22nd, lest Tycho be accused of falsehood. Nor, again, is any observation set down there from October 22 to November 4, because, as is said on page 14, through the 13 whole days that elapsed no observation could be made with instruments for lack of clear weather. Since, then, he found the motion from the 18th to the 20th to be 4° 29′, from the 20th to the 22nd 4° 3′, and from the 18th to the 22nd 8° 32′, he concluded that from the 18th to the 19th it was 2° 18′ — probably enough, but not evidently. For from the 18th to the 19th the comet could have been faster and covered almost three degrees, and then, having become slower from the 19th and 20th, covered a degree and a half, so that from the 18th to the 20th it completed 4° 29′. The motion from the 22nd to the 23rd is much less evident still; for what happened from October 22 to November 4 is not established by any observation, and from the 22nd to the 23rd the comet could have completed more than 1° 53′.

Second, as I have already said in chapter 17, number 5, proposition 19: from the fact that a phenomenon observed from a single place on the Earth at different seen altitudes appears to undergo a smaller difference of altitudes or of parallaxes than the Moon would undergo at the same altitudes, it is not evident that it has a parallax smaller than the Moon's. The reason is that the comet could, by reason of its motion in height, or in longitude, or in latitude, or of a mixture of all of these, be presented in such a position that the whole, or almost the whole, parallax was compensated by motion in the contrary direction. It would therefore be necessary either that the true motion be known beforehand, separately from the seen motion, or that the parallax be known;

but in a phenomenon whose revolutions do not recur, and which never passes through the vertex, this cannot be known from the seen motion through observations made at the same place on the Earth, as I said in the same chapter 17, number 5, propositions 20 and 21. Third, besides what Chiaramonti has demonstrated, I shall show in Book 10, the penultimate section, chapter 9, theorem 16, that from the equal distance of a phenomenon from the same fixed star, seen at different hours of the same night, the absence of a sensible parallax is not evidently inferred (through observations, that is, made at one place on the Earth). Therefore, although Tycho's arguments are very probable, they have not yet removed all misgiving, nor have they reached mathematical evidence.

Argumenta Rothmanni pro Cometa Anni 1585. eorumque Solutiones.

Rothmann's arguments for the comet of 1585, and their solutions.

IX. What Rothmann thought about the almost insensible parallax of this comet we have already learned from Tycho's letters, in number 5. But Rothmann himself published his opinion in writing, together with his observations, and it was published with Snel's dissertation. In the first chapter Rothmann treats the history of the observations; in the second chapter, the comet's longitude and latitude; in the third, its parallax. Chiaramonti, by more exact calculations and diagrams, detected his various errors in geometrical calculation, as well as his disagreements with Tycho, in the supplement to the *Antitycho*, part 2, from chapter 20 to 24, and proved himself to us the better geometer, as will be plainly clear to anyone who considers his demonstrations. He also refuted, and excellently, the method by which Rothmann boasted of having demonstrated the insensibility of the parallax from observations of the comet near the nonagesimal degree of the ecliptic from the horizon.

[Margin: Chiaramonti a better geometer than Rothmann.]

[Margin: Summary of Rothmann's argument.]

Rothmann's argument, drawn together in brief, was of this kind. "On October 21, at 9^h after noon," says Rothmann, "I observed the comet in 25° 27′ 40″ of Aries, with the 20th degree of Cancer rising; the comet was therefore only 5° 27′ 40″ from the nonagesimal. Again, the same night, at 15^h 6^m after noon, I observed the comet in 25° 56′ of Aries. Then, the following night, because vapours were in the way, I could not observe the comet near the nonagesimal; but I did observe it when it reached the same western altitude at which it had been the night before, namely at 15^h 6^m, and I found it in Aries, at degree 27° 43′." *Subtracting the preceding observation from this one, there remains the comet's true daily motion*, 1° 47′. Hence, for the 6^h 6^m that elapsed between the earlier and the later observation of the 21st, 27′ 12″ are due, keeping the proportion. Added to the place found near the nonagesimal, namely 25° 27′ 40″ of Aries, these give the comet's true place at the later observation as 25° 54′ 52″ of Aries. But by observation the comet's apparent place was found at 25° 56′, greater by about one minute. Yet if it had had a sensible parallax, the apparent place ought rather to have been smaller; for at 15^h 6^m the comet was in the

western quadrant of the ecliptic, where parallax diminishes the true longitude. This, in summary, is Rothmann's argument, which he tries to confirm by another, similar observation near the nonagesimal. He affirms that in both observations the comet, because of its nearness to the nonagesimal, lacked a sensible parallax in longitude.

[Margin: First refutation of Rothmann's argument.]

X. In the preceding argumentation, however, a manifold fallacy lies hidden. First, Rothmann himself, in that chapter 3, holds this view: *Without separating out the parallaxes, the true motion can in no way be established.* How, then, from comparing the places observed at $15^h\ 6^m$ on the 21st and the 22nd — which were certainly seen places, being found by observing — does he gather the true motion, so as to discern from it, compared with the seen motion, the parallax previously unknown? Is this not to contradict himself, or to seek the unknown through the unknown?

[Margin: Second refutation.]

Second, the insensibility of the parallax in longitude is not rightly concluded from a distance from the nonagesimal of less than 6 degrees. For if the parallax of altitude were 15° or 14°, the parallax of longitude could, at the polar altitude of Kassel, be greater than 69 minutes, as Chiaramonti demonstrates in the supplement already mentioned, part 2, chapter 24, problem 4. So long, then, as the parallax of altitude is unknown, the insensibility of the parallax in longitude cannot be concluded from a few degrees of distance of the comet from the nonagesimal, and consequently the true motion cannot be distinguished from the seen motion on the strength of such a position.

[Margin: Third refutation.]

Third, even if, as to longitude, it would have had no sensible parallax had it been in the ecliptic, this nevertheless holds of the comet's own arc, which was different from the ecliptic and inclined to it at an inconstant angle, as Rothmann himself admits in chapter 2.

[Margin: Fourth refutation.]

Fourth, the longitudes of this comet observed on November 4 at the same hours of the same nights, at Kassel by Rothmann and at Uraniborg by Tycho, differ so much that from the difference of longitude there follows a parallax of many degrees in the circle of altitude, as Chiaramonti showed above, in chapters 20 and 23. For, from Tycho's letters, page 15: on November 4, at $7^h\ 5^m$, the comet was seen by Tycho in 13° 12⅔′ of Taurus, and at $10^h\ 35^m$ it was seen in 13° 16⅓′ of Taurus. Therefore, keeping the analogy of motion that Tycho and Rorhmann [?] frequently use, with the motion due for a few hours, the place seen by Tycho at $7^h\ 37^m$ was 13° 13′ 13″ of Taurus. But at that same hour, $7^h\ 37^m$, Rothmann observed it in 15° 20′ 49″ of Taurus. The difference of longitude is therefore 2° 7′ 36″, if they had been under the same meridian. But since Chiaramonti holds that Uraniborg is east of Kassel by 25 minutes of an hour, then, taking account of the comet's motion in longitude due for

these 25 minutes, the difference of longitude will have been 2° 8′ 9″; or, if we put Uraniborg east of Kassel, with Tycho, by only a fifth part of an hour, as he often supposes in the letters, that difference was 2° 7′ 50″. The latitude observed at that time was 6° 18′ 26″ for Tyhco [?] and 6° 18′ 49″ for Rothmann, to the north. Now from this difference of the observed longitudes it follows that this comet had a parallax of altitude not less than 15 degrees, as Chiaramonti demonstrates in the supplement already mentioned, with calculations — and by no means erroneous ones — supposing, however, certain things: namely, that on November 4 of the year 1585, Old Style, at 7^{h} 37^{m}, the Sun was at Kassel very nearly in 21° 3′ of Scorpio; that on the meridian of Kassel the 10th degree of Pisces culminated, but on that of Uraniborg degree 11° 47′ of Pisces, because the 25 minutes of an hour by which he puts Uraniborg further east correspond to 1° 40′ on the equator, and the right ascension at Uraniborg was greater by so much; that the complement of the polar altitude is 37° 4′ at Uraniborg and 38° 41′ at Kassel; and that the distance between the poles of the equator and of the ecliptic is 23° 30′. But since the problem is very ingenious, I have decided to insert it here in clearer words.

Problema Claramontij.

Chiaramonti's problem.

Given the latitude and longitude of a phenomenon observed at the same moment at two places on the Earth whose polar altitude and distance between meridians are known, and given the moment of the observation, the point of the ecliptic culminating on each meridian, and the obliquity of the ecliptic: to find the parallax of altitude.

[Margin: Construction of the diagram.]

XI. In the following diagram, let AHPKL be the solstitial colure, in which A is the pole of the equator and H that of the ecliptic, from which the arc of the ecliptic PML is described. Let AC be a portion of the meridian of Uraniborg, in which the vertex of the place is B; and let AO be a portion of the meridian of Kassel, in which the vertex is X, and the culminating point of the ecliptic is O with respect to Kassel, but C with respect to Uraniborg. Let the beginning of Capricorn be at P, and of Cancer at L. Let the comet's ecliptic place observed at Uraniborg be at Q, marked off by the latitude quadrant HQ; but let the place observed at Kassel be S (because it was greater), marked off by the quadrant HS; on these quadrants let the seen northern latitude be IQ as observed by Tycho, and RS as observed by Rothmann. Let the comet's place seen by Tycho be I, and that seen by Rothmann R. Finally, from the vertices of the places let arcs of altitude be drawn through the seen places, namely BI and XR. These will necessarily intersect at F, since B is further east than X, and conversely S, the comet's ecliptic place seen from X, was further east, and Q, seen from B, further west. Therefore the comet's true place was F, by Lemma 1 of book 1 of *De tribus novis stellis*, chapter 27, demonstrated by the same Chiaramonti. And this is easily seen: because the true place of a phenomenon is always at some point of the vertical circle on which the seen place lies, and at one and the

same moment it is single, it follows that if at the same moment two different observers observe a phenomenon, the vertical cir- cles of each have one point in common, in which the true place lies; and this is none other than the point in which they intersect. The parallax of altitude, therefore, will be FI with respect to place B, and FR with respect to place X.

Diagram. The figure for Chiaramonti's problem, set in the left column. A large circle, the solstitial colure AHPKL, with H (the pole of the ecliptic) at the top and K at the bottom; A (the pole of the equator) on the circumference to the upper right. The arc of the ecliptic runs across the lower half of the circle from P at the left to L at the right, with O, C, M, N, Q and S marked along it from left to right. Arcs descend from H through X and B towards the ecliptic, and arcs from A descend to the left; the lines cross at F, inside the circle, below X and B. R stands above S, and a small I just above and to the left of Q.

- H (top of the circle): pole of the ecliptic
- A (upper right, on the circle): pole of the equator
- X (left of centre, inside): vertex of Kassel
- B (right of X): vertex of Uraniborg
- F (below B): the comet's true place, where the arcs of altitude intersect
- R (right, above S): the comet's place seen by Rothmann
- I (just above and left of Q): the comet's place seen by Tycho
- P (left, on the circle): beginning of Capricorn
- L (right, on the circle): beginning of Cancer
- O (on the ecliptic, left): culminating point for Kassel
- C (on the ecliptic, right of O): culminating point for Uraniborg
- M (on the ecliptic, bottom): foot of the arc HXM
- N (on the ecliptic, right of M): foot of the arc HBN
- Q (on the ecliptic): the comet's ecliptic place observed at Uraniborg
- S (on the ecliptic, right of Q): the comet's ecliptic place observed at Kassel
- K (bottom of the circle): a point of the colure AHPKL

Further, let the seen places be joined by the arc IR, and from the pole of the ecliptic H, through the vertices B and X, let the arcs HBN and HXM be drawn down to the ecliptic. With these laid down, we shall arrive in several steps at knowledge of the parallaxes FI and FR.

[Margin: Step 1.]

First, in the triangle HIR, the complements of the seen latitudes, HI and HR, are given, and the angle IHR, which is the difference of the seen longitudes. In it, therefore, seek both the arc IR and the angles HRI and HIR.

[Margin: Step 2.]

Second, in the triangle ABH, there are given AH, the distance of the poles, or the obliquity of the ecliptic; AB, the complement of the polar altitude of place B; and the angle HAB, which measures on the equator the difference of right ascension between the beginning of Capricorn and the culminating point of the ecliptic, C. In that triangle, therefore, find the arc HB and the angle AHB, the measure of the arc NL. From this subtract the arc of the ecliptic QL

— since L is the beginning of Cancer, and Q the point of the ecliptic known from the comet's longitude as seen by observer B — for there will then remain known the angle BHI and the arc of the ecliptic NQ.

[Margin: Step 3.]

Third, in the triangle BHI, from what has just been said, there are now given the side HB, the angle BHI, and HI, the complement of the latitude seen by observer B. Seek, therefore, the arc BI and the angle HIB; add this to the angle HIR, and the angle FIR will result.

[Margin: Step 4.]

Fourth, in the triangle AXH, there are given AH, the distance of the poles; AX, the complement of the polar altitude of place X; and the angle HAX, which is the difference of right ascension between the beginning of Capricorn, P, and the point O of the ecliptic culminating on the meridian of place X. In that triangle, therefore, find both the arc HX and the angle AHX, the measure of the arc ML. From this subtract the arc of the ecliptic SL, and there will remain MS, that is, the angle XHR.

[Margin: Step 5.]

Fifth, then, in the triangle XHR, from the arc HX and the angle XHR just found, and HR, the complement of the latitude seen at place X, find the arc RX and the angle HRX. Take this from the angle HRI, found by step 1, and the angle FRI will remain known.

[Margin: Step 6.]

Sixth and finally, in the triangle FIR, with the side IR now known by step 1, the angle FIR by step 3, and the angle FRI by step 5, find the arc FI, or the parallax of altitude with respect to place B, and the arc FR, that is, the parallax of altitude with respect to place X. Which was to be done.

EXEMPLVM Calculi Claramontani

Example of Chiaramonti's calculation.

	Value
XII. Q, longitude seen from B	♉ 13° 13′ 13″
S, longitude seen from X	♉ 15° 20′ 49″
QS, difference of longitude	0ˢ 2° 7′ 36″
QI, latitude seen from B	0ˢ 6° 18′ 26″
SR, latitude seen from X	0ˢ 6° 18′ 49″
HI, complement of latitude	0ˢ 83° 41′ 34″
HR, complement of latitude	0ˢ 83° 41′ 11″

[Margin: Data for step 1.]

	Value
Therefore the arc IR	0ˢ 2° 7′ 23″
And the angle HRI	0ˢ 90° 9′ 18″

	Value
And the angle HIR	0ˢ 89° 38′ 31″
AH, distance of the poles	0ˢ 23° 30′ 0″
AB, complement of the polar altitude	0ˢ 34° 7′ 0″
Right ascension of the beginning of ♎, P	0ˢ 270° 0′ 0″
C is the culminating point	♓ 11° 47′ 0″
Its right ascension is	0ˢ 343° 13′ 0″
Difference of the right ascensions, or BAH	0ˢ 73° 13′ 0″
Therefore the arc HB	0ˢ 34° 32′ 0″
And the angle AHB, or NL	0ˢ 71° 19′ 0″
But at L is the beginning of ♋	
Therefore the point N is	♈ 18° 41′ 0″
And Q is	♉ 13° 13′ 13″
Therefore NQ, or the angle BHI	0ˢ 24° 32′ 13″
And there are given HB	0ˢ 34° 32′ 0″
And the arc HI	0ˢ 83° 41′ 34″
Therefore the arc BI	0ˢ 52° 55′ 0″
And the angle HIB	0ˢ 17° 10′ 0″
But the angle HIR was	0ˢ 89° 38′ 31″
Therefore the angle FIR	0ˢ 106° 48′ 31″
Again AH, distance of the poles	0ˢ 23° 30′ 0″
AX, complement of the polar altitude	0ˢ 38° 41′ 0″
Right ascension of the beginning of ♎, P	0ˢ 270° 0′ 0″
O, culminating point of the ecliptic	♓ 10° 0′ 0″
Its right ascension is	0ˢ 341° 33′ 0″
Difference of the right ascensions, or HAX	0ˢ 71° 33′ 0″
Therefore the arc HX	0ˢ 37° 22′ 0″
And the angle AHX, or ML	0ˢ 77° 39′ 0″
But at L is the beginning of ♋	
Therefore the point M is	♈ 12° 21′ 0″
But S was	♉ 15° 20′ 49″
Therefore MS, or the angle XHR	0ˢ 32° 59′ 49″
But there are given HX	0ˢ 37° 22′ 0″
And the arc HR	0ˢ 83° 41′ 11″
Therefore the arc XR	0ˢ 53° 36′ 0″
And the angle HRX	0ˢ 24° 14′ 0″
But HRI was	0ˢ 90° 9′ 18″
Therefore XRI, or FRI	0ˢ 65° 55′ 18″
But FIR was	0ˢ 106° 48′ 31″
And the arc IR	0ˢ 2° 7′ 23″
THEREFORE the parallax FR	0ˢ 15° 47′ 51″
And the parallax FI	0ˢ 15° 3′ 0″

[Margin: Triangle HIR.]
[Margin: Data for step 2.]
[Margin: Triangle ABH.]
[Margin: Data for step 3.]
[Margin: In triangle BHI, step 3.]
[Margin: Data for step 4.]

[Margin: Triangle AHX, step 4.]
[Margin: Data for step 5.]
[Margin: Triangle XHR.]
[Margin: Data for step 6.]
[Margin: Triangle FIR.]
[Margin: The comet's distance from the Earth.]

XIII. With the parallaxes of altitude of the comet of 1585 now obtained — 15° 3′ as seen at Uraniborg and 15° 47′ 51″ at Kassel — the distance of the comet from the centre of the Earth is easily gathered, by the problem set out in book 3, chapter 7, number 3: 9324 miles from the former parallax and 8973 from the latter, provided that the Earth's semidiameter is, as Chiaramonti assumes, 3035 Italian miles. Hence the distance did not reach 4 semidiameters of the Earth, and the comet was far below the Moon, let alone below Saturn — if, that is, those observations of Tycho and Rothmann are accepted. For Tycho is accustomed, when he thinks they support him, to accept and commend the observations of the Landgrave and of Rothmann, and to disregard a difference of a few minutes between them and his own, as is clear from Tycho's letters, pages 37 and 38. Now the complement of the polar altitude at Kassel, as corrected in Tycho's letters, page 39, is 38° 41′, and at Uraniborg 34° 7′, or 34° 5′ 30″. And although the distance between the meridians is not 25 minutes of an hour, as Chiaramonti puts it, but 13, yet from these values the parallaxes FI and FR will still always come out at very nearly 15°; and there is no danger about its always being greater than 2 degrees, which is enough for it to be greater than any lunar parallax. Granted, then, that from other observations, whether taken separately or comparing Tycho's with Rothmann's, the parallax of this comet would be gathered to be smaller than the Moon's, while yet from others it is gathered to be greater, we are still uncertain which we ought rather to prefer; and consequently it has not yet been evidently demonstrated from these that this comet was either above or below the Moon, but only that on one hypothesis it was above the Moon, and on another below it.

Concluſiones ex Prædictis Deductæ.

Conclusions drawn from what has been said.

I. CONCLUSION. *It has not yet been demonstrated, so far as I know, that the comets of the years 1580, 1582 and 1585 were either above or below the Moon.*

This is clear from what has been said in [?] 1 to 13. For since each of the two opposites has been demonstrated on some hypothesis, but not absolutely, and it is not evidently established which hypothesis is to be preferred to the other, neither remains absolutely demonstrated. This I say of the comets of 1580 and 1585. For as to the comet of 1582, although it has not been demonstrated to have been above the Moon except on some uncertain hypothesis, it has nevertheless not been demonstrated on the other hypothesis, so far as I know, that it was below the Moon.

II. CONCLUSION. *It is nevertheless very probable that the comet of 1585 was above the Moon, and not improbable of the rest.*

For there are many more arguments of Tycho and Rothmann — besides the authority of very many who have subscribed to them — by which it is made persuasive that it was above the Moon; and the same is gathered from comparing several observations; and the measure of the proper motion gathered by Tycho, which measure is one of the principal foundations of his own arguments and of Rothmann's, is very probable.

CAPVT XX.

Chapter 20.

De Cometarum ab Anno 1590. ad Annum 1600. Parallaxi & Altitudine.

On the parallax and altitude of the comets from the year 1590 to the year 1600.

I. In the year of the Lord 1590 Tycho began to observe a certain comet, from the day[17] of February until March 6. It began from the northern Fish, at longitude 18° 27′ of Aries and northern latitude 18° 14′, and ceased to be seen on March 6 at longitude 3° 15′ of Gemini and northern latitude 20° 46′. Its motion was quicker at the beginning, since it then completed almost 8 degrees in its daily motion. The tail was at first thin but long, 10 degrees, directed opposite the Sun towards the zenith; the head at first barely 3′. Tycho wrote about it in the *Epistolae Astronomicae*, pages 173 and 174, but more fully from page 176 to 181, where he says that it described a great circle which, if produced as far as the equator, would have cut it at degree 339° 45′, with a constant inclination to it of 42°; from this intersection it was distant about 37 degrees when it first appeared to Tycho, and almost 80° at the end. We have already given its ephemeris, from Tycho himself, in chapter 3, for this year. From the observations of the 12 days he selects those made on March 1 and 2, on which the comet moved about the pole of the world without setting, and from them he tried to derive the parallaxes by a twofold experiment, or argument.

[Margin: Tycho's argument for the insensible parallax of this comet.]

First, on March 1, at 7^h 19^m after noon, the comet appeared 47° 35′ high, and its azimuth was distant from the meridian by almost 92 degrees; and then, with the ten-foot armillary [Armillam Decempedalem], its distance from the pole was taken as 51° 22½′, or its declination as 38° 37½′. Then, at 15^h 25^m, when it passed the meridian below the pole, its seen altitude, with refraction subtracted, was 4° 49½′; subtracted from the polar altitude of Uraniborg, 55° 54¾′, this leaves its distance from the pole of the world as 51° 5¼′. So in 8^h 6^m it approached the pole by 17′. On March 2, when it again appeared 47° 35′ high, at 7^h 34^m, he observed its declination as 39° 22′ and its distance from the pole as 50° 37½′. Its seen declination therefore increased in $24¼^h$ by 45 minutes; and so, if this motion had been uniform, 15 minutes would be due

[17]no numeral is printed

for 8^h 6^m. But since this change was greater at the beginning of the day than at the end, it is nearer the truth that 17′ are due for those first 8^h 6^m. The true change of declination was therefore as great as the seen one, and no other change of declination was seen after so many hours than that which the comet's own proper motion required. Therefore the parallax was nil, or insensible, and so this comet completed its course in the highest aether. But had it been on the border of the Moon, and distant from the Earth by at least 52° 15′, it ought to have been seen, at its lowest altitude, distant from the pole of the world by 15′ more than when it was 47° 35′ high; and that difference anyone would have observed with any instrument, however crude. And he confirms this very point from another, similar observation. He adds, on page 180, that he attributed to this comet the same refraction as to the fixed stars. Many, moreover, subscribed to Tycho's opinion, especially Froidmont, *Meteorologica* book 3, chapter 1, article 1, and placed this comet far above the Moon.

[Margin: Froidmont's opinion on the place of this comet.]

II. But the *Antitycho* — that is, Chiaramonti, book 2, reporting Tycho's arguments in chapter 23 and refuting them in chapter 24 — stands against Tycho.

[Margin: Chiaramonti's first objection against Tycho.]

First, he says that a refraction equal to that of the fixed stars ought not to have been attributed to this comet, since it is certain from Tycho himself and from the laws of optics that things nearer the Earth undergo a greater refraction; and, with the parallax unknown, it cannot be known whether this comet was as distant from the Earth as the fixed stars, for to suppose that would be to suppose the very point in dispute. The refraction could, moreover, have been great enough to correct that parallax of declination of 15′, which is sufficient to place the comet beneath the concave of the Moon, and indeed even a greater parallax; but that it was not so great cannot be known as long as the parallax is unknown.

[Margin: Second objection, Chiaramonti's and ours.]

Second, he denies that the regularity or measure of the motion estimated by Tycho is certain. And I add that this motion, taken from the observations, was the seen and not the true motion, and that by reason of its proper motion the comet could have been so presented as to keep apparently the same distance from the pole of the world, or to change its seen declination by only 17′ after 8^h 6^m, even though it had a large parallax; and that this cannot be known certainly and evidently as long as the parallax, or the true motion, is unknown.

[Margin: Chiaramonti's third objection.]

Third, he points out that one must not, in Tycho's manner, confuse the difference of the parallaxes of declination with the parallax of declination; and that at a great nearness to the Earth, such as this comet's was when it stood 4° 49′ high on the meridian, a large parallax of altitude can be gathered from a small difference of parallaxes. Therefore, although Tycho's argument carries

great likelihood, it does not yet seem to me to have attained mathematical evidence.

De Cometa Anni 1593. & 1596.

On the comet of 1593 and 1596.

III. Further, about the comet of 1590 and 1593 — and indeed not even about those of 1580 and 1585 — Antonio Santucci has nothing; and yet he observed, by order of Cosimo II, Grand Duke of Tuscany, as many comets as he could. This is a sign either that they were very low, towards the centre of the Earth, as Chiaramonti judges in the *Apologia* for the *Antitycho*, page 292; or that they were within the circle of perpetual occultation with respect to Tuscany; or that, because of cloudy nights, they could not be watched by Santucci. But it does not follow determinately that they could not be seen in Italy because of excessive nearness to the Earth. Therefore neither from this nor from anywhere else has Chiaramonti or anyone else, so far as I know, demonstrated that the comet of 1590, or of 1593, or of 1596 was below the Moon — just as no one has demonstrated that it was above the Moon; or, if anyone has tried to demonstrate it, his writings have not yet reached my hands. Granted that Froidmont, *Meteorologica* book 3, chapter 1, article 1, drew from Tycho — from where, I do not know — that the comet of 1593 was above the Moon.

[Margin: Froidmont's opinion on the comets of 1590 and 1593.]

An Cometa Anni 1597.

Whether the comet of 1597.

IV. Antonio Santucci, in his treatise on comets, from chapter 16, relates that in the year 1597 a comet appeared from July 16 to August 9, that it was observed by him diligently, and that at first it had a declination of 50° 30′. To show that it was in the heaven of the Sun, he adduces the following figure, in which, from D, the centre of the Earth, he describes the Earth's quadrant AP, and the quadrant of the heaven XE. Having drawn DE, the rational horizon, and AF, the natural horizon, and the vertical line DAX, perpendicular to the horizontal lines, he draws the straight line DG, cutting off the arc EG of 4 degrees — as large as he says the comet's least meridian altitude, observed on July 16, was. He further draws the straight line DK, cutting off the arc KX of 7 degrees, because he says the comet's highest altitude observed on the meridian was 83°. He then bisects the arc KG at S, and draws the straight line DS as though it were the axis of the world. Again, he takes the arc FH as 4 degrees, and draws to H the straight line AH, cutting DS at the point C; from this point he draws the straight line CL, parallel to the straight line DK. Then, taking DM as the Moon's least distance from the centre of the Earth, which for Ptolemy is 33⅓ semidiameters of the Earth, he describes the arc MO, cutting the straight line CL at B; and through the point B he draws the straight line ABN, and supposes the altitude of the pole at Florence to be 43° 30′. With these laid down, he tries to demonstrate that the comet, had it been so near the Earth, would have had

to be seen under N, since it would then have been on the line CL; and that, if it were placed in the lunar orb, for example at L, the arc KL would be 1° 58′, so that the comet would have appeared raised 81° 2′, and not 83° as was observed; whereas if it were in the sphere of Mercury it would have appeared 81° 58′ high, in the sphere of Venus 82° 33′, but in the sphere of the Sun 82° 57′; and hence that it was at least as high as the Sun.

Diagram. Santucci's construction for the comet of 1597, set in the left column beside the opening of paragraph V. A quadrant with its centre D at the lower left corner: the base DE runs to the right, the vertical DX rises at the left, and the arc of the heaven runs from X at the top through K, L, S, N, H, G and F down to E. A stands on the vertical between M above and D below; a second, smaller arc runs from M down through B to O on the base, with P on the base between D and O. Straight lines radiate from D and from A to points on the outer arc; a small lower-case c marks an intersection to the right of A and below B.

- X (top of the vertical): the end of the quadrant of the heaven XE
- K (on the outer arc, just right of X)
- L (on the outer arc, right of K)
- S (on the outer arc)
- N (on the outer arc, just right of S)
- H (on the outer arc, right side)
- G (on the outer arc, below H)
- F (on the outer arc, below G)
- E (right end of the base)
- M (on the vertical, upper)
- A (on the vertical, below M)
- D (lower left corner, the centre of the Earth)
- B (on the inner arc MO)
- c (lower case, at an intersection right of A)
- P (on the base, between D and O)
- O (on the base, where the inner arc ends)

V. But Chiaramonti, in book 2 of *De novis stellis*, chapter 5, and in the Supplement to the *Antitycho*, page 2, chapter 17, detected many errors in this method. First, the comet could not have been on the line CL, but on the line DK, for this is the line of the true place — granted that LC is drawn parallel to it. Second, the line of the seen place is not to be drawn from A to some point of the line CL, but to a point of the line DK: so that if the comet is at K, the line AK should have been drawn. Hence the comet's parallax will be the angle AKD, and not the arc KL; which is the third error. Fourth, the angle of 7 degrees of the observed distance from the vertex is not, as Santucci puts it, the angle XDK, but XAK, since the comet was seen with the instrument at the surface of the Earth; just as the angle of the least observed altitude is HAF, not GDE. Consequently the arc KH lay between the seen places of the comet, not between the true places — unless these are supposed to differ insensibly from

the true ones, which is, however, under dispute. Hence, once the arc KH is bisected at S, it is not rightly supposed that K and H are equally distant from the pole of the world, or that DS is the axis of the world. Finally, it is false that the comet could not have been seen 7° from the vertex, or 83° high, had it been at a smaller distance from the centre of the Earth than the Sun; for it could have been seen on the line AK however near the Earth it was, the angle XAK always remaining the same, of 7 degrees. But I am ashamed to detain the learned reader any longer with false demonstrations [Pſeudogramatiſmis] of this kind.

[Margin: Santucci's various errors, excised by Chiaramonti.]

VI. Further, I have so far found the place of no other comet, from this year 1597 to 1600, demonstrated. The present chapter is therefore to be concluded, in our usual manner, with the following theses.

Concluſiones ex Prædictis.

Conclusions from what has been said.

I. CONCLUSION. *It has not yet been demonstrated, so far as I know, that the comets of the years 1590, 1593, 1596 and 1597 were either above or below the Moon.*

II. CONCLUSION. *It is nevertheless probable that the comet of the year 1590, at least, was above the Moon.* This I concede to Tycho's authority and arguments, since it is probable that the motion by which that comet changed its declination was such as Tycho laid down, and that the refraction at an altitude of about 5° was not much greater than the solar refraction, which there is 14′, and was insufficient for inferring a parallax greater than the Moon's.

CAPVT XXI.

Chapter 21.

De Cometa Anni 1607.

On the comet of the year 1607.

I. Observations of this comet made by themselves are reported by Kepler, book 1 of *De cometis*, from page 25 to 36; by Longomontanus [?] in the Appendix to the *Astronomia Danica*, chapter 9; and by Antonio Santucci, *De cometis*, chapter 21. I have already presented the ephemeris of the comet's motion, from Kepler and Longomontanus, in the history of comets under this year. Now, since these three authors contend that the comet was above the Moon, and strive to show this chiefly from its parallaxes, we need examine only what pertains to those parallaxes.

Kepleri Obſeruationes pro Parallaxi Cometæ.

Kepler's observations for the comet's parallax.

II. On page 25 Kepler admits that on September 26, at Prague, the comet was seen by him with the help of telescopes [perſpicillorum ope], larger than the other stars but without a tail — which others nevertheless affirmed was seen;

and that his observations were not exact but rough. For he says: *"I noted the place as best I could"*; and soon after: *"We shall leave very little in doubt if, trusting our eyes, we place the comet at latitude 35° 30′"*; and on page 28: *"the comet, with the diagram compared mechanically with the sky, would have fallen at 29½° of Leo"*; and on page 30: *"For there is no need of subtlety, since the observation is rough. For Tycho's instruments require a platform, a free horizon, a skilled observer as companion, and keen eyes — all of which I lacked at the beginning"*; and a little below: *"From the daily and hourly motion, in passing, though from not very fine observations, it is nevertheless easy to show some parallax greater than the one the comet had, if it had any."*

So then, by the bare estimate of the eye, on September 26 at $8\frac{1}{2}^{h}$ after noon, he saw the comet as though on the line joining the knees of Ursa Major, but nearer the hind knee by about 3 degrees, and nearer the wheels than the knee itself is, and higher than the line already mentioned. Hence, from the longitude and latitude of those fixed stars, a diagram having been drawn roughly: the comet's longitude 18° 30′ in Leo, and its latitude 35° 30′, approximately. At 9^{h}, from its position with respect to the fixed stars as noted by eye, he deduced by trigonometry the longitude 18° 50′ in Leo. And the following morning, at 3^{h} after midnight, when the tail appeared clearly, fairly long and turned upward, he saw the comet distant from the left knee of the Bear's hind leg much less than it had been at $8\frac{1}{2}^{h}$ and at 9^{h}; and from its position drawn on paper, by the solution of triangles, he deduced the comet's longitude 21° 20′ in Leo and its latitude 35° 34′, approximately. From this he gathers that the comet, from 9^{h} after noon to 3^{h} after midnight — that is, in 6 hours — advanced from 18° 30′ of Leo to 21° 20′, and completed 2° 50′.

From here he proceeds to the 27th, at 9^{h} of which, he says, the longitude, observed as before, was seen at about 29° 30′ of Leo; from this he gathers a daily motion of 11 degrees. And on the 28th, at the same hour, at about 12° 0′ of Virgo, with latitude 40° 4′; from this he gathers a daily motion of 12½°. The 29th was empty of observations because of the cloudy air; and on the 30th, at $9\frac{1}{2}^{h}$, *through gaps in the clouds, by mechanical estimation*, the comet was seen at 7½ degrees of Libra, with latitude 38⅔°; and therefore from the 28th to the 30th it completed 25½ degrees, that is 12¾° a day.

Kepleri Argumenta pro Parallaxi Cometæ.

Kepler's arguments for the comet's parallax.

III. From the observations set out above, although they were taken rather roughly, he nevertheless hopes it can be shown that the comet was not even in the concave of the lunar heaven, but above the Moon. For from the moment of time he derives the right ascension of the midheaven; and from this, and from the altitude of the equator at Prague, 39° 54′, and the distance between the poles of the ecliptic and of the equator, 23° 31½′, he derives for the 9th hour of the 26th the comet's altitude, 9° 48′. And at this altitude above the

horizon — that is, in round figures, at an altitude of 10 degrees — the comet, had it been on the border of the Moon, that is, at a distance from the Earth of 60 semidiameters of the Earth, would have undergone a parallax of altitude of 59′. From this he gathers a parallax of latitude of 57′ and of longitude of 19′; and therefore the comet's true longitude would have been 18° 11′ of Leo, and its true latitude 36° 27′, since the seen longitude was 18° 30′ of Leo and the seen latitude 35° 30′. Then, by the same method, he investigates the comet's altitude at 15^h after noon on the 26th, and gathers it as 23° 30′; at this altitude, with the comet supposed on the border of the Moon, the parallax of altitude would have been 55′, of latitude 20′, and of longitude 63′. Hence the true longitude at 15^h would have been 20° 17′ of Leo, and the true latitude 36° 20′. In 6 hours, therefore, it would have completed by its true motion only 2° 6′ — as many as there are from 18° 11′ to 20° 17′ of Leo — and it would have completed much less had it been below the borders of the Moon. But if it had been higher than the Moon, it would have completed in 6 hours 2° 50′, or at least 2° 32′, as he gathers from the daily motion from 9^h on the 26th to 9^h on the 27th, which he says was 11 degrees: for a quarter of this is 2° 45′. Or else, because the daily motion from the 27th to the 28th grew by a twelfth of itself: if a twelfth part is taken from 2° 45′, 2° 32′ will remain. Otherwise, if you would have it complete 2° 6′ in those first hours and afterwards 11° in one day, *the comet would have been very slow in its true motion; then, from 3^h in the morning to 9^h the following evening, it would suddenly have become very fast, and on the next day slower again — which conflicts with the analogy of motions, even violent and sublunary ones.* And this is the first argument, from the motion in longitude.

[Margin: 1. Argument from the motion in longitude.]

The second is from the motion in latitude, and is as follows. If the comet had been on the border of the Moon, its true latitude at hour 9′ [?] on the 26th would have been 36° 27′, but at 15^h 36° 20′, and at 9^h on the 27th, because of an equal parallax of 57′, 38° 27′. Therefore the comet's true path — that is, as seen from the centre of the Earth — would have been tortuous, and the latitude in the morning smaller than on the preceding evening. If, by contrast, we consider the seen latitudes without parallax, the series of latitudes will be orderly, as you see in the following little table.

[Margin: 2. Argument from the motion in latitude.]

September	Hour	Seen latitude
26	9	35° 30′
26	15	36° 0′
27	9	37° 30′
28	9	40° 4′

He says this argument is firmer than the former. In the former, which was taken from the longitude and founded on a time determined from the town clock, someone could raise a dispute as to whether the clock ran regularly;

whereas in this argument, taken from the latitude, *nothing has been forced by me, or accommodated to my purpose by prosthaphaereses; but everything, even in a rough observation, comes of its own accord as one would wish: a proof of truth.* He therefore concludes, on page 33, in these words: *"Since, then, the Moon is distant sixty semidiameters of the Earth, that is, more than fifty thousand German miles, the comet was certainly higher; by no means, therefore, in any region of the air. For the region of the air ends at once with the extreme summits of the mountains, so that already on Olympus in Asia, and now on the snowy summits of Peru, one can no longer live, for want of that breath which we call air, and whose inhalation we cannot do without. And this, in passing, on the daily parallaxes of the comet."*

[Margin: Kepler's conclusion on the place of the comet.]

Afterwards, however, he constructed a table in which he gives the comet's distances from the Earth in parts of which the Sun's distance from the Earth is 100,000. He puts the comet's least distance, on September 28, at 13,135 such parts, and its greatest, on October 26, at 134,561 parts. But the Sun's mean distance, according to Kepler in the *Rudolphine Tables* and in the *Epitome Astronomiae*, was nearly 3410 semidiameters of the Earth. Therefore, for Kepler, the comet's least distance from the Earth was 449 semidiameters of the Earth, and its greatest 4588½.

Noſtra Solutio Argumentorum Kepleri.

Our resolution of Kepler's arguments.

IV. Although Kepler's arguments seem very involved, we shall nevertheless unfold them by reducing them to form. The first argument, then, taken from the motion in longitude, is this: *If the comet had been below the Moon or on the border of the Moon, it would have completed, on September 26, from 9^h after noon to 15^h, only 2° 6′ in longitude; but if it had been above the Moon it would have completed 2° 50′. But in that space of time it completed not only 2° 6′, but much more. Therefore the comet was not below the Moon, nor on the border of the Moon.*

[Margin: 1. Kepler's argument reduced to form.]

The major proposition is proved from the parallaxes of longitude due to the comet placed on the border of the Moon and having, at 9^h and at 15^h, those altitudes which the observations show it had. For at 9^h it was seen at 18° 30′ of Leo, nearly 10 degrees high; at a distance equal to the Moon's, a parallax of longitude of 19′ answers to that altitude, to be subtracted from the seen place, and so the true longitude remains 18° 11′ of Leo. And at 15^h it was seen at 21° 20′ of Leo, 23° 30′ high; at a distance equal to the lunar distance, a parallax of longitude of 63′ answers to that altitude, likewise to be subtracted from the seen place, and so the true longitude remains 20° 17′ of Leo. And from 18° 11′ to 20° 17′ there are only 2° 6′.

[Margin: 2. Proof of the major proposition.]

The minor proposition, or assumption, is proved, first, from the analogy of the daily motion. For from 9^h on the 26th to 9^h on the 27th the comet

completed 11 degrees, since at 9^h on the 26th it was at 18° 30′ of Leo, and at 9^h on the 27th at 29° 30′ of Leo. A quarter of 11 degrees is 2° 45′, and 6 hours are a quarter of the natural day; therefore, in the 6 hours that passed from 9^h to 15^h, the true motion ought to have been 2° 45′. And if you say that the comet was slower from the 26th to the 27th than from the 27th to the 28th by a twelfth part, because the former motion was 11 degrees and the latter 12, or 12½, then, when a twelfth part — which is about 14′ — is taken from 2° 45′, there remain 2° 31′. So great, then, and no smaller, ought the comet's motion in those 6 hours to have been. Second: if in those 6 hours the comet's motion had been only 2° 6′, its true motion would have kept a smaller analogy than violent and sublunary motions keep, whereas, being natural and supralunar, it ought to have kept a greater, or at least an equal one. For from 9^h to 15^h it would have been very slow, completing less than a quarter of the daily motion; and from then until the 9th hour of the following night, that is, through three quarters of the day, it would have completed more than three quarters of the daily motion — namely the remainder from 2° 6′ up to 11 degrees, which is 8° 54′ — whereas three quarters of 11 degrees are only 8° 15′. So, on the following day, because of equal parallaxes at an equal altitude, it would have been slow in the first 6 hours and then fast. Therefore this is Kepler's consequence: *it would have been very slow in its true motion, then suddenly very fast, and on the following day slower again.*

[Margin: First proof of the minor.]

[Margin: Second proof of the minor.]

I reply, however — granting, but not yet conceding, the major — by denying the minor proposition. To its first reason, I deny that the comet ought, by its true motion, to have completed 2° 45′, or 2° 31′, in 6 hours. For this is not proved from the true daily motion, since the daily motion of 11 degrees was gathered from two longitudes observed before the investigation of the parallax. Nor can Kepler say that, because of the revolution made in 24 hours back to the same point, the true motion was as great as the seen one; for, because of its proper motion in the order of the signs, the comet at 9^h on the 27th could not have been at the same altitude at which it was at 9^h on the 26th. To the second proof of the minor, I deny, first, that it did not keep the analogy; I likewise deny that it was first very slow, then suddenly very fast, and again slower. For it has not been shown that that change from slowness to speed was sudden, since it could have come about gradually, by increments, so that at the end of the 12th hour the comet had completed 4° 35′, at the end of the 18th hour 7° 30′, and at the end of the 24th hour 11°. Nor could it, or ought it to, have been shown from the seen motion what that daily motion of 11 degrees, and then of 12½ degrees, was; and from the true motion itself it could not have been shown except by supposing that motion regular, which can nevertheless be disputed. Next, the comet is supposed to move naturally, whereas it could have been moved by an Intelligence at will, or at the nod of divine Providence. It is likewise supposed to be supralunar, since the argument is *a fortiori* from sublunary things; but this

very point is still in question. Finally, if some violent or sublunary motions are so ordered as to keep an analogy in a perpetual decrease of speed, it does not necessarily follow that the same is owed to all comets, or to this particular comet — granted that this is not improbable.

[Margin: 1. Negative reply to the minor.]

Otherwise it would also have to be judged necessary — if some sublunary motions are uniform, as many certainly are — that the motions of the celestial planets, as seen from the centre of the Earth, must be uniform too, which is false. And he ought to have remembered the comet of 1472, observed by Regiomontanus, which was at first slow, then became so fast that it ran through 40 degrees in one day, and afterwards was slower again, as I said in the history of comets under that year.

I reply, second, that the first part of the major can be denied: both because that motion was derived from very rough observations, whereas in the matter of parallaxes very exquisite observations and computations are required, as we learned from Tycho in chapter 16, number 8; and because it is quite incongruous that the comet, at an altitude of 10 degrees, should have had a parallax of longitude of only 19′, and at an altitude of 23° 30′ a parallax of longitude of 63′, greater than the parallax of altitude. And Kepler himself does not deny that in this argument some things were forced and accommodated to his purpose.

[Margin: 2. Negative reply to the major, as to its first part.]

I reply, third, by denying the second part of the major, in which it was said that, had the comet been above the Moon, its motion in those first 6 hours would have been 2° 50′. First, that immediate jump from 2° 6′ to 2° 50′ is too great. For if, from its being on the border of the Moon, it made only 2° 6′, then, had it made not merely 26′ [?] but more — say 2° 30′ — it would have been above the border of the Moon; therefore it does not follow that, if it was above the Moon, it made 2° 50′. Second, this quantity of motion is proved only from the seen motion, namely because it is said to have been seen at 9^h at 18° 30′ of Leo and at 15^h at 21° 20′ of Leo; but this ought to have been shown of the true motion. For that Kepler is speaking of the true motion when he compares the motion the comet would have had on the border of the Moon with its motion had it been above the Moon is clear from these words of his on page 32: *"Therefore, by its true motion, freed from the parallax (which we attributed to it, referring it to the borders of the Moon), it would have completed 2° 6′ in six hours. But if it did not have so great a parallax, that is, if it was higher than the abodes of the Moon, then it completed 2° 50′ in 6 hours. Which of the two is true, the portion of the daily motion shows."* Third, because, even speaking of the seen motion, it is false that it completed 2° 50′ in 6 hours: it completed them in 6½ hours. For Kepler says in express and explicit terms, on page 25, that the first observation was made at $8½^h$; and from it he gathers the comet's motion on page 26, saying: *"It is nearest the truth that it was seen at 18° 30′ of Leo"*; and at once, examining the second consideration, on page 27, he says it was seen at 18° 50′ of Leo, adding: *"This last consideration, however, was made about half*

an hour after the earlier ones, at 9½ʰ." Then, speaking of the place observed at 15ʰ, on page 28, he says: *"the comet would have been seen beyond this line: in the morning, therefore, at 3ʰ, the comet was at about 21° 20′ of Leo, etc. The comet therefore advanced in six hours from 18° 30′ of Leo to 21° 20′ of Leo, at most 2° 50′."* That motion of 2° 50′, therefore, was not made from 9ʰ to 15ʰ, nor in precisely 6 hours, as he sets it down in contradiction with himself, but from 8½ʰ to 15ʰ, that is, in 6½ hours. From 9ʰ to 15ʰ, on the other hand, it went from 18° 50′ to 21° 20′, and so was 2° 30′; and this is the seen motion which, from his own observations, he ought to have assigned to the 6 hours. Fourth, because the true motion in those 6 hours, gathered from the daily motion, as Kepler thinks, ought to have been 2° 45′, or rather less, 2° 32′, as we saw above. Therefore it does not follow that, if it was above the Moon, the motion must have been 2° 50′. Hence, fifth, I turn the argument back thus: If it had been above the Moon it would have completed, on your own account, 2° 50′; but it completed only 2° 45′, indeed only 2° 32′; therefore it was not above the Moon.

[Margin: 3. Reply to the second part of the major.]

V. The other argument was from the motion in latitude, which Kepler calls much firmer; reduced to a syllogism, it runs as follows: *If the comet had been on the border of the Moon, or below it, its true path would have been tortuous, and its latitude in the morning smaller than on the preceding evening; but if it had been above the Moon, without parallax, the path would not have been tortuous, nor the latitude smaller in the morning than on the preceding evening. But the comet's true path was not tortuous, nor was its latitude smaller in the morning than on the preceding evening. Therefore the comet was above the Moon.* The major is proved from the parallaxes of latitude applied to convert the observed latitudes into true ones; for from these it follows that on the evening of the 26th, at 9ʰ, the true latitude was 36° 27′; in the morning, that is, at 3ʰ after midnight, the true latitude was 36° 20′; and on the evening of the 27th, at 9ʰ, the true latitude was 38° 27′. The minor, as to its second part, is proved because the latitudes without parallax were found in this order: namely 35° 30′, then 36° 0′, then 37° 30′, then 40° 4′.

[Margin: 2. Kepler's argument reduced to form.]

I reply [?], first — granting, as above, the first part of the major (which could nevertheless be denied by one who holds to astronomical rigour, since it was derived from rough observations in so slippery a matter) — by denying the second part of the major proposition. For the comet could have been above the Moon and yet its path have been tortuous because of some very sensible parallax, though one smaller than the Moon's.

[Margin: 1. Negative reply to the second part of the major.]

I reply, second, by denying the minor proposition, which is proved not of the true motion and from the true latitude, but of the seen motion and from the seen latitudes, taken as observed and reduced to true ones by no parallax. Even granting these, it is not proved that the comet's path was not tortuous; for the latitude could have increased continually and yet the path have been

tortuous, and bent now and again towards the same quarter, without being twisted by alternating bends towards opposite quarters of the world.

Solutio Argumentorum Kepleri & Santutij ex Claramontio.

The resolution of the arguments of Kepler and Santucci, from Chiaramonti.

VI. In the Supplement to the *Antitycho*, part 2, chapters 18, 26 and 27, Chiaramonti resolves the arguments brought both by Antonio Santucci and by Kepler for this comet's seat above the Moon: Kepler's, indeed, alone in chapter 26 — he too noticing the perpetual fallacy of arguing from the seen motion to the true, that is, of confusing these motions equivocally, and that the comet's motion could have been first slow, then very fast, and again slow, as happened in the comet observed by Regiomontanus. Santucci's argument alone, given by him in chapter 21, was taken from the stay of this comet above the natural horizon for $14^h\ 28^m$, in a northern parallel declining from the equator by 18° 20′, and this on October 8; to this parallel answers a diurnal arc of 217 degrees, that is, of $14^h\ 28^m$. From this he gathers that the comet's apparent stay was as long as its true stay above the true horizon ought to have been, and so that it had no sensible parallax.

[Margin: Santucci's argument.]

Chiaramonti, however, replies, first, in chapter 18, that this argument is not valid so long as the parallax is unknown. See what we said in chapter 15, problems 7 and 8, corollary 1.

[Margin: 1. Chiaramonti's reply.]

He replies, second, that the declination of this comet on October 8 asserted by Santucci disagrees by many degrees with the declination deduced from the longitude and latitude observed by Kepler on the same day, namely October 8. For Kepler says, in his book *De cometis*, page 34, that he saw the comet at 21⅙ degrees of Scorpio, with northern latitude 21⅚°. From this, by the usual problems of the primum mobile, with the obliquity of the ecliptic put at 23° 30′, the comet's declination is gathered as 3° 1′ 24″, differing, that is, from Santucci's by 15° 18′ 36″. And to a parallel declining 3 degrees answers a diurnal arc of 186° 14′, or $12^h\ 25^m$. Moreover, if, as Santucci affirms, on October 8 the comet's altitude at Pisa was 37° at $\frac{1}{2}^h$ of the night, and at $2^h\ 43^m$ before sunset the comet was on the meridian, Chiaramonti shows, and geometrically, that its declination was 10° 39′. Either way, then, it disagrees greatly with the declination of 18° 20′ asserted by Santucci himself. Therefore either both are false, or only one is true; and so we are still in uncertainty, since neither were Kepler's observations of this comet very accurate, nor are Santucci's consistent with themselves.

[Margin: 2. Chiaramonti's reply.]

But if anyone insists that the observations of both be admitted, Chiaramonti readily accepts this condition in chapter 27. From the difference of the comet's declinations on this eighth day — which difference was 15° 18′ 36″, as we have already said — and supposing the altitude of the pole at Prague, where Kepler

then was, to be 50° 6′, and at Pisa, where Santucci was, 43° 30′, he shows, by the problem set out by us in chapter 19, number 2, that this comet's distance from the centre of the Earth was less than 4276 Italian miles, of which 3035 make up one semidiameter of the Earth. Nor will there be any great disagreement from this if the altitude of the pole at Pisa is chosen as 43° 27′, as

[Margin: 3. Chiaramonti's reply.]

observed there, Dr Vincentius Reinerus [Vincenzo Renieri]. Therefore, if we are willing to admit some of the observations of both Santucci and Kepler, it follows necessarily that this comet was far nearer the Earth than the Moon is. For we have inspected Chiaramonti's calculations and found no error of great moment, except that he puts the difference of the declinations as 14° 18′ 36″ instead of 15° 18′ 36″.

Argumenta Longomontani pro Parallaxi Cometæ huius.

Longomontanus's arguments for the parallax of this comet.

VII. After some observations of this comet, made partly by estimation with the eye and partly by the sextant and by distances taken from the fixed stars, and reported by Longomontanus in the Appendix to the *Astronomia Danica*, chapter 9, we have these words of Longomontanus: *"Although in this comet it did not fall to us to have, in order, such observations as would let us extract its parallaxes, if it had any, with sufficient subtlety — according, that is, to what was taught in the doctrine of the sphere — nevertheless, among other things, even that single observation of the comet's position which we made on September 28 in the evening at 7¼^{h}, jointly as it were with Dr Johannes Kepler, who was then at Prague in Bohemia, will suffice for the one thing we aim at here."* He adds afterwards that the comet then appeared to him to be distant scarcely one diameter of the Moon, or 34′, below and to the east of the small unformed star placed behind the Bear; from which he gathers the comet's seen longitude as 11° 57′ of Virgo, with latitude 40° 5′. But Kepler too, for that evening, reports as follows in his little book *De cometis*, page 29: *"For on September 28 the comet was seen to stand below the small unformed star, the companion of the large unformed one, between the tails of the Bear and of the Lion: it is at 12° 16′ of Virgo, latitude 40° 30′. It was distant by a diameter of the Moon on the line that runs along the last star of the Bear's tail through the hand of Arcturus. The comet was therefore at 12° 0′ of Virgo, latitude 40° 4′."* From this Longomontanus supposes that in the vertical circle there was scarcely a difference of 3′ between the place seen by himself and that seen by Kepler. He likewise supposes that there was no sensible difference between the time at which the said observation was made by Kepler at Prague and that at which it was made by himself at Copenhagen, which he conjectures from its daily motion; that Prague is not more easterly than Copenhagen by more than 6 minutes of time; and that the altitude of the pole is 50° 6′ at Prague and 55° 43′ at Copenhagen, and therefore the difference 5° 37′. From this he gathers that these two places, as if on the same circle of altitude, are 5½° apart, and that the comet's altitude was then 20° at Prague and 25½° at Copenhagen. *"All*

things considered broadly," he says; for he thinks this latitude, and a somewhat rough construction from the data, suffice to convince the Peripatetics and to raise this comet above the Moon.

[Margin: In the new style the day was the 28th, but in the old style the 18th.]

With these things prepared, let the figure set out in chapter 19, number 2, be repeated here. In it let AT be the semidiameter of the Earth, of 860 German miles, and TL the altitude of the vapours, or of the highest region of the air, which is to be 873 German miles; let Prague be at A and Copenhagen at I, and let the arc of the vertical circle between them, AI, be 5½°; and let the distance of the comet, seen at L, from the zenith of Prague be LAR, of 70°, since its seen altitude there is put at 20°. Then the angle LAT, its supplement to two right angles, will be 110°. Therefore in the triangle TAL, given that angle and the sides AT 860 and TL 873, the parallactic angle ALT with respect to Prague is gathered, with trigonometry as guide, as 67° 46′.

[Margin: 1. Step.]

Diagram. Set in the left column below the paragraph above: a large semicircle on a base line, with a smaller concentric semicircular arc inside it and a small full circle centred on the base line at T, its lower half hanging below the base. A vertical rises from T to the top of the outer arc; straight lines run from points on the inner arc and the small circle out to points on the outer arc, and a horizontal line crosses through the inner arc to the right.

- R (top of the vertical through T): label
- D (just right of R, close against it): label
- Q (upper left, on the outer arc): label
- D (upper right, on the outer arc): label — a second D, as printed
- E, F (right, lower on the outer arc): labels
- m (lower-case, right end of the horizontal line): label
- M, O, P (left, on the outer arc, top to bottom): labels
- L, A, G, C (along the inner arc, left to right): labels
- I, B (on the small circle): labels
- H, N (where the inner arcs meet the base region): labels
- S, V, T, X, Z (along the base line, left to right): labels; Z is a Z-shaped sort with a curled foot
- u (lower-case, foot of the small circle): label

Second, in the triangle TIL are given TI 860 and TL 873, and the angle TIL, 115½°. For the comet's distance from the zenith of Copenhagen I, LIQ, was put smaller than at Prague by 5½°: it was put at 64½°; therefore its supplement to two right angles, that is TIL, is 115½°. Therefore by trigonometry the parallactic angle TLI with respect to Copenhagen comes out 62° 46′, and so the difference of the parallaxes, that is the angle ALI, or the arc PO, is 5°. So great, then, would have been the difference between the comet's distance from the unformed star of the Bear — say from the star Q — as seen by Kepler at A, and its distance from the same star as seen by Longomontanus at I, on this supposition. But the difference in the distances from the fixed star, placed

as if on the same vertical, was not greater than the Moon's diameter; indeed, probably it was not greater than 3′. Therefore the comet was not a mere 873 German miles high above the Earth, nor in the highest region of the vapours, as the Peripatetics would have it, but much higher.

[Margin: 2. Step.]

Having demonstrated these things — though with another diagram, which is corrupt in my copy — Longomontanus appends these words: *"Moreover, from the proportionate change of this comet's proper motion and latitude, and likewise from its constant progress under a great circle almost to the last days before it disappeared, we can gather, indeed plainly establish, that this comet was celestial; although the observations of it, taken less precisely and less timely, scarcely permit us to reason how far it climbed aloft through the aether. Therefore, breaking off the web here, we shall hasten on to the phenomena of the comet of the year 1618."* He admits, therefore, that the parallax or distance of this comet was not demonstrated and determined by him.

Solutio Argumentorum Longomontani.

Resolution of Longomontanus's arguments.

VIII. First, in the apparatus for the demonstration already mentioned, Longomontanus admits many errors not to be despised, but two above all. For he wrongly supposes that the observation was made by himself and by Kepler with the insensible difference of time already mentioned. He himself testifies that his own was made at $7\frac{1}{4}^{h}$ after noon, whereas Kepler records his observations of the 27th and 28th at 9^{h} after noon, and from them gathers the comet's daily motion at that time as 12 or 12½°. Therefore if Prague, as Longomontanus himself concedes, is more easterly than Copenhagen by only 6 minutes of time, when it was 9^{h} at Prague it was $8^{h}\ 54^{m}$ after noon at Copenhagen, and not $7\frac{1}{4}^{h}$. The difference of time between the one observation and the other was therefore $1^{h}\ 39^{m}$, which is very sensible; and to it there correspond, from the comet's daily motion in longitude observed by Kepler, about 50′, and not merely 3′, as Longomontanus supposed.

[Margin: 1. Error of Longomontanus.]

Second, from the fact that Prague is more easterly than Copenhagen by only 6^{m} of time, and from the difference of pole altitude of 5° 37′, he wrongly concludes that those places are apart on the same great circle — that is, on the vertical drawn through them — by an arc of only 5° 30′; for this distance had to be greater than the difference of the pole altitudes. For in the diagram below let AGQ be a portion of the equator, to which let quadrantal arcs be drawn from the pole of the world P: PBG, passing through B, Prague in Bohemia, and PKA, through Copenhagen, K; and let K and B be joined by KB, an arc of a vertical circle. Since, then, the altitude of the pole is given as 55° 43′ at Copenhagen and 50° 6′ at Prague, their complements will be PK, 34° 17′, and PB, 39° 54′; and the angle BPK is given as 1° 30′. Therefore by trigonometric calculation the arc KB comes out 5° 51′ 30″, and not 5° 30′, as Longomontanus

put it. But notwithstanding these errors, it is still gathered that the comet was above the altitude of the vapours.

[Margin: 2. Error of Longomontanus.]

Diagram. Set in the right column beside the lines describing it: a spherical triangle with its apex at P and its base an arc of the equator running from A through G to Q. From P two arcs descend, one through K to A and one through B to G; K is joined to B, and further lines run from K and B to a point C on the right-hand side.

- P (apex): the pole of the world
- K (upper left): Copenhagen
- C (right): label, not cited in the text of this page
- B (lower centre): Prague
- A (lower left): end of the arc of the equator
- G (base): on the arc of the equator
- Q (lower right): end of the arc of the equator

Third, however, I deny that the comet must plainly be established as celestial from the fact that it was seen to describe a great circle, and with a regular motion at that; for we have often taught already that both can belong to sublunary moving bodies. And I deny that this argument is evident: at most it is probable, for a place or motion above the Moon. Indeed, if Longomontanus's and

Kepler's observations are admitted, but reduced to the same moment of time, as is fitting, it can be demonstrated from the remaining data given by Longomontanus that the comet was below the Moon. For since the difference of longitude observed by himself and by Kepler (at the same moment of time, as he thought) was 3′, he inferred that the difference in the circle of altitude was about 3′. Therefore, since in reality it was 50′, he ought to have admitted a difference of the parallaxes of about 50′ in the circle of altitude as well. This being posited:

OSTENDITUR COMETAM ANNI 1607. EX DATIS À LONGOMONTANO & KEPLERO SED CORRECTIS, FUISSE INFRA LUNAM.

It is shown, from the data of Longomontanus and Kepler, but corrected, that the comet of 1607 was below the Moon.

IX. Go back to the figure shown here at number 7, and in chapter 19, number 2, in which let AT be 860 German miles, which are 3440 Italian; or, to be generous, let AT be 4139 Italian miles, according to our view shown in book 2, chapter 7. And let the arc AI between Prague and Copenhagen be 5° 51′ 30″, as stated under Longomontanus's second error. Then the chord AI will be 10,220 parts, of which the whole sine AT is 100,000; therefore, in units of which AT is 4139, AI is 423 Italian miles. The angle ATI, which measures the arc AI, is given as 5° 51′ 30″; therefore, by propositions 5 and 32 of the first book of Euclid taken together, each angle at the base of the isosceles triangle, that is both TAI and TIA, is 87° 4′ 15″. But the seen distance of the comet from the zenith of Prague, A, that is the angle RAL, is put at 70°; joined to the angle TAI of 87° 4′

15″, it makes a sum of 157° 4′ 15″. Therefore its supplement to two right angles, by proposition 13 of the first book of Euclid, that is the angle LAI, is 22° 55′ 45″. This angle is given in the same triangle LAI; but there are also given the difference of the parallaxes, that is the angle ALI, 50′, and the side AI, 423 Italian miles. Therefore the comet's distance from Copenhagen, that is the side IL, is by geometric necessity 11,332 Italian miles, in which AT, the semidiameter of the Earth (which we put at 4139), is contained 2 3054/4139 times. But let us put IL at 3 terrestrial semidiameters: with TI added to IL, TL, the comet's distance from the centre of the Earth, would come out at most 4 semidiameters of the Earth — much less, therefore, than the distance of the Moon at perigee, which in the Ptolemaic hypothesis is more than 33 semidiameters of the Earth from the Earth, and in Longomontanus's more than 51 terrestrial semidiameters, from what was said in book 4, chapter 14. But Longomontanus will perhaps object, and will not be willing to admit so great a difference of the parallaxes, namely 50′, which he nevertheless ought to admit. Since, then, he will not yield to us, let us yield to him in this and assume the difference of the parallaxes in the circle of altitude that he admitted, 3′. For from this too we shall forthwith demonstrate that this comet was still below the Moon. Let this, then, be a proposition most worthy of note in this matter.

Data differentia Parallaxium in circulo altitudinis 3′. tantummodo minutorum, & altitudine Cometæ viſa Pragæ Gr. 20. & arcu circuli maximi inter Pragam & Hafniam Graduum 5. Min. 51 1/2. Demonſtratur Cometæ Diſtantia à terris minor quàm Lunæ, & Parallaxis maior.

Given a difference of the parallaxes in the circle of altitude of only 3′, a seen altitude of the comet at Prague of 20°, and an arc of a great circle between Prague and Copenhagen of 5° 51½′, the comet's distance from the Earth is demonstrated to be less than the Moon's, and its parallax greater.

X. Keep everything else in the triangle ALI that was stated at the preceding number — namely the angle IAL, 22° 55′ 45″, and the side AI, 423 Italian miles in units of which AT is 4139 — but assume the angle ALI, that is the difference of the parallaxes, to be 3′. Then by the laws of trigonometry the comet's distance from the surface of the Earth, namely IL, is gathered as 188,848 Italian miles. Divided by 4139, the terrestrial semidiameter, these leave IL as 45 26/41 terrestrial semidiameters. If you add one semidiameter to these, that is, if you join the lines TI and IL, the comet's distance from the centre of the Earth, that is TL, would be at most 46 26/41 terrestrial semidiameters. But TL is less than the sum of the two sides TI and IL, by proposition 20 of the first book of Euclid; therefore the comet's distance from the centre of the Earth was less than 46 26/41 terrestrial semidiameters. It was therefore also less than the lunar distance, which for Longomontanus is at least 51 semidiameters 57′ of the Earth, from what was said in book 4, chapter 14. And, the Ptolemaic hypothesis excepted — which on this point, from what was said in the same place, is false

— there is no astronomer who does not put the Moon, however far at perigee, beyond 47 semidiameters of the Earth.

Corollarium Insigne in hac materia.

A notable corollary in this matter.

Gather from this that if the comet is distant from the zenith of each observer by 70° or more, and the places of the two observers are not more than 6° apart from each other, then even if the observed difference of the parallaxes is of a few minutes, say 3 or 4, the comet can nevertheless be below the Moon; indeed, this can sometimes happen even if the difference is of one minute. Chiaramonti showed this too in the *Antitycho*, book 3, chapters 19 and 20, and we with him in book 10, the penultimate section, chapter 11, problem 3, and in this very book, chapter 19, number 3; and finally we have thought it right to drive the point home with the example just given, so that it is not pronounced so readily, as most have done until now, that comets are above the Moon because of a slight difference of the parallaxes.

Conclusiones ex Prædictis.

Conclusions from what has been said.

I. CONCLUSION: *It has not so far been demonstrated that the comet of the year 1607 was above the Moon.* For none of the three authors who tried to demonstrate this has in fact demonstrated it: not Kepler, as is clear from what was said from number 4 to 6; not Santucci, as is clear from what was said at number 6; nor, finally, Longomontanus, as is evident from what was said from number 8 to 10.

II. CONCLUSION: *Although it is not improbable that the comet of the year 1607 was above the Moon, it is nevertheless more probable that it was below the Moon.*

The first part is established not only by the authority of Kepler, Santucci and Longomontanus, who assert it, but also by their arguments. The second, however, is clear from the observations made at two places on the Earth, which, other things being equal, prevail in this business over observations made at a single place on the Earth, as can be established from what has been said so far and will be shown in book 10, the penultimate section, chapter 9. And from the observations either of Kepler and Santucci, or of Kepler and Longomontanus, compared with each other, it follows that this comet was sublunary, as is clear from what was said at numbers 6, 9 and 10. It is more probable, moreover, that those observations — at least those of Kepler and Longomontanus, whatever they were like — are to be accepted rather than rejected; therefore it is also more probable that this comet was below the Moon, though far above the vapours and the twilight exhalations, as is certain from the distances of this comet deduced at numbers 6, 9 and 10, if they are compared with the altitude of the vapours and exhalations given at the end of chapter 14.

CAPVT XXII.

De Cometis Anni 1618. Diſtantia & Parallaxi.

Chapter 22. On the distance and parallax of the comets of the year 1618.

I. We come at last to the last comets seen, in our own time at least, since from the year 1618 down to this year in which we are writing no comet has shone upon our regions. Since, however, no comet of all those before had more observers skilled in mathematics, in different parts of the globe, than the last of this year; since none drew more votes for its celestial nature and place; and since none, finally, was made more famous by more booklets published, whether of the Peripatetics, who contend that it was born and died in the elementary region, or of the Tychonians (for so it pleases me to distinguish the sect of astronomers on this question from the Peripatetics, even granting that Tycho did not see this comet), who strive with all the engines of geometry and optics to carry it beyond the confines of the Moon — we too must discuss this comet more accurately. For if not even this one has been demonstrated to be above the Moon, there will be no reason for the astronomers, in this respect, to look down on the Peripatetics with such haughtiness.

Now, among those who raised this year's comet into the heavens, there are some who tried to show it by geometrical reasons: first of all Father Johann Baptist Cysat of our Society, professor of mathematics at Ingolstadt, in the *Mathemata Astronomica*, which he composed in 7 chapters on the one comet of this year; Kepler, in his book *De cometis*, from page 47; Longomontanus, in the Appendix to the *Astronomia Danica*, chapter 10; Snel, in his book on the comet of the year 1618, consisting of ten chapters; Father Orazio Grassi, both in the disputation on these comets held in our Roman College and in the *Libra Astronomica*, published under the name of Lotharius Sarsius; and Biancani, in the *Sphaera*, book 16, chapter 5. There are others who, drawing conclusions from the authors already mentioned that they think demonstrated, and adorning them with probable reasons, raised the comets of this year — but one above all — into the heavens. Chief among these were Thomas Fienus, Erycius Puteanus, Libert Froidmont and Descartes.

[Margin: Authors of different classes.]

On the other side, some have tried by mathematical demonstrations to keep these comets too below the Moon: first of all Scipione Chiaramonti, in his *Antitycho*, book 2, chapter 26, and book 3, chapters 20 and 21; in the *Apologia* for the *Antitycho*, book 2, chapters 11 to 13; and in the supplement to the *Antitycho*, part 2, from chapter 28 to the very end.

Others, again, did strive to draw them back below the Moon, but partly by using mathematical reasonings foreign to the matter, partly by merely probable reasons taken either from physics or from astronomy — above all Cottunius, on Aristotle's *Meteorology*, book 1, lecture 31.

II. We have already set out the history of these three comets in chapter 3. The first of them, which arose in the month of August, is said to have been very

faint through September, so that it could scarcely be made out by astronomers; and so Kepler says of it, on page 47: *"But since no proper observations of it exist, nothing can be brought forward to demonstrate its altitude and motions."* The second comet, for its part, was so confused by several observers with the third, or with the fiery beams seen in October and November, that we can have nothing certain from it as regards parallaxes.

[Margin: The first comet of the year 1618.]

[Margin: 2. Comet.]

The third comet was not observed in Europe, but in Persia and in the East Indies, and yet without any marks suited to extracting its parallaxes, or even to attempting them by conjecture; nor did it shine for more than 10 days, as I said in the History.

[Margin: 3. Comet.]

The fourth comet, then, which European writers call the third, is that most celebrated one which was observed by Father Giacomo Rho of the Society of Jesus at Goa; Father Antonio Rubino of the Society of Jesus at Cochin; the Fathers of the College at Rome; Father Biancani at Parma; Schickard in Württemberg; Chiaramonti at Cesena; Antonius Lucianus at Valencia in Spain; Gallus Prouenzalis at Paris; Gassendi at Aix; Snel in Holland; our Fathers at Antwerp; Father Cysat of the Society of Jesus at Ingolstadt; Dr Habrecht at Strasbourg; Gaspar Herspachius at Cologne; Froidmont at Louvain; Philippus Millerus [Philipp Müller] at Leipzig; Ambrosius Rhodius at Wittenberg; Benjamin Ursinus at Frankfurt an der Oder; Johannes Remus Quietanus at Rastadium and at Innsbruck; Longomontanus at Copenhagen; Kepler at Linz; Vuelperus at Lora and at Worms; Gotthard Arthus at Danzig; Herlicius in Pomerania; Nagellius at Magdeburg; and many others, most of whom raised this comet beyond the Moon. And so we must treat of it more accurately.

[Margin: 4. Comet.]

ARGUMENTA LEVIORA PRO CÆLESTI NATURA & LOCO COMETÆ ANNI 1618. ADDUCTA, & REIECTA INTER MERÈ PROBABILIA AUT INSUFFICIENTIA.

Lighter arguments for the celestial nature and place of the comet of the year 1618, adduced and rejected as merely probable or insufficient.

III. The first argument is from its motion, regularly unequal and along the same circle, sensibly a great circle — a motion such as seems to suit celestial rather than elementary phenomena. So it is used by Cysat, page 35; by the author of the *Libra Astronomica*, page 9; by Biancani, on the mathematical passages of Aristotle, page 93; by Froidmont, *Meteorology* book 3, chapter 1, article 3; by Cabeo, *Meteorology* book 1, text 37, question 1; by Franciscus Resta, treatise 1 on the *Meteorology*, chapter 2; and by Snel, in his first reason. But we have clearly taught, in chapter 11 from number 7 to 12, and in chapter 18 from number 2 to 4, that this argument is not evident and necessary for proving that the origin and place of the phenomenon are celestial rather than elementary,

and that in the end Tycho himself and Kepler, in other comets, acknowledged it as merely probable.

[Margin: 1. Argument, from the regular motion along the same great circle.]

The second argument is from the comet's proper motion towards the east, slower than the motion of the Moon: for, says Snel, it completed not a full three degrees. But this argument cannot properly be adapted to this comet we are dealing with, because its proper motion was not towards the east but towards the west, against the order of the signs, namely from the end of Libra to Virgo; and besides, it is not an evident argument, but at most a probable one, as we showed at length in chapter 11, numbers 5 and 6. Snel nevertheless uses it.

[Margin: 2. Argument, from a motion slower than the Moon's.]

The third argument is from the daily motion imitating the motion of the primum mobile, from which a celestial nature seems to be gathered; Biancani touches on this argument in the *Sphaera*, book 16, chapter 5. But this argument is not evident, because even the element of air and of fire may imitate this motion. See what I said in chapter 11, number 3.

[Margin: 3. Argument, from the imitation of the motion of the primum mobile.]

The fourth argument is from the comet's stay above the natural horizon being as great as the stay of the Sun, placed in the same parallel, above the rational horizon; from which it seems to be gathered that the comet's parallax is no greater than the Sun's; or at any rate, if the stay is not so great but is still of several hours, the comet's altitude seems to be gathered to be far greater than that of the vapours and of the highest region of the air. But the first part of the argument cannot evidently be fitted to this comet, because it was not seen to move within the torrid zone in such a way as to be visible all night from rising to setting. Besides, its true declination would have to be distinguished from the seen one, and the true parallel from the seen one, so that it could be known how great a stay above the rational horizon suited it, and how that stay differed from the stay above the sensible horizon; but this cannot be known so long as the comet's parallax is unknown, and it is still supposed unknown. On this see what we said in chapter 11, number 4, but especially in chapter 15, problems 7 and 8, corollary 1. Cysat, nevertheless, in chapter 3, proposition 2, shows from the stay of this comet above the horizon that it must have been higher than 2580 German miles, indeed than 8600, or 10 semidiameters of the Earth; and this, granting it to him, does not avail to advance the comet as far as the Moon.

[Margin: 4. Argument, from the stay of the comet above the horizon.]

As to the latter part of the argument, I admit that from the many hours during which the comet appears above the horizon its altitude is gathered to be greater than the altitude of the vapours, but not that it is concluded from this to be above the Moon, as I said in chapter 11, number 4. I therefore freely grant Father Cysat what he himself taught in chapter 3 in his first three propositions, of which the first is that the comet was above the highest region of the air, or higher than 13 German miles, because it was seen for more than two hours in

the morning above the horizon before it reached the meridian. This could not have been so had it not been higher than 13 German miles, or 52 Italian; for within two hours it would have flown from rising to setting — and all the more so, the more certain it is that this comet did not resist the motion of the primum mobile but rather complied with it, by its own proper motion towards the west. See on this matter the diagrams and problems 7 and 8 of chapter 15. But neither Cysat there, nor Froidmont, *Meteorology* book 3, chapter 1, article 4, contends anything else by this argument than that comets were above the vapours.

The fifth argument is from the comet's duration through a whole month and more, and the sixth from the comet's bulk, especially counting the tail; for neither of these does elementary matter, of itself fluid and ill-suited to so great a bulk, seem sufficient. But from this it is at most proved that the comet was not generated from a mere hot and fatty exhalation by ignition, etc.; and not even this is evidently shown, but is only urged by probable reasons, as may be seen from what was said in chapter 11, from number 13 to 18, where we weighed what Snel adduces for this comet.

[Margin: 5th and 6th arguments, from duration and bulk.]

The seventh argument is from the transparency of the tail, and the eighth from its being turned away from the Sun. But not even by these is it evidently shown that the comet was above the Moon; but if

[Margin: 7th and 8th arguments, from the transparency of the tail and its turning away from the Sun.]

anything is shown, this alone is shown: that it is not a meteor generated by mere ignition, but that its head is transparent and pervious to the rays of the Sun, which pass right through it and, in place of a shadow, paint a luminous tail on the comet. See more on these matters in chapter 11, numbers 1, 2 and 19, and below, in chapter 23.

The ninth argument is from the property of the telescope, which usually enlarges notably the objects that are below the Sun, and indeed the Sun itself, but usually does not enlarge the fixed stars, because of their immense distance. Yet the comet of the year 1618, viewed through excellent telescopes, gained no sensible increase; therefore it seems to have been far higher than the Sun, and near the sphere of the fixed stars. But we have already said more on this argument in chapter 11, from number 20 to 23, teaching that the telescope shows an increase even in the fixed stars, and that it showed in this comet as much increase as the real size of its apparent disc, shorn of adventitious rays, required.

[Margin: 9. Argument, from the almost null increase taken on through the telescope.]

The tenth argument is from the altitude this comet needed in order that it and its tail, which derive from the Sun's illumination, could be seen for many hours by night and not lie hidden within the Earth's shadow. But I reply to this that it is not necessary for the comet to be higher than the whole axis of

the Earth's shadow; it suffices that it be outside the shadow, to the sides, by reason of its latitude or declination from the ecliptic, along which that shadow is carried — in the way the Moon mostly is at full moon. For this, an altitude much smaller than the Moon's suffices, provided the latitude, or deviation from the ecliptic, is greater; and this argument proves no more than an altitude greater than the altitude of the vapours. See problem 9 of chapter 15.

[Margin: 10. Argument, from the altitude outside the Earth's shadow.]

The eleventh argument is from the altitude required for the comet to be seen at once from many places on Earth very distant from one another, or, in one place, very distant from the comet's parallel; and the twelfth is from parallax. Because these arguments carry greater weight in this matter, they must be discussed separately.

[Margin: 11. and 12., from the distance of the observers and from parallax.]

1. Argumentum ex Loci, in quo viſus eſt Cometes diſtantia à Parallelo, in quo verſabatur Cometa. Quod Argumentum eſt P. Cyſati.

1. The argument from the distance of the place in which the comet was seen from the parallel in which the comet was moving. This argument is Father Cysat's.

IV. This comet was seen here at Frankfurt an der Oder on November 19, and somewhat earlier at Danzig in Prussia; on that day it had a southern declination of at least 23°. But the latitude, or northern declination from the equator, of Frankfurt is at least 52°, and of Danzig 54°. When these are joined with the comet's southern declination, the result is an arc of 75° between Frankfurt and the comet, but of 77° between Danzig and the comet. Therefore, taking the Earth's semidiameter as 860 German miles, and the comet in the physical horizon, it follows, by problem 3 of chapter 15 of this section and its corollary 3, that the comet must have been 3322 German miles distant from the centre of the Earth, or, with respect to Danzig, 3823 miles — and much more, if the comet was seen above the horizon in both places. So Father Cysat on this comet, chapter 3, proposition 2.

[Margin: Argument of Father Cysat.]

But this argument supposes the true declination, which, so long as the comet's distance from the Earth and its parallax are unknown, cannot be distinguished from the apparent declination — such as was the one which Cysat, or others, had gathered from the observations. Moreover the apparent declination, when it is southern, appears greater to a northern observer, because of parallax, than it really is; and therefore the distance mentioned above ought to have been smaller. Next, from this it is shown only that the comet was much higher than the vapours and the boundary of the highest region of the air — namely, about 4 or 5 semidiameters of the Earth; but from there to the Moon a great deal still remains. Nor does Cysat wish to show from this that the comet was above the Moon, but rather by the following argument.

2. Argumentum ex Diſtantia duorum Locorum terræ, ex quibus eodem tempore obſeruatus eſt Cometa.

2. The argument from the distance between two places on Earth from which the comet was observed at the same time.

V. If any places were suited to this, they were surely Goa and Cochin, where Fathers of our Society — namely Father Giacomo Rho and Father Antonio Rubino — observed this comet on the same days on which it was also observed by our men at Rome, Parma and Innsbruck, and by others at Antwerp, Linz, Ingolstadt, Cesena, Valencia and elsewhere. But, as is clear from our problem 6 of chapter 15, the comet would have had to be observed at the same moment of the same day; or at least it would have had to be possible, from the hour exactly noted in each place and from the comet's true motion, to reduce the observation to the same moment. Further, the comet's distance would be needed, exactly observed from the zenith of each place, or from the same fixed star, first corrected for refractions; and finally the exact distance of the two places along the arc of a great circle drawn through both would have to be known, or knowable from an accurate polar altitude and the distance of the meridians. But in this case we lack all of these. For neither the true motion can be distinguished from the apparent, nor refraction from parallax, if one supposes, as is supposed here, that the comet's distance and parallax are still unknown. Nor is the polar altitude of Goa or of Cochin yet exactly known; nor is the distance of those meridians from the meridians of Europe determined, except only ὁλοσχερῶς [Greek: holoscherōs, "in the gross"], or roughly; nor the size of the arc of the great circle drawn through Goa and some place in Europe. Finally, in many of the European observations, and in all the Indian ones, the hour of observation is missing and only the day is noted, and the distances from fixed stars are not accurate throughout. So it is rightly said in the disputation on the three comets of this year held in our Roman College: *"I know that account should have been taken of the hours at which observations of this kind were made in different places, even though they were made on the same day; and besides of the refractions and other matters, whose investigation required far more diligent observers."*

Moreover, the Fathers already mentioned, in the letters they sent from Italy to Europe, in which they recorded their observations of this comet, excuse themselves in the very preface of those letters: through haste, and the lack of better instruments, they could not obtain more accurate observations. And one of them says, in the name of all: *"We had scarcely entered port, and were unprepared for this kind of observation, when it happened that the sky was suddenly seen to blaze with two comets in the morning hours. Among the Moors and the heathen, astonished by the novelty of the thing, this stirred up various rumours. Our baggage was still stuck in the ship; there were no instruments to hand [?] we could trust; to seek them elsewhere was not possible, since there were none there; and we ourselves were forced to move off, one here, another there. So we seized what was to hand — only the astrolabe and the astronomical radius of the College of Goa — and girded ourselves for the*

observations, as far as our other business allowed. Often with broken sleep, often on journeys by land and sea, often in places ill-suited to observing. These, nevertheless, such as they are, hastily copied, I send to the Europeans, meaning to satisfy everyone in the name of all; and since it could scarcely be done otherwise, it seemed good to commit them to print. I hope they will be welcome on this account, that they were sent from the East Indies. For I by no means offer them as exact. The instruments, the place and our occupations did not allow that: we gave what effort we could. We do not strive for a scrupulous count of minutes. If we do not err in whole degrees, that is enough." So he — or rather they — with the sincerity and candour that befit religious men.

[Margin: Excuse of the Fathers in India for their observations of the comets.]

Let our good Biancani go now and say, in the *Sphaera*, book 16, chapter 5: *"The observations made at Goa of the beam and of the comet of the year 1618 agree entirely with ours carried out at Rome, Parma and Antwerp — that is, both were seen by them near the same stars at the same time, etc. From which it may be affirmed with the utmost certainty that they showed no parallax, since from places separated by so great an interval of lands and seas they obtained the same apparent place in the firmament."* Had the observations been accurate, had they been made at the same moment of time, and had the comet's apparent place been the same in both places, Biancani would be arguing rightly; otherwise, not rightly.

[Margin: Biancani's excessive credulity about the diligence of the Indian observations.]

An Potuerit eodem momento temporis videri Cometa Goæ & Ingolſtadij; & ſi potuit aliquo die,quanta fuerit Diſtantia Cometæ.

Whether the comet could have been seen at the same moment of time at Goa and at Ingolstadt; and, if it could on some day, how great the comet's distance was.

VI. Therefore, since we cannot attain evidence from this, I wish, for the sake of conjecture, to investigate whether this comet could have been seen at the same moment of some day in the places of Europe and of India already mentioned; and, if it could, how great both the least and the greatest altitude of the comet must have been, or could have been, for it to have been visible above the horizon in both places while keeping to the observations. I choose Ingolstadt, because Cysat noted there carefully both the hour and the altitude of the comet. We shall take the difference of geographical longitude between Goa and Ingolstadt, established by many investigations in our geographical book, as 70°, that is, $4^{h}\ 40^{m}$; and the polar altitude of Goa as 16°, and of Ingolstadt as 48° 40′. Further, in the year 1618, on November 24, when the comet was first seen in both places, the Sun's place by Magini's Tychonic tables was at Sagittarius 2° 27′ at noon at Rome; but at the end of December, where the Indian observations stop, it was at Capricorn 9° 51′. From these places, from the declination of the parallels in which the Sun was moving, and from the polar altitudes already mentioned, the seminocturnal time set out below is gathered

by the usual problems of the primum mobile, or by the table of semidiurnal arcs published in our *Crux Geographica*.

Seminocturnal time [Seminocturnum Tempus]

Days	Ingolstadt	Goa	Sun's place
November 24	$7^h\ 42^m$	$6^h\ 25^m$	♐ 2° 27′
December 31	$7^h\ 56^m$	$6^h\ 28^m$	♑ 9° 51′

Now the comet is reported, at Rome and throughout Europe as well as at Goa, to have been seen from the beginning — that is, on November 24 and the days immediately following — in the morning hours, namely between midnight and sunrise. Now the difference of longitude, $4^h\ 40^m$, is smaller than the seminocturnal time in both places; so nothing, as far as this goes, prevents the comet from having been seen in both places in the morning hours, or between midnight and sunrise. Let us now examine the possible extremes, and suppose the comet seen at Rome at midnight itself, namely at $7^h\ 42^m$ or $7^h\ 56^m$ before sunrise. For, once $4^h\ 40^m$ are taken away, there remain for Goa, which lies further east, $3^h\ 2^m$ or $3^h\ 16^m$ before sunrise, at which time — and not before — it could first be seen there, if it was seen in the morning in both places. But if we suppose it seen much later at Ingolstadt — yet in such a way that it could not have been seen earlier — there would be a danger that at Goa, with the Sun already rising, the comet could not be seen: if, that is, on November 24 it began to be seen on the horizon of Ingolstadt only $4^h\ 40^m$ before sunrise, or $3^h\ 2^m$ after midnight; or if on December 31 it began to be seen $3^h\ 16^m$ after midnight at Ingolstadt, for then the Sun was rising at Goa. It could therefore be observed, but not later than $3^h\ 2^m$ or $3^h\ 16^m$ after midnight, and not at these very hours unless it was very close to the Ingolstadt horizon; and this was much less possible in the more westerly places of Europe. But if it were established from the record of observations that it was observed later than $3^h\ 2^m$ or $3^h\ 16^m$ after midnight at Ingolstadt, but very low and near the horizon, that would be proof that it could not have been observed at the same moment at Goa, or at any rate was not observed. Take now, for the conjecture, the hours of the Ingolstadt observations and the apparent altitudes of the comet; to these we shall add the hours before or after sunrise at Goa.

Hours before sunrise at Ingolstadt, but at Goa before or after sunrise [Horæ ante Ortum Solis Ingolſtadij, ſed Goę ante vel poſt ortum Solis.]

December, day	Ingol-stadt	Goa	Apparent altitude of the comet at Ingolstadt
1	$2^h\ 6^m$	$2^h\ 34^m$ after	12° 20′
2	$3^h\ 2^m$	$1^h\ 38^m$ after	5° 40′

December, day	Ingol-stadt	Goa	Apparent altitude of the comet at Ingolstadt
4	$3^h\ 42^m$	$0^h\ 58^m$ after	✠ ✠
4	$1^h\ 0^m$	$3^h\ 40^m$ after	28° 20′
7	$2^h\ 24^m$	$2^h\ 16^m$ after	32° 50′
7	$0^h\ 54^m$	$3^h\ 46^m$ after	44° 0′
8	$4^h\ 55^m$	$0^h\ 15^m$ before	12° 30′
Remainder of the preceding table [Reſiduum Tabulæ Præcedentis.]			
9	$6^h\ 10^m$	$1^h\ 30^m$ before	1° 0′
9	$5^h\ 9^m$	$0^h\ 29^m$ before	10° 12′
9	$1^h\ 33^m$	$3^h\ 7^m$ after	48° 58′
10	$1^h\ 30^m$	$3^h\ 10^m$ after	47° 30′
17	$7^h\ 39^m$	$3^h\ 0^m$ before	10° 45′
17	$4^h\ 0^m$	$0^h\ 40^m$ after	47° 25′
18	$5^h\ 0^m$	$0^h\ 20^m$ before	39° 40′
18	$1^h\ 0^m$	$3^h\ 40^m$ after	76° 56′
20	$4^h\ 6^m$	$0^h\ 36^m$ after	48° 30′
20	$1^h\ 21^m$	$3^h\ 19^m$ after	75° 20′
24	$1^h\ 41^m$	$2^h\ 59^m$ after	80° 45′
28	$2^h\ 56^m$	$1^h\ 44^m$ after	69° 30′
29	$3^h\ 30^m$	$1^h\ 10^m$ after	66° 20′
29	$1^h\ 24^m$	$3^h\ 16^m$ after	79° 30′
30	$1^h\ 54^m$	$2^h\ 46^m$ after	76° 25′

From what has been said so far it is clear that there were very few days on which the morning observations made at Ingolstadt could coincide at the same moment with the morning observations at Goa, since for the most part the Sun had already risen at Goa. Let us now choose December 9, when, $5^h\ 9^m$ before sunrise, Father Cysat saw the comet, as he himself reports in chapter 1, 6° 42′ distant from Arcturus and 10° 12′ high. On the same day the comet's

apparent distance from Arcturus at Goa is reported as 6° 46′. Let us suppose the comet seen at Goa at the same moment, namely 29^{m} before sunrise. With these posited, the task is to investigate, if the comet had been on the same great circle under which Ingolstadt and Goa lie, its least and greatest distance from the centre of the Earth, while keeping to the altitude of the comet then seen at Ingolstadt.

VII. First, by the problem and diagram given in chapter 21, number 8, let Ingolstadt be at K and Goa at B; let the complement of the polar altitude, PK, be 41° 21′, and PB 74° (since the distance from the equator is as great as the polar altitude, so that AK is 48° 40′ and GB 16°); and let the angle KPB be the difference of longitude assumed above, AG, 70°. From these the arc KB comes out as 64° 55′.

Diagram. A spherical triangle set in the right column beside the text of §VII: two arcs descend from the pole P at the apex to a base arc AQ; an arc runs from P down to G on the base; lines join K to C, K to B and B to C.

- P (apex): the pole
- K (left arc, upper): Ingolstadt
- C (right arc, middle): no reference in the text of this page
- B (inside, low): Goa
- A (lower left): end of the base arc
- G (base, between A and Q): foot of the arc through B
- Q (lower right): end of the base arc

Secondly, look at the other diagram, in which likewise Ingolstadt is I and Goa B, between which lies the arc IB already found, 64° 55′, which measures the angle IAB. Now let the straight line BV be drawn, touching the Earth at B, to be the physical horizon of Goa; and from the centre of the Earth, A, let the straight line AV be produced through I; and let VIC be the angle of the comet's apparent distance from the zenith of the place I. For the comet's distance will be least if it appears at C, a point of the sensible horizon of Goa — namely the distance AC. If, indeed, it is within CI, it will not appear, as being below the horizon BV; but if beyond, as at M, its distance from the centre of the Earth will increase. Let there therefore be sought in the triangle ABC, right-angled at B, the base AV, in parts of which AB, the Earth's semidiameter, is 4139, since the angle VAB, 64° 55′, is given in it; and AV will be found to be 9763 miles. Taking away AI, 4139, there will remain IV, 5624. With this seek IC in the triangle IVC: for in it, besides IV, every angle is given — namely AVB, the complement of the angle VAB to a right angle, 25° 5′, and VIC, the apparent distance of the comet from the zenith of Ingolstadt, 79° 48′; and therefore ICV, 75° 7′. There will

Diagram. A semicircle (the Earth's section) on the base line LG, set in the right column beside the second construction of §VII. From the centre A lines run to K, through O, and through I to V; the tangent at B rises to V; a line runs from V through C and M to G; further lines join L to the region of I and C, and R to O.

- K (on the arc, upper left): no reference in the text of this page
- V (above, right of centre): end of the line AV and of the tangent BV
- C (below V, right): point of the sensible horizon of Goa
- O (left of centre): no reference in the text of this page
- I (near the arc, left of C): Ingolstadt
- M (on the line toward G): a point beyond C
- L (left end of the base): no reference in the text of this page
- R (base, between L and A): no reference in the text of this page
- A (base): centre of the Earth
- B (base, right of A): Goa
- G (right end of the base): no reference in the text of this page

therefore be IC, 2466 miles. Third, in the triangle AIC, with the sides IC, 2466, and AI, 4139, given, and the angle AIC, 100° 12′, as the complement of the angle VIC, which is 79° 48′, you will find the least distance AC, 4872 Italian miles. Fourth, in the triangle IGA, in which AI, 4139, is given, with the angle IAG, 64° 55′, and AIG, 100° 12′, you will find AG, 15,860 Italian miles. This is less than 4 semidiameters of the Earth; and this is the greatest distance of the comet from the Earth that there could have been under the hypothesis already mentioned — namely, if it had appeared to the people of Goa at the zenith — when at Ingolstadt it appeared 10° 12′ high above the horizon, on December 9. But we have played enough at conjectures for indicating the comet's distance — not uselessly, however.

3. Argumentum, quod eſt Kepleri, ex obſeruationibus Romæ & Lincij factis.

3. The argument which is Kepler's, from observations made at Rome and at Linz.

VIII. Before Kepler brought out this argument, he set before it, on page 73, an excuse in these words: *"Here I ask the astronomers to forgive me if, for the sake of those who have learned to judge of the heavens in a popular way, I shall have used a demonstration that is popular too, not necessary to the astronomers themselves, and for that reason ridiculous."* Then, having brought forward many examples, he shows that most of the observations of this comet made in different places on Earth are rather coarse, indeed conflict with one another; and at last he lays it down thus, on page 75: *"I conclude therefore, on the whole, that no suitable suspicion of any sensible parallax can be elicited from the discrepancy of the observations. On the contrary, the observations of different places frequently agree very nearly, and furnish an argument that there is no parallax caused by the distance of the places; from which we are made somewhat more certain about the altitude of the comet from the centre of the Earth."* This mere probability, as you see, he tries to show from three pairs of observations; but he judges this one the most select, and so we put it forward, although in the end it will turn out a ridiculous demonstration.

[Margin: Kepler's excuse for the levity of the demonstration to be offered.]

He says that on November 29 the comet was observed at Rome a degree and a half from the northern Scale [Lanx Borealis], but by himself at Linz three

Moons, which he says is the same as 1½°; that the comet's distance from the Scale was then almost parallel to the horizon, and the comet very far down towards the horizon, yet with some altitude; and that the azimuth, or vertical circle, in which the comet appeared inclined towards the wind midway between Solanus and Auster, namely towards Notapeliotes, as the Roman observation has it. With these posited, let Rome be at A, says Kepler, and Linz at L, under very nearly the same meridian LG; and since Linz is more northerly, let the straight line AL point to the north; and, a semicircle LKB having been described from A, as of the eastern horizon, let there be cut off from it the arc LKI of 135°, the number of degrees by which Notapeliotes is distant from the north; and let the straight lines AV and LV be drawn to the comet V, raised a little above the horizon. For there will be formed the triangle ALV, in which AL is 100 German miles, because the difference of the polar altitudes of Rome and Linz is 6½°. The angle LAV, of 135°, is also given; and the angle LVA was very small, because in both places the comet was seen to stand equally distant from the Scale, to sense. Yet let the angle LVA be supposed, because of some error, to have been 15′, and the parallax of the apparent distance from the Scale to have been as great; for from these three givens it follows by trigonometric calculation that LV, that is, the comet's distance from Linz, is 16,000 German miles (of which, namely, he supposes 860 to be contained in the Earth's semidiameter). It was therefore the farther off, the smaller the angle LVA was than 15′. From this he at once boldly concludes: *"And beyond doubt not even one minute; so that this number of miles must be taken at least fifteen times over, so that by these observations we may raise the comet even up under the fixed stars: but we may not lower it below 16000 German miles."* Now if you take 16,000 fifteen times, you get 240,000 German miles, that is, almost 291 semidiameters of the Earth. From where, then, that leap all the way to the fixed stars?

Solutio Argument. 3.

Solution of argument 3.

IX. Having received Kepler's own admission, I could have spared myself this labour; for he calls the demonstrations of parallax to be drawn from the observations of different places ridiculous and popular, not worthy of astronomers; and by the very fact that he convicts some of them of grossness and uncertainty, he has no reason to be certain of the others — for they do not deserve certainty merely because they favour his opinion. Yet some things must be noted against this very kind of demonstration. First, on page 58, he had said of this observation of his of November 29: *"The comet's distance from the Scale, about three Moons. When I recorded this from memory of what I had seen in the same quarter of an hour, I began to doubt somewhat whether I had seen the comet above or below the Scale. But the Roman observer frees me from doubt, who gives its distance on this very day from the northern Scale as a degree and a half, towards the Sirocco wind, which are precisely my three Moons."* But how "about three Moons", and afterwards "three Moons exactly"? Next, if three Moons, from where "precisely one degree and

a half"? Was the Moon's apparent diameter then precisely thirty minutes? I do not think so: for it was to be at perigee on December 1, that is, the third day after quadrature; and yet, mind you, from what I said in book 4, chapter 16, the Moon at perigee in the quadratures, just as at full moons, is 32′ 44″, but at apogee 30′; at that time, therefore, at least 32′, so that three Moons were 1° 36′. How slippery that estimation of distances by Moons is, and how easily the eyes are deceived by several minutes, anyone can learn by experience if he asks several people at once, even experts, how many Moons two stars are distant from each other: from their disagreement, which has happened to us more than once, he will recognize the fallacy of this estimation. It was with excessive confidence, therefore, that you said that beyond doubt not even one minute's error or difference had entered your observation or the Roman one.

[Margin: Kepler's excessive confidence.]

Moreover, Chiaramonti, in the supplement to the *Antitycho*, page 149, denies that in the record of the Roman observation (which I do not now have to hand) the comet's distance from the northern Scale is set down for November 29 as 1½°, either in the writing or in the table, which begins from December 1; and he asserts that in the diagram for the 29th a distance of 2½° from the Scale rather appears, if you measure it carefully with compasses. Yet Chiaramonti errs in the same place when he thinks that in the preceding figure the angle LVA was posited by Kepler for the vertical parallax, since Kepler says expressly that it is the parallax of the distance from the Scale. But again, this diagram contains a false construction [pseudographia]: for since the plane of the triangle LAV would not be wholly parallel to both horizons — the Roman, that is, and that of Linz — but would be raised as much as the comet V, the distance from the Scale could not be seen on the same arc by both. Besides, in the Rudolphine Tables Linz is placed 10^{m} of time further east than Rome, that is, 2° 30′, by which the angle LAV must be diminished; nor is that turning of the comet towards Notolybicus to be taken so strictly that the comet strayed nothing this way or that from 135°. Finally, it is not established that the observation was made at the same moment of time; and that equal distance could have been seen at different times — a distance which, had it been seen at the same moment, would have turned out unequal because of the comet's motion. All these things considered, we may lower the comet below 16,000 German miles, which Kepler denies. Chiaramonti adds, moreover, that notwithstanding a large parallax the same distance from the same fixed star may be seen in different places — which he did in fact demonstrate in book 1 of the *Antitycho*, chapter 8.

[Margin: Chiaramonti's slip.]

4. Argumentum, quod eſt Kepleri, ex obſeruationibus in eodem loco terræ factis.

4. The argument which is Kepler's, from observations made at the same place on Earth.

X. After the kind of popular demonstration already mentioned, as he himself calls it, Kepler at once passes, on page 77, to another, saying: *"A demonstration becomes much more evident from observations made at one place but at different hours: for not all of them are untrustworthy; we have some placed beyond the hazard of error."* Having premised this, he lays down the following axiom: *"It is a well-worn axiom, and not to be demonstrated for the first time in this place (although earlier, in the example of the comet of the year 1607, already sufficiently declared in a diagram), that if any heavenly body [Sidus] whose two diurnal motions, following each other most closely, are equal, has in its whole diurnal revolution a motion corresponding proportionally to the number of hours elapsed, that body lacks parallax."* He shows that this happened with this comet from two comparisons of observations. The first is of the Ingolstadt observations, namely those of Father Cysat. For in the middle of the night preceding December 17, the distance from the extreme star of the Bear's tail, which the comet approached almost in a straight line, was 12° 30′; but at the following hour after midnight, $4\frac{1}{2}^{h}$, it became 12° 3′, so that the difference was 27′; and on the following morning, at the same hour, $4\frac{1}{2}^{h}$, that distance was 9° 54′. The diurnal motion was therefore 2° 9′, of which 24′ are due to 4½ hours; but because the diurnal motion was decreasing, it was rightly faster in those first 4½ hours of the preceding day, about 27′. No disproportionate inequality arising from parallax was therefore noticed — and yet one ought to have been noticed if the comet had had a very sensible parallax, both because within those 4½ hours the comet rose a good deal from the horizon, and because in the last observation it cut the vertical circle with its path at an angle nearer to a right angle. For these two reasons, then, its distance from that star ought to have appeared greater within 4½ hours than the diurnal measure gives; and since this did not happen, the comet therefore lacked sensible parallax.

[Margin: Kepler's axiom.]

[Margin: 1. Comparison of the Ingolstadt observations.]

The second comparison of observations, on page 78, is of those made by Schickard, from which he gathers that within three hours the comet increased in longitude by 20′ and in latitude by 21′, *"so that on the same great circle on which it was ascending from day to day it was also ascending from hour to hour, and was not moved off it to any extent that could be perceived: for about two or three minutes the observer himself does not wish to contend with anyone."* Then he concludes in these words: *"Let these things, then, suffice us thus far for this purpose: that the common people may be taught of some comprehensible sum of German miles by which the comet was certainly higher."*

[Margin: 2. Comparison of observations.]

Solutio Argumenti 4.

Solution of argument 4.

XI. Here again I could rest content with the last words of Kepler, who calls this not an absolute demonstration, but one by which the common people

may be taught of some sum of miles that must at the least be conceded to this comet's distance from the Earth. Yet a little earlier he called this very demonstration more evident, resting on observations not at all erroneous and on a well-worn axiom, and from it he twice pronounces that the comet lacked sensible parallax. With the greatest cunning, moreover, he wishes to insinuate it to the astronomers as a demonstration in such a way that, if they do not examine it, they accept it as beyond doubt; but if they call it back to geometrical examination and detect some defect in it, he can withdraw, as into a place of refuge, to his protestation that the demonstration was merely popular. It must therefore beyond doubt be examined by us, who are not pursuing mere probability in this place, but asking whether it has been evidently demonstrated that this comet was above the Moon. We shall weigh his propositions in the order in which he himself wove them together.

First, then, I deny that in the matter of parallaxes a demonstration from observations made at one place at different hours is more evident than one from observations made at the same moment at different places on Earth; for we shall show the opposite to be truer in book 10, the penultimate section, chapter 9, theorem 22. For by the very fact that the observations are made at the same moment of time, the demonstration does not depend on the comet's motion: the comet is considered as unmoved. And if the distance of those places from each other on a great circle is known, together with the comet's apparent distance from the zenith of each place, or from the same fixed star, or both, then from these alone the comet's distance from the Earth can be gathered most evidently, in parts of which the Earth's semidiameter is known — as can be gathered from what was said in chapter 19, number 2, and chapter 21, number 7, and from what is to be said in book 10, penultimate section, chapter 8. We are speaking, however, with other things equal, that is, given equal certainty in the observations. But Kepler has admitted that the observations of this comet were often coarse and uncertain. Why, then, did he hold as certain those from which no sensible parallax is gathered, rather than those from which a very sensible one is gathered? Surely because the former supported his opinion and the latter opposed it. Or at any rate, why did he not choose at least two observations, made in different places, that were trustworthy and not erroneous?

[Margin: 1. Refutation of Kepler's argument.]

Second, I say that these very Ingolstadt observations of the 17th, which he preferred to all others, are uncertain by his own judgement. For he had said on page 65: *"On the same day, the 16th of December, but at the 11th and 12th hours after noon, the distances taken at Ingolstadt from two stars of Draco were 28° 18′ and 35° 30′; from the extreme star of the Bear's tail, 12° 30′ and 12° 46′."* And a little after: *"On the following morning, December 17, at 4^h, distances were taken at Ingolstadt and in Alsace from a number of stars; but they do not agree with one another — not even the Ingolstadt ones — unless you add one degree to the one and subtract it from the other. The observation by straight lines [Rectarum] is therefore more certain; for the comet was seen at Ingolstadt at $6½^h$ between the hindmost stars of the Bear's tail and*

the hip of Boötes, etc." With what inconstancy, then — not to say effrontery — does he call the Ingolstadt observations of the 16th and 17th, by distances from the extreme star of the Bear's tail, *"trustworthy and beyond the hazard of error"* on page 77, if they do not agree with one another but disagree by a whole degree, and if the observation at $6\frac{1}{2}^{h}$, by straight lines drawn between the hindmost stars of the Bear's tail and the hip of Boötes, was more certain than the observation made at 4^{h} or $4\frac{1}{2}^{h}$ by distances from the fixed stars?

[Margin: 2. Refutation.]

Third, I say that the observations already mentioned, granted that they were conceded to be trustworthy, were nevertheless reported and used by Kepler in an untrustworthy way. For the comet's distance from the last star of the Bear's tail on the 16th, at 11^{h} after noon, was seen at Ingolstadt as 12° 30′, and at 12^{h}, that is, at midnight, it was seen as 12° 46′, as we saw from his own page 65. But on page 77 it is said: *"For at midnight preceding the 17th, the distance from the extreme star of the Bear's tail, which it approached almost in a straight line, was 12° 30′; at the following hour, $4\frac{1}{2}^{h}$, it became 12° 3′, a difference of 27′."* Here there is a twofold contradiction and untrustworthiness. First, the midnight observation ought to have been assigned the distance not of the 11th hour but of the 12th, namely 12° 46′; and, this done, if at $4\frac{1}{2}^{h}$ it became 12° 3′, the difference would have been 43′. But this was too damaging to Kepler, who, from the proportion of the diurnal motion, needed a difference exceeding 24′ by very few minutes. The other is that on page 77 the comet is said to have approached straight towards the star *with proportional inequality*, and yet on page 65 it is said to have receded from the 11th hour to the 12th, since the distance at the 11th hour was 12° 30′ and at the 12th 12° 46′.

[Margin: 3. Refutation of Kepler's argument.]

Fourth, therefore, from what has just been said, I turn the argument back against Kepler in this way. From $4\frac{1}{2}^{h}$ after midnight on the 17th to $4\frac{1}{2}^{h}$ after midnight on the 18th, the comet's motion was 2° 9′; and so, by your account, the motion of 4½ hours was 24′, or 27′ because of the somewhat greater speed at the start. Therefore, if at $4\frac{1}{2}^{h}$ on the 17th the comet stood 12° 3′ from the star of the Bear, it ought, those 4½ hours earlier, that is, at midnight, to have been distant from the same star only 12° 27′, or 12° 30′ at most — provided, of course, that the measure and proportion of the diurnal motion is kept and no parallax intervened. But at 12^{h} it stood 12° 46′ off, when the comet was lower, as Kepler admits. Therefore, by Kepler's own way of arguing, there follows a parallax of distance of 19′, or at least 16′, by which the comet, depressed towards the horizon, appeared to stand farther from the star of the Bear [vrſina ſtella] than it should have.

[Margin: 4. Refutation and retortion of the argument.]

[Margin: A parallax of distance of 16 or 19 minutes.]

Fifth, I say that the Ingolstadt observations which Kepler used are not those which Cysat later committed to the press after selecting the more certain

ones, but those which Io. Georgius Heruuartus [Johann Georg Herwart von Hohenburg], Councillor of the Duke of Bavaria and Chancellor of the Estates, had sent to Kepler before any selection — as is clear to anyone who reads Kepler on page 59 on the one hand, and Cysat on page 10 on the other, where in the Appendix Cysat testifies that his observations published by Kepler were mixed with uncertain and less accurate ones. This will be established even from his observations of this very day. For as Cysat himself relates on page 3: *"On December 16, about 9^h in the evening, the tail [Coma] of the comet was seen to pass through the middle of the quadrilateral of the Great Bear, the head lying hidden either below the horizon or in a cloud that lay close upon the horizon. It began to appear"* — that is, the head — *"at an altitude of 3° 23′, about 11^h of the night; at which time also the comet's distance from the extreme star of the tail of the Great Bear, or Benenaz, was 12° 36′."* Then, for December 17, thus: *"At 0^h 21^m, about the midnight that followed the 16th day, the comet's altitude was 10° 45′; it was 12° 30′ from Benenaz. At 4^h 7^m it was 12° 3′ from Benenaz, and the comet's altitude was then 44° 30′. On December 18, at 4^h 46^m, the comet's altitude was 49° 0′; it was 9° 54′, or 56′, from Benenaz."* But Kepler, in reporting the minutes of the hours, does not agree at all with this narrative, trusting an uncorrected writing, or

[Margin: 5. Refutation, and Kepler's disagreement with Cysat in reporting Cysat's observations.]

his own customary complacency: for 12^h 21^m he puts 12^h, that is, midnight itself; for 4^h 7^m he puts $4½^h$; and for 4^h 46^m of the following day he puts $4½^h$. But he needed precisely one diurnal revolution, and so the observation had to be drawn to $4½^h$ at both ends.

Sixth, as Chiaramonti also noticed in the supplement to the *Antitycho*, page 150, Kepler treats the comet's proper motion as if it had been made on the circle of distance from the Bear's tail, when in fact it was made on a different arc, since the comet approached that star not wholly in a straight line but nearly so. And what is worse, he takes this motion, which was of course the apparent motion, caught by immediate observation, for the true one. For his argument runs on this supposition: that if in 4½ hours the comet approached the star of the Bear by as much as the account of its motion required, there was no sensible parallax. Otherwise, if he were speaking of the apparent motion, there would be no argument at all; for it could be said that the apparent motion towards the star at midnight had been notably faster than at $4½^h$, and so had prevented a parallax that would otherwise have appeared from being seen. From here we step to the seventh refutation.

[Margin: 6. Refutation of Kepler's argument.]

Seventh, I deny that the axiom on which Kepler relies is true of the true motion taken by itself, or of the apparent motion taken by itself; it holds only, and at most, of the true motion compared with the apparent. For the true diurnal motion of two days can be equal, so that it keeps the proportion of equality even in any part of the day, and yet, because of a sensible parallax,

appear unequal — either the whole diurnal motion compared with the other diurnal motion, or a part of it with a part. Conversely, the true motion can be unequal, but so tempered by parallax alone, or by parallax mixed with refraction, that either the whole diurnal motion appears equal to the other diurnal motion, or one part appears equal, or proportional in that part to the whole diurnal motion. But if the measure of the true diurnal motion is established, and its equality, or the proportion due to a certain number of hours — say 4½ hours — and the apparent motion in those hours was also as great, and the comet did not stand farther from the same fixed star at those different hours than the true motion requires, then indeed it is a sign that the comet lacks sensible parallax, or, if it has one, that this was corrected by refraction. But Kepler, with the parallax still unknown, could not distinguish the true motion from the apparent; and the apparent motion of 4½ hours was compared with the motion due to the same time, gathered from the proportion to the apparent diurnal motion and from mere observation. This argument is therefore full of fallacy, and unworthy of Kepler's intellect.

[Margin: 7. Refutation of Kepler's argument.]

Eighth: the same axiom requires that two diurnal motions following each other most closely be equal. And yet Kepler gathers the quantity of the diurnal motion from the revolution of a single day; and that very motion is said to have been so much faster at the beginning that in the first 4½ hours it made 27′ instead of the 24′ it would have completed had it been equal. So it is being used as unequal.

[Margin: 8. Refutation of the same argument.]

Ninth: on the 16th, at 11^h after noon, the comet's apparent altitude was 3° 23′, and at 12^h 21^m its apparent altitude was 10° 45′, on Cysat's testimony, page 3. At 12^h, therefore, its apparent altitude was about 8°; and at that altitude the lunar refraction [Refractio Lunaris] is 12′, which could have contracted the comet's distance from the star of the Bear and so diminished the parallax. It is not rightly concluded, then, even if the rest were granted, that the comet lacked sensible parallax.

[Margin: 9. Refutation of the same.]

Tenth: there are many other observations of this comet, made at Ingolstadt and elsewhere, from which, by Kepler's method, a very sensible parallax is gathered. Therefore, even if some did not indicate one, it cannot be affirmed with certainty that this comet lacked sensible parallax. An example from the Ingolstadt observations, for a sensible parallax of distance of 19′ or 16′, we have already brought forward in the 4th refutation. Let another example be taken from Father Cysat, page 4, where he says that on December 20, at 4^h 40^m in the morning, the comet, at an altitude of 55° 42′, stood 26° 21′ from Arcturus, and at 6^h 39^m, at an altitude of 75° 20′, stood 26° 42′ from Arcturus; at 5^h 44^m, therefore, it stood very nearly 26° 33′ away. And on the 21st, at 5^h 44^m in the morning, at an altitude of 67° 10′, it stood 28° 18′ from Arcturus. The diurnal

motion was therefore 1° 45′, of which 9′ are due to the two hours between 4^h 40^m and 6^h 39^m. Subtract these from the apparent distance at 6^h 39^m, and there will remain a distance of 26° 33′ — the distance the comet ought to have stood from Arcturus, which lay lower than the comet, at 4^h 40^m. But it was seen to stand only 26° 21′ away. Therefore the parallax of distance, depressing the comet towards Arcturus, was 12′, which is very sensible, and is an argument for a much greater parallax of altitude, corresponding to an altitude of 56°.

[Margin: 10. Refutation, and an argument for a very sensible parallax.]

5. Argumentum Item Kepleri ex Motu per rectam lineam Traiectoriam.

5. An argument, likewise Kepler's, from the motion along a straight trajectory line.

To be done with Kepler, I note that on page 92, with no demonstration set out beforehand, he asserts that this comet at the beginning of its appearance was thirty times higher than the Moon and had a parallax of 2′, and at the end was between Jupiter and Mars. He takes the foundations of these altitudes from the annual motion of the Earth and from the comet's motion upwards along a straight line — neither of which, we shall show below, has been demonstrated or can be; and therefore this view about the comet's altitude is mere opinion, not demonstration.

6. Argumentum, Longomontani, ex proprijs Obſeruationibus Hafniæ habitis, diuerſis horis eiuſdem noctis.

6. The argument of Longomontanus, from his own observations made at Copenhagen at different hours of the same night.

XII. In place of the Tychonizing Kepler we put the Tychonic follower Longomontanus, who in the Appendix to the *Astronomia Danica*, chapter 10, page 35 in my copy, admits that observations of cometary parallaxes are the most subtle of all in astronomy, and are to be obtained not with childish instruments but with such as show aliquot parts of single minutes — as he professes his own to have been. On December 16, then, which was the 26th in the Gregorian style, in the evening at $7\frac{1}{3}^h$ after noon, when the comet was seen below the pole very nearly at its lowest altitude, from the comet's observed longitude, ♏ 12° 58′, and northern latitude, 60° 18⅙′, its northern declination is gathered by spherical triangles as 58° 41½′. Then, the same night, at $6\frac{1}{12}^h$ after midnight, when the comet appeared in the morning not far from the zenith above the pole, from the longitude then observed, ♏ 11° 9¾′, and the latitude, 60° 39⅔′, the comet's declination is gathered as 59° 35⅓′. In the space of 10^h 45^m, therefore, the declination advanced towards the north by 53⅚′. Now from the table he himself drew up, the motion in declination from December 26 to 27 was 1° 25′ or 26′, of which the proportional part due to 10^h 45^m is 38⅙′. *"Therefore the difference between the true and the apparent declination was 15⅔′,"* says Longomontanus. For between the observed declinations, 58° 41½′ and 59° 35⅓′, the difference was 53′ 50″; but between the true ones, gathered from the diurnal motion, it should have been 38′ 10″; the difference between these

was therefore 15′ 40″, or 15⅔′. With these preparations made, he girds himself to demonstrate the parallax by means of the following diagram, which, being mutilated in his work, we shall complete.

Diagram. A geometric figure for Longomontanus's demonstration of the comet's parallax from observations at two hours of one night, set in the right column. A large circle stands on a horizontal diameter R … L drawn through the centre A; a vertical line runs from A up to C at the top of the circle; two smaller semicircles stand on the diameter about A; straight lines run from A and from O to points on the arcs, and short arcs are drawn near the upper right. The text below names the meridian RCL, the semicircle IDS for the comet's places, TBV for the terrestrial meridian, the rational horizon RAL, the vertical ABC, the north pole of the equator P, and the equator ZAO with its axis AP.

- C (top of the circle): the upper end of the vertical ABC
- F, G (together near the top, right of C): not yet referred to on this page
- P (upper right): the north pole of the equator
- Q, H, K (down the upper right): not yet referred to on this page
- D (left of the vertical): on the semicircle IDS
- E, N (right of the vertical): not yet referred to on this page
- O (on the circle at the left): an end of the equator ZAO
- M (inside the circle at the right): not yet referred to on this page
- B (on the vertical, low): on the semicircle TBV and the vertical ABC
- R, L (ends of the horizontal diameter): the meridian RCL and the rational horizon RAL
- I, S (on the diameter, inner): ends of the semicircle IDS
- T, V (on the diameter, either side of A): ends of the semicircle TBV
- A (centre, on the diameter): the Earth's centre
- Z (on the circle at the lower right): an end of the equator ZAO

From the Earth's centre A, describe the meridian RCL, and under it the semicircle IDS for the places of the comet, and TBV for the terrestrial meridian; and let RAL be the rational horizon, and ABC the vertical line standing perpendicular to it. Let the north pole of the equator be P, and the equator ZAO, whose axis is AP. And let the comet be observed at its lowest meridian altitude at M, and at its highest at E. Through the comet, placed at each of these places, let there be drawn from A, the centre of the Earth, the lines of the true place, AMH and AEF; but from B, the eye placed on the surface of the Earth, let there be drawn the lines of the apparent place, BMK and BEG. Finally, from the pole P let the arc GNQ be described, touching the comet's path at N. With these marked out, the true declination in the first observation, when the comet was at M, was the arc HLZ, and the apparent declination KLZ; hence HK is the parallax of altitude, whose measure is the angle AMB. But in the other observation, when the comet was at E, the true declination was the arc OCF, the apparent OG, and the parallax of altitude FG, whose measure is AEB. Longomontanus says, moreover, that the arc QH is equal to the parallax FG, and that the arc KQ is equal to the sum of the parallaxes, and as great as

the difference between the apparent and the true declinations, that is, 15⅔′. From here he investigates the comet's apparent altitudes in this way. Since the apparent declination KZ of the comet at M was 58° 42′, its complement KP to the quadrant ZP was 31° 18′. Subtracting this from the altitude of the pole at Copenhagen, PL, which is 55° 43′, there remains the apparent altitude KL, 24° 25′; and so the angle CBK is 65° 35′, and ABM 114° 25′. Next, since for the comet placed at E the latitude was 60° 40′ and the declination 59° 35′, he gathers from these the comet's right ascension as 203° 40′, and its altitude at $6\frac{1}{12}^{h}$ as 80° 6′, which he says would have been 80° 8′ on the meridian; from this he gathers the arc FG, 2′, for the parallax. When this is subtracted from 15′, there remains the parallax HK on its own, that is, the angle AMB, 13′, in the triangle AMB, in which the angle ABM is also given — the complement, which was 114° 25′ — and AB, 1 semidiameter of the Earth. Hence AM, by the rules of triangles, comes out at 240 semidiameters of the Earth, or 206,400 German miles, of which there are 860 in AB.

Solutio Argumenti 6. ex Declinationibus in Altitudine Maxima & Minima Meridiana.

Solution of argument 6, from the declinations at the greatest and least meridian altitude.

XIII. First, Longomontanus supposes that the declinations of the comet that he drew up in his table are true, whereas they are apparent, being derived from the comet's apparent distances from the fixed stars. Thus he says, on page 34 of my copy, that on the 16th, or the 26th new style, at $7\frac{1}{3}^{h}$, he observed the comet's distance from Dubhe as 21° 28′, and from the antepenultimate star of the bend of the Serpent as 19° 45½′; and from the longitude and latitude of these fixed stars he gathers the comet's longitude at that time as ♏ 12° 58′, its northern latitude as 60° 18⅙′ [?], and from this its declination as 58° 41½′. But at $6\frac{1}{12}^{h}$ in the morning after midnight, he says, he observed the distance from Dubhe as 20° 42′, and from the same star of the bend of the Serpent as 19° 39′; from which he gathers the comet's longitude as ♏ 11° 9¾′, its latitude as 60° 39⅔′, and from this its declination as 59° 35⅓′. All these must certainly be understood of apparent, not true, values; nor does he set down any other observation for that day. From where, then, does he put in the table, for the 26th, a declination of the comet of 58° 10′, and for the 27th a declination of 59° 35′? Not from the apparent declinations: for if in 10¾ hours the declination increased by 53⅚′, then in 24 hours it ought surely to have advanced 2 degrees, and not only 1° 25′. Nor did he know it beforehand from the parallax of declination, since he seeks that parallax through these very declinations, as an unknown. That table, therefore, is fallacious and merely arbitrary, made from conjectures taken broadly — the table before which he himself, as if excusing himself, had said: *"There follows a synopsis of the motions of this comet, assigned from all the observations set out above, as in the earlier one, broadly, and as far as could be done and was needed."* Since, therefore, the whole structure of the problem

already mentioned rests on comparing the motion of the true declination with the apparent, and the true motion was not known beforehand, nor could it be known beforehand while the parallax and the comet's distance from the Earth were unknown, it follows that the conclusion of the problem is fallacious and uncertain.

Second: although he had said that this business is most subtle, so much so that not only one minute but not even aliquot parts of a minute are to be disregarded, he himself afterwards uses two altitudes outside the meridian as if they had been on the meridian itself — which could nonetheless introduce a discrepancy greater than one minute.

7. Argumentum, P. Cyſati, ex Differentia Parallaxium Cometæ in arcu proprij motus, collata cum differentia Parallaxium Lunarium in ijſdem altitudinibus.

7. The argument of Father Cysat, from the difference of the comet's parallaxes on the arc of its proper motion, compared with the difference of the lunar parallaxes at the same altitudes.

XIV. To unweave the web of this argument in order, let us first set down, from the *Mathemata Astronomica* that Father Io. Baptista Cysatus published on this comet — namely from chapter 2, problem 4, in which he teaches how to find the comet's diurnal and hourly motions — these words: *"We call the diurnal motion of our comet the arc of its path which it has advanced, from some one altitude of its own above the horizon, by the time it has returned to the same altitude, carried round in the revolution of the primum mobile; and this return of the comet happened in about 24 hours. And this is the comet's true motion for 24 hours, because, even if the comet had suffered the greatest parallax, yet since its altitude from the horizon is supposed to be the same, everything that follows from the parallax was the same as well. However apparent the endpoints may be, therefore, the motion intercepted between them is true; because by however much, and in whatever direction, the comet was pushed by the parallax at the earlier endpoint, by just as much and in the same direction it was pressed at the later endpoint also — by the same parallax, of course."* He then teaches, on page 17, how to find this motion, and indeed the apparent one as well, by the following method. Let the ecliptic be GEK, and its pole A, on its colure; from A let two quadrants, AF and AE, descend to the ecliptic; let the comet's path be DBC, etc.; and on the earlier day or hour let the comet be at B, whose longitude, and latitude BE, are known at that time through distances from the fixed stars. Then, on the following day or hour, let the comet be at C, whose latitude CF and longitude are known in the same way. For in the triangle BAC there will be given the sides AB and AC, which are the complements of the latitudes, and the included angle BAC, which is the difference of the longitudes. Therefore, he says, the arc BC will not be hidden — that is, the comet's motion completed either in one whole day or in some hours of the same day. And if it was completed in one day, it will be the diurnal motion, *which will indeed be true if, on the next day,*

while the comet is at C, its altitude from the horizon is the same as it had been on the earlier day, when the comet was at B; otherwise the motion will be apparent, if the comet's altitude was different when it was at B at the earlier hour from when it is at C at the later hour. On this foundation, from the observed longitudes and latitudes, he wove a table containing the diurnal motions of this comet, which we have already reported, from Cysat's ephemeris, in the history of the comets of this year. Afterwards, in chapter 3, lemma 8, he teaches that if the comet's true motion is subtracted from the apparent, there remains the difference of the comet's parallaxes belonging to its different altitudes from the horizon.

Diagram. Cysat's figure for finding the comet's motion BC from two observed longitudes and latitudes, set into the right half of the right column. A circle with a horizontal line G … K drawn across it below the centre, representing the ecliptic GEK; from the pole A at the top, a curved arc runs down to F and a straight line down to E (the quadrants AF and AE); a slanting line passes through C and B towards D (the comet's path DBC).

- A (top of the circle): the pole of the ecliptic
- C (left of centre): the comet's place on the following day or hour
- B (centre): the comet's place on the earlier day or hour
- G (left end of the horizontal line): on the ecliptic GEK
- F (on the horizontal line, left): foot of the quadrant AF; CF is the latitude at C
- E (on the horizontal line, middle): foot of the quadrant AE; BE is the latitude at B
- D (on the horizontal line, right): on the comet's path DBC
- K (right end of the horizontal line): on the ecliptic GEK

With these supposed, he now tries, in the same chapter 3, page 49, to demonstrate that the comet of the year 1618 was above the Moon from its parallax, which is smaller than the lunar parallax, by this method. On December 1, he says, the comet was observed at $5^{h}\ 45^{m}$ in the morning at an apparent altitude of 12° 30′, and at $6^{h}\ 56^{m}$ at an altitude of 22° 20′. The interval was $1^{h}\ 11^{m}$. The true diurnal motion from the ephemeris for that day was 3° 14′, of which 9. 34′ [?] are due to $1^{h}\ 11^{m}$. But the apparent motion in the same interval of time, gathered from the observed longitudes and latitudes by the preceding problem — that is, the arc CB — was 12′. Therefore, when the true motion is subtracted from the apparent by Cysat's lemma 8, there remains a difference of parallaxes of 2′ 26″ — namely of those two of which one belongs to the comet at an altitude of 12° 30′, the other at an altitude of 22° 20′. Now if, for Copernicus and Tycho, the greatest parallax of the Moon at perigee is 66′, the parallax due to an altitude of 12° 30′ will be 64′ 48″, and that of an altitude of 22° 20′ will be 61′ 36″; so the difference of parallax will be 3′ 12″, greater than 2′ 26″. Therefore the comet was above the Moon. The argument reduced to form, then, is of this kind: *That phenomenon is higher than another which, from one altitude to another on the same day, has a smaller difference of parallaxes.*

[Margin: Father Cysat's argument 7 in the form of a syllogism.]

But the comet of the year 1618, on December 1, had a smaller difference of parallaxes than the Moon from one altitude to another. Therefore the comet of the year 1618, on December 1, was higher than the Moon. The major premiss is certain, and was demonstrated by Father Cysat himself, chapter 3, proposition 4, lemma 3. The minor was proved from what was said just now, at number 14.

Solutio Argumenti 7.

Solution of argument 7.

[Margin: First refutation of argument 7.]

XV. The major proposition of argument 7 is granted; the minor is denied, because in its proofs some [?] things are supposed that are false, and some that are uncertain. First, it is supposed that the comet's true proper diurnal motion can be known before the parallax is known, from the comet's return to the same altitude. But this is false. For if a comet, besides the common diurnal motion of the primum mobile, has a peculiar motion in longitude that is sensible in one day — as this comet had, which within two months advanced against the order of the signs from the last degrees of Scorpio to about the 20th degree of Cancer — it is impossible that, when 24 hours have elapsed, or one revolution of the primum mobile has been completed, the comet should return to the same apparent altitude, whether it has a sensible parallax or not (except in the case in which the comet's path is parallel to the horizon). Rather, it will come back to the same altitude on the rising side later if it moves in the order of the signs, and earlier if it moves against the order of the signs. The first of these is evident in the Moon; the second in the five lesser planets when they become retrograde, especially in Mars, Mercury and Venus. Hence, when a revolution of 24 hours has been completed, the comet's altitude will be different from its altitude on the preceding day; and in that case Father Cysat grants that the motion gathered from two observations is apparent. But the motion of this comet was against the order of the signs: for according to his ephemeris, page 20, the comet on December 1 at $5^h\ 45^m$ in the morning was seen at 9° 24′ of Scorpio, but on December 2 at $4^h\ 48^m$ in the morning it was observed at 7° 56′ of Scorpio. Nor was its path parallel to the horizon, since at $5^h\ 45^m$ in the morning its apparent altitude was 12° 20′, as is said on page 2, or 12° 30′, as is supposed on page 49; and at $6^h\ 56^m$ its altitude, from the same pages, was 22° 20′; and on the following day, at $4^h\ 49^m$ in the morning, the apparent altitude was 5° 40′.

Moreover, even if the comet's apparent path were parallel to the horizon, though above the horizon, so that after 24 hours it returned to the same apparent altitude, the comet's true motion still could not be gathered from the difference of the longitudes and the complements of the apparent latitudes — nor indeed the apparent motion — because in such a case the arc of the comet's motion would be an arc of a circle in the sphere that is not a great circle; whereas in the problem already mentioned the arc BC is supposed to be an arc of a great circle, since otherwise it would run contrary to the laws of trigonometry that

Cysat applies here. It could not, I say, except in the case in which the ecliptic coincided with the horizon, and so the altitude of the pole was 66° 30′; for then after 24 hours the comet could return to the same apparent altitude, and the complements of the altitudes and of the latitudes would be the same. Likewise, if the equator coincided with the horizon, the true and the apparent motion could be gathered from the difference of right ascension and the complements of the declinations — which would be the same as the complements of the altitudes — because the comet's progress would be parallel to the horizon; and in either case the two motions would be equal, and so there would be no difference of parallax, however great the parallax might be — provided that in this case there is no proper motion oblique to the equator.

[Margin: Second refutation of argument 7.]

Second, even if we were to grant that the true diurnal motion could be known while the parallax is unknown (which nevertheless cannot be known except in the case just mentioned, in which the ecliptic coincides with the horizon and the comet's proper motion is parallel to the horizon, and so to the ecliptic), it still could not in fact be known by Father Cysat, as I shall show from his observations recorded for December 1 and 2, which are set out below from his pages 2 and 20.

December: day, time	Apparent altitude	Apparent distance from Arcturus	Apparent distance from Spica	Apparent longitude	Apparent northern latitude
1, 5^h 45^m morning	12° 20′	27° 6′	24° 48′	9° 24′ ♏	11° 37′
1, 6^h 56^m morning	22° 20′	26° 56′	24° 54′	9° 12′ ♏	11° 50′
2, 4^h 48^m morning	5° 40′	24° 0′	25° 20′	7° 56′ ♏	14° 31′
2, 6^h 52^m morning	✠	✠	✠	4° 56′ ♏	19° 18′

For 6^h 52^m of day 2 no altitude or distance from the fixed stars is given, because after the observation at 4^h 48^m it is said: *"When the observation was finished, the sky was covered with clouds."*

From these it is clear that Father Cysat could not have known the comet's true diurnal motion for certain, because between the first two observations, of the 1st and the 2nd day, not 24 hours elapsed but 23^h 3^m. And in the second observation of day 2 the observation of altitude and distances was either not made or doubtful — otherwise they would have been recorded — and the huge jump itself in the motion in latitude, from 14° 31′ to 19° 18′, makes the observation suspect. Further, from 5^h 45^m to 6^h 56^m of the first day there are 1^h 11^m, and in that space the apparent motion in longitude was 12′, as you see in the little table above; yet on page 49 he says that in 1^h 11^m the comet's apparent proper motion was 12′. He therefore takes the comet's motion just as though the motion were in the ecliptic, or in a parallel of the ecliptic. If so, then just

as in 1^h 11^m it completed 12′, so from 5^h 45^m of the first day to 4^h 48^m of the second day, that is, in 23^h 3^m, it completed 1° 28′, if you subtract 7° 56′ from 9° 24′ of Scorpio. But if this motion was gathered by proportion — whether one puts it: as 1^h 11^m to a motion of 12′, so 24 hours to 4° 35′; or: as 24 hours to 1° 28′, so 1^h 11^m to 4′ 20″ — you will never gather that diurnal motion of 3° 14′ that Father Cysat sets down; and the motion he gathered from the longitudes was not of 24 hours but of 23^h 3^m.

Third, the difference of the lunar parallaxes that Father Cysat considers at the altitudes already mentioned is the one gathered from parallaxes in circles of altitude; but the difference of the comet's parallaxes is not a difference of parallaxes in a circle of altitude but on the arc of the comet's path, which was indeed not parallel to the horizon, yet came nearer to a parallel than to a vertical circle. Now the difference of parallaxes on the phenomenon's own arc can be smaller than the difference of parallaxes in its circle of altitude, and so the latter larger; and the latter could have been larger than the difference of the lunar parallaxes of altitude. The major premiss of the argument therefore had to be distinguished: granted if the comparison is made within the same kind of parallax, denied if in a different kind.

[Margin: Two other examples of Father Cysat are refuted.]

From what has been said, the same argument, which Father Cysat confirms with two other examples on pages 50 and 51, remains refuted. For in these he errs in the same two ways. He supposes, namely, that the true motion is known distinctly from the apparent motion, without prior knowledge of the parallax, and from mere observed places, yet not at the same altitudes, as was required after a revolution of 24 hours. Then he compares the difference of the parallaxes of the arc that the comet described by its own progress with the difference of the lunar parallaxes viewed in a circle of altitude. For he says that on December 9, between the first observation, when the comet was seen at an altitude of 10° 12′, and the second, when its altitude was 44° 10′, the apparent motion was 39′ 30″, corresponding to 3^h 56^m, but the true motion 30′, because the diurnal motion was 2° 59′ 50″; and that the difference of parallaxes [?] was therefore 9′ 30″. But at the same altitudes the difference of the parallaxes of the Moon at perigee was 17′ 39″, much greater than the difference of the comet's parallaxes. He himself, however, on page 3, had said that the comet was seen at an altitude of 10° 12′ at 2^h 46^m, and at an altitude of 48° 58′ at 6^h 22^m; between these hours there are 3^h 36^m, not 3^h 56^m.

8. Argumentum,P. Cyſati, ex Differentia Parallaxium in diſtantijs ab eadem Fixa ſtella, Collata cum differentia parallaxium Lunarium.

8. The argument of Father Cysat, from the difference of parallaxes in the distances from the same fixed star, compared with the difference of the lunar parallaxes.

XVI. This argument is the same as the 7th, but the proof of the minor is different. Father Cysat's argument on page 49, then, runs as follows: *That star*

is higher than another which changes its parallax less from one altitude to another. But the comet of the year 1618 changed its parallax less than the Moon from one altitude to another; therefore it was higher than the Moon. He supposes the major as proved from his lemma 3 of chapter 3.

[Margin: First example of Father Cysat.]

The minor he tries to prove by two examples. For on December 1, at $5^h\ 45^m$ in the morning, the comet appeared at an altitude of 12° 30′ and at a distance from Arcturus of 27° 6′, or, on account of [?] refraction, 27° 9′; but at $6^h\ 56^m$ in the morning, at an altitude of 22° 20′, it appeared at a distance from Arcturus of 26° 56′. Therefore in the space of $1^h\ 11^m$ the apparent approach to the star was 13′. But the true motion, gathered from the comet's diurnal proper motion, was 9′ 34″ in that space, as was said in the proof of argument 7; therefore the difference of these, and so the difference of the parallaxes, was 3′ 26″. But the difference of the lunar parallaxes of altitude at the same apparent altitudes is 3′ 30″; therefore the comet's difference is smaller than the Moon's. Again, on the 9th, at $2^h\ 46^m$ in the morning, the comet appeared at an altitude of 10° 12′ and at a distance from Benenaz of 33° 36′; but at $6^h\ 22^m$, at an altitude of 48° 58′, at a distance from Benenaz of 32° 54′. Therefore within $3^h\ 56^m$ the apparent approach to the star was 42′, but the true approach, from the true motion, ought to have been 30′; hence the difference of the parallaxes was 12′. But at the same altitudes the difference of the parallaxes of the Moon at perigee is 17′ 39″; therefore the former is smaller than the latter. By a similar method he investigates the difference of parallaxes for December 1, 9, 17, 20 and 29; and from these, by his lemmas 5 and 6 (on which, our Book 10, penultimate section, chapter 2, problems 7 and 8), he investigates the comet's horizontal parallax; and finally from this, by the problem I have already given in Book 3, chapter 7, number 3, he seeks the comet's distance from the centre of the Earth, and finds it as in the table below.

December, day	Horizontal parallax	Distance of the comet from the centre of the Earth, in semidiameters of the Earth	Distance of the comet from the centre of the Earth, in German miles
1	47′ 34″	72	62139
9	35′ 33″	96	83172
17	26′ 3″	132	113456
20	22′ 15″	154	132921
29	13′ 39″	252	216625

Solutio Argumenti 8.

Solution of argument 8.

XVII. We reply by distinguishing the major proposition: granting it if the comparison of the differences is made within parallax of the same kind, and denying it if within parallax of a different kind; and so denying the minor, understood — as is fitting — of the difference of parallaxes of the same kind.

For the comet's parallaxes that Father Cysat uses are on the arc of distance, which was different not only from the circle of altitude but also from the circle of the comet's proper motion (and in this very point there is a flaw, nor is the approach to the star properly compared with the proper motion); whereas the lunar parallaxes and their difference are viewed in two circles of altitude. Moreover, the comet's true proper motion is supposed to be known distinctly from the apparent one before the parallax is known, which we have already shown to be false in the first refutation of argument 7. Finally, Father Cysat frankly admits, on page 10, that he used a wooden sextant: *"With this instrument,"* he says himself, *"we measured the comet's distances from the fixed stars, with an error, as is clear from the size and dimensions of the instrument, not greater than 6′."*

9. Argumentum, Villebrordi Snellij, ex Differentia Parallaxium Cometæ in diſtantia ab eadem Fixa, Collata cum differentia Parallaxium Lunarium in circulo Altitudinis.

9. The argument of Willebrord Snel, from the difference of the comet's parallaxes in its distance from the same fixed star, compared with the difference of the lunar parallaxes in a circle of altitude.

[Margin: Argument 9 in syllogistic form.]

XVIII. Snel's argument, from chapter 4 of his work published on this comet, extracted like marrow from the bone, is of this kind: *If the comet of the year 1618 had not been higher than the Moon from the centre of the Earth, then between the observation of the 13th day at 4½ʰ and that at 6½ʰ there would have been a difference of parallaxes equal to the difference of the lunar parallaxes, or smaller, or at least 23′ 37″. But no such difference occurred; therefore it was higher than the Moon from the centre of the Earth.* He proves the minor from the distances from the adjacent stars on the back of Boötes. But we reply in the same way as to argument 8. For here too, all things considered, Snel compares the difference of the comet's parallaxes on arcs of distance from the fixed stars (which nevertheless were different from the arc of the comet's path) with the difference of the lunar parallaxes in a circle of altitude. Chiaramonti also noted this in the supplement to the *Antitycho*, page 176, where he adds that a phenomenon can keep, at different altitudes, the same apparent distance from a given fixed star as its true one, and yet have a large parallax in the vertical circle — which he had demonstrated in the *Antitycho*, Book 1, chapter 8, proposition 1. Chiaramonti indeed detected many errors in Snel, but two above all. The first lies in his using Tycho's longitude of the fixed stars for the year 1600 as though nothing had changed by the year of this comet, 1618, although by Tycho's reckoning they advance 15′ 18″ in 18 years; on this error see Chiaramonti there, page 164. The second is that, from a disturbed order of the arcs in his trigonometric analysis, he gathers the comet's latitude towards the end of December as 72° 44′ 45″, whereas Chiaramonti, from the same data, gathers the latitude as 58° 33′ 37″. There is no need to unweave the calculations of either of them here;

but a sign of Snel's error can be the comet's greatest latitude, which the other observers — Kepler, Longomontanus, Cysat — put at no more than about 59°.

10. Argumentum, P. Blancani & commune Multorum, ex Aequali diſtantia Cometæ ab ijſdem Fixis viſa in locis Terræ valde diſtantibus eodem tempore.

10. The argument of Father Biancani, shared by many, from the equal distance of the comet from the same fixed stars, seen at the same time from places on Earth very far apart.

XIX. These, of course, are the feathers and wings with which the comet of this year 1618 tried to fly out beyond the bounds of the Moon and the Sun — wings that a truly Daedalian Rumour fitted to it, when, that is, the report spread from various regions of the world that this comet had been observed at the same time under the same fixed stars, or at a nearly equal distance from the same fixed stars; and astronomers hold it for certain that this could by no means happen if it had had a very sensible parallax. And the comet was observed not only at places in Europe very far apart — at Antwerp, Danzig, Linz, Strasbourg, Cologne, Leipzig, Wittenberg, Frankfurt, Ingolstadt, Innsbruck, Rome, Parma, Cesena, Paris, Aix, and Valencia in Spain — but also at Goa and Cochin, by Fathers of our Society in the East Indies. Why, then, should so great a distance between the places not supply an argument for the comet's distance from the Earth, splendid in appearance and celebrated with the applause of all? But if, by melting the waxen wings of this Icarus too, we show them to be too weak for so great a height of motion, I believe we shall obtain pardon from lovers of the truth — not to say no small gratitude. Further, this argument too, contracted into syllogistic form, is as follows:

[Margin: Argument 10 in syllogistic form.]

A phenomenon which, seen at the same time from places on the globe very far apart, appears equally distant from the same fixed star — or with a smaller difference of distance than the Moon would show, viewed from the same places on Earth — is above the Moon. But the comet of the year 1618 appeared, at the same time, from places on Earth very far apart, equally distant from the same fixed star, or with a smaller difference of distance than the Moon would show viewed from the same places on Earth. Therefore the comet of the year 1618 was above the Moon. Biancani uses a similar argument, though not framed in this way, in the *Sphaera*, Book 16, chapter 5. The major proposition is proved, first, because if the comet had a sensible parallax greater than the Moon's, it would appear unequally distant from the same fixed star from different parts of the Earth, as the Moon now appears — and indeed with a greater inequality; and second, because parallax is denied to the fixed stars precisely because, at any altitude above the horizon whatever, viewed from any place on Earth whatever, they appear to keep the same distance from the other fixed stars around them. The minor is proved from the history of the observations: for this comet was observed at distances from Arcturus, from Spica, and from certain stars of Ursa Major or of Draco,

and the difference seen between the distances was either none or of very few minutes.

Solutio Argumenti 10.

Solution of argument 10.

[Margin: First reply, distinguishing the major.]

XX. First, the major proposition must be distinguished. It is to be granted as to its second part — provided, that is, that the difference between the observed distances from the fixed stars was smaller than the one that belongs to the Moon. As to its first part, however, it must be subdistinguished: it is to be granted if the places on Earth, the phenomenon and that fixed star were under the same great circle, and the phenomenon is not within the zeniths of the two places on Earth but outside the arc enclosed between the zeniths, and if with exact instruments and observations the distance from the same fixed star was found equal. Otherwise it must be denied, because it is not true in itself, or universally, even if the instruments and observations are exact — namely, if the phenomenon, or the fixed star, or both, are outside the great circle under which the two places on Earth lie, or are indeed under that circle but on the arc enclosed between their zeniths. For in these cases the distance from the same fixed star can appear equal from both places, however near the Earth the phenomenon may be, as will shortly be shown; and so even though it has a parallax greater than the Moon's.

[Margin: Second reply, denying the minor.]

Second, the minor must be denied in the sense in which the major was granted. For from the history of the observations — those, at least, that are available to us — it is not established that this comet was observed at different places on Earth at the same moment of time (granted, on the same days), or in such a way that the phenomenon and the fixed star were with those places in the same great circle and outside the arc enclosed by the zeniths, or without any difference of distance at all. For it is not enough that the difference was small, since it can be of a single minute and the phenomenon nevertheless have a parallax much greater than the Moon's, as has already been shown in chapter 19, number 3, and will be shown again in Book 10, penultimate section, chapter 11, numbers 11 and 12, together with Chiaramonti, Book 2 of the *Antitycho*, chapter 20, and Book 3, chapter 19, theorem 1. Now, so that what we said in distinguishing the major proposition may be clear, the following figure must be inspected first. In it, from the Earth's centre A, let there be described the quadrant BC of a great terrestrial circle, beneath the celestial quadrant DS; under this let there also be the comet's circle VT; and let the two places on Earth be B and G, and their lines drawn from the centre of the Earth to their zeniths ABD and AGL. With these laid down, two cases bear on our matter.

Diagram. Riccioli's figure for the two cases of the solution of argument 10, set across the left column. A horizontal base line runs from S at the left through T, C and A to E at the right. From the Earth's centre A a vertical line rises

through B and V to D. An outer arc, the celestial quadrant DS, runs from D down to S; a middle arc, the comet's circle VT, runs from V down to T; an inner arc, the terrestrial quadrant BC, runs from B down to C through G. A further arc runs from B down to E. Straight lines from the two places B and G run to the comet's places K and R and on to the outer arc, and a line runs from A through G towards L. Star symbols mark points on the outer arc.

- D (top of the vertical): the zenith of B, on the line ABD; an end of the celestial quadrant DS
- Z (beside a star symbol near the top of the outer arc): not referred to on this page
- H (on the outer arc): the apparent place of the comet K seen from B, by the line BKH
- F (on the outer arc, beside H): the fixed star in the first case
- I (on the outer arc, beside F): the apparent place of the comet K seen from G, by the line GKI
- V (on the vertical, below D): an end of the comet's circle VT and of the arc QV
- K (on the middle arc): the comet in the first case
- L (on the outer arc): the zenith of G, on the line AGL
- M (on the outer arc, below L): not referred to on this page
- Q (on the middle arc): an end of the arc QV
- X (beside a star symbol on the outer arc): not referred to on this page
- N (on the outer arc): the fixed star in the second case
- O (on the outer arc, below N): the apparent place of the comet R seen from G, by the line GRO
- P (on the outer arc, below O): the apparent place of the comet R seen from B, by the line BRP
- R (on the middle arc): the comet in the second case
- B (on the vertical): the first place on Earth; an end of the terrestrial quadrant BC
- m (italic, to the right of B): not referred to on this page
- Y (beside a star symbol low on the outer arc): not referred to on this page
- G (on the inner arc): the second place on Earth
- S (left end of the base): an end of the celestial quadrant DS
- T (on the base): an end of the comet's circle VT
- C (on the base): an end of the terrestrial quadrant BC
- A (on the base, foot of the vertical): the Earth's centre
- E (right end of the base): not referred to on this page

[Margin: First case, for the negative part of the major.]

The first: if the comet is at K, a place enclosed within the vertical lines AD and AL (the sign of which is that for one observer it lies towards one region of the world, and for the other towards the opposite region), it can be equidistant from both zeniths as to both its true and its apparent place. The fixed star can be at F, and the comet's true place, marked by the line AKF, under it; while

its apparent place seen from B, by the line BKH, is under H, and seen from the place G, by the line GKI, under I; so that the arc HF, of apparent distance from F, is equal to the arc IF, likewise of apparent distance from the star F, and so the parallactic angles GKA and BKA are equal to each other. In that case, then, however great the parallaxes may be, the apparent distance of the comet K from the same fixed star F will nevertheless be equal, and the sum of the distances equal to the sum of the parallaxes.

[Margin: Second case.]

The second case is if the comet is outside the arc QV enclosed by the vertical lines AD and AL — for instance if it is at R, that is, towards the same region for both observers B and G. In this case, if the fixed star is at N and the comet R under it, the line of the true place being ARN, but the apparent place seen from B, by the line BRP, is under P, and the visible distance from the fixed star is PN, while the apparent place seen from G, by the line GRO, is under O, and the visible distance ON, then the difference of the distances, and likewise of the parallaxes — namely the angle BRG — will be equal to the angle PRO. But this difference can be of one minute, indeed less than one minute, even if the parallaxes ARB and ARG are very large, as was shown in chapter 19, number 3, and will be shown in Book 10, penultimate section, chapter 11, numbers 11 and 12.

XXI. From this you will understand that it can also happen outside the same arc that, however great the parallax of a phenomenon may be, it nevertheless appears equally distant from the same fixed star from different places on Earth. This may be explained, in more evident cases, by the following diagram. In it, A and B are two places on Earth under the same meridian ACB, equally distant from the equator GD, one towards the north and the other towards the south, and the comet D, with the fixed star E, is under the equator. Or the two places on Earth A, B are under the equator AB, and the comet D, with the fixed star E, under the same meridian GD, which lies equally between those places. Or, finally, the two places A, B are under some other vertical AB, and the comet D, with the fixed star E, under the same other vertical GD, which cuts at right angles the vertical AB — the circle of those places — and bisects at C the arc of the vertical AB intercepted between the two places; this can happen in countless verticals. With these laid down, the distance of the comet from the same fixed star could appear equal to both places and observers at the same moment, however near the Earth the comet might be. But besides these there are also other cases of this equality in the apparent distance from the same fixed star, even though the parallax is of several degrees. And let this be said for the negative part of the major proposition. And so that you do not doubt that in these cases the comet can be near the Earth, suppose that at D there is the top of a tower, lit by some lantern, so that at night, by some instrument, from the places A and B, equally distant from the vertical GD that cuts the vertical AB at right angles, the distance of that top from the star E, situated in the same

vertical, can be taken. It will be found equal from both, if they work exactly, and yet the top D will be very near the Earth.

Diagram. Riccioli's figure for the cases of §XXI, set in the right column beside the short lines of the text. A rhombus with A at the top, B at the bottom, G at the left and D at the right. A horizontal line G … D passes through F, C and E; a vertical line A … B crosses it at C. Straight lines run from A and from B to each of G, F, E and D. Star symbols mark G, F, E and D.

- A (top): one of the two places on Earth
- G (left end of the horizontal): an end of the circle GD (the equator, meridian or vertical of the three cases)
- F (on the horizontal, left of C): not referred to on this page
- C (centre, above the crossing): the point bisecting the arc AB
- E (on the horizontal, right of C): the fixed star
- D (right end of the horizontal): the comet, or the lit top of the tower; an end of the circle GD
- B (bottom): the other place on Earth

XXII. Now, so that it may be clear that we have denied the minor proposition of the argument put forward neither gratuitously nor without consulting the history of this comet, it has seemed good to gather faithfully into one synopsis this comet's distances from the same fixed star, taken at different places on Earth but on the same day, with the hour added if the observers noted it, or the comet's altitude. But if other booklets of observations are extant of this kind that have not come into our hands, from which the contrary may be clearly gathered, our negative reply to the minor is not to be understood of them: in that case we are ready to grant it. But so long as that is not established, it can rightly be denied, and the burden of proving the opposite falls on our opponents. Note, moreover, that several observers excuse themselves and do not present their observations as exact, since they say that, especially at the beginning, these were rough: so Kepler, Habrecht, Rhodius and Chiaramonti; nor would Cysat deny that an error of 6 minutes could have lain hidden in the distances. And Kepler said of these and similar observations, on page 61: *"These observations are not of the highest accuracy, nor is there any need to treat them by laborious calculation; a pair of compasses with a globe is enough, for nothing is to be suspected about parallax in so short a span of hours."*

[Margin: Kepler's excessive confidence in pronouncing.]

This last remark was made with Kepler's usual confidence — as though he himself and others had not used observations of the same night, taken a few hours apart, to investigate parallax.

Obſeruationes Diſtantiarum Cometæ Anni 1618. & 1619. à Stellis Fixis. Diebus Stylo Gregoriano notatis.

Observations of the distances of the comet of the years 1618 and 1619 from the fixed stars, the days given in the Gregorian style.

[Margin: Observations of November. Days 27 and 29. Day 30.]

XXIII. On November 27 the comet was seen by our men at Rome a little less than 4 degrees from the northern Scale. On the 29th it was seen at the same place 1½° from the same Scale. But at Goa Father Giacomo Rho saw it 34° 5′ from Arcturus and 25° 40′ from Spica. Finally, on the 30th, at Parma, Father Giuseppe Biancani saw the comet, at 14^h Italian civil time, 30° from Arcturus and 25° from Spica.

[Margin: Observations of December. Day 1.]

On December 1 Father Johann Baptist Cysat at Ingolstadt, at 5^h 45^m in the morning, saw the comet at an altitude of 12° 20′, 27° 6′ from Arcturus and 24° 48′ from Spica; and at 6^h 56^m in the morning at an altitude of 22° 20′, 26° 56′ from Arcturus and 24° 54′ from Spica. But Longomontanus at Copenhagen, at 5^h 45^m, saw it at an altitude of 18°, 24° 56′ from Spica and 32° 41½′ from the bright star of the Crown.

[Margin: Day 2.]

December 2. Cysat at Ingolstadt, at 4^h 49^m in the morning, saw the comet at an altitude of 5° 40′, 24° 0′ from Arcturus and 25° 20′ from Spica. But Biancani at Parma, at $13¾^h$ Italian civil time, saw it 24° from Arcturus and 25° from Spica.

[Margin: Day 3.]

December 3. At Rome it was seen 21° 0′ from Arcturus and 25° 25′ from Spica. But at Goa Father Rho saw it 21° 38′ from Arcturus and 25° 54′ from Spica; and at Parma Biancani, at $13½^h$ civil time, 21° 10′ from Arcturus and 26° 30′ from Spica. Longomontanus at Copenhagen, at $6½^h$ in the morning, saw it 25° 52′ from Spica and 27° 28′ from the bright star of the Crown.

[Margin: Day 4.]

December 4. At Rome it was seen 17° 40′ from Arcturus and 26° 33′ from Spica. At Goa Father Rho saw it 19° 0′ from Arcturus and 26° 50′ from Spica. At Parma Biancani, at 14^h civil time, saw it 18° from Arcturus. Chiaramonti at Cesena, at 12^h 40^m Italian solar time, saw it 17° 11′ from Arcturus and 27° 45′ from Spica. But Longomontanus at Copenhagen, at $6½^h$ in the morning, saw the comet at an altitude of 21°, 26° 52′ from Spica and 24° 54′ from the bright star of the Crown; then, when it appeared at an altitude of 24° 50′, he saw it 18° 13⅔′ from Arcturus. But Dr Habrecht at Strasbourg saw it 17° 0′ from Arcturus, and Ambrosius Rhodius at Wittenberg saw it 25° 21′ from Spica.

[Margin: Day 5.]

December 5. Our men at Rome saw the comet 23° 26′ from Arcturus; at Antwerp, however, 25° 10′.

[Margin: Day 6.]

December 6. At 13^h Italian civil time Biancani at Parma saw it 12° from Arcturus, 17° from the hip of Boötes and 27½° from Spica.

[Margin: Day 7.]

December 7. By Cysat at Ingolstadt, at $6⅛^h$ in the morning, it was seen at an altitude of 38°, 9° 48′ from Arcturus and 17° 46′ from the bright star of

the Crown; and at 7^h at an altitude of 44°, 1° 58′ from the star of the third magnitude in the leg of Boötes. But Habrecht at Strasbourg saw it 9° 20′ from Arcturus and 38° 30′ from the last star of the tail of Ursa Major. And Vuelperus at Lora in Alsace saw it 10° 30′ from Arcturus, 18° 30′ from the bright star of the Crown, and 39° 30′ from the last star of the tail of Ursa Major.

[Margin: Day 8.]

December 8. Cysat at Ingolstadt, at 3^h 0^m in the morning, saw the comet at an altitude of 12° 30′, 7° 57′ from Arcturus and 11° 42′ from the star of the third magnitude in the right elbow of Boötes. But Longomontanus at Copenhagen, at $5½^h$, saw it about 2° from the bright star in the knee of Boötes, and higher than that star, towards the west.

[Margin: Day 9.]

December 9. Cysat at Ingolstadt saw the comet rise in the morning at 1^h 45^m; at 2^h 46^m it stood at an altitude of 10° 12′, 6° 42′ from Arcturus, but 33° 36′ from Benenaz, the end star of the tail of Ursa Major; then at 6^h 22^m at an altitude of 48° 58′, 32° 54′ from Benenaz and 14° 30′ from the bright star of the Crown. At Rome it was seen 6° 25′ from Arcturus; at Goa Father Rho saw it 6° 46′ from Arcturus; and Johannes Remus Quietanus at St Johann in Tirol saw it about 6° from Arcturus, but a little less — and that at 5^h.

[Margin: Day 10.]

December 10. At Ingolstadt Cysat, at 6^h 25^m in the morning, saw it at an altitude of 47° 30′, 29° 26′ from Benenaz and 13° 44′ from the bright star of the Crown. At Goa Father Rho saw it 6° 5′ from Arcturus; he then saw that between the comet, at an altitude of 20° 24′, and Spica, at an altitude of 33° 10′, there lay an azimuthal arc of 34° 30′, from which Chiaramonti infers the comet's distance from Spica to be 33°.

[Margin: Day 12.]

December 12. At $5½^h$ in the morning Remus at Innsbruck saw it 1° 45′ from the belt, or hip, of Boötes. At Parma Biancani, at 13^h civil time, saw it 8½° from Arcturus and 3° from the hip of Arcturus.

[Margin: Day 13.]

December 13. At Antwerp our men saw it 11° 20′ from Arcturus, but in the Roman College 10° 55′; and Remus at Innsbruck 10° 53′ from Arcturus. Snel, at Leiden I believe, at $4½^h$ in the morning saw it 10° 47′ from Arcturus, 14° 3′ from the bright star of the Crown, 52′ 23″ from the upper of the adjoining stars on the back of Boötes, and 34′ from the lower. On the same day the comet covered, for the observers at Rome, the tenth star of Boötes, which it also partly covered for those at Cologne.

[Margin: Day 14.]

December 14. At Goa Father Rho saw it 12° 40′ from Arcturus; but at Parma Biancani, at 13^h civil time, 13° from Arcturus and 16° from the bright star of the Crown; while Willebrord Snel, at $5½^h$, saw it 13° 6′ from Arcturus and 15° 1′ from the Crown; and Remus at Innsbruck 13° 10′ from Arcturus.

[Margin: Day 15.]

December 15. Longomontanus at Copenhagen, at 5^h, saw it 43° 43′ from the bright star in the southern wing of ♍ [Virgo] and 48° 2′ [?] from Spica.

[Margin: Day 16.]

December 16. At Ingolstadt Cysat saw that around 9^h the head of the comet was still hidden, either in the vapours or below the horizon, but that it emerged around 11^h, when it appeared at an altitude of 3°; at that time it was seen 12° 36′ from Benenaz. But Remus at Innsbruck saw it 1° from the left shoulder of Boötes, while from the same star it was seen by Benjamin Ursinus at Frankfurt an der Oder to be one and a half diameters of the Sun away.

[Margin: Day 17.]

December 17. At Ingolstadt Father Cysat, at 0^h 21^m — almost in the middle of the night between the 16th and the 17th — saw the comet at an altitude of 10° 45′, 12° 30′ from Benenaz, and 16° 2′ from the unformed star of the second magnitude under the tail of Ursa Major; then at 4^h 7^m at an altitude of 44° 30′, 12° 3′ from Benenaz; and at 4^h 43^m at an altitude of 47° 55′, 19° 48′ from Arcturus and 19° 42′ from the bright star of the Crown; and at 5^h 30^m at an altitude of 57° 10′. Vuelperus at Lora saw it 19° 48′ from Arcturus, 19° 42′ from the bright star of the Crown, and 11° 3′ from the first star of the tail of Ursa Major; but Millerus at Leipzig saw it 20° 50′ from Arcturus, 18° 30′ from the bright star of the Crown, and 11° 30′ from the first star of the tail of the Bear.

[Margin: Days 18 and 20.]

December 18. At Ingolstadt Cysat, at 4^h 46^m, saw it at an altitude of 49° 0′, 21° 26′ from the bright star of the Crown and 9° 55′ from Benenaz. And on the 20th, at 3^h 54^m in the morning, he saw it at an altitude of 48° 30′, 26° 18′ from Arcturus, and 24° 42′ — or 48′ — from the bright star of the Crown; at 4^h 40^m at an altitude of 55° 42′, 26° 21′ from Arcturus and 24° 51′ from the bright star of the Crown; and at 6^h 39^m at an altitude of 75° 20′, 26° 42′ from Arcturus. From this day to the end, he says, the comet never set, but moved among the stars that are always visible.

[Margin: Day 21.]

December 21. At 5^h 44^m in the morning, at Ingolstadt, Cysat saw it at an altitude of 67° 10′, 28° 18′ from Arcturus and 26° 42′ from the bright star of the Crown. Remus at Innsbruck saw it 4° 20′ from the last star of the tail of Ursa Major and 2° from the wrist of Boötes; but Kepler at Linz, 4° 30′ from the last star of the tail and 50′ from the wrist of Boötes.

[Margin: Day 24.]

December 24. At Ingolstadt, at 6^h 19^m in the morning, Cysat saw the comet at an altitude of 80° 45′, 33° 30′ from Arcturus and 31° 6′ from the bright star of the Crown. But Ambrosius Rhodius at Wittenberg, at 6^h, saw it 46° 40′ from the tail of Ursa Major and 33° 50′ from Arcturus. Snel: distance from the last star of the tail of Ursa Minor 22° 3′, and 4° 9′ from the uppermost star of the quadrilateral of Ursa Major.

[Margin: Days 26 and 27.]

December 26. Longomontanus at Copenhagen, at 7^h 20^m in the morning, saw it 21° 28′ from Dubhe and 19° 45½′ from the third-from-last star of the coil of the Serpent; and on the 27th, at 6^h 5^m in the morning, 20° 42′ from Dubhe and 19° 39′ from the third-from-last star of the coil of the Serpent.

[Margin: Day 28.]

December 28. At Ingolstadt, at 5^h 4^m in the morning, Cysat saw it at an altitude of 69° 30′, 16° 39′ from the bright star of the quadrilateral of Ursa Minor, and 11° 42′ from the northern star among the front wheels of the Wain of Ursa Major. At Parma Biancani, at $12½^h$ civil time, saw it 5° from the second star of the tail of Ursa Major and 6½° from the third star of the tail of the Serpent. On this day Longomontanus stopped observing.

[Margin: Day 29.]

December 29. At 4^h 28^m, at Ingolstadt, Cysat saw it at an altitude of 66° 20′, 18° 49′ from the northern star among the posteribus [?] wheels of the greater Wain, and 15° 9′ from the bright star of the quadrilateral of Ursa Minor; and at 6^h 34^m in the morning at an altitude of 79° 30′, 18° 45′ from the already mentioned northern star of the Wain, and 15° 6′ from the bright star of the quadrilateral of Ursa Minor [?]. At Parma, however, Biancani, at $12½^h$ civil time, saw it 5½° from the second star of the tail of Ursa Major and 5½° from the third star of the tail of the Serpent.

[Margin: Days 30 and 31.]

December 30. At Ingolstadt, at 6^h 2^m, Cysat saw the comet at an altitude of 76° 25′, 17° 45′ from the northern star among the rear wheels of the greater Wain, and 13° 56′ from the bright star of the quadrilateral of Ursa Minor. Biancani at Parma, at $12½^h$, saw it 7° from the third star of the tail of Ursa Major and 5¼° from the third star of the tail of the Serpent; and on the 31st, scarcely able to make it out, he stopped observing.

[Margin: January. Day 3.]

January 3. At 7^h 30^m in the evening the comet stood in almost the same vertical as Benenaz, near the meridian of Ingolstadt; and Cysat saw the comet at an altitude of 28° 50′, but Benenaz at an altitude of 10° 50′. The comet was 10° 51′ from the first star of the tail of Ursa Major and 14° 15′ from the northern star among the rear wheels of the greater Wain.

[Margin: Day 7.]

January 7. At 8^h in the evening the comet was in conjunction with the penultimate star in the tail of Draco: its altitude was 35° 12′, while the star appeared at an altitude of 35° 31′. So Cysat at Ingolstadt. On this day Kepler stopped seeing the comet.

[Margin: Day 12.]

January 12. At 8^h 3^m in the evening, at Ingolstadt, Cysat saw it at an altitude of 40° 41′, 18° 12′ from the first star of the tail of Ursa Major and 12° 21′ from the bright star of the quadrilateral of Ursa Minor.

[Margin: Day 13.]

And on the 13th, at $3^h\ 45^m$ in the morning, he saw the comet almost on the meridian, at an altitude of 63° 33′; but at $7^h\ 51^m$ in the evening at an altitude of 41° 33′, 12° 45′ from the bright star of the quadrilateral of Ursa Minor and 18° 42′ from the first star of the tail of Ursa Major.

[Margin: Day 14.]

January 14. At $3½^h$ in the morning, at Ingolstadt, Cysat saw the comet at an altitude of 62° 57′, and in the same vertical the brighter star of the left foot of Cepheus at an altitude of 34° 36′; at $4^h\ 25^m$ the comet at an altitude of 61° 55′, 18° 57′ from the first star of the tail of Ursa Major and 12° 56′ from the bright star of the quadrilateral of Ursa Minor; at $5^h\ 56^m$ in the evening the comet at an altitude of 37° 40′, 19° 27′ from the first star of the tail of Ursa Major and 13° 20′ from the bright star of the quadrilateral of Ursa Minor; and at $9^h\ 15^m$, when its altitude was 48° 20′, 13° 25′ from the bright star of the lesser quadrilateral and 19° 31′ from the first star of the tail of Ursa Major.

[Margin: Day 15.]

January 15. At $8^h\ 3^m$ in the evening, at Ingolstadt, Cysat saw it at an altitude of 45° 20′, 19° 54′ from the first star of the tail of Ursa Major and about 13° 42′ from the bright star of the lesser quadrilateral. But at about 9^h the comet, at an altitude of 46° 53′, and the third-from-last star of the tail of Draco, at an altitude of 31° 31′, were in the same vertical.

[Margin: Day 16.]

January 16. At $6½^h$ in the evening, at Ingolstadt, Cysat saw it at an altitude of 39° 55′, 20° 28′ from the first star of the tail of Ursa Major and 14° 41′ from the bright star of the quadrilateral of Ursa Minor; and at $8½^h$ in the evening at an altitude of 46° 22′, 20° 30′ from the first star of the tail of Ursa Major and 14° 42′ from the bright star of the lesser quadrilateral.

[Margin: Day 17.]

January 17. At Ingolstadt, at $8^h\ 29^m$ in the evening, Cysat saw it at an altitude of 45° 50′, 21° 10′ from the first star of the tail of Ursa Major, 15° 24′ from the bright star of the lesser quadrilateral, and 54° 30′ from the left arm of Boötes.

[Margin: Day 18.]

January 18. At Ingolstadt Cysat, at $2^h\ 45^m$ in the morning, saw in the same vertical the comet at an altitude of 62° 36′ and the star of the 4th magnitude in the left shoulder of Cepheus at an altitude of 23° 9′; and at $6^h\ 44^m$ in the evening the comet at an altitude of 41° 31′, 16° 12′ from the bright star of the quadrilateral of Ursa Minor and 22° 0′ from the first star of the tail of Ursa Major.

[Margin: Day 22.]

January 22. At $4^h\ 30^m$ in the morning, at Ingolstadt, Cysat saw the comet at an altitude of 55° 59′, 16° 44′ from the bright star of the quadrilateral of Ursa Minor, 23° 40′ from the first star of the tail of Ursa Major, and 37° 55′ from the

left arm of Boötes — but this last with the radius. Eight cloudy days followed, after which the comet appeared no more.

These are the observations of this comet of the year 1618, which were spread abroad among the public with such fame and gathered by us with no less fidelity and diligence, so that we might at last extract some kernel or pith from so thick a rind, or so branching a trunk. If any observation has been passed over, it is because it either had no companion in an observation made elsewhere, or lacked features suited to this inquiry.

[Margin: The principal combinations of the foregoing observations are examined.]

XXIV. Among all the observations already mentioned, you see the greatest agreement as to the hour in the observation of the 13th made at Rome and at Cologne, for in both places the comet covered the tenth star of Boötes. But since the hour is not given, no certain argument can be derived from it? A great agreement also appears in the observation of December 3, between the Goa distance of the comet from Spica, 25° 54′, and the Copenhagen distance from Spica, 25° 52′; from which someone might at once declare, given how far apart the places are, that the comet had scarcely any parallax. And that could indeed be said, if it were established that this happened at the same moment of time. But Longomontanus says that it was then 6^h 30^m in the morning at Copenhagen; the Sun was then at 11° 17′ of ♐ [Sagittarius]; and the seminocturnal arc at Copenhagen, at a polar altitude of 55° 43′, was 8^h 27^m, while at Goa, at a polar altitude of 16°, it was 6^h 27^m. If, then, you subtract the 6^h 30^m from midnight from the Copenhagen seminocturnal time, there remains 1^h 57^m until sunrise. But Goa is more easterly than Copenhagen, according to our Corrected Geography, by 105°, or 7 hours. Therefore the Sun had already risen at Goa, and at Goa it was 5^h 3^m after sunrise at the time when Longomontanus observed the comet at Copenhagen. It was therefore impossible for the comet to have been seen at the same moment at a distance from Spica of 25° 52′ or 54′.

[Margin: Their agreement must be perfect.]

It was worth pointing this out, so that no one, because of the closeness of the distances, should at once declare that a sensible parallax is lacking, since the observers are not found to have agreed on the same distance from the same fixed star at the same moment of time. Much less ought Biancani to have asserted that lack from his own and the Goa observations, since for the same day of December he himself gives the distance of the comet from Spica as seen by him as 26° 30′ — that is, 36 minutes greater than the Goa distance — and that at $13\frac{1}{2}^h$ civil time, when the Sun had already risen at Goa.

Setting aside, then, this pairing [syzygia] of observations, another presents itself to us on December 1, which to all appearance is the best suited of all for investigating the comet's parallaxes. For at almost the same moment — namely at 4^h 45^m in the morning — Longomontanus at Copenhagen observed the comet at an altitude of 18° and at a distance of 24° 56′ from Spica. And Father

Cysat, likewise at 4^h 45^m in the morning, observed the comet at Ingolstadt at an altitude of 12° 20′ and at a distance of 24° 48′ from Spica, that is, 8′ less than Longomontanus. Now Copenhagen is more easterly than Ingolstadt, in my Corrected Geography, by 1°, or 4 minutes of time. Therefore either the hour noted by Father Cysat was correct, and so it was 4^h 49^m at Copenhagen; or, if the hour reported by Longomontanus was correct, it was then 4^h 41^m at Ingolstadt. If we wish both observations to have been made by chance at the same moment of time, or if we wish the moment of each to have been noted correctly, then it must be that either the moment of observation was different, or the meridian was not different, since each of them notes the hour as 4^h 45^m in the morning. Which of these was the case is uncertain, and for that reason we cannot demonstrate parallaxes from this. Next, even if we granted that the observation fell at the same moment, the apparent distances stand in the way. For at Ingolstadt, where the comet appeared at an altitude of only 12° 20′, the parallax of altitude must have been much greater than at Copenhagen, where the comet appeared at an altitude of 18°; therefore the parallax of distance from Spica was also greater there than here, and so the comet's distance from Spica ought to have appeared greater to Father Cysat at Ingolstadt than to Longomontanus at Copenhagen. But the reverse happened: Cysat observed it as 24° 48′ and Longomontanus as 24° 56′. Either, therefore, the moment was not the same, or some error lies hidden in the distances taken. If, however, anyone cares to waste effort on this business and to feign an observation made at the same moment, I shall show how he can do it by the following Problem.

Problema.

Problem.

To find the parallax of altitude, given the comet's apparent distances from the same fixed star at the same moment of time, from two places whose geographical longitude and latitude are known; and given, besides, the moment of time, together with the declination and right ascension of the same star.

[Margin: Problem of Father Francesco Maria Grimaldi.]

XXV. While I was working on these matters I put this problem to Father Francesco Maria Grimaldi, and he solved it with incredible speed and skill by the following figure, which is therefore to be credited to him. Let there be two meridians intersecting at the poles of the world, P and M: one drawn through I, the zenith of Ingolstadt, which is PIM; the other through H, the zenith of Copenhagen, which lies further east, which is PHM. Let the comet's true place be at C, through which arcs of vertical circles are drawn from the zeniths already mentioned to the comet's apparent places: namely, from the zenith of Ingolstadt the arc ICA, to the apparent place A, and from the zenith H the arc HCB, to the apparent place B. Let the fixed star be S, and the apparent distances BS and AS. Likewise let the vertical arcs IS and HS be drawn from the zeniths to the fixed star, and the arc of the star's circle of declination from the pole, namely the arc PS; and finally let the two zeniths be joined by the arc HI.

Diagram. Grimaldi's figure for the problem, set in the left column beside the short lines of §XXV. A large outer arc runs from M at the lower left up to I at the top and down to P at the lower right; a second arc from M, inside the first, rises to the right past H and runs down to P; a lower arc joins S and P. From I, lines run down to S and, through C, to A; from H, lines run to S and, through C, to B; a line joins H and I. Lines run from S to B and from S to A. A large dot marks C.

- I (top): the zenith of Ingolstadt
- H (right of I): the zenith of Copenhagen
- S (left): the fixed star
- B (centre, below): the comet's apparent place seen from H
- C (centre, with the dot): the comet's true place
- M (lower left): a pole of the world, where the two meridians meet
- A (bottom): the comet's apparent place seen from I
- P (right): the other pole of the world, where the two meridians meet

[Margin: First step.]

With these laid down: first, for the given moment of time, the distance of the fixed star from each zenith is to be found, together with its azimuth. For since the right ascension of the midheaven is given from the moment of time, and the right ascension of the fixed star is given too, from the difference of the two the angle SPH will be known in the triangle SPH, and the angle SPI in the triangle SPI. Moreover, in the same triangles SP is given, an arc which is either the complement of the fixed star's declination, or made up of a quadrant and the star's declination towards the other side; and the complements of the polar altitudes, HP and IP, are given as well. Therefore the bases HS and IS, which are the true distances of the fixed star from the zeniths H and I, will not be unknown. Nor will the complements of the azimuths, PHS and PIS, be unknown, and so neither will the azimuths themselves.

[Margin: Second step.]

Second: in the triangle BHS there are given HS, found by the first step; HB, the apparent distance of the comet for an inhabitant under H; and BS, the distance of the comet from the fixed star S as seen by the same observer. Therefore the angle BHS will be known.

[Margin: Third step.]

Third: in the triangle PHI, in which are given IP and HP, the complements of the polar altitudes, and the angle HPI, known from the difference in longitude of the places, that is, from the distance between the meridians, find the arc HI, that is, the distance between the zeniths of the two places, and the angles IHP and HIP. Now the supplement of the angle IHP to two right angles is IHM; compounding this with the angle SHM, found by the first step, and with the angle BHS, found in the second step, the whole angle CHI will be known.

[Margin: Fourth step.]

Fourth: in the triangle AIS three sides are given, namely AS, the apparent distance of the comet from the fixed star for the observer under I; AI, the

apparent distance of the comet from the zenith I; and IS, the distance of the fixed star from the same zenith, found in the 1st step. Therefore find the angle AIS; add to it the azimuthal angle SIM, known by the 1st step, and also the angle HIP, known in the 3rd step; subtract the sum of these three from 180°, and the angle CIH will remain known.

[Margin: Fifth step.]

Fifth, therefore: in the triangle CHI, given now the side HI and the angle CHI, by the 3rd step, and the angle CIH, by the 4th, find the comet's true distances from the zeniths, namely HC and IC; for their difference from the apparent distances HB and IA will show the parallaxes BC and AC. Which was to be done.

You see, I believe, that the apparent distance from Spica at Ingolstadt, namely AS, must be supposed in this problem greater than BS, seen at Copenhagen, on account of the parallax of altitude AC, which is greater than the parallax BC because it is owed to the greater distance CI from the zenith I; and consequently that we cannot infer anything with certainty from the observations mentioned for a determinate quantity of this comet's parallaxes. Much less can we do so from the other observations set out above, because it is not established whether they were made at the same time; nor can we reduce them to the same moment except by feigning a proper and uniform motion of the comet, and by altering the apparent distances with conjectures applied — all of which is full of uncertainty.

An PP. Collegij Romani demonſtrarint abſolutè Cometam Anni 1618. fuiſſe ſupra Lunam.

Whether the Fathers of the Roman College demonstrated absolutely that the comet of the year 1618 was above the Moon.

XXVI. I deny that this was demonstrated by them, or held as demonstrated. For the author of that disputation, who is said to have been Father Orazio Grassi, frankly admits that he lacked instruments reaching to single minutes, and that he could therefore have been less diligent in observing and in the calculations drawn from the observation; and he adds: *"For I know that account should have been taken of the hours at which observations of this kind were made in different places, though on the same day; and besides, of refractions and other things, the investigation of which required much more diligent observers."* Then, from the small difference in the distances of the comet from Arcturus, or from the tenth star of Boötes, observed at Rome, Parma, Antwerp and Cologne on the same day — but chiefly from the fact that on the 13th the comet's head covered the tenth star of Boötes at Rome and at Cologne — he infers that the comet was very high, and higher than the vapours, which he grants rise at most 100 Italian miles; otherwise, the least parallax between the Roman and the Antwerp observations would have been of 56 degrees, as if the comet had been on the horizon for one of the two cities. Indeed, he infers that the comet was either celestial or a little below the Moon, for these are his words: *"You have, then, from the parallax,*

observed after a fashion, that our comet was not sublunary but plainly celestial. But if anyone nevertheless thinks this should be little trusted, and that it is less certainly demonstrated, this at least certainly cannot be denied: since no difference is found in degrees, but only in minutes, then even if we were to grant that it was sublunary, it must nevertheless be judged not very far removed from the Moon itself." From this it is quite clear that the demonstration did not seem to him altogether absolute, nor fit to exclude all doubt about the opposite conclusion.

But even if, in view of the moderation and sincerity with which the Father already mentioned was endowed, he had not admitted what we have said so far, but had declared his demonstration certain and absolute, we could still by no means have admitted it. For he makes the least lunar parallax — that is, the difference of the parallaxes between Antwerp and Rome — 24 minutes, but in the circle of altitude; whereas the parallax of the comet's distance from Arcturus as seen at Rome and Antwerp, that is, the difference of the parallaxes, he says was 16′, and so less than the lunar. But the conclusion drawn from this is not sound, because the terms are exchanged, and a passage is made from the parallax in the vertical circle to the parallax in the circle of distance, which was different from the vertical circle. Moreover, even if it had been of the same kind, the difference of the parallaxes can still be of one minute, let alone of 16 minutes, and yet the absolute parallax of the phenomenon be of several degrees, as has already been demonstrated by us in chapter 19, number 3, and will be shown again in Book 10, penultimate section, chapter 11, numbers 11 and 12, following Chiaramonti. He also demonstrates in this place — that is, in the *Antitycho*, Book 2, chapter 28 — that if the parallax of distance between Antwerp and Rome had been 16′, as it is given to have been on December 5, the parallax in the vertical circle would have been 2° 25′; if the parallax of distance had been 25′, as it is granted to have been on December 13, again between Rome and Antwerp, the parallax of altitude would have been 3° 3′; and finally, if the parallax of distance between those places had been 59′, as it is given to have been on December 16, the parallax of altitude would have been 7° 51′ — that is, always greater than the lunar. These things, I say, were shown by Chiaramonti with a true and exact demonstration. But as for the comet's covering the same star at Rome and at Cologne on the same night: unless this is shown to have happened at the same moment of time, it is worth nothing. For it is very easy for a phenomenon, even one with a large parallax, to cover the same uppermost point for one more northerly place which it had already covered on the same day for a more southerly place, in a different vertical circle and at a different hour or moment, by virtue of its motion, as Chiaramonti showed in the same place, chapter 29. The remaining arguments, which are weaker and merely probable — namely those from the comet's bulk, from its regular path along a great circle, from its receiving no enlargement through the telescope, and from the tail's being turned away from the Sun — have already been answered in number 3.

An Claramontius Demonſtrarit Cometam Anni 1618. fuiſſe infra Lunam, ex incremento Caudæ orto ex Parallaxibus, & à ſe obſeruato.

Whether Chiaramonti demonstrated that the comet of the year 1618 was below the Moon, from an increase of its tail arising from parallaxes and observed by himself.

XXVII. Chiaramonti does indeed profess this, in Book 3 of the *Antitycho*, chapter 20, and in the *Apologia* for the *Antitycho*, Book 2, chapter 11. Kepler, however, opposes him in the *Hyperaspistes*, from page 171 to 176. It will be fair, then, for readers to weigh both the reasons of each of them and our own.

Chiaramonti's argument runs as follows. On December 4 the comet's tail was observed by him at Cesena to be 21°, when the comet's head appeared 26° high. On December 18 the tail was seen by him to be 28° long, when the comet's head appeared 53° high; and at both times it was $12^h\ 40^m$ by the Italian hour of the Sun, that is, reckoned from sunset. Therefore at the smaller distance from the zenith the tail appeared 7 degrees longer than at the greater. Therefore that increase was apparent, and arose from a greater parallax of the head than of the tip of the tail. He proves the last consequence in two ways. First, if the increase had been real, the comet's head would also have grown from the 4th to the 18th, whereas it rather shrank from day to day. Second, if this increase had not come from parallax, the tail would have appeared equal on both days, and would have appeared the same to an eye placed on the surface of the Earth as to an eye supposed to be at the centre of the Earth; and there is no doubt that, if the comet had a sensible parallax, the tail must have appeared longer than it really was, on account of the parallax of the distance of the head from the end of the tail.

[Margin: Construction of the figure for the position of the tail.]

To understand this and the rest, I shall propose a more suitable figure than Chiaramonti's. In it let the meridian be ABCD, with the zenith B in it, and the eastern horizon of Cesena ALC. Let the equator be EF, and let the southern parallel on which the Sun stood on December 18 — the Sun being very near the beginning of Capricorn, that is, to the Tropic of Capricorn — be GHD. Since at the time of the observation [?] it was $12^h\ 40^m$ by the Italian hour of the Sun, that is, $2^h\ 36^m$ before sunrise, let the Sun be at H, and the true place of the comet's head at I. Through I let the straight line HIK be drawn from the Sun, so that the true line of the tail, turned away from the Sun, is IK, which Chiaramonti holds throughout to have been 21°. From the zenith B let two quadrants of vertical circles be drawn, one through the comet's head I, which is BIL, the other through the extreme tip of the tail, which is BKM. On account of the parallax of altitude, let the apparent place of the head be at N and that of the tail's end at P; the parallax of the head is then NI, and that of the tail's end PK, which is smaller than NI because K was nearer the zenith than the head I.

Diagram. Riccioli's figure for the position of the comet's tail at Cesena on December 18, set in the left column beside the short lines of §XXVII. A circle,

the meridian ABCD, with B at the top, A at the left, C at the right and D at the bottom. A straight line from A to C crosses the lower half (the eastern horizon ALC), with L and M on it towards C. A line runs from E at the upper left to F at the lower right (the equator), and a line from G, below E on the left, to D (the southern parallel), on which the rayed Sun H stands. From B two lines run down to L and to M. A line from H runs up through I to K. Two small comet figures are drawn: one with its head at I and its tail reaching to K, the other with its head at N and its tail reaching to P. Q stands on the circle at the upper right, between B and C.

- B (top): the zenith
- E (upper left): one end of the equator EF
- G (left, below E): one end of the southern parallel GHD
- A (left): one end of the horizon ALC
- K (centre, upper): the true end of the tail
- Q (on the circle, upper right): the pole, the arc QC being the altitude of the pole
- I (centre): the true place of the comet's head
- P (centre, right of I): the apparent end of the tail
- N (centre, below P): the apparent place of the comet's head
- L (on the horizon, right of centre): foot of the vertical quadrant BIL
- M (on the horizon, near C): foot of the vertical quadrant BKM
- C (right): the other end of the horizon ALC
- H (lower centre, the rayed Sun): the Sun
- F (lower right): the other end of the equator EF
- D (bottom): the lowest point of the meridian, and the end of the parallel GHD

With these laid down, Chiaramonti's method proceeds through several steps, with another figure drawn in which there are the solstitial colure and the ecliptic. It seeks the parallaxes IN and PK: namely, how great they must have been for the tail, which appeared on the 18th 28 degrees long in the position NP, to have been really IK, 21°; and consequently how great the comet's distance from the Earth must have been.

[Margin: First step.]

First, then, assuming the altitude of the pole QC, 44°; the point of the ecliptic culminating, ♏ 8° 58′; the observed azimuth of the apparent head, LBC, 70° 28′; the apparent distance of the comet from the zenith, that is, the arc BN, 37°; and the apparent length of the tail, NP, 28° — he seeks, by seven resolutions of spherical triangles, the arc BP, that is, the apparent distance of the end of the tail from the zenith, which he finds to be 12° 4′.

[Margin: Second and third steps.]

Second, he seeks the angle NBP, which he finds to be 35° 52′. Third, he tried how great the comet's distance from the centre of the Earth would have to be, so that the parallaxes NI and PK, corresponding to the apparent distances BN, 37°, and BP, 12° 4′, when subtracted from BN and BP, would leave BK

and BI of such a size that from them and the angle IBK, already found to be 35° 52′, the true length of the tail IK, 21°, would follow. Chiaramonti does not, indeed, set out these trials; but where he came closest to it, he finds that the comet's distance from the Earth must be less than 11,500 Italian miles, of which 3035 make one semidiameter of the Earth.

[Margin: Fourth step.]

For fourth, by the problem and figure already set out by us in chapter 18, number 10, from three givens — namely the semidiameter of the Earth, 3035 miles; the distance of the comet's head from the centre of the Earth, 11,500 miles; and the apparent distance of the comet's head from the zenith, 37° — he seeks and finds the parallax NI, 9° 8′. When this is taken from the apparent distance BN, there remains the true distance of the head, BI, 27° 52′.

[Margin: Fifth step.]

Fifth, by the same figure and problem, from the sides 3035 and 11,500 and the apparent distance of the end of the tail, 12° 4′, he seeks and finds the parallax PK, 3° 10′. When this is taken from the apparent distance BP, there remains the true distance of the end of the tail, BK, 8° 54′. Sixth, in the triangle BIK, having found the angle IBK, 35° 52′, by the second step, the arc BI, 27° 52′, by the 4th step, and BK, 8° 54′, by the 5th step, he finds the true size of the tail IK to be 21° 15′. But it ought to have been only 21°, he says; therefore the parallaxes NI and PK must have been somewhat greater still, and so the comet's distance from the centre of the Earth still less than 11,500 Italian miles — which is what Chiaramonti had set out to demonstrate.

[Margin: Kepler did not understand Chiaramonti.]

XXVIII. What does Kepler say to this in the *Hyperaspistes*, from page 171? He confesses that he did not understand what Chiaramonti means, and complains of Cimmerian darkness in his method. Yet he tentatively seeks out his meaning, but only after this preliminary excuse: *"Scipio speaks obscurely, and at the start without a figure; let him forgive me, then, if I shake out of his words some senses that he himself perhaps did not intend."* After many attempts, however, it is plain that he did not understand Chiaramonti's method — which in this place is indeed more obscure than it should have been, for Chiaramonti does not set out his method with as much clarity as I believe I have achieved.

[Margin: Yet Kepler's conclusion is true.]

Nevertheless, Kepler rightly concludes, on page 176, in these words: *"It would have been enough to point this out alone: that there are very many causes, passed over in silence by Scipio, which vary the apparent length of the tail."* These are: 1. its natural generation and consumption; 2. inconstancy in its shooting forth; 3. the difference of eyes in making out the end of the tail; 4. the brightness of the nights from the presence of the Moon; 5. the inclination to the visual rays. When these five are weighed, it is easily seen that not everything is to be ascribed to the comet's approach itself — which indeed I too, elsewhere, accept among the causes of the tail's length. This tail of the comet, therefore, is worth nothing for thrusting the comet down into the air.

Solutio Argumenti Claramontij.

Solution of Chiaramonti's argument.

[Margin: Major premise.]

XXIX. Chiaramonti's whole discourse must be brought into syllogistic form, so that the solution of the argument may be clearer. The syllogism, then, is this: *If on December 18, at 12^h 40^m after sunset by the hour at Cesena, the true length of the comet's tail was 21°, and yet at an apparent distance of the head from the zenith of 37° it appeared 28° long, and that increase of 7 degrees came wholly from parallax, then the parallaxes of altitude of the head and of the end of the tail must have been so great that, for them, the comet's distance from the centre of the Earth had necessarily to be less than 11500 Italian miles, or less than 3 2395/3035 semidiameters of the Earth.*

[Margin: Minor premise.]

But on the day and at the hour already mentioned, at Cesena, the true length of the tail was 21°, and yet at an apparent distance of the head from the zenith of 37° it appeared 28° long, and that whole increase of 7 degrees came from parallax.

[Margin: Conclusion.]

Therefore the comet's distance from the centre of the Earth was necessarily less than 3 2395/3035 semidiameters of the Earth. And so the comet was below the Moon.

The major premise has been shown [?] from observations, which must be admitted for as long as they are not convicted of falsity, and from the trigonometric calculations which I indicated above in number 27, after the drawing of the figure, from the first step to the sixth. I have examined these and found no fault in them that would notably undermine that distance of the comet of about 4 semidiameters of the Earth.

I therefore grant the major premise.

[Margin: Proof of the minor.]

The minor premise was proved by Chiaramonti as follows. On December 4 the tail was observed by him to be of 21 degrees, and on December 18 of 28 degrees, and that increase was not real but optical and apparent. For if it had been real, the magnitude of the head ought to have grown as the tail grew; but the magnitude of the head decreased from day to day; therefore the increase was not real but apparent, and from no other source than parallax. And if from parallax, then the apparent length of 28 degrees was greater than the true, because it is the property of parallax to enlarge distances in appearance, and so the distance of the comet's head from the extremity of the tail, which is to enlarge the length of the tail. Therefore, if the apparent length of the comet's tail, 28°, was greater than the true, it remains that the observed length of 21 degrees was the true one, since it is smaller by 7°. So both parts of the minor seem to be proved: namely, that the true length of the tail was 21°, and that the increase of 7°, by which it afterwards appeared to be 28°, came wholly from parallax.

[Margin: Negative reply to the minor.]

In this discourse, however, some things are false and some are not necessary; and so the minor premise is denied. First, I deny that the true length of the tail was 21° if the length of 28 degrees was apparent and not true. For since both were observed by Chiaramonti, both were apparent; and since, according to Chiaramonti, the parallax was so great, neither could be true, but each was greater than the true. Otherwise, if the parallax had been insensible, both would have been true, and that increase would either have been real, or both would have been enlarged by something other than parallax. Indeed, the length of 21 degrees must have been less true than that of 28 degrees, because when it was seen as 21° the comet is taken to have been much nearer the horizon, namely 64° from the zenith. Further, since on December 4 the comet appeared with a larger head, Chiaramonti ought rather to have inferred from this a smaller distance from the Earth than on the 18th, when he asserts that the head had diminished; for both reasons, then, the parallax ought to have been greater. So if, at a distance of the head of 37 degrees from the zenith, the apparent length of the tail exceeded the true by 7 degrees on account of the parallaxes, then at a distance of 64 degrees the apparent length ought to have exceeded the true much more, and the true length ought thus to have been smaller by about 14 degrees — unless the inclination of the tail required otherwise, which, however, Chiaramonti would have had to show.

Second, I deny that the increase was not real on the ground that the head decreased. For the comet's head could have decreased only optically, on account of its distance from the Earth growing greater day by day, and the tail nonetheless have grown from another cause — for instance, by the arrival of matter behind the comet's back, suited to reflecting to us the rays of the Sun that pass through the comet's head. Or the head could indeed have decreased physically, but because part of the matter that was in the head was drawn out lengthwise into the tail.

Third, I deny that, if that increase was not real, it came wholly from parallax. For the smallest part of the increase could have arisen from parallax, and the greatest part from the other causes indicated above in number 28 by Kepler — except the difference of eyes, since the observer was the same.

[Margin: The comet's tail of varying length on the same day.]

Certainly, on the night of November 29, as I related in the history of this comet, the tail appeared to Kepler to be of 30 degrees, while at Rome and Vienna on the same night it appeared to be 58°. On November 30 it appeared to be of 23 degrees, then at once longer through a certain vibration. On December 9 it appeared to Kepler to be of 70 degrees, but repeatedly greater on account of the darting of its rays; the more constant and more certain length, however, was 70°. On December 14 Biancani at Parma measured the tail at 36 degrees, but noted that it was at times the greater, the darker the air was; and on the same day Millerus saw it likewise at 36 degrees, while on that same day Ambrosius Rhodius saw it now at 50°, now at 60°. And Longomontanus, in his observation of November 23, old style, that is, December 3, new style, has

these words: *"We also measured the length of the tail, about 20 degrees or a little more, although we could scarcely follow its extreme vibration with our eyes."* Finally, see Cysat's observations on this in chapter 4.

That enlargement, therefore, was in large part optical: it came from the vibration of the rays, from their inclination towards the eyes — although Chiaramonti says in the *Apologia* that this was the same — from matter fit to receive and arrest the rays of the Sun and reflect them to the eye, and from the darkness of the air. For on December 4 the Moon had shortly before been full of light, and at $12^h\ 40^m$ after sunset it was still above the horizon, lighting up from the opposite side the eastern quarter in which the comet stood, and so washing out the tail. But on the 18th the Moon, having been in conjunction with the Sun two days before, was below the horizon, and indeed further below it than the Sun; so the length of the tail then showed better in the darker air. How much the Moon hinders the seeing of a comet is clear from Cysat's observations reported in chapter 3; and from them it will also be clear that the head was not really diminished, had it been viewed through a telescope. And indeed, whether that tail is called a flame in inflammable matter or light from the Sun's rays, it is not probable that it really remained the same from December 4 to December 18. Finally, from the very fact of the increase at a smaller distance from the zenith, at which the tail ought rather to have appeared smaller, Chiaramonti should have suspected that the increase did not come wholly, or in large part, from parallax.

Although, then, he did not demonstrate by this method that the comet was below the Moon — as he had professed in the summary of chapter 20 of Book 3 of the *Antitycho*, using the word *"It is demonstrated"*, and saying in the conclusion of the whole problem *"and this is what was to be demonstrated"* — he nevertheless deserved great praise for a truly ingenious method, by which he taught how to find the parallaxes of the head and of the tail on the hypothesis that the whole increase of the tail came from parallax. Nor does Kepler himself, on page 173, deny that it is *"an ingenious and new invention."* Let us pass on to Chiaramonti's other demonstration.

An Claramontius demonſtrauerit Cometam Anni 1618. fuiſſe infra Lunam, ob Parallaxim maiorem,erutam ex collatione Obſeruationis ſuæ cum Romana Obſeruatione.

Whether Chiaramonti demonstrated that the comet of the year 1618 was below the Moon on account of a greater parallax, derived from comparing his own observation with the Roman observation.

XXX. Chiaramonti asserts that this too was demonstrated by him, in Book 3 of the *Antitycho*, chapter 21, and in the *Apologia*, Book 2, chapter 12; Kepler, however, denies it in the *Hyperaspistes*, page 177. Let us weigh to which of the two the case should be awarded. The problem Chiaramonti uses is the one whose figure and method we set out sufficiently, with his own example,

in chapter 19, number 11, to which we refer the reader. It is founded on the difference of latitude and longitude observed at two places. For this he here supposes an observation of the comet duly made at Cesena on the morning of December 4, at $12^h\ 40^m$ after sunset, and the comet's longitude and latitude derived from it by calculation, as you see in the following synopsis.

[Margin: Chiaramonti's first step.]

Quantity	Value
Altitude of the pole at Cesena	0ˢ 44° 0′ 0″
Obliquity of the ecliptic	0ˢ 23° 30′ 0″
Apparent altitude of the comet	0ˢ 26° 0′ 0″
Apparent altitude of Arcturus	0ˢ 43° 0′ 0″
Apparent altitude of Spica	0ˢ 26° 30′ 0″
Azimuthal angle between the comet and Arcturus	0ˢ 3° 0′ 0″
Between the comet and Spica	0ˢ 34° 0′ 0″
Therefore the distance of the comet from Arcturus	0ˢ 17° 10′ 40″
And the distance between the comet and Spica	0ˢ 27° 44′ 35″
And the distance between Spica and Arcturus	0ˢ 32° 26′ 56″
Therefore the apparent longitude of the comet	♍ 4° 33′ 36″
And the apparent latitude	north 21° 48′ 58″
But from the Ptolemaic and Copernican latitude and longitude of the fixed stars there is gathered:	
Distance of Arcturus from Spica	0ˢ 33° 41′ 0″
And the apparent longitude of the comet	♍ 3° 34′ 39″
And the apparent latitude of the comet	north 20° 42′ 36″

But in the *Antitycho*, page 354, having reckoned the matter more carefully, he says that the comet's longitude and latitude derived from the Ptolemaic and Copernican longitude and latitude of Arcturus and Spica is to be preferred to the one that had been derived in the first place from the observed distances of the comet from Arcturus and Spica, that is, from those deduced from the observation.

[Margin: Second step.]

Second, he supposes the comet's distance from Arcturus observed at Rome on the same night, 17° 40′, and from Spica, 26° 33′ — namely by our Fathers in the Roman College; from which, and from the Coperni- can longitude and latitude of Arcturus and Spica, he gathers the latitude of the comet, 19° 26′ 30″, and its longitude, ♍ 3° 7′; and so the difference from the longitude observed by himself, 27′ 39″, and from the latitude, 1° 16′ 51″ — that is, if the observation was made at the same hour. But since the Roman observer did not add the hour, he says that, taking into account the comet's daily motion in longitude and latitude, according to the method he has given elsewhere for reducing an observation to the same moment, the discrepancy cannot have been so great that the difference of the longitudes and latitudes could have been diminished

by more than 5 minutes. So if the observation happened to be made at the same moment of the hour, the difference of the longitudes observed at Cesena and Rome was, he says, 27′ 39″, and that of the latitude 1° 16′ 51″; but if it was not made at the same hour, the difference of the longitudes could still not have been less than 22′ 39″, nor that of the latitudes less than 1° 11′ — especially since he puts the difference in the altitude of the pole at no more than 2 degrees, and the two places lie almost under the same meridian, and likewise do not differ in this by more than 2 degrees, or 8^m of time.

[Margin: Third step.]

Third, then, he proceeds from the problem he gave in Book 1 of the *Antitycho*, chapter 7, problem 1, and from its working, given by him in Book 3, chapter 5, which we have set out in chapter 19, numbers 11, 12 and 13. He supposes it demonstrated by himself, from the difference of longitude of 10′ 15″ and of latitude of 24′ observed at Uraniborg by Tycho and at Kassel by the Landgrave of Hesse, that the comet of the year 1577 was not more than 12657 Italian miles from the centre of the Earth; on which see what we have said in chapter 18, numbers 34 to 37. With this supposed, he now reasons as follows, on page 355 of the *Antitycho*: if a difference of longitudes of 10′ and of latitudes of 24′, observed at Uraniborg and Kassel, shows that the comet was not more than 12,657 miles from the centre of the Earth, then a difference of longitudes of 22′ 39″ and of latitudes of 1° 11′, observed at Rome and Cesena, shows it much more. For the greater that difference is (refraction set aside), the greater the parallax it argues, and so the greater the nearness to the Earth; and still more so if the two places of the observers are nearer to each other and differ less in the altitude of the pole. Now Cesena is nearer to Rome, and differs less from it in the altitude of the pole, than Uraniborg does from Kassel: these cities differ by 4° 36′, whereas Rome and Cesena differ by only about 2 degrees. Therefore the distance of the comet of this year 1618 from the centre of the Earth was less than 12657 Italian miles, or 4 1/6 semidiameters of the Earth.

Solutio Argumenti Claramontij.

Solution of Chiaramonti's argument.

[Margin: Major premise.]

XXXI. This argument of Chiaramonti's likewise, bound to the rules of syllogism, stands thus: *If, for the comet of the year 1618 observed on December 4 at Rome and at Cesena at the same moment of time, the difference of longitude was greater than 10 minutes and the difference of latitude greater than 24 minutes, then the comet was less than 12657 Italian miles, or 4 1/6 semidiameters of the Earth, from the centre of the Earth, and so nearer than the Moon.*

[Margin: Minor premise.]

But for the comet of the year 1618 observed on December 4 at Rome and at Cesena at the same moment of time, the difference of longitude was greater than 10′, and the difference of latitude greater than 24′.

[Margin: Conclusion.]

Therefore the comet was less than 4 1/6 semidiameters of the Earth from the centre of the Earth, and so nearer than the Moon. So it was far below the Moon.

The major premise is plain from Chiaramonti's third step, on which see above, at the end of number 30. The minor is plain from his first and second steps, set out in the same place. For if we suppose, without any reduction, that the observations made at Rome and at Cesena happened to fall at the same moment of time, the difference of longitude was 27′ 39″ and that of latitude 1° 16′ 51″; and if they are reduced to the same moment, the difference was not less than 22′ 39″ in longitude, and 1° 11′ in latitude.

[Margin: Negative reply to the minor.]

I reply: granting the major (for the problem proceeds from those given differences by an evident calculation), I deny the minor. First, because in its proof the distance of Arcturus from Spica is assumed to be 33° 41′, whereas from Tycho's observations and our own, repeated many times with the sextant, it is 33° 2′, as is clear from what was said in Book 6, chapter 10. So Chiaramonti would have erred less had he used the distance deduced from his own observations, 32° 26′ 57″ — granted that not even this is correct, and that Kepler, in the *Hyperaspistes*, page 177, conjectures from this that Chiaramonti's observations were rather rough; and the frequent omission of minutes in them is no slight indication of it. Second, because he used the Copernican longitude and latitude of Arcturus and Spica, which are plainly erroneous, especially the longitude, as is clear from what was said in Book 6, chapter 14, and that by many minutes. Third, the comet's proper daily motion in longitude for December 4 is supposed too small, since Kepler and Longomontanus put it at about 1° 40′ or 1° 30′, of which 25′ or 22′ answer to 6 hours; taken from the difference of longitude, 27′ 39″, these leave 3′ or 5′. And 6 hours could have passed between the Cesena observation and the Roman one, since when Chiaramonti observed the comet it already appeared 53 degrees high. Whether, once Chiaramonti's assumptions are corrected, a distance of the comet from the Earth smaller than the Moon's still follows, it is not my task to show; it is enough to have shown that, from the assumptions already mentioned, it has not been demonstrated that the comet was only 12,657 miles from the Earth.

An Claramontius demonſtrarit Cometam Anni 1618. fuiſſe infra Lunam, ex Obſeruationibus ſuis collatis cum Obſeruationibus PP. Societ. noſtræ in India Orientali habitis.

Whether Chiaramonti demonstrated that the comet of the year 1618 was below the Moon, from his own observations compared with the observations made by the Fathers of our Society in the East Indies.

XXXII. Chiaramonti affirmed this too, in the *Apologia* for the *Antitycho* and in the supplement to the *Antitycho*. There, on page 2, he reports the distances of the comet from Arcturus and from Spica observed at Goa by Father Giacomo

Rho of the Society of Jesus, and from these, from page 4 on, he investigates the comet's longitude and latitude. But in the same place he supposes the distance of Arcturus from Spica to be 33° 41′, as he had supposed it in the *Antitycho*, Book 3, chapter 21, and takes the longitude and latitude of the fixed stars from Ptolemy and Copernicus, as he states expressly there on pages 4 and 5. From the difference between the longitude and latitude deduced from these and those he had gathered from his own observations on December 4, he deduces the parallaxes and the distance from the Earth. But that whole structure collapses, because it rests on false foundations: namely, the erroneous distance of Arcturus from Spica and the erroneous longitudes of Arcturus and Spica, as I taught a little earlier, in number 31, in the reply to the minor premise.

An ex Obſeruationibus aliquorum in diuerſis terræ locis factis demonſtrari poſſit Cometam Anni 1618. fuiſſe infra Lunam.

Whether it can be demonstrated, from the observations of some observers made at different places on the Earth, that the comet of the year 1618 was below the Moon.

XXXIII. This must be affirmed, whether we accept the observations of some observers which, on account of their excessive disagreement or other marks, appear rough and deceptive, or those of others which appear more accurate. Let us begin with the latter. Father Cysat, on this comet, page 20, derives from the distances of the comet from stars of Ursa Major and Ursa Minor, observed on December 28 at $5^h\ 4^m$ in the morning at Ingolstadt, the comet's longitude, ♏ 8° 31′, and its northern latitude, 60° 57′, using the Tychonic latitudes and longitudes of the fixed stars; and from the place observed on the following day, the daily motion in longitude is gathered as 2° 20′ and in latitude as 20′. On the same day, namely December 18 old style but the 28th Gregorian style, Longomontanus, from his own observations made at Copenhagen at $6^h\ 45^m$ in the morning, and likewise using the Tychonic places of the fixed stars, gathers the comet's longitude as ♏ 7° 30′ and its latitude as 61°; and from the motion of the immediately following day, as assigned by him, the comet's daily motion is gathered as 3° 25′ in longitude and 39′ in latitude, as is found in the Appendix to the *Astronomia Danica*, chapter 10, page 34 in my copy. The difference of apparent longitude, therefore, was 1° 1′, and of latitude 3′. The difference of the meridians of Copenhagen and Ingolstadt is at most 4^m of time; so when it was $5^h\ 4^m$ at Ingolstadt, it was $5^h\ 8^m$ at Copenhagen. Taking this from $6^h\ 45^m$ leaves $1^h\ 37^m$, by which the Ingolstadt observation preceded the Copenhagen one. Now the comet's motion was retrograde, as is clear from its ephemeris by Kepler, Lon- gomontanus and Cysat, and pointed out by us in the history of this comet. So in $1^h\ 37^m$ the comet's longitude should have decreased, but its latitude increased. Now the comet's daily motion in longitude was 3° 25′, and in latitude 39′. Conversely, if the Copenhagen observation is reduced to the moment of the Ingolstadt one, the latitude should be diminished but the longitude increased, in proportion to the daily motion corresponding

to 1^h 37^m; and that is 14′ in longitude and 3′ in latitude. Therefore, had the observation been made at Copenhagen at the same moment as at Ingolstadt — that is, at 5^h 8^m at Copenhagen but 5^h 4^m at Ingolstadt — the longitude observed at Copenhagen would have been ♏ 7° 44′ and the latitude 60° 57′; but at Ingolstadt it was then observed at ♏ 8° 31′, with latitude 60° 57′. So the difference of apparent longitude was 47′, and of latitude 0′. With these supposed, and given the altitude of the pole at Copenhagen, 55° 43′, and at Ingolstadt 48° 40′, and so a difference of 7° 3′, it is gathered by the problem given in chapter 19, numbers 11 and 12, that the distance of the comet from the Earth was much less than 41,390 Italian miles, or 10 semidiameters of the Earth.

You may learn this even without calculation, from the following. Chiaramonti, in Book 3 of the *Antitycho*, chapters 4, 5 and 6, demonstrated from a difference of longitude of 10′ 15″ and of latitude of 24′ 24″, observed by Tycho at Uraniborg and by the Landgrave of Hesse, that the distance of the comet of the year 1577 was not greater than 12657 miles; and from a difference of longitude of 15′ and of latitude of 14′, a distance not greater than 24,276 miles — demonstrations which we find to be sufficient. But in our case, although the difference of latitude is nil, that of longitude is three times greater, since it is 47′; nor is there a great difference between the differences of the altitudes of the pole. And it would be demonstrated much more strongly still from other observations, if they were accepted: for instance, from the observation of December 4, on which Cysat at Ingolstadt, at 6^h 52^m in the morning, observed the comet's longitude at ♏ 4° 56′, with latitude 14° 31′, as he himself has it on page 20. But on the same day D. Habrecht observed the comet at Strasbourg at ♏ 3°, with latitude 19° 40′; and Ambrosius Rhodius at Wittenberg saw the comet on the same day at longitude ♏ 1° 24′, with latitude 20°, as Kepler reports in *De Cometis*, page 60. For the difference between Cysat and Rhodius would be 3° 32′ in longitude and 5° 29′ in latitude, from which a huge parallax would be gathered, and an excessive nearness of the comet to the Earth. And finally from many other differences of longitude and latitude, which may be gathered from the first table placed at the end of chapter 3, where it treats of the comet of the year 1618.

But Kepler gathers that some of these observations are rough and deceptive, both from the fact that their authors themselves excuse themselves on this point — namely Rhodius, Habrecht and some others — and because the apparent quantity ought to have been intermediate at a place on the Earth lying in between, namely Wittenberg. I, however, gather the same from the observations of Cysat and Longomontanus, which I know to have been the more accurate; for they differ in the comet's longitude for this day by a single minute only. That the rest stray so far, therefore, must be judged to have come not from so great a parallax, but from some error of observation, or of calculation in reckoning the comet's hours.

Concluſiones ex Prædictis.

Conclusions from what has already been said.

I. CONCLUSION. *It has not yet been demonstrated absolutely that the comet of the year 1618 was above the Moon.* This is plain from what was said from number 1 to 26.

II. CONCLUSION. *But neither has it been demonstrated absolutely that the same comet was below the Moon.* This is plain from what was said from number 26 to 32, and from the fact that it is not established that anyone besides Chiaramonti has so far attempted to demonstrate it, or has actually demonstrated it; for if this should become established, our words will not have concerned him, and we shall change our view at once.

III. CONCLUSION. *Yet, on some probable hypothesis, and with some observations accepted, it has been demonstrated that this comet was below the Moon.* This is established from what was said in numbers 8 and 33, and towards the end of number 26.

IV. CONCLUSION. *It is probable that the comet of the year 1618 was above the Moon, yet in such a way that the opposite is not improbable.* The first part is established not only by the authority of very many astronomers who assert it, but also from their arguments. For although these are not evident, yet if they are considered attentively, many of them — those which take the true daily motion for the apparent one — cannot stray so far from the truth, especially when the observations of several days, made at several places on the Earth, are compared, that the comet would have been far below the Moon in the last days of November. And since from then until January 20 the comet's head decreased gradually but continuously, it is very probable that this came about from its ascent above the Moon: for before, it seemed to have a diameter of 7′ or 6′, but afterwards, at the end, scarcely 1′ or 2′ — granted that this could have come about from a failure of its matter and an extinguishing of its light. The second part is plain from the third conclusion.

CAPVT XXIII.

Chapter 23.

Collectis Opinionibus Aliorum in vnam Synopſim, Statuitur Quid ſentiendum de Loco, Parallaxibus, ac Diſtantijs Cometarum à Terra.

The opinions of others are collected into one synopsis, and it is settled what is to be thought of the place, the parallaxes and the distances of comets from the Earth.

I. I indicated the opinions on the place of comets in chapter 4, number 14, naming their authors, but without any other indication of their works or books. In chapter 6 I set out the same opinions more carefully, separated the doubtful ones from the certain, explained their sense, and vindicated some opinions

falsely attributed to certain authors; this I did partly in the body of that chapter itself, partly in its scholia. Now I must take a middle way, and list the authors of the opinions with the places from which they were gathered, but without any discussion of whether such an opinion was, or was not, that of the author to be named below. Then the opinions on the distances from the Earth and on the parallaxes of certain comets must be added; and finally various conclusions on these matters must be laid down according to our own view.

[Margin: Authors who place comets below the ☾.]

II. The first opinion, then, was that of those who held that all comets are generated and move below the Moon. So thought Aristotle, *Meteorology* Book 1, chapters 7 and 10, with very many interpreters and the whole Peripatetic school. And among the philosophers: the Coimbra commentators, tract 3 on the *Meteorology*, chapter 3; Cottunius, lecture 31 on the first book of the *Meteorology*; Julius Caesar Scaliger, exercise 79 against Cardano; Telesius, *De Cometis*, chapters 2 and 3; Carolus Piso, in the *Speculum physicum* of the comet; Raffaele Aversa, volume 2 of his *Philosophy*, from question 32 on; Laurentinus, in his little book on the comet of the year 1618; Bartolomeo Mastri and Bonaventura Belluto, disputation 4 *De Caelo*, question 4, article 2, number 36. And among the astronomers or astrologers, or the physicists who were, however, more skilled in mathematics: Johannes Regiomontanus and Johannes Vögelin, in their observations of the comets of the years 1572, 1532 and 1533; Giovanni Pontano, on the last aphorism of Ptolemy's *Centiloquium*; Girolamo Fracastoro, in the *Homocentrica*, section 3, ch. 23; Peter Apian, in the *Astronomicum Caesareum*, part 2, last chapter; Jakob Milich, on Book 2 of Pliny, chapter 25; Francesco Giuntini, in the *Speculum astrologicum*, in the note on comets; Petrus Surdus, in his treatise on comets; Ioannes Paduanus in the same place; Scipione Chiaramonti, in the *Antitycho*, Book 3, last chapter, and in its *Apologia* and *Supplement*. And besides these, many others listed by Tycho, volume 1, chapter 10, and volume 2 of the *Progymnasmata*, chapter 10: namely Andreas Nolthius, Nicolaus Winckler, Georg Busch, Conrad Dasypodius, Valentin Steinmetz, Nicolaus Bazelius, Georgius Henischius, Ioannes Huuernius, Theodosius Graminaeus, Thomas Erastus, and Adam Ursinus, in the little books they published on the occasion of the comet of the year 1577 or the star of the year 1572. Finally, all the ancient astronomers, as Angestus asserts in Book 2 *De proprietatibus formarum*, page 34. To this class, moreover, Heraclides, Metrodorus, Xenophanes, Strato, Epigenes, Boethus and most of the Stoics seem to be reducible, as is established from Plutarch, Book 3 of the *Placita*, chapter 2, and from Seneca, Book 7 of the *Natural Questions*, chapters 3 and 21.

[Margin: Aristotle. The Coimbra commentators. Cottunius. Scaliger. Telesius. Piso. Aversa. Mastri. Belluto. Laurentinus. Regiomontanus. Vögelin. Pontano. Fracastoro. Peter Apian. Milich. Giuntini. Surdus. Paduanus. Chiaramonti. Nolthius. Busch. Vinchler. Dasypodius. Steinmetz. Henischius. Bazelius. Huernius. Graminaeus. Erastus. Ursinus. Heraclides. Metrodorus. Xenophanes. Strato. Epigenes. Boethus. The Stoics.]

Two things, however, are to be noted here. The first is that Hájek is falsely counted in this class by Mastri and by Aversa; against them, see what was said in chapter 6, scholium 5. The second is that among the authors listed there are few Peripatetics, and fewer astrologers, who comets in the highest region of the air place them; but that the rest, especially the astronomers, place them above the highest region of the air, or above the vapours and exhalations — which Scaliger and Chiaramonti do not reject. Piso, however, places the bearded comets in the highest region of the air, but the tailed ones in the concave of the lunar heaven.

[Margin: Authors who place comets above the ☾.]

III. The second opinion was that of those who set comets above the Moon, either all of them or without qualification. Among them, of the ancients, were Democritus, Anaxagoras, Zeno, Artemidorus, Apollonius of Myndus, the Chaldaeans, the Italian Pythagoreans, and Seneca, as is clear from Seneca himself, Book 7 of the *Natural Questions*, from Aristotle, *Meteorology* Book 1, chapter 6, and from Plutarch, Book 3 of the *Placita*, chapter 2. Of the more recent: Cardano, Book 4 of *De subtilitate*; Oddus of Padua, Book 2 *De peste*, chapter 4; Cornelius Frangipanius, *De Cometis*; Tycho, volume 2 of the *Progymnasmata*, chapters 6 and 10, and throughout the *Epistolae*, especially pages 161 and 297; Longomontanus, in the Appendix to the *Astronomia Danica*, chapter 3; Marcello Squarcialupi; and, in Froidmont, Book 3 of the *Meteorology*, chapter 2, article 2, Ambrosius Rhodius, Erycius Puteanus and Thomas Fienus; Christoph Rothmann, in his little work on comets, chapter 3, and in Tycho's *Epistolae*, page 161; Galileo, in the first dialogue on the system of the world, and in the *Trutinator*; Mario Guiducci, in his treatise on comets; Giuseppe Biancani, in the *Loca mathematica* on *Meteorology* 1, chapter 6, and in the *Sphaera*, Book 16, chapter 3; Willebrord Snel, *De Cometa anni 1618*, chapters 1 and 4; Antonio Santucci, Ludovicus Columbus, Annibal Raymundus, Io. Echius and Paolo Aresi, in Resta, tract 1 of the *Meteorology*, chapter 1; Giordano Bruno of Nola, in the book *De innumerabilibus*; Descartes, Book 3 of the new *Philosophy*, number 132; Libavius, in a separate treatise, part 2, the chapter on comets; Philippus Millerus and Isaac Habrecht, in Kepler, Book 2 *De Cometis*, page 94.

[Margin: Democritus. Anaxagoras. Zeno. Artemidorus. Apollonius of Myndus. The Chaldaeans. The Pythagoreans. Seneca. Cardano. Oddus. Fringip an [?]. Tycho. Longomontanus. Squarcialupi. Rhodius. Puteanus. Fienus. Rothmann. Galileo. Guiducci. Biancani. Snel. Santucci. Columbus. Raymundus. Echius. Aresi. Bruno. Renatus. Libavius. Millerus. Habrecht.]

Here likewise two things are to be noted. First, that some of the authors already mentioned assign comets a sublunary origin, from elementary matter, but judge that they are afterwards carried up and move above the Moon: so thought Galileo, Guiducci, Aresi and, formerly, in his earlier opinion, Rothmann. Second, that Proclus, Nicephorus, Abū Maʿshar, Fracastoro and Piso are falsely counted in this class by some, as I taught in chapter 6, scholia 3, 4 and 5.

[Margin: Authors who place comets partly above, partly below the Moon.]

IV. The third opinion is that of those who placed most comets above the Moon, yet kept some below the Moon, and called or recognised the former as celestial, the latter as elementary: namely Michael Mästlin, in Tycho, volume 2, page 247; Cornelius Gemma, Helisaeus Röslin, Tadeáš Hájek and Bartholomaeus Scultetus, as may be seen in Tycho, volume 2 of the *Progymnasmata*, chapter 10 — granted that Hájek and Scultetus had once placed all comets below the Moon; Johannes Praetorius, in his catalogue of comets, at the year 1572; Philippus Apianus, in Tycho, volume 1, page 646; Tannerus, *De caelo*, from page 148 on; Froidmont, Book 3 of the *Meteorology*, chapter 1, from article 1 to 5, and article 6; Elias Camerarius, in Tycho, volume 1, page 694; Cabeo, on *Meteorology* 1, text 37, questions 2 and 4; Giovanni Camillo Glorioso, Book 5 *De Cometis*, chapter 1; Fortunio Liceti, Book 5 *De Cometis*, from chapter 41 and from 52; Franciscus Resta, tract 1 of the *Meteorology*, chapters 2 and 4; Balthassar Tellez, disputations 40 and 47 of the *Physics*, sections 1 and 2; Boulliau, Book 1 of the *Astronomia*, chapter 4. To this class can also be referred those who say that comets are made by God, either by himself or through the Intelligences, at will, out of any matter whatever: namely St John Damascene, Book 2 *De fide*, chapter 7; Gulielmus Hirsaugensis, in the *Institutio philosophica et astronomica*; Rodrigo de Arriaga, the single disputation *De caelo*, section 2, subsection 3; and Oviedus, the single controversy *De caelo*, point 3, paragraph 3. Kepler, too, was of a similar view, as is plain from his books on the comets of the years 1607 and 1618, page 45, and from the *Hyperaspistes*, page 149, where he affirms that the comet observed by Regiomontanus came near the Earth about the middle of its motion; and the motion of comets along a rectilinear trajectory, combined by him with the annual motion of the Earth (of which I shall speak in the last chapter), requires that many comets at some time approach the Earth, or the Earth them, granted that they afterwards withdraw into the depth of the aether and vanish. Cysat, too, in chapter 3, proposition 8, does not deny that the comet of the year 1618 advanced along a straight line in such a way that at the beginning it was lower than the Moon by 10 semidiameters of the Earth. Clavius also, and in his work Maurolico, in the *Sphaera*, pages 193 and 194, held that some comets can be generated in the aether above the Moon; and this was also the opinion of our Eusebius Nierembergensis [Juan Eusebio Nieremberg], Book 6 of his *Philosophy*.

[Margin: Mästlin. Cornelius Gemma. Hájek. Röslin. Scultetus. Praetorius. Tannerus. Philippus Apianus. Froidmont. Camerarius. Cabeo. Glorioso. Liceti. Resta. Tellez. Boulliau. Hirsaugensis. Arriaga. Oviedus. St Damascene. Kepler. Cysat. Clavius. Maurolico. Nierembergensis.]

The altitude and parallax of comets as determined by various authors

Comet of the year of Christ	Authors	Distance from the centre of the Earth: semidiameters of the Earth	German miles	Italian miles	Parallax at various altitudes
1472	Johannes Regiomontanus	9 ½	8200	32800	5° 40′ 0″
1532	Johannes Vögelin, in Tycho, vol. 2, p. 118	8 ⅗	7396	29584	6° 0′ 0″
1533	Johannes Vögelin	1 ¾	1597½	6390	35° 0′ 0″
1577	Tycho, vol. 2, p. 206 — minimum	173	148780	595120	0° 20′ 0″
	Tycho, as above — maximum	1733	1498180	5592720	0° 2′ 0″
	Mästlin	326	280360	1121440	0° 10′ 30″
	Gemma, in Tycho, vol. 2, p. 305	86	73960	295840	0° 40′ 0″
	Winckler	8	6880	27520	7° 0′ 0″
	Nolthius	8 ⅔	7820	31280	5° 32′ 0″
	Scultetus	9 ⅓	8047	32188	5° 22′ 0″
	Busch	24	20640	82560	2° 20′ 0″
	Chiaramonti, *Antitycho* Book 3, ch. 13, from the data of Tycho and the Landgrave of Hesse	2 23/30	2083	8333	19° 35′ 0″
1580	Tycho, vol. 2, p. 442	Beyond the sphere of Saturn, with Mästlin			
	Chiaramonti, *Antitycho* Book 3, ch. 18, from the data of Tycho and the Landgrave	1 25/86	1113	4452	42° 59′ 0″
	The same, from other data of the same observers	16 ½	14174	56696	✠
	And again from others	26 0	22388	89554	
1585	Tycho	In the sphere of the Sun			
	Chiaramonti, from the data of Tycho and Rothmann — or	2 61/86	2331	9324	15° 48′ 0″
	(the alternative)	2 52/86	2243	8973	15° 3′ 0″
1607	We, from the data of Kepler and Longomontanus	3 25/86	2833	11332	✠
	But from the same data corrected	46 0	39560	158240	✠
	Kepler, in his little table — minimum	449 0	386140	1544560	✠
	Kepler, as above — maximum	4588 ½	3945680	1578272	✠

[Margin: See what was said in chapter 20, number 3.]
The remainder of the preceding table

Comet of the year of Christ	Authors	Distance from the centre of the Earth: semidiameters of the Earth	German miles	Italian miles	Parallax at various altitudes
1618	Cysat, ch. 3, for the day of December 1	72 0	62139	248556	0° 47′ 34″
	December 9	96 0	83172	332688	0° 35′ 33″
	December 17	132 0	113456	453824	0° 26′ 3″
	December 20	154 0	132921	531684	0° 22′ 15″
	December 29	252 0	216615	866460	0° 13′ 39″

Comet of the year of Christ	Authors	Distance from the centre of the Earth: semidiameters of the Earth	German miles	Italian miles	Parallax at various altitudes
	Chiaramonti, Book 3, chs. 20 and 21, from his own observations	3 30/86	2875	11500	✠
	From his own and the Roman Fathers' observations	3 55/86	3134	12657	✠
	From his own again	1 1/15	1250	5000	✠
	We, from the data of Longomontanus and Cysat	10 0	10347	41390	✠
	Kepler, the minimum	291 0	2400009600000		✠
	The Fathers of the Roman College	Either above the ☾, or a little below the ☾.			

Concluſiones de Diſtantia & Loco Cometarum.

Conclusions on the distance and place of comets.

I. CONCLUSION. *Without violating the credibility of history, it cannot be denied that some comets were below the Moon.* This is against Tycho, volume 2, pages 247 and 441, who says: *So that I am entirely of the opinion that no comets have ever shone forth lower than the seats of the Moon.* He has the same in a letter of 1588 August 17 to Rothmann, and in a letter to the same man of 1589 November 24. It is likewise against Longomontanus, in the Appendix to the *Astronomia*, chapter 3, and the others listed in number 3. But it accords with the mind of the authors adduced in numbers 2 and 4. Such seems to have been the comet of the year 13 before Christ, of which Dio says, Book 54: *A comet, after appearing for many days hanging over the City, dissolved into torches.* So too the one which with its tail reached almost to the Earth and hung over Constantinople in the year of Christ 400, as I related in chapter 3, in the history, from Socrates and Sozomen; and the one which in the year 70 was seen to hang over Jerusalem for a whole year, as Zonaras narrates, volume 1; and, manifestly, the one which in the year 1450, passing beneath the full orb of the Moon, produced an eclipse, as I related in the same place from Georgius Phranza, Book 5 of his history, chapter 21; and the one which in the year 1543 drained a certain stream with its tail, another like which Scaliger narrates, exercise 79 against Cardano. Nor is there any ground for someone to say that these were not properly comets, or were not natural; for that very claim is against the credibility of the history, which reckons them among comets.

II. CONCLUSION. *No comet has yet been demonstrated absolutely to have been above the Moon, but only from a hypothesis which is probable indeed, yet uncertain.* This is against Tycho, Longomontanus, Kepler, Rothmann, Snel, Mästlin, Gemma, Cysat, Röslin and Hájek, at the places adduced in numbers 3 and 4, who thought they had themselves demonstrated that some comets were above the Moon; and against many others named in numbers 3 and 4, who affirmed

that it had been demonstrated by them, especially by Tycho. The first part of the conclusion is that of Chiaramonti, Cottunius, Mastri and Belluto, and it is established from the conclusions on the individual comets placed at the end of chapters 18, 19, 20, 21 and 22. From those chapters the second part is also plain. For on the hypothesis of a regular and true daily proper motion of the comet, known as if distinctly from its apparent motion, and on other similar ones which are assumed in the demonstrations of the authors already mentioned, their demonstrations hold — not all of them, indeed, but some of them; yet those hypotheses are not absolutely certain, and so the conclusion derived from them is not even certain, nor is it absolutely demonstrated.

III. CONCLUSION. *No comet has yet been demonstrated absolutely, by a mathematical demonstration, to have been below the Moon, but only from some probable hypothesis.* This likewise is clearly established from the conclusions of chapters 18, 19, 20, 21 and 22, and from the examination carried out in those same chapters. For the observations from which some comet has been demonstrated to have been below the Moon are not such that we can rely on them as certain; and there are other observations of the same comet from which, if they were admitted, that same comet would be shown to be above the Moon with as great certainty as it is shown from the former to have been below it. Nor do we have from elsewhere certain and evident indications for saying that either all these observations or all those were deceptive. I said "by a mathematical demonstration", because it has already been established from the credibility of history, in the first conclusion, that some comets were below the Moon.

IV. CONCLUSION. *It is probable that some comets were above the Moon, and some below it, even of those about whose place nothing is established from bare history.* Each part remains sufficiently proved, both by the authorities brought forward in numbers 2, 3 and 4, and by the arguments adduced for each opinion, partly in chapters 10 and 11, partly from chapter 18 to 23. For since, of the comets observed from the year 1577 to 1618, there is none which, on some hypothesis, is not demonstrated to have been above the Moon, and almost none which, on another hypothesis, is not demonstrated to have been below the Moon; and since both hypotheses are not without probability, each of the opposites is certainly probable. And what is said of these can also be said with probability of many others, on account of their likeness, even if very few of them were observed with astronomical instruments.

V. CONCLUSION. *But whether all comets were below the Moon, or some above and some below, it is certain that many moved above the height of the twilight vapours or exhalations, or above the highest region of our air.* This is established partly from what was said in chapter 14 about the small height of the vapours already mentioned; partly from all the measurements which astronomers or geometers have found by their measuring of the distances of comets, as in the table given above, for no one has obtained a distance in comets less than one semidiameter of the Earth; and partly, finally, from the stay of many comets

above the physical horizon, and from the small difference in their distances from the same fixed stars observed on the same night at different hours, as is plain from the arguments adduced in chapters 18, 19, 20, 21 and 22 to demonstrate that the parallax of some comets is smaller than the Moon's. For although those arguments do not demonstrate this absolutely, they do demonstrate, with at least physical certainty, that the comets could not have been so low that their distance was less than a thirtieth or a fortieth part of the Earth's semidiameter, which is the greatest height of the vapours that can be granted at most; otherwise they would have had a parallax of more than 35 or 40 degrees, that is, one perceptible without any instruments. Nor, in the comets observed from the year 1577 to 1618, can so great a leap in the comets' proper motion be feigned with any probability as to steal so enormous a parallax from the senses, by moving the comet towards the vertex; which will be manifest to anyone who considers the apparent motion of those comets in the tables shown in chapter 3.

[Margin: The author's final opinion.]

I add, moreover, that it is much more probable that the comets of the years 1577, 1585 and 1618 were, if not above the Moon, at least a little below the Moon. Therefore I judge that the region of very many comets is above our air, and that the whole space attributed to elemental fire, together with the whole ethereal air up to the fixed stars, is the field in which Divine Providence, at its own discretion, displays new phenomena of this kind to the eyes of mortals at certain times. There is therefore no reason why either the Peripatetics should insult the Tychonics, or the Tychonics the Peripatetics, with as much haughtiness and swagger as you see some have done. Rather, it would be fair from now on for each side to speak of the other with greater moderation, and in future to rely on more certain observations and demonstrations — if, however, there are any who, when new comets come to light, remember either our labours or our admonitions.

CAPVT XXIV.

Chapter 24.

De Vera Quantitate Capitis, & Caudæ Cometarum ex Obſeruationibus determinanda.

On determining the true size of the head and tail of comets from observations.

Problema 1. Capitis Cometici Magnitudinem veram inueſtigare.

Problem 1. To investigate the true magnitude of a comet's head.

I. To determine the size of a comet's head geometrically, three things must be assumed. The *first* is the apparent diameter of the disc of comets, which for the most part appear round, like stars — which is also why they have been

called "hairy stars" [stellae comatae]. But to measure this very diameter with the astronomical radius is most deceptive, and I am surprised that Father Cysat, chapters 1 and 4, observed the head of the comet of 1618 in this way, since he could have known how far Tycho, in the *Mechanica Astronomia*, and in volume 1 of the *Progymnasmata*, pages 247, 335, 343 and 671, and volume 2, page 31, had taught that this instrument is not to be trusted. But I am still more surprised that the same Cysat, chapter 5, on that comet, adopted the diameter of 7′ observed with the astronomical radius, and from it demonstrated the true magnitude, rather than the diameter of 8′ or 16′ observed with the telescope — not that I think this latter was itself exact, but because the telescope shows a more exact magnitude of the stars by rescuing it from the deception of the eyes. By what method the apparent diameters of the stars and of the lesser planets are to be observed, we have taught fully enough in Book 7, Section 6, chapter 9. All of this is to be understood here of the head of comets.

[Margin: 1. The supposition for measuring the head.]

[Margin: Father Cysat's observation of a comet's head is deceptive.]

Second, the distance of the comet's head from the centre of the Earth must be assumed; how uncertain it is, is plain enough from what has been said so far, from chapter 18 to chapter 23. Granting it, however, consider the triangle GBE, right-angled at E: for the straight line GE, drawn from the Earth's centre G and touching the comet's globe at E, makes with the globe's semidiameter BE a right angle at E, by proposition 18 of the third book of Euclid's *Elements*. In a triangle of this kind the base BG is given, namely the distance of the comet's head from the Earth, and so is the angle BGE, under which the comet's apparent semidiameter is seen, since the whole angle DGE measures the apparent diameter. Therefore, by the laws of triangles, the true semidiameter BE will not remain unknown, in the parts in which BG was known.

[Margin: 2. Supposition.]

Diagram. In the left column: a circle representing the comet's globe, with two straight lines rising from G at the foot and touching it on either side, and a vertical line from G through the circle up to H. A horizontal arc crosses the circle, with A and D at its left end and E and C at its right; F stands at the lower edge of the circle and B within it near the top.

- H (top): the upper end of the vertical line
- B (inside the circle, near the top)
- A, D (left end of the cross arc)
- E, C (right end of the cross arc)
- F (lower edge of the circle)
- G (foot): the centre of the Earth, where the tangent lines meet

Third, the comet's head is supposed to be spherical, or nearly spherical, by Tycho, volume 2, chapter 9; by Cysat, chapter 4; by Longomontanus in the Appendix to the *Astronomia*, chapter 10; by Snel on the comet of 1618, chapter 6; and by most others. With these posited, the threefold magnitude of comets

can be found by the rules we have given on the measurement of the circle and the sphere in Book 1 of this *Almagest*, chapters 4 and 6. But it is very uncertain whether a comet's head is spherical, and whether it is a single continuous body, as is plain from the observation of the comet of 1618 made with the telescope by Father Cysat and by Habrecht, which I reported in chapter 3, at the end of the history of that year's comet, before the first table.

[Margin: 3. Supposition.]

Problema 2. Caudæ Cometicæ Magnitudinem veram inueſtigare.

Problem 2. To investigate the true magnitude of a comet's tail.

II. The magnitude of the tail is much more uncertain than that of the head, because of the inconstant vibration and ejaculation of the rays, even on the same night — as was observed most recently, above all in the comet of 1618, and recounted by us in the history of that year. If, however, the tail's size is to be inferred from some determinate seen quantity of it, three things must be given: namely the distance of the comet's head and the distance of the end of the tail from the centre of the Earth, and then the apparent distance of the head from the end of the tail. For with these given, the true length of the tail will easily be found — and very easily indeed if the tail appears crosswise, so that the head and the end of the tail are equally distant from the eye. For in the triangle ADE, in which the head is D and the end of the tail E, and the given distances from the Earth's centre A are AD and AE, if the angle DAE is also given, measuring the arc DE — that is, the seen distance of the head from the end of the tail — then the chord DCE will certainly be found, in the parts in which AD and AE are known; and it will not differ much from the curvature of the tail DBE, if the tail is not enormously curved. As for how the distance is corrected from the seen distances from the vertex of both the head and the end of the tail, and from the parallaxes owed to the distance of each from the Earth — or the parallaxes from the distances — gather it from Chiaramonti's problem, expounded by us in chapter 19, number 2.

Diagram. In the right column: an arc DBE above a straight chord DCE, with straight lines from A at the foot to D and to E, and a line from B through C down to A.

- B (top): the midpoint of the arc of the tail
- D (left): the comet's head
- E (right): the end of the tail
- C (on the chord, below B)
- A (foot): the centre of the Earth

Problema 3. Longitudinem ac deinde Craſſitiem Caudæ Cometæ, Regiomontani methodo metiri.

Problem 3. To measure the length, and then the thickness, of a comet's tail by Regiomontanus's method.

III. Regiomontanus supposes, with the Peripatetics, that the tail of a comet is a flame, and so by its lightness tends upward in a straight line standing perpendicularly [?] on the Earth. On this assumption, in his treatise on the comet, Problem 15, he directs that from the Earth's centre A a circle be described, or at least a quadrant DE of the Earth, and another, BHC, in which the tip of the comet's tail ends. For he draws from A, through the comet's head F, the straight line AFH, so that FH represents the tail; and he draws from the eye D the visual lines DFG, through the head, and DH, through the end of the tail. He supposes three things given: *first*, the straight line DF, which can be found in the triangle ADF, given the distance of the head AF, the Earth's semidiameter AD, and the angle of parallax AFD; *second*, from the parallax AFD, the angle DFH, as being its supplement to two right angles; *third*, the angle HDF, which measures the apparent length of the tail. From these three, in the triangle DFH, he directs that the true length of the tail FH be sought.

Diagram. In the right column: a quadrant centred on A (lower left), with B at the top of its vertical radius and C at the end of its horizontal radius; a small inner quadrant from D down to E; H and G on the outer arc; F on the line from A towards H; lines from D to G and to H.

- B (top left): top of the vertical radius
- H (outer arc, upper): the tip of the tail
- G (outer arc, lower)
- D (left, on the vertical radius): the eye
- F (on the line AH): the comet's head
- A (lower left): the centre of the Earth
- E (foot, on the horizontal radius): end of the inner quadrant
- C (foot, right): end of the horizontal radius

Afterwards, in Problem 16, he teaches how to find the thickness of the tail: for if the tail is cylindrical, he directs that the true diameter

[Margin: To inquire into the thickness of the tail.]

of the comet's head be multiplied by itself, so that a square number is obtained; this he directs to be multiplied by the length of the tail, so that a square column that can be circumscribed about such a cylinder is obtained. Let it be, then, as 14 to 11, so the number of the column already mentioned to another, and the thickness of the cylinder will result. But if the tail is conical, he directs that a third part of the cylinder already mentioned be taken, because, from Archimedes, a cylinder is triple the cone inscribed in it. See an example of this calculation in chapter 11, number 15.

But if the head and the end of the tail are not equally distant from the Earth, and the distance of the end is not given but only that of the head, and the

comet's tail does not stand perpendicular to the Earth, its length will have to be investigated otherwise. Namely, if the distance from the Earth of the Sun, or of Venus or Mercury, is given — according as the comet's tail is supposed to be turned away from the Sun, or from Venus, or from Mercury — and if the apparent length of the tail is also given by observation, the true magnitude of the tail can still easily be investigated by another route. Before I teach this, I should like to set out first this problem from our Cysat, which is the third in his chapter 2.

Problema 4. Diſtantiam ſeu Digreſſionem Cometæ à Sole inquirere.

Problem 4. To find the distance, or digression, of a comet from the Sun.

IV. Let A be the pole of the ecliptic GEK, in which the Sun's place at a given moment of time is given as D; and let the longitude of the comet B be given, and also its latitude BE. Now let the portion BD of a great circle be drawn through the Sun and the comet, measuring the comet's digression from the Sun. For in the spherical triangle BDE, right-angled at E, the sides BE, the comet's latitude, and DE, the difference between the places of the Sun and the comet in the ecliptic, will be given; therefore, by the rules of triangles, the comet's digression from the Sun will become known — the Sun being that from which it is more probable that the tails of comets are turned away.

Diagram. In the left column: a circle with A, the pole of the ecliptic, at the top. A straight line GK across its lower part represents the ecliptic, with F, E and D just below it; arcs run from A down to E and from A through C to F; B, the comet, stands near the centre, on an arc rising from D.

- A (top): pole of the ecliptic
- C (upper left of centre)
- B (centre): the comet
- G (left end of the ecliptic)
- F (below the ecliptic, left)
- E (below the ecliptic, centre): foot of the comet's latitude BE
- D (below the ecliptic, right): the Sun's place
- K (right end of the ecliptic)

Problema 5. Veram Longitudinem Caudæ Cometæ aliter ac in Problemate 3. inueſtigare.

Problem 5. To investigate the true length of a comet's tail otherwise than in Problem 3.

V. This problem is Tycho's, volume 2, page 203, although there he uses the comet's digression from Venus; and it is Father Cysat's, on the comet of 1618, chapter 5. But we shall use Longomontanus's figure from his Appendix to the *Astronomia*, chapter 6, since it represents comets in various places with the tail turned away from the Sun.

Diagram. In the left column, across its full width: concentric circles about the Earth's centre A, with small circles for comets' heads and narrow cones for their tails pointing away from the Sun S. Heads are marked I (top), C (upper right), N (left), L and R (inside); a further head at the upper left carries a letter too small to settle on the scan. Tails run out to D, B, G, M, H and P. Lines join S to the heads.

- D (top): end of the tail of comet I
- I (top): a comet above the Sun
- B (upper right): end of the tail of comet C
- G (upper left): end of a tail
- [F?] (upper left, inside a small circle): a comet's head; letter not settled
- C (upper right): a comet above the Sun
- L (inside, upper left of A): a comet below the Sun
- H (left of L): end of the tail of comet L
- M (far left): end of the tail of comet N
- N (left): a comet above the Sun
- A (centre): the centre of the Earth
- S (right of A): the Sun
- R (lower right of A): a comet below the Sun
- P (foot): end of the tail of comet R

From the Earth's centre A, then, let the Sun's orbit SH be described, in which the Sun is at S; and a lower orbit LR, for comets below the Sun — for instance for the comet L, with tail LH, and the comet R, with tail PR. But for comets above the Sun let the circle NFI be described from A, in which there is a comet at C, with tail B; or a comet I, with tail ID; or a comet F, with tail FG; or a comet N, with tail MN. And let the heads of the comets be joined by straight lines to the Sun and to the centre of the Earth, and the ends of the tails to the centre of the Earth. Now let the comet be at I. *First*, in the triangle ASI, the angle SAI is given by Problem 4, namely the angle measuring the comet's digression from the Sun; the distances of both the Sun and the comet from the centre of the Earth, namely the sides AS and AI, are likewise supposed given. Find, therefore, the angle AIS, which is called the angle at the comet; for its supplement to two right angles will be the angle AID in the triangle AID, in which, further, the comet's distance from the centre of the Earth, AI, and the angle IAD, measuring the seen length of the tail, are supposed given. Therefore, by solving this triangle, the true length of the tail ID will become known. One must then proceed similarly in the other cases.

[Margin: 1. Step.]

[Margin: 2. Step.]

VI. What can be asked here about the apparent and true magnitude of the head and tail of comets has been set out by us sufficiently for the reader's instruction in chapter 4, where we treated the properties of comets gathered together from the history of observations: namely in number 6, on the apparent magnitude of the head; in number 7, on the apparent magnitude of the tail; and

in number 8, on the true magnitude of head and tail. Let the reader therefore turn back there.

[Margin: Examples of the true magnitude of comets.]

CAPVT XXV.

Chapter 25.

De Situ Caudæ Cometarum & Cauſis eius Phyſicis, & Opticis.

On the position of the tail of comets, and its physical and optical causes.

I. In chapter 4, number 9, we have already shown from observations that the tail of comets properly so called for the most part habitually points, continually, towards the quarter directly turned away from the Sun — granted that at times, because of nearness of position, it has been seen to go rather into the quarter opposite Venus or Mercury, and that some comet has been seen to stretch its tail right down to the Earth. What we said there, then, is to be recalled here; and, with it supposed, we must inquire what cause — whether physical, or optical, or mixed from both — accounts for that tail being seen to flash with so great a variety of shape and length, and with such inconstancy in the vibration and ejaculation of its rays, especially in the comet of 1618, on December 9, 10, 14 and 24. Here, therefore, we must investigate the cause of its turning away from the Sun, of its curvature, of its expansion or dilatation — at times, indeed, into the shape of a broom — and of its alternate lengthening and contraction, or vibration. For these four phenomena cause no small trouble, and one can scarcely find anyone who has brought forward a sufficient cause for all of them.

II. The *first* opinion was that of Aristotle, *Meteorologica* Book 1, chapter 7, and of his followers and interpreters on that passage, especially the Coimbra commentators: namely, that the tail or coma of comets is a kindled exhalation, or flame, but in a matter rarer and less compacted than the matter of the head; and that the figure of the tail — or coma, or tresses, or beard — differs according to the different disposition of the matter that feeds this flame. An argument for this is that the tail appears to us like a flame driven and tossed in one direction by the wind; Cardano uses this comparison for the comet of 1556, saying that its tresses glowed red as in a conflagration, as in torches when the wind blows on them. However, a flame tends upward by its natural lightness — and for that reason Regiomontanus, problem 15 of *De Cometa*, supposes the tail erected perpendicularly upward — whereas the tail or coma of a comet tends always, or for the most part, towards the side turned away from the Sun, and is not erected upward unless the Sun, below the horizon, stands so near the meridian that a straight line drawn from the Sun through the comet's head passes very nearly through the centre of the Earth; and it is established from the history of comets, observed more accurately in this century and the preceding one, that this rarely happens. The more recent Peripatetics therefore had, in order to sustain the ancient opinion, to devise a reason why

[Margin: The Peripatetics' opinion on flame.]

that flaming coma is scarcely ever raised upward. To resort to a wind blowing from the opposite side and driving the flame away from the Sun was to resort to an aerial contrivance devoid of all likelihood. For who would grant that, for whole months, during which comets often last and twinkle, a wind always blows from the quarter of the world in which the Sun is; that the source of the wind, so to speak, rises and sets with the Sun at an equal pace; and that it is carried by its impulse directly towards the part where the comet's head is — and this so constantly, and in so many, indeed in almost all, comets? And yet that this wind, raging so persistently in that direction, is not felt by us? Nor does it seem likely that the fuel and matter of the flame always lies along that line directly opposite the Sun, backward behind the comet's back or occiput, so that ever new matter of similar disposition, continuous with the earlier, is supplied to the flame for the whole time that the comet's tail lasts turned away from the Sun, and that it is never found around the head, above, below, or in front. Therefore, although flame does run to seize its fuel even downward — as when the smoke of a freshly extinguished candle is placed beneath the flame of another candle or lamp, it at once rekindles, and something similar happens in those fiery impressions we call falling stars or leaping goats — this still cannot rightly be applied to the tails of comets.

[Margin: A wind blowing from the opposite side is excluded.]

[Margin: A running-out to fuel is excluded.]

Carolus Piso, then, in his *Speculum Cometae*, chapter 17, granted that hairy comets are indeed in the highest region of the air, but that tailed or bearded [?] ones are in the concave of the lunar heaven, and that the flame of the coma therefore cannot be raised upward, because it is impeded by the solidity of the heaven already mentioned and is forced to bend crosswise — just as the flame of our own fire, shut in a furnace, is carried to the sides and takes a transverse position. He nevertheless also adds the motion of the comet, swept along by the motion of the primum mobile, as though the flame were bent in this way, as in torches carried by a runner; on which account that flame would have to be bent always towards the east, which is false. But since he was certain, both from his own observation and from that of Father Jean Leurechon, that the tail of the comet of 1618 had flowed continually into the quarter turned away from the Sun, he had therefore said in chapter 14 that the flame is indeed also in the front part, between the Sun and the comet's head, but is not seen, being obliterated and darkened by the greater brightness of the Sun. For it has been observed that at full Moon, because of the presence of the greater light, the comet's tail appeared much diminished, just as when the Sun approaches the horizon; and so the part turned away from the Sun, which the Sun's rays cannot reach because the comet's head stands in the way, appeared bright. As for why that flame does not end in a cone, as it would have to if the comet's head is smaller than the Sun and casts a conical shadow that guards the flame and the light, he thinks the cause is the great distance from the Sun — and so

he entangles himself in the difficulty he had foreseen rather than resolving it. The remaining things said against Piso, recall from chapter 11, number 2.

[Margin: Carolus Piso's opinion.]

Cottunius has something similar, in lecture 33 on Book 1 of the *Meteorologica*. For after he had said: *But the tail of comets, to Aristotle's ears, is the thinner, rarer and more spirituous part of the same exhalation in which the comet itself burns; and this part, lying next to the comet's head as to a denser matter, does not shine as the head does, because of its own rarity* — he then inquires why the tail of a comet always inclines towards the parts opposite the Sun, and says this happens because its light, being feeble as it is in rare matter, cannot be seen except in shadow. The part of the comet that faces the Sun, he says, is indeed inflamed and apt to shine of itself, but its light is overwhelmed by the Sun; whereas the part turned away from the Sun shines for us, in the shadow that the dense and opaque head of the comet casts. Finally, he refers the cause why that flame does not appear conical, as the comet's shadow is, to the brightness of the comet's head, which itself also overwhelms and washes out the parts of the tail nearer the head, but not likewise the more remote ones.

[Margin: Cottunius's opinion.]

Mastri and Belluto, on the other hand, disputation 4 *De caelo*, question 2, article 4, deny that the comet casts a shadow, and say that it shines on both sides — sometimes even on the part facing the Sun, because of the density of the kindled matter, as in hairy stars; but that this rarely happens, because the rarer matter and the feebler light are overwhelmed by the Sun's brightness, which does not happen on the part turned away from the Sun. Gemma, for his part, said in chapter 82 of the *Astrolabium* that it is fire in the comet's head, kindled by the Sun and expelled straight out by the Sun's violent action.

[Margin: The opinion of Mastri, Belluto and Gemma.]

III. But the opinion of these men does not seem one to be embraced. For, to say nothing of the solidity of the lunar heaven — less probable in this century — to which Piso attributes the impediment to the flame's rising: if only that part of the flame spread around the comet shines for us which is enclosed within the cone of shadow produced by the head, and is protected from the greater brightness, then it could never appear wider than the head itself, however far it were from the head, as the laws of optics require. Yet it often appears wider than the head itself: thus the tail of the comet of 1618 was seen on December 7 to be 2½° wide, and at times 3½°, although the head, even viewed with the telescope, never exceeded 16′. Next, either the back of the comet's head shines with its own light, or it is dark of itself. If it shines, as Cottunius would have it, it would not only make the lamp of the tail narrow but would cut it off from us entirely, and would cast no shadow; if, on the other hand, it is dark, it would not diminish the width of that flame. A reason therefore still remains to be given why the tail is sometimes seen so wide that it appears much wider than the head. And if it is true that the tail is sometimes so curved that a straight line drawn from the Sun to its convexity, or to the end of the bent tail, does

not pass through the comet's head, this difficulty certainly cannot be met on the hypothesis of these Peripatetics. Nor has Gemma ever explained what that violent action is which drives that flame in the opposite direction and does not do so in our own flames. And to resort to a hidden antipathy of that flame or light towards the Sun's rays, from which they flee away — as Franciscus Fernandez does, in Resta, *Meteorologia* Book 1, chapter 5 — is to resort to a contrivance and to the mysteries of nature, not to say to the dreams of those who philosophise arbitrarily.

[Margin: The preceding opinion is disproved.]

[Margin: Fernandez's opinion.]

IV. The *second* opinion was Seneca's, who held that the coma of a comet is not part of the comet, nor its flame, but rays. For in Book 7 of the *Natural Questions*, chapter 26, to someone objecting that a comet cannot be a celestial star because of its rarity, he says this: *Through the stars, he says, we do not discern what lies beyond; through comets we send our sight. First, if that is so, it happens not in the part where the star itself is of dense and solid fire, but where a rare splendour runs out and is scattered into tresses: you see through the intervals of the fires, not through the fires themselves.* For he holds that comets are stars appearing in the heaven from time to time, and that stars are fires, but eternal ones. And soon after, meeting another objection, taken from their shape, he says: *All stars, he says, are round; comets are stretched out; from which it is apparent that they are not stars. But who would grant you that comets are long? Their nature, like that of the other heavenly bodies, is a globe; but their brightness is extended, just as the Sun sends out its rays far and wide; but the Sun itself has one form, and the light that flows from it another; so the body of comets is itself rounded, but their splendour appears longer than that of the other heavenly bodies.* As for why the splendour of comets reaches farther than that of the other heavenly bodies, in chapter 27 he refers it to their very nature and to causes hidden from us. But surely, since most comets are of fainter light than the other stars, it is not very probable that comets emit so long a light while the stars emit none at all. Moreover, he gives no reason why comets shine towards the part opposite the Sun, nor what that matter is on which the rays of the head strike and are reflected to us; for light is not visible unless it is bounded by some opaque body. Nor, again, the old opinion of Rothmann, who said that the tail is part of the comet's body, of the same substance, and shining with its own light; for, as Tycho objects to him in the *Epistolae*, page 143, there would be no reason why it should always shine turned away from the Sun.

[Margin: Seneca's opinion.]

[Margin: Rothmann's opinion is improbable.]

V. The *third* opinion is that the coma or tail of a comet is the Sun's rays transmitted through the semi-transparent head of the comet as through a glass globe; an opinion which, however, has sprouted into various ways of expounding it. Peter Apian — who, just as he was the first of all to observe that constant turning of the comet's tail away from the Sun, so seems to have been

the first to refer the tail of comets not to the substance of the comet but to the rays of the Sun — said in the *Astronomicum Caesareum*, part 2, that their tail is rather the light of the Sun transmitted through their transparent head, and that comets are therefore much higher than the vapours that produce twilight; otherwise they would run into the shadow of the Earth and could not be lit by the Sun. But Philipp Apian, his son, in a letter written in 1572, December 6, to the Landgrave

[Margin: Peter Apian's opinion.]

[Margin: Philipp Apian's opinion.]

of Hesse, which Tycho reports in volume 1 of the *Progymnasmata*, page 643, held that comets are generated from the concourse of the Sun and the other planets, breaking against one another, and from their meeting in a definite part of the heaven — but chiefly from the Sun, since the tail of comets stretches out towards the quarter turned away from the Sun. Girolamo Cardano, in Book 4 of *De subtilitate*, teaches *that a comet is a globe set in the heaven, which appears lit by the Sun; and while the rays pass through, they form the likeness of a beard or a tail.*

[Margin: Cardano's opinion.]

Nor does Scaliger oppose him on this point, in *Exercitation* 79, although he would have the comet be below the Moon and an elementary body. His words are these: *For a comet is a body so dense that it can be illuminated, and so rare that it can transmit rays — which we also see in a glass globe. But that it is not in the region of vapours is false; it is far more remote from the truth that it is in the heaven.* Telesius, Tassonus and Aversa, question 42, section 1, judge the opinion of these men probable.

[Margin: The opinion of Julius Caesar Scaliger.]

Tycho followed these men. In his letter of 1589 February 21 to Rothmann, in that of 1590, on the Kalends of April, to the Landgrave of Hesse, and in volume 2 of the *Progymnasmata*, chapters 7 and 9, he at length concedes that the tail of a comet is turned away from the Sun rather, or more principally, than from Venus, and that it shines from the Sun's rays. Therefore, when comets are carried into the part opposite the Sun — that is, into the part such that our eyes stand between it and the Sun — they show no tail, or a very small one, because the tail then lies hidden wholly or almost wholly behind the comet's back; whereas when they head towards conjunction with the Sun, they put forth their beard more and more, as he relates happened with the comets of the years 1580, 1582 and 1585. The efflux of the tail, then, is the rays of the Sun penetrating the comet's head, bounded in some matter that is not wholly transparent, and reflected to us; for the substance of the aether is not wholly diaphanous, since the highest heaven appears blue to us because of some opacity after all. As for the cause why the tail of the comet of 1577 appeared curved, he says it is an aberration of sight, because the end of the tail, straight in itself, was nearer to us than the head, or beginning, of the tail. For it has been demonstrated by Ibn al-Haytham, *Optics* Book 2, proposition 28, by Witelo, *Optics* Book 4, theorem 31, and by Io. Pisanus the Englishman [John Pecham], *Perspectiva*, tract

1, proposition 69, that when sight takes in the extremes of some visible thing, whether a line or a surface, from differing distances, it appears oblique even if it is straight in itself. Biancani, in his *Sphaera*, Book 16, chapter 2, subscribed to Tycho, although he raises the difficulty about the curvature and breadth of the tail and does not solve it.

[Margin: Tycho's opinion.]

[Margin: Biancani's opinion.]

But we cannot wholly rest content with Tycho. He gives no reason for the tail being sometimes spread out into the shape of a basket or a clothes-brush; nor can that explanation by optical curvature be fitted to tails whose ends were seen to be farther from the Earth than their heads, as happened in the comet of 1618 and in many others. And that very theorem of Witelo, or proposition of Ibn al-Haytham, not only is not to the point Tycho is making, but actually fights against Tycho. For those two lights of optics say there that our sight distinguishes straightness from obliquity in lines and surfaces when they stand at so small a distance from the eye that it can perceive the differing distance of the extremities; but when the distance is so great that it cannot perceive the more remote of the extremities, it does not distinguish oblique things from straight ones — indeed it judges all things, however oblique, to be straight. This is the cause why the globe of the Sun and of the Moon appear to us as a flat disc. Since, then, for Tycho comets are higher than the Moon, the eye, because of the enormous distance, ought rather to judge their tail straight if it were curved and oblique, than to judge it curved if it were straight. Moreover, Tycho, as well as others who use the comparison of a glass globe or phial struck by the Sun's rays, ought to have explained why the Sun's rays passing through the comet's head do not always come together into a cone, as they come together behind a ball or a glass phial. Let us pass on, then, to Tycho's pupils.

[Margin: Tycho's opinion is refuted.]

VI. The *fourth* opinion was that of Johannes Kepler, who earlier, in the *Optica Astronomia*, chapter 6, number 13, or from page 264 on, tried in two ways to untangle the knots of this tail. He premises that the tail is scattered into the quarter contrary to the Sun, and is not hidden except when our eye is between the Sun and the comet's head opposite the Sun, or when the distance of the comets is immense; and that it is accordingly either kindled or illuminated by the Sun. Then he poses these doubts: *Why do the tails not look precisely towards the point opposite the Sun? Why do they almost always decline? Why are they curved into an arc? Finally, what is that which receives the Sun's rays and shines back by their impact, displaying the figure of a tail? For if you say it is matter belonging to the comet's essence, you will have fashioned a most monstrous prodigy; if the aethereal air, I ask why it is not lit by the Sun every day, so as to shine back, even without a comet?* Nor does he promise there a complete solution of these knots, but an attempt; for he says: *Since I cannot do as much as I wish, let me be allowed, then, to do as much as I can.*

[Margin: Kepler's opinion.]

He therefore supposes that the cause both of the curvature and of the declination from direct opposition to the Sun is the same, but that neither can come from parallax, since in chapter 9 of the same *Optics* he shows that parallax cannot represent curved things out of straight ones; recourse must therefore be had to refraction. He therefore judges it probable that the comet's head is not a bubble hollow within, as he had once supposed, but a solid globe, consisting throughout of the same matter — not hard like glass, indeed, since it is gradually dissolved, but nevertheless of such density, and so transparent, that, like a glass globe, it gathers a great many rays by a double refraction, one on entry and the other on exit, into one point behind the globe's back; and these rays, as the necessity of optics or dioptrics requires, are then dispersed and spread apart beyond that point, diverging again into contrary quarters. And since the aethereal air is not free of all opacity, if the rays, multiplied and coloured by their passage through the comet's head, strike upon it, it can be so strongly painted and illuminated from so near a place that it can shine for us from afar — granted that it cannot when the rays are not gathered together. That the heads and tails of comets radiate very strongly, he proves from the comet described by ʿAlī ibn Riḍwān, which was three times larger than Venus and of as much light as a quarter of the Moon; and from the comet that foretold the rise and death of Mithridates and, as Justin relates in Book 37, shone for seventy days in such a way that the whole heaven seemed to blaze, and occupied a fourth part of the heaven, so that four hours were required for it to rise and to set entire with its tail, and it outdid the brilliance of the Sun with its glare: see what we said in the history, chapter 3, at the year 119 before CHRIST. Since, however, the rays gathered into one beyond the point of common intersection spread apart, and decline from the straight line drawn through the centres of the Sun and of the comet's head, it comes about that the tail appears curved. With these things laid down, he says: *Grant now that one side of the head, by reason either of its figure or of its density, does not transmit the Sun's rays: then that part of the tail will be cut off, and the remainder, declining to the other side, will alone display the appearance of the whole tail, and therefore of a declining one. Since also, when the disposition of the comet's body or its situation relative to the Sun is changed, this inclination can be changed; as in 1596 the tail at first declined to the right, then fell in with the Sun's very diameter, and finally passed over to the left.*

[Margin: 1st mode.]

And this is Kepler's first mode. The second is that some matter proper to the comet be granted, breathing out from the comet's head and, as it were, expelled by the Sun into the opposite part, into which the Sun's rays are gathered from the head by the force of refraction, strongly illuminate such matter, and diverge beyond the point of intersection, etc. If this is granted, it is no wonder if a comet's tail sometimes touches the Earth and infects the air with poison. And the curvature of the tail in this second case can come from the position of the very matter expelled from the head.

[Margin: 2nd mode.]

VII. But to aid with a diagram the imagination of the reader, who is perhaps not well enough instructed in this part of dioptrics: in the following figure let ABCD be a glass globe, or rather the comet's head, through whose centre passes the Sun's unrefracted ray E F, issuing from the centre of the Sun (to be understood here as lying towards E). Now since rays radiating from one and the same luminous point onto a surface at a notable distance fall on it sensibly parallel to one another, as we partly demonstrated in Book 3, chapter 5, and more fully in our book on optics, let the Sun's rays radiating onto the convex surface of the globe already mentioned be GH, NO, SA, VC. First, then, let the ray GH radiate to the point H and there, because of the density of the transparent globe, be refracted towards the perpendicular εS to the point L; and again on exit, since it passes from a denser medium into a rarer, let it diverge from the perpendicular εf and be refracted to the point I, continuing on as far as M. In the same way let the ray NO, refracted at O towards P, again be refracted to the point I, tending as far as R. Likewise let the ray SA, refracted on entry at the point A and again on exit at the point B, come together at the point Q, proceeding from there to the point T; and finally let the ray VC, refracted once at C and again at D, come together at Q, tending towards X.

[Margin: Kepler's doctrine is illustrated by our own experiments.]

Diagram. A woodcut set in the left column: Kepler's refracting globe ABCD, with parallel rays of the Sun entering from the left and crossing beyond the globe, where a field of vertical dotted lines stands for the matter that receives them.

- S, G, E, N, V (left, top to bottom): the starting points of the parallel rays; the ray from E passes through the centre ε and out to F.
- A (top left of the circle), B (top right), C (bottom), D (bottom right): the points of the globe ABCD.
- L (just inside B), H (inside the circle, left, on the ray from G), β (left rim, on the diameter), ε (centre), μ (right of centre), γ (right rim), o (small, inside the circle lower left), P (inside the circle lower right).
- δ (Greek, at the head of the first dotted line, on the line rising from H through ε): the point of the text's perpendicular "εf" (p. 151 line 154, where the sort after ε is read f; not settled against δ).
- I (the first crossing of the rays), V (the upper crossing), a (the middle crossing), Q (the third crossing).
- [?] (a script sort below a; the text names a point Z in "DZM").
- R, X, F, T, M (right, top to bottom): the ends of the rays.
- [?] (a small mark near the foot of the dotted lines).

And in the same way let several other rays be gathered by the force of a similar refraction at the same points I or Q, or as near to them as possible, but most of them at the point *a*. For they will have the power not only of illuminating most strongly, but also of burning combustible matter lying at the point I or Q, because of the union of so many rays interpenetrating in the same place and receiving a formal or equivalent intension of their power in the same

part of space. We have found by experiment (to say this in passing) that there are three points at which burning takes place with crystalline spheres, glass ones, and glass-and-water ones — that is, glass but filled with water: one nearer, at I, and another more remote, at Q, at which the illumination and inflammation are slower and weaker; the third midway between these, at which it is very rapid and very strong. We therefore call this burning cone the primary one, but the cones PIL and BQD secondary. See their measures in the following little table, in parts of which one inch of the Roman foot contains a hundred, for the different sizes of the glass spheres we used.

It will be	If the semidiameter εγ of the sphere is, in parts:		
	334	444	600
γI	60	80	108
Axis μI	87	116	156
γQ	198	264	357
Axis μQ	225	300	405
γ*a*	124	164	221
Axis μ*a*	151	200	269
Base of the cone BD	124	164	221
The portion BγD containing the refracted rays is very nearly 50°.			

[Margin: The distance of the burning point beyond the glass sphere from the sphere, and the height of the axis of the burning cone.]

Moreover, we have found by experiment that if the experiments are made in a dark room, admitting the Sun's light through a hole equal to the diameter of the sphere, and behind the back of the glass globe the usual motes or atoms rise, or dust is scattered downward or thrown upward, or the steam of boiling water is put beneath, the rays appear, and a very great light in the part of space IQ, but a fainter [?] light in the farther part, lit by the rays IR, QX, QT, IM, already dispersed. Kepler therefore judges that the curvature of the tail arises from the figure DZM or BYR; but that if only half of the globe — say βAγ — transmits the Sun's rays, while the other half, βCγ, does not transmit them because of excessive opacity, nothing of the comet's tail appears to us but half, namely the portion BL*a*TM. This will therefore appear curved, because the rays emerging [?] between B and L are so refracted that they cut the straight line LM at one place and another, and LM itself is not the boundary of the tail on its lower side. He thinks, however, that the curvature of the tail can also come from the very matter breathing out, or expelled from the comet's head by the force of the Sun's rays, disposed in position so as to recede and decline from the line EF.

VIII. So far Kepler in the *Optics*, his judgement still in suspense; but afterwards, in the *Opuscula* on the comets of the years 1607 and 1618, Book 2, from page 101 on, he chose the second mode. For he denies *that the Sun's rays are*

seen in the pure aethereal air that is behind the comet; and in the air immediately surrounding it they are not seen unless there is some matter behind the comet onto which the Sun's rays, passing through the comet, strike. For neither with us can a ray of the Sun be seen except in matter that receives the Sun's ray — on walls, for instance, on clothes, on the surfaces of waters, on the earth, mountains, clouds, or in thick air. Next, it is established that the tails of certain comets are curved, but it is impossible for the Sun's rays to be curved in the free aethereal air; for in optics we are taught that the rays of luminous bodies are not arcs but straight lines. Since, then, it is established that the comet's tail is formed by the Sun's rays, it is consistent that the tails or tresses of comets have their own matter, out of the comet's body, expelled by the Sun's rays. He had said a little before: *As the Sun's straight rays strike and penetrate this transparent globe, I think that something of the comet's innermost matter follows and goes out by the same way by which the Sun's rays break through; and that in this manner the comet's body is washed through, strained, worn away and finally annihilated; and that, as silkworms by spinning out their thread, so comets are consumed by breathing out their tail, and finally die.*

[Margin: Kepler's later view.]

Given, then, a continual efflux of the matter already mentioned, it can be disposed in various ways, so that the tail appears now curved, now humped, now long, now short, and at times flickering with a sudden vibration like chasmata or fiery rods, as in the comets of the years 1577, 1607 and 1618 — a tail which, in the blink of an eye, from shorter appeared very long, by an alternate ejaculation of rays. Kepler testifies in the *Hyperaspistes*, pages 125 and 199, that this same second opinion — of a real efflux from the comet's head, or an expulsion made by the Sun's rays — seems to him the more probable; there he thinks not improbable the cause of a certain curvature adduced by Guiducci and Galileo, of which we shall speak below. And Longomontanus wholly subscribes to this second Keplerian opinion, in the Appendix to the *Astronomia Danica*, chapter 6, so that there is no need to report his view in other words.

[Margin: Longomontanus's opinion [?].]

IX. But although Kepler's opinion has much ingenuity, it nevertheless runs into many other difficulties. Those metaphorical words — straining, washing through, wearing away — do not sufficiently explain the Sun's proper action in this business, still less does the comparison with silkworms. For the Sun's rays are not bodies that would burst into the heads of comets with a violent local motion; our own heads, exposed to the Sun, would feel such a motion. It remains, then, that the Sun accomplishes this by an alterative action — heating and drying, and so thinning those parts of the head that are more disposed to it, and making them lighter and rarer. But if that happens, those parts will then be carried upward, not to the sides, and we shall be thrown back into the difficulty about the Aristotelian flame of which we spoke in number 2. Or, if it is granted that the matter does not at once ascend but is carried behind the comet with an oblique motion, the rays coming together at the points I,

a, Q by the force of refraction will at once dissipate and consume it; so that a little beyond there the tail will appear cut off. Or, if the matter is apt to resist inflammation, though not the most vehement illumination, then the tail ought to appear much more brilliant in that part than in the rest. But it is established that these phenomena have not yet been observed in comets; rather, the tail is continuous, and either uniformly bright or darker only at the end.

[Margin: Kepler's [?] Longomontanus's opinion rejected.]

X. The *fifth* opinion was Galileo's, published by Mario Guiducci and defended by Galileo in the *Trutinator* of the *Libra Astronomica*, from page 141 on: namely, that the tail of comets is straight by its nature, being produced by the Sun's rays, but appears curved to us when it is near the horizon and inclined to the horizon, because of the refraction of the species, or visual rays, made at the spherical surface of the air, which around the Earth is filled with dense vapours. Thus it came about that in the comet of 1577 — which Galileo diligently affirms he observed — the tail always appeared very curved, being always near the horizon and much inclined to it; whereas in the comet of 1618 it appeared bent, for the same cause, only in the first days, but afterwards, when it had gone up from the horizon and proceeded on a straighter path almost through a vertical circle, it lost its curvature. They add the comparison of an oar, which is straight but takes on an apparent curvature in water by the force of refraction.

[Margin: Galileo's [?] Guiducci's opinion.]

Against this opinion Lotharius Sarsius set three things in the *Libra Astronomica*, examination 2, question 3: first, that nothing has been said in this matter about refraction that Kepler had not said long before in the *Astronomia Optica*, chapter 6, number 13, though he left this phenomenon among the secrets of nature; next, that the comet of 1618 kept its curvature in its first days even far from the horizon; but that on the remaining days, especially December 20, while it was emerging from the Roman horizon, the run of the tail lay almost in the same vertical, but, carried higher, appeared so inclined towards the horizon

[Margin: Objections of Lotharius Sarsius [?] against Galileo [?].]

that it seemed almost [?] parallel to it, and yet no curvature appeared in it. Finally, that Galileo supposes the vaporous region to be spread spherically around the Earth, whereas it rather swells towards the poles and takes on an egg-shaped figure.

But Galileo replies that the reasoning of Kepler and of Guiducci was quite different. For Kepler spoke of the refraction of the Sun's rays made in the comet's head, and of a real curvature, either of the Sun's rays themselves or of the matter disposed behind the comet; whereas Guiducci was speaking of the refraction of the species and visual rays of the tail itself, supposing the tail straight in reality but apparently curved, because of the refraction made at the convexity of the vaporous air. Next, he replies that the very experience of the first days of the apparition, and of the 20th day, favours him: the tail appeared curved at first because those two conditions were joined — namely nearness and inclination to the horizon — whereas on the other days they were separated;

for when it was near the horizon it was upright and not inclined, and when it was inclined to the horizon it was nevertheless far from it and free of sensible refraction. Finally, that the figure of the vaporous air, whether it is perfectly spherical or egg-shaped, does nothing to hinder this optical demonstration; for all that follows from it is that the curvature of the tail also imitated an oval rather than a circular arc.

[Margin: Galileo's reply.]

Kepler, moreover, in the *Hyperaspistes*, page 199, having read through these quarrels between Galileo and Sarsius, denied that he had left this dispute undecided, since he had settled it in his little book on comets. The curvature apparent from refraction in vaporous air, he says, is very small, and does not account for the remarkable curvature of comets' tails, since it does not exceed half a degree; and even for that it is required that the head be very close to the horizon and that the end of the tail be not higher than 20° above the horizon. Nor can it be well fitted to the tail of the comet of 1618, whose curvature was [?] remarkable and occupied chiefly the extreme parts of the tresses, on which account it was compared to a Persian scimitar. And indeed refraction can show this curvature in no other way than by raising the parts nearer the horizon more and the remoter ones less; yet this can happen in such a way that all are kept in the same visible straight line. Of this we have an example in the stars of Orion's belt, which always appear in the same straight line, although near the horizon the belt is so inclined that the leading star of them, at rising, is higher than the middle one, and the middle one higher than the following one — and so the leading star is subject to less refraction than these.

[Margin: Kepler's objections against Galileo.]

XI. The *sixth* opinion was that of Father Johann Baptist Cysat, *De Cometa anni 1618*, chapter 7, where he asserts that the coma of the comet is not flame but a radiant cone or pyramid, formed by the Sun's rays transmitted through the comet's head, in much the same way as the Sun lights up a room through a hole or a convex glass lens, or produces radiant rods behind clouds. This, he says, can happen partly by refraction, partly by reflection from the many-angled sides of those corpuscles out of which, by an experiment with the telescope, he had reported that the comet's head coalesces, as we related at the end of the history, chapter 3; just as the solar faculae appear on the Sun's disc brighter than the Sun itself, when its rays are transmitted through diaphanous spots. Nor, he says, is the substance of the aether surrounding the comet so subtle that it cannot bound the Sun's rays, gathered by the force of refraction or even of reflection, and reflect them to us. Then, in number 10, he adds: *Why the comet's tail appeared curved, an explanation is given from theorem 11 of Euclid's Optics and axioms 8, 9, 10, 11. For things seen by lower [?] rays appear nearer; and one extremity of the comet (either the head or the end of the coma) was seen by lower rays than the other, since the comet stretched out in a straight line, by proposition 2, chapter 4; and either the head or the extremity of the coma was farther from the observer, by*

propositions 6 and 8, chapter 3. Some curvature, therefore, necessarily appeared, just as in a long gallery the upper ceiling near the exit is seen to slope notably downward.

[Margin: Father Cysat's opinion.]

In fact, however, Euclid does not teach there that the parts of straight surfaces or lines that stand on high, seen by lower rays, appear nearer, still less that they appear curved. For Euclid's theorem 11 runs thus: [Greek: τῶν ἄνω τοῦ ὄμματος κειμένων ἐπιπέδων τὰ πόῤῥω, ταπεινότερα φαίνεται (tōn anō tou ommatos keimenōn epipedōn ta porrō, tapeinotera phainetai, "of planes lying above the eye, the farther parts appear lower")] — that is, *of planes higher than the eye, those that are more remote are seen lower*. From this it does not, of itself, follow that they appear curved; for any points can appear lower the more remote they are from the eye, provided they lie in the same visible plane or in the same visible straight line. Besides, it would then be necessary that all comets' tails, at least those of a longer course, should appear curved — which, however, we do not find in the history; on the contrary, the tails drawn by Apian are without any curvature. And by the same reasoning the belt of Orion would have to appear curved to us in some position.

[Margin: Objection against Cysat.]

XII. The *seventh* opinion is that of Father Niccolò Cabeo, on Book 1 of the *Meteorologica*, text 37, question 7, where he holds that the tail of comets is the Sun's rays shining through the comet's head and refracted. But since rays, however much refracted, are propagated along straight lines, he says that a real curvature cannot be got from this, and that it is rather an apparent one. The cause of this apparent curvature he refers to the differing position, or plane, of the eye and of the tail, in these words: *If the comet's tail is not set straight before you, and is not wholly in the plane in which the eye also is, but is in another vertical, or even in a different plane that is not vertical, and bends to one side, then imagine for yourself also that the tail is, as it truly is, in a straight line; and draw in imagination, through the plane of the eye, another line parallel to the tail. Then the extreme parts of the tail, by the rules of optics, will approach that line, and that whole line of the tail will be seen curved, and the curvature will always bend towards that line.* Soon after, he admits that this imagining of his cannot be illustrated by any diagram — rather, a diagram would obscure it: *But by the example*, he says, *of things painted in perspective, and from the fact that the parts of the tail are not only moved farther and farther from the eye but are also raised, and are not in the same plane with our eye — from all these it comes about that that curvature is represented.* Finally he concludes that, because of the perpetual motion of the Sun, the plane of the comet — the plane in which the Sun and the comet's head and tail lie — comes out perpetually different from the plane of the observing eye; that the apparent curvature therefore varies perpetually; and that it appears greater the more the plane in which the tail lies is removed from the plane in which the eye of the beholder is. He had said above: *When the comet's head and its whole tail — that is, the rays issuing from the comet — are in one and the same vertical plane with our eye, then the tail will be seen not curved but straight.*

[Margin: Father Cabeo's opinion.]

So much for him. Had he explained these things better, with a diagram attached, or at least with the laws of optics he presupposes set out determinately, we could pass a clearer judgement on this opinion. Meanwhile, no efficient cause of the apparent curvature seems yet to have been brought forward, if the tail is in reality straight. For to say that the tail does not appear straight when it is not wholly in the plane of the eye is rather to offer a definition or description of curvature, or of a line that is not straight; for a line is not straight when not every three of its points are in the same plane, but two are in one plane and the third, with the second, in a different plane; and this property belongs to true lines as much as to apparent ones. Nor, from the fact that the tail moves and becomes transverse through one vertical after another, can it lose its straightness or be apparently changed into a curve; for the three stars in Orion's belt appear to us in the same straight line, even though the belt passes through one vertical after another in its oblique ascent and descent. By contrast, the three stars in the tail of each Bear never appear to us in the same straight line, unless perhaps, by the force of a differing refraction on some horizon, they are seen in a parallel of the horizon. Therefore, if someone says that the comet's tail, if it is straight — as the belt of Orion is thought to be — will also appear straight everywhere; and that if it is curved — as the tail of the Bear is thought to be curved, when its stars are joined in imagination — it appears curved because it is curved, he cannot be refuted from Father Cabeo's doctrine.

[Margin: Objection against Cabeo.]

XIII. The *eighth* opinion is that of Descartes, in the *Principles* of his new philosophy, part 3, from marginal number 136 on. He supposes that infinite corpuscles and globules are moved through various vortices, and he tries to account for almost all physical changes by motions of this kind; he also thinks that the Earth is moved annually. Accordingly, he says, the Sun's rays striking the comet's head are reflected towards all parts of the atmosphere surrounding the Earth — which he makes not spherical but spheroidal, and much more depressed towards the poles than towards the ecliptic — and some of those rays are scattered and fall obliquely onto the atmosphere, and are partly refracted because of the motion of the globules already mentioned, which perpetually fluctuate; and from this arises the curvature of the tail. But if

[Margin: Descartes's opinion.]

the foundations of this opinion were to be laid open, it would be necessary to expound many other things — some quite new, others old indeed but already antiquated by others and called back into the light again by him — a digression foreign to my purpose. It is enough for me to have pointed out the place for those who take delight in fictions of this kind. But this must also be noted: comets too are placed by him above the sphere of Saturn.

XIV. The *ninth* opinion is that of Giovanni Camillo Glorioso. In Book 3 of *De Cometis*, chapter 8, he acknowledges that the tails of comets which are not hairy all round are turned away from the Sun, and often do not taper into a

cone but take the form of a broom or of a basket. Then, in Book 5, chapter 2, from page 223, he teaches that a comet becomes hairy all round when the cometic matter has equal density in all its parts; for then the Sun's rays, entering and leaving uniformly, produce that corona. But if the bodies from which the comet's head is formed are several and close together, or a single one but very rare, then the Sun's rays run out beyond, behind the comet — through the space lying between those bodies, or through that rare substance — and generate a beard or a tail for it, in much the way that the Sun shines into a room through a hole, or runs out in luminous streaks through rarer clouds. Nor is the matter of the aether so subtle that it cannot reflect to us the Sun's rays, so intense and crowded together. And so he very nearly subscribes to Cysat. If this does not please, he says, we may philosophize with Kepler, *De Cometis*, page 100, and with Millerus, *De Cometa anni 1618*, page 52 — namely about matter expelled from the comet's head by the force of the rays, etc. When, however, the matter is so dense that it cannot be penetrated by the Sun's rays, then the comet is neither hairy nor tailed nor bearded. The cause, moreover, why the tail sometimes widens into the shape of a clothes-broom or of an inverted pyramid is, he says on page 226, that the Sun's light passing through a narrow opening always widens more and more. As if the comet's head were perforated, and transmitted the Sun's rays without any refraction — the refraction by force of which, on page 224, he had granted that they are gathered, crowded together and intensified! As to why the tail is curved, however, after setting out the opinions of Tycho, Kepler, Mario Guiducci and Cysat, he does not determine absolutely which of them is to be preferred, but only disjunctively: either it must be said that the tail is really curved, because various corpuscles behind the comet, or matter expelled by the Sun, are so disposed as to form a curved course, as Kepler and Millerus held; or it must be said that it is in truth straight but appears curved, either because of refraction, with Guiducci, or because the more remote parts appear more depressed, as Father Cysat taught. There is therefore no reason for us to repeat here what we said against these opinions.

[Margin: Glorioso's opinion.]

[Margin: 1st mode.]

[Margin: 2nd mode.]

XV. The *tenth* opinion is that of Libert Froidmont, *Meteorologica*, Book 3, chapter 4. In article 1 he affirms that the tail of comets is, for the most part, the Sun's rays driven through the brain of the comet's head; that at first it is neither very long nor tends exactly towards the quarter turned away from the Sun, because the depth and opacity of the head make that deviation vary; but that when the opacity has thinned out a little and the matter has been better digested, the rays go out straight and stretch into a longer tail; and that the rays of Venus too, when it is nearer the Sun, can concur with the Sun's rays in this effect. Further, that from the varied disposition of the parts of the head, and from agitation and motion, it comes about that the tail is sometimes split into several branches, as Cornelius Gemma reports in the comet of 1577, and

sometimes, by a sudden vibration, is drawn back into the head and soon darts out lengthwise again, as Cysat and Kepler observed in the comet of 1618. After this, in article 2, he teaches that the matter which sustains, as a subject, the Sun's rays forming the tail is the density of the very deep aether — which on that account shows us a blue heaven — and besides, many flocks of a rarer exhalation, or a fine dust breathed out from the head, lit up by the Sun; although at the end of the article he says: *Yet these things do not wholly satisfy my mind; but I must leave now, even half-starved.* Lastly, in article 3, anxiously seeking the causes of the curvature, he rejects Kepler's opinion about matter boiling out of the head or driven off by the Sun, since it would be of inconstant figure. He himself affirms that the comet of 1618, at the beginning of December, appeared to him sickle-shaped towards the south, and afterwards straightened itself and turned its concavity to the north, and so persevered thereafter to the end; and that a similar change happened in the tail of the comets of 1596, 1580 and 1577. He next reports Guiducci's opinion and refutes it, saying that if it were true, the tail would have to appear most curved when nearest the horizon, whereas in that position it was seen by him and by Cysat rising quite straight from the horizon. But he does not distinguish whether this was seen at a time when the head and the tail were in the same vertical; for in that position Guiducci too asserts that it is straight, since curvature requires inclination to the horizon, and not merely nearness to it. Afterwards he conflates the opinion of Tycho with that of Cysat, and teaches that neither shows the cause of the curvature, but only the cause why the part of the comet nearer the horizon appears more depressed and closer to the Earth. Finally, since he can rest neither on winds fanning and bending the tail, nor on an excessive rarity of the parts in the curved position, unsuited to reflecting the rays, he confesses that here the mind of mortals is blind — and that this is no wonder, since even about the beard of a man opinions are so varied and uncertain: very many think it is inanimate, others that it is informed by a vegetative soul, others even by a rational one.

[Margin: Froidmont's opinion.]

XVI. The *eleventh* opinion is that of Franciscus Resta, *Meteorologia*, Book 1, chapter 5. Having adduced various opinions, he too wavers and does not seem to find where to plant his feet. At last, however, he inclines in part to Kepler's view, and thinks it probable that thin parts are expelled from the comet's head by the Sun's rays into the opposite quarter; that near the head they are not yet dispersed, but at the end are scattered; and that the tail is therefore narrow near the head but looser at the end, and curved by that very declination. But besides what has been said against Kepler, there is the observation of many comets whose tail was not looser at the end than in the middle.

[Margin: Resta's opinion.]

XVII. The *twelfth* opinion was formerly that of the Marquis Sforza Pallavicino, which Cottunius reports in lecture 33 on Book 1 of the *Meteorologica*, page 309. The cause why a comet, the farther it is from the Sun and the more it is opposed to the Sun, shows the shorter tail, and sometimes none, he referred

to the comet's shadow. *Because*, he said, *an opaque body, the farther it is removed from a luminous body, the more it contracts its shadow; therefore a comet, the farther it is from the Sun or the more it is opposed to it, casts the smaller shadow, and accordingly has the smaller tail. Hence also it is that hairy comets are far from the Sun and come nearer to the Earth, whereas bearded ones are very high up.* He says that this reasoning was highly commended, and received with great applause by the learned men who had assembled in the house of Giovanni Ciampoli. But whether Sforza said this very thing at the time, or thought otherwise and was not sufficiently understood by Cottunius, the reasoning cannot stand. First, because it is utterly false that an opaque body contracts its shadow the farther it is removed from the luminous one, unless it is larger than the luminous body. For if it is smaller — as everyone admits the comet's head is, compared with the Sun — the farther it recedes from what illuminates it, the thicker and longer the shadow it casts, as is clear from what was said on eclipses in Book 5, chapter 3. Therefore if the Sun's rays, transmitted through the comet's head and shining in the head's shadow, form a tail as great as that shadow is, they will form a greater one the farther the comet's head has receded from the Sun. Next, he does not seem to have understood what that distance and opposition is in which comets' tails have been observed to appear either not at all or very small, since he judges it by nearness to the Earth. For they can be very remote from the Earth and yet opposed to the Sun, as Jupiter and Saturn are in opposition to the Sun; and so the tail lies hidden behind the head, not because the shadow is shorter there, but because our eye, set between the Sun on the one side and the comet's head on the other, cannot see the tail, however long, because the head stands in the way — especially at so great a distance.

[Margin: Opinion of Sforza Pallavicino.]

XVIII. Having considered so many and such varied opinions, then, we too had to apply some effort to devise various not improbable ways by which there can come about the aversion of the cometic tail from the Sun; the subject from which either the comet's own rays or rays carried over from the Sun are reflected to our eyes; the varied figure; and the motion of vibration. We would have done this by a surer method if observers agreed about the position of the tail and its manner as noted even in the most recent comets, or if we were certain of the place of comets — whether they arise and move above or below the Moon or the Sun.

[Margin: Partly our own, partly Father Francesco Maria Grimaldi's opinion.]

First, I say that it is not improbable that the tail belongs to the substance of the comet and shines with its own light, propagated from the head into the remaining part; but that the face of the comet's head is turned towards the Sun, as the heliotrope is, or as one face of a magnet is turned towards one pole, and is moved by a slow rotation about the centre of the head in such a way that the more perfect part of the comet is turned towards the Sun, while the remaining part, following the same motion, is turned away from the Sun and takes various figures, according to the different species of comets, from the very disposition

and configuration of the matter. Nothing stands in the way of this mode except the sudden vibration by which the tail of the comet of 1618 appeared at one moment short and soon after long — unless you attribute that very thing to little clouds or fumes running across in between.

[Margin: 1st mode.]

Second, I say that it seems very probable that the cometic tail is corpuscles, very subtle and very numerous, flying around the comet's head in the air or in the aether. They are not such as we see in a ray of the Sun let in through the opening of a darkened room, nor such as are in the exhalations that show us the twilight (if indeed not only the head but the whole tail must, on the strength of the observations, be placed above the sphere of the twilights); they are far more subtle and higher, and are not fit — partly by their smallness, partly by their lesser opacity, partly by their greater distance from the Sun — to be seen by us through the Sun's rays reflected from them, unless they are more strongly illuminated. This posited, I say that they are more strongly illuminated because of the Sun's rays gathered into one by force of refraction: not in precisely the way in which the rays come together into one, or into a few very close points, behind a glass sphere, as was stated at number 7, but in a somewhat similar way. For (as becomes probable from what Nicephorus relates of the comet of 392, and from what Cysat and Habrecht observed in the head of the comet of 1618, on which already in their place, pages 6 and 20) the heads of comets may be neither exactly spherical, nor made up of parts continuous and homogeneous in density and opacity, but of various bodies — polyangular, or spherical or spheroidal, or even having a surface bounded somewhere by hyperbolic or parabolic lines — so that, with the laws of both reflection and refraction preserved, many more rays leave the comet's head gathered, though not meeting in a single point, than would if they either did not pass through the comet's head or passed through it unrefracted. To understand this, the problem of Father Bonaventura Cavalieri in the *Speculum Ustorium* (his *Lo specchio ustorio*) can be of no small service, where he teaches how to reflect the Sun's rays from one parabolic mirror into another in such a way that almost infinite rays, gathered into one very slender cylinder, run on to infinity and have the power of burning. For if this can be done by reflection alone, with the help of only two mirrors, how much more if many reflections and many refractions, from many mirrors or dioptric bodies — and very large ones — concur to a similar effect and show us, in the corpuscles already mentioned, the varied form of tails! If to this there is now added a motion, either of trepidation or of rotation, by which the whole head of the comet nods this way and that or turns about its own centre; or a varied disaggregation or dissipation of the partial bodies of which the comet's head consists (of which dissipation the same Cysat likewise produced an example in the same comet of 1618); or an alternating fluctuation of fumes running across between the far end of the tail and our eyes — then it will not be difficult to understand why the tail could be seen now shorter, now longer, on the same night and at almost the same hours.

[Margin: 2nd mode.]
[Margin: The subject of the tail.]
[Margin: Collection of the Sun's rays.]
[Margin: Varied figure of the tails.]
[Margin: Vibration and inconstancy of the tail.]

Third, I say that the curvature of the tail — that is, of the Sun's rays issuing from the comet's head — can arise from one refraction after another, made away from the perpendicular outside the comet's head, in matter that belongs to the comet but is not as dense as the comet's head; indeed, matter having within itself various degrees of density, less and less the farther it is from the head. For if this is granted, and if the centre of the whole cometic matter, from which the perpendiculars that regulate the refractions are to be drawn, is not the very centre of the comet's head, those skilled in the science of dioptrics will easily understand how the lines of each whole refracted ray coalesce into a curve, so that it does not merely appear so but really is such. This perhaps also happens in the rays of the stars refracted in an atmosphere of one density after another, if they pass continuously from a rarer to a denser medium, as we shall say in Book 10, last section. And this, from the speculation of Father Francesco Maria Grimaldi while we were debating these matters together, it seemed good to show in the diagram below.

[Margin: Curvature of the tail: where does it come from?]

In this diagram the matter from which the comet is formed, spread out spherically, is represented by various circles described from the centre C — rarer and rarer, however, the farther it is from the centre C. The comet's head, however, is placed not in the very centre of that matter but outside the centre (whatever the cause of this position may in the end be). For neither are the piths of trees exactly in the middle of the trunk, as is clear from their veins, disposed as it were in circles, if one cuts tree trunks straight across; nor is the yolk always in the middle of the egg; nor do stones that have some principal kernel contain it exactly in their middle, etc. Let the head, then, be above, at AB, and let the Sun be towards V, irradiating the comet's head, and from the head let the ray AS go out, which, if it proceeded unrefracted, would go to G. But since the perpendicular drawn to the point S from which the ray goes out, and so running into the centre, is RSC, and the ray AS is supposed to go out of circle 3 into a rarer medium towards circle 4, it is therefore refracted away from the perpendicular, and would go on from there to H if no other refraction occurred. But because, going out from circle 4 towards 5, which is likewise rarer, it is refracted again, and once refracted would go to I if nothing else happened; and again, having gone out of circle 5, it is refracted away from its own perpendicular towards K; and so on. At length it comes about that the ray AS, refracted almost countless times, finally tends to Q; and so, to an eye placed towards F, it appears under the curved line ASQ. The same is to be understood of the other extreme ray, BDE, and of the rest enclosed between them and making up the comet's tail — provided that different perpendiculars

are understood to be drawn through the different points at which the refraction of the rays takes place.

Diagram. A woodcut set in the right column: nine concentric circles about the centre C, standing for the cometic matter, rarer outwards; the circles are numbered 1 (innermost) to 9 (outermost) along a radius running left from the centre. The comet's head, a shaded disc AB, lies a little above the centre; from it a shaded tail curves out to the left through the rings. From S, at the upper edge of the tail near the head, dotted lines fan out to a column of letters beyond the circles at upper left, and a dotted radius runs up through S to R at the top.

- G, H, I, K, L, M, N, o, P, Q (column outside the circles, upper left): the successive points towards which the ray AS would run after each refraction; the small lower-case o stands between N and P
- R (top): upper end of the perpendicular RSC
- S (upper edge of the tail, near the head): the point at which the ray AS leaves circle 3
- A, B (above and below the head): the head AB
- C (centre): centre of the circles
- D, E (lower edge of the tail): points of the other extreme ray BDE
- Z (below the centre): the alternative position of the head
- V (outside, right): the quarter of the Sun
- F (outside, below): the quarter of the eye
- 9, 8, 7, 6, 5, 4, 3, 2, 1 (along the left radius): the numbers of the circles

The eye, moreover, is supposed to be outside the whole of that matter of which the comet's head and tail have most immediately been formed; and so that curvature of the tail is aided in appearance by the rays of the species issuing, refracted, from the sphere of the matter already mentioned. But if the comet's head is below the centre C, say at Z, then, because of the refractions away from the perpendiculars to be drawn to the individual points of the circles from which the rays go out, the rays will be curved in such a way that the tail's concavity faces upwards and its convexity downwards, towards the eye at F. And similar things will happen if the comet's head were indeed at the centre of the matter already mentioned, but not at the centre of these circles, or quasi-circles, because the cometic matter would not be expanded in the manner of a perfect sphere about one centre. But if the head were at the centre of that matter and of all the circles already mentioned, the refractions already mentioned would not occur; and accordingly the tail would not even appear curved to us, so far as this speculation is concerned.

SCHOLIA.

Scholia.

I. It will not be amiss to indicate some ways, handed down by several authors, of representing to the eyes something like comets and their tails. Among them Cardano, in Book 1 of *De rerum varietate*, taught that a lighted candle should be

exposed to the Sun, so that the solar ray would appear through the candle's flame as though shot through a comet's head; but this, for him, not at all succeeded, as Tycho relates in volume 2, page 162. But he ought to have taken the Sun's ray into a darkened chamber through a hole, exposed the candle's flame to it, and made the Sun's ray transmitted through the flame visible behind it by throwing dust, or by smoke. Indeed, it is certain that the light and species of one lighted candle pass through the flame of another lighted candle or fire, and are seen both directly by us and in a mirror; how much more will the Sun's ray penetrate a candle's flame? Kepler, on the other hand, in the *Astronomia optica*, at the end of chapter 7, or page 267, gives this prescription: *Let a ray of the Sun enter*, he says, *through the opening of a darkened room; and set a globe of water — that is, a glass sphere full of water — half across its path, so that the Sun's ray strikes partly the glass and partly the wall; for you will see a comet on the wall.* Galileo, for his part, in his *Trutinator*, page 106, teaches another way, in roughly these words rendered into Latin: *Take a very clean glass sphere or flask, and set a lighted candle opposite it, not far off; for you will see on the surface of the glass a little image of that small flame. Darken that part of the glass, then, with the breath of the mouth or with some grease, until that image of the flame is almost extinguished; next turn the glass vessel so that the image comes out of the greased or breath-darkened place and appears at its contact, or outermost border. Afterwards wipe off the breath or the grease with a finger in a straight stroke, and you will see a ray propagated from the image of the flame, imitating the hair of a comet; and this ray will cut at right angles the track along which the finger rubbed and wiped the glass. For since the skin of the fingers is not altogether smooth, but is cut by little channels for perceiving the smallest differences of tangible things, by which certain very fine furrows have been ploughed in that breath or grease, from their ridges there arises a reflection of that light, which imitates the tail of comets.* Lastly Giovanni Ciampoli, as Cottunius relates in lecture 33 on Book 1 of the *Meteorologica*, page 309, took a lighted wax taper and, applying it in a dark place to the side of a certain mirror, made a comet with head and tail appear in the mirror, far better than Johannes Kepler prescribes; so says Cottunius, an eyewitness.

[Margin: 1. Cardano's way of representing a comet with its tail.]

[Margin: 2. Kepler's way.]

[Margin: 2. Galileo's way.]

[Margin: 4. Ciampoli's way.]

II. So far we have discussed the tail of a comet turned away from the Sun as if a straight line drawn through the whole length of the tail, or at least through the tail's end and the comet's head, would run into the Sun if it were produced; for Apian, Tycho, Kepler, Cysat and many others assumed this for us from their observations, because the comet's tail, with its head, appeared in the same straight line with the Sun. It does not, however, necessarily follow from this that these three were in the same straight line, but only that they were in the same plane meeting the eyes — as Galileo also notes in the *Trutinator*, page 39. Thus Mars or the Moon sometimes appear to us in the same straight line

with two fixed stars, and yet a straight line drawn through the two fixed stars does not on that account pass through Mars or the Moon. I add, further, that from the fact that, at the same moment, the altitude of the tail's end above the horizon, taken with some instrument, was greater than the altitude of the head, it does not follow that the tail's end was more remote from the Earth than the head. For imagine the comet's head to be where Sirius is when it rises above the horizon, and the tail's end to be at that moment where the Moon is when it crosses the meridian: the tail's end will certainly be much nearer the Earth than the head, even if it appears many degrees more elevated above the horizon than Sirius, and so than the comet's head. It will not have been useless to give this warning.

[Margin: Points to note on the position of the cometic tail.]

CAPVT XXVI.

Chapter 26.

De Cometarum Motu Curuilineo ac Rectilineo.

On the curvilinear and rectilinear motion of comets.

I. This final question about comets — namely, whether they move in such a way that by their motion they describe a great circle in the sphere of the world, or one close to a great circle; or whether rather they descend along a straight line towards the Earth and so come into view, and then ascend and little by little withdraw from the eyes of mortals — was pointed out long ago by Seneca, in Book 7 of the *Natural Questions*, chapter 29. For since he held that comets are celestial, and there was urged against him the slowness of their motion, as an argument of a heavy and earthy nature, and their being carried downwards, he replied that slowness of motion is not an argument of heaviness or of an earthy body, since Saturn, which runs its course most slowly, is neither heavy nor earthy. Next, he denies that either Saturn or the comets are slow, if the amplitude of the circuit they traverse is considered; for the comet last seen by him, namely under Nero, ran through half the heaven within the sixth month. Lastly, he denies that they are carried downwards, and says: *First, what is carried around is not carried down. Next, this most recent one began its motion from the north, came through the west to the south, and, raising its course, vanished. That other one, of Claudius's time, first seen from the north, did not cease to be borne straight on, continually higher, until it departed.* Thus he acknowledges a motion of circumlation in comets, but in such a way that he thinks it ends up rectilinear. The Peripatetics, however, and most astronomers who followed them in this, attributed to comets no motion other than a circular one — Tycho indeed, and many of his followers, a motion along a great circle — and from this they also drew an argument for their celestial nature. Others, by contrast, whether of their own accord or stirred up by Galileo and Kepler, have granted that a rectilinear motion must be attributed to comets, or at least can be.

[Margin: Seneca's opinion on the rectilinear motion of comets.]

II. Mario Guiducci, in his treatise on the comet of 1618, following Galileo's opinion — and Galileo himself, in his *Trutinator*, page 36 and from page 126 — teaches that it does not follow, from the fact that a comet has been seen to advance in the same plane, that its motion was along the circumference of a circle, but only in the plane of some circle; and this could equally have happened even if the comet's motion was along a straight line. Indeed, he teaches that it is not improbable that exhalations from our region, but lighter than the twilight vapours, ascend along a straight line perpendicular to the Earth, as far as the heaven of the Moon or above the Moon, and that from them a comet is sometimes formed, which keeps the same proper motion, or moves along another straight line in such a way that all its phenomena are saved; that for this the annual motion of the Earth is not necessary; and that it does not follow from that rectilinear motion that the comet must at some time reach the observer's zenith — granted that Lotharius Sarsius, in the *Libra Astronomica*, pages 9 and 34, attributed these two points to Mario Guiducci. But since the comet of 1618, by its motion in declination, not only crossed the vertex of the observers at Rome and Florence but, becoming more northerly, reached right up to the pole of the world, neither Guiducci nor Galileo denied that some other motion, besides that perpendicular ascent from the Earth, must certainly be attributed to it. And the same must be said of the comets observed by Seneca, as I said at number 1, and by Pontano — on which see our chapter 3, at the year 1471 — and of many others whose phenomena cannot be explained by a motion perpendicular to the surface of a resting Earth alone.

[Margin: Galileo's opinion on this motion.]

III. Father Johann Baptist Cysat, in *De Cometa anni 1618*, chapter 3, proposition 8, affirms that it is more probable that this comet, besides the motion of the primum mobile, was moved by a proper motion — a motion of rectilinear ascent, though not perpendicular to the Earth; for in this way its distances from the Earth are better reconciled with the observed digressions of the comet from the Sun. To show this he uses chiefly the comet's distances from the Earth and its digressions observed on December 1, 20 and 29, which are written below.

[Margin: Cysat's opinion on the rectilinear motion of the comet [?].]

December, day	Distance from the centre of the Earth, in Earth semidiameters	Digression from the Sun
1	72	32°
20	154	90°
29	252	104°

For, with these posited, the ecliptic described from the centre of the Earth, the Sun placed for the days already mentioned at the corresponding places of the ecliptic, and the comet placed on circles of latitude drawn through the ecliptic according to the comet's longitude observed on the same days, he draws

from the centre of the Earth three lines each, both to the comet's three positions and to the Sun. Of these, the lines drawn to the comet's head are its distances from the centre of the Earth; those drawn to the Sun are the distances of the Sun from the centre of the Earth, taken in Tycho's manner; and the angle at the centre of the Earth contained by them measures the comet's digression from the Sun. From these he gathers that the come- t had been, on those days, very nearly on one and the same straight line; that at the beginning of its appearance it was perhaps lower than the Moon by 10 semidiameters of the Earth, or was no more than 40 Earth semidiameters distant from the Earth; and that at the end, stretching upwards, it penetrated the sphere of the Sun, and indeed of Mars. From this he also gathers that by a trajection of this kind it can come about that stars fall from heaven, as Christ the Lord foretold. He says this not improbably; for nothing demonstrated is established in this matter, since not even the parallaxes or the distances of the comet from the Earth have been demonstrated, as we said in chapter 22.

IV. Kepler, however, built up this rectilinear motion in another way and to another end, in the *Astronomiae Pars Optica*, chapter 10, at the end of which he has this appendix: *Those who have demonstrated the motions of the comet of 1577 by circles took on a very difficult labour, and yet they did not wholly succeed. They will meet much greater difficulties if they undertake to demonstrate the same in the other comets. For my part, in several comets whose recorded observations I obtained, this way succeeded more easily: if, as the nature of things urges, I assigned them straight lines, which in equal times they would for the most part traverse equally — only somewhat slower at the beginning and the end, and nearer to rest, as other trajections usually are too. For the motion of the Earth, insinuating itself, readily lends circularity to such a line. Thus, if that comet of 1577, having risen on a straight line from the plane of the tropic of Capricorn, had ascended towards the northern pole — or on a line somewhat more inclined, but still a straight one — then the Earth, going round a resting Sun (for on this supposition the comet itself tends almost only in latitude), will lend it the same appearance of circular motion. So too the comet of Regiomontanus, carried on a straight line from the deepest aether past the Earth and passing very close to it, will find a most beautiful occasion to complete 40 degrees of a great circle in one day in the middle of its appearance, but very little before and after; and here the cause will also be plain why the tail, at that moment of time when the comet was so swift, matched 50 degrees in longitude, by 4.22 of Witelo.*

[Margin: Kepler's opinion on this rectilinear motion.]

Afterwards, in his books on the comets of 1607 and 1618, he tried to make the same opinion more and more probable by these arguments. *First*, he says on page 3, it is not likely that the motions of things which never return to the same place, as comets are, are circular. *Second*, it has been observed in many comets that, when they approach their heliacal setting and when they have completed a quadrant of the diurnal motion, they become stationary; and the cause of this is not given as well by a circular motion as by a rectilinear one. *Third*, if the annual motion of the Earth and a proper rectilinear motion of the

comet are admitted, the increases and decreases of the motions observed in many comets, especially of latitude and declination, are explained more easily than by a circular motion — especially if it is assumed that the comet's trajection proceeds at first with uniform speed, and that the daily journeys of the comet traversing it then gradually increase by the law by which the tangents of the arcs of a circle increasing by equal parts increase, or by some similar ordered path.

Moved by these considerations, he presents, from page 8 to page 24, 30 theorems, by whose aid he then investigates for the comet of 1607 the trajectory line and its quantity from the observations, as well as the perpendicular line and the distance of the comet from the Earth; and he draws up a small table, which he does from page 37 to page 45, through 13 conclusions. He does the same for the last comet, that of 1618, from page 79 to page 93. And finally, from page 96 to page 98, he professes that through the annual motion of the Earth, by which the Earth approaches the comet's trajectory line or recedes from it, and through the very motion of rectilinear trajection, reasons are given why some comets appear swiftest in their proper diurnal motion at the beginning, others after the beginning, and slow both before and after, and why after the beginning and before the end they appear larger and with a longer tail. 2. Why they are mostly hidden under the Sun's rays. 3. Why the motion of comets at the end of their appearance mostly becomes slow, or stationary, or retrograde. 4. Why it mostly happens at the same time that a comet is both hidden under the Sun and becomes stationary. 5. Why comets mostly appear to describe great circles in the sphere. 6. Why comets, once they have become slow, begin to curve their paths, as the comets of 1607 and 1618 did, and, as he imagines, many others. By these and similar things he was so enticed that on page 98 he said: *In short, as many comets as there are in the heaven, so many are the arguments (besides those drawn from the motions of the planets) that the Earth moves with an annual motion about the Sun. Farewell, Ptolemy; I return to Aristarchus, with Copernicus as my guide.* Finally, in the *Hyperaspistes*, pages 100, 149 and 193, he confirms this same opinion of his, as not certain but more probable: that circular motions suit perennial bodies, but non-circular motions suit perishing ones; that comets always tend towards the same region of the sky; and that, since this motion ceases, it is credible that it is adventitious, and similar to the violent motion impressed from outside on darts, javelins and all projectiles.

[Margin: Argument for the annual motion of the Earth from the motions of comets.]

V. It is not my intention to want all probability taken away from this rectilinear motion of comets introduced by Kepler; but I deny that it is so probable that, from its combination with the annual motion of the Earth, it becomes more probable that the Earth moves with an annual motion than that it rests.

[Margin: Critique of Kepler's opinion.]

First, because the properties of the motion of comets on which Kepler relies are not taken from the history of most comets — granted that he himself, gratuitously and out of his own invention, uses the word *Plerique*, "most" — but from very few comets, namely those of the years 1472, 1577, 1580, 1585, 1590, 1607 and 1618, as he himself expressly admits in the *Hyperaspistes*, page 194. There he also shows inconstancy in asserting these properties; for he admits *that he asserted the motion of comets to be slack at the beginning and end and swift in the middle, not on account of any geometrical demonstrations from the observations of the comet, but from mere contemplation of bolides, or artificial fires, which the Germans call Raketuli. For these, at the start, while the flame has not yet caught in all the chambers, hesitate and set themselves in motion slowly, and at the end of their duration languish again.* Then he adds: *now that I have treated the comets just mentioned,* namely from the year 1572 *to 1618, I find no very great reason why I should make this rectilinear motion slower towards either end; so I leave this inequality in doubt.* So he says — he who nevertheless so often tried to confirm, from this inequality, the combination of the trajectory line with the annual motion of the Earth.

[Margin: 1st argument against Kepler.]

Second: in his book on comets, page 85, speaking of the comet of 1618, Kepler says this: *Since it is now certain that, with the Earth at rest, or as it were even with the comet's motion rectilinear, the observations do not hold* etc. Yet Cysat defends the rectilinear motion of this very comet from his own observations without a motion of the Earth, as we said at number 3; and Cysat's observations were both more numerous and better than Kepler's, who admitted that his own and those of many others were rather coarse, as we said in chapter 22, and as he himself relates most expressly in that booklet, page 30. It is therefore not certain that, to defend his observations, the annual motion of the Earth must be joined with the rectilinear motion of the comet. But I prove this more strongly from his own admission in the *Hyperaspistes*, page 195, where, to Sarsius objecting that Catholics may not defend the motion of the Earth, he replies: *For as the straight trajectory has been propped up by me with the motion of the Earth, Tycho will prop it up just as firmly with the systematic motion of concomitance, which would carry along even the rectilinear trajectory itself — if anyone has oak and triple bronze about his breast for believing such things*; and in the same *Hyperaspistes*, page 98, he professes that he has done this very thing, having put on the person of Tycho. Therefore, if the rectilinear motion of the comet can be preserved just as firmly in the Tychonic hypothesis, which rests on a standing Earth, neither a certain nor a more probable argument for the annual motion of the Earth is drawn from the motion of comets. Father Cabeo too, on Book 1 of the *Meteorologica*, text 37, question 9, taught that this rectilinear motion can be accounted for with the Earth at rest.

[Margin: 2nd argument against Kepler.]

Third: Kepler professes that, with Copernicus as his guide, he withdraws from Ptolemy, the champion of a resting Earth, and returns to Aristarchus. But Copernicus admitted no motion in the heaven that was not circular and regular;

for he taught, in Book 1 of *De Revolutionibus*, chapter 8, and elsewhere, that this alone naturally suits the heaven. Kepler therefore works against Copernicus by introducing into the heaven a rectilinear motion, and one not so ordered as Copernicus requires in the celestial bodies.

[Margin: 3rd argument against Kepler.]

This argument is nearly that of Longomontanus, in the Appendix to his *Astronomia*, chapter 3, where he condemns the rectilinear motion of comets, adding that unless Kepler proves that comets are endowed with sense and recognise the end of such a motion, or teaches that some Spirit presides over governing it, no probable principle of such a motion appears; and he concludes: *These reasons, while with the preced-*

[Margin: Longomontanus's argument against Kepler.]

ing one combined, anyone wishes to weigh them rightly [?], he would say that Kepler accounts for the generation and motions of comets by the hypothesis of trajections more ingeniously — indeed more laboriously — than truly.

Kepler, however, asserted that this motion is rather a violent one, and from an external source, as we have just seen; and yet he did not attribute it to angels. Cabeo, by contrast, in Book 1 of the *Meteorologica*, text 37, page 211, concludes from the exclusion of the other causes of this motion that it comes from a directing Intelligence — a view which, as I have already shown in chapter 6, number 12, has also found favour with several others. With this posited as very probable, it will be easy for an Intelligence to move a comet, whether along a straight line or a curved one, in such a way that all the phenomena of cometic motion can be represented without any motion of the Earth. And this is the fourth argument against Kepler.

[Margin: 4th argument against Kepler.]

VI. Besides the authors I have adduced, there are few who have examined the rectilinear motion of comets indicated by Galileo and asserted by Kepler. It was acknowledged, however, above all by Glorioso, *De Cometis*, Book 5, from page 241, who, I regret to say, wrote on page 247: *It must surely be thought more probable and more likely that comets are carried freely along straight lines through the expanse of the aether, as Kepler thought* — if he says this while mixing in, with Kepler, the annual motion of the Earth. Next, Froidmont, in Book 3 of the *Meteorologica*, chapter 4, article 4, who said: *It is truer, as Kepler and Cysat have recently held, that the proper motion of comets is trajectory and transverse, not circular, nor straight upward.* And he added that the comet is carried upward by its own spirit, which in a certain way animates it, but that, with the weight of the head resisting, it is carried not straight but obliquely. By "spirit", however, he seems to understand the subtler and lighter parts, since he says: *But at the beginning comets are swiftest, because they are then more swollen with spirits and are of unimpaired strength; these then gradually breathe themselves out, as the exhalation is rarefied here and there by heat and the Sun's ray,* etc. At the end of that article, however, not resting content with any reason why most comets are carried towards the north, he concludes: *Unless here again you weep and implore the*

help of the angels. Franciscus Resta, by contrast, in treatise 1 of the *Meteorologia*, book 1, chapter 6, ranks the rectilinear motion of comets below the circular in such a way that he attributes it not to angels but to a multitude of stars flowing together to constitute the comet and mingling their motive force — hence that so varied motion; though he admits: *These things are indeed possible, but how the matter stands in fact could not be determined except by divining.*

[Margin: The opinion of Giovanni Camillo Glorioso and of Froidmont.]

In a matter so uncertain, then, let us at last be allowed, if not to walk out of this labyrinth, then certainly to fly out of it; and, just as we began chapter 1 with Seneca's words, to end this section too with his judgement. In Book 7 of the *Natural Questions*, chapter 25, since the motion of the planets was not yet sufficiently established, he bade that the motion of comets, still unknown, be borne with, in these words: *Why then do we wonder that comets, so rare a spectacle of the world, are not yet held by fixed laws, and that their beginnings and ends are not known, when their return is from enormous intervals?* Then, in chapter 26, he dared to prophesy — which I myself indeed do not dare; but I approve his counsel of admonishing posterity to observe comets more accurately, and with as much subtlety as we have shown to be necessary, so that their place, and everything else that depends on it, can be demonstrated absolutely. He says, then: *There will be someone who will one day demonstrate in which parts comets wander; why they go so far apart from the others; how great and of what kind they are. Let us be content with what has been found: let posterity, too, contribute something to the truth.*

[Margin: Seneca's prophecy or admonition.]

SECTIO SECVNDA

DE NOVIS STELLIS.

Section Two. On the New Stars.

PROHEMIVM.

Preface.

In this second section we have chosen a somewhat different order from that of the first. First we shall set out a short history, or chronology, of the new stars; then we shall present and weigh the observations of three new stars, so as to work out their place under the zodiac and their distance from the Earth and parallax — which we shall be able to accomplish more easily from what has already been said in the first section about the place of comets. This done, we shall investigate, as far as the weakness of human conjecture allows, what they are, and to what end God is believed to have shown these new stars to the world. Lastly, we shall discuss the admirable star divinely shown to the Magi.

[Margin: The order to be kept in the second section.]

CAPVT I.

Chapter 1.

Catalogus Chronologicus Stellarum Nouarum in Cælo obſeruatarum.

A chronological catalogue of the new stars observed in the heavens.

I. New stars, or those reckoned as new, are listed by Fortunio Liceti in his book *De novis astris*; by Cyprianus Leouitius [Cyprián Karásek Lvovický] in his writing on the star of 1572, and from him by Tycho in volume 1 of the *Progymnasmata*, page 331; by Io. Hechius in his disputation on the new star; by Redemptus Baranzanus [Redento Baranzano], part 2 of the *Uranoscopia*, page 112; by Chiaramonti, Book 1 of *De novis stellis*, chapter 1, and Book 3, chapter 6; by Froidmont, Book 3 of the *Meteorologica*, chapter 1, article 2, and chapter 2, article 7; by Franciscus Resta, treatise 1 of the *Meteorologia*, book 1, chapter 7; by Andreas Argolus [Andrea Argoli] in the *Pandosion Sphaericum*, chapter 62; by Antonius Maria de Rheita [Antonín Maria Šírek] in the *Radius Sidereomysticus*, book 4, chapter 1, member 1; and by Giovanni Camillo Glorioso, *De Cometis*, book 5, page 274. The rest, who have written on some of these stars in particular, we shall review below, when we deal with them separately.

II. *First* among the new or extraordinary stars, Liceti, Baranzano, Resta and Froidmont count the seventh of the Pleiades. For it is related that it appeared before the burning of Troy, but after that burning hid itself, and came into view again. From this arose the controversy whether the Pleiades, or Vergiliae, on the back of Taurus are six or seven; or whether they are sometimes six, and that ordinarily, but at other times

[Margin: The seventh of the Pleiades, held to be a new star.]

seven. Indeed Homer, in Book λ [Greek: λ (lambda), the letter that numbers the book, i.e. Book 11] of the *Iliad*, sings that only six were engraved on Nestor's cup; and only six are counted by Attalus and Geminus in their astronomical writings. But seven were recognised by Simonides, in Athenaeus, Book 11; by Marcus Varro, *De re rustica*; by Pliny the Elder, Book 2, chapter 41; by Aratus in the *Phaenomena*; and by Hipparchus. Indeed Ptolemy too counts seven, if you read the Greek codex, as Kepler reads it on page 115 of the *Rudolphine Tables* — and Kepler himself also counts 7, as does Bayer in the *Uranometria*, who adds that this asterism is called the Septistellium, but that because one of them is nebulous and appears only faintly, sometimes only six are seen. Moreover, all who have explored them with the telescope have discovered, besides the 7 usual ones, others too that are invisible to the naked eye. The mythologists, further, relate that they were the daughters of Atlas and Pleione. Of these, the four that form the quadrilateral are Maia — called the bright star of the Pleiades, and of the third rank — who is said to be the mother of Mercury; then Sterope, Taygeta and Celeno. The remaining three are Electra, Merope and Alcinoe. Among them Electra is nebulous and, as it were, sorrowful, because the fable

runs that after the burning of Troy she no longer wished to join the dances. Others, however, in Hyginus, say that the one that hides from time to time is Merope, because she married a mortal man, Sisyphus, whereas the rest married immortal gods — namely Electra, Maia and Taygeta married Jupiter; Sterope, Mars; Alcinoe and Celeno, Neptune. Ovid touches on these opinions, or fables, in Book 3 of the *Fasti*, in these verses:

> *The Pleiades will begin to relieve their father's shoulders, who are said to be seven, yet are usually six: whether because six of them came into the embraces of gods — for they say that Sterope lay with Mars, Alcyone and you, fair Celaeno, with Neptune, Maia, Electra and Taygeta with Jupiter; the seventh, Merope, married you, Sisyphus, a mortal; she repents, and alone hides for shame at the deed — or because Electra could not bear to watch the ruins of Troy before her eyes, and set her hand before them.*

After the invention of the telescope, Galileo, in the *Sidereus Nuncius*, page 17, has this about these stars: *In the second example we have drawn the six stars of Taurus called the PLEIADES (I say six, since the seventh almost never appears), enclosed within very narrow bounds in the heaven; next to them lie more than forty other invisible ones, none of which is elongated from any of the six already mentioned by much more than half a degree. Of these we have noted only thirty-six,* etc.

[Margin: Galileo's observation of the Pleiades.]

Afterwards Michaël Florentius Langrenus [Michael Florent van Langren], in a letter very kindly sent to me, described for me their position, accurately observed with the telescope, adding to two stars besides the seven more conspicuous ones the names of Atlas and Pleione. Very recently Don Vincentius Mutus [Vicenç Mut] did the same: in a letter of 1650 March 5, sent to me from Majorca, where he diligently observes the stars, he depicts and determines, relying on observation with the telescope, a most exact place of the seven Pleiades and their distance from one another; and he adds the following small table.

[Margin: And that of Langren.]

[Margin: And that of Don Mut.]

The position of the seven Pleiades.

Pleiades	Longitude in ♉	North latitude	Magnitude
Western, more southerly	24° 28′ 11″	4° 9′ 40″	5
Western, more northerly	24° 38′ 30″	4° 31′ 36″	6
Lowest, next to the western	24° 45′ 0″	3° 53′ 30″	6
Following the more northerly	24° 45′ 30″	4° 23′ 56″	6
Middle and bright star of the Pleiades	25° 6′ 0″	4° 0′ 0″	3
Eastern, more southerly	25° 31′ 30″	3° 51′ 20″	5
Eastern, more northerly	25° 32′ 30″	3° 54′ 0″	7

He regrets, however, that the position of some of them is set down so wrongly by Tycho, and also by Lansbergius [Philips van Lansbergen] in his tables, page 102; for he puts the distance between the bright star of the Pleiades and the more southerly of the western ones at 1° 18′, as great as that between

the first and the middle star of Orion's Belt, whereas even to the naked eye these two Pleiades appear to be much less distant. I have already said, in Book 6, chapter 3, that Father Zupi and I once detected with the telescope, together with the 7 Pleiades, nearly 50′ other stars belonging to that asterism. Finally, Wendelen is of the opinion that new little stars arise and perish in that asterism almost daily, as Antonius Maria de Rheita reports, from a letter Wendelen wrote to him, in Book 4 of the *Radius Sidereomysticus*, chapter 1, member 1.

[Margin: Wendelen's opinion.]

Let this be said so that the reason for counting the seventh Pleiad among the new stars may now be clear: namely, that it is either rather nebulous, or so close to the sixth that it can be distinguished only by keener eyes. For Baranzano clearly erred when he called this one, whose newness is in dispute, the brighter of the Pleiades.

[Margin: Baranzano's slip.]

III. The *second* new star was the one that appeared in the time of Hipparchus, about the year 125 before the birth of Christ. To it astronomy owes a very great deal — or rather to God, Best and Greatest, its maker. For on its occasion Hipparchus, so that posterity might thereafter distinguish new stars from old, and temporary stars from perpetual ones, set about observing the places of the fixed stars, and on the same occasion detected a proper motion in them. For this he most deservedly received from Pliny, Book 2, chapter 26, that encomium: *The same Hipparchus, never praised enough — since no one has done more to prove the kinship of the stars with man, and that our souls are a part of the heaven — detected a new star, and another one, in his own age; and from its motion, on the day it shone, he was led to doubt whether this happened more often, and whether those stars too that we think fixed were moving. He likewise dared a thing presumptuous even for God: to number the stars for posterity and to set the heavenly bodies out by rule, with instruments devised by which the places and magnitudes of each would be marked, so that from this one could easily discern not only whether they died or were born, but whether they passed anywhere at all or moved, and likewise whether they grew or diminished — the heaven being left as an inheritance to all, if anyone were found who would grasp that method.*

[Margin: The new star of Hipparchus, in his age [?], about the year 125 before Christ.]

[Margin: Pliny's impious opinion about God.]

The same Pliny had said in chapter 24 of the same book: *stars are suddenly born in the heaven. There are several kinds of them*; and soon after, in chapter 25, he treats of all the species of comets. When that chapter is finished, he then passes to chapter 26, in which he treats chiefly of the new star seen by Hipparchus. From this distinction, and from the whole tenor of Pliny's way of speaking, it is quite clear that the star already mentioned was not a comet, whatever some interpreters of Pliny have said. For neither was that whole age in which Hipparchus flourished free of comets, as is plain from what was said in section 1, chapter 3; nor was Hipparchus so unskilled that he could not distinguish

a comet from a non-cometic star, if it was distinguishable and displayed any mark of a comet. But if he had no indication of a comet, and from its new appearance, and from its motion in common with the fixed stars — that is, the motion of the primum mobile — together with some slow proper motion, he doubted whether other stars too, which we reckon fixed, had a proper motion, and whether some of them shone anew in the heaven at fixed times, then it is surely an almost evident argument that Hipparchus judged that star to be far different from the genus of comets. Accordingly Tycho, in volume 1 of the *Progymnasmata*, page 322, with truly sharp irony, taunts those interpreters of Pliny already mentioned, saying: *Hipparchus surely accomplished something outstanding, and Pliny commends him highly, if he saw some comet in his own age? Was he moved by such just and weighty reasons that, on account of some hairy star running swiftly about and quite often returning in like manner, he laid so great a burden on his shoulders, like a second Atlas, as to consecrate to posterity, observed with instruments, all the stars contained in the eighth orb that are in any way open to sight?* Hájek, in Tycho, volume 1, page 511, agrees with Tycho that a new star is to be understood in Pliny. But would that Hipparchus himself, or Pliny, or Ptolemy had recorded for us that star's place, time and duration! Since neither they nor others have done so, we know nothing else about this star except that it appeared new in the times of Hipparchus.

IV. The *third* of the new stars is said to have shone in the times of the emperor Hadrian, namely about the year of Christ 130. But Tycho, volume 1, page 331, suspects that it was made up to please Caesar, or that nothing certain about it is established among the historians.

[Margin: A new star under the emperor Hadrian, about the year 130 of Christ.]

[Margin: New star, or comet, of the year 389.]

V. The fourth among the new stars is the one that Hechius, Liceti and Resta, drawing on Cuspinianus [Johannes Cuspinian], place in about the year of Christ 389. Cuspinianus says that in that year, 389, a certain star rose from the north at about cockcrow, shone like Lucifer, the morning star, and vanished within three weeks. But Tycho (volume 1, page 605) suspects that it was a comet without a tail, similar to the one he himself observed with the Landgrave of Hesse in 1585. That year 389 was followed by the irruption of the Goths and the devastation of Rome by Alaric.

[Margin: New star, or rather comet, of the year 392.]

VI. The fifth new star, in the eyes of some, was the one of the year of Christ 392 — the year, that is, in which Valentinian was found strangled with a noose in his bed. It appeared near Venus, and a great many other stars gathered to it like a swarm of bees. But we listed it among the comets under this year, in Section 1 of this book, chapter 3 — and not without reason, as can be seen from Nicephorus, book 12 of the *Ecclesiastical History*, chapter 37, whose words we quoted there.

[Margin: New star of the year 398.]

VII. The sixth new *Sella* [?] on record is the one of which Claudian sang in *On the Fourth Consulship of Honorius*:

A bold star was seen even at midday, while the peoples marvelled — in nothing to be doubted, nor languid with blunted hair, but as great as Boötes is reckoned by night.

But Tycho (volume 1, *Progymnasmata*, page 542) suspects that it was Venus. Venus is sometimes, though rarely, seen not only by day but even at noon — as we have already related more than once, here in Section 1, chapter 4, in the scholium, and in Book 7, Section 6 — and so it could be believed a new star; for this has happened more than once in our own times, when inexperienced people looked up at it in astonishment as an unusual star. As to the year of this star's appearance, however, Tycho judges from certain words of Claudian that come before these that it shone rather in the year in which Honorius was born than in his fourth consulship; that year, he says, was 388 of Christ, or perhaps 389. If so, it seems to be the same as the 4th star, the one Cuspinianus reports. Honorius's fourth consulship, on the other hand, fell in the year of Christ 398.

[Margin: New star of the year 945.]

VIII. The seventh among the new stars is the one that Cyprianus Leovitius mentions in his writing on the star of 1572. He says that it fell in the year 945, when Emperor Otto I flourished, and that it was seen in almost the same place, between Cepheus and Cassiopeia, where the star of 1572 shone out. But he cites no historian from whom he drew this. Granted that Tycho (volume 1, pages 331 and 706), Liceti, Baranzano and Resta mention it on his authority; yet Cyprianus thought it was a comet without hair.

[Margin: New star of the year 1264.]

IX. The eighth new star shone in Cassiopeia in the year 1264, if we believe Cyprianus Leovitius, who says he had this from a certain manuscript, and adds that it was without hair and proper motion. On his authority Liceti, Baranzano, Resta and Tycho (volume 1, page 331) accept it as a new star — although Tycho also raises the suspicion that it was the comet assigned to that year by Camerarius and Pontano, which, however, they say had a tail; whereas Cyprianus, though he reckons this star among the comets, affirms that it appeared without hair.

[Margin: New star of the year 1572 in Cassiopeia.]

X. The ninth new star, and the most celebrated in the books of a great many writers, is the one that appeared in 1572 in the Chair of Cassiopeia and lasted until March 1574. Just like the star of Hipparchus, it gave Tycho the occasion to observe the fixed stars with new and very costly instruments, and to set a thousand of them in order. His observations of it concern partly its longitude and latitude and its distance from the fixed stars, and its altitude above the horizon — these we shall discuss from chapter 3 onward, in order to determine from them its parallax and its distance from the Earth; and partly its other circumstances, which for the learned reader's benefit we shall gather in chapter 3 from the authors who wrote about it. Tycho lists most of these authors in volume 1, *Progymnasmata*, chapters 8, 9 and 10, that is, from pages 489, 598 and

704. Among them Franciscus Vallesius wrongly says that it began to be seen in 1573, whereas it began in 1572, as will be clear from the history in chapter 2.

[Margin: New star of the year 1596 in Cetus.]

XI. The tenth new star is the one of the 3rd magnitude that David Fabritius [David Fabricius] is said to have observed in Cetus on 1596 August 13, as Kepler reports in *De Stella Serpentarii*, page 112, and in the *Astronomiae Pars Optica*, page 446, and as Froidmont and Resta report; it is said to have remained visible there for 3 months, and Liceti also mentions it. It was seen at Aries 25° 45′, with southern latitude 15° 54′, far from the Milky Way.

[Margin: New star in Cygn[?], year 160[?].]

XII. The eleventh new star appeared in the year of salvation 1600. It was seen first by Gulielmus Iansonius [Willem Janszoon Blaeu], as he himself declares in the inscription of a celestial globe published in 1601, and then by Iustus Byrgius [Jost Bürgi], Johann Bayer, Kepler and others. This star was of the third rank, and in position it lay next to the one that shines in the breast of the Swan, at the junction of the neck and the breast — so you need not think it was relegated among the unformed stars. Never before, however, was any such star recorded in the ancient catalogues or globes, not even in Tycho's; and so Kepler, who wrote an astronomical account of it, says, as Bayer does, that it was *by the tacit consent of all accepted as new*. Clavius, Griembergerus [Christoph Grienberger] and Biancani (*Sphaera*, book 18, chapter 3) also accepted it as new, and so did Baranzano, Liceti, Froidmont, Resta, and Argoli in chapter 62 of the *Pandosion*, where he says that it persisted in the sky until 1629, and that after its disappearance a black gap was seen in the sky in its place — unless this was invented in imitation of Tycho, who asserted something similar in the place of the star of 1572 after its departure. Chiaramonti, however, against Liceti (*De tribus novis stellis*, book 3, chapter 6), contends that it no longer appeared in 1626, nor indeed in '21.

[Margin: New star of the year 1602, or [?] 1603.]

XIII. Kepler reports, though doubtfully, a new star seen by him on 1602 December 21, in the *Astronomiae Pars Optica*, page 237; Froidmont and Resta take it from him. But what if this was the one that lasted into 1603, the year in which Father Philippus Plumeretus, as quoted by Piso in the *Speculum Cometae*, chapter 2, related that a new star had been seen by him at Rome?

[Margin: New star of the year 1604 in Serpentarius.]

XIV. The thirteenth new star was the one seen in 1604, at the beginning of October or at the end of September, in the right foot or heel of Serpentarius. It was larger than Jupiter and almost equal to Venus, and it lasted at least until October 1605, on the 8th of which month Kepler saw it for the last time, and never afterwards — partly because it set heliacally, partly because of clouds and the presence of the Moon. He therefore remained uncertain on what day between October 1605 and February 1606 it vanished; for in March, with every impediment removed, it was quite certainly no longer to be seen. We shall report more on its observations from chapter 14 onward.

[Margin: New star of the year 1612.]

XV. The fourteenth of the new stars is said to have been sighted in 1612 by Simon Marius [Simon Mayr] in the girdle of Andromeda, and another in Antinous by Bürgi, as Froidmont relates (*Meteorologica*, book 3, chapter 2, article 7), and as Simon himself does in the preface to the *Mundus Iovialis*.

[Margin: New star of the year 1618.]

XVI. The fifteenth new star is the one that Vincislaus Pantaleo [Václav Pantaleon Kirwitzer], in his observation of the comet of 1618, wrote that he had seen while sailing to the East Indies, resembling Venus; and he called into doubt whether it was a new star — although others seized on what he himself put forward in doubt as certain, and published it. Resta (treatise 1 of the *Meteorologia*, book 1, chapter 7) and Chiaramonti (book 3 of *De novis stellis*, chapter 6) also mention [?] it.

[Margin: The newest star, of the year 1638.]

XVII. The sixteenth and most recent of the new stars is the one that appeared in Cetus in 1638. Ioannes Phocilides Holuarda [Johannes Phocylides Holwarda] described it in a booklet on this phenomenon, together with the eclipse of the same year, and Wendelen also mentions it in a letter to Antonius Maria de Rheita, which Rheita himself quotes in the *Radius Sidereomysticus*, book 4, chapter 1, member 1.

So if we accepted all those enumerated above, there would be 16 new stars; but we have rejected the first and the fifth. Of the rest, some are held to be quite certain, namely 2, 9, 11 and 13; the others are not equally certain. Among these, however, we must examine 9, 11 and 13, since they have been illuminated by astronomical observations, so that from those observations a more certain judgement may be formed about their place and their distance from the Earth, and hence about their origin.

SCHOLIA.

Scholia.

[Margin: Mars and Venus taken for new stars.]

I. Mars, because of the size of its epicycle compared with the eccentric, appears much larger at the perigee of the epicycle than at the apogee or far from the perigee. This happens to it in alternate years, namely when, having become acronychal, it stands in opposition to the Sun. But it looks larger still by far when it is at the same time at the perigee of the eccentric and of the epicycle, so that some have taken it for a new and unusual star — as happened in 1529, 1561, 1593 and 1608, among people, that is, rather unskilled in astronomy. Indeed, for this very reason Andreas Perlachius [Andreas Perlach], professor of astronomy at Vienna, predicted that in 1529 a new star would popularly be believed to have arisen in the place of Mars in July and August; Tycho also reports this, from Hájek, in volume 1, page 745. He rightly adds that the same thing happens to Venus for the same reasons: because of the size of its epicycle

it sometimes comes so near the Earth that, if it is at once at the perigee of the epicycle and of the eccentric, it is swollen by an incredible increase of light and not only appears very large but can even seem a new star to those little practised in astronomy — as we said happened, in Section 1, chapter 4, scholium 2. And perhaps because of changes of this kind Venus was believed to be an extraordinary portent, as Saint Augustine reports from Varro in *The City of God*, book 21, chapter 8, whose words I have already cited in Book 7, Section 1, chapter 2, number 4.

[Margin: Why the star of the three Magi is omitted here.]

II. As for the star that God, Best and Greatest, raised up by divine power, bidding it shine forth to the Magi and go before the sacred birth of his Son and our Saviour, we could in no way count it among the new stars that we are dealing with here. At the end of this section, however, we shall discuss what others have thought or written about it, rightly or wrongly, and we shall show that some have mistakenly placed it among the comets, or among stars generated from a natural cause. Tycho (volume 1, page 601), and with him, in the same work, the Landgrave of Hesse, judge to be full of vanity what Cornelius Agrippa relates in chapter 30 of *De vanitate scientiarum*: namely that, according to a tradition of the Indians, there are still in the heavens two stars diametrically opposite each other, which complete their course against the order of the signs only in 144 years.

[Margin: Two stars falsely ascribed to the sky by the Indians.]

[Margin: A new star confused by the common people with ♂ and ♀.]

III. It sometimes happens that a new star has indeed appeared, and that a rumour about it, spread by the mathematicians among the unlearned public, has given people occasion to flock into the square or some other suitable place and gaze at the sky in the evening; and since Venus was then appearing as the evening star, it was believed — by the unskilled, as I said — to be the new star. So it happened at Vienna and elsewhere at the beginning of October 1604, when a new star had truly appeared in Serpentarius, as Kepler relates in *De Stella nova Serpentarii*, chapter 1. And because the common people believed that the new star was still lasting in 1606 as well, though it had in fact already been extinguished, they took it to be there in April, when it was Venus, and in June, when it was Mars at acronychal rising, as the same author relates in the same place.

CAPVT II.

Chapter 2.

Hiſtoria Obſeruationum generalium Stellæ Nouæ Anni 1572. in Caſſiopea viſæ : & Opinionum de illius Loco ac Origine.

History of the general observations of the new star of the year 1572 seen in Cassiopeia, and of the opinions on its place and origin.

I. We shall begin with Tycho — not because he was the first to notice this star, but because he is considered to have been the most diligent of all in observing it. As he himself relates, then, in volume 1 of the *Progymnasmata*, chapter 3, this star began to be seen by him on November 11; and although he settles nothing certain about the day of its first rising, he thinks it probable that its beginnings were around the new moon, which fell on November 5. Tycho was then staying with his maternal uncle Steno Bille [Steen Bille] at Herrevad; and as he was returning before supper on November 11 from the building where he did his chemical work, he saw near the zenith an unusual star in Cassiopeia, one he had never before observed, although he had been used to recognising the stars since boyhood. To rid himself of all doubt about the new star, in case his eye was deceiving him, he questioned both his servants and some peasants who were driving past in a cart; they answered that they saw a huge star in that same place of the sky, one never noticed before. Confirmed by them, he set about observing with instruments, from then on, both that star and the other stars of Cassiopeia, and at length a thousand other fixed stars. He says first, then, that it

[Margin: Duration of the star.]

[Margin: Motion and position.]

remained in the same place of the sky until March 1574, when it ceased to be seen, and that it had no other sensible motion besides the motion of the primum mobile; for with three other more conspicuous stars of Cassiopeia, of the third magnitude, it completed a figure like a rhombus or trapezium — namely with the one that is higher in the middle of the Chair, with the one in the breast called Schedar, and with a third near the flank. It was also very close to a star of the fourth magnitude that is in the middle part of the seat and is the eleventh of the asterism of Cassiopeia — that is, at the edge of the Milky Way, where its northern boundary near Cassiopeia faces the Arctic Pole. In this position it was seen for the whole of those 16 months and more. Tycho admits, however (page 304), that coachmen, couriers and sailors noticed this star in the sky and told others of it before learned men did.

[Margin: Shape of the star.]

The star's shape was perfectly round, without any hair, and it twinkled more than the other fixed stars.

[Margin: Apparent magnitude.]

Its apparent magnitude at first surpassed Lyra and Sirius, and even Jupiter somewhat, although Jupiter was then acronychal and moving at the perigee of its eccentric; so that it rivalled Venus at perigee very closely, was made out by day, too, around noon, by men of keener sight, and at night shone through dense clouds while the other fixed stars were hidden. In December, somewhat diminished, it equalled Jupiter; but in January 1573 it was smaller than Jupiter, though still larger than Sirius and Lyra, to which it became equal at the end of February and the beginning of March. In April and May it seemed equal to stars of the second rank; in June, July and August to stars of the third magnitude; in

September, October and November to stars of the fourth magnitude. Then in December and in January 1574 it appeared scarcely larger than stars of the fifth magnitude, and in February it resembled stars of the sixth magnitude, until in March it became so faint that at last it could no longer be seen.

[Margin: Colour of the star.]

Its colour at first was clear, whitish and brilliant; afterwards yellow; in March 1573 reddish, like Mars or Aldebaran. In May it took on a somewhat leaden whiteness, such as we see in Saturn, and it kept this colour to the end, though always growing duller. So far Tycho, chapter 3.

II. Next, in volume 1, chapter 4, Tycho goes on to describe the instruments as well with which he observed this star: namely a sextant requiring the work of one eye, an astronomical radius, and a great quadrant of 14 cubits, which he had had built at Augsburg; for that reason he regards the observations made with it there by Paul Hainzel as if they were his own, and he reports them too. From these and from his own observations he gathers, in chapter 5, the new star's longitude and latitude, right ascension and declination as seen, as in the following little table, and its culmination with Aries 0° 19′.

New star of the year 1572.

Longitude	North latitude	Right ascension	Declination
♉ 6° 54′	53° 45′	0° 26′	61° 47′

After this, in chapter 6, he tries to show by many arguments that this star was in the firmament itself, among the other fixed stars, without any sensible parallax. Finally, in chapter 7, digressing on the distances of the planets and the fixed stars and on their sizes, he assigns the fixed stars a distance of not less than 14,000 semidiameters of the Earth, and the new star, at its beginning, an apparent diameter of 3½′; and from these he gathers by geometrical calculation that its true diameter was to the Earth's diameter as 7½ to 1, and the star's bulk to the Earth's body as 361½ to 1, or at least 360, but to the Sun's body as 2⅔ to 1. On page 487, however, he holds that it diminished afterwards not only in appearance but in reality; otherwise, if it had shrunk from a diameter of 3½′ to an invisible disc because of an ascent, as some think, it would have had to rise higher by 300,000 semidiameters of the Earth to be seen as a little star of the sixth magnitude. He therefore sets its true magnitude as different in different months, as you see set out in the following table.

[Margin: Distance of the new star from the Earth, according to Tycho.]

[Margin: True magnitude of the star.]

True magnitude of the new star of the year 1572, from Tycho, page 486.

	It contained the Earth about this many times
1572 November	360
December	200

	It contained the Earth about this many times
1573 January	100
February and March	70
April and May	29
June, July, August	11
September, October, November	4½
December and 1574 January	1
February and March	0⅓

[Margin: Observations of the Landgrave of Hesse.]

III. After Tycho, Vuilhelmus [Wilhelm IV], Landgrave of Hesse, began to observe this new star and sent his observations to Tycho, who reports them in volume 1, chapter 8, from page 491 to 505. The Landgrave's account opens as follows: *In the year of the Saviour's Birth 1572, on December 3, at the prompting of the Elector of Saxony, I first saw and observed the new star, larger and brighter than Venus itself, in the asterism of Cassiopeia, etc.* He then gives its altitudes and azimuths, with the moments of the observations, up to 1573 March 14, when he gave up observing because he had detected no proper motion in it; and he admits that the altitude of the pole at Kassel that he assumed, 51° 16′, does not seem exact to him. In other letters sent to Tycho, and to Gaspar Peucerus [Caspar Peucer], which Tycho has on pages 599 and 612, he affirms that he detected no parallax in this star, or at any rate none greater than three minutes. From these he gathers a distance of 1062 semidiameters of the Earth, and he judged this star to be something supernatural, distinct from comets and from fixed stars.

[Margin: Observations of Hájek.]

IV. Tadeáš Hájek ab Hayck, a Bohemian, ordinary physician to the court of Emperor Rudolf, published a *Dialexis* on the unusual magnitude and the place of this new star, divided into fifteen chapters, in which he includes many general problems about parallaxes. He says that this star was seen by him around the Nativity of Christ the Lord, and not before, equal in magnitude to Arcturus, to the Dog Star or to Jupiter, and of a reddish colour. He adds many distances of the new star from the stars of Cassiopeia. In chapter 4 he teaches that it was new and set in the aether outside the order of nature; and in chapter 5 he contends, from the uniformity of the motion of the primum mobile and from the lack of parallax, that it was in the heavens — which in chapter 12 he attempts to establish by a safer inquiry, from its distances from the neighbouring stars of Cassiopeia when it was crossing the meridian.

[Margin: Observation and opinion of Paul Fabricius.]

V. Paulus Fabricius [Paul Fabricius], physician and mathematician to the Emperor, in a short writing on this star that he presented to the Archduke of Austria, relates that it began to shine at the end of October 1572, greater in magnitude than Sirius but similar in colour. He gives it a longitude of ♉ 6° 14′ and a latitude of 54° 12′, and says that it was [?] free of proper motion and kept

the same distance at its highest and lowest meridian altitudes. From this he argues that it had no sensible parallax, but was either in the sphere of the fixed stars or at least not below Jupiter, and much larger than the Earth — although he hesitates whether this star should be called a comet or not, and leans to the negative.

[Margin: Observation and opinion of Reisacher.]

VI. Bartholomaeus Reisacherus [Bartholomäus Reisacher], mathematician of the Vienna Gymnasium, had at first thought that this star was not new but the eleventh of Cassiopeia, enlarged by the interposition of vapours and of some transparent medium; afterwards, however, when the eleventh was seen distinctly apart from it, he accepted it as new. And from the fact that at both meridian altitudes it kept the same distance from the fixed stars, he concludes that it was not in the elementary region. Although he commits the origin of this star to the hidden counsel of God, or to the Intelligence that governs each sphere, yet if a physical cause must be adduced, he thinks it probable, with the Pythagoreans, that it was one of the ancient stars of the eighth orb, revealed once impediments were removed, and perhaps the same as the one that Cyprianus Leovitius reported as seen in Cassiopeia in 1264. Further, in determining its place at ♉ 6° 24′ with latitude 53° 49′, he uses the uncorrected Copernican places of the stars, the altitude of the pole at Vienna handed down by Purbachius [Georg von Peuerbach] and Regiomontanus, 48° 22′, and a least meridian altitude of the star of 20° and 5′, or 10′, where he admits that he is uncertain about the 5 minutes.

[Margin: Observation and opinion of Hainzel.]

VII. Paul Hainzel — whose observations of this star Tycho presents in volume 1, chapter 6, together with his reply to Hieronymus Vuolphius [Hieronymus Wolf], head of the Augsburg gymnasium, from page 536 — relates that this new light was seen by him on November 7 at 6^{h} after noon at Augsburg, and that from the meridian altitudes of this star the same altitude of the pole at Augsburg is gathered as from the altitudes of other circumpolar fixed stars; and that this is a sign that its apparent and its true place had no sensible difference. And yet he does not hesitate to count it among the comets, and indeed among the secondary ones, which are kindled below the Moon without tail or hair and are gradually consumed, because he noted that this one gradually diminished. In this matter he either shows himself unskilled in astronomy, since he admits the lack of sensible parallax, or the identity of the seen and the true place, and yet places this star below the Moon; or he certainly undermines his own observations — as Tycho rightly reproves him in volume 1, page 540. Yet Tycho reports on page 542 that Hainzel, once he had read through Tycho's treatise on this star, retracted his opinion and, with Tycho, raised this star to the highest sphere of the fixed stars.

[Margin: Observation and opinion of Mästlin.]

VIII. Michael Mästlin of Göppingen, in a short writing on this star, reports that he observed it in a little town of the Duchy of Württemberg called Backnang [Bachnang], where he was pastor — and with no instrument other than threads, in the way we shall describe in its place in Book 10 — and that in the first week of November. He relates that this star surpassed in magnitude not only Sirius but Jupiter and almost Venus, and that it was at first of a whitish colour, then flame-coloured or saffron, afterwards blood-red, and at length, in February 1573, was restored almost to its original colour. He adds that it was a new star, to be counted among the stars of the eighth orb, because its observed place, both on the meridian and near the horizon, was found exactly the same, without a difference of a single whole minute, and that it had no motion other than the daily revolution of the primum mobile. At so great a distance, then, since it surpassed all the fixed stars in apparent magnitude, it was many times larger than the Earth, and therefore no supply of exhalations was sufficient to generate that star. For this reason its cause was rather *hyperphysical*: *this new star was created by the supreme Creator in these last times, and as it began miraculously, so it will miraculously cease; the cause of both escapes all human understanding*. Mästlin wrote this in 1573, on March 4 [4. Nonas Martij], while that star was still shining in the sky. He assigned it a place different from Tycho's; but had he used Tycho's star places, its places would have come out for him as Tycho computes in volume 1, page 550. Here, then, are the computations of both: the first Mästlin's, the second Tycho's — though from the position of the new star relative to the fixed stars of Cassiopeia observed with threads by Mästlin.

Places of the new star of 1572	From the Copernican stars	From the Tychonic stars
Longitude	♉ 6° 35′	♉ 7° 3′ 2″
North latitude	54° 0′	53° 38′ 42″
North declination	61° 50′	61° 46′ 0″
Right ascension	359° 56′	0° 43′ 0″
Culminates with	♓ 29° 56′	✠ ✠ ✠

[Margin: Observation and opinion of Cornelius Gemma.]

IX. Cornelius Gemma, son of Gemma Frisius, wrote about this star both in a separate booklet and in his work *De divinis mundi characterismis*, book 11, chapter 3. In these he reports that its first appearance was, for him, on 1572 November 9; for Hainzel, on the 7th; and for others among the common people, at the end of October, from popular rumour. He says that it was like Phosphorus [the morning star] in strength of light and in magnitude, but more like the fixed stars in its twinkling, and that throughout the whole time of its duration it did not change its place in the sky:

from the positions he himself observed, he at first detected a parallax of not more than 4′, and afterwards none at all. He assigned the star a place in ♉ 6° 30′, with latitude 52° 40′; but from other, later observations of his there comes out a longitude of ♉ 5° 10′ and a latitude of 53°, as Tycho computes

(page 558). Tycho does not put much trust in them, first because Gemma took the distances of this star from the fixed stars with a radius, and second because they disagree greatly with his own. Gemma further denies that this star was a comet, and compares it with the star that shone for the three Magi, and with the portents which, as Holy Scripture testifies, have at times been produced outside the order of nature. Next, he held that it was in the eighth sphere and gradually descended, so that it could have been seen in March of 1572 or even earlier; that afterwards it rose again, and so vanished from sight; and because of its perpetual twinkling he denies that it was anything elementary. But what he fancies about apparitions of angels, or even of God himself, in the form of this star, and about its ascent and descent and a circular return familiar to divine minds, Tycho himself (volume 1, page 564) judged impious and unworthy.

[Margin: Cornelius Gemma's fictions, hardly pious.]

[Margin: Observation and opinion of Muñoz.]

X. Hieronymus Munosius [Jerónimo Muñoz], professor of the Hebrew language at the University of Valencia in Spain and a distinguished mathematician, wrote on this star in Spanish; his treatise was published by Cornelius Gemma at the end of his book *De Characterismis*, and from that by Tycho (volume 1, page 565). Muñoz states that on 1572 November 2 this star had not yet appeared, both from the report of shepherds and because during those days, while showing his students the positions and names of the stars, he did not notice it. He assigns it a place in ♉ 7° 48′, with latitude 53° 36′. In the meridian altitude of the new star — a maximum of 67° 30′ and a minimum of 11° 30′, from which Tycho (page 567) derives the pole height of Valencia as 39° 30′ — he differs greatly from Tycho; in the distances of the new star, however, only a little: namely, from Schedar by 30″, from the Seat of Cassiopeia by 1′, and from the flexure [flexura] by 8′. Yet from these very distances Tycho (page 569) derives a quite different longitude for the new star. Muñoz also disagrees with Hájek on many points; and in the distance of the new star from the Pole Star, which he puts at 26° 40′, he differs from Tycho, who puts it at 25° 14′, and from Hájek, who puts it at 25° 30′. On the strength of careful considerations, as he himself professes, he affirms that this star was above the Sun and had scarcely any sensible parallax; that it was indeed a comet, but one fixed in the heaven of the fixed stars; and that it was one of those unknown comets of which Lucan says:

Dark nights saw unknown stars.

[Margin: Opinion of Francisco Vallés.]

XI. Franciscus Vallesius Covarruviensis [Francisco Vallés of Covarrubias], in his books *De sacra philosophia* — that is, on the things written on physical matters in the sacred books — discusses this star in chapter 1. He says that it began to be seen in 1573 in the Knee of Cassiopeia; but he is badly mistaken on both counts, for it appeared both in 1572 and not in the Knee but in the Chair of Cassiopeia. He grants that it had no parallax and was above the Sun, but he denies that it was a new star: otherwise God would not have created all things at once, according to the text of Ecclesiasticus 18, *He who lives for*

ever created all things at once; nor would the adornment of heaven have been complete from the beginning, whereas Genesis 2 says: *Thus the heavens and the earth were finished, and all their adornment, and God completed his work on the sixth day.* And since this star did not last long, Ecclesiastes 3 would be saying falsely: *I have learned that all the works of God endure for ever.* Vallés therefore holds that it was an old star which, by a peculiar motion of its own, passed into lower and denser parts of the heaven, by which its size was optically enlarged. To this, however, Tycho (volume 1, page 574) replied that the words of Holy Scripture are to be understood of the things that were created at the beginning out of no pre-existing matter and belong to the ordinary adornment and the constituent parts of the heavens — not of adventitious things, whether stars or comets, generated from some matter or created by a miracle, as the star of the Magi was. Otherwise, by Vallés's reasoning, it would follow from those passages that God cannot work any new portent in the heavens, not even by a miracle.

[Margin: Observation and opinion of Clavius, and also of Biancani.]

XII. Christoph Clavius, as we have it in his commentary on the *Sphere* of Johannes de Sacrobosco (from page 191 in my copy), relates that this star appeared in 1572 and disappeared in 1573; that at first it looked larger than Venus, but afterwards smaller and smaller, so that to him in Rome in December 1573 it seemed equal to stars of the third magnitude. From this one may conclude that, had he persevered in observing it, and had neither bad weather nor perhaps the weakness of his eyes stood in the way, he would not have thought that it vanished so soon. He also teaches that it was not one of the 13 ancient stars of Cassiopeia, but a new and fourteenth one[18], distinctly known both to himself and to Maurolico and others, which completed a rhombus with three other stars of Cassiopeia. He further denies that it was a comet in the air, because it was seen to have no variation of view, and was seen by everyone near the point that lies 23° 30′ from the arctic pole, at which the equinoctial colure intersects the arctic circle. He therefore holds that this new star was in the firmament among the other fixed stars — above all because it had no motion other than that of the fixed stars — and that it was created by God, Best and Greatest, to portend something great; or, if this is not granted, that comets can be generated in the heavens, and that the heaven is a changeable body, though less corruptible than other bodies. This, he affirms, Plato and Saints Basil, Ambrose and Gregory of Nyssa, and almost all the other lights of the Church taught not obscurely.

[Margin: Observation and opinion of Maurolico.]

XIII. Francesco Maurolico, in a disputation on this star, has among much else the following words, which our Clavius selected from it and copied into his commentary on the *Sphere* (page 194 in my copy). They run as follows: *In this year 1572 an unusual and more wonderful sign appeared: namely a remarkable star, of exceptional brightness, in a place where no star was recorded. Nor does it*

[18] printed "nonam"; taken as a turned sort for "nouam", since "ninth" does not construe beside "fourteenth"

seem to me to be a comet of the kind generated in the air; for it appears higher, and belongs to the number of the fixed stars. Perhaps, as it began to shine, so it will cease — especially since certain philosophers, with whom Cardano agrees, hold that comets and new stars can be produced even in the heavens by an aggregation of light from the planets and the other stars. However that may be, I cannot wonder enough at this star's new blazing forth in our time. For it is certain that it is not one of the stars of the first magnitude that are recorded in the Ptolemaic and Alphonsine numbers, that have shone since the founding of the world, and that are fifteen; this new star so surpasses them in brightness that from now on they would have to be called of the second magnitude, provided it lasts. Observing this star here on the horizon of Messina, standing on the meridian at about the third hour of the night, I found its altitude to be 62°. From this I conjectured that it is located as if at the top of the arctic circle, so that it is 28° from my zenith here, and accordingly about 66½° from the equator, since the latitude of Messina is 38½°; and that it lies at the point where the equinoctial colure cuts the arctic circle, or very close to that point. But since he differs greatly in the pole height from the value that Don Carolus Ventimilius testifies he found, 38° 11′, some doubt seems to arise about the parallax; on which see below.

[Margin: Observation and opinion of Pridianus.]

XIV. Paulinus Pridianus, physician and astronomer, observed this same star at Antwerp and, as Clavius reports (*Sphere*, page 194), judged it equal to Venus in brightness and size. He wrote that it stayed fixed in the same place, moved only by the revolution of the fixed stars, contrary to the nature of fiery meteors; and that it was a new star which, with the star in the breast of Cassiopeia, the one above the seat near the legs, and a third in the middle of the Chair, formed a rhombus, and shone at the northern point of that rhombus.

[Margin: Observation and opinion of Peucer.]

XV. Gaspar Peucerus [Caspar Peucer], principal professor of the University of Wittenberg, pronounced on this star in summary, partly in a letter to Hieronymus Wolphius [Hieronymus Wolf], which is found in Tycho, volume 1 of the *Progymnasmata*, page 535, and partly in a letter to the Landgrave of Hesse, which Tycho likewise copied out in the same volume, page 605. He declared that it was plainly new, and surpassed the fixed stars and the lesser planets, Venus excepted, in size and light; that he first saw it on November 16, and that afterwards, around December 14, it appeared more like Mars in colour and size, yet always twinkled and had no motion other than that of the primum mobile. Its maximum meridian altitude, observed with instruments at Wittenberg, was 79° 56′, its minimum 23° 33′. He gives its apparent place as ♉ 6° 25′ when it was nearest the zenith, with a latitude of 54° 16′; but when it was nearest the horizon he puts it at precisely ♉ 6°, with latitude 54° 3′, and a declination near the zenith of 61° 58′ and near the horizon of 61° 39′; and from this a parallax of altitude of 19′ — a difference which, however, he admits was not exactly investigated. Yet if the parallax is no greater, the star's distance from the Earth was 164 semidiameters of the Earth. And although he brings many

arguments on both sides, he finally inclines to the view that it was contained in the elementary orb and had been set alight by Jupiter, which it resembled, and that as it had begun, so it would cease; for he was writing this on 1573 January 1 (the Kalends of January).

[Margin: Opinion of Schuler.]

XVI. Vuolfangus Schulerus [Wolfgang Schuler], a mathematician of Wittenberg, in a reply to a question put to him by Caspar Peucer the younger, which Tycho reports (volume 1, *Progymnasmata*, page 621), relates that he began observing this new star on 1572 November 6, at 6 in the morning, at Wittenberg; and that on November 27 he found its greatest distance from the zenith on the meridian to be 66° 27′, and its least altitude 23° 33′. From this, and from the pole height observed there by Crugerus and Rheinoldus, 51° 54′, he derives a declination of the new star of 61° 39′, in an observation made shortly before seven in the morning; and he derives that its greatest altitude should have been 80° 15′, whereas on the evening of December 4 he says he found it to be 79° 56′, and hence a parallax of 19′; and that it therefore appeared to have been above the Moon. He affirms that his quadrant had twice the radius of the Landgrave of Hesse's, and that he also used parallactic rulers of 10 feet. With this set out, he teaches that this star was not any of the planets, because it twinkled, because it was far from the zodiac, and because it lacked a motion of its own; but neither was it one of the fixed stars, because it gradually changed its colour and size. Consequently it was most certainly a comet — not, indeed, an ordinary one, formed and kindled from viscous exhalations, but an unusual one, placed in the heavens by God outside the order of nature as a kind of prodigy among the τέρατα [Greek: τέρατα (terata, "portents")]. After this, however, Tycho relates (volume 1, page 636) that Wolfgang confessed to him that with those parallactic rulers he had found in this star a parallax of scarcely one or two minutes; but to Tycho those rulers seemed by no means perfect in every respect, granted that Johannes Praetorius also used to employ them — of whom we must speak shortly.

[Margin: Imperfection of the parallactic rulers.]

[Margin: Observation and opinion of Praetorius.]

XVII. Johannes Praetorius of Joachimsthal, formerly mathematician at Wittenberg and later at Nuremberg, in his catalogue of comets, which I have read through in full, under the year 1572, listed this new star among the comets for the reason that it finally vanished. He reports that he too saw it from November 16; that it lasted 14 months; that with stars 2, 4 and 12 of Cassiopeia it formed a trapezium rather than a rhombus; that it was in ♉ 7°, with latitude 54° and declination 62°; that it resembled the fixed stars in colour and light, but at first was larger than Jupiter and smaller than Venus, and after two months appeared redder, so that it surpassed even the glow of Mars; moreover, that it stayed fixed in the same place and was above the orbs of the planets, because no parallax could be detected in it. And so, he says, this confirms the opinion of Io. Penna [Jean Pena]: namely, that the air extends from our sight up to the fixed stars

without any difference of substance; that there is no distinction between air and aether; and that there are no fixed bounds, below or above the Moon, within which comets are generated.

[Margin: Opinion and observation of Philipp Apian.]

XVIII. Philipp Apian, son of Peter Apian, in a letter to the Landgrave of Hesse which Tycho has rendered into Latin for us (volume 1, page 643), sets out what he discovered by observing this new star: namely, that it was equal to Jupiter or even larger; that with the three largest stars of Cassiopeia it formed a trapezium at the edge of the Milky Way, towards the north on the western side; that it was very close to the equinoctial colure; and that it was in ♉ at about 6°, with a latitude of about 53°. He himself saw it from November 10, but on the report of others it was seen earlier, though not before 1572 October 20; and it was driven by no motion other than the diurnal revolution of 24 hours. He further holds that it was a comet without hair or tail, of the kind he thinks are generated from a conjunction of ♃ and ♄, following the view of John the Englishman; yet that it was not in the elementary region, but almost in the sphere of the fixed stars, since comets can arise anywhere in the heavens, not so much from kindling as from the illumination of the Sun. So he wrote at Tübingen on 1572 December 26.

[Margin: Observation and opinion of Digges.]

XIX. Thomas Diggeseus [Thomas Digges], an Englishman, in his *Alae seu Scalae Mathematicae*, which he published on the occasion of this star and which Tycho summarised (volume 1, from page 653), found the distances of this star from the other stars of Cassiopeia, though taken with a radius, very nearly equal to Tycho's; for they differ from them by no more than 3 or 4 minutes. But he uses the Copernican places of the fixed stars. Later, in chapter 10, he affirms that this star was subject to no parallax greater than two minutes, and indeed that it is uncertain whether it had any. Tycho, however, refutes him (page 682), because from Digges's own observations the longitude of the new star comes out now as ♉ 9° 46′, now as 4° 22′. John Dee of London, on the other hand, thought that this star vanished by ascending along a straight line — which Tycho attacks on page 691.

[Margin: Opinion of John Dee.]

[Margin: Observation and opinion of Camerarius.]

XX. Elias Camerarius, professor of mathematics at Frankfurt an der Oder, in his writing on this star, asserts that this new star appeared in 1572, at the beginning of October, in the evening, at about the 10th hour, near the meridian, larger than Jupiter and smaller than Venus; and he assigns it a longitude of ♉ 7° 10′ and a latitude of 53° 44′. He denies, however, that it was a fixed star or a planet. Next, he affirms that the pole height at Frankfurt is, by observation, 52° 24′; from this, and from various meridian altitudes of the new star, upper and lower, he derives a parallax of altitude of sometimes 4½′, sometimes 10′ or 12′. From these he concludes that it was indeed a comet, but above the Moon, and that it vanished by gradually ascending.

[Margin: Opinion of Reinhold.]

XXI. Erasmus Reinhold, son of the Erasmus Reinhold who built the Prutenic Tables and deserved so well of astronomy, a doctor of medicine, in his Prognostication for the year 1572, derives from the Landgrave's observations — as Tycho maintains (volume 1, page 700) — a parallax of altitude of 2′ 12″, and hence a distance from the centre of the Earth of 1079 semidiameters of the Earth; and a true diameter of the star, on 1572 December 25, of 5385 German miles, with a solidity relative to the Earth as 31 to 1; but at the end of August 1573 a diameter of 1888 miles, and a solidity relative to the Earth as 67 to 50. In the end he denies that it was either a planet, or one of the fixed stars, or a comet, and concludes that it was a new work of God, produced outside the order of nature and not to be subjected to astrological rules.

[Margin: Observation and opinion of Karásek Lvovický.]

XXII. Cyprián Karásek Lvovický, a Bohemian, in his astronomical observations on this star, reports that he began observing it on 1572 November 25; that it looked larger than any of the lesser planets, and yellow and half-white in colour, but reddish towards the beginning of December and blood-red at the end of December. He therefore judges that it was kindled by Jupiter and Mars, and that it could be seen between the 2nd and 3rd hour after noon because of its great distance from the Sun and the brightness of the star. Its longitude was in ♉ 8°, and its latitude about 50°; it stayed fixed in the same place and had no proper motion for two months; afterwards, within one month, it was seen to advance a little, up to 3 degrees, northwards towards Cepheus, and to decrease in size daily. Finally he calls it a comet; and Tycho (volume 1, page 704) counts him among the authors who thought it a sublunary meteor, and (page 707) denies that it departed from its original place by even a single minute, let alone a degree.

[Margin: Opinion of Chytraeus.]

[Margin: Tycho's religion suspect.]

XXIII. David Chytraeus, a disciple of Philipp Melanchthon — a most pestilent man, condemned by the Church — (since Tycho, volume 1, page 712, warmly commends both of them in theological matters, just as he also defends Theodorus Beza [Théodore de Bèze] on page 327 and Luther on page 777, let prudent and Catholic men judge what is to be thought of Tycho; Tycho admits, however, that David was not a professor of mathematics) nevertheless counts this star among the comets, compares it with the comet seen under Nero, and from this concocts a great many dreams of astrological vanity. And although later, in his writing on the comet of 1577, he admitted that it had been in the aether, because he heard that no parallax could be observed in it, he nevertheless denied that it was generated in the aether, and thought it was produced in the elementary region.

[Margin: Opinion of Postel.]

XXIV. Gulielmus Postellus [Guillaume Postel], a Frenchman, put forward many astrological claims about this star, and noted, with Tycho, that it fell in

the seventh period of the greatest conjunctions, and on the colure of the vernal equinox, near which the greatest conjunctions take place, as we said in Book 7, Section 5, chapter 9. He adds that it was without parallax, and that it remained in the same state except for its size. The rest is full of the emptiness of astrology and the Cabbala, for which he was in the end expelled from our Society by our holy father Ignatius. On this, and on his death, Father Daniel Bartolus [Daniello Bartoli] says more in volume 1 of the *History of the Society of Jesus*, book 3, from page 371.

[Margin: Opinion of Raimondo.]

XXV. Annibal Raymundus [Annibale Raimondo] of Verona, as Tycho reports (volume 1, page 720), says that this star began to be seen in Italy in 1572, at the beginning of October, and by him on the 5th; and that it was by no means new, but the eleventh of the ancient stars of Cassiopeia. Tycho rebuked him, however, and exposed the occasion of the error: although this new star, to those of sharp sight, appeared distinct from the eleventh from the very beginning, it was more clearly distinguished once it had diminished; so it is not so surprising if, to duller eyes, it was taken at first for the eleventh, to which it was very close. And granted that Hyginus says that in the square of Cassiopeia there are certain stars shining more brightly, those are nevertheless of the 3rd or 4th magnitude, whereas this one surpassed the fixed stars of the first magnitude. Raimondo adds, however, that this star attracted a great quantity of exhalations, by the force of which, and of refraction, it appeared larger than usual. But why, then, do Sirius, Lyra and Arcturus not sometimes do the same? Of almost the same opinion was Cornelius Frangipanus, who thought that this star was the same as the eleventh of Cassiopeia, but that by some violent motion it had migrated two degrees northwards and had increased its light, as Tycho relates from Hájek (volume 1, page 741). He also — from what source I do not know — either invented, or took over the invention, that the Pole Star had hidden itself ever since the capture of Constantinople by the Turks, just as the poets invented of Electra after the burning of Troy.

[Margin: Opinion of Frangipanus.]

[Margin: Opinion of Nolthius.]

XXVI. Andreas Nolthius of Einbeck, in a treatise of eight chapters on this star dedicated to the Landgrave of Hesse, affirms that it was a sublunary comet of the kind that has neither tail, nor beard, nor hair, nor a motion of its own; he holds that a certain kind mentioned by Epigenes in Seneca was of this sort. He adds that it did not keep exactly the same distance from the same fixed stars, but swayed somewhat this way and that because of parallax; and that at an apparent altitude of 12° 21′ he detected a parallax of 39′, which will be discussed separately below. For him the longitude of the star was ♉ 5° 4′, the latitude 54° 48′, the declination 61° 49′, and the right ascension 357° 11′. Further, from that parallax he derives the distance of this star — or, as he himself calls it, this comet — from the Earth as 19, or nearly 20, semidiameters of the Earth, that is, 17,059½ German miles; and since he says that he found its apparent diameter with a

radius to be 10′, he concludes that its true diameter was 46 German miles, and its solidity relative to the Earth as 1 to 50,000. The rest, in which he soothsays astrologically, does not concern us.

[Margin: Opinion of Busch.]

XXVII. Georg Busch, painter and astronomer at once, of Erfurt, published two booklets on this star, which he judged to be a sublunary comet of the species they call the Rose, formed from dry and viscid fumes; he attributes to it a retrograde motion towards Cepheus, and confirms that it was a comet from the fact that it was seen by day at the beginning — as if Venus, which has been seen by day more than once by us and by many others, were therefore sublunary. He sets its longitude at ♉ 6°, and its distance from the Earth at 30 semidiameters of the Earth; and since on 1572 December 12 he assigns it an apparent diameter of 1° 28′, he derives from this and from the distance a true diameter of the head of 664 German miles, and its solidity relative to the Earth as 1 to 18. Later, on February 20, he affirms that this star had moved 2° 20′ away from a small star of Cassiopeia to which it had been close. But what he relates about this motion, and about an apparent size three times greater than the Sun's, runs contrary to the most trustworthy testimonies of very many others, as we have seen.

[Margin: Opinion of Graminaeus [?].]

XXVIII. Theodorus Graminaeus, professor of mathematics at Cologne, contends in a lengthy treatise that this new star was a tailed comet, but with a short and hidden tail, because, while the Sun was passing through ♐ and ♍, the tail was turned away from our sight towards the opposite parts and upwards. But since, turning aside to astrological fictions, he interprets the prophecies of Abbot Joachim and of Lichtembergius [Johannes Lichtenberger] against Luther and his followers, and for this purpose uses Luther's genethliacal theme, constructed long ago by Gaurico and Cardano, Tycho (volume 1, pages 776 and 777) is too much aggrieved that Graminaeus *unjustly and maliciously censures and mocks Luther and his fellow-tribesmen* — these are Tycho's words. Yet he immediately adds: *Nor do I say this because I wish to interpose a judgement on the controversies of religions. Let him judge these who alone knows how to distinguish the true from the false, to search the hearts of all, and to decide all things without prejudice.* But even from this he shows himself suspect of that heresy which denies to the Roman Pontiff infallible judgement in controversies of religion. And where does that iniquity and malice against Luther, charged to Graminaeus, come from, if not from Tycho's excessive affection for Luther? So too on page 712 he commended Philipp Melanchthon far too warmly — a man of the most pestilent doctrines, who bears in his very surname a token of the blackest faith. Io. Heckius, moreover, calls Tycho a Calvinist in his disputation on the new star of the year 1604, page 20; and indeed Tycho (volume 1, page 327) pours himself out excessively in praise of Théodore de Bèze.

[Margin: Tycho suspected [?] of heresy.]

[Margin: Opinion of Adamus Ursinus.]

XXIX. Adamus Ursinus of Nuremberg, in his Prognostication for the year 1574, lists this star among the sublunary comets, the *chrysoi* or golden ones, and says that it surpassed the fixed stars and the planets in size, Venus excepted, at ♉ 6°; but he admits that it kept the same distance from the three more notable stars of Cassiopeia, and had no revolution other than the common one about the pole of the world.

[Margin: Opinion of Andreas Rosa [?].]

XXX. Andreas Rosa Singfurdensis, a physician, in his prognostication, likewise for the year 1574, was in doubt whether he ought to count this star among the Plinian comets, and in the end said that he preferred to call it, with the common people, the New Star. And, on the authority of a certain Elias, a Talmudist, he says that roughly every two thousand years portents of this kind come forth from heaven: something of the kind happened in the first period, at whose end came the Flood, and again at the end of the second period, when the star appeared to the Magi. Of this vanity we shall say more below.

[Margin: Opinion of Antonio Santucci.]

XXXI. Antonio Santucci, mathematician of the University of Pisa, in his book *De Cometis*, chapters 6 and 7, says that this star appeared in 1572, at first of the sixth magnitude, and afterwards grew, so that in March 1573 it equalled Venus and at the end of May had become twice as large as Venus, and remained that large for 3 months; and that it shone so brightly that it cast a shadow from opaque bodies; then that from the end of October it diminished, until in April 1573 it vanished from sight; that it twinkled moderately and always stayed in the same place. How many other authors contradict him on its size, and how greatly, is clear enough from what has been said; but he contends that this star was above the Moon.

[Margin: Opinion of Giovanni Battista Benedetti.]

XXXII. Io. Baptista Benedictus [Giovanni Battista Benedetti], disputing about this star against Annibale Raimondo in a letter that is found among his others (page 371), tries to show that this star was above the Moon.

[Margin: Opinion of Chiaramonti.]

XXXIII. Scipione Chiaramonti, however, in books 1 and 2 of *De tribus novis stellis*, is wholly taken up with refuting the others, who had raised this star above the Moon. We shall weigh his arguments in their place, and report the various distances deduced from various hypotheses in chapter 10, numbers 4, 8 and 9.

XXXIV. Father Giuseppe Biancani (*Sphaera*, book 18, chapters 2 and 5) agrees with Tycho on this star's form, light, twinkling, immobility, place and distance from the Earth; but he thinks it more probable that it became visible by descending towards us, and then invisible by ascending, rather than that it was newly generated and perished.

Anyone who wants the opinions on distance and size gathered together should see chapter 16, to be given below.

CAPVT III.

Chapter 3.

Hiſtoria Obſeruationum Eiuſdem ſtellæ particularium quoad Numeros ac Menſuras.

A history of the particular observations of the same star, as to numbers and measures.

I. Since, for forming a judgement on the place and parallax of this star, a great deal is contributed by a comparison of its distances from the fixed stars, and of its altitudes — especially the meridian altitudes — and its azimuths, which, observed by various astronomers, are found scattered through volume 1 of Tycho's *Progymnasmata*, but collected together in Chiaramonti, book 1 of *De tribus novis stellis*, from chapter 2 to 9 — though in a different form from the one we would wish — we have decided to arrange these observations under definite heads and at the same time to gather them into the synopsis of a single table.

Diſtantiæ Nouæ Stellæ à Fixis obſeruatæ.

Distances of the New Star observed from the fixed stars.

[Margin: Dispute over the rhombus or the trapezium.]

II. First, it should be noted that the New Star, together with the stars of Cassiopeia — the second, called Schedir; the 4th, which is at the flank; and the 12th, which is the bright star of the Chair — formed a rhombus, as Clavius thought, and Paulinus Pridianus, an assertion Clavius calls the common one. Others, however, whom Praetorius leaves unnamed, held that it formed a rhomboid. Still others affirmed that it formed a trapezium, as did Tycho, Johannes Praetorius, Hájek and Philipp Apian — and rightly, if one accepts the distances of the New Star from the already-mentioned stars, together with the distance of the second from the fourth, which for Tycho is 4° 40′, and of the second from the twelfth, which is 4° 59′. For the side opposite this, that is, the distance of the New Star from the fourth, was 5° 2′, and the distance of the New Star from the twelfth, which stood opposite the former side, was 5° 19′.

Secondly, note the disagreement of some observers, expressed without numbers. Cyprián Karásek Lvovický said that the New Star, with its body, fell exactly on that part of the seat on which Cassiopeia's buttocks rest. Digges, by contrast, says that it was always in one and the same line with Cassiopeia's knee and with the star in the right side of Cepheus, below the girdle; and moreover that it lay in another straight line with the stars of Cassiopeia's hair and of Cepheus's left shoulder. Cornelius Gemma, for his part, relates that it was in a straight line with the more conspicuous star in Cassiopeia's loins, the bright star of Perseus (that is, as Tycho interprets it, p. 513, with Medusa's head) and the Pleiades; and likewise in a straight line with the Pole Star and the star in the haunches of Ursa Major.

As for definite distances of the New Star, Tycho reports his own from his earlier writing, *Progymnasmata* volume 1, p. 585. On pp. 593 and 341, however, he admits that they are uncorrected, because at that time he had not taken account of the parallax of the eye, that is, the distance of the pupil from the centre of the instrument, and had not arranged the sighting through parallel slits in the pinnules, but through holes. He therefore corrected them afterwards, as is plain from p. 342 itself, and gives the distances so corrected on pp. 344, 508, 554, 558 and 566. He also reports distances taken by Hájek, p. 508, but corrected by the same Hájek, p. 509; and those taken by Cornelius Gemma, pp. 554 and 558 — for these differ from the ones the same Gemma afterwards recorded in his *Cosmocritice*. The distances taken by Digges Tycho reports on p. 654, those by Muñoz on p. 566, and those by Camerarius on p. 694; and they are set out below.

[Margin: Distances of Tycho, Hájek, Gemma, Digges, Muñoz, Camerarius.]

Distances of the New Star of the year 1572 observed from the fixed stars, collected from various authors.

Fixed stars from which the New Star was distant	Order among the stars of Cassiopeia	Tycho	Digges	Gemma	Hájek	Muñoz	Camerarius
The middle star of the Chair	11	1° 31′	1° 28′	1° 24′	formerly 1° 24′; later 1° 26′		0° 50′
The bright star of the Chair	12	formerly 5° 21′; corrected 5° 19′	5° 15′	formerly 4° 40′; later 5° 4′	5° 15′	5° 20′	
Schedar	2	formerly 7° 55′; corrected 7° 50½′	7° 45′	formerly 6° 58′; later 7° 24′	7° 47′	7° 50′	
Cassiopeia's flank	4	formerly 5° 1′; corrected 5° 2′	4° 58′	formerly 4° 28′; later 4° 36′	formerly 4° 51′; later 5° 3′	5° 10′	
Cassiopeia's girdle	3	6° 53′		6° 36′	7° 0′		
Cassiopeia's knee-joint	5	8° 3½′	8° 5′				
Cassiopeia's leg	6	9° 48′	9° 45′				
The foot of the Chair	7	12° 58½′					
Cassiopeia's head	1	10° 22′		9° 36′			
The Pole Star		25° 14′		formerly 23° 48′; later 24° 40′	25° 30′		
Capella		42° 28′		42° 4′			
The bright star of Perseus		27° 22′		27° 7′			

Altitudines Meridianæ Stellæ Nouæ obſeruatæ cum Declinationibus inde, & ex Poli Altitudine deriuatis.

Meridian altitudes of the New Star observed, with the declinations derived from them and from the altitude of the pole.

III. It is well known that the declination of a star from the equator is obtained if the star's distance from the pole of the world is subtracted from 90°; and that this distance becomes known either if the altitude of the pole is subtracted from the greatest meridian altitude of the star itself, or if the star's least meridian altitude is subtracted from the altitude of the pole. Understand this of a star always visible toward the north, such as was the New Star we are dealing with here. We shall therefore report the altitudes of the New Star observed in the meridian, which Tycho also reports, with the altitudes of the pole, in volume 1 of the *Progymnasmata*: his own on pp. 347 and 351; Hainzel's on pp. 360 and 536; the Landgrave of Hesse's on pp. 491, 599 and 613; Muñoz's on p. 567; Peucer's on p. 605; Schuler's on p. 621; Camerarius's on p. 693; Georg Busch's on p. 769; Adam Ursinus's on p. 782; and Hájek's on p. 514. Maurolico's altitude, both of the pole and of the star, is reported by Clavius on p. 194 of the *Sphaera*, and Santucci's by Chiaramonti in book 2, p. 339. We shall add the declinations deduced from the already-mentioned elements. In the margin, moreover, the time after noon will be noted, unless the observers are silent about it.

Observers and places	Meridian altitude of the New Star	Northern declination of the New Star	Altitude of the pole
Hainzel at Göggingen, near Augsburg	greatest 76° 34′ 0″	61° 48′ 0″	48° 22′
	greatest 76° 33′ 45″	61° 48′ 15″	
	least 20° 9′ 40″	61° 47′ 40″	
	greatest 76° 34′ 0″	61° 48′ 0″	
	least 20° 9′ 30″	61° 47′ 30″	
The same, at the same place	least 20° 9′ 40″	61° 47′ 40″	48° 22′
	greatest 76° 35′ 0″	61° 47′ 0″	
	least 20° 9′ 30″	61° 47′ 30″	
	least 20° 9′ 20″	61° 47′ 20″	
Tycho at Herrevad	greatest 84° 11′ 0″	61° 47′ 0″	55° 58′
	least 27° 45′ 0″	61° 47′ 0″	
The Landgrave of Hesse at Kassel	greatest 79° 30′ 0″	61° 48′ 0″	51° 18′

Observers and places	Meridian altitude of the New Star	Northern declination of the New Star	Altitude of the pole
	least 23° 2′ 30″	61° 44′ 30″	
	least 23° 3′ 0″	61° 44′ 0″	
Peucer at Wittenberg	greatest 79° 56′ 0″	61° 58′ 0″	51° 54′
	least 23° 33′ 0″	61° 39′ 0″	
Schuler at the same place	least 23° 33′ 0″	61° 39′ 0″	
Camerarius at Frankfurt an der Oder	greatest 80° 30′ 0″	61° 54′ 0″	52° 24′
	least 24° 8′ 0″	61° 44′ 0″	
	greatest 80° 27′ 0″	61° 57′ 0″	
	least 24° 17′ 0″	61° 53′ 0″	
	least 24° 20′ 0″	61° 56′ 0″	
	greatest 80° 26′ 0″	61° 56′ 0″	
Busch at Erfurt	greatest 79° 20′ 0″	61° 50′ 0″	51° 10′
	least 23° 2′ 0″	61° 30′ 0″	
Muñoz at Valencia	greatest 11° 30′ 0″	62° 0′ 0″	38° 30′
	least 76° 30′ 0″	62° 0′ 0″	
Adam Ursinus at Nuremberg	greatest 79° 0′ 0″	60° 24′ 0″	49° 24′
	least 22° 0′ 0″	62° 36′ 0″	
Maurolico at Messina	greatest 62° 0′ 0″	66° 30′ 0″	38° 30′
Santucci at Pisa	greatest 79° 30′ 0″		
	least 7° 30′ 0″		43° 30′
Hájek at Vienna	least 20° 15′ 0″	61° 53′ 0″	48° 22′

The altitude of the pole is, however, more correctly: at Wittenberg 51° 46′; at Frankfurt 52° 6′; at Erfurt 51° 6′; at Nuremberg 49° 26′; at Messina 38° 10′.

[Margin: In the year 1572, November: day 14, 8^h; 21st, $7\frac{2}{3}^h$; 21st, $19\frac{2}{3}^h$; 26th, $7\frac{1}{8}^h$; 26th, $19\frac{1}{10}^h$.]

[Margin: 1573: January 2, 16 1/[?]^h; 4th, $4\frac{1}{4}^h$; March 2, $12\frac{2}{5}^h$; 12th, 11 11/12^h.]

[Margin: 1572, December: 26th, 4⅚^h; 26th, morning; 1573, January 2.]
[Margin: December 4.]
[Margin: November 26, 19^h.]
[Margin: November 23, evening and morning; January 5, 6, 7, and morning; April 7; August 7.]
[Margin: November 25, 7¼^h; 25th, 19^h.]
[Margin: May 6, 8^h 29^m.]

Altitudines & Azimutha Stellæ Nouæ Obſeruata.

Altitudes and azimuths of the New Star observed.

IV. Tycho reports, in volume 1, the azimuths and altitudes observed by the Landgrave of Hesse at Kassel, on p. 491; those observed by Nolthius at Einbeck, on p. 754; and those of Hájek at Vienna, on p. 514. But on p. 496 he concludes that the altitude at Kassel of 71° 20′ must be corrected, and 73° 20′ put in its place.

	After noon (day, hours, minutes)	Alti-tude	Western azimuth
Observations of the Landgrave of Hesse made at Kassel in the year 1572 — December	3, 7^h 52^m	75° 42′	144° 0′
	3, 8^h 16^m	73° 20′	138° 0′
	3, 8^h 34^m	71° 25′	135° 0′
	3, 9^h 16^m	66° 40′	131° 30′
	4, 8^h 11^m	73° 20′	138° 0′
	4, 8^h 16½^m	72° 45′	137° 0′
	10, 7^h 11¼^m	76° 32′	147° 0′
	10, 7^h 14^m	76° 19′	146° 0′
	10, 7^h 20½^m	75° 42′	144° 0′
	26, 4^h 21^m	78° 43′	161° 0′
	26, 4^h 26½^m	79° 0′	164° 0′
	26, 4^h 38^m	79° 22′	171° 0′
	26, 4^h 51^m	79° 30′	180° 0′
Remainder of the preceding table			
	After noon (day, hours, minutes)	**Alti-tude**	**Western azimuth**
By the same Landgrave, at the same place, in the year 1573 — January	11, 4^h 35^m	77° 37′	152° 30′
	11, 4^h 36^m	77° 33′	152° 0′
	11, 4^h 37½^m	77° 27′	151° 30′
February	26, 7^h 58^m	36° 45′	145° 30′
	8^h 3^m	36° 20′	146° 0′
March	11, 7^h 50^m	33° 30′	149° 30′
	11, 7^h 53½^m	33° 10′	150° 0′
	14, 8^h 2^m	31° 43′	152° 0′
	Day, hours, minutes	**Alti-tude**	**Eastern azimuth**
Nolthius, November 29		77° 0′	60° 0′

	After noon (day, hours, minutes)	Alti-tude	Western azimuth
Hájek, May	6, 10^h 7^m	22° 0′	12° 13′

I do not add the longitudes and latitudes, or the right ascensions, deduced from the already-mentioned observations: both because they were not observed directly, and because they were derived as differing values — some from the Alphonsine Tables, others from the Prutenic Tables, Tycho from his own places of the fixed stars. They will, however, be indicated below in their proper places, where needed.

CAPVT IV.

Chapter 4.

An Argumentis Parallaxin non inuoluentibus demonſtratum ſit, Stellam Nouam Anni 1572. fuiſſe ſupra vel infra Lunam.

Whether it has been demonstrated, by arguments not involving parallax, that the New Star of the year 1572 was above or below the Moon.

[Margin: Enumeration of the arguments.]

I. The arguments by which many have tried to show, independently of parallax, that this star was above the Moon, or even in the very sphere of the fixed stars, are six: namely, its twinkling; the intensity of its light or brilliance; the absence of a tail and of hair; its simple revolution with the primum mobile, without any motion of its own; its duration; and its aspect among the same stars — that is, its configuration with them — as seen from the whole of Europe. By contrast, the arguments by which the authors of the opposing opinion contended that it was generated below the Moon, like some meteor, are three: namely, its beginning and ceasing after so short a duration; the decrease of its apparent magnitude; and the variation of its colours. But neither kind of argument is demonstrative, as we shall shortly show.

[Margin: 1st argument: from twinkling.]

II. Aristotle (*De caelo* 2, text 48 or 158), and with him many Peripatetics — especially Philalthaeus on that passage, and Amicus, tract. 5, q. 7, d. 3, as well as Cardano in *De subtilitate* — hold that the cause of the twinkling that appears in the fixed stars is their distance. Our sight, they say, cannot reach that far without some tremor and wavering, and, being deceived, it supposes that its own tremor takes place in the stars themselves; the planets therefore do not twinkle, because they are near. Copernicus valued this sign for distinguishing the fixed stars from the planets so highly that in book 1 of the *Revolutions*, chapter 10, he thought it demonstrated from it that there is a vast distance between the wandering and the non-wandering stars, in these words: *"For that there is still a very great distance from Saturn, the highest of the wanderers, to the sphere of the fixed stars, their twinkling lights show; by this sign they are chiefly distinguished from the planets."*

Since, therefore, the New Star of the year 1572 was seen to twinkle no less than any fixed star — indeed more — it seems necessarily to follow that it was not only above the Moon but also above all the planets. Tycho used this argument not as merely probable but as apodictic and demonstrative, in volume 1 of the *Progymnasmata*, chapter 6, p. 401, saying: *"Moreover, its outstanding twinkling, flashing beyond the other lights of heaven, showed most plainly of all that it had chosen its place within the highest and most vast region of the fixed stars, and had surpassed the circuits of all the planets, not to speak of the lowest, elementary world."* And again on p. 405: *"Without any evasion it must rightly be admitted that this New Star — since by its continual twinkling, persisting as long as it was visible, it not only imitated the fixed stars but even surpassed them, especially at the beginning, by this vibration of its light — was not within those limits of heaven in which the circuits of the seven planets are contained, but stood among the most remote fixed stars."* With the same argument Kepler, in chapter 17, contended that the New Star of the year 1604 was among the fixed stars — using, however, an argument drawn from the Peripatos against the Peripatetics. Reduced to form, then, the argument is this:

[Margin: The argument in form.]

III. *Every star that twinkles as long as it is visible is above the planets, not merely above the Moon. But the New Star of the year 1572 twinkled as long as it was visible. Therefore it was above the planets, not merely above the Moon.*

The reply, however, is to deny the major premise, if it is put forward as necessary and infallible; for if it is put forward as highly probable, it is freely conceded. The reason for denying it is that twinkling, even continual twinkling, is neither an effect necessarily connected with a distance greater than the Moon's, nor an infallible sign of such a distance. For twinkling can arise from some cause other than the star's distance and the tremor of sight, as is clear from what was said on the twinkling of the fixed stars in book 6, chapter 2, number 6 — just as the absence of twinkling is not a necessary sign of nearness, since Tycho himself placed all comets above the Moon, although he knew that most of them shone with a faint light that did not twinkle at all. Chiaramonti adds, in book 2 of *De novis stellis*, chapter 2, that lamps twinkle from far off, and so do some comets — which, however, it has not yet been demonstrated were not sublunary, as we showed in section 1, chapter 22.

[Margin: 2nd argument: from light and colour.]

IV. Nor are the light and colour — in which this star closely resembled the light and colour of the fixed stars, and was far more splendid than any comet or sublunary meteor — a demonstrative argument for so great a distance of this star that it would necessarily have to be placed not only above the Moon but also above all the planets. For although Tycho uses such an argument, in volume 1, p. 401, saying: *"In its clear, pure, radiant light, too, it altogether rivalled the genuine stars of heaven, and had nothing in common with sublunary meteors or with the duller and less brilliant light of comets. Therefore it far surpassed the seats both of meteors and of comets, and so necessarily stood among the fixed stars"*; and a

little later: *"A colour plainly celestial was also in this star, so that it never took on any colour of such a kind that some star in heaven itself, very like it and endowed with the same colour, was not seen; from which it is also entirely consistent that it claimed celestial seats for itself as well"* — nevertheless the inconstancy of its colours, in which it was unlike the fixed stars, and the continual decrease of its light, take much away from that former likeness. Nor is it impossible for a sublunary comet to become so intense that it equals that light and colour; or at least, if this cannot happen by the powers of nature, it can by God working beyond nature. Tycho himself admits that this happened in the star of the Magi, volume 1, p. 399, saying: *"The star that appeared to the Magi, although it went before them in the air not far from the Earth, and was doubtless not unlike the other lights of heaven, was nevertheless a particular portent of God, shown to those Magi by singular grace, and not to be counted among natural things, however miraculous."* Now, that this New Star of the year 1572 was a natural work rather than a preternatural one, Tycho did not demonstrate, nor could anyone demonstrate it. On the contrary, many held, not improbably, that it was among preternatural portents — especially Reisacher, Mästlin, the Landgrave of Hesse, Schuler, Cornelius Gemma, Reinhold, Andreas Rosa, and our own Clavius; and those who said this of all comets would have said the same of this star much more, as we related in section 1, chapter 6, number 12. But if this star was made beyond the order of nature by God, whether by himself or through the Intelligences, it could certainly have had that vivid light and a colour rivalling the stars, however far below the Moon it is supposed to have been. Chiaramonti further adds, in book 2 of *De stellis novis*, chapter 2, that if a greater light showed a greater distance, the Sun would have to be placed above Saturn — which no astronomer would grant.

[Margin: 3rd argument: from shape.]

V. But Tycho will say, volume 1, p. 398: no comet was ever seen that was not either tailed, or bearded, or haired all round — unless perhaps, because of its opposition to the Sun and its immense distance, the appendage of the tail or the partings of the hair could not be discerned. Therefore this star, which in shape was most like the other fixed stars, and was neither rayed with any curls of hair nor imitated comets by any train of a tail or growth of a beard, either was not a comet, or, if it was, nevertheless concealed these features because of its almost immense distance from the Earth. And so that you may see with what lavish mouth Tycho bursts out into words of necessity where there is mere probability, here are his own words from volume 1 of the *Progymnasmata*, p. 398: *"Its very face and august form, most like the other stars shining in the world, plainly proved that this star had nothing in common with elementary and sublunary meteors."* But here too I flatly deny that this argument plainly proves this star to have been above the Moon. For if it was produced by divine agency, it could have been below the Moon and yet lacked all projection of rays and been perfectly round. Nor does it necessarily follow that, if it was not a comet, it was something above the Moon. Though I say this, I do not, however, reject

the probability of Tycho's argument. Chiaramonti too, in book 2 of *De tribus novis stellis*, chapter 2, denies that this argument is conclusive, because both perfectly round comets and new sublunary stars, likewise round, can exist — or at any rate are not shown to be impossible.

[Margin: 4th argument: from the common motion and the star's own immobility.]

VI. Perhaps greater weight seems to attach to the argument from the lack of a proper motion — that is, from this star's immobility, together with its revolution with the primum mobile. For in most observations it was always seen to keep the same distance from the same fixed stars, and to revolve with those fixed stars about the pole of the world only by the common revolution of 24 hours. For that reason Tycho said, in volume 1, p. 405: *"Its immobility, by which it stayed perpetually in the same place of the heavens like the rest of the fixed stars, gave sufficient testimony that it also held that seat in common with them"*; and on p. 406: *"That the New Star also complied so exactly with the universal motion — so that it expressed it most perfectly, and neither ran counter to it nor slackened, as happens in the planets and comets except when they appear stationary — proves that, observing the daily revolution as well so firmly and so scrupulously, it must be raised beyond all these, right up to the fixed stars."* And on p. 443, having said that this star did not vary its distance from the fixed stars during the whole time of its duration, he adds: *"And therefore it was placed altogether above the orbs of all the planets, within the sphere of those same fixed stars — which is what we undertook to set out and demonstrate."* Do you see how here too he reckons this argument convincing? Nor was Tycho alone: our own Clavius, too, in his *Sphere*, p. 193, was of the same opinion, for he says: *"Now, I am led to believe that that new star existed in the firmament, and not in any other celestial orb, chiefly by this argument: that neither I nor any astronomer at all (so far as I know) noticed any other motion in it besides the one we observe in the fixed stars. For it kept an ever constant motion, and plainly the same position among the other fixed stars, for almost two whole years. If it had been in the orb of some planet, then, since that orb certainly has a motion different from that of the fixed stars, the star itself would without doubt have had the same motion and course; but the astronomers found the matter to be otherwise. And this same argument evidently concludes that much less did that star exist in the elementary region, since there it could in no way have kept always the same position and distance with respect to the fixed stars. Since this is so, I am convinced* [?] *that that star was created at that very time by God, Best and Greatest, in the eighth heaven, to portend something great."* Biancani too, in book 18 of his *Sphaera*, chapter 2, judged this argument evident, as did Hájek in Tycho, volume 1, p. 513. Since, then, not probability alone — which we freely acknowledge in it — but also necessity and evidence are held by Tycho, Clavius and others to be present in this argument, it must be sharpened into syllogistic form, so that we may reply to it more closely, in this way:

[Margin: The 4th argument in form.]

VII. *Every star that has no apparent motion other than the motion of the fixed stars is in the heaven of the fixed stars. But the New Star of the year 1572 had no apparent motion other than the motion of the fixed stars. Therefore it was in the heaven of the fixed stars.* The major seems manifest from an induction made over all the phenomena that have, or ever had, the appearance of stars; the minor is established from the observation of almost all the astronomers and observers we reviewed in chapter 2. I said "every star", however, and not "every moving thing", since otherwise the Peripatetics, with Chiaramonti in book 2 of *De novis stellis*, chapter 2, would say that the air too is below the Moon and yet is moved with the motion of the primum mobile.

[Margin: 1st reply: distinguishing the major.]

I reply, first, by distinguishing the major premise. I concede it of stars that are already established to be ordinary stars, about which the only doubt is whether they are planets or fixed stars, and of whose nil or slight parallax we are certain from their having the same distance from the same fixed stars, observed at the same moment in several parts of the Earth, any parts whatever. But I deny the major premise of extraordinary stars, of disputed nature and origin, of whose parallax we are uncertain, or about which nothing is yet established — which is how we are using this argument here, in keeping with the title of this chapter: with all mention of parallax set aside for the time being. For insofar as this argument is connected with parallax, it will be examined separately below, and it will be shown not to be an evident argument if the already mentioned equality of distance from the same fixed stars was observed in a single place on Earth, or in different places but not at the same moment of time. In the sense, moreover, in which the major was conceded, the minor can be denied, because in that sense it has not been demonstrated by any of the astronomers.

[Margin: 2nd reply: denying the minor.]

I reply, second, by denying the minor — not only because Nolthius, Busch, Lvovický, Camerarius, Peucer, Schuler and Raymundus observed some variation in its distance from the same fixed stars, or some proper motion, as we related in chapter 2, but also because from its daily decrease an upward motion is inferred — not with certainty, indeed, yet not improbably — in the way it is inferred in the planets, especially in Mars and Venus, in which a marked decrease of the apparent diameter is a physically evident argument of recession from the Earth. It could therefore have happened that for five days at least it was below the Moon, and afterwards, ascending gradually above the Moon, finally vanished; for Cornelius Gemma and Biancani gave this as the cause of its disappearance. From this, then, arises an argument that could be turned back in the following way, and it will serve as an indirect reply.

[Margin: 3rd reply: turning the argument back.]

I reply, third, by turning a similar argument — that is, one taken from motion — back against the opponents, in this way. Every star that is above the Moon either lacks motion upward and downward, as the fixed stars do; or, if it has such motion, it also has a proper motion in longitude, as the planets have. But

the star of the year 1572 had upward motion and did not have a proper motion in longitude. Therefore it was neither among the fixed stars nor among the planets; that is, it was not above the Moon. And just as the opponents will say that this argument is not valid, because its major premise rests on an induction made over ordinary stars, so the same can be said of the major premise of the argument formed by them, or in their person, at the beginning of this number 7.

[Margin: 5th argument.]

VIII. Then there is the argument from its duration of a year and a half, by which Tycho, in volume 1, p. 407, confirms that this star was not a comet, nor any sublunary meteor, because no comet has lasted beyond 6 months. But not even Tycho himself regarded such an argument as demonstrative, since he says only that from so long a duration it is consistent that it was above all comets. And indeed, if some comet shone over Jerusalem for a whole year — whether it was a natural or a preternatural portent, as we related in section 1, chapter 4 — why could this star not have lasted a year and some months and yet have been below the Moon, without needing fuel on that account, being formed and preserved by God in another way? Chiaramonti, on the other hand, in book 2, chapter 2, of his work on these new stars, thinks this argument can be turned back against Tycho, saying: whatever is in the heavens is either perpetual or long-lasting; this star was neither; therefore it was not in the heavens. But the very probable rising and perishing of the solar spots and faculae cuts the strength and sinews of this argument of Chiaramonti's.

[Margin: 6th argument: from the distance of the observers from one another.]

IX. The last argument, taken from the fact that this star was seen on the same nights throughout Europe, is of little force unless something is added that bears on parallax. For we have shown in the first section of this book, chapter 15, that the same phenomenon can be seen at the same time not only throughout Europe but even from parts of the Earth much more remote from one another, even if it is below the Moon. Moreover, the argument itself is worth nothing unless it is shown that the New Star was observed in different parts of the Earth on the same day, or on the same night and at the same moment of the hour. But that cannot be shown from the historical record, as is clear from what was said in chapter 3, and more evidently from what will be said in chapter 6, toward the end of number 7. The configuration of the New Star with the fixed stars of Cassiopeia is also given differently by different authors, as I said in chapter 3, number 2.

[Margin: Is this star shown to be below the Moon?]

X. But neither has anyone demonstrated that this star was below the Moon on the grounds that it diminished until it vanished completely, or lasted only a short time, or changed its colours. For the solar spots and faculae are above the Moon and very close to the Sun, and yet they arise and perish and do not last long, as we taught in book 3, chapter 3; and the planets themselves change

colour according to the variation of the air and of the vapours running beneath them, and render their light to us now vivid, now dull and faint. Indeed, speaking universally, the dissolution of a superlunary body is not manifestly impossible.

CAPVT V.

Chapter 5.

Qua ratione Tycho demonſtrare conatus ſit, ex Parallaxis defectu Stellam Anni 1572. fuiſſe ſupra Lunam, immò ſupra Solem, ac Saturnum.

By what reasoning Tycho tried to demonstrate, from the absence of parallax, that the star of the year 1572 was above the Moon — indeed, above the Sun and Saturn.

I. In volume 1 of the *Progymnasmata*, chapter 6, in the second proof (found on p. 412), Tycho professes that from the nil or barely perceptible parallax of this star he will demonstrate its elevation not only above the Moon but also above the orbs of the planets, and will leave the Peripatetics *dumb and stupefied, not knowing where to turn.* To make his arguments clearer, let the centre of the Earth in the following figure be C. From it describe the Earth's quadrant AB, beneath the quadrant of the meridian VR, and beneath that let there be the quadrant HLN, in which the New Star was. Let the pole of the world be P, and let the zenith of the observer B be V. Now let the star be placed at L at the lowest of its meridian altitudes, and at M at the highest; through these two places let the lines of the true place, CLG and CME, be drawn from the centre of the Earth, and the lines of the apparent place, BLK and BMD, from the centre of the eye B. Let the line KLB be produced to O, until the perpendicular CO from the Earth's centre falls on it; likewise let the perpendicular CF fall on DMB produced to F. And let the Earth's semidiameter BC be, following Tycho, 860 German miles; but for the sake of a more precise investigation let it be reduced to 10,000,000 parts of the radius. On these assumptions he proceeds as follows, from p. 419.

Diagram. A geometric figure set in the left column beside the text. Three concentric quarter-circles stand about the Earth's centre C: the outer arc, the quadrant of the meridian, runs from V at the top to R at the lower right; a middle arc, the quadrant on which the New Star stands, runs from H through M and L to N; a small arc, the Earth's quadrant, runs from B down to A. A horizontal base line runs from F through C, A and N to R, and a vertical line rises from C through B and H to V. Star-marks stand at S, I, Q and T on the outer arc and at M and L on the middle arc. Straight lines run from C and from B through M and L out to the outer arc, and down to O and F near the base. All twenty labels are legible.

- V (top of the outer arc): the observer's zenith
- S (outer arc, beside V): the head of Cassiopeia at its greatest altitude

- E (outer arc): true place of the star at M
- D (outer arc): apparent place of the star at M
- I (outer arc, star-mark): the Pole Star at its least altitude
- P (outer arc): the pole of the world
- Q (outer arc, star-mark): the Pole Star at its greatest altitude
- G (outer arc): true place of the star at L
- K (outer arc): apparent place of the star at L
- T (outer arc, star-mark): the head of Cassiopeia at its least altitude
- R (lower right end of the outer arc)
- N (base line, foot of the middle arc)
- A (base line, foot of the small arc)
- C (base line): the centre of the Earth
- F (base line, left of C): foot of the perpendicular CF
- O (left, above F): foot of the perpendicular CO
- B (vertical line, top of the small arc): the observer's eye
- H (vertical line, top of the middle arc)
- M (middle arc, star-mark): the New Star at its greatest meridian altitude
- L (middle arc, star-mark): the New Star at its least meridian altitude

[Margin: 1st step.]

II. *First*, in the triangle BCO, right-angled at O, BC is given as 10,000,000 parts, and the angle OBC as 62° 15′, since it is equal to the vertically opposite angle HBL — namely the complement of the apparent lowest meridian altitude, the altitude Tycho affirms he observed in the New Star while he was at Herrevad, 27° 45′. Therefore, by the practice of triangles, the side CO is found geometrically to be 8,849,876 of the parts of which BC was taken as 10,000,000.

[Margin: 2nd step.]

Second, in the triangle CLO, right-angled at O, let the distance CL of the Moon at perigee from the centre of the Earth be taken from Copernicus as 52 17′ semidiameters of the Earth, or at least 52, of which BC is one — that is, 520,000,000 of the parts of which BC is 10,000,000. Since, then, CO is 8,849,876 of those parts, it follows by the analysis of triangles that the parallax, or the angle CLO, is 58′ 31″: the parallax the New Star would have had at its apparent altitude NL above the horizon, 27° 45′, if it had been distant from the Earth's centre by 52 semidiameters of the Earth.

[Margin: 3rd step.]

Third, in the triangle CBF, right-angled at F, CB is given as above as 10,000,000 parts, and the angle CBF as 5° 49′, since it is the complement of the highest of the meridian altitudes observed in the New Star, namely 84° 11′. Therefore, by trigonometry, the side CF is 1,016,351 parts.

[Margin: 4th step.]

Fourth, in the triangle CMF, right-angled at F, the side CF, 1,016,351, is given, and the distance CM, equal to the Moon's, 520,000,000, as above; therefore the parallactic angle CMF is found to be 6′ 43″.

[Margin: 5th step.]

Fifth, let the Pole Star be now at I, now at Q, equally distant in both places from the pole of the world P, at its least and its greatest altitude; or let the head of Cassiopeia be now at S, holding its greatest meridian altitude, now at T, holding its least, but equally distant in both places from the pole of the world P. For it was chiefly from these stars that Tycho took the distance of the New Star, at the meridian altitudes. On these assumptions it follows that the new star, placed at M, at its greatest altitude, was seen from the eye B under the point or apparent place D, granted that its true place would have been under E had it been seen from C. Hence the true distance of the new star in the heaven of the fixed stars from the Pole Star at I was EI, and the apparent DI; but from the head of Cassiopeia at S the apparent distance was DS and the true ES. Likewise, while the new star was at L but seen under K, its true place being under G, its apparent distance from the Pole Star at Q would be KQ, greater than the apparent DI, and the true GQ; but from the head of Cassiopeia, now revolved to T, the apparent distance would be KT, less than DS, and the true GT. Now the parallax, or the difference between the true and the apparent place, would have brought the New Star at L apparently closer to the head of Cassiopeia at T by the amount of the acute angle L, that is 58′ 31″, as is clear from the 2nd step, and would have moved it away from the Pole Star at Q by just as much, if the New Star had been distant from the Earth's centre by 52 semidiameters of the Earth. Conversely, the parallax would have moved the New Star at M away from S, and brought it closer to the Pole Star at I, by the amount of the acute angle M, which by the 4th step was found to be 6′ 43″. Therefore, combining these parallaxes — one producing an apparent approach, the other an apparent recession — the difference between the New Star's apparent distances from the same fixed star would have been 65′ 14″. Such a difference would surely have been noticed with the naked eye, let alone with any instrument, however coarse; and yet no distance — or a barely perceptible one — was observed in the New Star from the same fixed stars, namely from the Pole Star when it was at its greatest and when at its least altitude, just as none was observed from the stars of Cassiopeia. It cannot therefore be admitted, with the observations preserved, that the New Star was so near the Earth as to be distant from the Earth's centre by only 52 semidiameters of the Earth, which is roughly the Moon's perigee distance on Copernicus's hypothesis; and much less could it have been distant by only 33 Ptolemaic semidiameters. Therefore the New Star was above the Moon at perigee.

Comprobatio Tychonica Stellæ Nouæ ſupra Solem & Saturnum.

Tycho's proof that the New Star was above the Sun and Saturn.

[Margin: The New Star above the Sun.]

III. By exactly the same method Tycho tried, in volume 1, from p. 423, to show that the New Star was above the Sun at apogee, taking its distance from the centre of the Earth as 1182 semidiameters of the Earth — the amount he had gathered from his observations set out in chapter 3 — that is, 11,820,000,000 of the parts of which the Earth's semidiameter is 10,000,000. For in the triangle LOC (on which see number 2, in the 2nd step), with CO found, as there, to be 8,849,876 parts, and CL taken as 11,820,000,000, the parallax CLB is found to be 2′ 34″. And in the triangle CFM (on which see the 4th step), with CF found, as in the same place, to be 1,016,351, and CM taken as 11,820,000,000 parts, the parallactic angle CMB is found by trigonometry to be 0′ 18″. Hence the sum of the parallaxes is 2′ 52″. So great, then, would the difference have been between the New Star's apparent distances from the same fixed star — say from the Pole Star — when the New Star was, together with the Pole Star, at the greatest and when at the least of its meridian altitudes. But with the instrument no difference of even a single minute was observed between the two distances already mentioned; and yet the instrument, namely the sextant, as Tycho relates in chapter 4, had its degrees subdivided into single minutes.

[Margin: The New Star above Saturn.]

IV. In the same way Tycho, in volume 1, p. 425, takes from Copernicus, book 5, chapter 9, the distance of Saturn at apogee from the Sun as 11,077 semidiameters of the Earth, and from the centre of the Earth as 12,220; indeed, so as to raise the new star somewhat above Saturn, he takes the distance of the New Star as 12,300 semidiameters of the Earth. Using the remaining data, given or found, on which see number 2, he finds by the practice of triangles the parallax CLB to be 14″ and CMB 2″, whose sum is 16″. So great a difference, he says, would have appeared between the distances of the New Star from the same fixed star observed at both of the New Star's meridian altitudes, the greatest and the least, if it had been distant from the Earth's centre by only 12,300 semidiameters; but not even so tiny a dif- ference appeared. Therefore the New Star was higher than Saturn.

[Margin: A fourfold objection against Tycho.]

V. Tycho foresaw, however, that four objections could be raised against him. First, the division of the sextant, which either was not made with the utmost exactness, or at least could not show either single minutes with certainty and evidence, or a difference of 16″. Second, the width of the pinnules, which — even if the sextant, by reason of its division, had shown single seconds, let alone first minutes — did not allow an exact alignment on the centre of the star without an uncertainty of one minute, or at least of 16″. Third, that the New Star did not cross the meridian at the same moment as the Pole Star, or the other stars from which the distances were taken, crossed it — as would be required if the demonstration, and the diagram used in number 2, are to hold. The fourth is that he himself did not observe the New Star's greatest altitudes, but only its least.

[Margin: But resolved by Tycho.]

Tycho meets the first, however, by asserting that the angle of the sextant at which the New Star's distance from some fixed star had been taken at its least altitude was kept unchanged and fastened with screws, and that this same angle was applied to measuring the interval between the New Star and the same fixed star when the New Star was at its greatest altitude, and that by it exactly the same distance was found at both altitudes. Hence, even if the instrument had been faulty in the small minutes, its fault would have done no harm, since the angle was the same and remained so at both altitudes of the star. Indeed, even if it had had no divisions at all, it would have served the purpose in question: for there is no need to know by how many degrees and minutes the New Star was distant from some fixed star; it is enough to know that the distance appeared equal in every position of the new star. So Tycho, volume 1, pp. 347 and 425.

[Margin: Solution of the [?] objection.]

There he also resolves the second objection. For since each of the two pinnules was of the same width and fixed perpendicular to the plane of the sextant, and straight lines had been drawn through their middle from the same centre of the instrument, to which the eye was applied, those lines passed through the centres of the stars and cut the stars' apparent disc in two, so that through the edges of the pinnules an equal portion of the star was seen on either side — especially in the New Star, because of its size. Therefore, if there had been a difference of 16″ between the apparent distances, it would have appeared, notwithstanding the width of the pinnules. Tycho sets out the use and the form of this sextant in volume 1, from p. 336 to 350.

[Margin: Solution of the 3rd objection.]

He resolves the third objection from p. 426 to 436, showing, from the declinations and right ascensions of the New Star and of the stars from which it was observed to be distant, that if the New Star had been as far from the Earth as the Moon at perigee or the Sun at apogee, the difference between the distances at the New Star's two altitudes from stars standing off the meridian would have been much greater than the one detected with the same unchanged instrument. How he shows this we shall explain in the Scholia to this chapter, so as not to break the thread of this discussion — although, for those expert in observations, it would be enough that some of the fixed stars Tycho used crossed the meridian very nearly together with the New Star: especially the Pole Star, and the eleventh and twelfth stars of Cassiopeia, namely the one in the upright of the seat and the bright star of the Chair.

[Margin: Solution of the 4th objection.]

The fourth objection Tycho himself set out, by his own admission, in volume 1, p. 347, saying: *"Although I also took pains to measure the altitudes of the new star at the same time, especially the meridian ones, no suitable quadrant or other instrument properly assigned to this task was at hand, by means of which I might attain this with sufficient precision."* For he was a guest at Herrevad, not at Uraniborg. *"I therefore used this very instrument* — that is, the sextant — *to determine its nearest altitude, when it was placed on the meridian below the pole; for the highest, when it*

was approaching the zenith, I could not take with this instrument because of the star's excessive altitude, since it stood back only 6 degrees from the zenith overhead, and the greatest arc I had fitted to the instrument filled only a sixth part of a circle." But although he could not then take the greatest altitudes himself, he nevertheless used the altitudes taken at Augsburg — or at a place half a German mile from Augsburg toward the south-west, namely Göggingen — by Paul Hainzel, burgomaster of Augsburg, with a very large quadrant, which had a radius of 14 cubits; he describes it from p. 355 to 359, and Tycho himself was the author and director of its construction. He therefore uses the observations made with it by Hainzel as though they were his own. So, comparing the New Star's distances from the pole at both altitudes with the altitude of the pole at Göggingen, taken by Hainzel from the Pole Star, and with the star's least altitude taken by himself, Tycho found what greatest altitude the New Star ought to have had at Herrevad — and this by a method to be set out shortly.

De Altitudinibus Meridianis ſtellæ Nouæ Herritzuadij & Geggingæ obſeruatis; eiuſque Declinatione.

On the meridian altitudes of the New Star observed at Herrevad and Göggingen, and on its declination.

[Margin: The least altitude and the apparent declination of the New Star.]

VI. Tycho himself often observed the New Star's least meridian altitude at Herrevad, with the sextant set in the plane of the meridian, and on p. 351 gives it as established. It appeared to him as 27° 45′. Since the altitude of the pole at Herrevad was observed by the same man to be 55° 58′, subtracting 27° 45′ from it leaves the New Star's distance from the pole of the world, or the complement of its declination, as 28° 13′; and so its apparent declination was 61° 47′, as Tycho has in the same place. Paul Hainzel, for his part, as Tycho relates in volume 1, p. 361, observed with that very large quadrant at Göggingen the greatest and least altitudes on the meridian both of the Pole Star and of eight other stars belonging to Ursa Major, Cepheus and Cassiopeia. From half the difference of these altitudes, added to the least altitude, Tycho finds the altitude of the pole at Göggingen to be 48° 22′; and he confirms that those observations were well made from the distances of the Pole Star from the pole of the world taken afterwards by himself at Uraniborg with his largest instruments. The same Hainzel also observed in the same place, with the same quadrant, the meridian altitudes of the New Star, which I reported in chapter 3, number 3. From all of these, once delimited, Tycho on p. 360 derives the New Star's greatest elevation as 76° 34½′ and its least as 20° 9½′. Their difference is 56° 25′, and half the difference 28° 12½′, which, added to the least altitude, gives the altitude of the pole at Göggingen as 48° 22′ — exactly as it had been found from the altitudes of the circumpolar fixed stars. From this it follows, then, that the New Star kept the same apparent distance from the pole of the world at both its greatest and its least meridian altitude. Since, therefore, it was observed

at Herrevad to be 28° 13′ from the pole of the world at its least altitude, if this distance is added to the altitude of the pole at Herrevad, which is 55° 58′, the New Star's greatest meridian altitude at Herrevad comes out as 84° 11′. And so the 4th objection stands resolved; and at the same time it seems to have been shown that the New Star had no greater parallax than the fixed stars, since through both of its meridian altitudes it gave exactly the same altitude of the pole as the meridian altitudes of the fixed stars gave. This is Tycho's 3rd proof, which, however, I prefer to set out separately, together with Tycho.

Comprobatio Tychonica Stellæ Nouæ ſupra Saturnum ex Maxima & Minima ipſius Altitudine in Meridiano.

Tycho's proof that the New Star was above Saturn, from its greatest and least altitude on the meridian.

VII. Tycho uses this proof in volume 1, p. 443 and following, and his demonstration and method are exactly the same as the one he used when working with the New Star's distances from the fixed stars, as described in numbers 2 and 3. For what was said there of the distance from the fixed stars is said here of the distance from the pole of the world. Let the figure of number 1, therefore, be repeated here, and in it let there be, as we said, the pole of the world P, and the New Star now at L, having its least altitude above the horizon, now at M, having its greatest. For when placed at L it was seen under K, but was truly under G; and when placed at M it was seen under D, although it was truly under E. Hence the parallax L moved it away from the pole, whereas the parallax M brought it closer to the pole; the sum of the parallaxes M and L, therefore, was the difference between the New Star's apparent distances from the pole of the world. This difference was not sensi- ble, as was said at the end of number 6, since through them the same altitude of the pole was found as through the fixed stars. But if the New Star had been where the Moon is at perigee, or the Sun at apogee, or Saturn, Tycho tries to show that this distance from the pole of the world would have come out sensibly different. For this he assumes the following.

[Margin: Step 1.]

First, the New Star's greatest altitude observed by Hainzel, 76° 34½′; hence the angle HBM, that is CBF, is 13° 25½′. Therefore in the triangle CBF, right-angled at F, he finds the side CF to be 2,323,138 of the parts of which BC is 10,000,000. Likewise in the triangle CBO, right-angled at O, from the complement of the New Star's least altitude, HBL, that is CBO, 69° 50½′, he finds the side CO to be 9387,438 of the parts of which BC is 10,000,000.

Diagram. The figure of number 1 repeated, set in the left column at the right of the narrow lines. Three concentric quarter-circles stand about the Earth's centre C: the outer arc runs from V at the top to R at the lower right; a middle arc runs from H through M and L to N; a small inner arc runs from B down to A. A base line runs through F, C, A, N and R, and a vertical line rises

from C through B and H to V. Star-marks stand at S, I, Q and T on the outer arc and at M and L on the middle arc; E, D, P, G and K stand on or beside the outer arc. Straight lines run from B and C through M and L out to the outer arc, and O stands to the left of B. All twenty labels are legible.

- V (top of the vertical line)
- S, E, D, I, P, Q, G, K, T (along the outer arc)
- H (left end of the middle arc); M, L (star-marks on the middle arc); N (lower end of the middle arc)
- B (on the vertical, top of the inner arc); A (lower end of the inner arc)
- O (left of B); F (lower left, on the base line); C (centre); R (right end of the base line)

[Margin: Step 2.]

Second, he assumes from Copernicus, as above, the distance of the Moon at perigee, CL and CM, as 52 semidiameters of the Earth, that is 520,000,000 of the parts of which BC is 10,000,000; and by trigonometry he finds the parallaxes, at L 1° 2′ 4″, and at M 0° 15′ 21″, whose sum is 1° 17′ 25″. So great, then, would the difference have been between the New Star's distances from the pole, or between the altitudes of the pole derived from the New Star's greatest and least altitude, if the New Star had been where the Moon is at perigee — that is, if it had stood no more than 52 semidiameters of the Earth from the Earth.

[Margin: Step 3.]

Third, keeping all the data and results above, but changing the distance CL and CM from the lunar to the solar, and taking it as 1182 semidiameters of the Earth, he finds the parallax at L 2′ 43″ and the parallax at M 41″, and so the sum of the parallaxes 3′ 24″. That is how great the difference would have been between the New Star's distances from the pole of the world, or between the altitudes of the pole taken through its meridian altitudes, if the new star had been where the Sun is at apogee, or had stood no more than 1182 semidiameters of the Earth from the Earth's centre. Since, therefore, no difference was observed between them — and yet it would have been observable with an instrument as large as Hainzel's quadrant, which showed not only single minutes but even some seconds, if any had actually been there — it follows, says Tycho on p. 450, that the New Star was above the Moon and the Sun.

Per obſeruationes quaſdam in diuerſis Horizontibus ab alijs obſeruatoribus factas, Diſtantiam Stellæ Nouæ à terra, Lunari maiorem Tychonicè confirmare.

To confirm, in Tycho's manner, from certain observations made by other observers under different horizons, that the New Star's distance from the Earth was greater than the Moon's.

[Margin: Part 1 of the proof.]

VIII. This is Tycho's 4th proof, which he gives in volume 1 from p. 451 onward, and it runs as follows. If the New Star had been in the concave of the lunar heaven, that is, 52 semidiameters of the Earth from the Earth's centre, its parallax at the least altitude observed at Herrevad would have been 58′ 31″, and at the greatest altitude 6′ 43″, as was shown in number 2, steps 2 and 4. But at the least altitude observed at Göggingen the parallax would have been 62′ 4″, and at the greatest 15′ 21″, as was shown in number 7, step 2. Therefore the difference between the parallaxes of the greatest altitude at the two places would have been 8′ 38″, and between the parallaxes of the least altitude 3′ 30″. So great, then, would the apparent difference have been between the New Star's distances from the pole of the world as observed at Herrevad and at Göggingen; and yet scarcely half a minute was found. Therefore the New Star was not below the Moon.

[Margin: Part 2 of the proof.]

By a similar method Tycho, on p. 455, takes the New Star's greatest meridian altitude, 67° 30′, observed at Valencia in Spain by Hieronymus Munosius [Jerónimo Muñoz], and the Moon's distance of 52 semidiameters, and by the problem set out in number 2 finds a parallax of 25′ 18″; and from the least altitude, 11½°, observed in the same place, he derives with Muñoz a parallax of 64′ 47″. Hence the difference between the parallaxes of the greatest altitude as observed at Herrevad and at Valencia would have been 18′ 35″, and of the least altitude 6′ 16″. So great a difference in the New Star's distance from the pole of the world ought, then, to have been observed at one place or the other; yet by no means was even 6′ observed, let alone 18′. For Muñoz observed its distance from Schedar in Cassiopeia, with which the New Star crossed the meridian very nearly together, as 7° 50′, and Tycho as 7° 50½′; from which Tycho concludes that Muñoz does not differ from him in the New Star's distance from the pole by more than half a minute.

An Tycho putarit à ſe Demonſtratum Nouam ſtellam fuiſſe ſupra ☾ ☉ & ♄.

Whether Tycho thought he had demonstrated that the New Star was above the Moon ☾, the Sun ☉ and Saturn ♄.

[Margin: On the star above the Moon and the Sun.]

IX. If we are speaking of the New Star's place above the Moon, or even above the Sun, there is no doubt that Tycho thought he had demonstrated it with the greatest certainty. For in volume 1, chapter 6, p. 396, he says: *"Relying, therefore, on these aids, etc., we shall undertake to investigate the situation of this star with respect to the diameter of the universe, to show it by undoubted proofs, and to convince beyond all suspicion of error."* And on p. 422: *"It can therefore by no means be admitted that this star stood just within the bounds of the lunar revolutions, and much less in a place still nearer the Earth. This we have set out to demonstrate by this first induction."* Indeed, on p. 434: *"Therefore it is plain here too that this New*

Star must be set still higher above the Earth than the summit of the solar revolutions reaches, and consequently it will occupy a place far less in the space lying between the two luminaries. This we have striven to conclude demonstratively in this way, and no otherwise than before." And on p. 450: *"This star, therefore, by no means found its place just below the lunar revolutions, and much less in some region still nearer the Earth. This we undertook to show from the star's meridian altitudes alone."*

[Margin: On the star above Saturn.]

But if we speak of the New Star's distance above Saturn, Tycho is hardly consistent with himself, and seems to contradict himself within a few lines. For on p. 426 he says: *"It is fitting, therefore, and more probable — if we are to rest here on slender bounds — that the star itself passed beyond the sphere of Saturn, remote as it is."* And on p. 442: *"Hence, not even on this reasoning of ours concerning the nature and circuit of the heavenly bodies could this New Star have lingered at the bounds of Saturn, nor yet within the limits of the other planets — as we wished to infer, with probability, in this way."* And on p. 450: *"But since this does not furnish a sufficiently evident indication"* — where he speaks of the New Star's situation above Saturn; and he says similar things on p. 458.

[Margin: Tycho's inconsistency with himself.]

Notwithstanding all this, however, he steps beyond the limits of probability and does not hesitate, on those same pages, to assert the New Star's place in the sphere of the fixed stars above Saturn as something he has demonstrated. For on p. 426 he says: *"We conclude, therefore, since the star could not obtain any place — not in the elementary world below the Moon, nor from there up to the Sun, nor indeed from there up to the highest summits of Saturn — that would not be overturned when weighed by geometrical observations, that it necessarily follows that it claimed for itself a seat in the outermost eighth sphere of all, among the other fixed stars, above the revolutions of the secondary movables; and this we have set out to make plain and demonstrate by this second proof."* So on p. 451: *"It necessarily follows that it stood unmoved beside the highest lights of the eighth sphere, as we set out to establish by this third proof, from its revolutions about the pole."* Finally on p. 458: *"And so, since we have now confirmed by four invincible proofs that this New Star not only had no commerce with the elementary world, but did not even find a place within the vast revolutions of the seven wandering stars — its total lack of parallax bearing witness to this very fact — we shall conclude that it must necessarily be raised to the highest theatres of the fixed stars, where, like them, it stood unmoved."*

X. So far, then, Tycho works from his own observations, or from his own compared with those of Hainzel and Muñoz; after these, in chapter 7, he goes on to investigate the true magnitude of this New Star. But we shall speak more conveniently of the magnitude of this and of the other New Stars together in chapter 18.

SCHOLIA.

Scholia.

[Margin: From Tycho, volume 1, p. 428.]

I. In number 5 of the preceding chapter, in the solution of the 3rd objection, we promised to show the method by which Tycho, from the right ascensions of the New Star and of certain fixed stars, and also from their declinations, investigated the distances from those same fixed stars that would have been seen, at each of the New Star's meridian altitudes, if the New Star had been where the Moon is at perigee or where the Sun is at apogee — so that the difference from the observed distances might be established from them. To do this, Tycho's diagram must first be set out. Let, then, the meridian be AHQR, the horizon HYR, the eastern semicircle of the equator ÆYQ and the western ÆZQ. Let the pole of the world be A, and let the New Star at its greatest altitude be at B, and at its least at C. Nearest to it at its two altitudes, let there be, first, the Pole Star, at D and at E; then the eleventh star of Cassiopeia, the one that shines in the upright of the seat, now [?] at a, now at ε (but note that through the engraver's error this star is placed on the 2nd circle, AGV, and not, as it should have been, on the third) — the star to which the New Star stood closest; the bright star of the Chair, with which the New Star crossed the meridian very nearly together, at F and G; the head of Cassiopeia at L and S; the breast at K and I; the girdle at M and P; and finally the bend of Cassiopeia at N and O. Through these stars, placed in each of the two positions, let single quadrants be drawn from the pole of the world to the equator — namely AFT, AGV, ANX, AOZ, and so on — and let each star be joined by arcs of great circles to the New Star, placed now at B and now at C; although, to avoid confusion, only three are joined in the diagram: the Pole Star, by the arcs BD and CE; the eleventh of Cassiopeia, by the arcs Ba and Cε; and the bright star of the Chair, by the arcs BF and CG.

Diagram. Tycho's diagram, set in the left column. A large circle stands for the meridian, with A at the upper right, H at the left, R at the right and Q at the lower right. A horizon line runs from H through Y to R, and two arcs run from Æ at the left to Q, one through Y and one through Z. From A a fan of quadrants sweeps down and to the left across the figure to the arcs. Star-marks stand at B at the top, with D to its right, at F and at a below B, at L, K, M and N in a row in the upper left, at E and C on the right with ε below C, and at O, P, I, G and S in the lower right. T stands at the left beside Æ, with three small dots along the arc beyond it. One letter, on the left arc below the row L K M N, is not certain between X and Z on the scan.

- A (upper right, where the quadrants meet); B (top star-mark); C, E (right); D (right of B)
- F, a (upper left, below B); L, K, M, N (row of star-marks, upper left)
- ε (below C); O, P, I, G, S (lower right)
- H (left), R (right), Q (lower right), V (lower right, beside Q), Y (on the horizon line)
- Æ (left end of the arcs); T (beside Æ); Z (cursive, on the lower arc left of S)

• X [?] (on the left arc below L K M N; letter not certain between X and Z)

With these set down, Tycho's aim is to investigate the distances at which the New Star would appear if, at its highest or its lowest meridian altitude, it had been where the Moon is at perigee — that is, 52 semidiameters of the Earth from the Earth — or where the Sun is at apogee — that is, 1182 semidiameters of the Earth from the Earth. This he accomplishes by the following steps.

[Margin: Step 1.]

II. First, from the lunar distance of 52 semidiameters of the Earth, and from the New Star's greatest altitude observed or found at Herrevad, 84° 11′, and its least, 27° 45′, he had found that the New Star's parallax of altitude would be 6′ 43″ at its greatest altitude above the horizon and 58′ 31″ at its least, as is plain from what we said in the preceding chapter, number 2, in the first 4 steps. He had also found the New Star's declination, precisely, to be 61° 46′ 45″. Adding to this, therefore, the parallax of the greatest altitude, that is 6″ [?] 43″, and subtracting the parallax of the least altitude, that is 58′ 31″ — because the parallax, by depressing the star, would have increased its declination when it was above the pole, by bringing it down towards the pole, but would have decreased it when it was below the pole, by moving it away from the pole — the New Star's visible declination comes out as 61° 53′ 28″ above the pole and 60° 48′ 14″ below it.

[Margin: Step 2.]

Second, assuming in the same way the solar distance of 1182 semidiameters of the Earth, and the same altitudes of the New Star, 84° 11′ and 27° 45′, he had found the parallax at the former to be 18″ and at the latter 2′ 34″, as we showed in the preceding chapter, number 3. Adding 18″, therefore, to the New Star's true declination, and subtracting 2′ 34″, the New Star's visible declination comes out as 61° 47′ 3″ when it was above the pole, and 61° 44′ 11″ when below. From these are gathered four complements of the New Star's declination, two for the lunar and two for the solar distance supposed for the New Star, as you see in the following little table.

Complement of the visible declination.

If the New Star's distance is	Arc AB	Arc AC
☾ 52	28° 6′ 32″	29° 11′ 46″
☉ 1182	28° 12′ 57″	28° 15′ 49″

III. Third, now consider the spherical triangle ABD, in which the complement of the visible declination, BA, is given as 28° 6′ 32′ [?], if the New Star had stood 52 semidiameters away. The complement of the declination of the Pole Star D, that is the arc AD, was for Tycho in that year 3° 0′ 15″, and its right ascension 5° 3′ 30″; and the New Star's right ascension, true and apparent alike (since this star is placed on the meridian), was for Tycho 0° 26′ 25″. Subtracting this from the right ascension of the Pole Star leaves the difference

of right ascension, that is the angle BAD, as 3° 37′ 5″. Given, therefore, in the triangle ABD the sides AB and AD together with the included angle, the visible distance BD of the New Star from the Pole Star is found to be 25° 6′ [?] 56″. In the triangle ACE, on the other hand, he gathers the distance CE as 26° 12′ 10″. Hence the parallax of the distance, or the difference between the visible distances, would have been 1° 5′ 14″, whereas none was observed. Therefore the New Star did not stand only 52 semidiameters of the Earth away; otherwise the parallax of the distance would have been 7′ 4″, which would certainly have been observable with Tycho's instrument, especially with its interval kept unchanged.

By exactly the same method he solves the triangles ACE, ABa, ACε, ABF, ACG, and so on for the rest, using, that is, the complement of the visible declination of the star above the pole, AB, and below the pole, AC, under each supposed distance of the New Star, lunar or solar; and also applying the difference of right ascension from the right ascensions of the remaining six stars, together with the complements of their declinations. In this way he investigates the visible distances that would arise from those stars, and their differences, if the New Star had been where the Moon ☾ is, or where the Sun ☉ is — differences that would sometimes be greater than one whole degree; and he would certainly have perceived such a parallax or difference with his instruments, had one so great been present. Therefore, since no such parallax, nor any so great, was observed over a long time, he concludes that the New Star was not where the Moon is at perigee, nor where the Sun is at apogee, but stood farther from the Earth than 1182 semidiameters of the Earth, let alone 52. Here, then, from Tycho's volume 1, p. 436, is a little table of these visible distances with their difference.

A little TABLE showing the visible distances of the New Star from the fixed stars listed below, with which the New Star crossed the meridian very nearly together, in each of its two positions — that is, both above, at B, and below, at C — the New Star being supposed in the concave of the Moon or in the convex of the solar heaven, that is, 52 semidiameters of the Earth away in the former case and 1182 in the latter.

If the New Star of the year 1572 had been:

Names of the fixed stars distant from the New Star	In the concave of the Moon ☾, or 52 semidiameters of the Earth away: below, at C	Above, at B	Difference	In the concave of the Sun ☉, or 1182 semidiameters of the Earth away: below, at C	Above, at B	Difference
Pole Star	26° 12′ 10″	25° 6′ 56″	5′ 14″	25° 16′ 13″	25° 13′ 21″	2′ 52″
Eleventh of Cassiopeia	0° 57′ 36″	1° 36′ 33″	38′ 57″	29′ 9″	1° 31′ 22″	2′ 13″
Bend	4° 29′ 17″	5° 6′ 20″	36′ 35″	0′ 28″	5° 2′ 15″	1′ 47″
Bright star of the Chair	4° 26′ 2″	5° 25′ 22″	59′ 29″	16′ 46″	5° 19′ 26″	2′ 40″
Girdle	6° 1′ 38″	6° 58′ 43″	57′ 56″	50′ 28″	6° 53′ 0″	2′ 32″
Breast, or Schedar	6° 54′ 47″	7° 57′ 31″	2′ 16″	7° 48′ 8″	7° 50′ 53″	2′ 45″

Names of the fixed stars distant from the New Star	In the concave of the Moon ☾, or 52 semidiameters of the Earth away: below, at C	Above, at B	Difference	In the concave of the Sun ☉, or 1182 semidiameters of the Earth away: below, at C	Above, at B	Difference
Head of Cassiopeia	9° 24′ 32″	10° 28′ 35″	4′ 3″	10° 19′ 28″	10° 22′ 07″	2′ 49″

CAPVT VI.

Chapter 6.

Soluuntur Argumenta Tychonis ex Parallaxi deducta, & oſtenditur non fuiſſe ab eo demonſtratum, Stellam Nouam Anni 1572. fuiſſe ſupra Lunam, nedum in Octaua Sphæra. Detectis ſubtiliſſimis fallacijs, in Parallaxium profundo latentibus.

Tycho's arguments drawn from parallax are refuted, and it is shown that he did not demonstrate that the New Star of the year 1572 was above the Moon, let alone in the eighth sphere; the very subtle fallacies lying hidden in the depths of parallaxes are exposed.

I. Tycho's argument, resting on the last three proofs he brought forward in volume 1 of the *Progymnasmata*, chapter 6, and reduced to the form of a single syllogism, runs as follows.

[Margin: Major premise.]

If the New Star of the year 1572 had been just below the Moon, or where the Sun or Saturn is, it would not have kept the same visible distance from the pole of the world and from the same fixed stars at whatever altitude it was observed; rather, it would have shown at the observed altitude as great a parallax as the Moon, the Sun or Saturn shows at the same altitude; nor would the same altitude of the pole have been found from its greatest and least altitude as is found from the Pole Star's greatest and least altitude.

[Margin: Minor premise.]

But the New Star of the year 1572, at whatever altitude it was observed, kept the same distance from the pole of the world and from the same fixed stars; nor did it show as great a parallax as the Moon, the Sun or Saturn shows at the same altitude; and finally, as great an altitude of the pole was found from its greatest and least altitude as from the Pole Star's greatest and least altitude.

[Margin: Inference.]

Therefore the New Star of the year 1572 was not just below the Moon, nor indeed where the Sun or Saturn is, but above Saturn.

[Margin: Proof of the major.]

The *major* premise is taken as most certain by Tycho, in volume 1 of the *Progymnasmata*, pp. 422, 443, 450, 455, 458, and throughout that chapter 6 — and not by Tycho alone, but by most astronomers who discuss either this star or parallaxes in general. For the fixed stars always keep the same apparent

distance from one another, at whatever altitude above the horizon they are viewed, whereas the planets, especially the lower ones, even when viewed on the same night, show a distance from the same fixed stars that varies with their altitude. It seems to follow, therefore, that if some phenomenon always shows the observer the same distance from the same fixed stars, or one that varies less than the planets observed at an equal altitude, it is among the fixed stars, or above the planets; and conversely, that if it is where the planets are, it does not keep the same distance from the same fixed stars, or from fixed points (such as the poles of the world), at whatever altitude, but shows as great a parallax of altitude, or of distance, or as great a sum of parallaxes, or as great a difference of parallaxes, as that planet shows at the same altitude. Finally, who would not consider it an evident argument that a star is as high as the fixed stars, when from its altitudes precisely as great an altitude of the pole is derived as from the altitudes of the fixed stars?

[Margin: Proof of the minor.]

The *minor* premise seems to have been shown more than sufficiently by Tycho in volume 1, chapter 6, in proofs 2, 3 and 4, which we brought forward in chapter 4, from number 2 right to the end. For there Tycho appeared to show, on the one hand, how great a parallax, or variation in the distances from the fixed stars or from the pole of the world, the New Star would have shown when it was at its lowest and when at its highest meridian altitude, if it had been in the concave of the Moon, that is 52 semidiameters of the Earth from the Earth, or in the concave of the solar heaven, that is 1182 semidiameters of the Earth away, or in the convex of the heaven of Saturn, that is 12,300 semidiameters of the Earth from the Earth — and a parallax so great that it would have been sensible. On the other hand, through the observations of himself, of Paul Hainzel and of Jerónimo Muñoz, he taught that no such variation had been observed, and yet that it would have had to be observed with the instruments, had it actually been present.

[Margin: Reply 1 [?], to the major.]

II. First, however, I reply by denying the major premise of the syllogism above, if it is asserted as universally and necessarily true.

[Margin: Distinction 1 of the major.]

If this seems too harsh, I reply by distinguishing the major in many ways. First, by granting it if the New Star kept the same visible distance from the same fixed stars, or points of the highest heaven, when observed on the same night by several observers from different parts of the globe; but by denying it if it was observed on the same night by a single observer, that is, from a single place on the Earth's surface, or by several observers indeed, but each on a different night — and much more so if by no one on the same night.

[Margin: Distinction 2.]

Second, I distinguish and grant it if the New Star, in its daily revolution, in which it was seen to imitate the motion of the primum mobile, described a circle parallel to the equator and having its centre on the axis of the world; but

I deny it if it described a circle not parallel to the equator and eccentric to the axis of the world.

[Margin: Distinction 3.]

Third, I distinguish and grant the major if the New Star, in its daily revolution, described on the surface of a right cone, whose axis is drawn from the eye to the pole of the world, a circle or ellipse whose diameter is bisected by the axis of the world, exactly or nearly; but I deny it if the star described about the cone already mentioned a circle or ellipse whose diameter was not even nearly bisected by the axis of the world.

[Margin: Distinction 4.]

Fourth, I distinguish and grant the major if the New Star in its daily revolution did not vary its motion of declination, of latitude or of altitude with respect to the centre of the Earth; but I deny it if it truly varied one of these, or all of them. But so that the reasons for denying or distinguishing, in the ways already mentioned, the major premise may be laid open, and so that we may at the same time show that Tycho, by arguments drawn from parallax and resting on his own observations and those of Hainzel and Muñoz, by no means demonstrated that this New Star could not, with the phenomena saved, have been below the Moon, let alone below the Sun or Saturn, we must draw a diagram — and one fitted above all to the firmest foundation among Tycho's materials, namely the two observations of Paul Hainzel made on the same night, on November 21 and on November 26. On each night he took two meridian altitudes of the New Star with a huge quadrant, the highest in the evening and the lowest in the morning, as we related in the history in Chapter 3, number 3.

[Margin: Drawing of a diagram adapted to Hainzel's observation.]

III. From the centre of the Earth A, describe the quadrant BC of the terrestrial meridian, beneath the quadrant DE of the celestial meridian. On DE let P be the pole of the world, and let the Pole Star stand now at V, at its highest, and now at R, at its lowest meridian altitude, equally distant from the pole P on either side. Now Paul Hainzel, observing from B, saw the New Star at a distance of 13° 25′ 30″ from the zenith D at its highest meridian altitude, which Tycho, after limiting it by various observations, chose as 76° 34′ 30″; the least altitude observed by the same man was 20° 9′ 30″; and the altitude of the pole at Göggingen near Augsburg was found, both from these and from the altitudes of the Pole Star, to be 48° 22′. So, in the quadrant DE, take the arc EG of 76° 34′ 30″, EF of 20° 9′ 30″, and the altitude of the pole EP of 48° 22′. Draw two straight lines to the pole of the world: one, AP, from the centre of the world; the other, BP, from the centre of the eye. From that same point draw two straight lines BF and BG to the points G and F — these are the lines of the New Star's apparent place beneath the fixed stars at each meridian altitude — and join G and F by the straight line GTF. Now, since the New Star, as to its apparent motion, was seen by the eye at B to move with the same revolution as the primum mobile, keeping the same distance from the fixed stars and from the pole of the world, let us suppose that a right cone BGF moves about the

axis BP, with the New Star fixed in its surface; or else that, the cone staying still, the star itself de-

Diagram. A geometric figure set into the left column: the quadrant DE of the celestial meridian about the centre A, with the sides AD and AE; within it a smaller arc BC about A (the terrestrial meridian quadrant). On the arc DE stand G, the stars V and R (drawn as asterisks) and the pole P between them, and F. Lines run from B to G, P and F, and from A to P; GF is joined through T. About B lies a small circle through H, L, K and M with I on the axis, and an oval curve through O, S, Q and N.

- D (top left, end of the quadrant): the zenith
- G (top, on the arc): the New Star's place at its highest meridian altitude
- V (on the arc, asterisk): the Pole Star at its highest
- T (on GF, below V and P): intersection of GF with BP
- P (on the arc): the pole of the world
- R (on the arc, asterisk): the Pole Star at its lowest
- O (left, inside the quadrant): end of the ellipse's diameter
- S (centre): on the ellipse
- Q (centre right): on the ellipse
- F (right, on the arc): the New Star's place at its lowest meridian altitude
- H (left): on the small circle
- I (centre of the small circle): on the axis BP
- M (on the small circle)
- L (on the small circle)
- K (lowest point of the small circle)
- N (right of K): end of the ellipse's diameter
- B (left side, on AD): the observer's eye, apex of the cone
- A (bottom left corner): the centre of the Earth
- C (bottom, on AE): end of the terrestrial quadrant
- E (bottom right): end of the celestial quadrant

-scribes about the axis BP, in the surface of this cone, either a circle or an ellipse: a circle if at every altitude above the horizon it kept, on the same night, exactly the same apparent magnitude and distance from the eye B; an ellipse if it kept the same distance from the centre of the Earth in both places but a different distance from the eye, or if its distance differed both from the eye and from the centre of the Earth. But since, as is clear from the history in Chapter 2, the New Star did not vary its apparent magnitude on the same nights, but only on different nights, let us suppose that it described a circle; afterwards, for another case, we shall suppose that it described an ellipse.

[Margin: Case 1.]

Our aim is to show that all the phenomena indicated in the syllogism already mentioned, and observed by Hainzel — indeed by Tycho and by Muñoz as well — could have been saved even if the New Star had been distant from the Earth by only a single semidiameter of the Earth. From the centre B, then, where the apex of the cone already mentioned lies, with the radii BH and BK each

equal to the Earth's semidiameter AB, take two opposite points, and join them by the straight line HK, which cuts the axis BP at right angles at I. The plane HIK, cutting the cone GBF, will then describe in the section the circle HLKM, whose circumference will lie in the conical surface, as is very well known to geometers from the conic sections of Apollonius and Serenus. Suppose, then, that the New Star was at H at its highest meridian altitude and at K at its lowest, and that in its diurnal motion it described the circle HLKMH in the space of 24 hours. On these assumptions, this circle would not have been parallel to the equator, since it is taken to be at right angles to the axis BP, which differs from the axis of the world AP and is therefore inclined to it; whereas all circles parallel to the equator are at right angles to the axis of the world, or of the equator, as the equator itself is, by Theodosius, *Spherics* Book 2. Nor would its centre have been concentric with the axis of the world AP, but eccentric: for the centre of a circle described on a cone lies in the cone's axis, and the axis of the cone GBF, at the point I, because of its great nearness to the Earth and its remoteness from the pole of the world, is taken to be far removed from the axis of the world, namely from the point K.

For the other case, however, take AO and AN as equal distances from the centre of the Earth, and let the cone GBF be cut by the plane OSN, oblique to the axis BP. The conic section will be the ellipse OMNQO, on whose circumference the New Star could have moved with all the phenomena saved except its apparent magnitude: for on the same night it would have been seen larger at O than at N, since at N it is farther from the eye B — unless perhaps refraction at N produced the contrary. But neither the diameter HK of the circle nor the diameter ON of the ellipse would have been bisected by AP.

[Margin: Case 2 [?].]

IV. Before we go further, however, we want first to determine the parallaxes AKB and AHB which the New Star would have had at its least altitude K and its greatest H, if in both it had been equally distant from the eye B and no more than a single semidiameter of the Earth. This is very easy, and needs no laborious calculation. In the triangle ABK, which is isosceles since BK is taken equal to AB, the angles at the base AK are equal to each other, by I.5 of the *Elements*; and by I.32, the angle DBF — that is, the complement of the New Star's lowest altitude as seen by Hainzel, which was 69° 50′ 30″ — is equal to the angles BAK and BKA taken together. The parallactic angle AKB would therefore have been 34° 55′ 15″. Similarly, in the triangle ABH, isosceles because BH was taken equal to AB, with the angle DBG given by Hainzel's observation as 13° 25′ 30″, the parallactic angle AHB comes out at 6° 42′ 45″. Now this parallax, and any other of at least one minute, I call sensible in itself: for although by accident it might have been imperceptible to a single observer on the same night, or to several observers on different nights, it could nevertheless have been detected with an exact instrument, had both altitudes of the star been observed by the same observer on the same night.

[Margin: The greatest parallax of the New Star possible with the phenomena saved.]

[Margin: What a parallax sensible in itself and insensible by accident is.]

V. With what we laid down in numbers 3 and 4 now in place, I say that if the New Star, with respect to the eye B — that is, to a single observer on the same night — described in its diurnal revolution, on the surface of the cone BGF, the circle HLKMH in the space of 24 hours, with an apparently uniform motion, then all the phenomena reported of it are saved, even if it is supposed to be distant from the eye B by no more than a single semidiameter of the Earth, and therefore to have had, in the horizon of Göggingen, a parallax of 34° 55′ 15″ answering to its least altitude. For the equality of apparent magnitude and of distance from the eye B is saved not only at the lowest altitude K and the highest H, but at every altitude, because all the lines drawn from the vertex of the cone B to the circumference of the base, the circle HLKMH, are equal, the cone HBK being taken as a right cone. Moreover, just as the larger cone GBF, in its revolution about the axis BP — or a star or point of the highest heaven — was carried round about the cone from F through T to G in the space of 12 hours, and back again from G to F in the remaining 12 hours, so as to keep its apparent distance from the pole P and from any fixed stars whatever, say from the Pole Star at R or V, unchanged; so the New Star, carried round from K through L to H, and from there through M to K, in the space of 24 hours at an equal pace about the same cone, of which the smaller cone HBK is a part, could have been held in the same line of apparent place under the same fixed point F, carried round to G, and so, no differently from that point itself, have kept throughout that whole revolution an equal visible distance from the pole P and from any of the same fixed stars whatever. With this posited, the same altitude of the pole EP could not fail to be found from the lowest and highest altitudes of the New Star as had been found from the altitudes of the Pole Star, since the New Star is in this way supposed to be seen now under F, now under G, points equally distant from the pole P. Hence half the difference of the altitudes — that is, of the arc GF, bisected at P — namely FP added to the least altitude EF, or GP subtracted from the greatest altitude EG, gave the same altitude of the pole EP as half the difference VR of the Pole Star's altitudes, PR, added to its least altitude ER, shows.

What has been said of this circle can be said of any other circle that the New Star could have described about the cone GBF in its diurnal revolution, whether nearer to the Earth or farther from it, provided it cuts the axis BP at right angles. And what we have said of one night with respect to a single observer B could have held on other and yet other nights, but always different ones, with respect to different observers: as if Tycho observed on November 10, Paul Hainzel on the 11th, and Muñoz on the 12th, and so on for the other days and observers. Moreover, not only on circles described about the cone GBF, but also on innumerable ellipses — on the ellipse OMNQO, for example — all the phenomena of the New Star could have appeared, except the same

apparent magnitude within one and the same night, even though it was below the Moon: for it could have described those ellipses with a uniform diurnal motion, and have moved under no other points of the highest heaven than those under which a star describing the circle HLKMH, or GTF, would have moved, since they lie in the same surface of the same cone.

[Margin: On different observers on different nights.]

[Margin: The phenomena of the new star explained by an elliptical motion.]

This could not, however, have happened on the same night with respect to any number of observers at different places on the Earth. For by the very fact that the New Star is supposed nearer the Earth than the Moon, it would have been seen at one place more, at another less distant from the pole and from the same fixed stars, at the same moment. From all this, not to be too long, I think it is clear enough for what reasons I have conceded and denied the major premise of the syllogism set out at the beginning, according to the different senses in which it is understood.

It is clear, moreover, that with the star so near the Earth a parallax however large — one that would have been sensible in itself to different observers taking its altitudes or its distances from the same fixed stars at the same moment of time — could by accident have been insensible to a single observer, or to several observing it on different nights. For if, observed by any number of observers at different places on the Earth but at the same moment of time, it had been found equally distant from the same fixed points or stars of the firmament — as the distances of the fixed stars from one another and from the poles of the world are supposed to be observed or observable — that would be an infallible argument that the star lacked any parallax sensible in itself, and therefore was in the highest aether, and most certainly above the Moon.

Finally, it is clear that, on what we posit, the apparent declination could indeed have been seen as equal on the same night, because of the star's equal apparent distance on either side, under G and under F, from the pole of the equator D; but that its true declination was much smaller when it was at H than when it was at K, because at K it was little or not at all distant from the axis of the equator, but at H a great deal.

VI. You will say, however, on behalf of Tycho's opinion, that at so close a proximity to the Earth the New Star could not have appeared equally distant at the top and at the bottom of the meridian from any fixed star whatever, but only from a single one. But this very point is denied. For it would have been seen to be equally distant, at the top and the bottom of the meridian, or at any altitude of its own, not only from several but from all fixed stars whatever, indeed from all points whatever of the highest heaven, by the very fact that in its diurnal revolution about the cone GBF it was held under a single point of the heaven revolving with the motion of the primum mobile. For as to its apparent place it would have behaved just as if it had been in the highest heaven, or as if it had had no sensible parallax.

[Margin: Tycho's objection resolved.]

And yet — to say what actually happened — neither Hainzel nor Muñoz is reported to have observed distances of the New Star from fixed stars other than the Pole Star, in so far as they observed its distances from the pole of the world or from the zenith. Tycho did observe this star's distance not only from the Pole Star but also from several stars of Cassiopeia, but not on the same night, as the force of a demonstrative argument would have required. For when he observed the new star at Herrevad he had no other instrument with which to take either distances or meridian altitudes than the astronomical radius, which he did not trust, and a single sextant. When he had taken the New Star's distance from some one star with it, he fixed it with screws, so that, keeping the same angle of separation, he could test whether, when the New Star returned to the meridian together with nearly the same fixed star, it kept the same distance from that fixed star as in its former position on the meridian — as is clear from Tycho himself, *Progymnasmata* volume 1, pages 335, 342, 347, 349 and 416. Therefore on one and the same night he could not take the distance of the New Star twice, once when it stood at the bottom and once at the top of the meridian, except from a single fixed star.

VII. Second, I now reply by denying the minor premise in the sense in which I conceded the major. For in his arguments Tycho did not adduce any observations made on the same night by different observers, since he named no definite day on which he himself or Muñoz measured the New Star's distances from the fixed stars, or its meridian altitude. It could therefore have happened that, although Tycho often took them, his observations never fell on the same nights as those on which Hainzel observed its altitudes; for he took distances from other stars only indirectly, from the Pole Star. Nor could any follower of Tycho say, except gratuitously, out of a presumed opinion and excessive confidence, that he observed on the same nights as Muñoz and Hainzel. That can in no way be shown from the history that Tycho himself supplies, which we have carefully and faithfully compiled for this purpose and set out in Chapter 3.

[Margin: Reply 2, to the minor.]

Now, however, we are considering only the observations of this triumvirate, because the arguments of Tycho brought forward in *Progymnasmata* volume 1, chapter 6, set out by us in Chapter 5, and finally gathered into a single syllogism at the beginning of this chapter, rest on them alone. Indeed — to give this warning now in passing — none of the observers that he adduces in Chapters 8, 9 and 10 of the first volume is found either to have recorded definite days on which he obtained the New Star's distances from the fixed stars, or to have fallen on the same day on which some other observer took its meridian altitudes, as is plainly evident from the history given above in Chapter 3, numbers 2 and 3. But why do I speak of distances, or of meridian altitudes? In fact, no altitude whatever, no other observation of a determinate measure, presents itself in that history that is certainly established to have been made on the same night by two observers at different places, except on the day of December 4. Here, then,

is the whole parade of those observations displayed for you as in a mirror, as far as the days are concerned on which Tycho himself reports that they were made.

Determinate days on which the New Star of the year 1572 is reported to have been observed.

Days / Observers	Year 1572: November	December	January	February	Year 1573: March	April	May	August
Hainzel	14, 21, 26		2, 4		2, 12			
The Landgrave		3, 4, 10, 26	11	26	11, 14			

Remainder of the preceding table.

Days / Observers	Year 1572: November	December	January	February	Year 1573: March	April	May	August
Camerarius	23 and following		5, 6, 7, 8			7		7
Nolthius	29							
Hájek, Schuler	27	4					6	
Mästlin, however, as reported in Tycho, volume 1, page 545, observed this star on November 16 and on a number of following days.								

VIII. Third, I reply by again denying the minor premise, in so far as it extends to the distances of the Sun and of Saturn. First, because Tycho takes as certain a distance of the Sun at apogee of 1182 terrestrial semidiameters, and of Saturn of not more than 12,300 semidiameters. But neither is certain, as we have already shown in Book 3, chapter 7, on the distances of the Sun, and in Book 7, section 6, chapter 2, on the distances of the lesser planets. Indeed, from what is said there it is more probable that the Sun at apogee is distant from the Earth by about 7500 terrestrial semidiameters, and Saturn by 90,000. Now with the Sun's distance set at 7500 parts, of which the Earth's semidiameter is one, the horizontal parallax comes out no greater than 47″; and with Saturn's distance set at 90,000 terrestrial semidiameters, the parallax comes out no greater than 2″ 20‴. Of these the latter was imperceptible to Tycho's instruments, and the former not so evidently perceptible, given the imperfection of that sextant, which he himself admits.

[Margin: Reply 3, to the minor, and its first reason.]

Second, Tycho uses Muñoz's meridian altitudes (volume 1, page 456) as certain, and yet he reports them as having been taken rather coarsely, with a less exact instrument. Here are Tycho's words, volume 1, page 567: *"Muñoz puts the greatest altitude of the New Star in the horizon of Valencia at 67° 30′, the*

least at 11° 30′, and the declination of the star at 62° toward the north. But he set these down in too broad a manner, with a quadrant not large enough, on which he uses only half-degrees; so it is no wonder that he increased the declination too much, by about a fifth of a degree. From these, moreover, the elevation of the pole of Valencia is gathered to be 39½°, which does not differ much from the geographical maps, although the Canons of Apian and others disagree very greatly on it." In that place, however, he supposes that Muñoz erred in the declination of this star, because he thinks that the declination he himself took was both true — since there was no sensible parallax — and correct. But he did not demonstrate on other grounds that it lacked parallax. Tycho's declination, then, could have been an apparent one, and Muñoz's an apparent one too, but different because of the difference of altitudes arising from the difference of horizons and from parallax; or at least differing by 12′ for the reason that it was taken on different nights.

[Margin: Second reason for the reply.]

[Margin: Tycho's inconsistency and contradiction with himself.]

Third, from the meridian altitudes of the New Star taken by Paul Hainzel, and reported by us in Chapter 3, number 3, Tycho deliberately chose two from which there would follow an altitude of the pole at Göggingen of exactly 48° 22′, as much as had been deduced from the altitudes of the Pole Star. But he ought to have taken two altitudes taken on the same night: namely those of November 21, when the greatest was 76° 33′ 45″ and the least 20° 9′ 40″; or those of November 26, when the greatest was 76° 34′ 0″ and the least 20° 9′ 30″. Had he done so, the altitude of the pole on the 21st would have been smaller by 18″, or by 15″; and from this parallax a distance of the New Star would be gathered smaller than that of Saturn at apogee on his own hypothesis, or at any rate smaller than that of the fixed stars.

[Margin: Third reason.]

Fourth, as Chiaramonti noted (*De tribus novis stellis*, Book 2, chapter 7), Tycho in his demonstrations, in the fourth proof — which we treated in Chapter 5, in the Scholia — uses distances of the New Star from nine stars of Cassiopeia, whereas in his earlier writing he reports distances from only three stars of Cassiopeia, namely from Schedar, from the bright star of the Chair, and from the star at Cassiopeia's flank, as is clear from Tycho, volume 1, pages 343, 585 and 593. This led Chiaramonti to suspect that the remaining distances were invented rather than actually taken by Tycho. But this is not fully proved: for he could have taken them and yet not mentioned them in the earlier writing, as being from less notable stars, or as taken less often or less exactly.

[Margin: Fourth reason, from Chiaramonti.]

Fifth, Tycho admits (volume 1, pages 347 and 416) that in taking the distances of the New Star when it was at its least and at its greatest altitude he used the same sextant fixed with screws, keeping the same angle of the instrument, and from this he infers that the distances were the same. But Chiaramonti (*De stellis novis*, Book 2, chapters 5 and 6), having reduced Tycho's argument to this form — *"Whatever distance of two stars is observed everywhere under the same angle of the*

instrument always remains the same. But the distance of the new star from the same fixed stars was observed everywhere under the same angle of the instrument; therefore the distance of the New Star from the same fixed stars always remained the same" — denies that the major is true if the instrument is inclined to the horizontal plane now one way, now another. For then, he shows, because of the distance of the instrument's centre from the pupil, and because the cheekbone makes the eye's application to the angle of the instrument vary, the visual angle varies, and by several minutes at that. The distance of the New Star from the same fixed stars could therefore have varied between the highest and the lowest altitude, and yet the variation not have been detected by the instrument. But I do not make much of this subtlety of Chiaramonti's, because Tycho took the New Star's distances from the fixed stars of Cassiopeia when they were crossing the meridian nearly together with it, and so used nearly the same inclination of the instrument in both positions.

[Margin: Fifth reason, from Chiaramonti.]

[Margin: The imperfection of the sextant used with one eye.]

Sixth, Chiaramonti (*De novis stellis*, Book 2, from chapter 8 to 14) charges Tycho with a number of errors. These, however, are not so great that, if they were corrected, Tycho would then have demonstrated a large parallax of the New Star, or that they damage the substance of Tycho's assertion. Still, I am glad to point out the chief ones. Tycho, then, in volume 1, page 420, setting out to find the parallax the New Star would have had if it had been in the concave of the Moon, at its greatest altitude of 84⅙° and its least of 27° 45′, supposes that the pole of the world stands midway between these apparent altitudes. But this is not true unless the greatest and least altitudes are taken as true ones; otherwise, at the lower apparent altitude a greater parallax removes the star farther from the pole of the world, and makes its distance from the pole appear greater than when it is at its greatest altitude.

[Margin: Tycho's error 1.]

Moreover, Tycho, on page 429, supposes that the right ascension of stars, when they are on the meridian, is not subject to any parallax. But this is not true except when the meridian is the solstitial colure; otherwise a parallax of right ascension can arise from the parallax of altitude, as can readily be clear to experts, and as Chiaramonti shows (*Antitycho*, Book 1, chapter 1, and *De novis stellis*, Book 2, chapter 8, page 289).

[Margin: Tycho's error 2.]

Again, Tycho on page 429 says: *"The true declination, moreover, was found to be 61° 46′ 45″."* But this is the declination which, on page 331, he had deduced as 61° 47′ from the observed altitude of the pole at Herrevad, 55° 58′, and from the observed least altitude of the new star, 27° 45′. Yet a declination deduced from observation, so long as we are not certain of the parallax — which is the very point in dispute here — cannot be called true; it must be called apparent, and be used as such. So when Tycho adds in the same place, *"Since the parallax at the least altitude was found to be 58′ 31″ for the horizon of our observation, this,*

subtracted from the true declination, leaves the apparent declination as 60° 48′ 14″," the contrary

[Margin: Tycho's error 3.]

entirely ought to have been done, as Chiaramonti points out on page 291: the parallax ought to have been added to the apparent declination, to turn it into the true one. Or rather, since for the trigonometric calculation he was preparing Tycho needed the apparent declination, he ought to have used the declination of 61° 47′, or of 61° 46′ 45″, as the apparent one. For the rest, see Chiaramonti, Book 2, chapters 13 and 14 — points of no great moment, though true.

IX. It is clear, therefore, from Reply 1 to the major premise of the syllogism proposed at the outset, from Reply 2 to the minor, and in part from Reply 3, that Tycho — with his arguments drawn from parallax, resting on his observations of Hainzel and of Muñoz, which he chose in chapter 6 as the strongest — did not demonstrate that the New Star of the year 1572 was above the Moon, much less above the Sun or Saturn. And so this chapter must at last be closed with this conclusion of notable moment.

VNICA CONCLVSIO.

The single conclusion.

Neither by the arguments drawn from parallax and resting on his own observations, or on those of Paul Hainzel, or of Jerónimo Muñoz, nor by his other arguments — from twinkling, brightness and colour, from the lack of a tail, from the duration, from the diurnal revolution without proper motion, and from the sight of the star as viewed from many parts of Europe — did Tycho demonstrate that the New Star of the years 1572 and 1573 was above the Moon, much less above the Sun, above Saturn, or among the fixed stars of the eighth sphere.

The first part, on the arguments from parallax, is established from what has been said in this chapter; the latter part, on the other arguments, from what was said in chapter 4.

SCHOLIVM.

Scholium.

I. Tycho, and most others with him or before him, supposed that if a phenomenon keeps the same apparent distance from the same fixed star at its greatest and at its least meridian altitude, it necessarily follows that it lacks sensible parallax. Since that was their supposition, Chiaramonti demonstrated (*De novis stellis*, Book 2, chapter 4, and *Antitycho*, Book 1, chapters 9 and 10) that a star can keep an equal apparent distance from the same star as seen, even if it has a very large and highly sensible parallax; and we have shown this very thing in Book 10 of this volume, in the penultimate section, chapter 9, theorem 17. But not all the cases can be applied equally to the examples Tycho uses in

this place, and Chiaramonti did not show in particular the manner in which a phenomenon could appear to the same observer through the whole night at the same distance from the same fixed star — indeed from all of them — and yet have any given parallax whatever, and any distance from the Earth however small. How this can happen, however, is clear enough from our figure, set out in the preceding chapter from number 3; and that is why we used it in preference. I freely admit that formerly, when I was younger, overwhelmed by Tycho's arguments, I held the same view as Tycho and so many others about the location of this star — for I am not ashamed either to have erred with so many and such great men, or to confess the error — namely, that its place among the fixed stars had been demonstrated by Tycho. But afterwards, having examined Chiaramonti's *Antitycho* more deeply and looked into the doctrine of parallaxes, I re-read Tycho with a freer mind and found in him many things, either false or not demonstrated, which he himself had passed off as demonstrated. This I would perhaps not have done had Chiaramonti not tweaked my ear. Whether I have achieved more than he, or more clearly, let others judge.

[Margin: In commendation of Chiaramonti.]

CAPVT VII.

Chapter 7.

Whether Tycho, from other observations made in a single place on Earth and adduced in volume 1 of the Progymnasmata, *chapters 8 and 9, or the authors he reports there, or Santucci, demonstrated that the Star of the year 1572 was above the Moon.*

I. Up to this point our business has been with Tycho, but in weighing those arguments of his that are, in his own judgement, the strongest, since they rest on his own observations, or on Paul Hainzel's as though they were his own, because they had been made with a quadrant built by him. For Tycho says, in volume 1, chapter 6, page 451: *"It remains that we also take into counsel the observations made in different horizons and by different observers, weighing them in an equal balance — although, to say how the matter really stands, those experiments are far safer and more certain that are carried out by one single observer in the same horizon with the same carefully made instrument, provided each part is in order and subject to no sensible defect, than if the determination of the thing sought is looked for from different horizons and sighters. For both the variety of instruments, and the diligence and circumspection of observers, which is not the same everywhere in handling the business, can introduce some straying from the mark aimed at; and where the subtle investigation of parallaxes is concerned [?], this nonetheless errs a great deal in excess or in defect."* He presses this opinion on page 489 as well, and in volume 2, page 89; and with him Kepler, in the *Hyperaspistes*, page 129, and in *De Cometis*, page 77.

[Margin: Are observations from one place more certain than observations made in several places?]

[Margin: Error of Tycho and Kepler.]

But Chiaramonti (*Antitycho*, Book 3, chapter 3, and in his *Apologia*, Book 2, chapter 4) rightly held the opposite to be true. For, other things being equal, arguments from parallaxes resting on observations made in several places on Earth at the same moment are to be preferred: from them it can be established with certainty whether a phenomenon has a parallax, and how great, which cannot be established from observations made in a single horizon, as is clear from what was said in the preceding chapter, and from Section 1, especially chapter 22, number 11. Granted, if the operations of the individual observers are taken separately and do not agree, each may rightly keep the faith already pledged to his own observation and his own eyes, if he is evidently sure of the diligence he has sufficiently applied. Tycho, then, uses the observations of others in such a way that he accepts them as legitimate in so far as they favour his own opinion — that is, those that put this star back above Saturn, such as the ones he collected together in chapter 8; or at least those that do not put it below the Moon, of the kind he reports in chapter 9. But those that show this star was below the Moon he rejects or tries to refute — namely those he adduces in chapter 10 of volume 1. Yet this, in the end, is to reduce the force and weight of all observations to the observations of Tycho alone, made in one place on Earth, and to bend them, like a Lesbian rule, to his own norm. We, however, without any prejudice or prerogative of opinion or of author, shall consider how much weight there is in them for determining the parallaxes of the New Star, both in each separately and in all of them collected together.

Obſeruationes Vuilhelmi Haſſiæ Lantgrauij habitæ Caſſellis, ad Parallaxim ſpectantes.

Observations of Wilhelm, Landgrave of Hesse, made at Kassel, bearing on the parallax.

II. Tycho reports and examines these in volume 1 from page 491 to 505, and again on page 599, and we copied them out in chapter 3, numbers 2, 3 and 4. From them he hunts for the parallax of the New Star as follows. The Landgrave found the greatest meridian altitude of this star on December 26 to be 79° 30′, and the least, on the 29th, 23° 2½′ or 23° 3′. With the same instruments he had found, from fixed stars revolving about the pole, the altitude of the pole at Kassel to be 51° 16′, but afterwards 51° 18′. If you subtract this from the greatest altitude of the new star, there remains a distance of the New Star from the pole of 28° 14′; but if you subtract the least altitude of the New Star from it, there remains a distance of the New Star from the pole of 28° 13′ 30″. Conversely, if you add half the difference between the greatest and the least altitude of the New Star to the least, or take it away from the greatest altitude of the same New Star, you will gather an altitude of the pole of 51° 16′ 15″ — as much as was first gathered from the circumpolar fixed stars, but less by 2′ than the later

altitude of the pole. "Therefore," says Tycho, "it had no sensible parallax, nor any greater than the parallax of the fixed stars." But if the New Star had been in the concave of the Moon, that is, at a distance from the Earth of only 52 semidiameters of the Earth, it would have undergone a parallax of 12′ at its greatest altitude and of 1° at its least, which he gathers by the problem and figure already given in chapter 5, numbers 2 and 7; and so at the least altitude it would have appeared 48′ farther from the pole of the world than at the least [?], whereas in fact it appeared equally distant from it at both. But if one supposes the New Star to be where the Sun is on Tycho's reckoning, its parallax would have been about three minutes, and quite perceptible. Tycho's argument, then, reduced for brevity's sake to a single enthymeme, runs as follows:

At the greatest and the least altitude of the New Star, the Landgrave of Hesse detected no parallax, or none greater than the parallax of the circumpolar fixed stars; therefore it had no parallax, or none greater than the parallax of the fixed stars, and so it was above the spheres of the planets.

[Margin: Tycho's argument.]

One could reply, first, by denying the antecedent. On the one hand, if the calculations already mentioned are considered, a parallax is found — that is, a difference of 30″ between the distances of the star from the pole of the world, and of 15″ between the altitudes of the pole; while from the later observation of the pole's altitude a parallax of more than two minutes is gathered, as the Landgrave himself admits in his letters to Peucer, which Tycho reports on pages 600 and 613; and each of these is greater than the parallax of the fixed stars. On the other hand, the Landgrave did not observe both altitudes of the New Star on the same night, but on different nights. It could have happened, however, that had he observed on the same night, he would have found a much greater difference between the two distances from the pole of the world, and so a much greater parallax, which, because of the motion of declination correcting that difference, he did not find on different nights.

[Margin: Reply 1.]

I reply, however, secondly but principally, by distinguishing the consequent. I grant the consequence concerning a parallax sensible at Kassel on those nights, but I deny it concerning a parallax sensible in itself on the same nights in different horizons. For on the nights of December 26 and 29 (although in comparing the altitudes Tycho uses them just as if they had been observed on a single night) the star could have been very close to the Earth, so that, had it been seen on that night by different observers, it would have appeared much farther from the pole of the world and from the fixed stars than it appeared at Kassel; yet to the observer at Kassel it would appear the whole night under the same fixed point of the highest heaven, revolving together with it in the diurnal revolution of the primum mobile, as we set out and showed in chapter 6. The Landgrave adds, however, in a letter to Peucer dated 1572 December 14, that through the eight preceding days he had always seen this star motionless in the same place in the firmament.

[Margin: Reply 2.]

Thaddæi Hagecij Obſeruationes factæ Viennæ in Auſtria circa Parallaxin.

Observations of Tadeáš Hájek made at Vienna in Austria, concerning the parallax.

III. Tycho deals with these in volume 1, from page 505 to 528. He brings into a summary certain selected passages, comprised in 15 chapters, taken partly from Hájek's *Dialexis*, where much is taught about parallaxes in general, and partly from a manuscript copy sent to Tycho, in which Hájek had emended the distances of the New Star he had taken and corrected what he had said in the *Dialexis*. In chapter 5 of the *Dialexis*, then, there is this argument of Hájek's: *Every body that has either no parallax, or one smaller than the Moon's, is in no way to be reckoned to the elementary region, but to the ethereal. Our star has been found to have no parallax; therefore etc.* He proves the minor by this prosyllogism: *Every body that keeps an equal distance from the poles of the world and from the stars, perpetually and everywhere, can have no parallax. Our star keeps an equal distance from the pole of the world and from the stars at all times and places; therefore it can have no parallax.* Presently he adds: *The major of the prosyllogism is known from the doctrine of parallax; the minor is made evident by our observations and those of other learned men, especially Cornelius Gemma and Jerónimo Muñoz, and so by the following demonstration.* He then appends a figure by which he contends to show that the New Star — both at its lowest meridian altitude of 20° 15′, taken on May 6, and at the altitude of 22° which it had after $24^{m}\ 38^{s}$ of time, and at a complement of the eastern azimuth of 12° 13′ — stood equally distant from the pole of the world, namely 28° 7′, using an altitude of the pole at Vienna of 48° 22′. And he offers no other demonstration.

[Margin: Hájek's argument.]

We reply, however, first by granting both propositions of the first syllogism and denying the inference, because the syllogism has four terms: in the major the verb *has* is used, and in the minor the verb *has been found*. It is one thing to have a parallax, another for it to be detectable or sensible; and it could have had one, yet not have been perceived at Vienna on the night of that day, May 6, or on certain other nights too, on which it would nevertheless have been perceived elsewhere, had observations by several observers fallen on the same night.

[Margin: Reply 1.]

I reply, secondly: granting the whole of the first syllogism and the major of the prosyllogism, I deny the minor of the prosyllogism, which Hájek did not demonstrate. First, he puts the observed distance of the New Star from the pole at 28° 7′, which Tycho, on pages 351 and 509, affirms was 28° 13′. Either, then, both were true but at different times, and so the distance was not the same perpetually; or both were seen at the same time, and so a parallax intervened. Next, he uses the azimuth with too short a time, $24^{m}\ 38^{s}$, in which the star

appeared higher by only 1¾°; and even Tycho himself judged this slippery and insufficient for perceiving parallax, saying on page 516: *Indeed, to detect a parallax from so very slight a variation of elevation is scarcely granted, even with the largest instrument, even if the phenomenon had been a little below the Moon.* For Tycho shows there, from the data given by Hájek, that if the new star had been in the concave of the Moon, the difference of the parallaxes it would have had at the two altitudes observed by Hájek would not exceed 48″, which he affirms could not have been perceived except with a very large instrument — and much less from so short a time, and that gathered from the 4th star of Cassiopeia, which, being then very near the meridian, varied its altitude too slowly, as Tycho points out on page 517. And on page 518 he adds: *I say nothing of the fact that finding the azimuthal angle is uncertain, and more liable to deviation than the investigation of the altitude itself. From all of which it follows that Thaddaeus's givens were not exact enough, nor sufficiently suited to the solid investigation of so subtle an inquiry.*

[Margin: Reply 2.]

[Margin: First reason of the reply.]

[Margin: Second reason.]

But let us grant that these reasons are too subtle. I say, thirdly, that it does not follow, if the New Star was seen at two moments of the same night to be equally distant from the pole, that it therefore kept the same apparent distance the whole night. He ought at least to have shown this from the highest and the lowest altitude of this star, which he nowhere did, as Tycho himself attests on page 515: *And although he asserts that it was everywhere equally removed from the pole, he nevertheless sets down no greatest altitude of it, in its lofty position, the one least subject to parallax, which, compared with the least, would show that the difference of the interval from the pole was insensible at both, etc.* And presently he adds about Hájek: *He appeals, however, to the observations of others, especially of Cornelius Gemma and Jerónimo Muñoz — would that they had been so precise and reliable!*

[Margin: Third reason.]

But I say, fourthly: if Hájek had shown from the history of this star that this star, seen on the same night, was equally distant from the pole of the world both at Vienna and at Louvain, where Gemma was, and at Valencia, where Muñoz was at the time, the minor of the prosyllogism would have been sufficiently proved for that night; and if he had shown the same for all the nights on which the New Star appeared, with respect to several places on Earth, it would have been proved absolutely. Since he did not do this, we rightly deny that the minor of the prosyllogism was demonstrated. See also what I shall shortly say about the uncertain altitude of the pole at Vienna, at the end of number 5.

[Margin: Fourth reason.]

Pauli Fabricij Mathematici & Medici Cæſarei Obſeruationes ad Parallaxin ſpectantes.

Observations of Paul Fabricius, Imperial Mathematician and Physician, bearing on the parallax.

IV. Of him Tycho, on page 529 of volume 1, sums up the arguments by which he said that the New Star was not below the sphere of Jupiter, in these words: *Setting out to investigate its parallax, he dispatches himself too quickly and in too broad a manner. He says that it always kept the same distance from the fixed stars, both on the meridian above and below and when it was farthest removed toward the sides on either hand — though he does not add what that distance was, but only alleges the testimonies of others, whom he does not even name. And from this he rightly infers that this star had either no parallax or an entirely insensible one; but he does not, as was fitting, persuade us of it demonstratively from indubitable proofs — perhaps because he was too eager for brevity.* Tycho himself, then, acknowledges that this was not demonstrated by Fabricius; and although he had employed the manner by which Tycho believed this to have been demonstrated, he would in fact have demonstrated no more than that the New Star lacked a parallax sensible in some single place, or in several places on different nights; whereas he would have had to bring both together — observations in several places on Earth, but made on the same nights.

[Margin: Fabricius did not demonstrate the star above the Moon.]

Bartholomæi Reiſacheri Mathematici Viennenſis obſeruationes ad Parallaxin ſpectantes.

Observations of Bartholomäus Reisacher, Mathematician of Vienna, bearing on the parallax.

V. Although Reisacher wrote about this star more lucidly than Fabricius, not even he demonstrated what he asserted — namely, that it was not in the elementary region. For his whole reasoning, as Tycho reports in volume 1, page 531, rests on this: that at both meridian altitudes observed at Vienna it kept perpetually the same distance from the fixed stars of Cassiopeia. But he proves this by no measurements from observations, and so Tycho adds: *Nor does he add what distances from the fixed stars of Cassiopeia this star had, but simply reports that they were alike at both, which another could have denied just as easily, especially since he does not say how great they were. Nor, likewise, does he determine absolutely the quantity of the parallaxes that would arise in the lofty and in the lower position for some presupposed distance from the Earth — the two things I find wanting in this process of demonstration.* There is also the fact that, as Tycho reports on page 532, this astronomer took the least altitude at Vienna as mostly 20°, and sometimes greater by 5′ or 10′. From this it appears that he observed either with a less exact instrument or less diligently — if one admits with Tycho in the same place that the distance of the star from the pole of the world was 28° 13′, and that the

altitude of the pole supposed by Reisacher, and received from Peuerbach and Regiomontanus, was 48° 22′. For in that case the least altitude of the New Star ought to have been 28° 9′ [?]; so when he saw it at only 20°, or 20° 5′, either he was deceived, or the star underwent a parallax. Tycho adds, moreover, that the altitude of the pole at Vienna is not certain, because it was perhaps taken from the greatest altitude of the Sun at the summer solstice and the least at the winter one, without subtracting refraction from the latter, as was necessary.

[Margin: Reisacher did not demonstrate that this star was above the Moon.]

But I came upon a much greater occasion for suspicion about this altitude in my geographical book. For Clavius, in the *Sphaera*, on page 263 of my copy, relates that Regiomontanus observed at Vienna, at the two solstices, a greatest altitude of the Sun of 65° 30′ and a least of 18° 26′. Add to the greatest my solar parallax of 5″, and to the least 23″, but subtract from the least the refraction of 5′ 40″: the true greatest altitude will be 65° 30′ 5″, and the least 18° 20′ 43″. Their half-difference, 23° 34′ 41″ (too large, indeed, since the obliquity of the ecliptic was smaller), added to the least gives an altitude of the equator of 41° 55′ 24″, and of the pole of 48° 4′ 36″ — smaller, that is, than the old value by about 17′. On the other hand, Kepler affirms that the altitude of the pole at Graz was observed as 47° 2′; and in the map of Wolfgang Lazius in which Austria is described — a very exact one, as it appears — Vienna is put 1° 42′ farther north than Graz, and in the Ortelian map 1° 41′; so the altitude of the pole at Vienna would come out at 48° 44′ or 45′. Between these extremes the mean is 48° 20′. The testimonies, then, waver on both sides for this altitude of the pole, and consequently for the distance of the New Star from the pole.

Pauli Hainzelij Obſeruationes ad Parallaxin ſpectantes prope Auguſtam Vindelicorum.

Observations of Paul Hainzel bearing on the parallax, near Augsburg.

VI. Besides what we brought forward with Tycho in chapter 5 on Hainzel's observations, there remains what Hainzel himself wrote to Hieronymus Wolf in letters reported by Tycho in volume 1, page 536. In them he affirms that he found, both on 1572 November 21 and on 1573 January 4, the greatest altitude of the New Star to be 76° 34′ and the least 20° 9⅔′; and he adds: *That there is no difference between the true and the apparent place of this star seems to be made credible by the fact that its greatest and least elevations, joined together, make for us the same number that all the fixed stars which are always in view and never set make, compounded from their greatest and least altitude.* By this argument it is indeed proved that no parallax of this star was perceived by Paul Hainzel on those two nights at Göggingen, and so that it was insensible to him. But whether it was so by accident — because perhaps the star, with respect to that place and observer, was held through all those nights under the same fixed point of the highest heaven by the diurnal motion about a cone having its axis at one end in the eye and at the other in the pole of the world, although it was placed below

the Moon, as we showed in the preceding chapter could happen — or whether it was insensible in itself and in every place, if it had been observed on the same nights in different horizons: this he by no means demonstrated. Tycho, moreover, wonders at this man's inadvertence, in that, though he denied this star a parallax, he nevertheless believed it could be among the comets and perhaps sublunary; for these contradict each other — the lack of all parallax, and a distance of the phenomenon from the Earth less than the Moon's. But when he saw Tycho's writings, he placed this star among the fixed stars, as Tycho relates on page 542.

[Margin: Hainzel did not demonstrate the insensibility of the New Star's parallax.]

Michaelis Mœſtlini Gœppingenſis Obſeruationes, ad Parallaxin ſpectantes.

Observations of Michael Mästlin of Göppingen, bearing on the parallax.

VII. At the time when he observed this star, Mästlin was living in the town of Backnang, serving as parish priest. He lacked instruments, but with the help of a thread he observed the place of this star, as Tycho reports in volume 1, page 543, and from this he tried to demonstrate its lack of parallax. How the unknown longitude and latitude of the star is gathered by spherical triangles — by stretching a thread through the disputed star and two other fixed stars found in the same straight line, then stretching the thread again through the star of disputed place and through two other fixed stars, different from the first, lying in another straight line with the doubtful star, and using the known longitude and latitude of those four fixed stars — we shall teach in Book 10; and Mästlin himself teaches it in the writing he published on this New Star, in Tycho, page 547, as does Longomontanus in Book 2 of the *Spherics*, chapter 9, problem 5. Nor does it concern this place. What bears on our subject is Mästlin's argument, given in Tycho, page 545, in these words: *But we have been taught by accurate observations that this prodigious star is to be numbered not only not among the meteors, but not even among the planets, but among the stars of the eighth orb, or firmament. For on the sixteenth day before the Kalends of December* [November 16], *at* 7^h *after noon, we observed its place most diligently, when it was applying to the top of the sky on the meridian; on the following day, at* 5^h *before sunrise, when it was away from the middle of the sky and nearer the horizon, we noted its place again; on the same day, at the seventh hour before noon, we again considered its apparent place, when, having returned to the meridian, it occupied the lowest point of its circle. The same was done diligently by us on the following days. Yet in all these observations its place in the eighth sphere was found, throughout the whole time since it began to be observed, neither differing nor changed by a single minute. Some indeed assert that it advanced toward the pole by almost two degrees; but we have found nothing of the kind. Since, then, no discrepancy of motion can be perceived by sense in the diurnal revolution, it will easily be demonstrated from Ptolemy's* Almagest *that its true motion does not*

differ from the apparent, and hence that its distance is so great that the half-diameter of the Earth is incomparable to it: from which we necessarily gather that it is neither in the regions of the air nor of fire, etc. After this he shows that if the new star had been in the concave of the Moon, that is, at a distance from the Earth of only 52 17′ terrestrial semidiameters, as Copernicus puts it for the Moon at perigee, it would have come about that at its least altitude, which was about 20°, it had a parallax of 63′. Tycho extols all this with wonderful praise, although he corrects Mästlin's latitude, longitude, declination and right ascension of this star, because in computing them Mästlin used uncorrected places of the fixed stars; nor does he approve of Mästlin's liking for the immense distance of the fixed stars that must be asserted on the Copernican hypothesis.

[Margin: Mästlin's opinion, and his argument for the New Star's place among the fixed stars.]

We reply to Mästlin, however, that if it were established that on those nights on which he observed the New Star to stay fixed in the same place under the sky, it was seen in other horizons to remain under the same place of the sky, it would follow necessarily that it lacked all parallax sensible in itself, and so was above

[Margin: Reply to Mästlin's argument.]

the planets. If, however, this is not established, and one looks at the observations he made in a single place on Earth, that does not follow necessarily: the star could have been very close to the Earth and yet, on many nights — not merely on one night or another — have appeared to him always under the same place among the fixed stars, and so its parallax would, by accident, not have been perceived by him. But what we said had to be established is not established. For from the history of observations brought forward in the preceding chapter, toward the end of number 7, no one is known to have observed this star determinately in November, on the 14th, 21st and 26th, except Hainzel. Mästlin, for his part, could have observed on the 16th and the following days and yet not have observed on the 21st and 26th. And as long as we are not certain of an observation of the star under the same point of the sky made, at the least, on the same night in different horizons, we are uncertain whether the parallax was insensible in itself, because of a great distance from the Earth, or by accident, because of a diurnal revolution of the star attuned to a single eye. Granted, it is very probable that the spectacle of so great a prodigy was not attuned to a single eye — nor indeed to several, on different nights only, as we shall say later.

Cornelij Gemmæ Louanienſis Obſeruationes ad Parallaxin ſpectantes, Louanij initæ.

Observations of Cornelius Gemma of Louvain bearing on the parallax, undertaken at Louvain.

VIII. This author gives us no trouble. In his earlier writing on this star he denies that a parallax of it greater than 4′ could have been detected by him, and in the later one — that is, in the book *De divinis naturae characterismis* — he denies that it had any parallax at all. Yet, as Tycho says on page 553 of volume 1, *he brings forward no reasons, as befitted a mathematician, why he holds this view of the parallax; nor does he indicate sufficiently from what observations he assigned it such a place.* Indeed, reporting on page 554 the distances of the New Star from the fixed stars taken by Cornelius, and comparing them with his own, Tycho says: *Hence it appears how immoderately erroneous Cornelius Gemma's observations were, so that in the distance from the Pole Star he went astray by almost 1½°.* And although Cornelius later corrected them, Tycho says of those very observations, on page 558, that he confused the bright star of Perseus with the Head of Medusa, and adds: *From this comparison it is clear enough how far astray the intervals of the stars are, even as corrected by Gemma, although he thinks he sought them out duly with great and repeated diligence.* From this it follows either that Tycho's distances alone were seen correctly, and so the Cornelian ones are not to be trusted; or, if we wish both sets to be correct, we fall into a parallax — if, that is, we suppose them to have been observed by both men on the same nights. For if on different nights, we shall conclude nothing certain about parallax considered in itself. It remains, therefore, that Cornelius Gemma did not demonstrate that this star lacked all parallax, or that it was above the Moon — especially since Tycho shows on page 560 that the place assigned to this star by Gemma is inconsistent with his own givens.

[Margin: Gemma did not demonstrate that the New Star was above the Moon.]

Hieronymi Munosij Hiſpani Obſeruationes factæ Valentiæ, ad Parallaxin ſpectantes.

Observations of Jerónimo Muñoz, a Spaniard, made at Valencia, bearing on the parallax.

IX. What Muñoz wrote on this star in the Spanish language, Cornelius Gemma translated into Latin, and from him Tycho copied out a great deal, in volume 1 from page 565 on. But what serves to elicit the parallaxes are the distances of the new star from certain fixed stars and the meridian altitudes observed at Valencia; Tycho compares these distances with his own, as below.

Distance of the New Star from the stars of Cassiopeia	Muñoz	Tycho	Difference
from the Seat	5° 20′	5° 19′ 0″	1′ 0″
from the Bend	5° 10′	5° 2′ 0″	8′ 0″
from Schedar	7° 50′	7° 50′ 30″	0′ 30″
from the Pole Star	26° 40′	25° 14′ 0″	86′ 0″
Greatest altitude	67° 30′	84° 11′ 0″	
Least altitude	11° 30′	47° 45′ 0″	
Difference of the altitudes	56° 0′	56° 26′ 0″	

Distance of the New Star from the stars of Cassiopeia	Muñoz	Tycho	Difference

Muñoz affirms, moreover, that with frequently repeated observations he found no parallax in this star, and therefore that it was above the Moon, Mercury, Venus and the Sun — indeed, that it clung to the heaven of the fixed stars; and yet he calls it a comet. Now, even if we suppose with Tycho that the altitude of the pole at Valencia is 39½°, and therefore that this star was seen to keep the same altitude relative to the pole at both its meridian altitudes — indeed, even if we granted that on many nights Muñoz saw the same distances from the same fixed stars — these things still do not suffice to demonstrate that it lacked all parallax, or that it was above the Moon. For it could have been very close to the Earth, and yet its parallax could, by accident, not be perceived in the horizon of Valencia, though it would perhaps have been perceived by others had they observed its distances from the pole and from the fixed stars on the same nights. Add to this that Tycho, not without reason, suspects that Muñoz's instrument showed only whole decades of minutes, since he recorded all distances and altitudes only by their decades; or else that he had a quadrant that showed only half-degrees in taking an altitude. The astronomical radius with which he took the distances, especially the larger ones, Tycho thinks imperfect; and this, he holds, is how it came about that in the difference of the altitudes Muñoz differs from Tycho by 26′, and in the distance of the New Star from the Pole Star by 1° 26′. If anyone wishes to defend the observations of both, he will certainly be forced to ascribe that variation to a great parallax. Hence it is quite clear that Muñoz asserted, but did not demonstrate, that this star had no parallax greater than the parallax of the Moon, Mercury, Venus and the Sun — let alone that it was in the heaven of the fixed stars.

[Margin: Nor did Muñoz demonstrate that the new star was above the Moon.]

Franciſci Valleſij Couarrubiani Opinio potiùs quàm obſeruatio.

The opinion, rather than an observation, of Francisco Vallés of Covarrubias.

X. This author, in the book *De sacra philosophia*, chapter 1, speaks of this star not from any observation of his own but from the opinion of others, as follows: *You will understand that the thing was truly most worthy of wonder, even from the opinions into which very learned men were thrown. For since they knew that no star of the first magnitude, such as this one seemed to be, had ever existed there — namely in Cassiopeia — and yet it seemed, whatever it was, to be above the Sun, because it had no parallax, some affirmed that it was indeed a star, and a new one; others, thinking this easier, that it was a comet generated within the heaven itself, etc.* But he in no way professes a demonstration of what he says about the parallax. So let us pass on to the others.

Gaſparis Peuceri, & Vuolfangi Schuleri Obſeruationes initæ Vittebergæ in Saxonia ad Parallaxin ſpectantes.

Observations of Caspar Peucer and Wolfgang Schuler, undertaken at Wittenberg in Saxony, bearing on the parallax.

XI. Tycho brings forward both the observations and the letters of these men in volume 1, pages 605, 618 and 622. In them it is said that the least distance of this star from the zenith observed at Wittenberg was 10° 4′ and the greatest 66° 27′ — that is, that its greatest meridian altitude was 79° 56′, on December 4, and its least 23° 33′, on November 27 — and that its declination was 61° 58′ near the zenith but 61° 39′ near the horizon; hence a parallax of 19′, and a distance from the Earth of 164 semidiameters of the Earth. Schuler adds, from the observations of D. Cruciger and of Reinhold, the τοῦ πόλου ἔξαρμα [Greek: tou polou exarma, "the elevation of the pole"] — that is, that the altitude of the pole at Wittenberg is 51° 54′ — and goes on: *With these observations of ours (apart from tiny minutes) agree the* παρατηρήσεις [Greek: paratērēseis, "observations"] — *that is, the observations — which the most illustrious Prince Wilhelm, Landgrave, made at Kassel in Hesse, and certain others in Prussia and Silesia, and, as I hear, at Tübingen. For all agree in this, that the present comet has left the elementary region and migrated into the aethereal.* But from this that very parallax which they assert is by no means demonstrated, because both the greatest and the least altitude ought to have been taken on the same night, and it is not established that this was done: for why is one referred to November 27 and the other to December 4? Add that the altitude of the pole used by Schuler seems very uncertain; for Tycho, in volume 1, page 610, affirms that the altitude of the pole at Leipzig observed by Johannes Hommel is 51° 17′. Wittenberg, moreover, does not face Leipzig due north but deflects somewhat toward the east, by a third part of a quadrant, as is clear from the more correct geographical maps; and from the road distance of 8 German miles between these cities, and the angle of position of 30°, he gathers geometrically a difference in the altitude of the pole of 28′, to be added to the altitude of the pole at Leipzig, so that the altitude of the pole at Wittenberg comes out at 51° 45′. Hommel is to be relied on, because the altitude was taken with a quadrant that showed single minutes by the method of Nonius, and from two altitudes of the Sun, the greatest 62° 11′ and the least 15° 15′; from these, neglecting parallax and refraction, he gathers the altitude of the pole at Leipzig as 51° 17′. But Tycho, on page 631, applying both corrections, gathers an altitude of the pole of 51° 19′, and therefore that of Wittenberg as 51° 47′. But if, according to the elder Peucer's booklet on the distances of places, the equinoctial shadow at Wittenberg is to the gnomon as 19 to about 15, or as 76 43′ to 60, the altitude of the pole is gathered from this as 51° 58′. If, however, we use our own solar parallax of 5″ at an altitude of 62°, and of 22″ at an altitude of 15°, and the refraction of 7′ 30″ due to 15° 15′, the greatest altitude of the Sun was 62° 11′ 5″ and the least 15° 7′ 52″, and the difference 47° 3′ 13″. Half of this, 23° 31′ 36″, added to the least altitude of

the Sun, gives an altitude of the equator of 38° 39′ 28″, and of the pole of 51° 20′ 32″; and so, adding 28′, the altitude of the pole at Wittenberg becomes 51° 48′ 32″. And this is much more probable than the value obtained through the uncertain edges of shadows.

[Margin: Schuler's opinion, but not demonstrated.]

Ioannis Prætorij Ioachimici Opinio.

The opinion of Johannes Praetorius of Joachimsthal.

XII. We need not labour here with this astronomer, for he brings no observations of this star from which he determined its distance from the Earth, let alone any demonstration. But in a writing on the comet of the year 1577 he also reports, about this star of the year 1572, the opinion of others and his own, in these words: *This star had no motion except the one it had in common with the fixed stars, and it always kept thereafter the same place it first had. Its distance from the Earth was so great that it could in no way be perceived, except that it was found to have been placed within the orbs of the planets, or above them. For this has been confirmed by the observations of the experts, and those who were the most excellent and most diligent of all in this field affirm that it was stripped of all parallax: for that reason no certain seat could be assigned to it. For although Saturn too lacks all variation of view, yet its distance could be determined from its own proper motion and other accidents — aids that this star wholly lacked.*

[Margin: The opinion of Johannes Praetorius, but not demonstrated.]

Philippi Appiani, & Io. Dee Opinio.

The opinion of Philipp Apian and John Dee.

XIII. Nor did Apian demonstrate his opinion; but he asserted it in a letter written in the German language to the Landgrave of Hesse, who had asked for his view on this star — translated into Latin and printed by Tycho in volume 1, page 643. Apian's words selected on this matter, taken from John Dee the Englishman, are these: *John the Englishman writes in several places, and from other astrologers, that a conjunction of Jupiter with Saturn makes a star like the Moon, without a tail. All these things, he says, happen through aspects too, just as through conjunction, provided that the planets regard each other centrally. Next, that this comet, or second star, does not exist in the elementary region but comes to be numbered among the fixed stars; from which it can be gathered that it is much higher than the Moon, and produces no parallax or causes any variation of view, as the Moon does. And if it had to be placed below the sphere of the Moon, its parallax would greatly exceed the Moon's. There is this too: no motion could be detected in it, but it moves about the poles of the world with the firmament exactly like a fixed star. That it appears without a tail I would also gladly ascribe in part to its very height. For the reasons already mentioned, I judge that this star stands very high in the aethereal region. But in saying that this new star is a comet, and moves in the aethereal region, the physicists and Aristotelians will not side with me.* Moreover, Tycho testifies

on page 690 that John Dee brought forward no observation about this star, let alone a demonstration.

[Margin: The opinion of John Dee and Apian, but not demonstrated.]

Thomæ Diggeſei Cantienſis Angli Obſeruationes ad Parallaxin ſpectantes.

Observations of Thomas Digges of Kent, an Englishman, bearing on the parallax.

XIV. This noble writer, in his booklet which he titled *Mathematical Wings and Ladders* [Alas & Scalas Mathematicas], and in which he gives many subtle problems on parallaxes, reports the distances of the New Star from the fixed stars set down below, taken with an astronomical radius indeed, but in a more correct manner than that used by others. Tycho, on page 654, sets his own beside them, with the difference between them:

Distance of the New Star from the stars of Cassiopeia

Names of the stars	Digges	Tycho	Difference
from the Breast	7° 47′	7° 50½′	3½′
from the Bend	4° 58′	5° 2′	4′
from the Knee	8° 5′	8° 3½′	1½′
from the Leg	9° 45′	9° 48′	3′
from the Foot of the Seat	1° 28½′	1° 31′	2½′
from the Middle of the Ascent	5° 15′	5° 19′	4′

From these distances, kept the same at every altitude in the vertical, Digges concludes that it was detected even by sight alone that the star had no parallax greater than 2′. Let us hear him, in chapter 10, in Tycho, page 674: *Otherwise, too, anyone will be able by this art to discern by sight alone, without any instrument, the ravings of those who think that it is in the elementary region below the Moon and has a conspicuous parallax greater by a whole degree, if he will only compare its situation with that other little star which is in the foot of Cassiopeia's seat. For you will never find the phenomenon to depart from it by three times the Sun's diameter. If, therefore, it had a parallax one degree larger, it would be necessary, while the phenomenon was above it in one vertical, that it should appear nearer to it by two thirds of the former distance, and not differ from it by a single diameter of the Sun, namely thirty minutes or thereabouts — as is clear from what is demonstrated in the tenth and fifteenth problem. But there is no one who could not discern so great a variation, at so small a distance, by sight alone, without any instrument. And anyone who will consider the matter in this way will not only know most certainly that there is not so great a variation of view, but will find it so small and insensible that he will rather hesitate within himself whether he should believe it, being without parallax, to be truly fixed above the orbs of all the planets — so far is anyone (who is not blind, nor blinded by the darkness of the grossest ignorance) from affirming it to be below the Moon.* But this whole reasoning binds us in the end only to grant Digges that, whether he observed this star in Kent or at Ludinum [?], no parallax was

perceived, nor one equal to the Moon's; it is not shown that it could not have been perceived by others observing on the same night in a different place on Earth. Add that Tycho, on page 681, finds among the longitudes and latitudes of the New Star that Digges deduced from his observations a disagreement or discrepancy of 4⅓° in longitude and 55′ in latitude.

[Margin: Digges's opinion not demonstrated.]

Eliæ Camerarij Obſeruationes ad Parallaxin ſtellæ huius ſpectantes Francofurti ad Oderam initæ.

Observations of Elias Camerarius bearing on the parallax of this star, undertaken at Frankfurt an der Oder.

XV. Elias investigates the parallaxes of this new star by the following method, as it stands in Tycho, volume 1, page 693. On 1572 November 23 and the following days, in the evening, this star stood 9° 30′ from the zenith of Frankfurt on the meridian, and in the morning of the following day 65° 52′; the difference was 56° 22′. The altitude of the pole at Frankfurt an der Oder is, by observation, 52° 24′, and the distance of the pole from the zenith 37° 36′. Subtract 9° 30′ from this, and there remains the apparent distance of the new star from the pole, 28° 6′. Conversely, subtract 37° 36′ from 65° 52′, and there remains a distance from the pole of 28° 16′. Therefore, says

Elias, the parallax is 10′. Again, in the year 1573, on January 5, 6, 7 and 8, the distance of the new star from the zenith was 9° 33′ in the evening and 65° 43′ in the morning, from which he gathers a parallax of 4′ 30″. And on April 7 and the following days the greatest distance from the zenith was 65° 45′, while the least could not be taken because of the approach of the Sun; but on August 7 the least was found to be 9° 34′. Now, if the star had not been higher than the Moon but where the Moon is, it would have shown a parallax of 67′. It would therefore have come about, had it been in the elementary region, that the New Star would have stood off from the stars of Cassiopeia by a very sensible and greater interval when it was below the pole than when it was above. But even granting these data, still without any inquiry[19], it does not follow — if no greater parallax was found at Frankfurt — that none was seen, or was visible, elsewhere either, had the star been observed by others on the same nights. Moreover, the altitude of the pole that he supposes, 52° 24′, is uncertain; for Origanus, Kepler, Lansbergen and Wendelen put it at 52° 20′, and from the altitude of Leipzig and the road distance no more than 52° 6′ is gathered, as I said in my geographical book, where I treat the distance of the meridians, or the difference of longitudes.

[Margin: Elias's opinion not demonstrated.]

[19]printed *diquisitione*, for *disquisitione*

Eraſmi Reinholdi Iunioris Opinio & fortè Obſeruationes ad Parallaxin huius ſtellæ ſpectantes.

The opinion, and perhaps the observations, of Erasmus Reinhold the Younger bearing on the parallax of this star.

XVI. This Erasmus the Younger, son of the elder Reinhold, the author of the Prutenic Tables, and a professor of medicine, records in his prognostication for the year 1574 the observations set down below concerning this star, as having been made at Erfurt, as Tycho has it in volume 1, page 700.

	Value
Apparent greatest altitude	79° 30′ 0″
Its parallax	0° 0′ 34″
True greatest altitude	79° 30′ 34″
Apparent least altitude	23° 2′ 30″
Its parallax	0° 2′ 56″
True least altitude	23° 5′ 26″
Altitude of the pole	51° 18′ 0″
True declination	61° 47′ 0″
Apparent declination at the greatest altitude	61° 48′ 0″
Apparent declination at the least altitude	61° 44′ 30″
True latitude	53° 35′ 30″
Apparent latitude at the greatest altitude	53° 36′ 0″
Apparent latitude at the least altitude	53° 33′ 14″
True place, always the same	♉ 7° 12′ 26″
Apparent place at the greatest altitude	7° 12′ 45″
Apparent place at the least altitude	7° 10′ 33″
Distance from the centre of the Earth: 1079 semidiameters of the Earth	

Reinhold says this about so small a parallax, but he does not demonstrate it. Indeed, Tycho sees that he uses an altitude of the pole at Erfurt of 51° 18′ and the altitudes of the New Star observed at Kassel by the Landgrave of Hesse; and since the altitude of the pole at Kassel is 51° 18′, while the altitude of the pole at Erfurt is smaller in the geographical maps — 51° 10′ in the Prutenic Canons, indeed, and 51° 6′ in mine — he rightly suspects that Reinhold is speaking not of his own observations but of the Landgrave's, about which I have said enough in number 2.

XVII. So far, then, none of the authors whom Tycho brought forward in his support in volume 1, chapters 8 and 9, has demonstrated to us from his own observations, made in a single place on Earth, that this New Star lacked a parallax sensible in itself, or had a parallax smaller than the Moon's. For either they demonstrated nothing, but brought forward the opinions of others; or, if they demonstrated anything on this point, it showed only that a parallax was not perceived in their place, or that the parallax that was perceived was smaller than the Moon's. All of this could have happened by accident, so that, had several observers in different horizons observed the New Star on the same night,

they could have detected a parallax greater than the Moon's, as we taught in chapter 6. In this way the argument of Antonio Santucci is also answered, by which he contends, in his book *De Cometis*, chapter 7, that this star was among the fixed stars because at Pisa, at an altitude of the pole of 43° 30′, he observed the greatest altitude of this star as 79° 30′ and the least as 7° 30′, and so equally distant from the pole of the world on both sides, namely by 36°. But whether it can be demonstrated from the observations of the authors already mentioned — taken not separately but all together — that it was above the Moon, we shall see partly in the following chapter and partly in chapter 17. Meanwhile, let this be the

[Margin: Santucci's argument invalid.]

VNICA CONCLVSIO.

The single conclusion.

None of those whom Tycho brings forward in volume 1, chapters 8 and 9 — nor Antonio Santucci either — demonstrated that the New Star of the year 1572 had a parallax smaller than the Moon's, let alone that it had no parallax sensible in itself; and so none of them demonstrated that it was above the Moon.

CAPVT VIII.

Chapter 8.

Whether Galileo, from observations made in different places and compared with one another, demonstrated that the New Star of the year 1572 was above the Moon, or even among the fixed stars.

I. Chiaramonti contends, in Book 1 of *De novis stellis*, chapter 26, that from twelve pairs of meridian altitudes of this star, observed in different places, it follows that this star was below the Moon. But Galileo, in the third Dialogue on the system of the world, affirms that there are many more pairs of observations that raise it above the Moon: five, he says, that put it above the Moon, five others that put it among the fixed stars, and 30 that raise this star to an infinite distance. But in this he asks pardon if, by adding or subtracting a few minutes, he brings observations that do not agree into agreement, so as to raise the star to the firmament; and on page 287 he has these words, faithfully rendered into Latin by us: *So that it may be understood more fully, and so that it may be known readily how much more probably it is gathered, from the observations of the astronomers whom the Author uses, that the New Star was higher than the Moon:* and again on page 302: *From what I have shown so far you can learn how much more probably, and more clearly, it is gathered from them that the distance of the star is to be referred to the most remote fixed stars.* Since, then, Galileo does not contend in that place that the height of the New Star above the Moon has been demonstrated by him, but only that it has been asserted by more probable arguments — while we are still conducting the controversy about geometrical demonstrations, and are to speak about probability later — we could finish this chapter at once from his

own words already quoted, and lay down a negative conclusion. But someone might suspect from his calculations that geometrical certainty had been kept, and accordingly that it had been demonstrated, from some true hypothesis and from some correct observations, that the New Star was above the Moon; and since there are not a few errors in his calculations, they must be uncovered by us, for this will serve the examination of probability to be undertaken later.

Diagram. Set in the right column beside the opening of §II: three concentric quadrant arcs about the centre of the Earth, A, with the radius AO upright at the right and AG along the foot. The outer arc runs from O at the top to G at the bottom left, the middle arc ends at M on the upright radius and at H on the foot, and the inner arc ends near L on the upright and at K on the foot. Two radii run from A out to E and to F on the outer arc. The point C on the middle arc is joined by straight lines to B, near the upright radius, and to D, inside the figure; further lines run from C and from D to A. A small round mark beside the inner arc, left of D, is not identified as a label.

- E (top of the outer arc): point on the vertical ABE
- O (top right, end of the upright radius): end of the quadrant of the highest heaven, OG
- F (left on the outer arc): point on the vertical ADF
- M (on the upright radius): end of the New Star's quadrant, MH
- B (near the upright radius, at the top of the inner arc): the Landgrave of Hesse's station; end of the Earth's quadrant, BK
- L (beside B, on the upright radius): label only; not named in the text on this page
- C (on the middle arc): the New Star
- D (inside the figure, on the inner arc): Tycho's station
- G, H, K (along the foot): ends of the quadrants of the highest heaven, of the New Star, and of the Earth
- A (bottom right): the centre of the Earth

[Margin: 1. Galileo's first inquiry.]

II. In the present figure, with three quadrants of meridians described from the centre of the Earth — the Earth's, BK; the New Star's, MH; and the highest heaven's, OG — let the New Star be at C, seen by Tycho from D and by the Landgrave of Hesse from B, along the lines of the apparent place BC and DC; and let the vertical lines of the two places, drawn from the centre of the Earth, be ABE and ADF. With these laid down, Galileo, in his first inquiry, from the altitudes of the pole, and from the least altitudes of the New Star observed by Tycho and the Landgrave, as set out here, reasons as follows.

	Altitude of the pole	and of the New Star
Tycho	55° 58′	27° 45′
the Landgrave	51° 18′	23° 3′
Difference	4° 40′	4° 42′

Subtract the difference 4° 40′ from 4° 42′: a parallax of 2′ remains, that is, the angle BCD. The chord of the difference of the altitudes of the pole — that is, of the angle BAD, or the arc BD, of 4° 40′ — is 8142 in the tables of sines, in parts of which the radius AB is 100,000. Next, the angle BDC is given. For half the angle BAD, which is 2° 20′, added to a right angle makes the angle BDF, 92° 20′; and if you add to it the angle CDF, 62° 15′ — that is, the distance of the New Star from the zenith of Herrevad, where Tycho observed — BDC becomes 154° 45′ (says Galileo, in error), whose sine is 42,657. The sine of the parallactic angle C, 2′, is 58. Now in the triangle BCD the side DB is to the side BC as the sine of the opposite angle BCD is to the sine of the opposite angle BDC; therefore BC is 42,657 in parts of which BD is 58. But in parts of which DB is 8142, or BA 100,000, BC will be, by the golden rule, 5,988,160 ¼; and so BC is 59 88,160/100000 parts, of which AB is one semidiameter of the Earth. If one semidiameter is added to this, AC will be nearly 61 semidiameters of the Earth; and so the New Star C will be found above the Moon at perigee, which for Copernicus is not more than 52 17′ semidiameters of the Earth distant.

[Margin: 1. Galileo's error in geometry.]

Yet there are not a few errors in this reasoning. First, if the angle BDC is 154° 45′, as he expressly puts it (and that this is no printer's error is clear from the sine 42,657, which truly corresponds to an angle of 154° 45′), and the angle BCD is 2′, then these two angles together are 154° 47′. Now the angle BDF is, for Galileo, 92° 20′, and the other angle below the base of the isosceles triangle ABD, namely the angle EBD, is just as large; and the angle CBE — that is, the complement of the altitude of the New Star seen by the Landgrave at B — is 66° 57′, since the altitude was 23° 3′. Taking 66° 57′, then, from 92° 20′, the angle CBD remains, 25° 23′. Joining the angles of the triangle DBC, therefore, the sum will be 180° 10′, and so greater than two right angles. This is contrary to Euclid, Elements, book 1, proposition 32, and is plain even to beginners in geometry. Five days later I found this error noted also by Chiaramonti, in the *Apologia* in which he defends himself against Galileo's objections, part 3, chapter 22.

[Margin: 2. Galileo's error.]

The second error, which was the source of the first, is that he took the apparent altitudes of the New Star as though made in exactly the same meridian, and indeed at the same moment of time, and at the same time as exact. But it is false that Kassel and Herrevad lie exactly under the same meridian; and it is only gratuitously supposed that both observations fell at the same moment of time, since Tycho did not note the time at which he observed the altitude of the New Star. And although the meridian may be imagined as carried across, because the New Star is supposed to have had no motion other than that of the primum mobile, and both observations may be imagined as made at the same moment, the observations as made still cannot be reconciled with one another, nor accepted as exact. This is clear from the fact that, if the altitudes of the pole and of the New Star given by the observers and accepted by Galileo

are accepted, the triangle BCD is found to be of 180° 10′, or greater than two right angles, which is absurd.

[Margin: 2. Galileo's inquiry.]

III. The second combination, on page 292, is of the observations of Camerarius and Muñoz, from which he strives to find out the distance of the New Star from the Earth. The foundations of the calculation are of the following kind, applied to the preceding figure and with the same method employed as in the preceding number, supposing Camerarius observing at D and Muñoz at B.

	Altitude of the pole	of the New Star
Camerarius	52° 24′	24° 28′
Muñoz	39° 30′	11° 30′
Difference BAD	12° 54′	12° 58′
Difference of the parallaxes BCD		0° 4′

From these he gathers the angle BDC, 161° 59′, with its sine 30,930, and the chord of the angle BAD, 22,466, and the sine of the angle BCD, 116 parts. From this, then, he gathers BC as 59, and AC as about 60, in parts of which AB, the Earth's semidiameter, is one.

[Margin: 1. Galileo's error [?].]

But here too he errs in many ways. First, he assumes the apparent altitude of the New Star by Camerarius as 24° 28′, here and on page 288. You will find this nowhere in Tycho, but only 24° 8′ on November 23, 24° 17′ on January 5, and 24° 20′ on April 7, as is plainly established from the account which I reported with the greatest fidelity in chapter 3, number 3. Nor could he have taken that 24° 28′ from Chiaramonti himself, who in *De tribus novis stellis*, book 1, page 30, reports those least altitudes exactly as Tycho does — 24° 8′, 24° 17′, 24° 20′ — but nowhere 24° 28′. So, whether by guile or, more likely, by a slip of the pen or of the imagination, he founded his calculation on a false assumption.

[Margin: 2. Galileo's error [?].]

Secondly, here too Galileo either supposes these observations to have been made under the same meridian and at the same moment, so as to gather from them the difference of the parallaxes, or the angle BCD — yet Muñoz did not record the time at which he observed, and so it is only gratuitously imagined that both observed those least altitudes of the New Star at the same moment; and neither could that have happened, because, if the star was observed in the meridian at both places, the meridian of Frankfurt an der Oder, where Elias Camerarius observed, is more easterly than the meridian of Valencia, where Muñoz observed, by 17° 11′, that is, by more than the space of one hour. Or else he supposes that the New Star, carried from one meridian to the other with no motion other than that of the primum mobile, kept the same declination. But this is both assumed gratuitously and cannot agree with the altitudes seen by Camerarius and Muñoz, and with the altitudes of the pole and the parallax

accepted by Galileo; for from them it follows either that the triangle BCD has three angles together greater than two right angles, or that the difference of the parallaxes is not 4′, as he would have it.

For the angle BAD, or the difference of the altitudes of the pole, is given as 12° 54′. Therefore, by Elements, book 1, proposition 32, the angles ABD and ADB together are 167° 6′; by proposition 5 of book 1, each of them is 83° 33′; and by proposition 13 of book 1, the adjacent angle DBE, as well as BDF, is each 96° 27′. The angle CBE, the complement of the altitude of the star seen by Muñoz, is 78° 30′; subtracting this from DBE leaves the angle CBD, 17° 57′. Now if to the angle BDF, which is 96° 27′, you add the angle FDC, that is, the complement of the altitude seen by Camerarius — an altitude which was 24° and, first, 8′, then 17′ — the angle BDC will be 162° 19′ or 162° 10′. Adding to this the angle CBD, 17° 57′, the sum is 180° and 11′, or 2′; and if you add to these the parallactic angle BCD, the total will come out still greater than two right angles. But if you assume with Galileo the altitude seen by Camerarius as 24° 28′ (which is, however, falsely invented), BDC comes out 161° 59′, and with CBD 179° 51′; so the third angle, BCD, comes out 9′. And if you assume the last altitude, 24° 20′, BDC becomes 162° 7′, and with DBC 179° 59′, and accordingly the angle BCD is of one minute. This last case would favour Galileo, but he did not see it; for it would raise the New Star far above 60 semidiameters of the Earth. But since two or three other cases are in conflict with this one, it is clear enough that nothing has been solidly demonstrated by him for a seat of the New Star above the Moon — to say nothing of the uncertain altitude of the pole at Frankfurt, of which we spoke in chapter 7, number 15.

[Margin: 3. Galileo's inquiry [?].]

IV. The third comparison that Galileo uses is of the observations of Tycho and Muñoz of the greatest altitudes of the New Star observed in the meridian, as you see here.

	Altitude of the pole	of the New Star
Tycho	55° 58′	84° 0′
Muñoz	39° 30′	67° 30′
Difference BAD	16° 28′	16° 30′
Difference of the parallaxes, or BCD		0° 2′
Angle BDC	104° 14′	

And the method of investigating the difference of the parallaxes BCD, and from it the distance of the New Star from the Earth, — but of 478 semidiameters of the Earth — is exactly the same as the one we set out in the two preceding numbers by means of the figure of number 2; and there is no need to report anything but the foundations of the calculation that he assumes, and to suppose Tycho at D and Muñoz at B.

[Margin: 1. Galileo's error.]

But in this reasoning he assumes the altitude observed by Tycho as 84°, from Tycho's earlier writing — an altitude Tycho did not in fact observe, because he had only a single sextant, as Tycho himself admits in volume 1, pages 335, 351 and 420 taken together. There he also gives the greatest altitude of the New Star as 84⅙°, not observed but deduced from the least altitude, of 27° 45′, and from the altitude of the pole at Herrevad, 55° 58′, as he expressly states in the same place, supposing that the star had no parallax, and accordingly that the apparent distance of the star from the pole must have been equal in both meridian altitudes. This he laid down from a preconceived opinion, not from his own observation, and by assuming as certain the very thing that was in dispute. Therefore to assume the greatest altitude of the New Star as 84°, or 84⅙°, as though observed by Tycho, is to assume something uncertain and resting on nothing but a disputed supposition. Next, Tycho himself regarded Muñoz's observations as uncertain and rather rough, as we said in chapter 6, number 8; for if he accepted them as exact, a large difference of declinations would come out between himself and Muñoz. So not even by this inquiry has Galileo demonstrated that the New Star was above the Moon, except from a false or very uncertain hypothesis.

[Margin: 2. Galileo's error.]

Secondly, Galileo treats the observations as though the star, on passing from one meridian to the other, had not changed its declination — which is in dispute.

[Margin: 4. Galileo's inquiry.]

V. Galileo's fourth comparison is of the greatest altitudes of the New Star observed in the meridian by Muñoz and Peucer. From them, by the figure and the method already set out above, he gathers the distance of the New Star from the Earth as 358 semidiameters of the Earth, from a difference of the parallaxes of 2′. The foundations of the calculation are in the small table added here.

	Altitude of the pole	of the New Star
Peucer	51° 54′	79° 56′
Muñoz	39° 30′	67° 30′
Difference BAD	12° 24′	12° 26′
Difference of the parallaxes BCD		0° 2′
Angle BDC	106° 16′	

[Margin: 1. Galileo's error.]

But Galileo's first error is that he assumes the altitude of the pole at Wittenberg, where Peucer observed, as certain or more probable at 51° 54′, whereas by Tycho's reasoning it is 51° 47′, or by our parallax of the Sun 51° 48′ 32″, but by the observation of the elder Peucer 51° 58′; and so it is very uncertain. Tycho's value, however, or ours, is more probable, as in chapter 7, number 11; and from it there follows a difference of the altitudes of the pole of 12° 18½′, and of the parallaxes of 7½′.

[Margin: 2. Galileo's error.]

The second error, here and in all the preceding and following reasonings, is that he supposes either that the observations were made by both on the same night, at the same moment and under the same meridian — of which the first is uncertain and the second false — or else that the New Star, carried from the meridian of Valencia to the meridian of Wittenberg, and that after some days or hours elapsed between the two observations, had meanwhile been moved by no motion other than that of the primum mobile, and had not sensibly changed its declination, or its distance from the pole of the world. For unless he supposed this, he could not correctly deduce the difference of the vertical parallaxes from the difference between the difference of the altitudes of the pole and the difference of the altitudes of the New Star. But this very point is in dispute. Suppose someone were to say that the New Star, observed indeed on different nights by Peucer and by Muñoz, had those apparent altitudes; but that, had it been observed on the same night under the same meridian by some single observer placed on the parallel of Peucer or of Muñoz — that is, under the same altitude of the pole under which they lived — it would have had quite different apparent altitudes, and a quite different difference between them. If anyone, I say, were to say this, he cannot be refuted from these observations. Therefore Galileo's reasoning and conclusion are uncertain, and accordingly nothing has been solidly demonstrated for raising the star above the Moon.

[Margin: 5. Galileo's inquiry.]

VI. The fifth comparison, which Galileo uses in order to establish from it that the New Star was removed from the Earth by more than 716 semidiameters of the Earth, is of the greatest altitudes taken in the meridian by the Landgrave and Hainzel. The method and the figure are the same as were used so many times above, so it is enough to indicate the foundations of the calculation, and to suppose the Landgrave at D and Paul Hainzel at B.

	Altitude of the pole	of the New Star
the Landgrave	51° 18′	79° 30′ 0″
Hainzel	48° 22′	76° 33′ 45″
Difference BAD	2° 56′	2° 56′ 15″
Difference of the parallaxes BCD		0° 0′ 15″
Angle BDC	101° 58′	

[Margin: 1. Galileo's error.]

But first, Galileo either errs or, by a gratuitous choice, exposes himself to the danger of erring. For among the various greatest altitudes of the New Star given in our account in chapter 3, number 3 — namely 76° 33′ 45″, 76° 34′ and 76° 35″[20] — he chooses the least, 76° 33′ 45″, when it would have been better and more probable to choose the middle one among them, 76° 34′ 30″, as Tycho chose.

[20] so printed, with a double prime; the same altitude is set 76° 35′, with a single prime, further down this page

[Margin: 2. Galileo's error.]

Secondly, he supposes that the New Star, while it was carried from one meridian to the other (for Kassel, where the Landgrave observed, and Göggingen, where Hainzel observed, are not under the same meridian), so that by this fiction, as though the observation had been made under the same meridian at the same moment, he could fish out the diffentiam [?] of the parallaxes — he supposes, I say, that the New Star meanwhile changed its apparent declination not at all. But this very point, so long as it is not shown from the observation itself, is in dispute. For what if someone else, on the same parallel as Göggingen and under the meridian of Kassel, observing at the same moment, had found a meridian altitude of the star far different from Hainzel's? Moreover, Chiaramonti likewise condemns that choice of 76° 33′ 45″, in the *Apologia* for the books *De novis stellis*, part 3, chapter 27, and adds that if he had chosen 76° 34′ 30″, or 76° 35′, the sum of the angles CDB and CBD would be greater than two right angles. But for that very reason Galileo would perhaps say that he had set these aside and chosen 76° 33′ 45″. Still, he cannot avoid the fact that this choice, and the conclusion derived from it, rests on an uncertain foundation. Therefore what Galileo adds after these five combinations and inquiries — namely, that no difference of the parallaxes greater than 10′ 15″ was found in them, and that once these were removed the New Star would easily be placed in the firmament — is subject to the same uncertainty.

VII. After the five combinations already adduced, from which Galileo tried to show that the New Star stood distant from the centre of the Earth by semidiameters of the Earth as you see in the small table:

Combination of observations	Distance from the Earth, in semidiameters of the Earth
1	61
2	60
3	478
4	358
5	716

[Margin: 5. Other inquiries of Galileo.]

he selected five other comparisons of observations reported by Tycho and by Chiaramonti, in which, by comparing the altitudes of the pole and the meridian altitudes of the New Star with one another, he found the differences of the two to be equal, and the difference of the parallaxes to be nil, and so also no sensible parallax. Accordingly he concludes that, taking them as they lie and with no correction brought in to make them agree, it follows that the New Star was in the firmament. The observations are the following.

Observers	Altitude of the pole	Meridian altitude of the New Star
Camerarius	52° 24′	80° 26′
Peucer	51° 54′	79° 56′

Observers	Altitude of the pole	Meridian altitude of the New Star
Differences	0° 30′	0° 30′
the Landgrave	51° 18′	79° 30′
Hainzel	48° 22′	76° 34′
Differences	2° 56′	2° 56′
Tycho	55° 58′	84° 0′
Peucer	51° 54′	79° 56′
Differences	4° 4′	4° 4′
Reinhold	51° 18′	79° 30′
Hainzel	48° 22′	76° 34′
Differences	2° 56′	2° 56′
Camerarius	52° 24′	24° 17′
Hájek	48° 22′	20° 15′
Differences	4° 2′	4° 2′

[Margin: 1. An error of Galileo's, or a doubt about an error.]

VIII. But in all these combinations Galileo, first, supposes the observations to have been made at the same moment of time and under the same meridian — that is, even if they were not so made, then at least in such a way that, the New Star being carried from one meridian to the other, anyone who had observed the altitude of the star under the same parallel of the equator would have seen the same altitude as the other observer saw under that parallel but under a different meridian. For example: Peucer, at Wittenberg, which is more westerly than Frankfurt an der Oder, observed the altitude of the New Star as 79° 56′; the star was then carried, either on the same night or on any other night, to the meridian of Frankfurt, where Camerarius was observing; and if anyone under the meridian of Frankfurt, but at an altitude of the pole equal to that of Wittenberg, had observed the altitude of the New Star, he would have found it as great as Peucer did, namely 79° 56′. The reason is that there was a persistent rumour that the star was endowed with no motion other than that of the primum mobile [primi Mobili], and did not change its distance from the pole of the world or from the fixed stars, as Galileo affirms on pages 307 and 315. But this very point is in dispute. For it is not established that the pairs of observations selected by Galileo, nor any others, a single case excepted, fell at the same time; indeed it is established from the account in chapter 3 that they fell on different nights. And the star could have kept, or returned to, the same distance from the pole and from the fixed stars on those different nights, and yet on other nights have varied it so much that if, for example, Camerarius and Peucer had observed it on the same night, they would have found it very different; and so for the rest. Therefore that supposition of an unchanged declination, and that report spread among the astronomers, although not lacking in probability, is nevertheless uncertain, and insufficient for the certainty of a geometrical demonstration, which is what concerns us here.

[Margin: 2. A doubt about Galileo's foundation.]

Secondly, from the greatest altitudes of the New Star observed by Camerarius — 80° 30′, 80° 27′, 80° 26′ — which he himself had reported on page 288, Galileo then chooses, on page 304, the least, 80° 26′, so as to obtain no difference of the parallaxes, since from the first there follows 6′ and from the second 1′. And from the first, indeed, as Chiaramonti notes in part 3 of this *Apologia*, chapter 28, the comparison could not serve for finding the distance, because the three angles of the triangle BCD, as in the figure set out above, would come out greater together than two right angles; while from the second the distance of the New Star would be gathered as only 29 66/100 semidiameters of the Earth.

[Margin: 3. A doubt about Galileo's foundation.]

Thirdly, in the second and fourth comparisons he uses Hainzel's altitude of 76° 34′, which, however, he chose above, as we said in number 6, as 76° 33′ 45″. And in the third comparison he chooses Tycho's altitude of 84°, which, as is clear from Tycho's volume 1, page 420, should have been chosen as 84⅙°. But from what was said in number 4 it is clear that this very value is uncertain, and not observed by Tycho, but derived, from the presumed absence of papallaxeos [?], from the least altitude of the New Star and from the altitude of the pole at Herrevad. Finally, in the fifth comparison, out of the three least altitudes observed by Camerarius and acknowledged by Galileo on page 288 — namely 24° 28′ (where 24° 8′ should be read), 24° 17′ and 24° 20′ — he himself chose only one, 24° 17′, on an uncertain foundation, certainly.

[Margin: 4. A doubt about Galileo's foundation.]

Fourthly, the altitudes of the pole that he uses — namely of Frankfurt, 52° 24′, of Wittenberg, 51° 54′, and of Vienna, 48° 22′ — are very uncertain and probably in need of correction, as we taught in our geographical book and indicated in chapter 7, numbers 5, 11 and 15. But let us grant that they are certain: why, then, does Galileo not use the observations made at Vienna by Hájek and at Göggingen by Hainzel, these being at the same altitude of the pole, 48° 22′? Now the least altitude of the New Star observed by Hájek was 20° 15′, and by Hainzel 20° 9′ and 20″, or 30″, or 40″, as we related in chapter 3, number 3. So the difference of the parallaxes, or the parallax, would have been at least 5′ 20″, and the New Star accordingly far below the fixed stars.

[Margin: Galileo's error.]

IX. After this, Galileo says on page 306 that there remain 30 pairs of observations which raise this star to an infinite distance; but he neither shows this nor can show it. For, as Chiaramonti rightly argues in this *Apologia*, chapter 29: if some parallax is gathered from the observations, the phenomenon is below the fixed stars; if no sensible parallax, none of the astronomers places that phenomenon beyond the fixed stars, which are themselves also supposed to lack all sensible parallax; or at any rate, if the observations supply a triangle from which some parallax can at last be drawn, even if not a sensible one, that parallax itself requires a finite distance after all; but if the triangle gathered from the observations has three angles greater than two right angles, then no parallax can indeed be gathered from it, but it cannot be positively concluded,

or assumed, to be nil. Moreover, Galileo acknowledges on page 307 three other comparisons from which there would follow a distance of the New Star from the Earth smaller than one whole semidiameter of the Earth — namely those in which the altitude of the New Star taken by the one of the observers under the greater altitude of the pole is the smaller. They are set down here; but he rejects them as improbable.

Observers	Altitude of the pole	Altitude of the New Star
the Landgrave	51° 18′	79° 30′
Gemma	50° 50′	79° 45′
Busch	51° 10′	79° 20′
Gemma	50° 50′	79° 45′
Reinhold	51° 18′	79° 30′
Gemma	50° 50′	79° 45′

But who does not see that these combinations, since they posit nothing impossible, are much more tolerable than those from which it would follow that a triangle has three angles together greater than two right angles? And yet Galileo prefers these very ones, with the infinite distance which he asserts follows from them, claiming that these are corrected more easily, more probably and with less trouble, if even the slightest parallax is added. Nothing certain, therefore, can be concluded about the distance of the New Star from the Earth from all the comparisons selected by Galileo with so much industry. For in observations so different, some placing it above the Moon and others below, some uncertainty and deception must lie hidden in some of them — unless Galileo takes refuge in different times and says that it was now above the Moon, now below it; and this he by no means says.

[Margin: An error of these observations acknowledged by Galileo.]

Rather, he acknowledges some error in these observations, when on page 282 he says the following, rendered into Latin by us: *But I am willing to grant the Author* — namely Chiaramonti — *more than perhaps he himself requires: namely, that in each of the combinations of observations there is some error; which I think absolutely necessary.* That is, so that by emending these errors with some probable correction, as he afterwards does, he may bring the star back to no parallax and to the distance of the fixed stars. But it is not necessary that an error occurred in all of them; or, if one did occur, it is uncertain which of the two observers erred, and by how much, and how great a correction is to be applied; nor can we rely on anything but conjectures that are probable in a broad way. It is therefore manifest, as I said at the beginning of the chapter, that Galileo has not demonstrated from any certain hypothesis that the New Star was above the Moon, let alone among the fixed stars. Let this, then, be the conclusion:

VNICA CONCLVSIO.

The single conclusion.

In the third *Dialogue*, on the system of the world, Galileo did not demonstrate, either absolutely or from any certain hypothesis of observations made under different horizons, that the New Star of the years 1572 and 1573 was above the Moon, let alone among the fixed stars. Let it be granted that he demonstrated it probably, that is, from some hypothesis that may probably be accepted. This is clear from what was said from number 2 to the end.

CAPVT IX.

Chapter 9.

An vllus Authorum, quos Tycho tomo I. cap. 10. adducit contra ſe, demonſtrarit Stellam Nouam Anni 1572. fuiſſe infra Lunam.

Whether any of the authors whom Tycho cites against himself in volume 1, chapter 10, demonstrated that the New Star of the year 1572 was below the Moon.

I. In volume 1 of the *Progymnasmata*, chapters 8 and 9, Tycho reviewed the astronomers, or philosophers or theologians, who asserted that this star was above the Moon; he then closes that volume, in chapter 10, by citing the authors who affirmed that it was situated below the Moon. Let us see whether these men too demonstrated what they affirmed.

[Margin: Cyprián Karásek Lvovický, Chytraeus and Postel demonstrated nothing about this star.]

The first of them listed by Tycho, on page 705, is Cyprián Karásek Lvovický of Leonicia, a Bohemian. He brought forward no demonstrations — indeed, no observations fit for a demonstration. In his writing on this star, which he calls a comet, he says that it was at Taurus ♉ 8°, with a latitude of about 50°, that it was exactly joined in its body to the Chair of Cassiopeia, and that for two months, counted from November 25, it stayed in the same place, with no motion other than that of the primum mobile. He then adds: *But now, within a single month, that star seems to me to have advanced somewhat, by three degrees, toward the north.* Tycho censures him on this point, and in noting the star's place, since he disagrees with most of the observers. For us it is enough that he demonstrated nothing; just as nothing was demonstrated by David Chytraeus, or by Guillaume Postel the Frenchman, whose mere and empty astrological judgements on this star Tycho reports in volume 1, pages 712 and 716.

[Margin: The error of Raymundus and Frangipanus.]

Much less did Annibal Raymundus of Verona or Cornelius Frangipanus of Rome demonstrate anything. Indeed, only through a great hallucination — not to say ignorance of the stars — did they say that this star was not distinct from the old stars of Cassiopeia known before, but was seen as new either because the density of the medium had been removed or because it had moved away from one of them; and, as Frangipanus adds, that it had moved two degrees toward the north. As if any star of Cassiopeia had ever been seen equal to

Jupiter or Venus, as this star appeared at the beginning of its apparition; or as if, granting it was a fixed star, as these authors thought, any of the fixed stars had ever changed its place, while the others remained, or its distance from the other fixed stars, by two degrees. Tycho says more against them in volume 1, pages 720 and 740. Dismissing these, then, let us pass on to others, leaving aside also Bartholomäus Reisacher, who once followed the opinion of Annibal Raymundus and afterwards corrected himself, as is clear from what was said in chapter 7, number 5.

Obſeruationes Andreæ Nolthij Embeccenſis ad Parallaxim Stellæ huius ſpectantes.

The observations of Andreas Nolthius of Einbeck bearing on the parallax of this star.

II. Tycho reviews these in volume 1, from page 750 to 765. There Nolthius first states that he observed the altitude of the pole at Einbeck as 51° 42′. And in chapter 4 of his little book on this star he says that on November 29 its azimuth from the equinoctial east point toward the north was 60°, and its apparent altitude 77°; and that the bright star of the Lyre was then at an altitude of 41° 35′. From its Prutenic longitude, Capricorn ♑ 8° 29′, and its latitude, 62°, he gathers its right ascension as 275° and its declination as 38° 41′; and from these, by triangles, the right ascension of the midheaven as 344° 6′.

[Margin: Nolthius's method of investigating the parallax.]

Then, by another observation of the same night, made when the New Star — or, as he calls it, the comet — reached the meridian, he says that the altitude of the Lyre was 33° 51′. From this he gathers the right ascension of the midheaven, shared with the New Star, as 357° 11″. Subtracting from it the earlier right ascension of the midheaven, there remains an arc of the equator between the two observations of 13° 5′, or $52^{m}\ 20^{s}$ of time. From these data and the altitude of the pole he derives by trigonometry the true declination of the star as 61° 49′, and its distance from the zenith at the time of the first observation as 12° 21′, that is, a true altitude of 77° 39′. Therefore, since its apparent altitude was 77°, he concludes that the star's parallax in the circle of altitude corresponding to 77° was 39′. From this, in chapter 6, he gathers the distance of this star from the centre of the Earth as 19 52′ 12″ terrestrial semidiameters.

[Margin: What Tycho objects against Nolthius.]

But from page 755 Tycho raises many objections against Nolthius. First, that he used incorrect Prutenic places of the Lyre. Second, that even granting these, the arc gathered between the two observations is 13° 4′ 28″. Third, that he did not add any figure by which he demonstrated the parallax already mentioned. Fourth, that he does seem to have used problem 3 of Regiomontanus's *De Cometis*, but too unreliably, since he employed too short a time between the two observations, when he could have employed a longer time by observing the star in a lower position, or could have discerned the parallax more easily and more evidently from the greatest and least meridian altitudes of this star.

Fifth, that the altitude of the star and the azimuth in whole degrees, with no minute noted, are suspect to him, as is the observation of the altitude of the New Star and of the Lyre made without a companion, and therefore at different moments, when it ought to have been made at the same moment. Sixth, that by what Tycho had demonstrated in chapter 5, the right ascension of this New Star was not 357° 11′ but greater by 3° 15′. Seventh and last, if this star had a parallax of 39′ at an altitude of 77°, Tycho indeed demonstrates, by the problem set out earlier in chapter 5, number 2, that the parallax at the star's least altitude, which at Einbeck would have been 23° 30′, would have been 2° 42′; and so at its lowest altitude it would have appeared distant from the eleventh star of Cassiopeia by about three Moons, whereas in fact it kept sensibly the same distance from that star and from the other fixed stars.

[Margin: What Nolthius could answer to Tycho.]

None of this, however, evidently undermines Nolthius's conclusion. For Nolthius would say: that from Tycho's places of the Lyre, as Tycho himself concedes on page 758, a parallax of this star of 40′ 30″ follows after all, greater even than 39′; that the figure of the problem is available enough in Regiomontanus, and that he was addressing learned astronomers who would not be ignorant of it; that he observed carefully with some companion, and would not have concealed minutes in the altitude of the star and in the azimuth if there had been any, since he did not conceal them in the altitude of the Lyre; that a parallax of 39′ or 40′ found by such a method is not false or uncertain merely because it could have been gathered by another, easier and more evident way; that the great disagreement in right ascension arises from the difference of the days on which Tycho and Nolthius himself observed; and finally, that it is not established that on all the nights this star lasted its observed distance from the same fixed stars was the same, so that on some nights it could have been, and appeared, different.

[Margin: What we object against Nolthius.]

What therefore moves me not to acknowledge a demonstration in Nolthius is the uncertainty of the proper motion of this star. On that night it could have been such that, even though the star was above the Moon, the motion would apparently depress it toward the horizon, and an effect of parallax would be supposed which was in fact an effect of motion. For it has not yet been demonstrated — although it is repeated as probable — that this star had no other motion during the whole time of its duration than the motion of the primum mobile. See what we said in a similar case in section 1, chapter 18, against those who thought they had demonstrated absolutely that the comet of the year 1577 was below the Moon, without first demonstrating the comet's true motion.

Georgij Buſchij Erfurdienſis Obſeruationes ad Parallaxin Stellæ huius ſpectantes.

The observations of Georg Busch of Erfurt bearing on the parallax of this star.

III. He was a painter and an astronomer alike, and published two writings on this star in the German language; but what Tycho took from them bearing on our subject, in volume 1, from page 765, we shall pick out. Busch says, then, that he used an astrolabe, a quadrant and a geometric square, and that on November 25 at 7^h 14^m after noon he found the greatest meridian altitude of this star to be 79° 20′, and at about 19^h its least altitude to be 22° 40′; and he takes the altitude of the pole at Erfurt from Apian as 51° 10′. But, as one can see at once, and as Tycho saw on page 769: if the altitude of the pole is subtracted from the star's greatest altitude, the distance from the pole that remains is 28° 10′; while if the star's least altitude is subtracted from the altitude of the pole, the distance from the pole that remains is 28° 30′. The difference of the apparent distance from the pole, that is, the sum of the parallaxes, is therefore only 20′. Yet if this star had been in the concave of the lunar heaven, no more than 52 semidiameters of the Earth distant from the Earth, it would have had at such an altitude above the horizon a parallax, or difference of distance from the pole of the world, of one degree, as is clear enough from the diagram and from Tycho's problem, which we dealt with in chapter 5, number 2.

[Margin: Busch did not demonstrate what he asserted.]

So although Busch counts this star among the sublunary comets, not only did he not demonstrate this; rather, the opposite is gathered from his own observation. Yet even that is not evident, because the star could have had a greater parallax which did not appear greater to Busch; and one would need to consult observations made in different places on the same night, if they could be compared. For Busch's other errors, certain or doubtful, see Tycho if you wish.

[Margin: Graminaeus.]

IV. The remaining defenders of this star's place among the sublunary comets whom Tycho mentions in volume 1, from page 775 to 786, are Theodor Graminaeus, professor of mathematics at Cologne [Coloniæ Agrippinæ]; Adam Ursinus of Nuremberg; and Andreas Rosa of Schweinfurt, doctor of medicine. But Tycho denies that they brought forward any astronomical observation from which the distance of this star from the Earth could be conjectured, let alone demonstrated.

VNICA CONCLVSIO.

The single conclusion.

None of the authors whom Tycho reviews in volume 1 of the *Progymnasmata*, chapter 10, demonstrated that the New Star of the years 1572 and 1573 was below the Moon. This is clear from what has been said throughout this chapter.

CAPVT X.

Chapter 10.

An Scipio Claramontius abſolutè, vel ex certa aliqua hypotheſi demonſtrarit, Stellam Nouam Anni 1572.& 1573. fuiſſe ſublunarem.

Whether Scipione Chiaramonti demonstrated, absolutely or from some certain hypothesis, that the New Star of the years 1572 and 1573 was sublunary.

I. Scipione Chiaramonti is a writer of the first rank, noble not only by his knightly and afterwards priestly dignity but by the uprightness of his character, his talent and his learning; and — as must be openly admitted even by some who are unwilling — he has deserved excellently of the doctrine of parallaxes, as I think is already clear from what was said in section 1 of this book, from chapter 16. In book 1 of *De tribus novis stellis* he completed, in the first nine chapters, the history of the observations of this New Star of the year 1572. Then in chapter 10 he professes that he will demonstrate that it was below the Moon, *if its observations made at that time which can be compared, or at least the more important of them, are accepted*; for he states this limitation and condition at the beginning of the chapter. And he professes that he will investigate the distance of the star from the centre of the Earth by problem 8, given in chapter 2 of book 1 of the *Antitycho*, which is exactly the same as the one we already set out above in chapter 5, numbers 1 and 7. It rests on the parallax being given, together with the star's apparent altitude above the horizon and the semidiameter of the Earth, taken first in parts of which the radius is 100,000, and then taken as one.

[Margin: 1. Chiaramonti's method.]

For investigating the parallax he proposes five methods, or lines of inquiry. The first is from the meridian altitudes of the New Star observed under two horizons and in two places whose altitudes of the pole are given; for from these the difference of parallax is gathered, and from it the parallax and the distance from the centre of the Earth. He had already proposed this method in book 1 of the *Antitycho*, chapter 3, proposition 4.

Chiaramonti, moreover, supposes in book 1 of *De novis stellis*, chapter 11, Lemma 5, which runs as follows: *The New Star of the year 1572 kept the same parallax in all places perioecic [periœcis] to one another, and therefore the same apparent distance from the zenith; for it is posited as unmoved by any proper motion.* For in book 1 of the *Antitycho*, chapter 3, in the note on proposition 4, and in the supplement to the *Antitycho*, he had shown this very lemma in other words. Therefore, although the meridian altitudes were observed under different meridians, the star is posited as carried by the motion of the primum mobile from one meridian to another under the same parallel of the equator; so that, setting aside any other proper motion of the New Star, it is just as if two observers under the same meridian had observed the star at the same moment of time. Chiaramonti does this so as not to depart from the hypothesis that Tycho and most of those who assert this star's supralunar seat held as certain and established: namely, that during the whole time of its duration this star

had no other sensible motion at all than the common motion of the primum mobile.

II. Now, in the figure attached here, let LDK be a quadrant of the terrestrial meridian, under the quadrant OFG of the celestial meridian, and under the same let there be the quadrant MCH; all of them described from the centre of the Earth A. Let the New Star be at C, and let place B be the one that has the lesser altitude of the pole, and D the one that has the greater. Through these let the vertical lines AE and AF be drawn from A; let the same places be joined by the chord BD; and from them let the visual lines BC and DC be drawn to the star. There will thus arise the triangle BCD, in which the angle BCD is the difference of the parallaxes, as we shall show in book 10, in the penultimate section, chapter 8. Finally let AC be drawn, for the distance of the star from the centre of the Earth, which is proposed for investigation.

Diagram. A woodcut set into the right column beside the opening of §II: a quadrant with its centre A at the lower right, its horizontal radius running left through K and H to G and its vertical radius rising through L and M to O. Three concentric arcs are drawn — the outer from G through F and E to O (the celestial meridian), a middle one from H through C to M, and the inner from K through D and B to L (the terrestrial meridian). Lines run from A to E and to F (the verticals), from C to B and to D (the visual lines), with the chord BD and a line from A to C.

- E (outer arc, top, left of O): vertex of the vertical line AE
- O (top right, end of the vertical radius)
- M (vertical radius, below O)
- F (outer arc, upper left): vertex of the vertical line AF
- B (inner arc, near its upper end): the place with the lesser altitude of the pole
- L (vertical radius, beside B): end of the inner arc
- C (middle arc, left): the New Star
- D (inner arc, below B): the place with the greater altitude of the pole
- G (lower left, end of the horizontal radius)
- H (horizontal radius, right of G)
- K (horizontal radius, right of H): end of the inner arc
- A (lower right): the centre of the Earth

[Margin: 1. Step.]

With these things posited: First, in the triangle ABD the arc BD, or the angle BAD, which is the difference of the altitudes of the pole, is supposed given; and consequently each of the angles at the base BD is given, by propositions 5 and 32 of the first book of the *Elements*. For when the angle BAD is subtracted from two right angles, that is from 180°, and the remainder is divided in two, both the angle ABD and the angle ADB become known. From this the adjacent angles EBD and FDB become known, by proposition 13 of the first book of the *Elements*, since they are the complements of the angles ABD and ADB. Then, subtracting from the angle EBD the angle CBE, which is the complement

of the altitude of the star seen by the observer at B, the angle DBC becomes known. Conversely, if you add to the angle FDB the angle CDF, which is the complement of the altitude of the star seen by the observer at D, the angle CDB is composed; and the sum of this and the angle DBC, subtracted from 180°, shows the angle BCD, namely the difference of the parallaxes.

[Margin: 2. Step.]

Second, for the triangle CBD, in which all the angles are now known, the chord of the arc BD — that is, of the angle BAD, or of the difference of the altitudes of the pole — is sought from the tables of sines, in parts of which the radius, or the semidiameter of the Earth, AB is 100,000; and it is reduced by the golden rule to parts of which AB is one, or to miles, according as AB is known in miles.

[Margin: 3 and 4. Step.]

Third, then, BD being known in parts in which AB is known, the side BC is sought by the rules of triangles in the triangle BCD, whose angles are known. Fourth, in the triangle ABC, the sides AB and BC being now known, together with the angle ABC, which is composed of the angles ABD and DBC known above, the sum of the angles CAB and BCA is known; from it and from the sides already mentioned the parallax ACB is investigated, and when BCD is taken from it, the parallax ACD remains.

[Margin: 5. Step.]

Fifth and last, by means of the parallaxes and the sides AB and BC, the distance AC from the centre of the Earth is sought.

EXEMPLVM ex CLARAMONTIO.

An example from Chiaramonti.

III. The first comparison of observations that Chiaramonti uses, in book 1 of *De novis stellis*, chapter 12, is that of Francesco Maurolico, abbot, of Messina, who at Messina, at B, at an altitude of the pole of 38° 30′, observed the greatest altitude of the New Star on the meridian as 62°, and its complement as 28°, from what was said in chapter 3, number 3; and that of Paul Hainzel, who at Göggingen, at D, at an altitude of the pole of 48° 22′, observed the greatest altitude of the New Star as 76° 34′ 30″, as Tycho determined it from his various observations.

[Margin: 1. Step.]

With these things posited: First, the angle BAD, that is, the difference of the altitudes of the pole, is 9° 52′. When this is subtracted from 180°, there remains the sum of the angles at the base BD, 170° 8′; and half of it, that is, the angle ABD or ADB, is 85° 4′. Its supplement to two right angles, that is, both the angle DBE and the angle BDF, is 94° 56′. Subtract from it 28°, that is, the angle CBE, and DBC remains, 66° 56′. Add, on the other hand, to the angle FDB, of 94° 56′, the angle CDF, of 13° 25′ 30″, since the altitude seen by Hainzel is given as 76° 34′ 30″; and the angle CDB becomes 108° 21′ 30″.

Joined with DBC, of 66° 56′, this makes 175° 17′ 30″; therefore the angle BCD is 4° 42′ 30″.

[Margin: 2. Step.]

Second, the chord of the arc BD, of 9° 52′, is 17,200 parts, of which AB is 100,000. Therefore by the golden rule, from proposition 3 of Euclid's *Data*, in the parts of which AB is 3035 — namely miles — BD is 522.

Third, therefore: since, by proposition 1 of Clavius's rectilinear triangles, the sides of these triangles have the same proportion to one another as the sines of the opposite angles; and the sine of the angle BCD, of 4° 42′ 30″, is 8208, and the side opposite it is BD, of 522 miles; while the sine of the angle BDC, of 108° 21′ 30″, is 94,910, and the side opposite it is BC: if one makes, as the sine 8208 is to the sine 94,910, so the side BD, 522, to the side BC, the side BC will be found to be 6036 miles rather than 6035, the fractions being reduced to whole numbers.

[Margin: 4. Step, for the parallaxes.]

Fourth, in the triangle ABC, the angle ABC, composed of the angles ABD, 85° 4′, and DBC, 66° 56′, comes to 152°. Therefore the sum of the angles BCA and BAC at the base AC is the supplement of two right angles, namely 28°, half of which is 14°, whose tangent is 24,933. The sides opposite these angles are AB, of 3035 miles, and BC, of 6036 miles; their sum is 9071, and its half 4535½; and the difference of the sides is 1500½. If, according to proposition 12 of Clavius's rectilinear triangles, one makes, as the half-sum of the sides is to the tangent of the half-sum of the angles at the base, that is, to 24,933, so the difference of the sides, 1500½, to the tangent 8248, the angle corresponding to this tangent will be found to be 4° 43′ 52″, or in round figures 44′. Subtracted from the half-sum of the angles at the base, that is from 14°, this leaves the parallax, or angle ACB, of 9° 16′, corresponding to the distance from the zenith seen by Maurolico at B, that is, to the apparent altitude of 62° — although Chiaramonti's book has 9° 17′. When the angle BCD, which is 4° 42′ 30″, is taken from this, ACD remains, 4° 33′ 30″ — although Chiaramonti has 4° 34′ 30″ — that is, the parallax of the New Star at the altitude of 76° 34′ 30″ seen by Hainzel at D.

[Margin: 5. Step, for the distance of the star from the centre of the Earth.]

Fifth and last, in the triangle ABC, by proposition 1 of Clavius's rectilinear triangles, as the sine of the angle ACB, of 9° 16′, which is 16,103, is to the sine of the angle ABC, of 152°, which is 46,947, so the side AB, of 3035 miles, is to the side AC, of 8848 miles; in which the semidiameter of the Earth AB, 3053, is contained 2 2778/3035 times. But Chiaramonti, because he used the angle ACB of 9° 17′ and its sine 16,132, gathers AC as only 8832 miles.

IV. By the preceding method, therefore, Chiaramonti examines twelve combinations of observations in book 1 of *De novis stellis*, in as many chapters, from chapter 12 to chapter 25. He investigates both the parallaxes of the New Star belonging to the apparent altitudes and its distances from the centre of the Earth; and he finds them less than the distance of the Moon at perigee, even if

that were set, according to Ptolemy, at only 33⅓ terrestrial semidiameters. If you examine Chiaramonti's calculations — which is easily done, since he sets out all the numbers and opinions in order — you will not find in them any error of great moment, or one that would increase the distance he deduced by more than about a hundred Italian miles, or the parallax by more than a minute or two. Here, then, are the foundations of his calculation, reduced to a single synopsis, together with the parallaxes and distances of this New Star that he deduced.

Distances and parallaxes of the New Star of the year 1572, deduced by Chiaramonti from the observations written below.

Combination	Chiaramonti, book 1, chapter	Observers and places on Earth	Altitude of the pole	Apparent meridian altitude of the New Star	Difference of the parallaxes	Parallax of the apparent altitude	Distance from the centre of the Earth, Italian miles	Distance, semidiameters of the Earth
I	12	Maurolico, Messina	38° 30′	62° 0′ 0″		9° 17′ 0″		
		Hainzel, Göggingen	48° 22′	76° 34′ 30″	4° 42′ 30″	4° 35′ 30″	8832	2 2763/3035
II	13	Schuler, Wittenberg	51° 54′	80° 15′ 0″		0° 23′ 15″		
		Hainzel, Göggingen	48° 22′	76° 34′ 30″	0° 8′ 30″	0° 31′ 45″	76215	25 142/927
III	14	Tycho, Herrevad	55° 58′	27° 56′ 0″		2° 40′ 0″		
		Hainzel, Göggingen	48° 22′	20° 10′ 0″	0° 10′ 0″	2° 50′ 0″	57660	18 4895/4934
IV	15 and 16	Tycho, Herrevad	55° 58′	23° 3′ 0″				
		the Landgrave, Kassel	51° 18′	27° 57′ 0″	0° 14′ 0″	✠	29273	about 10
V	17	Cornelius Gemma, Louvain	50° 50′	79° 45′ 0″				
		Hainzel, Göggingen	48° 22′	76° 34′ 30″	0° 42′ 30″	✠	13389	4 1240/3035
VI	18	the Landgrave, Kassel	51° 18′	23° 17′ 0″				

Combination	Chiaramonti, book 1, chapter	Observers and places on Earth	Altitude of the pole	Apparent meridian altitude of the New Star	Difference of the parallaxes	Parallax of the apparent altitude	Distance from the centre of the Earth, Italian miles	Distance, semidiameters of the Earth
		Camerarius, Frankfurt	52° 24′	24° 8′ 0″	0° 8′ 0″	✠	13000	about 4
VII	19	Tycho, Herrevad	55° 58′	27° 57′ 0″				
		Hájek, Vienna	48° 22′	20° 15′ 0″	0° 6′ 0″	✠	97000	about 32
VIII	20	Hájek, Vienna	48° 22′	20° 15′ 0″				
		Adam Ursinus, Nuremberg	49° 24′	22° 0′ 0″	0° 43′ 0″	✠	4600	about 1½
IX	21	the Landgrave, Kassel	51° 18′	23° 3′ 0″			3090	1 55/3035
		Busch, Erfurt	51° 10′	22° 40′ 0″	0° 15′ 0″	✠	or 12400	about 4
X	23	Maurolico, Messina	38° 30′	62° 0′ 0″				
		Muñoz, Valencia	39° 30′	67° 30′ 0″	4° 30′ 0″	✠	3600	1 565/3035
XI	24	Cornelius Gemma, Louvain	50° 50′	79° 45′ 0″				
		Muñoz, Valencia	39° 30′	67° 30′ 0″	0° 55′ 0″	✠	39000	about 13
XII	25	Adam Ursinus, Nuremberg	49° 24′	79° 0′ 0″				
		Muñoz, Valencia	39° 30′	67° 30′ 0″	1° 36′ 0″	✠	21000	about 7

[Margin: Notes on the preceding table.] In the table already given, we have often put the miles as a round number, since that is enough for what is being sought. It should be noted, moreover, that Galileo, in the third Dialogue on the System of the World, more than once called "parallax" what we, with Chiaramonti, call in column 6 the "difference of parallaxes" — misusing the terms. Further, Chiaramonti, in Book 1 of *De stellis novis*, chapter 22, admits that from the observed altitudes of the New Star, of which there are at least 24 (as can be seen in chapter 3, number 3), 275 combinations can be formed; but that most

of them cannot be compared with one another for the purpose of gathering the parallax and distance of the New Star determinately and geometrically by the first method of inquiry. For observations by the same author cannot be compared with one another, nor the greatest altitudes with the least, or the least with the greatest; nor the observations of authors who have the same altitude of the pole; nor those in which the difference in the altitude of the pole is greater than the difference in the observed altitudes of the star; nor, finally, those from which, in the triangle BCD of the figure already given, three angles together greater than two right angles, that is, than 180°, result. This is why Chiaramonti reduced the combinations to 12. From these, in chapter 26, he concludes that this New Star was beneath the Moon, that is, below the Moon, and that one should rather abide by these 12 comparisons than by only two, which Tycho used to show that the star was above the Moon — namely, Tycho's altitude compared with Hainzel's and with Muñoz's. And it is true, as Chiaramonti asserts, that he demonstrated it from the hypotheses he had premised. But we must now see whether those hypotheses are to be accepted as certain.

V. In short, then, I deny that the hypotheses Chiaramonti assumed in the 12 comparisons and calculations already mentioned are certain; and so I deny that he demonstrated that the New Star of the years 1572 and 1573 was below the Moon — not only absolutely (which not even he himself claims), but also hypothetically, from certain and indubitable hypotheses, as against merely probable ones. For the altitude of the pole, or the altitude of the New Star, or the New Star's immobility as regards proper motion — which are assumed in those demonstrations — are uncertain. [Margin: That the hypotheses assumed by Chiaramonti are not certain.]

First, the altitude of the pole at Wittenberg, Frankfurt an der Oder, Vienna and Erfurt, as Chiaramonti assumes it, is uncertain, as is clear from what was said in chapter 7, numbers 5, 11, 15 and 16. [Margin: Uncertainty of the altitudes of the pole.] Likewise the altitude of the pole at Messina, which Maurolico gives as 38° 30′: although it was once observed to be that much from the shadows of gnomons, as Ptolemy too has it in his *Geography*, and was confirmed by Maurolico, so that it is very probable, nevertheless Don Carolus Vintimilius [Carlo Ventimiglia], who undertook a new and more correct chorography of Sicily, observed the altitude of the pole at Messina as 38° 11′, and at Palermo as 38° 10′. This disagreement may have arisen from not reckoning the Sun's apparent semidiameter in that old operation by shadows, as I have shown happened more than once in my book on geography. And although Chiaramonti, in Book 1 of *De stellis novis*, chapter 13, contends that the altitude of the pole at Wittenberg should be chosen as 51° 54′, as having been observed by Peucer and Reinhold, he will never bring it about that the value which Tycho corrected on other grounds is not probable, as I said in Book 7, number 11.

VI. Second, Chiaramonti assumes certain altitudes of the New Star while knowing that other and different ones were observed by the same authors, as is clear from chapter 3, number 3. [Margin: Uncertainty of the altitudes of the

New Star.] For Schuler, with his instruments, found the greatest altitude of the New Star to be 79° 56′; yet from the least altitude, 23° 33′, and the altitude of the pole, the same man computed it as 80° 15′. Chiaramonti, in Book 1, chapter 13, chooses this one rather than the other, because had he used the former, the three angles of the triangle BCD would have come out greater than two right angles. But given the parallax that Chiaramonti infers from it, the greatest altitude is not correctly gathered from the least apparent altitude and the altitude of the pole; or at any rate the uncertainty of what is assumed is evident from that disagreement. Further, in chapters 14, 15 and 19, Chiaramonti uses the altitude of the New Star of 27° 56′ that Tycho reported in his first writing, but uncorrected. He gives a threefold reason for choosing it: first, because it is the altitude Tycho observed while the star was still lasting, whereas the correction was made after the star had perished; second, because Tycho has it more than once in the second writing, in volume 1 of the *Progymnasmata*, pages 347 and 771; third, because if he chose the altitude of Tycho's second writing, namely 27° 45′, it could not be compared with Hainzel's except by running into a rectilinear triangle BCD having three angles greater than two right angles.

But these reasons are not sufficient for choosing the altitude of the star of 27° 56′, or at least for making it certain. [Margin: Chiaramonti's reasons rejected.] For while the star was still in being, Tycho used a sextant which, because of parallax and the cheekbone, gave incorrect intervals between stars and incorrect distances from the horizon and from the fixed stars. He corrected its defect, however, by a sure geometrical method, gathering how much smaller the angle of the true distance should have been — which he could do by examining the instrument, even when the New Star was no longer in being, as is clear from Tycho himself, volume 1, pages 341 and 593 — granted that in the second writing, where precision was not needed, he indicated the earlier value. And the fact that the later altitude of the star cannot be compared with Hainzel's, because of the three angles of the triangle being greater than two right angles, does not prove that the later altitude is less correct. It proves rather that either Tycho's or Hainzel's is erroneous, or that they were made at different times and that the star was altered by some motion in declination. As for Chiaramonti's saying, in combinations 8, 9 and 10, that some error lies hidden in the observations because a distance from the centre of the Earth of less than two semidiameters follows from them, I do not see on what foundation it is said. For that star could have been distant from the Earth's surface by only half a semidiameter of the Earth — indeed by only 565 miles, or even 55 — and yet be above the sphere of vapours.

VII. Third, Chiaramonti supposes, together with his very opponents, that the New Star never changed its apparent declination during the whole time it lasted, and had no motion other than that of the primum mobile; and therefore that altitudes observed under different meridians can be compared as if they had been made under the same meridian. But this is not certain, because it

is not established that the star was observed by the astronomers on each and every one of the same nights; indeed, from the history given in chapter 3, number 3, it is rather established that it was observed on different nights. If the observations of all, or of most, had fallen on the same night or nights, and on them the New Star had been seen everywhere on Earth to keep the same distance from the same fixed star, or from the pole of the world, then we would have an evident argument that it was altered by no motion in declination, nor by any other motion distinct from the motion of the primum mobile. And indeed, if it had a parallax as great and as varied as Chiaramonti gathers, how could it have happened that it seemed to all those observers equally distant from the same fixed stars? Finally, since Chiaramonti found, for the same time, parallaxes and distances from the Earth so different and so at odds with one another, even for one and the same astronomer — for example, from Tycho's observations a distance of 18, 10 and 32 semidiameters; from Hainzel's, 2, 25, 18 and 4 semidiameters; from Maurolico's, 2 and 1; from the Landgrave's, 10, 4 and 1 semidiameters; from Hájek's, 32 and 1 semidiameters; from Muñoz's, 1, 13 and 7 semidiameters — who does not see that they cannot be certain? For it is not enough that each stands on its own; what would be needed is that all should agree, very nearly, on the same measure.

VIII. Chiaramonti's second inquiry and method, which he proposes in Book 1 of *De stellis novis*, chapters 10 and 27, is the same one he had proposed in Book 1 of the *Antitycho*, chapter 5, proposition 3; and it is similar to the problem we shall give in Book 10, in the penultimate section, chapter 9, problem 5. [Margin: Chiaramonti's second inquiry.] In it an observation is supposed to have been made at the same moment of time and at two different places on Earth, whose altitude of the pole is known, with the star's distance from the zenith and its azimuth observed at both; or, if it was not made at the same moment of time, that it can nevertheless be used as if it had been made at the same moment, because the star neither changed its declination nor ever had any motion other than that of the primum mobile. But we have already shown, at number 7 and elsewhere above, that this very point is uncertain. Moreover, since Chiaramonti, using this inquiry from chapter 28 to 41, employs only the observations of the Landgrave of Hesse compared with Tycho's, and from one and the same comparison always gathers a different and conflicting distance of the New Star from the zenith, we cannot place sure trust in any of those observations. Or at any rate it must be said that they cannot be taken as made at the same moment of time by carrying the star from one meridian to the other, that is, to the perioecic place of the earlier or the later observer. But as for Chia- ramonti's attempt there, on pages 134, 138, 144, 177 and 179, to put that disagreement between the distances down to a refraction of uncertain quantity: this is indeed said not improbably, but it is still not demonstrated that it was so. Finally, as for his concluding, in Book 1, chapter 41, that — notwithstanding the disagreement between the distances derived from the same comparison — since both are nevertheless smaller than the Moon's distance, the New Star

is evidently shown by either of them to be sublunary: he is undoubtedly mistaken. [Margin: Chiaramonti's error.] For since it is uncertain where that disagreement arises from — whether from Tycho's observation, or the Landgrave's, or from refraction, or from some other cause, such as a motion in declination — we are also uncertain whether either of those two distances is to be accepted as the true one. For it could happen that, had no error intervened, or had we possessed elements sufficient for evidently investigating this distance, we would obtain by calculation neither of those distances, but a distance greater than the Moon's; and Chiaramonti has by no means demonstrated that this could not happen. So that the disagreement and conflict already mentioned, between the distances deduced by this second inquiry through Chiaramonti's calculations — which are otherwise correct enough — may be seen, and so that at the same time we may satisfy the reader's learning, here they are gathered into one view in the following table.

Chapters of Book 1	Observers	Parallax	Distance from the centre of the Earth, in semidiameters
28	the Landgrave	0° 43′ 4″	19 3/4
	Tycho	1° 3′ 48″	10 5/9
29	the Landgrave	2° 21′ 49″	7 3/4
	Tycho	0° 46′ 59″	17 9/14
30	the Landgrave	0° 36′ 29″	27 3/11
	Tycho	1° 17′ 14″	11 nearly
31	the Landgrave	0° 48′ 48″	16 3/7
	Tycho	1° 14′ 0″	8 1/3
32	the Landgrave	0° 34′ 39″	24 1/7
	Tycho	0° 55′ 12″	11 3/8
33	the Landgrave	0° 37′ 50″	23 1/29
	Tycho	0° 56′ 12″	11 3/8
34	the Landgrave	1° 4′ 20″	10 9/11
	Tycho	1° 17′ 0″	5 2/3
35	the Landgrave	1° 4′ 56″	10 2/19
	Tycho	1° 16′ 45″	5 5/22
36	the Landgrave	1° 49′ 51″	5 25/32
	Tycho	1° 55′ 41″	3 7/38

Parallaxes and distances of the New Star of the years 1572 and 1573, from Chiaramonti, Book 1 of De stellis novis.

IX. Chiaramonti's third inquiry and method, which he proposes in Book 1 of the *Antitycho*, chapter 5, propositions 1 and 4, and in Book 1 of *De stellis*

novis, chapters 10 and 42, relies on the apparent declinations at different places on Earth, the altitudes of the pole, the difference of longitude or distance between the meridians, and the difference of right ascension; and, besides, on the culminating point of the ecliptic at the same moment of time, or the place of the Sun at that moment. [Margin: Chiaramonti's third inquiry.] The fourth inquiry, which the same author proposes in Book 1 of the *Antitycho*, chapter 7, proposition 1, and in Book 1 of *De stellis novis*, chapters 10, 43 and 44, supposes the observed apparent latitude and longitude of the phenomenon at two places, and hence the difference of the parallaxes with respect to the ecliptic; and that the places on Earth differ either in longitude alone, or in the altitude of the pole alone, or, if in both, that the difference is known; and that the observation was made at both places at the same moment. [Margin: Chiaramonti's fourth inquiry.] Finally, the fifth inquiry, which the same author proposes in Book 1 of the *Antitycho*, chapter 9, proposition 1, and in Book 1 of *De stellis novis*, chapters 10 and 45, proceeds from the apparent distances from the same fixed star at two places on Earth whose altitudes of the pole are given, together with the time of the observations. [Margin: Chiaramonti's fifth inquiry.]

But grant that Chiaramonti's geometrical method and arithmetical computation are not fallacious: that the hypotheses on which they rest are not certain is still clear from this — that from them is gathered a distance of the New Star from the centre of the Earth that conflicts sharply with the distance drawn from the observations of the same astronomers using the first and second inquiries. Or at least it is clear from the fact that it cannot be denied that, from other observations compared with one another, a distance of the New Star from the Earth greater than the Moon's is probable, and rests on plausible, though uncertain, foundations, as is clear from what was said in chapter 8. Therefore the distance of the same New Star smaller than the Moon's, which Chiaramonti found by these five inquiries, can be no more than probable. Otherwise, if it were certain and evident, it would extinguish the probability of the opposite opinion: for a proposition remains probable to the same intellect for as long as neither it is proved false nor the opposite proposition is proved true. From this it follows that the foundations of neither opinion are certain; for if they were certain, the conclusion scientifically deduced from them would also be certain, and so true not merely hypothetically but absolutely.

Nevertheless, so as not to cheat the reader here of the parallaxes and distances of the New Star that Chiaramonti deduced with geometrical and arithmetical precision [acriuia] by the third, fourth and fifth inquiries — though only from single comparisons — let the following small table stand.

Chapters of Book 1	Observers	Parallaxes	Distance from the centre of the Earth, in semidiameters of the Earth
42	Tycho	1° 7′ 24″	9 193/197
	Muñoz	0° 51′ 0″	28 2/5
45	Tycho	0° 37′ 0″	9 4/11

Chapters of Book 1	Observers	Paral-laxes	Distance from the centre of the Earth, in semidiameters of the Earth
	Cornelius Gemma	1° 11′ 0″	9 14/21

Other distances and parallaxes of the New Star of the year 1572, deduced by Chiaramonti in Book 1 of De stellis novis.

From the fourth inquiry, however, no parallax or distance is given, because Chiaramonti teaches there that there are no observations of this star suited to the purpose. For from those which seemed most comparable with one another it followed that the whole is greater than its part, and that the New Star was at the centre of the Earth or below it with respect to the observers — which is absurd. And this confirms the uncertainty of those observations.

CONCLVSIONES.

Conclusions.

I. CONCLUSION. *Chiaramonti, in Book 1 of* De stellis novis, *from chapter 12 to the end, demonstrated that the New Star of the years 1572 and 1573 was sublunary — if many observations of that star are accepted and held to be the more reliable.*

He himself asserted this in Book 2 of *De stellis novis*, chapter 1, as he had promised in the preface and in Book 1, chapter 10; and he carried it out excellently, by a method plainly geometrical and arithmetical, as was clear to me when I considered his figures and problems and went over his calculations, and as will become clear to anyone who wishes to try the same — granted that he sometimes errs by a minute or two, and, in using observations from Tycho's first writing rather than his second, commits an error of 11′, as is clear from what was said at number 6.

II. CONCLUSION. *Yet the same Chiaramonti, in the same place, demonstrated neither absolutely, nor from a hypothesis to be accepted as certain and irrefutable, that the New Star of the years 1572 and 1573 was below the Moon, but only from a hypothesis to be accepted as probable.*

This is clear from what was said above, from number 5 to 9. Nor did Chiaramonti himself claim absolute demonstrations of the distance already mentioned, but only hypothetical ones. In this, however, he overstepped the limits of his demonstrations: he thought that one or other of the observations or hypotheses from which he had deduced two distances smaller than the Moon's, but conflicting with each other, was certain, and that from one of them at least it was evidently concluded that this star was below the Moon. Yet neither of them could be certain as regards this very effect, and once the falsity of both, or of either, was detected, a distance greater than the Moon's could have been gathered. And the same

I say about the comparison of the observations of Antonio Santucci with others, of which I shall speak shortly in the scholia.

SCHOLIA.

Scholia.

I. *Santucci, in his book on comets, chapter 7, contends that this New Star was probably in the eighth sphere, but most certainly above the Moon, because he himself, at Pisa, at a polar altitude of 43° 30′, observed its distance from the pole on the meridian, on either side, to be 36°. For the greatest altitude he saw was 79° 30′, and the least 7° 30′. But* Chiaramonti *refutes him in Book 2 of* De stellis novis, *in the Appendix. There, in chapter 4, using the first inquiry already set out by us in the preceding chapter, number 3, and comparing Santucci's altitudes with Hainzel's, he shows that if both are accepted, the difference of the parallaxes is 7° 48′, and that from it the New Star's distance from the centre of the Earth is gathered to be less than 3563 Italian miles, and from the surface less than 578 miles. Comparing Santucci's altitudes with those observed by Maurolico, by Muñoz and by the Landgrave of Hesse, a difference of the parallaxes of 8°, 4° 30′ and 7° 45′ arises, in the same order, from which follows a distance of the star from the Earth far smaller than the Moon's.* [Margin: Santucci's observation examined.] *Chiaramonti adds, however, in the same chapter, that he suspects the greatest altitude asserted by Santucci, 79° 30′, because it is greater than the altitude observed by Hainzel at Göggingen, whereas it ought to be, or to appear, smaller at Pisa, as a place more southerly than Göggingen by 4° 52′. And indeed, if at Göggingen, where the altitude of the pole is 48° 22′, the greatest meridian altitude of the New Star was seen as 76° 34′, as we related in chapter 3, number 3, and if the star kept the same distance from the pole of the world everywhere and had no proper motion, as Santucci affirms, then at Pisa, at a polar altitude of 43° 30′, it certainly ought to have appeared at an altitude of 71° 32′, not 79° 30′. Nor can so great a difference be put down to refraction, and much less to parallax, which could not have been so great at Pisa — and, if it were admitted, would bring the star close to the Earth, far below the Moon.* [Margin: Santucci's observation suspected of falsity.] *So nothing has been solidly demonstrated by such an observation regarding the distance of the New Star; nor is it shown from it that the star was above or below the Moon; it is only from an uncertain hypothesis that Chiaramonti concludes it was below the Moon.*

II. *In the same Appendix, chapter 6, Chiaramonti detects a manifest error of Santucci's in geometry. For Santucci had said, in his statement 21: "If the New Star had been in the orb of the Moon, and no higher, it would have undergone a parallax of up to 3° 40′." But this cannot be, because not even in Ptolemy — who of all astronomers brings the Moon closest to the Earth at quadrature [□] — does the Moon's horizontal parallax exceed 1° 44′; and in other astronomers the distance of the Moon at perigee is much greater than in Ptolemy, and so the parallax smaller.* [Margin: Santucci's error in geometry or astronomy.] *I add that if the parallax of the Moon, and so of a star placed in the Moon's heaven, were 3° 40′, at least on the horizon itself, the Moon would have to be distant from the centre of the Earth by only 16 semidiameters of the Earth, as will be clear to anyone who solves the right-angled rectilinear triangle in which, taking the Earth's semidiameter as 1 part, the opposite*

angle — that is, the horizontal parallax — is 3° 40′. For the base, that is, the distance from the Earth, will come out no greater than 16 parts. So Santucci erred either in geometry, if he thought that the distance of the Moon at perigee was not less than Ptolemy's and yet that so great a parallax followed from it; or in astronomy, if he believed that, with so great a parallax preserved, the Moon's distance could be reduced to 16 semidiameters of the Earth.

CAPVT XI.

Chapter XI.

The opinions of authors on the place of the New Star of the year 1572 and on its distance from the Earth are collected, and it is established what may be maintained about its distance — if not with certainty, then at least probably — or what is more probably to be asserted.

I. I now call "opinions" those, too, of the people who judged them to be not opinions but certain verdicts, because we have at last found that they are contained within the limits of mere probability. And although I reviewed them in chapter 2, nevertheless, since there the distance of this star was entangled with many other things observed in it, and we did not set out there on what grounds the opinions about the distance had been asserted, I would like now to gather those same opinions, on the distance alone, into one place, and to indicate the manner in which each was asserted by its authors; and afterwards to establish, in distinct conclusions, what can be asserted about it, or what ought to be.

II. The first opinion is that of those who affirmed that this star stood above the orbs of all the planets from beginning to end — either in the firmament, or in the eighth sphere of the fixed stars, or as near to it as possible. They were, first of all: Tycho, volume 1 of the *Progymnasmata*, chapters 6 and 7, that is, from page 396 to 489, though he also drives it home very often in chapters 8, 9 and 10; Longomontanus, in the Appendix to the *Astronomia*, chapter 5; Kepler, *De stella nova*, chapter 25; Tadeáš Hájek, in his *Dialexis*, chapter 5, and in Tycho, volume 1, page 513; Paul Hainzel, in his letter to Hieronymus Wolf, and in Tycho, volume 1, page 536 — though there he denies that it had a sensible parallax and yet counts it among the elemental comets; but Tycho relates there, on page 542, that he retracted this opinion and subscribed to Tycho's once he had seen Tycho's writings; Michael Mästlin, in a published writing on this star, and in Tycho, volume 1, page 545; Cornelius Gemma, in his booklet on this star, and in Book 11 of *De divinis naturae characterismis*, chapter 3, and in Tycho, volume 1, page 553 — for although he had earlier suspected a parallax of 4′, he afterwards excluded all sensible parallax and placed this star in the eighth sphere; as did Jerónimo Muñoz, in his writing translated from Spanish into Latin by Cornelius Gemma, and in Tycho, volume 1, page 570 — granted that earlier he had said only that this star was above the Sun; Francesco Maurolico, abbot of Messina, in his disputation on this star, and in Clavius's *Sphaera*, page 194 of my copy, and

Clavius himself in the same place; Biancani, in his *Sphaera*, Book 18, chapters 2 and 5; Redemptus Baranzanus [Redento Baranzano], in the *Uranoscopia*, page 112; Andrea Argoli, in the *Pandosion sphaericum*, chapter 62; Galileo, in the first and third Dialogues on the System of the World; Descartes, in the *Principia philosophiae*, part 3, marginal number 118; Liceti, *De novis astris*, Book 5, chapter 41; Froidmont, *Meteorologica*, Book 3, chapter 2, article 7; Antonio Santucci, *De cometis*, chapter 7, where he says that it is indeed evidently established for him that this star was above the Moon, but probably in the eighth sphere; Giovanni Camillo Glorioso, *De cometis*, Book 5, pages 275 and 281. To this group can be referred Annibal Raymundus and Cornelius Frangipanus, who, as Tycho reports in volume 1, pages 720 and 741, thought this star was one of the old fixed stars, namely the eleventh of Cassiopeia. [Margin: First opinion, placing this star in the heaven of the fixed stars [?]: Tycho.] [Margin: Longomontanus [?].] [Margin: Kepler.] [Margin: Hájek.] [Margin: Hainzel [?].] [Margin: Mästlin [?].] [Margin: Cornelius Gemma [?].] [Margin: Muñoz.] [Margin: Maurolico.] [Margin: Clavius.] [Margin: Biancani.] [Margin: Baranzano.] [Margin: Argoli.] [Margin: Galileo [?].] [Margin: Descartes.] [Margin: Liceti.] [Margin: Froidmont.] [Margin: Santucci.] [Margin: Glorioso.] [Margin: Raymundus [?].] [Margin: Frangipanus.]

III. The second opinion is that of those who said that this New Star, if not in the firmament itself, was at least among the orbs of the planets, and was not below the Moon. Of this kind were: Wilhelm, Landgrave of Hesse, in his second letter to Caspar Peucer, and in Tycho, volume 1 of the *Progymnasmata*, page 613, where he denies that it had a parallax greater than 3′ — adding, however, the words "if indeed it had any" — and assigns it a distance from the Earth of 1061 37/52 semidiameters of the Earth, concluding: "From this it appears that the comet is placed in the highest part of the sphere of Venus, scarcely nine semidiameters of the Earth below the sphere of the Sun; from which it is most certainly established that it plainly does not exist in the elementary region." For he calls it a comet, and did not yet know that Venus at its highest goes above the Sun. [Margin: Second opinion, placing this star among the orbs of the planets.] [Margin: the Landgrave of Hesse.] And Paul Fabricius, in a writing he published on this star, and in Tycho, volume 1, page 529; for he says that it was either in the aether of the fixed stars or not below Jupiter. [Margin: Fabricius.] Francisco Vallés, *De sacra philosophia*, Book 1, chapter 1, and in Tycho, volume 1, page 572, saying that because it had no parallax it was at least above the Sun. [Margin: Vallés.] Erasmus Reinhold the Younger, who in his prognostication for the year 1574 held the same view about the distance of this star, 1079 semidiameters of the Earth, as the Landgrave of Hesse, as can also be seen in Tycho, volume 1, page 700. [Margin: Reinhold the Younger.] Caspar Peucer, in his letter to the Landgrave of Hesse, which Tycho reports in volume 1, page 606; Wolfgang Schuler, in his reply to Peucer, in Tycho, volume 1, page 622 — for both assign this star a parallax of 19′ and a distance from the Earth of 164 semidiameters of the Earth. [Margin: Peucer the Younger.] [Margin:

Schuler.] Johannes Praetorius, on the occasion of a writing [Margin: Johannes Praetorius.]

of the comet of the year 1577, where he affirms that the star of the year 1572 *was placed either within the orbs of the planets or above them*. So too Philipp Apian, in Tycho, volume 1, page 643, together with John Dee the Englishman, judged that it was either to be counted among the fixed stars or at any rate had been above the Moon. [Margin: John Dee.] [Margin: Apian the Younger.] Thomas Digges thought no differently, in his *Alae ac Scalae Mathematicae*, chapter 10, and in Tycho, volume 1, page 674, where he says that it was most certainly above the Moon, but also doubts whether it was above all the planets. [Margin: Digges.] Elias Camerarius, in Tycho, volume 1, page 693, concludes from a parallax of 10′ that it was above the Moon. Giovanni Battista Benedetti, in his letters, page 371, likewise contends that it was above the Moon. [Margin: Giovanni Battista Benedetti.]

IV. The third opinion is that of those who asserted that this star was below the Moon. They are these. Andreas Nolthius, who in chapter 4 of his work on this star affirms that, from a parallax of 39′, it stood distant from the Earth by 19 52′ semidiameters of the Earth, as Tycho also reports, volume 1, page 755. [Margin: Third opinion, placing this star below the Moon.] [Margin: Nolthius.] Georg Busch, who counted this star among the sublunary comets — although it follows from the observations he himself gave that it had a parallax smaller than the Moon's, as Tycho shows, volume 1, page 769. [Margin: Busch.] Theodor Graminaeus, in Tycho, volume 1, page 776; Adam Ursinus, in the same place, page 781; Andreas Rosa, in the same place, page 783. [Margin: Graminaeus.] [Margin: Ursinus.] [Margin: Rosa.] For Cyprián Karásek Lvovický, David Chytraeus and Guillaume Postel, whom Tycho mentions in volume 1, pages 705, 712 and 715, said nothing about the distance of this star; nor did Annibal Raymundus, of whom the same author speaks on page 720, nor Cornelius Frangipanus, of whom Tycho speaks in the same place, page 741, and who rather calls this star a fixed star. Lastly, Scipione Chiaramonti, in Books 1 and 2 of *De stellis novis*, and Cottunius, on Book 1 of the *Meteorologica*, at the end of Lecture 31. [Margin: Chiaramonti.] [Margin: Cottunius.]

V. So much for the various opinions about the place and the distance of this New Star from the centre of the Earth. We must now come to gathering the conclusions scattered through the preceding chapters, and to substituting others for them, so that what we judge should be held about this New Star may be found here, collected and ready to hand.

CONCLVSIONES.

Conclusions.

I. CONCLUSION. *None of the authors named so far demonstrated that the New Star of the years 1572 and 1573 was above the Moon, much less above the other planets or in the eighth sphere of the fixed stars.* The authors I am speaking of are

those we reviewed a little earlier, in numbers 2 and 3. The conclusion has been sufficiently proved in chapters 6, 7 and 8, at whose end we set down conclusions, from which we have drawn this first one as their marrow. Granted that, first of all, Tycho, Hájek, Mästlin, Digges, the Landgrave, Hainzel, Gemma and Muñoz thought it demonstrated. [Margin: 1st Conclusion, on the star above the Moon.]

II. CONCLUSION. *Likewise, none of the authors already mentioned demonstrated, from a certain and irrefutable hypothesis, that the New Star of the years 1572 and 1573 was above the Moon.* For either the hypothesis rests on observations made in one place on the Earth — and we have already established in chapter 6 that from these alone, taken separately, it cannot be demonstrated that this star was above rather than below the Moon. For the star would have had to be observed in several horizons, but on the same nights, so that it showed the same distance from the pole of the world and from the same fixed stars to different observers at the same time; or else it would have had to show them no variation of distance so great that a parallax greater than the lunar parallax could be gathered from it. Or the hypothesis rests on observations made in different places on the Earth — and we have shown well enough that these are not certain, both in chapter 6, number 7, and in chapter 8, numbers 8 and 9, and from the scholia of chapter 10. This is clear even from the fact that, if those observations are accepted as reported, as certain and irrefutable, those very observations, compared with others that are certain on no lesser evidence, show this star to be sublunary, as is clear from what was said in chapter 10. [Margin: 2nd Conclusion, on the star above the Moon.]

III. CONCLUSION. *None of the authors named so far demonstrated that the New Star of the years 1572 and 1573 was below the Moon.* I am speaking of the authors just reviewed in number 4. The conclusion is clear from what was said in chapters 9 and 10, and from the conclusions asserted one by one at the end of those chapters. [Margin: 3rd Conclusion, on the star below the Moon.]

IV. CONCLUSION. *Likewise, none of the authors already mentioned demonstrated, from a certain and irrefutable hypothesis, that the New Star of the years 1572 and 1573 was below the Moon.* [Margin: 4th Conclusion. On the star below the Moon.] This conclusion is evident from what was said in chapters 9 and 10 and from their conclusions.

V. CONCLUSION. *From observations of the New Star of the years 1572 and 1573 made in a single place on the Earth, however fully they are accepted as certain, it cannot be demonstrated that it was below rather than above the Moon, or above rather than below it.* This was shown sufficiently in chapter 6, with a diagram also supplied. [Margin: 5th Conclusion, on observations made in one place.]

VI. CONCLUSION. *Among the observations of the New Star of the years 1572 and 1573 made in different places on the Earth and compared with one another, if they are accepted as good, there are some from which it is demonstrated that the star was above the Moon, and some from which it is demonstrated that it was below.* The former are established from what was demonstrated by Tycho in chapter 5,

number 8, and by Galileo in chapter 8, numbers 1 to 7. [Margin: 6th Conclusion, on observations in different places on the Earth.]

VII. CONCLUSION. *It is probable that the New Star of the years 1572 and 1573 was above the Moon for the whole time of its duration; yet it is also probable that the same star was below the Moon.* The first part of the conclusion is established by the authority of the men I reviewed in numbers 2 and 3 of this chapter, and by the arguments brought by Tycho, Galileo and others (on those arguments, our chapters 4 and 5, 7 and 8). The second part is evident both from the authority of the men we reviewed here in number 4 and from the arguments brought in chapters 9 and 10. Both parts, however, can hold together, from Conclusion 6 itself. [Margin: 7th Conclusion, on the probability of the star's place.]

VIII. CONCLUSION. *Yet it is more probable that the star of the years 1572 and 1573 was above the Moon than below it.* [Margin: 8th Conclusion: what is more probable.] It is proved, *first*, from the authority of very many, whom we cited in numbers 2 and 3: there are 32 authors, among whom many are distinguished for astronomical learning and erudition. Those who thought it was below the Moon, reviewed in number 4, are about 6, and of no great name except Chiaramonti — and, as far as philosophy goes, Cottunius. Where reason does not prevail to the contrary, the votes of authorities carry great weight; how much more, if reason is added? [Margin: 1st proof, from authority.] *Secondly*, then, the conclusion is proved by reasons. The *first* is the star's greater likeness to the fixed stars — in colour, light, twinkling, motion and figure (on these, chapter 4) — than its unlikeness to them; for it was unlike them only in its decrease and its disappearance. [Margin: 2nd proof, from reason; and Reason 1.] The *second*, and the strongest, is this. Suppose that on each and every night on which this star appeared it had appeared to all observers, or to very many observing at the same time, to stand equally distant from the same fixed stars and from the pole of the world — both when it was nearest the horizon and when it was highest, or higher above the horizon — and to keep the same figure of a rhombus, or of a trapezium close to a rhombus, with the three chief stars of Cassiopeia; and suppose this were certainly established. Then it would necessarily have to be said that it was above the Moon, and this would be physically certain. For it would have been naturally impossible for the star — observed at the same time from different parts of the Earth having different polar altitudes, now near the horizon, now far from it — to show the same distance from the pole and from the same fixed stars, and to keep the same configuration with them, if it had been below the Moon. This is established from the doctrine of parallaxes, and from the distance of the Moon from the same fixed stars observed on the same night in several horizons, both when the Moon is low and when it is high. [Margin: 2nd Reason.] Although, then, this is not established from the history of the observations reported in chapters 2 and 3, it is nevertheless much more probable that the star was observed on many nights besides those recorded there on particular days, so that the observations of many, though not reported in the history, fell on the same nights and hours.

First, because some of those observers affirm that they observed it not only on the days they record but on many others — above all Tycho, Mästlin and Digges. And all those who say that it always appeared in the same place indicate the very same thing — namely most of the authors of the first and second class, cited in numbers 2 and 3 of this chapter and earlier in chapter 2 — granted that they ceased observing with instruments once it seemed already ascertained to them that it remained in the same place beneath the fixed stars. For the Landgrave of Hesse expressly testifies that for this reason he refrained from observing the altitudes and azimuths of this star. For in comets, whose proper motion, manifestly distinct from the common motion of the primum mo- bile, is apparent, observations were made on continuous and very many nights, so that the size and direction of their motion might be established — especially in those that appeared towards the end of the preceding century and in this century of ours. But it is not probable that the astronomers already named did not turn their eyes again and again to so unusual a spectacle as this New Star was, especially in the first months — granted that, as I said, seeing that it had no proper motion, they rarely set their hands to the instruments. *Secondly*, even if we granted that there were no nights in common on which several observers in different horizons actually saw that star equally distant from the same fixed stars, or from the pole of the world, it is nevertheless more probable that this equality would have appeared to them had they observed it together on the same nights. For on all those nights, different though they were, on which they observed it, all who observed it agreed on the same equality, or at least none of them individually noticed a variation great enough to bring the star down below the Moon — except Nolthius alone, whose observations Tycho, for just reasons, renders suspect of deception and of rather crude workmanship, as we saw in chapter 9, number 2. For the observations of Busch, who out of inexperience placed it below the Moon, show that it was above the Moon, as is clear from what was said in chapter 9, number 3. Therefore, since the observations of all the nights, though different nights, attest that an equality of distance from the same fixed stars was kept, as great as suffices for a place above the Moon, and none of them — except one, to be disregarded beside the others — has dared to testify to the contrary, we have from this a more probable argument for affirming than for denying that the equality already mentioned would have appeared on all the remaining nights. *Thirdly* and lastly, it is not probable that God, Best and Greatest, who for the loftiest causes and ends roused mortals to this spectacle, universal to the whole world, would — had he given this star any proper motion — yet have willed that the motion lie hidden from mortals for the whole time of the star's duration; or that he would have so tempered it with respect to some observers that to them the star seemed to have no parallax, no proper motion, and no change of distance from the fixed stars, while to others it would have appeared otherwise, had they observed on the same nights. This being so, let us reduce this argument, already set out at greater length, to this very brief form. [Margin: 2nd Reason, reduced to form.]

If it is more probable that the New Star of the years 1572 and 1573, on all the nights on which it was observed or observable, lacked proper motion and kept the same distance from the pole of the world and from the same fixed stars, as seen or visible by all, or several, observers in different parts of the globe — or at least an equality of distance as great as suffices to preserve a place above the Moon — then it is also more probable that it was above the Moon. But it is more probable that it was so. Therefore it is more probable that it was above the Moon.

Next, whether it was among the fixed stars, or in the space between the fixed stars and Saturn, or between the Sun and Saturn — for I find no valid argument by which I could more probably assign this star a seat in one of these three seats or regions — granted, I lean towards the space between the fixed stars and Saturn, as though it had been assigned by God to these extraordinary stars, lest it be, or seem, idle. The remaining matters that pertain to the size and origin of this star will be treated below, together with the other new stars, so that we are not forced to repeat too often the arguments they have in common.

CAPVT XII.

Chapter XII.

History of the New Star seen in the breast of Cygnus from the year 1600; and whether it was truly new, and how long it lasted.

I. The history of this star has been set out, first of all, by Johannes Kepler, in an astronomical narrative about it published at Prague in the year 1606 and inserted into the book on the New Star of Serpentarius after chapter 27 — that is, at the end of the first part and before chapter 28, where the second part begins; by Scipione Chiaramonti, *De tribus novis stellis*, Book 3, chapters 1 to 6; by Biancani, *Sphaera*, Book 18, chapter 3; and by Liceti, *De novis astris*, Book 5, chapter 16.

The first witness to this novelty, as Kepler relates, was Willem Janszoon [Gulielmus Ianſonius], who, in an inscription on a celestial globe published in the year 1601, declares that he first saw it in the year of Christ 1600 at the top of the breast of Cygnus, or the Hen, or at the border of the neck and the breast. [Margin: The first observer of this star.] By the common consent of all who later observed it, it was of the third rank, or of the third magnitude: that is, like three other stars of Cygnus — the one in the breast, to which it was nearest, the one in the middle of the neck, and the one that shines in the beak; and indeed like three other stars of Cygnus too, of which one shines at the elbow of the upper wing, another at the elbow of the lower wing, and the last at the tip of the lower wing. So Cygnus, which until then had had six stars of the third rank, began in that year to have seven, besides the remaining stars of other magnitudes. [Margin: Magnitude of the star.] The longitude assigned to it by Kepler and the Braheans, and accepted by others, was Aquarius 16° 18′; the northern latitude 55° 30′, or 32′; the right ascension 300° 46′; the northern declination 36° 52′; and it culminated with Capricorn 28° 37′ — as Tycho's

pupils gathered from the star's distances set out below, taken with Tycho's sextant. [Margin: Place of the star in the heavens.]

Distance of the New Star of the year 1600

From the stars written below	Distance
the bright star of the Vulture	29° 49′ 0″
the bright star of Lyra	19° 43′ 20″
Scheat of Pegasus	35° 49′ 30″
the tail of Cygnus	8° 30′ 0″
the Pole Star	52° 0′ 0″

[Margin: Distance of the New Star from the fixed stars.]

This star lasted, most certainly in the same place, up to the year 1616 — indeed 1621 — years in which our Fathers at Parma observed it, and I with them, especially with Father Biancani, who attests this very thing in Book 18 of the *Sphaera*, chapter 3, for the year 1616, in which he published the *Sphaera*. For the year 1621 Fortunio Liceti affirms the same, from an observation by Corgneti, in Book 5 of *De novis astris*, chapter 16 — although Chiaramonti, Book 3, chapter 6, has doubts; and Glorioso, in Book 5 of *De cometis*, page 278, which he wrote in the year 1624, asserted that it was still lasting then. [Margin: Duration of this star.] Argoli, for his part, says in the *Pandosion*, chapter 62, that it lasted up to the year 1629. But on what day and in what year it vanished I do not find in any author. It is certain, however, that in these last years, from 1640 to 1650, it has not appeared to us, and that in duration it surpassed all the new stars whose duration is known to us so far. On the doubt whether it would perish, see Kepler's words in the following chapter, number 2.

II. There was a greater controversy over whether it was truly new, or rather one of the old stars of Cygnus — at least of the unformed ones — but omitted by Hipparchus, Ptolemy and the other authors before the year 1600 because of its smallness. The reason for doubting its novelty is the hesitation of very many, and especially of Mästlin, who leaned to the negative side. [Margin: Doubts about the novelty of the star.] For Kepler says, on page 154: *What I have said of myself I can say of almost all present-day professors of mathematics, who are spread everywhere through the universities of Europe: all of them raise the question of the new phenomenon; they keep their opinion to themselves, or confess that they do not know. Mästlin, indeed, having confessed that he is in doubt, still inclines to this: that the longer he examines his memory, the more he thinks that he too saw it long ago* — which he confirmed from its duration of so many years and from the other stars omitted by Ptolemy. [Margin: Mästlin's doubt.] Kepler himself, having said, *If I am asked whether I saw this star before the year 1601 or not, I shall have to answer that I do not know. For although I began to learn the heavens from the year 1591, with Mästlin pointing them out, I do not remember that we were told by our teacher that one star in the breast of Cygnus had been passed over by Ptolemy*, admits indeed that in November of the year 1601, when the beak of Cygnus

stretched towards the western horizon in the evening, there seemed to him to be in Cygnus as it were an image of the Crucified with bowed head — for the New Star represented the breast of the Crucified — but [Margin: Kepler's former hesitation.]

the star in the breast represented the head, those at the elbows on either side the arms, and the neck the upright of the cross. But whether he had seen it before this time or not, he does not recall, and he concludes about himself: *So here my own memory fails me; nor would I affirm from it alone that this is a new star.* To this is added that Santucci — who, with unceasing labour, by order of Ferdinand, Grand Duke of Tuscany, kept watch with observations of new phenomena from the year 1592 to 1607 — counts among the new stars the one that shone forth in Cassiopeia in the year 1572 and the one in Serpentarius in the year 1604, but nowhere mentions this one, which we are now discussing.

III. It must nevertheless be asserted that this was a new star, and different from the perennial fixed stars. [Margin: Assertion of the star's novelty.] *First*, because it vanished a few years ago. *Secondly*, because Hipparchus of Rhodes, as Pliny the Elder relates in Book 2, chapter 26, *detected a new star, and another, in his own age; and by its motion, on the day it shone, he was led to doubt whether this happened more often, and whether those stars too that we suppose fixed were moving. And he likewise ventured on a thing presumptuous even for God: to count the stars for posterity and to tally the heavenly bodies by rule, with instruments devised by which the places and magnitudes of each might be marked, so that it could easily be discerned from this not only whether they perish or are born, but whether any of them pass across at all, or move, and likewise whether they grow and diminish — the heaven having been left to all as an inheritance, etc.* But if Hipparchus had neglected this star, which is after all of the 3rd magnitude, how could he hope that posterity would be able to discern from his catalogue whether any new stars arose, or any of the old ones perished? Since, then, neither he nor Ptolemy — who nevertheless arranged Hipparchus's stars in a better order and form, not omitting even the nebulous ones, and counted the stars in the Milky Way with such diligence — ever entered this one in the table of stars, it is a very strong indication that it did not appear in the heavens in the age of either. Moreover, Hipparchus of Bithynia too (whom Kepler, through inexperience, in this place makes the same man as Hipparchus of Rhodes), in Book 1 on Aratus, reviews the stars of the Swan, or Hen, carefully and one by one, and criticises Aratus's ignorance about them, yet never mentions this star. [Margin: Kepler's error in the history.] And Tycho himself, who added to the northern stars two hundred others observed by himself, of roughly the fifth and sixth magnitude, passed over by the ancients, did not recognise this one, which is of the 3rd magnitude — granted that in his record he set down very many observations of the whole of Cygnus, the chief of which Kepler reports in this place. Nor, finally, did any of Tycho's surviving pupils, when they heard the report of this new star, dare to detract from its novelty. Weighing these things, Johann Bayer, in his *Uranometria*, said that this star was accepted as new *by the tacit consent of all*.

And Jost Bürgi, automaton-maker to His Sacred Imperial Majesty, built a silver globe at the expense and by order of Wilhelm, Landgrave of Hesse, constructed in such a way that he was told to look at the heavens again and again and to add any stars the ancients had omitted; the Landgrave later sent it as a gift to the Emperor Rudolf. When Bürgi was afterwards asked about this New Star, he appealed to that globe of his, conscious of his own diligence; and since the star was not found on it, *he pronounced most confidently on the novelty of this star* — so Kepler says in the same place. [Margin: Testimony of Bayer and of Jost Bürgi.] Kepler also adds, from a letter of Bayer's of the year 1602, these words about the celestial figure of the Swan, which Bayer had lately inspected: *Besides these, at the end of the neck and the beginning of the breast, I discovered another star, no longer doubtful to me, but in truth never seen until now, of roughly the third magnitude.* To these must be added, above all, Janszoon, of whom we spoke above. So, on account of these testimonies and the reasons indicated above, Kepler rightly laid aside his hesitation and said, on page 167: *The novelty of this star has, I think, been sufficiently attested; for if these things are not enough for anyone, it is credible that he would not believe even if whole nations affirmed it.* After him, all the authors listed, both at the beginning of this chapter and in chapter 1, accepted it as new — except a few, unnamed but indicated by Kepler above and in the words to be cited at the end of number 2 in the following chapter.

IV. Nor does Mästlin's argument stand in the way — namely, that Hipparchus and Ptolemy omitted the star seen above the middle one of the three in the tail of Ursa Major, which farmers call the rider but the Arabs call Alcor. For it is so close to the middle star of the tail, and smaller than its horse, that it can be distinguished from it only by the sharpest eyes; hence that proverb among the Arabs about a man who pretends not to see grosser things but boasts of a finer perception: *You saw Alcor, but not the full Moon.* But where the closeness was not so great as to obstruct the sight, they did not omit to count stars, however double and close, as is clear in the two of the horn and of the tail of Capricorn, in the stars of the shoulder of Sagittarius, in the horn of Aries, and in others like them. Nor could they have omitted this one on the ground that it was unformed: it lay within the very neck of Cygnus, and belonged more to the configuration of Cygnus than the bright star of the breast does. Nor could it have been confused with that star, since it was nearly two degrees distant from it, and was very nearly equal to it in magnitude. We therefore conclude that before the year 1600 it was not recorded because it was not shining in the heavens. Nor can Santucci's omission — whether it arose from forgetfulness or from hesitation — outweigh so many other witnesses and indications of this star's novelty, who observed it afterwards.

CAPVT XIII.

Chapter XIII.

On other observations concerning the New Star of the year 1600; and whether, or with what certainty, it is gathered from them that it was above the Moon.

I. Besides the distances of this star from five fixed stars, taken with Tycho's sextant by Tycho's pupils and reported in the preceding chapter, number 1, Kepler reports distances taken by himself in the year 1602, on August 20, in a very clear sky, at Prague, with Tycho's sextant and with the help of Ioannes Eriksen, who had once served Tycho in his astronomical work — but with the sextant corrected from distances between fixed stars taken anew and compared with the distances taken by Tycho, as below.

Distances taken, of star from star	By Tycho	By Kepler
of the Vulture from the tail of Cygnus	38° 3′ 30″	38° 2′ 0″
of Lyra from the tail of Cygnus	23° 53′ 0″	23° 51′ 30″
of Scheat of Pegasus from the breast of Cygnus	35° 12′ 30″	35° 11′ 0″

So, says Kepler, page 167, *our sextant fell short by 1½′, which will have to be added in the following distances*; and he immediately appends the distances of the New Star, as taken and then as corrected — namely these:

Distance of the New Star from the fixed stars	Taken	Corrected
from the *Vulture*	29° 51′ 0″	29° 52′ 30″
from *Lyra*	19° 44′ 0″	19° 45′ 30″
from *Scheat of Pegasus*	35° 50′ 30″	35° 52′ 0″

From these Kepler gathers the place of the New Star at Aquarius 16° 18′, exactly as the Braheans did from their own distances; but the northern latitude Kepler makes 55° 32′, the Braheans 55° 30′.

II. As regards the parallax, or the distance of this star from the Earth, we have nothing else from Kepler than the words of his letter written to Bavaria when the report about the star spread, which he himself inserted into the astronomical narrative on this star, page 166, where he says: *Whatever this star may be, then, old or new, it is established by geometrical reasons that it is certainly raised very high among the fixed stars themselves. This once found, when one also considers that so many months have already passed while it is always seen at the same size, never equalling its neighbour in the breast, never reduced to the faintness of the one in the beak, and of a constant and pale colour too — indeed it is no wonder if that very lingering, however fully one is convinced by the arguments above, still keeps one anxious about what is to become of it.* [Margin: Kepler's opinion on the parallax of this star.] That is, whether it would perish, and whether its novelty was to be confirmed by its perishing. For he wrote this letter in the year 1602; and of this doubt he had also spoken in the dedicatory letter of this astronomical narrative, written in the year 1606, in these words: *For it must be proved to mathematicians, shown to philosophers, and recorded for the universal memory of posterity, whether at this time there is some star in the heavens which, though it was not there formerly,*

has now for six years at least clung there motionless, unchanging, inextinguishable: so that, if it is one day to perish before the consummation of the whole world, men may understand the magnitude of the miracle that it lasted so many years; or, if it will endure with the rest of the fixed stars, then for as long as that is to be, those who are persuaded by the following arguments that it is new may always have before their eyes this most evident proof of a whole world created in time; while those who have stubbornly persisted in arguing from the star's long duration against the tradition of the astronomers, and in denying that it was lately born, may not be unaware by how great a weight of authorities they are pressed. [Margin: Kepler's notable admonition about this star.]

III. Since, then, he says that this star was motionless for six years — understand: by its proper motion — and indeed it was seen up to the year 1621 at least in the same place and position in Cygnus, viewed on however many nights and from however many parts of the world, it is as certain that it had no greater parallax than the other stars of Cygnus as it is certain that its distance from the same fixed stars was seen on the same nights at different horizons — and indeed both when it was highest above the horizon and when it was lowest. [Margin: On the distance of this star from the Earth.] But since Kepler produced no such observations — none other from which the equality of the distance already mentioned was seen on the same nights by different observers, at a great and at a small altitude of this star — Chiaramonti rightly holds, in Book 3 of *De novis stellis*, chapters 3 and 4, that neither the distance of this star nor its absence of parallax has been geometrically demonstrated. And he taunts both Kepler and Tycho's surviving pupils, because in a portent so new and so astonishing they were so negligent that they did not take its distances from the fixed stars accurately on several nights, nor, besides, its greatest and least meridian altitudes — since in the more northern regions it never set, and it shone for so many years. But why did Chiaramonti himself, who had observed the comet of the year 1618, not observe this star? Let him therefore grant to others too the indulgence he would wish for himself, according to that line of Horace: *We know it, and this indulgence we both ask and grant in turn.* Yet he too lacked observations by which he might show, at least on a hypothesis, that this star was below the Moon.

Although neither has been demonstrated by these authors or by others, so far as I know, it is nevertheless far more probable that the astronomers stopped observing because they judged such observations superfluous, seeing that this star had now clung to the same place in the heavens for so many years, let alone months — and for that reason many were slipping back into doubt whether it was one of the perpetual stars; and that there were many same nights on which different astronomers saw it equally distant from the same fixed stars. For the silence of all about any inequality and proper motion is the more probable testimony that this was never noticed. On this supposition it follows that it is far more probable that this New Star was among the fixed stars, or above

Saturn, and had no greater parallax than they, or none perceptible. Let there be, then, the

VNICA CONCLVSIO.

Single Conclusion.

Although it has not yet been demonstrated whether the New Star, which from the year 1600 up to the year 1621, and perhaps beyond, was seen at the junction of the neck and breast of Cygnus, was above or below the Moon, it is nevertheless more probable that it was either among the fixed stars or above Saturn.

The conclusion as to its place among the fixed stars is Kepler's, above — although he thinks it demonstrated; Biancani's, Book 18 of the *Sphaera*, chapter 3; Argoli's, chapter 62 of the *Pandosion*; Glorioso's, Book 5 of *De cometis*, page 275; Descartes's, part 3 of the *Principles of Philosophy*; and Froidmont's, Book 3 of the *Meteorologica*, chapters 2 and 7; and it now stands sufficiently proved from what has just been said.

CAPVT XIV.

Chapter XIV.

History of the New Star of the year 1604 and 1605, seen in Serpentarius.

I. The history of this star, too, was written above all by Kepler, in the book *On the New Star in the Foot of Serpentarius, and on the Fiery Trigon*, chapters 1 and 12; by Chiaramonti, Book 3 of *De novis stellis*, chapters 7, 8 and 9; by Ioannes Heckius, in a disputation on this star; by Liceti, Book 5 of *De novis astris*; by Biancani, in the *Sphaera*, Book 18, chapter 4; by Antonius Laurentinus Politianus, in a little book about it published in Italian; and by others mentioned by these authors — especially by Kepler, who says that many astrologers, following the doctrine of the Arabs, had predicted a comet for this year because of the Great Conjunction of the superior planets, and that for this reason many astronomers too had fixed their eyes on the heavens, to see whether they might notice some portent in them.

II. By the agreement of all, this star arose in the year of Christ 1604; but not all agree on either the day or the month of its first appearance. [Margin: First appearance of this star.] Georgius Spate, a Pomeranian priest, told Herlicius that it was seen on September 27, new style. Kepler contradicts him, thinking that Spate made this up because Magini and Origanus, in their ephemerides, assign the conjunction of Saturn and Mars to the night preceding September 27. Antonius Laurentinus Politianus, on the other hand, says that it was seen on October 8 in 18° of Sagittarius, in which sign, he affirms, the conjunction of Jupiter and Mars took place on about that day — as the ephemerides of Everhardus Belga also have it, on the faith of his tables, which he says were built upon the observations of 40 years — granted that Cyprián Karásek Lvovický places that conjunction of ♃ and ♂ on October 5, and Magini on the 9th, and

that Kepler affirms, both from calculation and from observation, that it took place around noon [?] of October 9. [Margin: ☌ ♃ and ☍ of the year 1604.]

But Kepler gives no credence to Laurentinus either, because nothing new was seen in the heavens by the Germans on September 27, or on October 7, 8 or 9, although they observed the heavens most attentively: on the 27th he himself; on October 7, Ioannes Schulerus; on the 8th, Ioannes Brunowsky at Prague, Helisaeus Röslin in Alsace, and David Fabricius in East Frisia; and finally on the 9th, which was the feast of St Michael the Archangel, Michael Mästlin at Tübingen. But on the following Sunday, namely October 10, after sunset, Brunowsky — who was devoted to the observation of the sky and by then trained in recognising the stars — when the clouds opened for a moment after sunset, saw in the place where Mars and Jupiter were three stars instead of two, and one of them not smaller than Jupiter; and not long afterwards the sky was wrapped in clouds, and at Prague it was overcast for six continuous days. This star was not seen before October 12 by Giovanni Antonio Magini at Bologna and by Röslin in Alsace, nor before the 13th by David Fabricius — all of whom said that it was very large from the beginning — and by others in Württemberg; on the 14th it was seen by Mästlin, though he was still in doubt about its novelty, but on the 16th it was manifest to him, and to Schulerus and Brunowsky; and on the 17th to Kepler. On that day, Bartholomaeus Crestinus, mathematician to the Prince of Savoy, says a letter had already reached his Prince from Spain announcing the rising of the new star. On the 21st, too, it began to be observed with instruments by Jost Bürgi, by Kepler, and by Franz Tengnagel with the Tychonic sextant in the Emperor's garden.

Ioannes Hechius says that it began to be seen around October 9, Arnerius, however, in the first days of October. Therefore, if it is true that at first it was seen small and shortly afterwards large, as Laurentinus and Arnerius affirm, it could have happened that it was not seen by others at that time, either because its faintness did not draw the eyes of observers to it, or because of intervening clouds. But Kepler stubbornly maintains that it appeared very large right from the beginning and was first kindled on the 10th, because the ☌ of ♃ and ☍ had taken place at noon on the day before — as if, indeed, in the application of the aspects, or a little before, their effects did not often precede the aspects themselves; or as if, granted that the rising of this star is to be attributed to the Great Conjunction, it could not have been born very small at first, immediately after the conjunction of Saturn and Mars, which took place on September 26 in the evening after 8^{h}, and then have grown, and, once reared and increased to its full size, have suddenly become very large after the ☌ of ♃ and ☍. We therefore have its beginning — not, indeed, later than October 10, but, not improbably, already from September 27. [Margin: Our opinion on the day of the star's rising.]

III. The star's duration is certain up to October 8 of the year 1605, on which day Kepler testifies that he saw it, adding that it is uncertain on what day between October 8 of the year 1605 and March of the year 1606 it disappeared.

For in March, although the Sun and the Moon were away, no trace of the New Star appeared to him, while on the preceding days it could not be made out, partly because of the nearness of the Sun and the New Star's heliacal setting, partly because of the presence of the Moon. Its duration, therefore, was not less than a year. Santucci says it lasted 13 months, but does not fix the day of its extinction. [Margin: Duration.]

IV. The apparent magnitude of this star was small at the beginning, if you think Laurentinus and Arnerius are to be believed; but large straight away, if Kepler and his friends — and indeed either equal to Venus or greater than Jupiter, Mars and Saturn, which were near the star and offered a convenient comparison. [Margin: Apparent magnitude of the star.] Kepler therefore affirms that by the account of all it was greater than ♃, ♂ and ♄, but by his own observation less than Venus at any time, and very like the star Tycho had observed in Cassiopeia in the year 1572. But Jupiter at that time — that is, at the beginning of October 1604 — was tending towards its apogee, where it would be about the end of December, and so appeared nearly at its smallest. The star, however, remained of the same size through the whole of October. Then at Prague it set heliacally on November 16, and on January 3 of the year 1605 it rose heliacally, plainly visible, but in magnitude greater than the Heart of Scorpius and less than Arcturus, although Arcturus held a position farther from the Sun. For on January 13, once the twilight had lifted, this new star was seen to be greater than Arcturus and than Saturn, which had risen heliacally on the 10th. But on March 20 it seemed less than Saturn, and not much greater than the stars of the third rank that are in the knees of Ophiuchus. On April 21 it seemed to equal the bright star in the right knee of Ophiuchus. On August 12 it was seen equal to the nearest preceding star in the shin of the same figure, or somewhat smaller than on April 21; yet it still twinkled more conspicuously than they and the others. On September 13 it was seen to be already smaller than the one in the shin; and on October 8, because of the evening twilight, it could scarcely be compared with the one in the shin, though it was better discerned because of its greater twinkling.

V. This star had a livelier light and a greater twinkling than the other fixed stars, so much so that Santucci affirmed that it was most brilliant, and more brilliant than the Dog Star itself, while Kepler said of it: *with a flashing or vibration so rapid that some denied that in all their lives they had ever seen anything in the heavens with so swift a motion, as they themselves put it* — that is, with a flickering of its rays. [Margin: Light, twinkling and colours.] He goes on to say that some compared it to a torch tossed by the wind, now stretching out its flames, now drawing them back, in restless alternation; others to the arms of swimmers flung about very rapidly; others to the pulse of people in a fever. On the colours let us hear the same Kepler, page 5: *I, who looked above all to the variety of the colours of the rainbow, which it threw out according to one state of twinkling after another, chose to use the example of a many-faceted diamond, which, as it is turned, flashes the Sun's rays back to the eyes of the onlookers with a changing*

brilliance, etc. On the colours Mästlin agrees with me entirely: that they changed from moment to moment — from yellow presently to saffron, and at once to purple and red; and that it appeared for the most part white when it had risen a little higher out of the vapours.

VI. The figure of this star was perfectly round, with no hair, no beard, no train of a tail cast in any direction. [Margin: Figure.] Its configuration with the planets ♃ and ♂, which were low towards the south, was such on October 10 that it formed an obtuse triangle with them; [Margin: Aspect.] this was resolved on the 13th into a straight line by the motion of the planets, not by the star's own motion. [Margin: Motion.] For throughout the whole time it lasted it moved with no other motion than that of the primum mobile, as Kepler and Johann Georg Brengger [Io. Georgius Brenggerus] observed: it never changed its distance from the fixed stars, and from Jupiter and Mars only exactly as much as their motion produced; it was always on the same parallel, and always rose from the same point of the horizon. Granted that one Crabbus, whom Kepler ridicules and dismisses on page 7, said that it had a proper motion, though a very small one, and that Herlicius agreed with him in this — yet this Crabbus, on the very first day of his observation, which was October 14, erred about the star's place, putting it at Sagittarius 17° 0′, when on that day Fabricius observed it at 17° 37′ or 40′.

VII. The position of the star was roughly at the heel of the right foot of Ophiuchus, or Serpentarius. [Margin: Position.] Below that heel, on October 10, was Jupiter, and Mars, more southerly, was so joined to it that it covered almost a quarter of Jupiter, while Jupiter all but touched the southern edge of the new star. Its place among the fixed stars is assigned by Kepler at Sagittarius 17° 40′; its latitude, quite certain, northern 1° 56′ or 57′; right ascension 256° 47′; southern declination 21° 1½′; it culminated with 17° 51′ of ♐. The star's meridian altitude observed at Prague in March and April by Kepler was 18° 48′ or 49′, but from the declination it comes out 18° 52½′. Fabricius, however, puts it at ♐ 17° 37′ or 40′; Santucci at ♐ 18°, with latitude 1° 44′; Hechius at ♐ 17° 41′ 30″, with latitude 1° 55′ 30″. It is noteworthy, moreover, as Kepler points out, that before it was extinguished this star was approached by all the planets and was in conjunction with them, at least broadly. But it will be useful to set out the distances of this star from the planets and fixed stars taken by Kepler and Tengnagel at Prague, by Fabricius in East Frisia, and by Brengger in Alsace. Kepler corrected the Tychonic sextant by distances of the fixed stars from one another already examined by Tycho, having found that it sometimes exceeded by 4′ because of a dislocation of the pinnule. [Margin: Distance from the planets and the fixed stars.]

Distances of the New Star of the years 1604 and 1605, observed and reported by Kepler, chapter 12.

Date and observer	Distance of the New Star from the old stars	Distance
In the year 1604	from Jupiter	3° 28′ 30″
October	Mars	8° 31′ 0″
day 17	Saturn	6° 13′ 0″
new style	the shoulder of Sagittarius	20° 0′ 0″
Kepler	the right knee of Ophiuchus	7° 37′ 0″
October	from Jupiter	3° 40′, twice
day 18	the shoulder of Sagittarius	20° 2′ 30″, twice
Kepler	the right knee of Ophiuchus	7° 31′ 30″
October	from Jupiter	4° 1′ 30″
day 21	the head of Ophiuchus	34° 2′ 30″
Tengnagel	the shoulder of Sagittarius	19° 34′ 0″
	the left knee of Ophiuchus	16° 52′ 0″
October	from the head of Ophiuchus	34° 0′ 0″
day 21	the right knee of Ophiuchus	7° 47′ 0″
Jost Bürgi	the bright star of Aquila	45° 43′ 0″
October	from Jupiter	4° 59′ 30″
day 27	the head of Ophiuchus	34° 1′ 30″
Kepler	Afterwards it set heliacally.	
In the year 1605		
January 29	from the left knee	16° 50′ 0″
Brengger	of Ophiuchus, or	16° 45′ 0″
February 2	from the Heart of Scorpius	14° 55′, three times
Kepler	from Aquila, at one time	45° 54′ 0″
	at another	45° 51′ 0″
	at another	45° 45′ 0″
	and at another	45° 44′ 0″
because of the awkward position of the sextant		
David Fabricius	from the head of Ophiuchus	34° 0′ 0″
	the left knee of Ophiuchus	16° 46′ 0″
	the bright star of Aquila	45° 45′ 0″

VIII. Kepler says that he defers to Fabricius, because he admits that he was much hindered by confused vision. From the distances already mentioned, Kepler, in chapter 13, investigates the place of the New Star in the zodiac by various lines of inquiry, and from many places chooses the one I set down above, in number 7; and in chapter 14 he partly corrects many stars of Serpentarius observed on that occasion, and partly adds stars passed over by the ancients and by Tycho himself. But since Kepler worked rather crudely, and compared the star with the planets just as if, because of their small latitude, they were on the ecliptic, Chiaramonti, in Book 3 of *De novis stellis*, chapter 11, criticises him on many points; and in chapter 12, from selected distances among those already mentioned, and from the Tychonic longitude and latitude of the fixed stars, he gathers trigonometrically the places of the New Star, which we set out here below in a small table, together with the places of the stars themselves.

From the stars	Distance of the New Star	Sign	Longitude of the fixed stars, from Tycho	Latitude of the fixed stars, from Tycho
from the head of Ophiuchus	34° 20′ 30″	♐	16° 53′ 30″	35° 37′ B
the shoulder of Sagittarius	19° 34′ 0″	♑	6° 54′ 30″	3° 31′ A
the left knee of Ophiuchus	16° 52′ 0″	♐	3° 42′ 20″	11° 30′ A
the bright star of Aquila	45° 43′ 0″	♑	26° 12′ 28″	29° 21′ B
the Heart of Scorpius	14° 53′ 0″	♐	4° 1′ 0″	4° 27′ A
From these is gathered, for the New Star:			Longitude	Northern latitude
from the longitude of the shoulder of Sagittarius		♐	18° 30′ 30″	3° 10′ 6″
from the longitude of the knee of Ophiuchus		♐	17° 26′ 30″	1° 34′ 23″
from the longitude of the head of Ophiuchus		♐	17° 10′ 30″	1° 34′ 23″
from the longitude of Aquila		♐	18° 46′ 15″	✠ ✠ ✠
from the longitude of the Heart of Scorpius		✠	✠ ✠ ✠	0° 43′ 44″
But from the New Star's distances taken by Fabricius — namely 34° from the head of Ophiuchus and 16° 46′ from the knee of Ophiuchus — there is gathered		♐	17° 52′ 30″	1° 37′ 15″

CAPVT XV.

Chapter XV.

On the parallax of the New Star of the year 1604, and its distance from the Earth.

I. Kepler's first preliminary argument [Prolusio] is found in chapter 1, in these words: *Since, then, this new star corresponded exactly to the other fixed stars, and to the new one in Cassiopeia, in magnitude, splendour, twinkling and purity of ethereal light, it came about that, as soon as astronomers came upon the sight of it, they at once concluded both its rest beneath the fixed stars and its place in the world, common to it with them as well — which the experience of a few days proved, since it kept exactly the same distances from the fixed stars.* But from chapter 15 to chapter 18 he tried expressly to establish this opinion of his. For it is quite clear from what was said in chapter 4 that the things he had brought forward about twinkling and other accidents distinct from immobility are not evident arguments but only probable ones. In chapter 15, therefore, he promises a demonstration in this formula: *First I shall demonstrate that this star was not only placed above the orb of the Moon, but was not even nearer than the Sun itself.* And in chapter 17 he says: *This immobility, etc., plainly proves that no place below Saturn can be assigned to the New Star.* This he takes pains to show by six arguments, though scattered here and there.

The *first* is of this kind. If the New Star had not been higher than the Moon, that is, than sixty semidiameters of the Earth, and yet had lacked a proper motion, then, observed in October at its setting and in March at its rising, it would have undergone a parallax of longitude of 52½′ and of latitude of 5′, as he demonstrates from his parallactic table and from a geometrical figure. [Margin:

Kepler's 1st argument for its seat above the Moon, from a parallax smaller than the Moon's.] But it lacked a proper motion, and, observed at both those times, it showed no parallax of latitude, and none of longitude beyond 6′ — indeed, once the defect of the instrument and refraction are taken into account, none beyond 2′. For in the year 1604, on October 21, from its distance from three stars taken by Tengnagel, Tycho's son-in-law, its place was found at ♐ 17° 39′, or 40′, or 42′, while it stood in its setting. And on February 12, when it was observed rising, it was found likewise, by means of Aquila, at 17° 40½′ of ♐, but by means of the Heart of Scorpius at 17° 44′ 20″ of ♐, and more correctly, from Fabricius's distances, at 17° 42′ of ♐. Indeed, because Bürgi in October and Kepler himself in February measured the New Star's distance from the bright star of Aquila as 45° 43′, and from this and other stars its place in both months was found at ♐ 17° 40′, 42′ or 43′, he concludes that it did not have a parallax of 3′; and therefore, if the Sun is 1200 semidiameters distant from the Earth and its parallax is 3′, as is commonly assumed, that the New Star had a parallax smaller than the Sun's.

II. The *second* argument, put forward on page 81, is this. If the Moon is observed at the same moment of time near the zenith in Bohemia and in Frisia — places about 6° of a great circle apart — it will be seen at different places of the zodiac, by about 6′. But Kepler in Bohemia and Fabricius in Frisia observed the New Star at the same place in the zodiac, without a difference of 6′. Therefore the New Star was not as low as the Moon. [Margin: Kepler's 2nd argument, likewise from a parallax smaller than the Moon's.]

The *third* argument, inserted on page 78 and in chapter 89 in the first exposition, is this. If the New Star had not been higher than the Moon, it could not always have been seen from different parts of the Earth equally distant from the head and from the right knee of Ophiuchus. But its distance from the head of Ophiuchus was always the same, both in the year 1604, when it was near its setting, and in the year 1605, when it was near its rising: therefore it was higher than the Moon. And he adds: and so its northern latitude too was always the same, between 1° 55′ and 1° 57′. [Margin: Kepler's 3rd argument, from the same distance kept from the same fixed stars.]

The *fourth* argument, indicated on page 6 but put into form by us — and in nearly the same way by Chiaramonti, Book 3, chapter 18 — is this. A star which is always observed by the same observer to rise from the same point of the horizon, and so always remains on the same parallel, does not change its distance from the fixed stars and lacks all parallax, so that it is in the eighth sphere. But such was this New Star, as Brengger observed. Therefore it lacked all parallax. [Margin: Kepler's 4th argument, from the star's rising at the same point of the horizon.]

III. The *fifth* argument is from the star's immobility, but by another route than through parallax. Kepler himself treats it in chapter 17, adapted to the Ptolemaic and Tychonic hypothesis of a stationary Earth, and we shall put it into form in this way. If the New Star had been in the region of the planets,

it would have imitated the motion of the wandering stars, and, besides the diurnal motion of the primum mobile, would have had some proper motion. [Margin: 5th argument, from immobility, or the lack of proper motion.] For if the orbs are solid, it could not have failed to be carried in the directions in which the orb of the planet giving lodging to this star was carried. But if, as Tycho holds, the orbs are not solid, then, since the planets follow the Sun by a natural sympathy, or at any rate are drawn by it as it carries them round, and cannot themselves resist, it is incredible, and would be said ἀναιτιολόγητον [Greek: anaitiologēton, "not accounted for by a cause"] — that is, without probable cause — that a New Star placed in a planet's orb could do what the planets cannot: namely, not follow the Sun, and not imitate its motion by a proper motion through the zodiac. But the New Star had no proper motion; rather, throughout the whole time of its duration, it stood in the same place beneath the fixed stars; and its configurations [ſchematiſmos] with the stars of the foot, shin and right knee of the Serpent-bearer [Serpentarij], and with the triangle that shines in the coil of the Serpent, it did not change by even the least amount that could register on the senses. Therefore it was not among the orbs of the planets, but above them, among the fixed stars. For it is consistent that the new stars of which Pliny the Elder said (Book 2, chapter 25), *Some move in the manner of the wandering stars, others stay fixed*, should be in the sphere of the wandering stars if they move like the wanderers, and, if they do not move, in the sphere of the immobile stars, that is, of the fixed stars [Affixarum].

IV. The *sixth* argument, which *Kepler* begins at the end of chapter 15 but completes in chapter 16, tells only against those who admit Copernicus's hypothesis of the annual motion of the Earth. It runs as follows. In the hypothesis of Copernicus, Saturn at apogee is ten times further from the Earth than the Sun is; and then, because of the annual motion of the Earth, it shows a parallax of the annual orb of about 6 degrees when the Earth is carried from the conjunction or the opposition of the Sun and Saturn to their quadrature, as is plain from the Prutenic Tables. Therefore, if the New Star had been in the place of Saturn at apogee, it would have incurred a parallax of 6 degrees; yet it never shifted its apparent place beneath the fixed stars by 6′, that is, not even by a sixtieth part of six degrees. Therefore, in this hypothesis, the New[21] Star must have been higher than the distance of Saturn taken sixty times, or than the distance between the Earth and the Sun taken six hundred times (since the distance between the Sun and the Earth is contained ten times in the distance of Saturn); otherwise, had it not been higher, it would have had a parallax of six minutes. [Margin: 6. The argument from the absence of annual parallax.] Indeed, since the most trustworthy observations all agreed to within 2′, and this star had no parallax greater than 2′, and two minutes are a third part of six minutes, the distance already mentioned — which we said was six hundred times greater than the distance of the Earth from the Sun — must be tripled. *It is certain, therefore,*

[21]printed *Nonam*, for *Novam*

says Kepler on page 82, *that according to the opinion of Copernicus this star was at the least three times higher, and so that two million one hundred and sixty thousand semidiameters of the Earth lay between it and the Earth.* For he assumes the distance of the Sun from the Earth to be 1200 terrestrial semidiameters; multiplied by 10, this gives the distance of Saturn, 12,000; this taken sixty times gives 720,000 semidiameters of the Earth; and this, finally, taken three times gives 2,160,000 semidiameters of the Earth. So he argues at the end of chapter 15. [Margin: The distance of the New Star from the Earth according to Kepler.] But in chapter 16, taking from this the occasion to determine the Copernican distances, he chooses a distance of the Sun from the Earth of 1432 terrestrial semidiameters, and of Saturn 14,320. Taking this sixty times, then, would give 859,200; and this taken three times would be 2,577,600 semidiameters of the Earth — namely the distance of the New Star required in the hypothesis of Copernicus, and by Kepler's measures, for it to show no parallax greater than two minutes. But if it is placed among the fixed stars, he assigns to it and to the fixed stars a distance from the Earth of thirty-four million seventy-seven thousand semidiameters of the Earth, and sixty-six more besides, that is, 34,077,066⅔. [Margin: The true size of the New Star, according to Kepler.] From this distance, he says, the whole sphere of the moving bodies, that is, the sphere containing everything below Saturn, appears no greater than three minutes; and therefore, if the New Star measured at least 4′, the figure he assigns to Sirius, it was of greater bulk than the whole sphere of the planetary system. So he writes on page 89.

V. I now reply to the First argument. First, I deny the minor premise, in which it was said that the star had no proper motion, nor a parallax of 52′ 30″ in longitude and 5′ in latitude. For Kepler has demonstrated neither: not the motion, as I shall show shortly in the reply to arguments 3 and 5; and as for the parallax, first, because the observations of the New Star's distances from the fixed stars vary, as is plain from the table set out under number 7 of the preceding chapter. [Margin: Reply to Argument 1.] On account of this variation, and of the fault in the Tychonic instrument, whose pinnules he admits had been dislodged, and of the dullness of his own sight, and of refraction, he does not deny that there is some uncertainty in them; for he says on page 81: *And although there is some uncertainty in the observations, for the reasons indicated*; yet from them the longitudes and latitudes of the New Star were deduced, and their differences in October and February. Second, because *Chiaramonti*, by careful demonstrations — as is quite clear to me on examining his geometrical and arithmetical procedure in Book 3 of *De stellis novis*, chapters 11 and 12 — gathered from the distances of the New Star from the fixed stars taken on 1604 October 21 and in February 1605, and finally from the distances taken by Fabricius, very various and differing longitudes and latitudes of the New Star, as may be seen in the table we drew up for this purpose in the preceding chapter, number 9. From this diversity a great uncertainty indeed arises; but if those distances are accepted, a parallax greater than 52′ ½ in longitude and 5′ in latitude follows, and indeed one so great that on Kepler's

own hypothesis the New Star would be demonstrated to have been below the Moon. For from the northern latitude of 3° 10′ 6″, which Chiaramonti derives from the distances of the New Star from the shoulder of Sagittarius and from the Knee of Ophiuchus, taken on the 21st day of October, when the New Star was near setting, and from the latitude of 0° 43′ 44″, which the same author derived from the distances of the New Star from the Heart of Scorpio and from Aquila, taken in February, when it was near rising — from those latitudes, I say, a difference of 2° 26′ 22″ is drawn; and from the longitudes in ♐ of 18° 46′ 15″ or 18° 52′ 30″, and 17° 10′ 30″, a difference of 1° 35′ 45″ or 1° 42′ is gathered. Therefore, if a difference of latitudes greater than 5′, and of longitudes greater than 52′ ½, would be an argument that the New Star was below the Moon, as Kepler asserts, then, since a much greater one is gathered from some of the distances Kepler admits — granted that it is not gathered from all of them — it follows that his hypotheses on this matter are uncertain, and that it is no more demonstrated that the star was above the Moon than below it. And so, as the truth of the calculations requires, we are compelled on this point to side with Chiaramonti (Book 3 of *De novis stellis*, chapter 14), who denies that Kepler has demonstrated a parallax of the New Star smaller than the Moon's, and who recognises a fallacy or uncertainty in his foundations.

VI. I reply likewise to the 2nd argument, denying the minor premise, with Chiaramonti above. For it is evident from the last table of the preceding chapter that from the distances of the New Star from the shoulder of Sagittarius taken in Bohemia, and from the Head of Ophiuchus taken in Frisia, the longitude derived there is ♐ 18° 30′ 30″ with a latitude of 3° 10′ 6″, but here 17° 52′ 30″ with a latitude of 1° 37′ 15″; and hence a difference in longitude of 38′ and in latitude of 1° 32′ 51″. [Margin: Reply to 2.] But if on both sides we use the head of Ophiuchus, the difference in longitude was 42′, that is, much greater than 6′ — which *Kepler* does not concede, seeing that if he conceded it the star would be placed below the Moon. But I add that it is not established that these observations were made, in both places, near the meridian and at the same moment of time, which was required, as even Kepler himself demands.

VII. I reply to the 3rd argument, again denying the minor premise: first, because the equality of the New Star's distance from the same fixed stars has not been demonstrated with respect to all observers, nor always even with respect to a few. [Margin: Reply to 3.] For how, from the observations of so few days, all of which we collected in the penultimate table of the preceding chapter, could it be affirmed with certainty that the star behaved so throughout the whole time of its duration? Second, in those very observations a variation appears that is not to be despised. So if he wished to escape the excessive parallax in longitude and latitude just mentioned, there certainly ought to have been more observations made, and the times recorded, and the vertical circles with the azimuths, so that it would be established whether the star kept an equal distance at every altitude and position.

VIII. I now reply to the 4th argument, denying the major premise, with Chiaramonti, Book 3, chapter 18. For any phenomenon, however near the Earth it may be and however great its parallax, will, if it lacks proper motion, always rise for the same observer from the same point of the horizon; for when it stands on the horizon it has the same parallax, because of its equal apparent distance from the zenith. [Margin: Reply to the 4th argument.]

IX. To the 5th argument I reply, *first*, that it is here already assumed as certain that the New Star was either in the spheres of the planets or above them; but neither is certain, and neither has yet been demonstrated. [Margin: Reply to the 5th argument.] *Second*, that it could have been in the sphere of some planet and yet have had within it a singular motion, by which its body, whether by its own epicycle or otherwise, was kept always beneath the same point of the fixed stars — especially since for Tycho the heaven is fluid; and that an argument from the ordinary motion of the planets does not hold for the motion of an extraordinary phenomenon, as is plain in the comets, some of which Kepler admits were within the region of the planets, and yet not the motion of the planet that gave them hospi- tality did they imitate, but one quite different. And so that whole argument moves within the mere bounds of probability.

X. Finally, to the 6th argument we reply, *first*, that the hypothesis of Copernicus is either false or improbable, as we shall show in Book 9, God willing. [Margin: Reply to the 6th argument.] *Second*, that, granting it, a phenomenon whose place is in doubt could be below the Moon and be carried along with the Earth, just as the Moon is carried in that hypothesis, so that it would undergo the parallaxes of the annual orb that Kepler speaks of there. In that argument he reasons about this star just as if it had already been demonstrated that it was either in the sphere of Saturn or above Saturn, when in fact neither has been demonstrated.

XI. After Kepler, Ioannes Heckius of Deventer [Deuentrienſis], Doctor of Medicine and Philosophy, in a disputation on this star published at Rome in the very year 1605, assumes as most certain a longitude for this star of 17° 41′ 30″ of Sagittarius and a northern latitude of 1° 55′ 30″; and because, when it was approaching the horizon, it was not seen in another place, nor at another distance from the fixed stars, than when it was far from the horizon and approaching the zenith, he concludes that it had no sensible parallax and was in the highest heaven. [Margin: Neither Heckius nor Biancani demonstrated that this star was in the firmament.] But we have already shown, in chapter 6 from number 3, that this argument is not demonstrative if the observations are made at one place on the Earth — indeed, not unless this has been observed on the same night at several horizons. Therefore, since Heckius brought forward only his own observations, he did not demonstrate absolutely that the star lacked all parallax sensible in itself, but only that it had none sensible at the place where he was observing. Nor did Biancani, in Book 18 of the *Sphaera*, chapter 1, where, from the fact that for 16 years it was seen in the same place beneath the fixed stars without any proper motion, he concludes that it was among the fixed stars

and stood off from the Earth at the Tychonic distance of the fixed stars, 14000 terrestrial semidiameters. For it would have to be certain, from the observations of determinate days, that on the same nights, viewed at different horizons, that is, from different places on the Earth, it kept the same place beneath the fixed stars, or the same distance from those same fixed stars.

XII. There remains Antonio Santucci, who in chapter 7 of *De Cometis* contends that this star was above the Moon, and not lower than the orb of Venus. [Margin: Santucci's argument is weighed.] His method rests on the stay of the phenomenon above the sensible, or natural, horizon, compared with the stay due to it above the rational, or astronomical, horizon. I have already given the problem for this method in Section 1 of this book, chapter 15, from number 12; and *Chiaramonti* also carries it out, in Book 3 of *De novis stellis*, chapter 30, much more clearly, fully and solidly than either *Giovanni Camillo Glorioso* — who, in Book 2 of *De Cometis*, chapters 3 and 4, restricted the problem to a phenomenon appearing only on the equator, to an observer likewise living beneath the equator — or *Santucci*, who restricted it to the case in which the phenomenon is on the equator or beyond it. But the chief point, which we too confirmed there, Chiaramonti shows in that Book 3, chapter 31: this notable theorem, namely, *When we are uncertain whether a phenomenon undergoes parallax, it is not permissible to deduce its distance from the centre of the Earth with certainty from its stay above the natural horizon.* The reason is that, to deduce this distance with certainty, the true parallel on which the phenomenon stands must be known with certainty beforehand, or (what comes to the same thing) its true declination from the equator, in the case where it is not on the equator, as this star was not; or at least the difference between the true and the apparent parallel must be known. Otherwise, since parallax apparently depresses the phenomenon onto a parallel more remote from the equator, a calculation that relies on that apparent parallel, taken as the true one, will always be fallacious, as is plain from the problem already given in Section 1, chapter 15, number 12. Therefore the parallax of altitude must be known beforehand; and once it is known, the distance of the phenomenon from the centre of the Earth is gathered much more readily by the problem already given in Book 3, chapter 7, number 3. But Santucci used the apparent declination and parallel of this star, that is, the one obtained from observation, as the true one. For, as *Chiaramonti* also reports (Book 3, chapter 32), taking the altitude of the pole at Pisa as 43° 30′, he subtracted from the altitude of the equator, 46° 30′, the meridian altitude of the New Star that he had observed, 25° 20′, and so obtained a declination of 21° 10′. [Margin: Santucci's error in misusing the apparent declination as the true one.] Then he uses the star's stay above the natural horizon observed on April 1, of $9^h\ 4^m$, and on April 14, of $9^h\ 4^m$, and on June 5, of $9^h\ 4^m$. Further, from the apparent declination and stay he gathers the true stay, that is, the diurnal arc of the star, of $9^h\ 4^m\ 40^s$; and hence that between the true and the apparent stay there was no sensible parallax. But, as *Chiaramonti* rightly insists (Book 3, chapter 33), this method is fallacious, because it uses the

apparent declination for the true one and the apparent parallel for the true one; and further, in the observed stay it takes no account of refraction, which, by raising the star, makes it rise apparently sooner and set later than it otherwise would — besides other fallacies which he detects, there and in chapter 35, in Santucci's calculation. Lastly, I note that the declination of this star observed by Santucci, 21° 10′, must be increased by three minutes, because the altitude of the equator observed at Pisa by Master Vincentius Renerius [Vincenzo Renieri], the chief mathematician there, and sent by him to me, is accurately 46° 33′, and the altitude of the pole 43° 27′; so the declination would rather have been 21° 13′. But Kepler, as we said in the preceding chapter, number 7, set it at 21° 1′ 30″; the difference between these is 11′ 30″. Santucci therefore did not demonstrate what he had promised, namely that this star was not lower than Venus — indeed, not even whether it was below the Moon.

XIII. So much for those who have tried, but in vain, to demonstrate that the New Star of the year 1604 was in the highest aether, or at least above the Moon. Now we shall see whether Chiaramonti, in Book 3 of *De stellis novis*, chapter 19, demonstrated from the observations Kepler adduces that it was below the Moon. [Margin: Chiaramonti's method and demonstration of the sublunary seat of this star is weighed.] His method, then, is this. *First*, comparing with one another the latitudes he deduced from the distances Kepler admits, which we set out in the last table of the preceding chapter, he gathers the difference of the apparent latitudes, the greatest 2° 26′ and the smaller 51′; and he assumes, from what was demonstrated in the *Antitycho*, Book 1, chapter 6, proposition 6, that the parallax of altitude is greater than the parallax of latitude when the circle of latitude is different from the circle of altitude, as in our case; and so that the parallax of altitude, from Kepler's data, is not less than 51′. *Second*, he demonstrates from John Dee's treatise on parallaxes, and from a theorem he himself demonstrated in the supplement to the *Antitycho*, part 2, chapter 35, that the sines of the complements of the altitudes have the same ratio to one another as the sines of the parallaxes of a phenomenon seen from the same place on the Earth and keeping the same distance from the centre of the Earth — the parallaxes, I mean, belonging to those altitudes. This we too, with Snel and Cysat, shall show in Book 10, the penultimate section, chapter 2. *Third*, he takes two distances of the New Star from the zenith, the smaller 56° 47′ 50″ and the greater 83° 5′ 47″; and from their sines and the parallax of 51′ he gathers, by the theorem set out above, the parallax due to the smaller distance from the zenith, 2° 1′, and that due to the greater, 2° 51′. *Fourth*, and finally, from the parallax of 2° 1′ he gathers a distance of the star from the Earth of 24 terrestrial semidiameters, and from the parallax of 2° 51′ a distance of 20 semidiameters. I pronounce these theorems and problems, together with the working of the calculations, correct, setting aside for the present the minutiae of the fractions. And so it has indeed been demonstrated by Chiaramonti, on a hypothesis, that this star was below the Moon — but not absolutely, nor on a certain hypothesis,

since other observed distances of this star show a much smaller parallax, as we saw from Kepler; and so each hypothesis is uncertain.

CONCLVSIONES.

Conclusions

I. CONCLUSION. *Neither Kepler, nor Heckius, nor Biancani, nor Santucci has absolutely demonstrated that the New Star of the year 1604 and 1605 was above the Moon, let alone among the fixed stars.* This is quite clear to anyone who weighs what we said from number 5 to 12. To me, moreover, it is not evident that anyone else has attempted to demonstrate this, although some of those already adduced in chapter 1 think it has been demonstrated universally that all New Stars were in the highest aether and far above the Moon.

II. CONCLUSION. *But neither has Chiaramonti demonstrated, either absolutely or on a certain and irrefutable hypothesis, that the New Star of the year 1604 and 1605 was below the Moon.* I showed this sufficiently a little earlier, at the end of number 13, and he himself professes nothing different.

III. CONCLUSION. *Yet on some probable hypothesis it has been demonstrated that the New Star of the year 1604 and 1605 was above the Moon, and on another that it was below the Moon.* The first part is plain from the arguments of Kepler adduced in chapter 2. 3., from number 1; the second from the demonstration of Chiaramonti set out in number 13.

IV. CONCLUSION. *Yet it is more probable that the New Star of the year 1604 and 1605 was far above the Moon than below it.* This is plain not only from the authority of more who assert it than deny it — in *Kepler*, Book 1; in *Froidmont*, Book 3 of the *Meteorologica*, chapter 2, article 7; and in *Glorioso*, Book 5 of *De Cometis*, page 275 — but also from the arguments from its colour, twinkling, light, immobility and shape, which are signs of a heavenly rather than a sub-heavenly place; but most of all from the fact that in so long a duration no one noticed that its distance from the same fixed stars varied very sensibly. It is therefore more probable that, had it been observed on the same nights from different places on the Earth, it would have appeared to each and every observer at the same distance from the same fixed stars; indeed, that the days of such observations were not recorded for this very reason, that even at a simple glance it always appeared equally distant from the same fixed stars, whether it was now grazing the horizon or was nearer the zenith.

CAPVT XVI.

Chapter XVI.

A synopsis of the opinions on the distance and size of the three New Stars which appeared in the year 1572 in Cassiopeia, in 1600 in Cygnus, and in 1604 in Serpentarius.

I. By what technique the apparent diameters of the stars are to be observed, and how the true diameter is to be gathered from their distance from the Earth

together with the apparent semidiameter, we have explained [?] sufficiently in Book 6, chapter 9, and in Book 7, Section 6, chapters 8 and 9; and we said something on this matter in Section 1 of this book, chapter 24. How, moreover, the circumference and the solidity of a star are derived from its true diameter, if it is supposed to be round and spherical, is plain from the common rules for the measurement of the circle and the sphere given in Book 1, chapters 4 and 6, and in Book 7, Section 6, chapter 8, from problem 3 to 9; from these passages taken together the reader will be able to draw the full doctrine on these matters. Here, however, I have decided to gather into one place the opinions on the distance and on the apparent and true size of the three New Stars — with the warning, however, that their authors were deceived about the apparent diameter, which they estimated as larger than was right, for the reasons adduced in Book 6, chapter 9, and Book 7, Section 6, chapter 9. The passages of the authors may be looked up again in chapter 2 for the star of the year 1572, in chapters 12 and 13 for the star of the year 1600, and in chapters 14 and 15 for the star of the year 1604. That Tycho and Reinhold make the star of the year 1572 smaller and smaller arises from his opinion: he thinks that it was seen as smaller because its body was in fact diminished, and not because it receded further and further from the Earth.

II. For the New Star of the years 1572 and 1573, then, *Tycho*, in volume 1, chapters 6 and 7, set its distance as great as that of the fixed stars, namely 14000 semidiameters of the Earth; but the *Landgrave* and *Reinhold* set it at 1062 or 1079 semidiameters — granted that the Landgrave does not deny that it could have been greater; *Peucer* and *Schuler* at 169 semidiameters; *Nolthius* at 19 or 20; and *Busch* at 30. [Margin: The distance of the star of the year 1572.] The rest set no determinate measures, but said only, indeterminately, that it was either above the orbs of the planets, like *Froidmont*, or within the orbs of the planets, or below the Moon, according to the three classes of opinions which I reviewed, with their authors, in chapter 11. But *Chiaramonti* deduces very many and differing distances from various hypotheses, which I reported in chapter 10, numbers 4, 8 and 9.

All testify that its apparent *diameter* was greatest at the beginning and afterwards decreased gradually to an insensible quantity — except *Santucci*, who said that it was smaller at first and afterwards grew, and then diminished. [Margin: Its apparent diameter.] That it was greater than Venus from the very start is affirmed by the *Landgrave* of Hesse, *Clavius* and *Mästlin*; that it was equal to Venus, by *Cornelius Gemma*, *Peucer*, *Ursinus* and *Pridianus*; and that it was smaller than Venus but greater than Jupiter, the Dog Star [Canicula] and Lyra, by *Tycho*, *Praetorius*, *Hájek*, *Fabricius*, *Maurolico*, *Apian* and *Camerarius*. Against them *Santucci* alone imagines that at the beginning it was equal to stars of the sixth magnitude, that it did not equal Venus until March of the year 1573, and that from the end of May to the end of August it was twice as great as Venus. But he is rightly believed to have imagined this in order to establish that it grew in its descent and decreased in its ascent, since so many other witnesses oppose

him. As for a determinate measure, the apparent diameter at the beginning was estimated by *Tycho* at 3′ ½, by *Nolthius* at 10′, and by *Busch* at 88′ — that is, three times greater than the Sun's diameter, unless in Busch there is an error of the printer or of the pen; the rest are silent about it.

The true *diameter* of this star was set by *Tycho*, volume 1, chapter 7, at 7 ½ terrestrial diameters at the beginning; by *Reinhold* at 3 ⅛, or 5385 German miles, but at the end at 1 ⅛ semidiameters of the Earth, or 1888 German miles; by *Nolthius*, again, at 46 German miles; and by *Busch* at 664 German miles. [Margin: Its true diameter.] But if it had been where the fixed stars are, and at least equal to Sirius, it would on our reckoning have been 34 semidiameters of the Earth, as we showed in Book 6, chapter 9, and its solidity would have been to the Earth's as 4913 to 1.

Finally, you have its *solidity* relative to the solidity of the Earth in the small table below; to which Tycho, volume 1, chapter 7, adds that at the beginning it was to the Sun as 2 ⅔ to 1. [Margin: Its solidity.]

The solidity of the star contained the globe of the Earth, times:

Authority	Period	Times	Period	Times
	In the year 1572, November	361 ½	June, July, August	11
	December	200	September, October, November	4 ½
For Tycho	1572, January	100		
	February and March	70	December, and 1573, January	1
	April and May	29	February, March	0 ⅓
For Reinhold	at the beginning	31	at the end	1 3/10
For Nolthius		0 1/50000		
For Busch		0 1/18		

III. As for the New Star that appeared at the boundary of the neck and breast of Cygnus from the year 1600 up to the year 1629, or at least 1621, no one has reported its distance from the Earth determinately; some merely contend, with Kepler, that it was among the fixed stars, but Chiaramonti that it was below the Moon, or at least that it has not been demonstrated that it was above the Moon. [Margin: The distance and size of the star of the year 1600.] Its apparent size from beginning to end was as great as in stars of the third magnitude; nor do I find anything else defined.

IV. Finally, the New Star of the year 1604 and 1605, which shone in the heel of the right foot of Ophiuchus, or Serpentarius, was seen by everyone at the beginning as greater not only than any fixed star but also than Saturn, Mars and Jupiter, to which it was very close in position throughout October; to some it seemed equal to Venus; to *Kepler* smaller than Venus in any state [omni Venere], and of 4 minutes. [Margin: The distance and size of the star of the year 1604.] Kepler places it among the fixed stars, with many others; but *Santucci* either among the fixed stars or not below the sphere of Venus; *Biancani* at a distance of 14,000 semidiameters of the Earth; whereas *Chiaramonti*, on the contrary, contends that it was below the Moon, and no higher than 20

or 24 terrestrial semidiameters. The other measures have been passed over by the authors — by Kepler, perhaps, so that its bulk should not seem incredible, given the enormous distance of the fixed stars from the Earth in the Copernican hypothesis; yet he affirms in chapter 16 that in bulk it was greater than the whole sphere of Saturn and of the planetary system. But mortal weakness labours, or plays, in vain over these numbers, since the distance and seat of these portents are uncertain — portents whose very origin and nature, too, GOD has willed to be unknown to us, as will become clear from the following chapter.

CAPVT XVII.

Chapter XVII.

On the matter, nature and generation of the New Stars.

I. In chapter 11 we reviewed three classes of authors and of opinions on the distance, or place, of the New Star of the year 1572. [Margin: Summary of the opinions.] The authors of the third class, who placed it below the Moon, mostly held that it was a meteor kindled or lit up from elementary matter, just like those comets which they call Roses. But the authors of the first and second classes, who held it as certain that it was above the Moon, then split into various opinions as to its matter, its nature, and the manner of its generation, or of its new appearance and disappearance; and the same happened over the stars of the years 1600 and 1604 — so much so that Ioannes Heckius and Fortunio Liceti collected more than 12 opinions. These are reduced, however, to 4 general heads: the novelty of these phenomena could be explained and defended either by local motion alone, of the star itself or of the heaven; or by alteration alone; or by generation and corruption; or by miraculous creation. Let us now come to the opinions one by one, and to their authors.

II. The *first* opinion is that of those who think that the New Stars were sublunary comets without a tail, generated from elementary matter, either by inflammation or by illumination. Of this kind, speaking of the star of the year 1572, were *Nolthius, Busch, Graminaeus, Ursinus, Lvovický*, and formerly *Peucer*, as reported in Tycho and in the passages reviewed in chapters 2 and 11. [Margin: 1. The opinion of sublunary matter.] [Margin: Of Nolthius. Of Busch. Of Graminaeus. Of Ursinus. Of Lvovický.] What could be said here of this opinion is what was said of comets in general in Section 1, chapters 10, 11 and 12, the questions concerning the tail excepted. But this opinion is opposed above all by the fact that it is far more probable that the New Stars of the years 1572, 1600 and 1604 were above the Moon, as we said in chapter 11, conclusion 8; chapter 13, the single conclusion; and chapter 15, conclusion 4.

III. The *second* opinion was that of *David Chytraeus*, as reported in Tycho, volume 1, page 712, who thought that the star of the year 1572 was produced in the elementary region and afterwards ascended into the heaven. [Margin: 2. The opinion of Chytraeus, of sublunary matter migrating into the heaven.] Against this ascent Ioannes Heckius protests, on page 14 of his disputation: for

if it had ascended, it would surely have done so by the force of lightness; but lightness has no place with respect to the celestial region — otherwise fire too would ascend into the heaven, or else perpetual violence would be done to it by whatever prevents its ascent. But Chytraeus is opposed more by the greater probability which, as we said against the 1st opinion, belongs to placing the new stars far above the Moon. Yet we adduced an opinion similar to this one, concerning comets, in Section 1, chapter 6, number 8.

IV. The *third* opinion was that of *Cornelius Gemma*, Book 11 of *De Characterismis Naturae*, chapter 3, and as reported in Tycho, volume 1, page 563; of *Antonio Santucci*, *De Cometis*, chapter 7; of *John Dee* the Englishman, as reported in Tycho, volume 1, page 690; of *Giuseppe Biancani*, Book 18 of the *Sphaera*, chapter 5; and of other unnamed authors, as reported in Kepler's *Optics* at the end of chapter 10. It holds that these stars were not new but ancient and perpetual, yet hidden in the deepest part of the aether and invisible because of their immense distance; that afterwards, by descending towards us, they became visible, and by ascending, invisible again. [Margin: 3. The opinion of the ascent and descent of the star.] [Margin: Of Cornelius Gemma. Of Santucci. Of John Dee. Of Biancani.] Concerning the ascent, *Elias Camerarius* also held this view, thinking that the star of the year 1572 was a celestial comet which vanished by gradually ascending, as Tycho has it, volume 1, page 695. [Margin: Of Camerarius.] Fracastoro says the same of comets, as do many others, ancient and more recent, whom we named in Section 1, chapter 6, number 4. Gemma adds, moreover, that the star of the year 1572, had it been observed, could have been seen in March of the year 1572, but of tiny size; it was not noticed, however, until November, when by its size it struck the eyes of mortals. So too Santucci says that in the year 1572 it was at first of the 6th magnitude; that afterwards, at the end of May of the year 1573, it became twice as great as Venus; and that at the end, in the year 1574, it vanished again through its own faintness. This opinion is supported by proposition 132 of Book 4 of Witelo; and it is well known in optics that nearer things are seen as larger, while things that are moved further away are seen as smaller.

V. But this opinion, though at first sight it has an air of likelihood, seems to be opposed by many things. [Margin: Objections against this opinion.] First, the star of the year 1600 appeared of the same size from beginning to end, whereas the stars of the years 1572 and 1604 appeared greatest straightway from the beginning; nor was the star of the year 1604 diminished gradually down to an insensible quantity, if we are to believe the many trustworthy witnesses whom I reviewed in the preceding chapter and in chapters 2 and 14. So the increase in apparent size is rightly judged to have been invented by Gemma and by Santucci to establish this opinion. If you grant that the stars appeared greatest straightway from the beginning because of the motion of a very swift descent within a single night, but that they were slower in their ascent, you will have invented a motion in the heaven that is hardly probable. Next, as Hechius says on page 17, either that motion was rectilinear — and such motion

is not to be attributed to celestial bodies without necessity — or it was circular, and through it the New Star would have been seen to vary its distance from the same fixed stars notably; whereas we have said it is more probable that none of the three new stars we are dealing with here varied its distance from the same fixed stars.

But the chief objection is Tycho's argument in volume 1 of the *Progymnasmata*, pages 332, 532, 563 and 691. For since the New Star of the year 1572 appeared to him at the beginning as of 3 ½ minutes, and stood 14000 semidiameters of the Earth from the Earth, then, if it had to diminish by ascending until it became equal to the stars of the sixth magnitude, it must have been moved away from the Earth by at least 300000 semidiameters of the Earth — a distance which he himself considers false and absurd. [Margin: Too great a distance of the fixed stars, if the New Star escaped by ascent.] *Kepler*, however, in his book on the New Star in Serpentarius, chapter 21, asking whether it withdrew into the depth of the aether, digresses to a question not necessary there: whether the sphere of the fixed stars is infinite. For however great it is set, whether by the Copernicans or by the defenders of the opinion we are now discussing, it will still always be set as finite; or, being finite, it will suffice to save all the phenomena whatever. What Tycho said about 300,000 semidiameters of the Earth, however, needs some correction; nor is it impossible or absurd, except ad hominem, against those who do not admit a distance of any fixed stars greater than about 14,000 or 20,000 semidiameters of the Earth.

I said that it needs correction because, according to Tycho, the apparent diameter of the New Star was smaller than Venus and a little greater than the diameter of Jupiter; and this diameter, viewed through the telescope, is no greater than 1′ 9″, as we showed in Book 7, Section 6, chapter 9, and its semidiameter is 35″. A star of the sixth magnitude, moreover, has a diameter of 4″ 24‴; but let 4″ be taken now, and a semidiameter of 2″. Now if we use the Tychonic distance of 14,000 semidiameters of the Earth, and problem 1, set out in Book 7, Section 7, chapter 8, the true semidiameter of the New Star will come out as 2 ½ terrestrial semidiameters. With this, and with the apparent semidiameter of 2 seconds, by no. 4 of our plane right-angled triangles, or by Cavalieri's, a distance of 245,000 terrestrial semidiameters is found. The thickness of the heaven of the fixed stars would therefore have had to be at least 225,000 semidiameters of the Earth, for the New Star, equal to Jupiter at the concave surface of that heaven, to appear equal to the smallest stars of the sixth magnitude at its convex surface. But if we used our own distance of the fixed stars, set in Book 6, chapter 7, at a minimum of 100,000 semidiameters of the Earth, and the star is supposed to have appeared at first at least of the size of Sirius, the true semidiameter of the New Star would have been 17 terrestrial semidiameters; and for the star's diameter to appear as 4 seconds in its ascent, it would have had to be raised as far as 1,750,000 semidiameters of the Earth. This is indeed a finite distance, but it will seem probable to few, since the heaven of the fixed stars would be set thicker than the sphere of all the planets more

than 17 times over. There will be some, however, to whom, considering the multitude of the fixed stars, so great an extent of region will not seem unfitting for so great a population. Finally, with the distance of the New Star, when it appeared equal to Jupiter, set at 100,000 terrestrial semidiameters, its solidity would come out to the solidity of the Earth as 4913 to 1. Yet one ought not to give the heaven of the fixed stars so great a thickness for the sake of the single New Star of the year 1572, if its increase and decrease can be explained otherwise than by descent and ascent.

VI. The *fourth* opinion counts the New Stars among those created of old, or at the beginning of the world, and known long since, but as seeming new because of an optical enlargement produced by vapours. [Margin: 4. The opinion numbering the New Stars among the ancient ones.] So *Annibal Raymundus* [Annibale Raimondo], astrologer of Ve- rona, and formerly *Reisacher*, thought that the New Star of the year 1572 was the eleventh star of Cassiopeia, which shines in the middle of the Chair, and that in the year 1572 it appeared larger and more brilliant because of thick and dense vapours and exhalations interposed between it and the sight of the observers, in which its rays were refracted so as to enlarge its appearance. [Margin: Of Raymundus. Of Reisacher. Of Frangipanus.] *Cornelius Frangipanus*, for his part, says that it was the eleventh star of Cassiopeia, which moved away from the little star nearest to it and enlarged its light, so that it was seen as new partly through motion and partly through alteration of its light. But all the other astronomers affirm that the New Star was distinct from the eleventh of Cassiopeia, although Tycho admits that towards the end, when it appeared smaller and less brilliant, it appeared better distinguished from that star. Yet no probable reason can be given through natural causes why at that time, for a year and a half, so great an abundance and so dense a mass of exhalations should have been beneath the eleventh star of Cassiopeia alone, and should not have been dispersed or carried elsewhere; or why, if that star had a peculiar power of attracting exhalations, it did not attract them at other times, and why many other fixed stars do not have that power.

Moreover, Giovanni Battista Benedetti, in a letter to the Raymundus already mentioned, written in the year 1573 and found among his works on page 371, contends as follows. If that exhalation had been at the concave surface of the Moon, 33 terrestrial semidiameters from the Earth, and the New Star had been seen at so great a size over the whole terrestrial hemisphere through the force of refraction occurring in that exhalation, then the exhalation would have had to occupy as much space in breadth as the Earth's diameter occupies. Given this, it would follow that, for eyes placed anywhere within that hemisphere of the Earth (whose pole would lie beneath the New Star, as beneath the zenith), that exhalation would subtend 6° 58′ in the heaven, and so would likewise enlarge all the stars that are no further than 3° 29′ from Cassiopeia; and since it did not do so, the opinion of Raymundus is false. But if the exhalation is supposed to be only 10 terrestrial semidiameters distant, he shows that it would occupy 23° 4′, and many more degrees if it had been in the highest region of the air. Yet

Benedetti went wrong against geometry on some points, as *Chiaramonti* shows in Book 2 of *De stellis novis*, in chapter 7 of the appendix. [Margin: Errors in geometry of Giovanni Battista Benedetti, noted by Chiaramonti.] He adds that if the exhalation had stood 10 semidiameters of the Earth from the Earth, and had occupied as much space as the terrestrial chord of the arc between the most northerly observer, Tycho, and the most southerly, Muñoz, it would measure only 160 German miles, and would occupy no more than 16′ in the heaven. But it is not worth dwelling on these things.

VII. The *fifth* opinion was that of *Franciscus Vallesius*, *De sacra philosophia*, chapter 1. On the one hand he grants that the star lacked all parallax and was therefore in the firmament; on the other he denies that any star is created anew by God. He gives three reasons: first, because, as is said in Ecclesiasticus 18, *He who lives for ever created all things together*; second, because the heaven would have been imperfect had it lacked this star, against that saying of Genesis 2, *Thus the heavens and the earth were finished, and all their adornment, and GOD completed his work on day 4*; and finally, because this star disappeared, so that, if it had been created anew when it appeared, it would have been annihilated when it disappeared — whereas it is said in Ecclesiasticus 3, *I have learned that all the works of GOD endure for ever*. [Margin: 5. The opinion of Vallesius, numbering the New Stars among the ancient ones and attributing their appearance to denser parts of the heaven.] Accordingly Vallesius affirms that the New Star of the year 1572 (and he would have said the same of the other New Stars) was created together with the rest from the beginning, but did not appear until the year 1572, because denser parts of the heaven passed beneath it, by which its size was optically increased. But so slow a motion of some heaven or other is hardly probable. Next, if that density is inherent in that heaven, why did it not, before and after, enlarge other stars close in position to that New Star? Or how did that denser part of the heaven last so long beneath the New Star of Cygnus? Nor is it necessary that, if GOD created all things in the heaven (for beneath the heaven he creates rational souls daily) by creation taken in the stricter sense, that is, from no pre-existing subject, he can create nothing further anew in the broad sense, that is, from some matter — or even in the strict sense, but as a work belonging to his extraordinary Providence and to miracles. Nor did the adornment of the heavens, according to ordinary Providence and the end intended by GOD, require this star for its completion, just as the earth did not require new rivers or monsters. Finally, it would follow from Vallesius's teaching that not even the Star of the Magi could have been made anew and placed in the heaven (granted that it is in fact thought to have been in the air), and at last extinguished — which is not to be denied to the divine Omnipotence. But that particle *simul*, "together", in Ecclesiasticus 18 does not necessarily signify sameness of the moment at which whatever has been created was created; it can signify the sameness of a creation made in time by one and the same Creator. So, as the Lateran Council explains in the chapter *Firmiter*, GOD at once, from the beginning of time, made from nothing both

kinds of creature, the corporeal and the incorporeal, namely the angelic and the worldly; and he did not create one in time and the other from eternity, nor one himself immediately — the angelic, say — and the other through the mediation of angels.

VIII. The *sixth* opinion was that of *David Fabricius* and others, as reported in Kepler's book on the New Star in Serpentarius, chapter 20: that the New Stars, as to substance and body, were created in the heaven from the beginning, but lay hidden until it pleased GOD to illuminate them anew and through them to signify something to the world — in much the same way as the Moon lies hidden at the interlunium and afterwards is illuminated towards us by the Sun. For since they were among the fixed stars, it is consistent that they were as ancient as the fixed stars as regards their body. [Margin: 6. The opinion of Fabricius, counting the New Stars among the ancient ones and ascribing to them only a new light.] And Fabricius adds that the gap, or hollow, which Tycho (volume 1, page 795) says he saw in the place where the New Star of Cassiopeia had been, after it was extinguished, was nothing other than the body of the star, now deprived of light. But if the New Stars must have been coeval with the other fixed stars as to the substance of their body, because they were in the sphere of the fixed stars, they ought also to have shone together with them, to have been brilliant from the beginning, and not to have lacked the chief accidents of stars. Or if their illumination was deferred in order to signify something extraordinary, why could not their creation or generation also have been deferred, so that they should not be idle in the heaven for so long? Of that gap — of Tycho's mouth rather than of the heaven — we shall speak below.

IX. The *seventh* opinion was that of *Ioannes Heckius* of Deventer, physician and philosopher, in his disputation on the New Star of the year 1604, dedicated to Master Federico Cesi, Marquis of Monticelli. [Margin: 7. The opinion of Heckius, associating the New Stars with the ancient ones and attributing their new appearance to rarer parts of the heaven.] This doctor affirmed that above the firmament there is another heaven; that every heaven, by Aristotle's pronouncement, exists for the sake of a star, and therefore that heaven too exists for the sake of a star and has its own stars created from the beginning, which are in the tract beneath which the Milky Way lies — because all the New Stars, namely those of the years 1572, 1600 and 1604, were seen in the Milky Way; and that those stars of the highest heaven are nevertheless not seen except when the rarer parts of the Milky Way run beneath them, since the denser parts prevent their being seen. But if this doctor thought that the heaven above the firmament is the crystalline or the Empyrean, Aristotle is not to be understood as having spoken of it. Secondly, not all the New Stars were seen in the Milky Way, if what is related is true: of the New Star seen in the year 1596 by Fabricius at 25° 45′ of Aries, with south latitude 15° 54′, a place far from the Milky Way; of the New one seen in the year 1612 by Simon Marius [Simon Mayr] in the girdle of Andromeda; and of the one seen in the year 1602 by Kepler in Pisces, outside the Milky Way. [Margin: Not all the New Stars

in the Milky Way.] Besides this, a motion of the firmament, or eighth sphere, beneath the ninth is invented without necessity, by which the rarer and denser parts of the Milky Way are to run alternately beneath those hidden stars; for this motion would have to follow the course of the Milky Way, obliquely from north towards south. Lastly, even if such a motion were granted, either those stars which are called New would be too few, or the rarer parts of the Milky Way would be too few in proportion to the multitude of the dense ones.

X. The *eighth* opinion is that of *Ludovicus Columbius* [Lodovico delle Colombe], in his book on the New Star of the year 1604, who thinks that the New Stars are in the tenth heaven above the firmament, and are seen when the denser parts of the starless ninth heaven pass beneath them. [Margin: 8. The opinion of Columbius, attributing to density what the 7th attributes to rarity.] Against him what we said against Heckius holds good; and Alimbertus Maurus attacked him.

XI. The *ninth* opinion was that of *René*, or *Reynatus*, *Descartes*, in Part 3 of his *Principles of Philosophy*, from marginal number 111. [Margin: 9. The opinion of René, attributing the appearance of New Stars to intervening atoms.] Since he explains the various changes of nature partly through the annual motion of the Earth and partly through the motion of atoms and corpuscles brought about by various vortices, he thinks that at times certain corpuscles are interposed between the stars and our eyes, blocking their rays so that they cannot move the eyes; but that at other times those corpuscles are driven off by the force of the vortices and the obstacle is removed, and that in this way, for us, New

Stars appear, and afterwards vanish from sight. But besides much in his philosophy that is a novelty of little solidity, it does not seem probable that, in that continual fluctuation of the vortices by which the atoms are moved, new stars should appear so rarely from the atoms' running between, or that the New Star of the year 1600 should have been seen for almost 30 years always of the same size. Nor, finally, do the vicissitudes of that tumultuous motion seem to have any cause except chance.

XII. The *tenth* opinion, indicated rather than asserted by *Maurolico*, as reported in Clavius on the *Sphere*, page 194, was that the New Stars are comets, but made above the Moon, in the heaven, from an aggregation of splendour from the planets and the other stars. [Margin: 10. The opinion of Maurolico, imagining the New Stars from an aggregation of splendour.] But since he does not explain whether the matter or subject of the splendour is new or old, and for what reason either that matter is so rarely gathered together, or the splendour from the stars is aggregated in it; nor the manner of this aggregation — this opinion must be left among the imperfect conceptions of the imagination.

XIII. The *eleventh* opinion was that of those who said that both New Stars and celestial comets are generated anew in the heaven, either by GOD, or by some secondary cause, or by a concurrence of secondary causes that is rare indeed but nevertheless ordained by GOD — and that from celestial matter. [Margin: 11. The opinion admitting the generation of New Stars in the heaven.] This our

Clavius admits above all, on the *Sphere* of Sacrobosco, page 193, granted that he also admits that the star could have been procreated by GOD. I should like to set down here the words of this truly prudent man: [Margin: The twofold opinion of Clavius.] *Since this is so, I am persuaded that that star of the year 1572 either was procreated then by GOD, Best and Greatest, in the eighth heaven, to portend something great (of what kind is still unknown), or else that comets can certainly be generated in the heaven itself, as in the air, although this happens more rarely. This indeed not a few of the ancient philosophers openly acknowledge, and many of the more recent ones adduce several authorities and histories by which to persuade us that stars of this kind have quite often arisen in the heaven, at long intervals of time, some to signify one thing and some another. If this is true, let the Peripatetics see how they can defend Aristotle's opinion on the matter of the heaven. For it will perhaps have to be said that the heaven is not some fifth essence but a mutable body, although less corruptible than these lower bodies — which indeed Plato held before Aristotle, with many other philosophers, and which after CHRIST not a few have taught, not obscurely: among them St Ambrose, Basil, Gregory of Nyssa, and almost all the other lights of the Church.*

Joining Clavius is *Johannes Praetorius*, who held both the comet of the year 1577 and the New Star of the year 1572 to be among the celestial comets, generated in the heaven, as did *Hájek*, *Muñoz* and *Philipp Apian*, as reported in Tycho, volume 1, pages 511, 571 and 643; but Apian thinks that the star was illuminated by the Sun. [Margin: The opinion of Praetorius, Hájek, Apian, Argoli.] Again, the New Stars were placed among the comets generated in the heaven by *Argoli*, chapter 62 of the *Pandosion Sphaericum*; by *Guntherus*, *Mollerus*, *Butenerus* and *Krabbus*, as reported in Kepler, chapter 25; and by *Liceti*, Book 5 of *De novis astris*, chapter 41, and Book 2 *De caelo*, chapter 7 — so much so that he would have it that even Aristotle does not oppose New Stars made in the heaven, because of that quasi-axiom of the Philosopher, *The stars are made of the body in which they have their motion*; and chapter 8, where he says, *That the stars are spherical is consistent for one who generates them from such a body*. [Margin: Of Liceti.] But in truth Aristotle is speaking of the ancient, and indeed eternal, stars. *Froidmont* too, Book 3 of the *Meteorologica*, chapter 2, article 7, thinks that new stars are made from celestial exhalations of the planets, and perhaps of the fixed stars; and these exhalations, being above Saturn, are illuminated by the nearer fixed stars and not only by the planets. [Margin: Of Froidmont.] *Elias Camerarius* counts the New Star of the year 1572 among the celestial comets, but says that it vanished by ascent, so that he seems rather to belong to the 3rd opinion. [Margin: Of Camerarius.] *Guillaume Postel* thinks that the New Star of the year 1572 was generated in the heaven, and refers its efficient cause to the Great Conjunction of Saturn and Jupiter, because this star shone in the equinoctial colure, that is, at the end of Pisces and the beginning of Aries, where the transition is made from the watery triplicity to the fiery, and because it fell in the seventh period of the great conjunctions; on these conjunctions see what we said in Book 7, Section 5, chapters 9 and 10. [Margin:

Of Postel.] Finally *Kepler*, in his book on the New Star of the year 1604, chapter 22, having rejected the 3rd opinion because of the excessive distance which the ascent and descent of the New Stars would require, determined that the matter of the New Stars is celestial and can be generated not only in the Milky Way but in any part of the heaven. [Margin: Of Kepler.] And in chapter 23 he teaches at length that the matter of the heaven is alterable, and that Aristotle would not have dissented: for he set the heaven down as unalterable for this reason, that it was established that in so many ages nothing new had been seen in it; he would therefore have thought otherwise had he seen or heard of that prodigious darkness of the year 1547, by whose force, on the testimony of both Gemmas, the Sun appeared blood-red for three or four days to the Belgians, Britons, French and Germans, as Scaliger confirms in *De emendatione temporum*, Book 5, and before him Bünting in his *Chronicle*. [Margin: Arguments for mutability and alteration in the heaven [?].] This darkness was so great that some stars of the heaven appeared at midday; and like it was the darkness which historians relate was seen for almost the whole year of the murder of Gaius Julius Caesar, and which is sung by Virgil and Lucan. For since the Sun was seen like this at setting and at rising, and in regions so far apart, and yet that matter occupied no other part of the heaven than the part beneath the Sun, it had almost the same parallax as the Sun, and was in the heaven.

Kepler adds those heads of comets which he thinks were demonstrated by parallax to have been in the heaven; but for that, a great probability at least suffices. He could also have added the argument from the matter of the sunspots, but he did not yet know the use of the telescope when he was writing this. The rest of what this author said in chapter 24 and elsewhere — about vapours and exhalations breathed out from the planets and at last collected into one by that celestial faculty whose task is to form that excrementitious matter into celestial monsters, so that, once the matter is at last consumed, the heaven may be purged of its dregs and keep its transparency — I have already reported in Section 1, chapter 6, number 10, and chapter 13. Kepler goes on, and in chapter 25 teaches that the New Stars, which lacked hair, tail and beard, ought not to be called comets — granted that many have counted them among the celestial comets — but stars. For stars seem to be named from the verb [Greek: στιλβεῖν (stilbein), glossed in the text as *scintillare*, "to twinkle"], and the New Stars, especially those of the years 1572 and 1504, twinkled vehemently; moreover, they were not movable by a motion of their own, and flashed not with a pale light but with a lively one; and to invent a tail for them, but one not seen because of their distance, is to rely not on the senses but on uncertain reason or imagination. [Margin: Whether New Stars can be called comets.] With this premised, Kepler inquires in chapters 26 and 27 whether the New Star of Serpentarius coincided by chance with the great conjunction, and how it can be called the star's efficient cause. He assumes, moreover, that the conjunction of Saturn and Jupiter took place in the same year; but we shall treat this question in the following chapter. I note only, with Giovanni Camillo Glorioso, that if

the New Stars were made from exhalations and vapours of the planets or of the fixed stars, kindled or illuminated, it would seem consistent that they should be carried this way and that with a wandering motion; and therefore, since they lacked a motion of their own, this motion of generation does not seem very probable. [Margin: Objection against [?] the generation of New Stars from exhalations of the heaven [?].]

XIV. The *twelfth* opinion was that of *Tycho*, volume 1 of the *Progymnasmata*, from page 794; and with him agree *Baranzano* in the *Uranoscopia* and *Giovanni Camillo Glorioso*, Book 5 of *De Cometis*, last chapter, page 281. It holds that the New Stars are made anew from the same celestial matter from which the fixed stars were once made, by condensation of its parts, especially from the Milky Way, where the matter is more nearly disposed to these novelties; and that they shine partly from their own transparency, having acquired intensity from that compaction, and partly from the Sun and the other nearer stars. [Margin: 12. The opinion [?] of Tycho and of Glorioso, resorting to condensation [?].] But because they do not have as great a perfection, nor as solid a cohesion, as the other fixed stars had, their structure was gradually dissolved and their light vanished. Tycho adds, on page 795, that on the border of the Milky Way, where the New Star of the year 1572 had shone, he noticed after its extinction, on winter nights, a certain gap and hollow about the size of a half-full Moon, as if that portion of matter, gathered into the body of the new star, had afterwards burned up, or, being dispersed, had left the trace of that gap. But this will rather have been an imagination or a deception of sight, which takes for cavities places surrounded by splendour but dark in themselves. Although this opinion agrees with the immobility of the New Stars, it does not give a probable reason why, in so vast an extent of the Milky Way, so nearly disposed to forming new stars, New Stars are nevertheless made in the heaven so rarely. And if Tycho has recourse to GOD as a cause operating outside the order of nature, it is just as easy for God to form these splendid portents by the creation of a whole star, or by the generation of a new substantial form with its accidents, as by condensation. Tycho, however, did not yet know that other New Stars would appear outside the Milky Way; and so he settled on the Milky Way, as on suitable matter, from which New Stars could be assembled.

XV. The *thirteenth* opinion — one I have so far found in no one, but have devised for myself as not altogether improbable nor unworthy of being reported, though I would not wish it to be taken as fully asserted — is this. The New Stars were created by God in the Firmament from the very foundation of the world, but among them were some not luminous on every side; in this they were like the planets, which shine from the Sun only on the side the Sun faces — granted that the stars we call New had a light of their own, in themselves and not from the Sun. So they had one half of the globe luminous and one half dark. When the time came at which God wills to raise and rouse the eyes of mortals to the heaven by these extraordinary signs, either some Intelligence, or a faculty implanted in the star, or God himself turns round with a sudden rotation the

part of the globe that had previously shone upward towards the Empyrean, so that it shines for the eyes of mortals for as long as he wills. Afterwards, either by a similar turning of the globe it suddenly withdraws all its light from men's eyes, or it steals that light away gradually, as happens in the waning Moon, until it seems to be entirely extinguished. [Margin: [?]. Opinion [?] … of the globe … splen[?] … New … stars … [?]ribing.] The vehement scintillation seen in some of them can be attributed not, indeed, to a complete rotation of these stars, but to a certain oscillation or vibration of the globe about its own centre; and if any appeared more splendid than other fixed stars, this can be put down to light doubled, or concentrated into half of the globe, which would otherwise have been dispersed over the whole globe. The remaining fixed stars, I judge it more probable, shine on every side, so that they may shine also for the eyes of the blessed, and *shine with gladness for Christ, who made them*, together with the Father of lights — for so may I borrow the words of the prophet *Baruch*, chapter 3, and turn to Christ God and his saints what Baruch referred to God not yet clothed in the eyes of flesh he was to assume. This opinion becomes highly probable to me — not because it is mine, but because it introduces the least possible change in the heaven, which I think is what should be done so long as no necessity constrains us to anything else. For here there is need of neither creation, nor generation, nor alteration, nor of a motion of the whole machine of any heaven, or of the star itself up or down, but only of the simplest turning of the star alone about its own centre. To make these stars coalesce out of many tiny stars, or to say that they are parts of the Empyrean which God willed at some time to display to mortals through openings in the lower heaven, as a sample of that splendid region, does not much commend itself to me, since the New Star of Cygnus was of the 3rd rank and not suited to providing that sample; [Margin: [?] other [?] in [?] [?].] and that assemblage of little stars or corpuscles is too laborious, nor does it yet receive the testimony of the telescope, as it does in certain comets.

XVI. The *fourteenth* opinion has recourse to God himself, who produced the New Stars outside the order of nature, either creating them from nothing or making them from whatever matter he pleased; it does not express the manner of this operation, as being hidden from us and uncertain. [Margin: 1[?]. Opinion [?] … [?]tion … God … [?].] The chief authors of this opinion were *Clavius, the Coimbra commentators, the Landgrave of Hesse, Mästlin, Bartholomäus Reisacher, Schuler, Reinhold, Rosa*, whose words and passages we shall adduce shortly, in the following chapter, number 5, where the efficient cause will be discussed. Rosa adds, moreover, from the Talmudists, that portents of this kind have regularly appeared by divine agency every two thousand years: one New Star appeared at the time of the Flood, another before the coming of Christ, and the last in the year 1572. But the history of so many other new stars, related in chapter 1, does not agree with these Talmudic fictions. Finally, when *Ioannes Heckius*, in his disputation on the New Star of the year 1604, page 15, had said:

The fourth view is that of those who say that these stars were created by God by a miracle,

he immediately added:

Of this opinion is the greatest part of the philosophers and mathematicians.

Why not of the theologians too?

CAPVT XVIII.

Chapter XVIII.

On the Efficient and Final Cause of the New Stars.

I. The opinions on the efficient cause of the New Stars, and the causes themselves, are reduced to three heads: namely, to the wandering or the fixed stars; or to a particular power and faculty in the heaven or below the heaven, whose proper function is to produce comets and New Stars; or to God, who produces these portents of the heaven either by himself or through the Intelligences. But among the opinions that have recourse to the ancient stars, as the agents by which the New ones are made or illuminated, none is more celebrated than the one that ascribes this to the greatest or great conjunctions of two or three planets; and so we shall begin from it.

II. The *first* opinion, then, is that of the astrologers who, following the tenets of the Arabs, attribute the New Stars to the Greatest or Great Conjunctions of Saturn and Jupiter, or of Saturn and Mars, or of Jupiter and Mars. [Margin: 1. The opinion of the Great Conjunctions.] By what conditions the Greatest and Great conjunctions are distinguished from each other, and how long their period and apocatastasis [apocataſtaſis] are, we have set out fully in Book 7, Section 5, chapters 9 and 10, and to that passage we refer the reader. Of this opinion was *Guillaume Postel* the Frenchman, who also noted that the Star of the year 1572 fell in the seventh period of the Greatest conjunctions, whose trigon makes its passage from the watery to the fiery signs, and that it lay on the border of Pisces and Aries. Likewise *Mollerus and Krabbus*, as Kepler reports in his book on the New Star of the year 1604, chapter 26: having predicted that a comet would arise in the month of October of the year 1604, they contended with all their might that the New Star of that year was generated by the Greatest Conjunction. For in the year 1603 the Great Conjunction of Saturn and Jupiter in 8° of Sagittarius fell, from Kepler's observations, on December 17, a little after noon — granted that in his Ephemerides Lvovický gave it for noon of December 29, Magini from the Prutenic Tables for the 24th, and Tycho's calculation, from his own papers, for the 19th. And in the year 1604, on September 26, the conjunction of Saturn and Mars took place after 8^h, in 10° of Sagittarius; and on October 9, at 1^h before noon, there was the conjunction of Jupiter and Mars in 19° 14′ of Sagittarius, which completed the Great Conjunction. So *Kepler* has it, from observations, in his book on the New Star of Serpentarius, chapter 11. Therefore, since it was near the beginning of Sagittarius, and seven periods of these conjunctions have been completed at the beginning of Sagittarius,

assigning about 800 years to each, the rising of this star is rightly said to have fallen in the seventh period of the Great Conjunctions held in the fiery signs. And what was singular to this star is the very place in the heaven in which the superior planets were conjoined; for the New Star was in Sagittarius 8°, which seemed truly marvellous and ominous.

III. But see how many things stand against the opinion of the Arabs! [Margin: 1. Objection against the force of the Great Conjunction.] First: how many Great or Greatest conjunctions have gone before without New Stars? Go over their periods, and then the events, which we treated in Book 7, Section 5, chapters 9 and 10: you will see a single example of a Greatest conjunction, in Sagittarius in the year 1604; and at the beginning of Aries, not indeed in the year 1572 but in the year 1583 — a succession of the conjunction 10 years after the rising of the New Star, which shows that the star was independent of it. As for what Kepler devises about the star of the Magi, it will be refuted below, in chapter 20. Nor is the outcome of a single star or comet of the year 1604, answering to the astrologers' prediction, enough to establish a sure law; for *Ioannes Vueidnerus*, a physician, replied in those learned verses which *Kepler* reports in chapter 27:

But if some idle astrologaster should contend, with mouth wide open, that the conjoined stars of the two planets, Jupiter and Mars, begot a beardless comet, and that it was foretold by him, we shall set against him a hundred meetings of the wandering stars that begot no comet, and that he spoke true by chance, having told a thousand lies.

Besides this, the Arabs bid us, so that a comet or a new star about to arise can be predicted from the Great Conjunctions, to consider the degree of the zodiac in which the conjunction takes place, and the exact day. [Margin: The vanity of astrology.] But meanwhile the astronomical tables have not yet received so final a polish that we can be certain of the day of the greatest conjunctions of Saturn and Jupiter, because of the slowness of the motion of both; and in the Great Conjunction of the year 1603 the difference among the authors of the tables was 13 days. Moreover, when one observes without a telescope, the disc of each planet looks larger, and for this reason, combined with the slowness of their motions, they seem to stay in conjunction for one whole day. Further, if the New Stars come into being in the Firmament, the aspect we are speaking of is not such absolutely, nor with respect to the fixed stars, but only relatively to us, for whom the circle of the zodiac is divided into the various aspects. For with respect to some fixed stars Saturn and Jupiter are perpetually in conjunction, and wherever they are, a straight line drawn through them falls on some point of the Firmament; so New Stars would have to come into being perpetually. Lastly, both the huge distance and the bulk of the New Stars make this opinion improbable. For if they are placed in the Firmament and yet appear no smaller than Jupiter, they are in reality much larger than Jupiter; how then can Jupiter and Saturn generate a star of such bulk, much larger than either of them? Or if they can, why so far away rather than near? For this reason, then, *Kepler*, in chapter 27, denies that the New Stars are of

themselves the offspring of conjunctions of the planets; although he argues learnedly against the Epicureans, who ascribe such effects to fortuitous chance, and teaches that the New Star of the year 1604 did not fall by chance into the conjunction and gathering of the three superior planets, and that it is not safe, with the Epicureans or the atheists, to belittle divine portents and drive all fear of the Deity out of human minds.

IV. The *second* opinion is that of those who attribute the making of the New Stars to some more secret Nature, or to a faculty residing in the heavens. [Margin: 2. The opinion of a hidden Nature and Faculty.] But this opinion scarcely explains the manner. Natural philosophers have been accustomed to regard as natural works the things that happen always or for the most part, and are compelled to ascribe the rest either to Chance or to Fortune or — if they wish to speak soberly — to God working beyond nature. And not even Kepler himself, who in chapter 24 had hinted at a certain faculty of a quasi-vegetative soul of the heavens, which preserves their transparency for the heavens and gathers the vapours and soots of the planets so that, kindled and consumed, they leave the heaven purged, afterwards rested in it, in chapter 27; rather, he referred the work of any more secret nature to God. Since, I say, these things are so, I do not care to wander any longer through uncertainties, but to pass to the third kind of cause and the third class of opinions.

V. The *third* opinion, then, is that of those who attribute the novelty of the stars — whether as to substance, or as to the accidents of illumination or of motion by virtue of which the New ones appeared — to God himself, working as above or beyond nature, either by himself alone or through the ministry of the Intelligences. [Margin: 3. The opinion that takes refuge in God.] So *Clavius*, in the *Sphere*, page 191, on the star of the year 1572, put this one first of two opinions, in these words: [Margin: Of Clavius.]

Since these things are so, I am persuaded that that star was either then created by God, Best and Greatest, in the eighth heaven, to portend something great, of what kind is still unknown.

So the *Landgrave of Hesse*, in Tycho, volume 1, page 600, and *Hájek* in chapter 4 of his *Dialexis*, and *Wolfgang Schuler*, in Tycho, volume 1, page 635, said that the same star of the year 1572 was *placed in the aether by God outside the order of nature; for he is not bound to secondary causes*, so as to be unable at times to work something beyond what they require. [Margin: Of the Landgrave. Of Hájek. Of Schuler.] So *Bartholomäus Reisacher*, in a writing of his, ascribed that same star to *the hidden counsel of God, or to an Intelligence governing the celestial spheres*. [Margin: Reisacher.] So *Mästlin*, in a short writing on the Star of the same year, determined that *its cause was hyperphysical, and that it was created by God in these last times miraculously, and beyond human comprehension*. [Margin: Mästlin.] So, in Tycho, volume 1, page 700, *Erasmus Reinhold the Younger* likewise affirms that the Star of the year 1572 was *a new work of God beyond the order of Nature, and not to be ascribed to astrological rules*. [Margin: Reinhold.] So too, concerning the New Star of the year 1604 and all the New ones, the

greatest part of the philosophers and mathematicians — as *Ioannes Heckius* attests on page 15 — hold that they were *created by God by a miracle*. So likewise *Tycho*, volume 1, page 799, acknowledges no other cause of the New Star *beyond the first and universal cause of causes*; [Margin: Most [?]. Tycho.] and *Kepler*, in his book on the New Star of Serpentarius, chapter [?] 27, after searching out its cause long and hard with marvellous anxiety, could at last rest neither in the Great Conjunction of the planets, nor in chance, nor in Nature alone; and so he concluded in these words: [Margin: Kepler.]

Since an apparent conjunction of the planets is at hand, which offers itself as the cause, this other thing must therefore be said, which I pronounce most securely and with full confidence: that this new celestial prodigy was associated by Almighty God himself with the three planets Saturn, Jupiter and Mars, then in conjunction, by a definite counsel directed to the salvation of men. To mark, with this as it were monument, the place and time of the Great Conjunction of the three superior planets, for a perpetual memory of the event, and to admonish the human race concerning the greatest matters, he needed no care and no labour to create such a thing — whether he used Nature as his minister, or put forth this as it were a ray of his extraordinary Omnipotence.

Finally, the *Coimbra commentators* taught this very thing, Book 1 *De Caelo*, chapter 3, question 1, article 4. [Margin: The Coimbra commentators.] We have seen, moreover, many others having recourse to God as the efficient cause of comets, as is clear from Section 1 of this book, chapter 6, number 12; they would doubtless have done the same for the New Stars, whose duration, form and place in the heaven, more probably asserted, make such prodigies more wonderful. Nor is this to summon, as they say, a god from the machine [Greek: θεὸν ἔκ μηχανῆς (theon ek mēchanēs, "a god from the machine")], as though from inexperience in untying the knot. For the cause of these effects, which transcend the order of Nature by such novelty, is not to be invented from one's own wit, nor searched out in the bowels of Nature; rather we must look up to God, who is above Nature, and who perhaps reserved certain works to this end: that in the order of Nature as well as of Grace we should be compelled to acknowledge, revere and adore the supreme Author of both.

VI. Hardly, then, could the efficient cause of the New Stars be inquired into without the final cause also being indicated. [Margin: The final cause of the New Stars.] And many authors indeed thought that they were created, or generated, or shown to mortals in order to signify to them something great, which was secret at that time but was to be made manifest afterwards. Others, driven by an itch for astrology and by a fever — not to say a frenzy — for predicting new things, filled the ears of idle men with a great many pamphlets published for the common people, with various rumours, but with empty sound; that lightness of leaves, however, should have been left as a plaything to the winds. For whatever changes in human affairs occurred around the times of the New Stars were not so great that either they deserved to be foretold by God through such unwonted tongues or characters from the heaven, or that the records of history have not handed down greater ones — which were

nevertheless preceded by no portents of comets or new stars. Much less did the catastrophe of all things take place which some had feared, or hoped for, from them. If, however, there is anyone who has the leisure and the liking to squander time reading through the prognostications of those who soothsay about these prodigies, he has, first of all, what *Tycho*, at the end of volume 1 of the *Progymnasmata*, from page 800, on the significations of the New Star of the year 1572, and what *Kepler*, from chapter 28 to 30, on those of the star of the year 1604, attempted to propose with conjectures gathered from every side, and to bring into suspicion: whether the Indians were to be converted universally to the Catholic Faith; whether the sect of Mahomet was to perish; whether some Monarchy was to arise that would rule over all the rest; whether a huge and universal change of religions was to come; whether, finally, the three new stars within a few years were the heralds of the second coming of Christ, and that last Fiery Trigon shone in advance of the ecpyrosis and the final conflagration of the world — just as, as Tycho and Kepler point out, the former on folio 802, the latter on 209, the first Fiery Trigon was distinguished by the creation of the world, and the fifth by the Nativity of Christ. However, when Kepler had shaken out all the pockets of divination, he seems to me to have said nothing more prudent than when, on page 208, he said of the end of the world: [Margin: Kepler's view.]

And so they cannot fit this Star in any particular way to the coming of Christ, and from it count the times and fix the limits — a knowledge God has reserved to himself. Meanwhile they certainly do not sin if, admonished by this Star, they renew a Christian life, and so prepare themselves, as men about to receive Christ the Lord at any moment. So far Kepler, who was certainly more sober and cautious than Tycho in defining the significations, and the times, of the events signified by the Star of the year 1572.

VII. For Tycho, volume 1, page 802, first set down these words: [Margin: Tycho's conjecture on the signification of the star of the year 1572.] *It is fitting that, just as that moving star of Hipparchus preceded the decline of the Monarchy of the Greeks and the growth of the Roman Empire, until almost the whole world was subjected to this one City, so now too some special and very great innovation in political administrations is pressing upon the following centuries. For just as this star displayed a most rare miracle, and stood forth from the heaven as it were unexpectedly, beyond the expectation of all men, so too it is likely that it will have most rare, huge and unforeseen effects; and what these will be in particular is, I judge, not sufficiently seen through by any mortal.* Shortly after this he ventured to carry these words over also to *a notable alteration of the religions, from those things which come from the stars and display a certain Jovial splendour and pomp*. Setting out to investigate the time of this change by the rules of astrological Directions and Progressions, he noted that the New Star of the year 1572 was the forerunner of the Greatest Conjunction of Saturn and Jupiter, which fell at the end of April of the year of Christ 1583; this conjunction put an end to the watery Trigon and began the seventh period of the fiery Triplicity. Since, then, it took place in 21° of Pisces,

he says that if a direction is made by right ascensions from that place to the place of the New Star, it will reach the point of the equator which the New Star marked after nine years, to be counted from the Greatest Conjunction, that is, in the year 1592; and that then the beginnings and seeds of those changes would be sown. But by setting up the direction to the place of the zodiac marked by the New Star, which was ♉ 7°, from the place of the Greatest Conjunction, which was ♓ 21°, he says that it will come about 48 years from the first shining of the Star, that is, about the year 1632; and that then the vigour and effect of this fiery Trigon would come out into the light. [Margin: 7. Period of the fiery Triplicity.] On these premises he lists the changes made at the restitutions of the fiery Trigon since the foundation of the world: the first in Enoch; the second in Noah and the Flood; the third in Moses and the liberation of the Israelite people from Egyptian servitude; the fourth in the Kings of Israel; the fifth in Christ and the summit of the Roman Monarchy; the sixth in Charlemagne and the translation of the Empire to the Germans. The seventh, therefore, indicates a kind of sabbath rest, and a change to a peaceful and most happy state before the end of the world comes, since those restitutions of the fiery Trigon that were completed in odd number — namely the third, in Moses, and the fifth, in Christ — were also the happiest. Having explained these things at greater length, Tycho, as if seized by enthusiasm, bursts out on page 807 in these words:

I augur, therefore, from this Star and its kinship with the new Trigon, that quite another overturning of worldly things is at hand, as regards the administration of Religion as much as of commonwealths.

We have come to the top; the highest things will suffer ruin. God, who rules in the heavens, and will rule all things on earth, etc.

Since, moreover, the New Star was in the zenith of Muscovy at the moment at which the new moon nearest to the rising of the new star took place — namely on November 5, at $7^{h}\ 31^{m}\ 40^{s}$ after noon at Uraniborg — he therefore devises, though by way of conjecture, that the first occasions of disturbances and changes would arise from Muscovy; and that Gog — of whom Ezekiel 38 and 39, Isaiah chapters 17 and 56, and Micah chapter 5 speak — and Magog, of whom Revelation 20 speaks, are, according to the interpretation of some, the Muscovite, because in Holy Scripture Gog is said to be the future prince of Mosoch, that is, of the Muscovites. These conjectures of his, however, Tycho calls general, and such as they are, on page 811, where he also strives to drag in to them the prophecy of the Tiburtine Sibyl found in the year 1520 among the Swiss, in the bowels of Mount Taurus laid bare by the rushing force of waters and snows, which Cornelius Gemma reports in his book *De divinis Naturae Characterismis*. [Margin: The forged oracle of the Tiburtine Sibyl.] It was in Latin, but in the old Oscan dialect, and was either written long ago or ingeniously forged by someone more recent, and carved in ancient characters on stone. But it does nothing for what Tycho wants, since in that oracle — a forged one, I think — the star is said to rise *over the Ubeni towards the Great*

House of the North; for Muscovy is no more designated there than any other kingdom of the North. Again, because a comet shone in the heaven about the fourth year after the disappearance of the star, of the year 1573 — that is, in the year 1577 — Tycho thinks that these verses of the Babylonian Sibyl refer to that star: [Margin: The Babylonian Sibyl's verses.]

Then shall a huge sign be given from on high; for a most brilliant star shall shine out, like the bright Crown, gleaming in the shining heaven beneath the setting Sun; the star, truly called a comet, shall shine out..... But from the fourth year after the huge Star has shone, from heaven a Star shall come which, hurling flame, shall flash across the great sea and the deep Ocean: .... Again God, the dweller in the great heaven, shall preserve the remnants of men everywhere, wholly. Then peace and the highest wisdom of the truth shall flourish, and the Earth shall bear more abundant fruitful crops, no longer divided, nor to be in servitude hereafter.

VIII. There were, however, those who said outright that the new star of the year 1572 was the herald of the second coming of Christ, such as Theodorus Beza [Théodore de Bèze] and the Landgrave of Hesse, whose words we shall report on another occasion below, chapter 20, number 18; for Beza thinks it the very same as the star of the Magi, and the Landgrave very like it. But *Tycho*, page 328, says that among the signs that are to precede the end of the world Christ does not number the risings of new stars, but rather the destruction of the ancient stars, and the fall and darkening of the Luminaries.

CAPVT XIX.

Chapter XIX.

On the Admirable Star which foreshowed to the three holy Magi the birth and cradle of Christ the Lord. What the astronomers, and what the holy Fathers and Doctors of the Church, have thought of it.

I. The Sun does not so lead the other planets, long since vanquished by the majesty of its splendour, perpetually round in triumph, as that admirable star surpasses all the New Stars. Made most illustrious long before by the oracles of the prophets, it became itself an oracle of the divine vicissitudes and the most sacred miracle of the celestial lights; with the same most eloquent tongue of its ray it commanded, by the authority of an almost divine brilliance, that the time and place of the Saviour — not about to be born, but born — be sought out; and to those who sought them it showed them most obligingly. What, then, the astrologers and what the astronomers have pronounced about it, sanely or insanely, I judge it my part to examine in this place, and to undertake to weigh their pronouncements against the canons of the holy Fathers and theologians, as against the weight and teaching of the Sanctuary. What was discussed most learnedly and eruditely about this star seemed sufficient: by *St Thomas* [Thomas Aquinas], part 3, question 36, article 7; *Abulensis* [Alfonso Tostado] on chapter 2 of Matthew, from question 11 to 16; *Thomas Beauxamis* in his Commentaries on the Gospels, from page 97; *Jansenius* on the Gospels, from page 67; *Salmeron*

[Alfonso Salmerón], volume 2 of his Commentaries on the Gospel history, tractates 38 and 39; *Io. Maldonatus* [Juan Maldonado] on chapter 2 of Matthew; *Suarez* [Francisco Suárez] on the third part, disputation 14, sections 5 and 6; *Sebastianus Barradius* [Sebastião Barradas], volume 1 on the Gospels, Book 9, chapter 9; *Cornelius a Lapide* on chapter 2 of Matthew; *Vincentius Regius* in his elucidations of the Gospels, Book 1, chapter 10; *Corderius* [Balthasar Cordier] in the *Catena* on the Gospels, from page 49; *Antonio de Escobar* on the Gospels, Book 6, section 3; *Salianus* [Jacques Salian], volume 6 of the *Annals*, page 787; *Tirinus*, volume 3 on Holy Scripture, page 5 — besides the holy Fathers and others, in homilies or sermons on the most sacred day of the Epiphany. And yet, although these seemed sufficient, I wish to set out their view together with the opinion of the astronomers.

[Margin: Matter and substance of the Star.] II. That the star which appeared to the Magi, then, was one of the ancient fixed stars, or one of the seven planets, which descen- ded to earth, was the view of *St Gregory of Nyssa* in his homily on the incarnation of Christ, and of certain unnamed writers cited by *Abulensis* on chapter 2 of Matthew, question 11, and by *Salmeron*, tractate 39. *St Augustine*, too, said in his third sermon on the Epiphany: *While God hangs at the breast and endures the poor wrappings of swaddling clothes, a new star suddenly shone out from heaven*; and in the same place he calls it *the tongue of the heavens*. But the air is not seldom called by the name of heaven.

The common view of the Fathers and the Doctors whom I named above, however, is that it was a New star, altogether different from the ancient stars of the heaven. For it would have portended nothing new to the Magi had it kept the course, place and figure of the ancient stars; but the very opposite happened. It was not carried round by the motion of the primum mobile, as the fixed stars are, nor was it borne eastward by a motion of its own; instead it moved from the East towards the West, bending somewhat to the South towards Bethlehem. Nor did it shine only by night: it shone by day as well, since it is more probable that the Magi made their journey by day rather than by night, or at least no less by day than by night. Nor did it move with a circular and continuous motion, but rather with a rectilinear and interrupted one, since it halted when the Magi rested and went on when they travelled. Nor did it last long: once it had shone motionless for some hours above the cave or hut of the newborn Child JESUS, its office discharged, it vanished — although it had also hidden itself earlier, when the Magi entered Jerusalem. [Margin: Place of the star.] Lastly, it was neither in the heaven nor in the highest part of the air; otherwise it would not have marked out exactly the house where the Child was — as *Tycho* rightly says, volume 1, page 564, and before him *Nicephorus*, *History* Book 1, chapter 13 — but the whole province of Palestine. For suppose it had stood at a height of 50 or 60 Italian miles, the height the older writers on optics assign to the highest region of the air: it could have been seen from a distance of 700 or 800 miles. If instead we set it at a height of 25 miles, it would have been seen from

a distance of 520 miles, as is plain from what was said in Book 2, chapter 10. So it was far lower.

None of the things we have listed so far belongs to the ancient stars. The Fathers therefore deny that it was one of the stars formed by GOD in the Heaven long ago, and above all *St Augustine*, Book 2 *Against Faustus*, chapter 5, who says: [Margin: The view of St Augustine.] *It was not one of those stars which from the beginning of creation keep the order of their courses under the Creator's law; but at the new childbearing of the Virgin a new star appeared.* And *St John Chrysostom*, in homily 6 on Matthew, pursues at length almost all the arguments we have indicated, beginning with these words: [Margin: The view of St Chrysostom.] *That this star was not one of the many — indeed, as I judge, not a star at all, but some invisible power shaped into the appearance of a star — is shown first of all by its very course.* He also thinks it almost touched the head of the newborn Child, when he says: *How, then, could a star point out so small a place, namely that of a hut and a manger, unless it had left its lofty place, come down to lower regions, and stood almost over the very head of the Child?*

The novelty of this star, and its difference from the other stars, was asserted as well by *St Basil* in his homily on the human generation of Christ; *St Ambrose*, Book 2 on Luke; *Origen*, Book 1 *Against Celsus*; *St Damascene* [John of Damascus], Book 2 *On the Faith*, chapter 7; *Saints Fulgentius* and *Leo*, Pope, in the sermon on the Epiphany; *St Thomas*, part 3, question 36, article 7; and before all of them *St Ignatius* in epistle 14, to the Ephesians; and finally *Nicephorus Callistus* [Nikephoros Kallistos Xanthopoulos], *Ecclesiastical History* Book 1, chapter 13. And although *St Augustine*, Book 2 *Against Faustus*, chapter 5, and sermon 3 on the Epiphany, held entirely the same view, *Maldonatus* and *Suarez* (part 3, disputation 14, section 5) nevertheless observe with him that the arguments we have touched on so far are not altogether conclusive, but only very probable. For the Maker of the stars could have called one of the ancient stars and commanded it to come down into Mesopotamia, Chaldea or Arabia, to suspend its ordinary course for a while, and to offer the Magi guidance by a new motion. Baruch, chapter 3, describes this power in these words: *And the stars gave light in their watches, and rejoiced; they were called, and they said, Here we are; and they shone with gladness for him who made them.* Yet the gentleness of divine Providence persuades us to think that he rather made a new star in the air, and out of the air, and heaped it with unaccustomed splendours, than that by a greater miracle he called an ancient star down from the heaven, filling its place in the heaven in the meantime and afterwards leading it back up again; for greater miracles are not to be asserted unless they are proved. I said "out of the air": for that it was made of celestial matter is denied both by St Thomas and Abulensis and by those who call it a comet, in keeping with the ancient opinion about the seat of comets.

[Margin: Was the star of the Magi a comet?] III. Whether it should be numbered among the comets, however, can be doubted because of these words of Origen, Book 1 *Against Celsus*: *We judge it to have been a new star, like none*

of those known ones that are either in the Firmament or in the lower orbs, but of the kind that comets are seen to be. Relying on this, our *Adam Tannerus* [Adam Tanner], question 7 *On the Heavens*, confirms from this star that some comets are sublunary; and *Beza*, in Tycho, volume 1, page 327, calls it a comet. But Origen said shrewdly that it was of the genus of comets, or like a comet — hinting, that is, at a remote genus, and at a likeness only in matter and place, since it was formed out of the air and in the air. To it there was probably added, besides, an appendage of a tail, or of a ray stretched out towards the hut at Bethlehem, as it is shown in many of the Church's pictures. In many other respects, however, it differed from the comets.

For most comets move with the motion of the primum mobile, or one or two of them, out of the history we gave in chapter 3 of the first section, have been found motionless; but this star moved, and with no other motion than from Mesopotamia, or Chaldea, or Arabia into Judea — not setting and rising again and completing a circle round the earth. Nor did it move by a natural motion, as they do, but by the will of a mover who carried it down or drove it on at his own nod, and halted it when the Magi halted. Moreover, the motion of comets is either circular or, according to Kepler, rectilinear indeed, but along a trajectory by which they are let down and afterwards withdraw upward again; whereas the rectilinear motion of this Star was transverse, and nearly parallel to the plane of the horizon, as *Salmeron*, tractate 39, teaches together with *Abulensis*, question 11 on Matthew 2.

Further, comets are visible only by night, and the mere light of twilight extinguishes them; but the star of the Magi, shining by day, was not overwhelmed by the light of the Sun — rather, it overcame it, as will be said shortly. Comets dwell either in the highest part of the air, or in the region of fire, or even in the heaven, as is plain from what was said in section 1, chapters 6 and 23; but the star of the Magi stood a few ells or cubits from the little house where the Child JESUS lay hidden. Comets, if they have a tail or a beard, cast it away from the Sun into the opposite quarter; but the ray of this star, which served as its tail, was turned towards its own Sun, JESUS. Comets do not always appear of equal size: mostly they are larger at the beginning or a little after it, and then, as their matter fails, they shrink and at last vanish. This Star, by contrast, was either of the same apparent size from beginning to end, as Abulensis holds, so as not to give the Magi an ambiguous sign by its inequality; or rather, when it was above the House of the Child Saviour, it bore witness to its joy at its task by a sudden increase of form and splendour and by a flashing of scintillating lights, and then, once they had entered the cave, it vanished all at once. Salmeron adds, with Abulensis, that comets hardly or never appear at the births of kings or princes, but are rather signs of their fall and destruction; yet it is established from our history of comets, given earlier in the first section, chapter 3, that comets shone out before the birth of Alexander the Great, of Mithridates, and of some other princes. St Thomas, however (part 3, question 36, article 7, reply to the third objection), notes that a cometary signification was not wholly absent: *Because*

the heavenly kingdom of Christ shall break in pieces and consume all the Kingdoms of the earth, and it shall stand for ever, as is said in Daniel 2.

A better argument, then, is taken from the natural origin and signification of comets, since the star of the Magi, on the contrary, was produced by a supernatural action and appointed for a supernatural end and signification, as we shall say shortly. Rightly, therefore, *St Thomas*, part 3, question 36, article 7; *Cottunius*, lecture 33; *Maiolus*, colloquy 1, page 15; the *Coimbra commentators*, tractate 3 on the *Meteorologica*, chapter 7; *Abulensis* on Matthew chapter 2, question 11; *Nicephorus*, Book 1, chapter 13; *Salmeron*, volume 2, tractate 39; and *Tycho*, volume 1, page 324, deny that it was a comet — although *Maldonatus*, on Matthew chapter 2, concludes thus: *I would call it either a comet or an angel*; and *Tannerus* thought it a comet.

IV. The truer and more common view, therefore, is that this Star was neither one of the ancient Stars in the heaven, fixed or wandering, nor properly a comet, nor a New Star produced by natural generation; but a New Star formed by divine power out of the air or meteorological matter, in a way ex- ceeding the order of Nature, and by a miracle — yet in such a way that it was a supernatural being as to its mode, if not as to its substance. [Margin: The supernatural character of the star of the Magi.] So, nearly, *St Thomas*, part 3, question 16 [recte: 36], article 7, when he says: *It seems more probable, however, that it was a Star newly created, not in the heaven but in the air near the earth, which moved according to the divine will.* And more clearly *Abulensis* on Matthew chapter 2, question 11: *It must be said that this Star was a certain impression produced supernaturally in the lowest air about the earth; and that body was very dense, having much of light, and because of its sphericity and its flames it was called a star. Such impressions are not usually produced naturally; God makes them for some special end.* Our *Salmeron*, volume 2, tractate 39, agrees with both, fusing the words of the two into one and calling this Star *a certain Impression made above the powers of nature in the lowest air about the earth.* The words of *Cornelius a Lapide* on Matthew 2 say the same: *For the angels formed it out of condensed air, putting splendour into it*; and a little later: *This Star, then, was a New Meteor, formed by angels in the air and informed with an immense light.* Before all of these, moreover, *St Fulgentius* had said plainly in his sermon on the Epiphany: *This Star had never appeared, because that Child created it then and appointed it to go before the Magi coming to him: that Child is laid, a little one, in the Manger, but he works wonders, a great one, in the heaven.* Tycho himself, in volume 1 of the *Progymnasmata*, comparing the New Star that appeared in 1572 in Cassiopeia's Chair, says this on page 324: *That star which appeared to the Magi in the East was not of the kind of the celestial stars, nor did it agree in any way with this New star that is mentioned here, or with the comets: it was rather a peculiar and admirable work of GOD.* And he hints the same on pages 584 and 784. Moreover, the *College of Coimbra*, on Aristotle's books of the *Meteorologica*, tractate 3, chapter 7, lays it down thus: *The already mentioned Star, then, was a certain new and unaccustomed Meteor, compounded out of subcelestial matter not by natural but by angelic or divine power,*

and sprinkled with the brilliance of an exceptional light; and it was carried along not by its own force but by the ministry of some angel. Its supernatural character, however, will be clearer from what is to be said about its splendour and form, about its mover, and about the way in which it signified Christ's birth to the Magi.

Meanwhile, the statements of *Suarez* on the third part, disputation 14, section 1, do not seem clear enough. For he says of Origen: *When he said that it was in the likeness of a comet, he does not mean that it was of the same nature as to splendour and motion, but only as to the matter of which it was made*; and a little later, asking whether that splendour belonged to a perfect mixed body and was connatural to that Star, he answers: *If one may guess, it must be said first that that body was such that that light was connatural to it*; and soon after: *It is probable that that body was some perfect mixed body.* For if the splendour of this Star was not like that of comets, which are reckoned imperfect mixed bodies, but connatural to some mixed body, and we know of no perfect sublunary mixed body to which so great a light is due, then the substantial form of that mixed body was supernatural, and hence supernatural as to its substance, and not only as to its accidents and mode: that, at least, is what seems to follow from what he says.

[Margin: Splendour and Size of the star.] V. That the splendour of this Star was very great, and more excellent than the light of all the Stars, is plain from the Fathers. First, *St Ignatius*, epistle 14, to the Ephesians: *A Star shone in the heaven above all that were before it, and its light was inestimable, and it appeared new and strange to those who saw it. The rest of the stars, together with the Sun and Moon, were a chorus to that star; but it surpassed them all by its splendour, and there was perturbation over where that novelty that had appeared came from. St John Chrysostom* teaches the like in homily 6 on Matthew, where he says: *But this star surpassed even the rays of the Sun with a brilliance of its own and a special one, appearing indeed brighter than they, and in so great a light of theirs shining out the more itself.* No less illustrious is that testimony of *St Leo*, Pope, in Sermon 1 on the solemnity of the Epiphany: *To the three Magi, then, in the region of the East, there appeared a star of new brightness which, being more brilliant and more beautiful than the other stars, easily turned the eyes and minds of those who looked upon it towards itself, so that it was at once perceived that what seemed so unaccustomed was not idle.* And in Sermon 3 on the same solemnity: *A star more brilliant than the other stars moves the Magi, dwellers in the further East; and these men, not unskilled in observing such things, understand from the wonderful brightness of its light the greatness of its signification — divine inspiration doubtless working this in their hearts, so that the mystery of so great a vision should not be hidden from them, and what was shown to their eyes as unaccustomed should not be obscure to their minds.* In that place he had taught, with a most beautiful observation, that GOD used a Star for this reason: that that innumerable succession promised to the most blessed Patriarch Abraham, and therefore compared to the multitude of the stars — so that from the father of all nations a progeny not earthly but heavenly might be hoped for — should also be shown to the Gentiles by a star. And accordingly

he adds at once: [Margin: Why a star, rather than some other sign, was given to the Magi.] *To bring forth, therefore, the promised posterity, the heirs designated in the stars are roused by the rising of a new star, so that the service of the heaven should attend on that in which the testimony of the heaven had been invoked.* Finally the Church sings with Prudentius: *Whom the star, which surpasses the Sun's wheel in beauty and light*, etc.

Notwithstanding this, *Suarez* in the place cited thinks it unnecessary to say that this Star had a light more intense and greater than the light of the Sun, because, he says, it is hard to believe that a more perfect light was connatural to some lower body, or was given it by God without necessity; for, for it to shine at midday and be visible, it was enough that it was near the earth and had a very perfect light and a quite dense matter. To me, however, it seems more consonant with what the Saints say if we say that it was equal or superior to the Sun, not indeed in bulk, but in splendour — especially since we hold that light to have been not connatural to it, but added supernaturally. [Margin: Size of the star.] *Cornelius a Lapide* on Matthew chapter 2 and *Barradius* teach, however, that it was not greater than the Sun, although, being near the earth, it would have looked very large — lest it be called a second Sun rather than a Star.

VI. On this occasion, however, *Suarez*, on part 3, disputation 14, section 5, asks: if that Star was endowed with such splendour and was so beautiful, how was it seen by no one but the Magi? [Margin: Was this star seen by everyone?] Or, if it was seen, how did it not carry everyone away in admiration of itself, and how did its fame not spread through the whole world? Answering the question, he says: *The reason perhaps is that, being near the earth, it shone only in nearby places, especially by day; at night it was perhaps raised higher, so that it could be distinguished only by those who devoted themselves to knowing the stars, as Nicephorus says, Book 1 of the History, chapter 13.* But Nicephorus's meaning is that it was so like the fixed stars that it could have been reckoned one of them, and was not distinguished from them except by skilled astronomers, as will be plain from his words, to be quoted in chapter 20, number 17. On the contrary, *Alphonsus Tostatus* on Matthew chapter 2, questions 14 and 41, replies: *It must be said that this star appeared for the manifestation of Christ's Nativity; it was therefore not fitting that the star should hide itself, so that no one but the Magi should know of this nativity; rather it was fitting that the star be seen by many, and that many should know of Christ's nativity, because this redounded to his glory. And in this way no one is unworthy to see such miracles, since they are done immediately more for the glory of God than for our benefit, although both concur there; the like was the case in the pillar of cloud of the Jews*, etc. And *Salmeron*, tractate 39, holds for the same reason of divine glory that it was seen by others too; *Cornelius a Lapide* subscribes to this, though, he says, few followed the Magi, because of their ignorance of the divine Mysteries and the hindrances of domestic cares.

Tycho, on the other hand, volume 1, page 326, dissents from Abulensis and puts forward a diametrically opposite view, which I shall not mind setting out

in his own words. He says: *Moreover, this star was seen by the Magi alone (for the Jews did not perceive it, however much it passed through their region; otherwise Herod would not have inquired so diligently about its appearance and state, since he could have learned the same from his subjects), either because GOD by some singular dispensation granted them this gift, or because they, endowed with some more recondite knowledge, perceived things that were hidden from other mortals, by the benefit of some excellent Magic.* As if Magic could make visible to them what was invisible to others, and do what the Telescope now does for little stars otherwise inconspicuous! Nor does it follow that, if it was not seen by the Jews because it hid itself near Jerusalem, it was not seen elsewhere by the servants of the Magi, and perhaps by some others. For *St Chrysostom*, homily 62 on the Epiphany, volume 2, says: *And this star was indeed seen by all, but not understood by all.*

VII. As for its form, since the star is said in the sacrosanct Gospel to be a star, it certainly had the form of a Star — whether of a tailed xiphias, or sword-shaped comet, with its point stretched out towards the earth, or of a perfectly round one. [Margin: Figure or Form of the Star.] Whether the image of the Child JESUS shone in that Star, however, is uncertain. *Pliny the Elder*, indeed, Book 2, chapter 25, setting out the various kinds of comets and prodigies, says: *There is also a white Comet, with silvery hair, so resplendent that it can scarcely be looked at, showing in itself, in human form, the likeness of GOD. Fromondus* [Froidmont], Book 3 of his *Meteorologica*, chapter 4, article 1, says that these words are cloudy, but that they can not improbably be understood of the star that was guide to the Magi to the cradle of Christ; for perhaps there was both a report and a fact, under the rule of Augustus, of a comet seen in the East and in Judea in which was expressed the image of a little boy, whom the Christians would call the man-GOD. The Author of the *Unfinished Work*, too, as cited by *Maldonatus* on chapter 2 of Matthew, relates an apocryphal story: that the likeness of a boy appeared in the Star, who spoke to the Magi and taught them that Christ was born. But it is more probable that they did not see the image of the boy before the Boy himself, in accordance with that prophecy of the Erythraean Sibyl in Book 8 of the *Oracles*:

And the Magi worshipped the divine new Star, And the Infant, shown to those who follow GOD's precepts, Is in the Manger, etc.

Granted, that Star may have signified Christ, or his Most Holy Mother, allegorically. That is where these words of *Blessed Peter Damian*, in his sermon on the Epiphany of the Lord, point: *There was a Star in the air, a Star on earth, a Sun in the manger: the Star in the air, that shining body; the Star on earth, the Virgin MARY; the Sun in the manger, our CHRIST*; and a little before he had said: *So the Sun rose from the Star*, namely CHRIST from MARY. So too that saying of *St Bernard*: *Look at the Star, call upon Mary*; and *St Ambrose*, Book 2 on chapter 2 of Luke: *Christ is the Star, for a Star shall rise out of Jacob, and a man shall spring up out of Israel; in short, where Christ is, there is the Star too: for he himself is the bright and morning Star; so he marks himself out by his own light.*

If anyone, however, wants also the words of the Author of the *Unfinished Work*, homily 2 on Matthew, he may have them, for all I care. For after relating, out of the books of the Sethians, that twelve successors of Balaam appointed in each generation some men to contemplate the heaven continually, or at least after the threshing of the harvest, to watch for when the star foretold by Balaam's oracle would rise, he added: *It appeared to them descending upon that Victorial mountain, having in itself the form as of a little boy, and above itself the likeness of a Cross; and it commanded them to set out for Judea. As they set out, the Star went before them for two years, and neither food nor drink failed in their wallets.* These fictions, however, were devised by the Sethian heretics, as is gathered from *St Epiphanius*, heresies 26 and 39. Who it was, then, that appeared to them hidden in the star, or who was the mover and director of the star, must now be related.

[Margin: Mover and Ruler of the star.] VIII. About the moving and directing power of this Star, two opinions are put about. The first attributes it to the Holy Spirit, who, just as he appeared to St John the Baptist above Christ in the form of a Dove, so appeared to the Magi in the form of a Star. The second is that it was God acting through an Angel, whether the Angel assumed that Star or merely carried or drove it. *St Thomas*, part 3, question 36, article 7, embraced both opinions, saying: *Some say that, just as the Holy Spirit descended upon the baptized Lord in the form of a Dove, so he appeared to the Magi in the form of a Star. Others, however, say that the Angel who appeared to the Shepherds in human form appeared to the Magi in the form of a Star.* He took both from the Author of the work *On the Wonders of Holy Scripture*, which is found in volume 3 of the works of St Augustine, Book 3, chapter 4; but about the Angel he says this: [Margin: Was the star of the Magi the Holy Spirit, or an Angel under its appearance?] *What objection is there if the Star in this place is held to be called an Angel? On this occasion an Angel is made the guide of the Magi, who, for astrologers, is transformed into the likeness of a Star. St Chrysostom*, homily 6 on Matthew, seems to hint at this with that form of words: *That it was not one star of the many, but a certain invisible power in the appearance of a star, full of reason.* No differently *Euthymius* [Euthymios Zigabenos], saying: *The Star had not the nature but only the figure of stars, and there was in it truly a certain divine power transformed into this figure. Theophylact*, however, uses plainer language when he lays it down thus: *Do not think it a star such as we see, but a divine and Angelic power appearing in the figure of a star. That the star was an Angelic power is plain from the fact that it both shone by day, and moved at the motion of the Magi, and rested at their rest. Abulensis*, on chapter 2 of Matthew, question 15, was of the same opinion; there he argues at length that God was accustomed to move this Star and similar things not by himself but through Angels, and above all the pillar of fire and of cloud, according to that verse of Exodus 14: *And the Angel of the Lord, who went before the camp of Israel, removing, went behind them, and with him the pillar of cloud.* The same is asserted by *Maldonatus* on chapter 2 of Matthew, *Barradius*, volume 1 on the Gospels, Book 9, chapter 9, and *Salmeron*, tractate 39, who denies that it

was the Holy Spirit and says that it was an Angel as mover, though not one transformed into a star.

[Margin: Time of the star's appearance.] IX. A greater controversy has arisen, and still continues, about the time at which that wonderful Star appeared to the Magi. Some say it was seen two years before Christ's birth; some, that it first appeared at Christ's birth and appeared again two years later, when the Magi left Jerusalem; most, that it appeared again only thirteen days after its first appearance (but that Herod nevertheless put off the slaughter of the infants to the 2nd year of Christ) — that is, the number of days the Church counts from the day of the Nativity of Christ the Lord to the day of the Epiphany. The foundations of these opinions are those words of St Matthew 2: *He learned diligently of them the time of the star which appeared to them*; and soon after: *he slew all the male children that were in Bethlehem and in all its borders, from two years old and under, according to the time which he had diligently inquired of the Magi.* Macrobius, too, speaks of this age of two, in Book 2 of the *Saturnalia*, chapter 10, reporting among the jokes of Augustus this one: *When he had heard that among the boys under two whom Herod, king of the Jews, ordered killed in Syria, his own son too had been killed, he said: It is better to be Herod's pig than his son.* For the Jews, under Leviticus 11, did not slaughter pigs, as being unclean to eat; whereas Herod had obtained leave from Augustus to kill his three sons — Alexander and Aristobulus, whom he put to death long before, and Antipater, whom he ordered killed five days before his own death, as Josephus reports, Book 17 of the *Antiquities*, chapters 10 and 11. Macrobius, however, as Salianus noted together with Deckerius under the year 1 of Christ, committed a paralogism if he thought Antipater a two-year-old because he was killed at the same time as the two-year-old infants.

[Margin: 1. Opinion: the star[?] two years before Christ's birth.] The *first* opinion, then, was that the star appeared to the Magi in the East two years before the Saviour's birth. So *St Augustine*, Sermon 7 on the Epiphany; *St Chrysostom*, homily 7 on Matthew; the Author of the *Unfinished Work* on Matthew, homily 2; *Theophylact* on chapter 2 of Matthew; *Nicephorus*, *History* Book 1, chapter 13; and *Kepler* in the *Silva Chronologica*. St Augustine, however, did not hold to this opinion, since he also proposed another, as St Chrysostom did too. For St Augustine had said in homily 4 on the Epiphany: *On the day on which Christ was born, they saw the Star in the East.* Of St Chrysostom I shall speak below, at the 4th opinion. So *St Leo*, Sermon 2: *The Magi by a Star and the Shepherds by Angels learned of the Lord's birth at the same time*; and *St Fulgentius*, Sermon on the Epiphany, indicates the same in the words already quoted: *This star had never appeared before, because that child has now created it.* And indeed, if the Magi saw the Star two years before Christ's birth, either they girded themselves for the journey at once — and then they would have arrived far too early, and would have asked after a king about to be born rather than one born, since no region of East India, let alone of Arabia, Persia, Chaldea or Mesopotamia, lies so great a distance of travel from Palestine — or, if they did

not set out at once, they would stand convicted of slowness and negligence. Nor was it fitting that the star should appear to them before the time needed for that journey.

[Margin: 2. Opinion: two years from the birth of Christ and of the star to the Adoration.] The *second* opinion was that Christ and the star arose on the same day indeed, and that it then first appeared to the Magi, but that the Magi did not reach Jerusalem and Bethlehem until two years later. So *St Epiphanius*, Book 2, heresy 30, who adds that they found the Infant not in the cave where he had been born, but in another house, because his parents returned to Bethlehem every year out of veneration for the Nativity. But this opinion both brands the holy Magi with the mark of slowness, since the distance of the places does not require so long a delay, and agrees less well with the letter of the Holy Gospel, which says that they found the Infant laid in the Manger — by which words an infant's age of a few days is sufficiently indicated, and so is the place in which he had recently been born, namely the Manger. And the very words *When he was born, etc., behold, Magi* indicate their speed, and an interval of very few days between the rising of the star and of Christ and the arrival of the Magi. And *St Justin*, in the dialogue with Trypho, says: *As soon as Christ was born, Magi coming from Arabia adored him*; and he refers to their speed those words of Isaiah: *Make haste, take away the spoils, etc.*

[Margin: 3. Opinion: on the deferral of the slaughter of the infants to year 2.] The *third* opinion was that the Star indeed rose on the same day as Christ, and that the Magi reached Bethlehem on the thirteenth day from its shining, but that the slaughter of the infants was deferred to the year 2 of Christ, so that Herod ordered all who did not exceed two years of age to be killed, secure that Christ was among them. For, having made a computation of the time that he had diligently inquired of the Magi, he knew that the Star had been seen by them a few days before their arrival; and, having likewise reckoned the time that had run on since their arrival, and the time by which, in his opinion, the star had perhaps risen after the birth of Christ, he judged that all this taken together could not exceed two years. *St Augustine* seems to have been of this opinion, Book 2 *On the Agreement of the Evangelists*, chapter 11, when he says: *I forbear to say with how many and how great occupations the royal care could have been distracted, and over very many days either turned aside altogether from that intention, or hindered. St Thomas*, part 3, question 36, article 6, reply to the 3rd, approves this one more, and with him *Salmeron*, volume 2, tractate 44, who says that Herod did not kill the infants until two years later, because he had been summoned to Rome by Caesar to defend himself against the accusation of his sons. Suarez, however, there, disputation 14, section 4, judges that this agrees poorly with Eusebius, Book 1 of the *History*, chapter 8, who says that after the slaughter of the infants Herod was afflicted with domestic calamities and seditions. *Cornelius a Lapide*, on Matthew chapter 2, verse 16, defends this very opinion tenaciously, adducing on his side Hugo Victorinus [Hugh of Saint Victor], St Antoninus, Lyranus [Nicholas of Lyra],

Abulensis, Dionysius the Carthusian, Barradius and others. He says that the slaughter of the infants was perpetrated about the 15th month from the birth of Christ, and that Herod put off the killing partly because of domestic cares, partly so as to learn better both the parents and the genealogy of Christ, and a sure way of registering the infants and afterwards gathering them for death, under the pretext either of a banquet or of some festival. *Nicephorus*, *History* Book 1, chapter 14, and *Cedrenus*, in his *Compendium of Histories*, favour this opinion, since they think that St John the Baptist fled into the desert because of this persecution of Herod's when he was two years old. For he had been born six months before Christ; if you add 15 months to these, they make 21 months, so that he would be completing the greater part of his second year. *Iacobus Tirinus* subscribes to it, volume 3 of the *Commentary on Sacred Scripture*, on chapter 2 of Matthew, and so do *Onuphrius Panuinus* [Onofrio Panvinio], under the year 1 of Christ, and *Genebrardus* [Gilbert Génébrard], Book 2 of the *Chronicle*. *Abulensis* too, on Matthew chapter 2, question 71, thinks this the more reasonable, and *Barradius*, volume 1 on the Gospels, Book 10, chapter 10, thinks it probable.

[Margin: 4. Opinion: on the enlarged age of the infants to be slain.] The *fourth* opinion is that the star appeared to the Magi on the day on which Christ was born, and that they arrived at Bethlehem after 13 days, whether because of the nearness of the places or because of the speed of the dromedaries indicated by Isaiah; and that the slaughter of the infants was carried out about the third month after the Nativity of Christ — when, that is, Herod saw that he had been deceived by the Magi, and heard the report of the King newly born and Presented in the temple. On this view, for the sake of greater security, and out of headlong fury and lust to retain his dominion, he enlarged the impious decree so that all who fell within the age of two should be killed; yet he ordered that those whom he was certain had been born after the rising of the star should not be killed, and this is what is signified by those words: *According to the time which he had diligently inquired of the Magi*. So he feared more for himself from those born before the star, or before the declaration of the star's rising, than from those born after it. So, namely, *St Thomas*, part 3, question 36, article 6, reply to the 3rd, who, reporting this opinion in the third and last place, seems to choose it; *Iansonius*; *Baronius*, under the year 1 of Christ; *Maldonatus*, on Matthew chapter 2; *Suarez*, part 3, disputation 14, section 4; *Salianus*, volume 6 of the *Annals*, under the Year 1 of Christ, from number 14. Barradius, volume 1, Book 10, chapter 10, thinks this the more probable, as do *Torniellus* and *Gordonus* under the year 1 of Christ; and *St Chrysostom*, homily 7 on Matthew, favours them, saying: *That he killed from two years old and under, let no one wonder, since fear and fury, for the sake of greater security and certainty, extend even the time, etc.* But we too have already shown in our *Chronology* that Herod died after the feast of the Passover in the same year in which Christ was circumcised, and that the slaughter of the infants was therefore perpetrated in that year. Perhaps Herod also feared lest the boy, whom the stars served, should transform his appearance

to somewhat above his age — as St Thomas notes, with St Augustine, in the place cited above.

[Margin: Did it always go before the Magi?] X. It is now asked by some whether the Star, after it appeared to the Magi in the East and roused them to undertake the journey, withdrew itself from their eyes and did not offer itself to be seen by them again until they had left Jerusalem; or whether, from its first appearance all the way to Jerusalem, it shone on them with an unbroken sight of itself.

Of the former opinion were certain persons left unnamed by St Thomas, part 3, question 36, article 7, in the body of the article, and by the Coimbra commentators [Conimbricenſes], tractate 3 on the *Meteorologica*, chapter 7. But *Cornelius a Lapide*, on Matthew chapter 2, verse 2, attributes it to Iansenius, Caietanus [Cajetan] and Clarius, and Barradius thinks it probable.

But the more common opinion is that the Star went before them from the East all the way to Jerusalem, and was visible at least at the time when they were awake and making their journey. In this too, then, it was like the pillar of cloud in the desert, which always appeared: it went when the Magi went, halted when they stood, and accommodated itself to their alternations. So teach *St Thomas*, part 3, question 36, article 7; *Abulensis*, on Matthew chapter 2, questions 13 and 14; *Suarez*, on part 3, disputation 14, section 5; *Salmeron*, volume 2, tractate 39; *Maldonatus*, on Matthew chapter 2; and *Cornelius*, in the place adduced, together with *Lyranus*, *Pererius* and the *Coimbra commentators*. Abulensis, however, judges that it differed from the Pillar of cloud in this: the Israelites moved at the motion of the pillar — for so great a multitude had to be governed in that way, to avoid the discord that would arise over the time of moving camp — whereas the Star moved at the motion of the Magi. Finally, the holy Fathers thought so. For *St Chrysostom* says, in homily 6 on Matthew: *But this star came from the North to the South, in which position Palestine faces Persia*; and soon after: *For until the Magi came to Palestine, it always appeared as the guide of their journey*; and again: *For that star did not then hold any course of its own; but when it saw them going on, it too went on, and when it saw them standing, it stood, ordering everything entirely to the will and the advantage of the travellers. St John Damascene* indicates the very same, Book 2 *On the Faith*, chapter 7: *The star that appeared to the Magi advanced now from the Rising to the Setting, now from the North to the South, etc.* But most eloquently of all *St Peter Chrysologus*, Sermon 156: *When the Magus walks, the star walks; when he sits, it stands; when he sleeps, it keeps watch.* And the Church sings in harmony with them in the hymn:

The Magi went on, following The star they had seen going before them.

These words are not to be restricted to the journey from Jerusalem to Bethlehem. And the phrase *We have seen the Star in the East* does not mean that it was seen only there, but that it began to be seen there, and was not seen at Jerusalem.

[Margin: Where was the star seen?] XI. It is disputed, however, where the Magi first saw the Star: above Judea, or above the place where they themselves

lived? For the *Ordinary Gloss* and *St Anselm* leave the dispute undecided, and *Barradius*, volume 1 on the Gospels, Book 9, chapter 9, thinks either view likely. For the former opinion stands Balaam's prophecy, *A Star shall rise out of Jacob* — as if to say that out of the house of Jacob, or out of Judea itself, a star will come into the East. Pagnino's version in the Royal Bible hints at this, when it says: *A star went forth out of Jacob*. Accordingly *Procopius*, treating of Balaam's prophecy, says: *When they saw a strange and unaccustomed star standing motionless perpendicularly above Judea.* And lest the text of the holy Gospel seem to conflict with this, where it is said *We have seen his star in the East*, the interpretation of *Haymo* is ready to hand, Sermon 3 on the Epiphany, which runs thus: We, placed in the East, saw his star shining in the West, over against Jerusalem. *Barradius* and *Cornelius a Lapide*, on Matthew chapter 2, verse 2, thought these should be reconciled thus: that the star was first seen above Judea and the town of Bethlehem, but from there flew off with swift motion to the East and stood above the place of the Magi until it moved them to undertake the journey towards Judea.

But against this stand the objections that Abulensis raised on Matthew chapter 2, question 18. For since Judea was distant from the place of the Magi by a journey of 13 days, that is, by about 400 miles, then if the star was near the earth it could not have been seen, because of the swelling of the earth; if, on the other hand, it was higher, it could not mark out a definite place, nor would it have seemed to hang above Judea any more than above the other neighbouring regions. And so he concludes that the star was seen by them hanging above their own head, and afterwards directed them into Judea. If, however, they had seen the star from a look-out on a high mountain, they could have seen it even though it was near the earth; or at any rate it was enough that it should show them roughly the region towards which they had to go, since afterwards it was to go before them *and, as it were with a finger, point out the place of the newborn Boy*, as Isidore of Pelusium puts it, Book 1 of the *Letters*, to Olympius, 378.

[Margin: What became of this star afterwards.] XII. Next, what became of this star after it had discharged its office and, as I was just saying, had pointed out the Lord's Manger with its ray as with a finger? Some have thought that the Star led the Magi back to their homeland. But *Abulensis*, on Matthew chapter 2, question 55, opposes this, because this miracle was not necessary, and had there been one the Evangelist would not have passed over it in silence; nor would there have been need of the Angel's warning in their dreams, since it would have sufficed to follow the star leading them by another way. The star therefore disappeared once they had entered the house where the Divine Boy was. *Haymo*, however, writes in Sermon 3 on the Epiphany, from a tradition of Gregory of Tours, that that star fell into a certain well at Bethlehem, and that of three men, one of whom was a virgin and the others not virgins, it was seen by him alone who was a virgin. But *St Anselm*, on chapter 2 of Matthew, thinks this frivolous, and with him *Barradius*, volume 1, Book 9, chapter 9. The star was therefore dissolved and vanished, since it had not been established for

any other end — like the pillar of cloud and the other signs of this kind. So it did not begin to shine in the Firmament, as some have imagined; *Abulensis* refutes them on Matthew chapter 2, question 16.

[Margin: How the star indicated JESUS.] XIII. But the chief question remains: by what means, namely, this Star taught the Magi that the King of the Jews was now born, and by what indications they could discern that it pertained to signifying the birth of this King rather than anything else — so much so that they could say *We have seen his star*: his, I say, and not another's. The answer is that three things concurred to their knowing this determinately and certainly: namely, a remote disposition in the Magi, arising from their skill in the stars, or from their astronomical profession, to which GOD willed to accommodate himself, though it did not demand it, by calling them through the splendour of an unaccustomed star; next, a particular knowledge of the oracle of Balaam; and lastly, but chiefly, an interior ray of divine illumination and of exciting Grace, which both accompanied and followed them up to the adoration and the return into their own region — indeed, all the way into heaven, to the contemplation of the form of the divinity. These three we shall show briefly from the holy Fathers and Doctors of the Church.

First, then, very many of the Fathers affirm that those Magi were philosophers and wise men, devoted to the study of Astronomy, or even of Astrology, and that GOD therefore willed in some way to speak to them through a Star. [Margin: The Author of the Unfinished Work.] The Author of the *Unfinished Work* on Matthew, homily 2: *Who are the Magi? Eastern men who came from Persia; for among them Magi are understood not as sorcerers but as wise men.* [Margin: St Cyprian.] Well known is that saying of *St Cyprian* in the Sermon on the Star and the Magi: *Not long afterwards the knowledge of this matter penetrated the borders of the Arabs, and to the inhabitants of Sabba a star of brilliant light indicated the heavenly godhead with new splendours. There were in those regions men accustomed to the inspection of the stars, who by the mathematical art knew the force and course of the Planets; for, measuring the reckoning of times from the nature of the elements, they had learned by sure experiments the ministries of the stars, assigned to their proper effects.* [Margin: St Leo.] *St Leo*, Pope, recognised the same science of contemplating the stars in them, Sermon 3 on the Epiphany, in words already adduced on another occasion: *A star more brilliant than the other stars moves the Magi, dwellers in the further East; and from the brightness of its wonderful light these men, not unskilled in observing such things, understand the greatness of its signification*; and in Sermon 4: *A nation that was strong in the art of observing the stars.* [Margin: St Anselm.] *St Anselm* agrees, on Matthew chapter 2: *They were not sorcerers, but wise Astrologers*; [Margin: Bede.] and the Venerable priest *Bede* said: *The Magi are so named not on account of the magic art, but on account of a certain Philosophy.* [Margin: Nicephorus.] No differently *Nicephorus*, Book 1 of the *Ecclesiastical History*, chapter 13: *These were wise men, distinguished by nobility of birth and by wealth.* [Margin: Innocent.] *Innocent III* himself too, the supreme Pontiff, in the Sermon on the Epiphany: *The Magi*

were not soothsayers, but philosophers, who were so called from the greatness of their knowledge; for those whom the Greeks call Philosophers, the Persians call Magi.

[Margin: Gloss.] It is no wonder, then, that the *Ordinary Gloss* said: *The Magi were not sorcerers, but belonged to the Chaldean Philosophers, who philosophise about each thing*; [Margin: Abulensis.] and *Abulensis* on chapter 2 of Matthew, question 6: *Magi, that is, great in wisdom; and so the wise are called Magi among the Persians, while among the Hebrews they are called scribes*; and a little later: *and those are most of all said to be Magi who are wise in Astrology; and these men appeared to be such, since God directed them by a star.* [Margin: St Jerome.] But *St Jerome*, on chapter 2 of Daniel, teaches most distinctly that the name of the Magi does indeed sound bad in common usage, but that among those who understand the force of the word the matter stands otherwise. For when he had said: *They seem to me, therefore, to be enchanters, who accomplish things by words. The Magi, who philosophise about each thing*, a little later he added: *Moreover, among the Chaldeans I think the* γενεθλιαλόγους *[Greek: genethlialogous, "casters of nativities"] are signified, whom the common people call Mathematicians. But custom and common speech take the Magi for sorcerers, though they are held otherwise among their own nation, since they are the Philosophers of the Chaldeans; and kings too, and princes of the same race, do everything according to the knowledge of this art.* The same writer, Book 2 *Against Jovinian*: *Eubulus relates that among the Persians there are three kinds of Magi, of whom the first, who are the most learned and most eloquent, take nothing more as food than meal and vegetables.* [Margin: Laertius, On the Lives of the Philosophers.] He seems to have taken this from Diogenes Laertius, Book 1, chapter 1, where, after asserting that the Magi devote themselves to the worship of the Gods, he describes their food and clothing in this way: *Their clothing is white, their bed the ground, and their food vegetables, cheese and coarse bread; they use a reed for a staff, and they eat the cheese fixed on its top by putting it to their mouths. Aristotle says that they were ignorant of magical divination.* [Margin: Strabo.] Not without reason, then, did *Strabo* say, Book 15: *The Magi follow an honourable kind of life*; [Margin: St Isidore.] and *St Isidore*, Book 8 of the *Etymologies*, chapter 9: *Interpreters of the stars were called Magi, as we read of those who announced in the Gospel that Christ was born.* To these let two followers of Platonic philosophy be added. [Margin: Philo.] The first is *Philo*, in the book *That every good man is free*, for he says: *Among the Persians, the Magi, investigators of nature, out of zeal for knowing the truth, who at leisure contemplate the divine powers more clearly.* [Margin: Chalcidius.] The other is *Chalcidius*, on Plato's *Timaeus*, who said on page 36: *There is another, holier and more venerable story, which tells that by the rising of a certain star were announced not diseases and deaths, but the descent of the venerable GOD, for the sake of human converse and of mortal affairs. When men of the Chaldeans, truly wise and well practised in the consideration of heavenly things, had looked up at this star on a night journey, they are said to have sought the recent birth of God and, having found that majesty in boyhood, to have venerated it, etc.* [Margin: Mantuan.] Why should we not also lend our ears

to *Io. Baptista Mantuanus* [Battista Spagnoli, called Mantuan], singing of the Epiphany:

He is a Magus among the Persians who knows the stars, Who knows the powers of herbs, and the worship of the Gods: At Persepolis this threefold wisdom makes men Magi.

[Margin: St Jerome.] But I return to *St Jerome* on chapter 2 of Daniel, who goes on to write of them thus: *They acquired this Magic from Physics and Mathematics, yet in such a way that from these they contemplated the things that were God's. These the Indians called Gymnosophists, the Gauls Druids, the Greeks Wise Men, the Hebrews Prophets.* Thus Magic either included Astrology among its other sciences, or was very closely akin to it. [Margin: Cedrenus.] Accordingly *Cedrenus*, in his *Compendium of Histories*, gathers from Josephus, Book 1 of the *Antiquities*, chapter 16, that Abraham taught the Egyptians Αστρολογίαν, καὶ Μαγεῖαν [Greek: astrologian kai mageian, "astrology and magic"]; and he so defines Magic that it depends on Astrology, adding: Αστρολογία γὰρ, καὶ Μαγεῖα, ἀπὸ Μαγουσαίων, τοὶ Περσῶν ἤρξατο [Greek: "for astrology and magic began from the Magusaeans of the Persians"] — that is, Astrology and Magic arose from the Magusaeans, or Persians. And indeed the Persians are called Magog by the natives. [Margin: Where did the name of the Magi come from?] The Syriac Gospel, too, calls them *Magusin*, and the Persian *Magusan*, that is, wise Astrologers. *Genebrardus*, however, on Psalm 1, thinks that this name springs from the root *Haga*, which in Hebrew means to Meditate; hence the Hebrew name *Magia*, that is, those who meditate or are deep in thought. For, as Ptolemy wisely said in the preface to the *Almagest*, *Meditation is the key of wisdom.* Or else this word also signifies the Silent, as the Author of the *Unfinished Work* on Matthew, homily 2, has it, because the Magi gave themselves, in the greatest quiet and silence, to the contemplation of Nature and of heavenly things, and afterwards came together among themselves to converse about the things they had learned by meditating; [Margin: Cicero.] as we have in Cicero, Book 1 *On Divination*: *And among the Persians the Magi take auguries and divine, who gather in a shrine for the sake of discussing and conversing among themselves. Nor can anyone be King of the Persians who has not first received the discipline and science of the Magi.* [Margin: The kings of the Persians steeped in Magic.] [Margin: Philo.] Not unlike this is what *Philo* attests in the book *On the Special Laws*: *That true Magic, the perspective science, by which the works of nature are seen more clearly, is pursued as honourable and worth seeking not only by commoners but also by kings, and the greatest of kings, especially the Persian ones, who are so curious of these arts that no one is permitted to reign unless he has been familiarly versed with the Magi.* [Margin: Eugubinus.] But I had almost forgotten *Eugubinus* [Agostino Steuco], Book 1 *On the Perennial Philosophy*, chapter 3, who says that the leaders of the Chaldean Theology were those whom they called in their native tongue Magi, as if to say wise men, or philosophers and Priests. There is also extant in *St Justin*, in the *Apology* against the Gentiles, that Oracle of Apollo:

Only the Chaldeans obtained wisdom, and the Hebrews with them.

Those whom the Babylonians called Chaldeans, the Persians called Magi, the Egyptians Hierophants, the Indians Brahmins, and the Etruscans Haruspices, as Strabo attests, Book 16, and Cicero, Book 1 *On Divination*. [Margin: Baronius. Salmeron. Maldonatus. Barradius. Suarez. Beauxamis.] Finally, very many of the more recent writers took this name in the better sense when speaking of these holy Magi, and above all *Baronius*, under the year 1 of Christ; *Salmeron*, volume 2, tractate 38; *Maldonatus* and *Cornelius a Lapide* on Matthew chapter 2; *Barradius*, volume 1 on the Gospels, Book 9, chapter 8; *Thomas Beauxamis*, volume 1 of the *Gospel Harmony*, from page 96; *Suarez*, on part 3, disputation 14, section 2; and very many others cited by them.

[Margin: The name of the Magi taken in a bad sense.] XIV. Granted, however, that the name of the Magi has had a bad reputation even in the sacred writings — since Magic is threefold, Natural, Artificial, and Superstitious, that is, infamous for commerce with demons, and it is easy to slip from the first to the second and from the second to the third unless one restrains, within the limits of sobriety and prudence, the curiosity of knowing secrets or of working marvels. So in Exodus 7, where those sorcerers of Pharaoh are called Magi by some translators. And in Leviticus 19 it is said: *Do not turn aside to the Magi, and do not inquire anything of soothsayers*; and in chapter 20: *The soul that turns aside to the Magi and to soothsayers, I will cut it off from the midst of my people.* Granted, too, that the Seventy Translators, at Exodus 7 and 8, Deuteronomy 18, Paralipomenon 33, Jeremiah 27, Daniel 2 and Malachi 3, were accustomed to call them φαρμακοῦς or φαρμακεῖς [Greek: pharmakous, pharmakeis, "sorcerers"]. For *Pliny the Elder*, Book 30, chapter 1, notes that the magical arts took their origin from Medicine, and grew through superstition and Mathesis to such a height that they gave commands to the kings of kings. And although our translator and the Seventy call the Magi observers of the stars, or interpreters of dreams, as in Daniel 1, 2, 4 and 5, they are nevertheless called by the same translators Pythons, or ἐργαστριμίθους [Greek: engastrimythous, "ventriloquists"], that is, ventriloquists. Munsterus [Sebastian Münster] too, in the Hebrew Gospel, chapter 2 of Matthew, put for Magi *Macassephim*, that is, enchanters or conjurors. [Margin: Saints Justin, Augustine, Basil, Ignatius, Thomas, Ambrose, Jerome, Hilary; Theophylact; Origen.] And *St Justin* in the Dialogue against Trypho, *St Augustine*, Sermon 2 on the Epiphany, *Origen*, Book 1 *Against Celsus*, *St Ignatius*, epistle 14, to the Ephesians, *St Basil*, homily on the human generation of Christ, *St Thomas*, part 3, question 36, article 3, reply to the 2nd, *St Ambrose* on Luke, the chapter on the Magi, *St Jerome* on chapter 19 of Isaiah, *St Hilary*, Book 4 *On the Trinity*, and *Theophylact* on Matthew 2, suspected that the three holy Magi had in fact once been such men. Of these, Origen relates that, having seen the portent of so unaccustomed a star, they consulted the demons; and when these, already struck dumb by Christ scarcely born, made no reply, they conjectured from this that he who was signified by the star was more powerful than the demons, and from that time resolved to adore him. So the demon was despoiled by the Infant JESUS of those very

persons whom he had plundered through wretched superstition, according to that saying of Isaiah: *Make haste to take away the spoils; hurry to plunder; for before the boy knows how to call his father and his mother, the strength of Damascus shall be taken away* — which St Justin, above, and St Augustine, Sermon 4 on the Epiphany, understood of the Magi, brought over from superstition to his worship. [Margin: Tertullian.] Noteworthy, however, is the sentence of Tertullian, in the book *On Idolatry*, chapter 9: *We know the alliance of Magic and Astrology with each other: it was the interpreters of the stars, therefore, who first announced that Christ was born.* For, as *Theophylact* says on Matthew chapter 2: *Since the Magi were Astrologers, the Lord for that reason drew them by a familiar sign*; and St Chrysostom, homily 6: *The Lord calls the Magi through the things that custom had made familiar to them, condescending to the salvation of men.*

XV. The *second* cause disposing the Magi to recognise through that star the recent birth of the King of the Jews was their foreknowledge of the ancient oracles: not only of the Samian Sibyl, prophesying of the Messiah:

Whom a Virgin, inviolate, will cherish in a human bosom. Heaven assents to this; the glowing stars show it.

nor only of the Erythraean Sibyl, in Book 8 of the *Oracles*, singing of the future as of a thing past:

And the Magi worshipped the divine new star, And to those who follow God's precepts the infant is shown In the manger.

— if indeed knowledge of these reached them — but also, either solely or chiefly, of that ancient Balaam, who had said, already 1500 years and more before, in Numbers 24: *I shall see him, but not now; I shall behold him, but not near: A STAR SHALL RISE out of Jacob, and a rod shall spring up from Israel, etc.* This Prophecy was handed down, either in written volumes or by tradition, to the descendants and successors of Balaam, who lived in Sabaea or Arabia, or Mesopotamia, or Persia (for those regions border on one another, and the Fathers and Doctors commonly say that the Magi set out from them). At length, when the fullness of time came and the Star rose out of Jacob — that is, the most holy Virgin MARY, and from her CHRIST, who were signified allegorically by that Star, while to the letter it was that Star which appeared in the East — the star was seen with such certainty to be the one Balaam had foretold that they said, as of a thing known among the Jews, *Where is he that is born King of the Jews? For we have seen his star in the East* — as if they were to say: For we have seen the star which is without doubt the index of no other than the King of the Jews newly born, and is no other than the one of which Balaam once prophesied. [Margin: Cyprian. Origen. Basil. Gregory of Nyssa. Jerome. Ambrose. Augustine. Leo. Anselm. Bede. Theophylact. Euthymius. Maximus. Remigius. Chrysostom. Thomas. Isidore. Abulensis. Lyranus. Maldonatus. Salmeron. Suarez. Maiolus. Chrysologus.] That the Magi were struck into this knowledge by Balaam's prophecy, and that they preserved it as his descendants, or as a particular memory in their records, very many Fathers teach most openly: above all *St Cyprian*, in the Sermon on the star and the Magi; *Origen*, homilies

13, 15 and 18 on Numbers; *St Basil*, in the oration on the generation of Christ; *St Gregory of Nyssa*, in the oration on the Nativity; *St Jerome*, on Matthew chapter 2 and on Isaiah chapter 47; *St Ambrose*, on Luke chapter 2; *St Leo*, Sermon 4 on the Epiphany; *Bede*, *Theophylact*, *Euthymius* and *St Anselm*, on Matthew chapter 2; and *St Maximus*, Sermon 3 on the Epiphany. The opinions of all these our Barradius recites one by one, volume 1 on the Gospels, Book 9, chapter 9, from page 149. The same is taught by *St Chrysostom*, volume 2, homily 62; *Blessed Remigius* on Matthew; *St Thomas*, part 3, question 36, article 5, reply to the 4th; *St Augustine*, Book 3 *On the Wonders of Holy Scripture*, and in the *Questions on the Testament*, question 39; *St Isidore*, in the book *On the Passion of the Lord*, chapter 13; *Abulensis*, on Matthew chapter 2, question 7; *Nycolaus Lyranus* [Nicholas of Lyra] in the *Gloss* on Matthew chapter 2; *Maldonatus* on Matthew chapter 2; *Salmeron*, volume 2, tractate 39; *Suarez*, on part 3, disputation 14, section 6; and *Simon Maiolus* [Simone Maioli], *Dog Days*, colloquy 1, page 15. Of all these, it suits our matter — which we are to treat afterwards, in the following chapter — to bring forward only the noteworthy words of St Peter Chrysologus, Sermon 156: *Not by Chaldean curiosity, not by the Magic art, but by Jewish prophecy*; and Sermon 157: *Not by the Chaldean art, but from the ancient tradition of the holy forefathers.* For granted that they were remotely disposed to judge that something great was portended by that star, they could nevertheless not divine from the rules of the casters of nativities [Genethliaci] what it signified, but only from their foreknowledge of the Prophecy of Balaam, with divine illumination above all supervening.

XVI. For the *third* and most powerful cause, which made plain to the minds of the Magi what that star signified, and drove their will to undertake the journey and to seek out the cradle of our Saviour, was without doubt the ray of divine illumination, as is most certain from the Fathers and from the common sense of Catholics: [Margin: St Leo.] *Divine inspiration without doubt working this in their hearts, so that the mystery of so great a vision should not be hidden from them, and what was shown to their eyes as unaccustomed should not be obscure to their minds* — these are the words of St Leo, Pope, Sermon 3 on the Epiphany. And the same writer, in Sermon 4: *But, as the event showed, the grace of God presided over this miracle, etc.*; and a little later: *O wonderful faith of perfect knowledge, which no earthly wisdom taught, but the Holy Spirit instructed! For from what source had those men — who, when they set out from their homeland, had not seen JESUS, nor had perceived by any sight of him anything to make them venerate him in so ordered a way — learned to observe this plan in the gifts they were to bring, unless it was because, besides that appearance of the star, which stirred their bodily gaze, a brighter ray of truth had taught their hearts through and through?* And in Sermon 5 the same Supreme Pontiff: *But this sign, etc., was without doubt the Sacrament of that grace, and the beginning of that calling, etc.* For in Sermon 1 he had said: *He therefore gave understanding to those who looked, he who supplied the sign; and what he made to be understood, he made to be sought.* [Margin: St John Chrysostom.] Before him, however, St Chrysostom, in Homily 62 on the Epiphany, which is found in

volume 2, attributes this knowledge first to the gift of divine condescension, and then to the prophecy of Balaam, and adds: *The splendour of this light, penetrating even the hearts of the Magi, flooded them with spiritual light, so that by the sign of a new star being born they should recognize the king of the Jews as the creator of heaven.* [Margin: Blessed Remigius.] Piously, too, and ingeniously, Blessed Remigius, on Matthew, teaches that they came from the East, that is, drawn by Christ through grace: *Beautifully are they said to have come from the East, because all who come to the Lord come from him and through him; for he himself is the East, according to the saying: Behold a man, the East is his name.* [Margin: Eusebius Emissenus.] To these let Eusebius Emissenus [Eusebius of Emesa] be added, in his homily on the Epiphany of the Lord: *It came about that, once the star was seen, they at once understood whatever was signified through it — not, however, by the certain demonstration of Astrology, but by that of the Holy Spirit. They were therefore seeking in the star Christ, whom they understood by divine inspiration to be signified through the star.*

[Margin: St Peter Chrysologus.] Yet at the first sight of the star they perhaps tried, like astrologers, to inquire what GOD signified by that sign; and at once, taught inwardly by heavenly light about the emptiness of their art, they received from God the occasion of their error as a remedy for their blindness. For that reason St Peter Chrysologus, in Sermon 157, himself answers someone questioning him: *Why Magi? Why a star? So that through Christ the very material of error might become the occasion of salvation, just as through Christ the cause of death was made the cause of life. To confound the enemy with his own sword is a singular mark of valour. Among the Magi the fault was ignorance, not will; and what lay under the charge was not their zeal but their error, in seeking and knowing God. To seek is a matter of necessity more than of guilt. It was therefore a loving act that God should both grant the seekers to find, and set the fault down to error. Finally Paul, by such a judgement, was made a preacher out of a persecutor, because through zeal for the law he was assailing the law, and was sinning against God out of love of God.* And just as St Paul became a preacher out of a persecutor, so these men were made Evangelists out of Magi. [Margin: St John Chrysostom.] This was the view of St John Chrysostom, Homily 16 of the various homilies on Matthew, addressing Christ the Lord thus: *You by your coming called the Magi from the East, and sent them back to their own people as Evangelists.* Thus that golden mouth of the Church. But the chapter must be closed with that other seal, golden as it were, namely that of [Margin: St Chrysologus again.] St Peter Chrysologus, Sermon 156: *From the East to the East the Magi come, so that he who had bidden them come might himself receive them as they came. For when would a Magus seek out God unless God bade him; when would an Astrologer have found the King of heaven unless God revealed him; when would a Chaldean, without God, adore the one God on earth — he who in heaven served as many gods as there were stars?* They had need, therefore, of God rising in their hearts; of God bidding, of God revealing.

CAPVT XX.

Chapter XX.

The Errors of Some concerning the Star of the Magi, but chiefly of the Casters of Nativities, are Reported and Refuted; where also of the Natal Horoscope of CHRIST erected with little piety by some, and of the Great Conjunction of the Planets that preceded this star.

I. If I am not mistaken, it is sufficiently established from the preceding chapter that the sense of the Catholic Church, or at least the more common opinion of the Fathers and Doctors, is that the origin and signification of the Star which appeared to the Magi must be acknowledged as plainly above the order of nature. Now we must refute those who, abusing the brightness and the gift of so great a light, have either impiously dared or imprudently tried to refer the causes of that star, or its force and signification, to some natural necessity. [Margin: 1st error, of the Priscillianists.] And in the first place the Priscillianists meet us. As St Gregory relates in Homily 10 on the Gospel, they said that this star had been the fate of Christ, and seized from it an occasion of confirming their error, namely that the Stars are the mistresses of nativities and of the things that depend on nativity. Yet GOD so ordered the appearance, motion, place and duration of this star that from these that error could rather be refuted — as in fact it was refuted by the holy Fathers. For the Stars from which the astrologers boast that they can predict the fortune or misfortunes of a newborn infant precede the infant's birth from the beginning of the world, and are placed in the heavens by the natural Providence of the Creator. They do not invert or change their course so as to move towards the place of an infant born or about to be born, nor do they accommodate themselves to his will or that of others; rather (as the astrologers indeed pretend) they incline or draw to themselves the wills and nature of others. Nor are they extinguished before they have worked out all the events of the child once born, up to his death; on the contrary, they persist in the heavens even after his death, so as to pour like characters of life, inclinations and morals into others who will be born under a like constitution of the stars. But everything was otherwise with this star of the Magi, as is clearly evident from what was said in the preceding chapter, where also, at number 4, we sufficiently showed that the star was supernatural, from Tycho himself, let alone from the Fathers. [Margin: 2nd error, of the Manichaeans.] A second error, akin to this one but tending elsewhere, was that of the Manichaeans. As is gathered from St Augustine, *Against Faustus the Manichaean*, Book 2, chapter 5, they abhorred Fate, and for that reason rejected the Gospel of St Matthew, because they thought that in its chapter 2, where the star and the Magi are treated, the fatal necessity of the stars is handed down and approved by him. Against both parties the arguments and opinions of the Fathers and Doctors of the Church must be set out distinctly, since they drew a medicine from this very star from which others drew the poison of error. [Margin: St Chrysologus's warning.] Otherwise, as St Chrysologus forewarned

on this very occasion in Sermon 156, *what is the material of life becomes the occasion of perdition*; and he at once adds: *We must take heed, brothers, that what has been divinely written for our advancement does not, through unskilfulness in hearing, turn to the detriment of souls. Do you think that the Evangelist today taught that Chaldeans, searchers of the stars; Magi wandering among the stars, investigating the affairs of heaven in the darkness of the nights; ascribing the causes of being born and of dying to the courses of the stars; establishing the goods or evils of men by the decision of the luminaries — that these found, with a star as guide, the rising of Christ hidden from the ages? Far from it: so the world thinks, so the gentiles understand, so the surface of the reading deceives. But the Gospel discourse speaks not human things but divine; not customary but new; not deceiving by art, but resting on truth; not playing to the eyes, but fixed in the hearts; not wavering on conjectures, but confirmed by authority; coming from God, not from fate; not gathered by numbers, but acquired by virtues.* [Margin: Enumeration of the arguments against the Priscillianists, etc.] Now we shall bring the arguments of the Fathers and Doctors against the Priscillianist casters of nativities and the Manichaeans under 8 heads. They are: 1. The independence of Christ, inasmuch as he is Lord and Creator of the star itself. 2. The service of the star towards Christ. 3. The compliance of the star, which moved at the nod of the Magi. 4. The place of the star. 5. The novelty and singularity of the star, without the company of others, or its supernatural origin. 6. Its signification of the nativity rather than of the events that follow from the nativity. 7. The miraculous Nativity and the deeds of Christ. 8. The supernatural force and manner of signifying by which the Star instructed the Magi.

II. [Margin: 1st argument: the independence of Christ from the Star.] The first argument, then, is taken from the Divinity of Christ, and so from his independence of that star and of all other creatures. For by the very fact that he was God, and Creator and Lord of it and of all the stars, no star could certainly be a cause influencing that person, which consists of both natures, the human and the divine; nor could it influence his operations, which theologians call θεανδρικὰς [Greek: theandrikas, "theandric"], that is, divine-human. [Margin: St Augustine.] St Augustine meant to indicate this to us in Book 2 *Against Faustus*, chapter 5. For when he had said: *And we indeed place the birth of no man under the fate of the stars, so that we may vindicate the free choice of the will, by which one lives well or badly, from every bond of necessity, on account of the just judgement of God* — for indeed, if a man sinned out of necessity, God would be unjust to condemn him by his own judgement — he at once adds: *How much less do we believe that the temporal generation of him who is the eternal creator and Lord of all things was brought about under the condition of the stars? Accordingly, that star which the Magi saw when Christ was born according to the flesh did not rule him by decree, but served him as a testimony; for it did not subject him by command, but pointed him out by its service.* [Margin: St Chrysologus.] The very same argument is that of *St Chrysologus* in Sermon 156. Having first said: *When Jesus is born, the origin of things arises; the bestower of kind is begotten; the author of*

nature is born, he then attends to the words *We have seen his star*, and teaches that through the particle *his* a peculiar dominion of Christ over this star is signified, saying: *He who made the whole creation, who holds and contains it, began to have one star. He who has a star is not had by the star; nor is he driven by the course of the star, but he himself drives the star's course.* And again, further on in the same sermon: *The star appeared not willing but commanded: not at the nod of heaven, but by the impulse of the divinity; not by the law of the stars, but by the novelty of Signs; not by the clime of heaven, but by the power of the one being born.* And in Sermon 157, driving home the same particle *his*, he says: *And what is not his? The star is his; he is not the star's: it received being, but did not give it. A Star whose rising its author held, not one that held the rising of its Author; coming not from a nativity but from the begetter, by command, not by fate; not from a calculator but from its author; not one that would give a rising, but one that would undergo a setting.* For, having discharged its office, it set, and appeared nowhere any more.

III. [Margin: 2nd argument, from the service of the star towards Christ.] The *second* argument, akin to the preceding one and following from it, is the motion and service that the star rendered to Christ, by which it professed itself a servant and handmaid to its Creator and Lord — so much so that it rather felt the power of Christ than exerted any on Christ's body, let alone on his soul or his divinity, which was utterly ἀδύνατον [?] [Greek: adynaton, "impossible"]. [Margin: St Thomas.] *St Thomas* uses this argument in Part 3, question 36, article 5, reply to 3. For when he has said: *The star, manifesting the nativity of Christ, removed every occasion of error* — since the other stars complete their courses and carry off with them, if it were right to believe the casters of nativities, the fates and fortunes of men, but it went otherwise with this star — [Margin: St Augustine.] the holy Doctor therefore aptly brings in the argument of *St Augustine*, taken from Book 2 *Against Faustus*, chapter 5, where he reasons thus: *What astrologers, pray, ever so placed the fates of men at birth under the stars as to assert that one of the stars, when some man was born, left the order of its circuit and went to the one who had been born? For they think that the lot of the one being born is bound to the order of the stars, not that the order of the stars can be changed on account of the day of a man's birth. Therefore, if that star was one of those that complete their orders in heaven, how could it decree what the newborn Christ was going to do, when it was itself ordered, at Christ's birth, to leave what it was doing? But if (as is more probably believed) it arose to point out Christ, having not existed before, then Christ was not born because it existed; rather, that new star arose because Christ was born.* Having concluded this most acute dilemma, he infers from it this conclusion: *Hence, if it had to be said, we would say not that the star was Christ's fate, but that Christ was the star's fate; for he brought it the cause of being born, not it him.* [Margin: Whether the Word of God can be called Fate?] On this occasion, with no less acuteness of mind, he suggests that Fate is nothing other than the Word of God — which is certainly most worth noting. He adds, therefore: *If, then, there are Fates, which are named a fando, that is, from speaking: since Christ is the Word of God, in whom all things were spoken before*

they existed, the company of the stars is not the fate of Christ, but Christ is the fate even of the stars. He also assumed the very flesh created under heaven by the same will by which he also created heaven; and he laid it down and took it back by the same power by which he also commanded the stars. [Margin: St Gregory.] From this source, indeed, *St Gregory*, Supreme Pontiff, seems to have drawn what he carried over into Homily 10 on the Gospels, which is on the Epiphany, when he said: *The Priscillianist heretics think that every man is born under the constitutions of the stars; and they take up in support of their error the fact that a new star came forth when the Lord appeared in the flesh, and they think that this same star which appeared was his fate. But if we weigh the words of the Gospel, in which it is said of that same star, Until, coming, it stood over where the child was — since the child did not run to the star, but the star to the child — then, if one may say so, the Star was not the child's fate, but the child who appeared was the Star's fate.*

IV. [Margin: 3rd argument, from the service of the star rendered to the Magi.] The *third* argument is from the service that the star rendered to the Magi, which *St Augustine* employed in Book 2 *Against Faustus the Manichaean*, when he said: *Accordingly this was not one of those Stars that from the beginning of creation keep the order of their journeys under the law of the Creator; but at the new birth from the Virgin a new star appeared, which rendered the ministry of its office even to the Magi themselves as they sought Christ, until, by going ahead of them, it led them to the very place where the Word of God was an infant.* [Margin: St Chrysologus.] Therefore, connecting the first argument with this third one, *St Chrysologus* says in Sermon 156: *Nor is he driven by the course of the Star, but he himself drives the Star's course; he so directs its course through the heaven, so governs its gait, so tempers its way, that it serves the step of the Magi and is sent to suit it. For when the Magus walks, the Star walks; when the Magus sits, the Star stands; when the Magus sleeps, the Star keeps watch. So the Magus perceives that those who share an equal condition of living share also an equal necessity of serving; and he no longer believes the Star a God, but judges it a fellow-servant, which he sees bound over in this way to his own services.* And no less acutely in Sermon 157: *This star is the minister of the way, not of life; the companion of the Magi, not the kinswoman of Christ; not the mistress of the Ruler, but the little handmaid of the servants.* Similar things are noted by *St Gregory of Nyssa* in the Sermon on the holy Nativity of Christ; by *Theophylact* and *Euthymius* on chapter 2 of Matthew, disputing against the casters of nativities; and by *Abulensis* on chapter 2 of Matthew, question 17.

V. [Margin: 4th argument, from the place of the star.] The *fourth* argument is taken from the place. For there was *a Star in the air, a Star on earth, a Sun in the manger*, as Blessed Peter Damian says, and as we have already shown in chapter 19, number 2. Accordingly Abulensis, on chapter 2 of Matthew, question 17, argues as follows. The star showing the nativity of Christ was not in the heaven but near the earth — a star of a kind by which the astrologers indicate nothing; indeed, astrologers never saw such a one, because it appeared only at the nativity of Christ, to lead the Magi to Bethlehem. Therefore this was

no occasion for thinking, against Christ, that his deeds came about naturally; rather it was a firm argument of his Deity. Thus far Abulensis.

VI. [Margin: 5th argument, from the singularity of the new star.] The *fifth* argument is from the singularity and novelty of the star, which appeared apart from the other stars and Planets, through whose company or mutual aspects the astrologers are accustomed to form a conjecture about the future. For Abulensis, on chapter 2 of Matthew, question 17, holds that if around the nativity of Christ, together with this star, some notable conjunction of the Planets had occurred in conspicuous places of the heaven, the Magi might perhaps have foreknown from it, by natural means, the Nativity or the deeds of Christ. But since nothing of the kind appeared then, and this star was seen near the earth apart from every other star, it could not give the Magi an occasion for judging such things about Christ by natural means — especially since no experience of such a star had gone before on which they could rely, as Cornelius a Lapide notes on chapter 2 of Matthew, verse 2.

VII. [Margin: 6th argument, from the signification, which presupposes the nativity.] The *sixth* argument is drawn from the Method that the casters of nativities use. For they presuppose that the moment of the child's birth is already known, and for that moment they investigate the Horoscope, so as to divine from it the other events that will befall the man; but they do not attempt to divine from the aspect of the stars that the one about whom they are concerned, or about whom they are questioned, is going to be born or is already born. *St Thomas* likewise uses this argument, Part 3, question 36, article 5, reply to 3, and took it from *St Chrysostom*, Homily 6 on Matthew. In that place, when he had said: *If Christ was born according to the law of the stars, how did he dissolve Astrology, destroy fate, take away this whole error entirely, and overthrow the whole workshop of this most vain art?*, after a few words he adds: *For it does not belong to the art of Astrology to recognize from the Stars those who are being born; on the contrary, it learns of the hour or point of those being born, and from that predicts what is going to befall them.* Therefore, since this star announced to the Magi one already born, they could not foresee the nativity of Christ from it by the astrological art.

VIII. [Margin: 7th argument, from the supernatural object.] The *seventh* argument we take from the supernatural object signified by this star, which involved many miracles: namely the Incarnation of the Word, a man to be born of a Virgin, and many other things done wonderfully at that time concerning Christ's body and soul, and afterwards done or to be done through Christ. But Astrology cannot exceed the limits of natural effects or events, since it does not reach with certainty even these. Our *Cornelius a Lapide* touches on this argument on chapter 2 of Matthew, verse 2.

IX. Finally, the *eighth* argument is to be sought from the supernatural manner by which the Magi learned from this star

[Margin: 8th argument, from the supernatural manner of knowing.] that Christ had been born. This manner was twofold: one depended on knowledge

of the prophecy of Balaam, which was certainly not natural, nor contained within the natural object of the Genethliac Art; the other proceeded from the inner revelation and inspiration of the Holy Spirit. Both of these remain sufficiently proved from the Fathers by what we said in the preceding chapter, numbers 15 and 16. [Margin: St Chrysologus.] For that reason, various sayings of *St Chrysologus* in Sermon 156 hold good of knowledge of this kind through this star: *Not deceiving by art, but resting on truth; not gathered by numbers, but acquired by virtues; not the science of the Astrologer, but the foreknowledge of the Maker; not by arithmetical reckoning, but by divine sanction; by heavenly provision, not by Chaldean curiosity; not by magic art, but by Jewish prophecy.* Nor does *St Augustine* stand in the way of this argument, in Book 3 *On the Wonders of Holy Scripture*, chapter 4 — if indeed the work is St Augustine's — where he seems obscurely to concede that the Magi could have attempted that knowledge by the art of Astrology; [Margin: St Jerome explained, and the objection solved.] nor does *St Jerome*, on chapter 47 of Isaiah, when he says: *The Magi came from the East, saying that they had seen the star of the Lord, either from the science of their art or from the prophecy of their prophet Balaam.* For he speaks only disjunctively, and in the first member he speaks to suit the understanding of the common people, or with regard to that remote disposition by which, through the use of Astrology, they could judge in general that the star portended something great; but he does not speak of the particular and determinate nativity of Christ, for this they learned partly from the prophecy of Balaam, partly from divine illumination. So, together with the holy Fathers, teach *Salmeron* [Alfonso Salmerón], volume 2, tractate 39; *Abulensis*, question 17 on chapter 2 of Matthew; *Thomas Beauxamis* in the *Gospel Harmony*, from page 99; *Suarez* on Part 3, disputation 14, section 6; *Cornelius a Lapide* on chapter 2 of Matthew; and others cited by them.

De illis & contra illos, qui Geneſim & Natalitium Thema Chriſti auſi ſunt erigere, ac de Chriſto ex legibus Aſtrologiæ iudicium ferre.

On those, and against those, who dared to erect the nativity and natal horoscope of Christ, and to pass judgement on Christ from the laws of Astrology.

X. We have now seen the decisions and Sanctions of the Fathers, which ought certainly to have moved every pious man so that, even if he knew the moment of the Nativity of Christ the Lord our King with more than certainty, and even if most splendid syzygies of the stars and concurrences of aspects had occurred at that moment, he would still think that he must refrain from passing judgement on the admirable prerogatives and works of Christ — lest he either offend pious ears, or offer the sect of the casters of nativities, which begs reputation from every quarter beyond measure to spend on the emptiness of its art, so noble a handle for erring and so specious an opportunity for soothsaying. But, alas, not a few have been found who tried to found a tower of Babylon in the very cave of the Lord's most holy Manger. And what kind of men, I ask? For I am suffused with a blush even at their purple, men who themselves stained their own purple with this blot of vanity: [Margin: 3rd error, of those

who configured the horoscope of the nativity of Christ. Cusanus. Alliacensis. Gauricus. Bacchon.] namely two Cardinals, in other respects most learned, *Nicolaus Cusanus* [Nikolaus von Kues] and *Petrus de Alliaco* [Pierre d'Ailly], Cardinal and Archbishop of Cambrai, in question 30 on Genesis and in the book *On Laws and Sects*. *Lucas Gauricus* [Luca Gaurico] dared the same in his *New Ecclesiastical Calendar*, page 25; he also brings forward the Cambrai cardinal's figure, or planetary constitution, as agreeing with the Nativity of Christ; and, as reported by him, *Rogerius Bacchon* [Roger Bacon] did the same, and finally *Hieronymus Cardanus* [Girolamo Cardano] in Book 2 of Ptolemy *On Judgements*, page 163. [Margin: Cardanus. Albumasar. Albertus Magnus.] In this vain curiosity, however, *Albumasar* [Abū Maʿshar] went before them, in his *Greater Introduction*, tractate 6, from which *Albertus Magnus* drew, in the book of the *Mirror*, the things we shall presently report. But against Abū Maʿshar and Alliacensis, in this horoscope of Christ, *Io. Picus Mirandulanus* [Giovanni Pico della Mirandola] rose up sharply, Book 5 *Against the Astrologers*, chapter 14, and *Sixtus Senensis* [Sisto da Siena] in his *Bibliotheca sancta*, Book 6, Annotation 10; and some things, though only in passing, are indicated by *Salmeron*, volume 2, tractate 39, page 357; *Alexander de Angelis* [Alessandro de Angelis], Book 4 *Against the Astrologers*, chapter 24; *Suarez*, Part 3, disputation 14, section 6; *Cornelius a Lapide* on chapter 2 of Matthew, verse 2; and *Tycho*, volume 1, page 326. But let us begin with Abū Maʿshar.

XI. [Margin: The opinion of Abū Maʿshar and Albertus Magnus on the Ascendant of Christ.] The words of Abū Maʿshar, as reported by Albertus Magnus, Sisto da Siena and Salmeron, are these: *There rises in the first face of that Sign a Virgin, beautiful and honourable, holding two ears of grain in her hand; and she nurses a boy, and a certain nation calls this boy Jesus; and with her rises the star of the Virgin.* From these words, and from the calculation of I know not whom, or from I know not what tables, Albertus Magnus in the *Mirror* — if indeed it is by Albertus (for learned men, says Salmeron, and Sisto too, deny this) — deduced that the Lord Jesus Christ was born under the ascendant of Virgo, namely with ♍ 8° 30′ ascending at the point of the nativity, as he himself computes. Then he adds: *Not that the most longed-for of those born was subject to the motion of the stars or to their judgement, he who had created the Stars themselves; but because, when he stretched out the heavens like a skin, forming the book of the universe, he did not wish to be lacking from its letters any of the things which, according to his providence, are written in the book of eternity — even that most elegant thing in nature, that he would be born of a Virgin; so that by this it should indeed be intimated that he was a man of flesh, and true, who was not born naturally. Not that the figure of the heaven was the cause why he was born, but rather its signification; indeed, and more truly than true, he himself was the cause why the manner of his wonderful nativity was signified by the heaven.* [Margin: Refutation of this opinion.] But not even so will whoever is the author of that book escape the censure of rashness or of vanity. For if before Christ the Lord some other person was born under that ascendant, namely under the 8th

degree or some other degree of Virgo, either he was born of a virgin — and this is false, for it would detract from the singular privilege that both Christ and his immaculate Mother had, of whom the Fathers and the Church sing in the Lauds of the Nativity of the Lord, antiphon 2: *She who gave birth bore the King whose name is eternal, having the joys of a mother with the honour of virginity: she is seen to have had none like her before, nor to have one to follow, alleluia.* And this is entirely Catholic dogma: that it was granted to the sole and only Virgin MARY to be a mother, and to this mother alone to be a Virgin before giving birth, in giving birth and after giving birth. As a signal miracle granted, or to be granted, to no other, the holy Fathers and the interpreters of the sacred codex everywhere proclaim and venerate the fruitfulness of motherhood together with the intactness of Virginity, especially at that oracle of Isaiah, chapter 7: *Therefore the Lord himself will give you a sign: Behold, a VIRGIN shall conceive and bear a son, and his name shall be called Emmanuel.* This sign would certainly not have been infallible, and notable above the rest, unless it had been unique and singular. But if no one born before Christ under the degree of Virgo was so born as to be born of a Virgin, then that Astrological rule likewise could be founded on no experience, and accordingly could have no rationale of the art in this case; nor could God, through that point of the heaven taken precisely and by itself, have signified determinately to astrologers, as if by some character, that the one born under that horoscope would be born of a Virgin, since he had decreed from eternity that no one else should be so born. I pass over now what *St Augustine* argued most eloquently, in Book 5 of *The City of God*, chapter 1, against that subterfuge of the astrologers by which they say that the parts of the Zodiac and the stars are not indeed causes, but signs and characters of future things; for our business now is not to show the fallacy of judicial Astrology in general.

XII. [Margin: The opinion of Petrus Alliacensis on the nativity of Christ.] After Albertus Magnus, or some other spurious author of that *Mirror*, Petrus Alliacensis — Archbishop of Cambrai, Cardinal of the Holy Roman Church, once Chancellor of the University of Paris, and teacher of Ioannes Gerson [Jean Gerson] — flourished and was distinguished at the Council of Constance in the year of the Lord 1416, 136 years after the death of Albertus Magnus. In question 30 on Genesis and in the book *On Laws and Sects*, he affirms that the Magi, as men most skilled in Astrology, could have foreknown from the position of the stars the future nativity and life of Christ, though not with the certainty with which they afterwards knew of Christ's rising when the New star showed it beforehand. For although, he says, the conception and Nativity of Christ surpass in most respects all the powers of the heavens and all the faculties of nature, yet in some respects the natures of the orbs and of the stars, serving their lord like handmaids, supplied the same Christ abundantly at his birth with as many friendly lights and benign influences as they had received from Christ, God. Accordingly he says that it was possible for the Magi, besides this appearance of the star, to have foreknown by two other indications that

a man illustrious in religion, empire and glory would be born. First, from a certain great conjunction of Jupiter and Saturn, which came together in the sixth year before Christ in the sign of Cancer, all the astronomers of those times predicting that a great change of religion was portended. Then, from the observation of the configuration of the heavens that Christ's nativity itself had. That nativity fell in the 42nd year of Octavius Augustus, on the eighth day before the Kalends of January [December 25], before the middle of the night. In it the Horoscope was the eighth part of Virgo, which signifies a change of religion; Saturn stood at the summit of the heavens, the Sun held the lowest point of the heavens, and the other planets were disposed in a certain order. These are precisely the words of Pierre d'Ailly, although here they are not marked in cursive type, as the saying goes. He sets the figure and configuration of the stars for the horizon of Jerusalem, to which he assigns a latitude, or altitude of the pole, of 36°, and for the 24th day of December at $10^h\ 4^m$ after noon, that is, $1^h\ 56^m$ before midnight. Luca Gaurico transferred this into his *Calendar*, page 26, more correctly than Sixtus Senensis did; and its characters and notes are as follows.

Christ's nativity theme from Pierre d'Ailly, corrected by Gaurico.

House	Beginning of the house	Planet	Place of the planet	Planet	House of the planet
1	♍ 7° 40′	☉	♒ 2° 46′	☉	4
2	♎ 3° 25′	☾	♉ 2° 2′	☾	9
3	♏ 2° 0′	☊ ☾	♐ 9° 3′	☊	4
4	♐ 5° 48′	♅ ☾	♊ 9° 3′	♅	10
5	♒ 10° 52′	♄ ℞	♊ 14° 10′	♄	10
6	♓ 11° 53′	♃	♎ 7° 41′	♃	2
7	♓ 7° 40′	♂	♈ 10° 22′	♂	8
8	♈ 3° 25′	♀	♓ 18° 43′	♀	6
9	♉ 2° 0′	☿	♒ 6° 50′	☿	4
10	♊ 5° 48′	Part of Fortune	♉ 7° 39′	(roundel of dots)	9
11	♋ 10° 52′				
12	♌ 11° 53′				

XIII. In this nativity scheme, however, besides the objections we shall raise a little further on against all such themes, the following faults are noted in particular. [Margin: Faults of this theme.] First, as *Pico* observes (book 5, chapter 14), the Lord JESUS was not born with Virgo ascending. Granting the tables of the astronomers, that could not have happened on the horizon of Bethlehem except about two hours before the midnight preceding December 25; and yet by the tradition of the Church he was born either at midnight, as St Anselm holds in the *Elucidarium*, and Pico too, [Margin: The hour of Christ's nativity.] or a little after midnight, so that the Lord's nativity belongs to the 25th day of December rather than to the 24th. That is the view of *Caietanus* [Cajetan] on chapter 2 of Luke, of *Baronius* [Cesare Baronio] in the *Apparatus*

to the *Annales*, of *Cornelius a Lapide* on chapter 2 of Luke, verse 8, and of *Suarez* on Part 3, question 35, disputation 12, section 4. Suarez adds that the nearer to the point of midnight the Lord is asserted to have been born — provided he is said to have been born after it — the better this agrees with those words of Wisdom 18: *While quiet silence held all things, and the night was in the middle of its course, your almighty word leapt down from heaven, from the royal seats, a stern warrior, into the midst of the land of destruction.* With very few changes from another version, the Church has adapted these words to this night.

Petrus Blesensis [Pierre de Blois], however, in Sermon 6 on the Nativity of the Lord, expounds that passage of Wisdom on the Lord's nativity by saying that he was born in the *conticinium* of the night, which is after cockcrow and just before dawn. For that is how *Macrobius* (book 1, chapter 3) and *Censorinus* (*De die natali*, chapter 10) take *conticinium*: namely, the time when the cocks fall silent and men are at rest. And *St Augustine*, on Psalm 109, and *Tertullian*, in book 5 *Against Marcion*, chapter 9, confirm that same hour of Christ's nativity, a little before dawn, from those words of Psalm 109: *From the womb, before the day star, I begot you.* And *Isichius*, as cited by Baronius, says that he was born at hour 3 — understand: after midnight, that is, at the beginning of the third watch of the night. Such were the watches the shepherds were keeping, to whom the angel appeared when the Lord was born, as is related in Luke 2. Granted that *Anastasius Nicenus*, a bishop, in his book of questions on Scripture, question 92, says that he was born at hour 7 — which Suarez interprets rather of the morning hour — and granted that the antiphon the Church sings on the vigil of the Nativity seems to favour Anastasius: *When the Sun has risen from heaven, you will see the King of kings proceeding from the Father, like a bridegroom from his chamber.* Nevertheless the Church, by celebrating the third Mass at dawn, indicates that this business was already accomplished by then, and it is certain that Christ was born at night. For where St Luke said, *And she brought forth her firstborn son, etc.*, he at once added, *And there were shepherds in the same region, watching and keeping the watches of the night over their flock.* Those words, then, *when the Sun has risen*, can be understood of the Sun having already passed midnight, in the way that in the hymn of Prime, which is very often sung before sunrise, *Now that the star of light has risen* is sung without falsehood.

Further, it is clear from what has been said that the moment at which Christ was born is uncertain — something GOD perhaps willed to remain hidden, so that, if by nothing else, by this at least the foundation of genethliacal construction might be withdrawn from the astrologers. It is clear, too, that it is more probable that his birth took place either at midnight or after midnight before sunrise, but in no way probable that it took place before midnight. For no one has followed Anastasius Sinaita, who says in book 7 of the *Hexaemeron* that he was born at evening — unless by evening he understood the whole night and any part of the night, according to that phrase of Genesis 1: *And there was evening and morning, one day.* So much for the first and chief fault of this

theme. In it, moreover, d'Ailly assumed the altitude of the pole at Jerusalem to be 36°, whereas it is only 32° 10′, as Gaurico too assumes.

XIV. Accordingly *Luca Gaurico*, on page 25 of his *New Calendar*, assuming the altitude of the pole at Jerusalem to be 32° and the longitude from the Fortunate Isles 66°, computed from the Alphonsine Tables the configuration of the heavens for the year of the World 5200, for December 24 and the 12th hour after noon of that day, which was the point of midnight. [Margin: Gaurico's nativity theme for Christ.] He drew up the nativity scheme of Christ, in which the cusp of the first house — the Ascendant and Horoscope — is not Virgo 8° but Libra 2° 35′. The rest stand as in the following small table.

Christ's nativity theme according to Gaurico.

House	Beginning of the house	Planet	Place of the planet	Planet	House of the planet	Planet	Planet	As-pect
1	♎ 2° 35′	☉	♍ 2° 52′	☉	4	♄	♂	✶
2	♎ 28° 37′	☾	♉ 2° 53′	☾	8	♄	♀	△
3	♏ 28° 32′	♄	♊ 14° 10′	♄	9	♃	♂	☍
4	♍ 2° 45′	♃	♎ 7° 41′	♃	1	♃	☿	□
5	♒ 6° 54′	♂	♈ 10° 24′	♂	7	♂	☿	□
6	♓ 6° 25′	♀	♒ 18° 43′	♀	5	☉	☿	☌
7	♈ 2° 35′	☿	♍ 6° 52′	☿	4	☉	☾	△
8	♈ 28° 37′	Part of Fortune	♍ 2° 37′	(roundel of dots)	4	☿	☾	△
9	♉ 28° 32′							
10	♋ 2° 45′							
11	♌ 6° 54′							
12	♍ 6° 25′							

XV. But *Girolamo Cardano*, commenting on book 2 of Ptolemy's *De astrorum iudiciis*, text 54 — where Ptolemy, speaking of hairy stars, had said among other things: *When they are matutine and have shone for a long time, they will signify swiftness; if vespertine, slowness* — takes occasion from these and other words to propose the nativity theme of Christ as the greatest and most notable example of the precepts he had set out at length in expounding that text. I shall report, almost in his own words, what he teaches there, partly rightly and partly not. First, then, he says that so many and such wonderful things shine forth in the nativity of Jesus Christ that, if one had to rely on natural reasons alone, the nature of Christ would have been as admirable as could be devised from the concourse of all the heavens; [Margin: Praises of Christ and of the Christian law, from Cardano.] and that, naturally, our law is a law of piety, justice, faith, simplicity and charity, excellently established, and will have no end except after the return of the ecliptics. Then he declares as follows: *Do not, however, believe that I mean to say that either the divinity in Christ, or his miracles, or the holiness of his life, or the promulgation of his law depends on the stars. Rather, just as nature adorned him with the best temperament, with beauty of body, perpetual health and gravity of countenance — all of which are gathered almost entirely from*

the testimony of Josephus in his book On the Jewish War — so also God, the Best and glorious, adorned his nativity with the best and an admirable configuration of the stars. Had Cardano kept within the natural endowments of the body, many of his statements might perhaps be tolerated. But not content with these, he adds: *Not everything in his geniture would have been so singular, magnificent, glorious and worthy of so great a concourse, and moreover so congruent with everything that followed — the holiness of his life, the gravity of his conduct, his wisdom, his persecution, the promulgation of the best law, the majesty and concourse of peoples, and finally his death and the time of his death — that nothing more exact could be devised.* Relying on this confidence, then, Cardano judged that this nativity should be published, as though complying with the divine will. He had constructed it twenty years and more before, but had not dared to publish it, out of religious scruple, lest he should seem to co-opt so great a majesty of light into the number of the others. Now, if anyone wishes to know that position of the stars and planets — to reject it rather than to approve it — for the 12th hour after noon of December 24 and an altitude of the pole of 32°, here is its synopsis in the following small table.

Christ's nativity theme from Cardano.

House	Beginning of the house	Planet	Place of the planet	Planet	House of the planet
1	♎ 2° 43′	☉	♑ 2° 53′	☉	4
2	♏ 3° 0′	☾	♈ 28° 5′	☾	7
3	♐ 3° 0′	♄	♊ 14° 17′	♄	9
4	♑ 2° 53′	♃	♎ 8° 3′	♃	1
5	♒ 3° 0′	♂	♈ 9° 56′	♂	7
6	♓ 3° 0′	♀	♒ 18° 36′	♀	5
7	♈ 3° 43′	☿	♑ 6° 38′	☿	4
8	♉ 3° 0′	(roundel of dots)	♑ 27° 55′	(roundel of dots)	2
9	♊ 3° 0′				
10	♋ 2° 53′				
11	♌ 3° 0′				
12	♍ 3° 0′				

Cardano goes on to say that there has been no more illustrious law in the whole world than Christ's, whether you consider its extent, which has embraced the whole world, or its duration, by which it had by then persisted for 1,520 years, or that it was promulgated against the Caesars, the lords of the world, or — what is greater than all — against vices. For it would have been easy for Mahomet to drag men prone to vice along with him to destruction, by attaching laws of laxity to them as if with a hook; *but to fight against the inclination of nature, and against human levity and greed — this alone is divine and the gift of God.* Having laid this down, he descends to the particular prerogatives of this nativity, drawn from the configuration of the stars. I would not be reluctant to report them, if it were permitted; but they are full of vanity, contrary to the

decrees of the Church and the sanctions of the saints, and put forward under the appearance of piety with too little piety or too little prudence. Such are these: that the Christian religion will last at least 36,000 years, because at that moment there was a conjunction of the heads of Libra of the ninth and the eighth orb at the intersection of the ecliptics; that the most brilliant comet — for so he calls the new star of the Magi — signifies the disturbance of the whole world and the most celebrated name of Christ; and that Jupiter in the Ascendant signifies wisdom, sweetness and eloquence, by force of which he disputed in the temple at twelve years of age. Finally, he affirms that from the position of the houses and planets one could have foreknown Christ's wisdom, his majesty, his endurance of evils, his zeal for overturning the law in which he was born — that is, the Mosaic — his poverty, his flight, his public but glorious death, and things like these.

XVI. But Tycho rightly says this in volume 1 of the *Progymnasmata*, page 325, speaking of the star of the Magi: [Margin: Tycho's censure of Cardano.] *There were indeed some who did not hesitate to assert that this star which appeared to the Magi was a comet. Among them is Cardano, who places it in the Ascendant for CHRIST at his birth, as though it were the same as what the Evangelist calls the star seen by the Magi in the East. But he and the rest of his followers assert this more impiously than with just reason, however they may in the end excuse it — to say nothing of the rest (for I am ashamed to report it) that he inserted on this matter into his astrological commentaries.*

It is not repugnant to the Faith to assert that Christ had either Virgo or Libra in the Horoscope, or Ascendant, together with splendid and benign aspects of the stars, as Salmeron teaches in volume 2, tractate 39, page 357, column 1, with *Cornelius a Lapide* subscribing, on chapter 2 of Matthew. [Margin: Salmeron's censure.] Yet, as the same Salmeron asserts in the same place, to wish to predict from them — whether as predominant causes or as infallible signs — things that depend on the freedom of God and of men, or that are supernatural, is foreign to the sacred Scriptures, and was condemned by the Church in the Priscillianists and others, and most recently by the holy Synod of Trent, at which he himself had been present. Suarez, moreover, on Part 3, volume 2, disputation 14, section 16, says that it is heretical to subject human freedom to the fatal necessity of the stars: [Margin: Suarez's censure.] *And it is much more absurd,* he says, *and more* intolenabilius [?], *to think this about the divine works and the supernatural mysteries, the order to which Christ's conception and nativity belong.*

For the birth of Christ from a Virgin; the most perfect organization of his most sacred body in the first instant of conception; the very many endowments of body and soul; the hypostatic union of the two natures, human with divine, in one and the same Person of the divine Word in the same first instant; the divine knowledge, beatific and infused of itself; the power of all miracles and of the supreme supernatural law; and the highest holiness, with the incapacity to sin, with which the Holy of Holies was anointed — [Margin: Our censure, and argument 1.] these were wholly the work of the Holy Spirit. No natural

disposition could contribute anything to them that would include any connatural claim or exigency, as I assume from what we have said in the treatise *On Grace* against the Pelagians and Semi-Pelagians. Next, either no one has ever had such a nativity, or someone has. If you say someone has had it, you are a blasphemer; [Margin: Argument 2.] for no one has had — I will not say all, but not even one — of those singular prerogatives that we have just pointed out in Christ, namely birth from a Virgin Mother, etc., granted that someone may have received by divine gift some small portion of some heavenly gifts. If no one has had it, then the rules of astrology could not have been founded on any experience, so that these things about Christ could be foreknown from the aspect of the stars. Further, it is most certain that judicial astrology, in the part in which it tries to predict, from the position of the stars, with certainty or with probability, future contingents that presuppose the freedom of God and of men, [Margin: Argument 3.] is opposed to the divine precepts and to the decrees of the Church, and so to Christ himself, who through his Church has so often condemned the casters of nativities. It is therefore not reasonable to believe that God so ordered the concourse of the stars that he would kindle the light for the Word to be made flesh from those half-dead sparks, and fetch splendour for that sacrosanct Majesty, as for the Sun, from those little fires. Otherwise St Chrysostom would exclaim here again, in Homily 6 on Matthew: *If Christ was born according to the law of the stars, how did he dissolve astrology and destroy fate, remove the whole error utterly, and overturn the entire workshop of this most vain art?* What of the fact that to this end too, unless I am mistaken, un- certain he willed the year of his nativity to remain even now [Margin: Argument 4.] (as will be clear from the opinions of all who have written on this matter, collected and weighed in book 3 of our *Chronology*), so that he might withdraw from the casters of nativities every foundation for building a nativity theme and for fitting it to the Nativity of his Son? Nor indeed did he who came to condemn the pomps of this world, who shrank from all pride of birthday display, and who sought out everything humble and most abject for his nativity, aspire to the nobility of a natural nativity drawn from the configuration of the stars. [Margin: Argument 5.] Rather, he either neglected it or concealed it, just as if he then had no stars serving him except one, new and soon to perish. In this he showed us a certain example of humility and poverty, as *St Peter Chrysologus* observes with most acute piety in Sermon 156. [Margin: St Chrysologus.] For, explaining the words *We have seen his star*, he draws out this point too among many: *Truly, as the Apostle said, though he was rich, he became poor: though he was rich in his Godhead, he becomes poor in our flesh; and he began to have one star — he who made the whole creation, and has it and holds it.* As for the arguments of Pico, book 5, chapter 14, against the astrologers who chatter about the nativity theme of Christ — arguments drawn from the astrologers' own rules — we have decided not to copy them out here, lest I should become too long-winded, or we should seem to prick with lighter goads those whom we judge must be slain and run through with more solid and stronger weapons.

CAPVT XX.

De Alijs Authoribus, qui Stella Magorum abuſi ſunt.

On other authors who have misused the star of the Magi.

XVII. Among the others who have thought and written about this star of the Magi less correctly than they ought, the first to present himself is *Cornelius Gemma*, in his writing on the star of Cassiopeia and in his book *De divinis Mundi characterismis*. [Margin: Cornelius Gemma's error about the star of the Magi.] He affirmed that this star too was similar to the new star seen in Cassiopeia in the year of the Lord 1572, and very high above the Earth; because, he says, if it had been in the air above Judaea, it could not have been seen in Persia, and it would rather have had the appearance of an ignited exhalation than of a star. But he also invents I know not what about the apparition of angels and of God in the form of a star, and about an ascent and descent and a circular return proper to divine minds, fabricated from Platonic fables — things which even *Tycho* himself, volume 1, page 564, thinks unworthy of being raked up again. He also adduces Nicephorus as favouring his fictions; but how wrongly will soon be made plain. Tycho, in volume 1, pages 555 and 564, criticizes him more than once, and not undeservedly. Tycho teaches that the star of the Magi had nothing in common with the new star of Cassiopeia except the appearance of a star; that it appeared to the Magi not in Judaea but in Persia, or in some other region lying to the east of Judaea itself; and that it led the Magi all the way to the house in which the child Christ was — which could by no means have happened had it not been in a place close to the Earth.

Further, Nicephorus Callistus, who flourished around the 600th year of the Lord, in book 1 of his *Ecclesiastical History*, chapter 13, [Margin: Nicephorus's opinion on the star and the Magi.] speaks thus of the Magi, whom he calls Persians, and of their star: *For they had seen a star in their own region — not one of those that from the beginning of things have been fixed to the firmament of heaven, nor one of those that by a prodigy, and as if by a riddle, foretell the successions of kings, and are named, according to the variety of their forms, casks, bearded stars, beams, javelins and comets; the origin of all of which, from exhalations, takes its substance above the air.* And shortly after: *They therefore did not see a star of that kind, but one quite new and unusual, inclining towards the Earth and shining very brightly, very like the other stars of heaven, and conspicuous and apparent to none but those who had devoted their efforts to knowing the stars.* Gemma ought therefore to have noted those words, *inclining towards the Earth*. This at least must be understood of the time when, after they had left Jerusalem, it pointed out to them the cave of the Saviour; otherwise it was so like the stars in form that to those unskilled in astronomy it could be thought one of the ancient stars of the first magnitude, whereas to the Magi, who knew the ancient stars and had counted them, it appeared plainly new. Nicephorus goes on, and after saying that they were greatly helped by the oracle of Balaam, he adds: *But above all, since GOD himself was rousing their minds to this, they therefore understood that by the providence of CHRIST, the supreme King of all, now born, the star was being moved from Persia towards Judaea just like some animate force participating*

in intelligence; and, using it as a guide for the way, they followed behind it, setting everything else aside. Here he by no means says that it was absolutely animate, or that some divine Intelligence slipped into their minds through that star; rather, he spoke metaphorically, or at most he thought that the motion of that star came from some Intelligence.

XVIII. In the second place we strike against *Théodore de Bèze*, the most wicked follower of Calvin. Tycho, in volume 1 of the *Progymnasmata*, page 327, calls him *a man noble in birth and learning, distinguished and excellent, who, if anyone in this age, has deserved splendidly of letters both sacred and philosophical*; in doing so Tycho deservedly renders himself suspect of Calvinism, [Margin: Tycho suspect of Calvinism and of Lutheranism alike.] although *Ioannes Heckius*, perhaps on other grounds and from more intimate knowledge, calls him an outright Calvinist in his book on the new star, page 20. Moreover, the same Tycho, in volume 1, page 712, pours himself out in praises of David Chytraeus and of Philipp Melanchthon — a most pestilent man, and, as his surname sounds, of the blackest faith — and extols them to the skies even in theological matters; and on page 777 he champions Luther with excessive zeal, regrets that he himself and the followers of Luther were reproached by Theodor Graminaeus, and says: *I wished at least to point this out in passing, so that it might be clear how unjustly and maliciously Graminaeus, even on a frivolous occasion of no moment, censures and cavils at Luther and his fellow-tribesmen.* In all this he shows himself a supporter of Luther no less than of Calvin. Nor did Tycho clear himself sufficiently of this suspicion by appending that protestation: [Margin: Tycho's protestation insufficient.] *Nor do I say this because I wish to interpose a judgement on the controversies of religions: let him judge them who alone knows how to distinguish the true from the false, to search the hearts of all, and to decide all things without prejudice.* For it is the part of a Catholic man not to praise heresiarchs in matters of religion or of sacred letters, but rather to depress their authority as far as can be done consistently with the truth. And then, why does he appeal to God alone in controversies of faith, as though in the Church there were no visible judge of these controversies, and God alone, and not the supreme Roman Pontiff and Vicar of Christ, were their infallible definer? But let this have been said by us in passing.

Tycho, then, in volume 1, page 327, as I said, reports the opinion of Bèze, who says that the new star of Cassiopeia of the year 1572 was a comet, and indeed the very same one that once led the Magi to Christ and that now shines before the second coming of Christ and the end of the World — granted that Tycho says Bèze did not affirm this seriously, but composed it with poetic playfulness. For Bèze played thus, boldly and with little piety: [Margin: Bèze's verses on the star of the year 1572.]

This new comet, with no furious hair, and radiant, whose splendour shines with pure fire — what it portends to the lands, that GOD of gods knows, and the fates will show it in their own time. But if human minds can praescissere [?] *anything, it is no sin for me to search out such signs: this is he who once led the Magi from the eastern*

world to the little city of David; and the same that shone before him at his birth, now returned, announces: Behold, GOD is here again, returning. To him, then, O happy throng of the pious, give applause; but you, bloody Herod, fear.

That the star of the Magi was not a comet is, however, clear enough from what was said in chapter 19, number 3; and no one fails to see that it cannot be asserted to have been the same as the new star of the year 1572 without the mark of rashness and of profane novelty, since that would be without the foundation of any authority or probable reason. Far more cautious was *Wilhelm, Landgrave of Hesse*, who compared the new star of the year 1572 with the star of the Magi in this respect, that each foretold the coming of Christ, but nevertheless said that it was different from it. [Margin: The Landgrave of Hesse's opinion on the new star of the year 1572.] For his words, in a letter of 1572 December 14 written to Caspar Peucer and reported by Tycho in volume 1, page 600, are these: *We cannot conclude that it is a comet, because of its great brightness and because it lacks motion — for it has so far stood still at one point of the eighth sphere — and then because it is set so high, outside the elementary region. Nor can we easily assert that God created some recent star, since nothing of the kind has been heard of since the founding of the world.* Understand this of a star properly so called, which is to be numbered among the Fixed Stars and is to last with them for ever afterwards. For he adds: *Moreover, in our judgement, its size and colour are falling away somewhat, little by little, so that it seems likely that it will soon come to an end.* He was writing this, as I said, on 1572 December 14; yet the new star did not vanish until 1574. He concludes as follows: *So far, then, as we are able to judge, we consider it a remarkable miracle, and indeed one of the number of those that are to precede the last day. For just as the Lord God willed that the first coming of his Son, our Lord Christ, should be signified beforehand by a star and announced to the Magi, so we hope that by this one too He will foretell the last coming of the Lord Christ.*

XIX. There is also an opinion, of I know not what delay [?], held by *Andreas Rosa* Suingfurdensis, physician and astrologer. As reported in *Tycho*, *Progymnasmata* vol. 1 p. 783, in his prognostication for the year 1574, speaking of the new star of 1572, which lasted until 1574, he asserted that miracles of this kind come forth from heaven roughly every two thousand years, and agree in threefold fashion with the ages of the world according to the saying of Elias the Talmudist. Thus before the Flood something of the kind appeared, or at least the pillar of fire shone forth before the Exodus of the Israelites from Egypt; then, about four thousand years after the founding of the world, a star appeared to the Magi in the East — that is, as he interprets it, near the beginning of the Earth — to show the beginning of salvation; and finally, in 1572, a new star appeared near the arctic pole to foreshow the end of the world, as though the revolution of the heavens about the poles were to cease not long after. But these stars were not distributed at intervals of two thousand years; nothing is established about a star seen before the Flood; and the region of the Magi was not the beginning of the Earth, except to the ignorant common people,

who think that the limit of the eastern horizon is the beginning of the Earth. [Margin: The opinion of Andreas Rosa on the star of the Magi and on the new star of 1572.]

XX. Finally we must contend with *Kepler*. In his book on the new star in Serpentarius of 1604, chapter 27, he denies that the star in Serpentarius was the offspring of the great conjunction begun in 1603 and completed in 1604, for the reasons we have already set out in chapter 18, number 5; but he nevertheless thinks that it was associated by God with that conjunction, by a definite design directed to the salvation of men, so as to signify something great to them. On the same reasoning he holds, in chapter 26 and in his *Silva Chronologica*, that the star of the Magi was not, indeed, produced from the greatest conjunction of Saturn, Jupiter and Mars by God using natural causes to that end; but that God nevertheless, to accommodate himself for a while to the rules of the Chaldeans — who expected some notable change of affairs at the transit of the greatest conjunctions from the watery trigon to the fiery trigon — kindled that star at the very moment of that greatest conjunction, so that the Magi, who were watching that greatest conjunction, might more easily catch sight of the new star; and that He so ordered the times of that greatest conjunction that it should serve this star and His Son, soon to appear in the flesh. [Margin: Kepler's opinion of new stars connected with the great conjunction.]

Now that conjunction took place in Julian year 39, and was indeed triple, one of its three falling in December, and it had remarkable astronomical characters, which we have already disclosed in Book 7, Section 5, chapter 10, number 3; and that year preceded the first year of the common Christian Era by 6 whole years. So, in order to establish that the star of the Magi was kindled in that year 39 at the moment of the greatest conjunction, he chose two positions in sacred chronology: that the star of the Magi appeared two years before the Nativity of Christ, and that the common Christian Era, which the Church and the Roman empire use everywhere, falls short by 4 years. Christ would thus have been born not in Julian year 45, as the common era supposes, but in Julian 41, and the star of the Magi would have appeared in Julian year 39. To confirm that the common era falls 4 years short, he gathers from the histories of Josephus, Dio and others the year of the death of Herod the child-slayer, in whose lifetime, as the Gospel of St Matthew makes certain, Christ was born; and he shows that Herod died no later than Julian year 42, which he tries to confirm also from an eclipse of the Moon. Kepler's opinion that the star of the Magi was connected with the greatest conjunction of Saturn and Jupiter is moreover favoured by Pierre d'Ailly, as is clear from what was said in number 12, although he says that the great conjunction made in Cancer preceded the Nativity of Christ by 6 years.

XX. My reasons for not assenting to Kepler are these. First, the opinion is far truer that the star of the Magi was kindled by God not before Christ's birth but at Christ's very birth, as I taught in chapter 19, number 9. Second, Christ's birth in Julian year 44 or 45 is likewise far more probable than in year 41, as I

showed at length in Book 3 of the *Chronology*, and that without prejudice to the eclipse of which Josephus the historian speaks. Third, both the rising of the star of the Magi and its signifying that Christ had now been born were plainly supernatural, and had no dependence on or connection with the Fixed Stars or the planets; nor did the star need, in order to be observed, to be observed on the occasion of watching the greatest conjunction of Saturn and Jupiter. For it was of such splendour and so near the Earth that it could not escape men's notice, especially of those who repeatedly explore the sky. Indeed the new star of Cassiopeia of 1572 too was observed by its own brightness right from the start, by many even who were unskilled in astronomy, and yet it preceded the great conjunction of 1583 by 10 years. It is, moreover, much more consistent with the sayings of the Fathers and Doctors of the Church, already adduced in chapter 19, number 16, and in chapter 20, numbers 1 to 10, to remove from that star all commerce and affinity with the rules of the casters of nativities [Genethliaci]. [Margin: Kepler's opinion rejected [?].]

Fourth, very many of the new stars recorded in history were kindled not only outside the moments of the great or greatest conjunctions; and conversely, by far the most of the great and greatest conjunctions from the beginning of the world down to this year in which we write have had no new star appear with them as companion, or follow them in the same year. This will be clear if you re-read, in our Book 7, Section 5, chapter 9, number 8, the table of all the great and greatest conjunctions and their years, and at the same time compare them with the years in which new stars appeared, whose chronicle we compiled in chapter 1 of this section. For although the new star seen by Hipparchus could have been kindled in the year 125, when there was a great conjunction at 11° of Pisces, its time is nevertheless not established by any history. With that one excepted, then, of the 15 new stars enumerated in chapter 1 only one is found, that of 1603 or 1604, whose rising fell at a great conjunction; all the rest are distant by several years from the year of the nearest great conjunction, and the greater number of them precede it. So God seems rather to wish deliberately to intimate to us their independence of the great conjunction, so that such miracles should be ascribed to Him alone. For here is a synopsis, drawn from the two chapters already mentioned, of the great or greatest conjunctions of Saturn and Jupiter that stood nearer to the beginnings of the new stars; conjunctions of Jupiter and Mars, or of Saturn and Mars, usually happen in the following year, or in the same year but some months apart from the conjunction of Saturn and Jupiter.

Years of the conjunctions of Saturn and Jupiter ♂	Years of the rising of the new stars
before Christ 125, great	before Christ 125, perhaps
in the year of Christ 135, great	in the year of Christ 130
393, great	389
393, great	392
393, great	398

Years of the conjunctions of Saturn and Jupiter ♂	Years of the rising of the new stars
948, great	945
1266, great	1264
1583, *greatest*	1572
1603, great	1596
1603, great	1600
1603, great	1603
1603, great	1604
1603, great	1612
1623, great	1618
1643, great	1638

Since, then, it is established by induction over about 1600 years that the risings of new stars have, rather, nothing to do with great conjunctions, and that from Christ down to us 80 great conjunctions and 2 greatest have taken place without the rising of any new star, the Arabs had no foundation for expecting new stars from those conjunctions; nor from the single new star in Serpentarius, which neither in the greatest nor in the great conjunction of Saturn and Jupiter, but only in the gre- at conjunction's completion — namely, in the conjunction of Jupiter and Mars — fell, ought Kepler to have dragged sacred chronology towards his own findings and his own ideas, so that the star of the Magi too would have been kindled by God at the moment of the greatest conjunction, in Julian year 39, which was the seventh before the first year of the common, generally used Era of the Christian world. On the contrary, from the opinion — no less, indeed more, probable — that the star was first seen on the night of Christ's Nativity, and that the Nativity of Christ took place at the end of Julian year 45, he should rather have argued that God put off by 6 whole years, after the greatest conjunction of Julian year 39, the kindling of the new star that was to be shown to the Magi, so that it should have nothing in common with this star and should in no way favour the errors of the Chaldeans and the Arabs. For this was not only more pious and more in harmony with the teachings of the holy Fathers and Doctors of the Church, but also consistent with experience and with the induction made through the other new stars.

This would have had to be maintained even if everything else on either side had been equal. For neither the Nativity of our Saviour, which was wholly a miracle, nor the splendours of that most holy night, nor that sacrosanct union of the divine Word with our human nature, needed that smoky little lamp of a great or greatest conjunction of Saturn and Jupiter, so as to seem from it either more illustrious and more evident, or more notable to observe. Finally, if anywhere at all, it is here above all that things which stand so far above nature must be separated from the rubbish of nature, and that miracles worked by God outside and above the order must not be forced into the order of human customs — even if the risings of new stars had usually accompanied the synods of the superior planets. That the star shown from heaven to the three holy Magi was a miracle of this kind, we have by now sufficiently proved.

FINIS LIBRI OCTAVI.

End of the Eighth Book.

LIBER NONVS DE MVNDI SYSTEMATE.

The Ninth Book: On the System of the World.

SECTIO PRIMA DE SVBSTANTIA ET ACCIDENTIB. QVIBVSDAM CÆLORVM AC CÆLESTIVM CORPORVM.

The First Section: On the Substance and Certain Accidents of the Heavens and of the Celestial Bodies.

CAPVT I. De Opere Primi Diei, ſeu de Creatione Cæli & Terræ Luciſq. ac Temporis ex ſacra Geneſi,& ad mentem Patrum ac Theologorum.

Chapter 1. On the Work of the First Day, that is, on the Creation of Heaven and Earth, of Light and of Time, from holy Genesis and according to the mind of the Fathers and Theologians.

I. It is surely right to prefer Moses to Plato and Aristotle, so that in searching out the foundations of this world we may be lit rather by that twin horn of divine light with which, from the face of the Lord, the countenance of that same Moses shone, than by the half-extinguished spark of profane philosophy. Yet, so that everyone may be equally satisfied, we must select from both lights those things that bear more closely on astronomy, deliberately passing over many things the knowledge of which matters almost nothing to us, at least for the purpose of the work we have undertaken. Of this kind are: whether there is a single world, or whether there now are, or once were, or will be several or infinitely many worlds; whether it was made in the beginning of time, as the Catholic faith teaches, or existed from eternity, as the Peripatetic school foolishly proclaims; whether it is perfect in every respect, or further perfectible; whether it was created or produced by God alone immediately, or through Intelligences subordinate to Him; whether this whole universal machine rests in some determinate part of the divine Immensity, or in some imaginary space, as they call it, or rather, without our knowing it, rolls on perpetually through that immensity as a whole; why this world was founded in this virtual part, so to speak, of the divine eternity rather than in another; and other questions like these, partly inscrutable, partly altogether foreign to the bounds of astronomy. For none of them can be discerned from the phenomena of the heavens, nor is it useful for explaining those phenomena. Passing over these, then, we shall turn [?] our thought and our pen to other matters suited to our purpose, first setting out some things that will seem more necessary for these very matters. [Margin: Controversies about the world that do not concern the astronomer.]

Question 1. *Whether by those words, "In the beginning God created Heaven and Earth, but the Earth was empty and void", the creation of any determinate body made on the first day is signified.* [Margin: Genesis 1.]

II. There is no doubt that by the names of Heaven and Earth the entirety of this whole universe is signified sometimes. That is how we must understand the saying of Genesis 2, *In the day that God made Heaven and Earth*; that of Exodus 31, *In six days God made Heaven and Earth*; that of Jeremiah 23, *I fill Heaven and Earth*; and finally that of the Apostles' Creed, *I believe in God, Creator of Heaven and Earth*. In this sense, then, some hold that Moses said that Heaven and Earth — that is, the universal world — were created in the beginning, not because the whole was created in the first moment of the first day, but because Moses set out in summary the argument of the divine works that were to be created successively thereafter. On this view he signified by these words nothing determinate made before the light, but set out only the aggregate of the things he was afterwards going to narrate as made in six days — just as if someone were to say: what God made in the first six days are Heaven and Earth. This interpretation is reported, though with hesitation, by St Augustine, *Confessions* book 12 chapter 17, and *De Genesi contra Manichaeos* book 2; and by St Cyril of Alexandria, *Contra Iulianum* book 2, in these words: *That, embracing everything comprehensively and as on a small tablet, he narrates the birth of the whole of nature.* St Thomas Aquinas, part 1, question 68, article 3, attributes the same view to St Chrysostom; Steuchus [Agostino Steuco] defends it in his *Cosmopoeia*; and so does Paulus Burgensis [Pablo de Santa María], addition 1, he too attributing it to Chrysostom.

But rightly, and following the more common teaching of the Fathers, *Ascanius Martinengus* [Ascanio Martinengo] in the *Glossa magna* p. 162, *Pererius* [Benito Pereira], *In Genesim* book 1 p. 26, *Suarez*, *De opere sex dierum* book 1 chapter 3, and *Tannerus*, *Summa theologica* vol. 1, disputation 6, question 2, doubt 1, teach that these words signify something determinate and actual, created by God in the beginning of the first day, that is, in the first instant of all time, before light was made. For by the particle *autem*, which is copulative, there is added at once: *But the Earth was empty and void, and darkness was upon the face of the deep, and the Spirit of the Lord was borne over the waters.* It is therefore supposed that the Earth had been made, but empty and void, and the deep and the waters — whether this was Chaos or something else, of which more shortly; and this is perhaps what Saints Cyril and Chrysostom meant to indicate. Keeping to the letter, then, we can accept neither the view of Plato — who, as St Justin noted in his *Admonitorium*, took his doctrine of the Ideas from these words of Moses — nor that of Philo, who says in *De mundi opificio* that before all things God made the ideas of heaven and earth in the intelligible world. For Moses did not set before an unlettered people ideas scarcely intelligible to the learned; nor are the divine Ideas separate from God, if they are taken formally and not objectively, nor are they something created or made. Rather, they are connatural with God from eternity and identical with Him, as St Thomas teaches, part 1, question 15, with St Augustine and most theologians. [Margin: Whether the Ideas of Heaven and Earth.]

Quæſtio 2. De notione nominis Cæli ac Firmamenti apud Latinos & Hebræos.

Question 2. On the meaning of the names Heaven and Firmament among the Latins and the Hebrews.

III. As for the meaning of the name: heaven is called οὐρανὸς [Greek: ouranos, "heaven"] in Greek because it is seen above, for ὁράω [Greek: horaō] is "to see" and ἄνω [Greek: anō] is "above". What if it comes from the Hebrew word *Or*, of which I shall speak in chapter 4, number 9? In Latin, *caelum* is so called because it conceals [celat] the air; or because, as Philo says in *De mundi opificio*, it is the ὥρος [recte: ὅρος] [Greek: horos, "boundary"], that is, the boundary, of all things; or because it was the first of the ὁρατῶν [Greek: horatōn], that is, of visible things, to be made; *or from its loftiness, or from engraving* [caelatura], as Bede says in his exposition of Genesis chapter 1; or because it conceals the higher things, or is, as it were, an engraved vessel, picked out with stars as with figures in relief, as St Isidore thinks, *Etymologies* book 13 chapter 10, and before him Varro, *De lingua Latina* book 5, and St Ambrose, *Hexaemeron* book 2 chapter 4. Or else because it conceals, that is, covers, all sensible things, as Petrus Comestor says, *Historia scholastica* chapter 4, together with *Abulensis* [Alonso Tostado] and *Hugo Carensis* [Hugues de Saint-Cher]. Varro, however, also thought that *caelum* was named from the Greek word κοιλὸν [Greek: koilon], which means "hollowed out"; for to us it appears hollowed out, or concave. [Margin: Etymology of heaven.]

But in this place the Hebrew reading has *Ha-samaim*, or *Haxamaim*, that is, those bodies worthy of stupor and wonder, from *Samah*, which means, among other things, to wonder and to be astounded; just as in Isaiah 53 the phrase *Many were astounded over you* is expressed in Hebrew by the verb *Samemù*. But if it is derived from the root *Sem*, which means "name", the name *Samaim* will signify the most named and most famous bodies. For the Hebrews too call famous men "men of name", that is, renowned men, which our Vulgate renders in Genesis 6: *These are the mighty ones of old, famous men.* [Margin: What is Samaim in Hebrew?]

Lusitanus, Martinengus and our Fernandius, on Genesis chapter 1, accept both interpretations; although Lippomanus [Luigi Lippomano] in his *Catena* interprets *Samain* as "there water", and Suarez, *De opere sex dierum* book 1 chapter 7, as simply all diaphanous bodies [?]. And although among the Hebrews this noun lacks a singular, and could have been rendered *He created the heavens*, the Chaldean version, that of the Seventy Interpreters, St Jerome and our translator nevertheless rightly render it in the singular. They do so either because a single heaven was created on the first day; or because it was created without the distinction of stars and motions, and with these set aside we have no sensible difference between heavens; or because, just as Moses wished to include all the elements under the name of the one Earth, so he wished to include the orbs of all the heavens under the name of the one heaven.

Finally, it must be noted, following Lusitanus and Martinengus, that in this place of Genesis the particle *He* or *Ha* is placed before *Samaim*; joined to a noun, it makes its meaning distinguished. So *Hasamaim*, as I said, denotes those distinguished and stupendous, or most renowned, bodies. In the same way, if you put *He* before the noun *Nabi*, which means "prophet", saying *Hanabì*, it will signify "that Prophet", that is, the distinguished one. This particle, as Fernandius says, when added also to the noun *Eres* or *Erez*, makes *Haarez*, which signifies that Earth, dry as well as fluid, or habitable as well as uninhabitable, as Lusitanus notes. Its root, however, is *Rus*, which means to rush and to run, as a heap of sand does; and so the earth, separated from its admixture with water, was called by God *Arida* [the dry land], Genesis 1 — or from *eruendo* [digging out], as I shall say more fully in question 3, towards the end of number 10. The name *Firmament*, further, will be explained far more suitably below, in chapter 2, number 2. [Margin: What is Hasamaim?] [Margin: What is Eres in Hebrew?]

Quæſtio 3. Quid Nomine Cæli ac Terræ Moyſes ſignificauerit creatum à Deo in prinmo inſtanti Mundi ?

Question 3. What Moses meant by the name of Heaven and Earth as created by God in the first [?] instant of the world.

IV. It is incredible in how many and how different ways the Fathers and Doctors of the Church have understood those two names, Heaven and Earth, with which Moses began to narrate the first creation of things, as may be seen in the authors who wrote books of *Hexaemeron* — that is, on the works of the six days — or glosses or postils, or commentaries, or questions on Genesis chapter 1. Many report their words, but Ascanius Martinengus most diligently of all, in his *Glossa Magna*; and we shall bring forward some selected ones below, from number 9 onwards and in questions 4, 5 and 6, where we judge that greater proof is needed. In this question, so as not to be too prolix on account of the many interpretations, it will be enough to indicate the places where the authors' words can be sought.

First, then, St Justin attests in his *Admonitorium* that Plato understood by those names the Ideas of Heaven and Earth, when he read them in Genesis and compared them with the words of Exodus 25 and 26, in which the pattern of the Tabernacle was shown to Moses by God on the mountain. [Margin: 1st interpretation.] Second, others understood formless matter, out of which Heaven and Earth could come to be, or which was Heaven and Earth in potency, not in act. So St Augustine, *De Genesi contra Manichaeos* book 1 chapter 7, *De Genesi ad literam imperfectus* chapter 3, *De Genesi ad literam* book 1 chapter 14, and *Confessions* book 12 chapter 20; St Gregory of Nyssa in his *Hexaemeron*; Hugo Carentis in his postils; and Alcuin, question 28 on Genesis. But St Augustine, in that chapter 14, holds that matter to be prior to the actual forms in origin, not in time, as I shall teach in question 5. [Margin: 2nd interpretation.] Third, some understood by the name of heaven and earth the formless matter not only of bodily but also of spiritual things. So St Augustine,

De Genesi ad literam book 1 chapter 1; Origen, homily 1 on Genesis; Bede on chapter 1 and in his *Questions on Genesis*; Strabus [Walafrid Strabo] in the *Glossa ordinaria*; Eucherius, *Commentary on Genesis* book 1 chapter 1; the Master, on book 2 of the *Sentences*, distinction 12; Hugo Victorinus [Hugh of Saint Victor], *De sacramentis* book 1 part 1 chapter 1; and also Hugo Carensis in his postils. [Margin: 3rd interpretation.]

Fourth, that by the name of Heaven the invisible created nature or substance — that is, the spiritual and intelligible — can be understood, and by the name of Earth the visible, bodily and sensible nature, was taught by St Augustine, *De Ge-* [Margin: 4th interpretation.]

nesi ad literam book 1 chapter 1, and in the unfinished work, chapter 3, and *De Civitate Dei* book 11 chapter 33; St Ambrose, *De Abraham* book 2 chapter 8; and Anselmus Laudunensis [Anselm of Laon] in the *Glossa interlinearis*.

Fifth, some, as though explaining the preceding interpretation, understood by the name of Heaven the angels, perfect naturally or supernaturally, and by the name of Earth bodily matter, as being imperfect. So St Augustine, *De Genesi ad literam* book 1 chapter 1; Albertus Magnus, part 1 of the *Summa de quattuor coaevis* [On the Four Coevals], question 2, article 9; Strabus in the *Glossa ordinaria*; and St Gregory the Great in his commentaries on Ezekiel and on Job, book 27, chapter 25. But St Augustine understands angels already blessed, whereas St Gregory understands them as not yet blessed; for afterwards he understands them under the name of the Firmament, when they were confirmed in grace and beatitude. [Margin: 5th interpretation.]

Sixth, some thought that under the name of Heaven the Empyrean together with the angels is comprehended, and under that of Earth the matter of the elements under one common form of corporeity, or the elements undistinguished. So St Bonaventure, on book 2, distinction 12; Lyranus [Nicolas de Lyre] in his postils; and Tostatus [Alonso Tostado] on Genesis chapter 1, who attribute the same view to Strabo and Bede; and Ambrosius Catherinus [Ambrogio Catarino Politi] in his exposition of Genesis chapter 1. [Margin: 6th interpretation.]

Seventh, some understood by the name of Heaven the Empyrean indeed, together with the angels, but by the name of Earth the elements constituted in act under their own forms, though confused. So Iunilius in his *Hexaemeron*; Hugo Victorinus in the *Sentences*, treatise 2 chapter 1; Rupertus Abbas [Rupert of Deutz], *Commentary on Genesis* book 1 chapter 6; Gulielmus Parisiensis [Guillaume d'Auvergne], *De Universo* part 1, from chapter 31 to 40; Alexander of Hales, *Summa* part 2, question 45, member 2; Comestor at the beginning of the *Historia*; Strabus in the *Glossa ordinaria*; our Molina [Luis de Molina], *De opere sex dierum*, disputation 2; Salianus [Jacques Salian] in the *Apparatus* to the *Annales*, chapters 6 and 11; and our Fernandius on Genesis chapter 1, section 2. But Hugo Victor understands the four elements as undistinguished and extending all the way up to the Empyrean in that beginning of the world; Iunilius, by contrast, the 4 elements arranged so that air and fire were inside the earth; Comestor, the 4 elements, saying nothing of their order. Rupertus,

Parisiensis, Halensis, Molina, Salianus and Fernandius, however, understand only two elements, Earth and Water. [Margin: 7th interpretation.]

Eighth, some understood the angels by the name of Heaven, and by the name of earth the elements in act, confused into one Chaos; the Master, on book 2 of the *Sentences*, distinction 12, mentions these without rejecting them. [Margin: 8th opinion.] Ninth, some included under the name of Heaven all the heavens with the angels, or all higher things, and under the name of Earth all lower and palpable things. So St Augustine, *De Genesi ad literam imperfectus* chapter 3; Anastasius Synaita [Anastasios Sinaites], *Commentary on the Hexaemeron* book 1; Petrus Riga [Pierre Riga] in the *Mundi Aurora*; and Petrus Comestor in the *Historia Scholastica*, on Genesis chapter 1. [Margin: 9th interpretation.]

Tenth, Gennadius, Acacius and Caietanus on Genesis chapter 1, and Hamerus and Lyranus in the same place, hold that under the name of Heaven all the heavens with the angels can be understood, and under the name of Earth the elements in act with their forms. [Margin: 10th interpretation.] Eleventh, Lippomanus in his *Catena* on Genesis supposes that by the name of Heaven the Empyrean and all the heavens with the angels — indeed fire and air as well — can be understood, and by the name of earth the Earth and water. [Margin: 11th interpretation.]

Twelfth, Albertus Magnus, part 1 of the *Summa de quattuor coaevis*, question 12, article 9, hints that by the name of heaven either the Empyrean alone or all the heavens can be taken, and by the name of earth formless matter or the elements. [Margin: 12th interpretation.] Thirteenth, Saints Basil, homily 1 on the *Hexaemeron*, Ambrose, *Hexaemeron* book 1 chapter 6, and Damascene, *De Fide* book 2, chapters 5, 6 and 10, teach that by the name of heaven the supreme Heaven is taken, and by the name of Earth the elements formed in act. [Margin: 13th interpretation.]

Fourteenth, by the name of Heaven, all the heavens, the Empyrean not excluded but rather included, and all the higher things above the concave of the lunar heaven, the stars excepted; and by the name of Earth, all simple bodies, or the lower or sublunary ones, or the four elements. So understood St Augustine, *De Genesi ad literam* book 1 chapter 1, and in the *Questions on Both Testaments*, question 106; St Athanasius, *De Incarnatione Verbi*; Hamerus, with many rabbis, commentary on Genesis; Lyranus and Tostatus in their first exposition; Comestor, *Historia scholastica* on Genesis chapter 1; the *Glossa interlinearis*; Hugo Carensis in his postils; Henricus, quodlibet 6, the last question; Pererius, *In Genesim* book 1, from p. 31; and Tannerus, question 2 *De caelo*, and *Summa theologica* vol. 1, disputation 6, question 2, doubt 2, who adds, with St Thomas, that air is comprehended under the name of water. [Margin: 14th interpretation.]

Fifteenth, by the name of Heaven, whatever is above us, including the Empyrean, starry, ethereal and aerial heaven; and by the name of Earth, whatever is below us, or the elements of water and earth. So take it Caietanus on

Genesis chapter 1, Steuchus in the *Cosmopoeia*, Vielmius [Girolamo Vielmi] on Genesis, lecture 3, and Suarez, *De opere sex dierum* book 1, from chapter 3 to 9. [Margin: 15th interpretation.] Sixteenth, by the name of Heaven, heaven taken as the fourth element, and by the name of earth the three remaining elements, yet in such a way that water and air were confused and mixed together: so Innocent III understood it, on the fifth penitential psalm. [Margin: 16th interpretation.]

Seventeenth, in an entirely opposite way, by the name of Heaven the three higher elements, confused together as the matter out of which heaven afterwards came; and by the name of earth the lowest of the elements, earth. So took it Strabus in the *Glossa ordinaria*; Hugo in his *Admonitiones* on Genesis chapter 1 and *De sacramentis* book 1 chapter 6; and Richardus, *Exceptiones* book 2 chapter 7. [Margin: 17th interpretation.]

Finally, eighteenth, Hugo Carensis in his postils admitted that by the name of Heaven the two higher elements, of fire and air, and by the name of Earth the two lower, of water and earth, can be understood. [Margin: 18th interpretation.] With these in view, scarcely anything could have been thought up that has not had some patron — granted that the Fathers who understood angels by the name of heaven either spoke in a mystical sense, or took the thing located or contained under the name of what contains it. It is now worth helping the reader's memory by gathering the preceding opinions in summary into a single view, setting out on one side what they understood by Heaven, and on the other what by Earth.

Order of the interpretations	Heaven [CÆLVM]		Earth [TERRA]
1	The Idea of Heaven,	and	Earth
2	Formless matter of Heaven,	and	Earth
3	Formless matter of spiritual things,	and	of bodily things
4	Spiritual nature, or invisible and intelligible,	and	bodily, visible and sensible
5	Perfect angels	and	imperfect matter of bodies
6	The Empyrean with the angels,	and	matter under the form of corporeity
7	The Empyrean with the angels	and	the elements formed in act
8	The angels,	and	the elements confused in Chaos
9	All the heavens with the angels, or all higher things	and	all lower things below the Moon
10	All the heavens with the angels,	and	all the elements in act
11	All the heavens with the Empyrean and the angels, and with fire and air	and	Earth with water
12	The Empyrean or all the heavens,	and	formless matter
13	The supreme Heaven,	and	the elements formed in act

Order of the interpretations	Heaven [CÆLVM]		Earth [TERRA]
14	All the heavens,	and	all the elements formed in act
15	All the heavens, with aether and air	and	Earth with water
16	Heaven as the fourth element,	and	Earth with air and water
17	Fire, air and water mixed together,	and	Earth alone
18	Fire and air,	and	Earth and Water

Noſtra Opinio.

Our Opinion.

V. Although several questions are included in the title of this question — namely, what God created in the first instant of this world, and what, by which persons, and for what reason or in what words Moses, or the Holy Spirit through Moses, wished to signify as created by God in that beginning — nevertheless with a single conclusion to all these we shall re- spond in summary, and afterwards confirm it by expounding it. It runs as follows.

CONCLVSIO.

Conclusion.

In the first instant of the world God, Best and Greatest, created the Empyrean Heaven with the angels, and the starry heaven, and all the elements with their primary qualities and substantial forms, in that order as to place or position which their nature required; but without light and stars, and without the secondary qualities and the substantial forms of mixed bodies. And this Moses, and the Holy Spirit through Moses, meant to signify — though not everything to everyone, nor in the same way, nor in a single way, but in diverse ways.

VI. In support of this Conclusion I suppose, in general, four things as certain, or claim them, with full right, as things to be conceded. The first is the propriety of the literal sense, the mystical or tropical sense not being excluded. For by the rule of St Augustine and of St Gregory the Pope, and afterwards the common rule of the theologians: as long as no manifest contradiction follows from the literal sense, that sense must always be kept intact, and sacred Scripture must be taken according to the sense the letter conveys, in its proper and least figurative meaning — granted that, once this sense has been laid as a foundation, it is then permitted to build other interpretations upon it, tropological, mystical or anagogical, and allegorical. And if this is to be observed in every part of sacred Scripture, much more so in history, of which kind sacred Genesis is; for there Moses, although he was a prophet, nevertheless chiefly and for the most part conducts himself as a historian, and therefore the literal and the historical sense are taken by the sacred interpreters as the same. [Margin: 1st postulate or supposition.]

Second, I suppose that Moses — although, as regards the execution of the promulgation and the order of time, he set out in sacred Genesis the manner and

order of the creation of this world to the Hebrew people, who were for the most part rude, accustomed to knowing bodily things and indeed to worshipping them in place of God, and scarcely capable of spiritual things — nevertheless, as regards the intention, either his own or also that of the Holy Spirit, wrote for the learned and the unlearned alike, and made himself a debtor to the wise and to the unwise. For he was established by God as the prince and summit of all historians, so that he surpassed all (excepting the most holy Evangelists) in antiquity and wisdom, having once been instructed in all the wisdom of the Egyptians and afterwards imbued with knowledge infused by God; so that he might impose silence on all the philosophers and theologians who boast empty and false things about the beginning of the world, and be the one whom so many and such illustrious lights of the Church were to read through with veneration and wonder. It was fitting, therefore, that he should serve the capacity of all: that he should wish to signify certain things to the unlearned in a plainer and more obvious sense, according to the propriety of the Hebrew language, but many other things to learned men, in more numerous and subtler, yet true, ways — if not to express them, then at least to indicate them. [Margin: 2nd postulate.]

Third: Moses must not be interrupted straight after the names Heaven and Earth, as though asking him unseasonably what those names signify. For the first period is not completed there, since there follows *But the Earth*, and the particle *autem* is copulative — indeed, as Vielmius notes, lecture 3, among the Hebrews it stands for *and* — and Moses at once explains the first part of the period by the second. Just so, it would be an importunate question if someone, after the half-period *In the beginning was the Word*, were at once to interrupt that divine narrator of the generation that is from within and eternal, by asking what kind of Word that was; for the whole period must be awaited and weighed: *In the beginning was the Word, and the Word was with God, and the Word was God.* In the same way, therefore, we too must wait patiently and reverently for the whole utterance of that period, pronounced in a single breath: *In the beginning God created heaven and Earth; but the Earth was empty and void, and darkness was upon the face of the deep, and the Spirit of the Lord was borne over the waters.* For in this way the unlearned will be able to understand that the sensible heaven, the Earth, the Water and the Spirit were then created; the more learned, however, the Empyrean too, and the angels, and in many ways the air, and perhaps fire, if it is granted as something distinct from Heaven. [Margin: 3rd postulate.]

The fourth is the integrity of the Mosaic history. For in a matter of such moment Moses must not, without evident necessity, be said to have been defective or to have fallen short. Since, then, nowhere in sacred Genesis did he set out the time or moment at which God created the remaining elements, or the things that had to be founded by creation taken strictly, but only added the things that God, by the word *Fiat*, commanded of the heaven and elements already created, so that they should work with Him as secondary and instrumental causes; and since in that first period he signified those things briefly, but sufficiently for the

capacity of each, it must be said that their creation was signified by the period already mentioned. This we shall shortly show by expounding that period part by part, selecting only those things that bear on our purpose. [Margin: 4th postulate.]

In Principio creauit Deus.

In the Beginning God created.

VII. I do not dwell here on explaining the particle *in principio* — in Hebrew *Beresith* — which literally, according to the more common interpretation, signifies not only "before all the works of God *ad extra*" (which could apply even to things created from eternity, if anything could be created from eternity), but also, and above all, the beginning or first instant of time, and of all the days and years that are counted from it to this day and have been completed. And so the creation of this world is signified as made not from eternity but in time; granted that in a higher and more recondite sense it is signified that God created all things in the Word, that is, in His Son. But what bears on our matter is this: I consider the word *Creavit*. Although among the Seventy Interpreters it is expressed by the verb ἐποίησεν [Greek: epoiēsen, "made"], that is, "effected", the Vulgate version and St Jerome more correctly have *Creavit*, because the Hebrew reading has *Barà*, which means to create, or to fashion from nothing. [Margin: What creating is.]

To create, strictly speaking, is to produce the whole substance from nothing; and so, if the substance consists of matter and form, it is to produce matter and form together, from no pre-existing subject and with no prior dispositions concurring with the Creator towards that first production. So teaches the greatest part of the Scholastics, with St Thomas, part 1, question 45, where in article 5 he teaches that to be created is proper to composite and subsistent things — granted that Vasquez [Gabriel Vázquez], vol. 2 on part 1, disputation 174, chapter 2, number 17, seems to oppose it; and almost all our writers subscribe to St Thomas in this.

To be created properly, but not strictly, belongs to the rational soul, because it is not the whole and complete substance of a man. By this word, then, Moses sufficiently signified — if not to all, at least to the more learned — that whatever things were made from nothing were made by God in that first instant. For afterwards he did not use that word alone, but the words *Fiat*, *producant*, etc., and, these being presupposed, the word *creavit* taken less properly: as when he said *Let the waters bring forth the creeping thing*, etc., and shortly after added *And God created great whales*, and, of man, *To the image of God he created him*. To those, then, who were going to take the word *Creavit* strictly, he signified that there were created both the things that could not be produced otherwise — of which kind is the angelic substance, and perhaps the Empyrean — and the things that it befitted God to produce in this way, supposing that the Earth was to be created from nothing; for the reasoning was the same for all equally simple bodies. And the very name GOD warns that nothing imperfect should

be attributed to Him in His first work, for in Deuteronomy 32 it is said: *The works of God are perfect.* It was fitting, therefore, that in that first instant of operation *ad extra* He should bring forth something perfect in its order; and this would not have been so had He created either formless matter or only one of the elements. That is why St Thomas, part 1, question 66, article 1, weighing those words of Deuteronomy and following the authority of St Augustine, asserts that heaven and the elements were created by God in that beginning under their proper form; since, according to St Augustine, their forms were created together with the matter at the same time, as we shall show in question 5. And in this sense almost all the more recent theologians take the verse of Ecclesiasticus 18, *He who lives for ever created all things together*: that is, by simultaneity of the beginning and of the manner of creating, so that whatever things were created by creation taken strictly and narrowly were created together, that is, in the same instant. So too they take the words of the Lateran Council in the chapter *Firmiter*, on the Triune and One God: *Who together, from the beginning of time, created from nothing both creatures, namely the angelic and the worldly* — understanding by "worldly" the visible world composed of heaven and the elements.

Cælum & Terram.

Heaven and Earth.

VIII. Of these names in Hebrew enough has been said in question 2. But who would doubt that Moses meant by them whatever was properly to be understood — whether as all those skilled in the Hebrew language were accustomed to understand it, or as more learned men too could understand it? So these names do not signify matter that is not Heaven and Earth in act; otherwise Moses would rather have deceived the people, or by an unnecessary equivocation have given occasion for error. They signify, rather, that heaven and that Earth of which he was going to say, in Genesis 2: *Thus the heavens and the Earth were finished, and all their adornment*; and a little further on in the same place: *These are the generations of heaven and earth, when they were created, in the day that God made heaven and earth*; and in the sense in which David was going to say, in Psalm 101: *In the beginning, O Lord, you founded the Earth, and the heavens are the works of your hands; they shall perish*, etc. — by which words chiefly the sensible heavens, which are to be dissolved, are signified.

Moreover, by the name of Heaven the common people generally understand whatever is above the Earth and the Water, because of the likeness in transparency and rarity; and so the Air and the Aether as well, as will shortly be proved separately — granted that the wiser also understand that supreme and invisible Heaven which sacred Scripture calls the Heaven of Heaven and the Scholastics the Empyrean, as we shall show in question 6. By the name of Earth, on the other hand, is understood the globe of the earth with the waters now enclosed in it; but it is sufficient that, in that beginning, the Earth taken properly — that is, with its substantial form — could and should have been

understood. Otherwise, why would it be called earth rather than anything else, if it was not earth in act, nor water, nor air, nor fire, but all of them in potency?

Aerem Cæli Nomine ſignificari.

That Air is signified by the Name of Heaven.

IX. I said that by the name of Heaven Air and Aether, or fire if there is such a thing, are signified, not only among the learned but also among the unlearned, because of their likeness in transparency and rarity. For the common people make no distinction within that whole most subtle substance which extends from the surface of the earth and the sea up to the outermost blue heaven. [Margin: Air everywhere called by the name of Heaven.] And therefore in Genesis 7 the clouds are called the floodgates [cataractæ] of heaven, and in Psalm 8 it is said *the birds of heaven and the fishes of the sea*; how familiar this is to sacred Scripture we shall shortly show with St Damascene. Moreover, heaven is said to be opened when it rains, and shut when it does not rain, as in Leviticus 26, Deuteronomy 28, 3 Kings chapter 8, 2 Paralipomenon chapter 2 and Luke 4; fire and sulphur are said to have rained from heaven, Luke 17; the heaven to glow red, or to be ruddy, Matthew 16 and Luke 12; and there is *the way of an eagle in the heaven*, Proverbs 30. That is why St Augustine, explaining this passage of Genesis according to the letter, in *De Genesi ad literam* book 1 chapter 4, says: *Nor is it in any way to be thought that any element of this world has been passed over in this scripture. Air, then, is understood to belong to heaven as well, if there are any most tranquil and peaceful spaces in its parts; or to earth, on account of this turbulent and murky place, which grows thick with moist exhalation — although this too is quite often called by the name of heaven.* [Margin: The air that goes by the name of Earth.]

Well known, too, is the division of Heaven into the first, that is, the aerial; the second, the starry; and the third, the Empyrean. We shall speak of it, following Anastasius Synaita and St Damascene, below, in question 6, number 25. For now let these words of Anastasius suffice, *Hexaemeron* book 1: *When you hear that Paul was caught up to the third heaven, think the first heaven to be the air*; and those of St Damascene, *De Fide* book 2 chapter 6: *It is usual and familiar to Scripture to call the air too heaven, namely because it is above.* How truly this is said is clear, because the *birds of heaven* are so named in Job 28 and 35, Psalms 8 and 103, Jeremiah 7 and 9, Ezekiel 38, Daniel 3 and 4, and Hosea 2 and 4; and the Saviour himself names them more than once — Matthew 8 and Luke: *The foxes have holes, and the birds of Heaven nests*; Matthew 13, Mark 8 and Luke 8: *And the birds of heaven devoured it*; and Matthew 13 and Luke 13: *So that the birds of heaven come and dwell in its branches.* [Margin: Birds of Heaven, often in Holy Scripture.]

Rightly, then, St Thomas says, part 1, question 66, article 1: *Under heaven air too is included*; indeed fire as well, in the opinion of Alexander of Hales, *Summa* part 2, question 45: *Heaven, that is, fire, for there is a fiery heaven; and air, for there is an aerial heaven.* Caietanus agrees with this in his commentary

on Genesis chapter 1, where, taking Moses to have embraced all the higher things under the name of heaven, he says: *Understand that, according to the sound of the letter, all simple bodies were set out for the Hebrew people: because by the name of the heavens they understand all the higher bodies, namely the celestial ones and the spheres of fire and air; and by Earth, the earth covered with waters.* On the same chapter Vielmius says, lecture 3: *It seems to me more probable that they come rather under the name of heaven, because in Psalm 8 the Air was, without ambiguity, called heaven*, etc. Lippomanus says no differently in his *Catena*: *By the name of heaven, moreover, he embraces all things that are above: air, fire, all the celestial bodies, together with the spiritual creatures that are in the heavens.* But in a narrower sense Hugo Carensis, in his postils: *Or heaven, that is, the two higher elements; Earth, the two lower.*

Indeed Steuchus, in the *Cosmopoeia* on Genesis, gathers three heavens from the Stagirite [Aristotle] himself, when he says: *For Heaven is spoken of in three ways, on Aristotle's authority. First, that which is the dwelling of the Gods, outside the celestial whirl, where there is neither place, nor void, nor time, but only most blessed things. Secondly, heaven is said of the substance of the celestial orbs, of the Sun [?], the Moon, etc. Thirdly, Heaven is whatever there is besides the earth, to which air and aether too are joined. It is this last, then, that is mentioned in Moses.* So, according to Steuchus and Hugo, so far is it from being the case that air and fire are not signified by the name of heaven, that rather these elements alone were signified by that name — provided you grant that the starry and Empyrean heavens were made on the second day; on that matter, see questions 6 and 8 below.

Tellez too, *Physics*, disputation 40, section 1, says that air and fire are signified by the name of heaven. And before all these Pliny the Elder, book 2 chapter 38, which is headed "On the air": *So much for the world itself and the stars. Now the remaining memorable things of the heaven. For our forebears also called this heaven — which by another name is air — all that, like a void, pours out this life-giving breath.* But the use of this name is much older. Plato says in the *Phaedo* that air is called heaven by us because it is above us; and not only do Homer, *Iliad* 1, as Eustathius notes, Aristophanes in the *Clouds*, and Callimachus take air for heaven, but so do the Latin poets everywhere — as Lucretius, book 4:

> *They are established in this heaven, which is called air.*

and Virgil, *Aeneid* 5:

> *Great-hearted Aeneas, not even if Jupiter himself were my guarantor*
> *would I hope to reach Italy under this sky.*

and *Aeneid* 6:

> *Daring to entrust himself to the sky on swift wings.*

Terra autem erat inanis & vacua.

But the Earth was empty and void.

X. Where the Latin has *Inanis et vacua*, the Hebrew says *Tohù Vabohù*, or *Thohù Bohù*, that is, solitude or vanity, and emptiness — granted that the translator rendered it in the concrete. The Chaldean has *Deserted and empty*;

Aquila, *Vanity and Nothing*; Symmachus, *Idle and undigested*; Theodotion, *Empty and nothing*; Pagninus [Santes Pagnino], *Desolate and empty*; Lippomanus and Steuchus, *Empty and Void*; the rabbis, *Formless and Empty*; Ionathas, *Empty of men and beasts*; and finally Caietanus reads, following learned Hebrews, *Empty to the bottom, and empty outside*. So the Earth was vain, or seemed to have been made in vain: for although it had been made to be inhabited by animals — according to the verse of Isaiah 45, *For he did not found it in vain: he formed it to be inhabited* — it had nevertheless not yet attained its end, but lay deserted, like a solitude, under the waters. And that "in vain", as also in Isaiah 49, *I have laboured in vain*, is in Hebrew *Iethohu*.

Nor did it only lack animals dwelling on it; it was also empty of all other mixed bodies, such as stones, plants, herbs and fossils. Nor did it then hold any waters within receptacles, nor had it yet drunk in waters through its outermost skin; for then it could not have been called simply empty, or empty to the bottom and outside, that is, empty on every side; nor could it have been called *Arida* in itself and without moisture — a name it deserved of itself. For below, in Genesis 1, it is said: *Let the waters that are under heaven be gathered into one place, and let the dry land appear; and it was so. And God called the dry land earth.* Here *Arida*, in Hebrew *Iabbasa*, from the verb *Iabas*, which means "to be dried up", is used as a substantive, not as an adjective: that is, let that which is dry appear; and it was such in itself, but was not seen, because it was covered by the waters. [Margin: When and how the Earth was Arida.] In the same way the same word is taken as a substantive when, in Mat- thew 23, Christ said: *You go round sea and dry land* [aridam], that is, the earth. And the words *he called the dry land earth* are to be understood in this way: that which was the Dry Land [Arida] he called Earth, just as he called the gathering of the waters seas. In Italian we would say *E nomino la Arida, Terra*; for in Hebrew the article stands before *Iabbasa*, thus: *Vaikra Elohim Laiabbasa erez*. Here the Chaldean, Pagninus, Vatablus [François Vatable] and the Vulgate translate *and God called the dry land earth*, and the Septuagint καὶ ἐκάλισεν ὸ θεός τῆν ξήραν, γήν [recte: ἐκάλεσεν ὁ θεὸς τὴν ξηρὰν γῆν] [Greek: "and God called the dry land, earth"]. Certain Hebrews say that the perfect is put for the pluperfect, and read *And God had called the dry land earth*; but one should not depart from the Vulgate. Caietanus too, and Lippomanus in the *Catena*, teach that the Lamed which is prefixed to the noun *Iabbasa* in the manner of an article serves the dative and the accusative, so that it could have been translated *and God gave the dry the name earth. And note*, says Caietanus, *that the dry is the thing named, and earth is the name.* Tertullian puts it elegantly indeed in his Poem on Genesis:

The Dry, a name soon set upon its nature: earth.

In the words *and let the dry land appear*, too, the noun *arida* is to be taken as a substantive, not as an adjective. For the sense is not "let it be seen dried and parched", but "let that mass whose name was Arida, and which did not appear because of its covering of waters, now appear, being uncovered by the waters". This is how Basil, Ambrose, Chrysostom, Cyril, Damascene, Theodoret and

very many Fathers and interpreters understand it, as to the primary sense. In a secondary sense it can be understood as if adjectivally: dried on the surface, and made firm and more solid by God — not by the Sun, which was not yet present — and made fit to bear the feet of animals and to sprout herbs.

For although in the first instant it was created entirely dry, as its nature demanded, and did not yet have waters within itself, afterwards, up to the third day, it gradually drew in waters as to its upper parts and its skin, so that it was muddy and marshy: its dryness attracted moisture, the upper parts of the earth drank in the nearest waters in the manner of a sponge, and God did not prevent this moistening, which follows naturally from that contact. Therefore God ordered the waters to be gathered into one place — that is, within the cavities and receptacles of the sea and of the abyss lying hidden in the depth of the earth — in such a way that by the same motion he drew out or extracted, by himself or through Angels, a great part of the water that had already penetrated the skin of the earth and made it muddy and stagnant like a marsh; and so he dried the Earth's skin. For it is one thing to consider the Earth in the first instant of creation, another after that instant up to the separation on the third day, and another after the separation of the waters. In the first instant it was simply dry, but immediately afterwards it began to be moistened, and over two continuous days it was soaked more and more by the waters poured into it. After the separation, however, it received a larger quantity of waters within channels and subterranean cavities, while less water remained on its surface and skin than there had been before.

The Fathers and interpreters who held that water was mixed with the earth are to be understood of the second state, not of the first. So St Gregory of Nyssa, in his book on the history of the six days, says that it was God's work *to separate the Water from its mixture with the earth, and to collect and gather it, confined in the caverns of the earth, pressing out of the earth the moisture implanted in it by condensation and constriction.* And St John Chrysostom, homily 5 on Genesis: *Since all things* — that is, every part of the surface and skin — *were filled with waters, he ordered the multitude of the waters to withdraw into one gathering.* And Iunilius on the Hexaemeron: *So that the earth, which, covered by waters, remained muddy and weak, might by their withdrawal be made dry and fit to receive shoots.* This is how I understand Hugo Victorinus, *De sacramentis* book 1, part 1, chapter 21, when he says of the earth: *At first indeed muddy and slippery, and bare, as one that had not yet brought forth any shoots*; and St Basil, who in homily 4 of the *Hexaemeron* calls the Earth, before it appeared uncovered by the waters, *unformed, uncomposed, full of mud, and mixed with waters*. Granted, Basil thought that those waters had penetrated very deep towards the depth of the earth, when he says: *Not only did the overflowing waters flow off the surface of the earth, but whatever water too was mixed in through the dimension of the whole depth gave way, obeying the inexorable command of the Lord.* For he does not contend that no part of the land was dry and immune from water, but that whatever water there

was anywhere in the depth of the earth was separated, saving the small portion that was required for the fertility of the earth.

Granted, Philo, in the book *De mundi opificio*, said — whether from his own opinion or by way of exaggeration: *After this, since all the water had been diffused through the whole earth and had penetrated all its parts, as a sponge drinks in moisture, so that, like some muddy marsh, it was confounded out of both elements macerated together and was in a manner fermented into one undistinguished and formless nature, God orders whatever of the waters was salt, and would hinder the fertility of sown crops and trees, to be collected and to flow together into one from all the openings of the earth; and then the dry land to emerge, sweet moisture being left in it for its perpetuity.* For if God had not created those two elements mixed in this way, but they had mixed themselves, the water by wetting and the earth by drinking the water, how could the water have penetrated within the space of two days through the whole depth of the earth, which is 4000 Italian miles and more? But Philo also errs when he thinks that the water was salt before the third day; for saltness was put into the Sea afterwards, produced or increased by the burning of the Sun. Let it stand, then, that in the first instant the Earth was entirely dry.

But you will say: why then was it afterwards called Earth by God, in Hebrew *Eres* or *Erez*? Does this name not come from the root *Rus*, which means "to rush down", and does it not therefore indicate the Earth as a heap of sand apt to rush downwards? [Margin: Why is Earth named from *Ruo* or *eruo*?] I reply that it is indeed named from *Rus*: first, because it rushes downwards of itself, as far as it lies in it, on account of its weight, if part of it is placed on top of a lighter element; second, because afterwards, on the third day, it was dug up [eruta] by God, and the part of it that was in the cavities of the Seas was heaped up into mountains; third, because a great part of it was to be dug up [eruenda] by ploughing, or by digging out stones, metals and the foundations of buildings.

You will say again: why then did Moses, speaking of that first instant, use the name Earth rather than Dry Land? I reply that he did so most prudently, because he wanted to set forth at once, with two names, the whole workmanship of God created in that instant; whereas if he had said Heaven and Dry Land, he would by no means have embraced everything. So, in order to comprehend earth and water, he preferred to use the name which God gave to the Dry Land on the third day — namely, when it received the waters within itself. [Margin: Why did Moses use the name Earth rather than Dry Land?] But lest anyone take occasion of error from this, he at once added, as if correcting or explaining the name earth, *But the earth was empty and void*, so as to distinguish its nature apart from the water, which was to be named shortly after.

Quid fuerit Terra Inuiſibilis & Incompoſita, ſeu Materia informis.

What the Invisible and Uncomposed Earth, or Formless Matter, was.

XI. Theodoret, question 1 on Genesis, reads in place of the words *inanis et vacua*: *Invisible and uncomposed*, following, that is, the Septuagint translators,

who have ἀόρατος καὶ ἀκατασκεύαστος [Greek: aoratos kai akataskeuastos, "invisible and unwrought"] — that is, inconspicuous or invisible, and uncomposed, or not yet prepared. [Margin: Why invisible and formless?] And in Wisdom 11 God is said to have created the world out of unseen matter, or, as the 70 translators have it and St Augustine reads, ἔξ ἀμόρφου ὕλης [Greek: ex amorphou hylēs, "out of formless matter"], that is, out of formless matter. Granted, some have taken occasion from this to assert a prime matter without any form, or under the common form of corporeity, created prior in time to the elements; nevertheless the truer opinion is that this matter did not precede the forms of the elements except in origin and nature, as we shall teach in question 5. It was called invisible and formless, then, for three reasons, as St Thomas notes, part 1, question 66, article 1. There, after saying with St Augustine that matter was prior to forms in origin only — *as sound is prior to voice, and voice to word* — he adds that Saints Basil, Chrysostom, Ambrose and the others who call it formless are not in conflict with St Augustine, and he continues: *But other Saints take the formlessness of matter not as excluding all form, but as excluding that beauty and comeliness which now appears in corporeal creation; and in this sense they say that the formlessness of corporeal matter preceded its formation in duration.* For, as the same author explains, it lacked light, visibility and the forms of mixed bodies, especially of herbs and plants; it was invisible, then, not of itself but by accident, first because it was covered by waters on every side, and second because light was lacking. This is taught in the clearest and plainest words by Saints Basil, *Hexaemeron* homily 2; Ambrose, *Hexaemeron* book 1 chapter 7; Augustine, *De Genesi ad literam* book 1 chapter 13, question 106 on both Testaments, and sermon 134 *De tempore*; Athanasius, question 80 on the Old Testament; and likewise Severian, Procopius and Theodoret on chapter 1 of Genesis, and Alcuin in the same place, question 30. But Saints Basil and Ambrose add that it was invisible also because there was not yet a man to see it, or for whose sake God might look upon it; and Procopius, because it was not yet worth looking at, since it was stripped of every ornament. It was uncomposed and formless, on the other hand, because it was not yet separated from the waters, not yet prepared for cultivation, and not yet clothed with shoots and other forms — as the same authors teach, in Martinengus [Ascanio Martinengo], *Glossa magna*, from page 329.

And although the water too lacked fish, and the air birds, yet because these animals eventually lie down and nest on land and on the bottom of the sea and the rivers, and so belong to the earth, and because the land holds many more ornaments than water and air, it was enough for Moses to say that the Earth was empty and void, so that the same could be understood of those two elements. For of their emptiness, and that of the heaven, as regards lights and stars, he was about to say that darkness was upon the face of the abyss — which must be explained next.

Et Tenebræ erant ſuper faciem Abyſſi.

And Darkness was upon the face of the Abyss.

XII. Darkness is called in Hebrew *Vehsech*, or *Chosech*, but in the singular number — as if the Latins had the word and could say *Tenebra*; "upon the face" is *Al pene*; and Abyss is *Thehom* or *Thethoh*. The Greek edition of the Seventy translators has καὶ σκύτος ἐπάνω τῆς ἀβύσσου [Greek: "and darkness over the abyss"], that is, *and darkness over the abyss*. The Chaldean version, keeping to the Hebrew reading, translates *And Darkness upon the face of the abyss*; but the *Biblia Regia* has *and Darkness over the abyss*. Lusitanus and Caietanus: *Obscurity upon the faces, or surfaces, of the abyss*. Pagninus and Vatablus: *Darkness was on the surface of the gulf*. Caietanus, however, says that the verb "was" is not set down a second time in the Hebrew, but that the one set down earlier serves, where it is said: *But the earth was empty and void, and darkness or obscurity upon the face of the abyss*. The Vulgate, by contrast, said for clarity: *And darkness was upon the face of the abyss*.

This darkness signifies the privation of the stars in the heaven, and of light in that whole transparent body which was above the waters — namely in the air, the aether and the heaven — because that body was and is simply transparent, and so the proper subject of transmitted light. The water, by contrast, although it lacked light, was not simply transparent; and for that reason darkness is said to have been upon the surface of the abyss, not in its depth. From this it follows that through the darkness itself, obliquely, as the schools put it, its proper subject is connoted and signified; and so, in another and subtler way too, the whole of that transparent body which is above earth and water is signified to learned men, granted that it had already been expressed once by the name of heaven.

Rightly, therefore, Rupertus said, *On Genesis* book 1 chapter 8: *Especially since by the darkness which was upon the face of the abyss the neighbouring air could be understood, which was as yet irradiated from above by no lights, and is therefore rightly designated by the name of darkness*. So too Procopius on Genesis: *Darkness does not subsist of itself, but is apt to be illuminated, and needs the presence of light; for air is receptive of light*; and St Ambrose: *There was darkness, because the air itself is dark*. In the same way Comestor, *Historia Scholastica* chapter 2, the Master on book 2, distinction 12, and others understood by the name of darkness the Air deprived of light. Caietanus, on the other hand, says on Genesis that the transparent body which was the subject of this darkness was the abyss. And later I also read in Suarez, *De opere sex dierum* book 1 chapter 7, that by the darkness the air is understood, as the proper subject of light, but not fire: for he thinks that fire naturally shines and would not lack light except by violence, but that even if it had had light, it would, because of its extreme rarity, neither have been seen nor have illuminated the air, much less the surface of the water.

Rabbi Moses, by contrast, as reported by Hamerus, says that fire is signified by the name of darkness, because of itself and in its own sphere it is not luminous. He tries to confirm this from the fact that Moses the Lawgiver, in Exodus, after saying *You have heard his words out of the fire*, said a little later *You have heard*

his voice out of the darkness. But in vain: he calls the fire dark because the flame that appeared on Mount Sinai was mixed with gloom and smoke. Rightly, then, St Thomas, part 1, question 66, article 1, judged this interpretation of Rabbi Moses forced. Hugo Victorinus, however, *De sacramentis* book 1 part 1, and Richardus, *Exceptiones* book 2, give the name Darkness to the three upper Elements confused together in the manner of a mist, and the name abyss to the earth itself and its depth. [Margin: What is the Abyss?]

The name of Abyss properly signifies the unsearchable depth of the waters, as St Basil says, *Hexaemeron* homily 2: *What does abyss signify? An abundant water, to whose bottom one could not easily penetrate downwards.* Eucherius subscribes to this, *On Genesis* book 1 chapter 1: *For the immense depth of the waters is called Abyss*; and St Augustine on Psalm 41: *The Abyss is a certain impenetrable depth, and is said above all of a multitude of waters*; and *Contra Faustum* book 22 chapter 11: *The Abyss is an inestimable depth of waters*. In exactly the same way the Abyss is defined by Theophilus of Antioch, *Ad Autolycum* book 2; Iunilius and Procopius on Genesis; St Damascene, *De Fide* book 2 chapter 9; St Isidore, *Etymologies* book 13 chapter 20; Rupertus, *On Genesis* book 1 chapter 7; Gulielmus Parisiensis, *De Universo* part 1 chapter 34; and Caietanus on Genesis chapter 1 — although a little later he extends it to all transparent bodies, as Vielmius also does, lecture 8, both saying that by the name of abyss the heavens and all the transparent elements, with their depth, can be understood. But St Ambrose, restricting the word to the waters, notes acutely in *Hexaemeron* book 1 chapter 8 that the swine driven by the demons threw themselves into the lake, so that the demons, who had begged Christ that they should not go into the abyss, nevertheless ran into what they had refused. And Suidas, in Hamerus's translation, explaining the words of Psalm 103, *The Abyss like a garment is its clothing* — that is, the earth's — says that the earth is called clothed with the abyss because it was filled with small and great depths of waters. Figuratively, however, the word is transferred to signify the depth of any gulf whatever — that of the underworld, for example — and the depth of anything unsearchable or scarcely intelligible, such as formless Matter, as Martinengus teaches in the *Glossa magna*, page 355, from St Augustine, Eucherius, Catherinus, Tostatus and others. The interpreters of Psalm 41 may also be consulted, on the words *abyss calls to abyss*.

Et ſpiritus Domini ferebatur ſuper aquas.

And the spirit of the Lord was borne upon the waters.

XIII. The Hebrew has *Veruah Elohim Merachephet al pene hamaim*, that is, *the spirit of the Lord bore itself upon the faces of the waters*; and the Septuagint καὶ πνεύμα θεοῦ ἐπιφίριτο ἐπάνο τοῦ ὕδατος [recte: ἐπεφέρετο ἐπάνω] [Greek: "and the spirit of God was borne over the water"], *and the Spirit of God was borne above the water*. The noun *Veruah* signifies in Hebrew both wind and spirit. Here, therefore, air and aether can again be understood, to whose thinness and mobility the wind is most similar — granted that the same air had also been

signified by the name of heaven. Indeed, I think that by the name of spirit is understood that whole body which, besides fire and air, also embraced the heavens; for because of their transparency, not yet bounded by any opaque body, they were invisible. The heaven which was made on the second day and called the firmament, out of water made solid and spread above it and below the Empyrean, was made visible; it is everywhere recognised as such by the Fathers, and is distinguished from the Empyrean, which is invisible to us, as I shall teach in questions 6 and 8. Since, therefore, they were then invisible not only through a lack of light but also through a lack of opacity, they were in a manner spiritual.

Pliny, however, restricting this name to the air, says, book 2 chapter 5: *Nor do I see that there is any doubt about the elements, that they are four: Fire highest, from which come those eyes of so many shining stars; next Spirit, which the Greeks and our own people call by the same word air — this life-giving breath, passing through all things and interwoven with the whole; that by its force the earth, suspended together with the fourth element, that of the waters, is balanced in the middle of space.* Indeed, in sacred Scripture too, wind, breeze and air are often signified by the name of the spirit of the Lord, by the common consent of the interpreters; as when it is said, Psalm 147: *his spirit shall blow, and the waters shall flow*; and Psalm 148: *Fire, hail, snow, ice, the spirit of the storms, which do his word*; and Isaiah 40: *The grass is withered and the flower has faded, because the spirit of the Lord has blown upon it*; and [Margin: Air and Aether under the name of Spirit.]

Isaiah chapter 49: *Like a violent river, which the spirit of the Lord drives*; and Psalm 47: *With a vehement spirit you shall break the ships of Tharsis*. Finally, in 4 Kings chapter 19, where it speaks of the vehement Wind, which indicated Elijah's excessive zeal, and of the whistling of a gentle breeze set against it, which signified zeal tempered with mildness, it is said: *a great and strong spirit, overturning mountains and overturning rocks before the Lord; the Lord is not in the spirit*, etc., *and after the fire the whistling of a gentle breeze*.

And so the words *And the Spirit of the Lord was borne upon the waters* must, or can, be understood of the air according to the proper sense of the letter: so affirm Tertullian, *Against Hermogenes* chapters 31 and 32; Origen, *Peri Archon* book 1 chapter 3; Theodoret, question 8 on Genesis; Diodorus of Tarsus, Procopius, Abulensis, Oleaster, Pererius and Ludovicus Stella on Genesis; Vasquez on the first part, disputation 108, chapter 1; and Suarez, *De opere sex dierum* book 1 chapter 7. Indeed St Augustine too admitted the same in the unfinished book *De Genesi ad literam*, chapter 4, saying: *A third opinion about this spirit can arise: that the element of Air should be believed to be expressed by the name of spirit, so that in this way the four elements from which this visible World arises have been expressed — namely Heaven, Earth, Water, and Air.* And Anastasius Synaita, setting out this explanation in *Hexaemeron* book 1, adds: *This is in keeping with the letter of Moses, as far as the history itself is concerned*; and St Athanasius prefers it, question 81, which is the second of the Old Testament.

Plato too, reading the history of Moses and not rising mystically to the Holy Spirit, of whom he knew nothing, understood air by attending to the sound of the letter, as Rupertus Abbas reports, *On Genesis* book 1 chapter 8. For he says: *This love, this goodness of the Creator, is the Holy Spirit. Not knowing this, Plato, when he had read this Scripture, judged it to be the spirit of the world, that is, the air, which in the order of the elements stands above the waters as being lighter; and because he suspected that by heaven too — of which it had already been said, In the beginning God created Heaven and Earth — fire was signified, he thought that only four elements, namely fire, Earth, Air and Water, were enumerated in this place.* [Margin: Plato read Moses; and how he understands him.] Rabbi Moses, who was certainly skilled in the Hebrew language, understood the very same, as Hamerus testifies on Genesis chapter 1, at the words *and darkness was upon the face of the abyss, and the spirit of the Lord*, etc., on which that Rabbi said: *The elements too were named according to the order of their natural position: since first he set down earth, and above it water, above which is air, above which fire stands.* That fire too is comprehended under the name of spirit — as also in that verse of Ecclesiastes 1 about the Sun, *Surveying all things in its circuit, the spirit goes on* — is affirmed by Abenezra [Ibn Ezra], David de pastris, and Vallesius, *De sacra philosophia* chapter 1; nor does Pineda [Juan de Pineda] dissent there. And by the name of spirit Hippocrates too understood fire, in the book *De flatibus*, and Aristotle, *Meteorologica* 1 chapter 4.

XIV. Nor is it an obstacle that the words *Spirit of the Lord* have been understood otherwise by many Fathers. [Margin: Other interpretations of the Spirit of the Lord upon the waters.] Caietanus, with Procopius, understood them of the Angels moving the Heavens; Aegidius, on book 2, distinction 12, question 2, article 3, of God moving the Heavens; of the good and well-disposed will, St Augustine, *De Genesi ad literam* book 1 chapter 5, and with him Tostatus and Hugo Carensis in the postils, Lyranus, the *Glossa ordinaria* and the *Glossa interlinearis*, Hugo Victorinus in the annotations, chapter 7, and Alexander of Hales, part 2, question 46, member 5; of the eternal Wisdom of God, Steuchus in the *Cosmopoeia*. But very many understood them of the Holy Spirit: namely Tertullian, *De baptismo* chapters 3, 4 and 5; Pope Clement, *Recognitions* book 6; St Cyprian, the sermon on the Holy Spirit; St Jerome, letter 83 to Oceanus, and in the *Hebrew Questions on Genesis*; St Augustine, the unfinished book *De Genesi*, chapter 4, and *De Genesi ad literam* book 1 chapter 7; St Ambrose, *Hexaemeron* book 1 chapter 8, and in the prologue to book 2 of *De Spiritu Sancto*; St Basil, *Hexaemeron* homily 2, and *Against Eunomius*; St Nyssenus [Gregory of Nyssa], in the book of testimonies on the Trinity against the Jews; St Athanasius, in the sermon against the heresies and in the letter to Serapion; St Cyril of Alexandria, *Against Julian* books 1 and 3; St Damascene, *De Fide* book 2 chapter 9; St Isidore, *De summo bono* book 1 chapter 10; Anastasius Synaita, *Hexaemeron* book 1; Procopius, Eucherius, Diodorus, and Claudius Victor, book 1 on Genesis; Philastrius, in the book *On Heresies*; Bede, in the book on the creation of the 6 days; Optatus of Milevis, *Against Parmenianus*

book 5; Rupertus, *On Genesis* book 1 chapter 8; Peter Comestor, in the history of Genesis, chapter 2; St Thomas, part 1, question 74, article 3, reply to 4; Albert the Great, *Summa* part 1, *On the 4 coevals*, question 12, article 11; and Albinus, question 29 on Genesis.

Their words are cited by Ascanius Martinengus in the *Glossa magna*, from page 372 to 385, who contends that this is the spirit by which the whole World is stirred and ruled, according to Trismegistus, Plato, and Virgil, *Georgics* 1, in the verses *Principio caelum* etc., *Spiritus intus alit* etc.; that this is the Mind of which Anaxagoras spoke; and that this is that Love which Hesiod and Parmenides said was the first of all the Gods. But these things, I say, are no obstacle, because, as Suarez notes, *De opere sex dierum* book 1 chapter 7, they can be understood to have spoken in a Mystical sense; and Martinengus himself, page 376, admits that it would not have been fitting for Moses to want to set before the uncultured Hebrew people, out of season, in the literal sense of the history, the most hidden mystery of the Most Holy Trinity, or the Holy Spirit. And indeed, if he did not want to set out to them either the Empyrean or the Angels, much less did he want to set out the Holy Spirit — nay, not even to learned men, as regards the literal sense; especially since there is no reason why he should either be silent about the divine Word or say that the Holy Spirit was borne above the waters only, and not rather poured over and into the whole world; whereas the reason why air and aether are above the waters is their natural lightness.

XV. Nor are the words *was borne upon the waters*, in Hebrew *Merachephet*, an obstacle, although they seem unable to be applied to the air, which neither moves naturally in its own place, nor had to be moved from outside at that time for any other cause. [Margin: What *was borne upon the waters* signifies.] For the Syriac version has *was cherishing the waters*, or others, *was brooding at rest upon the waters*; and so Vielmius on Genesis chapter 1: *This word indicates the site and place of the air where it was created, and not in fact a commotion and wind, but rather rest, as when a hen sits on eggs to make them living* — as Diodorus, Basil and almost all the rest write. St Jerome too asserts the same in that place, namely that it signifies not local motion but cherishing or brooding. Or, as Suarez says, *De opere sex dierum* book 1 chapter 7, it signifies the pre-eminence of a thing that requires a higher place, or that extends itself upwards above the water, or that contains the waters beneath itself. Or certainly it signifies the nature of the air, which is easily movable, as Theodoret teaches, question 8 — granted that others teach that a slight motion, after the manner of a breeze, put into the air by God for hidden reasons, can be understood; which, although it does not wholly displease Vielmius, does displease Suarez. Finally, all who understand the Holy Spirit there by the name of spirit are forced to deny it local motion: for God does not move locally.

So far, then, we have that Moses sufficiently signified, at least to learned men, the creation of the aether, or of fire and air, by the verb "create", because these elements were made out of nothing — whether under the name of heaven, or of darkness, or of spirit. Granted, St Basil, *Hexaemeron* homily 1; St Ambrose,

Hexaemeron book 1 chapter 6; St Jerome on Job chapter 38; Eucherius, Caietanus and the *Glossa ordinaria* on Genesis hint that these were indicated by the name of Earth, because air and fire, not to mention water, are enclosed in the bowels of the earth; but at that time they were certainly not enclosed.

Quæſtio 4. Quot & Quæ Elementa Priſci, ac præcipuè Patres ex Moyſe agnouerint ?

Question 4. How many and which Elements did the Ancients, and especially the Fathers, recognise from Moses?

XVI. That there are four simple bodies beneath the heaven, which are called elements — according to the Peripatetics, to say nothing of the Epicureans and Hippocrates — namely Fire, Air, Water and Earth; that they are called simple because, although they are composed of matter and form, they nevertheless have no mixture of the secondary qualities, but are held together by the properties of the primary qualities; and that they are called elements because all mixed bodies consist of them, either in act or virtually — all this is well known from Aristotle, *De caelo* 3 and 4, and his interpreters, and from most other Philosophers. [Margin: First opinion: four Elements distinct from the Heaven.] In their favour stands that very strong argument drawn from the combination of the 4 primary qualities: namely, that just as there is an Element dry and cold, which is Earth, and one cold and moist, which is Water, so there should be one hot and moist, namely Air, and one hot and dry, namely Aether or Fire; so that between the extremes of contraries there should also be means, and one element might be converted most readily into another that is akin to it [symbolum], or suffer as little as possible from the contrary extreme. This opinion was embraced, and elegantly set out, by St Gregory of Nyssa, *Philosophy* book 3 chapter 2 — or certainly by Nemesius, to whom some ascribe that work; by St Basil, *Hexaemeron* homily 4; St Ambrose, *Hexaemeron* book 3 chapter 4; St Anselm, *De imagine Mundi* book 1 chapter 3; Gregorius Pisides, *De Mundi opificio*; Hermias the Christian Philosopher, in the book entitled *Irrisio gentilium Philosophorum* [The Mockery of the Pagan Philosophers]; and many of the Poets, especially Ovid, *Metamorphoses* book 1.

Others, however, admitted the four elements in such a way that they counted the Heaven, that is, the aether, in fourth place, and called it fire, or fiery, or not distinct from the element of fire. [Margin: Second opinion: four Elements, the Heaven being taken for fire or for the fourth element.] For the Stoics called the heaven a mobile ardour; Pythagoras calls it flame, and Heraclitus fire; and Empedocles, according to Clement of Alexandria, *Stromata* book 5, enumerating the four parts of the World, said:

Earth, and the overflowing Sea, and the moist Air, Titan, and the Aether, which binds all things in a circle.

For if the aether is that in which Titan and the Titanian stars are, and which contains all things within itself, it is certainly the heaven. Pliny the Elder's saying in book 2 chapter 5 rather favours this: *Nor do I see it doubted that the*

elements are four; the fiery one highest, from which come those eyes of so many shining stars. Moreover, Plato too, when he read Moses, judged that Fire, one of the four elements, was signified by the name of Heaven, as Tostatus reports in the postils on Genesis 1, and Rupertus in book 1 of the Commentary on Genesis, chapter 8, in these words: *Plato, when he had read this Scripture*, etc., *because he suspected that fire too was signified by Heaven, thought that only four elements, namely Fire, Earth, Air and Water, were enumerated in this place.* Nor does St Augustine dissent, in the unfinished *De Genesi ad literam*, chapter 4, when he says: *So that in this way four elements have been expressed, from which this visible World arises: namely Heaven, Earth, and Water, and Air.* Nor, again, does Bede, in the *Exposition of Genesis*, speaking thus: *Nor should it be thought that air has been passed over in this scripture, since the World consists of four elements, that is, of Heaven, Air, Water and Earth. But the highest part of the air, which does not receive moist exhalations, is assigned to the heaven; while the lowest part of the air, in which are winds and rains, is assigned together with the earth.* And Pope Innocent III, on the 5th penitential psalm: *In the creation of the world Heaven and Earth were created together, the first and the lowest among the four elements, and with them likewise the two means, namely air and water, mixed and mingled together, but afterwards they were separated.* And Hugo Carensis in the postils: *Heaven, that is, the two upper Elements; Earth, the two lower.*

[Margin: Five Elements according to Pythagoras.] Pythagoras, however, seems to have posited five elements, that is, the heaven and the four commonly recognised, since Plutarch, *De placitis* book 2 chapter 6, says: *Pythagoras thinks that God [?] the creation of the world from fire and the fifth element* — on account of the five regular bodies, on which see number 18.

[Margin: Third opinion: of three elements only.] Next to these are those who recognised and asserted only three Elements, Earth, Water and Air. Franciscus Patritius [Francesco Patrizi], *Pancosmia* book 22, shows that Zoroaster, Hermes and Orpheus were of this kind; and among the more recent there is Copernicus, book 1 chapter 8; Cardano, *De subtilitate* book 1, who calls Earth and Water passive and Air active; and Tycho, *Progymnasmata* volume 1, pages 91 and 92, and frequently in the letters to Rothmann. There Tycho contends that there is no fire between the air and the aether, but that the air extends almost as far as the Moon and differs insensibly in transparency from the aether, or aetherial air; and yet that it is not, on that account, of the same nature and species as Air. Kepler holds the same in the *Optics*, in the *Mars*, and in the *Epitome of Astronomy*, calling the Aether, or Heaven of the Planets, aetherial air throughout. These views are strongly favoured by Claudius Marius Victor, Tertullian and Strabus, who are to be reported below at number 18. Add to these Io. Penna [Jean Pena] and Rothmann, who, as Tycho testifies in volume 1, page 92, recognised nothing besides Earth and Water except air extended as far as the highest Heaven.

[Margin: Fourth opinion: of two Elements only.] Many of the Ancients before Aristotle, on the other hand, recognised only two elements, namely

Earth and Water; for they will call [?] air a void. And so Paulus Burgensis [Pablo de Santa María], in the additions on Genesis, judges that only these two were expressed by Moses, so that he might accommodate himself to the less cultivated. Telesius, on the contrary, recognised only two elements, Fire and Earth, which were indicated by Moses under the name of Heaven and Earth, while Water and Air are by no means first bodies, but second.

Quæſtio 5. Quid fuerit Chaos apud Poetas, Philoſophos ac Theologos ?

Question 5. What was Chaos among the Poets, Philosophers and Theologians?

XVII. Laertius [Diogenes Laërtius] relates, in book 1, in the life of Epicurus, that Epicurus turned from teaching Grammar to Philosophy after neither the Grammarians nor the Sophists had satisfied him when he asked what Chaos was in Hesiod. A greater necessity is laid on us, however: that of distinguishing the Chaos which Moses indicated in sacred Genesis — and which sacred Scripture elsewhere, and the Fathers, indicated — from that profane one, teeming with errors, which some Philosophers or Poets dreamed of. Now Chaos is a Greek noun, called Χάος [Greek: chaos], not so much from Χάνειν [Greek: chainein, "to gape"], which is to gape from the depths, as from Χέω [Greek: cheō, "to pour"], which is to pour, and by pouring to confound many things into one — in the way that the fire of Corinth melted various metals and fused them into one bronze, called Corinthian from that. And so it signifies a confusion of many things mixed or heaped together. In Hebrew, however, it is called *Aphar*, that is, slime, as our Salianus thinks in the *Apparatus*, chapter 11; or rather it is signified by *Erer Thohu*, that is, empty Earth, and *Vabohu*, that is, void — or, as our Fernandius translates, *Solitude and emptiness*. From this, Steuchus thinks in the *Cosmopoeia*, Chaos was so named, with the letter Thau changed into Chet. Hence also, by metaphor or a certain analogy, the darkness and filth of the Underworld are called Chaos by Virgil, *Aeneid* 6:

Gods who hold dominion over souls, and you silent shades, And Chaos, and Phlegethon, places silent far and wide in night.

and by Valerius Flaccus, *Argonautica* book 7:

Through Chaos the blind shades come to meet them, without voices.

Indeed, even in the Gospel, Luke 16: *Between us and you a great chaos is fixed.* The same word is transferred to night, on account of the confusion of colours and of things, as in the Church's hymn for Sunday vespers:

You who command that morning, joined to evening, Be called day, Foul Chaos glides in,

that is, night comes on. Properly, however, it signifies that rude and undigested mass from which all bodies were afterwards made; but its better-known senses vary among Writers. [Margin: First and second notion of Chaos.] First, Hesiod and certain of the ancients feigned Chaos as unbegotten and as existing before all the Gods, or as unbegotten matter. Second, very many others said that it was a certain formless Matter, or one under a single rude form, which in act was neither heaven nor any of the elements, but only in potency, as the

seedbed of all corporeal things. Thus Trismegistus, in the ἱερολόγω [Greek: hierologō, "Sacred Discourse"], says that all things were enclosed in Chaos δυνὰμη [Greek: dynamei, "in potency"], that is, in power and virtue, before they were divided and separated. Hence Euripides, as quoted by Steuchus in the *Cosmopoeia*:

Heaven and Earth were of one form. But when, embraced together, they had been parted, Every thing came forth and emerged into the light.

And Ovid, *Metamorphoses* 1, said expressly: when

..... No thing kept its form.

and in the same place:

Before the Sea and the Lands, and the Heaven that covers all, Nature had one face in the whole world, Which they called Chaos, a rude and undigested mass, Nothing but inert weight, and heaped together in the same place The discordant seeds of things not well joined. And where there was earth, there too was sea, and air. Cold things fought with hot, moist with dry, Soft with hard, things having weight with things without weight. This strife God, and a kindlier nature, settled.

And St Gregory Nazianzen, in the poem *In Praise of Virginity*:

Night once covered all things, dreadful with darkness. There was no splendour of Dawn, etc. *But a reckless motion rolled all things, confused together, Bound by the heavy chains of darkness, Which ancient Chaos had spread over the whole world.*

So too Claudius Marius Victor sang, in the Commentary on Genesis:

And so that rich Wisdom might make this World, It brought forth all things together; but after this the seeds of things

Adorned, formless as they were, with forms laid over them.

Indeed, Diodorus Siculus too, as quoted by Eusebius in book 1 of the *Praeparatio Evangelica*, chapter 4, said: *For they report that before the beginning of all things there was one form of Heaven and Earth, the two being confused together; afterwards, when they had been separated*, etc.

[Margin: Unseen matter: what is it?] Taking Chaos in this sense, then, not a few have judged that it was signified by Moses under the name of Heaven and Earth, or at least under the name of the empty and void Earth; or also in Wisdom 11, where it is said: *For your Almighty hand, which created the world out of unseen matter, was not without power*; or finally by St Paul, Hebrews 11: *By faith we understand that the worlds were framed by the word of God, so that visible things might be made out of invisible ones.* That passage of Wisdom, in the Greek of the Seventy translators, reads ἔξ ἀμόρφου ὕλης [Greek: ex amorphou hylēs, "out of formless matter"], *out of formless matter* — of which I have already spoken, question 3, number 11.

And so, first of all, St Augustine, *Confessions* book 12 chapter 20, and book 1 of *De Genesi contra Manichaeos*, says: *The first matter was made confused and formless — the things which are distinct and formed — which I believe the Greeks call Chaos; for so too, we believe, it was said in another place in the praises of God: You who made the world out of formless matter, which some codices have as "out of unseen matter".* And in chapter 7: *That formless matter, then, which God made*

from nothing, was first called Heaven and Earth, and it was said, In the beginning God created Heaven and Earth — not because this already was, but because this could be. And in the unfinished book *De Genesi ad literam*, chapter 3, he calls this matter *the Seed of heaven and earth*; but he says this only by way of inquiry, not of assertion, for he says: *Let this much be inquired on this point; for none of these things could be rashly affirmed.* From book 1 of *De Genesi ad literam*, chapter 14, however, it is clear that he gave this matter a priority not of time but only of origin, and that it preceded the forms in the way that sound is prior to voice. For he says: *Not because formless matter is prior in time to formed things, since both were created together, both that from which it was made and that out of which it was made. For just as voice is the matter of words, and words indicate formed voice — and yet he who speaks does not first emit a formless voice which he can afterwards gather and form into words — so God the creator did not make formless matter at an earlier time and afterwards form it. For he created matter already formed. But because that from which something is made, even if not in time, is yet prior by a certain origin*, etc.

Philo too, in the book *De Mundi opificio*, after saying *that Moses reached the summit of Philosophy*, adds that, according to Moses' narration, a substance was made by God *having nothing good of itself, which yet could become anything; for of itself it was devoid of every quality, undigested, inanimate, full of rudeness, confusion and discord, but capable*, etc. St Gregory of Nyssa joins them, in *On the history of the six days*, where he says of God the craftsman: *But since he can do all things, he prepared beforehand, for the making of things, all things together by which matter is compounded, by a will equally wise and not [?] powerful, and gathered them together — light, heavy, solid, dense, rare*, etc.; *for none of these is matter by itself, but when they have concurred with one another they make up matter.* And not much later: *For when the whole nature of things had already been created in a heap, in one moment of the divine will, promiscuously and indistinctly, and all the elements had been mixed with one another*, etc. The other St Gregory, Nazianzen, indicates the same matter in the oration on the Holy Spirit: *It is agreed that God in six days set matter in place and formed it*, etc.; and St Gregory the Great, on Job, book 32: *For the substance of things was created at once, but it was not created at once in its species; and what came into being at once in the substance of matter did not appear at once in the species of form.* Philastrius expressly agrees, in the catalogue of heresies, heresy 63, saying: *There is another heresy which is in doubt about the earth, saying that this is the only one and that there is no other: not knowing that there is another, which is as it were the womb of all things and contains all things; which is found in the scriptures as* ὕλη [Greek: hylē, "matter"], *that is, matter; which is invisible and uncomposed, and which was made by God on the first day; whereas this one on which we live is shown to be founded upon water, as David said: You who founded the earth upon the water.* [Margin: Psalm 103.] No less clearly Tatian, in the oration against the Greeks: *This is how the matter stands. It is clear that the whole machine of the world, and whatever stands established in it, was constituted out of matter; but that matter itself was produced by God, so that it is understood partly as rude and formless, before it was separated, and partly as conformed and set in order, after its*

separation was accomplished. Theophilus of Antioch too, explaining the first and second verses of chapter 1 of Genesis in book 2 *To Autolycus*, concludes: *Sacred Scripture does this first of all, and admonishes us, so as to teach that matter was in some way made by God, from which he constituted the World and adorned it into a fitting order.* Very briefly, Alcuin, question 19 on Genesis: *He who lives for ever created all things together in formless matter*; and he repeats the same in question 28.

Let us come to later centuries. Peter Lombard, or the Master of the Sentences, book 2, distinction 12, explaining the first verse of Genesis, says: *Heaven, that is, the Angels, and Earth, namely the matter of the four elements, still confused and formless, which was called Chaos by the Greeks; and this was before every day*; and further below: *Some indeed have taught that all things were created together in matter and form, which Augustine seems to have held. Others have rather approved this, and asserted that first matter, rude and formless, holding the commixture and confusion of the four elements, was created; and that afterwards, through the intervals of six days, the kinds of corporeal things were formed from that matter according to their proper species. This opinion Gregory, Jerome and Bede, and many others, commend and prefer. It also seems to fit the scripture of Genesis better.* Afterwards, however, he seems to recognise Chaos, and that matter, as made of the Elements in act, but confused and not yet adorned, as we shall say in the 4th opinion, to be reported at number 19 — although St Bonaventure counts him in this second class, in which Bonaventure himself most evidently stands, on book 2, distinction 12, question 3, article 1, asserting: *There is another, more reasonable way of speaking: that that matter was produced under some form, but that this was not a complete form, nor one giving complete being to the matter; and therefore it did not inform it in such a way, because the matter would still be called formless; nor did it so far bring the appetite of matter to an end, because matter still desired other forms.* And he gives the example of the embryo in the mother's womb, whose form, however, he says is more perfect than that rude form under which matter was first created.

Alphonsus Tostatus, in the postils on Genesis, prefers this opinion, though he does not condemn the one we shall report in fourth place, at number 19. For after he had said: *The second position: that at the beginning of time incorporeal things, namely the Angels, were created, and the matter of all corporeal things under a certain common form of corporeity, not determined to any species of created things; and that afterwards, over the days, the distinction of things into species was made from that matter; and this position is held by Strabus, Bede and other doctors* — he added: *This position is very much in conformity with the letter.* Then, interpreting the words *upon the face of the abyss*, he clearly teaches the mode of the creation and distinction of things, saying that that matter lacked heaviness, lightness and the four first active qualities, and had only density mixed with rarity; that its upper part, which is now above the concave of the Moon, passed into the matter of the celestial bodies, and the lower part into the elements and the other sublunary bodies; and that from the part most condensed of all the earth was made, and then water, air and fire, with less density and greater rarity and

expansion. Lyranus likewise, in the postils, ascribes this to Strabus and Bede, and does not himself disapprove it. Let Ambrosius Catharinus be added to these. On the first chapter of Genesis, at the word *Earth*, relying on the words of Wisdom 11, *He created the world out of unseen matter*, and after adducing the authorities of the Poets, Philosophers and Fathers, he concludes with these words: *We must therefore confess that the matter from which the world was produced was prepared beforehand, unless we wish utterly to violate the words of scripture. For if matter had been created at the same time as some creature, how could it truly be said that the world was made out of it?* He then teaches that its act was very weak, confused, and more imperfect than the simple forms of the elements themselves.

XVIII. [Margin: Third opinion on Chaos.] The third opinion places Chaos, or the matter of the Heavens and of the elements, under the predominant form of one element or simple body. That body some called iron[22], as Xenophanes did; others watery and fluid, as Thales of Miletus and Xenocrates; others airy, as Anaximenes; others fiery, as Parmenides; or made of atoms of air and fire, as Epicurus. [Margin: The order of the separation of the Elements.] But as to the order in which these men — or those who held that the elements were in that chaos in act, but mixed — thought that the remaining elements and the heaven were produced or separated, some held one view and some another, as reported in Plutarch, *De Placitis* book 2 chapters 6 and 7. For very many of those called Physicists said that the beginning was made from the centre and from the earth, because the centre is the beginning of a globe:

Empedocles, however, held that air was separated out first, then fire, afterwards earth, and lastly water. Plato, for his part, judged that the visible world was made in the likeness of the intelligible world in such a way that, just as in it the soul is prior and then the body, and in the body the fiery and earthy portion comes first, so fire and earth came first, and then air and water. So Plutarch, chapter 6; but in chapter 7, reporting Plato's very words, he says that in Plato's view *Fire was made first, then aether, after it air, from there water, and afterwards earth.* In chapter 6, however, he speaks of Pythagoras a little differently; for after saying *Pythagoras: the beginning of the world from fire and the fifth element*, he adds that the world was made according to the figure and order of the five regular bodies. [Margin: The 5 regular bodies as a type of the fabric of the world, according to Pythagoras [?] and Plato.] It is worth bringing in Plutarch's words: *Pythagoras: since the solid figures, which are called mathematical, are five, the earth was made from the cube, fire from the pyramid, air from the Octahedron, that is, the figure of eight seats or faces; water from the Icosahedron, that is, the figure of twenty seats; and the universal globe from the Dodecahedron, that is, the figure sitting on twelve positions. In all these Plato Pythagorises.* Finally, St Gregory of Nyssa, *On the history of the six days*, teaches that fire leapt out of Chaos first,

[22] printed *ferreum*; the sequence of the elements, and Xenophanes' own doctrine, want *terreum*, earthy

and that all things were illuminated by it; for by the light that was made first he understood the separation of Fire from Chaos.

XIX. [Margin: Fourth opinion on Chaos.] The fourth opinion recognised in Chaos the elements in act, according to their simple forms, but mixed — either all of them or some. Such was that celebrated homoeomeria of Anaxagoras, and of Empedocles — except that Anaxagoras thought that other substances too lay hidden in it. But St Basil, homily 2 on the *Hexaemeron*, expressly rejects, first, the opinion of formless matter adduced in second place, saying: *But the depravers of the truth, not accommodating their own mind to Scripture but dragging the mind of the Scriptures to their own will and perverting it, say that matter is hinted at by these words. For this, they say, is invisible and unadorned by its own nature, inasmuch as it is separated from every quality, form and figure.* Then he explains the reason why the earth is called invisible and uncomposed: namely, because it was covered by waters. And in the first homily he had taught that the remaining elements were then created under their own forms, saying: *From two principles he indicates the essence of the whole universe: ascribing to Heaven the prerogative of dignity in generation, and to earth the second place. Whatever is intermediate between these took its beginning together with the very extremes. Therefore, although he said nothing of the other elements — fire, I mean, air and water — yet it is fitting that they be understood by your own skilful wit.* St Ambrose nearly copied these words and subscribed to them, *Hexaemeron* book 1 chapter 6; and most openly St Damascene, *De Fide* book 2 chapter 5: *Our God himself, who is glorified in Trinity and Unity, made Heaven and earth and all things that are in them, leading them from nothing into being. All these, indeed — Heaven, Earth, air, fire, water — not out of a subject matter.* And chapter 10: *Earth is one of the four elements, endowed with dryness, cold and heaviness, and unmoved; produced by God on the first day out of nothing into existence. For Scripture says, In the beginning God created heaven and earth.*

St Augustine taught similar things, as we saw above, since he teaches that forms were concreated with matter at the same time; and so did Junilius in his *Hexaemeron* — for he says expressly that the four elements, of Earth, water, air and fire, were created with the heaven, granted that not all were expressed by Moses, and that air and fire lay hidden within the earth, while water was above the earth. Alcuin likewise, question 20 on Genesis, enumerating all the things that were created or made from nothing, and so not from that unformed matter, says: *Which creatures were made from nothing?* and answers: *Heaven, Earth, Angels, light, air, water, the soul of man* — where by the name of light he understands fire, as St Nyssenus also does, as will be seen later when we treat the production of Light and of the stars. And the author of the *Glossa ordinaria*, Strabus: *All the elements at first sight mixed and confused, and this whole space of air full of waters — not such as they are now, but like thin clouds.* Very close to this view is the opinion of Hugo Victorinus, chapter 5 of the preface to Genesis, where, after asserting that the Empyrean, the Angels and matter were created at the same time, he says: *It was created formless, not wholly lacking form; but it can*

be called formless by comparison with the beauty and order that followed. Afterwards he describes the disposition and order of the elements in Chaos in this way: *The earth was in the middle, having in itself channels and veins, receptacles, that is, of the waters flowing both over the earth and within it. The three remaining elements, confused into one, after the manner of a thick cloud, were borne above the earth from every part and surface of the earth up to the highest Empyrean; and therefore they are called by the name of earth where it says: God created Heaven and Earth.* He confirms this more fully in book 1 of *De Sacramentis*, part 1, chapter 6; nor does the Master reject it, book 2, distinction 12. And Richard of Saint Victor, book 2 of the *Exceptiones*, thinks that the three upper elements, confused together, are designated by Moses under the name of heaven; for he says: *He calls heaven three elements, fire, air and water, which were mixed in one confusion, or confused in one mixture, floating around the fourth, which is earth; and the earth, balanced by its own weight, settled in the middle and at the bottom of the three.* Besides these, St Anselm of Canterbury, *De imagine mundi* book 1, calls the matter from which all things were created the four elements: *In matter, so that all corporeal things might be created out of four elements.*

Finally, that Chaos was the heaven and the four elements, complete in act with their substantial forms, is taught by Theodoret in the questions on Genesis; Rupertus, book 1 on Genesis, chapter 6; Hugo Carensis in the postils; Ascanius Martinengus, *Glossa magna* page 277; Suarez, *De opere sex dierum* book 1; Henricus, *Quodlibet* 6, the last question; Pererius, book 1 on Genesis, page 31; and Tannerus, question 2 *On the heaven.*

[Margin: The order of the Elements in Chaos.] Among the Authors of this Class, however, there were various dispositions of the Elements in Chaos. According to Empedocles, Saints Basil, Ambrose and Damascene and most of the Greeks, St Anselm, Alcuin, Caietanus and Pererius, the Earth was, in that first instant, in the lowest place; above it water, above the water air, above the air fire, and above the fire the aether or heaven. According to Junilius, however, fire and air were enclosed in the Earth, and water was above the earth. According to Strabus and Lyranus, Hugo Victorinus and Richard of Saint Victor, the Earth was at the bottom, and above it the three remaining elements, mixed with one another after the manner of a thick cloud and extended as far as the Empyrean, as Hugo says, or as far as the Heaven, as others say. But according to Pope Innocent the heaven was separate, in the place of fire, and so was the earth, while water mixed with air was in the middle — so he says on the 5th penitential psalm. How this matter was called formless, invisible and uncomposed was sufficiently explained in question 3, number 11.

XX. [Margin: Fifth opinion on Chaos.] The fifth opinion, besides the heaven placed outside Chaos, recognises in Chaos only Earth and Water in act, and holds that all corporeal things were afterwards made from these. Homer seems to have been of this opinion, when he brings in men imprecating utter destruction on their enemies with that verse:

But may you all return to earth and water.

Adducing this verse, St Justin says: *by vehement anger, dissolving them out of earth into their ancient nature*, as Eugubinus too noted, *De perenni philosophia* book 1. Orpheus holds the same: he called that primeval matter ἰλὺν [Greek: ilyn, "slime"], that is, Slime composed of earth and water, which the Hebrews call *Aphar*; although in Patritius he posited three elements. This opinion is learnedly defended by Gulielmus Parisiensis [William of Auvergne], chapters 31 and 39; Halensis, part 2, question 45, member 2; Rupertus, book 1 on Genesis, chapter 6; and our Molina, *De opere sex dierum*, disputation 2. Salianus subscribes to Molina in the *Apparatus* to the *Annals*, chapters 6 and 11, and so does our Fernandius on Genesis, chapter 1, section 2. Steuchus too says in the *Cosmopoeia*: *The beginning of all earthly things was formless hylen, and this was the best plan for creating the muddy earth, so that all things might be made from it; for we have shown that hylen is as it were ilyn and slime.* And he teaches that air and fire were afterwards made from these, and adds that Moses rightly called the Earth empty and void, that is, *Thohu* and *Bohu*; that this is the very thing which among the old Latins was *Chohum* — for so they called the World in its first origin, as Festus Pompeius notes — and among the Greeks Chaos, with Thau changed into Chet; and that the names Chaos and Chohum therefore flowed from that Hebrew root. Telesius, for his part, posits two elements, Fire and Earth, which were indicated by Moses under the name of heaven and of Earth. But three elements were posited by Zoroaster, Hermes, and — if you believe Patritius, *Pancosmia* book 22 — Orpheus; likewise by Cardano, *De subtilitate* book 1; Tycho, *Progymnasmata* volume 1, page 92; and Kepler, in the *Optics* and in the *Epitome*. Therefore, if Chaos consisted of elements, according to them it was compacted of these three.

XXI. [Margin: Sixth opinion on Chaos.] The sixth Opinion, midway between the 4th and the 5th, places in Chaos only three elements, namely Earth, Water and Air, inasmuch as these are expressly set down by Moses under the names of Earth, Water and Spirit. This is the view of Cyprian the Poet, in his poem on Genesis, as cited by Ascanius Martinengus at page 272 of the *Glossa magna*, and of Claudius Marius Victor, on Genesis, in these verses:

But, to touch at once on the first beginnings of things: The vault of the lofty sky and the Earth lying below Are the first part of the work; but the wave covered the lands, And the Air in the midst, dripping with a foul cloud, And the vast abyss veiled itself in black darkness.

And Tertullian, disputing against Hermogenes, denies that there are two earths, one first unformed and the other formed, saying in chapter 31: *But for us there is one God, and one earth, which God made from the beginning*; and he adds in chapter 32: *For if the abyss is below the earth, and darkness above the abyss, then without doubt both the darkness and the abyss are below the earth. The spirit and the waters, however, lay below the Heaven; for if the waters, which had covered it, were above the earth, and the spirit above the waters, then both the spirit and the waters were likewise above the earth. And what is above the earth is certainly below the heaven; and just as the Earth lay upon the abyss and the darkness, so the heaven too lay upon*

the spirit and the waters. From this it is quite clear that by the name of spirit he understood air, or took as one and the same thing the whole of what lay between the waters and the heaven. Strabus seems to favour this in the *Glossa ordinaria*, inasmuch as among the elements that were at first mixed together he places Earth, and water throughout the whole space in which air now is, but water then mixed with air and like a thick mist — unless he also thought fire was enclosed in it. Finally, this opinion is favoured by Zoroaster, Hermes, Cardano (*De subtilitate* book 1), Tycho (volume 1 of the *Progymnasmata*, page 92), and Kepler (in the *Epitome of Astronomy* and the *Optics*), who posited only three elements, namely Earth, Water and Air.

Lastly, there can be brought back to this opinion those who placed the heaven among the 4 elements, but posited as elements distinct from the heaven only Earth, Water and Air, and yet did not take the heaven for fire: such as St Augustine, in the unfinished *De Genesi ad literam*, chapter 4; Bede, in the *Exposition of Genesis*; and Innocent III, on the 5th penitential psalm, who sets water and air confused together — I quoted their words above; and also Io. Penna the Frenchman and Christoph Rothmann, who, as Tycho testifies in volume 1, page 92, infer from the same transparency that the heaven and the aether are of the same nature as air, and hence that there are only three elements.

XXII. [Margin: 7th opinion on Chaos.] The seventh opinion seems to hold that at the beginning of the World only the highest heaven and the Earth were created from nothing, while Water and the other bodies were produced afterwards. It is the opinion of St Gregory, as will be evident to anyone who reads his whole discussion in the Commentary on Job, chapter 38. On his view, therefore, Chaos enclosed the earth alone; for the remaining heavens and the other bodies were produced from water, and water from earth, as he teaches in the same place. But this is hard to grasp. Did the earth, then, occupy the whole space that now lies between the earth and the highest heaven, and occupy a smaller space after being condensed? Or was there a void between it and the heaven? Whatever you say will run against the other Fathers and the Philosophers, and will hardly agree with the Scripture of Genesis 1, which supposes that the waters were created at once with the earth.

XXIII. [Margin: 8th, and our own, opinion on Chaos.] The eighth opinion is ours. It has as its patrons the authors of the third, and some of the second — if, as we can, we interpret them more kindly — and some of the fourth and sixth, namely if fire does not exist, or does not exist as distinct from the visible heaven; although not all those of the third opinion agree with us as regards the order. For we assert that the World was not made by God from unbegotten or uncreated matter, as some of the 1st opinion hold, for that would be heretical; nor from matter unformed or crudely formed, which was not actually earth, nor water, nor air, but only potentially, as the 3rd opinion asserts — for one thing, because that imperfection would have been utterly unbecoming to God the craftsman; for another, because Moses must not be said to have fallen short by passing over in silence, in the first chapter of Genesis, the moment or the

day on which God afterwards drew the elements out of that matter; and for another, because if at that first instant heaven and earth had existed not actually but potentially, Moses would have used other names, and would not have given the uneducated people an occasion to go astray and to understand heaven and earth in the same sense in which they are understood in Psalm 101, where it is said: *In the beginning, O Lord, you founded the earth, and the works of your hands are the heavens.* Above all, because sacred Scripture, so long as no manifest contradiction follows from the literal sense, must be taken in its proper and historical sense, the sense the letter conveys; and sacred Scripture, in Genesis 1, says that heaven and earth were created by God in the beginning, and that before light or anything else was made there was water, and spirit over the water. We must therefore understand these things, created by God in the beginning, according to their proper forms — whether fire is then indicated by the common name of spirit, or by that of heaven, or there is no element of fire at all.

For these things are not in conflict with the words *The Earth was empty and void*, or with the words *from unseen* or *unformed matter*. On the contrary, they are reconciled excellently, and without any violence, with Earth, Water, Air and Heaven complete in their substance — and this in three ways. First, on account of the lack of light, which made dark the whole depth, not only of the waters and of the earth's surface but the whole depth of the air and the aether; and so St Augustine, *De Genesi ad literam* book 1 chapter 1: *The Earth was invisible and unformed, and darkness was over the face of the abyss* — by which words he seems to signify the unformedness of bodily substance. Second, on account of the covering of the Earth, beset on every side by water, so that even if there had been light and eyes, the eyes of animals could have seen nothing from the earth, much less above it. Third, and chiefly, on account of the lack of the forms of mixed bodies, of colours, and of the other accidents and secondary sensible qualities; for that reason it was unformed and unfit to be seen, as we also explained in Question 3, from number 11 on. Nor is it any obstacle that so many Fathers were adduced for the 2nd opinion; for many of them either must or can be understood of a matter unformed only in origin and in nature, preceding the formation of the elements, not in time — as St Thomas, with St Augustine, and the followers of Augustine understood it. And once it is granted that God created the Earth, no solid reason can be brought why he did not create all the elements from nothing, whether these be three or four. [Margin: The order of the elements in Chaos.] As for the order, I hold that God did no violence to those elements, and placed them as their natural order required before they became the dwelling-place of animals: namely the earth at the bottom and middle of the world, and all the water over the earth, since the separation of the waters and their enclosure in the vessels of the earth had not yet been made; and the spirit over the waters, and the aether, that is, fire, if it exists, over the air. For in this way the Earth could be called simply both

"empty" and "Dry", if it did not even have waters within it at that first instant — in keeping with what was said more fully at number 10.

Quæſtio 6. An detur, & quando factum, & cuiuſmodi ſit cælum Empyreum?

Question 6. Whether there is an Empyrean heaven, and when it was made, and of what kind it is.

XXIV. By the name of the Empyrean heaven is understood the highest heaven of all, invisible to us in this life and without motion, within which, or upon whose surface, are and will be the Saints and the blessed Angels and men. They called it Empyrean, that is, splendid, from the most pure light with which it is believed to be endowed — although it might seem possible to render it "flaming" or "fiery", from the Greek noun πῦρ [Greek: pyr, "fire"], which signifies flame tending upwards, or fire. For it is named not from heat but from splendour, as Strabus, Hugo and the Master teach on book 2, distinction 2; or else because, just as fire is the highest of the elements, so this heaven is the highest of the heavens, as Suarez says, *De opere sex dierum* book 1 chapter 4. We have various names and descriptions of it in sacred Scripture, for our consolation and for the hope of eternal life which GOD has "promised again" — that is, promised not once but many times — to those who love him. These names will serve us as the first argument that such a heaven is distinct from the others. For Caietanus alone, so far as we know, on the 2nd Epistle of St Paul to the Corinthians, chapter 12, has dared, against the torrent of the Fathers and Doctors, to deny such a heaven, and to say that God imparts his happiness to the Blessed in no bodily place. [Margin: Caietanus alone denies the Empyrean.] For he speaks thus: *"Even to the third heaven": that is, even to the highest heaven, even to the place in which God and the Blessed are said to dwell, even to the heavenly fatherland. And he called that place the highest, third heaven, because in sacred Scripture mention is made of three heavens. The lowest is the airy heaven, according to the saying "The birds of heaven"; the middle is the starry heaven, of which it is said at the beginning of Genesis, "And he set them in the firmament of heaven"; the third and highest* is all the waters that are above the heavens, which Scripture mentions many times, and which we call the watery heaven, while the Philosophers call it the primum mobile; for the Empyrean heaven, handed down by later writers, is found nowhere in the Scriptures. From this, Vielmius says (lecture 3 on Genesis), Caietanus infers that the ancient Fathers invented the Empyrean without cause and without any compelling reason, especially since it can be detected neither by sight nor by motion, and since God, who is said to dwell in it, and the Angels are incorporeal, so that it is superfluous to assign them a place of their own. [Margin: Censure of Caietanus.] This singular opinion of his is rejected by Vielmius in that same lecture 3; by Catherinus in the *Enarratio* on Genesis, under the word *caelum*; by Ascanius Martinengus in the *Glossa magna*, from page 235, who charges Caietanus with audacity, and at

page 244 with impudent rashness; and by Suarez, *De opere sex dierum* book 1 chapter 4, who asserts that the opposite is commonly received by the Church and altogether certain. With them I shall not be reluctant to prove the existence of this heaven by the authority of very many writers, and by reasons.

XXV. [Margin: [?] Argument: the authority of sacred Scripture.] We take the first argument from Holy Writ, where such a heaven is designated and described — above all where it is called the heaven of heaven, as in Deuteronomy 10: *To the Lord your God belongs heaven, and the heaven of heaven.* And 2 Paralipomenon chapter 2: *For our God is great above all gods. Who then can prevail to build him a worthy house? If heaven and the heavens of heavens cannot contain him.* And in the same book, chapter 6: *If heaven and the heavens of heavens do not contain you, how much less this house which I have built.* And David, Psalm 113: *The heaven of heaven is the Lord's, but the earth he has given to the children of men.* And in Psalm 67, when he had said of Christ the Lord, as St Paul interprets it in Ephesians 4, *You have ascended on high*, he went on to sing: *Sing to God, who ascends above the heaven of heaven to the East.* This same heaven is called the seat of God, and his dwelling, as in 2 Paralipomenon chapter 6: *You will hear from heaven, your most firm dwelling*; for so Solomon seems there to have understood that saying of Moses in Exodus 15: *You will bring them in and plant them on the mountain of your inheritance, your most firm dwelling [?] which you have made, O Lord; your sanctuary, O Lord, which your hands have established.* And Moses himself, in Deuteronomy 26: *Look down from your sanctuary and from your lofty dwelling*; and in chapter 33: *There is no other God like the God of the most upright; he who rides upon the heaven is your helper* etc., *his dwelling is above.* For from the sanctuary — the tabernacle and the earthly temple — each of them rose to contemplate the heavenly and immovable one, that is, the Tabernacle that can never be moved; of which Psalm 32 says: *The Lord has looked down from heaven, from his prepared dwelling*; and 102: *The Lord has prepared his seat in heaven*; and Isaiah 66 and Acts 7: *Heaven is my seat, and the earth my footstool.* Nor is it the house and dwelling of God alone, but of the Elect as well, according to that saying of the Apostle, 2 Corinthians 5: *We know that if our earthly house of this dwelling is dissolved, we have a building from God, a house not made by hands, eternal in the heavens.* And it is the Jerusalem that is above, or the heavenly Jerusalem, of which Galatians 4 says: *But that Jerusalem which is above is free, which is our mother*; and of which, surely, we think it was said in Psalm 147: *Praise the Lord, O Jerusalem*, etc., *who has made your borders peace* etc.; and Psalm 112: *Jerusalem, which is built as a city, whose sharing is in itself*; and Psalm 127: *And may you see the good things of Jerusalem all the days of your life*; and Psalm 146: *The Lord builds up Jerusalem; he will gather together the dispersed of Israel*; and Tobit 13: *I shall be blessed if there remain of my seed to see the brightness of Jerusalem. The gates of Jerusalem shall be built of sapphire and emerald* etc., *and through its streets Alleluia shall be sung.* From this it is evident that he looked, in a prophetic spirit, above all to that holy City of Jerusalem which St John saw in Apocalypse 22, *Prepared as a bride adorned for her husband.*

Of this same place it is said in Psalm 67: *God in his holy place; God who makes those of one manner of life dwell in a house*; and in the same place: *The mountain in which it has pleased God to dwell; for the Lord will dwell there to the end.* Finally it is called *the Region of the living* and *the land of the living*, Psalms 114 and 142; and *Paradise*, Luke 23: *Today you will be with me in paradise*; and Apocalypse 2: *To him who overcomes I will give to eat of the tree of life, which is in the Paradise of my God* — the Paradise to which, in the third heaven, the Apostle relates that he was caught up, 2 Corinthians 12; and St Francis of Assisi used to be caught up in ecstasy on merely hearing the name of Paradise.

XXVI. The second argument is the authority of the Fathers and Doctors — [Margin: 2nd argument: the authority of the Fathers, etc.] and indeed of the ancients as well, and of very many others — besides Philo, of whom I shall speak at the end of number 30. [Margin: St Clement, Pope.] For St Clement, in book 1 of the *Recognitions*, reports from the mouth of St Peter, disputing with Simon Magus: *This visible heaven was made, which is one day to be dissolved, for the sake of the present life, so that there might be a kind of partition and division, lest anyone unworthy should by chance see the dwelling of the heavenly beings and the seat of God himself, which is prepared to be seen by those alone who are pure in heart.* And again, when Simon contended that it was better to acknowledge and proclaim one heaven in the whole universe: *Not so*, says Peter; *rather it is fitting to confess one God, who truly is, but that there are heavens, which were made by him, as the law also says: that there is one higher heaven, by which the visible Firmament too is contained; that that one is perpetual and eternal, together with those who dwell there; but that this visible one is to be dissolved at the consummation of the age.* These words are like what St Peter says in Epistle 2, chapter 3. [Margin: Anacletus.] After him Pope Anacletus says at the end of his decretal Epistle: *The Lord came from the heavenly air and the Empyrean dwelling into the most pure womb of the most chaste Virgin, taking flesh for our redemption.* [Margin: The name of the Empyrean is very ancient.] From this you see that this name was not devised by Strabus and by Bede, as Pererius thought, on Genesis book 1, page 29, and Martinengus in the *Glossa*, page 248.

[Margin: St Justin and Plato.] That this heaven was known to Plato too is attested by St Justin, in the panegyric Oration, or Exhortation to the Gentiles, in these words: *And in the same way, concerning the heaven that came into being, he thought that the heaven which was made is that which, having been made, falls under the senses; but that the other is that which is grasped by intelligence and reason; of which the Prophet says, The Heaven of Heaven is the Lord's* — as though Plato believed that the Empyrean was not made, but either created or from eternity. [Margin: Clement of Alexandria.] Moreover, Clement of Alexandria, *Stromata* book 5, thinks that Orpheus and Homer had caught the scent of such a heaven; and, expounding that saying of Isaiah 66, *Heaven is my seat, and the earth my footstool*, he adduces those verses of Orpheus about God:

He himself stands fixed and firm on great Olympus. His throne is of gold, and the earth set beneath his feet, etc.

And similar verses of Homer, but more of them, in which he describes the House of God and of the Gods. [Margin: Origen.] But Origen says it more plainly, in Homily 1 on Genesis: *God made heaven first, of which he says, Heaven is my seat; but after it he made the firmament, that is, the bodily heaven* — by "bodily" meaning visible; and in *Peri Archon* book 2 chapter 9: *This whole is called the World, in which certain things are said to be supercelestial, placed in more blessed dwellings and in more heavenly and more splendid bodies.* [Margin: St Hilary.] That the opinion about the Empyrean was common even in his own times is shown well enough by St Hilary, in the *Enarratio* on Psalm 122. For when he had said: *This heaven, which lies beneath our sight by reason of its matter, and which, as being more firmly solidified, has received both the nature and the name of Firmament, will pass away and will not be; but the seat of the Lord remains for ever*, he added not long after: *And because, according to bodily understanding, that sense is available to us, that God is thought to dwell in the heaven which is beyond the nature of this firmament, let us not exclude that public opinion concerning the seat of God, that the heaven of heaven too is his throne* etc. He also gives the fittingness of creating this heaven, in the *Enarratio* on Psalm 135, saying: *When he who is infinite set the region of his knowledge in the Higher and first Heaven, which was likewise placed round as an orb, he enclosed a certain boundary for the powers which were to be begotten first for knowing him; for the creations to be begotten could not have borne the power of the unseeable God unless, by the interposition of a more tempered nature, they inhabited this seat in measure, to suit the weakness of their measure. For by raising the waters he tempered that very nature of the first heaven, which, kindled by the power of God on high, could by no means have been endured by lower natures; but the lower heaven he stretched out not uniform but manifold, the whole of which he named the firmament; and he solidified it with a firmness strong enough to sustain the upper waters and to temper our air.* Therefore, just as the damned in Gehenna inhabit a fire, but a smoky and dark one, and a dense gloom, so on the contrary the blessed, according to St Hilary, inhabit a most bright fire, or region, but one kindled by the power of God; and perhaps the bodily eyes of the Blessed see the Divine immensity tempering itself to them under the bodily appearance of the Empyrean — granted that they see it intuitively, and in a nobler manner, by the intellect raised through the light of glory. And God is said to dwell in this heaven because by sight and face to face he shows himself to those who dwell in it, as St Augustine excellently says on Psalm 122, where he also says that the Angels themselves and the Blessed can be called heavens, because God dwells in them by sight. [Margin: St Augustine.] And he adds: *For if we have understood this bodily heaven, which we see with our eyes, to be the dwelling of God, the dwelling of God will pass away, since heaven and earth will pass away.* [Margin: Matthew 24.] Since, then, the dwelling of God will not pass away, there is another heaven, now invisible to us, which the same writer assumed in book 12 of the *Confessions*, chapter 8, when he says: *Which firmament you called heaven, but the heaven of this earth and sea, which you made on*

the third day etc. — alluding to another Heaven, which does not belong to the use and sight of the inhabitants of Earth and Sea.

But whether St Augustine, in book 10 of *The City of God*, chapter 27, meant to take in this heaven under the name of Empyrean, or the starry heaven as well, or the starry heaven alone, is not evident. [Margin: The name of the Empyrean in St Augustine.] His words, with which he refutes Porphyry for having placed demons and evil spirits above the Moon, against the doctrines of Plato, and for having been worse in this than Apuleius the Platonist, are these: *But you learned this not from Plato but from your Chaldean masters: to raise human vices into the aethereal or Empyrean heights of the World and the heavenly firmaments, so that your Gods might pronounce divine things to the Theurgists.* But I ought first to have brought forward an older Father, of the second century, Theophilus of Antioch, who says in book 2 *To Autolycus*: [Margin: Theophilus of Antioch.] *In the first place, where sacred Scripture treats of the creation, or genesis, of the World, it does not speak of this visible Firmament, but sets forth some other heaven invisible to us; the second, which is visible, has been called this firmament.* Let there follow Diodorus of Tarsus, the teacher, as they say, of St Chrysostom, on Genesis chapter 1, saying: [Margin: Diodorus of Tarsus.] *After the visible substances and those endowed with intellect, God at once fashioned heaven together with the Earth — not the heaven that can be seen, but the one that is above all the heavens, and which David is accustomed to call the heaven of heaven.* Then, in the 5th Century, Junilius flourished and wrote a *Hexaemeron*, where, speaking of the heaven made on the first day, he says: [Margin: Junilius.] *For that is the higher heaven which, set apart from every condition of this revolving World, remains ever at rest in the presence of the divine glory. For of our heaven, in which are placed the luminaries needed for this age, Scripture declares in what follows how, or when, it was made.* Next, bringing in the authority of St Jerome, he most beautifully suggests the place in which Lucifer formerly was with the Angels not yet blessed, in these words: [Margin: And in him, St Jerome.] *St Jerome too mentions the higher heaven, writing thus, namely on that saying of Isaiah 14, "I will ascend into heaven, I will exalt my throne above the stars of heaven": And he said these things both before he fell from heaven and after he fell from heaven. If he was still placed in heaven, how does he now say, "I will ascend into heaven"? But because we read, "The heaven of heaven is the Lord's", while he was in heaven, that is, in the firmament, he longed to ascend into the heaven where the throne of the Lord is — not from humility but from pride. But if he speaks these words after he fell from heaven, we must understand arrogance: that even cast down he does not rest, but still promises himself great things — to be not among the stars but above the stars of God.* Add to these words that perhaps it is for this very reason that the pride of those who hate God, that is, of the demons, is said to be always ascending: because still and always, with most obstinate desire, they persist in that old but impious and vain resolve, *I will ascend.*

[Margin: Theodoret.] An author of the same century likewise, Theodoret, Question 11 on Genesis: *Just as this Heaven is a covering for us, so the heaven which we see has another heaven placed above it as its covering.* In the same fifth

century St Prosper also was illustrious, who says on Psalm 113: [Margin: St Prosper.] *But our God is in heaven above; he has done all things whatsoever he willed above the earth — not in the heaven where those who worship the creature see the Sun and the Moon, but in the heaven above, which surpasses all bodies, celestial and terrestrial.* I come now to two authors of the sixth century, of whom the earlier, Anastasius Synaita, in book 2 of the Commentary on the *Hexaemeron*, himself also acknowledges three heavens, when he speaks thus: *But if in the hymns of David you hear also of the heavens of heavens, this is a property of the Hebrew language, which often names the singular in the plural; but when you hear that Paul was caught up to the third heaven, consider the first heaven to be the air, in agreement with what is said, "I know all the birds of heaven", that is, of the air; then the second heaven, that of the stars; and the third, the highest heaven, that of the Angels.* [Margin: Procopius of Gaza.] Almost his contemporary, Procopius of Gaza more than once, and not inelegantly, depicted this heaven for us, on Genesis chapter 1; for he says: *God had decreed to found two most beautiful works: the one subject to change and corruption, which we behold lying beneath our eyes; the other he willed to be free and empty of the stain of the evils of the present age, and into it Christ, clothed in human nature, entered first of all*; and again: *Into this splendid work we shall enter after the second glorious coming of God* etc.; and he adds that this is that inner tabernacle of which Hebrews 6 speaks. Finally he concludes: *For with Christ we shall dwell in that Heaven, mingled with the invisible powers, or virtues; and the heaven which now serves as the roof will in the end be the foundation of the highest heaven at the common resurrection; for it is the first heaven, as it were the heaven of heaven; and indeed it is not without reason that David sings: The heaven of heaven is the Lord's, but the Earth he has given to the children of men.*

[Margin: Sedulius.] Closer to both these authors was Sedulius, though somewhat older, in the *Collectaneum* on the epistle to the Romans, on the words *Hope that is seen is not hope*: *Nothing at all, then, is to be hoped for in the future from the things that are seen. For eye has not seen what God has prepared for those who love him. But the eye has seen heaven and earth. It is not fitting, then, to believe that what is seen has been prepared by God for those who love him; rather, a heaven — or rather heavens — far more eminent and lofty than the firmament that can be seen, and an Earth, are to be hoped for: not, however, this dry land that lies beneath our eyes, but that land of the meek which eye has not seen.* [Margin: Bede.] I would add Bede at this point, but the words attributed to him in *De opere sex dierum* others ascribe, together with that whole work, to Junilius, and I have already quoted them as taken from Junilius. Yet it may have happened that, just as St Ambrose compiled his *Hexaemeron* from St Basil, so Bede compiled his from Junilius; and in that case we do not lack, for the Empyrean, a fit witness from the eighth century, in which Bede lived — just as we do not lack Alcuin, on Genesis chapter 1, who in the same century asserts the same heaven. [Margin: Alcuin.] In the ninth century, moreover, St John Damascene was eminent; [Margin: St Damascene.] he observed the distinction of three heavens fully in agreement with Anastasius Synaita, and asserted in book 2 *On the Faith*,

chapter 6: *Since, then, Scripture speaks of Heaven and of the heaven of heaven, and likewise of the heavens and the heavens of heavens, and the Divine Paul asserts that he was caught up to the third heaven: we say that in the creation of this universe that heaven was founded which those who flourished among foreign peoples with the praise of wisdom, claiming the doctrines of Moses for themselves, call the starless orb.* He goes on, and a little below says: *Indeed the heaven of heaven is that first heaven which exists above the firmament; so you now have two heavens, for God called the firmament by the name of heaven. Then it is usual and familiar to Scripture to call the air too heaven, because, of course, it is seen above; for it says, "Bless, all you birds of heaven", that is, of the air: there is the third heaven.* [Margin: Strabus. Rabanus.] In the same century Strabus, in the *Glossa ordinaria*: *In the beginning God created heaven and earth; by heaven he meant not the visible firmament but the Empyrean, that is, the fiery or intellectual heaven, which is so called not from burning but from splendour, and which was at once filled with Angels.* Rabbanus [Rabanus Maurus] agrees with him, on Genesis chapter 1. Let us pass now to the eleventh century, and hear St Anselm of Canterbury, *De imagine Mundi* book 1 chapter 28: [Margin: St Anselm.] *Above the firmament is the spiritual heaven, unknown to men, where the dwelling of the Angels is, arranged by all their orders*; and soon after: *This is the heaven which is read to have been created with the earth in the beginning; far above it the heaven of heavens is said to rise, in which the King of the Angels is said to dwell.* Therefore, on the view of this Doctor, just as the starry heaven is subdivided into several heavens according to the number and order of the Fixed stars and the Planets, so the Empyrean is subdivided into several heavens according to the hierarchical order of the Saints; and the highest among them, in which Christ dwells, he calls by antonomasia the heaven of heaven; and these he calls spiritual, because of their subtlety, and because they are invisible to us. [Margin: St Bruno.] St Bruno, Patriarch of the Carthusians, in the book *De Nouis*, chapter 2: *Moses posits two heavens; for he says, In the beginning God created heaven and earth: here is one; on the second day the Lord made the Firmament in the midst of the waters* etc.: *here is the other.* He adds that this second is the one in which the stars are, and which is seen by the eyes: *For it is impossible to see the first heaven from this World*, he says in the same place. [Margin: Rupertus.] And now the Fathers of the twelfth century invite us — namely Rupertus, book 1 of the Commentary on Genesis, chapter 1, on the words *In the beginning God created heaven*: *Surely by the name of heaven that invisible fatherland of the holy Angels is designated*; and in chapter 6 he says that by those words animal men do not understand heaven, but that by them alone who can say, *But our conversation is in heaven*; *where*, he adds, *the High Priest Jesus Christ, the Son of God, who has passed through the heavens, is locally present and is locally seated according to the true body of a man.* [Margin: Hugo Victorinus.] Next, Hugo Victorinus, in the *Summa sententiarum*, tractate 2, chapter 1: *Here we do not call heaven the Firmament, which was made on the second day, but the heaven which is called Empyrean, that is, splendid, which, as soon as it was made, was filled with Angels — namely that upper part, as far as which that fabric of the elements,*

still undistinguished, extended. [Margin: The Master.] From him the Master of the Sentences copied out almost word for word, in book 2, distinction 2, so there is no reason for us to repeat it.

But, to join this century to the thirteenth, the first to meet us is Alexander of Hales, [Margin: Halensis.] in part 2 of his *Summa*, question 47, members 1, 2 and 3, where he teaches that the Empyrean is luminous, and was fittingly made for the blessed spirits as well: because the spiritual souls themselves, joined then to incorruptible bodies, require the noblest body in the universe; and between them and the spirits separated from body, which of themselves do not require a bodily place, there is nonetheless a great affinity and fittingness. He adds that this heaven has the character of a container [rationem continentis] and is at rest. From this teacher St Bonaventure drew, in book 2, distinction 2, article 1, question 1, and St Thomas, in part 1, question 66, article 3. [Margin: Saints Bonaventure and Thomas.] For since bodily glory too is awaited in the bodies that are to be glorified, it was fitting that from the beginning, when spiritual glory was begun, there should be some place incorruptible, immobile and wholly bright; and for this reason, they say, it is called Empyrean — not from burning, but from splendour. But Gulielmus Parisiensis [Guillaume d'Auvergne], who also belongs to this century, teaches the same things about the Empyrean at greater length and brilliantly, [Margin: Gulielmus Parisiensis.] in the book *De Universo*, first part, from chapter 33 to 36; and he teaches that the Empyrean was founded in the beginning of time, and is most at rest, most bright, etc.

Finally, the authors of the 14th and 15th centuries expressly acknowledge it, and that it was created in the beginning of the World: [Margin: Lyranus. Tostatus. Catherinus. Picus.] Lyranus in the *Postills* on Genesis; Alphonsus Tostatus on Genesis chapter 1 and on Exodus chapter 23, questions 52 and 53; Picus Mirandulanus [Giovanni Pico della Mirandola] in the *Heptaplus*, chapter 1, citing Abraham and Isaac, most learned Rabbis; Catherinus on Genesis chapter 1; and among the more recent, Martinengus at length in the *Glossa magna*, from page 171 and from 236. So too all our own writers who have discussed this subject or the number of the heavens, [Margin: Molina. Valentia. Suarez. Pererius. Tannerus. Salianus. Cornelius. Tirinus. Fernandius. Coimbra.] but especially Molina and Suarez, *De opere sex dierum*, book 1, chapter 4; Valentia, in the first part, disputation 5, question 3, point 1; Pererius on Genesis, book 1, from page 27; Adam Tanner, *De caelo*, questions 1 and 2, and volume 1 of his *Summa Theologica*, disputation 2, question 2, doubt 2; Salianus, in the *Apparatus* to the *Annals*, chapter 6. And so too those who have published notes or commentaries on Genesis — especially Cornelius à Lapide, Tirinus, Fernandius and, learnedly, the Coimbra commentators, book 2 *De caelo*, chapter 1, question 1.

XXVII. [Margin: 3. Argument: from Reason.] The third argument is drawn from reason — not demonstrative reason, but reason resting on most fitting persuasions [suasionibus], which come down to two. The first is the perfection of the Universe: just as in the World there are simple bodies, corruptible of

themselves and mobile, so above them there should be a body incorruptible and immobile (for it is more probable that the starry and the watery heavens are of themselves corruptible from within, as we shall show below in chapter 2); and the whole World should be enclosed between two immobile extremes, namely the Earth and the Empyrean. The second is that it befitted God, from the beginning of the World, to prepare for the bodies of Christ, of the most Blessed Virgin and of the other saints, which were to be glorified in their own time, a special bodily place, to be the kingdom and seat of the Blessed, and in which God, as on a throne, might manifest his glory to them in a special way. For just as the eternal fire was prepared as a prison of the damned for the Devil and the Angels, so in Matthew 25 it is said: *Come, blessed of my Father, receive the kingdom which has been prepared for you from the beginning of the world.* Nor, as Suarez notes in *De opere sex dierum*, book 1, chapter 4, can this be understood of spiritual glory. For that glory was prepared, as to intention, not from the beginning of the world but by predestination before the foundation of the world and before eternal times [ante tempora saecularia], as the Apostle says; [Margin: Ephesians 1.] nor, in execution, was it given to men from the beginning of the world. The passage therefore speaks of a place suited to the bodily glory of men. And indeed, who would say that the body of Christ and of the most Blessed Virgin is in no bodily heaven at all, when the Apostle so often proclaims this of Christ, and the Catholic Church of the most Blessed Virgin? Or that they are above this mobile heaven and are swept round with it? Or that they have no fixed footing in any one part of heaven? Was it not fitting that a heaven of this kind be created for the sake and honour of Christ alone and of his most holy Mother, a heaven in which their bodies were to be for all eternity — if God founded so many other mobile heavens for the use of a very brief time and for the shadow of a momentary happiness? Now if it was fitting that the Seat of Christ and of the Queen of the Heavens, and the body of all the Saints, should be in this Empyrean heaven, then the Angels too, as ministers, ought to have been in it — granted that of themselves they do not require to be circumscribed or defined by a bodily place. These are all the reasons, or the chief ones, that Alensis, Saints Bonaventure and Thomas (cited above), Pererius (book 1 on Genesis, page 29), Suarez (*De opere sex dierum*, book 1, chapter 4), Salianus (in the *Apparatus* to the *Annals*, chapter 6) and Ascanius Martinengus (in the *Glossa magna*, from page 244) give for the Empyrean.

XXVIII. [Margin: Whether the Empyrean has been from eternity.] But when was this heaven created? From eternity? Or at least long before this World and its elements? Or rather in the first instant of this World? For Steuchus, or Augustinus Eugubinus, [Margin: The error of Augustinus Eugubinus about the Empyrean.] in the *Cosmopoeia* and in the treatise *De naturis incorporeis*, dared, among other paradoxes, to say that the Empyrean is something eternal and uncreated: that is, a certain light and brightness flowing from the essence of God, in which God is; a light of the divinity which has always and necessarily been with God, and to the participation and enjoyment of which both the

good Angels and just men are admitted. For he says: "This is that light and brightness of God, and the immense light of the divinity, and something most blessed which I dare not name, which is above heaven, which is neither a place nor a body, and which is so often promised in scripture to the just as their reward." Of this light, he holds, we must understand the verse of Psalm 103, *Clothed with light as with a garment*; and Paul's words, "Who dwells in light inaccessible"; [Margin: 1 Timothy 6.] and Christ's words in John 17, "Glorify me, Father, with the brightness which I had with you before the world was made." A certain specimen of this light, he says, was displayed in the bush of Moses, Exodus 3; in the same man's face, shining from his converse with God, Exodus 34; and in the Transfiguration of Christ, Matthew 17. He confirms this opinion of his from St Basil, Homilies 1 and 2 on the *Hexaemeron*.

But this wholly erroneous opinion is refuted, indeed hooted off the stage, [Margin: Censure of Eugubinus.] by Molina, *De opere sex dierum*, disputation 3, towards the end; Pererius, book 1 on Genesis, page 28; Catherinus, in his exposition of Genesis chapter 1; Vielmius, lecture 3 on Genesis; Ascanius Martinengus, in the *Glossa magna*, page 246; Suarez, *De opere sex dierum*, book 1, chapter 4; and Salianus, in the *Apparatus* to the *Annals*, scholium to chapter 6. Catherinus asserts that it is dangerous and a manifest error; Pererius, that it is to be refuted, utterly hooted down and blotted out by everlasting oblivion — indeed, that it is a perverse and impious dogma; Vielmius, that it resembles the heresy of Arnaldus de Brixia [Arnaldo da Brescia], who admitted that something besides God had been eternal, and that there is something which is neither creator nor creature; Martinengus and Salianus judge it erroneous and impious. Suarez, however, thinks it heretical — understand, materially; for Steuchus submitted this opinion of his to the judgement of the Church, and showed himself ready to condemn it if it were condemned by the Church. But it would have been much more prudent, say Catherinus and Pererius, not to put forward so dangerous and perverse a dogma, nor to give a handle for error to those who would read his work after his death and would not come to know the censures of the Church or of the Doctors that were to be passed, or had been passed, upon it. And against him could be urged what Cato said to someone or other: *Why did you prefer to beg pardon for a fault rather than to be free of it?*

XXIX. The error of Eugubinus is refuted by the following arguments. Either he thought that this Light of the Empyrean is in reality God himself, who, illuminating the minds of the Blessed objectively through the species of his divine essence, has the name of light, and by his immensity has the name of the Empyrean heaven — for so, it seems, he is to be understood, as it seems to Daniel Malonius, on book 2, distinction 2, disputation 7. In that case he wrongly explained its specimen by the bodily light of the bush, and of the face of Moses and of Christ transfigured; and moreover he thereby denied that the Empyrean heaven was created by God in the beginning of time, against the

sense and the common authority of the Fathers and Doctors of every century from Christ down to our own, as already in number

26 we have shown. Nor was he quite consistent with himself, for he says: *Whether this light is bodily or incorporeal I would not readily say; yet I would judge it to be such as to be visible to the mind and to the eyes — such as Christ showed when his face shone like the Sun, and such as he showed to Moses in the bush.* Or he thought that the Empyrean is something really distinct from God and yet uncreated; and that is manifest heresy, against the saying of John 1, *All things were made through him, and without him nothing was made*, and against the express article of the Creed by which we believe *in God, Creator of Heaven and Earth, of all things visible and invisible*. Or this Eugubinus judged that the Empyrean is something distinct from God and created by him, but by a necessary and eternal emanation; and in this a double heresy is involved — one which holds that God acts outwardly [ad extra] necessarily and not freely, the other which holds that some creature has really existed from eternity, against the common dogma of the Fathers and against the definition of the Lateran Council in the chapter *Firmiter*; indeed against Eugubinus himself, who in the same *Cosmopoeia* tried, and claimed, to demonstrate that the World not only did not exist from eternity but could not have existed from eternity. Moreover, he seems to savour of the error of those who say that the Blessed do not see God in his own substance but a certain light flowing from the divinity — an error which Suarez attacked in the first part, book 2, *De Attributis*, chapter 7.

Hence it is clear how ineptly he adduced the words of divine scripture to prop up his fiction. For the light with which, in Psalm 103, God is said to be clothed as with a garment, and the inaccessible light which, in 1 Timothy 6, he is said to inhabit, signify metaphorically either the holy Angels, in whose minds he dwells, or the divinity itself — since God is light, and his Word is Light from Light. But it is not something bodily, like the light of the bush or of Moses' face, nor should it be called the Empyrean heaven, for that name sounds of a creature. The brightness, moreover, which the divine Word and Son of God had with God from eternity was the truth of the Divinity, in which he was coequal and consubstantial with the Father, and according to which he wished to be known as the Son of God. Or, if that light is the very light of glory, or the expressed species of God, it is something created and in time; if, finally, it is the light of the Empyrean itself — whether connatural to it or emanating from the bodies of the Blessed — it is something bodily, visible, and likewise created in time.

But neither could St Basil support this error of Eugubinus. [Margin: St Basil misunderstood by Eugubinus.] For, often disputing against the Arians for the eternity of the Son of God, he teaches that it can befit no creature to have been eternal; and so, in book 2 *Against Eunomius*, alluding to the Apostle's words in Hebrews 1, *Through whom also he made the ages*, he said: *But since all the ages are creatures of the Son, they are understood as below the generation of the Only-begotten*; and a little after: *And some now honour even the aeon [aevum] with*

the title of eternal, because it took its name from the fact that it always is, as if ἀεὶ ὤν [aei ōn, "ever being"]. *But we assert that to attest to creatures what is eternal, and to deprive the creator of this claim, is the same madness.* Granted, he did think — not absolutely, but on a certain hypothesis not improbable to him — that the holy Angels were founded, before the creation of this visible World, in a certain invisible place suffused with a certain spiritual light, that is, one so subtle that by comparison with the light of the Sun it can be called spiritual, as Theodoret interprets him, question 6 on Genesis. And he called that place eternal and perpetual, that is, incorruptible, and eternal a parte post [at the later end], as Suarez explains — in the sense in which St Paul, 2 Corinthians 5, calls the Empyrean heaven *a house not made with hands, eternal in the heavens*. St Basil's words we shall cite shortly, in the next number.

XXX. [Margin: Whether the Empyrean is older than the visible World.] Although the Empyrean was not from eternity, some Fathers nevertheless believed — not absolutely, but with suspension of judgement and as a not improbable hypothesis — that it, taken at least as the place where the holy Angels were set from the beginning of their beatitude, existed long before the creation of this visible World, as an intelligible World. For St Gregory Nazianzen, in Oration 38, which is on the Nativity of Christ, says: [Margin: St Nazianzen.] *Therefore if, for these reasons too, the intellectual World was created by him — so far as I can philosophize and weigh great things by small: after he saw the first parts of creation splendidly established, he conceived another World, consisting of matter and visible — this concretion, namely, of Heaven and Earth* etc. And in Poem 4, which ex [?] on the World:

Now there are two Worlds, of which the one is older: Another Heaven, and the tranquil seat of pious men, Visible to souls alone, and bright on every side, etc. *But the other is in flux; then at length, for things in flux, a World Was founded,* etc.

[Margin: Nicetas.] At this place Nicetas, his interpreter, who flourished in the twelfth century, says that this intellectual World was founded *not from necessity but from goodness alone, and because that good had to be propagated*. [Margin: Anastasius Synaita.] Anastasius Synaita subscribed to this view, book 1 *Hexaemeron*, calling Gregory αὐτονομαστικῶς [recte: ἀντονομαστικῶς] [antonomastikōs, "by antonomasia"] the Theologian; for he says: *In the beginning God made heaven and earth: he names heaven, comprehending at once all the heavenly and intelligent beings of the holy Virtues, and the constitutions of the intelligent World, so that you may understand that the supernal World was fabricated first, in agreement with the discourse of the Theologian, who says* etc.; and he quotes the words of St Gregory Nazianzen. [Margin: St Basil.] Then St Basil, Homily 1 of his *Hexaemeron*: *Yet it is not unlikely that before the constitution of the World there was something intelligible to secret contemplation, though not committed to writing by Moses, as being little suited to infants and to those who need milk. There was indeed a certain state superior to the generation of the World, suited, as was fitting, to those powers abstracted from the concretion of the World — prior in time, and eternal and perpetual. In it the Creator and maker of all things perfected creatures,*

a spiritual light suited to their beatitude etc. And Homily 2: *We think indeed (if there was anything before the constitution of the sensible and corruptible World) that it was in light; for neither did the dignities of the Angels* etc. *live in darkness, but they had their state in this light, most suited to them, and in spiritual joy* etc. [Margin: Theodoret.] Theodoret transcribed these words, on Genesis question 6. He also teaches, in questions 11 and 14, that the first heaven was made from nothing and before light, the later one from water and after light; and in question 3, though he does not approve absolutely of saying that the Angels were founded before the World, he does not altogether oppose it, nor does he think it offensive to pious ears, provided they are said to be created, and in some determinate place. For he says: *For the divinity alone, as being in no way circumscribed, is not subject to place*; and at the end: *Moreover, it must be known that all things that exist, except the Holy Trinity, have a nature subject to creation; and this granted, if anyone says that the troops of the Angels were founded before heaven and earth, he will not offend the word of piety.* [Margin: Junilius, or Bede.] Junilius too, or Bede, in his *Hexaemeron*, reports the opinion of St Basil and does not refute it.

[Margin: The Empyrean was not created before the visible World.] Notwithstanding all this, it is now certain that the Angels, and therefore the Empyrean, were created only in that first instant of time of which it is said in Genesis 1, *In the beginning God created Heaven and Earth*. For so the Lateran Council defined in the chapter *Firmiter*, and so we taught, with the common view of the Theologians, in our treatise *De Angelis*; and it is gathered well enough from those very passages of the Fathers by which we showed in number 26 that there is an Empyrean — especially St Hilary, Theophilus of Antioch, Origen, Diodorus of Tarsus, Junilius, Hugo Victorinus, St Anselm, St Bruno, Halensis, St Thomas Aquinas, Gulielmus Parisiensis, Rupertus and the Master — so that there is no need to repeat them here. So too teach Molina, disputation 2 of *De opere sex dierum*; Pererius, book 1 on Genesis, page 27; Suarez, *De opere sex dierum*, book 1, chapter 4; Tanner, question 2 *De caelo*, and volume 1 of the *Summa Theologica*, disputation 6, question 2, doubt 2; and Salianus, in the *Apparatus* to the *Annals*, chapter 6 — granted that Ascanius Martinengus, in the *Glossa magna*, page 249, says that Moses, as one speaking with an uneducated people, handed down only the genesis of this sensible World and suppressed the creation of the Empyrean and of the Angels. But we have already said, in question 3, that Moses spoke not only to an uneducated people but also to all the wise of the World, though he did not express or indicate all things to all, nor in the same way; for he comprehended whatever can truly and properly be understood by the name of Heaven, by each according to his capacity. [Margin: Philo.] And so Philo, certainly most skilled in the Hebrew idiom, understood him in the book *De Mundi opificio*, where, expounding Moses, he says: *"In the beginning he made" means the same as "first he made heaven"; for it is indeed consonant with reason that this came forth first in generation, since it is the most excellent of the things that have been made and consists of the purest essence; for this*

reason it has been destined as the most sacred dwelling of the Gods, both those not apparent and those perceptibly manifest — of Angels, that is, and of Men.

XXXI. [Margin: The Empyrean incorruptible.] It remains for us to indicate briefly of what nature and condition the Empyrean is, and what its functions are. First, then, we say that it is incorruptible, or at least that it is not to be corrupted [?] nor dissolved at the end of the World; for so the Fathers cited in number 26 expressly teach. They also assert that it is invisible to us who live in this life, [Margin: And invisible.] because the Firmament and the watery heaven are interposed between us and it and by their opacity shut off our view of it; or because its light is rare, as is usually said of elemental fire. Next we say that it is uncertain whether it is an altogether simple body — indeed not so much a fifth as a certain sixth essence, in no way agreeing with material things, as Abulensis puts it on Exodus chapter 13, question 53 — or rather one consisting of matter and form; [Margin: Whether, and of what matter, it consists?] and, if the latter, whether of a matter different from the elemental (as Suarez, with St Thomas, holds in *De opere sex dierum*, book 1, chapter 5) yet the same as that of the other heavens, or rather of the same matter as the elements. It is far truer, however, that it was created not from elemental matter but from nothing, and of a matter nobler not only than the elemental but also than that of the other heavens, on account of its nobler and far more excellent end; and Suarez grants in the same place that this can be said with more probability. Nor is it absurd — indeed it agrees with the perfection both of the universe and of the Blessed themselves — that it should be altogether simple, if a body can be given that does not consist of matter and form really distinct from each other; on this see more in Suarez, *Metaphysics*, disputation 13.

All add that it is immobile [Margin: It is immobile.] — not because it could not be moved by God, or is from within incapable of motion, but because in fact it is not moved, as the eighth sphere or the lower orbs are said to be moved; for it was not made to carry the stars round. And perhaps on account of this immobility, or because it is incorruptible, or on account of the perpetual stability of the beatitude of the Saints, that city is said, in Apocalypse 21, to be laid out foursquare; for a square and cubic figure signifies stability. [Margin: The figure of the Empyrean.] Granted, perhaps on account of the excellence of the spherical figure it is round, as Aversa holds in question 31, section 4, of the *Physics*, and in this is a symbol of the divinity, as St Hilary indicates in the *Enarratio* on Psalm 135 in these words: *the higher and first Heaven, likewise set round as an orb*. Granted, too, that the Coimbra commentators, book 2 *De caelo*, chapter 5, question 1, article 2, think it is in fact square on the outside. In addition, Malonius, on book 2, distinction 2, disputation 7, section 5, holds it more probable that it is a fluid body, [Margin: Fluid or solid?] so that it may serve not only the motion of the bodies of the Blessed, which will run to and fro in it like sparks in a reed-bed, but also speech and breathing, and so that there may not be, without necessity, a perpetual penetration of bodies — though he admits that these reasons are not necessary. Suarez, however, in

De opere sex dierum, book 1, chapter 5, judges the opposite more probable. For in this way, he says, what the Fathers teach is truer, namely that it is wholly immobile; otherwise, with the arrival of one body of the Blessed after another, it would become denser, and the whole would almost fluctuate with their motion; and he thinks condensation and rarefaction unsuited to an incorruptible body. Now, just as the bodies of the Blessed can by their subtlety penetrate other bodies, they can also penetrate the Empyrean, according to the Apostle's words in Hebrews 4, *Jesus Christ, who has passed through the Heavens*. For speech, that most bright air suffices with which the inner parts of the viscera will be filled, in place of excrement and nourishment; nor will there be any need of breathing. He adds that the Blessed for the most part dwell on the convex surface of the Empyrean, as the same Suarez taught of the Body of Christ, volume 2 on the third part, disputation 51, section 1 and the last.

Light is believed to be connatural to the Empyrean, [Margin: The light of the Empyrean: of what kind?] granted that it is increased beyond belief by the light diffused from the bodies of the Saints, and above all of Christ, according to Apocalypse 21: *And the city has no need of Sun or Moon to shine in it; for the brightness of God has illuminated it, and its lamp is the Lamb.* Its light, however, does not reach us, [Margin: Why is it not seen by us?] whether because it is blocked by the opacity and density of the interposed watery heaven or of the supreme Firmament, as Ascanius Martinengus, with Basil, Ambrose, Theodoret and others, judges in the *Glossa magna*, page 249; or because it is of too rare a substance, or because that light is not proportioned to our eyes on account of its excessive splendour, as St Thomas thinks, part 1, question 66. For excessive distance, or the resistance of the waters that are above the heavens, and similar causes, which Richardus de Mediavilla adduces on book 2, distinction 2, and other Scholastics in the same place, do not seem sufficient — especially since the bodily eyes of the disciples saw the light from the face of Christ in the transfiguration, and that light ought not to be called spiritual absolutely, but only by comparison with the light that resides in dense bodies.

But does the Empyrean exert an influence on these lower things? [Margin: Whether the Empyrean exerts an influence on these lower things?] The affirmative answer is thought more probable by St Thomas, part 1, question 66, article 3, reply to 5, and in Quodlibet 6, question 11, article 19, retracting his earlier opinion; by Caietanus on that article 3; Durandus on book 2, question 2; Aegidius on book 2, question 2, article 4; Argentinas on book 2, question 2, doubt 1; Richardus on book 2, distinction 2, article 3, question 3; and Suarez, *De opere sex dierum*, book 1, chapter 5. Not that action is necessary to it in order that it be the place containing and conserving the heaven immediately subject to it — for that it suffices that it contain it and have no active qualities contrary to its qualities — but because every body naturally has some active power; because for the unity of the Universe some subordination is required, and an influence of the higher on the lower; and finally because, since the other heavens are moved, the stable diversity of colour, of inclinations and of complexions found

in the various regions of the earth seems unable to be referred to anything other than the different regions of the supreme immobile heaven, of which kind the Empyrean is. The contrary seems to me more probable, because the Fathers speak of it as of a heaven having no commerce or communication at all with natural things, and created for no other end than the supernatural beatitude of the Saints; and all that diversity among the regions of the earth is to be attributed to the different temperament of the soil. So Alexander of Hales holds of this heaven, part 2 of the *Summa*, question 47, member 1, and formerly St Thomas, on book 2, distinction 2, question 2, article 3; and before them Procopius of Gaza, on Genesis chapter 1, when he said: *The first heaven, indeed, on account of its immense remoteness, was in no way commensurable with the earth; therefore no part of it could have aided the earth's countless births.* Finally, many Fathers teach that the Sun and the stars were made on the fourth day for this reason: that it might be evident that the shoots of the earth do not depend necessarily on the stars but on God alone. But if the Empyrean naturally influenced these things, then for the same reason it ought not to have been created at the beginning.

Before we conclude this question: to some it does not seem improbable that either the Empyrean or the watery or crystalline heaven is a mirror [Margin: Whether the Empyrean has the colours and forms of the Earth?] in which all the most beautiful things that now delight the eyes of men on earth are represented in a more eminent manner and in far more vivid colours; and that this is perhaps signified in Apocalypse 21 by the jasper, the crystal, and the colours and appearances of other gems and stones, so that the eyes of the Blessed may not be deprived even of this charm and delight; and that it is in this way that a new heaven and a new earth are said to be created. Indeed Eusebius, book 11 of the *Praeparatio Evangelica*, chapter 19, thinks that there is no mixed body pertaining to the beauty of earth and sea that will not exist in that Land of the living, reserved as an inheritance for the meek, there expressed in better forms or painted in nobler colours; and he adduces the authority of Plato, speaking in the person of Socrates, who describes such a land of the blessed. But perhaps these are the dreams of men unable to tear their senses and imagination away from the figure of this world, which passes away.

Quæſtio 7. Quid fuerit Lux à Deo primo Die Mundi facta.

Question 7. What was the Light made by God on the first Day of the World.

XXXII. On that first day on which GOD created Heaven and Earth, when darkness was upon the face of the Abyss, *God said, let there be Light, and light was made.* [Margin: The light of the first day: bodily or spiritual?] In Hebrew it reads *Iehi Or, Vahc [?] or*, that is, "let there be light, and there was light"; or, as the Vulgate better has it, "let light be made, and light was made"; and the Seventy translators agree: γενηθήτο φῶς, καὶ ἐγένετω φῶς [recte: γενηθήτω φῶς, καὶ ἐγένετο φῶς] [Greek: "let light come to be, and light came to be"]. According to St Augustine — book 11 of *The City of God*, chapters 7 and 9;

book 1 of *De Genesi ad litteram*, chapters 3 and 9; book 22 *Against Faustus*, chapter 10; and often elsewhere — this light was spiritual, that is, the angelic nature. For on the one hand the Angels were created before those morning stars came to be which, in Job 38, are said to have praised the Lord while all the Sons of God shouted for joy; and their creation was indicated by Moses under no other name more fitting than in [?] that of light, since it was certainly not omitted. On the other hand, if it had been material and bodily light, it would either have been deferred to the 4th day, on which the stars were made, or else the Sun would have been created on the first day; but both of these are false. St Jerome seems to have agreed with this view on Psalm 46 — but allegorically — and so do Eucherius and Bede on Genesis, St Isidore in book 1 of *De summo bono*, chapter 10, and the ordinary and interlinear Gloss; and most constantly of all Rupertus [Rupert of Deutz], book 1 on Genesis, chapter 10, who adds that it was not fitting for that light to last only three days, since Moses is describing the production of permanent things. For the others cited above — as also Hugo in the *Summa sententiarum*, tractate 3, chapter 1; Halensis, part 2, question 49, member 1; the Master of the Sentences on book 2, distinction 13; and St Thomas in the *Disputed Questions*, question 4 — rather refrain from rejecting that opinion than approve it.

But the common view of the Fathers and Doctors is that it was bodily light, as Pererius teaches from them, book 1 on Genesis, page 45; Ascanius Martinengus in the *Glossa magna*, from page 470 to 490; Suarez, book 2 of *De opere sex dierum*, chapter 1; and Tanner, volume 1 of his *Theology*, disputation 6, question 2, doubt 3. For the literal sense demands it; and this is that light which was set against the darkness pre-existing over the face of the abyss, and which God, by dividing it from the darkness — that is, by assigning each of them a distinct time in which it should pass over the hemisphere of the earthly paradise — made into those three visible days, as patterns for the remaining days of our sensible time. Moreover, the reasons the Fathers adduce for light's having been made by God most fittingly first of all things hold only of bodily light. Namely: light is the first active quality, common to the higher and the lower bodies, and necessary for everything else to be rendered visible; but God had decreed to produce the plants and herbs, and to make the earth and the seas conspicuous, before the Sun and the Stars, *so that from things invisible visible things might be made*. [Margin: Hebrews 1.] Next, God wished to show that all that light which makes the day for us, measures the times by the motion of the Sun and Moon, and is the universal ornament of the Heavens and the elements and the vehicle of the species and of the influences, does not depend essentially on the stars, and therefore that the stars are not to be worshipped with divine worship; and for this it was required that it be of the same species as the light which he afterwards granted to the Sun and the Moon, as to servants carrying torches before God for the world. Finally, the Church presupposes this very thing in the hymn for Sunday Vespers, when it says:

Best Creator of the light, Bringing forth the light of days, Preparing, with the first beginnings of new light, The origin of the World; Who bid the morning, joined to evening, Be called day.

XXXIII. [Margin: Primordial light: was it Fire?] But what, then, or what kind of thing was that bodily light? That it was Elemental Fire which, separated out from the other elements as the lightest of them all, lit up everything, was the view of St Gregory of Nyssa in *On the History of the Six Days*; of Hermes, as cited by St Cyril, book 2 *Against Julian*; of St Damascene, book 2 *On the Faith*, chapter 7; and nearly that of Alcuin on Genesis and of Hugo, chapter 6. But elemental fire is judged too rare to be fit to illuminate and constitute the day. [Margin: Or a bright Cloud?] Others say it was a bright cloud, or something similar, set up either in the East or in the Meridian of that Horizon in which Paradise later lay, and that it played the part of the Sun. So Strabus in the ordinary Gloss; Hugo Victorinus; Richardus Victorinus [Richard of Saint Victor], book 1 of *De sacramentis*, chapters 9 and 10; Hugo Carensis [Hugues de Saint-Cher] in the *Postills*; Petrus Comestor, chapter 3 of the *Historia scholastica*; Lyranus in the *Postills*; Tostatus; the Master on book 2, distinction 13; St Bonaventure on book 2, distinction 13, article 1, question 1; and Alexander of Hales, part 2, question 46, member 1. [Margin: Or the Sun?] Others have said that it was the Sun, but without a determinate power for particular effects, or without the full density and figure of its globe, and endowed with only a little light: so St Thomas, part 1, question 67, article 4, adding St Dionysius the Areopagite, and question 70, article 1, reply to 2; Vielmius, lecture 10 on Genesis; Aegidius, part 2 of the *Hexaemeron*, chapters 5 and 6; Carthusianus on Genesis, article 9; Pererius, book 1 on Genesis, page 49; Valentia, disputation 5, question 3, point 1; and Suarez, book 2 of *De opere sex dierum*, chapter 2. [Margin: Or the Sun, with the motion of the primum mobile alone?] Or else the Sun indeed, but flashing with a fuller splendour than it does now, as Steuchus holds in the *Cosmopoeia*; or the Sun shining just as it did afterwards and does now, but moved at that time only by the motion of the primum mobile, and afterwards, from the fourth day, receiving its proper motion together with the other planets. [Margin: Or a splendour of the Empyrean?] Others hold that it was a splendour placed in the Empyrean, as St Bruno hints in the book *De novis*, chapter 2, and Gulielmus Parisiensis in the first part of *De universo*, chapters 40 and 41 — a view that greatly pleased Halensis, part 2, question 45, member 2. Others again hold that it was indeed the light of the Sun, but not yet joined to its globe, and implanted in it afterwards on the 4th day; and very many, who are to be cited below, have in fact thought so. This opinion especially pleases Molina, disputation 7 of *De opere sex dierum*; Eugubinus, book 7 of *De perenni philosophia*, chapter 8; Tanner, volume 1, disputation 6, question 2, doubt 3; and Salian in the *Annals*, at the 1st day of the world; and it appeals to me greatly — provided we say that that light was of the same species as the light of the Sun, and was sustained by the heaven and the air as by a subject, rather than that it was the same light in number. For in this way it is explained according to the letter how light

was made on the first day, but not the Luminary or the Sun; and on the fourth day the very globe of the Sun, which produced, dependently on God, light of the same species — light of a species that God alone had produced before, by himself or through an Angel, varying it in the same way as the Sun now does. For whatever the Sun does now, God alone could do, drawing that light successively out of the potency of the diaphanous bodies. Indeed Scripture here uses separate and different days and names. So light was made on the first day and the vehicle of light on the fourth, as St Basil puts it — or the lamp [lychnus], as Procopius says; and the Church certainly seems to hold this when, after those verses of *Lucis creator optime*, it sings:

Most holy God of heaven, Who paint the bright centre of the Pole With fiery brilliance, Enhancing it with comely light; Who on the fourth day, setting up The flaming wheel of the Sun, etc.

The reason why God alone willed to produce that very light — that is, light of the same species that he afterwards granted to the Sun to propagate — is the one we have already indicated: namely, that men should be turned away from idolatry, and from offering divine worship above all to the Sun. [Margin: Saints Chrysostom, Basil, Ambrose, etc.; St Dionysius the Areopagite.] This reason, above the rest, is touched on by St John Chrysostom, Philo, St Basil, St Ambrose, Severianus and Procopius, chapter 4, question 1, who are to be cited later. Nor does St Dionysius the Areopagite stand in the way, in the book *On the Divine Names*, chapter 4; for his words, faithfully rendered from the Greek, are these: *Light is the measure of the hours, of the days and of all passing time; for it is that very light — although it was then formless, in Greek* φῶς ἀσχημάτιστον [Greek: phōs aschēmatiston, "shapeless light"] — *which, as the divine Moses also says, defined and distinguished that first triad of our days.* So he holds that it was light of the same species as the Sun's light of today, and he calls it formless, or lacking figure, because it was not joined to any body having the determinate figure of a luminary. [Margin: St Gregory Nazianzen.] St Gregory Nazianzen says the same thing in clearer words, oration 43: *To the lower things* etc. *he imparted the first force of this luminary (for it was fitting that the Great Light should begin his work from light), by which he dispelled the darkness and that vast heap of things and the confusion that filled everything. Nor did he bring it forth bound to an organ from the beginning, nor — in my opinion at least — as solar, but without body and without Sun* — in Greek it reads ἀσώματον, καὶ ἀήλιον [Greek: asōmaton kai aēlion, "incorporeal and sunless"] — *and afterwards it was handed over to the Sun to illuminate the world. For since in other things he had kept to this order, first to found the matter and then to adorn it with form — order, that is, and figure and bulk being imparted to each thing — here, to give a specimen of some greater miracle, he brought forth the form before the matter (for light is the form of the Sun), and afterwards introduced the matter, fashioning this Sun, the eye of the day.* Here he recognises a miracle at least in this: that God alone produced light, the form of the Sun, before its own primary matter. For light is primarily in some luminous and opaque body, by which it is both sustained and produced, and from there it

is propagated into a secondary subject, which is the diaphanous body through which it is transmitted, such as air and water. In this sense I admit that that light was created by God independently of a sustaining subject — that is, of the primary subject — and without the concurrence of the air and the heaven producing it, granted that it was sustained by the air, the heaven and the water. For no greater miracle was needed for the end God had set himself, and we ought not to multiply miracles without necessity. [Margin: Miracles are not to be multiplied without necessity.] As St Augustine rightly says, book 2 of *De Genesi ad litteram*, chapter 1: *For it is now fitting for us to inquire how God instituted the natures of things according to his scriptures, not what he might will to work in them or out of them as a miracle of his power.* But scripture says: *God said, Let light be made* — the formula he used when he did not create in the proper sense, but from mat- ter or in matter produced something; therefore he created light neither properly nor absolutely, but relatively to its own subject. In this sense I accept that it was an accident subsisting without a subject. Nor do those words of St Basil bind us to anything else, in homily 2 on the *Hexaemeron* [Margin: St Basil.]: *The first word of God created the nature of light, dispelled the darkness, and lit up and gladdened the world*; and in homily 6, speaking of the Sun made on the fourth day: *These things are certainly not opposed to what was said above about light. For then the very nature of light was produced, whereas now the body of the Sun was founded, to be a vehicle for that primordial light,* etc. *And lest what I have said seem incredible — namely, that light and splendour are something other than the body that underlies the light and is its subject: all composite things are usually divided into the substance that receives and the quality that befalls it. So, just as whiteness and the whitened body are different, so too the things we have just spoken of differ, though they are united by the power of the Creator. So do not tell me that it is impossible for light to be separated from the body of the Sun. For I do not say that the separation of light from the body of the Sun is possible for me or for you; but I judge it must be asserted that things which can be disjoined from each other by the mind's thought alone can in reality be separated by the power of him who created both natures.*

[Margin: Theodoret.] Theodoret too speaks most beautifully to our point on Psalm 148. Weighing the verse *Praise him, Sun and Moon; praise him, all you Stars and Light*, he does not approve the version of Symmachus, who renders it *Stars of light*, as though the Stars were the same as light, or the principal causes of light; and he says: *Yet according to the Seventy it is to be understood thus. On the first day God made light, but on the fourth the luminaries; and for this reason the Psalmist made separate mention of light — not because it subsists by itself, but because it was assigned to the luminaries.* And in question 14 on Genesis, distinguishing creation taken in the broadest sense from creation taken strictly, he says: *The Lord God is accustomed to create both out of things that are not and out of things that are; for the earlier heaven he created out of nothing, but the later he made out of water,* etc. *Therefore he also created light as it pleased him,* etc., *and, distinguishing that light at his own will, he founded the luminaries great and small.* [Margin: St

Athanasius.] St Athanasius also teaches that that light was distinct from the body of the Sun and was founded first, and then gathered and assigned to the luminaries — question 83 on Genesis, at the words *Let light be made*, where he says: *That light is not the Sun. But hear what kind of light it is: this light is that morning light which rises at dawn and illuminates the world before the Sun rises. Since this light was great and most splendid, God divided it into those luminaries, the Sun and the Moon,* etc.; and in question 85: *That great light which God had made he divided into the Luminaries; and this light, scattered or distributed here and there, was to some extent diminished. Had this not been done, living creatures could not have borne the great radiance and splendour of so great a fire.* [Margin: St Ambrose.] St Ambrose urges the same distinction of light from the substance of the Sun, book 1 of the *Hexaemeron*, chapter 9, explaining from it how light was produced by God first and the luminaries afterwards: *The light of day is one thing, the light of the Sun and of the Moon and of the Stars another*; and he proves this from the light of Dawn, which makes the day without the presence of the Sun. [Margin: St Augustine.] And St Augustine, book 1 of *De Genesi ad litteram*, chapter 11, having raised the Question *Why then was the Sun made to rule the day, to shine upon the earth, if that light had been enough to make the day, which was also called day?*, answers that perhaps the Sun was made to illuminate the lands, since that first light perhaps illuminated only the higher regions; and then he adds: *This too can be said: that the brightness of the day was increased by the addition of the Sun, so that the day may be believed to have been less bright through that light than it is now.* For he is alluding to the words of Psalm 135, *The Sun to rule the day, the Moon and the Stars to rule the night* — or "for governance", as St Chrysostom reads it in homily 6 on Genesis: *That is, so that the Sun may make the day brighter with its rays*, as he himself says. [Margin: Procopius of Gaza.] Procopius of Gaza, too, speaks excellently on our side, book 1 on Genesis, and at length; but it is enough to have pointed out these words: *First light was brought forth by the intervention of the divine voice; then at last the receptacle of light was founded. For it is as fire is one thing and the Lamp another. For of the light that lacks matter the Sun was made, as it were, the body; since composite things, too, are divided into the subject and the quality that inheres in the subject.* He then shows that the power of God is so great that it can produce effects and qualities without substances. He confirms this both from the fire in Moses' bush, shining and not burning — for there the splendour was separated from the heat or from the heating, and the words of Psalm 28 were fulfilled, *The voice of the Lord dividing the flames of fire*; and he adds that perhaps, in the opinion of some, the fire is to be divided on the last day, so that the fiery Light is given back to the godly while the force of the fire, which burns, is assigned to the ungodly — and, finally, he calls the light of the Moon, now full, now waning, a quality.

[Margin: The other Procopius.] The other Procopius, however, in his commentaries on Genesis, chapter 1, says: *God, taking the purest part of the primordial light, transferred it into the Sun; the remaining part he distributed among the Moon and the remaining stars.* [Margin: Apollinaris.] Apollinaris holds this

too. Nor is the view of Iunilius on the *Hexaemeron* to be scorned. At the words *Let light be made* he teaches that that light was produced in the waters and the abyss of waters, which he thought then reached right up to the Empyrean; and that it is no wonder if light shines in water, since oil poured from the mouths of those who swim under water shines back in the water before their eyes. And at the words *Let them be for Signs and times* he adds [Margin: Iunilius.]: *For that whole three-day period had passed in an undivided progress of its course, with no measure of hours at all, since the primary light still filled everything in general and had no head — which it now has received from the Sun — and nowhere had more burning rays shone.* [Margin: Tertullian.] Tertullian also presupposes that light was made before the sun, in the book *Against Hermogenes*, when he says: *For he did not at once fill the light with the splendour of the Sun, nor at once mark the heaven with constellations and stars,* etc. Various poems of the Fathers on Genesis, and Sibylline verses, could be adduced; but, so as not to run on too long, I refer the reader to Ascanius Martinengus in the *Glossa magna*, from page 482. On page 488 he plainly agrees with us: namely, that that light was produced without the globe of the Sun, and the Sun only on the fourth day; and that light is called in Hebrew *Or*, but the Sun *Moor*; and by the distinction of these names and days he indicates the diversity of the things produced clearly enough. [Margin: Burgensis.] Burgensis also favours us, in his *Additions* on Genesis, where he notes that what was said was "let there be light", not "luminary"; that a bright cloud is not light but a luminous body and a luminary, and that what Moses expressed was therefore a quality without a substance; further, that if that light had been in the substance of the Sun, it ought to have been said "let the Sun produce light", just as, because the earth and the water existed, it was said in Genesis 1, *Let the earth bring forth grass, and let the waters bring forth the creeping thing*. And he observes most acutely that on the fourth day it was said to the luminaries that they should divide the light from the darkness, whereas on the first day this was not said, because they had not yet been made, but God himself divided the light from the darkness. In this, however, he dissents from us: he thinks that that light was of a different species from the light the Sun now produces, because he thinks it unfitting that the light of the Sun should then have been without its subject, and before its subject. But he does not notice that this goes against many of the Fathers already reviewed, who teach clearly enough both that it was of the same kind and that it was created, or produced, by a miracle of God alone, without the Sun sustaining and producing it. But, you will say with Pererius, the first day, on which the World was created and which the Church calls the Lord's day, was called the day of the Sun by the ancients; so it is a sign that the Sun existed on that day. Martinengus, however, replies that this is a very weak argument, since the day was called the Sun's by the heathen out of superstition or astrological vanity — the same from which they also called the Sabbath the day of Saturn; and yet Pererius would not grant that Saturn was made on the last day of that first week, on which God rested

from his work — and that the first day was ascribed to the Sun on account of its primacy in light.

Quæſt. 8. De diſcrimine trium primorum dierum à noctibus, ex illis verbis : Et diuiſit lucem à tenebris; appellauitq. lucem Diem, & tenebras noctem; factumq. eſt veſpere & mane dies vnus.

Question 8. On the distinction of the first three days from the nights, from the words: And he divided the light from the darkness; and he called the light Day, and the darkness night; and there was evening and morning, one day.

XXXIV. Day in Hebrew is called *Iom*, from the verb *Ham*, which means to make a noise — from the noise to come; Night, by contrast, is called *Hiela*, either from horror or from weariness after the day's waking, from which the night restores us, bringing back *sleep and the weary silences to the lands*, as Flaccus sings. Evening is called *Hareb* or *Ereb*, from *Harab*, which means to connect — that is, the day with the night; and morning, finally, is called *Boer* or *Boquer*, from the verb *Bacar*, which means to discern. With this premised, I suppose that the dark- ness that was over the face of the abyss preceded the primordial light not only in origin, or as it were in nature — as St Bonaventure maintains, as a problem, on book 2, distinction 13, article 1, question 2 — but also in duration, as St Thomas, with the common view of the Fathers, presupposes in part 1, question 74, article 3, and indeed in question 66, article 2, and on book 2, distinction 13, question 1, article 4. [Margin: Darkness prior in time to light, and everywhere.] Nor do I think it probable, whatever Suarez says in book 2 of *De opere sex dierum*, chapter 2, that it was in one hemisphere only — say, that of Palestine; rather, it was simply over the whole face of the abyss. [Margin: Was there time before light?] Further, I suppose that that whole duration up to the production of light can be called time, even if perhaps there was then no motion at all — neither of the waters, nor of the spirit borne over the waters, nor of the Sun, not yet founded, nor of the heavens. For there was a privation of that light which afterwards made the day, and it could be measured by time: just as that stay of the Sun made at Joshua's command was time, even though it could not actually be measured by the motion of the Sun. How long it lasted, however, is uncertain. Some say it was of 6 hours — namely those who think light was made at noon of the first day; others of 12 hours — namely both those who judge that light was first made on the eastern horizon of Palestine or of the earthly Paradise and those who say on the western horizon. For most of the more recent doctors suppose that that first artificial day was Equinoctial, and so of 12 hours, just like the night. [Margin: That light in a single hemisphere.] Finally, I suppose that the primordial light was not produced so as to illuminate the whole globe of lands and waters in a single instant, but only a single hemisphere of it — granted that Anastasius Synaita said: *The light of the three preceding days, expanded and dispersed through the whole*

universe, shows that the light of the uncreated Trinity reaches everywhere and leaves no place empty of it — unless you understand this of its successive propagation, by force of which no hemisphere was empty of light during those three days, as most of the Fathers understand it. For in this way it is better explained how God at once divided light from darkness, namely by place or subject and by time: he produced that light in a hemisphere different from the one where the darkness was, and willed that light and darkness should be in the same hemisphere at different times. With these points laid down, let us dispatch more briefly the principal doubts, and those more welcome to Astronomers.

XXXV. [Margin: Light produced in the upper hemisphere of Palestine.] I say, *first*, that that primordial light was produced not in the lower hemisphere of Palestine, or of the earthly Paradise that was to be founded later — as Hieronymus ab Oleastro held, and Catherinus in his exposition of chapter 1 of Genesis, who places it, or the Sun, as produced on the first day at its setting, so that the natural day should begin from the onset of night, that is, from evening — but in the upper hemisphere, as the one in which human life was to be first, and in which Christ was later to work salvation, as in the middle of the earth. For that this hemisphere was illuminated first is taught by Iunilius and Bede on the *Hexaemeron*, Comestor, chapter 3 on Genesis, Strabus in the ordinary Gloss, Abulensis on this passage, Molina, disputation 1 of *De opere sex dierum*, and Salian at the first day of the world. I said "of Palestine", or of Canaan, on account of Christ and of most of the Patriarchs and Prophets, which Molina, Tostatus, Martinengus and Salian also suppose — granted that St Augustine, book 1 of *De Genesi ad litteram*, chapter 10, and Suarez, book 2 of *De opere sex dierum*, chapter 3, number 12, seem to indicate that it was done in the land in which God was going to make Adam, or Paradise; while Steuchus places it in Armenia, Chaldaea and Mesopotamia, where the first men lived before and after the flood.

[Margin: That light applied successively to different horizons.] *Second*, I say that that alternation of days and nights took place with some succession. For what Iunilius seems to mean on the *Hexaemeron*, at the words *let them be for signs and times*, cannot stand, when he says: *For that whole three-day period above had passed in an undivided progress of its course, with no measure of hours at all, since the primary light still filled everything in general and had no head,* etc. — *no shadow was cool under rock or tree.* For granted that there was no gnomon to cast a shadow, no one to divide the day into hours, and no local motion of the light taken in the proper sense — for that, I think, is what he meant — still the time of night followed the day, nor was it day everywhere throughout those three days, as Iunilius himself had conceded earlier, and as I shall show below at number 36. Otherwise, how would God have divided light from darkness? Or how would the first, second and third day have come about from evening and morning? [Margin: Whether that succession [?] was by the local motion of the Sun or of the cloud [?]?] That succession, however, was not brought about by the local motion of the Sun or of a bright cloud, as those teach who said the

primordial light was the Sun or a bright cloud. For we have shown that that light was no such thing, granted that those Authors, consistently, teach this in the same places that we noted already at number 33. But among them Aegidius is singular. [Margin: The first day [?] shorter than [?] the 2nd and 3rd [?], and these than the rest.] He says that the first day contained only one revolution, as great as that of the primum mobile is now; that on the second and third days the Sun, besides the revolution of the primum mobile, was moved by as much as the proper motion of the Fixed stars, or of the Firmament now made; and finally that on the fourth day and after it made as much as the Sun's diurnal motion toward the East now requires; and so that the first day was somewhat shorter than the second and third, and these than the fourth and the rest. Martinengus, page 518 of the *Glossa magna*, indicates that this is fictitious; but it is certainly not improbable. St Gregory of Nyssa, on the other hand, since he judged that that light was a fire which by its lightness had been separated from the mass of the elements, held that those three days were completed by the rotation of the fire. [Margin: Whether by [?]traction and diffusion of the rays [?]?] Others explain that alternation by an alternating emission and contraction of light: namely St Basil, homily 2 on the *Hexaemeron*; St Damascene, following the teaching of Nazianzen, book 2 of *The Orthodox Faith*, chapter 7; Procopius of Gaza, chapter 1 on Genesis; and Burgensis on chapter 1 of Genesis. But St Bonaventure rejects this, on book 2, distinction 13, article 1, question 2; indeed St Augustine, book 1 of *De Genesi ad litteram*, chapter 10, fears that if this were said it would seem ridiculous. For he thinks it follows from this opinion that the day was on one Horizon only and persisted there with unmoving light for some hours, say 12, and then night in the same place for as many other hours; and, next, that no reasonable cause can be brought for such a contraction and emission, and no example. Yet the authors already mentioned can be brought round to the following opinion, which posits a successive diffusion of light; for they seem chiefly to have wished to exclude a succession of days and nights produced by the motion of the Sun. St Basil says there: *Night is the shadow of the earth, which follows the day when the Sun hides itself. Then, however, there came about a day, and night in turn followed it as it gave way — not according to the motion of the Sun, but as that primordial light diffused its splendour and drew it back again, and that by a measure defined by God.* Procopius, adhering to him, transcribes his words, adding only that that light was poured round on every side; and Burgensis, reporting the same doctrine of St Basil, adds: *This light indeed did not have motion of itself, and so could not of itself divide light from darkness, but did this through the emission and contraction of light by divine power.* St Damascene, however, says: *In those three days, by the light being poured out and drawn back, day and night came about by divine command.*

XXXVI. [Margin: Whether by successive transfer equivalent to the local motion of the Sun?] *Third*, therefore, I say that that alternation took place successively according to one Horizon after another, through a transfer or transfusion equivalent to local motion. For that division of light from darkness

was such that not only did light make and represent day, and darkness night, but also such that Morning followed Evening — a manner of speaking that signifies, in the proper sense, a day in the way it happens now, except for the Sun and the shadows bounded by the Sun's rays: God, that is, producing the daylight successively in the same way as the Sun now produces it. [Margin: St Augustine.] St Augustine indicated this, book 1 of *De Genesi ad litteram*, chapter 1, when he said that it happened then as now, so that when it was day in Palestine — that is, when day was beginning — it was still night in Africa: *That at the time when it is night with us, it illuminated with the presence of light those parts of the world through which the Sun returns from its Setting to its Rising; and that in this way, through all twenty-four hours, there was never lacking, around the circuit of the whole round, day in one place and night in another.* [Margin: St Justin. Strabus.] That those days were like ours, the Sun excepted, St Justin teaches, question 62; for he says of that light: *Days of the kind it now makes with the star of the Sun, such it made also before all the stars were founded.* Strabus agrees as well, in the ordinary Gloss: *As the light gradually set and, after the space of a day's length, went under the lower parts, evening came about, as now usually happens by the accustomed circuit of the Sun. And morning came about as the same light returned above the earth and began another day; and one day of twenty-four hours was completed.* [Margin: Hugo Carensis.] Next, Hugo Carensis in the *Postills* expressly affirms that by that light, carried round in the manner of the Sun, the upper and the lower hemisphere were illumi- nated. [Margin: Iunilius.] And before these two Iunilius did so too, in his *Hexaemeron*, adding: *He preferred to say "evening and morning" rather than "night and day were made", to suggest that the circuit of the primary light was then carried out as it is now established to be carried out, day and night, by the circuit of the Sun.* But he very shrewdly observes that in that three-day period the nights were darker than now, because the other stars, which usually temper the shadow of night, were missing, and the parts of the air lying outside the shadow of the earth were not lit by the Sun, as they are now lit even at night. So he goes on: [Margin: The three first nights very dark.] *This alone is evident: that after the stars were created, night too is suffused with a light of its own, though less than the day's; but in that first three-day period it remained dark and utterly obscure.* [Margin: Caietanus.] Most clearly of all, Caietanus on Genesis: *If the light had been fixed, then the day would have been fixed and perpetual in one hemisphere, and night continual in the other hemisphere; nor would there ever have been an evening ending the artificial day. Hence, from the fact that there was an evening ending the artificial day, and a morning ending the night, it is clearly signified that the light was made in motion; for evening comes about, and morning comes about, from the continuous motion of the light around the earth.* But if he means a local motion of the light, by which an accident would migrate from subject to subject, he multiplies a new miracle without necessity, and one of a kind that does not happen now. Therefore, just as now one light after another, numerically distinct, is produced by the Sun along straight lines, so then it was produced by God, through mere successive production rather than

through local motion. And so with us are Salian, at the 1st day of the World, and Tanner, volume 1, disputation 6, question 2, doubt 3, number 11.

Quæſtio 9. An Dies Nocti præceſſerit, An Nox Diei.

Question 9. Whether Day preceded Night, or Night Day.

XXXVII. [Margin: 1st opinion, giving precedence to Night.] We must repeat what we said in book 1, chapter 24: the ancients, and those more expert in the Latin tongue, called the Natural Day the stay of the Sun above the Horizon, which is distinguished from night, and called the day composed of light and darkness, or night, the Civil or legal day; but most of the more recent Astronomers and interpreters of sacred scripture call the Natural Day the one composed of light and night, and call the stay of the Sun above the Horizon the Artificial Day. With them, as being by far the more numerous, we shall speak from here on.

With this premised: that the Natural Day — evening being taken as the beginning of night — began from evening at the first beginning of the world was held long ago by the Manichaeans. [Margin: Manichaeans.] Of them St Augustine, book 1 of *De Genesi contra Manichaeos*, chapter 10, at the words *and there was evening and there was morning, one day*, reports as follows: *And here the Manichaeans cavil, thinking it was said as though the day began from evening.* [Margin: Thales.] The opinion also has as supporters Thales of Miletus, who, asked which came first, day or night, answered that night had come first; [Margin: Abulensis. Molina. Fernandius. Salian. Tirinus.] likewise Abulensis and Fernandius in their commentaries on Genesis at the words *and there was evening and morning, one day*; Molina, disputation 7 of *De opere 6 dierum*; Salian in the *Annals* at the first day of the World; and Tirinus in the *Chronicon sacrum*, chapter 1. These hold that that earlier darkness lasted 12 equinoctial hours and preceded the first artificial day as a first night, and that the first natural day was formed of the two together; and so that the natural day began from night, although the artificial day began from the rising of the light. For this, they say, is why God said *and there was evening and morning, one day*, naming evening, that is, night, in the first place. [Margin: Arguments for the precedence of Night, and for the beginning of the natural day from night.] They confirm this from the text of Leviticus, chapter 23: *From evening to evening you shall celebrate your sabbaths.* This seems an argument that the Hebrews, from the very origin of the world, began their natural days from the beginning of night, as most cities of Italy do now, and as the Athenians once did, counting 24 hours from sunset to sunset — witness Pliny, book 2, chapter 77; Gellius, book 3, chapter 2; Macrobius, book 1 of the *Saturnalia*, chapter 3; and Censorinus, chapter 10.

And indeed this opinion is highly probable, and I chose it elsewhere; and I could confirm it. *First*, because God divided the primordial light from the darkness; but as soon as this is heard, the reader of Moses meets that darkness which was over the face of the abyss. Therefore, when God called the darkness

night, he included that darkness too under that name; so if that darkness was night properly and literally, that natural day began from night. *Second*, in Genesis 2 it is said: *In the day that the Lord God made heaven and earth*; and in Exodus 20: *Six days you shall labour and do all your works; but the seventh day is the Sabbath of the Lord your God; you shall do no work on it, you and your Son,* etc. *For in six days the Lord made heaven and earth and the sea and all that is in them, and rested on the seventh day; therefore the Lord blessed the Sabbath day and sanctified it* — which is repeated in Exodus 31. Since, then, heaven and earth were created in the first instant of the time of this world, and before the production of light, that whole time from the first creation of heaven to the production of light certainly belonged to the first of those six days; and it seems quite absurd that there was some time which did not belong to those six days, or which belonged to them, but as an extraordinary appendix, making up together with the artificial day and the following night one natural day longer than the others and unlike them. *Third*, it is certain from Leviticus, chapter 23, that by God's command the beginning of the Sabbath the Hebrews were to celebrate, and of their rest from all work, was regularly taken from evening; and it is equally certain from the passage of Exodus just adduced that the reason for this precept was that they should remember and imitate that first Sabbath on which God rested from all the work he had accomplished. Therefore, just as they ceased from all servile work right at the beginning of night, and began the Sabbath from then, as they still begin it now, so God too seems to have ceased from all work at the setting of the Sun, that is, at the beginning of night, and so to have counted the six natural days from then and completed them there, and not from rising to rising of the light.

Hence it is clear that what Martinengus says in the *Glossa magna*, page 545, is neither certain nor more probable: that the Hebrews, just as they had a double year according to Philo and Josephus, so had a double day — one natural and ancient, which they began from sunrise, the other legal, which they began from sunset — and that the genuine cause of this change was that they went out of Egypt in the evening. For granted that the beginning of the Sabbath was confirmed for this reason as well, and explained on that occasion, since it could perhaps have come into question, still the prior and weightier reason was that they should imitate the Lord, who rested from working at the beginning of night. Nor does it seem negligible that not only the Athenians, as we said above, but also the Egyptians once began the day from sunset, as Alex. ab Alexandro teaches, book 4 of the *Geniales dies*, chapter 20 — as though they had learned this from the Jews; and that Hesiod in the *Theogony* says Night arose from Chaos, as though prior to day, just as Chaos was prior to the distinction of things. *Fourth*, if anything stands against this view, it is above all the opinion of many Fathers and Doctors. But they seem capable of being reconciled with it, by granting them their principal intent: namely, that the first artificial day, and the rest after it, began from the East — that is, from the rising of the light, first in the East and propagated from there into the other regions — against those

who thought it was produced at noon or at setting; granted that the natural day began from the first instant of the creation of the World, which was to be under darkness for some hours. Nor is it an obstacle that this opinion was the Manichaeans'. For it is not established that they said anything erroneous in this; rather, on this as on a foundation laid, they said there were two principles: one of evils, which preceded and produced darkness and evil as something positive and a kind of substance; the other of goods. And so the holy Fathers, with St Augustine, contended that evil and darkness are not a substance or a being that could be produced by a positive action, much less by a different principle — granted that St Augustine, to overturn their foundation as well, denied them that the day began from evening. Finally, St Jerome, on chapter 2 of Jonah, says: *For in Genesis too the night is not of the preceding day but of the following: that is, the beginning of the future, not the end of the past.*

XXXVIII. [Margin: 2nd opinion, giving precedence to day.] The second opinion gives precedence to the day. And indeed, if we follow the bare authority of the Fathers, I find hardly any who, St Jerome excepted, called that earlier darkness night, or began that first day from it; but very many who say that that day was begun from the rising of the first light, that night followed it, and that from these, up to the following morning, the first natural day was formed. Let St Basil be heard, homily 2 on the *Hexaemeron*, where, after he has said [Margin: St Basil.]: *Evening is indeed the boundary common to both day and night; and morning, equally, is the neighbourhood of night to day. So, in order to grant the day the prerogative of primacy in the order of creation, he mentions the end of the day itself in the first place; then he adds what is the end of the night, since night follows the day.* At once he added: *For that earlier state of the world, namely before the rising of the primordial light, was not called night but darkness. For night is the name given to that portion of time which is distinct from day and opposed to it; and it is as the successor of day that it acquired this new name.* Indeed, St Basil seems to hold that in the words *evening and morning, one day*, only two parts of the artificial day are expressed, the morning and the evening, while night is understood, as a less principal complement of the natural day — just as when Psalm 89 says *the days of our years*, and Jacob says in Genesis 47 *the days of my life are few and evil*, the nights too are understood, as will be clear to anyone who reads that homily. [Margin: St Ambrose.] And in all this St Ambrose subscribes to St Basil, book 1 of the *Hexaemeron*, chapter 10. Having raised this question — *Some ask whether, since scripture mentioned evening first and morning afterwards, it may not seem to signify night before day* — he answered: *They do not notice, first, that it put day first, saying: and God called the light day and the darkness night. Next, that evening is the end of the day and morning the end of the night; so, in order to give day the prerogative and primacy of birth, it first signified the end of the day, after which night was to follow, and then afterwards joined to it the end of the night. Scripture could not put the night before the day, so that it included the times of both day and night under the name of day, and claimed for it, as the principal, the authority of the name.* He confirms this, as St Basil does, from the psalm mentioned and from Jacob.

According to these holy Doctors, then, the sense of those words, *And there was evening and morning, one day*, resolves into this: *And one day was made of the evening and the morning time*, the night being understood as having elapsed between the completion of the day and the beginning of the following artificial day. [Margin: St Chrysostom.] St John Chrysostom agrees, homily 3 on Genesis: *He pointedly named the end of the day and the end of the night as one, so as to establish a certain order and sequence of visible things*; and homily 4: *and one would rightly call evening the end of light, morning the end of night, and the completion the day; for this is what divine scripture means to make plain when it says: and there was,* etc. [Margin: St Nyssen.] St Gregory of Nyssa understood the names of evening and morning in the same way, in the book on the history of the six days: *He therefore named the withdrawal and setting of the light evening; and again, when the fire had run through the circle of the lower regions and restored its splendour to the upper parts, he called that daybreak, once it had come about, morning.* Remember that in his view the light was fire, segregated first of all from the other elements. [Margin: St Athanasius.] St Athanasius too, expressly, question 90: *From the primitive creation of the world,* etc., *day was prior; just as divine scripture also says that God said let there be light, and light was made, and that God called the light day, but the darkness he called night. Light was set first, for through it the first day came to be. And beginning first from light, we afterwards came to darkness.* [Margin: Procopius.] Procopius discusses this controversy, once a celebrated one, at greater length in the Commentary on chapter 1 of Genesis, and sets it out thus: *There is a great dispute whether day or night came first; but those who argue that day is older than night, as is commonly believed, proceed thus. God himself gave the primacy to day: for he first set the end of the day, namely evening, as though day had preceded night, and only when it had ended in evening was night born from its end. For night, as the rest from the day's work, is brought to us from the preceding day. But that face which the World had before the beginning of light is not properly called night, but rather gloom or darkness.* And since the Greeks call the natural day ν υκθήμερον [recte: νυχθήμερον] [Greek: nychthēmeron, "night-day"], as it were *Noctidium*, that is, consisting of night and day, Procopius there judges it should rather be called ήμερόνυκτον [Greek: hēmeronykton, "day-night"], as it were *Dienoctium*, because in this compound the day comes first. [Margin: St Damascene.] In addition, St Damascene, book 2 of *The Orthodox Faith*, chapter 7: *It was not night that was called first, but day. So day is first and night last: night therefore follows day, and from the beginning of one day up to the next day is one — so to speak — night-day; for scripture says: and there was evening and morning, one day.*

So much from the Greek Fathers — although, because of the affinity of interpretation, we have already attached St Ambrose to St Basil. Now, then, to the remaining Latins. [Margin: St Augustine.] Thus St Augustine, book 1 of *De Genesi contra Manichaeos*: *After this operation* — that is, of the light made — *evening came about as though the day were ended; but because night too belongs to its day, one day is not said to have passed unless the night too has passed, when morning*

came; and so thereafter the remaining days are reckoned from morning to morning. He repeats this in the unfinished book *De Genesi ad litteram*, chapter 7, and in book 1 of the finished work, chapter 10. [Margin: Eucherius.] Eucherius too, book 1 of the commentary on Genesis, chapter 2, uses almost the same words as St Ambrose above; [Margin: Iulianus.] and so does Iulianus, Archbishop of Toledo, book 3 *Against the Jews*: *In six days God made heaven and earth, and founded every creature formed by him on distinct days, beginning each day with light and ending it with evening, as scripture says: And there was evening and morning, one day.* [Margin: Iunilius.] Iunilius too supports them, in the *Hexaemeron*: *There was evening, as the light gradually set and, once the space of the day's length was complete, went under the lower parts of the World,* etc. *There was morning too, as the same light gradually returned above the lands and began another day; and up to this point one day was completed, namely of twenty-four hours. For it was fitting that the day, beginning from light, should be extended into the morning of the following day.* [Margin: Strabus.] Strabus has these very words in the ordinary Gloss. [Margin: Hugo Victor.] And Hugo Victorinus understood those words, *Evening and morning, one day*, in entirely the same way, adding that the primordial light arose in the East without a preceding Dawn, and that day naturally precedes night — so he writes in book 1 of *De Sacramentis*, part 1, chapter 9. [Margin: Richardus.] So too Richardus Victorinus, book 2 of the *Exceptiones*, chapter 7: *That primary light is believed to have been made in the place where the Sun now rises every day, and to have circled by the same path as the Sun, and so to have accomplished evening and morning; and this was the work of the first day.* [Margin: Hugo Carensis.] Add to these Hugo Carensis in the *Postills*: *For when the light,* he says, *made in the East, proceeded to the West in the manner of the Sun, evening came about; and as the same light migrated under the earth and came to the Rising, morning came about; and so one natural day of twenty-four hours was made.* So too, entirely, thought and spoke [Margin: Comestor.] Comestor, chapter 3 of the *Historia Scholastica* on Genesis; [Margin: Master.] the Master of the Sentences, on book 2, distinction 13; [Margin: Lyranus.] Lyranus in the *Postills*; [Margin: Albertus Magnus.] Albertus Magnus, part 1 of *De quatuor coaequaevis*, question 12, article 11; [Margin: St Thomas.] St Thomas, part 1, question 74, article 3, reply to 6; [Margin: St Bonaventure.] St Bonaventure, on book 2, distinction 13, article 1, question 2, in the second exposition; [Margin: Dionysius Carthusianus.] Dionysius Carthusianus, Commentary on Genesis, article 9; [Margin: Caietanus.] Caietanus on chapter 1 of Genesis; [Margin: Vielmius.] Vielmius, at the same place, lecture 3; [Margin: Lippomanus.] Lippomanus in the *Catena*; [Margin: Pererius.] Pererius, book 1 on Genesis, page 54; [Margin: Martinengus.] Martinengus in the *Glossa Magna*, from page 540; [Margin: Suarez.] and Suarez, book 2 of *De opere sex dierum*, chapter 3, number 12. Their words are ready to hand, and I have judged they should not be quoted, to avoid prolixity. Nor does Tostatus reject this opinion, in his commentary on Genesis; he reports it as probable.

XXXIX. [Margin: 3rd opinion, intermediate between those already mentioned.] The third opinion begins that first natural day from Evening, taking Evening to begin from Noon. Placing the Sun, or the primordial light, on the Meridian, it holds that that first day had its morning not before it but after it, and that this is why Moses said, *there was evening and morning, one day*; so that the first artificial day had only six hours, from noon to setting, while the six preceding hours, in which the World was in darkness, belonged not to the first artificial day but to the completion of the natural day. This opinion is therefore intermediate between the two extremes already mentioned. With the former it begins the day from Evening — which leans and declines rather towards setting — and begins the natural day from the preceding darkness; but with the latter it begins the artificial day from Noon, which belongs to the day rather than to the night. [Margin: St Bonaventure.] [Margin: Steuchus.] [Margin: Aegidius.] [Margin: Cornelius.] [Margin: Ficinus.] This is the view of Augustinus Steuchus, or Eugubinus, in the *Cosmopoeia* on chapter 1 of Genesis; of Aegidius, part 2 of the *Hexaemeron*, chapters 2 and 6; and of Cornelius à Lapide on chapter 1 of Genesis; and St Bonaventure reports the same opinion and does not think it improbable, on book 2, distinction 13, article 1, question 2. Speaking of the fourth day, Marsilius Ficinus [Marsilio Ficino] supports them, in the book *De Sole et lumine*, chapter 10, where he teaches that the Sun was produced on the Meridian, and that with respect to our hemisphere the day began from noon. From this view, however, it follows that the remaining days, both artificial and natural, began from the rising of the light or of the Sun above the horizon. For to fill out the first natural day it counts the 6 hours in which the world was in darkness; and by the following morning it does not understand the time from the rising of the light to noon — which, it teaches, belongs to the following day — but the moment at which the first night was completed and the light, or the Sun, arrived from noon, by way of setting and midnight, at its rising. Besides that reason — that Evening was named first by Moses — these Authors add this one too: that the works of God are perfect, and that the Sun or the light on the Meridian illuminates the hemisphere more perfectly and evenly in every direction; and that most Astronomers usually begin the day from Noon, as from a more certain point of the day. [Margin: This opinion is rejected.] But if we wish to speak with most of the Fathers and Doctors, according to what was said at numbers 37 and 38, the light or the Sun began the first artificial day from the East; and it is more probable that that [?] first day, both artificial and natural, was the norm for the rest, or at least was not unlike the two that followed it, and so had 12 hours of light and as many of night — not only six of light and eighteen of darkness, as this opinion supposes. And the greater perfection of an equinoctial day is that it be whole and fill a time of 12 hours, rather than that the Sun should shine more ardently from the summit of the sky at its beginning than in its progress, and at once begin to decrease — in keeping with the text, *The path of the just is like a shining light, growing up to the perfect day*, Proverbs 4. Moreover, the way of beginning the day from noon is

too subtle to serve the common people and ordinary use, because finding the meridian line calls for no common skill; for the common people are used to observing the rising and setting of the Sun, as the more certain sign.

XL. [Margin: 4th opinion, beginning the day from night in another way.] The fourth opinion, already indicated at the beginning of number 35, was that of Catherinus and of Lusitanus, that is, Hieronymus ab Oleastro, on Genesis. [Margin: Catherinus. Oleaster.] They think that Evening was put first, or named first, by Moses before the morning time because the Sun or the primordial light was placed first on the western horizon with respect to Palestine, and began from there to rise for the hemisphere opposite that region; and, when a night of 12 hours was completed, reached the east of Palestine and made the first morning; and then, after another 12 hours, returned to its setting, and so completed one natural day. So this opinion agrees with the first in beginning the natural day from the beginning of night, but differs from it in not counting, in the first natural day, the time of the darkness that preceded the primordial light. Catherinus confirms his opinion both from the fact that the Hebrews celebrated their sabbaths from evening to evening, from Exodus 20, and from the fact that the Egyptians used to begin the day from sunset, as perhaps did many of the ancients — which is why Thales of Miletus, when asked, answered that night was prior to day.

But what more improbable thing could be said than that the light, as soon as it was produced, was hidden, or withdrawn, from that hemisphere which was to be the principal dwelling of men, for which Moses chiefly wrote, and which was to receive first the light of divine knowledge and of the Mosaic and Gospel law, and was shown first to that hemisphere which was to come much later to civil culture and to the knowledge of the true religion? Next, since heaven and earth are said to have been created on the first day under darkness, either that darkness which, after that first instant, was over the face of the abyss was over the hemisphere of Palestine only, or it was there too, but its time was a supernumerary appendix to the first day, and that day was longer than the rest and unlike them; and neither of these fits the letter of Moses well. If, on the other hand, that time belonged in no way to the first day, it follows that heaven and earth were created not on the first day but before the whole first day. Since, then, of the four opinions already mentioned, the third and the fourth have far fewer patrons and are much more improbable than the first and the second, it remains for us to choose one of those two earlier ones.

XLI. [Margin: Five acceptations of Evening.] Before I put forward my own view, however, I note that Evening can be taken in five ways, and Morning, its opposite, in as many. *First*, so that evening signifies the indivisible moment at which the Sun sets, and morning the moment at which it rises. *Second*, so that evening signifies the twilight that follows sunset, or the earlier part of that twilight, in which the star of Venus appears when it follows the Sun. At that time this star is called Vesperugo by Plautus, Vesper by Ennius, and Hesperus and Vesper by Virgil, as Censorinus notes in *De die natali*, chapter 10, and St

Isidore in book 5 of the *Etymologies*, chapter 31; for they allude to those lines of Virgil:

Ante diem clauso componet vesper olympo ("Before that, Vesper will close Olympus and lay the day to rest"), and

Ite domum satur e, venit Hesperus, ite capellae ("Go home, well-fed [?]; Hesperus comes; go, she-goats").

Morning in this sense will signify either the morning twilight or the part of it nearer sunrise, that is, Dawn; and Lusitanus and Vielmius, lecture 11 on Genesis, take it in this way. They confirm it both from the Hebrew word *Ereb*, which signifies Evening, from the verb *Harab*, which means to bind or tie together, since it is a kind of bond between day and night; and from Plautus, who called this twilight the first evening — Livy calls it the first darkness, Macrobius the first torch. From this they explain the text of Exodus 12, where the Jews are commanded to slaughter the lamb between the two evenings, that is, about the middle of this twilight, which divides it in two: an earlier part nearer the sunset just completed, and a later part more like night. Morning, in turn, is called in Hebrew *Boquer*, from the verb *Bacar*, which means *to discern*, because in the morning twilight visible things begin to be discerned. *Third*, Evening can be taken for the whole time in which the Sun, descending from the meridian, slips down to its Setting, and Morning for the time in which the Sun climbs from its Rising to Noon. In this sense we are said to lunch in the morning and dine in the evening, even in high summer; or else so that Morning is the time from midnight to sunrise. *Fourth*, Evening can be taken for the time in which the Sun descends from Noon to midnight, and Morning for the time in which it climbs from midnight to noon. *Fifth*, finally, so that Evening signifies the whole night — but the night following the day that has elapsed — and morning the whole artificial day. In this way the text of Exodus 12, *between the two evenings*, could be understood of the moment of sunset. The text of Genesis, *Evening and Morning, one day*, could also be understood in as many ways: whether those words are taken for the two boundaries, indivisible or divisible, between which day and night are enclosed, as in the 1st and 2nd acceptations; or for opposed parts, making up the day and the night adequately or inadequately.

XLII. [Margin: Our opinion.] With this premised: since the arguments brought at number 37 for the first opinion are strong, and the authorities adduced for the second at number 38 are more numerous and more ancient, the two opinions seem to me to call for reconciling, as follows. The first natural day — properly so called, indeed, but taken broadly, for a day formed of night taken broadly and of time taken broadly, that is, without actual motion — began from the darkness lasting 12 hours, or a duration commensurable with them and equivalent, which preceded the light first produced in or about the East of Palestine; whereas the first artificial day took its beginning from that first production of light, and when its evening, taken in the first, second or third way, was complete, the first day, natural as well as artificial, was complete

— although only the artificial day is expressed by *Evening and morning*. And it was in this way that the Hebrews understood that day, both in sanctifying the sabbath and before. From this it follows that the second, third, fourth and remaining days, and so the sixth, which are counted in sacred Genesis and within which God completed his work, were completed at Evening, that is, at the setting of the Sun or of the light that was the Sun's deputy; and that God began to rest from work in the night preceding the sabbath day taken artificially. In saying this, however, I do not on that account say that evening was named by Moses before morning; for that darkness can be called evening neither strictly nor broadly. Nor did he mean to express night by the words *evening and morning*, but only parts of the artificial day, composed of evening and morning time, so as to signify to us that those days were not unlike ours but had the same alternations — granted that he named first that part of the day which contained the consummation, maturity and perfection of the day, and of the works accomplished on that day, as its fruit, before the part which contained only its beginning and, as it were, the flower of the day. Taking the Natural Day more strictly, however, for the aggregate of the artificial day and of night taken strictly — that is, night as the privation of light once produced — and of time taken strictly, as the measure of the motion of the Sun or of the light that was the Sun's deputy, I say that it began from the rising of the light, from which the artificial day began, and that the artificial Day preceded night taken strictly, as the second opinion holds. So, on this view, the same moment of the light's rising was the beginning of both the natural and the artificial day, but the end of the two was not the same; whereas, according to the first opinion, the same moment was the end of both days, but their beginnings were different moments. I have not enumerated, however, a conceivable fifth opinion: namely, that the first natural day was composed of six earlier hours of darkness, of the 12 following hours of light, and of the remaining six hours from setting to the middle of the night, because I have found no author who has asserted it, even though the Romans and some Astronomers have reckoned days from midnight to midnight.

XLIII. [Margin: Light produced and the Angels beatified at the same time.] Supposing, finally, a time under darkness from the first instant of the creation of the world down to the first production of light, whether of 12 hours or thereabouts, it could be asked whether it is probable that that time of darkness was the one in which the battle took place between the good and the bad Angels, and that the first instant of light was the one in which the holy Angels were confirmed in grace and raised to the light of glory — so that that corporeal light is to be understood literally, but was, symbolically and mystically, a sign of the spiritual light produced in the minds of the Angels. Abbot Rupert [Rupert of Deutz] certainly held this and stated it in express words, in book 1 of his commentary on Genesis, chapter 10, and so did Aegidius on the Hexaemeron, part 2, chapter 2. But these matters are nothing to us now.

CAPVT II.

Chapter II.

De Opere Secundi Diei. Seu quid fuerit ac ſit Firmamentum illo die factum, & ex qua materia; Quidue & quales Aquæ ſuper cælos.

On the Work of the Second Day; or, what the Firmament made on that day was and is, and from what matter; and what the Waters above the heavens are, and of what kind.

I. We set out three questions in one, because they are so entangled with one another that none of them can be solved without the other two. We must inquire whether the starry and visible heaven was made out of elemental water by condensation and solidification; next, whether every starry heaven, or the whole of it, was so made, or at least the eighth sphere, the highest of the visible ones; lastly, whether the waters above the heavens are liquid and distinct from the solid heaven, or enclosed within it and liquid in potency, not in act. For once these points are settled from sacred Scripture and from the Fathers and Doctors of the Church, we shall dispatch many other questions more easily and more briefly, and shall be able to pass judgement on the opinions of others with greater and more solid calm. Otherwise, it is incredible how great a confusion of opinions and questions is found among writers on this passage of sacred Genesis. Amid this multiplicity we shall choose the view that so accords with the letter of sacred Scripture that it reconciles as many of the Fathers and Doctors as possible with one another, conflicts least with astronomical observations, and, finally, presents the workmanship of God more reasonably. This we shall try to accomplish in the following Questions and Conclusions.

Quæſtio 1. An ex Aqua denſata & ſolidata in modum glaciei aut cryſtalli, aliquod Cælum ſecunda Mundi Die factum ſit?

Question 1. Whether some Heaven was made on the second Day of the World out of Water condensed and solidified in the manner of ice or crystal.

II. [Margin: 1st Conclusion, and its proof from the name Firmament and from sacred Scripture.] The Conclusion is affirmative. It is proved, first, from the name that sacred Scripture gave this Heaven, namely Firmament — in Hebrew *Rachiagh*, or *Rakiaa*, or *Raquagh*, for different people pronounce it differently — from the verb *Rcah*, or *Rakaa*, or *Raquagh*, which means both to spread out or extend and, by extending, to make firm and consolidate, as Lippomanus notes. Further, Pererius and Martinengus, with Caietanus on this passage, teach that a thing is extended in four ways: [Margin: 5 modes of extension.] namely, 1. by the unfolding of a thing folded up, as when sails and curtains are spread out, or when a sponge first squeezed by the hand strives to restore itself to its former occupation of space; 2. by beating, as when metal is

beaten out into plates with a hammer; 3. by rarefaction, as when water swells into bubbles with heat or with blowing, or boils over out of a vessel; 4. by increase of substance, as when plants and infants grow into trees and men. But St Jerome and Vielmius have a fifth mode, namely fusion, by which metal, or anything that can be melted, is cast in such a way that it keeps the solidity and stability of the shape once received.

Now those who favour the fluidity of the heaven, or who understand by the name Firmament the air, or a mere stability of the aether in bounding the waters on either side, without hardness or solidity strictly taken, contend that the name *Rachiagh* ought to have been translated according to the first mode of extension, and that the text should say *Let there be*, or *Let it be*, an *Expansion* or a *Spreading-out*. So Calvin, on Tanner's testimony, Caietanus, Steuchus, Vielmius and Pererius; and they adduce the text of Psalm 103, *Stretching out the heaven like a skin*, or like a curtain and a tent; and Isaiah 40, *Who stretches out the heavens as nothing, and spreads them out as a tent to dwell in*. But these examples do not altogether favour them, because they suppose at least that the thing unfolded has some solidity.

[Margin: The name Firmament is defended.] Next, St Jerome and the Vulgate version have translated *Firmament*, and the Septuagint translators στερέωμα [Greek: stereōma, "firmament"], which signifies a solid body; Velleius in Cicero, *De natura deorum* book 1, said that solid and firm bodies were called στερέμνια [Greek: steremnia, "solid things"] by Epicurus on account of their firmness; and even now the Hebrews who write on Geometry, where solid and compact bodies are treated, call them στερεώματα [Greek: stereōmata, "solids"], and the treatment of them στερεωμετρίαν [Greek: stereōmetrian, "stereometry"]. Since this is so, we too ought to take this name in the same sense, so that it does indeed signify extension or expansion, but either according to the second mode, for beating or, as it were, hammering — as Pagninus takes it, and Oleaster, or Lusitanus, who explains in this way this passage of Genesis, and that of Isaiah 42, *Who spreads out the earth and its offshoots*, where *Rocah* stands, that is, *spreading out*, or *who spreads out*; and Jeremiah 10, *Silver spread out with hammers is brought from Tarshish*, where for *spread out* the Hebrew edition has *murecah*; and that passage of Ezekiel 6, *Strike with your hand, or stamp with your foot*, where in Hebrew, for *strike* or *stamp*, stands *Vrechah*, that is, spread out thus, as one who spreads out metal or clay by pounding it with hand or foot. Or else, certainly, the name Firmament is to be taken in the 5th sense, so as to signify an expansion by force of which a solid and stable shape is nevertheless left in, or given to, some thing: as when metals are cast into statues, or — what pleases me more, and sets before the eyes, as it were, both forces of this name and God's manner of proceeding in this work — when glass, first melted in the furnace, is blown by the mouths of glassworkers making very large flasks out of a little glass; for they spread that material out in such a way that they nonetheless make something solid and firm out of it.

And indeed several other passages of sacred Scripture favour this fifth mode: namely Psalm 32, *By the word of the Lord the heavens were made firm*, where the Septuagint has ἐστερεώθησαν [Greek: estereōthēsan, "were made solid"], that is, *were solidified*; and 2 Paralipomenon 6, where Heaven is called *a most firm dwelling*; and Proverbs 3, *The Lord by wisdom founded the earth; he established the heavens by prudence*; and Job 14, *Until the heaven be worn away* — for what is worn away supposes solidity; and Hebrews 4, *Who has penetrated the heavens* — for that, as St Thomas teaches, part 3, question 57, article 4, reply to 3, is said more properly of solid bodies than of fluid ones, since if they were all fluid there would be no need of a penetration strictly taken; and that text of Isaiah 51, *The heavens shall melt like smoke, and the earth shall be worn away like a garment* — for what will become liquid was surely solid before. As for what some translate with the Septuagint, *The heaven was made firm like smoke*, it agrees neither with the Hebrew nor with the Vulgate edition, nor with St Jerome, nor with Pagninus, nor with Arias Montano, nor with the comparison drawn from smoke — granted that St Basil, homily 1 on the Hexaemeron, accepts the Septuagint version.

But the most celebrated passage of all is the one in Job 37, where Elihu says: *Perhaps you fashioned the heavens, which are most solid, as if cast of bronze.* For although that saying is not a matter of Faith, since it is not the sacred historian's own nor uttered from the mouth of God, nevertheless, so long as Elihu's sayings contain no falsehood or contradiction and are not found rebuked by God, they have great authority, like certain moral sayings, as the sayings of a very expert man speaking from the ancient tradition about the structure of the heavens set out by Moses. For the sayings of the two remaining friends, his and Job's, are also adduced by Sts Peter and Paul as having great and irrefragable authority. Thus in 1 Corinthians 3 the saying of Eliphaz the Temanite is brought from Job 5, *Who catches the wise in their craftiness*; and in 2 Peter 2, from the same Eliphaz, *Behold, those who serve him are not steadfast, and in his Angels he found wickedness.* So too the holy Fathers u- se that saying of Bildad the Shuhite from Job 23, *The stars are not clean in his sight*; and those words of Elihu from Job 34, *Who does not accept the persons of princes*, and in the same place, *Who makes the hypocrite reign because of the sins of the people*.

Finally, very many Fathers and Doctors understood this name in this sense. They are to be brought forward presently for the 2nd proof, in selected words of their own, so that readers may not think that we drag them to this sense forcibly, or from too remote a conjecture or too general a phrase — especially since this question is of the greatest moment and, because these authorities have been passed over, has not been discussed by many as thoroughly as was fitting, Martinengus, Salian and Tanner excepted, whom I have found the most diligent of all in this matter.

III. Secondly, then, the conclusion is proved by the Authority of the Fathers and Doctors of the Church. Before them let me be allowed to bring forward Josephus, [Margin: Josephus.] who in book 1 of the *Antiquities*, chapter 1,

has this: *After this, on the second day, he placed the heaven above all things and, distinguishing it from the others, commanded it to be set in itself; and, fixing crystal round it, he tempered it with moisture for watering the earth.* From the mouth of St Peter, again, St Clement, Pope, [Margin: St Clement.] in book 1 of the *Recognitions*: *The water that was within the World, in the middle of the space of that first heaven and of the earth, is stretched out as if congealed by frost and solidified as crystal; and by a Firmament of this kind the middle spaces of heaven and earth are, as it were, shut off; and the creator called that Firmament heaven, naming it with the word of that older one. And so, though the fabric of the whole world was one house, he divided it into two regions.* [Margin: Tertullian.] Whether the book *On the Trinity* ascribed to Tertullian is Novatian's, as Pamelius [Jacques de Pamèle] contends, or Tertullian's own, in it those words of Ezekiel 1, *The firmament as the appearance of terrible crystal*, are explained thus: *All things being covered over from above by the crystal, that is, by the heaven covering all things; so that what had been solidified into the firmament at God's command out of the flowing matter of the waters, like strong ice, dividing in half the waters that had long covered the earth, might sustain on a kind of back the weights of the upper water, its strength reinforced by the frost.*

How much water was spent on this work is declared by St Hippolytus of Rome [Margin: St Hippolytus.] — perhaps that Bishop of Portus and Martyr — in Martinengus, page 571: *A third part of the waters he fixed by his power in the middle; a third he raised above the firmament; and a third, finally, he willed to be below, namely for the use of men.* [Margin: St Hilary.] Although St Hilary, in his exposition of Psalm 135, does not explain this solidity, he nevertheless says: *For with the waters raised up he tempered that very nature of the first heaven, which, kindled by the power of God on high, could by no means have been endured by the lower natures. The lower heaven, however, he stretched out not uniform but manifold; and, calling the whole of it the Firmament, he solidified it with a firmness strong enough to sustain the upper waters and to temper our air.* So, between the kindled heaven, or Empyrean, and our air, he recognizes heavens that are all solid, and waters above them. [Margin: St Caesarius.] But St Caesarius, brother of St Gregory Nazianzen, in the dialogues, questions 4, 5 and 6, teaches that the firmament set between the upper and the lower waters is different from the heaven that Moses mentioned in the first place, and is of a crystalline nature, congealed by divine power so that it can carry and separate the upper waters.

[Margin: Acacius.] Let us now hear Acacius, who flourished in the same century; for he, as is found in the Catena of Lippomanus and the Gloss of Martinengus, says: *The first heaven was named heaven, not firmament; but the second received the name both of heaven and of firmament — of heaven because it is above us and renders us the office and use of the earlier heaven; of Firmament because it consisted of the fluid essence of the waters, and that fluid and liquid nature was made most dense and most solid by God, the maker of all things. Divine scripture, therefore, recognized only two heavens.* And a little later he seems to put its solidity on a par with the solidity of the earth, when he says: *But this too is to be known,*

that the earth also is called a firmament in scripture, as when it says, Psalm 135, Who made the earth firm above the waters, and Isaiah 51, The Lord who made the heaven and fixed it, and made the earth firm. [Margin: Severianus.] But plainer words are used by a writer of the same century, namely the fourth, Severianus, as is found in the same Catena and Gloss: *This water rose above the earth by thirty cubits (for example). God said, Let there be a Firmament in the midst of the waters, and without delay it was compacted. A kind of crystalline congealing in the midst of the waters is this very heaven, which, suspending half of the waters with itself, left the rest below.* And again: *So God made this heaven — not that highest one, but the one that is visible — out of the waters, and fixed it like crystal. See the power of the works, that you may admire the craftsman: he willed half of the waters to be above and half below.*

[Margin: St Chrysostom.] Next, St John Chrysostom, homily 4 on Genesis, though he does not dare to define how the Firmament was made, holds this for certain: that it was made fit to carry the upper waters on its back — which can hardly be conceived as done without solidity. For he says: *What is "Let there be a Firmament"? As if one were to say in human language: let there be a kind of wall and partition, dividing in the middle and making a separation.* And a little lower: *He ordered some waters to be borne below the Firmament, and others to be above the back of the Firmament. But what, after all, shall we say this Firmament is? Condensed water, or air spread out, or some other essence? No prudent person would rashly assert this. For what is said must be received by us with great modesty and gratitude, and we must not go beyond our nature and search into the things that are above us; but only know and hold this among ourselves: that by the Lord's command the Firmament was produced, making a separation of the waters, able to contain some beneath itself and to carry others on its back.* Further on, however, we shall see that he is not speaking of the heaven where the Planets move, since he thinks it immobile; or that he does not recognize in it solidity with the hardness that Claudius Marius Victor nevertheless supposed, [Margin: Claudius Marius.] book 1 of the commentary on Genesis, in these verses:

..... Cum se firmissima moles Fluctibus e medijs concreto corpore cedens Extulit, ac late duro solidata rigore, Divisas suspendit aquas, gelidumque profundum Axibus obiecit calidis, seque aequore septo Nixa superfudit rebus, quas circite vasto Contegit aethereum, deducto hinc nomine, caelum.

("…When the most firm mass, yielding from the midst of the waves with congealed body, raised itself up, and, solidified far and wide with hard rigidity, held the divided waters suspended, and set the icy deep against the hot axes; and, resting on the enclosed sea, spread itself over the things that the aethereal heaven — its name drawn from this — covers with its vast circuit.")

[Margin: St Jerome.] With Tertullian, too, expounding Ezekiel as I showed above, St Jerome agrees in the Letter to Oceanus: *Between heaven and earth the Firmament is built, which, according to the etymology of the Hebrew tongue, takes the name heaven, that is, Samaim, from the waters; and the waters that are above the heavens are set apart for the praises of God. Hence also in Ezekiel the*

Prophet crystal is seen stretched out above the cherubim, that is, compacted and denser waters. [Margin: St Augustine.] St Augustine, book 2 of *De Genesi ad litteram*, chapter 1, points to the opinion of these authors before him and does not refute it, saying: *The Firmament, as it seems to some, is ice of waters; and it is called Firmament because it was made firm by waters congealed by divine power, which is called heaven from hiding, or concealing.* Yet further on he holds this much at least for certain: *This Firmament was made from the substance of water mentioned above*; and a little after: *The Firmament, then, that is, heaven, was made in the midst of the waters, to divide between the water that was above the firmament and the water that was below the firmament, so that by the power of God the firmament stood in the midst of the waters, having water beneath itself, within itself and around itself.* See him also at question 106 in the *Questions on both Testaments*, which are found in volume 4 of the works of St Augustine.

[Margin: Gennadius.] Let Gennadius, an author of the fifth century, be counted with these. On Genesis, as the Catena of Lippomanus and the Gloss of Martinengus have it, he says: *So he named it Firmament from the very workmanship of the thing, because, whereas it was before of a pliant, fluid and soluble nature, it received a solid and in no way mobile compaction.* [Margin: Theodoret.] Theodoret, almost his contemporary, set out his mind thus in question 11 on Genesis: *For it was necessary to know, from the reckoning of the time and from the very manner of creation, the diversity of the heavens: that one was created before the light and the other after the light; and that the former was created not out of any matter, the latter out of water. For, he says, Let there be a Firmament in the midst of the water, to separate the waters from the waters through the middle. For thus the heavens obtained their name from the thing itself.* He alludes to the Hebrew name *Samaim*, that is, *there waters*, and goes on: *Since it was composed of the fluid substance of waters, and, being fluid by nature, was condensed and consolidated, it was named Firmament.* [Margin: Eucherius.] More briefly Eucherius, book 1 on Genesis: *By the name Firmament he signified heaven, because it is strongly and firmly composed.* [Margin: Iunilius.] Let Iunilius close this fifth century; in his Commentaries on the Hexaemeron he discourses thus: *Here is described the creation of our heaven, in which the stars are fixed: which, it is agreed, is the firmament in the midst of the waters. For that there are waters placed beneath it we also see for ourselves in the air and on the lands; that there are waters placed above it we are taught not only by the authority of this scripture but also by the words of the Prophet, who says: Stretching out the heaven like a skin, who cover its upper parts with waters. The starry heaven, then, is agreed to be the Firmament in the midst of the waters. Nor does anything prevent its being believed to have been made of waters too. For we know what the firmness of crystalline stone is, and what its transparency and purity — and it is certain that it is produced from a gathering of waters. What prevents us from believing that the same disposer of natures solidified the substance of the waters into the firmament of heaven?* I have already said that there are some who take Iunilius and Bede for the same man, although Bede belongs to the 8th century. First, then, from the 6th century we shall bring forward Procopius and Olympiodorus.

[Margin: Procopius.] Thus Procopius of Gaza, in his commentaries on Genesis, on the Firmament made not in vain on the second day, says: *Nor would anyone say that its creation was superfluous. For while it received on its back, as it were, no small mass of waters, it came about that the waters left behind flowed together more readily and more quickly into one collection, and that the earth, the flood removed, emerged the more quickly, fit for habitation.* But this is remarkable: he teaches that this is the common opinion of Theologians — namely, that this heaven was made of congealed water — since he says: *There are those who maintain that its nature is little known and little ascertained by us; although, by the well-trodden way of all Theologians, it is handed down that it was compacted of water, and that it is called by the Hebrews Samaim, a word which denotes crystal or frozen water.* [Margin: Olympiodorus.] After him Olympiodorus, in his Catena on Genesis: *So, he says, I myself made the most tenuous nature of heaven solid, as if one had fitted a cube together with stone* — so, that is, that solidity signifies not only threefold dimension, as in a cube, but also hardness, as in a cube of stone.

[Margin: Strabus.] Let us pass now to the ninth century, in which Strabus, in the ordinary Gloss, expounding the work of the second day, has the scent of Iunilius's opinion: for after copying out his view almost word for word, and using the comparison with crystal, he then added of his own: *If anyone is troubled how waters, fluid by their nature and apt to slip downwards, can stand above the heaven, whose shape seems to be round, let him remember the one of whom it is written by Job:* [Margin: Job 20 [recte: Job 26].] *Who binds up the waters in his clouds. For he who below the heaven binds up waters, held back for a time in the vapours of the clouds, can suspend waters above the sphere of the heaven, so that they do not slip, not by vaporous thinness but by icy solidity — he who also hangs the mass of the earth upon nothing. But what kind they are, and of what use, he knows who founded them.*

[Margin: St Anselm.] St Anselm of Canterbury lived at the same time, and accordingly wrote things not unlike these, in *De imagine mundi*, book 1, chapter 25: *The upper heaven is called the Firmament, because it is a firmament in the midst of the waters: that is, spherical in form, watery in the nature of waters, adorned with stars on every side, solidified out of waters like ice, indeed like crystal; hence it is also called the Firmament.* With these words he certainly seems to describe the eighth sphere, or heaven of the Fixed stars; for this is the upper one, and adorned with stars on every side. The tenth century has been called the iron century, because it had scarcely one or two writers of any name. But at the end of the eleventh century Anselm of Laon said in the interlinear gloss: [Margin: Interlinear Gloss.] *The upper part of the waters was congealed in the manner of crystalline stone; the lower was reduced into matter.* [Margin: Hugo Victorinus.] Not long after him, Hugo Victorinus, *De Sacramentis* book 1, part 1, chapter 17: *The firmament was drawn out and made into such a form that, coming in between as a kind of middle with its circuit, it separated and divided from one another that great and immense heap of waters, and made of the waters waters and waters: that is,*

waters which it would embrace within its circuit and contain beneath itself, and waters which it would leave outside and above the round of its boundary.

[Margin: Comestor.] More plainly the Master of the Scholastic History, on Genesis chapter 4, namely Petrus Comestor: *So on that day God made the firmament in the midst of the waters, that is, a certain outer surface of the world out of frozen waters, consolidated like crystal and translucent, containing the rest within itself, after the likeness of the shell that is on an egg; and in it the stars are fixed. And it is called firmament, not however because of its own solidity, but because it is the boundary of the waters that are above it, firm and impassable. It is also called heaven [cælum] because it conceals [cælat], that is, covers, all sensible things.* Who does not see that in this passage only that outermost heaven is designated in which the fixed stars are, and which is the outermost shell and surface of the visible world? [Margin: Master.] Petrus Lombardus, on book 2, distinction 14: *Here is described the creation of that heaven in which the stars are fixed; beneath which are placed the waters in the air and on the earth, and others placed above, of which it is said, who cover its upper parts with waters. In the middle, then, is the firmament, that is, the starry heaven, which can be believed to have been made of waters. For crystalline stone, whose firmness and transparency are great, was made of waters.*

[Margin: Richardu[s?].] In very few words Richardus, book 2 of the *Excerptions*, chapter 7, indicated that the stars are fixed in the heaven like nails and revolve with it, and hence that the heaven is solid: *The Firmament is proved to be revolving from the stars fixed in it above.* [Margin: Hugo Car[?].] Hugo Carensis, in the *Postills*, held to the Master of the Scholastic History, as is clear from these words: *The Firmament, the surface of the sensible world, made of waters congealed into crystal, as they say, containing all other sensible things after the likeness of the shell of an egg; and it is called firmament because it is firm.* [Margin: Dionysius Carthus.] Dionysius Carthusianus does not depart much from this, commentary on Genesis, article 10, where he teaches that the Firmament was made on the second day not as to its substantial form but as to certain qualities, especially solidity; and he adds: *The Firmament, that is, the heaven of the stars, which is the eighth sphere, because beneath it are the seven spheres, or orbs of the heaven, of the planets.*

But although not all of the 27 authors adduced express a solidity of the starry Heaven — at least one that includes a hardness like that of ice or stone — but only that it was made of water, most of them nevertheless express it, or indicate an equivalent. Its Solidity, moreover, was taught expressly and absolutely by the more recent Theologians, [Margin: Martinengus. Molina. Delrius. Cornelius. Serarius. Salianus. Tannerus. Sà.] especially Martinengus in the *Glossa magna*, page 590, and, from our Society, Molina, *De opere sex dierum*, disputation 2, last chapter; Martinus Del Rio [Martín Antonio Delrío] and Cornelius à Lapide on Genesis, at the 2nd day; Serarius on the 2nd Epistle of St Peter; Salian in the *Annals of the Old Testament*, at the 2nd day; Tanner, volume 1 of the *Theology*, disputation 6, question 3, doubts 1 and 2; and Sà [Manuel de Sá] on that text of Apocalypse 4, *And in the sight of the throne, as it were a sea of glass like crystal*.

Quæſtio 2. An aliquod Cælum Sidereum ſecunda die factum aut relictum fuerit fluidum, vel ita ſolidum, vt duritiem tamen non habuerit nec habeat, ſed alia de cauſa firmum ac ſolidum dictum ſit ex Scripturis & Patribus: Vbi & de motu Planetarum in cælo.

Question 2. Whether some starry Heaven, made or left on the second day, was fluid, or so solid that it nonetheless neither had nor has hardness, but was called firm and solid for another reason — from the Scriptures and the Fathers. Where also on the motion of the Planets in the heaven.

IV. I suppose here, from Genesis 1 and from the Fathers to be cited in section 3, chapter 1, number 9, that the Firmament is that whole space in which the wandering and the non-wandering stars were placed by God, or in which they were produced.

[Margin: 2nd Conclusion.] Let this CONCLUSION likewise be affirmative. Although it cannot be shown positively from the sacred Scriptures, it will become manifest from not a few Fathers and Doctors — indeed from the scriptures as well, but negatively, in number 5. For some think it is called the Firmament because it is something corporeal, or a body having threefold dimension, and think this suffices for solidity; some because it is an impassable boundary of the waters; some because it is incorruptible; some, again, expressly affirm that it is penetrable by rains or by stars, and of itself immobile; some, finally, ascribe solidity not to the whole starry heaven but only to its highest part, or the eighth sphere — as will presently be made plain. [Margin: Philo.] Philo, book 1 *De mundi opificio*: *He aptly called the heaven firmament, as a corporeal thing. For a body is naturally firm and solid, having three dimensions. And what other notion can there be of a solid and of a body than dimension in every direction?* [Margin: Origen.] The same was the notion in Origen, homily 1 on Genesis: *After that he makes the Firmament, that is, the corporeal Heaven. For every body is without doubt firm and solid; and this is what divides between the water that is above the heaven and the water that is below the heaven.* [Margin: St Basil.] In almost the same sense St Basil, homily 3 on the Hexaemeron, understood the Firmament — namely as a body having threefold dimension, but fit to resist, yet without hardness. For when he had said that on the second day a Heaven different from the first and of a more Solid nature was made, he explains himself more fully in this way: *Outsiders call that body firm which is, as it were, solid, dense and full, to distinguish it from the Mathematical body, which consists in dimensions alone — in breadth, I mean, length and depth. But firm and solid is that which, with the di- mensions, can push back and resist. Sacred Scripture, however, is accustomed to call firmament whatever is very strong and does not yield, so much so that it often uses this very word even of condensed air, as when it says: Who makes the thunder firm. For Scripture has called the firmness of the thunder that solidity and resistance of the wind shut up in the hollow bosom of a cloud which, bursting out violently,*

produces the sound we call thunder. In this place too, therefore, we judge that this word was established by Scripture for a firm and solid nature, fit to hold back and restrain the waters, which by their own nature slip away. Then, rejecting the hardness of ice or crystal, he adds: *Yet the firmament, although according to the common and widespread opinion it seems to have arisen from water, is not to be thought similar either to water congealed into ice, or to any such matter made from water as the nature of crystalline stone is agreed to be, or as is the nature of the specular stone, which grows among the metals, etc. For it is surely the mark of a childish and simple mind to hold such opinions about the celestial bodies.* And again, expounding the name Firmament: *Plainly, he intimates a nature in no way resisting; he does not say that the firmament is of the kind that is solid, heavy with its own weight, or resistant, etc.; but Moses called it firmament by comparison with the most tenuous bodies, and because it may easily be grasped by the senses.* In homily 1 on the Hexaemeron, however, he had taught that the nature of the heaven is declared by Isaiah to be tenuous and not at all solid, in these words: *The heaven was made firm like smoke, etc.* — words I shall report in chapter 7, number 13.

[Margin: St Nyssen.] With him his brother St Gregory of Nyssa is in remarkable agreement, in *De historia sex dierum*: *I for my part judge that no body dense, thick, solid, resisting and hard, etc., is to be considered in the firmament, but the extremity of sensible nature; and that it is called firmament by Scripture by comparison with the everlasting, incorporeal property that escapes touch. For who does not know that whatever is solid and firm has been thoroughly condensed and consolidated by a certain hardness? But what is dense and hard is not empty and free of the quality of weight and heaviness; and what is heavy cannot naturally be carried to the regions above.* [Margin: St Ambrose.] St Ambrose subscribed to the same St Basil, book 2 of the *Hexaemeron*, chapter 2, but has these points of his own: *Firm is everything that God has established*; and further on he asks: *How was the firmament solidified among the waters? They flow forth, it is bound fast; they run, it remains*; and he answers that this was done by divine power and might. And in chapter 4: *From firmness, then, the firmament was named, or because it was made firm by divine might, as Scripture teaches us, saying: Praise him in the firmament of his might.* But in chapter 6, speaking of the heaven which Isaiah compares to smoke, he says: *wishing to declare its nature subtle and not solid.* [Margin: St Augustine.] And St Augustine too, book 2 of *De Genesi ad litteram*, says: *For one may understand the firmament to be so called on account of its firmness, or on account of the impassable boundary between the upper and the lower waters.* These very words were copied out by Eucherius, book 1 on Genesis; by Bede, in the book on the creation of the six days; by the Gloss, both the ordinary and the interlinear, that is, Strabus and Anselm; by Hugo Carensis in the *Postills*; and by the Master, both of the *Sentences*, on book 2, distinction 14, and of the *Scholastic History*, chapter 4 — granted that these authors do not deny any heaven all solidity.

[Margin: Albertus Magnus.] In another way, namely as incorruptibility, Albertus Magnus interpreted this firmness, part 1 of the *Summa de quatuor coaevis*,

question 4, article 19: *The firmament is so called from the firmness of its nature, because it is neither generated nor corrupted.* And he takes the harmony of the heaven for a tempering of qualities that are not at all in conflict, and adds: *For this harmony prevents the harmfulness of each one by itself; and on this account it is called firm, because that harmony is indivisible, since it is drawn out beyond the action and passion of contraries.* [Margin: Tostatus.] Likewise Tostatus [Alonso Tostado], in his Commentaries, or notes, on Genesis: *That whole heaven is named from firmness, because the celestial substance is firm and lasting.* And although he adduces that saying of Elihu in Job, *Which are most solid, as if cast of bronze*, he nevertheless adds: *But in Hebrew that whole celestial body is called Raquin, which signifies expansion, not firmness, etc.; and it is called the expanse because it is spread out over the whole world, covering all things.*

[Margin: St Damascene.] But I return to the more ancient authors. Among them St Damascene, book 2 *On the Faith*, chapter 6, makes no mention of solidity; for he had said: *Then God called heaven the firmament, which he commanded to be made in the midst of the waters, bidding it divide the half of the water that is above the firmament from the half that is below the firmament. Its nature St Basil, taught by divine scripture, said is tenuous, like smoke; others, however, say it is watery, because it was produced in the midst of the waters; others, that it is of the four elements; others, a fifth body different from them.* [Margin: St Cyril of Jerusalem.] Before him, St Cyril of Jerusalem, Catechesis 9, assumes the heaven to be pervious and penetrable to waters, saying: *He set the waters above the heavens, so that when the earth needed the watering of rains, this heaven might be ready by nature for it.*

I admit, however, that, St Basil and St Gregory of Nyssa excepted, the following testimonies are clearer for the fluidity of the heaven, resting as they do on the immobility of the whole heaven, at least of the planetary one, and on the motion of the stars within it. [Margin: St Justin.] St Justin, question 39 of the *Questions to the Orthodox*: *How is it that those who say this speak truly, when the stars are carried not in the heaven but below the heaven? If we say they are in the heaven, how do they have the power and action of moving, when the celestial body is immobile?* [Margin: Eusebius of Emesa.] In what sense he is to be understood we shall learn from Eusebius of Emesa, in Lippomanus, in the Catena on chapter 1 of Genesis and the work of the fourth day: *It is asked whether the luminaries and stars complete their course at a distance below the heaven, or are fixed in the heaven, or make their journey in the heaven like travellers.* And he answers that they are in the heaven, but not fixed, because they themselves move while the heaven is unmoved; and he concludes: *Sacred and divine scripture therefore testifies that the heaven is firm and immobile, but that the Sun and Moon move in the heaven and complete their course. For in Joshua it is said: Sun and Moon, stand still; it does not say: Heaven that moves the Moon and the Sun, stand still. Likewise to Hezekiah it is said: Let the Sun go back; it does not say: Let the heaven that moves the Sun go back.*

[Margin: St Chrysostom.] I find that St John Chrysostom was plainly of the same opinion, homily 6 on Genesis: *He set the stars in the firmament of heaven. What does 'set' mean? Is it as if one were to say 'fixed'? Far from it; for we see*

them cross a great space in a single moment of time, and never stand in one place, but complete their own course, which the Lord prescribes for them. And homily 14 on the Epistle to the Hebrews: *Which God fixed, and not man: where are those who say that the heaven moves? etc.* And homily 12 to the people of Antioch: *The heaven indeed has remained immobile, as the Prophet says, who set the heaven like a vault, and stretched it out like a tent over the earth; but the Sun, with the other stars, revolves daily.* [Margin: Philastrius.] Philastrius, however, presses the same opinion in weightier words, in the *Catalogue of Heresies*, heresy 46: *There is a heresy which has the stars fixed in the heaven, and does not acknowledge that, from the treasury and the hidden places of light and disposed by God, they come forth suddenly towards evening at the divine command, and at the appointed hours fulfil with their light the course and ministry enjoined on them. This is found contrary and alien to the Catholic faith, if anyone should think so, and it will be shown to have fellowship with pagan vanity and the empty wisdom of the Philosophers rather than with Christian knowledge.*

[Margin: Procopius.] Let Procopius add his vote to these, on chapter 2 of Genesis, where, to the question put forward — *whether the luminaries revolve fixed in the heaven, or complete their revolutions while the heaven stays stable and immobile* — he gives a satisfactory answer: *As it seems to me, the luminaries proceed through the space of the heaven, extending all their light to us. For not in vain does Scripture have, God set them in the firmament; nor is the word 'set' to be understood, as some will have it, for 'fixed'. Therefore also Jesus son of Nave cries out: Sun, stand still; and he did not say: Heaven, stand still, which moves it.* [Margin: Diodorus.] The mind of Diodorus of Tarsus, on Genesis, was without doubt the same; Lippomanus also reports it in the Catena: *Let no one, he says, when he hears 'and he set them in the firmament of heaven', suppose that the Sun, Moon and Stars are fixed in the heaven and the heaven moves, bringing a gentile dogma into the Church.* And he adduces the example of Adam, whom God set in paradise but did not fix; and he goes on: *Therefore it must be understood that, just as men are set on the earth, so the luminaries are set in the heaven, completing that celestial journey.*

[Margin: St Augustine. Isidore.] To these must be joined those who thought that the non-wandering stars are indeed fixed in the firmament, but that the Planets move in it, or in the aether contained beneath it, by themselves, and not with the motion of the orb. St Augustine seems to approve this, book 2 of *De Genesi ad litteram*, chapter 10, and Isidore, book 3 of the *Etymologies*, chapter 49: *They say that the Sun moves by itself. For if it remained fixed to the heaven, all days and nights would be equal; but since we see that it will set tomorrow in one place and set yesterday in another, it is apparent that it moves by itself, and does not turn with the world.* Nor indeed, if anything was mistaken by him or by others in astronomical subtlety, does it harm us; for it is enough if we prove what we intend from the authority of the Fathers — namely, that the Planets do not move with the motion of the or- b; [Margin: Richardus Victorinus.] and so does Richardus Victorinus, book 2 of the *Excerptions*, chapter 7: *On the fourth day he made the luminaries, which are shown to be fixed in the firmament, except for*

the seven Planets, whose names are these: Moon, Venus, Mercury, Sun, Mars, Jupiter, Saturn. By the name of luminaries he means the lights of the stars. [Margin: St Anselm.] But St Anselm of Canterbury, book 1 of *De imagine mundi*, chapter 24, speaks admirably to our purpose: [Margin: Fire, or aether, as the heaven of the Planets.] *Fire, which is enrolled as the fourth element, is so called as if "not begetting"; it extends from the Moon up to the firmament, and is as much subtler than air as air is thinner than water and water rarer than earth. This is also called aether, as if pure air; it rejoices in perpetual splendour. From it the Angels take bodies when they come, sent to men. In it seven stars are carried, each in its own circle, against the world, and on account of their wandering course they are called Planets, that is, wanderers. These are swept along by the immense speed of the firmament from East to West, yet are shown to go by their natural course against the world, like a fly on a millwheel.* Yet the same author, in chapter 25, as we reported in number 3, teaches that the firmament in which the non-wandering stars are is solid, like crystal. [Margin: Iunilius.] Iunilius had taught the very same thing long before, in his *Hexaemeron*: [Margin: Aether for heaven.] *It must be understood that the birds fly along the firmament of heaven because the aether too is indicated by this name: that is, that upper space of air which reaches from this turbulent and murky place, in which the birds fly, up to the stars, and which is believed, not without reason, to be wholly tranquil and full of light. For the seven wandering stars, which are said to be carried wandering in this space of aether, scripture has said are set in the firmament of heaven.*

[Margin: Dionysius Carthusianus.] Dionysius Carthusianus likewise reports another opinion and does not disapprove of it, in his exposition on Genesis, article 12: *Here the firmament is taken for the starry heaven, or Eighth sphere*; and presently: [Margin: Plato. Ptolemy.] *Plato in the Timaeus says that the stars are not fixed in the firmament but travel and lead a dance in it; and in this Ptolemy followed Plato. Hence they say that it is not the heaven but the stars of the heaven that move and are carried around.* [Margin: Rupertus.] Finally Rupertus, book 1 on Genesis, chapter 23: *Nevertheless we do not go so far in agreeing that there are waters above the firmament as to grant that they are also congealed.*

[Margin: 2nd argument: from the fact that the motion of the Planets is never granted to the heaven.] V. The second argument from the scriptures is taken from negative authority, which has great force in this place: on the contrary, motion is attributed to the Sun and to the Moon, whereas the motion of any of the Planets is never read in sacred scripture as ascribed to the heaven. For whenever there is talk of the rising or setting of the Sun, it is always attributed to the Sun, as in Genesis 19: *The Sun was risen upon the earth*, and chapter 32: *And immediately the Sun rose upon him.* So too the Sun itself is said to go down or set: Leviticus 32, Judges 19, 2 Paralipomenon 18, Tobit 2, and Joshua 10: *Sun, do not move against Gabaon, and you, Moon, against the valley of Aialon; and the Sun and the Moon stood still, etc.* Of this same miracle, 1 Paralipomenon 4: *Who made the Sun stand still*; and Ecclesiasticus 48: *In his days the Sun went back.* Of that other miracle, worked on the dial of Achaz, book 4 of Kings, chapter 20,

and Isaiah 38: *And the Sun returned ten lines by the degrees by which it had gone down.* So in Job 9 we have of God: *Who commands the Sun, and it does not rise.* And Psalm 103: *The Sun is risen, and they are gathered together.* And Ecclesiastes 1: *The Sun rises and sets, and returns to its place, and there, rising again, circles through the South and bends towards the North; surveying all things round about, the wind goes on, and returns to its circles*; and Matthew 5: *Who makes his Sun rise upon the good and the bad.* Search diligently, then, and turn over the sacred pages: you will never find motion attributed to the heaven, such that the Sun or Moon might be understood to move with its motion, but only to the Sun and the Moon.

[Margin: Mutual penetrability of the celestial bodies.] Either, therefore, the celestial globes penetrate one another together with the heaven, and do not have the impenetrability that the other bodies below the Moon have — which some have conceded, as reported, and not refuted, by Albertus Magnus, part 1, question 4, article 19, and by St Bonaventure on book 2, distinction 14, part 2, article 1, question 2: *As light passes through air*, says Albertus; or because *A body of light can be together with another light*, as St Bonaventure says; and which others admit, who are to be reported in chapter 7, number 6. Or else it must certainly be said that, the heaven being for the most part unmoved, the Planets move through it as birds move through the air, the part of the heaven through which they travel giving way; and that part at least must be admitted to be fluid. Or it must be said that sacred scripture expresses the motion of the Luminaries as being the one that is more manifest and strikes the eyes, as Martinengus, Pererius and Amicus say, who are to be adduced below, in chapter 7, number 13.

Quæſtio 3. An & quæ Aquæ ſint ſupra cælos ſidereos, & in quem finem vel vſum & quantæ molis ?

Question 3. Whether there are Waters above the starry heavens, and of what kind; for what end or use; and of how great a mass.

VI. Not a few have denied these waters, among them Clavius in the *Sphere*, page 45, Arriaga in the single disputation on the heaven, section 4, and all those who are to be reported below, in numbers 11 and 12. Let the CONCLUSION nonetheless be this: [Margin: 3rd Conclusion.] *Some Waters, once of the same species as the elemental waters, have been above some starry heavens from the second Day of the founding of the World down to these times.*

It is proved, first, from sacred Scripture. In Genesis 1 the Abyss is supposed to have been made on the first day, that is, a depth and great mass of waters, as is the common consensus of the interpreters, with one or another dissenting — whether those waters were then mixed with the other elements or not. Then, of the work of the second day, it is said: *Let there be a Firmament in the midst of the waters, and let it divide the waters from the waters. And God made the Firmament, and divided the waters that were under the firmament from those that were above the*

Firmament. And God called the firmament heaven. Here, in the strictness of the Hebrew language, the word *were* is not present, at least in the second place; it was added in the second place by a certain consequence or correspondence, since it could have been rendered: *those under the Firmament from those above the Firmament*: *those that were under the Firmament, from those above the Firmament*, with *were to be*, or *were*, or *are* understood. For before the Firmament was made there were no waters above the Firmament. These things premised, it is sufficiently clear that there are some waters above the firmament, and that they are of those which were joined and continuous with the waters that remained below the Firmament. Then in Psalm 148 it is said: *Praise him, heavens of heavens, and let all the waters that are above the heavens praise the name of the Lord.* And earlier, in Psalm 103: *Stretching out the heaven like a skin, who cover its upper parts with waters.* And again in the Canticle of the three youths: *Bless the Lord, all you waters that are above the heavens.* Hence those lines in the hymn for Vespers of Monday:

Immense caeli Conditor, Qui mixta ne confunderent Aquae fluenta dividens Caelum dedisti limitem, Firmans locum caelestibus, Simulque terrae rivulis; Ut unda flammas temperet, Terrae solum ne dissipent.

("Immense Founder of the heaven, who, dividing the flowing waters lest, mingled, they should throw all into confusion, gave the heaven as their boundary, making firm a place for the celestial waters and likewise for the streams of the earth, so that the wave may temper the flames, lest they waste the soil of the earth.")

[Margin: 2nd proof from the Fathers.] VII. It is proved, secondly, by the Authority of the Fathers, who are so many that Delrío says in his notes on Genesis that they have the force, as it were, of a single Council. And certainly in the sixth Synod of Constantinople the letter of Sophronius was read in action 11 and approved in action 13; in it, among the other errors of Origen, this one was reported: [Greek: ὑπερουρανίων ὑδάτων κακίζοντες (hyperouraniōn hydatōn kakizontes, "disparaging the supercelestial waters")], namely, rejecting the creation of the supercelestial waters. Moreover, we have already brought forward many of the Fathers, in number 3: namely Tertullian, Hilary, Hippolytus, Caesarius, Severianus, Cyril, Chrysostom, Claudius Marius, Procopius, Anselm, Theodoret, Iunilius, Strabus, Hugo, Comestor, and finally all who think that the heavens were made of water and made firm or solidified. [Margin: St Augustine.] But besides the passages already adduced, St Augustine, book 2 of *De Genesi ad litteram*, chapter 1, after saying: *Many assert that the nature of these waters cannot be above the starry heaven, because they have their weight so ordered that they either flow upon the earth or are borne as vapour in the air next to the lands*, then concludes, in chapter 5: *But in whatever way, and of whatever kind, the waters there may be, let us in no way doubt that they are there; for the authority of this scripture is greater than the whole capacity of human wit.* [Margin: Bede. Eucherius.] Eucherius and Bede on Genesis bring forward and approve this very opinion in as many words. But again St Augustine, book 11 of *The*

City of God, chapter 34, teaches that this is no more to be wondered at than if someone should say that the seat of Phlegm, or pituita, is in the head of man: if we did not know this, and scripture said it, it would not be believed by certain Philosophers who are moved by the weights of the elements, and whom he there calls *weighers of the elements*. And expounding Psalm 103, on the words *who cover its upper parts with waters*, he says: *And this is read, and according to the let- ter, is understood: for when he commanded that a firmament should be made between waters and waters, it came about that there are lower waters, which flood the lands, and upper waters, removed from sight yet commended to faith.* [Margin: St Prosper.] St Prosper copied out and adopted this opinion, on Psalm 103: *Therefore, although the eye of the body does not discern them, the eye of faith can reach them.* [Margin: St Clement.] Indeed St Peter, as Clement has it in book 1 of the *Recognitions*, after saying that part of the waters was solidified into heaven, added: *After this, what was left of the waters in the lower regions the place of the sea received, by the command of the eternal will.* He therefore supposes that those waters are of the same nature as the other waters. [Margin: St Hilary.] So too does St Hilary, on Psalm 135, when he says: *For by the waters raised up he tempered that very nature of the first heaven.*

[Margin: St Basil.] And St Basil too, homily 3 on the Hexaemeron, after teaching that an abundance of the upper waters was necessary for tempering fire, and after reporting the fictions of Apollinaris and Origen, who understood by the waters above the heavens either Angels or something of that kind, says: *Setting aside, then, these opinions and those of that kind, as being like dreams and old wives' tales, let us understand water as water*: that is, according to the letter, with this name taken in its common sense. In the same place, to those asking how fluid waters can stand upon the spherical surface of the heaven, he had answered that, although the concave surface of the heaven is, or seems, round, the convex surface is not necessarily round, but can have the shape of a vessel containing waters. [Margin: St Ambrose.] St Ambrose took this very point from Basil, book 2 of the *Hexaemeron*, chapter 3, against those who, as he himself puts it, *want to destroy what has grown habitual in our minds and been impressed on them by frequent reading of the scriptures, because waters cannot be above the heavens, saying that that orb of heaven is round, etc.* And a little after, having dismissed them, he adds: *But we follow the sequence and order of the scriptures, and contemplate the work by the judgement of its author, etc. I hear that the firmament is made by a precept by which the water should be divided and the upper separated from the lower. What is clearer than this? He who commanded the water to be separated by the Firmament set between them in the middle provided for how, divided and separated, it could remain so.* Then, pouring himself out in praise of the divine omnipotence, he concludes: *When you hear these things, why do you wonder if, above the firmament of heaven, water could be suspended by the working of so great a majesty?* He confirms this from the waters of the Red Sea and of the Jordan, poised on either side in heaps and held suspended until the people of God passed over dry-shod; from the waters bound in the clouds; and from the swelling of the sea, which does not

flood the land. And in chapter 4, after adducing the opinion of Origen, who interpreted the name of Waters as an intellectual, or Angelic, nature, because those waters are invited to praise God, he added: *To us, however, it seems neither alien nor absurd if we understand true waters, for the reason we have given. For dew and frost, and cold and heat, bless the Lord according to the prophetic hymn, and we do not refer these to intelligible natures, but to reality.*

[Margin: St Epiphanius.] Epiphanius recounts the same error of Origen in his letter to John, Bishop of Jerusalem, and refutes it, saying: *And how do we read that in the flood the cataracts of heaven were opened, and the waters of the flood overflowed?* [Margin: St Chrysostom.] We heard St John Chrysostom above, in number 3, but we must hear him again, homily on Psalm 148, on the words *And all the waters that are above the heavens*: *You have heard Moses too saying that, of the waters, he left these below, but made those stand on high upon the celestial ridges, when he had established the firmament in the midst of the abyss.* [Margin: Theodoret.] To him I join Theodoret, question 11 on Genesis, who speaks thus: *God divided the nature of all the waters in two: some he placed above, which by their moisture and coldness would not allow the firmament to be corrupted by the fire of the luminaries; the remaining waters stayed below, to nourish with their vapours the air, scorched and dried by the fire above.* [Margin: St Damascene.] St Damascene, book 2 *On the Faith*, chapter 6, should also be read; he teaches expressly that these waters are above the firmament. [Margin: And Chlichtovaeus.] So does Iudocus Chlichtovaeus [Josse Clichtove], his commentator, who, although he takes them for the ninth heaven, nevertheless admits that Damascene thought otherwise, when he adds: *Or there really are thin and transparent waters, standing above the firmament that is adorned with so great a multitude of stars, by the precept of the founder, and for a reason which the founder himself knows, though it is hidden from us — as the sacred writings seem to hold in very many places, and Damascene here.* [Margin: Procopius.] Moreover, having reviewed the opinions of many, Procopius, chapter 1 on Genesis, concludes: *They say that the Firmament is called heaven either because it is concreted and compacted from the most simple nature of water, or so that it may distinguish and separate from the lower ones a most tenuous and most light nature, comprehensible by no sense. In saying "in the midst of the waters", he teaches clearly enough that the division of the waters was equal on each side.*

[Margin: Bede.] We named Bede above as subscribing to St Augustine; now let us adduce him again, from the book *De natura rerum*, chapter 8, differing thus: *The waters set above the heaven are lower indeed than the spiritual heavens, that is, the Empyrean and the Angels, but nevertheless higher than every corporeal creature. Some affirm that they are kept for the flooding of the deluge; others, more rightly, that they are suspended to temper the fire of the stars.* And in the *Questions on Genesis*: *Let the heretics at last cease their madness and, confounded, acknowledge that he who could create all things out of nothing could also establish that nature of the waters in the heaven with an icy solidity. For since they themselves say that the orb glittering with burning stars revolves, did not divine providence necessarily see*

to it that water should overflow within the orb of heaven and above the orb, to keep apart those blazing fires of the axis? [Margin: Cassiodorus.] Let Cassiodorus follow him, although he is older, on Psalms 103 and 148, where, supposing these waters, he adds the cause: *So that the tenuity of the heavens may stand firm, and the waters placed above the firmament, though they are heavier, may not flow down except when they are commanded.* [Margin: Haymo. St Bruno.] But most acutely Haymo and St Bruno, on the same two psalms, have those waters praise God — that is, rouse us to his praise — for this very reason: that, though they are true waters, they are nevertheless held there, even if their use is hidden from us. [Margin: Remigius.] Most simply, however, Remigius of Auxerre, on Psalm 148: *That there are waters above the firmament is clear from Genesis, where it is said: he divided the waters, etc.*

[Margin: Rupertus.] Now let Rupertus the abbot come forward for us, book 1 on Genesis, chapter 23: *Others of the Doctors want nothing else to be understood by the name of the waters that are above the firmament than waters, simply. Their opinion is without doubt the more just and the safer, namely, that the sense should not disagree with the letter of the sacred history.* Yet he denies that they were congealed; and in chapter 24 he teaches that the firmament is like the roof of the vault of heaven; and in chapter 25: *And it is indeed a wonderful work of the magnificent God that the subtle nature of the firmament, spread beneath, sustains the mass of the waters.* He confirms this by the example of the air and the water that sustain the mass of the earth on every side; and in chapter 31: *The waters that are below the firmament are surely remnants, or a part, of the waters that are above the firmament.* [Margin: St Anselm.] St Anselm of Canterbury, book 1 of *De imagine mundi*, chapter 28, thought them attenuated after the manner of a cloud, saying: *Above the firmament are waters suspended like a cloud, which are said to encircle the heaven round about; hence it is also called the heaven of waters; above it is the spiritual heaven, etc., and the Paradise of paradises.* In chapter 25, however, he set the firmament itself as solidified like crystal, as I reported in number 3. [Margin: Hugo Victorinus.] But Hugo Victorinus chose half of this opinion, in his annotations on Genesis, chapter 6: *That there are waters above the firmament is held both in Genesis and in the Prophet, etc. But what those waters are like is not certain to us. Some, however, say that they are solidified there in the manner of ice; but it has seemed more likely to me that they stand there suspended as vapour, after the likeness of a vapour, that is, of smoke or cloud.*

[Margin: Comestor.] Petrus Comestor, in the *Scholastic History*, chapter 4, plainly holds the contrary, but under a disjunction: namely, that both the Firmament and the waters above it congealed in the manner of ice. For after asserting: *Truly the firmament divides the waters that are below it from the waters that are above it*, he added: *and they, like it, are congealed like crystal, so that they cannot be melted by fire; or else they are vaporous in the manner of a cloud.* [Margin: Hugo Carensis.] Hugo Carensis used almost the same words in the *Postills*, saying of the waters above the firmament: *And these waters are congealed after the manner of crystal, so that some call them the crystalline heavens; or they are vaporous after the*

manner of a cloud. [Margin: Oleaster.] Prudently, Lusitanus, or Hieronymus ab Oleastro, on chapter 1 of Genesis, on these waters: *Since scripture everywhere cries out that they are there, many Theologians prefer to follow the blind Philosophers rather than the authority of scripture, etc.* And having adduced other authorities of scripture, he concludes: *I would rather, then, err with scripture (if it could err) than speak the truth with the Philosophers.* And presently: *I think, however, that they are elemental and of the same nature as the lower ones; for before, as is clear from the text, they were continuous with the lower ones; and once the firmament, or the division, had been made, they do not seem to have changed their nature.* It is my pleasure on this occasion, against those who neither the Divine scriptures nor the interpretations of the Fathers hold in reverence, to report a judgement full of gravity from Bishop Vielmius, lecture 14 on Genesis. [Margin: Vielmius.] There, after setting out the objections against waters of this kind — as though they were above the heavens both to no purpose and by violence — he says: *Rightly indeed, etc., were it not that the word of God stood in the way, especially as interpreted by nearly all the ancient saints, who understood from these words that waters of one kinship and nature with ours are placed and exist above the heavens.* And after bringing forward the words of Genesis he adds: *But we Christians are at this time in such a calamity that many of our own (if indeed they deserve to be called our own) grow used, from the first flower of youth, not to stand in awe of holy antiquity.* Moreover, Claudius Marius Victor, whom we have already cited for a solid heaven in number 3, judged that the quantity of those waters is great; for he sings:

And that a great sea is borne above the stars and constellations, prove it by the reasoning of those very men who refuse to believe it, and want the World to hang.

For brevity's sake I now pass over many more recent writers on Psalm 103, at the verse *You who cover its upper parts with waters*, and on Psalm 148, at the verse *And all the waters that are above the heavens*. [Margin: Authors who place in the heaven waters different from the elemental ones.] Granted that St Gregory of Nyssa, in the *Hexaemeron*, said that these waters are of a different nature from the elemental ones, lighter than fire and devoid of all sensible qualities; and granted that many who assert the incorruptibility of the heaven appear to recognise that difference from the elemental waters — and with them Alensis [Alexander of Hales], Albertus, Scotus, Durandus, Bonaventure, Aegidius, Catherinus, Petrus de Tarantasia, and St Thomas in the *Disputed Questions*, q. 4 ad 5, on book 2 [of the *Sentences*], d. 14 q. 1, and in the First Part, q. 68 art. 3 ad 1. For he teaches that these waters resemble crystal in their solidity and water in their transparency, but are of a different species. With them our Tanner agrees, tome 1 of his *Theology*, disputation 6, q. 3, doubt 1, where in assertion 10 he concludes that by the name of the waters above the heavens is most probably to be understood the Empyrean, or a simple celestial body, transparent, liquid and breathable, destined for the dwelling of the Blessed. Gulielmus Parisiensis held the same, part 1 of *De Universo*, chapter 39. But surely he would rather have called it air than water, since air is better suited to breathing.

VIII. From the authors adduced, then, one may gather four opinions on the condition of these waters. [Margin: 1st opinion, on the quality of these Waters.] The first view — later in time, however, than many — was that they differ in substance from the elemental waters. We adduced its authors a little earlier, and it is indeed hardly consistent with the literal sense of Sacred Scripture, or with most of the Fathers already cited in numbers 3 and 7. [Margin: 2nd opinion, of waters thinned into vapours, and its authors.] The second holds that those waters are indeed of the same kind in substance, but rarefied and spread above the Firmament after the manner of a vapour or cloud — much as we see, from high mountains, a misty air besieging cities and towns, or thin vapours and smokes lying over valleys and standing waters, or the whole surface of the horizon covered with dewy mists. As a result of this lightness, even if they had no solid firmament beneath them, they could nevertheless remain suspended there, in the same way as vapours and mists above our air. So thought St Augustine, *De Genesi ad litteram* book 2, chapter 4 — especially on the supposition of a rarefaction possible finitely to infinity, and of the view of those who assert: *All things are divided infinitely, since every part of a body is a body, and every body must have half of its quantity. And therefore, if water can, as we have seen, reach droplets so minute that it is borne as vapour above this air, which is by nature lighter than waters, why could it not also remain above that lighter heaven in more minute droplets and lighter vapours?* Subscribing to him were Eucherius, book 1 on Genesis, chapter 3, who says: *For just as these clouds, which are everywhere, are borne above the air, which is the third element, so I think it can happen that thin and subtle waters are borne above the heaven*; the Master, on book 2, d. 14, using St Augustine's very words; and also St Anselm, both Hugos, and Comestor, as is clear from their words cited a little before, toward the end of number 7. To these add Anselm of Laon in the interlinear gloss, who distinguishes the upper waters from the lower by this mark: *In the midst of the waters suspended as vapour, and of those gliding over the earth* — if he is to be understood of vapours above the starry heaven. St Thomas, however, in the *Disputed Questions*, q. 4 art. 1 ad 5, refutes this opinion on the ground that water cannot exceed the degree of rarity proper to water: *Nor could it*, he says, *naturally go beyond the spaces of fire and air, unless it had lost the nature of water and surpassed their rarity.* And certainly clouds and vapours, however thin, have exhalations and spirits mixed into them, and with these they make up a body lighter than air taken in a bulk equal to their own magnitude; and that is why they fly above the air. But St Thomas spoke of what happens naturally. For God could have spread out the waters, thinning them so that they were rarer than the ethereal air itself, and yet have willed not to withdraw his conserving concurrence from the original substantial form of water — and indeed from its coldness and moisture as well. Of the Doctors already mentioned, however, St Anselm, Comestor, Hugo Carensis and the Master admitted a Firmament solid like ice or a vault, as is clear from what was said in number 3.

[Margin: 3rd opinion, of liquid waters above a solid Firmament, and its authors.] The third opinion was that these waters are liquid and flowing, in the way the other elemental waters flow, and that they do not slip down because they are held up by the back of the firmament. Of this opinion were Tertullian and Saints Basil, Ambrose, Hilary, Caesarius, Cyril, Chrysostom, Hippolytus of Portus and Anselm; likewise Clement, Theodoret, Iunilius, Procopius and Strabus. Nor did St Augustine, Hugo Victorinus and Comestor reject it, as will be plain to anyone who rereads their words quoted by us from number 3 to number 8. Of these, some think those waters do not flow away because they are enclosed in vessels and outermost hollows of the highest heaven; others attribute this to the power of divine Omnipotence. But it could be added that, for this, it suffices or contributes both that they are at an equal distance from the centre of the World, and that there is a rapidity — either of the Firmament, lest they be carried round by it, or their own, if that watery heaven is taken as the primum mobile. For we see that water in a vessel spun very fast up and down does not flow out. Or else they do not gravitate there, because they are outside the sphere of the elements, as Oviedus says in his *Physics*, page 466.

[Margin: 4th opinion, of waters merely congealed, and its authors.] The fourth opinion seems to have been that of those who judged both the waters above the heavens and the Firmament itself to be solid after the manner of crystal or ice, as though admitting twin crystalline heavens; or who certainly recognised no waters other than the Firmament itself, consolidated like ice, or at least its upper part. In that way both things are true: that through it the waters are divided from the waters, and that they do not fall down because they have congealed into that hardness. So, it seems, thought Josephus the Hebrew, Pope Clement and Severianus, adduced in number 3. Strabus admitted it in the ordinary Gloss, in these words: *He was able to suspend the waters not by bodily thinness but by icy solidity*; and Anselm of Laon in the interlinear gloss, saying: *The upper part of the waters is congealed like crystalline stone; the lower is gathered into a womb.* And Bede, in the book *De natura rerum*, chapter 7: *We think, however, that the nature of the waters there now hangs not by vaporous thinness but by icy solidity, in order to temper the natures of the stars.* But no one, so far as I know, set out two crystalline heavens more distinctly than Comestor, *Scholastic History* chapter 4, when he said of the firmament and the waters above it: *And they, like it, are congealed like crystal* — granted that he did not deny that they could be thinned out like a cloud. Into this class can be placed those who, as Hamerus reports in his *Commentary on Genesis*, understand by the upper waters the crystalline heaven, so called for its hardness and transparency; by the lower waters, the heaven of Saturn; and by the firmament, the Aplanes [the sphere of the fixed stars]. Our Clavius too, on chapter 1 of the *Sphere*, formerly page 45, says that no one of sound judgement understands that there are flowing and falling waters above the heavens, like these lower ones; rather, the ninth and tenth heavens are to be understood, which are called waters for their transparency and uniform rarity, and by others crystalline or glacial. So

too Cajetan, on 2 Corinthians chapter 12, says that the watery heaven is the primum mobile, which he calls the third heaven; and Io. Antonius Delphinus [Giovanni Antonio Delfino], *De caelestibus globis* page 62, that those waters are the ninth, crystalline heaven, or primum mobile.

IX. [Margin: The quantity of the Waters above the heavens.] But you will ask how great that quantity of waters above the heaven is. I answer that this is most uncertain. Some, however, moved by the phrase *let there be a firmament in the midst of the waters*, judged that half of the whole abyss — that is, of the water created on the first day — was left on the earth, and the remaining half passed into the heaven by expansion or solidification. So hold Severianus, Damascene and Procopius, already adduced from number 3 to number 7, and Theophilus, to be reported in number 10. But

St Hippolytus thinks that a third of them was left on the earth, a third driven together into the firmament, and a third placed as waters above the heavens.

X. [Margin: The end and use of these waters.] The end for which the waters already mentioned were carried up into the heaven is said by some to be unknown, and known to God alone. Others think they are reserved there for the dew that drips upon the earth, as Comestor reports from the opinion of others, *Scholastic History* chapter 4, saying: *Why they are there, God knows; except that some hold that dew comes down from there in summer.* Others think they are reserved for rains, especially at the time of the flood, and that these are the Cataracts of heaven which are said to have been opened then, Genesis 7, and of which it is said in Malachi 3: *If I will not open to you the cataracts of heaven.* So the Master, on Psalm 103, says of these waters: *They flow down through rains only when commanded; as is said, they are sent down through rain when the cataracts of heaven are opened.* The same is held by St Epiphanius in the letter to John, Bishop of Jerusalem; by Cyril of Jerusalem, Catechesis 9; by Cassiodorus, Richardus and Haymo on Psalm 148; by Lusitanus on chapter 7 of Genesis; before these by Josephus, *Antiquities* book 1, chapter 1; and by Theophilus of Antioch, book 1 *To Autolycus*, who says: *The second heaven, this visible one, has been called the firmament; it carried up half of the water, so that for the use of men it might pour out rain, showers and dew.* Some hold that the waters were placed there to make the heaven firm, unshaken by the storms of the winds — for St Justin too counts this among the other causes, q. 93 of the *Orthodox*; some, to chill the star of Saturn and to preserve cold influences in the World, from the same Justin in the same place; others, to drive the rays of the stars back downward: *For fire by its nature tends upward*, says Procopius in his *Commentary on Genesis*; *therefore water is joined to the highest heaven, to thrust and drive the brightness down to the lowest parts.* Others hold that they blunt the heat of the first, or Empyrean, heaven: so St Hilary on Psalm 135, and Bede, in the book *De natura rerum*, chapter 7. Others, that by the variety of visible forms which crystal and water can receive from the light of day, as appears in the Rainbow, they refresh the eyes of the blessed — as Cornelius à Lapide teaches, on no slight conjecture, on the Pentateuch at the second day of the world, and Salian in

the *Annals* at the same day, number 20. Very many, however, hold that those waters were carried up into the heaven for this end: to temper the heat of the stars and of the aether, and to preserve heaven and earth from burning. This is taught by St Justin as well, question 93 of the *Orthodox*; St Basil, homily 3 on the *Hexaemeron*; St Ambrose, *Hexaemeron* book 2, chapter 3; St Damascene, *On the Faith* book 2, chapter 9; Theodoret, question 11 on Genesis; Procopius on Genesis, chapter 1; Bede, *De natura rerum* chapter 8, who prefers this opinion to the others; St Thomas in the postills on Genesis; Claudius Marius Victor, commentary on Genesis; and the Church herself, in that hymn:

That the wave may temper the flames, lest they scatter the soil of the earth.

But perhaps they are there so that by their own motion they may perform the office of the 24-hour motion, that is, of the primum mobile, as we shall say below, section 2, chapter 3.

Quæſtio 4. An Aquæ ſuperiores ſint Angeli Sancti ?

Question 4. Whether the upper Waters are holy Angels.

XI. From what has been said so far it is sufficiently clear that the Waters which sacred scripture teaches are above the heaven are some watery body: either of the same nature as elemental water, as we taught with the more common opinion from number 6 onward, or at least agreeing with it analogically in their transparency, and with the hardness of crystal. [Margin: Origen's error.] From this — and above all from sacred scripture understood, as is right, according to the letter — it is clear that the fiction of Origen deserves to be hissed off the stage. He said that the waters above the heaven are good Angels, and those below the heaven evil demons, because it was said of the upper waters that they should praise the name of the Lord. But this error, destructive of the literal sense and of sacred history, is refuted by St Basil, homily 3 on the *Hexaemeron*; St Ambrose, *Hexaemeron* book 2, chapter 4; St Epiphanius in the letter to John of Jerusalem; St Augustine, *De civitate Dei* book 11, chapter 34; St Thomas, 1 p. q. 68 art. 2; likewise Procopius, chapter 1 on Genesis; and Rupertus in the same place, book 1, chapter 23; and, among more recent writers, Pererius, book 1 on Genesis, page 63; Suarez, *De opere sex dierum* book 2, chapter 4; Martinengus in the *Glossa magna*, page 626; and Tanner, tome 1 of his *Theology*, disp. 6 q. 3 doubt 1. Without prejudice to the sense of the letter, however, those waters can be taken mystically, or figuratively, for Angels or spiritual creatures, as Saints Nyssenus [Gregory of Nyssa] and Jerome did on Psalm 76, adducing *The waters saw you, O God*, and that of Apocalypse 17, *Many waters, many peoples*. So did St Augustine, in the unfinished book *On Genesis*, chapter 8, and *Confessions* book 13, chapter 32 — although, lest this give anyone a handle for error, he retracted it, *Retractations* book 2, chapter 6.

Quæſtio 5. An Aquæ ſuperiores ſint Nubes ac Vapores ſupra Aerem vel in Aere ?

Question 5. Whether the upper Waters are Clouds and Vapours above the Air, or in the Air.

XII. It has seemed to some that elemental Waters are unfittingly placed above the heavens, especially the incorruptible ones, since they are a heavy, gross body, less noble than air; indeed, that they are placed there to no purpose and for no probable use, or even against their own nature, against the order of the elements and the heavens, and finally against the seemliness of the divine workmanship; and lastly that, if they are asserted to be non-elemental so as to avoid these difficulties, it is maintained without any valid foundation that the waters there are of a different species. They therefore went so far as to say that by the name of the waters above the heavens are to be understood the waters contained, as to their substance, in the clouds, vapours and mists that are in the highest or the middle region of the air; and that the Firmament, in this place, is the aerial heaven — the whole region composed of air and aether up to the stars, spread between those waters and the waters of the sea and the rivers as a firm and stable boundary. This opinion is indicated, and judged not wholly improbable, by St Thomas, 1 p. q. 68 art. 2 and 3; by Durandus on book 2, dist. 14 q. 1; and, following St Augustine, *De Genesi ad litteram* book 2, chapter 4, by Eucherius, Bede and the ordinary Gloss on Genesis. Indeed Abulensis, in the postills, and Vielmius, lecture 14, attribute it to the Rabbis and to Abenezra [Ibn Ezra]. It is defended tooth and nail by Rupertus the Abbot, *De operibus Trinitatis* book 1, chapter 22; Eugubinus [Agostino Steuco] in the *Cosmopoeia*; Arias Montanus [Benito Arias Montano], Lorinus, Pineda, Pererius, book 1 on Genesis from page 65; Gregorius de Valentia [Gregorio de Valencia], 1 p. disp. 5 q. 3 point 2; Suarez, *De opere sex dierum* book 2, chapters 4 and 5; Torniellus in the *Annals* at the 2nd day of the world; and Kepler in the introduction to the *Commentaries on Mars*.

For they try to show that the air can be called the firmament from Proverbs 8: *When he prepared the heavens I was present; when he made firm the aether above, and balanced the fountains of waters.* In the Hebrew, in place of "aether", there stands *Sekakim*, which properly means clouds, so that the Septuagint translated it τὰ ἄνω νέφη [Greek: ta anō nephē, "the clouds above"], that is, the upper clouds. Consequently, they say, the fountains of waters which the Wisdom of God then balanced were the upper fountains of the clouds, or the cataracts of heaven, of which Genesis 7 and Malachi 3 speak; and they say that the passage of Job 26 is to be understood of the clouds: *Who binds up the waters in his clouds, so that they do not burst out downward all together* — especially since Pliny the Elder himself, book 31, chapter 1, said: *What is more wonderful than waters standing in the heaven?* Finally they try to drag in here the passage of Jeremiah 10: *At his voice God gives a multitude of waters in the heaven, and raises mists from the ends of the earth.* Finally, they hold that the space which is now between the earth and the Moon

was formerly full of watery and misty matter, according to Job 38: *Who shut up the sea as with doors, etc., when I made the cloud its garment, and wrapped it in darkness as in swaddling-bands?* and Wisdom 24: *I made in the heavens that an unfailing light should rise, and like a mist I covered all the earth*; and that this matter was easily converted into air. Nor are Fathers lacking who champion this opinion: for St Augustine, *De Genesi ad litteram* book 2, chapter 4, said that it was *praiseworthily devised, and not against the faith*; and Bede approved it, *De elementis Philosophiae* book 2. [Margin: Authors against this opinion.] But Catherinus rejected it, on chapter 1 of Genesis; and so did Martinengus in the *Glossa magna*, page 566, saying that it is supported by no arguments; on page 585, adhering to the Fathers, he judges erroneous the opinion of those who say that by the name of Firmament only the air is understood, and on page 589 he calls it not safe enough. Molina, disputation 2 *De opere sex dierum*, says that it is plainly false; and Tanner, tome 1, disp. 6 q. 3 d. 1, says that the opposite is the common view of the Fathers, and in the dissertation *On the Heaven*, q. 8, page 179, calls the opposite certain and established. [Margin: Arguments against this opinion.] Nor has it ever seemed to me very probable: first, because on the 2nd day the Sun was not yet present to raise clouds and mists; and because it was more fitting for the divine workmanship that on the first day he should create pure elements, not elements mixed with clouds; indeed, for several days there had been no rains, as Moses says in Genesis 2: *For the Lord God had not rained upon the earth, but a spring went up.* Next, the whole of that Firmament spoken of in Genesis 1, verse 4, divides the upper waters from the lower; but our air is not such, nor is it a firm interval dividing the waters of the clouds from the lower waters, since we often see clouds and mists lying upon the sea and the marshes — so much so that the common people think the clouds suck up water from there. Nor is it consistent with sacred Genesis that the firmament made on the 2nd day should be different from the Firmament spoken of on the fourth day, that is, the one in which the Luminaries and the other stars were placed, which are certainly not located in the air. Or, if the whole space from the earth to the Fixed stars is the Firmament, it would not have told us distinctly in which part God placed the stars — the upper or the lower? And if you say that they were placed in the Firmament of heaven, that is, in the part called heaven, the objection stands: for on this opinion the waters above the heavens are above the middle or the lowest region of the air; so, if this region is heaven, someone could think on Moses' authority that the stars are located in the air. But it was a far more probable undertaking, and one more worthy of God as craftsman, to create all the heavens and all the elements on the first day; then, on the second day, to make the Firmament out of condensed water — that is, at least the heaven of the fixed stars — and so to separate the Empyrean heaven from the aether and the air by a stable boundary and interval, so that this work alone was the entire business of one day; and on the third day to bring about the divisions of Land and Sea.

CAPVT III.

Chapter III.

Quot & Qui Cæli à Deo Prima & Secunda die Mundi Conditi fuerint ?

How many Heavens, and which, were founded by God on the First and Second day of the World.

I. [Margin: All the heavens created as to their substance on the first day.] FIRST CONCLUSION. *On the first day God created all the Heavens as to their substance: namely the Empyrean, the Starry and the Aerial.*

This conclusion has been sufficiently proved from scripture and the Fathers in chapter 1, question 3, from number 6 to 17, and in question 6, where we dealt specifically with the Empyrean, that is, from number 24 to 31; and from what was said there it is clear that this is more probable than that only the Empyrean and the elements were created. Nor is it against the conclusion if someone were to say that the starry heavens were made on the second day, provided he says that they were made not by creation or substantial generation, but by an expansion of rarefied water into the Planetary heaven, and by a condensation into the heaven of the fixed stars or into the crystalline and watery heaven. For by the very fact that all that water was created on the first day, and is not held to have been changed substantially afterwards, those heavens are held to have been created, as to their substance, on the first day.

II. [Margin: Which heavens were made from water on the 2nd day?] SECOND CONCLUSION. *On the second day God made from the elemental waters both the Firmament, that is, the Starry heaven, and the Crystalline and Watery Heaven — whether these two are really distinct from each other and from the Firmament, or not.*

This conclusion likewise becomes manifest from the sacred scriptures and from the very many Fathers and Doctors adduced throughout chapter 2, up to number 11. For from the eleventh onward we refuted two false opinions in particular: one about the Angels, the other about an aerial heaven and clouds founded on that day.

III. [Margin: The distinction of the Watery and the Crystalline Heaven.] THIRD CONCLUSION. *It is more probable, having regard to the authority of the Fathers, which rests on Sacred Scripture, that the watery Heaven, or the Waters that are above the Heavens, are liquid or fluid; or at least that they are really distinct from the crystalline Heaven, taken either for the whole Firmament or for its highest part.*

For in chapter 2, from number 2 to 4, we showed from Scripture and the Fathers that the Firmament was made on the second day out of water consolidated like ice or crystal; and from number 6 to 8 that there are true waters above this Firmament — whether thinned into vapours, or, as more affirm, not thinned but flowing and liquid like the elemental waters, or consolidated into ice, as we taught from number 8 to 9. Therefore those waters, which

some Doctors have also called the Watery heaven, are really distinct from the Firmament.

IV. [Margin: The upper part of the starry heaven solid, the lower fluid.] FOURTH CONCLUSION. *It is more probable that the heaven which is solid like ice or crystal is the heaven in which the Fixed, or non-wandering, stars are, or else that other heaven which God placed immediately above it; but that the heaven of the Planets is fluid.*

This conclusion is one of those that we have so far investigated most of all, with so great a gathering of authorities; and it must therefore be established by distinct arguments — and, unless I am mistaken, very strong ones.

[Margin: 1st argument.] First, then, many very weighty Fathers and Doctors say that the Firmament is solid, solidity being taken for such hardness as ice has, or crystal, or an eggshell, or the hide of a tent, as is clear from what was said in chapter 2, from number 2 to 4; and not a few recognise the same solidity in the waters placed above this heaven. On the other hand, many others no less weighty — indeed some of those who had asserted the solidity already mentioned — nevertheless teach that the heaven is not solid with a solidity that includes hardness; and others that it is fluid and permeable, as is established from what was said in chapter 2, numbers 4 and 5. Those who asserted solidity with hardness assert it chiefly so that the heaven may be fit to sustain the fluid waters above the heavens and to separate the invisible heaven from the visible. Those who contend that the heaven is fluid, and stand more evidently for its fluidity, rely chiefly on the motion of the Luminaries through a motionless heaven — a motion that Scripture never attributes to the heaven, but only to the Planets. Therefore they can be reconciled with one another in no better way than by distinguishing: granting Solidity and hardness to the highest heaven of the visible things, or to the highest part of the Firmament — since on it immediately, and on it alone, lies the burden of sustaining the hypercelestial waters and of bounding and separating the Empyrean and every invisible heaven from the visible — and granting Fluidity to the Heaven in which the Luminaries travel, and so the other Planets with the Sun as their coryphaeus, or leader. It is better to reconcile so many Fathers, since the sacred text fully permits, and indeed indicates, such a distinction, than to suppose them at odds with one another, or to abandon them, or to swerve without necessity into one of the extreme opinions.

[Margin: 2nd argument.] Second: by the same conclusion not only are very many Fathers and Theologians reconciled with one another, but also with the more recent and more expert Astronomers, as will become clear from what is to be said in chapter 7, especially toward the end. Why, then, should we not foster this alliance with every effort?

[Margin: 3rd argument.] Third: Sacred Scripture itself, as I was saying, indicates clearly enough that some heaven is hard, being fit to divide the upper waters from the lower and to preserve that separation perpetually; and we showed in chapter 2, number 2, that this is gathered from the Hebrew name

itself. But the same Scripture, when it says that the Luminaries move freely through the heaven, and never mentions any motion of their heaven, as was said in chapter 2, number 4, also indicates that at least the part of the heaven through which they are carried does not move — supposing the impenetrability of bodies, celestial ones included, which the Theologians likewise draw from the sacred scriptures, as I here suppose. Therefore the conclusion already mentioned is very consonant with the sacred scriptures, and so the more probable; and the passage of Proverbs 8, *When he made firm the aether above*, may without impropriety be understood of the highest part of the aether, solidified.

[Margin: 4th argument.] Fourth: among the Fathers who discuss the solidity and fluidity of the heaven, many speak only indefinitely; but those who speak distinctly about some part of the Firmament ascribe Solidity and hardness only to the highest starry heaven, that of the non-wandering stars, or grant to the Planetary heaven only fluidity and permeability by the Planets. Although I have already brought forward some of them, it is my pleasure nevertheless to repeat here their select words on this matter, and those of others. [Margin: St Bonaventure.] St Bonaventure, on book 2 [of the *Sentences*], d. 14 p. 2 art. 1 q. 1, calls it the common view of Physicists and Mathematicians that *the Planets move in the heaven in one continuous body, subtle and apt for motion, such as air and water are.* [Margin: St Anselm.] Clearly, too, St Anselm, *De imagine mundi* book 1, chapter 21, where he says: *The higher Heaven is called the firmament because it is a firmament between the waters in the middle. It is spherical in form, watery in nature, adorned on all sides with stars, consolidated out of waters like ice, indeed like crystal.* If it is higher and adorned on all sides with stars, then it is the one called the eighth sphere. But he himself discloses his mind in chapter 24, when he adds about the Planetary heaven: *Fire, the fourth element, extends from the Moon up to the firmament. It is as much subtler than air*

as air is thinner than water, and water rarer than earth. Here too the aether is called pure air, as it were, etc.; in it the seven stars are carried, each in its own circle, against the course of the world, and because of their wandering course they are called Planets, etc. We cited the whole passage in number 4.

[Margin: St Isidore.] With the same distinction St Isidore, *Etymologies* book 3, chapter 49, affirmed *that the Sun moves by itself, is not turned with the World, and is not fixed in its heaven*; [Margin: Richard of Saint Victor.] and Richard of Saint Victor, book 2 of the *Exceptiones*, chapter 7: *On the fourth day he made the Luminaries, which are proved to be fixed in the Firmament, except the seven Planets, the Moon, etc.* [Margin: Gulielmus Parisiensis.] The teaching of Gulielmus Parisiensis also favours us in part, where in part 1 of *De universo*, chapter 34, he says that after the first heaven, that is, the Empyrean, which he calls most at rest, there was established on day 2 a heaven which is at the extreme limit of mobility, and that this is the Firmament; and in chapter 37 he says that the Firmament made out of the waters is a heaven containing nine heavens. Here he seems to admit eleven heavens, namely the Empyrean, the Firmament or primum mobile, the eighth sphere, that of the Fixed stars, and the seven orbs

of the Planets. He therefore favours us insofar as he does not recognise the heavens of the Planets as the Firmament made out of the waters.

[Margin: St Thomas.] No less — indeed more — St Thomas supports us in his postils on Genesis, saying: *But that Firmament which divides these waters is the heaven itself in which the Fixed Stars are, under which are the seven Planets and their orbs.* He said "under which", not "in which". [Margin: Hamerus.] Hamerus, moreover, reports and tacitly approves the same opinion in his commentary on Genesis, in these words: *There are also those who borrow their understanding from Astronomy. For they interpret this firmament as the eighth sphere, fitted with innumerable torches of stars. By the Greeks it is called Aplane, by the Latins inerratica* [both meaning "the unwandering"]. [Margin: Catherinus.] And clearly Ambrosius Catherinus, in his exposition of chapter 1 of Genesis: *It is therefore far more fitting and more correct that the heaven we call starry, namely the eighth sphere, should be understood by this name.* [Margin: Carthusianus.] And Dionysius Carthusianus too, in his Commentaries on Genesis, article 10: *What is to be understood by the name of Firmament is clear from the fact that a little later the stars are said to be placed in the firmament. Therefore at God's command the firmament was made, that is, the Heaven of the stars, which is the eighth sphere, because under it are the seven spheres, or orbs and heavens, of the planets, etc. And this firmament is now taken as including the seven spheres already mentioned — including them, I say, not in itself but under itself, so that the Firmament solidified out of the waters is the eighth sphere, like a vessel or the shell of an egg containing in itself, or within its own circuit, the heavens of the Planets.* And he adds: *It has already been shown before that the exposition of those is more reasonable, and more consonant with the text and with the ancient and Saintly Doctors, who say that in the beginning the celestial bodies and the elements were produced together under distinct and proper substantial forms. Therefore the Firmament and the Orbs of the Planets were created in the beginning as regards their whole substance, etc.* He therefore distinguishes the Firmament and the eighth sphere from the orbs of the Planets.

[Margin: Hugo Carensis.] The words of Hugo Carensis in his postils say the very same: *The Firmament is the surface of the sensible world, made of waters frozen into Crystal, as they say, containing all the other sensible things after the likeness of an eggshell; and it is called the firmament because it is firm.* [Margin: Comestor.] The Master of the *Historia scholastica* on Genesis, that is, Petrus Comestor, chapter 4, plainly agrees with him: *On that day, namely the second, God therefore made the Firmament in the midst of the waters; that is, a certain outer surface of the world, consolidated out of frozen waters like crystal, and translucent, containing within itself the other sensible things, after the image of the shell that is on an egg; and in it the stars are fixed.*

[Margin: Chrysostom, Procopius, Iunilius.] Moreover, from number 4 of chapter 2 it is established that the heaven of the Planets is not moved but is passed through by the Planets, in the opinion of Chrysostom, Anselm, Procopius, Iunilius, Carthusianus and others; and from number 3 it is established likewise that the highest heaven, which divides the upper waters, is solid. From these,

therefore, it is gathered as the more probable view that the highest heaven is solid, but the Planetary heaven fluid and permeable.

[Margin: 5th argument. The heaven of the Planets the same as Fire, or Aether.] Fifth: according to not a few Doctors, the heaven of the Planets is the same as the Aether, or Fire; but this is without doubt fluid, and yields to bodies moving within it more than air does. [Margin: St Anselm.] The major premise is proved first of all by the authority of St Anselm of Canterbury, adduced a little before and in number 4; [Margin: Iunilius.] and by that of Iunilius in his *Hexaemeron*, who says: *It is to be understood that the birds fly along the firmament of Heaven.* [Margin: Innocent III.] So too Pope Innocent III counts the heaven in the place of the element of fire, saying on the fifth penitential psalm: *In the creation of the World Heaven and Earth were created together, the first and the lowest among the four elements.* [Margin: Bede.] And Bede, in his exposition of Genesis, likewise judged the same when he says: *Since the World consists of four elements, Heaven, Air, Water and Earth.* [Margin: Hugo Carensis.] Almost the same is Hugo Carensis in his postils: *Heaven, that is, the two upper elements; Earth, the two lower.* Nor do Tostatus, on chapter 1 of Genesis, and Rupertus, book 1 of the Commentary on Genesis, chapter 8, refuse it — though as the view of Plato, who understood sacred Genesis in this way; [Margin: Plato.] for they say: *When Plato had read this Scripture, etc. Because he suspected that fire too was signified by "heaven", he thought that only four elements, namely fire, earth, air and water, were enumerated in this place.* [Margin: St Augustine.] To this belongs that saying of St Augustine in the unfinished book *De Genesi ad litteram*: *So that thus four elements have been stated, from which this visible world rises: namely heaven, earth and water, and air*; and in book 2 of *De Genesi ad litteram*, chapter 3: *Therefore above the air pure fire is said to be heaven.*

[Margin: Pliny the Elder.] Well known, too, is the opinion of Pliny the Elder, book 2, chapter 5, who says: *Nor do I see any doubt about the elements, that they are four: the fiery one highest, and from it the eyes of so many shining stars.* [Margin: The Stoics.] Long ago, too, the Stoics called the heaven a moving blaze, [Margin: Heraclitus.] and Heraclitus called it fire; [Margin: Empedocles.] and Empedocles, as quoted by Clement of Alexandria in book 5 of the *Stromata*, enumerated the four parts of the world in these verses:

Earth, and the overflowing Sea, and the moist Air, and Titan Aether, who binds all things round in a sphere.

Anaxagoras and Empedocles asserted that the starry heaven is Fiery, as Plutarch reports in *De placitis* book 2, chapter 13; [Margin: Pythagoras. The Egyptians.] and Pythagoras and the Egyptians held the same, if we believe Albert the Great, *De caelo*, tractate 1, chapter 4. For this reason I judge it neither new nor absurd, but indeed highly probable — [Margin: Delrio. Tellez. Paracelsus.] with Paracelsus, Delrio on chapter 1 of Genesis, and Tellez, disputation 40 of the *Physics*, section 1 — that the aether, or heaven of the Planets, is Fire, and not merely fiery; and that the luminaries are placed in the Firmament in the way in which something would truly and properly be said to be placed

in a room or in a vessel, even if it were not fixed into the roof itself, or into the shell or crust. For the Firmament contains the heavens of the Planets as the shell of an egg contains the white and the yolk, and together with them it makes up one heaven, which in Genesis 1 is called the firmament of heaven. Suarez, Valentia [Gregorio de Valencia] and Pererius ought to grant this all the more willingly, since by the name of Firmament they themselves understand that whole diaphanous body which extends from the earth up to the highest heaven.

CAPVT IV.

Chapter IV.

De Opere diei Quartæ, ideſt de Luminaribus alijſq. Stellis conditis, & in cælo poſitis à Deo.

On the Work of the Fourth Day, that is, on the Luminaries and other Stars created and placed in the heaven by God.

I. We pass over, as if at a leap, the works of the third day — that is, the gathering into the hollows of the earth of the waters that had remained under the heaven, and the plants born from the earth and grown to maturity on that same day by God's command — because they do not bear much on the system of the World, in which the heavens and the celestial bodies are chiefly considered. We therefore move on to the work of the fourth day, which is contained in these words of Genesis: *And God said, Let there be Luminaries in the firmament of heaven, and let them divide the day and the night; and let them be for signs and seasons, and days and years; that they may shine in the firmament of heaven and give light to the earth. And it was so. And God made two great luminaries: the greater luminary to rule the day, and the lesser luminary to rule the night, and the stars; and he set them in the firmament of heaven, to give light upon the earth, and to rule the day and the night, and to divide the light from the darkness.* With these set down, the following questions have to be dealt with.

Quæſtio 1. Cur Siderum Productio in diem quartum dilata?

Question 1. Why was the Production of the Stars postponed to the fourth day?

II. If God had not clothed the earth with grasses and plants on the third day, a physical cause of this postponement could have been given from the dignity of the heaven: for since in six days the whole

business of this work of fashioning was to be completed by divine decree, the first three were assigned to the creation of the great parts and to universal place: namely, the first day to creating the heavens and the elements, the second day to separating the heavens from one another and from the elements, and the third to separating the lower elements. The remaining three days were destined for the adornment of the heavens and of the elements. Therefore the first of these days, namely the fourth from the very beginning of creation, had to be granted to adorning the heaven, as the noblest of bodies and superior among

the simple ones, and to making it illustrious with the most excellent creatures. [Margin: 1st cause of the production of the stars on day 4.] So, setting aside for now the adornments of the plants, this First cause of creating the Sun and the stars on day 4 is put forward by St Augustine, *De Genesi ad litteram* book 2, chapter 13; by Bede with him, in his exposition of Genesis; by Iunilius on the *Hexaemeron*; by the ordinary and interlinear Gloss; and by the Master, on book 2, distinction 14: namely, that the higher and nobler region of the World should be adorned before the lower.

[Margin: 2nd cause.] Closest to this is the Second cause, given by St Thomas, part 1, question 70, article 1; by Hugo Carensis in his postils; by Comestor, chapter 6; by Albert the Great, part 1 of the *Summa*, question 14, article 14; and by Lyranus and Tostatus in their postils: namely, that the heaven had to be adorned first, before the elements, as being more worthy and as already mentioned in first place in the creation of the world. But the same authors teach that plants, as being attached to the earth and immobile, do not belong to that adornment of which Genesis 2 says, *Thus the heavens and the earth were finished, and all their adornment*; whereas the things that have motion do belong to the adornment, because by the very fact that they move from one place to another it is apparent that they are not intrinsic parts of the earth. Sprouts and plants, by contrast, since they are intrinsic to the earth, as hairs are to the skin and flesh of an animal, do not come under the name of the adornment of the earth. Martinengus quotes the words of these Authors in volume 2 of the *Glossa magna*, from page 846 to 848. But some of these same authors, and others, suppose that grasses and plants too belong to the adornment of the earth — for instance Origen, homily 1 on Genesis; Chrysostom, homily 6; and Anastasius Synaita, book 6 of the commentary on Genesis. And certainly that universal particle, *and all their adornment*, that is, of heaven and earth, seems to embrace sprouts and plants as well.

[Margin: 3rd cause.] The Third cause, then, seems to have been that the middle day of this Week, and therefore the nobler, should be adorned with the most excellent things. St Augustine reports this very cause, *De Genesi ad litteram* book 2, chapter 13, but presses against it as if disapproving: for this day was not the middle one among those on which something was produced by God, since on the Sabbath he rested from all work; nor is the Sun more excellent than the animals produced on the fifth and sixth days. And the same can be said against Philo, who thinks that the excellence of the number four was regarded, because four is potentially ten: for if to four you add the preceding numbers, namely 1, 2, 3, ten results.

[Margin: 4th cause.] St Athanasius on Genesis, question 85, and Theodoret in the same place, question 16, pointed to a Fourth cause: namely, that the light of the first three days, being excessive and intolerable to animals, should be tempered and diminished by being distributed among several stars — as though the whole heaven had previously shone in the way the Sun now does, or as though it were certain that that light was then more intense than afterwards.

[Margin: 5th cause.] The Fifth cause, according to Procopius on Genesis chapter 1 and others cited in Martinengus, page 845, is said to have been that lamps should be lit for the banquet being prepared for man, and that the less noble things should be sent ahead of man, as attendants and forerunners go before an emperor.

[Margin: 6th cause.] But to me the most probable of all seems the Sixth cause. On the one hand, it was to be clear that light — which is the universal vehicle of influences and the soul of colours — and the grasses and plants, which seem to depend most of all on the heaven, do not depend essentially on the Sun and the other stars; and men would therefore be turned away from idolatry and from offering worship to creatures that are otherwise most beautiful. On the other hand, the stars were to exercise from the beginning, together with God the principal founder, some power in producing animals, and were not to seem inferior to the elements. Nor ought their production to be deferred to the last day, lest God should seem to have come to rest in it as the most excellent work and the end of the others. [Margin: Philo.] The first part of this cause was indeed handed down by Philo in his book *On the Creation of the World*, whose words very well deserve to be copied here. This, then, is how he speaks of God the craftsman: *For he foresaw the opinions of men not yet created: that they would follow probable conjectures, which have much appearance of reason, but not likewise pure truth; that they would believe their own eyes more than God, admirers of sophistry rather than of wisdom; and that they would credit to the courses of the Sun and Moon — to the summer, winter, spring and autumn changes — and to the circuits of the heavenly stars the causes of the yearly produce of the earth and of all things that spring up. Lest they should dare to ascribe the first causes to any creature, whether through impudence or through ignorance, let them go back in their minds* (he says) *to the first generation of this universe, and remember that before the Sun and the Moon plants of every kind and fruits of every kind came forth from the earth. And so let them persuade themselves that hereafter too the earth will bring them forth by the will of the heavenly Father, whenever it pleases him, without waiting for the favour of the heaven, to which he gave power indeed, but not free power, etc. And this is the reason why the earth sprouted first and brought forth grass.*

[Margin: St Basil.] St Basil, having pursued the very same point at length and eloquently in homily 5 of the *Hexaemeron*, condenses it in homily 6 into these few words: *Heaven and earth came first; light was created after heaven and earth; night and day were separated; then the firmament was brought forth; next the dry land emerged, and the waters flowed together into one determined place. At last the earth was filled with its sprouts, both with innumerable kinds of grasses brought forth and abounding with various forms of shrubs and trees that sprang up in a moment. But there was not yet a Sun, nor a Moon — lest those who do not know God should call the Sun the prince and father of light, or judge it to be the creator of the things that spring from the earth. That is why the fourth day came: and then God said, Let there be Luminaries.*

[Margin: St Ambrose.] But sweeter than all honey, and even than ambrosia, are the words of St Ambrose on this matter, *Hexaemeron* book 4, chapter 1: *Beware therefore,* he says, *lest the rising ray of the Sun confound your sight too. And so look first at the firmament of heaven, which was made before the Sun; look at the earth, which began to be visible and composed before the Sun came forth; look at its sprouts, earlier than the light of the Sun. The bramble is earlier than the Sun; the grass older than the Moon. Do not, then, believe that to be God before which you see God's gifts were preferred. Three days passed, and no one looked for the Sun, and the brightness of light abounded.* And a little later: *What, then, did the height of the wisdom and knowledge of God foresee, that trees should begin to exist before those two luminaries of the world, and certain heavenly eyes of the firmament, unless that all should know, by the testimony of the divine reading, that the earth can be fruitful without the Sun?*

[Margin: St Chrysostom.] Let that golden mouth, namely Chrysostom, homily 6 on Genesis, join this ambrosial mouth. He teaches that the grasses were sent ahead of the Sun, *so that you may not ascribe the produce of fruits to the Sun, but to God, the maker of all, who said from the beginning, Let it sprout*; and below: *What we have said of the seeds may also be said of the day: that there were three days before the formation of the Sun*, etc. [Margin: Severianus.] Severianus, in Lippomanus's *Catena*, offers things similar to these: *Why did God adorn the earth before the heaven? Because of the error that was to come about many Gods, such as the Sun, the Moon and the Stars.* [Margin: Procopius.] And Procopius, in the Commentaries on Genesis, chapter 1: *The Sun followed the plants, grasses and sprouts in the order of creation, lest anyone should believe it to be the author of the things just mentioned. So too the creation of light long preceded it, lest anyone should perhaps tacitly judge the Sun to be the bestower of light, and divine worship be decreed to it on that account. For what reason allows the Sun to be the origin of light, when the creation of light preceded it?*

Quæſtio 2. Fueritne Sol cum reliquis ſtellis quarta die productus quoad ſubſtantiam; An illis prima die productis indita ſit lux tanquam forma, & addita vis influendi, ac motus proprius diſtributus?

Question 2. Was the Sun, with the other stars, produced on the fourth day as regards its substance? Or, the stars having been produced on the first day, was light put into them as a form, and a power of influencing added, and a proper motion distributed?

III. You see the whole variety of opinions on this matter indicated in the title of the question. [Margin: 1st opinion and its authors.] For the First opinion was that the Sun and the stars were created on the first day, together with the heaven, as regards their substance; but that on the fourth day light was gathered and poured into the Sun and the stars, a determinate power of influencing added, and finally a motion proper to each star assigned toward the East, so that they might be signs of the seasons, etc. So taught St Thomas, on book 2 [of the *Sentences*], distinction 13, and part 1, question 67, article 4, and question 70, article 1, reply to 1; Dionysius Carthusianus on Genesis, articles 9 and 12;

Aegidius, part 2 of the *Hexaemeron*, chapters 5, 6 and 31; Catherinus on chapter 1 of Genesis; Vielmius in the same place, lecture 20; Augustinus Eugubinus in the *Cosmopoeia*; Valentia, disputation 5, question 3, points 1 and 4; Pererius, book 1 on

Genesis, page 49; Suarez, *De opere sex dierum* book 2, chapter 8, from number 16, and chapter 9; Tannerus, volume 1 of the *Theology*, disputation 6, question 4, doubt 2. Among these, Catherinus and Vielmius think that the Sun was produced on the first day and perfected on day 4 accidentally, with light, power and motion; but that the Moon and the other stars were produced on day 4 as regards their substance — for they cannot understand the primeval light as anything other than the Sun. Eugubinus, however, by a plainly arbitrary fiction, affirms that the Sun was founded on the first day, the Moon on the second, the other Planets on the third, and the Fixed stars on the fourth. Pererius, moreover, says that light was given to the Sun as an essential form — a view supported by St Nazianzen, oration 43, and supported also by the Hebrew name of the Luminaries, Meoroth, and by the Greek of the Seventy, φωστήρες [phōstēres, "luminaries"]; he also mentions a determinate power for influences, and a proper motion. Suarez, on the other hand, teaches that the Sun and all the stars were produced on the first day as regards substance, light and power of influencing, and that on day 4 they received nothing other than their proper motion; Tannerus, a more intense and stable light, and their proper motion. The foundation of these views — though not of all of them — is twofold: first, that the heavens are incapable of new generation, of condensation or rarefaction, and that therefore all the celestial bodies had to be created on the first day; and second, that they think it very difficult to explain what kind of light that was which was made on the first day, and how the first three days consisted of morning and evening, unless it is said that the Sun and the Stars existed as regards substance, and as regards motion — but the common motion, that is, that of the primum mobile.

IV. [Margin: The 1st opinion attacked.] But the opinion already mentioned must without doubt be rejected by anyone who wishes, as he ought, to understand Sacred Scripture literally, since in this there is no conflict — indeed, as will be said later, there is the greatest agreement and an admirable order of the divine works. Next, the opinion already mentioned has none of the ancient Fathers as its patrons, and most of them against it. Moreover, the authority of Sacred Scripture by itself, and much more with the interpretation of the Fathers joined to it, ought to have moved the mind of every prudent man to understand that the Luminaries and stars were made anew on day 4, at least as regards form and shape; to resolve those very slight difficulties that had presented themselves in the production of light on the first day outside its proper subject, and in the production of the Sun on day 4; and not to measure their production by merely Physical rules, but by the divine power made manifest to us by God himself in such clear words. For in what clearer and simpler words, I ask, could Moses have explained to us the new production of the Luminaries and stars, and their

placing in the heaven, and then the distribution of their charge and office, than in those he used? [Margin: 1st and 2nd argument.] And let this be the First Argument, namely the authority of Sacred Scripture understood literally; for the Second, from the Fathers, will be adduced shortly, to confirm the second opinion.

[Margin: 3rd argument.] The Third argument is that Sacred Scripture said not only that the Luminaries were made on that day, but that they were placed in the heaven and ordered to divide the night and the day, and to be for Signs and seasons, etc. Now if they were made on the first day, where were they placed? Or if they were placed in the heaven on the first day, why is this passed over in silence on that day and ascribed to day 4? Does this not give occasion for manifest error to the uneducated people for whom, they hold, Moses wrote above all? Besides, if the first day had the motion of the primum mobile, and the day was divided from the night by it, why is that office repeated, or insisted upon, on the fourth day? For these words, *and let them divide the day and the night*, contain not the proper motion toward the East but rather the common motion.

[Margin: 4th argument.] Fourth: the proper motion of the planets and of the Fixed stars toward the East is very remote from the common notion of ordinary people — indeed the proper motion of the Fixed stars was detected only by Hipparchus — nor is it in itself an accident worthy of God's assigning a whole day to arranging it alone, if one considers the works of the other days, on which either great bodies of the world were founded or separated, or substances of different kinds were produced with their accidents, and not merely a motion, or one accident or another. So on days 5 and 6 animals were produced, and not merely their manifold motions.

[Margin: 5th argument.] Fifth: the end would be lost which, in the opinion of the most weighty Fathers and Doctors, God had in deferring the production of the Luminaries and stars until day 4 — namely, that it might be apparent that the Light of the heaven, and the plants of the earth, could exist and be brought forth without the Sun and the stars, and that in this way men might be turned away from offering divine worship to the Sun, the Moon and the host of heaven; we have already spoken of this end in the preceding question. I therefore cannot let pass unreproved, above all, Steuchus, or Eugubinus, who wishes to attribute to the Sun what neither Scripture nor the Fathers attribute to it. For he says in the *Cosmopoeia*: *The Sun, gathered into a globe and kindled, itself did the remaining works on earth. Once made, it made the day, made the air, made the uncovering of the earth, drained the waters, created the plants and the animals.* Is this deterring men from adoring the Sun? But how gratuitously, and out of his own free invention rather than from the light of the divine writings, he imagines that the Sun was founded on the first day, the Moon on the 2nd, the lesser Planets on the 3rd, and the stars on the 4th! In this he errs more than Catherinus and Vielmius, who grant that the Moon and all the Stars, the Sun excepted, were founded on the fourth day.

But that all the Luminaries were produced by God on the same day, as things of the same order, is taught by [Margin: St Athanasius.] St Athanasius, sermon 3 against the Arians: *For all things alike obtained their rise and origin together with all; although among themselves some differ from others in glory. For not one of the stars, or of the great luminaries, so appeared that this one emerged into the nature of things earlier and that one later; but all were called into being on the same day and by the same command*, etc. [Margin: Theodoret.] In a similar way Theodoret, expounding that verse of Psalm 71, "His name endures before the Sun", discourses thus: *This particle, Before, declares him to be older than the Sun and the Moon. For it is not simply that he endures before the Sun; rather, he placed the generation of generations before the Sun and the Moon. For the Sun and the Moon are of the same time: they were made at the same time. For God said, Let there be Luminaries in the firmament of heaven; and God made two great Luminaries.*

V. [Margin: 2nd opinion.] The Second view, then, and one wholly certain in my judgement, is that on the fourth day God made the Luminaries and the Stars anew — whether by producing them out of some matter (for we have already taught, in chapter 1, number 7, that whatever was created out of nothing and by creation strictly taken, he created on the first day), or by a condensation of parts of the heaven, as some would have it. For even if the second way does not include the production of a new substance, and no generation physically taken occurs through it, nevertheless, regarding God here as the supreme craftsman, a certain new artefact is posited as made, which is like a substantial production — so much so that Aristotle brought forward the forming of a statue as an example of substantial generation, because something is said to be made a being simply, and not made such-and-such with some accidental addition. For this reason we shall also adduce for this view those Authors who think that the stars were made by mere condensation and configuration of parts of the heaven. This is the opinion of Ascanius Martinengus in the *Glossa magna*, page 867, who calls it a quasi-ecumenical definition; of Molina, *De opere sex dierum*, disputation 15; and of Salianus in the *Annales Ecclesiastici* of the Old Testament, at the 4th day of the World.

[Margin: 1st proof, from the Scriptures.] And it is proved, First, from Sacred Scripture taken in its plain and literal sense. By the words *Let there be* and *He made*, Scripture signifies the production of the Luminaries and stars by God on this fourth day in the same way as on the first day it had signified the production of Light; it then expresses the place in which they were produced and placed, and finally recalls the offices of the stars. And it can be understood of a new production as regards substance without any conflict or difficulty. Indeed, this view not only makes the sequence of the text clear and separates the works of this day clearly and distinctly from the works of the other days, but also displays a most fitting disposition of divine providence. For what is more fitting than that God should found no mixed or particular body before he had fashioned the universal and greater bodies and had divided and separated them perpetually by fixed limits and regions — which was done in the first three days, as is clear from

what has been said so far; that he should then, beginning from the vegetative grade, produce plants and grasses; then step up to the things that require a power of moving from place to place, whether intrinsic or extrinsic, of which kind are the stars; afterwards, proceeding to the sensitive grade, produce on the fifth day the animals of the water and the air, which are the ornaments of the two elements nearer to the heaven and are moved by the rowing of wings or fins; and finally, on the sixth day, make the earth inhabited, first by brute animals and then by rational ones?

Secondly, it is proved by Authority: for rightly Martinengus says: [Margin: 2nd proof, from the Fathers.] *This opinion has stronger supporters, superior both in number and in authority. It is better fortified by antiquity, since it had taken root in the Church, received by the common votes of the Fathers, almost twelve hundred years before the others.* And above all the Church assents to it in the hymn:

> *Who on the fourth day, setting up the flaming wheel of the Sun, the ministering order of the Moon, and the wandering courses of the stars.*

[Margin: Tertullian.] We also have that poem of Tertullian:

> *The fourth day begets the Moon together with the lamp of the Sun, and fashions the stars, radiant with trembling light.*

[Margin: St Basil.] St Basil, homily 6 on the *Hexaemeron*: *For then, that is, on the first day, the very nature of light was produced; but now this body of the Sun was founded for this purpose, that it might be a vehicle for that firstborn light.* [Margin: St Ambrose.] St Ambrose, who almost always supports him, in book 4 of the *Hexaemeron*, chapter 1, where he begins his explanation of the works of day 4, says: *In our reading the Sun must now rise, which had not existed before. We have already passed the first day without the Sun; we spent the second without the Sun; we completed the third without the Sun. On the 4th day God commands the Luminaries to be made: the Sun, the Moon and the Stars.* [Margin: St Chrysostom.] So too St Chrysostom, homily 6 on Genesis: after the earth had been adorned with grasses on the third day, he teaches that the heaven was adorned on the fourth day, *above all by the formation of the two great luminaries, the Sun, I say, and the Moon*; and he adds: *For this reason he made the Sun too on the fourth day, lest you should think it the author of the day.* [Margin: Severianus.] Severianus, most briefly: *After the earth had sprouted, then God made the Sun and the Moon.* [Margin: Procopius.] In no other sense Procopius, chapter 1 on Genesis: *First light was brought forth into the open; then at last the receptacle of light was founded*; and he compares the Sun to a lamp, and the light to fire, etc. I have put these five Fathers first because they are the same who teach, with Philo, that the production of the Sun was deferred to the fourth day for this reason: lest the Sun be thought the first author of light and of plants, as we showed in question 1 of this chapter.

[Margin: St Nazianzen.] St Gregory Nazianzen, in the oration on New Sunday, says that the first light was founded by God *not through some instrument, or the Sun, but incorporeally and without the Sun*; and a little later: *But, in order*

to make something worthy of greater admiration, here he placed the form before the matter: for the form of the Sun is light. Then he also brought in the matter, founding this Sun as the eye of the day. So he made the body of the Sun on this day — granted that he made its form, if its form is light (as Nazianzen and our Pererius think), on the first day, at least as regards its species.

[Margin: St Augustine.] St Augustine, book 1 *On Genesis against the Manichees*, chapter 14, answers those who ask how the stars — that is, the Sun, the Moon and the stars — were made on the fourth day, and how the three earlier days could have existed without the Sun, in this way: *To them we reply that it could have come about that the three former days were each reckoned by as long a span of time as the Sun takes to go round, from the moment it proceeds from the East until it returns again to the East. For men could perceive this span and length of time even if they lived in caves, where they could not see the Sun rising and setting. And so it is understood that this span could have taken place even without the Sun, before the Sun had been made.* How much more readily he would have answered the Manichees, had he thought that the Sun was made as regards its substance on the first day and had the motion of the primum mobile! Eucherius, as usual, subscribes to him, book 1 on Genesis, chapter 5.

But since the matter is clear, and undoubted among the ancient Fathers, I have no need to review them one by one. If anyone wants their words, let him read them at their sources, or in Martinengus, *Glossa magna*, from page 859 to 873, or in Salianus at the 4th day of the world; let it suffice to have indicated the places. The following, then, may be seen: St Damascene, book 2 *On the Faith*, chapter 7, who calls a Luminary the receptacle of Light; St Athanasius, question 74; St Nyssenus [Gregory of Nyssa], the book *On the History of the Six Days*; Theodoret, question 14 on Genesis, and on Psalm 148; Anastasius Synaita, book 4 on Genesis; Apollinaris, in Steuchus; Didymus, in Lippomanus's *Catena*; St Bonaventure, on book 2, distinction 14, where, on St Basil's opinion of the Sun as a vehicle and vessel of light made on this day, he adduces that verse of Ecclesiasticus 46: *An admirable vessel, the work of the most High*; Burgensis, in the *Additions*; Hugo Victorinus, in the notes on Genesis, chapter 6, and book 1 *On the Sacraments*, part 1, chapter 15; the Master, on book 2, distinction 13; Comestor, chapter 6 of the *Historia Scholastica*; Lyranus in the *Postills*, who chooses this view; Tostatus on Genesis; and almost all the others before the 13th century.

Quæſtio 3. Quo modo Producta ſint Luminaria & ſtellæ quarto die? Num Creatione ex nihilo, an Condenſatione & Rarefactione; An generatione per productionem formæ ſubſtantialis?

Question 3. How were the Luminaries and stars produced on the fourth day? By creation out of nothing, or by condensation and rarefaction? Or by generation, through the production of a substantial form?

VI. [Margin: 1st opinion: that the stars were created out of nothing.] The First opinion was that of Burgensis in the *Additions* on Genesis. He held that

the primeval light was miraculous and produced in a supernatural way; and, once natural provision had then been made, he says that the Luminaries and stars were created out of nothing on the fourth day, because God said *Let there be luminaries*, etc., just as he had said *Let there be light*. He did not say *Let the heavens bring forth luminaries*, or *Let a bright cloud bring forth the Sun*, as he said *Let the waters bring forth*, *Let the earth sprout*, etc. — and this, to show us that no subject concurred in their production. On that day, moreover, the firstborn light was wholly extinguished: just as Joshua 5 says, *The manna ceased after they had eaten of the fruits of the land*, so that extraordinary light ceased when the stars, the sources of ordinary light, were created. Suarez, book 2 of *De opere sex dierum*, chapter 8, number 2, would have subscribed to this opinion had he thought that the Sun, etc., was produced on day 4 as regards its substance; for, he says, *since the luminaries and stars are incorruptible bodies, they cannot, in keeping with their nature, be produced except by creation out of nothing.* Yet he says that this opinion has no weighty supporters — for he mentions none — and that it is to be rejected, because whatever God produced by creation properly so called, he created at once and all together from the beginning, as is the accepted view of the Theologians and as we have already taught in chapter 2, number 7. It does not follow, however, that if the Stars are incorruptible intrinsically, they require to be created out of nothing. For they can consist of matter and of a form such that the matter is wholly filled and seeks no other form, and that there is no natural agent able to introduce contrary dispositions and to assail them with that corruptive alteration; and so they can be incorruptible of themselves and yet be produced out of such pre-existing matter. I add, moreover, against Burgensis: by the very fact that Moses did not say *he created*, but that God said *Let there be*, God surely commanded some creature to concur in that production, at least in the order of a material and sustaining cause; and so he also drew the first light out of the potency of the diaphanous body, and did not create it out of nothing, as Burgensis supposes. But to the earth and the waters he said *Let it sprout*, *Let them bring forth*, etc., because he willed that they should concur in the production of those mixed bodies not only as a material and sustaining cause, but also as an instrumental efficient cause — through the remote dispositions of the primary qualities, which they already had, and the proximate dispositions of the secondary qualities, or of the temperament proper to each of the mixed bodies, which he put into them.

VII. [Margin: 2nd opinion: of condensation and rarefaction.] The Second opinion is that the Sun and the other stars were made from the same matter of which the starry heaven consists, but by mere condensation or rarefaction. This opinion is mentioned by Molina, *De opere sex dierum*, disputation 15, and by Suarez, book 2, chapter 8, number 5, with no Author named; and they suppose it follows from this opinion that the Luminaries and stars do not differ substantially among themselves or from their heaven, but only in quantity of matter, which makes density, or in scarcity of it, which makes rarity. All those, however, who said that the Sun and the stars were made from that primeval

light, gathered into these bodies, or globes of the stars, and distributed into them, might seem reducible to this opinion — for I do not believe that they took the light to be an accident alone, without a subject. Among them were Gregory of Nyssa, Didymus, Apollinaris, Athanasius, Aegidius, Albert the Great and some others, named at the end of number 5. Singular among them is Nyssenus in the *History of the Six Days*. He says that the stars were made from a collection and heaping together of light, or fire — yet in such a way that its parts were already, beforehand, some denser than others and of different natures — and that on the 4th day all the particles that were of the same nature, say solar, flowed together to constitute the Sun, and those that were lunar to the Moon, and so on for the rest. Furthermore, Albert the Great, part 1, question 4, article 21, and St Bonaventure, on book 2, distinction 14, but from

the mind of Aristotle, teach that the stars were made from a condensation of the heaven. Finally Aegidius, in the *Hexaemeron*, part 2, chapter 31, imagines a threefold condensation of light: the first unfit for illuminating; the second fit for it, when light is said to have been made dividing the day from the darkness; the third when it was condensed into the body of the Sun. But since that light was diffused through all diaphanous bodies — that is, through the whole extent of the heavens and of the waters above the earth — if it had been gathered by condensation, together with its subject, into the stars, then the whole matter, at least the celestial matter, would have passed into the stars, and it would have been necessary to produce the starry heaven anew, at least by rarefaction, so that the void might be filled. This has no appearance of probability, nor does it agree with Scripture, which supposes that the stars were placed in a firmament and heaven already made — in fact, on the first and second day. Suarez, however, refutes this opinion on other grounds. He thinks that the celestial bodies are incorruptible, and indeed that in their first production they must have been optimally disposed and must connaturally have required the whole perfection of their substance, and so their due density and rarity; whereas condensation and rarefaction befit only corruptible bodies, since such changes are by their nature ordered to corruption and generation. But he proves this with no valid arguments, and he seems not to recognise any natural condensation and rarefaction other than that which comes about by the force of heat and cold. Besides, the things that the Author of nature made in the beginning are not to be judged by the same law as those that are now made by other natural agents.

So for my part this opinion does not satisfy me, on a different, twofold ground. First, because the perfection of the Sun, the Moon and the Stars — that is, of bodies so noble, and of so great and so varied a power for influencing such varied sublunary mixed bodies — certainly seems to require that at least the Planets have a substantial form differing among themselves and from the heaven, one that virtually contains the lower mixed bodies; and even if the whole heaven of the Sun were condensed into one globe, one Sun would never be forged out of it. Secondly, because if any of us had been present at the

production of the stars on the fourth day, he would have had all those signs of a new substance produced that we have now when we see some substantial composite produced — namely a new shape, colour, light and active powers — and these moreover persisting together, with such constancy, up to this very day. Therefore Scripture is to be understood in a plain and simple sense, of stars made simply, that is, produced as regards their substance. Otherwise we could refer most of the things that are made anew to condensation and rarefaction.

VIII. [Margin: 3rd opinion: of substantial generation.] The Third, then, and true opinion is that the Sun and the other Stars were produced on the fourth day as regards their substance — not from nothing, but from some matter, of which shortly — in such a way that they received a proper and distinct substantial and specific form. This was the view of those who hold that the Luminaries and stars were produced from air, or from a bright cloud, or from water, or from fire, of whom we must speak in the following question. This mode of production is defended by Molina, *De opere sex dierum*, disputation 15; by Salianus at the 4th day of the world; and, before them, by Hugo Victorinus, chapter 6 on Genesis and book 1 *On the Sacraments*, part 1, chapter 15 — for he says that the form of the Sun was made from a bright cloud or from fire, as wine was made from water at the wedding at Cana; by Lyranus in the *Postills*, in the second exposition, who says: *Here the work of the fourth day is described; and it is understood that the Luminaries are made here as regards their specific substantial forms*; and by Tostatus, or Abulensis, on Genesis, who says: *Hence it is believed that the Sun was made from that little cloud, advanced substantially into another species, and accidentally in brightness and in the power of influencing.* It remains for us to investigate, from the Fathers and Doctors of the Church, from what matter the Luminaries and stars were produced on the fourth day.

Quæſtio 4. Ex qua Materia facta ſint ſeu producta quarto die Mundi, Luminaria & ſtellæ, Ex quintane eſſentia An ex materia Elementari?

Question 4. From what Matter were the Luminaries and stars made, or produced, on the fourth day of the World: from the fifth essence, or from elemental matter?

IX. [Margin: 1st opinion: of the matter of the simple heaven.] The First opinion was that all the Stars and Luminaries were made from the celestial body itself, or from the matter of the heavens condensed and intensified — but in such a way that the celestial body is simple, and not composed of any elements. So thinks Albert the Great, in the first part of the *Summa de quatuor coaevis*, question 4, article 21. [Margin: The opinion of Albert the Great and St Bonaventure.] This was the opinion of Aristotle and of almost all the Peripatetics, and so St Bonaventure, on book 2, distinction 14, in the Commentaries, said from that opinion: *It can be said according to the Philosopher that the luminaries were produced from an aggregation of light, or of their orbs. For just as scattered heat, if it were gathered into one, would become more intense and stronger, so, when the nature of light found in the orbs is gathered into one by the power of its own fashioning, a luminary is constituted; and accordingly a luminary is greater or lesser according as*

the aggregation of light in it is greater. The Peripatetics may be seen on book 2 of *De caelo*, chapter 7, or from text 41.

X. [Margin: 2nd opinion: the stars from earthy matter.] The Second opinion was that the Sun and the stars are clods of earth, or stones, snatched up and set alight by the whirling of the heaven. So Theodoret, in the book *On Matter and the World*, and Plutarch, book 2 *On the Opinions of the Philosophers*, chapter 13: [Margin: Thales.] *Thales said that the stars are earthy, but fiery.* [Margin: Anaxagoras.] *Anaxagoras said that the surrounding aether is fiery in its own essence, but that, snatching up rocks from the earth by the force of its whirling, it set them alight into stars with its own fire.* [Margin: Diogenes.] *Diogenes thinks the stars are like pumice, and are the breathing-holes of the world; and he also holds that invisible stones often fall from heaven and are extinguished — just as once a stony star, very like fire, fell at the river Aegos.* But Theodoret adds that Anaxagoras, accused by Cleon of having said that the Sun is a glowing plate, was condemned by the Athenians as one of the atheists. Diogenes Laërtius reports similar things here and there in the lives of the philosophers already mentioned. Further, in the same Theodoret and in Plutarch, book 2 of the *Placita*, chapter 20: *Anaxagoras, Democritus and Metrodorus said that the Sun is a globe of glowing iron, or a fiery rock; and Epicurus, an earthy compaction, like pumice, fiery within its eaten-out passages.* [Margin: Euripides.] So Euripides, the disciple of Anaxagoras, in the tragedy *Phaethon*, called the Sun a golden clod. It is of these authors, I believe, that Seneca speaks in book 7 of the *Natural Questions*, chapter 1. For when he had said, *No one could inquire into anything more magnificent, or learn anything more useful, than the nature of the stars and constellations — whether it is contracted flame, as our sight too affirms, and as the very light flowing from them and the heat coming down from there affirm*, he went on: *Or whether they are not fiery orbs, but certain solid and earthy bodies which, gliding through fiery tracts, draw their splendour and heat from there, bright not of their own; in which opinion there have been great men, who believed that the stars are compacted from something hard and feed on a fire not their own.* [Margin: Philo's censure of this opinion.] Philo, however, sharply rebukes them and condemns authors of this kind to eternal fires, in the book *On Dreams*. For he says: *What are the stars themselves? Are they fiery masses of earth? For some say they are fiery hills and glades. They themselves deserve the mill and the prison, where these instruments exist for torturing the impious who are like them.* These, then, we pass over — above all because they have no Fathers or Doctors of the Church as their patrons.

XI. [Margin: 3rd opinion: the stars from water, or air, or vapours.] The Third opinion was that the Sun and the stars are of the matter of exhalations and vapours, and hence partly watery, partly airy. The Stoics held that the stars are fiery and feed on vapours. [Margin: Pliny.] So Pliny, book 2, chapter 9, especially about the Moon; for he says: *The stars undoubtedly feed on earthly moisture, and the Spots of the Moon are nothing other than the dregs of the earth, snatched up with the moisture. For things consist of those things by which they are*

nourished. [Margin: Xenophanes.] And in Plutarch, book 2 of *De placitis*, chapter 13: *Xenophanes [held] that the stars are compacted from fiery clouds, extinguished by day, and at night glow again like coals; and that their risings and settings are kindlings and extinctions.* [Margin: Zeno.] Zeno Citticus [Zeno of Citium], in Laërtius, book 7, said that the Sun draws from the sea the nourishment of intelligible — that is, most subtle — fire. But, to consult the sacred authors: [Margin: Comestor.] Petrus Comestor, chapter 6 of the *Historia Scholastica*, says: *Of that bright cloud it is handed down either that it returned into the matter from which it had been made — like the star that appeared to the Magi, and the dove in which the Holy Spirit was seen — or that it always accompanies the Sun, or that the solar body was made from it.* [Margin: Tostatus.] Similarly Tostatus on Genesis: *Hence it is believed that the Sun was made from that cloud*, etc. [Margin: Hugo Victorinus.] Further, Hugo Victorinus, in the notes on Genesis, chapter 6, says: *But all the Stars, like the Moon too, were made of airy matter, and only reflect light; they do not shine — that is, they do not shine by their own light, but by light received from the Sun, which is held to have been made from light, or from a bright cloud.* [Margin: Eugubinus.] But more safely Steuchus, in the *Cosmopoeia*, saying: *This is most true, and asserted also by the most ancient Theologians, that in the same way there was drawn out of the waters the ethe-*

real substance, and the heaven itself, and the heavenly breathings. For we see water changed into air, into clouds, into torches and fiery vapours. It must therefore be thought that this was done at that same time, when the wisdom and genius of God was borne over the waters and drew purer substances out of them — first the Sun, whose elaborate size and beauty surpass all things celestial and terrestrial. Stirring, then, and shaking the waters he had created, he separated them, and kindled fire, and the purest lights, as of mirrors and of clouds, etc.; and he adds: *Flame is nothing other than kindled air.*

XII. [Margin: 4th opinion: that the stars are fiery.] The Fourth opinion, and the most celebrated, was that the Sun and all the other stars are fiery and produced from fire, or consist of a fiery form. Among more recent writers it has been embraced by Ascanius Martinengus in the *Glossa magna*, page 875; Steuchus in the *Cosmopoeia*; Salianus in the *Annales Ecclesiastici*, at the 4th day of the World, from number 4; and Christoph Scheiner in his *Rosa Ursina*, book 4, part 2, chapter 17; Boulliau, book 1 of the *Astronomia*, chapter 7; Franciscus Resta, tractate and book 1 of the *Meteorologica*, chapter 10; Tellez, disputation 44 of the *Physics*, section 3, number 14 — although he thinks here that certain stars consist of other elements. [Margin: Empedocles, Zeno, Cicero, Trismegistus, Hermes, Xenarchus, Plato.] So, long ago, Empedocles declared the stars to be fiery and to consist of fire, as Plutarch relates, book 2 of *De Placitis*, chapter 13; and so did Zeno with the Stoics, whom Cicero joined on this point, book 2 of *De natura Deorum* and the book *On the Dream of Scipio*. Trismegistus held the same in the *Pimander*, chapter 10; Hermes, in Cyril, book 2 *Against Julian*; Xenarchus, in Simplicius, book 1 on *De caelo*; Seneca, book 7 of the *Natural Questions*; Germanicus, the interpreter of Aratus, who says: *The Sun meanwhile,*

while it is fiery, grows hotter still because of the excessive motion of its revolution; and the Philosophers say that its fire is nourished by water; and Manilius, book 1, from the verse *It marks out fiery forms, and fires answer to fires*, etc. Plato too held them to be fiery for the most part, but sharing also in the other elements, after the manner of a glue. Granted that Aristotle argues against him in book 2 of *De caelo*, chapter 7.

Sacred Scripture certainly favours this opinion when, in Ecclesiasticus 43, the Sun is likened to a furnace of burning: *The Sun when it appears, announcing at its going forth, an admirable vessel, the work of the most High. At noon it burns up the earth, and who shall be able to endure the face of its heat? Keeping a furnace in works of heat; the Sun three times as much, burning up the mountains, breathing out fiery rays, and shining with its rays it blinds the eyes.* Taken literally, these words signify that the Sun is a kind of furnace of heat, and so hot formally. So too the Church, in the Vespers hymn for the 4th feria:

> *Who on the fourth day, while you kindle the flaming wheel of the Sun*
> —

unless you think this said poetically. So Juvencus sang in the prologue of the first book of the *Evangelical History*:

> *Nothing is held immortal in the frame of the World, not the sea, not the land, not the fiery stars of heaven.*

And in book 3 he calls the Sun *Ignicomus*, "fire-haired"; and Proba Falconia, in the *Centos*:

> *For there were not yet the fires of the stars, nor the bright aether.*

And Dracontius, book 1, *Hexaemeron*:

> *Soon he bids the globe of the Sun shine with the fire of salvation, and bids the Moon go second from its fire.*

But the Fathers certainly seem to have spoken not poetically but properly. [Margin: Philo.] The most ancient of them, Philo, in the book *On Dreams*: *There are certainly some who say that the Moon is not a globe of pure aether, as the other stars are, but mixed of aether and air; and that the spot appearing in it, which some call a face, is nothing other than the admixed air, which by its own nature is believed to be black all the way up to the heaven.* [Margin: St Justin.] St Justin, in the book on the refutation of the doctrines of Aristotle, teaches that the Sun cannot heat unless it is itself hot. [Margin: Tertullian.] Tertullian, in the book *On the Soul*, teaches that the Sun is endured by the eyes of the eagle, while it is invisible to owls — *not because it is incorporeal, but because the visual power fails*; and he adds: *For the Sun is a body, since it is fire; but what the eagle would confess, the owl would deny.* [Margin: St Basil.] As for those who deny that the Sun is hot, see with what a brand St Basil marks them, homily 3 on the *Hexaemeron*: *Although*, he says, *they do not even grant that the Sun is hot: such abundance of time overflows for them for ravings and old wives' trifles*, etc. [Margin: St Ambrose.] St Ambrose, following him closely, book 2 of the *Hexaemeron*, chapter 3: *We often see the Sun moist and dewy, by which it gives evident sign that it has taken to itself a nourishment of waters for its own tempering. But there is in them such zeal*

for attacking the truth that they deny the Sun itself to be of a hot nature, etc. And he goes on to say that the Sun is therefore now in one place, now in another, lest, if it rested in one place, it should burn up everything there.

[Margin: St Cyril of Jerusalem.] St Cyril of Jerusalem joins these, catechesis 9: *The heaven is water, and in it the Sun is fiery, and the Moon and the stars; and how do fiery things run in water? But if anyone doubts this because of the contrary natures, let him remember that fire in Egypt, in the time of Moses, burning in the hail, and consider the most wise workmanship of God.* [Margin: St Cyril of Alexandria. Hermes.] The other Cyril too, that is, of Alexandria, book 2 *Against Julian*, on the opinion of Hermes, has this: *Hermes, in the books of the Diexodica, in the first discourse, said: The Lord of all said, by his holy and intellectual Word, which has the power of founding: Let there be a Sun; and at once fire appeared, which by nature is borne upward — pure, I say, and most brilliant, and most fertile.* [Margin: St Nyssenus.] Most clearly of all, St Gregory of Nyssa, in the *History of the Six Days*, teaches that the light made at the first was fire, separated from the mass of the other elements, but having diverse qualities: one for constituting the Sun, another the Moon, another the other stars. [Margin: Caesarius.] Add to these Caesarius, who says expressly, in questions 68 and 69, that the Firmament is crystal, which was to receive the Sun and the Moon and the company of the other stars, *which has its nature from fire*; so, lest what is fixed should be dissolved by their heat, by means of it he separated that boundless water from it, placed as it were upon its shoulders to cool the inflammation. [Margin: St Chrysostom.] No less absolutely St John Chrysostom, in the homily to the Neophytes: *The nature of those stars is fiery, and these are taught to obtain a fiery nature*; and in homily 6 on Genesis he calls the Sun the element of fire: *For the Sun not only heats, but also dries; and not only dries, but also scorches, and furnishes us with many other and varied uses. And so this element is a great wonder.* And on Psalm 134 he teaches that fire is mixed in everywhere: *And there is fire: this in the Sun, another in the Moon, another in lightning, another in the air, another that in lamps, and that which is made from wood*, etc. [Margin: St Augustine.] And St Augustine, in the appendix, sermon 59: *Sun, heat and brightness are three words, and the three are one. What is bright is hot, and what is hot is bright*, etc. [Margin: Arnobius.] Likewise Arnobius, *Against the Gentiles*, book 2: *Can you, when asked, make plain what the Moon is, what the stars are?* etc. *Why have these little fiery fragments come together through the whole body of the World?* And book 8: *Although fire kindled the stars*, etc. [Margin: Lactantius.] And Lactantius too, inveighing against the gentiles who adore the stars, book 2, chapter 6: *Now how contradictory and absurd it is that, while they affirm the heavenly fires and the other elements of the world to be Gods, they likewise call the World itself God!*

[Margin: Anastasius Synaita.] Anastasius Synaita, book 2 of the commentary on the *Hexaemeron*, indicates clearly enough that he held the same opinion, in these words: *For he who made the heaven firm out of waters, and hung the earth upon the waters, could also have kept the firmament indissoluble without water, even from the fire of the Sun.* [Margin: Theodoret.] And what of Theodoret, most

expert in Greek opinions? Having related, in the book *On Matter and the World*, the opinions of the Philosophers on the fiery nature of the stars, he then says, in book 1 *On Providence*: *For this reason, then, although so great a fire revolves around this — namely the Firmament — that is, the fire of the Sun, the Moon and the other stars, yet in so many revolutions of years it is neither melted and poured off, nor dries up, nor burns, unconsumed.* For by fire, as he continues in the same place, metals are melted, mud and marshes dried, stones baked, wood and straw set alight. *But the nature of the heaven suffers none of these things; for what appears icy in it is not melted and poured off, although so great an abundance of fire is present to it*, etc. And on Psalm 148: *He commanded the immense fire of the luminaries to run between the waters of the Firmament compressed into a cloud. Water and fire are opposed to each other; and yet neither is the fire extinguished by the water, nor is that compressed water consumed by the immense fire.* [Margin: Procopius.] Procopius, chapter 1 on Genesis, subscribes to this entirely. For when he had said, *The Sun, Moon, Stars, whose substance is fiery*, he says: *Lest they should dissolve the congealed heaven, an abundance of waters was placed by God in the upper region of the heaven, so that it might keep it unharmed by the heat of the fire.* [Margin: Alcuin.] [Margin: Bede.] In the same way Alcuin, in the questions on Genesis, and Bede, in the book *On the Nature of Things*, chapters 5 and 19, reply to the question of what nature the heaven and the stars are, that they are *of a fiery and subtle nature*. [Margin: St Anselm.] And St Anselm of Canterbury, book 1 *On the Image of the World*, chapter 23, says: *The Sun is spherical in form, fiery in nature, surpassing the earth eight times in size* — unless perhaps we should read *eighty times — the body of the Moon is globular, fiery in nature, but mixed with water.* [Margin: Isidore.] Further, St Isidore, book 3 of the *Etymologies*, chapter 48: *The Sun, while fiery*

, grows hotter still from the excessive motion of its revolution. And Blessed Peter Damian, sermon 1 on the Epiphany: *For a star is of a fiery nature.* [Margin: The Master.] As for the Master's statement, in book 2, distinction 14, that St Augustine agrees with those who say that the stars and luminaries were made of fire, it is perhaps drawn from the book *De Genesi ad litteram*, chapter 4, and from book 2 of *De Genesi ad litteram*, chapter 3. And certainly St Thomas and St Bonaventure, on book 2 of the *Sentences*, distinction 14, question 1, article 1 or 2, and Tannerus, *On the Heavens*, questions 2 and 3, say that St Augustine and a number of the Saints held, following Plato, that the celestial bodies were made of the elements, or of fire.

It is clear, then, how many weighty authors this opinion has on its side; and so we can by no means oppose it, if we ought to revere antiquity and the chief masters in the Church more than certain more recent writers who are too much devoted to the Peripatetic school. Had these men seen the many faculae which the Sun continually throws out — as our age has seen by the benefit of the telescope, according to what was said in book 3, chapter 3, and is to be said more fully here in section 4, chapter 12, numbers 19 and 20 — they would surely have changed their opinion. And indeed, as Scheiner rightly argues in book 4 of the *Rosa Ursina*, part 2, chapter 22: since Scripture says, in Ecclesiasticus

17, *What is brighter than the Sun?*, and the faculae that appear in the Sun are brighter than the rest of the Sun's surface, then if they were not parts of the Sun, something would be brighter than the Sun. And no one who has looked at the Sun through a telescope will deny that these faculae are formally flames.

This is supported, in the first place, by the Hebrew name *Or*, or *Ur*, which signifies the Sun and light, as Pagninus teaches in the *Thesaurus linguae sanctae*, from page 46. From the word *Or* is derived *Maor*, which signifies a luminary, as we have it in Genesis 1, verse 16, and Psalm 74, verse 16. Hence the Greeks probably took the name οὐρανὸς [Greek: ouranos, "heaven"] as from a first root. But *Ur* signifies the flame of fire, and not only light, as is clear from Isaiah chapter 31, verse 9, and chapter 47, verse 14. So in Genesis 15 it is said, *I brought you out of Ur of the Chaldeans*, that is, out of the fire of the Chaldeans, as Cornelius à Lapide, with others, interprets it on chapter 11 of Genesis. The Hebrew names of the Sun also strongly support it: *Chamah*, which signifies heat; *Schemes*, which signifies "There is fire"; and *Cheres*, which signifies dryness. That the Sun is hot formally, and not only virtually, is taught by Tellez, Resta, Scheiner and Salianus, cited above.

Quæſtio 5. Cur Moyſes Solem & Lunam Luminaria Magna; & illum luminare maius, hanc luminare minus appellarit.

Question 5. Why Moses called the Sun and the Moon great Luminaries, and the one the greater luminary, the other the lesser luminary.

XIII. This question is not useless to the astronomer, since the Copernicans misuse the words of Moses to establish that sacred Scripture speaks about physical things not with the rigour of truth but to suit the understanding of the common people, or according to appearance [?]; and so, when it says that the Sun rises, moves, and so on, it is to be understood according to appearance. "For look," they say, "Scripture calls the Sun the greater luminary, although it does not subtend more minutes in the sky, in its diameter, than the Moon; and it calls both a great Luminary — that is, according to appearance, since otherwise the Moon is in reality smaller than many Fixed Stars and than the other Planets."

But although we grant that the Moon is in reality of smaller bulk than other stars, and that it appears great on account of its nearness to the earth, it nevertheless deserves the name of a great Luminary. [Margin: Why the Moon is a great Luminary.] *First*, in its character as a Luminary, because it fills and illuminates the whole sublunary hemisphere with its light far more fully than all the Fixed Stars together. For this reason St Basil, in the *Hexaemeron*, homily 6; St Ambrose, book 4 of the *Hexaemeron*, chapter 6; Iunilius on the *Hexaemeron*; Procopius on Genesis; Comestor, chapter 6 of the *Scholastic History*; and Steuchus in the *Cosmopoeia* affirmed that these Luminaries are to be called great not so much comparatively as absolutely. Above all St Augustine, book 2 of *De Genesi ad litteram*, chapter 16, concludes in these words: *For certainly let them grant this much to our eyes: that it is manifest that it shines over the earth more fully than the rest, and that the day does not grow bright except by the light of*

the Sun; nor does the night, with so many Stars appearing, shine so, if the Moon is absent, as when it is illuminated by her presence. For this reason they were called φωστήρες [Greek: phōstēres, "luminaries"] by the Seventy translators — that is, not great bodies, nor great stars, but specifically great Luminaries. *Secondly*, it is great on account of its efficacy and its power, greater and more perceptible to sublunary nature, as St Thomas, part 1, question 70, article 1, reply to 5, and Vielmius, lecture 21 on Genesis, teach together with St Chrysostom. *Thirdly*, from its fellowship and likeness with the Sun, which is absolutely great: the full Moon appears equal to it, so that if the Sun is great, the Moon too can be called great relatively to the Sun. *Lastly*, the Moon cannot be understood to be great in real magnitude if it is compared with many Planets and Fixed stars, because that conflicts with the plain demonstrations of the Astronomers; and therefore we are compelled to take the word "Great" here according to appearance, with St Thomas on book 2 of the *Sentences*, distinction 15, question 1; Carthusianus on Genesis, article 12; and Cajetan and Tostatus on chapter 1 of Genesis.

But when sacred Scripture says that the Sun moves and the Earth stands still, no inconvenience follows from these words taken literally, nor any manifest absurdity; and therefore it must be understood not merely according to appearance but according to truth: that the Sun really moves and the Earth really stands still. One is ashamed, indeed, to report the fables of the Talmudists, who say that the Moon was at the beginning of equal bulk with the Sun, but was afterwards diminished because she dared to ask God that the Sun's splendour be lessened; or because an Angel flying through the air touched the Moon with his wing and darkened her — as Vives [Juan Luis Vives], book 3 *On the Truth of the Christian Faith*, Lyranus in the *Postills*, Tostatus on Genesis, and Carthusianus in the same place, article 12, report.

XIV. God, moreover, called the Sun the greater Luminary: *First*, by reason of its bulk, since the Sun is in reality the greatest of all the stars, and is not merely twenty-seven times greater than the earth, as Anaximander thought (on the report of Plutarch, book 2 of *De placitis*, chapter 2, and of Theodoret, book 4, which is *On Matter and the World*), but much greater, as is clear from what was said in book 3, chapter 11. *Secondly*, inasmuch as it is a Luminary — that is, by reason of the splendour with which it rules the day and surpasses the splendour of all the stars even taken together. So the Sun is named *Sol* by the Latins, because it alone [ſolus] shines and, having overwhelmed the other stars with its brilliance, alone appears. *Thirdly*, because it is the fountain of light, and therefore greater not only than the other planets but also than the Moon — that is, more independent, because the other Planets receive their primary light from the Sun, as we showed in book 7, section 1, chapter 5. *Fourthly*, by reason of its position, because it stands in the middle of the Planetary system, or in the fourth place. *Fifthly*, by the simplicity of its motions, since it never departs from the Ecliptic, and is for that reason the rule of the other motions and the regulator of the times. [Margin: Psalm 18.] Therefore [?] David rightly likens

it not only to a bridegroom coming forth from his chamber but also to a giant, and for a number of reasons it was called the greater Luminary by God.

[Margin: Encomia of the Sun, and its encomiasts.] Accordingly we read those rivers of sacred eloquence poured out in its praise — praise which redounds at last to the praise of its maker: Gregory Nazianzen, oration 2 *On Theology*, which is the 34th; Basil, homily 6 on the *Hexaemeron*; Ambrose, book 4 of the *Hexaemeron*, chapter 1; Chrysostom, homily 6 on Genesis; Tertullian, book 2 *Against Marcion*; to whom add Cicero, *On the Dream of Scipio*, and Macrobius on it, and Pliny, book 2, chapter 6. Gathered together from these authors, the Sun is called the eye of the world, the heart of the heaven, the delight of the day, the beauty of nature, the prince of the Planets, the King of the stars, the regulator and norm of the times, the fountain of lights; and by Pliny, though not very aptly, the soul and mind of the whole World. More correctly and more magnificently Ecclesiasticus, chapter 43, calls it *a wondrous vessel, the work of the Most High*.

[Margin: The chariot of the Sun taken from the chariot of Elijah.] As for the Poets and Painters having given it a four-horse chariot, listen to St Chrysostom, who for that reason compares it with Elijah — or compares Elijah with the Sun, which in Greek is called Ηλιος [Greek: hēlios, "sun"] — and says that the occasion of this fable was taken from the rapture of Elijah. For in the homily on the ascension of Elijah he says: *Hence, I believe, the Poets and painters took their models in depicting the image of the Sun: he who, himself glowing and radiant, in a chariot with shining horses, is raised from the wave of the Ocean and, escaping between the steep crags of the mountains, seems to ascend as it were to the heavens, is brought in in the likeness of its light. For the Sun in the Greek tongue is called Helios. Hence Elijah is truly Helios, since in a chariot with horses shining with fire he ascends from the wave of the Ocean, that is, from the turmoil of the World, advancing by the crags of the mountains, that is, by the hardships of great labours, borne up to the heavens.*

[Margin: The horses of the Sun.] The names and offices of the four horses of the Sun, according to Fulgentius, book 1 of the *Etymologies*, are taken from the four seasons of the year, or rather from the division of the ancient hours. For *Erytraeus* in Greek means "reddening", because in the morning the Sun rises ruddy; *Acteon*, again, means "shining" and "purged of vapours", which happens about the third hour of the day;

Lampos, when, the sixth hour completed, it reaches the navel of the day; and finally *Philogeus*, when at the end of the ninth hour it inclines downward towards the earth — for *Philogeus* means, as it were, "loving the earth".

Quæſtio 6. An Sol quarta die poſitus ſit in puncto cæli Solſtitiali, an in Aequinoctiali, & cuius Aequinoctij? Luna verò in Plenilunione, an in Nouilunio?

Question 6. Whether the Sun, on the fourth day, was placed at the solstitial point of the heaven or at an equinoctial point, and of which Equinox? And the Moon: at Full Moon, or at New Moon?

XV. [Margin: The star was made in the heaven, not outside the heaven.] I assume here that the words *He set them in the firmament of heaven* do not mean that God made the Luminaries and stars outside the heaven and then carried them over into the heaven — as Anastasius Synaita alone among the Fathers, so far as I know, said, book 4 on the *Hexaemeron*. They mean that he set them by making them, or made them by setting them, in the heaven itself, as St Augustine, *On Genesis, the unfinished book*, chapter 13, and Procopius and Catherinus on Genesis learnedly explain; and that this *set* is to be referred not so much to their placement as to the end and office for which they were made, and is to be joined at once with the words *that it might shine*, etc. This assumed, the question whether in the first instant in which it was established the Sun was at the summer Solstice, as Mercator and Kepler hold, or at the equinoctial point of the vernal Equinox, as many hold, or rather at the point of the Autumnal Equinox, belongs to the controversy whether the World was created in Spring, in Autumn, or in summer. We treat that at length in book 2 of the *Chronology*, chapters 4 and 16, and to those places we refer the Reader.

[Margin: Whether the Moon was established at Full Moon or at New Moon?] As to the second controversy, about the Moon: Anastasius Synaita, in the Commentaries on the *Hexaemeron*, book 4, says: *When God had made these two Luminaries, he at once fixed the great one, namely the Sun, at the East of the firmament, and the Moon at the West* — from which it follows that the Moon was opposite the Sun and full of light. Moreover the Fathers of the synod of Palestine, as Bede reports in volume 2, in the short work *On the Vernal Equinox*, meeting under the presidency of Theophilus by the authority of Pope Victor, concluded that the World was made in Spring, because of the words of Genesis 1, *Let the earth bring forth green grass*; and at the Equinox, because for the words *and he divided the light from the darkness* they read *He divided between light and darkness equal parts*; and that on the fourth day of the World there was a Full Moon, because God made both a great Luminary — that is, perfect in its apparent magnitude — and ordered the Moon to rule the night; but it would not have begun to rule the night unless it had shone the whole night, and so had had full light. Of the same opinion were not only Bede himself, on the *Hexaemeron* and in the book *On the Reckoning of Time*, but also Procopius on chapter 1 of Genesis; St Damascene, book 2 *On the Faith*, chapter 7; Abulensis on chapter 1 of Genesis, question 25; and Lyranus in the *Postills*; and St Thomas thinks the same probable, part 1, question 70, article 2, reply to 5.

On the other side St Augustine, book 2 of *De Genesi ad litteram*, chapter 15, although he defines neither absolutely, and affirms that *whether God made the Moon at its first day or full, he made it perfect*, nevertheless inclines to the New Moon. For it ought to begin at the beginning of its month; otherwise it would have begun to move not on the first but on the fourteenth day of the Moon, and, once made, it would at once have begun to wane, when it ought rather to wax. But Pererius, book 1 on Genesis, page 93, and Suarez, book 2 of *De opere sex dierum*, chapter 9, number 10, touch on this question and settle on

neither side. Suarez adds, however, that some, taking a middle way, said that the Moon was made at quadrature, that is, as it usually shines on the seventh day from its synod with the Sun. [Margin: Suarez's slip in Astronomy.] But the reason which he at once adds contains a blunder and a slip of the pen, or of the imagination, in Suarez. For he says: *In the disposition which it usually has on the seventh day: because it then rises above our horizon when the Sun sets, and so could illuminate the whole night.* Yet on the seventh day the Moon is on the Meridian, or near it, when the Sun sets, and illuminates then not the whole night but half of it.

[Margin: The Author's opinion.] For my part, if we look to authority, I incline to the Full Moon; indeed, I add other reasons too. Just as God willed that this should be the first month of the spring season, whose full moon fell either on the day of the vernal Equinox or after it — as I taught, with the Fathers and the Catholic Church, in book 1 of the *Chronology*, chapter 28 — so it is very probable that the first year of the World began at the vernal Equinox, and that he placed the Sun either at the beginning or in the fourth degree of Aries, so that together with that beginning he joined the fourteenth-day Moon, that is, the Full Moon. Thus afterwards, in the waning of its light, the Moon would show itself to be the lesser luminary by this title too, or would be *the Luminary that wanes at its consummation*, that is, when it has reached the highest perfection of its apparent light. And it was fitting that the aspect of the Moon to the Sun at the beginning of the World should be such as it was when the World was restored through the death of CHRIST.

Now, with the cycle of sabbatical years carried back to the beginning of the World: if we wish the World to have been established in one of those years that were Sabbatical, and in such a way that the Full Moon or the New Moon fell on the day of the vernal or the autumnal Equinox; and the World to have been created in the century that preceded the Epoch of Christ by 4000 years, as we showed in book 2 of the *Chronology*, chapter 15 — then from astronomical calculation only four years present themselves to me as probable for the beginning of the World, on account of the characteristics already mentioned. I have chosen to show them in the following small table, in which we have computed the years and months in the Julian manner, adding the weekday on which the Equinox and the new or full Moon fell together.

Years before the first year of CHRIST, all Sabbatical	Concurrence of the lunar syzygies with the day of the Equinox, and their time	D.	H.	Week-day
4042	☌ New Moon, October	23	15	1
4049	☍ Full Moon, October	26	6	4
4056	☍ Full Moon, April	19	0	7
4063	☌ New Moon, April	21	14	1

CAPVT V.

Chapter V.

CAPVT V.

Sitne Cælum Simplex corpus, an Compoſitum ex Forma & Materia diuerſa ab elementari, an eiuſdem rationis cum elementari? & ſi ex hac, Ex quonam elemento?

Whether the Heaven is a Simple body, or Composed of Form and of Matter different from the elementary, or of the same kind as the elementary? And if of the latter, of which element?

Quæſtio 1. De Simplicitate & Compoſitione Cæli.

Question 1. On the Simplicity and Composition of the Heaven.

I. [Margin: 1. Opinion: on the simplicity of the heaven.] The FIRST Opinion was that every heaven is a simple body, that is, not composed of matter and form really distinct. First, because Aristotle seems to have asserted it; next, because we have no argument from change by which to infer that there is in the heaven matter receptive of contraries, or of an alteration tending to the corruption of one substantial form and the generation of a new form; again, because that matter, if it were truly matter, would seek one form after another, and since these are denied it, that appetite would be in vain, and it would be violently and perpetually deprived of them; finally, the perfection of the Universe requires that there be in it this grade of substance too, that of a fifth essence and of an altogether simple body. For it cannot be demonstrated that a created corporeal substance which is simple is impossible. [Margin: Authors for the simplicity of the heaven.] So judged Averroes, in the book *On the Substance of the Orb*, chapter 2, and on book 8 of the *Metaphysics*, comment 12, on book 12, comment 20, and on book 1 *On the Heavens*, comment 20; Zimara [Marcantonio Zimara], theorem 107; Faber, theorem 59; Antonius Andreas on book 8 of the *Metaphysics*, question 4; Iandunus [Jean de Jandun] in the same place, question 7, and on book 1 *On the Heavens*, question 23; Saxonia [Albert of Saxony] on book 1 *On the Heavens*, question 4, article 1; Caietanus Thienensis [Gaetano da Thiene] on book 1 of the *Physics*, question 21; Piccolomini [Francesco Piccolomini], *On the Heavens*, chapter 106; Achillinus [Alessandro Achillini], book 1 *On the Orbs*, question 1; and, among the Scholastics on book 2 of the *Sentences*, Durandus [Durand de Saint-Pourçain] and Gabriel [Gabriel Biel], distinction 12, question 1 or 2; Maironus [François de Meyronnes], distinction 14, question 2; and Bassolius, question 1, article 4. And certainly Aristotle, if he is understood in the plain sense and without forced interpretation, teaches in book 1 *On the Heavens*, from text 20 and from text 121, that the heaven is ingenerable and incorruptible, like the Intelligences, because it has no matter; and in the same place, chapter 2, he infers from simple and circular motion that it is a simple body; and in book 12 of the *Metaphysics*, text 10, where he treats this question on purpose, he concludes: *But all things that change have matter, but a different matter; for even of the everlasting things themselves, those that are not generable but movable by locomotion have matter — not, however, matter apt for generation, but for motion from a place and to a place.*

II. [Margin: 2. Opinion: on the composition of the heaven.] The SECOND Opinion was, and is, that the heaven is composed of Matter and substantial form. To some, an argument for such a composition has seemed almost necessary: both because it falls under the senses, and because of quantity, rarity, density, local motion and, in short, physical change. Thus, finally, besides the philosophers and the Fathers who say that the heaven consists of one or more elements, [Margin: Authors of the 2nd Opinion.] this is the view of St Thomas, part 1, question 66, articles 1 and 2, on book 1 *On the Heavens*, lecture 6, and on book 8 of the *Physics*, lecture 20 — granted that earlier he seems to have inclined to the former opinion, on book 2 of the *Sentences*, distinction 12, question 1, article 1, on book 8 of the *Metaphysics*, chapter 14, and in the *Disputed Questions*, the question *On Spiritual Creatures*, article 6, reply to 2. So too Avempace [Ibn Bājja], as reported by Albert on book 1 *On the Heavens*, tractate 1, chapter 3; Avicebron [Solomon ibn Gabirol] in the book *Fons vitae*; Plotinus, book 1, Ennead 2, question 1; Simplicius on book 2 *On the Heavens*, comments 3 and 35; Philoponus on the 6th and 13th of Proclus; Avicenna, book 1 of the *Sufficientia*, chapter 2; Niphus [Agostino Nifo], *On the Substance of the Orbs*, comments 39 and 42; Soncinas on book 12 of the *Metaphysics*, question 7; Iavellus [Crisostomo Javelli] on book 8 of the *Metaphysics*, question 12; Soto [Domingo de Soto] on book 2 of the *Physics*, question 1; Flandres [Dominic of Flanders] on book 8 of the *Metaphysics*, question 7; Paulus Venetus [Paolo Veneto], *On the Heavens*, chapter 1, who attributes it to Themistius and Theophrastus; Aegidius, in the treatise *On the Matter of the Heaven*; Gianninus, *On the Nature of the Heaven*, part 1, chapter 21; the Coimbra commentators, *On the Heavens*, book 1, chapter 2, question 4; Rubius [Antonio Rubio] in the same place, question 5; Suarez, disputation 13 of the *Metaphysics*; Hurtadus [Pedro Hurtado de Mendoza], disputation 1 *On the Heavens*, section 1; Moris., disputation 2 *On the Heavens*, doubt 2; Amicus [Francesco Amico], tractate 4, question 2, doubt 2; Tannerus, *On the Heavens*, question 3; Arriaga, disputation 2 of the *Physics*, section 11, subsection 2; and Oviedus, the single controversy *On the Heavens*, part 2.

Among the Scholastic Theologians, moreover: Capreolus and Ocham [William of Ockham] on book 2, distinction 12, question 1; St Bonaventure on book 2, distinction 12, article 2, question 1; Aegidius on book 2, distinction 12; Alensis [Alexander of Hales], part 2 of the *Summa*, question 44, member 2; Hervaeus [Hervaeus Natalis], *Quodlibet* 4, question 3; Albert the Great, part 1, question 4, *On the Four Coevals*, article 3; and Carthusianus on book 2, distinction 14, question 1, who says that the opposite is erroneous, indeed contrary to Aristotle. For they try to draw Aristotle to their opinion, on the ground that in book 8 of the *Metaphysics*, texts 3 and 10, he says that every sensible substance consists of matter; and in book 1 *On the Heavens*, chapter 9, from text 93 to 95, that there cannot therefore be another heaven, because this one that we see consists of the whole of matter; and again, in book 12 of the *Metaphysics*, text 22, and book 2 *On Generation*, text 51, that the principles of all sensible things

are the same, matter and form. But in these places Aristotle takes matter for a corporeal and sensible entity which is the subject of sensible accidents, not for matter really distinct from the form, which he usually calls the matter of Generation. And when in book 7 of the *Metaphysics*, text 5, he seems to say that the heaven and the stars are from the elements, in fact he does not say this in the Greek text, but that these are substances, like the elements, the mixed bodies and all natural bodies.

III. [Margin: 3. Opinion: problematic, or distinguishing.] The THIRD opinion, then, either thinks each of the two former probable, as Zerbus [Gabriele Zerbi] does on book 8 of the *Metaphysics*, question 3, or distinguishes. For Scotus on book 2, distinction 14, question 1, Aversa, question 33, section 5, and Mastrius and Bellutus, disputation 2 *On the Heavens*, question 2, article 1, say that, if one looks to the authority of Aristotle, the Heaven is not composed of matter and form; but, if one looks to the authority of many theologians and to the truth, it is composed of matter and form. Ioannes Poncius [John Punch], on the other hand, himself also a Scotist, disputation 22 of the *Physics*, question 1, concludes that if reason is regarded it is uncertain whether the heaven is simple or composite, but if authority is regarded it is more probable that it is composite. I, for my part, use a threefold distinction, for which let the following stand.

I. CONCLVSIO.

Conclusion I.

[Margin: 1. Conclusion.] *The visible Heaven, if the authority of Aristotle is regarded, is a Simple body, that is, one having matter, but matter not really distinct from the substantial form.* This is clear enough from the passages of Aristotle cited at the end of numbers 1 and 2.

II. CONCLVSIO.

Conclusion II.

[Margin: 2. Conclusion.] *The visible Heaven, whether one regards the authority — resting on Sacred Scripture — of many Philosophers, ancient and recent, and of many Fathers, Doctors of the Church and Theologians, or the more probable reason, is not simple, but consists of matter and form really distinct from each other.*

For nearly all the Philosophers before Aristotle held that the heaven consists of one or more elements, or was generated from their matter, as will be clear from what is to be said in the following chapter. And very many of the Fathers affirmed that the heaven — either all of it, or indefinitely, or at least the Eighth sphere — was made of elementary water solidified in the manner of ice or crystal, and moreover that above the heaven there is an aqueous heaven consisting of true, elementary water. I have already cited them at length for this purpose in chapter 2, questions 1 and 3, where we also showed that this agrees closely with Sacred Scripture understood literally (as indeed it ought to be understood, since no inconvenience follows from it). Not a few, too, affirm that the starry

heaven is Fiery, indeed that it is the fourth element itself, fire; these I cited both in chapter 1, question 4, and in chapter 3, conclusion 4, argument 5. Finally, several assert that the Planets and the other stars are either fiery, or of earth, or of fiery vapours; these I reviewed in chapter 4, question 4, from number 10. To say that these authors speak metaphorically, or analogically, or because the stars are hot only virtually and not formally, is not to take their meaning in its plain and obvious sense; indeed it conflicts with the reasons of some of them, who thought that waters were kept in the heaven and above the heaven to temper the stars' heat. A distinction of this kind arises only from a presumed opinion about the simplicity of the heaven, whether on account of the authority of Aristotle or on account of the reasons indicated in number 1, which are presently to be resolved. Therefore the first part of the conclusion remains sufficiently proved.

[Margin: 1. Argument for the affirmative [?] side.] The second part of the conclusion is proved, *First*, from the new Stars and from certain comets, or hairy stars, some of which were more probably in the heaven, that is, above the Moon, as we showed in book 8 in the conclusions of the chapters — and indeed generated in the same way as the spots and faculae of the Sun are usually generated, according to what was said in book 3, chapter 4. [Margin: 2. Argument.] *Secondly*, because either the grades of beings are not to be multiplied without grave necessity and solid foundation (and this multiplication is greater than that of two really distinct entities within the order of composite being); or, if they are to be multiplied, and a simple substance is required for perfecting the Universe, it suffices that the Empyrean Heaven, which is invisible to us, be of this kind; nor is it necessary that every heaven, or any of the visible ones, be such. [Margin: 3. Argument.] *Thirdly*, although sensible Quantity, rarity, density, opacity, transparency, light, colour and the other accidents of this kind are not necessary indications of composition from matter and corporeal form, yet so long as we do not know of the simplicity from elsewhere, they are rather indications of such matter and composition. This is on account of the kinship they have with the other sensible accidents that accompany such a composition — indeed with themselves, taken in kind and considered in so many elements and mixed bodies, by reason of which the presumption and possession stand in favour of composition and of corporeal matter and Form.

Nor is a physically evident substantial transmutation required for this probable indication; otherwise it would no longer be a merely probable argument for composition, but an evident one. And those who think that the heaven consists of elementary matter, or of some element, will say that a sufficient indication of this change is had in that part of the element which is near the earth, and that it is not necessary that such changes occur in every part of it. For if there is some part of the earth, or of the air, never substantially transmuted since the beginning of the world, it is not for that reason to be called simple. [Margin: Solution of the objections.] And so one of the reasons usually brought for the first opinion is resolved. The reason drawn from the incorruptibility of the heaven will be resolved from what is to be said in chapter 6. As for the appetite

for other forms, which seems to be a property following from matter in the fourth mode — since otherwise, if matter sought only one form, it would not seem to have potentiality and indifference to several — it can be replied that such an appetite is in it only on the supposition that it is deprived of its former form, and that there is an agent which intends, and is able, to introduce another form; and God is undoubtedly an agent of this kind.

Quæſtio 2. De ratione Materiæ, ex qua Cælum conſtat.

Question 2. On the nature of the Matter of which the Heaven consists.

IV. [Margin: 1. Opinion: the Matter of the Heaven is different from the elementary.] The FIRST Opinion is that the Matter of the celestial Bodies is of a different kind from the matter of the bodies that are subcelestial or lower. Aristotle would certainly have defended this opinion (book 1 *On the Heavens*, chapter 2, and book 1 of the *Meteorologica*, chapter 3) if he had thought that the heaven consists of matter distinct from form. Taken absolutely, however, the holders of this opinion were Simplicius, Theophrastus, Alexander of Aphrodisias, Avicenna and Algazel [al-Ghazālī]; likewise Albert the Great on book 2 of the *Sentences*, distinction 14, question 2, and in book 2, part of the *Summa*, tractate 11, question 53; Alensis, part 2 of the *Summa*, question 50; Gulielmus Parisiensis, book *On the Universe*, part 1, chapter 39; St Thomas, part 1, question 66, article 2, and on book 2 of the *Sentences*, distinction 14, question 1, and in book 2 *Against the Gentiles*, chapters 18 and 20. He is followed there by Ferrariensis [Francesco Silvestri], Caietanus, Capreolus, Durandus, Bannes [Domingo Báñez] and Zumel [Francisco Zumel]; and likewise by Riccardus [Richard of Middleton] on book 2, question 8; Hervaeus; Zabarella [Jacopo Zabarella] in the short works *On the Matter of the Heaven*; Gianninus, *On the Nature of the Heaven*, part 1, chapter 21; Suarez, disputation 13 of the *Metaphysics*, section 11; Valentia [Gregorio de Valencia], part 1, disputation 5, question 2, point 1; Pererius, book 2 on Genesis, question 1; Fonseca [Pedro da Fonseca], book 5 of the *Metaphysics*, chapter 4, question 2; Amicus, tractate 4 *On the Heavens*, question 3, doubt 1, who adduces for it Flandres, Achillinus, Soncinas, Niphus, Mersennius, Pavesius and Piccolomini; Tannerus, *On the Heavens*, question 3; Rubius, book 1 *On the Heavens*, question 5; Aversa, question 33, section 5; and Poncius, disputation 22 of the *Physics*, question 2.

Among the Astronomers, too, Tycho Brahe, in the letter to Rothmann of the year 1588, August 17, page 106: although he admits a fluid heaven, he does not admit that it is of air, as Io. Penna [Jean Pena] and Rothmann contended. Indeed he says: *But that it is of air, or shares in any elementary nature, I shall by no means admit; for far be it that anything elementary, corruptible and variable, etc., be stitched onto the most pure and most perfect celestial nature, which is subject to no changes.* And shortly after: *for all unanimously and most certainly lay down that the heaven is wholly exempt from the number and nature of the four elements, and displays a certain more excellent nature; and therefore that it is a certain fifth essence, wholly different from those four elementary ones.* Then he taunts the Theologians

who, understanding Moses too little aright, said that the heaven consists of watery matter; and he concludes that, although it is uncertain of what nature the heaven is, and inscrutable to us, it is certain that it shares in no elementary nature. He confirms this in the letter of the year 1589, February 21, from page 137.

V. [Margin: Arguments for the 1st opinion.] The arguments for this opinion are these. *First*, the authority of St Dionysius, book 1 *On the Divine Names*, chapter 4, who says that the celestial bodies *are of such a nature and essence as can neither grow nor diminish, nor be changed in any part*; and of St Basil, homily 3 on Genesis, who says: *But we are not pleased to assert that the Firmament is made either of one of the simple bodies or as a mixture of them all: for we have already been so instructed by divine scripture itself that we may not think or imagine anything beyond what has been granted to our mind.* But from these passages nothing more is gathered than that the heaven is incorruptible, and not actually composed of elements.

Secondly, the same thing seems to be proved by the simplicity of celestial motion, the nobility of the spherical shape, the eminence of its place, the predominance of its influences, its independence from lower things, the perfection of the Universe, which requires this grade of matter, and above all the natural ingenerability and incorruptibility of the heaven. For it is to these heads that the reasons, set out at greater length by the champions of this opinion, are finally reduced. But some of these reasons are topical and have very slight probability; others prove only the nobility and excellence of the form, to which incorruptibility would be due. Therefore, although the incorruptibility of the heaven is defended better, and with greater dignity to the heaven, if it is said to have a different matter, as Suarez and Amicus say above, it still does not follow from natural incorruptibility that the matter of the heaven is of a different species from sublunary matter — as Arriaga notes, together with Molina [Luis de Molina], disputation 2 of the *Physics*, section 11, subsection 2. Granted, Suarez (disputation 13 of the *Metaphysics*, section 11, from number 14) contends that the heaven would of itself be corruptible no less than the elements if it consisted of the same matter: for that, he holds, it is enough that the matter have the privation of some form which it is proximately or remotely apt to have; and so the heaven would be generable *per se* from air, and air from the heaven, even if *per accidens* an agent capable of introducing such a form were lacking. The solution of this controversy therefore depends on the question of the incorruptibility of the heaven, which is to be treated a little below.

VI. [Margin: 2. Opinion: on matter of the same kind.] The *Second* opinion is that the visible heaven has matter of the same kind as the matter of sublunary things. This opinion was held not only by the ancient Philosophers, or by those more recent thinkers who supposed that the heaven and the celestial bodies consist of one or more elements, but besides them by Avicenna, book 1 of the *Sufficientia*, chapter 4; Avicebron, as reported by St Thomas; Scotus on book 2, distinction 14, question 1; St Bonaventure on book 2, distinction 12,

article 2, question 1, and distinction 14; Ocham in the same place, question 22; Aegidius in the treatise *On the Matter of the Heaven*, and on book 2, distinction 13; Gabriel on book 2, distinction 12; Abulensis on chapter 1 of Genesis; Moris., disputation 2 *On the Heavens*, doubt 3; Angestus, book 1 *On the Properties of Matter*, proposition 9; Molina, the tractate *On the Work of the Six Days*, disputations 2, 3 and 5; Scaliger, exercise 61; Mastrius and Bellutus, disputation 2 *On the Heavens*, question 2, article 2; Baltazar Tellez [Baltasar Teles] on the *Physics*, disputation 40, section 1, number 22; Salian in the *Annals of the Old Testament*, on the second day of the world; and Francisco Oviedus, the single controversy *On the Heavens*, point 2.

The arguments for this opinion are these. *First*, because a specific diversity of matters seems impossible, since every distinction of species and genera comes from the Form, according to Aristotle's saying in book 7 of the *Metaphysics*, text 49: *It is the act that separates, or distinguishes.* This saying, however, admits many explanations, and the opponents will say that substantial Form constitutes the Physical species, but not every species, or the metaphysical essential nature; and Suarez, disputation 13 of the *Metaphysics*, section 11, number 8, teaches at length that a specific diversity of matters involves no contradiction. *Secondly*, because if anything compels us to posit a different matter, it is the incorruptibility of the heaven; but this incorruptibility can stand even if the matter of the heaven is of the same kind. For its natural incorruptibility can belong to the heaven either by reason of a form that does not have contrary qualities capable of being overcome by the qualities of other bodies, or because it lies outside the hazard and the sphere of the sublunary agents — although that incorruptibility has not yet been demonstrated, nor is it demonstrable. *Thirdly*, and most of all: very many Fathers and doctors of the Church teach that the heavens or the celestial bodies are made of water or of fire. I have cited them at length, partly in chapter 1, question 4, partly in chapter 2, question 1, and partly in chapter 3, conclusion 4, argument 5; and St Thomas, on book 2, distinction 14, article 2, says that Saints Basil and Augustine and several of the Saints, following Plato, held that the heaven is of the nature of the elements.

VII. [Margin: 3. Opinion: suspending assent.] The *Third* opinion, on account of uncertainty, or of the equal weight of reasons and authorities on both sides, either teaches that each of the preceding opinions can be held problematically, or ventures to affirm nothing definitely. Thus the Coimbra commentators, book 1 *On the Heavens*, question 6, think each opinion probable as regards the matter (for as regards the nature taken from the form, they had already determined in question 5 that it differs from the sublunary); so too Hurtadus, disputation 1 *On the Heavens*, paragraph 15, although he inclines to the second opinion; and Arriaga, disputation 2 of the *Physics*, section 11, subsection 2.

Philo, moreover, in the book *On Dreams*, puts it thus: *The heaven, however, has an incomprehensible nature, and no true knowledge of it comes down to us. For what can we pronounce about it? That it is congealed ice, as some hold? Or the*

purest fire? Or a certain fifth body, movable in a circle, akin to none of the four elements? What of that outermost sphere of the fixed stars — has it solidity, or only a surface without depth, like figures painted on a plane? With equal restraint St Basil, speaking of the Firmament in homily 3 on the *Hexaemeron*, says that he is not pleased to assert that the heaven is made of one element or mixed from all, because he has learned from divine Scripture to think nothing beyond what has been granted. And St Damascene, book 2 *On the Faith*, chapter 6, after reviewing various opinions, concludes: *But there is no need to search into the substance of the heaven: it is unknown to us.*

But let us hear St John Chrysostom, homily 2 *On the Incomprehensible Nature of God*: *Of what substance the heaven consists we do not know. Let him who contends, and claims to know, say of what substance the heaven is. Is it some moisture congealed into frost? Is it a condensed cloud, drawn together into a thicker substance? Is it a gross and more corpulent air? No one, surely, can teach plainly what it is.* Let us also repeat the words cited elsewhere from his homily 4 on Genesis. *But what, then,* says Chrysostom there, *shall we say this firmament is? Condensed water, or compacted air, or some other essence? No prudent man would rashly assert this: for what is said must be received by us with great modesty and gratitude; and we must not go beyond our nature and search into the things that are above us, but only know and hold among ourselves this: that the firmament was produced by the Lord's command, making a separation of the waters, so that it can hold some beneath itself and carry others on its back.* Hugo [Hugh of Saint Victor] expressed altogether the same opinion in other words but with the same sense, book 1 *On the Sacraments*, part 1, chapter 18, concluding:

But now it does not seem to me that I should labour much over questions about these things, which no reason comprehends and no authority to which faith must be given proves. Having weighed these passages, Ascanius Martinengus in the *Glossa magna*, although on page 605 he admits that God made the starry heaven on the second day from the elements or from chaos — if the plain sense of the Mosaic history and the views of the Fathers are regarded — nevertheless concludes on page 607 that the substance of this heaven is unknown and imperceptible to us. Let this, then, be the

CONCLVSIO.

Conclusion.

[Margin: 3. Conclusion.] *Although it cannot be known by us demonstratively and evidently what the substance and nature of the visible heaven is, it is nevertheless more probable that it consists of matter of the same kind as the elementary.*

VIII. The first part of the conclusion is established both by the authorities adduced for the 3rd opinion and by the mere consideration of the arguments which we said are usually adduced for the 1st and 2nd opinions; for there is none of them that exceeds mere probability. And certainly, if we once admit that corruptible and incorruptible bodies alike can have the same matter, and that corruptibility is to be sought not only from the indifference of the matter

but also from the properties of the form — according as it has or does not have qualities corruptible by contraries — or even from the nearness and power of an agent, then no way remains for us of gathering with certainty, from any change or absence of change, that the matter is different rather than not different.

The second part is proved thus. That conclusion is more probable which is gathered from sacred scripture taken literally, and according to the interpretation of several Fathers, than its opposite, so long as the opposite is not deduced from valid arguments and demonstrative reasons, or from reasons that come close to demonstrative ones. But that the starry heaven consists of matter different from the sublunary is neither demonstrated nor urged by valid arguments approaching demonstration; on the other side, that it consists of watery or fiery or some other elementary matter is gathered from sacred Scripture taken literally, and from the Fathers, as I have already taught in chapter 2, questions 1 and 3, in chapter 1, question 4, and in chapter 3, conclusion 4, argument 5. Thus Carthusianus, on book 2, distinction 14, question 2, rightly pronounces it safer, on the basis of sacred Genesis and many Fathers, to assert that the heaven is of an elementary nature. And certainly, if above the Firmament there is a watery heaven, that is, one of water of the same matter as the elementary waters, and if the Firmament itself, taken for the 8th sphere — if not the whole starry heaven — is of elementary water, as we gathered from sacred Scripture and the Fathers in chapter 2, questions 1 and 3, who then would deny that the rest of the starry heaven below the eighth sphere of the Fixed stars is of the same matter? Nor ought either the authority of Aristotle in this matter, or one or another reason of fitness drawn from Physical or Metaphysical subtlety, move us to interpret Scripture and the Fathers violently. This is all the more so since Aristotle's opinion on the diversity of celestial matter, and on a certain fifth essence, sprang from an infected root — namely from the opinion of the eternity of the World, and of the necessity by which he thought the heaven and the World were produced by God — and since it tends to insinuate to us that the heaven cannot be destroyed even by God. [Margin: St Ambrose's warning against a fifth essence.] St Ambrose gravely and prudently warned us of this, in book 1 of the *Hexaemeron*, chapter 6, in these words: *Others, therefore, considering that these things could not be stable, introduced a certain fifth nature of body, by which they supposed that the substance of the heaven would long endure. But that opinion could not withstand the Prophetic sentence, which the divine Majesty of our God, the Lord JESUS Christ, also confirmed in the Gospel.* He then brings the words of Psalm 101, *The heavens are the works of your hands; they shall perish*, etc., and those of Christ in Matthew 24, *Heaven and earth shall pass away*; and he adds: *Those, therefore, accomplish nothing who think that a fifth, ethereal body must be introduced in order to assert the perpetuity of the heaven: for they see equally that a portion of one member, joined on unlike the rest, usually brings a blemish to the body rather than otherwise.*

QUÆSTIO 3. EX QUO AUT QUIBUS ELEMENTIS CÆLUM CONSTET.

Question 3. Of what Element, or Elements, the Heaven consists.

IX. Matching the number of the four Elements, we adduced four opinions on the nature of the stars as well, in chapter 4, question 4, from number 10. Now we shall report as many on the nature of the heaven itself, to be compared with those. Further, the opinions of the ancient Philosophers on the nature of the heaven are reported by Diogenes Laertius in their lives; by Plutarch, book 2 of the *De placitis philosophorum*, chapters 11 and 13; by Theodoret in the book *On Matter and the World*; by Clement of Alexandria in the books of the *Stromata*; by St Justin in the *Admonition*; and by Mirandulanus [Gianfrancesco Pico della Mirandola], book 1 of the *Examination of the Vanity of the Gentiles*, chapter 12.

[Margin: Authors for the 1st opinion.] The *First* opinion was that the heaven is fiery — indeed that it is that very element which we call fire, though pure, and because of the rarity of its substance not suited to burning. For this opinion, in chapter 3, conclusion 4, argument 5, I reviewed the Stoics, Heraclitus, Empedocles, Pythagoras, the Egyptians, Plato and Pliny the Elder, and, among our own, Saints Augustine and Anselm of Canterbury, Innocent III, Bede, Junilius, Hugo Carensis, Delrio [Martín Antonio Delrío], Paracelsus and Balthassar Tellez. But since I see that Plato's view is doubted, what I said there must be confirmed. [Margin: Plato's opinion on the fiery heaven confirmed.] Plato, then, says in the *Timaeus*: *God first created fire and earth; but two things alone cannot cohere well without some third, and both require some middle bond.* Then, after a little: *By this conjunction the World was so compacted that it can be seen and touched. Hence the body of the World was forged out of things of this kind, four in number.* And shortly after: *The World therefore received each of these four things entire; for its maker composed it from all the fire, air, water and earth, in such a way as to leave no part or power of any of them outside.* From these passages Plotinus, Porphyry, Taurus, Marsilio Ficino and very many Platonists gather that the World consists of these four bodies alone, and that, since the highest of them is fire, the heaven is nothing other than Fire. So too Philoponus, disputation 13 against Proclus, chapters 15 and 18, teaches that for Plato the heaven is the same as fire; and Nemesius in the book *On the Nature of Man*, and St Gregory of Nyssa, book 3 of the *Philosophy*, chapter 4, say that Plato asserts that the heaven which Aristotle calls the fifth essence consists of Fire and Earth.

Nor is it an obstacle that Plato counts five kinds of souls — earthy, watery, airy, fiery and ethereal — among which in the *Epinomis* he distributes the five regular bodies, or their five figures; or that in the *Timaeus* he says that that Fire is not a burning flame but a light that illuminates gently. From these passages Io. Picus, Maynettus and Simplicius (books 1 and 2 *On the Heavens*) try to reconcile Plato with Aristotle, as though they differed only in the word, and Plato in reality acknowledged a certain fifth essence in the heaven. For, as Philoponus replies, Plato does not distinguish the fifth body of the World from the four Elements adequately, but inadequately, as a whole from its parts taken

separately, and to this whole he assigns the fifth figure; and he calls Fire ether, as being a purer air, and because of its rarity least apt for burning. Add to these the very many whom I named in chapter 4, number 12, who assert that the stars are Fiery, or are fires; and Tycho in the *Epistolae Astronomicae*, page 137, who thinks indeed that the heaven is a certain fifth essence, but that if it did consist of any element, it should rather be called fiery, because fiery stars appear in it.

[Margin: 2. Opinion: on the watery heaven.] The *Second* opinion was that the starry heaven — either all of it, or at least the Firmament, that is, the eighth sphere — is watery, with waters placed above it. How many Fathers and Doctors held this opinion we made clear in chapter 2, questions 1 and 3, setting out and expounding their own words honestly and without violence; and in the same chapter, at the end of question 1, we said that Martinengus, Molina, Delrio, Cornelius à Lapide, Serrarius [Nicolaus Serarius], Salian and Tannerus were also of this opinion. Add to these those whom I adduced in chapter 4, number 11, who affirm that the stars consist of watery vapours and are nourished by them.

[Margin: 3. Opinion.] The *Third* opinion affirms that the heaven is of air — and not only that heaven in which the birds of the sky fly (for our Tellez, disputation 40 of the *Physics*, section 1, asserts that this heaven is of air, and is truly and properly called heaven), but also that in which the stars are. For that this heaven is of the same substance as pure air, by reason of its very similar rarity and transparency, was asserted by Io. Penna the Frenchman and by Giordano Bruno in the book *On the World* against the Peripatetics, and for several years by Christoph Rothmann, as Tycho reports in volume 1 of the *Progymnasmata*, page 92, and in the *Epistolae* to Rothmann dated 1587 January 20, page 60, and again in the Letter of the year 1588, August 17, page

107; and in Rothmann's letter of the year 1588, September; and in the letter of the year 1589, February 21. Rothmann, indeed, drew his argument from the refraction of the rays of the stars, which is the same in the ether and in pure air. But Tycho teaches in the same place that this argument has no force at all; and indeed it often happens that wine and water, or some other liquid, are of different species and yet agree so closely in density that they refract rays equally. Johannes Kepler, too, seems to be reducible to this opinion. In book 4 of the *Epitome of Copernican Astronomy* he explains the Creation of the World described by Moses in such a way that the heaven of the fixed stars is a crust or wall of the Universe consisting of water or crystal, while inside it, all the way to the Sun, the centre of the world (as he himself thinks), everything is full of pure air. This air he elsewhere calls in many places the ethereal [?] aura, and the expanse, from the Hebrew name *Raquiah*; it is contained in the same way as air is contained within watery bubbles produced by blowing. These things laid down, since according to what was said in conclusion 2 it is more probable that the visible heaven consists of elementary matter, I briefly express my opinion in the following Conclusion.

CONCLVSIO.

Conclusion.

[Margin: 4. Conclusion.] *It is more probable that the Heaven in which the Fixed stars are is Watery, but that the heaven in which the Planets are is Fiery.*

X. For that proposition is more probable than the others which, if it cannot be proved by valid reasons, rests on more authorities, reconciles more authors, accords better with Sacred Scripture taken together with the interpretation of the Fathers, and does not make the Workmanship of God unseemly or ill-proportioned. Such is our conclusion. For that some visible heaven solidified out of water hardened in the manner of ice or crystal is confirmed both by the Hebrew name of the Firmament and by a great multitude of Fathers and Doctors, as we showed at length in chapter 2, question 1. Yet many Fathers and Doctors likewise assert that some starry heaven is fluid, in which the Planets move by themselves and not with the motion of the heaven, as is clear from what was said in chapter 2, question 2. I taught sufficiently in chapter 3, conclusion 4, that the former, who introduce solidity, are to be understood chiefly or only of the orb of the Fixed stars, and the latter of the heaven of the Planets, and that in this way those Fathers are best reconciled with one another — just as almost all the more recent Astronomers, in order to save the celestial Phenomena, grant solidity to the sphere of the Fixed stars alone, but affirm that the heaven of the Planets is fluid and permeable. But since the heaven of the Planets could be fluid and yet be either watery, or airy, or fiery, we are persuaded in the end by the following reasons that it is rather fiery.

[Margin: 1. Reason.] *First*: since some of the Fathers said, without distinction, that the heaven is watery, and others that it is fiery, they cannot be better reconciled than if you interpret the former of the sphere of the Fixed stars and the latter of the heaven of the Planets, and do not award the whole dispute to one party. For the reasons they adduce for that solidity apply solely to that highest heaven: namely, that solidified waters should be, as it were, the outermost walls of the world, bounding our sight, or like the shell of an egg or the vault of the world, able to sustain the upper waters on their back; and that they should contain the innumerable fixed stars scattered through the whole heaven and temper their heat, preserving that heaven until the last day of the world. Conversely, the reasons that others adduce for the fluidity of the heaven apply only to the heaven of the Sun, the Moon and the Planets. Therefore, if one of these heavens must be called watery and the other fiery, it is surely the heaven of the Fixed stars that must be called watery, and that of the Planets fiery.

[Margin: 2. Reason.] *Secondly*, some of the Fathers and Doctors speak not without distinction but distinctly, and say expressly that the heaven which they gather from the scriptures to have been made of water made firm and consolidated is the eighth sphere, or the highest of the visible heavens, which for that reason has for many centuries usually been understood by the name of Firmament. In the same way not a few teach expressly that the heaven of the

Planets is fiery, indeed is the fourth element itself, fire, as I showed by quoting the words of those Authors themselves in chapter 1, number 16, and in chapter 3, conclusion 4, argument 5; and so they suppose that this heaven — in which the Sun, or Titan, stands, and the Titanian stars, that is, the Planets, receive from the Sun their light and the law of their motion — is called Ether, that is, pure fire.

[Margin: 3. Reason.] *Thirdly*, there is the fitness of the divine workmanship. For if only the good of the elements is regarded, their natural order ought to have been such as it was at the first instant of creation: namely, that the earth should be lowest, with the abyss, that is, water of great depth, surrounding it above, and that above the waters should be both of those elements which earned their name from the thinness of spirit, namely air, and, in the highest place, fire (for we showed in chapter 1, number 13, that both are signified by the words *the spirit was borne over the waters*). Yet the good of the remaining natural bodies required another order, which God accordingly established on the second and third day: namely, that part of the earth, uncovered by the waters, should become fit to hold men and the other animals with ample lodging and dwelling, and to nourish and refresh crops, flowers, grasses and foliage; that the moister air should be left for the breathing of animals around the earth and the water, and for the flight of birds, which for the most part nest and rest on the earth; and that above it should be fire.

Moreover, a great mass of waters had been created from the beginning, as its proportion to the earth demanded — so great that it deserved the name of abyss, that is, of water having an almost unsearchable depth — and neither all of it, nor even the greater part, could fittingly be enclosed within the hollows of the Earth or in underground caverns. The supreme maker therefore, most prudently, lest he should seem to have made anything superfluous and to be destroyed immediately after creation, obtained five advantages from that superabundant mass of waters. For he raised a part of them and placed it immediately beneath the splendour of the Empyrean, so that from that mixture of light and watery opacity he might form a most beautiful and perpetual Rainbow, and refresh the eyes of the Blessed with an incredible variety of colours far more delightful than ours; and so that there might, besides, be a kind of mirror, in which and from which the bodies of the Blessed and their splendours could be reflected in an admirable and manifold spectacle. Thus that watery heaven beneath the Empyrean may be imagined as a certain vast Rainbow set around the whole firmament, whose blue and darker part is lowest, toward us, the red in the middle, and the yellow or golden highest. Next, he consolidated the remaining part into the crystalline heaven, and by its density and opacity established a boundary between the invisible heavens — namely the Watery and the Empyrean, which belong to the other life — and the visible or sensible heavens, namely the fiery and the airy. Thirdly, he fixed the non-wandering stars in this Firmament, or crystalline heaven, so that they keep perpetually the same distance from one another and move at an equal pace with the motion of

that orb. Fourthly, he immediately tempered the heat of the innumerable fixed stars by the cold and moisture of the waters already mentioned, because the fixed stars, each of them, had to stay in the same place of that heaven, and were scattered through all that heaven. This immediate tempering was not necessary for the heaven of the Planets. These are only seven; they move only within the Zone of the Zodiac; and they do not remain in the same place of their heaven but roll very swiftly into one part after another, so that they do not threaten one and the same part with destruction or consumption by excessive heat; and it was enough to restrain and temper the elements of fire and air, enclosed by waters on both sides. Fifthly and lastly, he perhaps used the Watery heaven as the primum mobile — not so that that water should be a clepsydra measuring universal time by its downward flow, but so that by its perpetual whirling in a circle it might sweep the sphere of the Fixed stars along with it, and, carried round on the poles of the Equator in the space of 24 hours, might revolve that sphere along with itself — yet in such a way that the sphere of the Fixed stars should advance little by little toward the East on the poles of the Ecliptic. I said *perhaps*, because the motion of the Fixed stars toward the West could perhaps be reconciled in another way with their motion toward the East, as we shall say in section 2.

Thus for us the number of the elements and of the heavens, visible and invisible, is preserved, without a superfluous heap of orbs and without any incongruity in the works of God; the authority of the divine scriptures is preserved; the interpretation of the Fathers and Doctors of the Church, agreeing as though in a single council, is preserved; and finally the consensus of the more recent Astronomers is preserved. To my mind these considerations far outweigh the subtleties of certain Peripatetics, and their conjectures, which are topical rather than physical or metaphysical. Therefore the saying of our Oviedus, on page 463 of his *Course of Philosophy*, pleases me greatly: *In a Question whose resolution is destitute of all natural reason* — that is, of valid or forceful reason — *one must take refuge in the authority of the Fathers and Doctors*, etc.

CAPVT VI.

Chapter VI.

An Cælum Sit Generabile & Corruptibile.

Whether the Heaven Is Generable and Corruptible.

I. [Margin: 1. Opinion: for incorruptibility.] The FIRST opinion was, and still is, that the heaven is ingenerable and incorruptible intrinsically and by its own nature. This was held not only by Aristotle (*On the Heavens*, book 1, chapters 3 and 4, and book 2, chapter 1) and by all those Peripatetics who affirmed that the heaven is a certain simple substance — that is, one not composed of matter and form really distinct from each other — and indeed that it is an eternal and necessary being (I have already listed the chief of them, as regards

simplicity, in chapter 5, number 1). It was held also by many of those who said that the heaven consists of matter and form really distinct from each other. Chief among these were St Thomas, part 1, question 66, article 2, and on book 2, distinction 14, question 2; Albert the Great, part 1 *On the Four Coevals*, question 4, article 1; Alensis, part 2 of the *Summa*, question 50; St Bonaventure on book 2, distinction 14, article 1, questions 1 and 2; Pererius, book 2 on Genesis, question 1; Molina, part 1, question 9, article 2, disputation 2; Suarez, disputation 13 of the *Metaphysics*, section 11, from number 13; the Coimbra commentators, book 1 *On the Heavens*, chapter 3, question 1, article 2; Adam Tanner, question 5 *On the Heavens*, assertion 2; Amicus, tractate 5 *On the Heavens*, question 1; Hurtadus, disputation 1 *On the Heavens*, section 5; Arriaga, the single disputation *On the Heavens*, section 3, number 42; Oviedus, the single controversy *On the Heavens*, point 2, number 20; disputation 22 of the *Physics*, Ioannes Poncius [John Punch], question 4, last conclusion; and Tycho, in the letters cited in chapter 5, number 4 — whom, however, Tellez wrongly adduces for the contrary view. Some of these, however, such as St Bonaventure and Molina, grant the visible heaven matter of the same kind as the sublunary, but attribute its incorruptibility to the perfection of the form, or to the incapacity of a natural agent. Some, again, add that it will not be destroyed as to its substance even at the end of the World — so Pererius, the Coimbra commentators, and Tannerus.

[Margin: Arguments for incorruptibility.] Their foundations are these. *First*, the authority of Saints Dionysius, *On the Divine Names*, chapter 4; Gregory of Nyssa, the book *On the Making of Man*, chapter 1; Jerome on chapter 65 of Isaiah; Augustine, *City of God*, book 4, chapter 24; and Gregory the Great, *Moralia*, book 11, chapter 5 — who seem strongly to favour this opinion. *Secondly*, that this grade of incorruptible bodies is required for the perfection of the universe, since such a grade is not otherwise impossible. *Thirdly*, because no trace of substantial generation or corruption appears in the heaven as a whole or in its parts: all the novelties of the heaven — new stars, say, and Comets, and the spots and faculae of the Sun — can be accounted for by mere alteration, and by condensation or local motion, or at any rate by a fresh creation made by God. *Fourthly*, because the Fathers who say that the heavens are corruptible say so because neither the destruction of their substantial form nor even the annihilation of the whole composite is repugnant to them with respect to divine Omnipotence; and in fact the heavens are to be destroyed, or at least greatly changed as regards their accidental qualities.

II. [Margin: 2. Opinion: on the corruptibility of the heaven.] The *Second* opinion was, and still is today, that this visible heaven is corruptible by its nature — indeed, if you speak of our Theologians, that it is to be destroyed. Its ancient authors are reported by Plutarch, *De placitis*, book 2, chapter 4; Theodoret, in the book *On Matter and the World*; St Jerome on Isaiah, chapter 51; and Eusebius, book 15 of the *Preparation for the Gospel*. Thus, besides Heraclitus and Cratylus, who held that earthly and celestial things are in continual flux, those who judged that the heaven would at some time be dissolved were Anaxagoras,

Democritus, Epicurus, Chrysippus, Cleanthes, Posidonius, Antipater, Panetius, and their chief Zeno, and Seneca, a follower of the Stoics. As for what Plutarch says above — *Pythagoras and the Stoics hold that the world was begotten by God and is liable to corruption as far as its own nature goes, since it is sensible and corporeal; yet it will not therefore perish, but will endure for ever, since divine power holds it together; Epicurus holds that it will perish, since it was begotten, like an animal or a plant* — this belongs to the opinion of Pythagoras, not to that of the Stoics, since they thought that the world would at last be burned up in an ecpyrosis, or conflagration. That the Sibyls also sang of this is gathered from book 2 of the *Oracles*, from these verses:

Then a burning, fiery river shall flow from high heaven, and shall consume all places utterly, etc. And the celestial pole, etc.

Plato too, in the *Timaeus*, asserts that the heaven is by its nature dissoluble and perishable, but by the favour of God immune from [?] for the meantime[23]; and Aristotle also attributes this view to him (*On the Heavens* 1, text 102). Cicero held the same (*On the Nature of the Gods*, book 2), as did Lactantius (*On False Religion*, chapter 12) and Macrobius (book 1 on the *Dream of Scipio*). But that the heaven is corruptible, and is in fact to be substantially destroyed, is taught by the Master of the Sentences on book 2, distinction 14, and in the same place by Scotus, question 1, and Carthusianus, question 7; by Catharinus on the Epistle to the Hebrews, chapter 1, and on 2 Peter 3; Vallesius, *De sacra philosophia*, chapter 89; Hieronymus Magius [Girolamo Maggi], book 2 *On the Burning of the World*, chapters 6 and 7; Vielmius, lecture 18 on Genesis; Ascanius Martineng[?]s in the *Glossa magna*, from page 592; Salmeron, volume 1, last prolegomenon, and on 2 Peter 3, disputation 4, and in the same place Cornelius à Lapide and Serrarius; Martinus del Rio on chapter 1 of Genesis; our Molina, *De opere sex dierum*, disputation 3; Cornelius à Lapide both in the places cited and on Isaiah, chapter 34, from page 294; Leo Castrius [León de Castro] on chapter 51 of Isaiah; Emmanuel Sà [Manuel de Sá] in his notes on the 2nd Epistle of St Peter; Fabricius Paulucius on 2 Peter, chapter 3; Marinus Mersennius [Marin Mersenne] on Genesis, page 12, column 2; Io. Bapt. Folengus [Giovanni Battista Folengo] on Psalm 101; Genebrardus on Psalm 101; Martinus Becanus, *On the Work of the Six Days*, from page 403; and Io. Ludovicus de la Cerda [Juan Luis de la Cerda], *Adversaria sacra*, chapter 154. Suarez, too, thinks it probable, volume 2 on part 1, disputation 58, section 2; so Eusebius Nieremberg [Juan Eusebio Nieremberg], book 6 *On the Life of the Stars*, chapter 13; Tannerus in the dissertation *On the Heavens*, question 5, and volume 1 of his theology, disputation 6, question 2, doubt 6; Scheiner in the *Rosa Ursina*, book 4, part 2, chapters 24 and 25; Mastrius and Bellutus, disputation 2 *On the Heavens*, question 3; Kepler, *On the New Star*, chapter 23; Descartes, page 193 of his *Philosophy*; Franciscus Resta, the tractate, and book 1 of the *Meteorologica*, chapter 10; Balthassar Tellez, disputation 40 of the *Physics*, section 1; Fromond[?]s, book 3 of the *Meteorologica*,

[23] the print reads "ab interim interim immune", with "interim" set twice; the noun governed by "ab" is not printed — "interitu", "destruction", is the likely intended word, a conjecture

article 6; and Bullialdus [Ismaël Boulliau], book 1 of the *Astronomy*, chapter 5. Salian, however, although he had championed this opinion in the *Annals*, at the 4th day of the world, number 9, later seemed in the scholia to retract it and return to the Peripatetics. But I had nearly forgotten Lucretius, book 6, who sings thus:

And since I have taught that the temples of the world are mortal, and that the heaven consists of a body that was born, and that whatever things come to pass in it, and will come to pass, must be dissolved, etc.

The foundation of this opinion is threefold: namely the authority of sacred Scripture, the testimonies of the Fathers, and arguments drawn from experience — from the Spots and faculae, which arise and perish near the Sun's disc, as the Telescope bears witness, and from some Comets born and dying above the Moon, whose vicissitudes are explained more connaturally by generation and corruption than by other, more violent modes, or modes that involve miracles.

III. [Margin: 1. Argument, from Holy Scripture.] *First*, then, Psalm 101 says: *In the beginning, O Lord, you founded the earth, and the heavens are the works of your hands: they shall perish, but you remain; and they shall all grow old like a garment, and like a covering you shall change them.* And Job 14: *Man, when he has fallen asleep, shall not rise again until the heaven be worn away; he shall not awake from his sleep.* But it is certain that men will one day rise again; therefore the heaven too will one day be worn away. Isaiah, chapter 24: *All the host of the heavens shall waste away, and the heavens shall be rolled up like a book, and all their host shall fall away*; and chapter 51: *The heavens shall melt away like smoke, and the earth shall be worn out like a garment*; and chapter 65: *For behold, I create new heavens and a new earth.* And Matthew 24: *Heaven and earth shall pass away.* And 2 Peter, chapter 3: *But the day of the Lord will come like a thief, in which the heavens will pass away with great violence, and the elements will be dissolved with heat, and the earth and the works that are in it will be burned up. Since, then, all these things are to be dissolved*, etc.; and in the same place: *the heavens, being on fire, will be dissolved, and the elements will melt with the heat of fire; but we look for new heavens and a new earth according to his promises.* And earlier in the same place St Peter had said: *But the heavens are laid up, reserved for fire.* These authorities may not be interpreted, without necessity, by destroying or abandoning the literal sense. And it was soundly said by our Maldonatus, on the words *and the stars shall fall from heaven*: *I judge that Christ, who affirms it, is to be believed rather than Aristotle, who denies that it can happen*; and there he also holds, with Chrysostom and Euthymius, that the stars will truly fall.

[Margin: 2. Argument, from the Authority of the Fathers.] *Secondly*, several Fathers teach that the waters are above the heavens for this reason: to protect them meanwhile from the excessive heat of the stars; and they assert that the heavens are watery or fiery, as we have already shown in chapter 2, questions 1 and 3, and in chapter 3, conclusion 4, argument 5. Therefore, just as the elements are corruptible of themselves, so too are the heavens. Indeed, they teach in express words, which no evasion can escape, that the heavens are in fact

to be dissolved and, their former form being destroyed, transformed into a better state — as anyone may see who reads their words in Ascanius Martinengus, *Glossa magna*, and in Scheiner, book 4 of the *Rosa Ursina*, part 2, chapter 24. So, first of all, St Justin, *Responses to the Orthodox*, questions 93, 94 and 95; St Basil, homilies 1 and 3 on Genesis, and 6 of the *Hexaemeron*; St Ambrose, *Hexaemeron*, book 1, chapter 6; St Chrysostom, homily 10 to the people of Antioch, homily 23 on chapter 9 of Matthew, and homily 14 on chapter 8 of the Epistle to the Romans, and in the same place Theodoret; Oecumenius, in the collectanea on chapter 3 of the 2nd Epistle of Peter; St Damascene, book 2 *On the Faith*, chapter 6; Gennadius, the book *On Ecclesiastical Dogmas*, chapter 7; St Jerome on chapter 51 of Isaiah; St Hilary on Psalms 122 and 148; St Epiphanius, book 2 *Against Heresies*; St Irenaeus, book 4 *Against Heresies*, chapter 6; St Cyril of Alexandria, book 4 on Isaiah, chapter 51; Andreas of Caesarea, sermon 22 on the Apocalypse; Zacharias Scholasticus, bishop of Mytilene, in the disputation *On the Making of the World*; and Caesarius, *Dialogues*, question 71.

[Margin: 3. Argument, from reason and examples.] *Thirdly*, that the Spots and Faculae of the Sun are newly generated and perish is proved by the arguments already indicated in book 3, chapter 3, and by those to be adduced from Scheiner in book 9, section 4, chapter 12, numbers 19 and 20; and the same is proved of the Comets that are above the Moon, of which enough was said in book 8, section 1, chapter 6, as also of the new stars in section 2, chapter 17. Granted [?] that Tannerus (dissertation *On the Heavens*, question 7, and volume 1 of his theology, disputation 6, question 3, doubt 3) contends either that the observations of the Mathematicians who assert these novelties are uncertain, or that these things are done by God in a supernatural manner, as portents, or that the spots are carried round the Sun on special Epicycles and that Comets and new stars descend towards us in the course of time — of these and the other modes devised to explain these Phenomena enough has been said by us, in the places indicated a little earlier. Let this, then, be the

CONCLVSIO.

Conclusion.

It is more probable that the visible heavens are corruptible by their nature and intrinsically, although they are incorruptible per accidens and extrinsically.

IV. The first part of the Conclusion is proved from what was said in chapter 5, number 10, for conclusion 4. For by the very fact that we proved that the heaven of the Fixed stars is watery and that of the Planets fiery, and showed this to be the more likely view, it follows that these heavens are intrinsically and by their nature capable of generation and corruption; and that their matter is not so bound to the celestial form that, if a natural agent apt to introduce another form were given, that matter would not be positively apt to receive it. Yet — whether on account of their distance, or of their huge bulk, or of the remarkable tempering of the secondary qualities with the primary that God has put into the heaven — there is no created natural agent that could transform the heavens

substantially; and for that reason I said that they are incorruptible per accidens, in the way that neither the whole earth nor the whole air is transmutable as to the whole or the greatest part of itself. The Conclusion is confirmed, moreover, because the new Phenomena of the stars and of the Comets, and those of the Solar spots, and the dissolution and renewal of the heavens at the end of the world, are to be explained preferably without a miracle, or at least with a lesser miracle, and in a more physical manner, similar to that of the other natural bodies that come to be anew. And this is achieved if it is admitted that the heaven is now subject to generation and corruption in some small part of itself, and that at the end of the world Fire is to be taken up by God as the instrument for the substantial renewal of the heavens. Nor do I care, or have I leisure, to linger longer over a controversy of this kind, which has neither evident reasons nor authorities that cannot be explained away.

[Margin: The incorruptibility of the Empyrean.] I said in the Conclusion "the visible heavens"; for that the Empyrean heaven is incorruptible is sufficiently proved from the Fathers' manner of speaking, on which see chapter 1, question 6, where that heaven and its conditions are treated at length. And this is the body that was required for the perfection of the Universe — but for that state and order of things in which our bodies too will have incorruptibility and eternal soundness.

[Margin: The destruction of the heavens at the end of the World.] How the heavens are to be dissolved at the end of the world is treated learnedly, above all, by Pererius, book 2 on Genesis, questions 1, 2 and 6; Tannerus, *On the Heavens*, question 11, and volume 1 of his theology, disputation 6, question 4, doubt 5; Salmeron on 2 Peter 3; and others cited by them.

CAPVT VII.

Chapter VII.

An Cæli Solidi ſint, an verò Fluidi omnes vel aliqui.

Whether the Heavens Are Solid, or Rather All or Some of Them Fluid.

1. THERE is no doubt that the heaven of the Planets is solid, if solidity is taken broadly for the threefold dimension of a body having length, breadth and depth — the sense in which Philo (book 1 *On the Making of the World*), Origen (homily 1 on Genesis) and St Basil (homily 3 of the *Hexaemeron*) said that every body is solid; I gave their words in chapter 2, question 2. Nor is it disputed among others whether the heaven of the Fixed stars has some depth or height: Philo, in the book *On Dreams*, is the only one I have found calling this into doubt, in these words: *Does that outermost sphere of the Fixed stars have solidity, or only a surface without height, like figures painted on a plane?* For Physicists and Astronomers commonly attribute to the Fixed stars some corpulence and bulk, like that of globes or quasi-globes, and so attribute to the heaven in which they are fixed some thickness or depth as well, which Philo, as I said, calls solidity. The controversy, therefore, concerns solidity taken strictly: solidity which,

besides the threefold dimension, has joined to it a hardness opposed to softness — the sense in which we say that marble is solid, and metals so long as they do not melt, and ice itself before it is dissolved. With these points noted in advance:

II. [Margin: 1. Opinion: on the Solidity of all the heavens.] The *First* opinion was, and endures to this day, that all the starry heavens are Solid. It was held by Aristotle, *On the Heavens*, book 2, from text 43 to 52, where he contends that the stars do not move in the heaven as fish do in the sea — that is, they do not move of themselves — but with the motion of the orbs, some of which, the rotating ones [voluentes], he devised and named with Eudoxus and Calippus, and others, the Revolving ones [Reuoluentes], from his own ingenuity (*Metaphysics* 12); by Plato in the *Timaeus*, though on the interpretation of Eusebius, book 1 of the *Preparation for the Gospel*, chapter 16; and by Anaximander, as Plutarch reports, book 2 *De placitis philosophorum*, chapter 16. It was held by Dionysius Carthusianus on book 2, distinction 14, question 3, who says: *By the inviolable authority of canonical scripture, the heavens are most solid, as if cast of bronze.* The Master indicates the same on book 2, distinction 14; and it is taught by St Thomas on book 2, distinction 14, question 1, and part 1, question 68, article 2, and by many Scholastics, St Bonaventure excepted; and also by Gulielmus Parisiensis, part 1 *On the Universe*, tractate 3; Lippomanus in the *Catena* on chapter 1 of Genesis; Fracastoro in the *Homocentrica*; Alfraganus [al-Farghānī], *dist.* 14 and 17; Sacrobosco and Clavius on the *Sphere*, chapter 4; Purbachius [Georg von Peuerbach] in the *Theoricae*, and his followers; Fernelius [Jean Fernel] in the *Cosmotheoria*; Vielmius on Genesis, lecture 14; Pererius, book 2 on Genesis, question 9; Martinus Del Rio on chapter 1 of Genesis, number 36; the Coimbra commentators on book 2 *On the Heavens*, chapter 8, question 1; Martinengus in the *Glossa magna*, page 1018 in my copy; Suarez, book 1 *De opere sex dierum*, chapter 5; Salian in the *Ecclesiastical Annals of the Old Testament*, at the 2nd day of the World; Adam Tanner in the dissertation *On the Heavens*, question 6, assertions 1 and 2, and volume 1 of his theology, disputation 6, question 3, doubt 2, likewise assertions 1 and 2 — although Tellez did not hesitate, wrongly and ill-advisedly, to adduce him for the opposite view; Amicus, tractate 5 *On the Heavens*, question 5, article 3; Io. Poncius, disputation 22 of the *Physics*, question 5; and Aversa, question 33, section 6. And although we ourselves think otherwise about Copernicus, as will appear from what is to be said in section 3, chapter 1, number 14, Tycho nevertheless attributes this very reality and distinctness of the orbs to Copernicus, though with hesitation, in volume 1 of the *Progymnasmata*, page 439, and to Ibn al-Haytham and Witelo in the *Letters*, page 61. The foundations of this opinion will be refuted a little below.

Secunda opinio de Fluiditate Cælorum Vniuerſaliter aut indefinitè.

The Second Opinion: on the Fluidity of the Heavens, Universally or Indefinitely.

The SECOND opinion affirms that all the heavens are fluid, either expressly or implicitly, inasmuch as it asserts indefinitely that they are fluid, that is, freely

permeable by the stars. [Margin: Most of the ancients.] So held certain of the ancients reported by Aristotle, *On the Heavens* 2, texts 45 and 56; to them are referred all who said that the stars are fed by moisture — namely Homer, Anacreon, Virgil, Lucretius, Pliny, Manilius, Seneca, Metrodorus and others adduced by our Lorinus [Jean Lorin] on 2 Peter 3. [Margin: Cicero.] Cicero held the same, together with those learned in physical matters, when he said in book 2 *On the Nature of the Gods*: *The stars, moreover, occupy the ethereal region; and since it is most subtle, and is always in motion and in vigour*; and soon after: *Nor do they have ethereal courses, nor do they inhere in the heaven, as most say who are ignorant of physical reasoning. For the nature of the aether is not such that it could embrace the stars and whirl them round by its own force: the aether, thin and transparent and suffused with an even heat, does not seem apt enough to hold the stars. The fixed stars, therefore, have their own sphere, separated and free from any conjunction with the aether.* [Margin: Pliny.] Pliny seems to have been of the same opinion, book 2, chapter 5, where by the name *spiritus* he understands both air and aether; in which sense he says in chapter 6: *Between earth and heaven, hanging in the same spiritus and separated by fixed intervals, are the seven stars which from their motion we call wandering.* [Margin: Vitruvius.] Vitruvius too, book 9, chapter 4, indicated clearly enough that at least that part of the heaven through which the Planets travel is free and fluid, when he said that Mercury and Venus go round the Sun, crowning it as their very centre, stand still and retrograde, and that the seven planets make their journeys as if through channels, against the diurnal motion. [Margin: Hyginus.] Hyginus likewise, in book 1 of the *Poetica Astronomica*, listing the sum of what he had written, says: *Next we have set out whether the Sun, being fixed, was turned with the World, or moved of itself; and, if it moves of itself and goes against the risings of the twelve signs, why it rises and sets with the World.* Then in book 4 he tries to prove that the Planets are not moved as if fixed in the heaven — by an argument hardly necessary, but one that nonetheless reveals his opinion — saying: *But, as I said before, we shall now speak straightway of the course of the Sun. For it is necessary either that the Sun moves of itself, or that it turns with the world while remaining in one place. If it remained, it would necessarily set and rise in the same place in which it had risen the day before, just as the signs rise and set in the same place, etc. If, therefore, it rises and sets in different places, it is necessary that it moves and does not stand still* — that is, does not move with the World, but by itself and freely. [Margin: Germanicus.] Germanicus, the translator of Aratus, agrees with him, saying: *It is established that the Sun moves by itself, not that it is turned with the World.* [Margin: Manilius.] And Manilius indicates it in book 1 of the *Astronomica*, when he says of the motions of the stars: *All things are passable*; and again: *Since the Moon and the stars fly through the voids of the world.* [Margin: Seneca.] Seneca teaches most plainly that Comets and stars travel freely through the liquid aether, and rejects the opinion of Artemidorus, which ascribes hardness to the heaven, as may be seen in book 7 of the *Natural Questions*, chapters 12, 13, 14, 15, 23 and 24. [Margin: Ptolemy.] Ptolemy, moreover, is counted in

this very class by St Thomas, part 1, question 70, article 1, in these words: *It must be said that according to Ptolemy the luminaries are not fixed in the spheres, but have a motion separate from the motion of the spheres.* Here the holy Doctor leaves everyone free to hold what he will about the hardness and liquidity of the heaven. So Dionysius Carthusianus, on chapter 1 of Genesis, article 10, on the words *Let there be Lights in the firmament*, says: *Plato in the Timaeus says that the stars are not fixed in the firmament, but travel and lead a dance in it, and in this Ptolemy followed Plato; hence they say that it is not the heaven but the stars of the heaven that are moved and carried round.* Likewise Vielmius, *On the Six Days*, lecture 20: *It is certain that Ptolemy, the prince of Astronomers, affirmed that the Eccentrics and Epicycles by no means really exist.* And Cornelius à Lapide, on the 2nd Epistle of St Peter, chapter 3, and Scheiner, book 4 of the *Rosa Ursina*, part 2, chapter 29, hold the same of Ptolemy, as favouring the liquidity of the heaven. Scheiner, about to bring Ptolemy himself forward, says aptly: *Let Ptolemy come forward and speak for himself; for he is 1498 years old, and, grey with much wisdom, he lives still — lives, I say —* because Ptolemy himself says in the preface of the *Almagest*: *A man's learning is the companion of his intellect and his intercessor among men, and he who gave life to knowledge did not die.* Now Ptolemy, in book 13 of the *Great Composition*, chapter 2, seeing that the paths of the Planets would seem difficult to many, says: *Those are divine bodies, which suffer no hindrances either from others or from themselves*; and soon after: *Especially since their nature has no power whatever of resisting, but accommodates itself with the greatest proportion and ease, and, as is according to nature, makes room for their motions by yielding place, even if these are found to be contrary to one another, so that all flows can pass through and shine through any bodies whatever.* Raffaele Aversa, volume 2 of his *Philosophy*, question 33, section 1, affirms, however, that Ptolemy held that the heaven consists of all the elements.

[Margin: Rabbis.] The fluidity of the heavens, moreover, is asserted expressly by many Rabbis, as reported by Iacobus Naclantus [Giacomo Nacchianti] in the *Medulla*; [Margin: Gilbert.] by William Gilbert, book 6 *On the Magnet*, chapter 3; [Margin: Neander.] Michael Neander, *Elementa astronomica*, chapter 4; [Margin: Peucer.] Caspar Peucer, in Tycho's *Letters*, page 157, and in the same, Christoph Rothmann, page 149; [Margin: Tycho.] Tycho himself, as we shall see presently, in the *Letters*, page 149, and in volume 1 of the *Progymnasmata*, page 794, and volume 2, page 268 — whose words we shall report under the 5th opinion, in which some place him; [Margin: Longomontanus.] Longomontanus, chapter 5 *On Comets*, and in the *Astronomia Danica*, book 1 of the *Theorica*, chapter 1, folio 161, and chapter 4, folio 220, teaching that the Fixed stars too, and not only the Planets, are balanced in the most limpid aether no differently from the Earth in the air, and that they are not moved, lest they be drawn apart from one another in the expansive stereoma, or firmament — which would not have to be feared if they were fixed in a solid sphere; [Margin: Kepler.] Kepler, in the *Mars*, chapter 4, and in the *Epitome of Copernican Astronomy*, pages 53, 142 and 495, saying that the single heaven of

all the stars consists of a most thin ethereal breeze; [Margin: Galileo.] Galileo in the *Sidereus Nuncius*, in the *History of the Spots*, and in Dialogues 1 and 2 *On the System of the World*; [Margin: Scheiner.] Christoph Scheiner, book 4 of the *Rosa Ursina*, part 2, from chapter 26 to 30, and, cited by him, [Margin: Mästlin.] [Margin: Gemma.] [Margin: Vuitichius.] [Margin: Raymarus.] Cornelius Gemma, Vuitichius [Paul Wittich], Mästlin, Nicolaus Raymarus [Nicolaus Reimers Ursus], and Orazio Grassi, who adds on page 765, [Margin: The Chinese Mathematicians.] from the letters of our Fathers among the Chinese, that the opinion of the hardness and multiplicity of the Planetary heavens is held there to be foreign, new and vain, and that they will not let themselves be moved from the opposite view. [Margin: Ciccus.] Likewise Mersennius on Genesis, pages 836 and 843; and Ciccus Esculanus [Cecco d'Ascoli] on the *Sphere*, saying that the orbs of the Planets are neither continuous nor contiguous, but stand in a middle way, and that between them there is a body that admits compression. [Margin: Michael Scotus.] So too Michael Scotus [Michael Scot], in the *Questions on the Sphere*, page 105, says that "solid" is said in three ways: in one, of what is hard, like earth; in the second, of what is continuous, as are the remaining elements and the supercelestial bodies; in the third, of what has the threefold dimension. [Margin: Origanus.] [Margin: Mariana.] [Margin: Celada.] [Margin: Renatus.] [Margin: Aversa.] So also Origanus in the preface to the *Ephemerides*; Mariana [Juan de Mariana] on Genesis, chapter 1, verse 12; Celada [Diego de Celada] in the same place; Descartes, part 3 of the *Philosophy*, number 24; Raffaele Aversa, volume 2, question 34; [Margin: Bullialdus.] [Margin: Tellez.] Bullialdus, book 1 of the *Astronomia Philolaica*, chapters 2 and 4; Tellez, disputation 40 of the *Physics*, section 1, numbers 16 and 21, and disputation 44, section 3, number 5; the *Aristarchus* published at Paris, page 23; [Margin: Hevelius.] [Margin: Snellius.] [Margin: Stevinus.] [Margin: Gassendus.] [Margin: Crugerus.] [Margin: Eichstadius.] [Margin: Linemannus.] Io. Hevelius [Johannes Hevelius], chapter 7 of the *Selenographia*, and, cited by him, Snel, Stevin, Gassendi, Crugerus [Peter Crüger], Eichstadius [Lorenz Eichstädt] and Linemannus [Albert Linemann]; [Margin: Pena.] likewise Ioannes Pena in the preface to the *Optics*; [Margin: Cysatus.] and Io. Baptista Cysatus, *On the Comet of the year 1618*, affirming that Comets pursue their courses in a most limpid expanse of aether. [Margin: The Professors of Parma.] [Margin: Bellarmine.] This same opinion was first defended by our Professors in the theses published at Parma in the year 1613, theses 742, 743 and 783; and Federico Cesi, in a letter to Io. Faber [Johann Faber], cited by Scheiner in the *Rosa Ursina*, book 4, part 2, chapter 27, testifies that Bellarmine was of the same opinion.

[Margin: 3. Opinion, positing the same heaven partly solid, partly fluid.] The *Third* Opinion divides the same heaven into a solid part and a fluid part, positing channels in every Planetary heaven, full of air or of a subtle and thin breeze, through which the Planets can travel — as Hurtadus posits, disputation 2 *On the Heavens*, section 1, though disputation 1, section 5, paragraph 29, should

be consulted. But St Bonaventure, on book 2, distinction 14, part 2, article 1, question 1, teaches that the Luminaries are placed in several orbs, but orbs that are continuous, subtle and suited to motion, and that the diversity of motions in them does not remove the continuity of those orbs, just as it does not remove it in water and in air. From these words Tannerus and Amicus gather that he recognises the Planetary heavens as being in part soft and fluid.

[Margin: 4. Opinion: on the outermost heavens alone being [?].] The *Fourth* Opinion is that of our Father Niccolò Cabeo, on book 1 of the *Meteorologica*, text 37, question 6, and on book 2, text 11, question 4, where he thinks it more probable that the heaven of the Fixed stars and the heaven of the Moon are solid, so that the starry heaven is thus enclosed on either side by a stable boundary; and because we have no argument for fluidity in these heavens from the celestial phenomena, such as we have for the fluidity of the other heavens enclosed within them. He is moved to suspect this chiefly lest the spirits of the earth and the sea, and the volatile salts, flying upward without limit, should be consumed and never return to ferment the earth; rather, they are kept by the hardness of the lunar heaven from travelling further upward, and are driven back down.

[Margin: 5. Opinion, positing the heaven of the Fixed stars solid, that of the Planets fluid.] The *Fifth* opinion, and now the most celebrated, is that the heaven of the Fixed stars is solid, and that the non-wandering stars are set in it like nails or knots, while the heavens of all the Planets are fluid. The first to hand down this distinction, so far as I know, was Empedocles; for Plutarch says in book 2 *On the Opinions* [*de Placitis*], chapter 13: *Empedocles holds that the non-wandering stars are bound to the crystal, but the wandering ones loose*; and Anaximenes seems to agree with him, for Plutarch goes on thus in chapter 14: *The Stoics hold the stars to be globular, like the world and the Sun and the Moon; Cleanthes, cone-shaped; Anaximenes, fastened like nails to something crystalline. There are some who think them fiery plates, like painted ornaments.* There he seems to report opinions about the stars as distinct from the Sun, the Moon and the other wanderers. Plato, too, called the non-wandering stars *Haerentes* ("the clinging ones"), as Plutarch has it in book 2 *On the Opinions*, chapter 15. Therefore what Carthusianus says on Genesis, article 10 — *Plato in the Timaeus said that the stars are not fixed in the firmament, but travel and lead dances in it* — is to be understood of the Planets. After these, Richardus Victorinus, in book 2 of the *Exceptiones*, chapter 7, said expressly: *The lights of the stars are shown to be fixed in the firmament, except the seven Planets*; and St Anselm, in book 1 of *De imagine Mundi*, chapter 24, says that the aether in which the seven Planets move is as much thinner and subtler than air as air is thinner than water, and that in it the seven stars move about freely, although they are swept round by the Firmament, like a fly on a mill-wheel, etc. Junilius too, on the *Hexaemeron*, distinguishes the Firmament, in which the Fixed stars are, from the aether, which he declares to be like pure air; he says that the aether reaches up to the stars of the firmament, and that in it the seven wandering stars are carried about

freely. His words, like Anselm's, I reported in chapter 2, question 2, at the end of number 4.

Among the more recent writers the same distinction is indicated by Philalthaeus on book 2 *On the Heavens*, text 46; Io. Antonius Delphinus, *De caelestibus globis ac motibus*, page 80; and Antonius Mizaldus [Antoine Mizauld] in the *Planetologia*. Tycho too is cited for it by some. But he calls the heaven liquid indefinitely, in his letter to Rothmann of the year 1587, January 20, page 60, and in that of the year 1588, August 17, page 106, and in others, pages 109, 120, 137 and 152. From these it is clear that Rothmann too was of the same opinion — namely, that the heaven in which the Planets move is most liquid and not at all hard — granted that Rothmann holds it to be of air, while Tycho holds it to be of aether and of a nature different from the elementary one. Tycho drives home the same opinion in volume 1 of the *Progymnasmata*, pages 92 and 641, and in all these places he rejects real orbs. But in volume 1 of the *Progymnasmata*, page 794, treating of the matter from which the new star of the year 1572 was compacted, and which he had often said was in the eighth sphere, he says: *For even though the whole heaven is something most thin, and everywhere open to the motion of the stars without any obstacle, it is nevertheless by no means wholly incorporeal (otherwise it would also be infinite and without place). The very matter of the heaven, therefore, most subtle as it is and open to our sight and to the powers of the Planets, could nevertheless, condensed and compacted into one globe and lit by a light if not its own then at least the Sun's, form this star*; and soon after: *and although in the whole vastness of the celestial World there is, in my judgement, matter abundantly available for the formation of any adventitious star, yet nowhere more copiously and fully than near the Milky Way, etc.* Although, then, he once said that the heaven is open to the motions of the Planets, yet because he said universally that it is open to the motions of the Stars, and called the whole heaven such — and pointedly that part in which the stars of the Milky Way are — he acknowledged the heaven of the Fixed stars too to be liquid, and is therefore to be counted among the authors of the second opinion.

In this 5th opinion, however, were Tassonus, book 2, question 3; Blancanus [Giuseppe Biancani] in the *Sphaera*, who assigns spiral and free motions to the Planets, but to the Fixed stars a motion according to the motion of the Firmament; Fortunio Liceti, book 5 *De novis astris*, chapter 50; Io. Camillus Gloriosus [Giovanni Camillo Glorioso], book 4 *De Cometis*; Froidmont, book 3 of the *Meteorologica*, chapter 1; Paulus Aresius [Paolo Aresi], book 1 *On Generation*, question 23, section 8; Hugo Sempillius [Hugh Sempill], book 10 *De mathematicis disciplinis*, chapter 1; Franciscus Oviedus, the single controversy *On the Heavens*, point 3, paragraph 4; Rodericus Arriaga, the single disputation *On the Heavens*, number 41; and Io. Bapt. Zupus [Giovanni Battista Zupi], as reported by Amicus, tractate 5 *On the Heavens*, question 5, article 2; and Mastrius and Bellutus think it very probable, disputation 2 *On the Heavens*, question 1, number 38. All these, I say, assert that the heavens of the Planets are fluid; as for the heaven of the Fixed stars, they either affirm that it is solid, or bring only

arguments for fluidity that suit the Planetary heavens alone — which holds also of many authors adduced for the 2nd opinion. It must be noted, however, that most followers of the Copernican sect assign no motion to the Fixed stars; and because they think the Earth, as a Planet in the heaven, is moved around the Sun, the centre of the world, they accordingly hold the whole Planetary system to be thin and permeable — which Aristarchus and certain Pythagoreans also posited long ago, for the same reason. [Margin: Ptolem[?], asserter of fluidity [?].] That Ptolemy, however, is to be counted in this fifth class we shall show in section 2, chapter 2, number 3; it is also clear enough from book 13 of the *Almagest*, chapter 2, and St Thomas teaches it expressly, part 1, question 70, article 1, reply to 3, as does Carthusianus on Genesis, article 12.

EXAMINANTUR ARGUMENTA PRO SOLIDITATE OMNIUM, AUT ALIQUORUM CÆLORUM; & SOLUTIONES EORUM.

The Arguments for the Solidity of All, or of Some, of the Heavens Are Examined, with Their Solutions.

III. [Margin: 1. Argument, from Holy Scripture.] The *first* argument is drawn from Holy Scripture. For the heaven, in Genesis 1, is called the Firmament, from the word *Raquiah*, which signifies a firmament — or an expansion indeed, or extension, but with firmness and solidity; and therefore the Seventy Interpreters rightly called it στερέωμα [Greek: stereōma, "firmament"], that is, something solid, as I taught at greater length in chapter 2 from number 2. There I also taught that the saying of Elihu in Job 37 — *You perhaps made the heavens, which are most solid, as if cast of bronze* — has great authority, because he is not rebuked for it by the Lord, and other sayings of Job's friends were adduced by the Apostles to confirm what they said. Reread what is said there [?][24]. Moreover, Job 14 says: *Until the heaven be worn away*; Isaiah 51: *The heavens shall melt like smoke*; and Hebrews 4: *Who has penetrated the heavens*. But what is worn away, melts and is penetrated is presupposed to be solid.

[Margin: 1. Response.] It can be answered, however, that the name *Raquiah*, according to not a few who are skilled in the Hebrew language, sometimes signifies mere expansion; and that some Fathers and interpreters understood by the name Firmament not something hard, but a stable, durable and impassable boundary between the upper and the lower waters, as I taught in chapter 2, number 2 and number 5, confirming it by the authority of Saints Basil, Nyssenus and Augustine, and likewise of Albert the Great and Tostatus. And it can be answered that the words of Elihu are not those of the canonical writer himself, since many of his sayings are rebuked by the Lord. For when Elihu had spoken from chapter 32 to 37 inclusive, it is said immediately in chapter 38: *Then the Lord, answering Job out of the whirlwind, said: Who is this that wraps up sentences in unskilful words?* Here the Lord, although he is speaking to Job himself, nevertheless pointed at Elihu and at his many sayings spoken unskilfully, even if mixed with true opinions. Nor is that saying of Elihu found confirmed

[24]the print reads "di-/cat", not "dicta"; the sense "what is said" is kept, the construal unsettled

by any canonical writer; Carthusianus, therefore, on book 2, distinction 14, question 3, wrongly called it an inviolable authority of canonical Scripture. That particular rebuke of the Lord against the sayings of Elihu suffices, granted that in chapter 42 he was not rebuked again, but only Job's three earlier friends, namely Eliphaz, Bildad and Zophar, in these words: *And after the Lord had spoken these words to Job, he said to Eliphaz the Temanite: My wrath is kindled against you and against your two friends, because you have not spoken rightly before me, as my servant Job has.* Indeed, it suffices that those words were not uttered in the person of the canonical writer himself, nor approved by any other canonical writer, for them to be set aside as outside the canon of infallible authority. By the verbs *to melt* like smoke and *to be worn away* is signified the corruption of the original form, which will vanish and be consumed at the end of the World. Nor indeed do solid things alone melt or suffer wearing, since wax, oil and clouds melt, and air itself and water give out a sound when friction is made by the winds. Finally, impenetrability, of which the physicists speak, belongs to fluid bodies too. Hence, if Christ penetrated the heavens using the gift of a glorified body, it was a great miracle whether the heavens he penetrated were fluid or solid; but a greater one, without doubt, if an- y heaven too is hard — and it is enough that the heaven of the Fixed stars is of that kind. Therefore it is not necessarily inferred, either from these places of Holy Scripture, which are the principal ones, or much less from others, that all the heavens are solid.

IV. [Margin: 2. Argument: the authority of the Fathers.] The *second* argument is drawn from the authority of many Fathers and Doctors of the Church, who assert in the most explicit words that the Firmament in the midst of the waters was made by God in such a way that it grew solid out of waters consolidated in the manner of ice, or of crystalline stone, or of an eggshell, or of a vault and a wall, as is evident from their words faithfully recited in chapter 2 from number 3. And these are not only Josephus the Hebrew, but Pope Clement, speaking according to the mind of St Peter; Tertullian, Caesarius, Jerome, Chrysostom, Acacius, Severianus, Claudius Marius Victor, Gennadius, Theodoret, Junilius, Procopius, Olympiodorus, Strabus, Anselm of Canterbury, Anselm of Laon, the author of the interlinear Gloss, Petrus Comestor, the Master of the Sentences, Hugo Carensis, Carthusianus — how many men, I ask, and of what quality! Nor does St Augustine, reporting this opinion in book 2 *De Genesi ad litteram*, chapter 1, dare to condemn it.

[Margin: Response.] The response, however, is that many other Fathers and Doctors of the Church, adduced by us in chapter 2, question 2, stand for the fluidity of the heaven, as we made plain from their words reported there. One of two things must therefore be concluded: either one is free to follow whichever opinion one likes, or rather the Fathers are to be reconciled so that the former are understood of one heaven only, namely the eighth sphere, and the latter of the heaven of the Planets — as we have already done in chapter 3, conclusion 4, where the foundations of this distinction were adduced.

V. [Margin: 3. Argument, of Aristotle, and Response.] The *third* argument is Aristotle's, book 2 *On the Heavens*, chapter 7, where he says that it is consonant with reason that the heaven be of the same body as the star, and that every star is solid; therefore the body of the heaven from which it is made is solid too. But granted that the heaven and the Star that is in it were conceded to be of the same matter, it is nevertheless denied that they have the same conditions; otherwise, since a star is luminous and opaque, the whole heaven too would have to be luminous and opaque.

VI. [Margin: 4. Argument, of the Coimbra commentators.] The *fourth* argument. If the heaven in which the stars move were fluid, and neither a vacuum nor the interpenetration of bodies is admitted, it would follow that, at the motion of a star, either the whole heaven would fluctuate, or the part of the heaven pushed by the star would be condensed and the part left behind by the star rarefied. But condensation and rarefaction are incompatible with the heaven, and are a sign of corruptibility. The same holds of that fluid body with which the channels would be filled that some admit in that portion of the heaven through which the stars move.

[Margin: 1. Response, admitting the interpenetration of celestial bodies.] Some would answer that in luminous and celestial bodies mutual penetration can be admitted, whether they are fluid or not. Ioannes Provenzalis admits this, quodlibet 3, question 7; Poncius does not refuse it, disputation 22 of the *Physics*, numbers 52 and 55; nor does Amicus, article 5, number 25; nor do certain Mathematicians cited by St Bonaventure on book 2, distinction 14, part 2, article 1, question 2, who says: *Others say that that body is not cut, and that the Planets pass through it; for the body of light can be together with another body*; and cited by Albert the Great, part 1 of the *Summa*, question 4, article 19, where he says: *Certain ancients said that the Eccentrics and the stars pass through the body which is between the spheres, and yet do not divide it; and this happens because of the formality of those bodies, just as light too passes through the air.* And not a few of the ancients thought that light is a corporeal substance, which nevertheless penetrates diaphanous bodies. [Margin: 2. and 3. Response.] Others, again, could respond that that whole tract of the heaven, or Zone, in which the path of the planet lies, fluctuates. Others will concede rarefaction and condensation in the heaven, and will deny that this is a property belonging to corruptible body alone: for granted that the kind which arises from heating and cooling may seem to be such, yet the kind which comes from mere local motion is common to corruptible and incorruptible body, since density of itself includes nothing but a multitude of parts in the same space, and rarity a fewness of them. [Margin: 4. Response.] Others, finally, granting but not conceding this sign of corruptibility, will deny that the starry heaven is an incorruptible body, according to what was said in chapter 6, number 4.

VII. [Margin: 5. Argument, of Tannerus.] The *fifth* argument is drawn from sound. For it could hardly be avoided, says Tannerus, that a huge sound would be raised by the stars, if such vast bodies broke through a liquid body by

collision at so incredible a speed; for it is established that our air hisses horribly when stones are hurled by the whirling of a sling, and that when lashed by thongs swiftly bent this way and that it resounds. [Margin: 1. Response.] Tycho would answer this argument, in the *Letters*, page 106, that if the celestial orbs were solid, then whether they collided with one another by immediate friction or air were enclosed between them, a huge sound would be produced by the most rapid motion of the stars, one that would strike our hearing too. [Margin: 2. Response.] But Rothmann would answer better, in the same letters, page 121, by denying that the sound, if there were one, would reach our ears, both because of the rarity of the ethereal breeze and because of the huge distance. For he had been, he says, in a place five miles distant from another place in which a most violent wind had overturned buildings and trees, and yet he had not heard the noise; whereas the air near us, because of the watery thickness mixed into it and because of its nearness, makes a sound perceptible to us. And indeed, which of us perceives the crash and roar that accompany lightning from a distance of 10 or 20 miles? Oviedus adds that, just as no sound is made in water by the motion of fish, because water is not thought a medium suited to the species of sound, so neither are the heavens suited to it. Finally, the asserters of solid orbs must themselves answer why fire or air does not give out a perceptible sound from the revolution of the lunar heaven.

VIII. [Margin: 6. Argument, of Aristotle and Pererius.] The *sixth* argument, indicated by Aristotle and driven home by Pererius, is taken from the multiplicity of motions. For it is impossible for the same body to be moved with several motions, and contrary ones at that — as the Planets, and indeed the Fixed stars too, are established to be moved — unless they are moved by one motion of their own in one direction, and with the rest are moved according to the motion of a solid body to which they are fixed or on which they sit. [Margin: Response.] The answer will be to deny the antecedent: for this can happen through a spiral or quasi-spiral motion, in the way indicated elsewhere, book 7, section 1, last chapter, and to be expounded below in this book, in section two. Moreover, even if another body were required for this, it would not necessarily be solid; for fish too, carried downstream by a rapid river, can nevertheless strive upward toward the river's source. Clavius proposes this argument in another way, on chapter 4 of the *Sphere*, page 449. He says that if the planets were moved like fish and birds in a fluid, their motion would be free and too wandering, and so there would be no certain science of their motions — as though they could not be moved by an Intelligence in a fluid while the laws of motion are still observed.

IX. [Margin: 7. Argument, of Tannerus and Poncius.] The *seventh* argument, which Tannerus and Poncius use, is of this kind. It is impossible, or too difficult, for the Intelligences to move the stars with such speed and with so perpetual and constant a uniformity except by means of solid orbs; otherwise, since they cannot move bodies distant from themselves, they would have to be moved together with the stars at equal speed. But this seems absurd. [Margin:

Response.] The answer is to deny the major and the minor. For it is not incompatible that an Intelligence, remaining in a single place, should impress on the star such and so great a motion that it lasts in its vigour and intensity until the star revolves back to the same place, where it receives a new impetus from the Intelligence. Nor, likewise, is it incompatible that the Intelligences be moved together with the stars; for there is no danger at all that they should either tire or at length disdain to serve their Creator in this ministry. In which of the two ways they move the Stars is to be investigated in the following section.

X. [Margin: 8. Argument, of Tannerus.] The *eighth* argument. If solid orbs are not granted, it will be necessary, in order to move the celestial bodies, to multiply innumerable Intelligences — as many, says Tannerus, as there are Fixed stars, and as many as there are spots on the Sun. [Margin: Response.] The answer is to deny the antecedent. For the heavens of the Planets can be posited liquid, and the heaven of the Fixed stars solid and movable by one Intelligence or a few; and the spots around the Sun, since they are bodies very close to it and erupting from it, are easily carried round by the Sun's own rotation; or else a single Intelligence could preside over several of them that are near one another, to move and direct them — if an Intelligence is said to be needed for this at all. As for the Planets, it is the asserters of solidity who multiply far more entities, those of the Eccentrics and Epicycles.

XI. [Margin: 9. Argument, of Tannerus.] The *ninth* argument. If the solidity of the orbs is admitted, the Divine craftsmanship shines out more in the machine of the heaven, out of many and various orbs, like a clock, skilfully com- posed, and in which the stars are set like gems in rings. Likewise, that subordination of influences and motions in the lower heavens with respect to the higher, which St Dionysius the Areopagite and our holy Patriarch Ignatius [Ignatius of Loyola], in the letter on Obedience, supposed, is better preserved. [Margin: Response.] The answer, however, is that this craftsmanship shines out no less if GOD from the beginning infused into the Angelic minds the species and idea of the motions, by virtue of which they can move the Planets certainly and infallibly and direct them to their ends — yet in such a way that the lower look to the higher and strive to imitate something in them. For this craftsmanship is rational and intellectual, and that is surely nobler than mechanical craftsmanship; just as a painter who could draw a perfectly straight line without any rule, or a perfectly round circle without compasses, would be far more excellent than one who could do neither without the aid of rule and compasses. Indeed, if the Orbs of the Planets consisted of solid Eccentrics and Epicycles, either the craftsmanship would be poorer, if the orbs were moved by themselves without an Intelligence, or the Intelligences would have no need of excellent knowledge, but only of motive force. But the good Father Tannerus, a German, too devoted to those clockwork machines perfected in Germany, seems to have been unable to imagine anything more excellent in this matter. I said "perfected", because it is known to us that Iannellus of Cremona [Gianello

Torriano], architect to Charles V — or, as they call it, *Organopoeius* (maker of instruments) — began to sketch out the machine of a striking clock in his house at Cremona, and that afterwards some German or other, having inspected it, perfected it and brought it to a better form; and this deserves not the least praise.

XII. [Margin: 10. Argument.] The tenth and last argument is indicated by Palingenius [Marcello Palingenio Stellato] in the *Aquarius*. Reduced to form, it is this: the longer something lasts in its state, the harder and more solid it is, and conversely; but the heaven lasts in its state from the beginning of the world to its end; therefore the heaven is supremely hard. [Margin: Response.] The major premise, however, is denied unless other things are equal. For earth is not more durable than water and the whole air, nor an apple than oil, nor the flesh of dead animals than water drawn off by distillation. The verses of Palingenius, moreover, are those written below:

> *But the heaven is eternal, and so is consumed by no age; But since things that are harder are more lasting, And are harmed less, the aether is therefore hardest, Harder than adamant, so that it scorns iron and fire And every force but the Lord's, by whom it was itself created.*

Expenduntur Argumenta pro Fluiditate Cælorum, & Reſponſiones ad illa adducuntur.

The Arguments for the Fluidity of the Heavens Are Weighed, and Responses to Them Are Adduced.

XIII. [Margin: 1. Argument: the authority of Holy Scripture.] The *first* argument is the authority of the divine writings. For it is said in Isaiah 51: *The heavens shall melt like smoke*; where the Septuagint reads: *The heaven has been made firm like smoke*. Following this version, St Basil says in homily 1 on the *Hexaemeron*: *But concerning the substance of the heaven, what was said by the Prophet Isaiah satisfies us; in common words he disclosed to us a fitting sense of its nature when he said, who made the heaven firm like smoke — that is, who produced a thin nature, neither solid nor thick, for the composition and substance of the heaven.* Furthermore, Holy Scripture never attributes motion to the heaven except by miracle, as when it is said in Isaiah 34: *The heavens shall be folded up like a book*; and in Joel chapter 2: *Before his face the earth trembled, the heavens were moved*; and chapter 3: *The heavens and the earth shall be moved*. But whenever the discourse is of the natural motion of the stars, of the Sun and the Moon for instance, that motion is never attributed to the heaven, as is clear to anyone who reads Holy Scripture, and from the places I have already reviewed in chapter 2, number 5.

[Margin: 1. Response.] It can be answered that that version of the Seventy does not outweigh the Vulgate, or the versions of St Jerome, Pagninus, Vatablus and Arias Montanus; that the likeness of smoke is not suited to explaining firmness; and that from the Hebrew word *Racha* it should rather have been rendered *they were expanded*, or *they shall be expanded*. Or else the sense is that the heavens, according to the form and conditions of the present time, have no greater duration than smoke has, but that they too are to be transformed very

swiftly; as it is said in Psalm 36: *Like smoke they shall vanish*; and in Psalm 101: *My days have vanished like smoke*. Further, Holy Scripture expresses only the motion of the Sun and the Moon, because that motion alone is manifest to the common people and strikes the eye, and because the Planets are the bodies for whose sake the heavens are moved, if they are moved: just as a horseman, or someone carried in a chariot, is said to arrive in a city, even though his horse or chariot has arrived too. It can happen, therefore, that the heavens too are moved, and yet Scripture says nothing of their motion, as being less perceptible and less principal. This response is that of Pererius on Genesis, book 2, question 9; of Martinengus in the *Glossa* on Genesis, page 1019; and of Amicus, above, article 5.

[Margin: 2. Response.] It could be answered, secondly, that even if the stars moved while the heaven was unmoved, it would not follow from this that the heaven is liquid. For someone could admit, with Arriaga, single disputation *On the Heavens*, number 26, a vacuum in the heaven through which not only the stars but also the Epicycles are carried — for he says that a vacuum is prevented by God only in sublunary things; or, if this is too bold, he could admit the penetration of the stars with the heaven, which others already named in number 6 have admitted.

XIV. [Margin: 2. Argument: the authority of the Fathers [?].] The *second* argument is the authority of 30 Fathers and Doctors cited by Father Schombergerius. Some of them denied that the Firmament is solid with a solidity that includes hardness; others affirmed that the Planets move of themselves in the heaven, the heaven itself being unmoved, and so move like birds in the air and fish in the water, and that the heaven is therefore liquid. Their words I have already reported in chapter 2, question 2. For to say, as Amicus says, that they deny to the heaven only such solidity as elementary bodies have, is forced and arbitrary; otherwise the same could be said of the solidity of the heaven that he strives to confirm from the Fathers. [Margin: 1. Response.] It could therefore be answered better that many other Fathers stand for solidity, and that therefore either one is free to follow whichever side one likes, or they are to be reconciled so that some asserted the solidity chiefly or solely of the supreme heaven, that is, the heaven of the Fixed stars, and others the fluidity of the Planetary heaven; how much foundation this distinction has in the Fathers I have already made plain in chapter 3, conclusion 4. [Margin: 2. Response.] The answer, 2, is that from the statements of those who say that the Planets move while the heaven is unmoved, the fluidity of the heaven does not follow, because one could admit either a vacuum in the heaven or the penetration of bodies, according to what was said in number 6 and at the end of number 13.

XV. [Margin: 3. Argument, from the superfluity of motions and instruments.] The *third* argument. So many real and solid orbs of the Planets, and their motions, are multiplied in vain — indeed not only in vain but with the danger of mutual collision and hindrance, given so great a variety of motions; or at any rate we are compelled, without necessity, to weary the imagination

in conceiving so many real and solid Epicycles, Eccentrics, Concentrics, EccentrEpicycles, *circitores*, *anticircitores*, *vecti*, *contravecti*, etc. Finally, it seems unbecoming to the divine Wisdom that, for the sake of the motion of one Planet — Saturn, say — which can very easily be moved by itself or by an Intelligence, so great and so vast a machine should be moved as the whole heaven of each Planet is. For the Planet, compared with its heaven, is no more than a point, and is smaller than a drop is with respect to the Ocean; just as it would seem unbecoming if, for the motion of one droplet of water, the whole Ocean, which would otherwise be immovable, were moved; or if, to move an ant, the whole earth had to be shifted. Thoroughly moved by this argument, which concerns a fittingness partly physical and partly moral, Arriaga, single disputation *On the Heavens*, number 53, conceded so far as to assert that it is more probable that the heavens of the Planets are liquid.

[Margin: Response.] Tannerus and Amicus nevertheless respond that in the structure of these Machines and the subordination of their motions the craftsmanship of the supreme Maker, and his Omnipotence, appear the more, as does the power divinely communicated to the Intelligences for moving such vast bodies perpetually.

XVI. [Margin: 4. Argument, from the trajection of the Comets.] The *fourth* argument is taken from the multiple and wandering motion, or free and oblique trajection, of those Comets which Tycho and others are held to have demonstrated to have been generated and moved above the Moon. For this was thought to have been demonstrated of many Comets by the Astronomers: both by those whom we adduced in book 8, section 1, chapter 23, and, on the occasion of this controversy about solidity, by Scheiner in the *Rosa Ursina*, pages 740 and 765; Oviedus, number 15; Tellez, disputation 40, number 7, and, cited by him, our Father Ignatius Stafford, a distinguished Mathematician, in manuscripts; and, cited by Oviedus, our Eusebius, book 6 of the *Philosophy*. Now if the heavens were solid, their whole mass would have gone into the Eccentrics, Concentrics and Epicycles of the Planets, and neither matter nor place would remain for the Epicycles of the Comets. Therefore, from the

trajections of the Comets, Kepler, in the *Epitome of Astronomy*, book 4, page 442, held that the fluidity of the heaven had been demonstrated by Tycho, saying: *Tycho Brahe refutes solid orbs by three arguments: one is from the motion of the Comets, the second from unrefracted Light, the third from the proportion of the orbs. For if the orbs were solid, Comets would not be seen passing from one orb into another, since they would be impeded by the solidity; but they do pass from one into another, as Tycho demonstrated.*

[Margin: 1. Response.] Tannerus, Amicus and Hurtadus nevertheless *respond* that it has not been demonstrated that any Comets were above the Moon. They dared to say this without any discussion of Tycho's arguments, which are drawn chiefly from parallax, and not without the risk of earning ridicule or indignation from those skilled in Astronomy and Geometry. For this reason I judged that I should undertake, to this end as well, all the labour which, God

favouring, you have already seen carried through in book 8, section 1. There, in chapter 23, I finally concluded that it has not indeed been demonstrated so far, absolutely, that any Comet was above the Moon, but that it has been demonstrated on a certain hypothesis that is highly probable.

[Margin: 2. Response.] Secondly, therefore, Tannerus, Oviedus, Arriaga and the others named in the same book 8, section 1, chapter 6, number 12, *respond* that Comets, as well as new stars, are portents created or made in the heaven divinely, and not without a miracle, to rouse, terrify and warn mortals, etc. They can therefore also pass from one orb into another in a supernatural way, whether by penetration or by reproduction, however solid the orb were granted to be.

[Margin: 3. Response.] Thirdly, Fracastorius [Girolamo Fracastoro] *would respond*, in the *Homocentrica*, section 3, chapter 23, that there is in the heaven of the Moon a certain orb that serves the motion of the Comets in latitude; though he says that these Comets are moved in the air, and he acknowledges the difficulty of his response. In the same way, then, someone could feign within the orbs of the Planets still other orbs, really distinct and assigned to the motion of the Comets. How difficult that would be, let him judge who knows what we related about the motions of Comets in book 8, section 1, chapter 3.

XVII. [Margin: 5. Argument, from the motion of the satellites of ♄ and ♃.] The *fifth* Argument arises from the motions of the satellites of Saturn and Jupiter, on which see what I set out in book 7, section 1, chapters 1 and 3, and section 6, the last chapter. For there is so great a variety in them that the solidity of the heaven seems inadmissible around Saturn and Jupiter, lest it impede such motions. To say with Tannerus and Arriaga that it is not yet certain whether these are Fixed stars, visible only through the telescope, rather than wandering ones, is surely to show oneself less skilled in astronomical matters. [Margin: Response.] The same authors therefore *respond* better that all their motions, since they are after all regular, can come about through particular Epicycles around Saturn and Jupiter, hollowed out within the greater Epicycle of Saturn and of Jupiter.

XVIII. [Margin: 6. Argument, from the motion of ♂.] The *sixth* Argument arises from the motion of Mars. For we have already described, with Kepler, in book 7, section 1, chapter 8, its Spirals and windings as well, wonderfully tangled and intricate. Next, we showed that Mars is indeed carried above the Sun when it is near its Apogee and its conjunction with the Sun, but descends below the Sun and penetrates the Solar heaven when, having become acronychal, it tends towards its Perigee and its opposition with the Sun. We showed this both from the increase of its apparent size at perigee and from the parallax, which must be greater in it than in the Sun if the place of Mars in longitude and latitude, deduced from the tables or from the geometrical construction of the hypothesis, is to agree with its place as then observed with precise instruments. This, I say, we showed — or taught that it had been shown by Copernicus, Tycho and Kepler — already in book 7, section 2, chapter 3,

Scholium 4, and section 6, chapter 4, Scholia 3 and 4, and chapter 10, Scholium 1. There we also reproved Tannerus, Carolus Malapercius [Charles Malapert] and Bartholomaeus Amicus [Bartolomeo Amico], because, out of excessive love for the solidity of the Planetary heaven, they dared to deny that descent of Mars below the Sun without any examination of the Tychonic or Keplerian observations or demonstrations. Yet this descent can be gathered by those skilled in Astronomy and Geometry even from the mere comparison of the apparent diameter of Mars near apogee with its apparent diameter near perigee, as may be seen from what we said in book 7, section 6, chapter 10, Scholium 1. Certainly Lansbergius [Lansbergen], Maginus [Giovanni Antonio Magini], Galileo, Boulliau, Gassendi and all the most expert in astronomical subtleties have yielded to Tycho and Kepler on this point — except Rothmann, who, although he holds the heavens to be fluid, thought that the order of the heavens would be disturbed if he conceded that Mars is carried now above, now below the Sun; though this must be conceded of Venus and Mercury, as we shall shortly say. Just as Tannerus defends the ascent and descent of Mercury and Venus by Epicycles around the Sun, therefore, so he could have defended that of Mars too, as *Oviedus* rightly argues against him, single controversy *On the Heavens*, point 3, number 22.

[Margin: Response.] *Tannerus* therefore *responds*, in his dissertation *On the Heavens*, question 7, page 158, and in volume 1 of the *Summa theologica*, disputation 6, question 3, doubt 3, number 78 — and *Amicus* with him, treatise 5 *On the Heavens*, question 5, article 5, number 30 — granting such a descent without conceding it: *Perhaps it is not impossible, even if the solidity of the heavens stands, to save such Phenomena, if the heavens of Mars and of the Sun, interlocked with each other, are revolved together; and if it is needed for saving the Eccentricity of Mars, let its own Epicycle be added to it besides; or else let the Eccentrics of the Sun and of Mars be so disposed that Mars at perigee [hypogaeo] is nearer the earth than the Sun at apogee. Some more recent Mathematicians add that this could also come about if Mars and the Sun were moved in certain zones, as it were, or rings, interlocked with each other in a certain way, so that each in turn could rise above the other and again pass beneath it. This I leave to the Mathematicians to explain in their own time.* But it was wrong to set these doubtful modes, not yet demonstrated by any diagram, against the observations and demonstrations of the Astronomers. Nor ought one, with obscure hypotheses feigned in confusion of mind, to build up with a vain or superfluous effort a heaven of the Sun intersected by the heaven, the Epicycle or the Eccentric of Mars, when, once the liquidity of the heaven is admitted, the motion of each Planet can be explained far more easily and more fluently, without the danger of an inconsistent and self-contradictory system. The more recent Astronomers, therefore, and posterity, will rightly judge — for as long as the contrary has not been demonstrated — that the solidity of the Solar heaven is shattered by this argument, as by a most powerful weapon of Mars and, as it were, a thunderbolt, especially in view of the most tortuous spirals of that planet.

XIX. [Margin: 7. Argument, from the motion of ☿ and ♀.] The *seventh* Argument rises from the motion of Venus and Mercury. We showed in book 7, section 1, chapters 2 and 4, that it has now been made plain by the Telescope and by the most evident phenomena that each of them circles the Sun, and is carried sometimes above it, sometimes sinks below it. Since, moreover, they do not digress from the Sun beyond 50°, their Epicycle must intersect the orb of the Sun. Although this seems able to happen with the solidity of this heaven kept intact, yet there is so great a difficulty in it — especially in view of the spirals and windings of Mercury, which surpass every most tortuous coiling of the serpents he holds twined round his caduceus — that it is better to allow a free and permeable heaven for explaining them than to be entangled in them so as to defend, tooth and nail, a solidity that is otherwise of no use or convenience.

XX. [Margin: 8. Argument, from the spots of the Sun.] The *eighth* Argument is drawn from the spots and faculae that bubble up around the Sun and, with varied motion, seem to riot as it were — in alternating surge, or in rotation, and in running together into one, or in scattering and dispersal into various smaller spots. Although this motion could be accomplished by innumerable Epicycles, it is better that it should come about freely in a fluid heaven.

XXI. [Margin: 9. Argument, from the roughnesses of the ☾.] The *ninth* Argument is drawn from the roughnesses of the Moon, namely the valleys, mountains and caverns on its surface, made manifest by the Telescope just as on the earth, as is now established from what was said in book 4, chapter 7. Lest a vacuum be admitted within them, therefore, it seems that a liquid and yielding aetherial air must be admitted around the Moon. The assertors of solidity, however, *will respond* that the globe of the Moon is so set into an Epicycle that the hollows of those roughnesses are filled by solid parts of the Epicycle, just as the kernel of a nut, or the seeds of a pomegranate, are set into their rind.

XXII. [Margin: 10. Argument, from the Refraction and Reflection of rays.] The *tenth* Argument Tycho took — as I reported from Kepler in number 16 — and so did *Bullialdus*, in book 1 of the *Astronomia Philolaica*, chapter 4, from the multiple Refraction of the rays of the Sun and of the other stars that would take place at the surfaces, convex as well as concave, of so many Epicycles and Eccentrics. By reason of it rainbows or wonderful colours would be seen in the heaven, and the figures of the stars and their mutual distances would be varied everywhere. Indeed Boulliau thinks that, from the reflection of the Sun's rays, there would never be night, but that in the opposite part of the heaven an image of the Sun, as in a mirror, would by night

be seen. All these things conflict with the daily Phenomena of the stars.

[Margin: Response.] *Amicus responds* that all the heavens are of the same density and rarity, and behave like several lenses, or several spectacle-glasses, concave on one side and convex on the other, laid one upon another; for if they are of the same material they do not vary a refraction once made. From this, however, it would follow that the apparent sizes of the stars are made uncertain

— unless someone feigns that the refraction made in the heavens towards the perpendicular is compensated by the refraction made in the air away from the perpendicular, and that the images of the stars are thus restored to the original size that would appear to us if no refraction intervened. *It could*, however, *be answered* that solidity and hardness are not an accident that necessarily has density joined to it in the heavens. For even among sublunary things some are harder than others and yet rarer, as ice is with respect to water not yet frozen: ice rarefies and swells, and holds lighter exhalations within itself, by virtue of which its body is made lighter than water of equal bulk, and for that reason ice floats on water. Conversely, lead is denser than silver, and yet less hard. It can happen, then, that the aether has a rarity as great as that of the most purified air, and yet a hardness and solidity greater than a crystal's. And certainly those who posit the sphere of the Fixed stars as solid must, for their part, clear it of the variety of refractions and reflections already mentioned.

VNICA CONCLVSIO.

Single Conclusion.

XXIII. *It is much more probable, though not yet evident Mathematically or Physically, that the heaven of the Fixed stars is solid, but that of the Planets Fluid.*

This is established from the arguments and responses on both sides adduced so far. For by this distinction — both of probability from evidence, and of the heaven of the Fixed stars from the heaven of the Planets — very many opinions of the Fathers and Doctors are reconciled with one another and with the observations of the more recent Astronomers. With less violence, and less multiplication of motions and machines, and also with less danger of physical conflict among the so varied motions of the Planets, the Phenomena of the Comets, of Mars, of Venus and of Mercury are explained, which are the chief ones in this business — and indeed the motions of the Companions of Saturn and Jupiter, and of the Solar Spots as well. Conversely, if the sphere of the Fixed stars is posited as solid, a readier account is given of why they perpetually keep the same distance from one another, and innumerable movers of the Fixed stars will not have to be multiplied. Indeed, since almost that whole heaven is crammed with stars, as the Telescopes show, there is nothing unfitting in God's having ordained the motion of so vast an orb in order to move so many stars, which taken together occupy a great part of it. But to posit the heaven of the Moon as solid is to make it impervious to Comets, if any happen to be formed in it or to ascend upward through it. Nor is it probable that the volatile salts or spirituous parts of the earth reach up as far as the heaven of the Moon, or, if they did arrive there, that they could rebound downward with such force as to descend 50 and more semidiameters of the earth and return to ferment the earth; for, being light, they would rather wander round about the concave of the Moon, like smoke about the vault of a room. The heaven of the Moon is therefore not to be posited as solid for that reason, as our Cabeo wanted.

CAPVT VIII.

Chapter VIII.

VTRÙM CÆLI & AſTRA ſINT ANIMATA.

Whether the Heavens and the Stars Are Animate.

I. [Margin: 1. Opinion, making the stars Gods.] The First Opinion, and the most ancient, held by very many, was that the heavens and the stars are animated by an intellective soul that is blessed, and indeed even divine. [Margin: Egyptians, Stoics, Athenians.] For the Egyptians and the Stoics numbered the stars among the Gods, as Theodoret reports in book 3 *On Evil Gods* [de Dijs malis], and Eusebius of Caesarea in book 1 of the *Praeparatio Evangelica*, chapter 6. Indeed the Athenians did so too: for St Augustine, in book 18 of the *City of God*, chapter 41, and Laertius [Diogenes Laertius], in the life of Anaxagoras, report that this very Philosopher, namely Anaxagoras, was condemned for impiety by the Athenians because, in asserting that the Sun is a fiery stone, he made it out to be neither a God nor animate. [Margin: Lucillus.] *Lucillus* the Stoic certainly argued as follows, in Cicero and in Lactantius, book 2, chapter 5: *This constancy, then, in the stars, this so great agreement of times in such varied courses through all eternity, I cannot understand without mind, reason and counsel; and since we see these in the stars, we cannot but place the stars themselves in the number of the Gods*; and a little before: *It remains*, he says, *that the motion of the stars is voluntary; and whoever sees this would act not only ignorantly but also impiously if he denied that the Gods exist.* [Margin: Cleanthes [?].] *Cleanthes* believed the same, in Cicero, book 2 *On the Nature of the Gods*; [Margin: Plato.] and so did *Plato*, in the *Epinomis* and in the *Timaeus*, whose words addressing the stars as Gods St Augustine reports in book 13 of the *City of God*, chapter 16; and St Cyril testifies the same of Plato in book 2 *Against Julian*. But Plato and the Stoics also added a vegetative soul, as I shall say in number 4.

II. [Margin: 2. Opinion, that of Philo.] The *Second* Opinion was that of those who thought the stars animated by an intellective soul, yet without placing them among the Gods. For *Philo* says in the book *On Dreams*: *that the individual stars are called not only living beings but also most pure minds.* And in the book *On the Giants* he says: *It is necessary that the whole World have a soul in all its parts, etc. The heaven likewise, and the stars. For these are wholly immortal and divine souls, and for that reason they are moved in a circle, a motion akin to mind. For the mind of each one of them is most perfect.* [Margin: Of Rabbi Moyses.] This very view is ascribed to *Rabbi Moyses* [Maimonides] by Martinengus, page 881, and by the Coimbra commentators [Conimbricenses], book 2 *On the Heavens*, chapter 1, question 1. [Margin: Of Origen.] *Origen*, however, in homily 28 on Numbers, and more clearly in book 1 of the *Peri Archōn* [Greek: περὶ ἀρχῶν (peri archōn, "On First Principles")], chapter 7, and in volume 1 of the *Commentary on John*, considering the words of Isaiah chapter 45, *I have commanded the host of heaven*, concludes after much discussion: *But since the stars*

are moved with such order and such reason that their course has never at any time been seen impeded in anything at all, how is it not beyond all stupidity to say that so great an order, and so great an observance of discipline and reason, is demanded of or fulfilled by irrational things? Next, because of the saying in Job 25, *The stars are not clean in his sight*, he judged that the stars are capable of vice and virtue. Finally, because in Romans 8 the creature is said to be subject to vanity and to be freed from bondage, he held that the souls of the stars are to be freed from the body of their globe, and that the souls of the stars were once outside their globes but were afterwards drawn down into them, as into a prison. There are also those who suspect that *Athanasius* partly adhered to Origen, because, expounding the words of Matthew 11, *All things have been delivered to me by my Father*, he adds: *The Angels did not keep their order, and the stars are not held pure before him, and the devil fell from heaven, and Adam transgressed.* [Margin: Of Caietanus.] Among the more recent writers, too, *Caietanus*, on Psalm 133 and in the treatise on indulgences, expounding the words *Who made the heavens in understanding*, and those in the Preface of the Canon of the Mass, *The heavens and the powers of the heavens*, says: *What does he understand by the powers of the heavens? The Angels, the movers? But he listed the Angels themselves above, and ought not to have repeated them. The forces of the heavens themselves? But he ought not to have mixed insensate things with the Angelic orders, and the speech would then have been inept if the forces and accidents of the heavens were coupled with the most excellent substances. Therefore by the powers of the heavens he understood the very souls of the heavens and of the stars.* [Margin: The hesitation of Cassiodorus and of Haymo.] *Cassiodorus*, likewise, shows himself doubtful on this question, on Psalm 148, when he says: *Therefore the Prophet commanded the Sun, the Moon and the Stars to praise the Lord, whether through their own reason or through other substances, sensible and able to be declared.* So too *Haymo*, on the same psalm, showed his hesitation, saying: *If the Sun is said to have a spirit, and the Moon and the other Stars can praise God of themselves: if they do not praise you well because they are matter, why should we praise you?* [Margin: What of St Jerome?] To some, *St Jerome* also seemed doubtful on this, because in book 9 on Isaiah, explaining the words *The light of the Moon shall be as the light of the Sun, and the light of the Star shall be sevenfold*, he says: *That the Moon and the Sun may obtain the rewards of their labour and of their course.* And on chapter 1 of Ecclesiastes, expounding of the Sun the words *The spirit goes on*, among other expositions he gives one according to the mind of Virgil and of Plato, saying: *Or because a spirit within nourishes both the shining globe of the Moon and the Titanian stars, and mind, poured through the limbs, moves the whole mass, etc.* But he himself expressly condemns Origen's opinion, both in letter 59, to Avitus, and in book 13 on Isaiah. [Margin: What of St Augustine?] Likewise *St Augustine*, in book 2 of *De Genesi ad litteram*, chapter 18, seemed to doubt, as also in book 13 of the *City of God*, chapter 16; but

afterwards he retracted it, in book 1 of the *Retractations*, chapters 5 and 11. [Margin: What of St Thomas?] The other light of the Theologians, *St Thomas*,

following Aristotle, teaches in the *Disputed Questions*, the question *On the Soul*, article 8, in the second book *Against the Gentiles*, and in the *Opusculum on the Angels*, that the celestial bodies are informed by a certain intellective soul, which strives in the motion of the heaven to imitate the divine likeness; which, according to the Philosopher, from book 2 *On the Soul* and book 11 of the *Metaphysics*, is purely intellective, and not also imaginative, as Avicenna would have it; and which, finally, could have sinned but did not sin, and belongs to the company of the Angels. Nevertheless, in the first part, question 70, article 3, he concludes that the soul of a celestial body is not united to it as a form, because it is helped by the body in no operation, but only as a mover to a movable thing, by contact of power. His last words are: *It remains, therefore, that it is united to the celestial body for the sake of motion alone* — understand this — *but for it to move, it is not necessary that it be united to it as a form, but by contact of power, as a mover is united to a movable thing*; and he adds the example of the first mover that moves itself, composed, according to Aristotle in book 8 of the *Physics*, of a moving part and a moved part, which are united by contact; and finally, *The Platonists too*, he says in the same place, *held that souls are united to bodies only by contact of power, as a mover to a movable thing.* From this passage Caietanus and many Thomists judged that St Thomas changed his opinion, as one who had left his last will declared in the *Summa*, as in a testament; but Ferrariensis [Francesco Silvestri], book 2 *Against the Gentiles*, chapter 70, and Vielmius think that he adhered constantly to neither side. [Margin: Vielmius's opinion.] Vielmius himself, the teacher of Ascanius Martinengus, judged in lecture 19 on Genesis that this is a problem, notwithstanding the decree of the University of Paris. For he says, in the question whether the heaven is animate: *And let this much be said for the negative opinion, which I do not in the least doubt is the more probable — so much so that the affirmative was long ago placed among the articles condemned at Paris, as Richardus writes in the place cited. But nonetheless I think that the arguments brought on both sides can be answered in the diatribes in such a way that it appears with certainty both that those who thought otherwise were not unlearned, and that it is not wicked even now to discuss it.* [Margin: Aureolus, Scotus.] *Aureolus* [Pierre Auriol], on book 2, distinction 14, question 2, article 2, and question 3, article 1, and *Scotus* in the same place, question 2, likewise judged the affirmative side, concerning an intellective soul of the heaven, to be probable in view of the arguments, Scotus saying: *If the stars are not animate, that will have been believed rather than demonstrated.*

III. [Margin: What of Aristotle?] So much for the ancient Philosophers before Aristotle, and for the Theologians. What Aristotle thought, however, and after him the Peripatetics and the other Philosophers, is disputed. For in book 2 *On the Heavens*, texts 13 and 61, and in book 2 of the *Metaphysics*, text 35, he seems to attribute to the heaven an intellective soul as its form; and so he is interpreted by Themistius, Averroes, Simplicius and Philoponus, and also by *St Thomas*, book 2 *Against the Gentiles*, chapter 50, and in the *Disputed Questions*, the question *On the Soul*, article 8, reply to 3; by *Vasquez* [Gabriel Vázquez]

on the first part, question 183; *Niphus* [Agostino Nifo], book 1 *On the Substance of the Orb*, chapter 29; *Pomponatius* [Pietro Pomponazzi], *On the Immortality of the Soul*; *Achillinus* [Alessandro Achillini], book 3 *On the Orbs*, doubt 1; and *Balduinus* and *Mercenarius* in the question *On the Soul of the Heaven*. But from book 8 of the *Physics*, texts 40, 41 and 52, and book 12 of the *Metaphysics*, texts 30 and 43, it seems to be gathered that Aristotle does not attribute an Intelligence to the heaven as an informing form, but only as a pure act that assists it, like a sailor his ship — and indeed not even as moved along with the motion of the heaven. So he is interpreted by *Scotus*, quodlibet 7; *Piccolomineus* [Francesco Piccolomini], *On the Heavens*, chapter 17; *Iandunus* [Jean de Jandun], book 2 *On the Heavens*, question 4; and *Alexander*, as cited by Simplicius. An intellective soul was likewise attributed to the heaven by *Iamblichus*, *Porphyry* and *Proclus*, as cited by Psellus on the oration of St Nazianzen, 38 and 42; by *Julius Firmicus*, book 1 of the *Mathesis*, chapter 3; by *Bellantius*, as cited by Alexander de Angelis, book 1 *Against the Astrologers*, chapter 13; by *Marsilius Ficinus*, in the dialogue *On the Laws*, argument 10; and by *Carpentarius*, himself a Platonist, chapter 12.

IV. [Margin: 3. Opinion, on a vegetative and sensitive soul of the heavens.] The *Third* Opinion was that of those who attributed to the heaven, besides an intellective soul, a sensitive one as well, and indeed a vegetative and a locomotive one too. Among them are those who thought that the stars are nourished, of whom *Plutarch* speaks thus in book 2 *On the Opinions*, chapter 17: [Margin: Heraclitus, Plato, the Stoics.] *Heraclitus and the Stoics hold that the stars are nourished by vapours rising from the earth. Aristotle, that the celestial bodies do not need nourishment, for they cannot be corrupted, but are eternal. Plato and the Stoics, that the stars, like the whole universe, take nourishment*; but Plato should be read in the *Epinomis*. [Margin: Pliny.] Of this opinion, without doubt, was *Pliny the Elder*, book 2, chapter 8: *Not so great*, he says, *is the fellowship of the heaven with us that the brightness of the stars too should be mortal by our fate. Through excessive nourishment of the moisture drawn up, they give back its fiery force in abundance when they are believed to fall*; and in chapter 9 he refers the spots of the Moon to this, saying: *But the stars are without doubt fed on earthly moisture, since at half orb the Moon is never seen without spots — its force, that is, not yet being sufficient to draw up beyond its due. For the spots are nothing other than the dirt of the earth carried up with the moisture.* [Margin: Avicenna, Simplicius.] *Avicenna*, moreover, book 9 of the *Metaphysics*, chapter 2, besides an intellective soul moving the heaven remotely, gave it a sensitive soul produced by the intelligence, so that it might move it proximately by the imagination of a sensible motion[25]. *Simplicius*, on book 1 *On the Heavens*, text 50, added three external senses, namely sight, hearing and touch. [Margin: Tycho.] *Tycho*, for his part, in the *Epistolae* of the year 1590 to Rothmann, recognized a divine, and moreover a vital, spirit in them, saying: *But these things happen in the celestial bodies in a certain celestial manner, livelier and more refined than in terrestrial or marine animals; since that divine*

[25]the print reads "mo ûs", the t not printed

philosophy of the Platonists seems, not without reason, to have held that the heaven too is animate, and that the celestial bodies themselves are certain living creatures of the heaven, endowed with a vital spirit. [Margin: Kepler.] Lastly *Kepler*, in the book *On the New Star* and in the *Physiologia* of comets, attributes to the heaven a certain soul whose office it is to gather, at certain times, the soot and filth of the heaven, which would taint the splendour of the Sun and the stars and the pellucidity of the aether if they were not kindled into Comets. For when a great quantity of such filth has arisen, *then*, he says on page 99, *there is need of defecation and purging, which that faculty performs which is in the very substance of the aetherial air, a faculty similar to an animal or vital one.* And in the introduction to the commentaries on Mars he will have it that the Sun is endowed with a soul, which preserves for the Sun its light and its power of moving the other Planets; he calls it a motive soul, one that gives the Sun a corporeal life. [Margin: Baranzanus.] And *Baranzanus* [Redento Baranzano], in part 1 of the *Uranoscopia*, question 2, doubt 3, attributes to the celestial bodies a certain soul intermediate between the intellective and the brute soul, agreeing with these only analogically.

V. [Margin: 4. Opinion, the Negative.] The *Fourth* opinion, the true and sound one, is that the Heaven and the stars are inanimate, and wholly lack any soul that would be a form informing the celestial bodies; but that if they are moved by an Intelligence, they are moved by it not as by an informing form but as by an assisting Mover. For this opinion there contend, first of all, *Catherinus*, on chapter 1 of Genesis, where he denies that Caietanus's opinion can be defended by a Christian; *Ascanius Martinengus*, in the *Glossa magna*, pages 887 and 890, who, though he thinks it a problem — setting aside Origen's error — whether the stars are animated by a rational soul, nevertheless confirms by arguments and authorities that the negative side is far more probable; *Mastrius and Bellutus*, disputation 2 *On the Heavens*, question 2, article 2, from number 62; and *Io. Poncius*, disputation 22 of the *Physics*, question 3. Among the writers of our own Society there are *Pererius*, on Genesis, book 2, question 7; the *Coimbra commentators*, book 2 *On the Heavens*, chapter 1, question 1, article 2; *Toletus* [Francisco de Toledo], book 2 *On the Heavens*, chapter 5; *Rubius* [Antonio Rubio], book 1 *On the Heavens*, chapter 2, question 8; *Tannerus*, dissertation *On the Heavens*, question 4, and volume 1 of the *Summa theologica*, disputation 6, question 2, doubt 5; *Amicus*, treatise 3 *On the Heavens*, question 4; *Arriaga*, single disputation *On the Heavens*, section 2; *Oviedus*, single controversy *On the Heavens*, point 1; and *Balthassar Tellez*, disputation 43 of the *Physics*, in whom many scholastics are cited. Let this, therefore, be the

VNICA CONCLVSIO.

Single Conclusion.

The Celestial Bodies are animated neither by an Intellective or rational soul, nor by a sensitive soul, nor by a vegetative soul.

VI. [Margin: 1. Argument.] It is proved, 1, by the Authority of the Fathers: especially of *St Cyril*, book 2 *Against Julian*; of *St Basil*, homily 3 on the *Hexaemeron*; and of *St Ambrose*, book 2 of the *Hexaemeron*, chapter 4 — the two of whom refute Origen even on this ground, that dew and frost, cold and hoar-frost would then also have to be called animate, because they are invited to praise God; of *St Gregory Nazianzen*, orations 38 and 42, where, as Psellus interprets him, he condemns the impostures of the Platonists about the soul of the heavens; and of *St Chrysostom*, in the homily on Psalm 4, where among other things he pronounces: *For the Sun has neither reason, nor mind, nor thought.* Likewise by that of Theophilus of Alexandria, in Paschal letter 1, where he says that Origen *builds up dogmas contrary to the faith*; and of *St John Damascene*, who says: *Let no one think the heavens or the Luminaries animate, for they lack soul and sense. Procopius* teaches the same, with St Basil, in the *Commentary on Genesis*; and *Lactantius*, book 2 of the *Divine Institutes*, chapters 5 and 8, sharply attacks the Gentiles with the argument that if, on account of their constancy in motion, the stars seem fit to be ranked among the gods, one ought rather to conclude from this that they lack not only divinity but also intellect and will. For if they were gods, endowed with intellect and will, they would wander freely here and there as they pleased; and since that does not happen, but they are compelled to weave the same path perpetually, like beasts fastened to a mill, their motion is not voluntary but necessary. Yet this argument would also strike at the moving Intelligences. For the remaining Fathers, see Ascanius Martinengus.

[Margin: 2nd argument.] It is proved, second, from the definition of the Church. In the Fifth General Council, which was the Second of Constantinople, among the other errors of Origen — as Nicephorus reports, book 17 of the *Ecclesiastical History*, chapters 27 and 28; Ioverius in the *Summa Conciliorum*, volume 2; and Liberatus in the *Breviarium*, chapter 23 — this one too was condemned by the decree: *If anyone says that the heaven, and the Sun, and the Moon, and the stars, and the waters that are above the heavens, are certain animate beings and material powers, let him be anathema.* Here "the material powers of the heavens" is taken in Origen's sense, as souls informing the bodies of the stars. Origen's error about the souls produced before the heaven, by contrast, the Council condemned separately, as Nicephorus reports in that same chapter 27; hence by this other decree a distinct error is condemned, namely the one by which an intellective soul is attributed to the heaven and the stars. Moreover, in the chapter *Firmiter*, on the Supreme Trinity and the Catholic Faith, no rational creatures are listed except Angels and Men; St Jerome counts this opinion among the heresies; and St Epiphanius, book 11 of the *Panarion*, and St Irenaeus, book 1 *Against Heresies*, list the opinion on the soul of the heavens among the condemned doctrines of the Marcosians.

[Margin: Censure of the opposite opinion.] On account of these and similar authorities, therefore, the opinion asserting that the heaven and the stars are animated by an intellective or rational soul is judged erroneous in faith by St Bonaventure, on [the *Sentences*] 2, distinction 14; by Petrus Tarantasius [Pierre

de Tarentaise], cited by the Carthusian on 2, d. 14, q. 2; by the *Directorium Inquisitorum*, from page 239 on; and by Tannerus, Tellez, Poncius and the Conimbricenses in the places listed above. That it is not consonant with the faith is taught by Capreolus [Jean Capréolus] on 2, d. 9, q. 1, art. 1, by Gabriel [Gabriel Biel] at the same place, q. 1, and by Pererius, book 2 on Genesis, q. 7; and Catherinus, on chapter 1 of Genesis, says that it cannot be defended by a Christian. Pineda [Juan de Pineda], on chapter 1 of Ecclesiastes, page 124, calls our opinion *a Doctrine of the Church and next to Faith*; Lorinus [Jean Lorin], however, at the same place, verse 6, denies that the opposite is an open error, granted that he holds, against Paulus Ricius [Paolo Ricci], that the heavens are not animate. But if anyone attributes to the heaven or the stars only a sensitive soul, or a vegetative one, or a locomotive one, that proposition is judged *absurd indeed and erroneous in Philosophy* by Tannerus, above, assertion 1, and by Tellez; and fit to be utterly hissed off and rejected by Martinengus — though not contrary to Faith. Granted, not even the opinion that gives them an intellective soul was thought erroneous in Faith by Vielmius, lecture 19 on Genesis, or by Martinengus in the *Glossa*, page 890, but rather to be a problem on the path of the Fathers [in via Patrum]. Yet this same Vielmius admits, on the authority of Ricardus, that it was condemned by the Academy of Paris.

[Margin: 3rd argument.] It is proved, third, by theological arguments. If those intellective souls were blessed and holy, it would be lawful to adore them with some cult and veneration — if not of latria, then at least of hyperdulia or dulia, as the theologians say. But this is against Deuteronomy, chapters 4 and 17, where the cult and adoration not only of foreign gods but also of the Sun, the Moon and all the host of heaven is forbidden; nor has the Church ever permitted this cult, even of dulia, towards the stars. On the contrary, it is execrable, as the sacred interpreters teach on that passage of Job: *If I saw the Sun when it shone, and the Moon advancing brightly, and kissed my hand in my bosom; which is the greatest iniquity, and an abomination before the most high God* — that is, if I adored the Sun and the Moon. And if those souls are damned, then the place of the damned would be in heaven itself, or the heavens and the stars would at some time have to be thrust down into hell and the eternal fire, which is the common place of all the damned, as is gathered from Matthew 25, verse 41. Since, however, someone could pretend that those souls are not capable of merit and demerit, I therefore add a fourth proof.

[Margin: 4th argument.] It is proved, fourth, because neither in the heaven nor in the stars do we have any indication a posteriori, from their operation, of a sensitive, vegetative or rational soul; nor do any organs appear in them that would be necessary, mediately or immediately, for these operations. Therefore an informing soul is posited in them rashly and without foundation. And if you say that those bodies do not need organs or dissimilar parts, this very kind of soul, animating a non-organic body, is nonetheless invented gratuitously; and for their regular motions the external direction of an assisting and moving Intelligence is sufficient.

[Margin: Objections resolved.] Nor does that verse of Psalm 135 stand in the way: *Who made the heavens in understanding*. It is to be understood of the wisdom of God himself, by which he founded and ordered the heavens and their motions, as St Augustine, Euthymius, Cassiodorus and Haymo expound it there; or of the Word, which is the Son of God, as St Bruno takes it. As for Psalm 18, *The heavens declare the glory of God* etc., Psalm 148, *Praise him, Sun and Moon*, and Job 38, *When the morning stars praised me*: these are understood objectively, not formally — that is, because the stars are an object highly suited to rouse Angels and men to praise God. Likewise Isaiah 45, *My hand stretched out the heavens, and I commanded all their host*, and Habakkuk 3, *They were called, and they said, Here we are* etc., are to be taken metaphorically; or else they indicate the obediential potency by which every creature, even an inanimate one, is subject to the command of God and serves it. Finally, the verse of Ecclesiastes 1 about the Sun, *Surveying all things round about, the spirit goes on its way*, can be understood either of the Intelligence governing the Sun or in other ways, of which below, in Section 2, where we treat of the movers of the heavens. [Margin: A soul lower than the rational may, without prejudice to the faith, be attributed to the stars.] It should be noted, however, with St Thomas, *Contra Gentiles* [book] 2, chapter 7, that it matters little to the Faith whether the stars are called animate or not, provided they are not said to have an intellective soul.

CAPVT IX.

Chapter 9.

An Cælum ſit Infinitum, & an Figuræ Sphæricæ.

Whether the Heaven is Infinite, and whether it is of Spherical Shape.

On the former question, so far as it concerns the astronomer, I shall set out the choicer points in Section 4, chapter 5, number 19. On the latter, that is, on the shape, see the same chapter 5, number 2, and Pliny's arguments, book 2, chapter 2, whose words we shall quote in that chapter 5, number 20. Meanwhile let us move on to other matters.

SECTIO SECVNDA DE MOTORIBVS ET MOTIBVS CÆLORVM.

Section Two: On the Movers and Motions of the Heavens.

CAPVT I.

Chapter 1.

CAPVT I.

An Cæli aut Sidera Moueantur ab Intelligentijs, An verò ab intrinſecò à propria Forma vel Natura.

Whether the Heavens or the Stars are Moved by Intelligences, or rather from within, by their own Form or Nature.

I. [Margin: 1st opinion: that their own form moves the heavens.] The first opinion was that the heavens are moved by their own form, that is, from within. Otherwise that form would be idle and inferior to the forms of the elements, since each thing exists for the sake of its own operation, as the Philosopher says, *De caelo* book 2, chapter 3, text 17; and otherwise that perpetual circular motion would be violent, since, from *Ethics* 3, chapter 1, that is violent whose principle is outside. And certainly there seems to be nothing to prevent God from making some body that has a natural appetite for perpetual circular motion, and so a power of moving itself perpetually in a circle — just as an impetus once impressed on a stone for motion in a circle would perhaps last perpetually, as some say, if it were not gradually destroyed by a contrary impetus produced by gravity. Since, therefore, there is no gravity or levity in heaven, what wonder if that impetus, once driving the celestial body in a circle from its origin by the force of its own form, revolves it perpetually? [Margin: Major, Parisiensis, Albert of Saxony, Gabriel, Trallianus, Strato.] Of this opinion, then, were Io. Maior [John Mair], *De caelo* book 1; Parisiensis [Guillaume d'Auvergne], *De Universo* part 2, chapter 152, who thinks the opposite vain and frivolous; Albertus de Saxonia [Albert of Saxony], *Physics* 8, last question; and others cited by St Bonaventure and by Bassolius on 2, d. 14, art. 3, q. 2; and Gabriel thinks it probable, at that place, q. 1, art. 3, doubt 2. In former times too, as the Conimbricenses report, *De caelo* 2, chapter 5, q. 1, [Margin: Pererius.] and Pererius on Genesis, book 2, q. 5, Trallianus and Strato of Lampsacus said that the motion from right to left, that is, from east to west, comes from an Intelligence, but that the motion from left to right comes from its own form.

[Margin: Longomontanus.] Among the more recent writers, Longomontanus, in the preface to book 1 of the *Theorics*, says that they are moved from within; [Margin: Vallesius.] Vallesius, by contrast, *Controversies* 11 and 26 on the *Physics*, teaches that they are moved partly by their own form and partly from without, since an Intelligence regulates the speed; [Margin: Camerarius.] and so Camerarius, disputation 25, who was moved by that verse of Psalm 18, *He exulted as a giant to run his course*, to think that the Sun is moved from within — although it is said metaphorically — and by that phrase in the Preface of the Mass, *the heavens and the Powers of the heavens*, where the powers of the heavens are counted as distinct from the Angels; as if the word "powers" could not there mean forces of influence, or indeed some order of Angels, since the Preface goes on to say, *and the blessed Seraphim*.

[Margin: Eusebius Nierembergensis.] Eusebius Nierembergensis [Juan Eusebio Nieremberg] inclines strongly to the same opinion in his *Philosophy*, book 6, on the life of the stars. He concedes, however, that for stirring up extraordinary motions, or for halting their motions, some Intelligence is appointed,

at one time or another: as when, at the death of Christ, the full Moon was brought back to conjunction with the Sun; and when the Sun's course was held back, not only at Joshua's command but also at the prayer of St Francis Xavier, [Margin: St Francis Xavier halts the Sun.] so that a ship being tossed by a violent storm might reach port before sunset — as Velasquez, Sherlogus and Oviedus (the single controversy on the heaven, point 1, number 7) relate of St Xavier. [Margin: Tellez.] Baltazar Tellez, disputation 44, section 3, affirms without qualification that the stars are moved by their own form. In disputation 40, section 3, he had also taught that if the stars are considered in themselves, without regard to lower things, circular motion is not connatural to them but comes from without; but if they are considered as what they are, the principal part of the universe, possessing natural powers of influence, it is natural to them.

The author of the *Philolaus* (book 4, chapter 2), [Margin: Bullialdus.] Ismaël Bullialdus [Ismaël Boulliau], likewise concludes, in book 1 of the *Astronomia Philolaica*, chapter 12, and in the *Philolaus*, book 4, chapter 2, that both the Sun, about its own axis at the centre of the world, and the other planets (among which he counts the Earth) are moved by their own form.

[Margin: Kepler.] Kepler, on the other hand, in the introduction to the *Mars* and in chapter 33, and in book 4 of the *Epitome of Copernican Astronomy*, parts 2 and 3, that is, from page 499 to 530, teaches that the Sun is at the centre of the world and has a motive soul with a kind of bodily life, by which it is moved perpetually about its own axis. *For although this motion*, he says on page 514, *could be continued by some other means, yet the duration and perpetuity of this motion, in which the life of the whole world consists, is more correctly secured by the support of a soul.* He then says that the Sun emits from itself a species similar to the one a magnet emits, and through it grasps the other planets as if by a hand; and while it turns in rotation about its own axis, it carries the other planets around. It does so, however, in such a way that when that magnetic virtue falls on the friendly part of the planets, according to the disposition of their magnetic fibres, it draws them towards itself, and they become perihelial. But since the planets have a power of holding themselves in a position parallel to the one they first acquired at the beginning with respect to their situation in the universe, it comes about that gradually their hostile part, the face contrary to the Sun, is turned towards it; and so they are driven off by the Sun's magnetic species or quality and become aphelial — just as a magnet draws to itself the friendly part of a magnetic needle and drives off the hostile part. Finally he says that in the other planets there is a certain inertia towards motion, and a resistance that is greater the farther they are from the Sun. Thus he teaches that the Sun is moved from within and the other planets from without, namely by the Sun — but by no means by an Intelligence (page 508). Bullialdus refutes these fictions of his, ingenious as they certainly are, with many arguments in the place cited. Yet the same Kepler, in the *Hyperaspistes*, page 6, admits that the Christian schools ascribe these motions of the heaven to Angels; and, on

page 7, that according to Tycho the Star of the Magi and the comets were moved either by a Genius or by an innate knowledge.

[Margin: What of Tycho?] Tycho seems equally to belong here. In volume 2 of the *Progymnasmata*, page 268, he says: *It is therefore proved, even from these observations alone of the two ethereal comets, namely those of the years 1577 and 1585, on account of their motion peculiar beyond the other planets, that no orbs really exist in the aether, nor does the heaven itself consist of hard and impervious matter; but that the stars themselves possess a certain natural and innate — or rather divinely implanted from the beginning, and perpetually preserved — knowledge of regular motion, by which, driven or supported by no orbs, they complete their courses most perfectly and most constantly.* But from these very words, and from what was said in the preceding section, chapter 8, number 4, it is clear enough that he is to be assigned to the second class, which teaches that the stars are moved by a soul that is not only sensitive but also intellective; and Kepler too belongs there in part. [Margin: What of Suárez and Raynaudus?] Suarez [Francisco Suárez], moreover, *Metaphysical Disputations* 29, section 1, number 16, and earlier in disputation 18, section 7, number 36, nevertheless concedes, in the course of disputation, that the motion of the heaven can be attributed to its own incorruptible form; and Theophilus Raynaudus [Théophile Raynaud], in his *Natural Theology*, distinction 2, q. 1, art. 1, says that the opinion of those who assert that the heaven or the stars are moved by an intrinsic form of their own is not to be condemned as rash.

II. [Margin: 2nd opinion: on a moving soul.] The second opinion was that of those who said that the heavens and the stars are indeed moved by their own form, but that this form is an intellective soul, or else a sensitive and vegetative one. I have already listed these individually in Section 1, chapter 8, from number 1 to 4, and so we do not repeat them. [Margin: 3rd opinion: on God as mover.] The third opinion, then, was that the heavens are moved by God himself, immediately. So judged Albert the Great, on 2, distinction 14, art. 6; Alpetragius [al-Biṭrūjī], *On the Physical Cause of the Celestial Motions*; and, if you believe the Conimbricenses, Ptolemy himself; and Leonardus Lessius [Leonard Leys], in the book *De Providentia*, from number 20 on. This was also judged probable by

Gabriel, on 2, d. 14, the single question, and Gregorius Ariminensis [Gregorio da Rimini], on 2, d. 8, q. 3. For although Abulensis, on chapter 22 of Exodus, q. 5, denies that local motion can be produced by God immediately — both because it is the act of a being in potency, and something imperfect, and because he would then move in an instant — it is nevertheless an error to deny this, as St Thomas teaches, part 1, q. 105, art. 2. Whether the supreme heaven is moved immediately by God, according to Aristotle, I shall say at number 11. Nor, if it is denied that God immediately produces the motion of the supreme heaven as a particular cause, will it follow that there is an infinite regress among movers and causes, or that the means of demonstrating the divine existence is taken away. For it suffices that one can infer from creatures and effects that

God exists as a universal cause, who nonetheless concurs, by an immediate concurrence, with the efficient secondary causes.

[Margin: The existence of God is demonstrated by a new argument.] There are, moreover, other ways of demonstrating the divine existence, and the one that strikes me as the most evident of all is this. We judge evidently that a supremely perfect being is possible, one having all the simply simple perfections that are compossible (for if some perfection is mixed with imperfection, or conflicts with another that is more necessary and more to be chosen, we are not speaking of it). But among the other simply simple perfections, which conflict with no such perfection, is necessary existence — that is, so to exist that it never can, and never could, not exist — as any intellect that is not badly disposed judges. Therefore, if such a being is possible, it also exists in fact, since that perfection includes not the mere possibility of existing but actuality, and indeed necessary actuality. And in this case alone the argument from the possible to being, that is, from being able to be to actually being, is valid. Many years afterwards I found this argument indicated, though in a different form of words, by Renatus Cartes [Descartes] in the *Principles of Philosophy*, part 1, number 14.

[Margin: Some have said that the heaven moves by chance.] Let this be said against the atheists. But those who wish to show God from the motion of the heaven are diametrically opposed by those who said that the heaven moves by chance, namely Democritus and Epicurus.

III. [Margin: 4th opinion: that they are moved by Angels.] The fourth opinion, and the most common, is that the heaven and the stars are moved by Intelligences, or Angels, as assisting and immediate efficient causes, not as souls informing the heaven; and so that they are moved by an extrinsic principle, but a created one. In this opinion are all those already enumerated in Section 1, chapter 8, from number 5 on, and those who treat this controversy expressly. [Margin: Authors of the 4th opinion.] They are: St Thomas, part 1, q. 70, art. 3; q. 6 *De potentia*, art. 3; opuscule 10, art. 3; opuscule 11, art. 2; and *Contra Gentiles* 2, chapter 92, with Ferrariensis at that place. Likewise the Scholastics almost generally on 2, dist. 14, especially St Bonaventure, art. 3, q. 2; Scotus, q. 1; Durandus, q. 2; Argentinas [Thomas of Strasbourg], q. 1 and 2; Ariminensis on 2, d. 1, q. 1; Capreolus, dist. 9, q. 1, art. 3; Albertus, *De quatuor coaevis*, part 1, q. 4, art. 26; Nicolaus Cusanus [Nikolaus von Kues], book 7 of the *Exercitationes*; Soncinas, *Metaphysics* 12, q. 36; Iandunus, *De substantia orbis*, q. 2, 13 and 14, and *De caelo* 2, q. 92. Among the more recent: the Conimbricenses, *De caelo* 2, chapter 5, q. 5; Rubius, at the same place and *De caelo* 1, chapter 2, q. 8; Toletus [Francisco de Toledo], *De caelo* 2, chapter 5; Suarez, *Metaphysics*, disputation 18, section 7, number 36, and d. 35, section 1, number 20; Pererius, book 2 on Genesis, q. 5, and *Physics* book 7, ch. 7, q. 2, who says that it was handed down as if from hand to hand by the philosophers and received by the theologians; and Bubalus, *De Angelis*, q. 1, diff. 2, art. 15, who judges the opposite dangerous; Arriaga [Rodrigo de Arriaga], the single disputation on the

heaven, section 5, number 64; Hurtadus, disputation 2 on the heaven, section 4; Oviedus, the single controversy on the heaven, point 1, number 7; Mastrius and Bellutus [Bartolomeo Mastri and Bonaventura Belluto], disputation 2 on the heaven, q. 4, art. 3; Georgius Polaccus [Giorgio Polacco], in the *Anticopernicus*, assertion III; Aversa, q. 34 of the *Physics*, section 7; Io. Antonius Delphinus [Giovanni Antonio Delfino], Franciscan, the opuscule *On the Celestial Orbs*, from page 80; Io. Poncius, disputation 22 of the *Physics*, q. 9; Amicus, treatise 5 on the heaven, q. 6, doubt 2; and Chiaramonti, book 2 *De Universo*, chapter 17.

[Margin: Notable sayings of St Thomas.] But I am glad to record the view of St Thomas, who says, in q. 6 *De potentia*, art. 3: *But it is the teaching of the Faith that Angels move locally by their command not only the celestial bodies but also other bodies, God ordaining and permitting.* He had reported the opinions of the philosophers who granted the Angels the motion of the heavens but denied them that of other bodies. And in opuscule 10, art. 3: *But that the celestial bodies are moved by a spiritual creature, I do not remember having read denied by any of the Saints or the philosophers.* Finally, in opuscule 11, art. 2: *But it seems to me that it can be proved demonstratively that the celestial bodies are moved by some intellect, either by God immediately or by the mediation of Angels. But that he moves them by the mediation of Angels befits the order of things, which Dionysius asserts to be infallible, that lower things are administered by God through intermediaries according to the common course.* That this was the view of the ancients was taught by Trithemius in the book *On the Seven Intelligences that Move the World*, whose words I have already reported in book 7, section 1, chapter 1, where I listed many things, not unworthy of a learned reader, about the seven spirits that move the seven planets.

VNICA CONCLVSIO.

Single Conclusion.

Although it cannot be demonstrated either metaphysically or mathematically, but at most physically or morally, that the heaven or the stars are moved by Intelligences, yet, in view of authority both sacred and profane, it must be said that they are moved by Intelligences.

IV. The first part of the Conclusion is evident, because neither from the principles of metaphysics nor from those of mathematics can a necessary connection be deduced between the motion of the heaven or the stars and the Intelligences — whether their motion is considered according to its substance or its essentials, or as to its accidental perfections: especially the most perfect order and harmony, that is, the uniformly difform anomaly, and the infallible rules that they keep in their periods and revolutions, and their perpetuity and constancy over so many centuries. For it is not repugnant either to the principles of being as being, or to terminated quantity as such, whether continuous or discrete, that God could make bodies which are moved in this way from within, by a virtue and qualities connatural to them and implanted by God at the beginning of the world, as we said at number 1 in reporting the first opinion; and that they are moved by such

motion for the good of lower things in such a way that this very thing is their good — to exert their virtue and communicate their influences to others. In the same way it is the perfection of a master to teach others, and of a physician to heal others; granted that this is an extrinsic perfection, it nevertheless reveals an intrinsic one. For what some have attempted against this is too slight to need a reply: as when they say that circular motion is reflexive upon itself, and that reflexion comes from a soul; and that every mover presupposes an immobile part on which it leans, as when an animal leans on its foot — for what will they say[26] of the fins of fishes and the wings of birds? — or when they say that the same thing cannot be at once in potency and in act, and that therefore every mobile thing must be moved from without.

Nor is a moving Angel necessary in such a way that its existence could be demonstrated from this. For either its existence is known from elsewhere, and so it is not necessary on that account; or it is unknown, and so the thing about which the question is raised here, as the less known, is not shown from what is better known. [Margin: Whether the existence of Angels is demonstrated from the motion of the heavens.] Moreover, we have already shown with many arguments in the treatise *On Angels*, while we were publicly professing theology, that there is no natural middle term by which the existence of Angels may be demonstrated. For all effects either include a determination to moral fault or to falsity, and can then be attributed to separated souls; or they do not include it, and can be attributed either to those same souls or to God. And so on, reasoning through the rest.

[Margin: 2nd part of the Conclusion.] The second part, about physical or moral demonstration — conceded rather than asserted — is proved as follows. On the one hand, we have no indication in the stars, or from the stars, of animal operation, since neither organs nor anything similar appear in them. On the other hand, the admirable harmony of the celestial motions, and their proportion and order, constant through so many centuries — far greater than in the humours of the human body, indeed than in any machines devised by human skill — is an argument for a soul or mind that constantly preserves and directs such motions. Therefore it is physically, or at least morally, evident that the heaven or the stars are moved by some mind, which is nevertheless not properly their soul. This holds all the more since there is such variety and subtlety in the motions of the planets that not even the most skilled of the astronomers have so far fully comprehended it. Add to this that we have it by sufficient induction that whatever is moved locally is moved in order to be better off in a different place, and that to be moved within its own place is proper to living things; but neither of these fits the heaven or the stars.

The third part is confirmed, first, from the multitude of autho- rs of both classes; and then from Sacred Scripture. For many Fathers interpret as referring to the Angels the text of Job 9, *Under whom they stoop that bear up the world*, and

[26]the print reads "d-" / "cent", the i not printed

Job 26, *The pillars of heaven tremble*, and 38, *When the morning stars praised me, and all the sons of God shouted for joy*, and the text of Matthew 24, *The powers of the heavens shall be moved*. Lastly, St Dionysius (*Celestial Hierarchy*, chapter 5), St Augustine (*On the Trinity*, book 3, chapter 4) and St Gregory (*Dialogues*, book 4) teach that it belongs to the sweetness of divine providence to move and govern lower things through higher ones, and corporeal things through spirits; and by ascribing this motion to the Angels all difficulties are more easily avoided. Nor, as Lessius seems to fear, is it absurd that some of the Angels should be held perpetually — that is, until the end of the world — occupied in this ministry, so that they can neither ascend to the Empyrean nor descend to the earth to men. For they obey God most willingly, and their beatitude is not diminished by it; and if it were expedient to send some of them to earth — say, to adore the Most Holy Sacrament of the Eucharist more closely, or to instruct someone in some account of celestial motion — it is very easy for God to put others in their place.

[Margin: The objections of Kepler resolved.]

V. But now the objections of Johannes Kepler must be resolved, by which, in his *Epitome of Copernican Astronomy* from page 508, he contends that created Intelligences do not move the planets, but that the planets are moved by the Sun with a certain material and natural necessity. For, he says, solid orbs cannot be admitted, since it has been shown that the heaven of the planets is fluid; and without solid orbs a created Intelligence cannot move the planets. First, because a mind lacking animal and locomotive power cannot produce motion in a body other than its own by a mere nod, or by the desire or command of its will. Second, because if a Mind were the mover, it would certainly move the planets in a perfect circle, which has mental beauty and perfection; but from astronomical observations properly treated we are convinced that the path of a planet is not a perfect circle but either an ellipse or a similar figure, which savours rather of the nature of the steelyard [statera] and of material necessity than of the conception of a mind. Again, although the tempering of the extreme motions — that is, of the slowest and the fastest in each planet — is most exquisitely harmonious, and the work of the supreme and adorable creating mind, yet the lengths of the periodic times have no mental beauty, because they do not have expressible or rational proportions, such as double, triple and so on, but inexpressible or irrational ones that thus run on to infinity; nor are the times of a single period gathered from equal delays [morae], but from unequal delays in different parts of the circle, according to the ratios of the steelyard. Lastly, no place can be assigned in which the Intelligence might reside and from which it might recognise the path along which it is to direct the planet. Not at the centre of the circle, or at the centres or foci of the ellipse, for in the ethereal air those places do not differ from other points; nor from there could the mind perceive whether the planet, at so great a distance, was travelling along the path it must, unless you give it eyes, or a cord to which the planet is tied and led round. For it is not enough to have a mental idea of

the planetary orbit: some instrument is needed by which the real course of the planet, having a definite quantity, may be carried into execution. Nor can the Intelligence be placed on the circumference of the orbit. For either it will rest at one point — and the argument just made recurs when the planet is at very distant points — or it will move from place to place, as the soul does with the body. But since this mind lacks a body, and is not of itself mobile or capable of position, it will have no means by which to measure its distance from the centre of the world (since this centre lies outside the centre of the planetary orbit), or to keep its position in relation to that centre. Yet Kepler grants that the figure of the planetary orbit and the periodic motions are the work of the creating Mind, that is, of God. This is precisely Kepler's reasoning, and it seems to me as much the duller as it seemed to him the sharper: first, because it assumes as granted certain things that cannot be granted without injury and discredit redounding upon the Intelligences; second, because it entangles reasons that contradict one another. It will be worth the effort, however, if I put these arguments into form and blow them away.

[Margin: First argument from Kepler.] [Margin: Proof of the [?]]

VI. The first argument is of this kind. *If a Mind or created Intelligence moved the planets, it would move them in a perfect circle, and in such a way that the periodic time of a planet would have a rational proportion, one that can be gathered from equal delays.* For such a figure and motion have a beauty and perfection worthy of a mind and befitting an Intelligence. But the motion of a planet does not take place in a perfect circle, nor does its periodic time have a rational proportion, nor one gathered from equal delays. Therefore …

[Margin: First reply.]

I reply, first, by denying the major premise, for there is no necessity in such a proposition. The circle is not the only perfect figure — especially with respect to the end to which the motion is ordered: the ellipse and many other figures are perfect too. Nor are only the proportions expressible by us beautiful, but also those that are inexpressible by us, yet rational to God, who ordained them, and expressible at least to his divine Word, to whom he speaks all things, finite and infinite. Nor is it necessary that he should have communicated the idea and knowledge of them distinctly to the Angels: it is enough if he commanded them to move the planets along such a determinate path through the ethereal air, with such a velocity, and within such a time to bring them round to the same, or nearly the same, point — whatever proportion finally arises between the parts of that motion, or between the whole period and the period of another planet.

[Margin: Second reply, *ad hominem*.]

I reply, second, by turning the argument back, thus. An imperfection of figure and motion that is unbecoming to an intellectual nature, and unworthy of a created mind because of the perfection of such a nature, is much more unbecoming to the divine nature and unworthy of that supreme Mind. But the figure and periodic motion of the planets are of this kind, on your own

account, O Kepler. Therefore they are not the work of the divine mind — which it is impious to say, and not even you say it; on the contrary, you have expressly professed the opposite. Nor will you escape by saying that that figure and motion are not immediately from God but from the Sun, which moves the other planets. For since the Sun does not determine it freely but by natural necessity, this whole determination is in the end from God, and it is not to be ascribed to him if it contains an imperfection unworthy of God as its author.

[Margin: Second argument from Kepler.]

VII. The second argument is of this kind. The motion of the planets takes place along an ellipse according to the ratios of the balance and the lever — that is, according to a material necessity arising from the nature of steelyards — rather than according to an intellectual ratio or a voluntary purpose. But if they were moved by an Intelligence, they would not move so. Therefore, etc.

[Margin: First reply.]

One could reply by denying the major premise, if the discussion concerns the ratio intended per se and primarily by the motive faculty of the planets. For granting that some similarity or analogy were conceded between such a motion and the motions produced by a steelyard and a lever, it is still not worthy of God that he should have willed to represent this analogy per se and primarily; nor is that text of Wisdom 11, *He has disposed all things in weight, number and measure*, to be confined to these mechanical narrows, but extended to higher ends and to the determinate proportions of means to their ends.

[Margin: Second reply.]

I reply, second, granting the major, by denying the minor. For the knowledge of such ratios could have been infused into the moving Angels, so that they would balance the planets and move them in such a way that that analogy would be preserved — as being better suited to us, who are of grosser understanding — and yet not contrary to the end of these motions.

[Margin: Third argument from Kepler.]

VIII. The third argument is as follows. If an Intelligence moved the planets, it would need some body by which to perceive by what path they were to be directed, and how their distance from the centre of the world was to be preserved in such motion. But the Intelligences have no such body: for there are no solid orbs of the planets by whose motion that path and distance might be kept; nor have they cords or similar aids by which to lead the planets round; nor are they forms informing or animating the planets — or, if they were, the planets still contain no sense organs through which the path and distance already mentioned could be perceived. Therefore …

[Margin: Reply.]

I reply, first, by denying the major, as being uttered unworthily against the most perfect mode of understanding with which the Angels are endowed. For they understand all sensible things without sensations, and all corporeal motions, and the figure that those motions can describe, through impressed species alone and divinely infused ideas — and far more perfectly indeed than

Kepler understands those ellipses and librations of his. Unless he is so dull as to think that an Angel, lacking eyes, hands and so on, does not understand those figures and ratios of motions, and the intervals between the planets and the centre of the world, which Kepler himself believes he understands — although not immediately through the senses but deduced by reasoning from observations; or else that the Angel does understand them, but cannot, without corporeal organs, impress upon the planets such a motion as shines forth in their intelligible Ideas:

What could be more stupid than this statement, or this thought? Could a painter be found who would draw a perfectly straight line and a perfectly round circle without ruler and compass, and yet an Angel will be unable, without any corporeal instrument, to push a planet, or lead it round, along that orbit along which Kepler's Sun — and its soul, which is neither intellectual nor sensitive — drives the planets round by some blind direction? But now I am ashamed, in Kepler's name, and weary of lingering any longer over refuting these frivolous trifles; and it is better to dispatch three other questions belonging to this chapter.

De Modo, quo Intelligentiæ Cælum vel Sidera mouent.

On the Manner in which the Intelligences Move the Heaven or the Stars.

IX. It is commonly asked whether the Intelligences move the heavens — or, more generally, whether Angels move bodies — by intellect and will alone, or rather through a power distinct from both of these. For Averroes, Durandus, Bassolius, Argentinas, Capreolus, Soncinas, Parisiensis, Hervaeus [Hervaeus Natalis], Aquarius, Victoria [Francisco de Vitoria] and certain other Thomists — whom the Conimbricenses report (*On the Heavens* 2, chapter 5, question 7), and Amicus (treatise 5 *On the Heavens*, question 6, doubt 6) — thought that the intellect and the mere desire, or command, of the will are sufficient; and among our own authors they are followed by Vasquez (disputation 102, chapter 5), Delrius [Martin Delrio] (*Disquisitiones magicae* 2, question 6) and Molina (part 1, question 54, article 5). St Thomas strongly favours them in the same place, and in Quodlibet 2, article 2, in Opusculum 11, articles 3 and 13, and in *De malo* question 16, article 1, reply to 14. So does Aristotle (*On the Soul* 3, text 49), where he said: *Both of these, then, are motive with respect to place, intellect and appetite.* For it seems pointless to multiply another power, especially in a purely intellectual nature, in which it is fitting that what is dispersed among lower beings should be united, and that all its powers should be purely intellectual.

[Margin: The power of an Angel to move in place is distinguished from its intellect and will.]

On the contrary, however, it has been taught more correctly that the power by which Angels move themselves and bodies in place is distinct from their intellect and will — so taught Scotus (on Book 4, distinction 10, question 7, and on Book 1, distinction 45), Henricus Gandavensis [Henry of Ghent], Maironus, Aureolus, Caietanus, Godophredus and others, whom the Conimbricenses and Amicus, cited above, adduce and follow; Suarez (*Metaphysical Disputations* 35,

section 2, number 21, and *De Angelis* book 4, chapter 29), Raynaudus (*Theologia naturalis*, distinction 4, question 3, article 4), Ruvius (*On the Heavens* 2, chapter 5, question 11), Tannerus (part 1, disputation 5, question 5, doubt 1), Meratius (disputation 21 *De Angelis*) and Alarcon (disputation 6, chapter 8). For both the objective concept of these powers and their proper object are different; nor can the intellect elicit any operation other than intellection, nor the will any other than volition, insofar as each is such a power; but neither of these is local motion, or the production of the impetus that effects it. Otherwise an Angel could move in an instant, and to any distance, and any body however large, because an Angel can understand and love these things. The intellect, therefore, by directing and proposing, and the will, by commanding, can only be the moral cause of this motion; an executive power is needed, to which the command is given, as St Thomas also taught (*De potentia* question 6, article 7, and Quodlibet 9, article 10). Indeed, in part 1, question 25, article 1, he teaches that the divine power carries the character of a principle that executes what the will commands and what knowledge directs. When, therefore, he teaches that God and the Angels move by intellect and will alone, he must be understood to exclude a motive power that is really distinct, or one that adequately includes bodily organs, such as exists in animals. I suppose, moreover, from what I have said elsewhere in the treatise *On Angels*, that the Angels impress a certain quality which carries the mobile body from place to place, which is called impetus, and from which motion follows immediately — as Molina also teaches (part 1, question 110, article 3), and Vasquez (disputations 128 and 218) and Tannerus (above, doubt 2) — at least when the mobile body is separated from the Angel. Granted that Suarez (disputation 35, section 6, number 24, and *De Angelis* book 4, chapter 31) and Amicus (above) teach that only motion is produced by them, at least when the Angel is immediately present; and this holds whether that impetus is spiritual, as coming from a spiritual principle, or rather corporeal, as ordered to moving a body and received in a corporeal subject.

An & vbi quieſcant Intelligentiæ Motrices.

Whether, and Where, the Moving Intelligences are at Rest.

X. It is further asked whether the Intelligences, while they move the heaven or the planets, are at rest — and in what part of the heaven — or rather fly round together with them. That the Intelligence resides in one determinate place of the heaven, and from there impresses motion on the heaven, to which it is immediately present, the motion being afterwards diffused into the other parts of the heaven, was taught by Aristotle (*Physics* 8, last chapter, text 84, and *On the Heavens* book 2, chapter 2), St Thomas (part 1, question 52, article 2), St Bonaventure and Scotus (on Book 2, distinction 2, question 5), the Conimbricenses (*On the Heavens* 2, chapter 5, question 8), Tannerus (volume 1 of the *Theology*, disputation 5, question 5, doubt 2) and Amicus (treatise 5 *On the Heavens*, question 6, doubt 6). The Peripatetics teach that this part is on the equator, where the motion is fastest, and in the east, where the motion

begins. Others, however, draw a distinction. They say that the Intelligence which moves a whole heaven, or one entire heaven — say, the primum mobile — rests in one determinate part of it, but that those which move the planets are carried along with the motion of the epicycle, so as to impress a different motion on it and on the planet. So Bartholomaeus Mastrius and Bonaventura Bellutus (disputation 2 *On the Heavens*, question 4, number 148), granted that they say Scotus considered it unfitting for Angels to run round and be moved with the motion of the heaven.

[Margin: The Angels that move the planets are moved along with them.]

But I judge that one should answer otherwise. For, speaking of the planets: since they move in a fluid aether, as was said in Section 1, chapter 7, number 21, it is consistent that the planets are led round by the Intelligences. For the Intelligences are not everywhere, nor can they produce impetus or motion in a body enormously distant, as I suppose from Saints Damascene (*On the Faith* 2, chapter 3), Ambrose (*On the Holy Spirit* book 1, chapter 10), Chrysostom (Homily 3 on Hebrews) and Gregory (*Moralia* book 2, chapter 2); for their place is circumscribed and limited, and their power finite. And this conforms with Sacred Scripture, if that text of Ecclesiastes 1 — *The Sun rises and sets, and returns to its place, and being reborn there it circles by the south and turns towards the north; surveying all things round about, the spirit goes on and returns to its circuits* — is understood of a Spirit, that is, of an Intelligence moving the Sun, with St Thomas (opusculum 10, article 6). Pineda and Lorinus, on that passage of Ecclesiastes, do not reject this, although they also bring forward other interpretations; nor indeed does St Jerome. For since he grants that a spirit and mind driving the Sun can be understood there, and elsewhere censures Origen for granting a soul to the stars, the text seems capable of being taken of an Intelligence that does not inform the Sun but moves it from outside. This being posited, if Sacred Scripture is taken literally, as it can be, it can be sufficiently confirmed from it that the Angel who moves the Sun himself also goes round, and accompanies the Sun that he moves. I later saw the same opinion, about Angels carrying the planets round, defended by Io. Antonius Delphinus in a truly learned little work *On the Celestial Globes*, page 80, and in Arriaga (single disputation *On the Heavens*, number 54). But if we are speaking of the heaven of the fixed stars, whether it is moved by one Intelligence or by several, it can be granted that they reside in a single place of the heaven. But if there is one, and any point of the heaven on the equator is rising with respect to some horizon, it will be necessary to fix on some particular region — say Palestine — on whose eastern horizon that Angel would be; or else, to avoid this singular position and the difficulty of moving so vast a machine, this could have been done by several Intelligences arranged in a circle. For the rest, it is not for our weakness to divine about these things; and we must from time to time remember that divine rebuke in Job 38: *Do you know the order of heaven, and will you set down its reason on Earth?*

Quot ſint Intelligentiæ Motrices Cælorum & An inter eas DEVS ?

How Many Moving Intelligences of the Heavens there are, and whether GOD is among them.

XI. There were some, left unnamed by Fracastoro (*Homocentrica*, section 1, chapter 7), who said that all the celestial orbs are moved by a single Intelligence; among them Alpetragius is counted. Just as all the vital operations of one body are carried out by a single soul — especially since the heaven resists no motion once impressed, nor is it necessary that the contact of the angelic power be immediately applied to all the parts to be moved; it is enough for it to be present to one part, from which the motion is diffused into the others. But, as the Conimbricenses note (*On the Heavens* book 2, chapter 5, question 8), and Mastrius and Bellutus (disputation 2 *On the Heavens*, question 4, number 147), it is the common opinion of the theologians, on Book 1, distinction 37, that the sphere of angelic activity is bounded by definite spaces, and the Angels' power is limited, so that they cannot move one whole element from its place — much less impart with a single impulse such varied and almost contrary motions to so many and such vast heavens, really distinct from one another, if solid orbs were posited. But since the heaven of the planets is fluid, and is not itself moved in such a way that the planets are moved with its motion, it is clear enough that it follows from this that, besides one or more Intelligences moving the sphere of the fixed stars, there are at least seven others for as many planets. And this fits very well with the order and distinction of offices — just as individual men, provinces, cities, and perhaps churches each have their own Guardian Angel, as Albertinus teaches in his book *On the Guardian Angel*.

Furthermore, Aristotle (*Metaphysics* 12, chapter 8, from text 47) posits as many moving Intelligences as there are partial celestial orbs, adding that each Intelligence is so commensurate with its own orb as regards motive power that it cannot move a higher sphere. This the Conimbricenses nevertheless refute, referring the reader to Capreolus (on Book 2, distinction 3, question 1, article 3) and to Maior (on Book 2, distinction 14, question 2).

[Margin: Is God among the moving Intelligences?]

It is, moreover, disputed whether Aristotle counted God among these Intelligences, as the immediate mover of the highest heaven — as the Conimbricenses (*On the Heavens* 2, chapter 5, question 6), Piccolomineus, Pendasius, Ruvius, Aversa and Amicus, with Gregory (on Book 2, distinction 1, question 1, article 1), hold — or rather placed him outside and above this number, inasmuch as he does not move that heaven except through some created Intelligence, as St Thomas (*Contra Gentiles* 1, chapter 13) and the Thomists with him hold, and likewise Iandunus (*Metaphysics* 12, question 17), Abulensis (on Exodus chapter 23, question 3), Scotus (on Book 1, distinction 2, question 1, and distinction 8, question 5, and Quodlibet 7), Contarenus (*On the Heavens*, in the question on this matter), Suarez (*Metaphysical Disputations* 35, section 1), and Mastrius and Bellutus (disputation 2 *On the Heavens*, question 4, article 4). And indeed, since Aristotle thought that God necessarily acts outwardly (as St Thomas teaches

in *De malo* question 16, article 10, and on *Metaphysics* 12), and so exerts his whole effort, and moreover that God is of infinite power, it would follow from Aristotle's reasoning (*Physics* 8, texts 78 and 79) that God moves with infinite velocity and in no time; or, if anything resisted him, that a finite power could prevail over or equal an infinite one — both of which are absurd. Furthermore, in *Metaphysics* 12, text 36, he taught that the first mover moves as lovable and desirable, inasmuch as the created Intelligences, contemplating his supreme goodness in communicating being to themselves and to other things, strive to make themselves like him by moving the heavens, and through their motions and influences to communicate very many goods to the lower natures. Hence God moves the heavens in the manner of a good and an end, or also of an Idea — without, however, excluding the effective concurrence by which, as universal cause, he concurs immediately in all effects. And so Aristotle must be understood in the same place, text 38, where he says that the primum mobile is moved by the first Intelligence, and in text 43, where he counts as many Intelligences, the first not excluded, as there are orbs.

Therefore, since he had learned from Eudoxus that the whole inequality of the planets in their motion could be demonstrated by 25 [?] orbs, and from Calippus by 33, Aristotle himself, believing that the heaven consists of solid orbs, added 22 other orbs, ἀνελίττοντας [Greek: anelittontas, "unrolling"], that is, Revolving orbs. The purpose was that each lower orb should be freed from the drag which, because of the contiguity of the surfaces, it would otherwise have suffered from the one above it, by means of an interposed Revolving orb, straining in the opposite direction in an equal time and by that counter-effort presenting the appearance of an orb at rest; so that the lower or inner orb, resting on that orb as on one immobile, would complete its own proper period.

[Margin: The Aristotelian number of the Intelligences.]

So Kepler explains it, in his work on Mars, chapter 2, and in the *Epitome of Copernican Astronomy*, book 4, page 505; but in those places he wrongly says that the movers established by Aristotle are 49, or according to Calippus 53 or 55. For below, in Section 3, chapter 5, number 5, we shall show that in Aristotle's mind they are 55, and with the primum mobile 56; but under another hypothesis 47, and with the sphere of the fixed stars 48; and we shall expose the errors of many concerning this Aristotelian number. Again, in Section 3, chapter 7, we shall treat the 70 solid orbs that Fracastoro devised in the *Homocentrica*, composed of *circitores*, *anticircitores*, *circumducentes*, *contravecti* and so on.

[Margin: A conjecture on the number of the Intelligences.]

XII. Further, if I were permitted to settle anything about this number by some conjectures, I would say that, besides the Intelligences — either 4, or 12, or 24 — that move the highest orb of the fixed stars, there are seven principal Intelligences, assigned to revolving the seven planets; and that these are those seven Spirits of whom we spoke, with Trithemius and Cornelius à Lapide, in Book 7, Section 1, chapter 1, setting out many things worth knowing, which it will help to re-read at this point. I would add, however, other, less principal

Angels, either for the less principal motions in altitude and latitude, or at least for driving round the companions one by one — two of Saturn and four of Jupiter. For I would by no means assign individual Angels to the spots of the Sun, which come into being and perish from time to time. But I fear that God and the Angels themselves, if it is right to speak so, may laugh at these efforts of ours, as of people who wish to investigate things that are above us.

CAPVT II.

Chapter 2.

De Inſtrumentis Motuum Cæleſtium, ſintne Orbes ſolidi, an meri Circuli in fluido æthere deſignabiles, ijque Excentrici cum Epicyclis, an meri Concentrici.

On the Instruments of the Celestial Motions: whether they are Solid Orbs, or mere Circles that can be designated in a fluid aether; and whether these are Eccentrics with Epicycles, or mere Concentrics.

I. An admirable variety of motions has been detected in the planets by observation — indeed one as manifold as we indicated in Book 7, Section 1, chapter 7. Yet a threefold variety, corresponding to the threefold dimension of bodies, was more manifest to the ancient physicists and astronomers: namely, in longitude, in latitude, and in depth or altitude. But because, of these, the variation in altitude was not evident to all, or was seen not as real but as apparent, arising from some deception of our sight or from a difference in the interposed medium, two very ancient sects arose from it.

[Margin: Who the Eccentricists and Concentricists are.]

One was that of the Eccentricists, who said that the motions of the planets are carried out by orbs having their own centre, different from and distant from the centre of the World, together with smaller orbs set into the larger ones, which they called epicycles; by their help they explained the inequality in motion of longitude and latitude, and also of altitude. The other was that of the Homocentrists or Concentricists, who asserted that these motions are completed by orbs concentric with the centre of the World, so that the planets in reality always kept the same distance from the centre of the World that each once had; but since their orbs moved on different poles, some orbs accelerating and others retarding, it came about that the motion of the planets, which in itself was equal, appeared unequal. What eccentrics, epicycles, concentrepicycles and so on are, and in what way they are imagined to be interwoven with one another, we have explained more than enough, with a diagram added — for the Sun in Book 3, chapters 19 and 21; for the Moon in Book 4, from chapter 25; and for the other planets in Book 7, Section 2, chapter 1 — and so we refer the reader there.

II. The authors of each sect, however, are further subdivided into two classes. Some thought that all these motions are made by solid orbs; others, without solid orbs — that is, in a liquid aether — whether by an intrinsic faculty

and principle implanted in the planets, or by an informing soul, or by an Intelligence that merely presides and drives the planets round along circles that can be designated, and described only in the imagination or the intellect. But this part of the controversy, concerning the solidity and fluidity of the heavens, setting aside the concentricity and eccentricity of the orbs, we have already examined sufficiently in Section 1, chapter 7. There, among the defenders of solid orbs, we placed not only Alfraganus with many Arabs, and Sacrobosco and Purbachius with their followers, but also, among the ancients, Eudoxus, Calippus and Aristotle, inasmuch as they attributed the inequality of motions to different orbs on different poles revolved upon them, which pushed forward or rolled back the orb in which the planet was set. Indeed, we placed there Anaximander too, of whom Plutarch says (*De placitis*, book 2, chapter 16): *Anaximander carries the stars round by the circles and orbs in which each star is set.* For neither Purbachius nor the Arabs were the first to introduce that solidity of the orbs. Rather, what Aristotle, with Eudoxus and Callippus, had tried to achieve through concentric orbs, and afterwards Fracastoro and some more recent writers through Homocentrics, he himself, following Ptolemy, achieved by stuffing Ptolemy's Eccentrics and Epicycles with solid matter. So neither Alfraganus nor Purbachius was the author of solid orbs as such, but of solid Eccentrics and Epicycles.

[Margin: Those who held solidity and eccentricity together.]

III. For Ptolemy and Hipparchus, and indeed the Pythagoreans, who introduced Eccentrics and Epicycles, or designed more perfectly those already introduced into astronomy, by no means bound them to the solidity of orbs. Either abstracting from solidity, or rather presupposing fluidity, they set out the motions of the planets by mere circular lines, and explained their hypotheses by bare circles, that is, by the peripheries of circles. This is how they, or at least Ptolemy, were judged by St Thomas (lecture 10 on *Metaphysics* 12), Piccolomineus in the *Theoricae Planetarum*, Vielmius (lecture 21 on Genesis), Martinengus in the *Glossa*, page 1023, Kepler in his work on Mars, chapter 2, and in the *Epitome of Copernican Astronomy*, book 4, part 2, Cabeo (on *Meteorologica* 1, text 38, question 2), and Blancanus [Giuseppe Biancani] in the *Sphaera*.

[Margin: Ptolemy does not support the solidity of the orbs, but rather their fluidity.]

But what need is there of a gloss, when Ptolemy's own meaning is plain and clear enough? For Ptolemy, in book 13 of the *Great Work*, or *Great Composition*, chapter 2, speaking of the manifold motion of the planets, especially in latitude, says: *Let no one think hypotheses of this kind too laborious when he considers the manifold artifice required in carrying them out. For it is not fair to put human things on a level with divine ones, and to seek belief about sublime things from examples of the most dissimilar things. For what is more unlike than things that always stay in the same state and things that are never constant to themselves, or than things that can be impeded by any cause whatever and things that cannot be impeded even by themselves?*

Rather, one must strive that hypotheses as simple as possible be fitted to the celestial motions; and if this does not succeed, at least such as are possible for us. For if by such hypotheses it follows that everything that appears in the heavens is accounted for, what wonder is it if this variety can befall the celestial motions? Especially since there is no nature there that impedes the motions, but one fitted by nature to yield to the natural motions of each globe, even if they seem contrary; so that all of them can pass through those simple bodies, spread wide and fluid, and show through them, or be seen through them. And this can go forward correctly not only in the circles but also in the spheres themselves and in the axes of the revolutions — whose variety, and the alternating diversity of their motions, it is so laborious and difficult for us to represent in constructed images and in models of Theoricae, that the motions seem to impede one another. In the heavens, however, that variety of motions does not impede itself in the least. One must not, therefore, judge the simplicity of celestial things from what can seem simple among us, since there is nothing among us that seems equally simple to all men. For if anyone wished to judge in that way, he will think nothing in the heavens simple — not even the stable and simple nature of the primum mobile itself, because something like it among men is not only difficult to find but altogether impossible. Judgement must therefore be passed not from these things but from the very nature and unchangeableness of the celestial motions. For in this way it will come about that all those motions appear simple, and indeed much simpler than the things that seem simplest among us, since we can conceive no difficulty and no labour in their periods.

Although Ptolemy, then, names spheres and axes, yet since he acknowledges the greatest possible simplicity of motions — indeed, through a single continuous line — and posits the planets as going through the substance of the aether, which yields and impedes nothing, who does not see that he favours the fluidity of the heavens rather than their solidity? And so he was understood by St Thomas (part 1, question 70, article 1, reply to 3), by Carthusianus (article 12 on Genesis), and by Kepler in the *Epitome*, page 504 — granted that there Kepler censures him for making the motion intrinsic and natural to the planets, for the divinity attributed to them, and for excluding every figure, image or model drawn from our own things. For Kepler thinks that by an ellipse, and by examples of the balance and the magnet and the like, the motions of the planets can be represented very well. But below we shall speak again of this simplicity of motions according to Ptolemy's mind.

Now we must come to the other part of the proposed question: namely, whether — in a solid or in a liquid aether — the motions of the planets can be carried out in the heavens, and explained on earth, by concentric orbs or circles alone; or whether Eccentrics, or equivalents of Eccentrics, are required.

[Margin: The first opinion asserting [?] concentrics.]

IV. The first opinion was that of the Homocentrists. Some of them thought the motions of the planets equal; others thought them unequal, indeed, but their distances from the centre of the world equal. Of these we may understand that saying of Plutarch (*De placitis*, book 2, chapter 16): *Anaximenes [holds] that*

the stars turn in the same way both around the earth and above the earth. Plato and the mathematicians [hold] that the Sun, Venus and Mercury are moved with equal motions.

[Margin: Anaximenes.]

But above all Eudoxus of Cnidus and Callippus of Cyzicus, who held with Plato that the celestial motions are circular, uniform and perpetual, tried to explain the whole real variety of the motions by orbs concentric with the centre of the universe; and Aristotle laboured at this very thing by adding other orbs, as is clear from Simplicius on books 1 and 2 *On the Heavens*, and from Aristotle himself, there and in *Metaphysics* 12. He was followed by Averroes, there, by Achillinus in his book *De orbibus*, and by the Conciliator [Pietro d'Abano] in Differentia 1.

[Margin: Eudoxus. Callippus. Aristotle. Averroes. Achillinus.]

Sosigenes, Hipparchus and Ptolemy rejected this opinion, calling in Eccentrics and Epicycles, and all astronomers thereafter followed Ptolemy. Yet a very few called it back into the light out of the rubble: namely Io. Baptista Amicus of Cosenza [Giovanni Battista Amico], in his opusculum *De motibus corporum caelestium*, published in 1537; Io. Baptista Turrianus [Giovanni Battista della Torre], and his heir in this, Hieronymus Fracastorius [Girolamo Fracastoro], in the *Homocentrica*, published the following year, that is, 1538; Lucillus Philalthaeus [Lucilio Maggi] (on *On the Heavens* 2, text 51); Andreas Caesalpinus [Andrea Cesalpino] (*Quaestiones Peripateticae*, book 3, question 4); and finally Io. Antonius Delphinus, a Franciscan from Casale Maggiore, in his book *De caelestibus globis et orbibus*. Alpetragius sides with them in many points in his celestial physics.

[Margin: Amicus. Turrianus. Fracastoro. Delphinus. Philalthaeus. Caesalpinus.]

Our own Bartholomaeus Amicus [Bartolomeo Amico], however, is mistaken (treatise 5 *On the Heavens*, question 5, doubt 2, article 2) when he counts among these authors all the astronomers who posit a fluid heaven, with the stars going in it like birds in the air or fish in the sea. For neither Tycho, nor Kepler, nor Boulliau, nor indeed any of the more recent astronomers who assert that the heavens of the planets are fluid, has used mere concentric circles in his hypotheses.

[Margin: Bartholomaeus Amicus's slip.]

[Margin: The second opinion posits eccentrics.]

V. The second opinion was that of the Eccentricists — that is, of those who, in setting out the motions of the planets, used circles, or ellipses, or quasi-circular spirals, having however a centre different from the centre of the world; or else used concentric ones, but carrying various Epicycles equivalent to eccentrics. The inventors of this opinion were the Pythagoreans, as Simplicius reports from Nicomachus (on *On the Heavens* 2). He also reports that Aristotle's opinion of concentric orbs was rejected by Sosigenes, because it could not

save the greater part of the motions, especially the stations, the retrogradations, and the elevations from the earth and the depressions — although he there describes no definite hypothesis of Sosigenes. Hipparchus, however, and Ptolemy throughout the *Almagest*, and after them Albategnius [al-Battānī], Alfraganus, Geber [Jābir ibn Aflaḥ], Thebit [Thābit ibn Qurra], Purbachius with his commentators, Copernicus in his work *On the Revolutions*, Maginus [Giovanni Antonio Magini] and Piccolomineus in their *Theoricae*, and from Tycho onward all astronomers, persisted in the same view. Those who argued for it, and asserted it with distinct arguments, were above all Maior (on Book 2, distinction 14, question 4), Petrus Alliacensis [Pierre d'Ailly] (question 13 on the *Sphere*), Christophorus Clauius [Christoph Clavius] (on chapter 4 of Sacrobosco's *Sphere*), and Bartholomaeus Amicus (treatise 5 *On the Heavens*, question 5, doubt 2). And these very things had long ago been confirmed by many indications by Pliny the Elder (book 2, chapters 15 to 17) and by Martianus Capella (book 8 of *De nuptiis Philologiae et Mercurii*, in the chapter *That the Earth is not the centre for all the planets*).

VNICA CONCLVSIO.

Single Conclusion.

The motions of the planets do not come about through circles concentric with the centre of the world, but through eccentric circles or quasi-circles, or through equivalents of eccentrics.

VI. The first argument could be drawn from the authority of almost all astronomers from Hipparchus down to this century of ours, who for seventeen centuries have held to this way of setting out the celestial motions, with very few resisting — and those few, if you except Fracastoro, unskilled in celestial observations. But in an astronomical question it is better to confirm our assertion by reasons.

[Margin: 1. Argument from the inequality of the distances.]

The first reason, then, is taken from the unequal distance of any planet from the centre of the earth: for within one period the planets are now farther from the earth, now nearer, now at some middling distance. We proved the Moon's unequal distance from the earth from the variation of the lunar parallaxes, in Book 4, chapter 14. Given its varying distances, a varying distance of the Sun from the earth necessarily follows, because of the astronomical connection that the Sun's distance has with the phase of the dichotomous Moon. This is clear from the problem of Aristarchus, and from our careful and legitimate use of it, often put into practice and set out in Book 3, chapter 7. So although the variation of the distances cannot be shown so evidently from the variation of the solar parallaxes, it can nevertheless be gathered from the Moon's distances and from the time between the appearance of the half Moon and the moment of quadrature. Once the varying distance of the Sun from the earth has been obtained, the distances of the other planets become known, because they have no less a connection with the Sun's distance than their motions have with the

Sun's motion, as can be established from what was said in Book 7, Sections 2 and 3, and Section 6, chapter 2. For equations agreeing with the true motions and the observed places of the planets cannot be derived except by supposing in their hypothesis a varying distance from the Sun and from the earth.

Moreover, in all seven planets a varying apparent magnitude is observed: in the luminaries, both outside eclipses and in eclipses, whose magnitude varies with their varying recession from the earth or approach to it. In Mercury, Venus and Mars, that variety too has been observed with telescopes, so that they appear sometimes full of light, sometimes gibbous, sometimes falcate, as we showed in Book 7, Section 1, chapter 2. Finally, how much larger Mars and Venus appear at perigee than at apogee we taught clearly in Book 7, Section 2, chapter 3, in the scholia; in Section 6, chapter 4, likewise in the scholia; and in chapter 10, likewise in the scholia. If the reader has understood these properly, no doubt will remain for him about that variety of apparent magnitude.

[Margin: The true cause of the varying apparent magnitude.]

Now the variety already mentioned cannot be put down either to vapours of the air, now thicker, now thinner, or to parts of the heaven, some denser and some rarer, as Fracastoro, Amicus and Delphinus dared to put it down. For, first, the most invincible argument drawn from the parallaxes stands against it: it shows the variation of distance, from which the variation of apparent magnitude necessarily follows. Besides, the same planets often appear larger when either there are no thick vapours in the air or the planets are far from them. And finally there is the wonderful agreement, in the proportion of the increase and decrease of this apparent magnitude, with a determinate situation relative to the Sun. For the three superior planets always appear very large in opposition to the Sun and in the acronychal situation, having become retrograde, and very small around conjunction with it; and Venus and Mercury always appear largest when falcate. And these phenomena are seen whatever the season of the year, and in whatever part of the zodiac. Besides, if the varying density of the heaven running beneath them were the cause of this variety, the magnitude of the fixed stars, and the clarity and brightness of the planets, would also change from time to time: for it would either be blunted by the density, or would seem more intense, the rays converging by force of refraction. Lastly, it would be necessary to multiply so many heavens, or celestial zones gliding beneath the seven planets, with denser and rarer parts — and indeed one heaven below the Moon. This would be done under no necessity, indeed with scarcely the thinnest probability, since it can be done far more naturally and simply by the alternating elongation of the planets from the earth and their nearness to it: that is, by the very means by which the inequality of motion in longitude is also said to come about. But if anyone is unwilling in this business to use Eccentrics or Concentrepicycles, and prefers concentric circles, now larger, now smaller, yet in such a way as to make them continuous, as the observations demand, then these circles, ever wider, will have to follow the form of a spiral gyration

rather than that of a perfect circle, returning into itself and concentric with the world.

[Margin: 2. Argument from the inequality of motion.]

VII. The second argument is taken from the inequality of the motions in longitude and latitude, which we have already described in Book 3, chapter 19, on the Sun; in Book 4, chapter 18, on the Moon; and in Book 7, Section 1, chapter 7, on the other planets; but above all from the inequality of the stations and retrogradations, on which see Book 7, Section 5, chapter 2. This argument makes our conclusion more probable; by eccentrics and epicycles, or by courses of lines equivalent to them, the variations already mentioned are set out better, and more neatly and beautifully according to the laws of geometry; and the physical cause of such inequalities is in some way placed before the eyes. Yet — to confess candidly what I think — it is not an evident argument. If the variations of the parallaxes and the alternations of the apparent magnitudes did not stand in the way, we could set out that whole change of speed and slowness by mere concentric circles, and by laws of motion that are numerical and logistic rather than geometrical, as we shall say below. It remains for us to dispose of the arguments of the Homocentrists.

[Margin: 1. Objection disposed of.]

VIII. First, it is objected against Eccentrics and Epicycles that the centre of the earth would not be at the centre of the heaven, or the centre of the heaven would not be at the centre of the world; and that this is absurd. The reply is to deny the major premise as regards the highest heaven, that of the fixed stars, and so too as regards the whole heaven of the planets, which is single and fluid and whose convexity is bounded by the concavity of the Firmament. If, however, the orbs are solid, the minor premise is denied when taken of any heaven whatever, and is granted only of the highest.

[Margin: 2. Objection.]

Second: simple motion upward and downward belongs to the elements alone, whereas to the celestial bodies belongs not straight motion but simple circular motion. But if they were moved by the force of Eccentrics and of an Epicycle, upward motion would belong to the celestial bodies. Therefore … The reply is to grant the major premise as regards motion upward and downward along a straight line perpendicular to the globe of the earth; in this sense the minor premise is denied.

[Margin: 3. Objection.]

Third: according to Aristotle, any heaven is perfectly spherical; but it would not be, if Eccentrics were posited. Reply: the major premise is granted and the minor denied. For if the heaven of the planets is fluid, it is single in reality and spherical with respect to the centre of the universe; and if it consists of solid orbs, every Eccentric orb is spherical as to its outermost surface, and every Epicycle with respect to its own centres.

[Margin: 4. Objection.]

Fourth: if Eccentrics and Epicycles existed, they could not be moved without penetration or splitting of the heavens, nor without rarefaction and condensation, since the less deep part would enter into the deeper; but this is unfitting. The reply is to deny the major premise. On the hypothesis of a fluid heaven the objection falls away. On the hypothesis of a solid heaven, too, the two orbs that are eccentric only in a qualified sense are not moved in such a way that the less deep part of one succeeds into the place of the thicker and deeper part, as the Averroists imagine out of inexperience. Rather, they are rolled proportionally, so that the thicker part of the lower eccentric always lies under the less deep part of the upper one, and the narrower part under the deeper, so that the orb that is eccentric simply has no motion other than that of the whole heaven of the planet.

[Margin: 5. Objection.]

Fifth: according to Aristotle, the farther a planet is from the highest heaven, which has a simple motion, the more motions it needs to attain its perfection; but on the hypothesis of Eccentrics and Epicycles the Sun has fewer motions than the three superior planets. The reply is to deny the major premise; for even according to the opinion of the Homocentrists, if they wish to save the phenomena of the Sun, fewer motions must be attributed to it.

[Margin: 6. Objection.]

Sixth: all the φαινόμενα [Greek: phainomena, "appearances"] of the heaven can be saved by concentrics and a plurality of motions, as Aristotle affirms (*On the Heavens* 2); therefore Eccentrics and Epicycles are multiplied in vain — especially since the two eccentrics carrying the planet's aux seem superfluous, and one seems to suffice, as Augustinus Niphus says. The reply is to deny the antecedent. For the motion upward and downward cannot be saved by mere concentrics; and, solid orbs being posited, both Eccentrics are necessary in the Ptolemaic hypothesis — not precisely to carry the aux, as Niphus thinks, but so that the whole heaven of the planet may have its own motion about the centre of the world. For in other hy- potheses these motions are arranged differently.

[Margin: 7. Objection, of Fracastoro.]

Seventh, Fracastoro objects. If the Sun is farther from the earth — or farthest — when it is at the beginning of Cancer, and least far when at the beginning of Capricorn, then either it will describe, in both places, parallels equally distant from the Equator, and so the Sun's greatest declination will not be equal on both sides, that is, 23½°. This is contrary to the observations: for the angle made at the centre of the world, or at the surface of the earth, by two lines, one ending at the periphery of the Equator and the other at the centre of the solar globe, will be narrower when the Sun is higher than when it is lower, as the laws of geometry and optics require. Or, if the Sun's greatest declination is equal on both sides, the Sun at the beginning of Cancer will necessarily describe a parallel more distant from the Equator than when it is at the beginning of Capricorn; and so, in an oblique sphere, the diurnal arc of the longest day will

not be equal to the nocturnal arc of the longest night. This too is contrary to experience and to astronomical observations.

The reply is that the Sun's greatest declination is indeed equal in both places, and that the diurnal arc of the longest day is unequal when compared with the nocturnal arc of the longest night in the same oblique horizon; but that this inequality is not perceived, because of the Sun's huge distance from the earth in both cases and its insensible parallax, which for us does not exceed 30″. Besides, the varying refractions, which hasten the Sun's rising and delay its setting, quite often compensate that inequality. Remember, however, that the Sun's apogee does not always lie at the beginning of Cancer.

[Margin: 8. Objection, of Fracastoro.]

Eighth: it would follow from Ptolemy's view that the epicycle of Venus is so large that it would reach almost to the earth; for its semidiameter contains 43°, and if it contained 45° it would pass through the centre of the earth. The reply is to deny the antecedent. The semidiameter of Venus's epicycle does not contain 43 degrees of the heaven of Venus, as Fracastoro falsely assumes, but 43 parts of such a kind that 60 of them are contained in the semidiameter of its eccentric. Further, if you consult the least Ptolemaic and Alphonsine distance of Venus from the earth, which we set out in Book 7, Section 3, chapter 1, and Section 6, chapter 2, you will see that it is 179 2′ semidiameters of the Earth according to Fernelius [Jean Fernel], but 167 57′ according to Maurolico and Clavius. But in our hypothesis, which we think truer, the least distance of Venus is at least 1917 terrestrial semidiameters.

[Margin: 9. Objection, of Averroes and Fracastoro.]

Ninth: if the Moon were rolled round on an epicycle, we would not always see the same face of the Moon, but one face at the apogee of the epicycle and another at the perigee. But we always see the same one. Therefore … I reply: if no other motion were attributed to the Moon, I grant the major premise. But I deny it if there is attributed to the Moon either a turning about its own centre, in the direction contrary to the motion of the epicycle, as Fernelius attributes in his *Cosmotheoria*; or if you transform the epicycle into a second eccentric, as Maginus does; or if you use other equivalent hypotheses, on which enough was said in Book 4, from chapter 26.

The remaining objections rest either on false observations or on the mere authority of Aristotle defending concentrics. Yet just as Aristotle subscribed in this to Eudoxus and Callippus, he would certainly have accepted other hypotheses had he lived after Ptolemy and learned astronomy from him; for he himself professed that in these matters the more skilled astronomers are to be consulted. In book 2 *On the Heavens*, chapter 7, text 34, he says: *When, therefore, someone has attained more certain necessities, then thanks must be given to those who discover them; but for now what seems so must be said.* And in *Metaphysics* 12, text 45, on the lations of the planets, he says: *As to how many these are, we for now state what some of the mathematicians assert, for the sake of understanding, so that we may grasp in the mind some determinate plurality. Further, some things we ought*

to seek out ourselves, and others to inquire of those who investigate these matters, if anything beyond what has so far been handed down seems to be so to those who deal with these things; and to love both, but to adhere to the more certain.

[Margin: The modesty of Aristotle's mind.]

And a little later he admits that he has spoken of these things only probably, and that necessary demonstrations are to be sought from those more skilled in these matters, when he says: *Therefore it is reasonable* — in Greek λογικῶς [Greek: logikōs, "reasonably"], that is, topically and with likelihood — *to judge that the substances and principles, both the immobile and the sensible, are so many; for what is necessary may be left to the more powerful to say.* Let those Peripatetics hear this, who contend that Aristotle, even against his will, always pronounces infallible oracles; and who, instead of letting him walk in the Peripatos ready to follow better things, try to carve him for posterity as a stone statue, fixed immovably to opinions once conceived.

CAPVT III.

Chapter 3.

An detur Motus Primi Mobilis diſtinctus à motu Fixarum ac Planetarum, & in quo ſubiecto, & An illi contrarius ſit motus ſecundorum Mobilium, Quaq. ratione hi duo motus ſimul Conciliari poſſint.

Whether there is a Motion of the Primum Mobile distinct from the motion of the Fixed Stars and of the Planets, and in what subject; and whether the motion of the Secondary Mobiles is contrary to it; and in what way these two motions can be reconciled with each other.

I. No motion among the celestial motions is better known than the motion of the primum mobile, by which, namely, we see all the stars, and especially the fixed stars, carried round every day in 24^{h} from one and the same point of the heaven back to the same point, towards the west. And yet nothing is more obscure, whether as to the subject of this motion, or as to its distinction from the other motions of the stars, and the way of combining them with one another without any physical incompatibility. I touched on this controversy in Book 6, chapter 18, in setting out the Theorica of the eighth sphere and of the fixed stars. But since I have afterwards come upon other authors, and repeated contemplation of this motion has yielded some new reasons not unworthy to be known; and since this, after all, is its proper place, where we treat of what is universal and common to all the stars, I am glad to bring it back to the anvil.

[Margin: The first opinion, placing the primum mobile in the single motion of the fixed stars.]

II. The first and most ancient opinion was that the fixed stars are moved by no motion other than the diurnal and common one towards the west. Their own peculiar motion upon the poles of the zodiac, with a change of declination, had not yet been detected by certain observation, since it is perceptible only

after many years and cannot be evidently demonstrated within the ordinary span of one observer's life. On this supposition the first motion was nothing other than the motion of the fixed stars themselves, or of the eighth sphere, about the axis of the Equator and upon its poles; the eighth sphere itself was the Primum Mobile and the highest astronomical heaven; and the universal visible time, and the best-known measure, was nothing other than the revolution of any single fixed star — the Dog Star, say — to the same point of the same fixed meridian.

[Margin: The authors of this opinion: the Babylonians, the Egyptians, Eudoxus, Callippus, Aristotle.]

That the Babylonians, and some of the Egyptians and Greeks, but especially Eudoxus and Callippus, and Aristotle too, held this opinion is gathered from Aristotle himself, *On the Heavens* book 2, from chapter 9, that is, from text 57 to 70, and *Metaphysics* 12, from text 42 to 51. There, together with Eudoxus and Callippus, he attributes a single and most simple lation to the sphere of the non-wandering stars; and among the other causes for which it is filled with innumerable stars, he adduces this one: that the simplicity of its motion is compensated by the multitude of its stars, and that, just as the spheres of the wandering stars have a single star but several motions, so the sphere of the non-wandering stars has a single motion but several stars. So he says in *On the Heavens* 2, text 67. And he reports, in *On the Heavens* 2, text 60, that this account of astronomy flowed from the Egyptians and Babylonians to the Greeks. For this reason the most ancient Babylonians and the Egyptians, as Albategnius reports (chapter 27 of *De scientia stellarum*), defining the year, said that it was *the revolution of the Sun to the same fixed star* — just as if the fixed stars were fixed points, movable by no motion other than the diurnal. And Nembroth the Chaldean, in his astronomical work as cited by Augustinus Riccius [Agostino Ricci] in his treatise *De motu octavae sphaerae*, ascribes only a single motion to the signs of the heaven. Nor were Hesiod and the other most ancient poets of the Greeks of a different opinion when they placed the nine Muses in the heaven: for to the highest, namely Calliope, they ascribed no peculiar voice and no motion, as Glareanus [Heinrich Glarean] notes (*Dodecachordon* book 2, chapters 8 and 13), but the highest and shrillest voice, on account of its very great speed of motion — the voice which for Glareanus himself is Μέση [Greek: Mesē, a note-name, "the middle"] — they gave to Urania, who governs the sphere of the fixed stars. Calliope, therefore, as mistress of the celestial harmony, presided over the other Muses and over keeping the motions of all eight heavens together in concord, and contained the voices of all, as Macrobius expressly teaches from Hesiod (book 2 on the *Dream of Scipio*).

There are some who doubt what Plato's view was. But in book 10 of the *Republic* he brings in a certain Pamphilius, killed in battle and returning to life from the underworld, who asserts that *he saw the tracks of eight celestial motions, of which the eighth was driven in the swiftest course*; and in the *Epinomis* he says: *The philosopher ought not to be ignorant of how the seven circuits turn beneath the*

first. Nor, surely, would his pupil Aristotle have kept silent about it if Plato had taught some motion in the fixed stars other than the diurnal; nor would *Cicero*, who in the *Dream of Scipio* carries a great deal over into Latium from Plato's books on the *Republic*. For there he mentions nine globes, of which he assigns the lowest to the immobile earth and the highest, the heaven of the fixed stars, with a single motion. He says:

[Margin: Cicero's opinion.]

All things are bound together for you in nine orbs, or rather globes, of which one is the celestial, the outermost, which embraces all the rest, itself the supreme God, enclosing and containing the others; in it are fixed those everlasting courses of the stars which revolve. Beneath it are seven, which turn backwards with a motion contrary to the heaven. And after listing the seven globes of the planets he adds: *for the one that is in the middle and ninth, the earth, does not move, and is the lowest*; and a little later: *but those eight courses, of which two have the same force, produce seven sounds distinguished by intervals*; and indeed a little before he had said: *and nature brings it about that the extremes sound low at one end and high at the other. For this reason that highest course of the star-bearing heaven, whose revolution is swifter, moves with a high and rapid sound, but this lunar and lowest one with the deepest.*

[Margin: Macrobius's opinion of Cicero's opinion.]

Granted, Macrobius (book 1 on the *Dream of Scipio*, chapter 17) thought that the proper motion of the fixed stars was known to Cicero and indicated by him. For he says: *Apart from the two Lights and the five stars which are called wandering, some have said that all the others are fixed in the heaven and are moved only with the heaven; others (whose assertion is nearer the truth) have said that these too advance by their own motion, because [?] they are carried along with the revolution of the heaven; but that, because of the immensity of the outermost globe, they spend ages exceeding any credible number on a single circuit of their course, and that therefore no motion of theirs is perceived by man, since the span of human life is not enough to detect even a brief point of so slow an advance.* Hence Tully, ignorant of no school approved by the ancients, touched on both opinions at once when he said *in which are fixed those everlasting courses of the stars which revolve*: for he both called them fixed and did not leave unsaid that they have courses. But who would not see that Cicero is being dragged here by force? From the whole context of those words it is plain that he understood no course in the fixed stars other than the course of the highest heaven, by whose running they run.

[Margin: Philo's opinion.]

Philo too, a Platonist to be sure, says in his book *On the Cherubim*: *The heavenly orbs have motions contrary to each other; one of them carries the fixed stars to the right, the other the wandering stars to the left. But that outermost orb of the fixed stars is one, which is turned from east to west; while the inner seven, namely those of the wandering stars, whose course is at once compelled and voluntary, have two motions contrary to each other: the compelled one being like that of the fixed stars, etc.*

[Margin: The opinion of St Ambrose and St Damascene.]

St *Ambrose* also expressed this opinion (*Hexaemeron* book 2, chapter 2), whose words we shall report in Section 3, chapter 1, number 9; and so did Damascene, when in book 2 of *On the Orthodox Faith* he granted motion towards the east to the seven planets alone, saying: *For they assert that there are seven planets, the Sun, the Moon, Jupiter, etc. They have called these planets, that is, wanderers, because they perform their motion contrary to the heaven: for the heaven and the other stars are moved from east to west, but they alone have their motion from west to east.* Some scholastics seem to have held this opinion, especially *St Bonaventure* (on book 2, distinction 14, article 1, question 1) and *Aegidius* [Giles of Rome] (*Hexaemeron*, part 2, chapter 36), in these words: *Therefore the orbs of the planets, because they abound in motions, are lacking in stars; but the eighth sphere, because it abounds in stars, is lacking (so to speak) in motions, since it has only one motion. Ruuius* too (*On the Heavens* 2, chapter 5, question 1) and *Hurtadus* (disputation 1 on the heavens, section 1) have called the proper motion of the fixed stars into doubt.

There is also extant a little book by *Aben-Ezra* entitled *Misphate hamazaloth*, that is, *On the Judgements of the Stars*, where it is said: *The great and glorious orb, where the glorious hosts of God are, is called the heaven of heavens, because beneath it are the orbs of the seven planets.* So this author does not seem to recognise any heaven above the seven planets other than the single heaven of the fixed stars — which, however, he wrongly confuses with the Empyrean. Nor did *Pliny the Elder* (book 2, chapter 8) recognise any motion of the fixed stars other than this one, nor *Vitruvius* (book 9, chapter 4).

As for the contention of *Iosephus Scaliger* [Joseph Justus Scaliger], in book 4 of *De emendatione temporum*, page 283, and in the *Diatriba de aequinoctiis*, chapter 1, that Thales of Miletus, Meton, Euctemon, Aristyllus or Aristarchus, and Timocharis came, from the difference between the tropical year of the Sun and the sidereal year of the fixed stars, to suspect some proper motion of the fixed stars by which they gradually receded from the equinoctial points in the order of the signs — that is a suspicion of Scaliger's rather than a proven assertion. For *Ptolemy* (*Great Syntaxis* book 3, chapter 2, and book 7, chapters 2 and 3) attributes that first conjecture and recognition of the proper motion of the fixed stars — made, it is true, from places of the stars observed by Aristyllus and Timocharis — to Hipparchus, who compared his own observations with theirs; and Ptolemy himself afterwards made it more certain by collating his own observations with those of Hipparchus.

[Margin: 2. The opinion that distinguishes two motions really, and heavens for them.]

III. The *second* opinion was that of those who, following the lead of Hipparchus and Ptolemy, recognised two apparent motions in the fixed stars: one upon the poles of the Equator, from east to west, very swift, and completing its revolution in the space of 24 hours; the other upon the poles of the ecliptic, from west to east, very slow. They recognised them in such a way that they held it necessary, for the diurnal motion, to have a heaven really distinct from the heaven of the fixed stars, that is, from the eighth sphere — a heaven by

which all the lower spheres would be swept to the west; and they accordingly called this motion the motion of *Raptus*, of sweeping along.

[Margin: 1. Class of the 2nd opinion, positing 9 mobile spheres.]

Within this opinion, however, three classes of authors must be further distinguished. The authors of the *first* class posited only nine celestial spheres, and so recognised the ninth as the primum mobile: Rabbi Moyses; Rabbi Iosue in the *Book of Demonstrations*; Haly on the *Quadripartitum*, chapter 11; and many Jews — indeed *Abenezra* himself too, in the book *Taamin*, that is, *Of Reasons*, as Augustinus Riccius reports (*De motu octavae sphaerae*, chapter 4). *Sacrobuschius* [Johannes de Sacrobosco] on chapter 1 of the *Sphere*; *Scotus* on book 2, distinction 14, question 2, and on *Metaphysics* 12, comment 44; *Caietanus* on 2 Corinthians 2; *Delphinus*, *De caelestibus globis*, chapter 30. Finally *Mastrius and Bellutus* [Bartolomeo Mastri and Bonaventura Belluto] (disputation 2 on the heavens, question 1, article 2) distinguish the primum mobile from the starry heaven, but posit a single starry heaven. Of these, Caietanus and Delphinus say that the primum mobile is the crystalline heaven.

[Margin: 2. Class: 10 spheres.]

The *second* class posits 10 mobile spheres: namely, the tenth for the primum mobile; the ninth for imparting the motion from west to east to the fixed stars and to the auges of the planets, the Moon excepted; and the eighth for the motion of access and recess from the equinoctial points, that is, the libration or trepidation in longitude. In this class were many Arabs, Moors and Jews, but especially *Alphonsus*, King of Spain [Alfonso X]; *Georgius Purbachius* with his followers; *Io. Regiomontanus* in the *Theorica* of the eighth sphere and in book 7 of the *Epitome of the Almagest*, proposition 6; *Petrus Apianus* [Peter Apian] in the *Opus Caesareum*; *Io. Baptista Amicus* [Giovanni Battista Amico], *De motibus caelestibus*; and *Petrus Alliacensis* [Pierre d'Ailly], question 2 on the *Sphere*, who reports the opinion of 10 mobile spheres and does not refute it; *Maurolico*, dialogues 1 and 3 of the *Cosmographia*, pages 24 and 89; *Fernelius*, *Cosmotheoria* book 2, chapters 1 and 7; *Iosephus Langius* in the *Elementa astronomica*, chapter 4; *Clavius* in the earlier editions of the *Sphere*, pages 44 and 72; the *Conimbricenses* [the Coimbra commentators] on book 2 *On the Heavens*, chapter 5, question 1. To this class can be referred Tebith, Arzachel [al-Zarqālī] and Isaach Israelita [Isaac Israeli of Toledo], in the second treatise of *De fundamento mundi*, inasmuch as they admitted a motion of trepidation of the fixed stars in longitude — granted that they ascribed its cause to the ninth heaven rather than to the eighth, or left this arrangement of the orbs in doubt.

[Margin: 3. Class: 11 spheres.]

The *third* class posits eleven mobile spheres: the eleventh for the primum mobile; the tenth for the trepidation or libration in latitude, to be imparted to all the lower spheres so that they follow the variation of the obliquity of the ecliptic; the ninth for the trepidation or libration in latitude, by force of which the anomaly of the motion of the fixed stars and of the precession of the equinoxes is produced; and finally the eighth for the equal or mean motion of

the fixed stars towards the east. So, namely, *Ioannes Vernerus* [Johannes Werner], as cited by Erasmus Osualdus [Erasmus Oswald Schreckenfuchs] on Peuerbach's *Theorica* of the eighth sphere; *Io. Antonius Maginus* in the *Theoricae*, book 1, and in the Secondary Mobiles, canon 18; *Leupoldus de Austria* [Leopold of Austria] in his *Compilatio*; *Clavius* in the last edition of the *Sphere*, page 36 — which is why Scheinerus [Christoph Scheiner], in the *Disquisitiones mathematicae*, number 22, calls it the Clavian system. The same number of mobile spheres, according to the old opinion, was set out by *Antonius Deusingius* [Anton Deusing] and *Petrus Gassendus* [Pierre Gassendi] in their *Institutiones astronomicae*. Some of the authors of the second class, however, especially Clavius and the Conimbricenses, admit an eleventh, immobile heaven, that is, the Empyrean. Indeed Clavius thinks that the aggregate of the 9th and 10th heavens is the glacial and crystalline heaven, which has earned the name of *the waters above the heavens* in the sacred writings, because of its transparency and because in it no parts are denser than others.

[Margin: 3. The opinion of Turrianus and Fracastoro, on 14 mobile heavens.]

IV. The *third* opinion was that of *Io. Bapt. Turrianus* [Giovanni Battista della Torre] and *Girolamo Fracastoro*, in the *Homocentrica*, section 2, from chapter 11 to 17. There he posits above the Aplanes, that is, the sphere of the non-wandering stars, other mobile orbs really distinct from it, but differing in number, order and function from those that the authors of the second opinion posited; and he gives different hypotheses for these motions, according as one may wish to recognise a motion of trepidation and of the fixed stars both direct and retrograde, or only direct. But since he himself in the end thinks it more probable that the motion of the fixed stars is always retrograde, though unequal, he concludes in chapter 17 that above the Aplanes there are 14 orbs.

The first or highest is the *Primum Mobile*, which with marvellous speed, in 24 hours, sweeps with it from east through west back to the same east all the orbs enclosed within it. The second is the *Circumducens* [the Leader-round], which cuts the Equator of the primum mobile at right angles; carried round in latitude through the upper colure from north to south in 3600 years, it carries all the lower orbs with it, except in so far as some counter-carried orbs hinder it. The third is the *Circitor* [the Circuiter], which proceeds through the lower colure, that of the equinoxes; it is moved accidentally by the *Circumducens*, in 3600 years, from south to north, but of itself it is moved towards the opposite quarter. The fourth orb is the one *Contravectus* [Counter-carried] *both to the Circitor and to the Circumducens*, and it is borne through the upper colure from south to north with twice as swift a motion, that is, in 1800 years. The fifth is the *Anticircitor*, which is carried round by the Contravectus always towards the part opposite to the Circitor. The sixth is the *Contravectus to the Anticircitor*, yielding to it insensibly, so that the Aplanes is not carried round with a sensible motion in latitude. And beneath it, finally, is the *Aplanes*, the orb bright with so many lights and marked out by the Milky circle, which of itself completes one degree in 100 years and fulfils its revolution in 36000 years, but within

that revolution is now slower, now faster. Thus above the eighth sphere these authors posit six mobile orbs, so that there are 14 in all.

[Margin: 4. The opinion of a primum mobile that is distinct but does not sweep along.]

V. The *fourth* opinion does admit a ninth heaven really separate from the eighth sphere, to be the primum mobile; but it denies that the lower orbs are swept along by it — that is, physically drawn or carried — and it does not grant absolutely that the participation in the first motion is compounded of two motions really distinct and contrary. But it does not explain in one and the same way how the sphere of the fixed stars and the spheres of the seven planets are moved by that primum mobile.

[Margin: 1. Class of this opinion.]

For *Alpetragius*, in his *Theorica physica* of the celestial motions, and more or less Achillinus in his book *De orbibus*, deny that the primum mobile sweeps the lower orbs along by identity, continuity or immediate contact. He says rather that the first Mover, or first Intelligence, impresses on the separate orbs a revolving motion from east to west, just as an impetus is impressed by someone throwing a stone or shooting an arrow; and that this impetus grows weaker and weaker the farther the spheres of the secondary mobiles are from the primum mobile. Accordingly all of them, of themselves, are carried round from east to west by a motion that is simple as regards longitude, but they do not reach the speed of the primum mobile by their own motion; and therefore the fixed stars and the planets appear accidentally to recede towards the east and to strive against the primum mobile, whereas in reality this motion is a single one, towards the west — slower, however, in the fixed stars, slower still in Saturn, and slowest in the Moon. But because he knew that this motion is oblique, and runs along helices, that is, a gyrating line, which the Arabs call *Lacalabinum*, and that the fixed stars change their declinations, he said that this motion takes place upon various poles that go round the poles of the world by force of little circles. And because, following Hermes, Theon and Arzachel, he suspected that the motion of the fixed stars is sometimes retrograde, or at least unequal, he posited an unequal motion of the poles of the eighth sphere in circles about the poles of the world. In a proportional way he explains the anomalies of the planets, their directions, stations and retrogradations, without epicycles and eccentrics.

Against him rise Augustinus Riccius (*De motu octavae sphaerae*, chapter 10), Fracastoro (*Homocentrica*, section 2, chapter 3), *Clavius* (on the *Sphere*, page 48) and other more recent astronomers. But while Alpetragius is rightly criticised in that by this hypothesis he neither explains nor maintains the motion of the planets in altitude — thinking, I believe, that it is apparent and not true — he is unjustly criticised in that he maintains the oblique motion of the fixed stars, or the anomalies of the planets in longitude and latitude, as though he had explained that motion upon the same poles and along the same line.

[Margin: 2. Class of this opinion.]

Another form of this opinion is that of Petrus Alliacensis, Cardinal and Bishop of Cambrai. In questions 2 and 9 on the *Sphere* he posits a tenth, immobile orb, to keep the different properties of the terrestrial regions unchanging through its influences; and he posits the ninth heaven as the primum mobile, in question 4. But he too denies that the lower orbs are swept along or drawn by it, or swept along properly and physically by the primum mobile. For he says that the individual orbs have their own Intelligences, and that from the primum mobile, or mover, a certain power flows into the lower ones which inclines them to a motion similar to the motion of the primum mobile — just as nature inclines heavy things to motion upwards, so that a vacuum may be avoided — and that such a motion is therefore neither purely natural nor violent, but would be violent and impossible if they were swept along by the primum mobile. He grants, moreover, that this motion takes place upon different poles and axes, and in question 13 he admits epicycles and eccentrics, or a motion equivalent to them, on account of the elevation and depression of the planets. Hence on this opinion the fixed stars and the planets are really moved with a single motion towards the west, and strive to imitate the primum mobile; but because they do not imitate it perfectly, they seem, by their slowness, to travel back towards the east.

[Margin: 5. The opinion positing the primum mobile as the ninth heaven, but crystalline, and without sweeping along.]

VI. The *fifth* opinion is that of *Ioannes Antonius Delphinus*, of the Franciscan family. In his book *De globis ac motibus caelestibus*, chapters 19, 29 and 30, he affirms with Alpetragius that there is a primum mobile and that it is the ninth heaven, but that this is the crystalline or watery heaven, which sacred Scripture calls by the name of *the waters above the heavens*; and that all the orbs are concentric with the world. He says, however, that each is moved by its own Intelligence, that is, by Angels, always from east to west, yet so that they move more slowly than the first Intelligence moves the ninth heaven, and moreover now towards the south, now towards the north; and this not by natural necessity but by free will, because they willingly serve the Creator who commands such a motion. Lastly, he calls the name "ninth mobile" metaphorical, inasmuch as that heaven does not move the lower spheres or globes physically, but only as the first exemplar of the swiftest motion towards the west; and he says that the apparent motion towards the east comes from their being left behind by the swifter motion of the primum mobile. To this opinion can be referred *Chiaramonti* (*De universo*, book 2, chapter 18, and book 6, chapter 1), inasmuch as he posits as the primum mobile a ninth heaven which the other spheres, moved by the Intelligences with the diurnal motion alone, do not keep pace with, and which they therefore appear to fall behind towards the east.

[Margin: 6. The opinion denying a primum mobile distinct from the 8 spheres, but positing two motions.]

VII. The *sixth* opinion denies that there is a primum mobile really distinct, by an adequate distinction, from the eighth sphere and the remaining seven; it

says that it is distinguished only as a whole is from its parts taken one by one. It admits, however, two motions. For the whole world, except the globe of the earth, is moved from east to west — that is, the eighth sphere, the seven orbs of the planets, and the element of fire and of air. And the primum mobile, under one aspect, is the aggregate of all eight spheres, which by priority of nature is moved in this way before its parts are; under another aspect it is the eighth sphere, inasmuch as the Intelligence moves it principally, as the nobler heaven — just as the soul is said to move first and principally the heart, and then the other members. Yet just as the heart does not move the other members, so neither does the sphere of the fixed stars sweep the other spheres along; it is moved together with the whole by the same Intelligence, just as the whole body is moved by the same soul. And it holds that there is in each heaven a twofold appetite, one for the motion of the whole towards the west, the other for the motion proper to it, towards the east; and that these motions do not conflict with each other, just as a particular motion of the hand, sometimes towards the opposite side, does not conflict with the motion of the whole animal body towards one side. So holds *Augustinus Riccius* in the treatise *De motu octavae sphaerae*, chapters 1, 13, 14, etc.; and so *Orontius Finaeus* [Oronce Finé], *Sphere* book 1, chapter 5, saying: *The primum mobile, then, is to be called the world itself, and not some particular celestial orb. The second difference of motion, however, belongs to the particular orbs themselves, which are parts of the whole heaven; for each of the particular orbs strives against it with its own motion from west through south towards east, etc.* Riccius, however, denies (chapter 22) that there is one celestial body moved by a single lation, although in the same place he grants to the theologians that there is one immobile body, namely the Empyrean.

To this class can be referred those unnamed authors who, as reported by *Fracastoro* (*Homocentrica*, section 2, chapter 10), said either that there is a single Intelligence in all eight spheres, moving them westward, and another Intelligence in each of the seven orbs, so that there would be eight in all; or that there are two in each orb — say two in the eighth and two in the seventh — so that there would be sixteen, of which one would move its orb westward and the other eastward, obliquely. *Oviedus* too points to this opinion about one or two Intelligences (the single controversy on the heavens, point 4). *Clavius*, moreover, refutes the opinion of Riccius and Orontius (on the *Sphere*, page 49) as impossible: for just as it is impossible, he says, for a whole animal to advance westward at the same time as each of its parts, without exception, goes back eastward, so it is impossible for each of the eight heavens to be carried eastward at the same time as the whole aggregate of them is rolled round to the west. Fracastoro, for his part, says that it follows that the Intelligences never move according to their nature, since one always resists the other and hinders it from moving its heaven with as much speed as it otherwise would; and this he thinks violent and unfitting for powers and Intelligences so excellent.

[Margin: 7. The opinion denying two motions, and a primum mobile above the 8th sphere.]

VIII. The *seventh* opinion is similar to the opinion of Alpetragius set out in number 5, but differs from it in that it denies that there is above the eighth sphere another heaven which is the primum mobile. For it says that the primum mobile is the eighth sphere, inasmuch as it revolves the fastest of all towards the west, and in the space of 24 hours restores the same stars to the same point of the same meridian or horizon from which that motion began on the first day of the world; but that this sphere does not sweep along the other, lower spheres, as certain authors of the first opinion, explained in number 2, held. For it says that the remaining lower spheres, moved either by their own power or by an Intelligence, are of themselves carried towards the west and aim at the same kind of motion that the eighth sphere performs, but cannot attain its speed; and that each day they reach the same point of the same meridian or horizon somewhat more slowly than it does, and the more slowly the farther each is from the eighth sphere. The result is that, because the slower planets are left behind each day by the faster fixed stars and are separated from them more and more, the planets appear to go back towards the east — granted that in reality they do not go back, and are not moved with a twofold motion but with a single one towards the west, though a slower one; and it is called a motion towards the east not absolutely or truly, but comparatively, with respect to the faster one, and in appearance.

[Margin: Anaxagoras, Democritus, Cleanthes.]

Plutarch (*De placitis* book 2, chapter 16) indicates fairly ancient authors of this opinion, though he adds the contrary opinion, when he says: *Anaxagoras, Democritus and Cleanthes think that all the stars are carried from east to west; Alcmaeon and the mathematicians, that the wandering stars are rolled with a motion contrary to the non-wandering stars, namely from west to east.*

[Margin: Ptolemy's criticism of this opinion.]

And at the same time — though somewhat younger than Plutarch — *Ptolemy* (*Almagest* book 1, chapter 8) indicates that the same view could stand if the apparent motion of the planets always took place in circles parallel to the Equator, without change of declination; but that, since it happens otherwise, it cannot. For he says: *Thus, if the progress of the wandering stars took place in circles equidistant from the Equinoctial, that is, about those poles about which the first revolution takes place, one might quite rightly suppose that the revolution of all is one and the same, following the first. For it would seem credible that their progress comes about not through an opposite motion, but because they are left behind by the first. But as it is, together with their progress towards the east, they also approach the north or the south, and not even an equal quantity of this approach is seen, so that this accident seems to come about through certain impulses in them, etc.* From this he concludes that two distinct great circles are necessary: namely the Equator, for the primum mobile carrying the fixed stars and the planets round to the west, and the Zodiac, within which the planets proceed obliquely to the east. So, since in book 7, chapters 2 and 3, he recognised that the fixed stars too advance slowly eastward upon the poles of the ecliptic, he seemed to some to suppose a

ninth heaven as the primum mobile, and to others both motions in the eighth heaven. But this about Ptolemy in passing.

Martianus Capella attributes the same opinion to the Peripatetics (*De nuptiis Philologiae* book 8, in the chapter on the orbs of the planets), saying: *Finally, the doctrine of the Peripatetics also contends that these stars are not moved forward against the world, but are passed by the speed of the world, which they could not follow.*

[Margin: Fracastoro's criticism of the same.]

Fracastoro, moreover, mentions this seventh opinion without naming anyone (*Homocentrica*, section 2, chapter 3), and refutes it on these grounds: if all the orbs are carried of themselves westward in longitude, they would be moved in latitude accidentally, and so by the force of some higher orb or Intelligence. This, he says, is unfitting, because it would follow that the planets are deflected by that orb which moves them sideways in such a way that on one and the same day the Sun, rising at the northern summer rising on this side of the Equator, would set beyond the Equator at the southern winter setting — which is contrary to observations. But that inference is not necessary. For they can be carried of themselves, by the same motive force, at once westward and obliquely, along spirals or helices, gradually deflecting from the parallels to the Equator, so that they appear to rise and set on nearly the same parallel, with a difference that is the smaller, the smaller the sideways bending by which they vary their declination from the Equinoctial and their latitude from the ecliptic. Therefore neither Fracastoro's refutation, nor Ptolemy's, nor that of Clavius — who attacks this opinion, which is in part that of Alpetragius and Achillinus, on page 49 of his *Sphere* — is solid as regards the oblique motion of the planets. There remains, however, a difficulty arising from the proper and oblique motion of the fixed stars which appears to us: for if they too appear to advance towards the east because they move rather slowly, then their motion is not the first motion, and it seems that another, swifter motion towards the west must be assigned, in comparison with which the fixed stars would be slower. We shall come back to this matter below.

[Margin: 8. The opinion of Iosephus Scaliger, denying the motion of the eighth sphere.]

IX. The *eighth* opinion was that of Iosephus Scaliger, in his *Diatriba de aequinoctiorum anticipatione* and in book 4 of *De emendatione temporum*, from page 284. He says that the motion of the eighth sphere eastward upon the poles of the ecliptic is a kind of monster, an old wives' and trifling tale. For the fixed stars, he says, are neither moved with a motion in longitude, or εἴς τὰ ἑπόμενα [?] [Greek: eis ta hepomena, "towards the following (signs)"] — that is, not in the order of the signs — nor do they change their declinations or distances from the poles of the world, which he posits as immobile; but the equinoctial and solstitial points are moved εἴς τὰ προηγούμενα [Greek: eis ta proēgoumena, "towards the preceding (signs)"], that is, against the order of the signs, towards the west. It is not the stars that recede from the Sun, but the Sun that recedes from the stars and keeps the equinoxes and solstices each year at

points of the Equator farther and farther to the west; and this is why the fixed stars appeared to Hipparchus and Ptolemy to move eastward, and why they persuaded a credulous posterity of it. Next he says that the poles of the Equator are distant from the poles of the world, and move westward along small circles described about the poles of the world; and that from this motion it comes about that the stars appear to change their declinations and their distances from the poles of the world. Further, that from this it comes about that all meridians are mobile, and that the equinoctial circle is not perpetually at right angles to the poles of the world, so that the position of meridian lines gradually varies, and sundials laid out on them become deceptive after some years. He professes to have observed outstanding evidence of this in the monuments of antiquity, and he thinks that the changes of the magnetic declinations can be referred to this. Thus by this fiction Scaliger frees himself from the trouble of reconciling two motions in the eighth sphere, since he grants it the single motion of the primum mobile and attributes the other to the equinoctial points, against the order of the signs.

The foundation of this opinion is that the pole star, which shines in the tail of Ursa Minor, was seen to be equally distant from the pole of the world — that is, 12° 24′ — both in the time of Eudoxus, namely Olympiad 103, or the year 367 before Christ, and in that of Eratosthenes, or the year 227 before Christ, and in the time of Marinus of Tyre and Ptolemy, who in the year

130 after Christ flourished. So from Eudoxus to Ptolemy there are about 500 years, in which that star certainly ought to have changed its distance from the pole of the world. In the same way, says Scaliger, the southernmost star of the same Bear stood equally distant from the pole of the world at both times, that of Eudoxus and that of Ptolemy, and stands so even now. In his letter to Dauid Riualtus [David Rivault de Flurance], too, which is at the end of the *Diatriba*, he relates the risings and settings of the stars that he collected from Hipparchus of Bithynia, and says that they cannot be understood unless a globe is constructed whose pole is distant from the tail of Cynosura by 12° 24′. When he communicated this to Tycho Brahe, Tycho, he says, was astonished at the novelty and strangeness of the ἐπιχειρήματος [Greek: epicheirēmatos, "of the undertaking"] — and rightly so. Copernicus, moreover, he says, in large part favours him, since Copernicus said that the equinoxes move forward [in anteriora procedere], and not that the eighth sphere moves in the order of the signs. In the end Scaliger admits that much concerning the construction of the sphere is still hidden from us, and he implores the help of more learned mathematicians to weigh and discuss these things more accurately. They did so indeed; but no one has been found willing to rear this offspring of his. Rather, as a monster born for the ruin of astronomy, it was smothered at once by four most learned men of our Society: above all the most erudite *Dionysius Petavius* [Denis Pétau], in his books *De ratione temporum* and in the dissertations of the *Uranologion*, book 3; and also *Paulus Guldinus* [Paul Guldin], book 2, chapters 9 and 10, *Against Calvisius*, the enemy of the Gregorian calendar; *Georgius*

Furnerius [Georges Fournier], book 10 of the *Hydrographia*, chapter 26; and *Clavius* on the *Sphere*, page 62. Outside our Society, *Ismaël Boulliau*, book 5 of the *Astronomia Philolaica*, chapter 4. We, for our part, have sufficiently overturned his foundation concerning the distance of the pole star from the pole of the world, partly in book 3, chapter 28, number 10, and partly in book 6, chapter 4, where we treat of Ursa Minor and of the pole star's approach to the pole of the world. I now add this: although at first sight it may seem that the motion of the fixed stars in longitude can be attributed to the equinoctial points advancing westward, yet, since according to Scaliger their advance comes about[27] only through the Sun, which keeps the equinoxes at points ever farther west — while the Sun is seen each day to become not more westerly than the fixed stars but more easterly — it is too difficult, and almost impossible, to defend this very thing.

[Margin: 9. The opinion attributing the primum mobile and all motion of the fixed stars to the Earth.]

X. The *ninth* opinion denies to the fixed stars all motion, both the diurnal one westward and the proper one eastward. To the planets it grants a proper motion eastward, but denies to them too the diurnal one westward. For the diurnal motion which appears in the fixed stars and in the planets, it says, is in reality the motion of the Earth about its own centre and axis, by which, in a rotation of 24 hours, it is spun round towards the east; and so it is we who go to meet the stars towards the east, and we who rise to them rather than they to us. The proper motion of the fixed stars, on the other hand, it attributes to the equinoctial points on the terrestrial equator, which precede westward by the force of a certain libration about the Earth's axis, indicated in book 3, chapter 29.

[Margin: 1st class, attributing an annual motion to the Earth.]

But the authors of this opinion split into two classes. The first attributes the annual motion, which produces for us the alternation of the four seasons, not to the Sun but to the Earth: the Earth not only turns daily about its own centre, but is also carried annually, together with its centre, under the ecliptic eastward around the Sun, which rests in the middle of the world, or at any rate is not carried from its place.

[Margin: Philolaus.] [Margin: Aristarchus.] [Margin: Copernicus and his followers.]

So held Philolaus long ago, according to *Plutarch* (*De placitis philosophorum* book 3, chapter 13), and after him Aristarchus of Samos, as *Archimedes* relates in the *Arenarius* [*Sand-Reckoner*]. This opinion was revived by *Nicolaus Copernicus*, not as a hypothesis, as some excuse him, but absolutely, as is clear from the preface to Paul III prefixed to the books of the *Revolutions*, and from book 1, chapters 8, 10 and 11, and book 5, chapter 2. It was embraced by Mästlin, Rheticus, Caelius Calcagninus [Celio Calcagnini], Rothmannus [Christoph

[27]the print reads "f t", one sort not printed

Rothmann], Galileo, Didacus Astunica [Diego de Zúñiga], Paulus Anton. Foscarinus [Paolo Antonio Foscarini], Antonius Laurentinus Politianus, but most zealously of all by *Johannes Kepler*, in his *Mars*, in the *Epitome of Copernican Astronomy*, in the *Mysterium Cosmographicum*, and in the *Harmonice*.

[Margin: 2nd class, denying an annual motion to the Earth.]

The *second* class, however, grants its own proper motion to the Sun as well as to the other planets — to the Sun, indeed, the annual one — but leaves to the Earth the diurnal motion, a rotation of 24 hours about its own immobile centre, and does not carry the Earth from one place to another.

[Margin: Heraclides, Ecphantus.]

So held, long ago, Ecphantus and Heraclides, according to Plutarch (*De placitis* book 3, chapter 13), and Nicetas of Syracuse, according to *Cicero* (*Academica* book 2 and *Tusculan Disputations* book 1). They were followed later by *Longomontanus*, in his *Astronomia Danica*, book 1 of the *Theorica*, chapters 2 and 4; by *William Gilbert* [Gulielmus Gilbertus], book 6 of the *Philosophia Magnetica*, chapter 3; by *Origanus*, tome 1 of the *Ephemerides*, in the dedicatory epistle; and by *Andrea Argoli* in the *Pandosion sphaericum*, chapter 3. The same opinion is ascribed to Plato in the *Timaeus* by *Aristotle* (*De caelo* book 2, text 75), *Cicero* (*Academica* book 2) and *Laertius* in his life of Plato. But we shall make it plain from Plato's *Timaeus* itself, in section 4, chapter 2, number 2, that Plato stood for the immobility of the Earth.

[Margin: 10. The opinion leaving [?] a proper motion to the fixed stars [?], but attributing the diurnal motion to the Earth.]

XI. The *tenth* opinion attributes to the Earth both the diurnal and the annual motion, but nevertheless does not take away from the fixed stars their proper motion eastward, still less from the planets. So, in the end, *Philips van Lansbergen*, in his *Tables*, precept 11; *Jacob van Lansbergen*, physician, in the *Apologia* for Philips, against Froidmont and Morinus [Jean-Baptiste Morin]; and *Ismaël Boulliau*, book 1 of the *Astronomia Philolaica*, chapter 2, for the reasons I set out in book 6, chapter 18, number 14. They differ, however, in that Lansbergen makes the motion of the fixed stars upon the poles of the ecliptic unequal, because of the unequal motion of the equinoctial points, whereas Boulliau makes it equal. But neither this opinion nor the ninth may we follow, by the divine Scriptures or by the decrees of the Church.

[Margin: 11. The opinion setting out [?] everything by a simple motion along spirals and gyres.]

XII. The *eleventh* opinion is very similar to the seventh, but it does not make the primum mobile the eighth sphere. Instead it says that all the proper motions which appear towards the east, in the fixed stars as well as in the planets, can be reconciled with their diurnal motion towards the west by a single oblique motion towards the west, made screw-wise [cochleatim] along helices and spirals, or helicoid spirals. Along these the stars do not complete exactly the same diurnal circle, nor return today to the same point of the meridian or horizon at which they stood yesterday, but bend somewhat towards the sides of the world,

towards the poles — yet in such a way that the fixed stars come round to the same meridian more quickly than Saturn, and Saturn more quickly than Jupiter, and so for the rest. Understand this of the planets when they are direct; for if they become stationary, they then appear just as swift as the fixed stars, and when they are commonly called retrograde they are then swifter than the fixed stars, and slower only when direct. Again, the planets are carried along this gyrating line in such a way that they move now higher, now lower. This is taught, though in other words, by *Scotus*, in 2, distinction 14, question 2; *Hurtadus*, disputation 2 on the heavens, section 3; *Mastrius and Bellutus*, disputation 2 on the heavens, question 4, article 1; *Cabeo*, on book 1 of the *Meteorologica*, text 38, question 2; *Amicus*, treatise 5 on the heavens, question 6, doubt 9, article 3, who in article 2 had adduced for this view Philoponus, Alexander, Vicomercatus, Galtus, Alvernia and Maior; *Arriaga*, the single disputation on the heavens, sections 4 and 5; *Oviedus*, the single controversy on the heavens, point 4; *Ioannes Poncius*, disputation 22 of the *Physics*, question 9, conclusion 3; and *Balthasar Tellez*, disputation 45 of the *Physics*, section 3, number 6. Of the same opinion, for the most part, were Alpetragius, Achillinus, Aliacensis and Delphinus, cited in numbers 5 and 6, and before them Anaxagoras, Democritus and Cleanthes, of whom number 7 speaks. Scotus, Mastrius and Bellutus, however, speak in such a way that they seem to admit, with Alpetragius, a primum mobile distinct from the eighth sphere. Tellez, on the other hand, does not recognise that first motion as really taking place in any body, but holds it to be merely imaginary, and that neither the fixed stars nor the planets exactly attain its measure. Others do not explain clearly enough whether there is any motion of the primum mobile, real or imaginary, which the fixed stars emulate but do not attain by their diurnal motion; and those who retain solid heavens, such as Amicus and Mastrius, can defend the spirals just mentioned only with difficulty.

[Margin: 12. The opinion of imperceptible [?] brief pauses.]

XIII. The *twelfth* opinion is the one I set out in book 6, chapter 18, number 16, as devised by me; but after many months I found it indicated by Amicus, treatise 5 on the heavens, question 6, doubt 9, article 3, in these words: *Fourth, others, thinking that those motions are two, say that they are not exercised simultaneously but by turns: while the Intelligence moves the heaven towards the poles of the Zodiac, it does not move it from east to west; and because that lateral motion takes place in the twinkling of an eye, the orb does not appear to cease from its diurnal motion. But this way of speaking, as it lacks a suitable author, so it cannot be probable; it is therefore more reasonable that there is really one motion arising from a diverse impetus, or a single gyrating one, but not perfectly gyrating.* Yet if that interruption by imperceptible brief pauses [per morulas insensibiles] cannot be refuted by any observation, it is not altogether improbable. *Poncius* touched on the same idea above, when he said: *It is manifest that something can be moved at once, at the same time, without any intermission perceptible to our senses, by two motions, one from east to west and the other from west to east.* How I explained this mode is clear from what was said in that number 16.

So many ways, then, and such remarkable ones — some worthy of human ingenuity, some unworthy of it — have been attempted for reconciling these two motions. What our own opinion is, although it was indicated in book 6, chapter 18, from number 15, must nevertheless be set out more fully here, through the following conclusions.

PRIMA CONCLVSIO.

First Conclusion.

[Margin: 1st conclusion.]

Both the wandering and the non-wandering stars can be moved westward by the motion of a primum mobile, or after the pattern of its motion [à motu vel ad motum], really distinct from their own heaven or from the stars; and meanwhile they can at the same time be moved, truly or apparently, by a proper motion eastward, without any physical contradiction.

XIV. For if there were any contradiction, it would lie either in the moving and movable bodies themselves — that they could not exist, or could not be joined or assigned together for this purpose — or in the very manner of producing both motions. But the contradiction arises on neither head; therefore what the conclusion asserts can come about. As to the first head, various celestial bodies can be assigned to perform the office of the primum mobile.

[Margin: 1st body for the primum mobile.]

First, this can be a watery heaven, made of the waters which, as we taught in section 1, chapter 2, question 3, and chapter 3, conclusion 3, are above the heavens, according to the more probable opinion of many Fathers, consonant with the letter of the divine books. This heaven does not flow away this way and that, because of its equal distance from the centre of heavy things, or because there it is outside the place of heavy and light things; and yet the whole of it may be moved in the space of 24 hours by one Intelligence, or by several if need be.

[Margin: 2nd body.]

Second, this can be an icy or crystalline heaven above the sphere of the fixed stars, as Delphinus and Clavius posit. This agrees with those Doctors and Fathers who believed that the waters above the heaven of the fixed stars were consolidated by God in the manner of ice or crystal, according to what was said in section 1, chapter 2, number 8 — such were Iosephus Hebraeus [Flavius Josephus], Pope Clement, Severianus, Anselmus Laudunensis, Strabus, Bede and Petrus Comestor. This heaven too could be revolved westward in the space of 24 hours by one Intelligence or by several.

[Margin: 3rd body.]

Third, it could be the heaven of the fixed stars itself, but fluid, which would move the fixed stars westward immediately, and the planets by means of the heaven of the planets, whether fluid or solid.

[Margin: 4th body.]

Fourth, it could be the heaven of the planets itself, whether fluid or solid, which would itself move the planets immediately, and indeed the fixed stars too, which it would touch immediately with its convex surface; or would move them and all the rest by moving the heaven of the fixed stars, whether fluid or solid.

[Margin: 5th body.]

Fifth, some body could be assumed by one Intelligence — or a body carried round, though not properly assumed, which would not be a heaven — and the Intelligence could lead it round the Firmament in the space of 24 hours, as a sensible exemplar of universal time, in imitation of which the fixed stars and the planets would have to be moved by their Intelligences, either immediately or through intermediate orbs. In the same way, some say that in the first three days that primordial light, which made three natural days, was a luminous cloud.

[Margin: 6th body.]

Sixth, without the motion of any body round the heaven, there could be various clocks measuring a time of 24 hours, far more excellent than ours, which the movers of the fixed stars and of the heavens would likewise have to imitate in their motion westward.

[Margin: 7th body.]

Seventh, above the heaven of the fixed stars, or in it, there could be a kind of equator, like a girdle of the world, made of solid matter and really distinct from that heaven, which would be moved westward by an Intelligence in the space of 24 hours. For among these bodies listed there is nothing that could not have come about physically. And the immediate conjunction of the first two with the heaven of the fixed stars neither involves a contradiction nor is absurd in astronomy. For no sphere is required between that heaven and the primum mobile to account for trepidation, or for a libration in longitude or in latitude; for we have already taught, as much the truer view, that the obliquity of the ecliptic does not vary (namely in book 3, chapter 27), that the equinoctial or tropical year is not unequal (namely in book 3, chapter 30), and that the precession of the equinoxes, or the motion of the fixed stars, is neither retrograde nor anomalous (namely in book 3, chapter 28, and book 6, chapter 17).

[Margin: 1st mode.] [Margin: 2nd mode.]

XV. As to the manner of moving other heavens or the stars westward in such a way that they nevertheless seem, truly or apparently, to move somewhat towards the east: the *first* could be by imperceptible brief pauses, already set out in book 6, chapter 18, number 16, and indicated in this chapter, number 13. The *second* could be by a continuous sweeping-along [raptus] — whether it should be called a drawing, a pushing, a carrying or a leading-round — which, however, because of the resistance of the heaven or star swept along, would not move it westward as swiftly as it would have moved it if nothing resisted it; and so it could not, within precisely those 24 hours in which the primum

mobile itself would complete its whole revolution, revolve the lower globes as well to the same meridian or horizon.

[Margin: 3rd mode.]

The *third* is if the movers of the sphere of the fixed stars and of the planets try to imitate the motion of the watery or crystalline heaven, or of a body or clock equivalent to those very heavens, but do not revolve their globe to the same meridian in the space of 24 hours, and reach it each day a little later; and this along spiral helices, as we said in number 12 when explaining the 11th opinion.

[Margin: 4th mode.]

The *fourth* is if the heaven both of the planets and of the fixed stars is fluid and is moved by one or more Intelligences in the space of 24 hours, while in imitation of it other Intelligences move the planets and the fixed stars westward indeed, but later each day, always falling short towards the east. And so that there need not be as many Intelligences multiplied as there are fixed stars, the fixed stars could be bound to one another as if in a kind of solid network invisible to us, as we said in book 6, chapter 18, number 17, under the 2nd mode. For in none of these 4 modes is there any contradiction, nor two continuous and mutually contrary motions, but a single motion — slower, however, or less swift, than the motion of the primum mobile.

SECVNDA CONCLVSIO.

Second Conclusion.

[Margin: 2nd conclusion.]

Yet without any mobile body really distinct from the star-bearing heavens, or from the stars themselves, that twofold motion which appears in all the stars — one westward, the other eastward — can come about and be explained, the other motions of the heavenly bodies also being preserved, without any physical contradiction.

[Margin: 1st mode.]

XVI. This is proved by the modes indicated. The *first* of them is if one or more Intelligences, moving the sphere of the fixed stars from east to west, have in their impressed species, infused by God, a perfect idea of a time of 24 hours — not hours such as make up the days we usually measure by the revolution of the Sun or of the fixed stars, but somewhat shorter ones, such as there would be if a primum mobile distinct from the star-bearing heavens and from the stars really existed. In moving the orb of the fixed stars, however, they exceed the perfect quantity of that intellectual measure, and so revolve it to the same meridian more slowly, that is, in a time a little longer than that of the 24 intellectual hours — such, perhaps, as was the time of the first three days. The Intelligences of the remaining orbs or globes, on the other hand, would take as the measure of the motion they are to perform not that intellectual day but the day made by a single revolution of the fixed stars — yet in such a way that they too would revolve their globes somewhat later, and the later the farther they are from the fixed stars. Accordingly, with respect to the planets, the sphere of the fixed stars

would be a primum mobile not physical, but after the manner of a moral, or as it were exemplary, cause. All these motions, however, would have to take place along the spirals and gyres of which I spoke in number 12, so that the variety of declinations and latitudes would be preserved; and when the planets are called stationary they would have to be moved just as swiftly as the fixed stars, but when retrograde they would have to be driven by their Intelligences somewhat more swiftly than the fixed stars.

[Margin: 2nd mode.]

The *second* mode would be if all the Intelligences, in the motion already mentioned, of the fixed stars as well as of the planets, westward — yet along spirals, etc. — looked to that single time, imaginary for us but mental or intellectual for them, shining in their impressed species and infused by God from the beginning of the world.

[Margin: Imitating the motion of the fixed stars by a mechanical experiment.]

XVII. But since to some the motion of the planets in a fluid heaven along the spirals just mentioned seems easy, but that of the fixed stars difficult, because they are accustomed to their own me- chanical globe, resting on poles fixed in the meridian — let them, I ask, use a star-bearing globe or Aratean sphere that is not fixed by its poles, or by any axis, to the horizon or the meridian, but is supported by some concave vessel, or rests on water or sand. Let them fix a small nail in any one point of it, or in a star painted on the globe, and, grasping this as a handle, lead the globe round towards the east, but bending it spirally and imperceptibly towards the poles. They will see that at the motion of one point or star all the other points and the other fixed stars are moved likewise, and change their declinations. And if someone has a small clock which meanwhile, by a kind of fiction, he uses as the measure of one revolution, let him move the globe in such a way that a star once chosen does not reach the meridian of the sphere until somewhat later. From this he will learn how much easier it is for one Intelligence, or several, to move all the fixed stars together spirally, or following the motion of one of them — whether because they are fixed to a solid firmament, or because, if the firmament is supposed fluid, they are nevertheless bound together by some indissoluble bond.

TERTIA CONCLVSIO.

Third Conclusion.

[Margin: 3rd conclusion.]

It is more probable that there is no body which is the primum mobile, nor two motions produced in the stars at the same time towards opposite quarters of the world, but a single motion towards the west along helicoid spirals — that of the fixed stars in a solid heaven, that of the planets in a fluid one; and that the office of the primum mobile is performed by an intelligible time, that is, an idea of the diurnal motion infused into the mind of every moving Intelligence.

XVIII. The first part is proved as follows. As is commonplace in the schools, what can be done by fewer means is done in vain by more; and, as *Ptolemy* advised (*Almagest* book 13, chapter 2, whose words I quoted in chapter 2, number 3), that motion is to be attributed to the celestial bodies which answers the phenomena while being as simple as possible and exposed to the fewest possible hindrances. Such is the motion we stated in the conclusion. For it takes place along a single spiral line, adaptable to all the motions of the stars, and capable of being traced by Intelligences that have the highest skill concerning the path and course along which they must carry the stars round by God's prescription. Nor can any violence from a sweeping-along, or any resistance from a medium or surrounding body, be feared, since the fluid heaven of the planets yields both to the planets and to the concave surface of the heaven of the fixed stars, above which the watery heaven is, and is fluid as well. Next, even if some body were posited as the primum mobile, whose motion at least the Intelligence would have to imitate, its motion would nevertheless have to be understood by the Intelligence, and so the Intelligence would have to have at least expressed species of it in its mind; and if it contemplated this by another, as it were reflex, act, it could use it as an impressed species and idea of that motion. Therefore, if through these species, which we cannot do without, the Angels can achieve whatever they would do with them and with that mobile body, then that motion of a distinct orb or body is certainly multiplied in vain. Indeed, not only in vain, but also against the dignity of the angelic nature. For if that musician is more excellent who knows how to sing, or to strike the strings, in harmonic measure, using the time he has in his mind, and has no need of any sensible beating of the hand or external sign to regulate his voice, then certainly the mode we have ascribed to the Intelligences is more excellent too, since they need no corporeal clepsydra, as it were, in order to move skilfully and harmoniously the stars and globes committed to them, and to perform their dances, so to speak, to this secret harmony of the world.

[Margin: 2nd part of the conclusion.]

The *second* part, on the distinction of a solid heaven from a fluid one, has already been proved in section 1, chapter 7, number 21. It is confirmed by this: in this way it is much easier for the fixed stars to keep their distances from one another perpetually, and to be moved uniformly, following the motion of one or a few of them, by one or a few Intelligences, than if they were moved in a fluid heaven.

[Margin: 3rd part of the conclusion.]

The *third* part, on the intellectual time known by each Intelligence and used as the immediate measure of the motion to be performed, is proved because it cannot be denied that this time is known by them. They would therefore be supposed in vain to need that sensible time by which the heaven of the fixed stars is moved; and it is better that there be one measure common to all of them, to which they may adjust their motions and revolutions. For just as they also know that time which did not exist before the world but could have

existed, so they know that diurnal time by which the ninth heaven would be moved if it were the primum mobile, and how much more swiftly it would reach the meridian than the fixed stars or the planets. They can therefore use this immediately; and it is more fitting that they should need no other sensible motion and time, granted that they must move sensible bodies. Nor is it absurd that the first and most universal time, with respect to an intellectual nature, should really be only intellectual — or, as they say, exist intentionally — and exist only objectively, in the angelic mind or in ours. For this is what happens with very many other ideas which we use in artefacts and sensible works. *For the house which is outside the soul comes from the house which is within the soul*, says the Philosopher (*Metaphysics* 7, text 23) — that is, from the idea and impressed species, which shines objectively in our mind and is the image of the house to be made. Therefore, if that architect would be more perfect who knew how, and was able, to direct the builders without a mechanical and external model or exemplar, or to build with his own hands, then the motion of an Intelligence will also be more perfect if it needs no external and sensible exemplar to perform the diurnal motion.

[Margin: What if the primum mobile is some sensible body?]

XIX. But if someone nevertheless requires that the time which is the universal measure of all motions, and as it were the norm of the rest, should be — or should at least once have been — the motion of some sensible body, or its number or measure, I would urge on him three more probable modes.

[Margin: 1st mode, by a simple equator.]

The *first* is the one I indicated in number 14, in the seventh place: namely by a kind of equator in the celestial globe, or above it, divided into 360 degrees and subdivided into minutes — or rather directly into minutes — not such as ours are, but into far finer particles, fit for discerning all the minutiae and little differences of the motions to be performed in the fixed stars and planets from the beginning of the world to its end. These would nevertheless be very large, on account of the vastness of that girdle, if they were compared with the minutes, or even the degrees, of our spheres or astronomical instruments. For if that motion of 24 hours could be represented by the motion of this girdle, wrought of solid matter, the whole machine of some ninth, tenth or eleventh heaven would be supposed in vain to be moved in order to provide this single motion. Supposing, then, that this motion is produced by a single Intelligence revolving that equator to the same meridian, the remaining Intelligences would have to move the fixed stars and the planets westward spirally, or screw-wise, in such a way that, when the beginning of the equator reaches some one meridian — say, that of Jerusalem — a fixed star which on the previous day had been under the initial point of the equator would on the following day not yet have been brought with its centre to the same meridian, and much less Saturn when direct, etc.

[Margin: 2nd mode, by the crystalline heaven.]

The *second* mode is that we choose that watery or crystalline heaven which is above the Firmament as the body which, by its own motion produced by an Intelligence, represents the motion of the primum mobile. In this way it will be confirmed that those waters were not placed above the Firmament by God on the second day in vain — granted that others have assigned many other purposes, concerning which, section 1, chapter 2, number 10. The *third* mode could be through the mere memory of the primordial light made by God on the first day of the world, and of its motion or successive production, by virtue of which the first three days of the world were completed. For there have been those who said that this light, or even the Sun made on the first day, had no other motion than that of the primum mobile, and that on the fourth day a proper motion was added to the Sun and to the other stars — according to what was said in section 1, partly in chapter 1, number 34, in reporting the opinion of Aegidius, and partly in chapter 4, number 3. In that case that first natural day, or even the first three days, would have been the most perfect and the shortest of all, since that light revolved most swiftly — no less swiftly than the equator of the primum mobile is now said to be moved in the treatises on the sphere. Therefore the memory of that day, or of those days, would suffice for the Angels to be able to revolve their spheres or stars to the same meridian in a time not so short, but a little longer. Whatever else seems to need adding to this subtle point I shall indicate shortly, in the scholia.

SCHOLIA.

Scholia.

[Margin: Whether two contrary [?] motions can be given.]

I. In connection with reconciling the motion of the primum mobile with the proper motion of the stars towards the east, physicists customarily dispute whether there can be two continuous and contrary local motions — that is, motions by which one and the same mobile body, with respect to the same part of itself, is carried towards one quarter of the world and at the same time towards the opposite one: if not *per se* and by the force of one and the same moving principle, then at least *per accidens* and by the force of different moving principles. This controversy is touched on by Maior, on book 2, distinction 24, question 1; Mayronus, in the same place, question 3; Girolamo Fracastoro, *Homocentrica*, section 1, chapter 8; Clavius, on the *Sphere*, page 52; Rubius and the Coimbra commentators [Conimbricenses] on *De caelo* book 2, chapter 5, question 4, article 2; Mastri and Belluto, disputation 2 on *De caelo*, question 4, article 1, numbers 129 and 130; Amicus, treatise 5 on *De caelo*, question 6, doubt 7, article 3; Hurtadus, disputation 2 on *De caelo*, section 3, paragraph 12; Arriaga, the single disputation on *De caelo*, section 5, number 56; Oviedus, the single controversy on *De caelo*, point 4, number 6; Poncius, disputation 22 on the *Physics*, question 9, number 94; and Tellez, disputation 40 on the *Physics*, section 2, numbers 4 and 5.

Of these, Poncius and Tellez seem to grant two contrary motions, one *per se* or from within, the other *per accidens* and from without. They use the now hackneyed example of an ant or a fly crawling on a wheel that is turned round in the contrary direction, or of a man walking in a ship towards the east, from the prow to the stern, while the prow of the ship is carried towards the west. What Tellez adds about a stone that would naturally descend along a straight line but is carried sideways by an external impulse, and so is moved by two motions, is not to the point, since these are not two contrary motions, that is, motions towards contrary termini. More to the point is a stone drawn by a rope from prow to stern by someone sitting on the bank of the river, while the ship is meanwhile carried by the river in the opposite direction.

The rest of the authors already mentioned, however, and finally Poncius himself, reply that it is naturally impossible for the same part of a subject to be moved by two simply contrary motions — that is, in such a way that at the same time it approaches the same fixed point of the world and recedes from it. For although metaphysically the contradiction seems to be removed by distinguishing two different moving principles, physically this is nevertheless incompatible with the nature of local motion. Local motion is an action by virtue of which either the whole mobile body or a part of it recedes from one place as its *terminus a quo* and approaches another as its *terminus ad quem* along the same line, and acquires another *where* [ubi] distinct from the former one. But it is naturally impossible for the same mobile body to be in two adequate places at the same time, and to approach, actually and in reality, the terminus from which it recedes: for receding includes or connotes the privation of approach and of local presence, and a privation and the form opposed to it cannot physically exist together in the same instant of time. Granted, if a body were replicated by divine omnipotence, it could, so replicated, be moved in opposite directions. The same thing can, however, be moved with respect to the same part of the subject by a twofold impetus, one of which by its nature pushes or draws towards the east and the other towards the west — but in such a way that only one of them prevails and actually moves it in a single direction, though more slowly than if it were not retarded by the other impetus. For if neither prevailed, the mobile body would be moved in neither direction. And so they deny that there are two motions: there is a single one, only slower. In the motions of animals, however, which lift their legs as they walk, a motion upward and downward is mixed in, which is neither contrary nor directly opposed to the motion of the ship or the wheel. Although this reply is good, and one I have accepted elsewhere, it can nevertheless be explained by a still more perfect distinction, as will be done shortly in the following scholium.

[Margin: Explanation of two opposite motions that are compossible.]

II. Two things, then, must be noted and distinguished in the motion either of animals, moved from within, or of inanimate things, moved from without, that are moved not over an immobile place but over a mobile one, or within it as within a vessel. *First*, in such a motion a twofold *terminus ad quem* can

be considered: one fixed and immobile, in a determinate part or point of the world; the other mobile, in the vessel and place itself — a terminus that is moved with the motion of the vessel, as a part with the motion of the whole. *Secondly*, motion towards some terminus, or in its direction, that is relative or respective, or comparative, or even hypothetical, must be distinguished from absolute motion, by which that terminus is approached.

Therefore, suppose some body contained within some vessel is carried with the motion of the vessel towards the west, and meanwhile that body is carried by another motive principle towards the eastern part of the vessel. If each impetus is equal, it will indeed approach the mobile eastern terminus by an absolute motion; but it will neither recede from the immobile eastern terminus nor approach it, and will remain in the same place, absolutely. But if the impetus by which the vessel is carried to the west prevails, that body will absolutely recede from the fixed eastern terminus and absolutely approach the mobile eastern terminus. It will not approach the immobile eastern terminus except respectively, inasmuch as its proper motion is directed towards the east; or comparatively, because it recedes from it less than if it were moved by no proper motion within the vessel or upon it; or, finally, hypothetically, inasmuch as it would have approached it if it had used its own motion alone and the vessel had stood still. Finally, if the impetus of that body prevails, it will either approach or pass beyond the fixed eastern terminus absolutely, but will recede from it only respectively, or comparatively, or hypothetically.

To assist the imagination, it is convenient to set this out in the accompanying diagram. In it, let AEB be a stretch of river, bisected at E; let its half AE be equal to the length of the ship CCDC,

Diagram. In the right column: a horizontal base line, the stretch of river, carries five points; above it a row of ship-positions, joined to the base line by dotted verticals and to one another by three curved arcs, concave upward; above these, a row of fixed points of the world, each of the first four marked with a small dot. The man O stands below H.

- G (top row, far left): a fixed point of the world, the eastern terminus
- Q (top row, second): a fixed point
- H (top row, centre): a fixed point
- L (top row, fourth): a fixed point
- K (top row, far right): a fixed point, the western terminus
- O (below H): the man
- C (middle row, far left, above A): the stern
- C (middle row, second, above M): the stern
- DC (middle row, centre, above E): prow and stern
- D (middle row, fourth, above I): a ship-point
- D (middle row, far right, above B): the prow
- A (base line, far left): end of the river stretch
- M (base line, second): midpoint of AE
- E (base line, centre): the point of bisection

- I (base line, fourth): a point on EB
- B (base line, far right): end of the river stretch

and the other half EB to the length of the same ship DC DD, now moved from the place under the fixed points of the world CQH [?] to the place under the fixed points HLK. But when the midpoint of its hull has covered half the space AE, namely the space ME, let the same ship be under the fixed points QHL; and let G be the eastern terminus, and K the western.

[Margin: 1st case.]

Now, *first*, at the beginning of the whole motion let the man O be at the prow D, under H, and let him begin to walk towards the stern C — towards G — so that in one minute of time, in which the ship, carried by the river, has traversed the whole space equal to its own length and has come under the points HLK, he has reached the stern C. For he will still be under the same fixed point H under which he was at the beginning of the motion, because the stern of the ship, which he has reached, is under it. Therefore, although he has moved absolutely and has approached the mobile terminus, that is, the stern, he has approached the immobile terminus G absolutely not one whit more — the terminus he would have approached had the ship not been moved.

[Margin: 2nd case.]

Secondly, imagine that the man O, while the whole ship in one minute of time leaves the whole space AE, which lies under the fixed space GQH, moves twice as slowly, and so in one minute gets from the prow to the middle of the ship. For he will be at D, under the point L. Therefore, although he has approached the stern DC absolutely, and indeed his walking was respectively directed towards G, he has nevertheless absolutely receded from G, the fixed eastern point, and approached K, the western terminus, since at the beginning of the motion he was under H. But if the ship had meanwhile not been moved forward at all, he would have come from DC to C, under Q; this motion, however, is only hypothetical. Conversely, if only the ship had been moved forward, he, not walking at all, would at the end of the motion have been at the prow D, under K. Therefore, since because of the walking just mentioned he is at D, under L, he has indeed receded from G absolutely; but comparatively — with respect to the recession that would have put him at D, under K — he has approached it, because he has receded less.

[Margin: 3rd case.]

Thirdly, while the man O runs through the whole ship from prow to stern in one minute of time, let the ship, twice as slow, cover half of its own length, the space ME, so that, whereas the prow was under H and the stern under G, at the end of the motion the prow is under L and the stern under Q. That man will then be under Q; and so he has indeed approached both termini absolutely — the mobile one, of the stern, and the immobile one, G; but from the immobile G he has receded only comparatively or hypothetically.

The motions of an ant on a wheel turned round in the opposite direction, and of water stirred towards the left with a finger in a vessel revolved towards

the right, and similar cases, are to be understood proportionally. For in truth, in the ant and in the water there are two motions fused into one, which are not contrary, since their *termini ad quos* are not the same, one being fixed, the other mobile; nor is there, with respect to the fixed one, an absolute approach and recession of the same part, etc.

[Margin: Whether we have any example of the first motion?]

III. You will ask whether, of the motion of the primum mobile — which in our Idea we assign to the rational equator, and which, as we said above, shines in the minds of the Intelligences that move the heaven of the fixed stars and the planets — we can have some sensible speci- men? For commonly we have no other exemplar than the motion of a single fixed star revolved once from a meridian back to the same meridian; and still more commonly we use the revolution of the Sun to the same meridian or horizon. I reply as follows. By what I said in book 6, chapter 16, any fixed star, in my view, completes one degree of its own apparent proper motion in 72 years: that is, when 72 Julian solar years have elapsed, a fixed star — Sirius or Spica, for example — stands one degree towards the east, in its parallel to the equator, from the meridian under which it stood at the noon from which the year, and the start of the 72 years, was taken. It follows that in one day it completes, of such apparent motion, $8''' \; 13'''' \; 9^{v}$, with a further small fraction of 3/13. Now, to complete $8'''$ of the equator a time of 32 fourths is required, that is, very nearly half of a single third. We would therefore need a pendulum that by one simple vibration would show us the time of half a third; we would measure with it the revolution of one and the same fixed star from a meridian back to the same meridian, and subtract a single vibration from that number of vibrations, so as to have some specimen of that ideal day. But a pendulum of this kind is not possible for us, as you will gather from what we said in book 2, chapters 20 and 21; for the vibrations of a pendulum each of which shows ten thirds of time can scarcely be counted. In vain, therefore, would anyone try to construct a clock that would show us such a day and distinguish it from the day produced by the revolution of a single fixed star to the meridian. Granted, we can gather the differences of such days and years by reasoning and with the help of logistic; and for astronomical practice and the most subtle observations it is more than enough to use the day that a single fixed star shows us by its revolution, since it is imperceptibly longer than one day of the rational primum mobile. But we have treated the differences of these days from one another and from the solar day with sufficient subtlety in book 3, chapter 21.

CAPVT IV.

Chapter 4.

Whether the proper motion of the stars is, per se and by the primary intention of nature — or rather of God and of the moving Intelligence — equal, or rather unequal; yet ordered and regularly unequal, and sufficient to manifest God and His Providence.

[Margin: Restriction and state of the question.]

I. I am not raising the question of the motion of the primum mobile, for all admit that it is most equable and uniform, as being the most universal rule of all the motions that occur in this sensible world. Of it hold those words of the Philosopher (*De caelo* 2, chapter 6, text 35): *Next we must go through the question of the motion of the heaven, that it is regular and not irregular. I say this of the first heaven and of the first motion [latio].* And speaking of this same motion (*Physics* 8, text 76): *For since*, he says, *circular motion is the measure of motions, it must itself be first; for all things are measured by what is first; and because it is first it is the measure of the others; moreover, it can also be regular.* Nor do I raise a doubt about the proper motion of the fixed stars in longitude, for I already suppose it to be equal, from what was said in book 3, chapters 28 and 30, and book 6, chapter 17. There remains, therefore, the controversy about the motion of the planets, which is compounded of several mixed kinds — namely in longitude, latitude, and depth or altitude. Yet about this, too, there is no doubt that it appears unequal to us, and that, with respect to the centre of the terrestrial surface, it passes through unequal parts of the heaven in equal times. For I believe this is now established by the clearest experiments of observation, and that the reader has been persuaded of it by what was said — of the Sun in book 3 from chapter 13 on, of the Moon in book 4 from chapter 18 on, and of the remaining planets in book 7, section 1, chapter 7, and sections 2 and 3.

We therefore ask about the motion of the planets *per se* and from the nature of the mobile planets — or rather from the intention of God, the Founder and first institutor of such motions, and hence from the intention of the Intelligence that moves the planet and that, of course, complies with the divine intention: namely, whether it is unequal as it appears; or whether it merely seems unequal to us *per accidens*, placed as we are outside the place from which it ought to be viewed, while *per se*, and viewed from where it ought to be viewed, it is equal and uniform. Just so a rule may be straight in itself and yet, like an oar, appear bent because of refraction in water; or a man may in reality be hairy and have a rather long beard, yet, seen from far off, appear smooth and beardless, because at too great a distance the hairs do not stand out or do not appear.

[Margin: 1st opinion [?], for the equality of the celestial motions.]

The first opinion was that the motions of all the stars are equal in their circles. For Anaximenes *said that the stars turn in the same way both above and around the Earth*, as Plutarch reports (*De placitis* book 2, chapter 16); and Aristotle (*Physics* 8, text 76, and *De caelo* 2, from text 35 on) adduces such arguments for the regularity of the first motion of the highest heaven that they hold equally for every circular motion that occurs in the heaven.

[Margin: Anaximenes.]

[Margin: Aristotle.]

For he says that it is regular for this reason: being circular, it has no beginning or end by approaching which it would have to be carried faster. For he says in that text 76: *Moreover, it is the circular alone that can be regular; for things that come*

about along a straight line are carried irregularly from the beginning and towards the end — for all things are carried faster the farther they are from that which is at rest. Of the circular alone, however, neither the beginning nor the end is by nature within it, but outside. There he is speaking of the purely natural motions of simple bodies. But in *De caelo* 2, text 35, speaking more universally, he says: *For if it were moved irregularly, it is clear that there would be in it an intensification, an acme [status] and a remission of its motion; for every irregular motion has remission, intensification and acme. And the acme is either at the terminus from which the mobile body is carried, or at the terminus to which it is carried, or in the middle. In things carried by nature, rest is at the terminus to which; in things carried violently or against nature, rest is at the terminus from which; and in projectiles, in the middle. But circular motion has no terminus from which or to which, nor a middle: for it has simply no beginning, no end and no middle; in time it is everlasting, and in length it is drawn back into itself and in no way interrupted. Therefore, if there is no acme of its motion, there will be no irregularity either; for irregularity arises through intensification and remission.* And this is Aristotle's first argument.

[Margin: 2nd argument of Aristotle.]

The second he adds in text 36, taken from the exclusion of the causes of irregularity. For since, he says, everything that is moved is moved by another, the irregularity of a motion must arise either from what moves or from what is moved; but in a celestial body it can arise from neither source, because both what moves and what is moved are incorruptible, simple and wholly immutable.

[Margin: 3rd argument of Aristotle.]

The *third* argument is adduced in text 37, thus: either the whole period of the celestial motion would be unequal, now slower, now faster, or its parts would be. But its parts have not yet been observed to have become faster or slower; for since an infinite time has preceded in which this motion has existed, the distance of the stars would by now have become infinite. Nor could the whole period be changed and become slower: for remission arises through incapacity, and incapacity is against nature; but in the first bodies there is nothing against nature (for they are simple and unmixed, and in their proper region, and nothing is contrary to them). Therefore there will be no incapacity, and consequently neither remission nor intensification, which connotes remission before it, if not after it. Besides, says Aristotle in text 38, it is absurd that what moves should be incapable for an infinite time and in turn capable for another infinite time; for all things in the infinite are indeterminate.

It is therefore clear that Aristotle's arguments apply not only to the motion of the primum mobile, or to that of the highest heaven, but to every motion of the planets, which he held to be circular and eternal (*Metaphysics* 12, text 43). But since he had said in *De caelo* book 1, text 23, *For the motion of a simple body must be simple*, and since astronomers use "simple motion" for one intermediate between fast and slow, that is, for equal motion, there spread from this into the schools of the physicists and astronomers this quasi-Peripatetic axiom: *the*

motion of a simple body, such as the heaven is, must be simple and equal, that is, regular.

[Margin: Simple and regular motion are not the same.]

But this, certainly, in that place is not what Aristotle meant; for in the same text he at once adds: *But we say that these alone are simple: the one that is circular, and the one that comes about along a straight line; and of the latter there are two parts, one away from the middle, the other towards the middle.* Therefore the motion of the elements, which are themselves simple bodies too — whether it is away from the middle of the universe, as is the motion of light things, or towards the middle, as is the motion of heavy things — is a simple motion. And yet the same Philosopher teaches (*Physics* 8, text 76, and *De caelo* 2, text 35), as we have already said, that this motion is irregular, and becomes faster and faster, and more intense from more remiss, the nearer they come to the place of their natural rest — granted that Plato, in the *Timaeus*, denied that the motion produced by the elements along a straight line is simple. Although, then, it was an axiom among most Peripatetics that a motion regular and uniform *per se* belongs to the heaven, as a simple body, it was nevertheless wrongly derived from the name "simple motion".

[Margin: Plato.]

Plato too, as Theon of Smyrna reports in his *Astronomy*, adhered to the Pythagoreans and held that the celestial motions are circular, regular and equal.

[Margin: Ptolemy's view.]

II. Among the astronomers, their chief, Ptolemy (*Almagest* book 3, chapter 3), expressly affirms that equality *per se* belongs to the celestial and everlasting bodies; for that chapter begins thus: *Since it follows that we must demonstrate the apparent inequality in the motion of the Sun, it must be stated in advance, in general, that the motions of the wandering stars too, in the order of the signs, no less than the motion of the whole universe against the order of the signs, are all by their nature equal and circular: that is, all the lines that are understood to carry the stars or their circles round intercept, in all simply equal times, equal angles at the centres of each circulation. The inequalities that appear in them arise from the positions and arrangements of the circles by which they are moved and in whose spheres they are enclosed; and nothing alien to their everlastingness in any way actually happens on account of the confused order of the appearances.* He attributes the apparent inequality of the Sun either to our eye being placed outside the centre of the circle that carries the Sun — a circle described from a centre other than the Earth's — or to the composition of a concentric with an epicycle carrying the Sun. Hence, if our eye were at the centre of the circle carrying the Sun, it would see equal portions of the ecliptic traversed by the Sun in equal times. And in book 9, chapter 2, he declares: *Since it is our purpose (as we did for the Sun and the Moon) to demonstrate for the five planets too that all their apparent inequalities arise through equal and circular motions — for such motions befit the nature of divine bodies, from which disorder and irregularity [?] are far removed — we must think highly of whatever we achieve in*

this matter. Ptolemy was followed thereafter by the Arabs and by very many astronomers, whether upholders of eccentrics and epicycles or of homocentrics.

[Margin: Fracastoro's opinion.]

For Girolamo Fracastoro himself, too, in the *Homocentrica*, section 2, chapter 20, has these words: *The revolutions of this inequality are not equal to one another, neither the whole nor their parts, etc. But this is per accidens; for per se they are all equal, and if they are measured by the mean motion of the first circitor.*

[Margin: Copernicus's view.]

How tenaciously Copernicus asserted the equality of the celestial motions is gathered from this: in Ptolemy he could not stomach that the Moon and the five lesser planets should have, besides the epicycle, two eccentrics — one of which would carry the centre of the epicycle, but unequally, while the other would not carry it, and yet the centre of the epicycle would be moved equally with respect to that eccentric's centre. This second eccentric the astronomers call the equant, and of it we have said enough in book 7, section 2, chapter 1, scholium 1. For Copernicus thought it absurd that a celestial motion should beg its equality from a foreign centre, that is, from a centre from which the circle of such a motion was not described. Copernicus's words in book 4, chapter 2, on Ptolemy's lunar hypothesis, are these: *The motion of the epicycle, then, is unequal on its own eccentric, which it describes. If this is so, what shall we answer to the axiom that the motion of the celestial bodies is equal, and seems equal only as to appearance, if the motion of the epicycle, apparently equal, is in reality unequal, and the very opposite of the principle laid down and assumed comes about? But if you say that it moves equally about the Earth's centre, and that this is enough to preserve the equality, what kind of equality will that be, in a foreign circle in which its motion does not take place, rather than on its own eccentric? Thus indeed we wonder, etc.* And a little later: *So too, with the Moon running through its epicycle unequally, if we now wish to prove the inequality of the appearance from unequal ones, one may observe what kind of argument it will be. For what else shall we do but give a handle to those who disparage this art?* — namely, astronomy.

III. With Copernicus, then, it was an axiom and a principle already established and assumed as certain that *the motion of the celestial bodies is equal* — that is, *per se*, and with the motion viewed from the circle's own centre. Indeed, this author was so devoted to the equality of the celestial motions, and so averse to that kind of inequality which Ptolemy had admitted in the five lesser planets as well, by introducing the equant circle into their hypothesis, that it served him as a spur to asserting the motion of the Earth. For so he himself professes in book 5, chapter 2: *The ancient mathematicians, who held the Earth immobile, imagined in Saturn, Jupiter, Mars and Venus eccentrepicycles, and in addition another eccentric with respect to which the epicycle would move equally, and the planet in the epicycle.* After this he describes their hypothesis, which we have set out with a clearer diagram in book 7, section 2, chapter 1. Then he infers against them: *But it is agreed that the equality of the epicycle ought to have been with respect to the centre of its deferent, etc. They therefore concede here too that the equality of a*

circular motion can occur about a foreign centre and not its own; and similarly that this happens even more in Mercury. But this has already been sufficiently refuted in the case of the Moon. These and similar things gave us occasion to think about the mobility of the Earth and about other ways by which the equality and the principles of the art might remain, and the account of the apparent inequality be rendered more constant. Therefore in this place likewise he assumes, among the first principles of the astronomical art, the equality of celestial motion [?] about the point that is the centre of the circle on whose circumference the motion takes place — whether of the planet or of the centre of the circle carrying the planet.

[Margin: Reinhold's opinion.]

Nor did his follower Erasmus Reinhold the Elder think otherwise, in his preface to Purbachius's *Theoricae*; since his words embrace the view of many astronomers, I shall not mind quoting them. So, after pointing out the inequalities that appear daily in the motions of the planets, he adds: *Since, then, the variety of the celestial motions and appearances, which the Greeks call* φαινόμενα*, is so manifold, the astronomers have investigated with the greatest diligence, with very long night-watches and labours, the causes of such dissimilar appearances. For the whole periods or revolutions of the orbs, which are agreed to be equable among themselves, plainly protest against and refute the notion that so great a diversity in the motions of the planets arises from some irregular motion of the celestial orbs themselves that carry the bodies of the planets, as the unskilled imagine. Therefore the astronomers hand down a learned and plain cause of this great irregularity, which is seen in the parts of the periodic motions: namely, that motions equable and uniform by their nature appear dissimilar to us, either because they take place in eccentric orbs, or because, when many simple motions are variously joined together, as it were, one single irregular motion results from them all.* This same author, further on, in the theory of the Sun and in the scholium added to Purbachius's *Theorica*, page 31, repeats things similar to those just quoted: *We said above*, he says, *that when the observations of whole revolutions are compared, reason concludes that the motions of the celestial bodies are altogether equable, constant and fixed; but if the parts of each revolution are compared with one another, a varied and manifold anomaly is detected.*

[Margin: And of Clavius.]

With much weightier words, however, and with a sterner brow, our Clavius rebukes the men of the contrary opinion (on chapter 4 of the *Sphere*, page 434), so much so that he does not hesitate to burst out into this statement: *But later men, of sounder mind, when they had begun to look at celestial things more correctly, more subtly and more scrupulously, came to the view that they pronounced it the height of madness to think that any irregularity, difformity or inequality is found in the motions of the celestial bodies; on the contrary, the highest equality, uniformity and regularity must be posited in them.* He goes on to confirm this, both *a fortiori* from some motions of sublunary things, which, though perishable and mutable, nevertheless keep a fixed law and order in their motions, and by Aristotle's arguments: namely, that in circular motion there is no beginning or end, approach to which must become faster and faster, and therefore irregular;

and that in the celestial movers there is no corruptibility or incapacity, from which, more remiss than at other times and so slower, a motion could come about; and finally, by the fixed law of periodic motion returning into itself, by force of which astronomers predict with the utmost certainty the future synods and syzygies of the planets with one another.

[Margin: Other authors for the first opinion.]

The same was the opinion of the Coimbra commentators (*De caelo* 2, chapter 6, question 2), of Suessanus (*De caelo* 2, text 40), of Philalthaeus (in the same place, text 35), of Iandunus (question 8), of Albertus de Saxonia [Albert of Saxony] (question 13), of Paulus Venetus [Paolo Veneto] (chapter 15), of Achillinus (*De orbibus* 1, doubt 3) and of Amicus (treatise 5 on *De caelo*, question 6, doubt 6, and question 5, doubt 3, article 1), though they use various distinctions; for some distinguish irregular motion from difform and unequal motion. But all of these agree that if the whole periods of one and the same planet are compared with one another — for example, the tropical solar years with one another, or the revolutions of Saturn with one another — they are entirely equal, and their whole circle is completed in an equal time. If, however, the parts of one period are compared with one another — say the northern semicircle, in which the Sun is, with the southern semicircle — they admit that the motions are indeed unequal, since they do not take place in equal times; but they refer this very inequality to appearance, owing to the position of the eyes observing these motions, or to the composition of several circles or partial orbs, each of which, taken separately, has equal motions, but which together compose a single unequal motion.

[Margin: Tycho's opinion.]

Finally, Tycho was carried off into Copernicus's opinion when he said (*Progymnasmata*, tome 1, page 11): *That all the celestial motions are regular and equable per se, and are carried round circularly by a constant law, has long since been received by all astronomers as an axiom.*

[Margin: Boulliau's opinion.]

And what may surprise you more: Ismaël Boulliau, although he contends that the motion of the planets takes place along an ellipse and not along a circle — and in this would seem about to side with the physical inequality of the motions that Kepler, the inventor of this elliptical path, had established, as we shall shortly say — has nevertheless tried to reduce this very hypothesis to equality. Accordingly, in book 1 of the *Astronomia Philolaica*, chapter 13, he sets out, among other principles, these six propositions:

1. *That the planets are moved by a single motion along a single line, as we have already shown.*
2. *That the revolutions of the planets are equal, perpetual, constant and uniform, and each like all those before and after it; and the observations of all ages confirm them to be such.*
3. *That they must therefore be regular, and composed of regular ones,*

4. *Because they are perpetual motions, they can otherwise only be circular, or along a line returning into itself.*
5. *That they are composed of circular ones, because they are completed according to the laws of circles.*
6. *That the revolutions themselves have a principle of equality, which the planet regards by its own nature.*

[Margin: Second opinion, for the inequality of the celestial motions.]

IV. The *second* opinion, however, is that the motions of the planets have a physical inequality, and not only an optical one arising from mere appearance: so that, even with respect to some proper centre of the orbit along which the planet proceeds, unequal portions of that orbit really correspond, by the nature of planetary motion, to equal times.

[Margin: Kepler.]

So, namely, Kepler, in the *Commentaries on Mars*, chapters 22, 32, 33, 34, 39, 44 and 57 taken together, and in the *Epitome of Copernican Astronomy*, book 4, part 3, and book 5, especially page 711. Indeed, earlier, Ptolemy admitted for the Moon and the five lesser planets the equant: that is, a circle with respect to whose centre the planet or the epicycle would move equally, but by which it would not be carried. Hence in the eccentric that carries the planet he admitted a motion unequal with respect to its own centre, and so, to that extent, a physical inequality. But concerning this equant, and the distinction between physical and optical inequality, I here take as already explained much that was set out in book 3, chapter 22, and book 7, section 2, chapter 1, scholium 1.

[Margin: Cabeo.]

Our Cabeo, too (on *Meteorologica* 1, text 38, question 2), teaches that the figures of the astronomical hypotheses, by which the celestial motions are reduced to equality, were devised for our use and as a foundation for geometrical calculation rather than being actually carried out in the heaven. For he says on page 221: *For just as there is in truth no mean motion, although it is necessarily supposed for calculation, so neither is it necessary that there be a motion of the eccentric or of the deferent.* And on page 222 he leans wholly towards a true and physical inequality of the motions. We, however, need a distinction in a controversy of this kind in order to set out our own opinion; and in support of it let there stand the

PRIMA CONCLVSIO.

First Conclusion.

[Margin: First conclusion, on the whole period [?] of the motions.]

Although it is probable, it is nevertheless not yet certain that the periods, or whole revolutions, of the celestial motions are equal to one another; indeed, of the Moon it is certain that they are unequal.

V. What I have said of the Moon is already known to astronomers, and indeed to those skilled in the ecclesiastical computus. For the Moon's period,

whether it is taken as the periodic month, in which it runs through the zodiac, or as the synodic month, in which it is joined to the Sun again, is sometimes greater than 30 days and sometimes less; the variety of these months becomes manifest more than once every year, and enough and more than enough has been said of it in book 4, from chapter 18 on. And so the third part of the conclusion stands proved.

[Margin: Third part of the conclusion is proved.]

[Margin: First part of the conclusion is proved.]

The first part is proved both by the authority of very many who hold this view about this equality — some of whom I have already brought forward in number 3, speaking from their own opinion and that of many others — and by reason. For an equality of this kind has in itself a beauty to be desired; and the very small difference between the periods of the other planets as observed by different astronomers is rather an indication of equality, though of an equality not yet known so perfectly that all astronomers must admit the same measure. If God had willed their periods to be unequal, and this were more fitting to the nature of sublunary things and to the perfection of the universe, they would surely at length differ from one another by some notable difference, one manifest after some centuries, as has happened in the lunar periods. Thus the motion of the solar apogee, which was unknown to Ptolemy because the interval between his own observations and Hipparchus's was small compared with the slowness of such a motion, nevertheless at length became evident after many further centuries.

[Margin: Second part of the conclusion is proved.]

The second part of the conclusion, moreover, is proved as follows. Although the Sun's period, that is, the equinoctial years, are more likely equal to one another, as I taught in book 3, chapter 30, probable reasons for an inequality of the years are nevertheless not lacking, and I adduced them there with their authors; and the widely differing length found for a year of this kind, which we displayed in the synopsis in book 3, chapter 15, number 9, also bears somewhat on the point. Once a probable inequality of the solar period is posited, the inequality of the period of the other planets becomes much more probable, since their motions are bound up with the motion of the Sun and depend on it, as is clear from book 7, section 1, chapter 7. It is made probable, besides, by the differing opinions about their periodic motions, as can be seen in the tables I set out in book 7, sections 2 and 3, where I displayed, according to the different astronomers, the various measures that must be known beforehand for the theory of the planets. Not inappropriately, therefore, Io. Antonius Delphinus, in his book *De globis ac motibus caelestibus*, page 53, infers from the uncertainty of the solar motion the uncertainty of all the celestial motions. Likewise, although the period of the fixed stars is more probably equal, according to what I have already said in book 6, chapter 17, the diversity of this period asserted by various astronomers, which I reported in the same book, chapter 16, and represented in a table, makes its inequality probable; and yet, for a particular reason, its

equality or inequality still remains uncertain. For not one of its periods has yet been completed, and so neither alternative has yet been established by observation: it requires many more than twenty thousand solar years, and yet from the beginning of the world not seven thousand years have yet elapsed.

SECVNDA CONCLVSIO.

Second Conclusion.

[Margin: Second conclusion, on the parts of the revolutions.]

Although the motions of the planets are ordered, and regularly unequal in the parts of their periods, they are nevertheless absolutely unequal — not only per accidens, owing to mere appearance or to composition from several equal motions, but per se, from the primary intention of God and from the end to which they are ordered.

VI. The former part of the conclusion, which is affirmative, is proved from the admirable symmetry of the partial motions with one another, as far as it has been possible to detect it by observation over so many centuries. For they have such fixed laws that practitioners have so far been able to reduce the motions of the planets to ephemerides and to predict them long in advance without an error that would be notable in most kinds of motions, or that could exclude astronomy from the number of the practical sciences or arts — especially in these later centuries. Nor is it the case that they lack fixed laws because small errors are still made in predicting the eclipses of the Moon and the Sun. For this art, in matters so abstruse and so far removed from our regions, it is enough if such motions and aspects of the stars as the astronomers predict from the hypotheses and astronomical tables come about as nearly as possible and for the most part — how much more if they always do so, so that for the uses of human life we can rely on predictions of this kind with the greatest security? From this, moreover, we gather that this art is far more perfect in the minds of the Intelligences that preside over such motions, and so that the rules of these motions are more certain in the nature of things than those that exist among us.

VII. The assertion of the same first part is confirmed, moreover, by the fact that not only the writers of the true religion but the heathen too rose — from the order of the celestial motions, and from the admirable harmony by which at fixed times that inequality and discord of the motions is brought back to concord — to recognising the divinity of the Godhead and the providence of God, and to extolling it with wonderful praises, though never worthy enough.

[Margin: Plato's opinion.]

Thus Plato said in the *Timaeus*: *We must assert that God begot eyes chiefly for this reason: that, beholding the circuits of mind that are carried out in the heavens, we might turn them to the use of our own mind, and recall the discursive motions of our understanding — akin to them, but in some way disturbed — to their tempered order. And when we have come to know them and, endowed with right reason according to nature, have perceived the order of each, we may imitate the revolutions of God, which proceed without any error, and compose the wandering and erratic discursive*

motions of our thought after the example of the gods. He calls the Intelligences that move the heavens gods; and their motions he calls the revolutions of God himself, inasmuch as they were first instituted by him and ordered with the highest harmony.

[Margin: Plutarch's opinion, and again Plato's.]

But Plutarch sets out Plato's view more clearly in *De placitis philosophorum*, book 1, chapter 6. There, after he had said of mortals in general, *They took their notion of God first from the beauty of the things that are seen. For nothing beautiful comes about rashly or by chance, but by the use of some fashioning art; and that the world is beautiful is evident from its form, colour and size, and from the variety of the stars that surround the world*, he added, in accordance with the mind of Plato, whom he praises there: *The things conspicuous in the heaven also complete its beauty; for the oblique circle is marked out with various images*; and, having enumerated the signs of the zodiac, he concludes: *Innumerable other things of this kind the same God scattered through the vaults of the heaven; hence, says Euripides, the starry splendour of the heaven, Chronon, finely adorned, the work of a skilled craftsman. Truly we have taken our knowledge of God chiefly from this. For with a perpetual tenor the Sun, the Moon and the other stars, having sunk beneath the earth, rise again with one colour, with equal size besides, from the same places, and at exactly the same time.* What Cicero says on this argument in book 2 *On the Nature of the Gods* is splendid, but rather long.

[Margin: Seneca's opinion.]

Let us therefore hear Seneca, who opens *De providentia*, in its very first chapter, thus: *You have asked me, Lucilius, why, if the world were governed by providence, many evils befall good men. This I should prove more conveniently in the course of a work in which I were proving that providence presides over all things and that God is concerned with us.* And soon after he lays this down beforehand as most familiar: *It is superfluous at present to show that so great a work does not stand without some guardian; that this fixed course of the stars is not the work of a chance impulse; that this unimpeded velocity proceeds by the command of an eternal law, bearing so much on land and sea, so many most brilliant lights, shining in such an arrangement; that this order is not that of wandering matter, and that things which have come together at random do not hang together with such art.*

But nothing more magnificent could be said than what is said in Psalm 18, *The heavens declare the glory of God*, etc. — where our Lorinus should be consulted first of all — and by God himself in Job 38: *Do you know the order of heaven, and will you set down its reckoning on earth?* And a little further on: *Who will declare the reckoning of the heavens, and who will make the harmony of heaven sleep?* — on which see Pineda. Not undeservedly, then, does St John Chrysostom (homily 10 *To the People*) call the heaven [Greek: βιβλίον μέγιστον καῖ ιδιῶταις, καῖ σοφοῖς (biblion megiston kai idiōtais kai sophois, "a very great book both for the unlearned and for the wise")] — a very great book open to the unlearned and the wise. And Minucius Felix says in the *Octavius*: *You see the heaven itself: now you will know how wonderful and divine is the balance of the*

supreme ruler in it. For more on this book written in golden letters, see St Basil (*Hexaemeron*, homily 11) and Nikephoros Kallistos Xanthopoulos (*Ecclesiastical History*, book 11, chapter 43).

[Margin: First argument for the second part of the conclusion.]

VIII. The latter part of the conclusion is shown thus. *First*: although inequality without any order, rule or rhythm would be a kind of imperfection, an inequality that is ordered, regular, and has rhythms and recurrences of motions at fixed times is a great perfection, and has far greater beauty than simple equality. Therefore it is this that rather befits the celestial motions; or at least this too ought to have existed in the heaven, and not equality only. So in the fixed stars, which most closely imitate the primum mobile, there would be no inequality as to the motion in longitude, nor as to latitude from the ecliptic, nor as to their distance from one another or from the centre of the world, but only some variation, though a very slow one, in the motion of declination. In the Moon, by contrast, which is nearest to the earth and closest to the sphere of mutable things, there would be the greatest inequality. In the Sun there would be only a moderate inequality — namely in the motion of longitude, of altitude and of declination, but not in the motion of latitude, since it has no latitude, nor in a motion of libration about its axis, nor that variation or second anomaly which is in the Moon and the other planets. Among these, the two nearer the fixed stars, Saturn and Jupiter, have been allotted a smaller inequality than those that are below the Sun and come [?] nearer the Moon; and of these latter, Mercury and Venus, which are bound more closely to the Sun and do not digress from it through a whole semicircle, have obtained a smaller inequality than Mars.

The antecedent is evident by induction from all the most beautiful things, whether made by nature or by art. As to nature, neither the bodies of animals nor the branches, foliage, leaves and flowers of plants preserve equality of members or parts, whether in shape, in size or in colour, but only a symmetry and agreement of unequal parts, with incredible variety. Nor are all fruits — indeed, hardly any — spherical or cubic, or aim at the mathematical beauty of some regular figure; and among things dug from the earth you would hardly find one or two that have a regular figure from nature. The tracts of the lands, the beds, as it were, of this great garden, and the tracts of the seas within them — did God from the beginning of the world confine them within the limits of a circular, quadrangular or triangular figure? Certainly not. As for artefacts, although the beauty of some requires a certain equality in figures or motions, the greater part requires inequality and disparity proportioned by fixed rules. This can be seen in architecture, in sculpture, in painting and the like; it shines out in the liberal arts, and finally, above all, in every kind of music, which bears on our subject. For if you try to reduce its motions and sounds to equality, or to derive their laws from the equality of the parts, as though they looked to that per se and primarily, you would surely rob them of all the sweetness of their harmonies and kill the soul of harmony. For what is more unpleasant, and usually borne with greater tedium, than monotony and uniformity in singing

and in modulation? Or what is more ungraceful than if dancers keeping time, or players striking the cithara, intended per se and primarily to describe a circle or some figure with equal motions?

[Margin: Second argument, from the figures of the stars, and from the asperity [?] of the Moon and the planets and the spots of the Sun.]

IX. *Second*: if equality of motions per se, and the simplest uniformity, were due to a celestial body because it is supposed to be simple, then there would also be due to it the beauty of regular figures in the arrangement of the stars, a most exactly spherical figure of the planets without any asperities, the most lustrous splendour and whiteness without any spots, and finally a perpetual light without any alternation of shadowing. But we see the opposite, all of us who look up at the heaven with eyes either naked or armed with the telescope. For the configurations of the stars are so far from regular figures that, if you except one or two triangles — and those not equilateral but isosceles — the other stars might seem to have been sown by chance and at random across the fields of the heaven, and most of them to be counted among the [Greek: σπόραδας (sporadas, "scattered")] and [Greek: ἀμόρφας (amorphas, "formless")], had it not at length become clear to those who contemplate these things deeply that this incredible variety serves agriculture, navigation, medicine and astronomy itself in a wonderful way. The asperities on the globe of the Moon; the asperities or spots on the face of the Sun, on the bands of Jupiter and on the boss of Mars; and finally the eclipses of Jupiter's satellites and the shadowings of Saturn by its attendants — already in our century were disclosed, not by a fiction of Prometheus's wand, but by the most truthful experiment of the telescope. And yet from these very things, no less than from the eclipses of the Moon and the Sun, there arises a greater elegance and beauty than if everything in the heaven had been arranged in a simple and uniform way.

[Margin: Third argument, from the end.]

X. *Third*: the end to which the celestial motions are ordered — namely, the diversity of effects in sublunary things, and the incredible variety of influences and of changes to be brought about for the sake of living things and animals, and above all for the sake of man, who was to obtain from it many more goods and much richer matter for praising and loving his Maker, who is the End of all things — requires in them some inequality, though one that is regulable and ordered. That the inequality I have described is a more apt means to this variety than mere equality is too manifest to need proof.

[Margin: The heaven and the stars were made for the sake of men.]

But that the visible heaven, and whatever stars in it shine for us, were founded for the sake of men is most true — whether the Epicureans like it or not, and certain Copernicans too, who would rather drive the Earth round the Sun than the Sun and all the stars round us, the inhabitants of a motionless Earth, as round a centre: whether they like it, I say, or not. For it is not Aristotle alone who says so — in *Physics* 2, text 23, *for we too are in a way the end of all things*, and in *Politics* 1, chapter 5, *that nature has made all things for the sake of men* —

and Cicero, *De legibus* 1. Divine Scripture too, of infallible authority, tries by this most powerful argument to deter men from offering divine worship to the stars, when it says in Deuteronomy 4: *Lest perhaps, lifting up your eyes to heaven, you see the Sun and the Moon and all the stars of heaven, and, deceived by error, adore and worship the things that the Lord your God created for the service of all the nations that are under heaven.* For what is more foolish than that man, to whom the supreme Lord granted a certain useful dominion over visible creatures, should offer worship and adoration to his own servants and handmaids? For God, as Genesis 1 has it, placed the stars in the firmament of heaven not to shine for one another or for the heaven, but *to shine upon the earth*. On this, rightly, Lippomanus in the *Catena*: *But if they were placed by God for this purpose, to illuminate the earth, then not to rule over men; for it is the Lord who created them for the use of men.*

The sacred interpreters may be consulted on that passage of Psalm 8 where David, having said *For I shall see your heavens, the works of your fingers, the Moon and the stars which you have founded*, at once, astonished that God had subjected these too to man, exclaimed: *What is man, that you are mindful of him, or the son of man, that you visit him?* etc., and *You have set him over the works of your hands.* These words are to be understood chiefly of Christ, at least in the mystical sense — for it belongs to him more properly to be the son of man, that is, of the Virgin Mary alone, since the others are sons either of no human being, like Adam and Eve, or of two — and St Paul understood them so; yet not a few Fathers teach that literally they can be taken of any man whatever, as may be seen in Ioannes Lorinus on that psalm, and some hold that the former part, *What is man* etc., applies to every man, and the latter, *or the son of man*, peculiarly to Christ. And certainly that saying of the Apostle in 1 Corinthians 3 applies to men: *All things are yours, whether the world, or life, or death, or things present, or things to come.*

[Margin: St Chrysostom's opinion.]

That those who deny that the stars were made for us favour the error of the Gentiles, I gather from St John Chrysostom, who says in homily 6 on Genesis: *Let us teach with all gentleness those who are taken up beforehand by the Gentile error, so that they do not confound the order, nor, abandoning the Creator, adore the creatures, which were made for our salvation and benefit. And if what you teach displeases the Gentiles, cry out nevertheless with a clear voice that all these things were made for the sake of the human race. For the Maker, wholly sufficient to himself, had no need of any of them; but he produced all these things to show his indulgence and kindness towards us — showing with how great honour he regards the human race — and so that, led by the hand by these things, we might ascend to the adoration that befits him.* There this golden Doctor was explaining the work of the fourth day and the formation of the luminaries and the stars.

[Margin: Lactantius's opinion.]

Lactantius too teaches, in his book *De ira Dei*, chapters 13 and 14, that the heaven was made for us, as St Nyssenus also does in his book *De opificio hominis*, chapters 1 and 2; and in book 7 of the *Divine Institutes*, chapters 3 and 4, he

inveighs not only against the Epicureans, who deny that the world was made for us, but also against the Platonists, who did assert the providence of God but did not expressly teach that the world was made for us — which, however, the Stoics did teach. *Plato, therefore*, he says, *and those who held the same view, ought to have taught and explained what the cause, what the reason was for fabricating so great a work: why, or for whose sake, he made it. The Stoics likewise say that the world was made for the sake of men.* And a little later, against Plato, who asserted that the world would be eternal, he argues in this way: *If the world was made for the sake of men, and was made such as to be eternal, why then are they themselves, for whose sake it was made, not everlasting? If those for whom it was made are mortal, then it too is mortal and dissoluble; for it is not of more worth than those for whose sake it was made.* And again: *But whoever denies that it was made for the sake of men holds to no reason at all. For if he says that the maker himself contrived such great works for his own sake, why then were we born? Why do we enjoy the world itself? Why do we intercept advantages belonging to another?* etc. *God, doubtless, wished to be seen, and to fashion figurines, as it were, of his various images with which to delight himself; and nevertheless, if this were so, he would have care for living beings, and above all for man, to whose rule he subjected all things.* But what the same Lactantius says in chapter 4 — *The world, therefore, was made by God, certainly not for his own sake; for he needs neither the heat nor the light of the Sun*, etc., *and God himself does not use all these things that are in the heaven and that are generated* — must be corrected; for *the Lord has made all things for himself*, as is said in Proverbs 16.

Rightly, therefore, Froidmont teaches in his *Vesta*, from Tertullian, *that both the stars and the elements serve man*; and Io. Bapt. Morinus [Jean-Baptiste Morin], against Lansbergen, observes that it befitted God to have regard in these motions to the utility of men, and not only to the ease or economy of nature; and Father Nicolaus Zucchius [Niccolò Zucchi], in his *Nova de machinis philosophia*, part 5, section 11, is indignant with those who strive to suppress this truth, though Scripture and the arrangement of the whole universe resist them. Nor should it seem too much to anyone that bodies so vast and sublime were made for men. For since they were made so that a contemplative creature might recognise and adore the Creator from them, and an angel, being itself without senses, would not need the beauty of the sensible world for that purpose, it remains that all these sensible things were made for men alone, and man himself and all things for God.

[Margin: Andreas Hierosolymitanus.]

Pointedly, therefore, Andreas Hierosolymitanus said in his sermon on the Angelic Salutation that man is the one *for whose sake God* [Greek: ουρα[ν?]οὺς ἐκαμαίρωσαι (the fifth letter of the first word faintly printed, probably ν; the Latin that follows renders the sense "founded the heaven")] etc., that is, *founded the heaven, made firm the earth, spread out the air, widened the seas, and in short made the whole mass and fabric that lies before our eyes to be looked up to*. And so elsewhere he calls man [Greek: μέγα τῆς ἐργασίας κατόρθωμα (mega tēs

ergasias katorthōma, "a great achievement of the work")] — that is, a huge specimen of the divine operation.

[Margin: St Augustine.]

And St Augustine (sermon 225, *De tempore*) says of him: *Man, the dear and beloved possession of God; man, for whose sake the heaven was made firm.* Nor should you say that it seems incongruous that this greatest world and the immense mass of the heavens were made for a manikin three cubits tall. For although man was called by the heathen philosophers [Greek: μικρὸς κόσμος (mikros kosmos, "little world")], and is smaller than the heaven in bulk, he is nevertheless more august and greater in the perfection of his nature and in his rational nobility.

[Margin: Arnobius.]

Arnobius touched on this, indeed, in book 2 *Against the Gentiles*, in these words: *This is that precious man, endowed with most august reasons, who is called the lesser world, and fashioned wholly in the form of a likeness.*

[Margin: Hermias.]

Indeed, in Protagoras's opinion (Hermias, *Mockery of the Gentiles*), man was called [Greek: ὅρος καὶ κρίσις τῶν [recte: τῶν] πραγμάτων (horos kai krisis tōn pragmatōn, "the boundary and judgement of things")] — *the boundary, judgement and norm of things*.

[Margin: St Gregory Nazianzen.]

But St Gregory Nazianzen, in oration 38, expressly denied that man is a [Greek: μικρόκοσμον (mikrokosmon, "microcosm")], *a small world*, and rather called him [Greek: μέγαν κόσμον ἐν μικρῷ (megan kosmon en mikrō, "a great world in a small one")], *a great world in a small one*. For the rest, to whom should it seem wonderful or incredible that the heaven and the stars were made for the sake of men, if the Son of God — as the Church professes in the Creed, and, bowed down under the weight of the immense benefit, sings: *Who for us men, and for our salvation*, etc.; and in that sweetest hymn: *Given to us, born for us*?

[Margin: Synesius.]

Aptly, therefore, Synesius (letter 57) on man: [Greek: τίμιον ζῶον ὁ ἄνθρωπος (timion zōon ho anthrōpos, "man is a precious animal")], etc. — *man is a precious animal, if for his sake Christ was raised upon the cross*.

[Margin: St Ambrose.]

And St Ambrose, on Psalm 118, calls him *the precious work of God*; and in his book *De bono mortis*: *The whole world is small against the loss of a single soul.* Finally, St Gregory of Nyssa (homily 4 on Ecclesiastes) affirms that not even the whole world is [Greek: ἄξιον τῆς ψυχῆς ἀντάλλαγμα (axion tēs psychēs antallagma, "a worthy exchange for the soul")], *a worthy price for the soul*.

Let Lucretius, then, be gone — he of the herd of Epicurus, more used to grunting than to singing — when he dares to say:

To say, further, that for the sake of men they willed to prepare the glorious nature of the world, etc. *is folly: for what is it to the immortal and blessed that they should set about doing anything for our sake?*

I grant, indeed, that there will be some who think I am now wandering off the subject, and playing the preacher or the churchman rather than the astronomer. But from what has been indicated they ought to consider that these weapons of ours are aimed at two kinds of enemy. The first consists of those who attribute an annual motion to the Earth and will not have it be the centre of the celestial motions; nor will they have the stars revolved for the sake of the living things placed on it — not even for the sake of man — in such a way that the Earth too, and man with it as the inhabitant of a planet, is not revolved around some other, more principal body in the world. For this reason they make light of this doctrine that the Heaven and the World were made for man. The second kind consists of those who are too much attached to the equality of the motions: seeing that the motions are unequal with respect to the Earth, or to the centre of the world from which we observe them, they deny that these motions are ordered per se to this centre, or exist for our sake.

[Margin: Summary of the third argument, and its twofold force.]

Against these, then, we hurl this twin argument — set out rather loosely before, but now gathered into a few words, as though by a throwing-thong: *The motion of the heavens and the stars is unequal with respect to the men who observe it; but the motion of the stars per se — that is, from its primary end and from God's principal institution — is unequal; therefore it is unequal per se, and not merely per accidens.* And again: *The motion of the heavens and the stars, in relation to the variety of effects which sublunary things were to enjoy for the sake of man, and man himself above all, had to be unequal; but this same motion is ordered per se to the variety of those effects, and above all to the benefit of man; therefore this motion had per se to be unequal.*

[Margin: 4th argument, from the relation of the parts to the whole.]

XI. *Fourthly.* It is admitted among the astronomers who champion the equality of the motions that if the whole compound motion of the planets is considered, it is unequal, since it is carried out through different circles described from different centres; but that if the motions of the individual circles are considered separately and one by one — for instance, the motion of the centre of the epicycle on the eccentric, and the motion of the planet's body on the epicycle — they are simple and equal about their own centres. Now I argue thus. The motion of the parts is not for the sake of the parts themselves, nor is it per se when compared with the whole; rather, it is per se for the sake of the whole. Or at any rate, when the parts are compared with the whole, what holds per se is rather the whole itself, as the end and as something complete. But the whole motion arising from these partial motions is unequal, even granting that the partial motions are equal. Therefore the motion of the planets is unequal per se.

[Margin: 5th argument.]

XII. *Fifthly.* If the motion of the stars were equal per se — that is, considered in relation to its own centre — it would have had to be ordered by God to that centre as to some good, something more excellent than the moving body itself. For if you say that equality belongs to them for their own sake and not for the sake of the centre, that is the very point in dispute, and it must not be assumed as granted. Indeed, the bodies themselves are of themselves indifferent to motion about this or any other point of the universe. Therefore, if in their motion they depend per se on a single point, the centre of their circle, they certainly acquire perfection from it, either as from an end or as from an efficient cause that exerts influence. But neither the centre of the eccentric nor the centre of the epicycle has any dignity in the universe, or any power of exerting influence on the stars — unless someone invents it gratuitously and rashly. They are unstable points in the universe, of no magnitude, not to say of no excellence. Inappropriately, then, is the quality [?] of the motion of the heavens judged by its relation to such points; and so equality per se, founded on them, has an unstable foundation of no weight.

[Margin: 6th argument, from the position of the spectators and from sense.]

XIII. *Sixthly.* If the apparent, or optical, inequality is per accidens because our eye is now outside the centre of the circle along which the planet moves, and no eye is at the centre of the eccentric or of the epicycle from which it could see the motion as it is per se and as it ought to be seen, then that perfection lacks a spectator, and only the imperfection — that is, what is per accidens — has had a spectator. This seems to detract from divine omnipotence. And if you say that God gave man not only the eyes of the body but also those of the mind, so that by reason he might infer from the inequality of the apparent motion the theory of a motion equal per se, against this stands the fact that, in this judgement, possession lies on the side of the senses. Unless the intellect is evidently or certainly compelled from some other source to correct the judgement it formed about these motions in reliance on the senses, it ought to pronounce for the senses, and to judge these motions to be such as, by the constant experience of observations through all ages, they are established to have been found — namely, unequal. Granted that, for ease of calculation and because of our weakness, we imagine mean and equal motions, and use perfect circles to set them out — that is, circular motions made equally about their own centres. But this equality, or rather this fiction of equality, is per accidens in this art; the inequality, therefore, is per se.

[Margin: 7th argument, from rectilinear motion.]

XIV. *Seventhly.* Some celestial motions take place along straight lines, or chords of circles, according to Copernicus, Maginus, Longomontanus and Lansbergen — certain librations, for example. And yet, to preserve equality, they take their law from a circle, and those motions are imagined to be made in a circle, although in fact they are not. Therefore, if one considers the path along which they are really accomplished, those motions are unequal per se. The same argument holds for motion along an elliptical circumference, according

to Kepler and Boulliau. For in reality, as these two astronomers hold, the planets are carried unequally along the circumference of an ellipse — granted that, as Boulliau adds, circles can be drawn through any two extremities of the ordinates in the ellipse, and some rule of equality taken from them; for the planet does not travel along those circles. Moreover, it is one thing to be moved equally per se, which we deny of the stars, and another to take from an extraneous circle laws — not of equality, but of some regular increment or decrement of motion — which could be granted: as if a moving body were carried along a straight line, but with the variation that the sines, or the secants or tangents, require, taken for an equal number of degrees in equal time. But from this it would rather follow that the inequality itself is intended per se, since to preserve it — though by some rule — a fictitious equal motion through a circle is taken from outside. In the same way, if an instrument were made for describing an ellipse or a parabola, which, by some part of itself being carried round in a circle, were fitted to describe an ellipse or a parabola, the ellipse and the parabola would certainly be intended per se, as the end, and the motion in a circle as the means. So too, even if the choirmaster, or chief musician, measures out the beats for the singers in equal time by equal raisings and lowerings of the hand, what he intends per se is that they, following the musical notes, should raise and relax their voices, lift or lower them, quicken or slow; and he looks per se to the inequality of the harmony, not to the equality of the beat.

[Margin: 8th argument, from the motion of the fixed stars.]

XV. *Eighthly.* The motion of the fixed stars in declination, and the variation of their right ascensions, is unequal, and does not vary equally in equal time, as is known from the problems of the primum mobile. Yet it is such in relation to its own centre, and follows necessarily from the equal motion in longitude; nor can it be said with any foundation that this motion is intended per accidens and the motion in longitude per se. Just as this motion, then, is unequal per se, so there can be others in the heaven.

[Margin: 9th argument, from reconciling the motion of the planets with the primum mobile.]

XVI. *Ninthly.* There is no better or simpler way of reconciling the proper motion of the planets with the motion of the primum mobile than by spirals and helices, as we showed in chapter 3. But these are not ordered to a single centre: they loosen and tighten like a screw, and rise and fall in various ways. Therefore that equality taken from perfect circles is not intended per se by Nature or by God.

[Margin: An objection refuted.]

XVII. Further, the opponents assert that it is an axiom long received in the schools of physics and astronomy that a simple body — and every celestial body is of that kind — must also have a simple, that is, an equal, motion. The reply would be that this is true of the motion of the primum mobile, or indeed of the motion of the secondary movables, but with equality taken as it is opposed to irregularity and randomness, that is, to disorder of motions. Otherwise it is

denied to be an axiom; or at least it is granted, but of a body that is simple not only in its being but also in its character as a cause, or in its effects — and in neither way are the heaven or the stars simple. Moreover, Aristotle's arguments are only topical, and do not contain the adequate cause of inequality in motion; for that inequality can arise from a source other than the end and terminus in which the moving body desires to rest, or than from the impotence of the mover, or from the resistance of the subject that is moved, as is evident.

CAPVT V.

Chapter 5.

Whether the Intelligences that move the heaven and the stars intend per se the description of some geometrical figure, or depend on it as on a rule; or whether, by the power of logistic alone, they accomplish the numbers of the motions; and whether men should attempt the same in setting out these motions.

[Margin: The opinion of Petrus Ramus on astronomical hypotheses.]

I. The occasion for this controversy was given us by Petrus Ramus [Pierre de La Ramée], whose opinion is best heard from Tycho, in his letter to Rothmann [Christoph Rothmann] of 1587 January 20, where he writes and judges as follows: *That the most celebrated philosopher of our age, Petrus Ramus, thought that astronomy could be established without hypotheses, by logical reasonings, has no foundation. He did indeed put this opinion to me sixteen years ago, when we were together at Augsburg, and at the same time he urged me that, once I had reduced the course of the stars to exact order by means of hypotheses, I should aim to attempt the same without them. That this could be done, he added, rested on this reason: that he had read that the Egyptians once had a very easy knowledge of astrology; and since the method of hypotheses seems difficult and intricate, they must have known the courses of the stars by some other, shorter and plainer way, and so without any hypotheses.*

[Margin: Tycho's verdict on P. Ramus.]

So Tycho writes on page 60, and he appends his judgement and verdict in these words: *But I resisted him, showing that without hypotheses the celestial phenomena cannot be reduced to certain knowledge, nor be excused [?] so that they may be understood; and that that Egyptian ease existed only in the planetary equatoria, by which they freed themselves from tedious computation when the convenient method of ephemerides was not yet in use. But since this man, though otherwise endowed with a penetrating intellect and a lover of truth if anyone ever was, seemed to me not to have thoroughly seen into the inner recesses of this art, and not to have noticed the variation in the motion of the stars, which by no means recurs at fixed times of the year, I neither could nor would obtain anything from him on this point. He still has a great many followers, who hope that the same can be done; but they neither understand the matter itself nor will ever bring it to effect. For since all things consist in numbers, weights and measure, without these nothing even in the invisible world can be explained. Hypotheses, moreover, show nothing other than the measure of the apparent motion through a circle and other figures, which arithmetic resolves into*

numbers. Anyone who wants to comprehend the motions of the stars without these must invoke Fortune, as the saying goes, and imagine a supramundane reason, beyond human understanding, wholly incorporeal and more than angelic.

[Margin: The opinion of Patrizi, and Kepler's verdict.]

So far Tycho against Petrus Ramus. And Kepler says much the same against Franciscus Patritius [Francesco Patrizi], in the *Commentaries on Mars*, chapter 1, where, having pointed to the ancients' observation of a double motion in the heaven, he adds: *This first sketch of astronomy — which consists in no explanation of causes but only in the very slow experience of the eyes, and which can neither be explained by diagrams or numbers nor drawn forth into future times, since it perpetually disagrees with itself, so that no spiral equals another in its duration of time, and none passes into its neighbour by a bend of the same size — this, I say, some people nevertheless try to restore today, trampling on two thousand years of labour, diligence, learning and knowledge, and thrusting admiration of themselves on the common people — an effort not in vain among the unskilled. The more skilled judge them, with full right, to be talking nonsense, or, if they wish to be heard as philosophers, like that Patrizi, to be mad with reason.*

Add to this that God is said (Wisdom 11) to have disposed all things in measure and in number and weight, and by Plato in the *Timaeus* to do geometry; and finally, that if it can be done that the phenomena of the celestial motions are represented by some figure having the beauty [pnlchritudinem] of a geometrical figure, this is better than to set out those motions without this form.

[Margin: The defence of Ramus and Patrizi.]

II. Yet reasons not to be despised present themselves in favour of the opinion of Ramus and Patrizi. *First*: of all the many geometrical figures that astronomers have so far tried to fit to the celestial motions — whatever they may boast, Lansbergen especially — none yet fully satisfies all the properly observed phenomena of a single planet without some discrepancies coming to light every year, both in eclipses and in the other vicissitudes of such motions; and not a few attribute this to the hypotheses being fabricated rather than to a defect in the observations. Now what has not been achieved through these hypotheses hardly seems achievable hereafter through others, since every combination of eccentrics, epicycles and concentrics, and every form of hypothesis through circles or ellipses, seems to have been tried, as is clear from what was said in Book 7, Sections 2 and 3. And the multiplication of small circles is liable to more errors the more differences of motion it has been compounded from.

[Margin: 2nd reason.]

Secondly: that necessity of a geometrical hypothesis seems to have arisen from our weakness or our habit rather than from the nature of such motions. For since we have been versed in the mathematical dust from our youth, and can hardly do without the apparatus of geometrical demonstrations — or can only with difficulty imagine the causes and rules of the celestial motions without it — we seem to need these in order to understand and to teach their theory. This is especially so because we do not have every one of the observations

that would show us one whole period, explored day by day part by part, and display its constant ratio. Therefore we are compelled, in a way, to hunt out the remaining equations of the mean motions — which we ourselves devised for easier use in calculation — from the necessity of some geometrical figure by whose laws the planets are bound. And this, evidently, is why we never attain the subtlety of astronomical truth: we hunt the planets' route along a path that we ourselves have devised. But if we had sufficient observations, taken on every single day, we could perhaps give an account of these motions without the laws of geometry, through purely logistic laws — that is, through very recondite ratios and proportions of numbers.

[Margin: 3rd reason.]

Thirdly: the motions of the stars are most probably carried out along spirals, as we said in chapter 3, and these are unequal per se, as we showed in chapter 4. But these two conditions do not seem to fit easily within the bounds of any geometrical figure; rather, they indicate that the planets' motions are freed from those bounds and bound by other laws, outside the jurisdiction of geometry.

[Margin: 4th reason.]

Fourthly: it befits the dignity of the Intelligences that move the heavens and the stars that in these motions they should follow the ideas of numbers, which abstract from sensible and imaginable matter, rather than those of continuous quantity, which does not abstract from it; and that they should not need, to direct those motions, the sight of circles or ellipses or similar figures. And if perhaps they trace any figures in the heaven by those motions, these are something posterior and a consequent effect, but not an exemplary cause or idea that they need. In the same way, no figures of the various motions are set before singers, or cithara-players, or those who dance in time, for them to imitate — figures which they do trace with tongue, hand or feet, like a passing track in the air.

[Margin: 5th reason.]

Fifthly: the proportions of the motions by which various pendulums are swung to and fro, together with their intervals, follow the ratio of evenly odd [?] numbers, so that the heights of any two pendulums are to each other as the squares of the times; and the increments of speed in the natural motion of heavy bodies keep a similar proportion, as we showed in Book 2, chapters 20 and 21; and in these the law of no figure is attended to. In the same way, it seems probable that there are similar, or more sublime, ratios in the minds of the Intelligences, by which they infallibly lead the stars round and present them in their places at the appointed time, without the aid of any figure.

[Margin: Conclusion.]

III. Let us conclude, then, that geometrical figures are indeed to be used, as long as nothing better or more certain presents itself; but that they are not to be passed off as the true causes of the anomaly of the celestial motions, nor that one should pass from the name of a hypothesis to the real neces- sity of such a figure. Meanwhile, however, let us conclude that what was said under

number 2 is probable: namely, that the Intelligences use purely logistic ratios for the harmony of these motions, without the sight of a figure directing them, or of one set before them, as their primary intention, to imitate. Yet it is not improbable that they follow those logistic ratios in the archetype of their mind in such a way that they also wish at the same time to represent the beauty of some geometrical figure, joined with the niceties of arithmetic — but of a figure to which that spiral motion, made as it were like a screw, can be fitted. However that may be, whatever geometry and harmony may be imagined in this must both agree with the phenomena and be taken rather from the end that divine Providence set before itself in motions of this kind. Therefore, just as it would be the mark of an unskilled man to hold that God practised geometry in the human body — geometry taken properly and not metaphorically — and bound the motion of the heart, of the arteries and of the animal spirits to some regular figure, or assigned to the heart the figure of a perfect pyramid, to the brain a spherical one, to the liver a cubic one and to the kidneys a cylindrical one; so, if figures of this kind conflicted with the end of the various effects to which these motions are ordered, they would certainly have to be banished from the heaven to the mathematical dust. But it is not yet established that they evidently conflict with this end.

CAPVT VI.

Chapter 6.

Whether the proportions of the celestial motions are knowable by us in this life, and expressible; and whether they are all rational, or rather some irrational; where also of the revolutions of all of them into the same.

I. What Plato thought on this matter will become clear from his *Epinomis*. There he calls those who were occupied with mere observations of risings and settings, and with the prognostics to be drawn from them, [Greek: τοὺς ἀστρονομοῦντας (tous astronomountas, "those who practise star-observing")]; but he calls [Greek: ἀστρονόμους (astronomous, "astronomers")] those who search into the celestial motions themselves.

[Margin: Plato's opinion.]

And he says: *You do not know astronomy. For he who is truly an astronomer must be most wise — I do not mean the one who practises astronomy after the manner of Hesiod and all those like him, as one who contemplates the risings and settings of the stars, but the one who studies the seven periods of the eight periods, each of which runs through the same circle in such a way that all nature is scarcely, scarcely equal to contemplating them, unless it shares in that wonderful nature.* I believe, however, that by "the seven periods of the eight periods" — for so the Greek text has it, [Greek: τῶν ὀκτὼ περιόδων τὰς ἑπτὰ περιόδους (tōn oktō periodōn tas epta periodous, "of the eight periods, the seven periods"; the breathing on ἑπτὰ is printed smooth)] — he means not the simple revolutions of the fixed stars and of the planets separately, but the revolutions of all of these together to the same

point, which they usually call the Platonic year. This accords with that passage of Cicero in the *Dream of Scipio*: *But when all the stars have returned to the point from which they once set out, and have brought back the same configuration of the whole heaven over long intervals, then that can truly be called the turning year; and I hardly dare say how many generations of men are contained in it.* Of this year, then, both the proportions and the commensurations of the celestial periods no one can perfectly attain, in Plato's opinion, unless he has been allotted a heavenly and admirable intellect. Aristotle, however (*De partibus animalium* 1, chapter 5), notwithstanding this difficulty, which he admits is met with, says that even a modest knowledge of these things is more delightful than the knowledge of certain others.

[Margin: Delfino's opinion.]

Indeed, Io. Antonius Delphinus, in his book *De caelestibus globis et motibus*, chapter 23, contends that no one, while he remains shut in the prison-house of this mortality, can naturally have indubitable knowledge of the proportions and measures that are precisely inherent in the celestial motions. For those proportions consist in an indivisible, and even the least difference alters them; and because of the distance of the stars, differences of this kind remain insensible within the life of one man, but afterwards, with the passing of the ages, become sensible.

[Margin: Clavius's opinion.]

Our own Clavius [Christoph Clavius], too, toward the end of his *Commentary on the Sphere of Sacrobosco*, speaking of Copernicus's attempt to correct the celestial motions, concludes thus (p. 452): *It is very difficult to define the periods of the motions so that they do not deviate from the truth over many centuries of years, since no mortal has ever been able to determine the period of a single planet so exactly that some small fractions are not left over or lacking, which over a great interval of years introduce a notable error. So that it is truly wonderful that God, Best and Greatest, willed to obstruct the motions of the planets with such difficulties that no man can attain them perfectly, but always finds something to admire in so great an artifice of such noble bodies, and in so great a harmony and concord of their motions — celebrating their founder and mover with perpetual praises. So that it seems to have been written by Ecclesiastes, chapter 3, chiefly on account of the constitution of the heavens and their motions — in which there always seems to remain something to be investigated with the greatest diligence by the most skilful searchers of celestial things: And he delivered the world to their disputation. That is, lest, if men should at some time perfectly understand the number, order, constitution and motion of the heavens, they should cease to investigate and admire the works of God, and their minds, the reason for exercising them taken away, should grow torpid through idleness.*

[Margin: Job 38.]

II. But we learned long ago, from the divine writings and the sacred writers, how arduous this astronomical business is. For what else does that thunder of the divine voice mean, speaking to Job out of the whirlwind: *Do you know the order of the heaven, and will you set down its plan on earth?* And again in the

same chapter 38: *Who will declare the plan of the heavens, and who will make the harmony of heaven sleep?* That is, if I am not mistaken: who will describe and explain the motions of the heavens so perfectly that men, now unconcerned about their course, would cease from observing the motions of the stars in future, and so sleep? For just as the heavens are said to praise the Lord because they rouse men to the praises of the Lord and are an object that moves them to praise him, so they would also be said to sleep if they were so well known that they now invited men to sleep.

[Margin: Solomon's knowledge of these things, infused.]

And although Solomon proclaimed that the wisdom of these things had been infused into him by God, saying (Wisdom 7): *For he himself gave me true knowledge of the things that are, that I might know the disposition of the world and the virtues of the elements, the beginning and the end and the middle of times, the alterations of the vicissitudes and the changes of the times, the courses of the year and the dispositions of the stars*; yet the same writer (Wisdom 9), considering the purely natural powers of human ingenuity, declares: *And hardly do we judge the things that are on earth, and the things that are before us we find with labour; but the things that are in heaven, who will search out?* And indeed the Fathers press the judicial astrologers from this: that it is impossible to know the course and aspect of the stars exactly — above all Saints Basil (homily 6 on the *Hexaemeron*), Ambrose (*Hexaemeron* 4, chapter 4), Augustine (*City of God* 5, chapter 3) and Gregory (homily 10 on the Gospels). Add to these Origen, in Eusebius, *Praeparatio Evangelica* 6, chapter 9;

[Margin: Philo's opinion.]

and Philo, in his book *On Dreams*, who, after recounting various opinions about the stars, concludes: *What is reported of them and of the heavens is uncertain and incomprehensible, and rests on probable conjectures rather than on certain and true reasons, so that one might lawfully swear that no mortal can ever rightly perceive any of these things.*

[Margin: Martinengo's opinion.]

III. Whether the periods of the celestial motions are commensurable with one another and consist of rational proportions, or rather of irrational ones, astronomy has not yet advanced far enough for this controversy to be settled with certainty, and it will be enough to have indicated the opinions. For, as Ascanius Martinengus says in the *Glossa magna* on Genesis, p. 1045: *Certain very learned mathematicians have judged, on the best reasons, that the revolutions of the heaven are incommensurable with one another, and that for this reason the same face of the heaven and the same position of the stars can never return; and they strive to confirm this with no weak reasons. Others have held that the configurations of the stars return to the same only after thirty-six thousand years, or, according to others, forty-nine thousand years, in which the revolution of the supreme and slowest sphere will be completed.*

[Margin: Io. Antonius Delphinus.]

So too Io. Antonius Delphinus, in his book *De globis ac motibus caelestibus*, chapters 24 and 25, defends both sides as a problem, and adds that it is not sufficiently proved from this that these motions are asymmetrical — from the fact, that is, that astronomers have so far disagreed among themselves about the measure of the solar year and of lunations. This, he says, happened because of the shortness of life and because the restitutions [apocatastases] of these periods went unobserved, or were badly observed; and for that reason the astronomical tables must be restored every two hundredth year, so that the fault of the old observations may be corrected. And certainly, if the commensurability of any motions at all ought to be established, it would surely be that of the Sun and the Moon with each other, because the observations of them are frequent and more evident. And yet, as we showed in Book 4, chapter 19, no lunisolar cycles have so far been found at whose completion the luminaries are restored to the same point in which they were once conjoined together. That very period of 3400 Julian years which Franciscus Vieta [François Viète], in his *Calendar*, proposition 31, put forward as the most suitable of all for this purpose, is not such a period, if we believe Kepler in the *Rudolphine Tables*, precept 135: for it falls short of an exact restitution by 1′ 19″. Nor can it be such a period, says Clavius in his *Calendar*, chapter 24, unless the synodic month is $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 10‴\ 43⁗$ — and Vieta has not yet proved that it is so much.

[Margin: Boulliau's opinion.]

Indeed Boulliau, in Book 2 of the *Astronomia Philolaica*, chapter 3, after saying, *But that measure is false, for the motion of the Sun does not depend on the lunar one. Nor, certainly, did the ancients succeed any better in making the annual motion of the Sun commensurable with the lunar synods*, says a little later: *For who has so far asserted and proved that the annual motion of the Sun is commensurable with the lunar cycles?* To this Amicus (*De caelo*, treatise 5, question 6, doubt 10) teaches that these proportions can remain unknown, but that even if they are irrational, this is no imperfection in the heaven, because they are commensurate with the end appointed by God.

[Margin: Kepler's opinion.]

Kepler, however, contemplated these matters with a deeper effort of mind. In the *Epitome of Copernican Astronomy*, Book 4, p. 512, he drew this distinction: *The tempering of the extreme motions, the slowest and the fastest, in any one planet is most exquisitely harmonic, and is the work of the supreme and adorable Creating Mind. But the lengths of the periodic times, if they were the work of a mind, would have something of beauty, of the kind that effable proportions have — double, triple and the like. As it is, the proportions of the periodic times are ineffable (commonly called irrational), and so share in infinity, in which there is no mental beauty, because there is no limit.* He confirms, moreover, that these times cannot be the work of a mind, in these words: *Secondly, these times cannot be the work of a mind (I do not speak of the Creator, but of the nature of the mover), because the times of one period are gathered from unequal stays in different parts of the circle; and those unequal stays, as will be said below, arise from material necessity, and as it were*

from the principle of the steelyard. So he writes, wholly intent on showing that the planets are moved not by Intelligences but by the Sun, according to the principles of the balance and the lever and by magnetic tractions, and the Sun in turn by a certain material soul. In the *Mysterium Cosmographicum*, chapter 23, however, investigating the end of the astronomical world and the Platonic year, he concludes in these very words: *I shall prove from a single postulate that no end can be set to motion with reason, and that there will be no Platonic year. For let it be granted that the eccentricity is in a rational proportion to the orb: then the radii of the orbs will be irrational to one another, because they stand to one another as the radii inscribed in and circumscribed about the bodies, and these are irrational, since they follow from the ratio of the subtense in the square and of the section in extreme and mean ratio, which are the two examples of irrationals in geometry. But the motions are in proportion to the radii; therefore the motions are irrational to one another, and so they will never return to the same beginning, even if they lasted for infinite ages: because never, not even in an infinite division of time, would there occur a common measure which, repeated often enough, could be set up as the one terminus of all the motions and the goal of the Platonic year.*

[Margin: The Platonic year impossible, for Kepler.]

This established, he exclaims with Pliny the Elder (Book 2, chapter 1): *The world is sacred, immeasurable, whole in the whole, nay rather itself the whole, finite and like the infinite*; and with Copernicus (Book 1, chapter 10): *So great, truly, is this divine work of the Best and Greatest.*

[Margin: Criticism of Kepler's opinion.]

IV. But Kepler's argument already mentioned, drawn from the comparison of the orbs and of the eccentricities, does hold in a hypothesis that employs the figure of an eccentric orb, or in one equivalent to it as far as geometry is concerned; but not in every hypothesis that is equivalent only as far as the numbers are concerned — for example, in a spiral motion made along helices, or if Angels move the heaven and the planets using purely logistic rules, without regard to circles, ellipses or similar figures. Nor is it evident that proportions which are ineffable to us, or at any rate are so because they tend to infinity, are devoid of all beauty, since the divine Immensity and Eternity possess every perfection of the infinite and yet do not lack their own beauty. Just so, it is not evident either that it is not more beautiful for the celestial motions to have proportions of both kinds. Finally, I do not approve of his making God the author of the effable proportions while denying that He is the author of the ineffable ones.

CAPVT VII.

Chapter 7.

Of sound and concord, or of the harmony of the celestial motions.

I. Since this harmony arises not only from the celestial motions but also from the intervals of the distances and from other proportions, and does not behave

in the same way in every system, it has seemed best to defer this controversy to the last section of this book, in which the harmonic and geometric system will be treated, so that with that last sound, as it were, the ears of the learned reader may be refreshed.

SECTIO TERTIA DE SYSTEMATE MVNDI CIRCA TERRAM IMMOBILEM.

Section 3. On the system of the world about an immobile Earth.

Authores Præcipui qui de Syſtematibus Mundi tractant.

The principal authors who treat of the systems of the world.

Ptolemy, *Almagest* Book 9, chapter 9, and there Theon and Regiomontanus; Macrobius, Book 1 on the *Dream of Scipio*, chapters 19 and 21; Copernicus, Book 1, chapter 10; Fracastorius [Girolamo Fracastoro] in the *Homocentrica*, sections 1 and 3, chapter 25; Io. Baptista Amicus [Giovanni Battista Amico] in his short work *De motibus caelestium corporum*; Io. Antonius Delphinus [Giovanni Antonio Delfino] in his book *De globis ac motibus caelestibus*; Valentinus Naiboda [Valentin Naibod], Book 1 of the *Astronomicae institutiones*, chapters 10 and 16; Clavius, Book 1 on the *Sphere*, from p. 64; Franciscus Barocius [Francesco Barozzi], *Cosmographia* Book 1, chapter 2; Ascanius Martinengus, *Glossa magna* on Genesis, p. 968; Galileo, in the *Dialogues on the Two Systems of the World*; Longomontanus, *Astronomia Danica*, Book 1 of the *Theorica*, chapter 1; Sheinerus [Christoph Scheiner] in the *Disquisitiones mathematicae*, from number 13; Andreas Argolus [Andrea Argoli] in the *Pandosion sphaericum*, chapter 3; Petrus Gassendus [Gassendi] in the *Institutio astronomica*, Book 2, chapter 14, and all of Book 3; Io. Hevelius [Johannes Hevelius], *Selenographia*, chapter 7; Baranzanus [Redento Baranzano] in the *Uranoscopia*, pp. 103 and 115. But first we must deal with the number of the heavens, as with the matter of the system.

CAPVT I.

Chapter 1.

Of the number of the total heavens.

[Margin: A single heaven for Aristotle?]

I. The first question usually asked is whether there is a single heaven, or whether several must be asserted. The occasion for this question was given both by Moses, who names a single heaven in the beginning — on which see St Thomas [Thomas Aquinas], part 1, question 68, article 4, and ourselves below, numbers 2 and 3 — and by Aristotle, *De caelo* Book 1, chapter 6, or text 76, where he says: *But let us say why it is not possible either that there be several heavens.* In the same place, however, he understands the world by the name of heaven, the denomination being taken from the principal and by far the greatest part of the world; for he concludes in text 80: *Since this is absurd, it is impossible that there be more worlds than one.* He tries to show this, first, from the name of

Universe, which is commonly attributed to the world: if there were several worlds, the name of Universe would be attributed to one of them wrongly or equivocally. Second, because if every world were of the same species, the earth of one would try to tend toward the earth of another by its weight and its own nature, and fire toward fire; or else the individual elements would be held within the individual worlds by perpetual violence. But the elements of any world must be of the same species — fire, for example, compared with fire — because simple motion, which is due to simple bodies, is either circular, and this belongs to the heaven, or straight; and straight motion is either upward simply, or upward in a certain respect, or downward simply, or downward in a certain respect; and the element that tends simply upward we call fire, and that which tends downward, earth, and so on. Therefore, since no other division of simple motion according to species is possible, it seems impossible too that there should be other elements of a different species. From this discourse it appears that he is speaking of the unity of the world composed of the heavens and the elements. Earlier, however, in chapter 4, text 22, he had distinguished the heaven from the elements, saying: *And so, as though there were some other first body besides earth and fire and air and water, they called the highest place aether, giving it its name from its always running through everlasting time. Anaxagoras, however, misuses this name, and not rightly; for he names aether in place of fire.* Hence it came about that he entitled four books *De caelo*, although in Books 3 and 4 he treats only of the elements. But in the heaven taken strictly, as the place of the stars, or that simple body to which circular motion belongs, Aristotle acknowledges several spheres (*Physics* 8 and *Metaphysics* 12), as we shall show below when explaining the Aristotelian system. Now we must treat of the opinions of others.

[Margin: 1. Opinion: of a single heaven.]

II. The first opinion posited a single heaven, but not in a single sense. For St John Chrysostom held as follows. Because Genesis 1 says, *In the beginning God created the heaven and the earth*; because he knew besides that it is called *Samaim* or *Hasamaim* in the plural number for this reason, that among the Hebrews "heaven" lacks a noun of the singular number, as among the Latins do *Athenae* and *Venetiae*; because he thought besides that this heaven named on the first day was not distinct from the heaven, or firmament, made on the second day; and because he finally held that the whole heaven is immobile, and that the stars are moved not by the motion of the heaven but by themselves within the heaven — for these reasons, I say, he thought that there is a single heaven, taking heaven, however, more strictly, and not including the aerial heaven.

[Margin: St John Chrysostom.]

Accordingly, in homily 4 on Genesis he concludes: *Who, then, after so much teaching, will bear those who dare to speak out of their own brain and, against divine Scripture, to say that there are many heavens?* He has similar passages in homily 12 to the people of Antioch, and in homily 25 on the Epistle to the Hebrews.

[Margin: Tertullian.]

Tertullian was of the same opinion, in his book *Against Hermogenes*.

[Margin: St Bonaventure.]

Speaking, however, of the starry heaven, St Bonaventure (on Book 2, distinction 14, article 1, question 1) acknowledges it as single — and indeed with a unity of continuity — although he admits that several tracts can be marked out in it, according to the plurality of the motions. For he says: *According to the common opinion, whether of the natural philosophers or of the mathematicians, the luminaries are placed in different orbs. But this distinction or diversity of the orbs, according to those who understand better, does not come from a distinction of forms, as air and water are distinguished; nor does it come from a discontinuity of surface, as stone is distinguished from stone; but it comes from a diversity of motions. And a diversity of motions does not take away continuity in what is subtle and fit for motion, as appears manifestly in water, and likewise in air, etc.* Therefore the conclusion is this: *According to this position, then, it must be said that the luminaries of the heaven are placed in several orbs, yet in one continuous body, which Scripture calls by the name of firmament.*

[Margin: Tellez. Tychonics. Copernicans.]

And indeed, whoever hold the starry heaven to be fluid throughout have no reason to divide it into several heavens, but must admit that it is single, as Tycho teaches in the *Epistolae Astronomicae* (p. 149). Oviedus also notes this (the single controversy *De caelo*, point 4, conclusion 5), asserting: *If the heavens are liquid, only one heaven of the wandering and the unwandering stars is to be admitted.* For this reason Tellez (*Philosophy*, disputation 44, section 3, number 5) affirms a single star-bearing heaven. And such are Tycho, Longomontanus, Kepler, Boulliau and, probably, Copernicus, as we shall say in number 14.

[Margin: A single heaven, even though solid.]

But even if the whole starry heaven were held to be solid, Aegidius and Hurtadus nevertheless assert that it is single.

[Margin: Aegidius.]

The former, in part 2 of the *Hexaemeron*, chapter 32, since Scripture says that the stars were placed not in the firmaments but in the firmament, says: *If, then, the luminaries and the stars are in the firmament of heaven, and the planets are counted among the luminaries of heaven, it seems that all the planets are in the firmament of heaven; which could not be, unless the spheres of the planets made one sphere with the firmament, or with the starry heaven.* And a little later: *Yet, if eccentrics and epicycles are posited, nothing prevents all the spheres already mentioned from being one and the same sphere.* And finally he concludes that the deferents of the planets are seven, and that in respect of these the place of the fixed stars can be called the eighth sphere; but that all of them are enclosed in a single heaven — nay, that they are a single sphere — and that it is vain to do through more what can be done through one.

[Margin: Hurtadus.]

But Hurtadus (*De caelo*, disputation 2, section 1) defends a single heaven, notwithstanding its solidity, in such a way that he admits in it, with some older authors, vari- ous channels through which the planets are carried up and down,

to the right and to the left. He does not admit that these are empty, because a vacuum, he says, is a place lacking body but apt to hold a body, whereas those channels are not apt to be filled with body. Against these channels Pererius rises up at length (Book 2 on Genesis, question 9), as do Oviedus (above) and Arriaga (the single disputation *De caelo*, section 4, number 49). And certainly, if one considers the innumerable windings and turnings of the planets, especially of Mars, Mercury and Venus, their whole heaven would have to be hollowed out; nor is it apparent how light could be transmitted to us from the whole hemisphere of the planet's body through those cavities, if light is an accident needing a subject; and so we would see a single point of them.

[Margin: Cisalpinus, Aversa, Tannerus.]

In another way, a single heaven was asserted by Andreas Cisalpinus [Andrea Cesalpino] (*Peripateticae quaestiones*, Book 3, question 4) and Raphael Aversa (*Physics*, question 32, sections 6 and 7), and Tannerus did not think it improbable (*Theology*, volume 1, disputation 6, question 3, doubt 3, number 78): namely, that the starry heaven is a single sphere, in which, however, there are different circles interlocked with one another like rings, or zones within the zodiac carrying the planets forward and back, up and down, without fluidity or vacuity. Finally, all who accept three heavens taken broadly — namely the Empyrean, the starry and the aerial, of which we shall speak in number 5 — would seem to favour the unity of the starry heaven, for by that very fact they would seem to make the starry heaven single; and Tellez too uses this argument. But, as we shall see partly in number 5 and partly in number 7, many of them subdivide the starry heaven into several really distinct heavens.

[Margin: 2. Opinion, for a plurality of heavens.]

III. The second opinion affirms absolutely that there are several heavens. Against these — some mathematicians or other — St Basil, in homily 3 on the *Hexaemeron*: *Since those*, he says, *who are accustomed to use demonstrations are of much greater weight, and have confirmed by the force of geometrical proofs that conclude necessarily that nature cannot bear to establish another heaven besides this one, then indeed we shall laugh all the more freely at the linear trifles of those mathematicians, contrived with whatever art and ingenuity, etc.* But perhaps they were using the Aristotelian arguments put forward in number 1 to show that there are not several worlds. Certainly divine Scripture indicates that there are several heavens, when it says in Book 1 of Paralipomenon [Chronicles], chapter 2: *If heaven and the heavens of heavens do not contain you*; and in Psalm 113: *The heaven of heaven is the Lord's*; and in Psalm 148: *Praise him, you heavens of heavens*; and in Ecclesiasticus 16: *Behold the heaven and the heavens of heavens.* Moreover, sacred Genesis mentions two heavens, according to the more common opinion of the Fathers (on which see section 1, chapters 1 and 2): namely the Empyrean, or even the starry heaven, created on the first day, and the firmament made on the second day; and St Paul, in 2 Corinthians 12, mentions three heavens.

[Margin: St Athanasius.]

Pointing to these arguments, St Athanasius said in the *Questions to Antiochus*, question 5: *How many are the heavens? I answer: this question is in doubt among many. For the book of Genesis mentions two heavens; the divine Paul, however, says that he himself saw even a third heaven, and David the prophet speaks of a fourth heaven: Praise him, you heavens of heavens. But it must be understood that, just as human nature is called one and man is called one, and yet there are many men, so too the nature of the heavens is one, and in divine Scripture both one heaven and many heavens are spoken of.* Nor, indeed, do these writers differ in substance from those who posit one heaven, but only in name, as St Thomas teaches (part 1, question 68, article 4): for St Chrysostom takes by the name of heaven the whole space that lies above the earth and the water, while the others distinguish several heavens within it.

[Margin: The error of Basilides concerning 365 heavens.]

It is not permitted, however, to multiply heavens at will; and so the error of Basilides, who posited as many heavens as there are days in the year, namely 365, is numbered among the heresies by St Irenaeus (*Against Heresies*, Book 2, chapters 21 and 22), by Tertullian (in the book *On the Prescription of Heretics*), by St Epiphanius (in the *Panarion*, Book 1, heresies 24 and 26) and by St Augustine (in the book *On Heresies*, heresy 4) — granted that this heresy consisted not so much in the size of the number as in the production; for, as St Irenaeus relates, he imagined the second heaven procreated from the first, the third from the second, and likewise the others by successive birth. So our Tellez wittily said (disputation 44 of his *Philosophy*, section 3, number 4): *Although he admits so many heavens, into none of them, as I rather think, will he be admitted; for he was a heresiarch, and therefore deserved to be banished not into heaven but into infernal Tartarus.* Leaving this aside, then, let us see how many heavens others have counted.

IV. The third opinion counts only two heavens, but not in the same sense.

[Margin: 3. Opinion [?], of 2 heavens [?].]

For St Clement (Books 1 and 2 of the *Recognitions*), Acacius (in Lippomanus, in the *Catena*) and Theodoret (on chapter 9 of the Epistle to the Hebrews, and question 11 on Genesis) affirm one heaven, the Empyrean, or the one created on the first day, and another made on the second day.

[Margin: St Clement [?]. Acacius. Theodoret [?].]

Let us hear Theodoret: *Since divine Scripture teaches that in the beginning God created the heaven and the earth, and then says that after the creation of light the firmament was made on the second day, a question of this kind (about the number of the heavens) seems full of ignorance. For one ought to have known the diversity of the heavens from the reckoning of time and from the very manner of creation: the one before light, the other after light; and the former made not from other matter, the latter from the waters, etc. Whoever, then, does not believe that there is a second heaven strays from the straight path; but whoever tries to count more clings to fables, setting aside the teaching of the divine Spirit.*

[Margin: Claudianus Mamertus [?]. Suidas.]

In the same sense Claudianus Mamertus (*De statu animae*, Book 2, chapter 13) said that there are two heavens. Likewise Suidas, in his historical work: *There are two heavens: one procreated together with the earth, the other which was commanded to stand in the midst of the waters, which they also called the firmament.*

[Margin: Procopius [?].]

And Procopius, speaking of the heaven made on the second day, says on Genesis chapter 1: *Some deny that the generation of a second heaven is depicted here. For they think the present sequence of words is a rather brief repetition of the heaven spoken of above. These and such things they dare to babble, although Scripture brings forward another name and another use for this second heaven.*

[Margin: St Bruno [?].]

The same understanding of these two heavens is found in St Bruno, in the book *De novis*, chapter 2.

[Margin: Anastasius [?].]

But Anastasius Synaita puts it most elegantly, joining the mystical sense with the literal (*Commentary on the Hexaemeron*, Book 2): *On the first day was made the veil of the upper heaven, which draws near to the Holy of Holies, where Christ entered as forerunner for us. For that reason, moreover, on the second day is made the second and outer veil, which is called the firmament in the midst of the water, and which is indeed the firmament of faith in Christ.* Then, after a few words: *We have learned from sacred Scripture, then, of the making of only two heavens, as a type and figure of the two veils of the Temple, which signify the two natures of Jesus, and the two peoples, and the two Churches, etc.*

[Margin: Mastrius and Bellutus [?].]

Lastly, Mastrius and Bellutus (*De caelo*, disputation 2, question 1, article 2, number 30) admit a primum mobile really distinct from the starry heaven, but in such a way that they say the starry heaven is single and continuous, and they account for the various motions of the planets not through whole heavens or spheres but through circles or rings and zones contained within the zodiac.

[Margin: St Gregory of Nyssa [?].]

In another way, St Gregory of Nyssa, in his *Hexaemeron*, especially toward the end, teaches that a single heaven was made on the second day, once the fiery element had been separated from the others, and that this heaven was divided into two regions, one for the unwandering stars, the other for the wandering.

[Margin: St Justin [?].]

And finally, in yet another sense, St Justin (*Questions of the Orthodox*, question 57) says that there are indeed two heavens, but that these are divided into various spaces, which Scripture calls heavens; he is speaking, however, of the aetherial heaven and the aerial. For he asks: *If Moses had set forth to us the creation of a twofold heaven, how does Scripture teach that there are more, saying at one time "the heavens of heavens", at another "the heavens were opened to him"?* Then he answers: *Moses indeed spoke of heavens, but in number he handed down neither one, nor two, nor more. Divine Scripture, moreover, is accustomed to call the upper intervals of the parts heavens — as the birds of the heaven, and the eagles in the heaven, and the*

stars of the heaven. From these sayings, then, it follows that we are to understand the heavens as two in substance, but more by their intervals. And if we take heavens in this way, there will be no contradiction in the words. Two heavens, then, are held by Theodoret, Claudianus, Suidas, Procopius, Bruno and Anastasius — the Empyrean and the starry; by Nyssenus, the heaven of the fixed stars and that of the planets; but by St Justin, the aetherial and the aerial; and by Mastrius and Bellutus, the primum mobile and the starry heaven. But if you bring in the aerial heaven, St Nyssenus will acknowledge a third heaven, as we shall show in the next number.

[Margin: 4. Opinion [?], of 3 heavens [?].]

V. The fourth opinion, and the most celebrated among the sacred writers, was and still is that of three heavens. It arose from that saying of St Paul in 2 Corinthians chapter 12: *I know a man in Christ fourteen years ago — whether in the body or out of the body I do not know, God knows — caught up thus to the third heaven*; words which St Thomas expounds most learnedly (II-II, question 175, articles 3 and 4, etc.). Although, as Oviedus notes at the end of the controversy *De caelo*, some have believed that "three heavens" were put for many or for all, in the way in which, in Amos 1, it is said, "for three crimes of Damascus", that is, for very many; and in 2 Corinthians 11, "Three times I besought the Lord", that is, several times.

[Margin: Three metaphorical heavens [?] — three visions.]

Nevertheless the common opinion is that the three heavens enumerated are really three in their whole extent, speaking properly. For metaphorically, as St Augustine interprets (volume 3, Book 12, chapters 29 and 34), they can be understood as three kinds of supernatural vision, namely corporeal, imaginary and intellectual; and so St Paul, passing beyond corporeal and imaginary vision, was raised to intellectual, and indeed intuitive, vision of God — which some call the heaven of the Trinity. It was to this heaven, that is, to the beatific vision, as though it were owed to his natural excellence, that Lucifer strove not to be raised but to raise himself, when he said, "I will ascend into heaven", as Martinengus thinks (*Glossa magna*, page 623), and before him St Thomas (part 1, question 68, article 4). But literally these three heavens are either the aerial, the starry and the Empyrean; or the first, made on the first day, the second, made on the second day, and the third, to which St Paul was caught up. But let us recite the words of the Fathers.

[Margin: St Basil.]

St Basil, in homily 3 on the *Hexaemeron*: *In the second place there offers itself for inquiry whether this firmament is different from the heaven that was made in the beginning, since it too has been allotted the name of heaven; and accordingly whether two heavens are to be posited at all.* And at length he replies: *We, on the contrary, are so far from doubting at all about two that we would even inquire into a third, of whose contemplation that admirable Paul was held worthy. Moreover, when the Psalm expressly mentions the heavens of heavens, it has surely implanted in us a clear understanding not of one heaven only, but of several.*

[Margin: St Ambrose.]

St Ambrose, as usual, agrees with him (Book 2 of the *Hexaemeron*, chapter 3), where he finally lays it down thus: *Therefore we cannot deny that there is not only a second but also a third heaven, since the Apostle confirms by the testimony of his writings that he was caught up to the third heaven. David, too, set the heavens of heavens in that choir of those who praise the Lord.*

[Margin: Cassiodorus.]

With them Cassiodorus also agrees, on Psalm 148.

[Margin: St Damascene.]

But what these three heavens are, St Damascene taught more openly (Book 2 of the *Orthodox Faith*, chapter 6). For when he had said: *Since, then, Scripture speaks of heaven, and of the heaven of heaven, and of the heavens of heavens, and affirms that blessed Paul was caught up as far as the third heaven, we say that in the creation of the whole world the making of heaven is understood by us to be that sphere which certain of the wise (who are not of ours) call starless, making their own those doctrines which are Moses'; and then that God also called the firmament heaven* — he added: *The heaven of heaven, then, is the first heaven, set above the firmament. Behold two heavens, for God also called the firmament heaven. It is also the custom of divine Scripture to call the air heaven, from its being seen above. For "Bless," it says, "all you birds of heaven", meaning the air; for the air is the path of flying creatures, and not heaven. Behold the three heavens of which the divine Apostle spoke.*

[Margin: St Gregory of Nyssa.]

St Gregory of Nyssa, however, distinguished these three heavens under another notion, in *On the history of the six days*, in these words: *I think, then, that the outermost part of the sensible world is called the third heaven by St Paul, who divides whatever appears into three parts, etc. One heaven Scripture names the boundary of the denser air, as far as the winds and clouds reach and the nature of high-flying birds is borne, etc.; then it names another both heaven and firmament — that which is seen within, beneath the sphere of the unwandering stars, in which the wandering stars move. And the very extremity of the sensible world, which is the border of his creation that is perceived by thought, it names firmament and heaven.* For Nyssenus, therefore, the first heaven is the aerial, the second the planetary, the third the firmament of the unwandering stars.

[Margin: St Thomas.]

[Margin: Caietanus.]

Differently again, St Thomas (part 1, question 68, article 4) says that these three heavens are the Empyrean, which is wholly luminous; the aqueous or crystalline, which is totally transparent; and the starry, partly luminous and partly transparent. Differently once more, Caietanus (on Genesis, and on 2 Corinthians chapter 12) took these three heavens: in the third place he substituted the aqueous heaven for the Empyrean, saying: *In sacred Scripture mention is made of three heavens. The lowest is the aerial heaven, according to the words "the birds of heaven"; the middle is the starred heaven, of which it is said at the beginning of Genesis, "and he set them in the firmament of heaven"; the third and*

highest is the waters, all those that are above the heavens, which Scripture mentions many times, and which we call the aqueous heaven, while the philosophers call it the primum mobile. For the Empyrean heaven, handed down by later writers, is found nowhere in Scripture. But how falsely he excludes the Empyrean we have said sufficiently in section 1, chapter 1, from number 24.

Most of the more recent writers, however, subscribed to Damascene's understanding: first of all Suarez (*De opere sex dierum*, Books 1 and 2), Oviedus (the single controversy *De caelo*, point 4, number 12), Tannerus (in the dissertation *De caelis*, question 10), Christophorus Borrus [Cristoforo Borri] (folio 260 of his *New Astronomy*) and Genebrardus [Gilbert Génébrard] as cited by him, and Hurtadus (disputation 2 *De caelo*, section 1): namely, that heaven is divided into the Empyrean, the starry and the aerial. Not all of them, however, deny the subdivision of the starry heaven into several heavens; on the contrary, Saints Basil, Ambrose and Damascene subdivide it into eight heavens, as we shall see below. That the air too is designated by the name of heaven among heathen and profane writers, I have already shown (section 1, chapter 1, number 14).

[Margin: 5th opinion, of 4 heavens.]

VI. The fifth opinion, of four heavens, is hinted at by St Athanasius (question 5 to Antiochus), whose words I have already copied out in number 3. He seems to understand the Empyrean, the firmament of the unwandering stars, the heaven of the planets and the aerial heaven; but he does not express his meaning clearly enough.

[Margin: 6th opinion, of five heavens.]

VII. The sixth opinion is of 5 heavens: namely that of Oviedus (the single controversy *De caelo*, point 4, number 10), where he says, not absolutely but on the supposition that the heaven of the stars is solid, that it must be divided into five heavens, namely those of the fixed stars, of Saturn, of Jupiter, of the Sun and of the Moon. For since Mars, Venus and Mercury move now above the Sun, now below it, he thinks they are enclosed within one and the same heaven of the Sun, as though it were common to these four planets. Absolutely, however, he inclines to fluid heavens, and therefore in number 12 he posits three heavens: the Empyrean, a single starry heaven without subdivision, and the aerial. Hence, if he had added the Empyrean and the aerial to the five solid heavens, he would indeed have made 7 heavens, assembled from solid and fluid.

[Margin: 7 heavens.]

But we shall assert 5 heavens below in another way: namely the Empyrean; the aqueous or crystalline; the solid firmament of the fixed stars; the aether, or fluid heaven of the planets; and the aerial.

[Margin: 7th opinion, of 6 or 7 heavens.]

[Margin: Philastrius.]

VIII. The seventh opinion is of 6 heavens, which Philastrius heaps together with other opinions in his book *On Heresies*, saying: *Concerning the diversity of the heavens there is a heresy which is in doubt*; and a little later: *If, then, anyone wishes to accept six heavens according to David, and this firmament as the seventh, he*

does not err. For Solomon speaks of three heavens thus: heaven, and the heaven of heaven; Paul the Apostle likewise declares that he was caught up as far as the third heaven. If, then, anyone accepts seven, as David does, or three, or two, he does not err, etc.

[Margin: Bede.]

[Margin: Rabanus.]

Otherwise Bede (on Genesis chapter 1), having quoted the verse

The golden-hued aether of the sevenfold heaven is parted,

at once adds: *For these are the names: air, aether, Olympus, the fiery space, the firmament, the heaven of the Angels, the heaven of the Trinity.* The same order of the heavens, but reversed, is given by Rabanus [Hrabanus Maurus], as it is also given by St Thomas (on Book 2 of the *Sentences*, distinction 14, question 4, and part 1 of the *Summa*, question 68, article 4): namely the Empyrean, the crystalline, the starry, the fiery, the Olympian, the aethereal and the aerial. And in the same place he says, following Rabanus on Genesis chapter 1, that the space from the Earth to the Moon is divided into four regions, as though into four heavens, and that the upper region of fire is called the fiery heaven; the lower region of fire, the Olympian heaven; the upper region of air, the aethereal heaven; and the lower region of air, the aerial heaven. And perhaps someone might draw in here the "heptachord of the world" which Virgil indicates in these verses:

[Margin: Eclogue 2.]

I have a pipe, joined from seven unequal hemlock stalks, etc.

And this:

He sounds the seven distinct notes against the measures.

[Margin: 8th opinion, of 8 heavens — but of the stars [?].]

IX. The eighth opinion was of eight starry heavens. All those held it who thought that the fixed stars are moved by no motion other than that of the primum mobile. Hence, since they posited as many heavens as there are planets, 7, the sphere of the fixed stars, or the Aplanes, turned out for them to be the highest and eighth sphere, which was afterwards called the Firmament by many Catholics. Not all Catholics, however, excluded the Empyrean or crystalline heaven, or even the aerial; but, speaking of the subdivision of the starry heaven, they distributed it into 8 total spheres.

[Margin: The Babylonians. The Egyptians. Eudoxus. Callippus. Plato. Aristotle. Cicero. Philo. St Damascene. St Bonaventure. Carthusianus. Abenezra.]

Of this opinion were the most ancient Babylonians and Egyptians, Eudoxus, Callippus, Plato, Aristotle, Cicero, Philo, St Damascene, St Bonaventure, Dionysius Carthusianus and Abenezra, as I have already shown (section 2, chapter 3, number 2), where I adduced the words of Plato, Cicero, Philo and Damascene. He, indeed, in Book 2 of the *Faith*, chapter 6, having accepted three heavens — the Empyrean, the starry and the aerial — adds concerning the

starry heaven of the planets: *If, however, you wish to take seven circles for seven heavens, you will in no way offend against the account of truth.*

[Margin: St Basil.]

Similarly St Basil, concerning the seven heavens of the planets (homily 3 on the *Hexaemeron*), asserts the most common opinion: *Nor is it right that this should ever be believed with less faith than that with which we believe that there are seven orbs, in which the seven stars are proclaimed, by the almost unanimous voice of all, to run their course; and these heavens, indeed, they assert are neatly fitted one into another, as if jars or small vessels were set one inside another.* He had, however, admitted three heavens on the authority of St Paul, and among them the Empyrean and the Firmament.

[Margin: St Ambrose.]

So too had St Ambrose (Book 2 of the *Hexaemeron*, chapter 2), and he then adds: *David, too, set the heavens of heavens in that choir of those who praise the Lord. Imitating him, the philosophers introduced the harmonious motion of the seven stars, of the Sun, the Moon, etc., etc.; and they think that these, linked and as it were inserted into one another, turn backward and are carried with a motion contrary to the rest.*

[Margin: Remigius.]

To these must be added Remigius of Auxerre [Remigius Antisiodorensis], on Psalm 148, explaining the words "Praise him, you heavens of heavens": *Heavens, that is, the firmament and the aether, taken collectively [?]; of heavens, that is, of the seven circles, where the aplanes and the planets are fixed: may they be for you matter for praising the Lord.*

[Margin: St Thomas.]

On the same principle St Thomas, who asserts three heavens — the Empyrean, the crystalline and the starry — speaks thus of the starry heaven (part 1, question 68, article 4): *The third, partly transparent and partly luminous in act, which they call the starry heaven; and it is divided into eight spheres, namely into the sphere of the fixed stars and the seven spheres of the planets, which can be called seven heavens, or seven spheres.*

[Margin: Lyranus.]

[Margin: Tostatus.]

Here Lyranus joins them, in his *Postills* on Genesis, saying: *The firmament was made according to the specific form of the starry heaven. In this production the seven orbs of the planets, which are comprehended under the name of firmament, are also understood to have been produced.* Nor does Tostatus say otherwise, on chapter 1 of Genesis: *By the name of Firmament is taken the whole celestial magnitude, or mass, containing eight orbs: namely the starry heaven, and the heavens of Saturn, Jupiter, Mars, the Sun, Venus, Mercury and the Moon.*

[Margin: Dionysius Carthusianus.]

[Margin: Burgensis.]

Likewise Dionysius Carthusianus, in his *Commentaries* on Genesis, article 10: *The firmament was made, that is, the starry heaven, which is the eighth sphere, because beneath it are the seven spheres, or orbs, or heavens of the planets.* With him

Burgensis agreed, in his *Additions* on Genesis chapter 1: *The firmament with which we are dealing here is the starry heaven, which is divided into the eighth sphere, that of the fixed stars, and into the seven orbs of the planets.*

[Margin: Riccius.]

[Margin: Orontius.]

Augustinus Riccius, on the other hand (the treatise *On the motion of the eighth sphere*, chapters 13 and 14), and Orontius [Oronce Finé] (Book 1 of the *Sphere*, chapter 5) do acknowledge an apparent motion of the fixed stars toward the east, but teach that it arises from the aggregate of 8 heavens.

[Margin: Arriaga.]

[Margin: Amicus.]

[Margin: Aversa.]

[Margin: Cremoninus.]

[Margin: Rubius.]

[Margin: Hurtadus.]

And our own Rodericus Arriaga [Rodrigo de Arriaga] (the single disputation *De caelo*, section 4, number 52), on the hypothesis that the starry heavens are solid, teaches that there are eight of them; for he thinks the eighth sphere can be moved by the Intelligences in such a way that there appears to be a twofold motion in it, one to the west, the other to the east. Amicus also thinks this the more probable view (treatise 4 *De caelo*, question 5, doubt 2), and so does Aversa (question 32 of the *Philosophy*, section 4). Eight heavens are likewise affirmed by Cremoninus [Cesare Cremonini] (*De motu caeli*, section 2, chapter 13), Rubius (Book 2 *De caelo*, chapter 5, question 1) and Hurtadus (disputation 1 *De caelo*, section 1), because they think it uncertain whether the fixed stars have a proper and peculiar motion. Arriaga, however, acknowledges a ninth heaven above the starry ones, namely the Empyrean.

[Margin: 9th opinion, of 9 heavens.]

X. The ninth opinion, of nine heavens, does not count them all in the same way. For Arriaga, as I have just said, counts the Empyrean and eight starry heavens. But others held that the ninth heaven is the primum mobile, which carries along with itself the eight lower spheres, of the fixed stars and of the 7 planets; and to each of these they attributed its own proper motion toward the east — to the fixed stars a very slow one.

[Margin: Macrobius. Alpetragius. Rabbi Isaac. Rabbi Moyses. Zagutus. Haly. Scotus. Sacrobosco.]

So Macrobius (Book 1 on the *Dream of Scipio*, chapter 17); so Rabbi Isaach, Alpetragius (in his *Celestial Physics*), Abraham Zagutus [Abraham Zacuto] and Rabbi Moyses, as cited by Riccius (the treatise *On the motion of the eighth sphere*, chapter 4); Haly (chapter 11 of the *Quadripartitum*); Scotus [John Duns Scotus] (on Book 2, distinction 14, question 2, and Book 12 of the *Metaphysics*, comment 44); and Io. de Sacro Boscho [Johannes de Sacrobosco] (on chapter 1 of the *Sphere*). And Mastrius and Bellutus, Scotists, say that it was the common opinion in Scotus's time (disputation 2 *De caelo*, question 1, article 2, number

30). They indeed admit a primum mobile distinct from the starry heaven, though they posit the eight starry heavens as a single solid and continuous heaven, with various circles or zones carrying the planets, following Aversa and Cesalpino; as does Chiaramonti too (Book 2 *De universo*).

[Margin: Whether Hipparchus [?] and Ptolemy [?] … this [?] … — the note's ends are lost into the binding.]

Averroes (on Book 2 *De caelo*, comment 67), Albert the Great (Book 2 *De caelo*, treatise 3, chapter 11), Clavius (on the *Sphere*, page 43), the Conimbricenses [the Coimbra commentators] (Book 2 *De caelo*, chapter 5, question 1) — who attribute the same opinion to Menelaus, or Mileus, to Agria, to Alexander and to Alfraganus — Scheinerus (*Disquisitiones mathematicae*, page 50), Tannerus (dissertation *De caelo*, question 10), and Mastrius and Bellutus (disputation 2 *De caelo*, question 1, article 2, number 19) think that the authors of this opinion were Hipparchus and Ptolemy. That is to say: Hipparchus and Ptolemy were the first who, by comparing their own observations of the fixed stars — Hipparchus with those of Timocharis and Aristyllus, Ptolemy with the observations of Hipparchus — detected and confirmed that the fixed stars, besides their apparent motion upon the poles of the equator toward the west, are slowly advanced in the order of the signs toward the east, upon the poles of the ecliptic. But if you read Ptolemy (Book 1 of the *Almagest*, chapter 8, and Book 7, chapters 2 and 3) and Alfraganus (differences 5 and 18), where they treat of these motions, you will nowhere find that they posited two distinct spheres, but only distinct poles about which those motions take place. And Tannerus himself concedes that the argument for a distinction of heavens drawn from the distinction of motions is not necessary, but that a heaven can be moved by one Intelligence or by several with a single real motion in such a way that the motion nevertheless appears twofold. Indeed, Ptolemy and Alfraganus seem to describe the equator and the ecliptic in the same heaven.

[Margin: 10th opinion, of 10 heavens.]

XI. The tenth opinion asserted 10 heavens: namely seven for the seven planets; the eighth for the motion of trepidation, or of access and recess, of the fixed stars toward rising and setting; the ninth for the motion of the fixed stars and of the auges, or apogees, of every planet except the Moon; and the tenth for the motion of the primum mobile.

[Margin: Alphonsus. Amicus. Fernelius [?]. Purbachius. Regiomontanus. Apianus [?]. Maurolycus. Langius. Thebith. Arzachel. Isaac.]

So King Alfonso [Alfonso X] in his tables; Io. Baptista Amicus (*De motibus caelestibus*, the last chapter); Io. Fernelius (Book 2 of the *Cosmotheoria*, chapters 1 and 7); Purbachius in the *Theory of the eighth sphere*, with his followers; Regiomontanus (in the same place, and Book 7 of the *Almagest*, proposition 7); Maurolico (dialogue 1 of the *Cosmographia*, page 24, and dialogue 3, page 89); Apian (in the *Opus Caesareum*); and Ioseph Langius [Joseph Lange] (*Elementa astronomica*, chapter 4). The same number, but in another way, was accepted by Thebit, Arzachel and Isaac Israelita [Isaac Israeli]; but they assigned the motion

of trepidation in longitude rather to the ninth sphere, and the continuous motion in longitude, of itself, to the eighth. To this class could be reduced those authors who posited 11 heavens in such a way that they called the eleventh the Empyrean — namely the Conimbricenses, Martinengus, Clavius and, before them, Alliacensis, as we shall say shortly; for in the number of 10 mobile spheres they agree with the Alphonsines.

[Margin: Parisiensis.]

[Margin: Delphinus.]

But Gulielmus Parisiensis (first part *De universo*, chapters 34 and 37) admits a first heaven, most perfectly at rest and immobile, or Empyrean, and nine other mobile heavens, namely the primum mobile and the eight remaining spheres; and the same view is defended by Io. Antonius Delphinus (the book *De caelestibus globis ac motibus*, chapters 30 and 32).

[Margin: 11th opinion, of 11 spheres.]

XII. The eleventh opinion is of eleven heavens, but not in the same way. For some count, with the Alphonsines, ten mobile spheres, according to what was said a little earlier in number 11; but above this they acknowledge an eleventh, immobile sphere: namely the Empyrean, or supreme immobile heaven, which pours diverse and stable properties into the different regions of the Earth.

[Margin: Alliacensis. Clavius. Conimbricenses. Martinengus. Polaccus.]

So Petrus Alliacensis (question 2 on the *Sphere*), Clavius (on the *Sphere*, page 45), the Conimbricenses (Book 2 *De caelo*, chapter 5, question 1), Ascanius Martinengus (*Glossa magna*, page 1021) and Georgius Polaccus [Giorgio Polacco] (*Anticopernicus*, assertion 172). But among these, Alliacensis reports the opinion of 10 mobile heavens and does not reject it, and adds that besides them an eleventh, immobile heaven is required because of the diverse influences — which Clavius too thinks probable. Again, Clavius thinks that this same heaven is that Empyrean of which Strabus [Walafrid Strabo] and Bede speak; but he thinks that the aqueous or crystalline heaven is the aggregate of the ninth and tenth heavens of the Alphonsines.

[Margin: Io. Vernerus.]

Others, however, posited eleven heavens, but all mobile: namely the eleventh, or primum mobile; the tenth for the first trepidation, or libration, in latitude, by force of which the obliquity of the ecliptic varies by turns; the ninth for the second libration, or trepidation, in longitude, by force of which the equinoctial points seem to advance and to recede [?], and so the motion of the fixed stars in longitude seems to be quickened and retarded; the eighth for the proper and uniform motion of the fixed stars in the order of the signs; and the remaining seven for the motion of the seven planets in longitude, etc. So Io. Vernerus [Johannes Werner], as cited by Erasmus Osualdus [Erasmus Oswald Schreckenfuchs] in the *Theory of the Eighth Sphere*;

[Margin: Leopoldus, Maginus, Clavius.]

Leopoldus de Austria [Leopold of Austria] in his *Compilatio*; Ioannes Antonius Maginus in his *Theorics*, Book 1, and on the second movables; and Clavius,

in the last edition of the *Sphere*, on account of the motions already mentioned, asserted by Copernicus but transferred to the hypothesis of a resting Earth — which is why Scheinerus calls this system the Clavian (*Disquisitiones mathematicae*, page 36). This same number is set out, not on their own opinion but on that of others, by Antonius Deusingius [Anton Deusing] and Petrus Gassendus [Pierre Gassendi] in their *Institutiones astronomicae*.

[Margin: 12 heavens.]

Hence, since Clavius admitted the Empyrean heaven, he certainly admitted 12 heavens.

[Margin: Tannerus's slip.]

XIII. But since the assertion of eleven heavens is attributed to Copernicus by Tannerus (dissertation *De caelis*, question 10, and volume 1 of the *Summa theologica*, disputation 6, question 4, doubt 4, number 6), it should be understood that Copernicus acknowledged this number not on his own opinion but on that of others. For he says in Book 3 of the *Revolutions*, chapter 1: *On account of these, some devised a ninth sphere, others a tenth, through which they thought these things came about in this way; and yet they could not deliver what they promised. And now an eleventh sphere too had begun to come to light — a number of circles that we shall easily refute as superfluous by means of the motion of the Earth.*

[Margin: 2 heavens in the hypothesis of Copernicus, or a single one.]

Copernicus [?], then, attributes all the apparent motions in the fixed stars, the very motion of the primum mobile, and the variation of the obliquity of the ecliptic to the motions of the Earth, as I shall show more fully in section 4. Nor are the heavens distinct in his hypothesis except into two classes: namely into the immobile sphere of the fixed stars, and into the fluid heaven of the planets, among which planets is the Earth with the sphere of the elements. For although he did not state that fluidity expressly, it is gathered by no slight conjectures from what he teaches in Book 1, from chapter 8 to 11. In chapter 8 he teaches that not the whole air is carried round together with the Earth in the annual motion, but only the air that is near us. Just as the Earth, therefore, which is one of the planets, revolves with the nearby air within the rest of the air, which is fluid but unmoved, so it is fitting that the same should happen to the other planets too — that is, that they move in a fluid aethereal breeze. Again, in the same place he says that immobility befits the world as the place of the stars and planets, and that it is absurd to ascribe motion to what contains, or locates, and not rather to what is contained and located, which is the Earth. But if this reasoning holds, it holds also of any heaven with respect to its own planet, whose place it is; nor can the sphere of the fixed stars alone be understood by the name of the world. Finally, in chapter 10 he names the sphere of the fixed stars; but while he inquires into the order of the celestial orbs he never names the spheres of the planets, only the planets themselves and their revolutions — indicating clearly enough that they revolve not at the motion of orbs but of themselves. What indicates this most of all, however, is that the Earth with the Moon is carried through the annual orb, and yet the Moon does not cut it:

from this argument Boulliau (Book 1, chapter 8) concludes that the Moon and the planets move freely through a fluid aether. Accordingly, in the Copernican system we recognise two heavens in reality, one of the fixed stars and the other of the planets, the latter, however, being distinguishable in reason and designation into seven heavens — granted that Longomontanus (Book 1 of the *Theorics*, chapter 1) thinks that Copernicus, having removed the Ptolemaic epicycles, nevertheless retained the eccentricity of the orbs with their reality intact, and that Tycho (volume 5 of the *Progymnasmata*, page 439) thought the same of Copernicus, though timidly.

[Margin: Corollary from Pererius.]

From what has been said it is clear enough how prudently Pererius said (Book 2 on Genesis, question 4) that none of the opinions already mentioned, except the error of Basilides, is repugnant to Scripture or to the Fathers.

CONCLVSIO.

Conclusion.

XIV. *Heavens taken broadly are five: namely the Empyrean, the aqueous or crystalline, the firmament of the fixed stars, the aether of the planets, and the air. Taken strictly, however, they are only two, if we are speaking of heavens really distinct: namely the firmament and the aether, or the heaven of the fixed stars and the heaven of the planets.* For that the Empyrean is above all the heavens we have shown sufficiently in section 1, question 6, from number 24. That there is, besides, an aqueous or crystalline heaven above the whole firmament, whether it be fluid or solid, I taught in section 1, chapter 2, questions 1 and 3, and chapter 3, conclusion 4. Further, that the firmament of the fixed stars is solid and the aether of the planets fluid, I likewise taught in section 1, chapter 3, conclusion 4, and chapter 7, number 21. And so the heaven of the fixed stars is indeed to be distinguished from the planetary heaven; but for the planetary heaven, since it is continuous, there is no reason to distinguish it really, any more than to distinguish the air or water into several individuals because different birds or fish move in them. Finally, that the aerial heaven, or the air, is called heaven by both sacred and profane writers — broadly indeed, but properly — is clear from what was said in section 1, chapter 1, number 9, and in section 3, this chapter 1, number 5. But if you take heaven strictly for a simple visible body in which the stars move, it follows that there are only two: namely the solid heaven of the fixed stars and the aerial [?] heaven of the planets; and these are also distinguished from each other in their species as heavens, since the firmament is of water and the aether of the planets of fire, as I taught in section 1, chapter 3, conclusion 4. If anyone, however, wishes to distribute the fluid heaven into one region for the new phenomena, into the heaven of Saturn, the heaven of Jupiter, the heaven of the Sun with its satellites Mars, Venus and Mercury, and the heaven of the Moon, I do not object; indeed I admitted this in Book 7, section 6, chapter 4, scholium 5.

[Margin: The specific difference of the heavens.]

On the specific difference of the heavens, however, others hold differently, as may be seen in Suarez (disputation 13 of the *Metaphysics*, section 11), Amicus (treatise 4 *De caelo*, question 7, doubt 1), the Conimbricenses (Book 2 *De caelo*, chapter 5, question 3, and chapter 7, question 1), Pontius (disputation 22, question 7), Mastrius and Bellutus (disputation 2 *De caelo*, question 2, article 4), Arriaga (the single disputation *De caelo*, section 1) and Tannerus (volume 1 of the *Theology*, disputation 6, question 4, doubt 4).

XV. Now, since among other writers this dissertation on the number of the heavens, and their distribution, is very incomplete or confused, it seemed good to put together a brief synopsis of the preceding opinions and to gather it into the following catalogue, adding a brief little explanation for the subdivision of the opinions into their classes. Their authors, however, I have already listed in their places from number 1 onward.

SYNOPSIS Numeri cælorum iuxta diuerſos Auctores.

Synopsis of the number of the heavens according to various authors.

Number of heavens		Number of the paragraph
I.	A single heaven, that is, a single world. *Aristotle.*	1
	A single starry heaven, as being fluid. *St Chrysostom, Tertullian, St Bonaventure, Tycho, Longomontanus, Kepler, Boulliau, Tellez.*	2
	A single starry heaven, though solid. *Aegidius, Hurtadus, Cesalpino, Aversa.*	2
II.	Two heavens: that is, the Empyrean, or the heaven created on day 1, and the Firmament, made on day 2. *St Clement, Acacius, Theodoret, Anastasius Synaita, Procopius, Suidas, St Bruno, Claudianus Mamertus.*	4
	Two heavens: the starry and the aerial. *St Justin.*	4
	Two heavens: of the fixed stars and of the planets. *St Gregory of Nyssa.*	4
	Two heavens: the primum mobile and the starry. *Mastrius and Bellutus.*	4
III.	Three heavens: namely the Empyrean, the starry and the aerial. *Saints Basil, Ambrose, Damascene; also Cassiodorus, Genebrardus, Suarez, Tannerus, Hurtadus, Oviedus, Tellez, Borrus.*	5
	Three heavens: of the fixed stars, of the planets, and the aerial. *St Gregory of Nyssa.*	5
	Three heavens: the Empyrean, the aqueous and the starry. *St Thomas Aquinas.*	5
	Three heavens: the aqueous, the starry and the aerial. *Caietanus.*	
	Three heavens, but mystically three supernatural visions: corporeal, imaginary, intellectual. *St Augustine.*	
IV.	Four heavens: the Empyrean, that of the fixed stars, that of the planets, and the aerial, hinted at by St Athanasius.	6
V.	Five starry heavens: of the fixed stars, of Saturn, of Jupiter, of the Sun (with the inclusion	7

Number of heavens		Number of the paragraph
	of Mars, Venus and Mercury) and of the Moon, if they are solid. *Oviedus.*	
V.	Five heavens, taken broadly: the Empyrean, the aqueous, that of the fixed stars, that of the planets, and the aerial. *Ourselves.*	7
	Seven heavens, confusedly. *Philastrius.*	8
VII.	Seven heavens: that is, air, aether, Olympus, fire, the Firmament, the heaven of the angels, and the heaven of the Trinity. *Bede.* Or: the lowest and the highest region of the air, the lowest and the highest of fire; the starry, the crystalline, the Empyrean. *Rabanus.*	8
VIII.	Eight starry heavens: namely the Aplanes, or sphere of the fixed stars, and the 7 spheres of the planets. *The Babylonians, the Egyptians, Eudoxus, Plato, Callippus, Aristotle, Cicero, Philo; Saints Basil, Ambrose, Damascene, Bonaventure, Remigius, Thomas; also Avenezra [Ibn Ezra], Carthusianus, Lyranus, Tostatus, Burgensis, Riccius, Orontius, Cremoninus, Philalthaeus, Amicus, Ruvius.* Of these, however, Augustinus Riccius, Orontius and Amicus attribute a proper motion to the eighth sphere.	9
	Nine heavens: namely the Empyrean and eight starry heavens, if the starry ones are solid. *Arriaga.*	10
IX.	Nine heavens: that is, the primum mobile and eight starry heavens. *Macrobius, Haly, Alpetragius, Rabbi Iosue, Rabbi Moyses, Scotus, Abraham Zagutus [Avraham Zacuto], Sacrobosco, Chiaramonti*; it is attributed by many to *Hipparchus* and *Ptolemy*, but not safely enough.	10
	Ten heavens: namely the primum mobile; a sphere for the motion of the fixed stars and of the auges; the eighth sphere, for the trepidation in longitude; and seven planetary spheres. *The Alphonsines, Fernelius, Purbachius, Regiomontanus, Giovanni Battista Amico, Apian, Maurolico, Langius.*	11
X.	Ten heavens: that is, the primum mobile; a sphere of trepidation in longitude; a sphere of the fixed stars; and seven planetary spheres. *Arzachel, Thebit, Isaac Israelita [Isaac Israeli].*	11
	Ten heavens: namely the Empyrean, the primum mobile, and eight starry heavens. *Gulielmus Parisiensis and Io. Antonius Delphinus.*	11
	Eleven heavens: namely the Empyrean and the ten mobile spheres of the Alphonsines. *Petrus Alliacensis, the Conimbricenses, Martinengus*, and formerly *Clavius*.	12
XI.	Eleven heavens: namely the primum mobile; a sphere of the first libration, in latitude; a sphere of the second libration, in longitude; a sphere of the fixed stars; and seven spheres of the planets. *Io. Vernerus [Johannes Werner], Leopoldus de Austria [Leopold of Austria], Giovanni Antonio Magini*, and later *Clavius* — though for him, since he admits the Empyrean, the heavens come to 12.	12
XII.		
XIV.	Fourteen: *Giovanni Battista della Torre and Fracastoro*; on them see what we shall say below, in the Fracastorian system.	

CAPVT II.

Chapter II.

Quid & Quotuplex ſit Syſtema Mundi. Et de Syſtematibus vetuſtiſſimis, ac præcipuè de Pythagoræ, Archimedis, & Plinij, quod Ptolemaicum dici ſolet; viguitq. vſq. ad Clauium.

What the system of the world is, and of how many kinds; and on the most ancient systems, and chiefly on those of Pythagoras, of Archimedes and of Pliny — the last usually called the Ptolemaic, which prevailed down to Clavius.

[Margin: What a system is.]

I. The system of the world is nothing other than the co-ordination, or composition, of the great parts of the world — namely of the elements and of the heavens. Its matter, as it were, is the number of the elements and of the heavens, both total and partial; its form is their order and position with respect to one another and relative to the centre of the universe. Considered by its form, therefore, the system of the world is divided into two highest genera: the one constructed around the Earth, immobile at the centre of the universe; and the other constructed around the Sun, immobile at the centre of the universe — that is, free from τῆς φορᾶς κατὰ τόπον [Greek: tēs phoras kata topon, "motion from place to place"], namely from translation — while moving the Earth through an annual orb.

[Margin: How many kinds of system there are.]

Again, each of the two, but chiefly the first, is subdivided into the homocentric, the eccentric, and the mixed, made up of some orbs concentric and some eccentric. Now, since enough was done in the preceding chapter on the number of the heavens, as the remote matter of the cosmic system, it follows that we should treat of the order of the heavens and the planets, and of the number of the partial orbs. And we take our beginning from the most ancient.

[Margin: The alternating rise and fall of systems.]

II. It is, moreover, most worthy of observation that two sects of physicists and mathematicians have, like Castors and Polluxes, alternately died away and then, revived, prevailed once more in the schools. For first Pythagoras, with some of his sect[28], placing the Earth at the centre of the world, used eccentrics and epicycles and placed the Sun in the middle of the planets. Others of the Pythagoreans, however, keeping the epicycles and eccentrics, nevertheless placed the Sun at the centre of the world, and the Earth in the middle of the seven planets. Then Plato restored the Earth again to the centre of the universe, and placed the Sun in the middle of the planets — in such a way, however, that he thought Venus and Mercury were for the most part carried above it. Following him, Eudoxus, Callippus and Aristotle tried to explain the whole motion of the planets by concentric circles; and so they held that neither Venus nor Mercury goes above and below the Sun on epicycles, but that they are either always above the Sun or always below it. When these had gone the way of fate, others again embraced eccentrics and epicycles, with the Earth in the

[28]the page prints "rectę", for "sectę"

middle of the world — such as Archimedes, Hipparchus, Sosigenes, Cicero, Vitruvius, Pliny, Macrobius and Capella, though with a varying order of the planets. Others, who moved the Earth from its place, such as the Pythagoreans Aristarchus and Philolaus, either did not set out what they thought about eccentrics and epicycles, or it has not been handed down to posterity. Ptolemy, again, reviving the most ancient opinion of the Pythagoreans, so established eccentrics and epicycles that he kept the Earth in the middle of the world and the Sun in the middle of the planets; and by persuading the greatest part of astronomers of this opinion he extended it down to the fourteenth century. That order of the planets, however, Alpetragius and Geber judged should be changed. But Alpetragius, Amicus and Delphinus, with a few others, also took up homocentrics again; by contrast, Turrianus and Fracastoro, keeping the Ptolemaic order, nevertheless brought homocentrics back to light — and indeed more of them than Aristotle's, let alone Eudoxus's. About the same time, moreover, Copernicus not only called up from the tombs the fiction of Philolaus and Aristarchus about an annual motion of the Earth around an immobile Sun, but adorned it with arguments so plausible that he has persuaded very many of it down to this day; Venus and Mercury, however, he carried round now below, now above the Sun, with Plato, Vitruvius, Macrobius, Capella and Bede. From this Tycho, and Longomontanus with him, having found a most beautiful handle, not only bade Venus and Mercury, but Mars too, and Jupiter and Saturn, go round the Sun, and the Sun and the Moon, with the fixed stars, round the Earth, which rests in the middle of the world free from all motion of translation; and, keeping the eccentrics, he did away with the Ptolemaic epicycles. Finally Argoli, dislocating the Tychonic system, tried to make the three superior planets move around the Earth, but Mercury and Venus around the Sun, the Earth being left in the middle of the world. We, for our part, judge that Saturn and Jupiter, the Sun, the Moon and the fixed stars can be arranged around the Earth, but Venus, Mercury and Mars around the Sun. Lastly, Signor Giovanni Battista Baliani suspected that the Moon could be placed at the centre of the world. It is worth the effort, then, to discuss one by one so many and such diverse attempts and endeavours, and to weigh the reasons brought forward on either side.

[Margin: Ancient but absurd systems.]

I have, however, passed over as of no consideration those three opinions which Plutarch sets out in Book 2 of *De placitis*, chapter 15, in this way: *Xenocrates thinks that the stars move on one and the same surface. The other Stoics, at heights and depths that vary among themselves. Democritus puts the fixed stars highest, then the wandering stars, and below these the Sun, Lucifer, the Moon, etc. Metrodorus of Chios and Crates set the Sun highest of all, after it the Moon, and below these the wandering and the unwandering stars.* So true is it that nothing is absurd that does not fortify itself with some patron.

[Margin: The system of Pythagoras.]

III. The most ancient of the tolerable systems, then, was the Pythagorean. In it Pythagoras arranged the intervals by harmonic ratios in such a way that, starting from the Earth, he ascended from there to the Moon, then to Mercury, then to Venus; then on to the Sun, and from the Sun to Mars, then to Jupiter, then to Saturn, and finally from Saturn to the fixed stars — as Pliny relates expressly in Book 2, chapter 22. For he says: *Pythagoras sometimes, on a musical reckoning, calls the distance of the Moon from the Earth a tone; from her to Mercury, half of that space, and from him to Venus about the same; from her to the Sun, one and a half times as much; from the Sun to Mars, a tone, that is, as much as from the Earth to the Moon; from him to Jupiter, a half, and from him to Saturn, a half; and from there, one and a half to the zodiac [Signifer].* With this Censorinus agrees entirely, in his book *De die natali*, chapter 11, near the end, in these words: *Pythagoras therefore thought that from the Earth to the Moon there are about 126,000 stadia, and that this is the interval of a tone; from the Moon to the star of Mercury, which is called* στίλβων *[Greek: stilbōn, "the Glittering"], half of that, as it were a* ἡμιτόνιον *[Greek: hēmitonion, "semitone"]; from there to* φόσφορον *[Greek: phosphoron, "Light-bringer"], which is the star of Venus, about the same, that is, another semitone; then on to the Sun three times as much, as it were a tone and a half; and so the star of the Sun is three tones and a half from the Earth, which is called the diapente; and from the Moon two and a half, which is called the diatessaron. From the Sun to the star of Mars, whose name is* πύροεις *[Greek: pyroeis, "the Fiery"], there is as much interval as from the Earth to the Moon, and this makes a* τόνον *[Greek: tonon, "tone"]; from there to the star of Jupiter, which is called* φαέθον *[Greek: phaethon, "the Shining"], half of that, which makes a semitone; from there to the highest heaven, where the signs are, likewise a semitone.*

[Margin: The Pythagoreans the originators of eccentrics and epicycles.]

So much for the order and symmetry of the heavens according to Pythagoras. That the Pythagoreans first devised eccentrics and epicycles, Nicomachus is the authority, in *Simplicius*'s commentary on Book 2 of *De caelo*, where he carefully sets out the hypotheses of the ancients; and, following him, *Giovanni Battista Amico* (*De motibus corporum caelestium*, chapter 6), and *Io. Antonius Delphinus* (*De caelestibus globis et motibus*, chapter 4), where he attributes this very thing to Pythagoras.

[Margin: The system of Pythagoras not received by all the Pythagoreans.]

From what has been said it is clear that Paulus Antonius Foscarinus [Paolo Antonio Foscarini] is not right to call "the system of Pythagoras", or simply "the Pythagorean", the one in which the Sun is at the centre of the world and the Earth revolves around the Sun above Mercury and Venus — granted that some disciples of Pythagoras held so.

[Margin: The system of Archimedes.]

IV. Archimedes subscribed to the Pythagorean system as regards the order, if what *Macrobius* relates of him (Book 2 on the *Dream of Scipio*, chapter 3) is true: *And Archimedes believed he had found the number of stadia by which the Moon is distant from the surface of the Earth, Mercury from the Moon, Venus from Mercury,*

the Sun from Venus, Mars from the Sun, Jupiter from Mars, and Saturn from Jupiter. He also thought he had measured by reasoning the whole space from the orb of Saturn up to the starry heaven itself. This dissent of Archimedes, however, was rejected by the Platonists.

[Margin: And of Cicero.]

And not Archimedes only, but Cicero too, in the *Dream of Scipio* — whose words we shall report in a more suitable place, in the following chapter, number 4, so as to separate him from Plato.

[Margin: The system of Pliny.]

V. *Pliny* himself also, in Book 2, chapters 5, 6, 8, 15, 16 and 17 taken together, expressed the same order, and acknowledged the apsides of the eccentrics and epicycles, that is, the perigees and apogees of the planets — doubtless from Sosigenes, from whom he was accustomed to draw much. So in chapter 5 he says: *that the Earth is lowest and in the middle of the whole, and that it stands suspended at the pivot of the universe, balancing the things by which it is suspended; and so that it alone is immobile, with the universe revolving around it.* Then chapter 6: *Between it and the heaven, suspended in the same air, hang seven stars separated by fixed intervals, which from their gait we call wandering, though none wander less than they; in the middle of them the Sun is carried.* But with greater distinction, chapter 8: *Now, leaving the body of the world itself, let the rest, between the heaven and the Earth, be treated. It is certain that the highest is the star they call Saturn's, and that for this reason it appears smallest, goes round in the greatest circle, and returns in the thirtieth year to the shortest beginnings of its seat, etc. The star of Saturn is of a cold and stiffening nature; and the circle of Jupiter, much below it, is for that reason carried round with a swifter motion in twelve years. The third, that of Mars, which some call Hercules's, fiery and burning from the nearness of the Sun, turns round in about two years; and so, from the excessive heat of this one and the cold of Saturn, Jupiter, set between the two, is tempered by both and becomes beneficent. Next, the course of the Sun is of 360 degrees; but, so that the observation of its shadows may return to its marks, five days are added to the years, and above that a fourth part of a day. For this reason one intercalary day is added in the fifth year, so that the reckoning of the times may agree with the Sun's journey. Below the Sun goes round a great star called Venus's, wandering with an alternating course, and in its very surnames a rival of the Sun and the Moon. For when it goes before and rises ahead of the morning, it takes the name of Lucifer, as a second Sun hastening the day; when, on the contrary, it shines out from the west, it is named Vesper, as prolonging the light and taking the Moon's turn. This nature of it Pythagoras of Samos first detected, about the forty-second Olympiad, etc. It completes the circuit of the zodiac in 348 days, never standing off from the Sun by more than 46 degrees, as Timaeus holds. In a similar way, but by no means next to it in size, the star of Mercury, called by some Apollo's, is carried in a lower circle, in a circuit nine days quicker, shining now before sunrise, now after sunset, never more than 23 degrees distant from the Sun, as this same author and Sosigenes teach, etc.* From here he goes on in chapter 9 to speak of the Moon, and from chapter 15 to 17 he treats of the stations and regressions, the elevations and depressions,

the speed and slowness of the planets, and refers their causes to the position of the circles, according as they approach their apsides or recede from them.

[Margin: Pliny accepts eccentrics.]

Let these words of chapter 15 be noted: *All these things happen for several causes. The first is that of the circles which the Greeks call apsides in the stars — for we shall have to use Greek words. Each of them has its own, and these are other than those of the world;* [?] and a little later: *Therefore the apsides of each rise from a different centre, and for that reason they have different orbs and unlike motions, etc.* Do you see how Pliny has marked out eccentric orbs?

[Margin: The system of Ptolemy.]

VI. As Pliny was growing old, *Ptolemy* began to flourish. He willingly took up the system of Pythagoras, which he knew had certainly pleased Archimedes, Hipparchus and Sosigenes, and fortified it with several arguments: both as regards the eccentrics and epicycles, which he used in Books 3, 4 and 5, and from 9 to 13, of the *Almagest*; and as regards the order of the planets and the position of the immobile Earth at the centre of the universe. His arguments on the position and immobility of the Earth, from Book 1 of the *Almagest*, chapters 5 and 7, although we did not neglect them in Book 2, chapters 2 and 3, will nevertheless have to be treated more carefully below, in section 4, and weighed together with the solutions of the Copernicans. As regards the order of the planets, here are his own words from Book 9 of the *Great Construction*, chapter 1: *The sphere of Saturn, which is the greater, and that of Jupiter, which is second and nearer the Earth, and that of Mars beneath it, are more remote from the Earth than the rest. That of the Sun, too, is described in almost the same way by all the first mathematicians. The spheres of Venus and Mercury, however, are placed by the ancients below the solar sphere; but by some of the more recent they too are, for this reason, placed above it: because the Sun has never been seen to suffer eclipse from these planets. But this reasoning seems weak to us. For some planets can be below the Sun and yet not be set against it, because they are not in the same plane passing through them and our sight — just as in the conjunctions of the Moon, for the most part, no failure of the Sun occurs. But since an understanding of this matter cannot be had otherwise, because none of these stars makes a sensible parallax, from which alone distances are taken, the order of the ancients seems to me plausibly to proceed more naturally, since, with the Sun placed in the middle, it separates those planets which can digress from the Sun by any distance whatever from those which cannot so digress but are always carried round close to it — although it does not remove them so far from the Sun towards the Earth that a parallax worth taking into account can arise.*

[Margin: Ptolemy's reasoning defended against the objections of Geber and Copernicus.]

With these words he certainly forestalled well enough the objections that were later to be hurled against him by Geber and Copernicus. For he does not deny Venus and Mercury all parallax, since by the very fact that he places them below the Sun he is compelled to grant them a somewhat greater one than the Sun's — to Venus at perigee about 3′, and to Mercury 7′, as *Geber* gathers

in Book 7, chapter 1, of his *Astronomy*. What he denies is that this parallax needs to be taken into account in this business, or that it is so evident from observations that their distance could be established from it, known in advance. Besides, he does not deny absolutely that some very small part of the Sun can be eclipsed by Venus or Mercury — for Geber shows in the same place that this follows from the Ptolemaic hypothesis — but he says that, if this has not been observed, it happened because the observations of the conjunctions of the Sun with Venus or Mercury were made at a time when either of them, on account of its latitude from the ecliptic, appeared on this side of or beyond the Sun's disc. Finally, both Geber, above, and *Copernicus*, Book 1, chapter 10, argue thus: *That argument of Ptolemy's too* — the words are Copernicus's — *that the Sun must be carried in the middle, between the planets that digress from it in every way and those that do not, is shown to be unpersuasive by the fact that the Moon, which also digresses in every way, betrays its falsity.* But Ptolemy thought this argument plausible not absolutely, but on the supposition that knowledge of this matter cannot be had otherwise; whereas the Moon's place below the Sun is manifest from elsewhere, both from eclipses and from the lunar parallaxes, which very evidently exceed the solar ones.

[Margin: The system of Regiomontanus.]

VII. To Ptolemy, moreover, as regards the order of the planets, there subscribed in turn all the Arabs, except Geber and Alpetragius; all the Greeks, except Theon; and finally all the Latins down to Clavius, before the use of the telescope had taken hold — with the exception of Martianus Capella, Macrobius, Apuleius, Sammerius, Bede, Copernicus and the Copernicans; but chiefly *Ioannes Regiomontanus* [Regiomontanus], in the *Epitome of the Almagest*, Book 9, proposition 1. He kept Venus and Mercury below the Sun all the more willingly because he thought it the more likely that the space between the Sun at perigee and the Moon at perigee — namely of 1006 terrestrial semidiameters, as he himself computes — ought not to be empty of every star. He adds, and so do *Clavius* (in the *Sphere*, page 71) and the *Conimbricenses* College (on Book 2 *De caelo*, chapter 7, question 4, article 2), that the eclipse the Sun suffers from Venus passing beneath the Sun's disc is not sensible, because the disc of Venus is about a hundredth part of the Sun's disc, as I too computed in Book 7, section 1, chapter 4, number 4; and that the eclipse by the tiny Mercury is much less so — or, as *Copernicus* puts it (Book 1, chapter 10): *so small a spot is not easily seen under the most excellent light.*

[Margin: Regiomontanus defended against the attacks of Copernicus.]

But these things, as I said in the same place, are to be understood of the naked eye, and before or apart from the use of the telescope; otherwise, I related in Book 3, chapter 3, scholium 3, that Mercury has been seen beneath the Sun with the help of the telescope; and there, in scholium 2, I also showed that the blackish thing seen beneath the Sun by Averroes, or Avenrodan, or by others in the time of Charlemagne, was not Mercury but one or more spots of the Sun. But *Copernicus* (Book 1, chapter 10) refutes the first reason of Regiomontanus —

whom, however, whether from forgetfulness or from swollen pride, he passed over unnamed — on the ground that between the Moon and us there are air and fire, and a space of 52 semidiameters empty of every star. Regiomontanus, however, would reply that the case is not alike, because air and fire are not heavens, nor a place fit to contain a star. Copernicus adds that the semidiameter of the epicycle of Venus, by which she digresses from the Sun about 45 degrees on either side, occupies a greater space than there is from the Earth to Mercury at apogee, and yet is empty of any star; but Regiomontanus would reply that the space within that epicycle is not empty, since Venus runs through it from apogee to perigee.

[Margin: The system of Clavius.]

VIII. Finally our *Clavius* tried to confirm the order of the Pythagorean and Ptolemaic system in chapter 1 of the *Sphere*, from page 65 in my copy, and from him *Franciscus Barocius* [Francesco Barozzi], Book 1 of the *Cosmographia*, page 9. These reasons must be weighed.

[Margin: Clavius's first reason, from parallax.]

The *first* is from parallax: for the greater it is, other things being equal, the nearer to the Earth is the star that has it, as is clear from what was said in Book 1, chapter 35; but the parallax of the Moon is far greater than that of the Sun, as is plainly established from what was said in Book 3, chapter 8, and Book 4, chapter 14; likewise the parallax of Mercury is greater than that of Venus, and that of Venus than that of the Sun, says Clavius — for he admits that this argument does not tell for the three superior planets. But in truth, as Regiomontanus admits (Book 9 of the *Epitome*, proposition 1), and before him Ptolemy (Book 9 of the *Almagest*, chapter 1), the motion of Venus and Mercury is not known so scrupulously that their parallax can be discerned evidently from it, especially when using the Ptolemaic or Alphonsine tables; rather, their distance from the Earth must be established from elsewhere, and the parallax then gathered from it by reasoning. Further, since it is now established, the telescope showing it, that Venus and Mercury, when they are nearer the apogee, move above the Sun, it is false that their parallax is then greater than the Sun's. Again, since they go round the Sun in such a way that Mercury digresses less from the Sun, it follows that Mercury at its perigee is higher than Venus at perigee, and so undergoes a smaller parallax.

[Margin: Second reason, from shadows.]

The *second* reason is drawn from the shadow: for the smaller the shadow of a gnomon, the higher (other things being equal) is the star by whose rays, carried through the tip of the gnomon, the shadow is cast, as is clear from the diagram brought in at Book 4, chapter 10, number 1. But at an equal elevation above the horizon the solar shadow is shorter than the lunar; indeed, says Barocius, the shadow of a ray of the Sun is shorter than that of a ray of Venus, Venus's than Mercury's, and Mercury's than the Moon's; and he testifies that this has been proved by experience. For the rest, we grant this of Venus at perigee with respect to the Sun, but deny it of her with respect to Mercury; and of either

planet when it is near the apogee we likewise deny it — quite apart from the fact that the experiment of Mercury's shadow is suspect to me, since Mercury near perigee usually lies hidden under the glare of the Sun, and even at its greatest digression does not appear without the twilight washing out its shadows.

[Margin: Clavius's slip.]

This argument could certainly have been applied much more clearly to Jupiter, whose shadow is more evident than Mercury's. And so I could wish that these words had not escaped Clavius (page 68 of the *Sphere*): *What we have said of the Moon with respect to the Sun can be applied with respect to the other planets; for although the other planets do not shine so as to cast a shadow, it can nevertheless be known how far their rays are cast through the tips of gnomons.* For the rays of Venus and Jupiter do cast a manifest shadow. Then, how slippery is that application and supplementing of the shadow! — namely by placing the eye in the plane and looking back at the planet through the tip of the gnomon; and how difficult it is, in these planets, to know in advance the moment at which they stand at such and so great an altitude above the astronomical horizon, independently of their distance from the centre of the Earth. But it is all too easy to invent experiments from a presupposed hypothesis, and to suppose that experiments never made would turn out so, if they were made.

[Margin: Third reason, from eclipses.]

The *third* reason is sought from mutual eclipses: for that star is nearer to the Earth, and to us observing from the Earth, which by the interposition of its body hides the other from us; but the Moon hides from us not only the Sun but all the other planets; therefore the Moon is the lowest of all. *By the same reasoning Mercury will be below Venus, and Venus below Mars, and so on*, says Clavius — that is, from the presumption of this system, not from any experiment concerning Mercury; for Mercury has never been seen to have hidden Venus; indeed perhaps it has never even been able to hide her, nor will be, but rather Venus Mercury. See what we set out about these occultations of the planets, established from observation, in Book 5, chapter 2, scholium 2, and Book 7, section 6, chapter 14.

[Margin: Fourth reason, from the swiftness of motion.]

The *fourth* reason is taken from the speed of the proper motion and the slowness of the common motion: for the farther a planet is from the primum mobile and from the fixed stars, the more quickly it completes its period, tending eastward; and so it comes about that, returning daily later to the same meridian, it appears slower than the motion of the primum mobile and of the fixed stars towards the west. Since, therefore, the fixed stars cannot complete their period except after many thousands of years, while Saturn completes it in about 30 years, Jupiter in 12, Mars in about 2 years, the Sun in a single year, and the Moon within one month, and Venus and Mercury complete it more quickly than Mars and more slowly than the Moon, it is established that below the fixed stars there stands, immediately, Saturn; below him Jupiter; below Jupiter, Mars; below Mars, the Sun, Venus and Mercury; and below these, finally, the

Moon. From this argument, however, Clavius denies that anything certain can be gathered about the order of the Sun, Venus and Mercury among themselves: for the mean motions of the Sun, Venus and Mercury are equal, while the apparent motion on the epicycle lasts longer for Venus than for the Sun, and for the Sun than for Mercury. By this argument Alpetragius, as we shall relate below, contends that Venus must be placed below Mars and above the Sun, and Mercury below the Sun. Further, as we said in Book 7, section 3, chapter 11, towards the end, the revolution of Venus, reckoned with the motion of the Sun which she accompanies, is of very nearly 584 days, and that of Mercury of about 116; but about a Sun at rest the revolution of Venus is of about 225 days, and that of Mercury of about 88.

[Margin: 5. Reason from the interval between the Sun and the Moon not being empty.]

The *fifth* reason is the one I have already brought forward from Regiomontanus in number 6: namely, that the space from the Moon to the Sun should not be empty of a star. This reason tells much more strongly in the hypothesis of Kepler, in ours and in that of Vendelinus [Wendelen], in which many more semidiameters of the Earth than 1006 intervene between the Moon and the Sun: for Kepler there are about 3400, for me 6036, for Wendelen about 14,600. In the Ptolemaic system, moreover, the intervals of the planets are so ordered that nothing superfluous intervenes between heaven and heaven, but the lowest part of Saturn, or its perigee, touches Jupiter at apogee, and so of the rest, as we taught in Book 7, section 6, from chapter 1 to 4. But this reason now ceases in the more recent hypothesis. In that hypothesis, if, as the telescope shows, we carry Venus round the Sun in a wider circuit and Mercury in a narrower one, and wish to keep intact the magnitude of the epicycle of Venus derived from her greatest digressions from the Sun and from the Sun's distance from the Earth, then a great space must intervene between the Moon at apogee and Venus at perigee, empty of every ordinary planet. This may be seen by anyone who compares the lunar distances (on which see Book 4, chapter 14) with the least distances of Venus from the Earth on the more recent hypotheses (on which see Book 7, section 6, chapter 2, in table 2).

[Margin: 6. Reason from the multiplicity of motions.]

The *sixth* reason is taken from the multiplicity and irregularity of the motions: for Mercury has more motions than Venus and the Sun, and therefore the Ptolemaics assign Mercury five orbs and an epicycle, but Venus three. But this reason is very weak, for on that ground the Sun would have to be placed above all the planets, because in the simplicity of its motion it is more like the motion of the fixed stars and of the primum mobile than all the planets are. For this reason Aristotle said, in Book 2 *De caelo*, chapter 10, or text 60: *It is no less a marvel for what cause those bodies which are farther from the first motion are not always moved with more motions, etc. For the Sun and the Moon are moved with fewer motions than some of the wandering stars, although these are farther from the centre, and nearer the first of the bodies, than they are.* And yet the latest age has

detected many more motions in the Moon, according to what was said in Book 4, chapter 18. Clavius could therefore have neglected this paltry little reason.

[Margin: 7. Reason from the dependence of the planets on the Sun.]

The *seventh* reason is drawn from the dependence of the planets on the Sun in light and motion. For the Sun is like the King and Heart, the rule and moderator of all the planets, since all the planets are bound in their motions to the motion of the Sun by a certain admirable harmony: this is established for the synodic revolutions of the Moon, and for the periods of the other planets, whether on the eccentric or on the epicycle, it is clear from what was said in Book 7, section 1, chapter 7. The Sun therefore ought to proceed in their midst, so as to direct their motions evenly and to illuminate them, since we see Mars and Venus more strongly illuminated because of their nearness to the Sun. Nor did Clavius refrain from the likeness and analogy of the things customary in human polities, so as to set the Sun in the middle place as a King. For he says that Saturn, being an old man, is the Sun's counsellor; Jupiter, for his magnanimity, the judge; Mars the general of the army; Venus the mother of the household; Mercury the scribe or chancellor; and the Moon, finally, the swiftest messenger — *as most people jest*, says Clavius, for he did not wish these things to be recounted by himself in earnest; granted that Stœflerus [Johannes Stöffler], on Proclus, page 44, relates them in earnest. But these things are said too much in the manner of commonplaces; and by almost the same argument the Copernicans contend that the Sun ought to have been placed in the middle of the whole universe, and Tycho in the middle of the five lesser planets that surround it. Nor must a King or Master or Leader always be in the middle of his people; often he follows behind them all, or sometimes even goes before them.

[Margin: 8. Reason from the activity of the Sun.]

The *eighth* reason, which *Albumasar* [Abū Maʿshar] brings forward in his *Great Introduction*, tractate 3, difference 3, rests on the Sun's activity: for it ought to be placed in the middle of the planets, lest, if it were higher, it should act too little on these lower things, and, if lower, too much. Hence Clavius thinks that Ovid too alluded to this in *Metamorphoses* 2, where he brings in Phoebus guiding with his counsel Phaethon as he drives the solar four-horse chariot:

Going too high, you will burn the signs of heaven; too low, the earth; in the middle you will go most safely.

But if this reason held, the Sun would have to be in the middle of the interval from the Earth to the fixed stars, precisely as to quantity; whereas in no hypothesis, much less in the Ptolemaic, can this be true, as is established from the distances of the planets from the Earth reduced by us into a table in Book 7, section 6, chapter 2.

[Margin: 9. Reason from the days of the week and the dominion of the planets.]

The *ninth* reason is taken from the order and naming of the seven days of the week. Xiphilinus, from Book 36 of Dio [Cassius Dio], in the *Pompey*, reports

that this order was established by the Egyptians: beginning from Saturn and assigning him the dominion of the first hour of the day, and running through the 24 hours in this order of the planets, ♄ ♃ ♂ ☉ ♀ ☿ ☾, the dominion of the first hour of the following day falls on the Sun, of the next on the Moon, then on Mars, afterwards on Mercury, then on Jupiter, and finally on Venus, according to what was said more fully in Book 1, chapter 30. But Dio in the same place, and Xiphilinus, indicate another reason too for this order, taken from the harmonic intervals, which Pliny the Elder and Censorinus, brought forward in number 3, also intimated to us from Pythagoras. And that distribution presupposes the order of the days already mentioned, but does not produce it; for it is not true — indeed it is a most empty fiction — that the Sun rules the first hour of the Lord's day more than any other planet, and so of the rest. The rest concerning the harmonic intervals, which each man tries to draw over to his own system, is to be discussed in section 5 of this book.

For these reasons, then, and chiefly because of its antiquity and authority, this system prevailed over all the others from Pythagoras right down to our own age; and so for about 20 centuries it had on its side the greater part of the astronomers, and many of the theologians and natural philosophers. It is pleasing to review the chief of them.

Auctores qui Pythagoræ & Ptolemæo ſubſcripſere quoad Ordinem Planetarum.

Authors who subscribed to Pythagoras and Ptolemy as regards the order of the planets.

IX. I said "as regards the order": for as regards the number of the heavens, total or partial, some disagreed, as is clear from what was said here in chapter 1; and as regards the hypothesis of eccentrics, the homocentrists disagreed, as we have already said in section 2, chapter 2. But as regards the order of the planets, this system was favoured by *Archimedes* (in Macrobius, Book 2 on the *Dream of Scipio*, chapter 3); the *Chaldaeans* (in Bede, in the book *De elementis philosophiae*); *Cicero* (*On the Dream of Scipio*); *Pliny the Elder* (Book 2, chapters 6 and 7); *Cleomedes* (Book 1 of the *Cyclical Theory*, chapter 3); *Sosigenes* (in Simplicius, on Book 2 *De caelo*); *Albategnius* [al-Battānī] (*De scientia stellarum*, chapter 50); *Alfraganus* (difference 12, where he says he follows the opinion of the ancients); and *Albumasar* (in his *Introduction*, tractate 3, difference 3). These were followed by the greater part of the Arabs, and by the Alphonsines in their tables; by *Io. Fernelius* (Book 1 of the *Cosmotheoria*); by *Purbachius* in the *Theoricae planetarum*, and by his commentators, chiefly *Petrus Nonius* [Pedro Nunes], *Erasmus Reinholdus* [Erasmus Reinhold], *Erasmus Osualdus* and *Christianus Vurstisius* [Christian Wurstisen]. Besides these: *Regiomontanus* (in the *Epitome of the Almagest*, Book 4, proposition 1); *Io. Stoeflerus* (on Proclus's *Sphere*, page 44); *Iulius Firmicus* [Firmicus Maternus] (Book 3 of the *Mathesis*, from chapter 3 to 9); *Iovianus Pontanus* [Giovanni Pontano] (Book 7 *De rebus caelestibus*); *Io. de Sacrobosco* [Johannes de Sacrobosco] (chapter

1 of the *Sphere*), and there his interpreters, enumerated at the beginning of Book 1 of our *Almagest*; *Orontius* (in the *Sphere*, Book 1, chapter 3); *Io. Baptista Amicus* (the little work *De motibus corporum caelestium*); *Io. Antonius Delphinus* (*De globis caelestibus*); *Fracastorius* [Girolamo Fracastoro] (in the *Homocentrica*, sections 2 and 3); *Egnatius Dantes* [Egnazio Danti] (in the tables, from table 12); *Petrus Apianus* [Peter Apian] (in the *Astronomicum Caesareum*); *Gemma Phrysius* [Gemma Frisius] (*De principiis cosmographiae*, chapter 1); *Maurolycus* [Maurolico] (dialogues 1 and 3 of the *Cosmographia*, and at the end of the same); *Io. Antonius Maginus* (in the *Theoricae planetarum*); *Clavius* (on the *Sphere*, from page 64); *Barocius* in the *Cosmographia*, Book 1, page 8; *Rodulphus Goclenius* [Rudolph Goclenius] in the *Uranie*, chapter 4; *Michaël Neander* [Michael Neander] in the *Elementa doctrinae sphaericae*, chapter 4; *Cardanus* [Girolamo Cardano], *De varietate*, Book 1, chapter 1; *Ascanius Martinengus* in the *Glossa magna*, page 968. Finally, to add one or two Fathers: *St Damascene*, in Book 2 *De fide orthodoxa*, chapter 7, supposes that *in the first circle of the heaven, that is the highest, is Saturn; in the second Jupiter; in the third Mars; in the fourth the Sun; in the fifth Venus; in the sixth Mercury; and in the seventh and lowest the Moon.* And Bede, following Pliny, set out the same order in the book *De natura rerum*, chapter 13, and in *De ratione temporum*, chapter 24 — although in Book 2 of *De elementis philosophiae* he teaches that Venus and Mercury go round the Sun.

If anyone wants a figure, though it is not necessary, of the system of Pythagoras[29] and Ptolemy etc. — which may be called the common and most ancient system, and the Chaldaean or Babylonian, as we shall learn from Bede in chapter 4, number 4 — here it is; in it, however, the eccentrics and epicycles are to be understood, according to the diagrams given in Book 7, sections 2 and 3.

Diagram. *I. The most ancient and common system, of Pythagoras, Ptolemy and very many others.* A woodcut in the left column: an outer band, between two circles and divided by radial strokes, carries the twelve signs of the zodiac; small star marks stand in the band next inside it and between the circles; within, a series of concentric circles, with the planetary signs set one above another on a vertical line from the top downward; at the centre, a small disc with a figured interior, ringed by small star-like marks.

- Outer band, clockwise from the top: ♋ (upper left of the top), ♊ (top), ♉ (upper right), ♈ (right), ♓ (lower right), ♒ (lower right), ♑ (bottom right), ♐ (bottom), ♏ (lower left), ♎ (left), ♍ (left), ♌ (upper left) — the signs of the zodiac
- Vertical line, from the top: ♄ Saturn, ♃ Jupiter, ♂ Mars, ☉ the Sun, ♀ Venus, ☿ Mercury, ☾ the Moon
- Centre: an unlabelled disc with a figured interior

[Margin: Clavius's last word on the system, once the telescope had been seen.]

[29] the page prints "Pythagotæ"

X. Thus far the astronomers down to Clavius, as long as the heaven was explored with unarmed eyes. But when the Belgian telescope had been brought to Rome, and Venus had been seen to be illuminated by the Sun like the Moon, in such a way that she must go round it, and other things like these, the good and candid old man Clavius exclaimed — indeed, like a swan near death, he sang of the change of the ancient system in these words, which Scheinerus [Christoph Scheiner] (number 22 of the *Disquisitiones mathematicae*) and Tannerus (*De caelo*, question 10) quote from the last edition, on chapter 1 of the *Sphere*, page 75: *I do not wish, however, this to be hidden from the reader: that not long ago a certain instrument was brought from Belgium, like a kind of oblong tube, in whose ends two glasses, or lenses, are fitted, by which objects far removed from us appear very near, and indeed far larger than they are in reality. With this instrument very many stars are discerned in the firmament which can in no way be seen without it, etc. The Moon too, when it is horned or half full, appears in a wonderful way broken and rough, so that one cannot marvel enough that such great inequalities are present in the lunar body. But on this matter consult the little book of Galileo Galilei which he entitled* Sidereus Nuncius, *printed at Venice in the year 1610. Among the other things seen with this instrument, this holds not the last place: namely, that Venus receives her light from the Sun, like the Moon, so that, horned, she appears now more, now less, according to her distance from the Sun — which I have observed here at Rome, with others, more than once.*

[Margin: Galileo praised by Clavius.]

Saturn too has two stars joined to it, smaller than itself, one towards the east and the other towards the west. Jupiter, finally, has four wandering stars, which vary their position in a wonderful way, both among themselves and with respect to Jupiter, as Galileo diligently and accurately describes. SINCE these things are so, let the astronomers consider in what way the celestial orbs must be constituted so that these phenomena can be saved. Thus Clavius. But let us proceed to the other systems.

CAPVT III.

Chapter 3.

De Syſtemate Platonis, Platonicorum, Gebri, ac Theonis. Qua occaſione diſputatur de Ciceronis Syſtemate.

On the system of Plato, of the Platonists, of Geber and of Theon. On which occasion the system of Cicero is discussed.

I. Just as discord not seldom brings forth concord, so concord has brought forth discord. For there was a wonderful concord between Pythagoras and Plato on this point: that the intervals and order of the planets were to be established from the laws of music. But at once discord was born from this, through the differing opinions on the harmonic interval [diastema] among the Platonists and the Pythagoreans. Nor do all write alike even about the Platonic system itself. For Plutarch — certainly most learned in Greek matters — in Book 2 *De*

placitis philosophorum, chapter 15, after relating the opinions on the order of the stars (on which I spoke in chapter 2, at the end of number 2), continues thus: *Plato, after the position of the fixed stars, puts first Phaenon, which is called Saturn's; second Phaethon, which is Jupiter's; third Pyroeis, which is Mars's; fourth Phosphoros, which is Venus's; fifth Stilbon, which is Mercury's; the Sun, finally, sixth, and the Moon seventh. Some of the mathematicians agree with Plato; some put the Sun in the middle of all.*

[Margin: Plato's system, from Plutarch.]

[Margin: And from Macrobius.]

In a not dissimilar way *Macrobius*, Book 1 on the *Dream of Scipio*, chapter 19, teaches that according to Plato the Sun holds the sixth place among the planets and the Moon the seventh; and in chapter 21: *It is to be noted in this place that, at the birth of the world, either the providence of things itself or the genius of antiquity gave the stars this order which Plato assigned to their spheres: that the Moon should be first, the Sun second, above him Mercury, Venus fourth, then Mars, next Jupiter, and Saturn last.* And from these two sources, I believe, all those drew who ascribe this order to the Platonic system: namely *Copernicus*, Book 1, chapter 10; *Valentinus Naiboda* [Valentin Naibod], Book 1 of the *Institutiones astronomicae*, chapter 16; *Clavius* on the *Sphere*, page 64; *Barocius*, Book 1 of the *Cosmographia*, chapter 2; *Orontius*, Book 1 of the *Sphere*, chapter 3; and *Argolus* [Andrea Argoli] in the *Pandosion sphaericum*, chapter 3 — although only Clavius and Argoli express the position of Venus above Mercury, while the rest count both of them above the Sun, according to the opinion of Plato and the Egyptians.

[Margin: Another system of the Platonists.]

II. Some Platonists, however, varied this order somewhat, and raised Mercury above Venus; for so the same *Macrobius* reports of them, Book 1 on the *Dream of Scipio*, chapter 3: *This measurement of Archimedes, however, was rejected by the Platonists, as not keeping the double and triple intervals. And they laid down that this is to be believed: that as great as the distance is from the Earth to the Moon*[30]*, twice that is from the Earth to the Sun; and as great as it is from the Earth to the Sun, three times that is from the Earth to Venus; and as great as it is from the Earth to Venus, four times as much is from the Earth to the star of Mercury; and as great as it is to Mercury from the Earth, nine times as much is from the Earth to Mars; and as great as it is from the Earth to Mars, eight times as much is from the Earth to Jupiter; and as great as it is from the Earth to Jupiter, twenty-seven times as much is from the Earth to the orb of Saturn.* This Platonic persuasion *Porphyry* inserted in the books in which he shed some light on the obscurities of the *Timaeus*. The same order is asserted by the author of the book *De mundo*, addressed to Alexander, by *Apuleius* in his book *De mundo*, and by *Hamerus* on chapter 1 of Genesis. It was not, then, the author of the little book *De mundo* alone who

[30]the page prints "Lunum"

placed Mercury above Venus and Venus above the Sun, as Clavius said (on the *Sphere*, page 42).

[Margin: The opinion of Marsilius Ficinus.]

Marsilius Ficinus attributes the same intervals and the same order to Plato, in almost the same words, in the *Compendium in Timaeum*, chapter 34; but he adds: *Here you see the heavier planets, Jupiter and Saturn, designated by solid numbers; and although elsewhere, following the opinion of some Pythagoreans, I have reviewed other measures of the intervals, I nonetheless judge the Platonic ones more probable.* And by these he thinks it can be explained what is said rather obscurely about these spheres and proportions in Books 8 and 10 of the *Republic*. I should think, moreover, that the handle for these was given by those words of Plato in which he brings in Timaeus of Locri, the Pythagorean, speaking thus: *God placed the Moon in the first circuit above the Earth; the Sun in the second; then the globe of Lucifer, and the star sacred, as it is called, to Mercury, he fixed in circles equal to the Sun in speed but contrary to it in power; so it comes about that these stars — the Sun, Lucifer and Mercury — overtake one another and are in turn overtaken by one another.* But from the fact that he named Venus before Mercury it does not follow that Plato, or Timaeus, placed Venus lower than Mercury.

[Margin: The motion of Mercury undiscovered.]

For in the *Epinomis* he names that planet after the Sun and Lucifer because its motion was undiscovered, or because formerly it had no settled name among the Egyptians. For so Plato speaks there, in the person of the Athenian: *Of the Sun, I say, and of Lucifer, and of that third one which, since it is unknown, cannot be named; and this came about because the first observer of these things was a barbarian: for an ancient region nourished those who, on account of the clearness of the summer season, were the first to look into these things. Such were Egypt and Syria, where all the stars, so to speak, are always seen clearly, because neither clouds nor rains hinder the view of the heaven.* Therefore, because its nature and motion were undiscovered, it received its name later than Venus. And he adds: *But they have also received names from some people; for Lucifer, which is also Vesper, has indeed its reason; while the star which runs equally with the Sun and together with it is called Stilbon from its brightness.*

III. The order of the first system, in which Mercury stood above the Sun and Venus above Mercury, was defended by *Theon* (Book 9 on the *Almagest*, chapter 7); by *Summartus* (in Io. Picus [Giovanni Pico della Mirandola], Book 10 *Against the Astrologers*, chapter 4); and by *Geber* of Seville (Book 7 of his *Astronomy*, chapter 1), where he censures Ptolemy for having said, in Book 9, chapter 1, that the parallaxes of Mercury and Venus are to be neglected, although the parallax of the Sun is 2′ 51″, that of Venus 3′ and that of Mercury 7′, from the Ptolemaic commensurations; then, likewise from the Ptolemaic latitudes, he shows that the Sun can sometimes be partly hidden by Venus and Mercury. But we have already defended Ptolemy against Geber's objections in the preceding chapter, number 6. Geber concludes, however, that it is more probable that Mercury moves above the Sun, and Venus above Mercury, together with the three other

planets, because of their likeness in the composition of the orbs, in the double anomaly, and in stations and retrogradations.

De Syſtemate Ciceronis, eiuſq. Diſſidio à Platonico.

On the system of Cicero, and its disagreement with the Platonic system.

IV. Thus far, then, we should have two forms of the Platonic system: one in which Mercury lay below Venus and the Sun below Mercury; the other in which the Sun lay below Venus and Venus below Mercury — had not *Macrobius* suggested a third form, in Book 1 on the *Dream of Scipio*, chapter 19, where he inquires what order Cicero, and what order Plato, assigned to the Sun among the wandering stars. Cicero had said in that dream, in the person of the elder Scipio: *All things are bound together by nine orbs, or rather globes, of which one, the outermost, is the celestial, which embraces all the rest, itself the supreme god, confining and containing the others; in it are fixed those everlasting courses of the stars that revolve. Beneath it are seven, which turn backward with a motion contrary to the heaven. Of these, one globe is possessed by the star which on earth they call Saturn's. Then comes that brightness, prosperous and salutary to the human race, which is called Jupiter's. Then the ruddy one, dreadful to the lands, which you call Mars. Then, below, the Sun holds nearly the middle region, leader and prince and moderator of the other lights, the mind and tempering of the world, of such magnitude that he lights up and fills all things with his light. Him the courses of Venus on the one hand and of Mercury on the other follow as companions. And in the lowest orb the Moon revolves, kindled by the rays of the Sun. Below it there is now nothing but what is mortal and perishable, except the souls given to the race of men by the gift of the gods. Above the Moon all things are eternal: for the Earth, which is the middle and ninth, does not move, and is the lowest, and towards it all weights are borne by their own tendency.* This is fully and plainly the Ciceronian system, differing in nothing at all from the Pythagorean and the Archimedean.

[Margin: Macrobius's opinion on the system of Cicero.]

But because Plato assigns the Sun the sixth place among the planets, and Cicero the fourth, *Macrobius*, in Book 1, chapter 19, as I said, investigates the causes of this disagreement; for he says: *Plato followed the Egyptians, the parents of all the disciplines of philosophy, who want the Sun to be placed between the Moon and Mercury in such a way that they nevertheless detected by reasoning, and drew out, why the Sun is believed by some to be above Mercury and above Venus; for neither do those who think so stray far from the appearance of truth.* And a little after: *Now next to Venus is the star of Mercury, and the Sun is near to Mercury, so that these three go round their heaven in an equal space of time, that is, in a year, more or less. Hence Cicero too calls these two courses companions of the Sun, because in an equal space they never withdraw far from one another.* And, after interposing a few words about the Moon, he goes on: *The nearness of these three neighbours, Venus, Mercury and the Sun, confused their order — but among others; for the reason did not escape the skill of the Egyptians, and it is this: the circle through which the*

Sun runs is surrounded, as the lower, by the circle of Mercury; and that circle in turn the higher circle of Venus encloses; and so it comes about that these two stars, when they run through the upper tops of their circles, are understood to be placed above the Sun, but when they pass through the lower parts of their circle, the Sun is thought to be above them. Those, therefore, who said that their spheres are below the Sun took this impression from that course of the stars which, as we said, sometimes appears lower, and which is truly the more noticeable, because they then appear more freely; for when they hold the upper parts they are more hidden by the rays. And so that persuasion prevailed, and this order was received into use by almost everyone. A more perspicacious observation, however, detected the better order. Having said this, and interposed some remarks on the light of the planets, he concludes: *What order Plato or his authorities gave to the Sun, or whom Cicero followed in assigning the fourth place to its globe, or what reason introduced the persuasion of this diversity, has been sufficiently stated.*

Cicero, therefore, followed the more common opinion about Venus and Mercury. Because these planets are discerned [?] longer and better when they digress most from the Sun, and are not so hidden by the light of twilight — and at that time they are in the lower semicircle of their epicycle — they are therefore believed to move below the Sun. And the Egyptians followed that very reasoning in the distribution of the planetary hours, as we said from Dio[31] and Xiphilinus in the preceding chapter, number 7, reason 9; granted that, reasoning about these digressions, or else because, enjoying a clearer air, they sometimes saw Venus and Mercury much diminished in apparent magnitude, they judged that this happened because they rose far from the Earth above the Sun, and so described circles about the Sun.

[Margin: Did Plato follow the system of the Egyptians?]

V. If, therefore, Plato followed the Egyptians, as Macrobius says, and the Egyptians taught that Venus and Mercury move not only above but also below the Sun, Plato might seem to have carried this very thing over to the Greeks, although he expressed only the higher position, because he thought that in the harmonic intervals it was rather their greatest distance from the Earth, and their apogee position, that should be kept, than their least distance and perigee place. Indeed Cicero himself too might seem not to have dissented from Plato, although he expressed only the common opinion, since Cicero's book *On the Dream of Scipio* is part of Book 6 of the *Republic*, and in the books on the *Republic* he imitated Plato, who indeed, in Books 8 and 10 of the *Republic*, gives the reason of the harmonic intervals between the seven orbs, attributed to seven sirens — though in a rather involved way. For that reason I elsewhere judged it probable (Book 7, section 1, chapter 4) that the Platonic and Ciceronian system was in reality the same as the Egyptian, to which Vitruvius, Macrobius, Capella and Bede subscribed. But having considered the matter better, I think it more probable that Cicero held with Pythagoras, and, having consulted

[31] the page prints "Dioue", for "Dione"

Plato's *Epinomis*, that Plato dissents from the Egyptians on this point. For Cicero [?] nowhere indicated that order of Plato's; and although he admitted a concert of the heavens, he subscribed rather to the Pythagorean intervals, and followed the opinion which Pythagoras had taught in Italy and which had passed from Numa Pompilius in Latium to Varro and Cicero. Plato, on the other hand, judged that everything received from the barbarians — for so he calls the Egyptians and Syrians and all foreigners — had been made better by the Greeks; and in this business of the system of the world he judged it better that Venus and Mercury should always be above the Sun and should move uniformly on concentric circles, and not by epicycles now below, and now above, ranging about at changed distances. I gather this from his own *Epinomis*, where he says:

[Margin: Plato's opinion on the motion of the stars.]

That what proceeds in order in the heaven has mind is sufficiently shown by this sign: that it always goes by the same path and in a like manner. The necessity of a soul possessing intellect is the greatest of all necessities; for it governs, leading and not led. When the soul, which is best, conducts itself by counsel according to the best intellect, then what is truly accomplished by the power of intellect will come about of necessity, without persuasion, and not even adamant could hold more solidly and more immovably, etc. It ought therefore to have been a proof to men that the stars and that whole circuit have mind, because they always do the same things; for over a very great and wonderful span of time they have already deliberated beforehand what they are to do, and they do not, going up and down, approve now this and now that by their counsel. Hence they neither wander nor revolve outside their ancient order.

Plato therefore judged that the planets are driven by a moving Intelligence along the same path with uniform motion, and are not carried up and down; and that this is the work of a mind which has once chosen what is best and preserves it immovably. In a word, he judged that they are rolled round on circles concentric with the world. And he thought it very probable that in this the Greeks had judged more perfectly than the barbarians. For further on in the same *Epinomis*, speaking of the knowledge of the stars, which became known to the Greeks later than to the barbarians because of the hindrances of a less purified air, he says:

[Margin: Astronomy older among the Egyptians than among the Greeks.]

But since we are further than the barbarians, as we said, from summer clearness, we have come to understand the order of these deities (for so he calls the stars) *more slowly. But whatever the Greeks received from the barbarians, they rendered better. And this we must bear in mind also in the matters we are now treating.*

It remains to represent in one scheme the twofold form of the Platonic system: that is, Plato's, which Theon, Samerius and Geber supported; and that of certain Platonists, namely Porphyry, Apuleius and Ficino.

II. The Platonic System.

Diagram. A woodcut in the left column: concentric circles about a central Earth, drawn with a wavy outline inside a stippled ring, with small six-pointed

stars set on the outermost circle. Two lines of planetary signs run from the outer circles down to the centre, one on the upper left, headed *Plato*, and one on the upper right, headed *Porphyrius*. Each line reads, top to bottom, Saturn, Jupiter, Mars, the two small signs of Venus and Mercury, the Sun and the Moon; in the left line (Plato) Venus stands above Mercury, in the right line (Porphyrius) Mercury stands above Venus.

- Upper left, above the outer circle: Plato
- Upper right, following the curve: Porphyrius (Porphyry)
- Left line, top to bottom: ♄ Saturn; ♃ Jupiter; ♂ Mars; ♀ Venus; ☿ Mercury; ☉ Sun; ☾ Moon
- Right line, top to bottom: ♄ Saturn; ♃ Jupiter; ♂ Mars (drawn with its arrow to the lower left); ☿ Mercury; ♀ Venus; ☉ Sun; ☾ Moon

CAPVT IV.

Chapter IV.

De Syſtemate Aegyptiorum, ſeu Vitruuij, Martiani Capellæ, Macrobij, Bedæ, & Argoli.

On the system of the Egyptians, or of Vitruvius, Martianus Capella, Macrobius, Bede and Argoli.

I. It might seem that the system of the Egyptians has been sufficiently demonstrated from what was said in the preceding chapter, number 4. Yet because its form is very different from the Pythagorean and the Platonic, since it contains the extremes of both, and because what the telescope has at last made manifest in our own age they foresaw by the acuteness of their wit, it is right to discuss it again here, as in its proper place. When they were investigating, then, why Venus and Mercury digress from the Sun not by a whole semicircle — indeed Venus by scarcely half a quadrant of the circle, and Mercury not even by a twelfth part of the circle of the zodiac — and why about their greatest digressions they appear larger, they judged that this came about from an epicycle described about the Sun, on which they moved sometimes below the Sun, sometimes above it, sometimes at distances equal with it; and that they digress from the Sun in proportion to the size of the epicycle, and appear larger because of nearness to the Earth and smaller because of remoteness. Therefore they prepared for us, even then, a most beautiful system and, for this part at least, a most true one. Let Macrobius be heard again, if you please, Book 1 on the *Dream of Scipio*, chapter 19, telling it thus:

[Margin: The system of the Egyptians, from Macrobius.]

Among the ancients there was no disagreement about the order of the three upper planets, which an immense distance distinguishes manifestly and clearly, nor about the region of the Moon, which has withdrawn far from all the rest. But the nearness of these three that are close to one another — Venus, Mercury and the Sun — confused their order; but among others. For the reason did not escape the skill of the Egyptians,

and it is this. The circle through which the Sun runs is surrounded, as the lower, by the circle of Mercury; and that circle in turn the higher circle of Venus encloses. And so it comes about that these two stars, when they run through the upper tops of their circles, are understood to be placed above the Sun; but when they pass through the lower part of the circle, the Sun is thought to be above them. Those, therefore, who said that their spheres are below the Sun took this impression from that course of the stars which, as we said, sometimes appears lower, and which is truly the more noticeable, because they then appear more freely; for when they hold the upper parts, they are more hidden by the rays. And so that persuasion prevailed, and this order was received into use by almost everyone. A more perspicacious observation, however, detected the better order. Truly more perspicacious: an observation which, so many centuries beforehand, by the sagacity of wit alone, anticipated the use of the Belgian spyglass.

[Margin: Macrobius's system.]

From these same words, moreover, it becomes plain that Macrobius gave his vote to this system — especially since in the same chapter he goes on to confirm that these two planets must also be carried above the Sun, because, like the others, they have their own light (for so he held), whereas the Moon is always below the Sun, since it shines with borrowed light and only from the Sun.

[Margin: Vitruvius's.]

II. Nor do I doubt that *M. Vitruvius Pollio*, Book 9 *On Architecture*, chapter 4, recognised the same order. For although he enumerates the planets thus — *the Moon, the star of Mercury, of Venus, the Sun itself, and likewise of Mars, of Jupiter and of Saturn* — following the common opinion, a little later he says: *The stars of Mercury and Venus, however, crowning the Sun itself as a centre with their paths about the Sun's rays, make their regressions backward and their retardations. Also, by their stations, because of that circling, they linger in the spaces of the signs.* In this sense *Scheinerus* too interpreted him, in the *Disquisitiones Mathematicae*, number 22, towards the end.

[Margin: The system of Martianus Capella.]

III. No less plainly *Martianus* Mineus Felix, surnamed *Capella*, in Book 8 of *On the Marriage of Philology and Mercury*, in the chapter whose title is *On the orbs of the planets*, says: *But Venus and Mercury do not go round the Earth.* And in the following chapter, having asserted eccentrics by saying, *It must be known that, in general, the Earth is eccentric to all the orbs of the planets; that is, it does not hold the middle of the circles, which is without doubt the centre of the world*, a little later he sets out what the centre of the circles of Mercury and Venus is, in these words: *For although Venus and Mercury show daily risings and settings, yet their circles do not go round the Earth at all, but circle about the Sun with a looser compass. In short, they set the centre of their circles in the Sun, so that they are carried sometimes above it, but mostly below it, nearer to the Earth. From it Venus is separated by one sign and a half. But when they are above the Sun, Mercury is nearer to the Earth; when below the Sun, Venus, as being curved in a chaster and more widely spread orb.* This African writer could not have painted the Egyptian system in clearer

words. And further on, in a chapter specifically on Mercury, he says: *So much for the Sun. Now it is fitting to look at the courses of the planets, and chiefly of those which revolve about the Sun. For Stilbon, completing a circle of nearly a year, runs through 8° of latitude, driven by an alternating variation. I recalled above that the circles of this planet and of Venus are epicycles, that is, not within their own compass do they enclose the roundness of the Earth.*

[Margin: Bede's system.]

IV. But from Africa we must sail to England, and seek out the opinion of that most learned and venerable priest, namely Bede. In his book *On the Nature of Things*, chapter 13, he speaks thus: *The highest of the planets is the star of Saturn, cold by nature, going through the zodiac in 30 years. Next, Jupiter's, temperate, in 12 years. Third, that of Mars, burning, in 2 years. In the middle the Sun, in 365¼ days. Below the Sun, Venus, which is also Lucifer and Vesper, in 368 days, never standing off from the Sun further than 46°. Next to it the star of Mercury, swifter in its round by 9 days, shining now before the Sun's risings, now after its settings, never more than 22° removed from it. Last the Moon, completing the zodiac in 27 days and a third part of a day.* But Bede may have said this either when he was younger, or while indulging the common opinion for the moment. For otherwise, in the book *On the Constitution of the Celestial and Terrestrial World*, which is found in the first tome of his works, in the chapter on epicycles and intersections — page 383 in my copy — he speaks thus of Mercury and Venus: *That they are carried both above and below the Sun is shown by conjectures in three ways: either by the intersection of the circles; or because they are epicycles, that is, supercircular, so that, not having the Earth as their centre, they make the Sun as it were the centre of their course; or because they measure out the height of the Sun with circles of obtuse or of acute windings.* That is, as I at least interpret it, with ellipses described about the Sun rather than circles. And in the book *On the Elements of Philosophy* too, judging that the Platonic and the Egyptian system are the same, and busying himself to reconcile it with the Chaldean, he says: *Next it must be said why the Chaldeans say the Sun is fourth, but the Egyptians and Plato sixth. It is true that the Sun is below Venus and Mercury, next to the Moon*; and he gives the reason, namely that the cold and moisture of the Moon may be tempered by the hot and dry Sun, and that the Moon may receive light from the Sun from nearer. Soon he adds: *To the Chaldeans, however, it seemed otherwise*; he puts the cause in the intersection of circles that are nearly equal, and: *They intersect one another, then, in such a way that the circle of Venus in its lower part cuts the upper parts of the circles of Mercury and of the Sun, taking in more of Mercury's than of the Sun's. The circle of Mercury in its upper part cuts that of Venus, and in its lower part the Sun's. The circle of the Sun in its upper part cuts those of Mercury and of Venus — more of Mercury's, less of Venus's.* At length, after a few words, he concludes: *Since, therefore, the circle of the Sun is surrounded in the upper parts of those circles, it is rightly said to be below them; but because it sometimes happens that the Sun runs through the upper parts of its circle while those stars run through the lower parts of theirs, and they then appear more freely — for the Sun does not obscure what lies beneath it as much as*

what lies above it — it has been reckoned above them. This hypothesis, however, is very involved, and the one he had pointed to in the first place, by epicycles described about the Sun, is much neater and more convenient.

[Margin: Argoli's system.]

V. Finally, *Andrea Argoli*, in his *Pandosion sphaericum*, chapter 3, chose this very system, ordering Mercury about the Sun and Venus about Mercury on epicycles, but the orbs of Mars, Jupiter and Saturn about the Earth. He adds, moreover, that what Vitruvius and Capella had left unpolished and simple he has fortified with geometrical demonstrations, adding the size of the eccentricities and of the epicycles, and that on these measures he has published tables of the secondary motions, which he affirms agree to a hair with the Tychonic ones, except for the Moon. These measures he sets out in the same *Pandosion*, from chapter 50 to 59. But in asserting, in chapter 3, that Tycho rests on a very weak foundation when he makes Mars at its acronychal rising nearer to the Earth than the Sun — namely, with a greater parallax than the Sun's — and that this observation is almost impossible, he speaks too boldly, as will be plain to anyone who considers the Tychonic and Keplerian foundations in this matter, which we have already treated in Book 7, section 2, chapter 3, scholium 4; section 6, chapter 4, scholia 3 and 4; and chapter 10, scholium 1.

[Margin: Argoli's boldness against Tycho.]

But here is the scheme of this Egyptian, Vitruvian, Capellan, Macrobian, etc. system. Although someone among the more recent writers ascribes it to Martianus Capella, it is nevertheless far older, and from its first origin I judge that it should rather be called Egyptian.

III. The Egyptian System.

Diagram. A woodcut at the head of the right column: concentric circles about a central Earth drawn as a small globe, with small six-pointed stars on the outermost circle. Saturn stands at the top of an outer circle, with Jupiter and Mars on the circles within it. Within these, a small circle is centred on the rayed Sun, carrying Venus at its top; Mercury stands just below the Sun on a second small circle about it; the Moon stands on a small circle about the Earth.

- Top, outer circle: ♄ Saturn
- Below it: ♃ Jupiter
- To its left, on the next circle: ♂ Mars
- Top of the small circle about the Sun: ♀ Venus
- Centre of the small circles: ☉ Sun
- Just below the Sun: ☿ Mercury
- Small circle about the Earth: ☾ Moon

CAPVT V.

Chapter V.

De Syſtemate Eudoxi, Calippi, & Ariſtotelis.

On the system of Eudoxus, Callippus and Aristotle.

[Margin: Which system Eudoxus and Callippus followed.]

I. The first controversy concerns the order of the planets — namely of Mercury, Venus and the Sun (for there is no doubt about the three upper planets, nor about the lowest, the Moon) — as asserted by Eudoxus, Callippus and Aristotle. And first, Eudoxus, since he was Plato's disciple in philosophy (as Laertius attests, Book 8, in the *Life of Eudoxus*), seems to have followed the Platonic system as well; especially since he chose Plato's opinion on concentric orbs, as we shall soon see. But since Callippus did indeed correct the system of Eudoxus, yet kept the concentrics and only added certain other starless orbs, and Aristotle did the same, adding further revolving orbs beyond Callippus's number, and was moreover Plato's disciple, it seems consistent too that Callippus, equally with Aristotle, retained Plato's system as to the order of the planets.

[Margin: The system of the author of *On the World to Alexander*.]

Add to this that Aristotle, in the book *On the World to Alexander*, chapter 2, has this: *Next to this, that is, to the globe of the fixed stars, the circle called that of Phaenon, and also of Saturn, always has its position contiguous; nearest to it is that of Phaethon, which is also called Jupiter's. After this follows Pyrois, called likewise Hercules' and Mars's. After this again is Stilbon, which is believed to be sacred to Mercury, and by some to Apollo; next to it is the orb of Lucifer, which some call the orb of Venus, others of Juno. After it is the orb of the Sun, and finally that of the Moon, nearest to us, extending its bounds right down to the Earth.* And if the author of this little book is called into doubt, these are certainly the words of Aristotle, Book 1 of the *Meteorology*, chapter 4, where he says: *Moreover, if, as is shown in the astrological theorems, the size of the Sun is greater than the Earth, and the distance of the stars from the Earth much greater than that of the Sun, as that of the Sun from the Earth is greater than that of the Moon, then the cone which the Earth casts from the rays of the Sun is not raised very far from the Earth, nor will the shadow of the Earth, which is called night, be up among the stars; but the Sun must look round on all the stars, and the Earth stand in the way of none of them.* He supposes, then, that no stars can fall into the shadow of the Earth because they are much more remote from the Earth than the Sun — granted that so great a distance is not necessary; but he supposed it from the hypothesis of certain astronomers which places all the planets above the Sun. Finally, that Aristotle supposed Venus and Mercury to move above the Sun is affirmed by *Hamerus*, chapter 1 on Genesis; *Valentinus Naiboda*, Book 1 of the *Institutiones Astronomicae*, chapter 16; *Clavius* in the *Sphere*, page 64; Ba- rocius, Book 1 of the *Cosmographia*, chapter 2; *Longomontanus* in the *Astronomia Danica*, Book 1 of the Theorics, chapter 1; *Argoli* in the *Pandosion sphaericum*, chapter 3. Nor can it be denied that their opinion is probable.

[Margin: The order of Eudoxus called into doubt.]

II. These things notwithstanding, it seems, first, that Eudoxus, being a Cnidian and belonging to the Assyrians rather than to the Greeks, must have followed the opinion of the barbarians, namely of the Assyrians and the Egyptians — all the more because, as Laertius narrates in his Life, he heard Plato at Athens for only two months. Being, that is, both pressed by poverty and thrown out by Plato, he was compelled to return home from there; afterwards, supported by the generosity of friends, he set out for Egypt, and after staying there 16 months he at length returned to Athens with many disciples, to vex Plato, who had dismissed him at the start. Therefore, although he chose concentrics, it is nevertheless not certain which form of the Egyptian system he chose: the one which carried Mercury and Venus above the Sun, or the one which carried the Sun above them.

[Margin: And of Callippus.]

I say the same of Callippus, who, being an Asian, is believed to have been devoted to the Chaldean hypotheses, and whose writings were known to Ptolemy (and, I believe, those of Eudoxus too), as is plain from Book 2 of the *Almagest*, chapter 2; and yet in Book 9, chapter 1, Ptolemy puts Venus and Mercury below the Sun on the opinion of the ancients, and attributes the opposite only to more recent writers.

[Margin: And of Aristotle.]

As regards Aristotle: since he himself says, in Book 2 *On the Heavens*, chapter 10, *But now the contrary happens; for the Sun and the Moon are moved by fewer motions than some of the stars, although these are further from the centre and nearer to the first body than they are*, he seems to have spoken not of all the planets but only of some, as being higher than the Sun and the Moon and yet having more motions. Then, having related in the same place that he had seen Mars occulted by the Moon, in order to show that the Moon is the lowest of all, although it has fewer motions than certain planets nearer the fixed stars, he adds: *And similarly they speak of the other stars too — the Egyptians and the Babylonians, who observed them long ago over very many years, and to whom we give much credence about each of the stars.* It is therefore more credible that Aristotle, as to the order of the planets, either followed the system of the Egyptians, or — because he chose concentrics and placed no planets now above and now below the Sun — adhered rather to the Babylonian or Chaldean system, which put Venus and Mercury below the Sun, as we learned from Bede in the preceding chapter, number 4. For Alexander the Great, having taken Babylon, had already brought back into Greece the observations kept there and recorded over very many years, as may be gathered from Q. Curtius. And, as Pliny relates in Book 7, chapter 56: *Epigenes, a weighty authority of the first rank, teaches that among the Babylonians there were observations of the stars over 720 years, inscribed on baked bricks; those who give the least, Berosus and Critodemus, 480* — understand [?] years of observations. Moved, then, by these conjectures, and by the authority of certain Peripatetics on Aristotle's books *On the Heavens* and Book 12 of the *Metaphysics*, I once held that Eudoxus, Callippus and Aristotle, keeping the concentrics, put the

Sun above Venus and Mercury — granted that I now confess the opposite to be probable. Let us now come to the other part of this system, namely to the number and office of the orbs by which they set out the motions of the planets, while keeping the planets' distance from the Earth always at the same interval.

[Margin: The concentrics of Eudoxus.]

III. On the orbs of Eudoxus, Aristotle, Book 12 of the *Metaphysics*, chapter 7, or texts 45 and 46, reports with wonderful brevity as follows: *Eudoxus, then, held that the motion of each — of the Sun, I mean, and of the Moon — takes place in three spheres: the first being the one which accomplishes the motion of the non-wandering stars*, that is, of the primum mobile; *the second for the motion which takes place along the circle that runs through the middle of the signs*, namely along the ecliptic, in longitude; *the third for the motion oblique to the zodiac, in latitude; and that the circle along which the Moon is carried is inclined at a greater latitude than the one along which the Sun is carried.* As though the Sun did change its latitude somewhat. *The motion of the remaining wandering stars he held to take place in four spheres each: the first and the second undergo the same arrangement as for those; for both the one which has a motion like the motion of the fixed stars and common to all, and the one placed below it, whose motion is along the circle through the middle of the signs, are common to all. The poles of the third sphere of each lie in the circle which passes through the middle of the signs; and the motion of the fourth cuts this circle obliquely and in two. The poles of the third sphere are proper to each of the other stars, but those of Venus and of Mercury are the same.* From this it appears that Eudoxus assigned one sphere to the fixed stars, three to the Sun, three to the Moon, and four to each of the lesser planets — that is, 27 in all; and that he did not think the lower spheres were swept along by the sphere of the fixed stars, but that each planet was rolled by its own sphere with the daily motion through the meridian to the west. Moreover, that he attributed the variation of the obliquity of the ecliptic and of the Sun's greatest declination at the solstices, which some assert, to the motion of the Sun in latitude: for he did not think, says *Simplicius* on Book 2 *On the Heavens*, that the Sun always proceeds under the ecliptic. Each planet, therefore, had one orb for the common motion from east to west; a second for its proper motion under the zodiac towards the east; and a third for the motion of latitude, from south to north. This third orb, as Io. Bapt. Amicus gathers from Simplicius and Theophrastus in his little work *On the Motions of the Celestial Bodies*, chapter 1, completed its motion in the time in which each planet completes its [Greek: Μετάβασιν (metabasin, "passage")], that is, its transit through the aspects to the Sun; and this time, according to Eudoxus, was 18 months for Venus, 110 days for Mercury, 8 months and 20 days for Mars, and 3 months and 10 days for Jupiter and Saturn. The fourth orb, as the same writers and Delphinus (chapter 5 *On the Celestial Globes*) explain, carried the body of the planet, and, by resisting the third orb, prevented the planet from reaching the poles of the zodiac; and therefore the planet described with its centre a certain line which Eudoxus called the [Greek: ὑποπέσην (so printed; the term meant is ἱππoπέδην, hippopedēn, "horse-fetter")]. Eudoxus

also wrote a little book [Greek: περὶ ταχυτήων (peri tachytēōn, so printed; the title meant is περὶ ταχυτήτων, "On speeds")], and other works which have not come down to us. *Fracastorius*, however, interpreted Eudoxus and Callippus otherwise, from mere conjectures, in the *Homocentrica*, section 3, chapter 25; let anyone who cares to waste his effort consult him.

[Margin: The homocentrics of Callippus.]

IV. Callippus of Cyzicus, a contemporary of Aristotle, added to the homocentrics of Eudoxus some other orbs, likewise concentric with the world. For of him Aristotle, Book 12 of the *Metaphysics*, text 47, after setting out the orbs of Eudoxus, writes thus: *But Callippus posited the same position of the spheres as Eudoxus, that is, the order of the intervals, and the same number for Saturn and Jupiter; but he thought that two further spheres must be added to the Sun and two to the Moon, if one wished to give the causes of the things that appear to the senses; and to each of the other wandering stars, one.* Alas, how meagrely Aristotle puts it! Now Eudemus, as Simplicius reports, said that Callippus did this because he wished to save, in the Sun, the delay of the equinoxes and solstices observed by Almeon and Meton; but others, says Delphinus in the place cited above, devised other causes: in the Moon a certain new inequality, and in Mars, Venus and Mercury one sphere each, because of their stations and retrogressions. Moreover, from what Aristotle has said, it becomes manifest that Callippus's spheres were 33, and 34 with the sphere of the fixed stars: namely one of the fixed stars; four each for Saturn and Jupiter; five each for Mars, Venus and Mercury; and finally five each for the Sun and the Moon. Aristotle, however, counts 8 star-bearing spheres and 25 starless ones, that is, 33, as we shall soon see.

[Margin: Aristotle's revolving spheres.]

V. Finally, Aristotle himself, in the same Book 12 of the *Metaphysics*, text 47, after setting out Callippus's opinion on the number of the concentric spheres, wrapped up rather than unfolded his own opinion in these words, and the common translator poured in no little cuttlefish ink; but we translate that text more clearly thus: *It is necessary, however, if the spheres already mentioned, put together, are to restore each planet to its former place, to set under each of them as many other spheres, less one, which revolve them back and bring the first sphere of the star that is ordered below into the same position. For only in this way can it come about that all of them accomplish the motion of the wandering stars. Since, then, the spheres in which the stars are carried are eight, and the remaining ones twenty-five, of these only those do not need revolving back which carry the body of the planet, or which are the lowest in their order. Those which revolve back the spheres of the first two will be six; those which revolve back the spheres of the four after them, sixteen; and the number of all of them, both of those which carry and of those which revolve these back, fifty-five. But if one does not add to the Moon and the Sun the motions we spoke of, all the spheres will be forty-seven.* These words still seem like a riddle; but we shall solve this riddle if we lay open Aristotle's meaning point by point. First, then: among the partial spheres whose number he is inquiring into, Aristotle does not count the first, although he presupposes it, because,

being endowed with a single simple motion, it needs no other orb. Hence it is that he does indeed count eight orbs of Eudoxus provided with stars, but 25 that are ἀνάστρους [Greek: anastrous, "starless"], that is, lacking a star, or not carrying the body of a star; and the sum of these is 33. Second, Aristotle adds no revolving spheres to the Moon. For although at the end he may seem to say that he has added revolving spheres to the Moon and the Sun, he by no means says this; what he says is that if revolving spheres are not added to the Moon and the Sun, of the kind, namely, that he had added to the first two planets and to the four remaining ones, the number of spheres is 47. As for why he did not fear that the Moon would be swept along by the higher orbs, and so why it had no need of revolving spheres, the reason was perhaps the wonderful speed of the Moon in its own proper motion; or rather, that the Moon has no orbs beneath it to which it might communicate its motion. Third, he thinks that one revolving sphere apiece must be added to all the other spheres of the planets, but only to the starless ones, to prevent or compensate the sweep of the higher ones; but to the spheres carrying the body of the planet he adds no revolving sphere. Fourth, reviewing Callippus's number of starless spheres, he counts as many revolving spheres for them: namely, 6 for Saturn and Jupiter together, and four apiece for Mars, Venus, Mercury and the Sun, and so 16 together, and none for the Moon. In Callippus, however, there were 5 lunar spheres, and 5 for the Sun, 5 for Mercury, 5 for Venus, 5 for Mars, 4 for Jupiter and 4 for Saturn. When these are brought into one sum, the planetary spheres come to 55. Fifth: since the stations and retrogradations[32] of Mars, Mercury and Venus were more evident in that age than those motions on account of which Callippus added two spheres apiece to the Sun and the Moon, if someone wishes to mix the Eudoxian number with the Callippic — following Eudoxus as regards the Sun and the Moon, since he gave the luminaries three spheres apiece, six taken together, but following for the remaining planets the Callippic number increased by the revolving spheres — the planetary spheres come out at 47. To make this reckoning more evident, we set out the numbers of the spheres according to these three authors.

Number of the spheres.

Planet	Eudoxus	Callippus	Aristotle: starless	Aristotle: all, with the revolving spheres (Callippic)	Aristotle: all, with the revolving spheres (Eudoxo-Callippic)
♄ Saturn	4	4	3	7	7
♃ Jupiter	4	4	3	7	7
♂ Mars	4	5	4	9	9
♀ Venus	4	5	4	9	9
☿ Mercury	4	5	4	9	9
☉ Sun	3	5	4	9	3
☾ Moon	3	5	4	5	3
Sum	26	33	26	55	47

[32]printed *tetrogradationes*

Planet	Eu-doxus	Cal-lip-pus	Aristotle: starless	Aristotle: all, with the revolving spheres (Callippic)	Aristotle: all, with the revolving spheres (Eudoxo-Callippic)
With the fixed stars	27	34		56	48

[Margin: Errors of others on the Aristotelian number of spheres.]

VI. From this you now see how great the number of spheres really was for Eudoxus, Callippus and Aristotle, and how variable, according as the sphere of the fixed stars was or was not counted in, and according as the Callippic hypothesis alone, or one mixed from the Callippic and the Eudoxian, was used. The numbers some writers give therefore by no means stand, nor can they rest on Aristotle's text. Take Kepler, who says of Eudoxus, Callippus and Aristotle in the *Commentaries on Mars*, chapter 2: *For since those authors employed 25 orbs to demonstrate the whole inequality of the planets, Aristotle, believing the heaven to be packed with solid orbs, judged that 24 others, revolving ones, should be interposed, so that each lower orb should be freed from the sweep which, through the contiguity of the surfaces, it would suffer from the higher. Therefore, having accumulated 49 orbs in all (or, according to Callippus, 53 or 55), he added a mover to each one.* Nor do Io. Antonius Delphinus, in his book *On the Celestial Globes and Motions*, chapter 8, or Fracastoro, section 3, chapter 25 — where both expound this passage of Aristotle with the greatest diligence — give the correct number. For Delphinus says that Aristotle has 55 by the Callippic reckoning but 49 by the Eudoxian; yet he correctly counts the Callippic spheres separately as 34 and the Eudoxian as 27, with the fixed stars. Fracastoro, for his part, does not count 47, but leaves out Aristotle's second computation. Hevelius too, in chapter 7 of the *Selenographia*, wrongly counts the Aristotelian orbs as 53 or 54. The Coimbra commentators [Conimbricenſes], in Book 2 of the *De caelo*, chapter 3, question 8, article 2, give 47 or 50; and Mastrius and Bellutus [Bartolomeo Mastri and Bonaventura Belluto], disputation 2 on the *De caelo*, question 1, number 35, give 55 or 57. But all of them stray in some respect from the true number. For Aristotle has 55, and with the sphere of the fixed stars 56; but if he subscribes to Eudoxus as regards the Sun and the Moon, and to Callippus in the rest, there are 47, and with the sphere of the fixed stars 48.

VII. Sosigenes, as Simplicius relates, uncovered many imperfections in this system, inasmuch as it did not save many of the phenomena of the planets. But it certainly errs most of all in this, that it uses homocentric spheres; for from the variation of the lunar parallaxes, and of the apparent magnitude of the planets, it has now become evident, physically and optically, that the planets are at one time higher and at another lower [?]. Rightly, therefore, he rejected the homocentrics and took refuge in the Pythagorean eccentrics and epicycles.

CAPVT VI.

Chapter VI.

De Syſtemate Auerrois, Alpetragij, Delphini, & Io. Bapt. Amici.

On the system of Averroes, Alpetragius, Delphinus and Io. Bapt. Amicus.

[Margin: The system of Averroes.]

I. How doubtful and inconstant Averroes was about the system of the heaven is clear from *De caelo* 2, commentary 35, from *Metaphysics* 12, commentary 45, and from his commentaries on the *Almagest*; for there, as to the order of the planets, you cannot tell whether he subscribes to Ptolemy or rather to others, although he leans towards Ptolemy's order. He nevertheless strives with many arguments to overthrow the eccentrics and epicycles and to substitute the ancient concentrics in their place. In that commentary 35 he promises that, if God prolongs his life, he will treat of that theory of the celestial motions which was cultivated in Aristotle's time, and which he reckons agreeable to physical principles; but in *Metaphysics* 12, commentary 45, acknowledging himself worn out with old age, he despairs of the matter, exhorts posterity to it, and hopes that someone will not be wanting to take up this task. And not much later Alpetragius appeared, who turned his mind to it. His work is entitled *Theorica Planetarum, physicis rationibus probata* [*Theory of the Planets, Proved by Physical Reasons*], because he suspected that Ptolemy's hypotheses, while not indeed impossible as theory, conflict with physical principles and get in each other's way; although, as to the numbers of the motions and the periods of the times, he follows Ptolemy to the letter. Moreover, at the end of this work he himself admits that he has passed over many other particulars bearing on the motion of the planets which are in Ptolemy's *Almagest*, because he saw that he was now worn out with old age.

[Margin: The system and hypotheses of Alpetragius.]

II. First, then, Alpetragius teaches that there is a ninth sphere, which is moved by one Intelligence with the single, most perfect motion of the primum mobile, in the space of 24 hours, from east through the meridian towards the west; and that the same Intelligence impresses its motion on the eight lower spheres, but uniformly difformly, ever slacker and slacker. Hence the sphere of the fixed stars is moved somewhat more slowly towards the west and does not revolve wholly back to the same meridian in 24 hours, but in a somewhat greater small addition of time; the sphere of Saturn, again, is moved more slowly still, and the sphere of Jupiter more slowly than this, and the sphere of Mars more slowly than this, and Venus than Mars, and the Sun than Venus, and Mercury than the Sun, and the Moon than Mercury. This is because, through an imperfect participation in that impulse, they are more sluggish; and so observers were deceived into thinking that the fixed stars and the planets have a proper motion contrary to the motion of the primum mobile, whereas in reality they are not moved by a contrary motion, but by a motion of the same kind, though slower and slower the further they are from the sphere of the primum mobile.

III. Second, he establishes in chapters 8, 9 and 11 that Venus must be placed above the Sun, but Mercury below it, because it revolves back to the same meridian more quickly than the Sun and is therefore nearer the highest sphere. In the same place he affirms that Venus and Mercury have a light of their own, and that therefore, although Mercury comes between our eye and the Sun, it does not darken any part of the Sun. Further, he ascribes the slowness and speed of the motions, and all their inequality, as well as the stations and regressions, to the motion of the poles of each sphere, which describe various small circles around the poles of the world or of the ecliptic. But it follows from his Theory that the planets always keep the same distance from the centre of the world; and yet he gives no reason why they appear smaller and larger than themselves. The planets, moreover, describe a kind of spiral, in Arabic *laulabina*, but on the same spherical surface.

[Margin: The system of Io. Antonius Delfinus.]

IV. Io. Antonius Delphinus of Casalmaggiore, of the Franciscan order, in his book *On the Celestial Globes and Motions*, from chapter 30 on, posits, besides the Empyrean, a ninth sphere or crystalline heaven, to which he assigns the office of the primum mobile; but he holds that each sphere has its own Intelligences moving it westward along spiral lines. As to the order of the planets, he follows Ptolemy, but he uses concentrics; and the cause for which the planets appear to have different magnitudes he attributes to the varying density and opacity of the air, through which the apparent disc of the stars is enlarged by refraction, and to the varying disposition of the eyes.

[Margin: The system of Io. Bapt. Amicus.]

V. Io. Baptista Amicus, by contrast, in his short work *On the Motions of the Celestial Bodies according to Peripatetic Principles, without Eccentrics and Epicycles*, defends Aristotle's revolving spheres and uses homocentrics; the cause of the varying apparent magnitude of the same planet he lays partly on the deception of sight and of instruments, partly on the varying temper of the air. In the eighth sphere, however, he admits a proper motion towards the east, and an unequal one at that. He therefore posits ten spheres, of which the tenth is the Primum Mobile, and the ninth and the eighth serve for the motion of the fixed stars, which is equal in itself but unequal by accident, through a titubation or trepidation, exactly as the Alphonsines held; and he also follows their order in placing the planets. Nor does he recognize any superior planets other than Saturn, Jupiter and Mars. The systems of these men, however, are so obsolete that it is not worth drawing a figure of their own to explain them; let it be enough to have indicated their hypotheses.

CAPVT VII.

Chapter VII.

De Syſtemate Io. Bapt. Turriani & Hieronymi Fracaſtorij.

On the system of Giovanni Battista della Torre and Girolamo Fracastoro.

I. I shall begin with the letter which Girolamo Fracastoro of Verona, outstanding in the adornments of astronomy, medicine and poetry, wrote to Pope Paul III in order to dedicate his *Homocentrica* to him. In it he affirms, among other things, that all philosophy, indeed nature itself and the celestial orbs themselves, had always protested against the eccentrics and epicycles of Hipparchus and Ptolemy; and that many had therefore striven, but with vain labour, to explain the causes of the celestial motions in another way. He then relates as follows:

[Margin: Praise of Io. Bapt. Turrianus for his astronomy.]

In our age Io. Bapt. Turrius, our fellow citizen, a man of the greatest and almost divine genius, and a cultivator of a certain more secret philosophy, discovered two wonderful motions in the spheres — those, I believe, which both al-Battānī and most other astronomers divined lay hidden in the stars. Through them he seemed to himself to have found an entry to many things, but chiefly to demonstrating without any eccentrics the things that are seen about the stars. He had begun to turn this matter over in his mind with great hope when, fortune deciding otherwise — he would better have said, God — *he was overtaken, at its very beginnings, by an untimely and unexpected death: a young man in age, most flourishing and admirable in talent, in studies, in virtue and in every discipline. When he was now about to die* (just as they say Mercurius Trimegistus [Hermes Trismegistus], dying, asked the friends standing by to remember that ship which he had taught to rise and descend perpetually in the midst of the aether)*, he turned to us, his friends who stood by, and after he had said many other things, then, looking at me, he said: "There is something, Fracastoro, that I would ask of you last of all." When I urged him to say it, he said: "I would have you remember those circles which Timaeus first cuts in the shape of the letter X and then bends back so that the ends of the lines meet"* (he meant the motions he had discovered)*; "for if I may hope that you will accomplish what death now begrudges me, and complete the business I have begun, it will be a great consolation to me as I die." And I to him: that I would remember, etc.*

[Margin: Fracastoro, reviver of the homocentrics.]

II. This, then, was what led Fracastoro to think out how he could explain all the celestial motions by homocentric orbs alone, that is, by orbs concentric with the centre of the world. But to accomplish this he had to give a reason why the Moon and the other planets appear at times larger than themselves and at times smaller; and since he could not refer this either to eccentrics or to epicycles, which had been banished from his hypothesis, he tried, in section 1, chapter 8, and in section 3, chapter 23 — which is entitled *On the variation of view in the quadratures, and in fast and slow motion* — to put the cause down to a variation in the medium: if not in the air, at least in a heaven rarer in some places and denser in others; a heaven, I mean, lying beneath the Moon, which moves in such a way that whenever the Moon is in the quadratures it

shows its disc larger in appearance than usual, because of the refraction of the solar rays in the denser but transparent part of that zone which glides between, beneath the zodiac and beneath the Moon. But this man, otherwise so great, does not seem to have understood what that variation of view is from which Ptolemy shows the variation in the Moon's distance from the Earth. For it is not merely a difference of apparent magnitude, but the difference between the true place and the place seen by us from the surface of the Earth — as we have often set out, but chiefly in Book 1, chapter 35, Book 3, chapter 7, and Book 4, chapters 13 and 14. When these last passages are read again it will be clear that the difference of the lunar parallaxes is so great, and of such a kind, that its cause cannot be put down to the Fracastorian refraction, nor yet to the nearness of the greater light, which is the other cause he adduces. Moreover, Fracastoro presupposes the Ptolemaic order of the planets. Finally, he devises a great many homocentrics, by whose means the inequality of the proper motion of the fixed stars and of the planets is produced. I shall list them briefly, useless though they are.

[Margin: The orbs of the Aplanes: 7.]

III. First, then, for the Aplanes, or sphere of the non-wandering stars, he employs 7 orbs, in section 2, chapter 17. The first and highest is the *Primum mobile*, carrying all the lower ones with it; its motion, the simplest of all, is made in 24 hours from rising to setting along the equator. The second is the *Circumducens* [the leading-round orb], which, carried round in latitude along the upper colure from north to south in 3600 years, rolls all the lower orbs with it, except in so far as some, carried the contrary way, hinder it; the circle of its motion cuts the equator at right angles and has its poles on the equator. The third is the *Circitor* [the circling orb], whose circle, subordinate to the lower colure of the equinoxes, is carried round in latitude by the upper circle in 3600 years; but of itself it moves everything in the opposite direction, and it completes 4° in the time in which the Circumducens completes the quadrant of its circle. The fourth orb is called the *Contrauectus* [the counter-carried orb], carried contrary to the Circitor and to the Circumducens in latitude; moving twice as fast as the Circumducens, it is carried along the upper colure from south to north. The fifth is the *Anticircitor*, which is rolled by the fourth in latitude, always to the side opposite to the Circitor, and proportionally; from this it comes about that by its proper motion it is always opposed to the Circitor in latitude, but agrees with it in longitude and drives in the same direction. And so that the Aplanes is not moved in latitude, that is, so that the fixed stars do not vary their latitude, a Contrauectus follows the Anticircitor and prevents this motion. Last, in the seventh place, is the Aplanes, that is, the sphere of the fixed stars itself, which of itself, by its mean motion, completes one degree in 100 years, but by accident now more quickly, now more slowly; and the period of its equality is completed in 3600 years.

[Margin: The orbs of the planets.]

IV. To Saturn, moreover, Fracastoro assigns 10 orbs, in section 2, chapter 21; to Jupiter 11 orbs, in chapter 24; and to Mars 9, in chapter 27. In section 3, chapter 10, he gives the Sun 6 orbs, or at least 4; in chapter 15, Venus 11 orbs; and in chapter 19 to Mercury 11 orbs; and finally, in chapter 24, he assigns 7 orbs to the Moon. These orbs he calls Circitores, Circumducentes, Contrauecti, Anticircitores, and so on; and to them he adds 7 orbs carrying the bodies of the planets. There is no need to shake out the superfluous furniture of this hotchpotch of solid orbs, since we have already rejected solid orbs in section 1, chapter 7, and have established the variation of the distances from the Earth in each planet by far stronger arguments in Book 7, section 6, chapter 1. For the sake of a little erudition only, therefore, it is enough to know that the sum of Fracastoro's homocentric orbs rises to the number 77 or 79; but if you count the starless orbs, Scheinerus rightly said that there are at least 70, or else 72, as is clear from the following small table.

Fracastoro's orbs.

Orbs of	Number
the Aplanes	6
Saturn	10
Jupiter	11
Mars	9
the Sun	4 or 6
Venus	11
Mercury	11
the Moon	7
Star-bearing	8
Sum	77 or 79

CAPVT VIII.

Chapter VIII.

Et Syſtemate, Tychonis, Longomontani, Blancani & multorum Tychonis ſectatorum.

And [?] the system of Tycho, of Longomontanus, of Blancanus and of many followers of Tycho.

I. The Tychonic system is proposed and expounded by Tycho himself in volume 1 of the *Progymnasmata*, from page 477, and in the letter to Rothmann dated 1589 February 21, old style; but more fully in volume 2 of the *Progymnasmata*, chapter 8, from page 185. This system, but with the diurnal motion of the Earth taken on, was chosen by Longomontanus in the *Astronomia Danica*, Book 1 of the Theorics, chapter 1, and by Blancanus in the *Sphaera*, together with most members of our Society in their writings on the heaven; and Scheinerus inclines to it in the *Disquisitiones Mathematicae*, number 22. Io. Antonius Maginus afterwards commended it in a letter to Tycho dated 1590 September

13 [Idibus Septembris], which is found on page 52 of Tycho's own *Mechanica Astronomica*; and on it he built the tables of the *Supplementum Ephemeridum*. Tycho, then, judged the assumption of epicycles together with the Ptolemaic eccentrics to be inelegant, indeed superfluous, and held that the Ptolemaic equants likewise sin against the first principles of this art, since they suppose that the planets move with some physical inequality — that is, that they do not move uniformly about the centre from which they are described, but about another centre. On the other side he was offended by the Copernican system, by which the absurdities of the Ptolemaic system are emended only in such a way that one falls into a greater absurdity: because of the diurnal and annual motion of the Earth, contrary to the principles of physics and to Holy Scripture, and because of the immense vastness between Saturn and the fixed stars. For these reasons, as it were, he turned his mind to another, new system, and set it out for us in the following scheme.

[Margin: Exposition of the Tychonic system.]

II. In this system the Earth rests at its own centre in the centre of the universe, wholly free of annual motion; and from the same centre is described the outermost or highest sphere of the fixed stars, advancing slowly eastward with an equal proper motion. Further, the centres of the orbs of the Sun and the Moon turn about the centre of the Earth, since these serve to distinguish the times. But the centres of the orbs of the five lesser planets, he denies, turn about the centre of the Earth or look to it of themselves; rather, they turn about the Sun and measure their eccentricities from it. For in this hypothesis all these five planets have the Sun as the centre, so to speak, of their revolutions, so that the Sun, like Apollo in the midst of the Muses, governs the whole harmony of the planetary choir. From this, therefore, he hopes that not only the cause of the inequalities, the stations and the regressions can be accounted for, but also that of the admirable connection which the other planets have in their motions with the motion of the Sun; and he promises to account for it in his work *On the Restoration of Astronomy*. What is most worth noting, moreover, appears in this scheme: Mars when acronychal — that is, when it appears at both ends of the night, being opposite the Sun — descends so far towards the Earth that it becomes nearer the Earth than the Sun itself. This must happen both on the hypothesis and observations of Copernicus and on Tycho's own observations, according to what was said in Book 7, section 2, chapter 3, scholium 4, and section 6, chapter 10, scholium 1. The result is that the circle on which Mars is carried intersects the orb or circle on which the Sun itself is carried; but no physical absurdity arises from this, since for Tycho the whole heaven is liquid and permeable, as I taught in section 1, chapter 7.

IV. The system of Tycho and of Blancanus, and in some respects of Longomontanus.

Diagram. A woodcut in the right column showing the Tychonic system. An outer double circle, studded with small stars, encloses the whole; at the centre is a dark disc, the Earth, which carries no sign. Inside the ring of the

fixed stars stand the circle of Saturn and, within it, the circle of Jupiter, each marked by its sign near the top and again near the bottom. About the Earth run a small circle carrying the Moon, set as a crescent just above the Earth, and a larger circle carrying Mars below the Earth. Above the Earth the rayed disc of the Sun carries two small circles, with the signs of Mercury and Venus on them. The circle of Mars intersects the circle of the Sun. A dark smudge to the upper right of the figure carries no sign.

- Top of the outer planetary circle: ♄ Saturn
- Below it, top of the next circle: ♃ Jupiter
- Upper small circle about the Sun: ♀ Venus
- Inner small circle about the Sun: ☿ Mercury
- Centre of the small circles: ☉ Sun
- Just above the Earth: ☾ Moon
- Below the Earth: ♂ Mars
- Near the bottom: ♃ Jupiter
- Near the bottom, lowest: ♄ Saturn

[Margin: The theft of Nicolaus Dithmarsus, and his error about Tycho's system.]

III. In the letter to Rothmann indicated in number 1, Tycho complains that this system of his was passed off by Nicolaus Raymarus Ursus Dithmarsus [Nicolaus Reimers Ursus] as his own; but he convicts him of theft or plagiarism, from the fact that this Nicolaus had been Tycho's servant and had fled from him, and had copied this system for himself from a certain imperfect scheme in which, however, through carelessness, the orb of Mars enclosed the whole orb of the Sun without any intersection. So, since he was unable to correct that error, he boasted of the Tychonic system, though stained with this fault, as his own before Wilhelm, Landgrave of Hesse. Tycho therefore affirms there that by this single indication alone the man's imposture is manifest; and so does Longomontanus in the preface to Book 1 of the Theorics. He adds, moreover, that it was not the Copernican hypothesis inverted that gave occasion to these hypotheses[33] of his — for he had seen no such inversion — but both the approach of acronychal Mars to the Earth and the phenomena of comets, which he judged could not be explained otherwise if the annual motion of the Earth were excluded. Granted, Rothmann did object this to him, namely, that his hypothesis was the Copernican one inverted, as appears from Tycho's *Epistolae*, page 128.

[Margin: The agreement and disagreement of the Tychonic and Copernican systems.]

IV. As for Scheinerus's assertion, in the *Disquisitiones Mathematicae*, number 22, that the Tychonic system shares nothing with the Copernican, it must be taken with a grain of salt. It is true as regards the annual motion of the Earth, the rest of the fixed stars, their distance, and the other things that depend on

[33] printed *hypotheſibns*, a turned u

these; otherwise there are many things common to both systems. Namely: the Sun is the centre of the planetary system, the Earth the centre of the lunar motion; Mars in opposition to the Sun nearer the Earth than the Sun.

Whether Tycho admitted the diurnal motion of the Earth is commonly doubted. For he nowhere taught by what means the fixed stars and the planets are moved with the diurnal motion; and in volume 2, page 187, he seemed to repudiate only the annual motion of the Earth, in these words: *I judge that it must be laid down beyond all doubt — with the ancient astronomers and the received opinions of the natural philosophers, and with the sacred Scriptures moreover attesting this very thing — that the Earth which we inhabit occupies the centre of the universe and is not turned round with any annual motion, as Copernicus wished*; and Kepler thought that he was not averse to this motion.

[Margin: Tycho accepts neither a diurnal nor an annual motion of the Earth.]

But if what he himself contrives against Copernicus's hypothesis in the letters to Rothmann is read attentively, it will without doubt be judged that Tycho denied every motion to the Earth, and indeed that he brought a particular argument against the diurnal motion, both from the opinion of St Augustine and from the perpendicular fall of a lead ball. For in the letter of 1589 November 24, old style, page 156, he says: *The question was raised whether the mobility of the terrestrial globe — and that a threefold one, as asserted by Copernicus — can really hold, whether it ought to be preferred to our discovery, and whether the sacred Scriptures are contrary to that imagination of his or not. If, therefore, you have anything from the sacred oracles or from their interpreters, such as Augustine or other Fathers, that makes for this Copernican assertion and for your agreement with it, cite it from their writings. Provided the reasons are sound, I shall not resist. This much I know well: that Augustine, whom alone you name, never granted the Earth either an annual or a diurnal motion, inasmuch as, being less of a mathematician, he detracted from its roundness by denying the Antipodes. This, however, must be forgiven him. Therefore, since he did not perceive that it is round, he could far less admit a motion proper to roundness.* And at the end of the same letter he adds: *But since I see that the Copernican opinion about the threefold motion of the Earth pleases you greatly, I shall propose on each of these at least one doubt, and not a very laborious one, though more could be given. First, as regards the diurnal motion about its own axis, by which the Earth is imagined to be turned round in 24 hours, and the universal course from east to west is thus accounted for: tell me how it could happen that a lead ball let fall in the proper way from a very high tower should touch exactly the point of the Earth placed perpendicularly below it. For geometrical computation will teach you that this could by no means happen while the Earth is meanwhile carried round, since its course is very swift: in a single second of time the Earth would have to turn, even in these northern regions, approximately 150 greater paces. From this, reason out the rest. For the falling lead does not accompany the air, but passes through it violently.*

When Rothmann replied to this, in a letter of 1590 April 18, that according to Copernicus's view heavy and light bodies are endowed with two motions

— one by which they follow the motion of the whole in a circle, the other by which they tend upward or downward — Tycho pressed against him again, adducing a further argument from balls fired from a cannon, one towards the east and the other towards the west, as may be seen in his letters from page 190. Again, in volume 1 of the *Progymnasmata*, after confirming from its diurnal motion, by which it complied with the motion of the primum mobile, that the new star of 1572 was celestial, he says: *Unless perhaps we were willing to assent that that universal motion takes place in the Earth, as pleased Copernicus and certain of the ancients; which, however, must in truth by no means be conceded, as we shall make abundantly plain elsewhere.* It must be concluded, therefore, that neither an annual nor a diurnal motion of the Earth ever appealed to Tycho.

[Margin: Tycho's system defective in the diurnal motion.]

From this, however, it is gathered that his hypothesis is incomplete, since he never taught how, in a liquid heaven, the stars can perform both the motions that appear to us without physical contradiction — an imperfection that Galileo also noted in his *Trutinator*, page 25; nor did Kepler sufficiently wash out this stain in the *Hyperaspistes*, page 190.

[Margin: Censure of Tycho's system by Rothmann and others.]

V. Rothmann detected another imperfection, too, in Tycho's system, as he himself reports in his letter of 1588 September 19 [13. Calendas Octobris]: namely, a confusion of the spheres, chiefly because Venus, Mercury and Mars are made to wander up and down through the heaven of the Sun. For he says: *Moreover, this inverted scheme of Copernicus introduces confusion into the spheres of the planets. For although no collision of the celestial bodies can follow from it, as you strive to plead, yet no true or determinate distinction of the spheres remains, but all are confused with one another. The Copernican hypothesis, by contrast, assigns to each planet its own determinate space, from which it cannot depart in its motion. And you know that God is the author not of confusion but of order, and that he so founded Nature that she abhors confusion.*

[Margin: The censure of Herigonius and others.]

The same objection is made against him by Malapertius [Charles Malapert], Tannerus and Amicus, in the places cited in Book 7, section 6, chapter 4, scholium 4, and by Herigonius [Pierre Hérigone] in volume 5, page 548; and further, that it is not likely that the larger circles of 5 planets are carried round by the smaller circle of the Sun. But the first objection is answered by Tycho, page 149, in the letter of 1589 February 21, in these words: *Nor is any confusion committed in my hypotheses; for the heaven is one and like itself from the Moon to the eighth sphere, and in it the planets ascend and descend freely, according as they go round the Sun, which walks in their midst, in the most beautiful harmony. For I do not believe that you wish to introduce once more any real distinction of the celestial orbs, against which we agreed between ourselves long ago.* The second objection also falls, because the Sun's circle carries the other circles not effectively but objectively, and is the centre to which their motions look.

Accordingly Longomontanus, in Book 1 of the Theorics, chapter 1, extols Tycho's system and hypotheses with wonderful praise, because in the three superior planets the orbs described about the Sun play the part of the Ptolemaic epicycles — that is, because the single annual orb of the Sun serves them as a single great epicycle — while, conversely, the epicycles of Venus and Mercury described about the Sun serve them as orbs; and so each and every one of the lesser planets has its own orbs eccentric to the Sun.

[Margin: In what Longomontanus's system differs from Tycho's.]

VI. Longomontanus himself, however, thought that the diurnal motion of the Earth should be introduced into the Tychonic system, as Origanus had done: on the one hand, so that by this single motion, and a uniform one, the apparent motion of the stars towards the west might be represented, while in reality they have a single and simple motion towards the east; on the other, so that the incredible speed of the diurnal motion might be taken away from the fixed stars. Indeed, he does not attribute any real proper motion at all to the fixed stars towards the east; for he fears that this would lead to their being drawn apart from one another and to a variation of their distance, which has nevertheless never been found to vary. That fear would be empty, and would vanish altogether, if he made the heaven of the fixed stars solid and the stars fixed in it; but he makes that heaven, not to mention that of the planets, fluid and most limpid, and the single globes of the stars hanging in balance in the free aether, in the way that the Earth hangs balanced in the air. Accordingly he attributes the apparent motion of the fixed stars to a precession of the equinoxes taking place in the Earth. So Longomontanus has it in Book 1 of the Theorics, partly in chapter 1, page 161 of my copy, partly in chapter 4, page 220 of my copy.

Moreover, when he says on the same page 161 that the centre of the Earth is the centre of the Sun and the Moon, and indeed that their orbs are homocentric with the Earth, he alludes to his own hypothesis of the Sun and the Moon. For he explains their motions not by eccentric orbs, as Tycho did at least for the Sun, but by a single epicycle of the Sun carried by a concentric, and by two epicycles of the Moon — yet in such a way that the centre of the lunar orb is carried round the Earth on a small circle concentric with the Earth, as we have set out sufficiently in Book 3, on the Sun, chapter 19, and in Book 4, chapter 26, number 8, on the Moon.

[Margin: The system of Longomontanus is Semi-Tychonic.]

Although Longomontanus, then, follows Tycho in the fluidity of the heaven, in carrying the five lesser planets round the Sun and the Sun and the Moon round the Earth, and in the very order of the planets, he nevertheless dissents from him: both because he attributes a diurnal motion to the Earth and transfers to it the apparent motion of the fixed stars towards the east, and because for the Sun he uses an epicycle rather than an eccentric. It is therefore manifest that the system of Longomontanus ought to be called *Semi-Tychonic*, not Tychonic.

CAPVT IX.

Chapter IX.

De Syſtemate Noſtro.

On our own system.

[Margin: Summary of our system.]

I. After the various systems of the ancients and the moderns that rest on the immobility of the Earth, two at last seem to me more probable, after the Tychonic: one semi-Ptolemaic, the other semi-Tychonic, as regards the centre from which the eccentrici- ties of the orbs are to be measured. For as regards the order of the planets it is plainly Egyptian, and as regards the number of the heavens it differs from both. In it we place the Earth in the centre of the world, free from all motion, and around it the orb of the Moon, above which the circuit of the Sun turns in such a way that Mercury is nevertheless carried round the Sun, and Venus round Mercury, and they are found sometimes above the Sun, sometimes below it. Next, Mars encircles Venus in such a way that in opposition to the Sun it comes nearer to the Earth than the Sun does. Above Mars rises Jupiter, carrying round four satellites, which describe their epicycles about it; and above Jupiter, Saturn, he too attended by two companions, or attendants [lateronibus]. All of these are carried round obliquely by the Intelligences, freely, in a single heaven that is in fact fluid, along helical spirals, or in a screw-like course, with a single motion towards the west. Above Saturn is the solid sphere of the fixed stars, which, by a single spiral motion likewise, from one Intelligence or several, apparently exhibits a threefold motion: namely in longitude towards the west, in longitude towards the east, and in latitude on account of the variation of declination — a motion which is nevertheless in reality single, towards the west. Above the sphere of the fixed stars is the watery or crystalline heaven, for the use that is known to God; and above that the immobile Empyrean, flashing with admirable splendour, the most happy and everlasting dwelling of the Blessed.

We shall bring the arguments for the immobility of the Earth in the following section. For the solidity of the heaven containing the fixed stars, and for the fluidity of the planetary heaven, I have already brought them in section 1, chapters 3 and 7; for the number of the heavens being five, the air included, see what was said in section 3, chapter 1; and for the Intelligences that move the heaven, section 2, chapter 1. Further, for the order of the planets, see what I said in Book 7, section 1, chapter 4; and for the ascent and descent of the planets, Book 7, section 6, chapter 1. Again, for the exclusion of different orbs or circles, and for the single spiral motion, see what we said in this book, section 2, chapter 3, scholium 4, and in Book 6, chapter 18. Finally, for the approach of Mars to the Earth, see what was said in Book 7, section 2, chapter 3, scholium 4, and section 6, chapter 4, scholia 3, 4 and 5, and chapter 10, scholium 1. But if, as some would prefer, a distinct heaven must be given to the new stars between

the planetary heaven and that of the fixed stars; and if a heaven of their own must be assigned separately to the planets that have their companions, and the watery heaven is reduced to the Empyrean, there will be 6 heavens and 7 great bodies of the world, according to the economy set out in Book 7, section 6, chapter 4, scholium 5.

[Margin: The first form of the new system.]

II. These things being laid down as general elements of the world system, the first form is Semi-Ptolemaic: namely, the one which uses the centre of the Earth as the term from which it measures the eccentricities of the orbs, but in such a way that it describes the epicycles of Venus and Mercury about the Sun, and supposes the eccentricities of five or at least four planets, and indeed the epicycles too, to be not always of the same quantity, according to the hypotheses already explained in Book 7, section 2, chapter 6, and section 3, chapter 6. For this form, indeed, the figure of the Egyptian system set out in chapter 4 could serve, if Mars in opposition to the Sun came nearer to the Earth than the Sun does.

[Margin: The later form of the system.]

The second form is the Semi-Tychonic: namely, the one which I devised formerly at Parma, while I was dictating a treatise on the heaven, having professed philosophy there — that the Sun should be the centre of the motions of Mercury, Venus and Mars, and the Earth the centre of the lunar motion and of the fixed stars, and also of the motions of Saturn and Jupiter. I was driven to this by the following reasons. *First*, because I saw that Saturn and Jupiter had their own planets about them, but Mars, Venus and Mercury none; I therefore thought it probable that Saturn and Jupiter exercise their own monarchy in the heaven and are not satellites of the Sun, but that Mars is the greatest satellite of the Sun, inasmuch as it embraces the Earth in its circuit, while Venus and Mercury are the inner and nearer attendants of the Sun. *Second*, because I saw in the variation of eccentricity a greater connection of Mars, Venus and Mercury with the Sun than of Saturn and Jupiter. *Third*, because I judged that Saturn and Jupiter, as the slower and heavier planets, have a greater affinity with the very slow sphere of the fixed stars, and therefore with the centre of the Earth, since the fixed stars too look to the Earth rather than to the Sun in their motion. *Fourth*, because, since Mars, like Venus and Mercury, enters the heaven of the Sun as regards designation, it seemed far more likely that these three have the Sun as the centre of their motion, and that distinct regions of the aether are not to be distributed to them, but a single one, whereas distinct realms are to be ascribed to Jupiter and Saturn; and so a confusion of the heavens is avoided, even of heavens distinguished only by designation. I see that this inclusion of Mars, Venus and Mercury in the single heaven of the Sun found favour with Amicus, *De caelo*, page 285, and with Oviedus, in the single controversy *De caelo*, page 468. Therefore, if we wish to divide the visible heaven into several regions, we could distribute the first to the fixed stars, the second to Saturn, the third to Jupiter, the fourth to the Sun together with Mars, Venus

and Mercury, and the fifth to the Moon. *Fifth*, it is very probable that there is found among the planets one that is intermediate between the superior and the inferior planets and that has in its motions many things in common with Saturn and Jupiter and many also with Venus and Mercury: namely Mars. All of this, once more observations have been collected and weighed in volume 3, will be subjected to examination, so that it may be established whether this form satisfies the phenomena. Meanwhile let no astronomer take it ill that it is reviewed by us in this place among the other systems. The scheme of this system, as regards the visible heavens, is given below; for around it must be understood the watery or crystalline heaven, and around that the Empyrean.

V. Our system.

Diagram. A woodcut in the right column showing Riccioli's own system. An outer double circle, studded with small star sorts, encloses the whole; at the centre is a dark disc, the Earth, which carries no sign, with a small dot standing to its left. Near the top stand the sign of Saturn beside a small double circle and, below it, the sign of Jupiter beside a set of small concentric circles; below these the sign of Mars stands on a large circle. Lower down, the sign of Venus and a smaller sign stand above a rayed disc drawn with a face, the Sun. A crescent stands just above the Earth, and the sign of Mars again below the Earth. Two large circles, one about the Sun and one passing through the lower sign of Mars, intersect.

- Top, beside a small double circle: ♄ Saturn
- Below it, beside small concentric circles: ♃ Jupiter
- On a large circle below them: ♂ Mars
- Above the Sun: ♀ Venus
- Just above the Sun, below Venus: [?]
- Centre of the small circles: ☉ Sun (a rayed disc with a face)
- Just above the Earth: ☾ Moon
- Below the Earth: ♂ Mars

[Margin: On the interval and thickness of the primary heavens.]

III. It is further to be noted that in this system of ours the intervals are not so constructed that the lowest part of Saturn's concave heaven touches the highest part of Jupiter's convex one, or the lowest of Jupiter the highest of Mars, and so on, as happens in the Ptolemaic hypothesis; rather, some interval is left empty of stars between the fixed stars and the highest point of Saturn, namely of at least 9824 semidiameters of the Earth, and between the lowest point of Saturn and the highest of Jupiter an interval of 10,045 semidiameters of the Earth. Between the lowest point of Jupiter and the highest of Mars the interval is 5310 semidiameters of the Earth; and between the lowest point of Venus and the highest of the Moon, 1850. Further, the thickness of the whole heaven of Saturn, its satellites included, is 32,454 semidiameters of the Earth; that of Jupiter, its satellites included, is 21,361; that of the Sun, Mars, Venus and Mercury, taken as a single heaven, is 19,091 semidiameters of the Earth. Therefore the thicknesses of the heavens of these three monarchs with their

satellites stand in such proportion that, of units of which Saturn has 32, Jupiter has 21 and the Sun with its three satellites 19; while the interval between Saturn and Jupiter has 10, and that between Jupiter and Mars, and so on, 5. But you have more on these measures at the end of Book 7, arranged in their tables.

SECTIO QVARTA

Section Four

DE SYSTEMATE TERRÆ MOTÆ.

On the system of the moving Earth.

CAPVT I.

Chapter I.

De Tractationis huius Dignitate ac Neceſſitate. Deque Authoribus, qui hoc Syſtema Propugnandum aut Oppugnandum ſuſceperunt.

On the dignity and necessity of this treatment; and on the authors who have undertaken to defend or to attack this system.

[Margin: On the dignity of this subject.]

I. We now at last approach the controversy that is by far the most celebrated among astronomical controversies, especially in this century, and that, by the nobility of its subject, is supremely suited to rousing minds and sharpening judgements. So Seneca, in Book 7 of the *Natural Questions*, chapter 2, having raised the question why comets, if they consist of thin matter, last so long and are not dispersed by the diurnal turning of the heaven, in these words: *But if they are nothing other than pure fire, and remain for six months at a time, and neither the turning of the world nor its speed dissolves them, those others too can consist of thin matter and yet not be shaken apart on that account by the unceasing revolution of the heaven* — rightly and most shrewdly observed that another question, most worthy of contemplation, bears on the solution of this one, the question of the motion of the Earth; and he at once added: *It will also be relevant to this to have examined whether the world goes round while the Earth stands still, or whether the Earth turns while the world stands still. For there have been those who said that it is we whom the nature of things carries along without our knowing it, and that risings and settings do not come about by the motion of the heaven, but that we ourselves rise and set. It is a matter worthy of contemplation, that we may know in what state of things we are — whether the seat we have been allotted is the most sluggish or the swiftest; whether God moves all things around us, or moves us.* Nothing more excellent than these words could have been said of the excellence of this treatment.

II. Moreover, the necessity of settling this controversy is laid upon us — chiefly in view of the subject of the work we have undertaken — by the two

supreme governors and arbiters of our intellect, REASON and AUTHORITY. For, to touch briefly on Reason: nowhere has astronomy, which is subordinate to mathematics and to physics, clashed more sharply with the rest of physics than on the occasion of this controversy. On this side, the Philolaics, the Aristarcheans and the Copernicans have armed arguments drawn from every kind of phenomena, ancient and recent, against the constancy and stability of the Earth; on that side, the Peripatetics, and indeed almost all the natural philosophers, have summoned from every quarter the forces and remarkable properties of the elements to fortify and defend the citadel of the Earth. The former, so that they might secure victory and triumph more freely for the novelty of their hypothesis, have also contrived a new physics of the motion of the elements and of elementary bodies; the latter, on the contrary, have built upon the most ancient foundation of Nature a new mass of more solid reasons against these novel assaults — so that it now seems to be no longer astronomers and philosophers contending with one another, nor those fabled gods with the Giants, but the heavens themselves with the elements. Reasons have been devised on both sides of such subtlety at times, and stratagems of sophisms or paralogisms dressed in such an appearance of truth, that they can deceive even the most cautious, unless he examines himself and them again and again. So it is not at all surprising that not only those lightly imbued with the principles of astronomy, but even men otherwise most expert in this sublime science, have not penetrated to the depth of the Copernican hypothesis, and have repelled its falsity not with necessary reasons but with arguments lighter than was fitting. As a result, the spirits of that sect were raised, and it dared to sing that triumphal "Io" everywhere throughout Germany, England, France, and even Italy itself, and to reproach philosophers and some theologians with ignorance of the celestial and the terrestrial revolutions alike.

III. But these things might perhaps be tolerable, had these disagreements not gone so far at last as to bring into danger the authority — I do not say of Vesta or of the Vestal priests, but of the Catholic Church in defining controversies of this kind, and indeed even of Sacred Scripture. For if we grant the Copernicans the licence they have assumed for themselves of interpreting the divine Scriptures and evading the decrees of the Church, that licence will perhaps not be contained within the bounds of astronomy alone, or of natural philosophy, but could be extended by others to other, more sacred dogmas as well — if, that is, it were once permitted to deny the literal sense of the divine book without manifest necessity. That there is no necessity of departing from it, whether from the principles of astronomy or from those of physics, we shall have to show hereafter, however much ingenuity and artifice the hypothesis of the moving Earth may display. Before I set about doing this, I think the principal authors should first be reviewed: both those who have written for the suppositions of Copernicus and those who have written against them, and whose works have so far come into my hands — besides those who have built their own opinions on this hypothesis, taken as already supposed.

Authores pro Copernici Hypotheſi.

Authors for the Copernican hypothesis.

IV. Copernicus [Margin: Copernicus.] himself, in the letter to Paul III, and in Book 1 of the *De revolutionibus* from chapter 5 to chapter 11, Book 3 from chapter 3, and Book 5, chapters 2 and 3. Georgius Ioachimus Rheticus [Georg Joachim Rheticus] [Margin: Rheticus.], in the *Narratio prima* on the books of the *De revolutionibus* of his teacher Copernicus, with the preface and additions of Mästlin [Margin: Mästlin.]. Johannes Kepler [Margin: Kepler.], in the *Prodromus*, or *Mysterium Cosmographicum*, with the appendix of Michael Mästlin to the same *Prodromus*; in the book *De stella nova*, chapters 15 and 16; in the introduction to the *Commentaries on Mars*; and in the *Epitome of Copernican Astronomy*, especially Book 1, part 3, and the whole of Book 4. Christophorus Rothmannus [Christoph Rothmann] [Margin: Rothmann, formerly.], in his letters to Tycho, especially that of 1590 April 18, which is found on page 284 — although he afterwards gave way to Tycho, as Tycho reports in the *Epistolae Astronomicae*, page 92. Galilaeus de Galilaeis [Galileo Galilei] [Margin: Galileo.], in the four dialogues on the two chief systems of the world, the Ptolemaic and the Copernican, published first in Italian in the year 1632 but afterwards rendered into Latin and published at Lyon in the year 1641; where, although he professes to set out the reasons on both sides without deciding [indefinitè], he nevertheless concludes absolutely that, if one must stand by reasons alone, one must adhere to the Copernican hypothesis. Gulielmus Gilbertus [William Gilbert] [Margin: Gilbert.], in Book 6 of *De magnete* — an assertor, however, of the diurnal motion only. Paulus Antonius Foscarinus [Paolo Antonio Foscarini] [Margin: Foscarini.], a Carmelite, in the letter to Sebastianus Fantonus [Sebastiano Fantoni], the General, on the mobility of the Earth and the stability of the Sun, which David Lotaeus rendered into Latin. Didacus Stunica [Diego de Zúñiga] [Margin: Zúñiga.], in the *Commentaries on Job*. The *Aristarchus redivivus*, published at Paris with the author's name suppressed; the *Philolaus*, published in the same place in the year 1634, whose author Ismaël Boulliau [Margin: Boulliau.] declares himself to be in his *Astronomia Philolaica*, Book 1. Iacobus Lansbergius [Jacob van Lansbergen] [Margin: Lansbergen.], in the *Apologia* for the *Commentationes* of Philips van Lansbergen on the diurnal and annual motion of the Earth, against the *Vesta*, or *Anti-Aristarchus*, of Libertus Fromondus [Libert Froidmont] and against Io. Baptista Morinus [Jean-Baptiste Morin]. Pierre Hérigone [Margin: Hérigone.], in volume 5 of the *Cursus Mathematicus*, Book 2 of the Theorics; and Pierre Gassendi [Margin: Gassendi, formerly.], both in the *Institutio Astronomica*, Book 3, and in two letters, but chiefly the second, *De motu impresso a motore translato*, at the end of which, however, he submitted his intellect captive to the decrees of the Church. Descartes [Margin: Descartes.], too, inclines to this opinion in part 3 of the *Principles of Philosophy*.

Auctores qui contra Hypotheſim Terræ Motæ ſcripſerunt.

Authors who have written against the hypothesis of the moving Earth.

V. Aristotle [Margin: Aristotle], in Book 2 of the *De caelo*, from chapter 12, or from text 71, and again from text 96; and there Simplicius, Philalthaeus, the Conimbricenses, Ruvius and very many other Peripatetics. Ptolemy [Margin: Ptolemy.], in Book 1 of the *Almagest*, chapters 5 and 7; and there Theon of Alexandria [Margin: Theon.]; and Regiomontanus [Margin: Regiomontanus.] in the *Epitome of the Almagest*, Book 1, conclusions 3 and 5. Alfraganus [Margin: Alfraganus.], difference 4. Macrobius [Margin: Macrobius], Book 1 on the *Dream of Scipio*, chapter 22. Cleomedes [Margin: Cleomedes], Book 1 of the *Cyclic Theory*, chapter 9. Petrus Aliacensis [Margin: Aliacensis], question 3 on the *Sphere*. Georgius Bucanhanus [George Buchanan] [Margin: Buchanan.], Book 1 of the *Sphere* set out in verse. Maurolico [Margin: Maurolico], dialogue 1 of the *Cosmographia*, from page 9. Clavius [Margin: Clavius.], in the *Sphere*, from pages 143 and 195. Barocius [Margin: Barocius.], Book 1 of the *Cosmographia*, page 33. Michaël Neander [Michael Neander] [Margin: Neander.], in the *Elements of Astronomy*, page 40. Telesius [Bernardino Telesio] [Margin: Telesius.], Book 2 *De caelo*. Martinengus [Margin: Martinengus.] in the *Glossa magna*, from page 320. Iustus Lipsius [Justus Lipsius] [Margin: Lipsius.], Book 2 of the *Physiologia*, dissertation 19. Christoph Scheiner [Margin: Scheiner], in the *Disquisitiones mathematicae*, from number 13. Tycho [Margin: Tycho.], in the *Epistolae Astronomicae*, pages 129, 147, 149, 157, 166, 185 and 188; in volume 1, the *Progymnasmata*, pages 406, 549, 658, 660 and 684; and in volume 2, chapter 8. Alexander Tassonus [Alessandro Tassoni] [Margin: Tassoni.], Book 4 of the *Diversarum cont.*, question 25. Scipio Claramontius [Scipione Chiaramonti] [Margin: Chiaramonti.], in the *Antiphilolaus*, and in the *Defensio Antitychonis* against the author of the two systems, namely Galileo. Melchior Incofer [Melchior Inchofer] [Margin: Inchofer.], in the *Tractatus syllepticus*. Libertus Fromondus [Margin: Froidmont], in his *Vesta*. Iacobus Acarisius [Giacomo Accarisio], Bishop of Vieste, in the fourth lecture, on the rest of the Earth and the motion of the Sun. Iulius Caesar Lagalla [Giulio Cesare Lagalla] [Margin: Lagalla.], *De phaenomenis in orbe Lunae*, chapter 7. Adam Tanner [Margin: Tanner.], *De caelo*, question 9, and volume 1 of the *Theologia*, disputation 6, question 4, doubt 3. Bartholomaeus Amicus [Margin: Amicus.], *De caelo*, from page 284. Antonius Rocco [Antonio Rocco] [Margin: Rocco.], in philosophical exercitations 6 and 7. Marinus Mersennius [Marin Mersenne] [Margin: Mercurius.][34], on chapter 1 of Genesis. Georgius Polaccus [Margin: Polaccus.], in the *Anticopernicus*. Athanasius Kircher [Margin: Kircher.], in the *Magnes*, from pages 539 and 581. David Spinellus [Margin: Spinellus.], in his *Jupiter among the Ethiopians*. Io. Pineda [Juan de Pineda] [Margin: Pineda.] and Io. Lorinus [Jean Lorin] [Margin: Lorinus.], on chapter 1 of Ecclesiastes.

[34]the margin note against Mersenne's name is printed "Mercuri⁹" (Mercurius).

Bartholomaeus Mastrius [Margin: Mastrius.] and Bonaventura Bellutus [Margin: Bellutus.], disputation 1 *De caelo*, question 2 and question 4, article 3. Io. Poncius [John Punch] [Margin: Punch.], disputation 22 of the *Physics*, question 9. Io. Antonius Delphinus [Margin: Delphinus], *De globis caelestibus*, chapter 18. Io. Elephantutius [Margin: Elephantutius.], in the *Universi orbis structura*. Those, however, who contended against that hypothesis at greater length were Tycho, Chiaramonti, Scheiner, Froidmont, Tassoni, Inchofer and Polaccus.

CAPVT II.

Chapter II.

De Auctoribus, qui Diurnam Vertiginem Terræ circa ſuum centrum & axem attribuerunt.

On the authors who have attributed a diurnal rotation to the Earth about its own centre and axis.

I. The diurnal revolution, completed in 24 hours, by which all the stars, whether fixed or wandering, are seen to be carried round from the east through the south into the west, and to return from there to the east through the midnight point — this, the most ancient authors once asserted, appears indeed in the stars, but in reality comes about through the motion of our Earth, turned about its own centre and axis toward the east and going round to meet the stars themselves. Copernicus speaks of these authors in Book 1, chapter 5, as follows: *Of this opinion, indeed, were Heraclides and Ecphantus the Pythagoreans, and Nicetas of Syracuse in Cicero, who turned the earth in the middle of the world. For they thought that the stars set by the interposition of the earth, and rose when it gave way.* He does not, however, indicate the sources from which he drew these things; but they are, of course, these. Cicero, in Book 2 of the *Academic Questions*, after saying: *Do you not also say that there are people over against us on the opposite side of the earth, who stand with their footprints set against ours, whom you call Antipodes? Why are you angrier with me, who do not scorn these things, than with those who, when they hear them, think you out of your minds?* — at once added: *Nicetas of Syracuse, as Theophrastus says, holds that the heaven, the Sun, the Moon, the stars — in short all things above — stand still, and that nothing in the world moves besides the earth; and since it turns and twists itself about its axis with the greatest speed, all the same effects are produced as would be if the heaven moved while the earth stood still. And some think that Plato too says this in the Timaeus, but rather obscurely.* About Plato, however, we shall inquire a little below. Again, Cicero likewise, in Book 1 of the *Tusculan Questions*: *Nicetas of Syracuse was the first to hold that the Earth moves.* And in Book 3 of the *De placitis philosophorum*, chapter 13, Plutarch reports the following: *Heraclides Ponticus and Ecphantus the Pythagorean do indeed move the earth, but by no means from place to place; rather, like a wheel resting on its axle, it turns about it from west to east. Democritus holds that from the very beginning it wandered widely, both because it was small and because*

it was light; but that as time went on it grew dense and came to rest, balanced by its own mass. And in chapter 17: *Seleucus the mathematician, who himself also moves the earth, says that the revolution of the Moon runs counter to its rotation and motion; and that by this impulse, as the wind gathers itself between these bodies and rushes into the Atlantic, the seas are, as is probable, thrown into turmoil.* That Plato was once of this same opinion we have already heard above from Cicero; but a much more ancient witness, Aristotle, seems to have said the same, in Book 2 of the *De caelo*, text 75: *But some say*, he says, *that it, though lying at the centre*, namely the Earth, *rotates and moves about the pole that is always immobile, as is written in the Timaeus.* And this is how Rheticus, in his *Narratio prima*, Gassendi in Epistle 2, *De motu impresso*, and others understand Plato.

[Margin: The opinion of Heraclides, Ecphantus, Democritus and Seleucus.]

[Margin: The opinion of Nicetas.]

II. Yet when I read Plato's *Timaeus* through, I could never be persuaded that this opinion of the rotation of the earth was Platonic, since Plato speaks in the person of Timaeus. For in that dialogue the first part in discussing the generation of the world and of its parts is assigned to Timaeus by Critias, with Socrates' consent. Critias had said: *Consider, Socrates, whether we have arranged the return banquet for you suitably. It seemed good to us that Timaeus, as the most expert of us all in astronomy, and the most versed in learning the nature of things, should speak first, so that, beginning from the generation of the world, he should come down to the nature of the human race*, etc. And a little after, Socrates replies: *You seem to me to have prepared an abundant and splendid table of discourse; it is therefore your office, O Timaeus*, etc. Timaeus, then, beginning from the generation of the world and setting out the intervals of the eight spheres according to harmonic proportions, recognizes two motions in the heaven: one outer, from east to west, which he attributes to the outermost sphere and not to the Earth; the other inner, from west to east, which he ascribes to the seven spheres of the planets. His words about God are these: *He made one of the circles outer, the other inner. The outer motion he named that of the Same nature, the inner that of the Other; and the one that was akin to the nature of the Same he bent sideways to the right, namely to the west, but the one of the Other diagonally to the left; and he gave the sovereignty to the motion that was alike and of the same nature. This alone he left undivided*, that is, simple and uniform. *But the inner, when he had divided it six times and made seven unequal orbs at double and triple intervals — each of them being three, that is, endowed with three motions: lengthwise to the right, lengthwise to the left, and in latitude toward the poles of the world — he ordered the orbs to travel in courses contrary to one another; and of the seven inner ones, he ordered three to turn with equal speed, namely the Sun, Mercury and Venus, and four with a speed unequal both to one another and to the other three, yet duly measured.* And after a few intervening words he follows the same motions further, saying: *When God had made the bodies of such stars, seven in number, he applied them to seven orbs, which revolve in the circuit of the Other and diverse nature. The Moon he placed in the first circuit above the earth, the Sun in the second. Then the globe of*

Lucifer and the star said to be sacred to Mercury he fixed in circles equal to the Sun in speed but with a power contrary to it; and so it comes about that these stars — the Sun, Lucifer and Mercury — overtake one another and are overtaken by one another in turn. If anyone should wish to run through the orders, progressions and reasons of the other stars in full, it will be more than the work itself which is undertaken for the work's sake. And presently, setting out how the planets, by their oblique motion through the zodiac, are seen to be carried with a motion contrary to that of the eighth sphere, he says: *They kept this course, namely, that according to the oblique motion of the Other and diverse nature, subordinated to the motion that is uniform and of like nature,*

[Margin: The genuine opinion of Timaeus and Plato.]

those orbs traverse partly a wider, partly a narrower circle, and those with the smaller circle revolve more swiftly, those with the larger more slowly. But according to that common and uniform motion itself, those that run through it most swiftly seem, while overtaking, to be overtaken by the slower. For that supernal rotation, turning all their circles round with a varied circuit — because they are driven by two contrary motions at once — showed as nearest to itself the one that falls back from it most slowly, swiftest as it is: namely Saturn. And so that there might be a most certain measure of their speed and slowness, and the dance of all eight motions might come forth into the light, God kindled a most brilliant light in the second circle from the earth, which we now call the Sun.

Could Timaeus, and so Plato, have declared his mind to us in clearer words as to what we are asking — namely, whether the apparent motion of the primum mobile is accomplished by the rotation of the earth, the lowest globe, or rather by the eighth circle, the outermost of all? Not at all, I say. But neither in the *Epinomis* does Plato recognize any other proper motion in the eighth sphere, but only the common one toward the west. Therefore, if he had ascribed this motion to the Earth, he would not have described for us a simple motion, and one of the same nature, in the eighth sphere, nor would he have called the motion of the planets contrary to it. But why say more? Did not the same Timaeus — that is, Plato speaking in the person of Timaeus — when he had undertaken to distribute the five regular bodies to the heaven and the elements, say in the plainest words: *Let us assign the cubic form to the Earth; for of all four the earth is the most immobile, and the most compacted of all bodies, and that must necessarily become such which possesses the firmest and safest bases.* [Margin: Timaeus assigns the cube to the Earth.] And therefore Plutarch, in Book 2 of the *De placitis*, chapter 6, reporting Pythagoras' opinion on attributing the cube to the Earth, said: *Plato in all these things Pythagorizes.* From this, perhaps, came the rite by which the cube was the symbol of the motionless earth, and priests sacrificed seated to Ops, that is, to the resting earth. Accordingly Marsilius Ficinus, certainly most expert in Platonic doctrines, in the *Compendium on the Timaeus*, chapter 25, lays down that in Plato's view the Earth is immobile.

But from where, you will ask, did that suspicion arise of a motion of the earth asserted by Timaeus or, as Cicero says, indicated somewhat obscurely?

I should think, from these words: *He willed the Earth, our nurse, bound about the pole stretched through the universe, to be the maker and guardian of day and night, and also the first and most ancient of all the gods*, etc.; *but the dances of these gods, and their meetings among themselves*, etc. By these last words, however, he does not place among the mobile gods the earth — which a little later he lays down as sitting motionless on the cube — but the souls and intelligences of the heavenly bodies. Nor does he call it the maker of day and night on the ground that by its own turning it goes to meet the Sun, but because, casting by its opacity a shadow into the part opposite the Sun, it makes night in that hemisphere of itself. And perhaps Aristotle, in that text 75 of the second book of the *De caelo*, adduces Timaeus not for the motion of the earth but for the immobility of the axis and pole about which others said it moved. What Cicero attributes to Timaeus therefore still remains in doubt, and it is far more probable that neither Timaeus nor Plato, in this passage at least, gave any motion to the Earth. Laertius must therefore be corrected, who in Book 3, in the life of Plato, having misunderstood the passage of Timaeus already mentioned, says of Plato's opinion: *That the Earth is the most ancient of all the gods there were in heaven; that it was made so as to vary the times and alternations of day and night; and that, being in the middle, it moves about the middle.*

III. Besides the ancients, however, not a few of the more recent writers have attributed to the Earth this single motion — not, that is, the motion τῆς φορᾶς [Greek: tēs phoras, "of carrying"], that is, of translation, but τῆς δινήσεως [Greek: tēs dinēseōs, "of whirling"], that is, of spinning, or diurnal rotation. Nicolaus Cusanus [Margin: The opinion of Nicolaus Cusanus.], the Cardinal, in Book 2 of the *De docta ignorantia*, chapter 11, where he says: *The earth, therefore, which cannot be the centre, cannot lack all motion.* And again: *From these things it is evident that the earth moves; and since from the motion of the comet we have learned by experience that the elements of air and fire move, and that the Moon moves less from east to west than Mercury or Venus, or the Sun, and so by degrees: hence the earth itself moves still less than all.* And after a few words: *Although the earth, as a star, is nearer to the central pole, yet it moves, and does not describe the smallest circle in its motion.* And in chapter 12: *The ancients did not reach these things now said, because they fell short in learned ignorance. It is now evident to us that this earth truly moves, although this does not appear to us, since we do not apprehend motion except by a certain comparison with something fixed. For if someone did not know that water flows, and did not see the banks, how, being on a ship in the middle of the water, would he apprehend that the ship moves? And for this reason, since it would always seem to anyone, whether he were on the earth or in the Sun or in another star, that he himself is at a centre, as though immobile, and that all other things move, he would certainly always set up for himself one set of poles after another: being in the Sun, some; on the earth, others; others in the Moon, and in Mars, and so for the rest. Hence the machine of the world will be as though having its centre everywhere and its circumference nowhere; for its circumference and centre is GOD, who is everywhere and nowhere.*

This author nevertheless seems, in these same passages, to attribute to the Earth some motion of translation as well; but elsewhere — in Book 7 of the *Exercitationes*, in the sermon that begins *Debitores sumus* — he seems either, forgetful of himself, to contradict himself, or to have come to a better view, where he says [Margin: A sounder opinion of the same author.]: *For prayer is more powerful than all creatures: for the Angels, or Intelligences, move the orb, the Sun, the stars; but prayer is more powerful, because it stops motion, just as the prayer of Joshua made the Sun stand still.* From him, however, Copernicus seems to have taken the simile of people on a ship, and from it the argument for the diurnal motion of the earth. In Book 1, chapter 5, he had said [Margin: The opinion of Copernicus.]: *Although it is generally agreed among authors that the earth rests in the middle of the world, so that they think it inconceivable, and even ridiculous, to hold the contrary, yet if we consider the matter more attentively, this question will be seen to be not yet settled, and therefore by no means to be despised. For every change of place that is seen comes about either through the motion of the thing observed, or of the observer, or certainly through an unequal change of both; for between things moved equally in the same direction no motion is perceived — between the thing seen, I mean, and the one seeing. But it is from the Earth that that celestial circuit is viewed and reproduced to our sight. If, then, some motion is assigned to the earth, it will appear in all things that are outside, but toward the opposite part, as though they were passing by; and such, first of all, is the daily revolution.* Afterwards, in chapter 8, from the argument drawn from the earth's spherical shape, which is most apt for motion, he flies off altogether into the opinion of Nicetas, and says: *Whether, then, the world is finite or infinite, let us leave to the dispute of the natural philosophers, holding this as certain: that the earth, enclosed by its poles, is bounded by a globular surface. Why then do we still hesitate to grant it the mobility that nature has made fitting to its form, rather than that the whole world, whose limit is unknown and cannot be known, should glide round? And why should we not admit that the appearance of the daily revolution is in the heaven, and its truth in the earth?* Copernicus, therefore, asserted this motion of the earth not merely by way of doubt, or speaking hypothetically, but absolutely [Margin: Copernicus affirms the motion of the Earth not merely hypothetically but absolutely.], and tried to confirm it with many arguments throughout that chapter 8 — arguments which we shall set out and refute below. And so he concludes that chapter by saying: *You see, then, that from all these things the mobility of the earth is more probable than its rest, especially in the daily revolution, as being most proper to the earth.*

IV. Next, as regards the diurnal motion of the earth, William Gilbert, physician of London, subscribed to Copernicus, in his new physiology *De magnete magneticisque corporibus*, Book 6, chapter 3, where, having praised Heraclides Ponticus, Ecphantus, Nicetas, Aristarchus, Philolaus and Copernicus, he says: *It is therefore an old opinion, handed down from ancient times, but now augmented by great thoughts, that the earth is carried round whole by a diurnal revolution in the space of 24 hours.* And below he concludes: *And so we are carried by the*

diurnal rotation of the earth (a motion, that is, more fitting), and as a boat moves upon the waters, so we are turned round with the earth, and yet we seem to ourselves to stand still and be at rest. He then contends, in chapter 4, that this motion comes about from the magnetic virtue of the earth; in chapter 5 he refutes, or tries to refute, the reasons of those who deny the motion of the earth; and in chapter 6 he ascribes the cause of the definite time of 24 hours in which this revolution is accomplished to a magnetic alliance of the Earth and the Moon, these globes being driven forward by the Sun in proportion to their orbs. Finally, Copernicus and Gilbert were followed in this by David Origanus, in volume 1 of the *Ephemerides*, in the dedicatory epistle; by Christianus Severini Longomontanus, in the *Astronomia Danica*, Book 1 of the Theorics, chapters 1 and 4; and by Andrea Argoli, in the *Pandosion sphaericum*, chapter 3 [Margin: Origanus, Longomontanus and Argoli, assertors of the diurnal motion of the Earth.]. Not, however, by Tycho, whatever some have imagined; those people, however, we have refuted sufficiently in Section 3, chapter 8, number 4. Besides these, moreover, all those who attributed an an- nual motion to the Earth also freely granted it the diurnal one; we shall review them in the following chapter. And although Seneca, taking occasion from the diurnal motion of comets, raised a doubt about this motion of the earth in particular, as is clear from his words quoted in this Section at the beginning of chapter 1, the same author nevertheless, in the book *De providentia*, chapter 1, followed the sound and common opinion and said [Margin: Seneca, an assertor of the stability of the Earth.]: *That this order is not the work of wandering matter, and that things which came together at random do not hang with such art that the heaviest weight of the lands sits motionless and feels the flight of the heaven hastening round it.*

CAPVT III.

Chapter III.

De Auctoribus, qui præter Diurnam Vertiginem, Annuam quoq. tranſlationem Telluri adſcripſerunt. Vbi obiter de Veſta, fueritne Terra an Ignis.

On the authors who, besides the diurnal rotation, also ascribed an annual translation to the Earth. Where, in passing, on Vesta: whether she was Earth or Fire.

I. At this point I am pleased to borrow that saying of Pliny's [Margin: Pliny, Book 2, chapter 23.]: *It is wonderful how far the shamelessness of the human heart goes, invited on by some tiny success.* The Pythagoreans had seen that one of their own sect, namely Ecphantus, had successfully represented by a diurnal motion of the earth the same phenomena that others had accounted for by a diurnal revolution of the stars. From there, therefore, taking a step further and with greater daring, they added to it an annual translation as well, which is commonly ascribed to the Sun, placing the Sun, as the fount, so to speak, of heat and light, immobile at the centre of the universe. The first of them, I find,

was Philolaus, of whom Plutarch, in Book 3 of the *De placitis philosophorum*, chapter 11, reports the following [Margin: Opinions of Philolaus on the Earth.]: *The followers of Thales said the Earth is in the middle. Xenophanes, that it is first fibred down into the infinite. Philolaus the Pythagorean, that fire is in the middle, being the hearth of the universe; second, the antichthon, that is, the earth of the Antipodes; third, the one we inhabit, which is both set opposite the antichthon and turns round it — which is the reason why those who live on that one are not seen by us.* And in chapter 13: *The others said that the earth stays still; but Philolaus the Pythagorean, that it turns about the fire in an oblique circle, like the Sun and the Moon.* Diogenes Laërtius, in the life of Philolaus, agrees with Plutarch, but indicates only the motion of the primum mobile: *He holds that all things come about by harmony and necessity; he says that the Earth moves according to the first circle; others affirm that it was Nicetas of Syracuse who held this.* From these things one better understands what Aristotle relates in Book 2 of the *De caelo*, text 72, in these words: *While most of those who say that the whole heaven is finite say that the earth lies in the middle, those who live around Italy, and are called Pythagoreans, say the contrary. For they say that fire is in the middle, and that the earth is one of the stars and, carried in a circle around the middle, makes night and day. Moreover they construct another earth opposite to this one, which they name* ἀντίχθονα [Greek: antichthona, "counter-earth"], *not fitting reasons and causes to the phenomena, but dragging the phenomena themselves toward certain opinions and reasons of their own.* Among these reasons he mentions in first place, in text 73, the one drawn from the dignity of place: namely, that the most honourable region befits the most honourable of bodies, and that fire, or the Sun, is nobler than the earth; and he adds: *Moreover the Pythagoreans, because it is most fitting that what is most principal in the Universe be guarded, and the middle is such, said that fire holds this region, which they call the prison, or guard-house, of Jove.*

II. Not only in Italy, however, but also in the island of Samos, from which we know that Pythagoras came, Aristarchus, himself a Samian, taught the same opinion on the motion of the Earth [Margin: The opinion of Aristarchus of Samos.], more than a hundred years after Philolaus; for it is gathered from Ptolemy, Book 3, chapter 2, that this Aristarchus flourished in year 44 from the death of Alexander the Great, that is, about the year 280 before Christ. Now Archimedes, in the *Arenarius* [*The Sand-Reckoner*], speaks of Aristarchus thus: *Aristarchus of Samos, refuting these things in what has been written by the astrologers* — namely, about the Earth resting in the middle of the world — *published certain hypotheses from which it follows that the world is many times the world just described. For he supposes that the fixed stars and the Sun remain immobile, but that the earth is carried round the Sun along the circumference of a circle which is set in the middle of the course; and that the sphere of the fixed stars, situated about the same centre as the Sun, is of such size that the circle in which he supposes the earth to be carried round has the same proportion to the distance of the fixed stars as the centre of a sphere has to its surface.* But Archimedes at once rejects this [Margin: Archimedes' censure

of Aristarchus.], saying: *But this is impossible; for since the centre of a sphere has no magnitude, it must not be supposed to have any ratio to the surface of the sphere either.*

With these words of Archimedes, what Plutarch says in the person of Lucius in the little work *De facie in orbe Lunae* agrees excellently. He speaks thus: *Come now, would you make us guilty of impiety, in the way that Cleanthes thought Aristarchus of Samos ought to have been indicted by the Greeks for violating religion, as if he had moved the Lares of the Universe, and Vesta, from their place — because that man, trying to save the phenomena of the heaven by certain reasonings, had supposed the heaven to be at rest, and the earth to be rolled round along an oblique circle and meanwhile to turn about its own axis.* [Margin: Cleanthes' censure of Aristarchus.] From this passage of Plutarch it came about that Kepler, fearing for himself the ecclesiastical censure raised against him by Chiaramonti for asserting the motion of the earth, said in the *Hyperaspistes*, Book 3, chapter 22, page 183: *The hypotheses that I follow — if he should seize the bolt of Jove and thunder; if he should bring a charge against me, as Cleanthes once did against Aristarchus, of violated religion because Vesta's sacred things were moved from their place — the astronomers will be abundantly satisfied by the equipollence demonstrated in the Commentaries on Mars, inculcated in the Harmonics, and used in this very booklet.*

III. But Plutarch now invites me to take this occasion to inquire what kind of deity Vesta was among the pagans, and in what place she stood — something bearing on this Pythagorean fiction about the place and motion of the Earth [Margin: Vesta: what and where?]. In this very question we shall find two opinions, arising, unless I am mistaken, from a twofold opinion about the place of the earth. For all held that Vesta rests at the centre of the world. So those who thought that the Earth sits at this centre also held that Tellus was the same as Vesta; whereas those who believed the centre of the Universe to be occupied by Fire or by the Sun likewise took Vesta to be perpetual and immobile fire. Of the first opinion were those who said she was the sister of Ceres and Juno, born of Rhea Ops and Saturn — as Apollodorus in Book 1 of the *Bibliotheca*, with whom Ovid agrees in the *Fasti*:

They record that Juno and Ceres were born of Ops, From Saturn's seed; the third was Vesta.

He also sings, in Book 1 of the *Metamorphoses*:

The earth stands by its own force; by standing with force it is called Vesta.

But before him, Plato in the *Phaedrus*, having said that eleven gods proceed following the chariot of Jupiter, their chorus-leader and leader of the dance, added, making an exception of Vesta [Margin: Plato's view of Vesta.]: μένει μὲν Εστία ἔν θεῶν οἴκω μόνη [Greek: menei men Estia en theōn oikō monē, "Hestia remains alone in the house of the gods"] — *for Vesta remains and stays alone in the house of the gods.* And again in Book 12 of the *Laws*: *Earth and Vesta are a sacred dwelling for all the gods.* To this point are those words of Arnobius, Book 3, *Against the Gentiles*: *Some declare the Earth to be Vesta, because it alone stands still in the world, its other parts being set in perpetual mobility.* Servius too, on

Book 1 of the *Aeneid*, thinks Vesta was so called because she is clothed [vestita] with various things; *for she herself*, he says, *is said to be the Earth.* Some Greeks too, as Lipsius notes in his book *De Vesta*, chapter 1, think Vesta was so called παρὰ το ἑστᾶναι ὅ ἐστιν ἐν ἑνὶ τόπω [Greek: para to hestanai ho estin en heni topō, "from standing, that is, being in one place"], because she remains in one place. And in Festus, some call her for this reason *Stata mater* [the Standing Mother]. Lipsius also adduces these lines of Pindar:

And Earth Mother, and by the wise You are surnamed Vesta, a seat in the very air.

Nor does it stand in the way that Fire too is attributed to Vesta. That can be understood of subterranean and perpetual fire, even though it exhales through various chasms, or of fire preserved in the earth and by the earth, as Servius notes, saying: *For Vesta is said to be the Earth, which no doubt has fire, as is given to be understood from Etna and Vulcan.* To this point are the words of Phurnutus [Cornutus] in Lipsius: *Because the fiery force that is in the world is nourished from here, and subsists through it; or because the fertile Earth is also the mother of animals, for whom the fiery force is the cause of life.* Or certainly, as Dionysius of Halicarnassus relates in Book 2 of the *Antiquities*, fire is sacred to Vesta, *because, since Vesta is the Earth and holds the middle place of the world, she kindles from herself the su- perior fires.* From this arose the opinion, which Martianus Capella indicates in Book 1 of *De nuptiis Philologiae*, that *Vesta was the nurse of Jupiter*. Others, however, take Vesta simply for living and perpetual Fire, according to that line of Ovid's:

And understand Vesta to be nothing other than living flame.

and of Virgil in the *Moretum*:

And while Vulcan and Vesta perform their parts,

And Cicero, Book 3 of *De legibus*: *And since Vesta, so called by a Greek name, has embraced, as it were, the hearth of the city*; and Book 2 of *De natura deorum*: *The name of Vesta is taken from the Greeks, for she is the one whom they call* Εστία [Greek: Estia, "Hestia"], *and her power pertains to altars and hearths.* For just as, by the addition of the letter V, *Er* became *Ver* and *Eneti* became *Veneti*, so *Esta* was called *Vesta*, as Servius notes on Book 1 of the *Aeneid*. But more to our point is Plutarch in the *Numa*: *Numa is said to have built round the perpetual fire, to guard it, a temple of Vesta of circular form — not to imitate the shape of the Earth, as though the Earth were Vesta, but the shape of this whole universe, at whose middle the Pythagoreans hold that Fire stands; and this they call Vesta, and the Unity. The Earth, on the other hand, they hold to be neither immobile nor set at the middle of the circuit, but suspended in an orb round the fire, and not to be reckoned among the excellent or first parts of the world. They would have it that Plato, when he was already old, thought the same of the Earth: that it is set in another place, and that the middle and most worthy region belongs to some other, more excellent body* [Margin: The opinion of Plato, when already old, on the place and motion of the earth.]. But so long as no passage from Plato is brought forward, I hold to the view that Plato always thought the earth immobile — on the strength of his own words in the *Phaedrus* and in Book 12 of the *Republic*, already cited,

and of the words of the *Timaeus* quoted in the preceding chapter, number 2. Granted that he came to Italy to see Pythagoras and Philolaus, and there came to know Timaeus of Locri and Archytas of Tarentum, as we have learned from Cicero, Book 1 of the *Tusculans*, and from Laertius. For this view I appeal from Plutarch to Plutarch, who says in the *De placitis philosophorum*, Book 2, chapter 6, that Plato, with Pythagoras, assigned the cube to the Earth on account of its immobility; in chapter 15, that he did not number it among the planets; and in Book 3, chapter 13, excepting Heraclides and Philolaus, he says: *The others said that the earth stays still.* I therefore do not agree with Gassendi, who in Letter 2, *De motu impresso*, asserts on the authority of Plutarch that Plato in his later years granted both motions to the Earth. Plutarch does not assert this; he reports the conjecture of others, when he says: *Some would have it.*

IV. The opinion of Philolaus and Aristarchus on this motion of the Earth had, however, fallen asleep: lulled into the deepest silence through many past centuries, it had lain in almost universal oblivion, when Nicolaus Cusanus, as I indicated in chapter 2, number 3, in a lowered voice, and the other Nicolaus, namely Copernicus, in a loud voice, roused it. Nor did Copernicus put forward this motion of the Earth as a mere hypothesis, as some think — above all Scipione Chiaramonti, in the preface to his *Antiphilolaus*, and before him the anonymous author who prefixed the notice to the reader to Copernicus's works. On the contrary, he asserted it absolutely, either as more probable or even as demonstrated [Margin: Copernicus asserts the motion of the Earth absolutely, and not only on a hypothesis.]. That is the sense of the words of Copernicus himself in his preface to Paul III, where he says: *Therefore, when I considered with myself how absurd an* ἀκρόαμα [Greek: akroama, "a thing heard", a doctrine] *those would think it who know that this opinion has been confirmed by the judgements of many centuries — that the Earth is placed immobile in the middle of the heaven, as its centre — if I, on the contrary, were to assert that the Earth moves, I hesitated for a long time whether I should bring to light my commentaries, written to demonstrate its motion, or whether it would be better to follow the example of the Pythagoreans and of certain others, who used to hand down the mysteries of philosophy not in writing but from hand to hand, and only to their relatives and friends, as the letter of Lysis to Hipparchus testifies.* Not much later he uses the word "demonstration" again, and says: *that the more absurd this doctrine of mine on the motion of the Earth now seemed to most people, the more admiration and gratitude it would win once they saw, through the publication of my commentaries, the fog of absurdity removed by the clearest demonstrations.* And again, repeating the same word, he says: *Nor do I doubt that ingenious and learned mathematicians will agree with me, if — as this philosophy above all demands — they are willing to learn and weigh, not superficially but thoroughly, what I bring forward in this work for the demonstration of these matters.* So firmly, indeed, did he set his mind on this view of both motions of the Earth that he even contrived to brand with rashness and calumny those who tried to assert the Earth's immobility from Sacred Scripture. For when he had said that he dedicated his work to Paul III, as one versed in mathematics, so

as not to seem to shun anyone's judgement, he added: *so that by your authority and judgement you may easily check the bites of slanderers, although the proverb says there is no remedy against the bite of a sycophant.* And soon after: *If perhaps there are* ματαιόλογοι [Greek: mataiologoi, "idle talkers"] *who, although ignorant of all mathematics, nevertheless take upon themselves to judge of these things, and who, on account of some passage of Scripture badly twisted to their purpose, dare to censure and attack this undertaking of mine, I do not care about them at all, so much so that I even despise their judgement as rash.* [Margin: Copernicus's arrogant pronouncement.] With such haughtiness, to be sure, did that man dare to rise up who a little earlier had seemed almost to dread the judgement of men.

But it must now be shown from his own words in what way he assigned the annual translation to the Earth, as to one of the planets, rather than to the Sun. Having begun chapter 9 of Book 1 of the *Revolutions* thus: *Since, therefore, nothing prevents the mobility of the earth, I think we must now see whether several motions also befit it, so that it may be considered one of the wandering stars*, he finally concludes: *If, therefore, the earth too makes other circuits, for example about the centre, they must be those which appear in like manner from outside in many bodies, among which we find the annual circuit. For if it is changed from a solar to a terrestrial one, immobility being granted to the Sun, the risings and settings of the signs and of the stars, by which they become morning and evening stars, will appear in the same way. The stations, retrogradations and progressions of the wandering stars, too, will be seen to be a motion not of theirs but of the earth, which they borrow for their appearances. Finally, the Sun itself will be thought to occupy the middle of the world. All this is taught us by the reason of the order in which those bodies succeed one another, and by the harmony of the whole world, if only we look at the matter itself with both eyes (as they say).* And when in chapter 10 he had related many things about this order, bearing on the various systems of the world, he added: *Accordingly we are not ashamed to confess that all this which the Moon encircles, together with the centre of the earth, passes through that great orb among the other wandering stars with an annual revolution about the Sun; that near it, namely the Sun, is the centre of the world; and that, the Sun also remaining immobile, whatever appears of the motion of the Sun is rather verified in the mobility of the earth.* [Margin: Copernicus places the Sun not in the very centre of the world, but round it, or near it.] In what way the annual motion of the Earth's centre about the centre of the world is combined with its diurnal revolution about its own centre, and indeed with a certain third motion of the earth's axis, which keeps its parallelism to the axis of the Equator, he explains at greater length in chapter 11, with a diagram attached — which we shall supply in the following chapter. In Book 5, chapters 1, 2 and 3, moreover, he teaches how the annual motion of the earth's centre, referred to the motion of the five lesser planets, can be called the motion of commutation, because it either exceeds the mean motion of Saturn, Jupiter and Mars or is exceeded by the mean motion of Venus and Mercury; and how their stations and retrogradations are explained by that motion of the earth.

V. In the end, there adhered to Copernicus, and made the Earth turn with a threefold or at least a twofold motion, his own pupil Georg Joachim Rheticus, who expounded those motions in the very long *Narratio* usually appended to Copernicus's works, and commended them highly [Margin: Asserting the annual motion of the earth: Rheticus. Mästlin. Kepler. Galileo. Boulliau. Jac. Lansbergen. Hérigone. Gemma. Naiboda. Reinhold. Calcagnini. Zúñiga. Foscarini. Baranzano. Politianus. Bruno. Descartes. Philips Lansbergen. Schickard. Hevelius.]; Michael Mästlin, and Mästlin's pupil Johannes Kepler; Galileo; Boulliau, Jacob van Lansbergen and Hérigone, in the places cited in chapter 1, number 4; and besides these Gemma Phrysius [Gemma Frisius] in his letter to Io. Stadius [Johannes Stadius]; Valentinus Naiboda in Book 1 of the *Institutiones astronomicae*, chapter 16; Erasmus Reinhold in the *Prutenic Tables* and in his exposition of the Copernican hypothesis; Caelius Calcagninus [Celio Calcagnini]; Didacus Stunica [Diego de Zúñiga] in his commentary on Job; Paulus Antonius Foscarinus, the Carmelite, in his letter on the mobility of the earth and the stability of the Sun to Sebastianus Fantonus [Sebastiano Fantoni], General of the Carmelites; Redento Baranzano in part 2 of the *Uranoscopia*, page 42, and, in that work, Antonius Laurentius Politianus; and also Giordano Bruno in Book 3 of *De maximo et immenso*, the last chapter; Descartes in the *Principles of Philosophy*, part 3, from number 26 to 34; Philips van Lansbergen in his *Uranometria*, Book 3, and in the controversies on the annual motion of the earth; Schikardus [Wilhelm Schickard] and Johannes Hevelius in Books 5 and 7 of the *Selenographia*. Among these, however, Lansbergen and Boulliau do not admit that third motion of the earth by which the proper motion apparent in the fixed stars is accounted for by Copernicus; for they say that the fixed stars in reality move slowly eastward. Nor, on the other hand, does Giordano Bruno accept any other motion of the earth besides the diurnal and the annual: in place of the third he substitutes a motion of the Sun in a circle bounded by the Tropics, from which the annual inequality, and the variation of the eccentricity and of the magnitude, are to arise. Rothmann, however, who had once clung tenaciously to Copernicus, afterwards surrendered to Tycho [Margin: Rothmann's palinode against Copernicus.] — if we believe Tycho himself, in the *Epistolae Astronomicae*, page 192; we shall report his words below, in chapter 29, number 19, and chapter 30, number 7.

SCHOLIA.

Scholia.

Vbi de alijs quibuſdam Motibus Terræ affictis.

Where certain other motions ascribed to the Earth are discussed.

I. Among the motions attributed to the Earth, the most celebrated are the three already mentioned, according to the view of Copernicus; and besides these there is the motion of trepidation [Margin: The motion of trepidation.]

— on which Petrus Alliacensis, treating it as a problem, in question 3 on the *Sphere*, and others, cited and rejected by us in Book 2, chapter 3, number 1 — a motion that Athanasius Kircher nevertheless favours in Book 2 of the *Magnes*, page 489. Andreas Cisalpinus [Andrea Cesalpino], moreover, in Book 3 of the *Peripateticae quaestiones*, question 4, imagined in the Earth a certain very slow motion toward the east, such as others recognize in the fixed stars as their proper motion; of this there is no need to say more.

II. Ioannes Pena, too, introduced a certain new rectilinear motion of the Earth, upward and downward, in the preface on the use of optics addressed to Cardinal Charles of Lorraine and prefixed to Euclid's *Optics*, which he himself translated from the Greek into Latin; and so did Io. Stadius, in the history of astronomy placed before the *Tabulae Bergenses*, page 23 [Margin: A motion of the Earth upward and downward, asserted by Io. Pena and Stadius.]. With these authors the basis of this motion is the apparent inequality in the motion of the fixed stars, which he attributes to optical causes, and to the Earth's approach to the fixed stars and its recession from them. But it will be best to draw this opinion out of his own words. He says, then: *It is a rule of optics, as true as it is brief, that of bodies carried with equal speed, those which seem to be carried more slowly are farther away. Now the fixed stars are carried with equal speed (for the hypotheses of astronomy teach that the celestial motions are uniform, although they appear non-uniform to us); yet they seem to advance unequally, as the observation of the ages teaches. For at the beginning of the periods of Callippus, that is, in the times of Alexander the Great, the fixed stars travelled one degree of the heaven in seventy-two years; in the age of Ptolemy, in a hundred years; in the time of Machometus Aracensis [al-Battānī], in sixty-six years; and in this century they advance at almost the same rate as in those first times of Callippus* [Margin: The first basis of Io. Pena.]. *From this*[35] *it is evident that the globe of the Earth was farthest from the heaven in the century of Ptolemy, came closest to the heaven in the times of Machometus, and in our age stands midway between the two extremes. So you see, most illustrious Prince, that it is concluded from optics that the Earth must be now nearer to the heaven, now farther from it: that is, that it has some motion by which it changes place. I do not wrap the earth in a threefold motion; I do not assign to it the diurnal and the annual motion, the motion both of the centre and of declination; I only assert, from optics, that the Earth advances by some motion from place to place — and that with a very slow progression of time, since scarcely in four hundred years or more can any such inequality of the motions be perceived. By this the other question, too, about the middle of the world is resolved: for if the earth changes place, how will it be the centre of the world? Thus the old controversy of the wisest men will be illuminated by the light of optics, and settled with very little trouble.*

III. So far Pena. In the same preface, a little earlier, he had built up the same motion of the earth from the unequal apparent magnitude of one and the same fixed star [Margin: The second basis of Io. Pena.], supposing that the

[35]printed *qno*, a turned u

diameters of the fixed stars are now seen larger than in Ptolemy's time, because of the earth's approach to them. But with the same ease of invention — not to say futility — he could have asserted that the eighth sphere itself moves to and fro, up and down along its diameter, approaching the immobile Earth and in turn receding from it. That this inequality of the motion of the fixed stars is improbable, however, or at least that their uniformity, even apparent, is more probable, we have shown sufficiently in Book 6, chapter 17. And how falsely it is supposed that the diameters of the fixed stars now appear larger than in Ptolemy's age is clear from what was said in Book 6, chapter 9, and Book 7, section 6, chapters 9 to 13. This writer ought also to have noticed how much greater and more frequent an inequality would appear in the motion and magnitude of the planets, especially of the Moon, if that approach and recession of the Earth were admitted; indeed, that it would become necessary either for the Earth to rise above the heaven of the Moon and the Sun, or at least for the centre of the planetary system, and with it the heavens of the planets, to rise together with the centre of the earth from the centre of the world toward its outer limit by as much as about a tenth part of the distance of the fixed stars. This will be demonstrated by the following diagram.

IV. Let AB be an arc of one degree of the heaven of the fixed stars, described from the centre of the world C [Margin: How far the Earth would have to rise, if the inequality of the motion of the fixed stars arose from its motion.]; and in that centre let the centre of the earth likewise stand in the age in which the fixed stars are supposed to complete one degree by their mean motion in 72 years. Let AC, or BC, the distance of the fixed stars according to Ptolemy or al-Battānī, be taken as 19,000 semidiameters of the earth. Then let the earth be carried to D in the age in which the fixed stars are supposed to complete one degree in 66 years; for then, by the rule of proportion, they will be seen to complete in 72 years 1° and 5′ 26″ besides. Therefore the arc AB, seen from C under the angle ACB, will appear as 60′; but seen from D under the angle ADB, it will appear as 65′ 26″. Therefore, by proposition 32 of the first book of the *Elements*, the angle CDB in triangle BCD will be 178° 54′ 34″, and the angle CBD will be 5′ 26″. Since, then, all the angles in that triangle are given, and besides them the side BC, of 19,000 semidiameters of the earth, the side DC too will become known with the help of trigonometry; and by the second case of oblique-angled triangles DC will be found to be 1900 terrestrial semidiameters and more. This measure exceeds Ptolemy's distance of the Sun from the centre of the world, let alone that of the Moon. Therefore the Earth's centre would have had to rise, between the time of Callippus and the age of al-Battānī, from the centre of the world to above the Sun; or at least the whole system of the planets, together with the earth, would have had to rise from the centre of the world by 1900 semidiameters of the earth and more [Margin: Either an error of Io. Pena in geometry, or audacity.]. Let Pena see, therefore, either how greatly he erred in geometry and in optics itself, or how much further he would have

had to venture, and what hypothesis he would have to swallow, if he wished to defend his opinion.

Diagram. A small woodcut set in the right column beside the demonstration of § IV: a tall, narrow figure. A slightly curved line at the top joins A and B, the arc of one degree of the heaven of the fixed stars; a vertical line descends from A to C, the centre of the world, at the bottom; D, the displaced centre of the earth, stands on that vertical about two-thirds of the way down from A; straight lines run from B to D and from B to C, so that the angles ACB and ADB stand at C and D under the same arc AB.

- A (upper left): end of the arc AB
- B (upper right): end of the arc AB
- D (on the vertical, lower part): the earth's centre carried to D
- C (bottom): the centre of the world

V. But I also find another slip of Pena's, in the history of astronomy [Margin: A slip of the same author in the history of astronomy.], when in the same preface to Euclid's *Optics* he says: *Let Aristotle have judged that the Earth is at rest; let Ptolemy, most experienced in celestial matters, judge so; let Theon judge so; let common credulity have judged the same. But those noble Pythagoreans judged otherwise, and pronounced that the earth moves. The same is asserted by Plato's Timaeus, by Philolaus, Ecphantus and Seleucus* [Margin: Seleucus asserts the motion of the Earth.]; *the same is taught by Aristarchus of Samos, by Archimedes and, in this age, by the most illustrious Copernicus.* But granted that Timaeus judged that the earth moves — which is, however, by no means certain, as is clear from what was said in this chapter 3, number 3 — it is clear from the same chapter, number 2, that Archimedes cannot be counted among those who move the Earth, since he rejected Aristarchus's opinion, even though, granting Aristarchus's hypothesis and the immense distance of the fixed stars, he showed in the *Arenarius* that the number of grains of sand contained in the whole capacity of the world can be computed.

VI. On the motion of the Earth in the meridian, by force of which the northern altitudes of the Pole increase continually [Margin: A motion of the Earth changing the altitudes of the pole.] — a motion devised by Dominicus Maria [Domenico Maria Novara] and followed by Giordano Bruno, page 306, and by Maginus in canon 8 of the *Secundorum mobilium* — we shall say more in chapter 11, from number 7; and there, at number 9, on the motion of libration from Africus to Caecias asserted by Alexander Calignonus.

CAPVT IV.

Chapter IV.

Explicantur diſtinctè tres quatuorue Motus Telluri à Copernico, & ſectatoribus attributi, eorumq. proprietates, & conditiones admirabiles.

The three or four motions attributed to the Earth by Copernicus and his followers are explained distinctly, together with their remarkable properties and conditions.

I. Whoever wishes either to defend or to attack the Copernican hypothesis must certainly first have explored it most thoroughly and looked deep into all its parts, so as not to expose himself — as has happened to not a few until now — to the ridicule and contempt of the Copernicans. Above all this holds for the man who thought that the diurnal revolution of the earth takes place from east to west, whom Galileo rightly mocks in the second Dialogue. It must therefore be known that three motions are attributed to the Earth by this sect. The first is called the Diurnal, or the motion of diurnal revolution, because in the time of 24 hours, in which one natural day is completed, the whole earth revolves about the centre of its body from one point to very nearly the same point. The second is called the Annual, because its revolution is not completed except in one solar year. The third is called the motion of declination, or of parallelism. We shall now explain these distinctly.

Explicatur Diurna Terræ reuolutio.

The diurnal revolution of the Earth is explained.

II. First, Copernicus, in Book 1 of the *Revolutions*, chapters 5 and 8, denies that the fixed stars and the planets move from east to west with any motion of the primum mobile, as astronomers commonly suppose. In place of this motion he substitutes a diurnal rotation of the terrestrial globe — a globe composed of earth, the water enclosed in it, and the air poured round it and akin to it — by which it revolves about the centre and axis of its body in 24 solar hours, from the west through the meridian toward the east [Margin: The direction in which the diurnal motion of the Earth tends.]. For all things that seem to move with respect to place seem to move so either because they really change place; or because, while they are unmoved, the sight or eye of the observer moves; or because both the object seen and the eye of the observer move unequally. For if the sight and the object moved with equal speed toward the same part of the world, no motion would be perceived between the thing seen and the observer's eye, as Copernicus argues admirably in that chapter 5. Whether, therefore, the diurnal revolution toward the west is assigned to the stars, or that revolution is attributed — but toward the opposite side, namely toward the east — to the Earth and to our sight, which views the stars from the earth, all the phenomena of the diurnal motion will appear to us in just the same way. As we go to meet them, the stars will seem to rise and gradually ascend above the Horizon as far as the Meridian; and as we for our part pass on beyond them and leave them behind our backs, they will seem to descend to the Horizon

and at last to set. Nor will the stars themselves pass through the vertex or the Meridian by a diurnal motion of their own; rather, the vertical points of the Earth themselves, and the meridian circles themselves, conceived on the globe of the Earth, will pass from the western quarter to the eastern; and so the Sun, now covered, now uncovered, will bring about the alternations of night and day. Since, then, from what has been said, the phenomena of the diurnal motion can arise in either way, it seemed more probable to Copernicus to ascribe that motion not to the whole world, nor to the huge spheres of the fixed stars and the planets, but rather to the Earth, inasmuch as it is spherical and small in comparison with the heaven, and is, as it were, the thing contained and placed. He indicates these reasons in these words of chapter 5 [Margin: The bases of Copernicus for the diurnal motion of the Earth.]: *But if you grant that the heaven itself has nothing of this motion, and that the Earth turns from west to east, you will find, if you consider it seriously, that this is how matters stand as regards the apparent rising and setting of the Sun, the Moon and the stars. And since the heaven is that which contains and conceals all things, the common place of all, it is not at once apparent why motion should not be attributed to the contained rather than to the container, to the placed rather than to the placing. Heraclides and Ecphantus the Pythagoreans, and Nicetas of Syracuse in Cicero, were indeed of this opinion, turning the earth round in the middle of the world. For they thought that the stars set through the interposition of the earth, and rose through its withdrawal.* Then in chapter 8, having established as certain that the Earth, enclosed by its poles, is bounded by a globular surface, he went on: *Why, then, do we still hesitate to grant it a mobility that by nature suits its form, rather than have the whole world slide round, whose limit is unknown and cannot be known; and why do we not confess that the appearance of the daily revolution is in the heaven, and its truth in the earth?* Finally, in Book 1, chapter 11, asserting a threefold motion of the earth, he calls this diurnal one the *First*, which he says the Greeks call νυχθημέρινον [Greek: nychthēmerinon, "of a night and a day"]: the circuit proper to day and night, turning about the axis of the earth from west to east, as the world is thought to be carried in the opposite direction, and describing the equinoctial circle.

III. That the stars, however, are commonly thought to move toward the west rather than the Earth toward the east, Copernicus ascribes to a popular error very like the one by which sailors sometimes think that their ship stands still and that the shores and banks flee in the opposite direction [Margin: By the simile of sailors the opinion of those who think the Earth does not move is set out.]. For he says in that chapter 5: *This is just as if Virgil's Aeneas were saying:*

We are carried out of the harbour, and lands and cities recede.

For as a ship floats along in calm weather, everything outside is seen by the sailors to move according to the image of that motion, and they in turn think that they themselves are at rest, with everything that is with them. So, indeed, it can happen in the motion of the earth that the whole world is thought to go round. I showed in chapter 2, number 3, that this simile was used before Copernicus by Nicolaus Cusanus

[Margin: The same simile used by Cusanus and Buchanan.]. Indeed, not a few years before Copernicus's *Revolutions* were published, Georgius Bucanhanus [George Buchanan], a Scot, in Book 1 of his *Sphere*, written in most elegant verse, acknowledged the same simile, speaking in the person of the ancient Pythagoreans, but rejected it when he sang:

Since there is nowhere any violence of a mass so great That it could heave from its seat, and move by force, The heap of the Earth. Nor again does it turn itself Round in a circle, as a great part of the ancient sages falsely Believed, sworn to the words of the Samian master: For it is the eyes that watch the fires of the stars that are deceived, Which think the heaven is whirled round in a swift vortex While the earth stands unmoved; as when a ship flees the shore And skims the salt shallows with a following wind, And the hills flee, and the woods and cities recede, So night dulls the bodily senses with its darkness.

Kepler too, in Book 1 of the *Epitome of Copernican Astronomy*, page 130, heaps up other similes besides this one [Margin: Kepler's similes.]; for he adds: *Whoever sails downstream, unless he has been warned beforehand, will think that the nearby banks are moving the opposite way; and if the ship is carried past a stake that stands submerged with the current washing over it, the passengers will cry out that an otter is coming up to meet them. Whoever rides forward in a carriage between hedges will swear that the hedges on both sides are running at him; whoever rides backward will swear that the hedges are fleeing — an impression that the eyes conceive and retain, deeply imprinted, even when the man comes to a pause. And whoever is carried backward in a carriage away from some tower of remarkable height, along a road leading straight from the tower, will dread the fall of the tower bearing down on his head. So too, having watched a cloud breaking open* [?] *and widely spread, moving gently from south to north, a man will swear that the stars which fall into its rifts and shine out from them are moving with the contrary motion, from north to south.* Pierre Gassendi, for his part, in Letter 2, *De motu impresso a motore translato*, uses the example of two ships [Margin: Another simile of Gassendi's for this matter.]. Whether another ship moves toward ours while ours is unmoved, or ours moves toward another unmoved ship, he says that the motion nevertheless always appears to be not that of our ship but that of the other. So much so that, if we are in mid-sea, with nothing but sky on every side and sea on every side, we seem to ourselves to stand quite unmoved, however much we are driven by a following wind and carried toward another ship held fast by its anchors; and that ship will therefore appear to approach us exactly as much as we in reality approach it. This would happen above all to a man either born and brought up on a ship, or carried onto one while asleep and waking afterwards, and having no other means of correcting the illusion of his eyes. Having set these things out first, he adds: *Now the globe of the Earth is that onto which we have been carried as if asleep, or rather on which, not aware of it, we were born. And from the time we opened our eyes and, besides this globe, contemplated another, namely the Sun, in the same expanse, as it were, as the Earth, or in the space of the world, the question has been which of these globes rest would belong to, and which motion. And*

we, without hesitation, pronounced that rest undoubtedly belongs to the Earth and motion to the Sun. Pythagoras, Plato, Aristarchus and other ancients, and likewise Copernicus, Galileo, Kepler and many more recent writers, have warned that perhaps we are deceived, and that the Sun is at rest and the Earth moves. But he draws Pythagoras and Plato into this opinion too absolutely, as is clear from what was said in chapter 2.

IV. [Margin: Why this motion is commonly attributed to the stars rather than to the Earth.] Someone may ask why that motion, which could equally have been attributed to either while saving the phenomena, has usually been assigned to the stars rather than to the Earth. Kepler replies, in the *Epitome of Copernican Astronomy*, page 130, that motion is not the proper object of sight but is judged by the common sense, and that the common sense is deceived in this for two reasons. First, a man supposes that his eyes are at rest whenever he knows that the motion by which his eyes are carried does not come from his own internal motive faculty, or when he does not recognise that motion by any evidence of jolting; and therefore he judges that the object, rather than his own eyes, is moving. Second, we are inclined to attribute rest to things that are much larger than the eye and are widely spread out, as being hard to move — especially if they keep the same position relative to the eye — and to ascribe motion to smaller things. Now the fields and seas of the Earth appear much larger than the stars taken singly; so from childhood we have grown used to attributing motion to the stars rather than to the Earth. Galileo, moreover, in Dialogue 2 on the World System, affirms that we must necessarily fail to perceive this motion by sense, and that it is imperceptible to us, because we inhabit the Earth and share equally in the same motion. But let these things be said in passing [Greek: παρεργῶς (parergōs, "in passing")]; we must return [?] to unfold the other conditions of this diurnal motion.

V. [Margin: The kind of the Earth's diurnal motion.] The motion of the Earth, then, which we have so far said tends eastward according to Copernicus, is a kind of whirling or spinning [vertigo … seu turbinatio], by which the Equator of the terrestrial globe itself, together with all the terrestrial parallels, is turned round the axis of that same Equator in the space of 24 hours. This Equator lies in the plane of that imaginary Equator which writers on the sphere commonly employ to explain the motion of the primum mobile. The axis of the Earth, if it were produced beyond the Earth, and if the Earth were moved by no other motion than this diurnal one, would meet the poles of the World at either end. What happens to the axis so produced when the annual motion of translation, and the motion of the axis itself, are mixed with the diurnal motion of whirling, I shall say more fittingly below. Meanwhile we may know that on the globe of the Earth there are two poles, or immobile points, which stand at rest, to sense, beneath two opposite points of the heaven; that the parts of the terrestrial surface nearer those points describe small parallels and therefore move more slowly, while those farther from them describe larger parallels and turn more swiftly; and that the greatest circle, the terrestrial Equator, turns

most swiftly of all. In Copernicus's view, moreover, there is a single Equator, and it is terrestrial, and its motion alone suffices to explain everything that others explain by the celestial Equator and its parallels, as is plain from Book 1 of the *Revolutions*, chapter 11, and Book 3, chapter 1. Finally, that this motion is uniform and circular is to be established above all by this sign, as Kepler notes in the *Epitome of Astronomy*, page 111: those who, following Tycho's example, use armillae — that is, perfect circles — to measure differences of right ascension and the motion of the primum mobile set them so that their axis is perpendicular to their planes and elevated by as much as the elevation of the Pole of the World; and sighting on the stars through the sights fixed to the rim of the armillae, they see the stars go round the rim of the armillae uniformly. Besides[36] this, the spherical figure of the Earth itself demands that this whirling be circular rather than of any other shape.

VI. [Margin: The diurnal motion common to the Earth, the Water and the lowest air.] Nor, in Copernicus's view, is it only the element of Earth that is turned eastward by the diurnal motion. So too is all the water enclosed in the Earth's hollows, and that part of the air which is contained within and around all the highest mountains, or rises a little above them, inasmuch as it is akin to the Earth and the Water and consists of the vapours and exhalations of both — even though, because of its thinness, it may seem to us to be elemental air. Hence it happens that, because of this common whirling, neither those sailing westward, nor birds or clouds travelling toward the same quarter, experience any resistance from the water or from this air, as if it were striving against them. Indeed all terrestrial and aquatic bodies, and so all birds as well, and finally all mixed bodies, so long as they have not left this lower region of the air, share in this common motion, which they therefore do not feel, because it is common. But whether this motion of the water or of the lower air comes from some nature common to it and to the Earth, or rather from a seizure [raptus] made by the Earth, Copernicus did not determine. For in Book 1, chapter 8, he says: [Margin: By what force the water and the air are moved by this motion?] *What then shall we say of the clouds and the other things that hang in the air in any way, or sink, and again tend upward? Only that not the Earth alone, with the watery element joined to it, moves in this way, but also no small part of the air, and whatever things have a kinship with the Earth in the same way. Either because the nearby air, mixed with earthy or watery matter, follows the same nature as the Earth; or because the motion of the air is acquired, and it shares it from the Earth through contiguity, by perpetual revolution and without resistance.* Of the upper portion of the air, in which he thinks comets are formed, he adds these words: [Margin: The upper region of the air is exempt from the diurnal motion.] *We can say that this part of the air, because of its great distance from the Earth, is free of that terrestrial motion. Hence the air that is nearest the Earth will appear tranquil, and so will the things*

[36]printed *Ptæterquam-*, for *Præterquam-*

suspended in it, unless they are driven to and fro by the wind, or by some other force, as happens.

So you see that in this hypothesis everything is reversed. Those who assign the diurnal motion to the heavens think that the element of fire and the upper region of the air are carried westward with it, the lowest region remaining unmoved; whereas those who with Copernicus impose this whirling on the Earth think that the lowest region of the air is carried eastward with the Earth, the highest being immobile. But a further distinction must still be drawn within the lowest air: whether some part of pure air, distinct from the vapours and exhalations, is intermixed with them. For this pure air is carried by force by the very whirling of the terraqueous globe, and by the very vapours and exhalations that surround it; while the vapours and exhalations themselves are moved by nature itself, and by the same faculty by which the Earth itself and the water are moved — a faculty of which we shall have to speak below. [Margin: A perpetual breeze within the Torrid zone toward the West.] But if some part of the air around the Earth, because of its distance from the poles, is less subject to vapours and exhalations — such as the part that lies within the Tropics and is spread widely over the most open and vast expanses of the Ocean — that part, as Galileo held in Dialogue 4 on the World System, page 327, and Pierre Gassendi in Letter 2, *De motu impresso a motore translato*, as we shall say in Scholium 5, complies less readily with the seizure of the terrestrial rotation. Therefore in that tract, while the Earth rolls eastward and the air is moved more sluggishly in that direction, a perpetual breeze is felt, passing through from the East with so constant a course that by its help too, not only by that of the running waters, ships are carried happily to the West Indies; and ships setting out from the same Indies, from the Mexican shores, carried forward by the same favour, plough the Pacific Sea toward the Molucca and Philippine Islands. Voyages in the opposite direction, eastward, are by contrast more difficult and of slower course.

VII. [Margin: What gravity and levity are in this hypothesis.] Heavy bodies thrown upward, or let fall through the air from a tower or from a ship's mast, are seen to fall perpendicularly onto the same place of the Earth, so that the Earth does not withdraw itself from under them; and all heavy bodies universally tend by their own nature toward the centre not only of the Earth but of the world, while light bodies rise from that centre. So that this should not stand in the way of the terrestrial motion, Copernicus lays down in Book 1, chapter 9, that gravity is not a principle of tending toward the middle and centre of the Universe, but an innate propensity to tend toward its own whole, so that the parts may be preserved together with it and may arrange themselves in a spherical figure about the centre of their whole. For he says: *I for my part think that gravity is nothing other than a certain natural appetency, implanted in the parts by the divine providence of the Maker of all things, so that they may gather into their unity and wholeness, coming together in the form of a globe. And it is credible that this affection is present also in the Sun and the Moon and the other wandering lights,*

so that by its efficacy they remain in the roundness in which they present themselves; and yet they make their circuits in many ways. Therefore, if any part of the Moon or of the Sun could be torn away from their globe, once left to itself it would fly back not to the centre of the universe but to the globe of the Sun or of the Moon. For it is not the centre of the Universe, be- ing a point or nothing and lacking all appetibility, as Kepler notes in the *Epitome of Astronomy*, page 95, that is rightly set as the terminus toward which all heavy bodies tend; it is rather the Earth itself with its centre — not insofar as it is a centre, but insofar as it is the middle of that whole to which the kindred parts are borne. Kepler adds there, however, that this tendency of the parts toward their whole does not occur without a certain magnetic attraction; this, however, is excellently refuted by Cabeo in Book 1 of the *Philosophia Magnetica*, chapter 19. And if you urge that light bodies are carried upward, Copernicus will reply, in Book 1, chapter 8, that there is no fire other than this earthly one, or burning smoke, whose property is to extend whatever it has invaded, and that extensive motion runs from the centre to the circumference; but that this earthly exhalation or smoke is snatched upward and thrust out of its own place, and therefore at once grows faint, as though confessing the cause of the violence done to the earthly matter; and that, for this reason, levity does not exist, or is not connatural to these bodies.

Therefore, from the motion of heavy bodies toward the centre of the Earth, or of light bodies away from it — a motion that would have to take place along the shortest line, and so a straight one — and from the motion of whirling which these same bodies share with the Earth toward the East, it follows that things projected or thrust upward, and things falling downward, ascend and descend by a certain motion mixed of straight and circular. Granted, this common motion, precisely because it is common to the eye of the onlookers, who are carried along with the Earth, is not perceived, and so the motion appears to be only rectilinear and made along the perpendicular. But that it is mixed of straight and circular Copernicus taught in Book 1, chapter 8, and that in it the circular prevails, because the circular is unfailing and uniform, is intended per se by the nature of these bodies, and is common to the whole world; whereas the straight does not belong to them unless they are badly placed and set outside their own place, and it is unequal, and it is not distinguished from the circular in reality but only in thought, as a point is distinguished from a line and a line from a surface. But we shall report these very things in Copernicus's own words in chapter 5, number 17. Here let these words suffice: *We must admit that the motion of falling and rising bodies is twofold in comparison with the world, and altogether compounded of straight and circular;* and below: *Since, then, circular motion belongs to wholes, while parts have straight motion as well, we can say that the circular remains with the straight, as the animal does with the sick. Indeed, Aristotle's division of simple motion into three kinds — from the middle, to the middle, and about the middle — will be thought to be only an act of reason. In the same way we distinguish line, point and surface, although one cannot subsist without another, and*

none of them without body. Therefore in this hypothesis the motion of heavy bodies and of light bodies, upward or downward, does not take place along a line that is truly straight, but along one mixed of straight and circular, which are nonetheless distinguished only by reason. What kind of line this is will be discussed more conveniently in the scholia of this chapter (so as not to break the thread of this account too much), and more fully in chapter 17, from number 6.

VIII. [Margin: Kepler's doctrine on the principle of the Earth's diurnal motion.] So far we have set out with Copernicus the nature and conditions of the diurnal revolution of the whole globe, composed of earth and water and the atmosphere poured round it, and of terrestrial and aquatic bodies as well, inserting a few things from Kepler and Gassendi. Now certain other things must be added which Kepler devised, either of his own or from Gilbert's work on the magnet. In the *Epitome of Copernican Astronomy*, Book 1, from page 116 to 128, investigating the efficient and the subjective cause, together with the dispositions toward this diurnal motion, he holds that the terrestrial globe, and the whole Earth insofar as it is a whole, has no natural motion with respect to its matter, but rather an inertia toward motion and a certain resistance, especially in its denser parts and those nearer the centre. If, however, one considers its form, which is a certain faculty and soul moving it round in a circle, this whirling is natural to it, since nothing is more natural to matter than its form, or to a body than its soul. Thus the nature of the magnet tends downward because of its gravity, but by the species of its form ascends to another magnet, and that ascent is not violent but natural. Thus too the courses of animals poising themselves in the air, and the leaps and pounces of cats and of serpents, are not violent for the whole animal, if you consider the soul that demands such motions, although with respect to bodily gravity they may seem violent in a certain respect [secundum quid]. But before this soul moved the Earth, [Margin: God the first author of this whirling.] he says on page 120, God in the beginning impressed this diurnal whirling on the Earth, and from it all the subsequent rotations have flowed on by continued vigour, just as boys spinning a top impress on it an impetus for a great many gyrations; and to this day more than two million [vicies centena millia] diurnal whirlings have been completed. This vigour persists still because no external roughness or density of the ethereal air opposes it. Nor can any cause be given why the Earth revolves toward this quarter, namely toward the East, rather than toward another, except this: that it was set rotating toward this quarter by the Creator at once, in the beginning. Since, however, the vigour of this motion could be weakened by the inertia of the resisting Earth, or by the ebbing of an impetus gradually growing faint, two things contributed to the perpetuity and uniformity of this whirling: the impression of the motion, made according to the disposition of the Earth's parts, and a faculty and soul, motive and conservative of the original impetus, implanted by God.

IX. [Margin: Magnetic fibres of two kinds in the Earth's globe. [?]] As regards the disposition of the parts, Kepler, on pages 116 and 121, recognises

in the Earth's globe fibres or filaments of two kinds, just as in a great magnet. Those of the first kind are rectilinear fibres parallel to its axis; by virtue of them, if the globe is moved, it keeps itself in its original position, just as the Earth's axis, wherever it is carried, directs itself toward the same quarter of the world in which it was from the beginning, and always remains parallel to itself, and so has the character of something at rest, or retaining the same position. Moreover, because of such fibres parallel to the axis, the terrestrial globe has a faculty of directing magnets, and all magnetic things, toward itself; and magnets in turn have a faculty of turning themselves toward this quarter of the Earth and of conforming to its axis, or directing themselves toward the poles of the Earth — as do magnetic tongues or needles too, unless they are forced to decline toward great continents or nearby mines of magnets, where the motion is mixed in proportion to the moving principles. The fibres of the second kind are circular, like circular threads, into which those rectilinear fibres parallel to the axis are woven, standing circularly round the axis. This could be illustrated in some measure by the example of certain spiders, which weave their filaments with rectilinear rays and circular tracts. But Kepler, on page 122, preferred to use the example of fibres not merely twofold, as in the terrestrial globe, but threefold, found by anatomists in the substance of the stomach; the anatomists likewise recognise, among those three mutually interwoven orders of fibres, three faculties of the stomach: the attractive, along the straight fibres, upward; the retentive, along the transverse ones; and the expulsive, along the oblique ones, downward. But we have a more fitting example in trees. If their trunk is cut by a plane orthogonal to the pith, or axis, veins appear arranged in a circle in such a way that the ducts of the pores both tend upward — which is why the wood comes out fissile — and diverge transversely, like rays, from the pith toward the bark. Therefore it is by virtue of these circular fibres that a motion impressed along them is one of circular whirling about the axis, describing circles at right angles to the axis. Accordingly Kepler says on page 121 that it is likely that that very species of the first rotation, impressed by God on the Earth and continued, insinuated itself into the body of the Earth along the course of the rectilinear fibres, but arranged circularly about the axis, and was transformed as it were into a special bodily form that conserves this diurnal motion — no longer as a guest, as the impetus is in a top spun by boys, but as an inhabitant, and now the conqueror and tamer of matter, which of itself is inert; or at any rate that this impetus, with the rectilinear fibres demanding rest in the same position and the circular fibres demanding circular motion if any is impressed, is the proximate disposition required by the soul and the motive faculty. For he concludes on page 122 that even if this bodily form of the fibres were alone the cause of motion, what is moved and what moves would still not be the same; and that the same globe, by reason of the rectilinear fibres, rests in the same position and underlies the motion, while by reason of the circular fibres it continues the impetus it has received into circular motion.

X. For many reasons, however, besides the impetus once impressed on the Earth by God, and besides its species, conformed according to the position and disposition of the fibres already mentioned, a certain soul and a faculty, not intelle- ctive, not sensitive, not vegetative, but purely motive, and forming certain figures, must be added to the Earth, as Kepler held from page 123 on. [Margin: Foundations to be laid for a motive soul in the Earth's globe.] For just as bones, nerves and muscles are instruments and dispositions for motion rather than its moving cause, so too are the magnetic fibres of the Earth and the impression made along them. Next, the vigour, speed and uniformity of this motion will be more securely preserved if it comes from a soul, as from an internal principle and source, which reproduces its acts for itself like an Entelechy and restores itself, than if it comes from an extraneous principle, or from an impetus sojourning as a stranger in a foreign subject; and it will be the more like the first mover, God, who is the supreme Intelligence, if it is a soul than if it is not. And that very direction of the axis, brought into second act with so great a uniformity of circular motion, and one so perennial, is an argument for a soul which, if not intellective, at least governs this motion by some instinct. To these are added other indications of this soul: namely the perpetual and sensible subterranean heat — for just as cold is proper to matter, so heat is a trace of soul — and likewise the faculty that forms fossils, minerals, and the hexagonal stones in certain crystals and salts, similar to the faculty that forms hexagonal snow in the air, flies and locusts, and so many monsters in the water. And in the introduction to the *Commentaries on Mars* he says that if the Moon and the Earth were not each held back in its own circuit by an animal force, or by some other equivalent force, the Earth would ascend to the Moon by a fifty-fourth part of the interval, and the Moon would descend to the Earth by the remaining fifty-three or so parts of the interval, and there they would be joined by mutual magnetic attraction — provided, however, that they were of the same density.

XI. [Margin: The Earth a great magnet, according to Gilbert.] What Kepler devised concerning the soul of the Earth and the course of the magnetic fibres had, however, been anticipated by William Gilbert. For in Book 5 of *De Magnete*, chapter 12, he had asserted that the magnetic force, because of motions so varied and wonderful, is either animate or an imitator of soul; and by experiments with the terrella he had tried to show, especially in Book 6, chapter 4, that the globe of the Earth is a great magnet. The terrella is nothing other than a magnet turned into the form of a globe or sphere. In it the experimenter will be able to detect two poles and faces terminating its axis; and magnetic needles placed on it always conform to its axis in their length, but in altitude are inclined more or less toward the poles of the terrella, according as they are more or less distant from them. Since, then, similar things happen to magnetic needles with respect to the axis of the Earth, he thought he could gather from this, with fair probability, that the Earth is a great magnet. And just as the globe of a terrella, enclosed in a wooden box so that it can float, and set on

water so that its north pole points to the South, turns itself by a circular motion about its centre until its north pole faces the North, so he judges that the Earth is fitted for a magnetic revolution, so as to preserve the position it once had toward the poles of the World. This would be truer still if, as Petrus Peregrinus [Pierre de Maricourt] affirms, the globe of a terrella, suspended on its poles in the meridian, revolved in 24 hours. But our Cabeo, in Book 1 of the *Philosophia Magnetica*, chapter 19, taught by excellent arguments that the Earth is not a great magnet; nor, as we shall show below in chapter 7, numbers 10 and 11, is the Earth's diurnal turning rightly deduced from the turning of a terrella toward the poles. Further, Gilbert, in Book 6, chapter 5, himself also held that gravity is an appetency of the terrestrial parts to tend toward their primary globe; and he placed this appetency also in the parts of the celestial globes, with respect to their primary globes, namely the Sun, the Moon, and so on.

XII. [Margin: A fourth cause of the Earth's diurnal motion according to Kepler, namely the Sun.] Besides the causes already mentioned by which the diurnal revolution of the Earth comes about, Kepler recognises yet a fourth, in Book 4 of the *Epitome of Copernican Astronomy*, from page 550: namely the Sun. He does so in order to give a physical account of the equation of natural days, which he had set out in Books 1 and 3, that is, on pages 108 and 286, and which we treated in Book 3, chapter 32, number 4 — the equation, that is, which, as Tycho found by experience in eclipses, supposes the Earth's summer revolution to be a little slower than its winter one. This, he says, arises from the interval between the Sun and the Earth being greater in summer than in winter, because of which the Earth's faculty is then less strengthened by the Sun than in winter. This inequality of the diurnal revolution is also recognised by Longomontanus in Book 1 of the *Theorics*, the chapter on the Sun, and by Boulliau in Book 2 of the *Astronomia Philolaica*, chapter 6. Kepler therefore assumes that the archetypal number of the Earth's revolutions within one year ought to have been round, namely of 360 days, rather than composed of unarticulated, ignoble and broken numbers, namely of 365¼ days; and that the Earth, accordingly, if it were not stimulated by the Sun, would be moved more slowly in its diurnal revolution, since it would then revolve 360 times within the year. Therefore the fact that it adds five supernumerary revolutions and a quarter of a day is to be attributed to the speed of the Earth, stimulated by the Sun's light the more strongly the nearer it is to the Sun. How that supernumerary portion is to be distributed into the parts of the physical equation of natural days, we have already taught from Kepler himself in Book 3, chapter 32, number 4. [Margin: The diurnal revolution of the Earth is the cause of the Moon's proper motion, according to Kepler.] Moreover, just as the Earth is stimulated by the Sun in its diurnal revolution, and just as all the primary planets, by their whirling about their own centre, emit a certain species by which, as though with a hand, they grasp their secondary planets — the Sun its six lesser planets, and Jupiter its satellites — and drive them round in a circuit, so the Earth, by gyrating about its axis, emits from itself a certain species by which it seizes the Moon and whirls

it round toward the same eastern quarter, as far as the Moon's inertia toward motion allows it to be carried round. And so, just as he attributes the motion of the other planets and of the Earth — but the annual motion, eastward — to the magnetic drawing-round by the Sun, through the species emanating from it while it turns about its centre, so he attributes the Moon's proper motion eastward to the diurnal revolution of the magnetic Earth, drawing the Moon round in a circle; and from this, he says, it comes about that the Moon is the swifter the nearer it is to the Earth. He confirms this from the kinship between the Moon and the Earth: through the telescope the Moon is seen to be another earth, roughened with mountains, seas and valleys, and the Moon produces the reciprocal tide of the sea by its passage over the vertices. But let it be enough, and more than enough, for our present purpose to have sampled these things from Kepler, in order to explain, illustrate and confirm Copernicus's hypothesis of the Earth's diurnal revolution.

Explicatur Secundus Telluris Motus, qui eſt Tranſlatio Annua Centri terreſtris per Orbem Magnum circa centrum Vniuerſi.

The second motion of the Earth is explained, which is the annual translation of the Earth's centre along the Great Orb about the centre of the Universe.

XIII. The second motion, which Copernicus, with Philolaus and Aristarchus, attributes to the Earth, is called the annual translation. By it the centre of the terrestrial globe is carried uniformly in the plane of the Ecliptic round the Sun, which is placed at or near the centre of the World, in the order of the signs, or toward the East; and it completes its mean period in 365 days and 6 hours. With the centre of the terrestrial globe, moreover, not only the whole Earth but also the whole elementary sphere, and the very body and system of the Moon, are carried round, since the Moon is the Earth's attendant and, as it were, its secondary planet. The Earth, then, becomes one of the six planets, and is not at the centre of the universe. The Sun, on the other hand, ceases in this hypothesis to be a planet, since it is not moved by the motion of translation, nor even by the diurnal motion of whirling, but remains immobile near the centre of the universe. The orb or circle which the centre of the Earth describes by this motion in the plane of the celestial Ecliptic is commonly called by Copernicus the Annual Orb [Orbis Annuus], but sometimes, as in Book 1, chapter 10, the Great Orb [Orbis Magnus], by comparison with the terrestrial Equator which the diurnal revolution of the Earth describes on the terrestrial surface, and which is indeed far smaller than this annual circle; for the annual circle, though distinct from the Sun's, is as large as the orb of the Sun is in the Ptolemaic hypothesis. It can also be called great because it has great equipollence and power, since it alone accomplishes what scarcely many other orbs can accomplish in the Ptolemaic system — although with respect to the highest sphere of the Fixed stars it is like a point. To this Great Orb, which we may also call the annual orb and the terrestrial Ecliptic, the terrestrial Equator is inclined at an angle of

23° 30′: as great, that is, as the obliquity of the celestial Ecliptic to the celestial Equator in the hypothesis of a standing Earth, and as great likewise as the inclination of the axis of the Ecliptic to the axis of the Equator, or the distance between the Poles of the Zodiac and of the Equinoctial circle. These things, which in summary concerning this motion we have touched on, must now be explained from what Copernicus and the Copernicans add to it.

XIV. [Margin: Copernicus's prelude to the annual motion of the Earth.] First, then, Copernicus, in Book 1, chapter 5, having mentioned the diurnal motion of the Earth, at once added: *If anyone should deny that the Earth occupies the middle or centre of the world, and yet not admit that its distance is so great as to be comparable to the sphere of the non-wandering stars, but hold it considerable and evident with respect to the orbs of the Sun and the other stars, and should think that their motion appears irregular for that reason, as though they were regulated about some other centre than the centre of the Earth, he could perhaps offer a not inept account of the irregular apparent motion. For the fact that the wandering stars are seen now nearer the Earth and now, the same ones, more remote necessarily argues that the centre of the Earth is not the centre of their circles. It is all the less settled whether it is the Earth that nods toward them and away from them, or they toward and away from the Earth. Nor would it be so very surprising if someone, besides that daily revolution, were to hold some other motion of the Earth: namely, that the Earth revolves, and wanders with several motions, and is one of the stars — as Philolaus the Pythagorean is said to have held, a mathematician of no common rank, for the sake of visiting whom Plato did not hesitate to go to Italy, as those who wrote the life of Plato report.*

With these words he paves the way for himself, by a kind of prelude, to the motion of the Earth along the Great Orb. And first he requires a distance of the Earth from the centre of the universe so great that it bears an evident proportion to the distances of the Planets from the same centre, but an unevident, or imperceptible, one to the distance of the Fixed stars. Next, from the fact that the diameters of the Planets appear now larger, now smaller, he infers that the Earth is not the centre of their motions; and that therefore that diversity, or second inequality — but the greater and more frequent one, which they call the anomaly of the orb, or the Argument, or which they attribute to the motion of the planetary orbs, referred to the Sun's approach or recession — can not ineptly be attributed to the motion of the Earth as it approaches them and recedes from them annually. And he fortifies himself with the authority of Philolaus. After this, in the same Book 1, chapter 9, having said that in the Sun and the Moon and the other Planets there is an affection and inclination of the parts toward their whole, so that they arrange themselves in a globe about their centre and are preserved together with the whole, and that this propensity is of the same kind as the one heavy bodies have toward the centre of the Earth; and that nevertheless the centre of those Planets is not the centre of the universe, but that they perform their own circuits — he added: *If, then, the Earth too should perform other circuits — for instance, about a centre — these*

must necessarily be those which likewise appear externally in many bodies, among which we find the annual circuit. For if it is changed from a solar into a terrestrial one, immobility being granted to the Sun, the risings and settings of the signs and of the Fixed stars (he is speaking of rising and setting with respect to the Sun's rays, that is, the heliacal) *by which they become morning and evening stars will appear in the same way; the stations, retrogradations and progressions of the wandering stars, too, will be seen to be not theirs but a motion of the Earth, which they borrow for their own appearances. Finally, the Sun itself will be thought to occupy the middle of the world.* See how he teaches that an annual motion can be granted to the Earth and immobility to the Sun — and in such a way that not only can all the appearances which used to be explained in the Sun and the other Planets by the annual motion of the Sun take place, but they can take place without the imperfection of a real retrogradation and station of the planets.

XV. Having touched on these things first, Copernicus, in the following chapter, that is chapter 10, treats of the order of the Planets in the system of the World. He argues both from their light, received from the Sun, and from the fact that the three superior planets become nearer the Earth about their evening rising, that is, when they are opposite the Sun, but are most remote when, having become morning stars, they are in conjunction with the Sun; and from the fact that Venus and Mercury, in the opinion of some of the ancients as well, go round the Sun. From this he concludes that not the Earth but the Sun is the centre of their motion, and that about it revolve Mercury, then Venus, after that the Earth with its attendant the Moon, then Mars, Jupiter and Saturn. And the annual motion of the Earth, which he had indicated in chapters 5 and 9 and had called not inept for saving the Phenomena of the heavens, he now, his shame at last overcome, introduces outright into the system of the World, saying: [Margin: Copernicus now introduces the annual motion of the Earth outright.] *But since all these rest upon one middle, it is necessary that the space left between the convex orb of Venus and the concave orb of Mars also be marked out as an orb or sphere, concentric with them on both its surfaces, which receives the Earth with its attendant the Moon and whatever is contained beneath the lunar globe. For we can in no way separate the Moon from the Earth, since beyond dispute it is the nearest to it — especially as in that space (that is, between the convex of Venus and the concave of Mars) we find a place for it that is suitable enough and ample. We are therefore not ashamed to confess that this whole which the Moon encircles, together with the centre of the Earth (that is, all the spheres of the Elements and of the Moon), passes round the Sun along that Great Orb, among the other wandering stars, with an annual revolution; that about it (that is, near the Sun) is the centre of the world; and that, the Sun also remaining immobile, whatever appears of the motion of the Sun is rather verified in the mobility of the Earth.* There follows, then, the order of the spheres as laid down and described by Copernicus, as you see in the system attached.

Diagram. The Copernican order of the spheres, set in the right column. The Sun, a rayed disc, stands at the centre. Round it run concentric circles: the

two innermost (dotted) carry the orbits of Venus and Mercury; next comes the Earth's circle, on which a dark globe (the Earth) carries a small epicycle with the Moon's sign; outside it the circles of Mars, Jupiter and Saturn; and outermost a circle set all round with star sorts, for the sphere of the Fixed stars. The labels run round the circles:

- Outermost labelled orbit (Saturn's sign): "Revolution of 30 years"
- Next (Jupiter's sign): "Revolution of 12 years"
- Next (Mars's sign): "Revolution of 2 years"
- Round the Earth's circle: "Of the Earth … and Lunar … annual revolution … orb … Annual" — the word set sideways on the right of this circle is unread [?]
- Second dotted circle (Venus's sign): "Revolution of 9 months" [?]
- Innermost dotted circle (Mercury's sign): "Revolution of 80 days"

In it the Sun is to be placed not, indeed, at the very centre of the World, if you listen to Copernicus, but about or near the centre of the world, at as great a distance from that centre as, among the Ptolemaics, is the eccentricity of the Sun's orb with respect to the centre of the Earth. For according to the Ptolemaics the Planets, in their second inequality or anomaly, look to the mean motions of the Sun, that is, the equal ones; and these the annual motion of the Earth here supplies about the centre of the world. About that centre Mercury, nearest the Sun, is carried round with a revolution of about 80 days; then Venus, in a [?] — that is, a looser and more remote — orb, with a revolution of about 9 months. About it the globe of Earth [?] and water, with the whole elementary sphere and with the Moon and the lunar heaven, turns in a period of 365 days and a quarter of a day; and above this orb Mars, in about two years, Jupiter in twelve, and Saturn in a period of three years [?], all revolve toward the eastern quarter; while the outermost sphere of the Fixed stars, separated from the circuit of Saturn by an almost immeasurable interval, rests immobile.

XVI. With this annual motion of the Earth substituted for the annual revolution of the Sun, two kinds of τῶν φαινομένον [Greek: tōn phainomenon, so printed for φαινομένων, "of the phenomena"], that is, of appearances, are simply explained: the first in the Sun and the Moon, the second in the five lesser Planets. On the Phenomena concerning the Sun let us hear Copernicus discoursing thus in Book 1, chapter 11: [Margin: Effects of the annual motion of the Earth with respect to the Sun alone.] *The second is the annual motion of the centre of the Earth, which describes the circle of the Signs about the Sun, likewise from west to east, that is, running forward in the order of the signs, between Venus and Mars, as we have said, together with the things lying upon it — that is, with the elements and the lunar heaven, resting on the centre of the Earth. Hence it is that the Sun itself seems to traverse the Zodiac with a similar motion: as, for example, when the centre of the Earth passes through Capricorn, the Sun seems to pass through Cancer; from Aquarius, Leo; and so on, as we have said.*

For, for clarity's sake, in the diagram immediately following let the Sun be near the centre of the Universe, at S. From this centre describe the celestial

Ecliptic with its four cardinal points, which the appropriate letters indicate: namely A, the beginning of Aries; C, of Cancer; L, of Libra; and K, of Capricorn. Then in the plane of this Ecliptic, and from the same centre, describe the inner circle EFBD, which the centre of the Earth traces annually by its motion, from E through F and B to D, until it returns again to E. This will become plain if the centre of the Earth is at E, under the beginning of Aries: then an eye at G, looking from the surface of the Earth at the Sun S along the line EGSBL, will see the Sun itself appear under L, the beginning of Libra. When the centre of the Earth has advanced from E to F, under the beginning of Cancer, an inhabitant of the Earth will see the Sun S, along the line FSK, under the beginning of Capricorn. Then, as the Earth proceeds to B, that is, to the beginning of Libra, the Sun will seem to be under the beginning of Aries, at A; and when the Earth has been carried on to D, under the beginning of Capricorn, the Sun will appear to it under C, which is the beginning of Cancer. And everything is to be understood in the same way of the other Signs, by opposition.

Diagram. The annual orb and the apparent place of the Sun, set in the left column. A large circle (the celestial Ecliptic) is lettered at its four cardinal points — C at the top, L at the left, A at the right, K at the foot — with a sign sort beside each. Inside it a circle centred on the Sun S carries four small circles, the Earth at the four quadrants of its annual orb, centred at F (top), B (left), E (right) and D (bottom); each small circle is lettered P, G, I, H about its centre. Straight lines through the Sun join opposite stations to the Ecliptic.

- Top: sign of Cancer; C; small circle with P, G, I, F, H
- Left: L, with a sign sort below it [?]; small circle with G, B, I, H, P; a crescent Moon sort and N between this circle and the Sun
- Centre: S, the Sun, drawn as a rayed disc
- Right: small circle with P, G, E, I, H; A, with the sign of Aries
- Bottom: small circle with P, G, D, I, H; K, with a sign sort below it [?]

Therefore, when for us it is the Vernal Equinox, the Earth is in the autumnal section, that is, under the beginning of Libra with respect to the Fixed stars, or rather with respect to the Rational Zodiac; when it is the Autumnal Equinox, the Earth is under the vernal section; and when it is the Solstice, the Earth is under the beginning of Capricorn, but when it is midwinter [Bruma], under the beginning of Cancer. So, just as the revolution of the Earth's diurnal whirling produces for us the alternation of days and nights, the annual revolution brings back the four seasons of the year, and all the variety within them, which the Sun brings in the Ptolemaic hypothesis — supposing, however, that the axis of the terrestrial Equator is inclined to the plane of the Ecliptic by 23½ degrees, at right angles to the Equator, and always parallel to itself, as we shall say below.

[Margin: How the Earth's aphelion and perihelion answer to the Sun's apogee and perigee.] Moreover, the centre of the annual, or great, orb is distant from the centre of the Sun's body, along the line of the Apsides, by as large a portion as the Eccentricity of the Sun's orb, or its distance from the centre of the universe, is supposed to be in the other hypotheses; and in this age the

greatest distance of the Sun from the Earth, that is, its Apogee, is about the beginning of Cancer, or about its 7th degree, and its Perigee about the 7th degree of Capricorn. It follows that the Sun is at apogee, that is, most distant from the Earth, when the Earth itself by its motion has come under the 7th degree of Capricorn; for then it will see the Sun under the 7th degree of Cancer, and the Earth in its turn will be at aphelion, that is, most distant from the Sun. Conversely, the Sun will be at perigee, and the Earth at perihelion, when the Earth has come to the 7th degree of Cancer, from where it will see the Sun under the 7th degree of Capricorn. [Margin: Effects of the annual orb with respect to the Sun, taken together with the Moon.] And since the Earth carries the lunar heaven with it, and the Moon goes round the Earth — now set between the Earth and the Sun, at new moons, now having the Earth between itself and the Sun, as at full moons — not only eclipses but also the other aspects of the Moon to the Sun are presented to the Earth itself just as if the Earth were at rest in the centre of the world and the Sun were carried round through its annual orb.

XVII. We should like, moreover, to set before the eyes in a special diagram what we have said about the annual revolution mixed with the diurnal, and about the Moon. From A, then, as centre — where the centre of the Sun is approximately — describe the Earth's annual orb BECD; and on its circumference, taking a centre at C, describe from it the terrestrial globe, whose Ecliptic FGOH is to lie in the plane of the great orb BECD, while its Equator FIOL is inclined to it at the angle GFI, or HFL, of 23½ degrees. Let the portion of the Earth illuminated by the Sun, that is, the diurnal hemisphere, be HFI, and the nocturnal one HOI, casting into the part turned away from the Sun the terrestrial shadow HXZI; although the diagram shows this truncated, you are to conceive it extended as far as the apex of its cone. Next draw through C, to the surface of the terrestrial globe, several straight lines marking great circles of the Earth — at right angles to its Equator if you want the position of a right sphere, or oblique if you want that of an oblique one — to serve now as Meridian, now as Horizon. Let such a line, for example, be FCO, representing the meridian of an inhabitant at F, who has midday, and of an inhabitant at O, who has midnight; and let their Horizon, at right angles to the Equator, be HCI. [Margin: The succession of the hours from the diurnal revolution of the Earth.] Since, then, the Earth turns about C, the centre of its body, toward the East, from O to H, and completes its whole revolution in the space of 24 hours, it happens that for an eye at H the Sun rises and for an eye at I it sets, while for an eye at F it is midday and for an eye at O midnight. But after three equinoctial hours the Meridian FO is turned round to the position PQ, and the Horizon HI to the position MN; and neither actually performs the office of Meridian or of Horizon, but each retains only the character common to a circle of Declination and an Hour circle. So for the eye at O, carried over to Q, it will be the 3rd hour after midnight; for the eye at Q, brought to H, the Sun will rise; for the eye that was at H, carried to M, it will be the 3rd hour

after sunrise; for the eye at M, which before had the 3rd Babylonian hour, that is, 3 hours before midday, now brought to F, it will be midday; for the eye at F, which at first enjoyed midday, turned round to P, it will be the 3rd hour after midday; for the eye at P, brought to I, the Sun will now set; and for the eye that three hours before was at I, carried to N, it will be the 3rd hour after sunset, or 3 hours before midnight. Finally, for the eye that had been at N, turned round to O, it will be midnight. And so with the other hours of the day, and the alternations of the days, according as the Horizons marked on the terrestrial globe are more or less right, or more or less oblique, to the Equator.

Diagram. The annual and diurnal motions of the Earth together with the Moon, set in the right column. A large circle, the annual orb, is lettered D (left), E (right) and B (foot); C, the centre of the terrestrial globe, stands on its upper part. The globe at C, half shaded, is divided by lines and lettered round about; from it a hatched, truncated shadow cone rises, with X and Z at its top corners. Round the globe runs the Moon's circle, with Moons, half dark, at R (in the shadow cone), V (left), T (right) and S (below). Inside the annual orb, round the Sun A (a rayed disc), run two circles carrying four small bodies.

- Shadow cone: X, Z; the Moon at R within it
- Above the globe: Q, O, N
- On the globe: H (left), I (right); L, G within; C at the centre; M, F, P below
- On the Moon's circle: V, T, S
- Inner circles round the Sun A: n and u on the outer of the two, r and m on the inner; q and p below the Sun
- Annual orb: D, E, B

Now, for the lunar system, describe from the centre of the Earth C the circle, or orbit, of the lunar heaven STRV. [Margin: The lunar phases and eclipses with the Earth placed outside the centre of the world.] In it the new Moon, set between the Earth C and the Sun A, illuminated toward the Sun but dark toward the Earth, will be able — if its latitude from the great Ecliptic, of itself or on account of the parallax of latitude, does not prevent it — to cover the Sun for the Earth in a total or partial Eclipse. At the first quarter the same Moon will appear διχότομος [Greek: dichotomos, "cut in two"], that is, bisected, at T, and will appear so to an eye at I at sunset, and nearly so to eyes not far removed on this side of I or beyond it. Then, the semicircle completed, the Moon will be at R, and will appear full of light to the Hemisphere HOI, unless a small latitude from the plane of the Ecliptic has carried it within the sha- dow of the Earth; for then it will undergo an Eclipse. Finally, at V, at the time of the second quarter, it will appear halved, and that indeed to an eye at H, at sunrise. From these phases it is very easy to understand the others. You see, then, that for these phases, or for the Eclipses of the Luminaries, it is not necessary that the Earth be at the centre of the universe, A; rather, the Sun can be there, and the Earth can be removed from that centre by a due interval, provided it carries with

it through the annual orb the system of the lunar heaven and the elementary sphere, resting on the same centre C.

[Margin: A foretaste of the phases of ♀ and ☿.] If one likes to describe from A the inner orbs of Venus, *upn*, and of Mercury, *mqr*, which the great orb contains within itself, it will at once appear how, for the eye at I, for which we have supposed the Sun to be setting, Venus at *u* appears as an evening star at its greatest digression from the Sun, and nearly horned, not yet fully halved; as does Mercury, an evening star, at *m*. But to the eye at H, for which the Sun rises, Venus at *n* will appear as a morning star, and Mercury at *r* as a morning star, both with a falcate light. In other places of their orb they will show one phase after another; but at *p* and *q* they will be wholly hidden behind the Sun.

[Margin: Station, direction and retrogradation of a kind in terrestrial bodies, by force of the Diurnal and the Annual motion.] Lastly, from these two motions — the Diurnal, by which the Earth turns about its own centre, and the Annual, by which the centre of the Earth, with the whole atmosphere and the lunar heaven, is carried toward the East — it follows that the motion which these bodies undergo daily by force of the annual motion becomes unequal by force of the diurnal motion. For the Diurnal motion of the points of the terrestrial surface, through the whole arc IOH, which is nearly the nocturnal arc, is direct and swift, and toward the same eastern quarter as the motion of the centre C; it therefore adds to the Annual motion, most of all at the point O, of midnight, where the greatest speed occurs. But in the arc HFI, which is nearly the diurnal arc, the diurnal motion is, as it were, retrograde, toward the western quarter, and subtracts from the annual motion of the centre — especially at F, the point of midday, where the greatest slowness is. Near the points H, for which the Sun rises, and I, for which it sets, the motion is nearly stationary, and the diurnal motion neither adds anything to nor takes anything from the annual motion of the centre C. For this inequality consult the Tables to be set out in chapter 22, number 1. Therefore all the points of the Earth placed outside its centre, and all earthy and watery bodies, or bodies of the thicker air, that follow the motion of the Earth, become every day once direct, once retrograde and twice stationary, and would appear so to an eye placed at A, the centre of the annual orb. [Margin: The Moon stationary and retrograde by force of the same motions.] The Moon too, in its circle STRV, is compelled every month to be once Direct, in the arc TRV — namely from the first quarter T to the second V — and swiftest at the full moon R, as regards its monthly motion combined with the annual; but from the second quarter to the first quarter of the next Lunation it becomes Retrograde, and slowest at the new moon S, while at the quarters it is Stationary. But on these stations, directions and retrogradations of the Earth and the Moon, see, if you like, what we shall say more fittingly below, in chapter 14 from number 4, and in chapter 19 from number 12.

XVIII. [Margin: Effects of the annual motion of the Earth with respect to the lesser Planets.] Furthermore, the effects of the Annual motion of the Earth with respect to the five lesser Planets, and the phenomena represented by it in a truly

wonderful compendium, are very many; and they were indicated in summary by Copernicus in Book 1, toward the end of chapter 9, where he says of the annual circuit: *For if it is changed from a solar into a terrestrial one, immobility being granted to the Sun, the risings and settings of the Signs and of the stars*, namely the heliacal, *by which they become morning and evening stars, will appear in the same way; the stations, retrogradations and progressions of the wandering stars, too, will be seen to be not theirs but a motion of the Earth, which they borrow for their own appearances.* But in the same book, chapter 10, and in Book 5, chapters 2 and 3 and chapters 21 and 26, he explains many more of their phenomena by means of the annual orb, so as to represent their second inequality, or anomaly of the orb. These have been carefully reviewed by Georg Joachim Rheticus in his *Narratio prima*; by Kepler in the *Mysterium Cosmographicum*, chapter 1, in the *Epitome of Copernican Astronomy*, Book 4, from page 542, and in the introduction to the *Commentaries on Mars*; by Galileo in Dialogue 3 on the World System; by Pierre Gassendi in Letter 2, *De motu impresso a motore translato*, and in the *Institutio Astronomica*, Book 3; and by *Philolaus* revived, published at Paris. From these I wish to list the chief ones briefly.

[Margin: The second anomaly of inequality of the Planets explained by the single Great Orb.] *First.* It has been observed that the five lesser Planets, besides the first equality [?] or anomaly, which they have in their periodic motions independently of the Sun, also have a second, and that a greater and more frequent one, depending on the Mean motion of the Sun according to the ancients, or on its True motion according to the more recent Astronomers, as was said in Book 7, Section 1, chapter 7. And since in the ancient hypothesis that diversity arises from the revolution of the Sun relative to the Planets — which is swifter than their own proper motion in Saturn, Jupiter and Mars, but slower in Venus and Mercury — that whole anomaly is accounted for by the annual approach of the Earth itself, from which [?] we observe the Planets, to the Planets, or its recession from them; and the Sun becomes the centre of the planetary system, as it is the source of motion and of light. This equipollence Copernicus indicates in Book 1, chapter 5, from the words *Quo assumpto*, etc., and in Book 5, chapter 1, where he calls the motion of the Earth relative to the Planets the *motion of commutation.* [Margin: What the motion of commutation is in Copernicus.] For he says: *Each Planet, then, has privately its own revolution of commutation — I mean the motion of the Earth relative to the Planet — which they explain between themselves. For we say that the motion of commutation is nothing other than that by which the equal motion of the Earth exceeds their motion, as in Saturn, Jupiter and Mars, or is exceeded by it, as in Venus and Mercury.* Accordingly, the Anomaly that in Ptolemy is called the motion of inequality, or of diversity, among the Alphonsines the Anomaly of the argument, by Longomontanus the Motion of the Anomaly, and by many the Anomaly of the orb, is called in Copernicus and in the Prutenic Tables the Anomaly of Commutation; and what the ancients explain by five orbs, together with the Sun's Eccentric, Copernicus represents by the single Great Orb of the Earth's annual motion. In this way

the three epicycles of the superior planets, and the two Eccentrics of the inferior ones, which Ptolemy employed to explain that second diversity, are removed.

[Margin: The annual motion of the Earth the cause of the identity of the motion of ☉, ♀ and ☿.] *Second.* A reason seems to be given why the Sun, Venus and Mercury are equal in mean motion of longitude, that is, have equal revolutions of longitude, according to that saying of Plutarch in *De placitis philosophorum*, Book 2, chapter 16: *Plato and the Mathematicians think that the Sun, Venus and Mercury are moved with equal motions.* Hence it is that the table of the Sun's mean motion also serves for computing the mean motions of Venus and Mercury. For the reason, according to Copernicus, is that these planets do not have three distinct orbs, as in Ptolemy; rather, the single annual motion of the Earth accomplishes their motion in longitude, since they are enclosed within the great orb. But because the Earth revolves round these three Planets, the dwellers on the Earth judge that the planets are carried round about them while they themselves stand immobile, and so out of one motion they make three. Pressing this point against the ancient system, Copernicus says in Book 1, chapter 10: *But what cause will those allege who place Venus beneath the Sun, and then Mercury, or separate them in some other order, for these planets not likewise making circuits separate and distinct from the Sun, as the rest of the wandering stars do?*

[Margin: The appearance of stations and retrogradations from the single annual motion of the Earth.] *Third.* A reason seems to be given why the five lesser Planets appear Stationary and Retrograde, and do not always advance Direct — which seems the greatest imperfection in celestial bodies, and which made the ancients very anxious, as may be seen in Vitruvius, Book 9, chapter 4, and in Pliny the Elder, Book 2, chapter 16, and before them in Apollonius, as we related more fully in Book 7, Section 5, chapter 3. For in the Ptolemaic system, because the motion of the superior planets on their Epicycles is in reality impeded, or that of the inferior Planets on their Eccentric orbs is tempered by the motion of another orb, the planets sometimes do not move at all beneath the Fixed stars as regards their true motion, but seem to stand still, and sometimes they go back toward the West. In the Copernican hypothesis, by contrast, a Phenomenon of this kind is a mere appearance, arising not from the object itself, that is, from the real motion of the Planets, but from the carrying of our eye along with the Earth. For since the Earth, overtaking the superior and outer Planets by its annual motion, is swifter than they are, it afterwards leaves behind it, toward the opposite quarters of the world, those it used to see ahead of it; whereas it is slower than Venus and Mercury, the inner Planets, enclosed within the great orb, and lets itself be overtaken by them against the order of the signs. So the planets seem at times to go back, and, in the passage from direct course to retrograde or from retrograde to direct, to stand still — when in reality it is the Earth that makes these changes of aspects and motions by its single motion. How this comes about we have already set out, with figures suited to the purpose, in Book 7, Section 5, chapter 4.

[Margin: The annual motion of the Earth frees the Sun and the Moon from Stations and Retrogradations.] *Fourth.* From this a reason seems to be given why the Luminaries do not appear stationary or retrograde. The cause is that the Sun is at rest, while the Earth is always moved in the order of the signs with an equal motion; and so the Sun too, into the opposite par- ts, always seems to the Earth itself to advance. The Moon, however, seems neither to stand still nor to go back, because the annual motion of the Earth is common to the Earth and to the lunar heaven; and two movable bodies that have the same motion toward the same quarters seem to be at rest with respect to each other. Hence the motion of the Earth is not discerned in the Moon, as it is in the other planets. In the Ptolemaic hypothesis, by contrast, the reason for this difference is given more laboriously.

[Margin: The variation of the Arcs of Station and Retrogradation, from the annual motion of the Earth.] *Fifth.* From the same source is drawn the reason why the progression appears greater, and the regression smaller, in the Planets nearer the Earth than in the more remote ones: that is, why the arc of direction is greater in Mars than in Jupiter, and in Jupiter than in Saturn, and likewise greater in Venus than in Mercury, while the arc of retrogradation in the same planets is smaller, as is clear from the small table set out in Book 7, Section 5, chapter 2, number 4. Conversely, it gives the reason why the alternations of stations and retrogressions appear more frequently in ♄ than in ♃, and in ♃ than in ♂, and likewise more frequently in Mercury than in Venus. The reason, namely, is that the Earth by its motion overtakes Saturn more swiftly than Jupiter, and Jupiter than Mars, whereas Venus overtakes the Earth more slowly than Mercury does. Copernicus touched on these alternations in these words, Book 1, chapter 10: *For from this it may be noticed by one who contemplates not idly why the progression appears greater in Jupiter than in Saturn, and smaller than in Mars, and in Venus than in Mercury; and why such a reciprocation appears more frequently in Saturn than in Jupiter, more rarely still in Mars, and in Venus than in Mercury.*

[Margin: The reason for the Digressions from the Sun, from the annual orb.] *Sixth.* From this it likewise appears why Saturn, Jupiter and Mars can digress from the Sun through a whole semicircle, whereas Venus and Mercury cannot digress through a whole semicircle, but far less. For the orbs of the three superior Planets lie outside the great orb of the Earth and are therefore larger, while those of the inferior Planets lie within it and are smaller, as will be plain to anyone who contemplates the system set out in number 15. But if you hold to the nature of the Ptolemaic hypothesis, then, observations set aside, no cause appears why an orb as large as the Epicycle of Mars, of Jupiter or of Saturn could not be assigned to Venus and Mercury, so that, embracing the Earth in their circuit, they could digress from the Sun through a whole semicircle.

[Margin: The size of the Epicycles and of the Equations, from the annual orb.] *Seventh.* From the same situation and motion of the Earth the reason seems to be established why in the larger orbs the Epicycles are smaller, and in

the smaller orbs larger. For in Ptolemy, if he is to preserve the prosthaphaereses of the orb, the Epicycle of Saturn is smaller than that of Jupiter, and Jupiter's smaller than that of Mars, although the Eccentric deferent carrying the Epicycle of Saturn is larger than Jupiter's, and Jupiter's larger than Mars's. That is to ask why the prosthaphaeresis of the orb of Mars is greater than that of Jupiter, Jupiter's greater than Saturn's, and Venus's greater than Mercury's. The reason is this: the nearer each of the superior Planets is to the orb of the Earth, the greater the proportion that the annual orb of the Earth bears to it, and the larger that orb appears. The true orbs of Mercury and Venus about the Sun, on the other hand, are those which the ancients took for Epicycles; and Mercury's, as the swiftest planet, is also the smallest orb. In this way the vastness of the Epicycle of Venus is removed, and that of Mars, which in Ptolemy is certainly excessive with respect to the Deferent.

[Margin: The Direction of the superior planets in conjunction (☌) and their Retrogradation in opposition (☍) with ☉, from the annual motion of the Earth.] *Eighth.* The annual motion of the Earth seems to bring with it the cause why the Superior Planets are always direct in conjunction with the Sun and retrograde in opposition — a cause Ptolemy cannot give a priori, if he is asked why he does not move the Planets, together with the Epicycle, always against the order of the signs. But through the single annual motion of the Earth toward the same parts toward which the Planets move in their orb, it follows that the Earth, when it is placed between them and the Sun — that is, at the oppositions with the Sun — leaves them behind by its speed, while at the conjunctions it has them ahead of it, slower though they are. So much for the motion of longitude.

[Margin: The annual motion of the Earth makes the Inclinations of the Planets to the Ecliptic constant.] *Ninth.* The variation of the latitudes arising from the inclination — which comes from the approach of the Planets to the Earth, or from their recession, on the Ptolemaic Epicycle — requires five different librations and variations of the inclinations. But if the Earth itself is supposed to approach them and recede from them by its annual motion, all the orbs will be inclined to the Ecliptic with complete constancy, and those superfluous librations will be removed; and so the Phenomena of the latitudes, especially in Venus and Mercury, are saved much more simply, and through an a priori cause. But see what we said in Book 7, Section 4, chapter 5, Scholium 3.

[Margin: The annual motion of the Earth is the cause why ♄ ♃ ♂ appear larger and nearer in opposition (☍) with ☉.] *Tenth.* As to the motion in altitude: through the single annual orb of the Earth, which approaches the lesser Planets and recedes from them, the reason at once appears why the three superior planets always appear largest and are perigee in opposition with the Sun, since the Earth then approaches them; and why in conjunction they appear smallest and are apogee, namely because the Earth is then at its greatest distance from them. From this it also becomes manifest why Venus and Mercury, around their evening conjunction, appear both perigee and larger — Venus especially.

Hear, moreover, what Copernicus says of Mars, Book 1, chapter 10: *For from this it may be noticed*, etc. *Moreover, that Saturn, Jupiter and Mars are nearer the Earth when acronychal than around their occultation and apparition. Most of all, Mars, when he shines all night, seems to equal Jupiter in size, distinguished only by his ruddy colour; but there,* that is, around conjunction with the Sun, *he is scarcely found among the stars of the second magnitude, recognised only by those who track him with diligent observation. All of these proceed from the same cause, which lies in the motion of the Earth.* For these and similar causes, therefore, and also in order to remove the imperfection of the Ptolemaic Equant, which begs the equality of the motions from a foreign circle, Copernicus turned to the annual motion of the Earth, as he himself admits in Book 5, chapter 2, saying: *These and similar things gave us the occasion to think about the mobility of the Earth, and about other ways by which equality and the principles of the art might remain, and the reason for the apparent inequality be rendered more constant.*

[Margin: Once the annual motion of the Earth is supposed, the motion of the Planets is simple, and does not pass through the heaven of another.] *Eleventh.* When all the motions of the Planets in longitude, latitude and altitude are considered, and the solid orbs are removed, as they must be, then in the hypothesis of a resting Earth the Planets must necessarily be led round through extremely intricate spirals — such as the one we sketched for Mars, after Kepler, in Book 7, toward the end of Section 1 — and Mars, Venus and Mercury must enter the heaven of the Sun. But once an annual motion of the Earth is supposed besides the diurnal one, the Planets describe very simple orbs about the Sun, and none of them intrudes into the region of another. No wonder, then, that the Copernicans have loved this hypothesis so dearly for so many allurements of a most beautiful and most compendious equipollence, and that Copernicus called the annual orb of the Earth, as we said, the Great Orb. It would be of greater excellence still if, as some think, it alone gave the reason for the motion of the Solar spots, the libration of the Moon, the passage of Comets, and the ebb and flow of the sea — of which below.

XIX. [Margin: Effects of the annual Motion and Orb with respect to the Fixed stars.] But although the Phenomena of the Sun and of the other Planets are saved by the annual orb of the Earth, which carries the Moon with it, the Phenomena of the Fixed stars do not seem able to be saved. For it would be necessary that, as the Earth receded from the Fixed stars through the whole diameter of the great orb (which is as great as the distance of the Sun from the Earth doubled), and again approached them by as much, the Fixed stars should appear smaller there and larger here; indeed, that the meridian altitudes of the same Fixed stars, observed from the Earth when it stands under Cancer, should appear much different from those observed when the Earth is under Capricorn. Yet no such difference has so far appeared in any phenomena. But Aristarchus — as Archimedes reports at the beginning of the *Arenarius* [*Sand-Reckoner*] — and Copernicus very shrewdly provided against this inconvenience, when they set the distance of the Fixed stars from the centre of the world so great that

the semidiameter of the great orb, compared with it, has no sensible ratio to it, and therefore shows no perceptible parallax in the Fixed stars; or (which comes to the same thing) by making the whole great orb so tiny, and the sphere of the Fixed stars so vast, that with respect to it the annual orb is like a point — by roughly the same reasoning by which the Earth itself is said to be a point with respect to the highest heaven. And this, no doubt, is the reason why Copernicus, meeting these objections in Book 1, chapter 5, said that one does right *if one denies that the Earth occupies the middle or centre of the world, and yet does not admit that the distance is so great as to be comparable to the sphere of the non-wandering stars*; and again in chapter 10: *Therefore we are not ashamed to admit*, etc., *that the magnitude of the world is so great that, while that distance of the Earth from the Sun has a magnitude clearly evident with respect to any other orbs of the wandering stars, in proportion to their sizes, compared with the sphere of the non-wandering stars it is not such as to appear: which I think should be conceded more readily than that the intellect be distracted into an almost infinite multitude of orbs, as those have been compelled to do who kept the Earth in the middle of the world.* And finally, at the end of the same chapter, confirming that distance of the Fixed stars from the constant appearance of their size and from their twinkling, he concludes: *That nothing of these things appears in the fixed stars* — that is, no variation in the apparent diameter — *argues their immense height, which would make even the orb of the annual motion, or its image, vanish from the eyes, if indeed the annual orb were viewed from the Fixed stars; for every visible thing has some length of distance beyond which it is no longer seen, as is demonstrated in optics. For that there is still a very great interval from Saturn, the highest of the wandering stars, to the sphere of the Fixed stars, their twinkling lights demonstrate; by this sign above all they are distinguished from the Planets. And it was fitting that there be the greatest difference between the moved and the unmoved. So great, no doubt, is this divine fabric of the Best and Greatest.* [Margin: Copernicus's closing exclamation on the immensity of the World.] In chapter 8, moreover, he had said that the end, or boundary, of the world cannot be known, and therefore it cannot be determined whether it is finite or infinite. And in chapter 11 he says: *Provided you remember that the distance of the Sun and the Earth has already exceeded our sight in the sphere of the fixed stars*; and in Book 3, chapter 15: *If there were a distance between the Sun and the Earth which could not be reckoned against the immensity of the sphere of the fixed stars.*

XX. [Margin: What Kepler and Boulliau added to the Copernican annual Orb and motion.] So much for the tenets of Copernicus on the annual motion of the Earth. To them Kepler and Boulliau added two other characteristics, and Kepler a third besides. For Kepler, in the *Epitome of Copernican Astronomy*, pages 538, 718 and 763, and in the *Commentaries on Mars*, chapters 5, 6, 33 and 52, and Boulliau, in Book 11 of the *Astronomia Philolaica*, theorems 2, 6, 11 and 12, taught that the line of the apsides of the Planets passes through the centre of the body of the Planet and through that of the Sun, and hence that the apsis of the Earth too is drawn through the same centre of the Sun; and that the centre of

the Sun is the centre of the universe, while the centre of the great orb is distant from it not, indeed, by the whole, but by half of the Eccentricity of the Solar orb asserted by Ptolemy or Copernicus. Next, both tried to show that the path of the Planets, and hence the orbit of the Earth, is not perfectly circular but Elliptical. Lastly, Kepler maintains that the Earth, no differently from the other five Planets, is carried round by the Sun by a magnetic virtue: while the Sun, rolled about its own centre with a roughly monthly rotation, emits from itself a certain species, or quality, which seizes the Planets as if by a hand and carries them round, according to the measure of their distance and interval — which, however, Boulliau denies. These matters have been explained elsewhere and are not to be repeated here out of place. But we must come to the third and fourth motion of the Earth.

Explicatur Tertius Terræ Motus, Declinationem Axis Aequatoris, in præcedentia se conuertentis, includens.

The Third Motion of the Earth is explained, which includes the Declination of the Axis of the Equator, turning itself against the order of the signs.

XXI. We have not yet exhausted the whole depth of the Copernican hypothesis; and the deeper one descends into it, the more talent and precious subtlety one may dig out of it. Copernicus therefore saw that a third motion (which includes within itself a fourth and almost a fifth) had to be adopted, so that, by transferring to the single globe of the Earth, from the celestial spheres, the unequal alternations of the artificial days and of the diurnal arcs; at the same time the anomaly of the precession of the Equinoxes, which others call the unequal motion of the Fixed stars, with a trepidation or libration toward East and West; and finally the anomaly of the obliquity of the Ecliptic, which others call a second trepidation, or a libration toward the sides of the world, he might represent them through a librating and nodding conversion of a single terrestrial axis. This motion he called the motion of declination, or of the convertible Inclination of the terrestrial axis, or of the reflexion of the axis against the order of the signs, with a libration joined to it. This motion is indeed most difficult for our apprehension, especially as it is entangled with the diurnal revolution of the Earth and the annual circuit of its centre, as Galileo himself admits in Dialogue 3 on the World System, Latin page 289 (for we have said that it has been translated from Italian into the Latin language) — granted that he admits that the system of Copernicus, which is arduous for our imagination, is very easy in its effect for nature itself. Nevertheless I shall try to set out this motion too as little obscurely as possible, from what has been handed down to us about it by Copernicus, Book 1, chapter 11, from the words *To this circle, which runs through the middle of the Signs, and to its surface, the Equinoctial circle and the axis of the Earth must be understood to have a convertible inclination*, up to the end of the chapter, and in Book 3, chapters 3 and 4; [Margin: Authors who have explained the Third motion of the Earth [?] more lucidly.] by Georg Joachim Rheticus in his *Narratio prima*, from the words *Third, that the equinoctial*

and the axis of the Earth have a convertible inclination to the plane of the Ecliptic, up to these: *But concerning the apogee of the Sun* — or, if you read its old edition, from column 31 to 40; by Kepler in the *Epitome of Copernican Astronomy*, from page 113 to 118; by Galileo in Dialogue 3 on the World System, from Italian page 372 to 394, but from Latin page 280 to 297; by *Philolaus* revived; and by Pierre Gassendi in Letter 2, *De motu impresso a motore translato*, from page 128 to 132 and again from page 137 to 141. Taken together, these seem to me to have embraced everything necessary for understanding this motion.

But lest the involved and disparate, rather than contrary, mixture of these motions should seem impossible to anyone, the imagination must be prepared and disposed by some likeness drawn from better-known motions. Let there be, then, in the first place, a Top AB, whose axis is CD. Suppose this top, wound round with a lash [?], as boys do, and then thrown onto the floor, describes in a slower turn, by the force of the impetus impressed by the boy's arm, the arc of a larger circle CEF; but by the force of the motion impressed by the cord it conceives a very swift spin, and before it rolls from C, through E, to F, it marks on the floor very many little circles, spirally or in the manner of a snail-shell — say at C, O, E, P, F — and indeed turning itself toward the same direction, namely from G, through H, toward K.

Diagram. A woodcut in the right column showing a spinning top drawn as an inverted cone, its point resting on a small circle at a hatched floor line; a dotted arc runs from the top leftward and down through four ringed little circles, each lettered, marking the path of the top's point across the floor.

- M, D, N (above the head of the top; D set higher): the ends of the axis as it appears multiplied by the wobble — DC, ML, NI in the text
- A (left of the rim), B (right of the rim): the top AB
- O, E, P, F (in the ringed circles along the arc, from the top leftward and down): the little circles traced on the floor
- H (lower right of the cone), K (left of the bottom circle), G (right of the bottom circle): the sense of rotation, from G through H toward K
- I, C, L (inside the bottom circle): the point C of the axis, with I and L of the multiplied axes
- R (on the floor line): in the second illustration, the axis of the wooden disc

[Margin: In this figure the little circles about C, O, E, P, F ought not to be concentric, as the engraver wrongly made them, but a spiral, or snail-shell.] Next, observe the wobbling of the axis DC, by which it totters this way and that so much that at times it seems to threaten itself with a fall; and owing to that vibration of the nodding point C and of the head, or handle, D, it seems to flicker with three-forked tongues, so that the axis can seem not single but multiple, or split into three — namely DC, ML, NI — or into more. Finally, when gravity prevails and it has fallen over, inclined to one side from the line of its centre of gravity, you will see it, as far as appearance goes, revolved in the contrary direction by its contact with the floor. By this example, then —

dissimilar in some conditions, but similar in many — you will easily understand how it can come about that the globe of the Earth slowly describes with its centre the circle of the annual, or great, orb, and yet is revolved upon itself, top-wise, toward the same eastern direction, in a diurnal spin repeated 365¼ times, while its axis nevertheless wobbles about by a certain third motion of libration and can describe either little circles or twisted garlands.

You will conceive the same thing readily in another way. Let there be a round wooden table, or wooden disc, half of which the preceding figure CEF meanwhile represents; and let a small vessel full of water be fixed to its rim, say at C. On this water let a little wooden globe float, having an iron point below and a wooden one above, opposite the iron, as if both were produced from the axis of the globule, so that the iron one is under the water and the wooden one stands out above it. Then, putting your finger into the water along the rim of the vessel, drive the globule round, and the water with it, from G, through H, to K, so that the water conceives a very swift spin; and, at once withdrawing your finger, turn the wooden disc about its axis R, but slowly, from C to O. For you will meanwhile see the water, with its many vortices, carry the wooden globe round with it in the same direction, and at the same time its wooden point nod somewhat and waver tremulously, describing a certain little orb or garland. By this example and others like it one is given to understand how easily nature, or some Intelligence, could turn the terrestrial globe round with three similar motions.

XXII. With this preparation laid down — though it is hardly necessary for stronger minds — let us now turn to Copernicus, who, in Book I, chapter II, having said that, with the plane of the great orb set in the plane of the Ecliptic, it is necessary that the Equa- tor of the Earth and its axis have a convertible inclination, at once adds the reason: *For if they remained fixed* — namely the terrestrial Equator and the axis — *and simply followed nothing but the motion of the centre, no inequality of days and nights would appear; it would always be either solstice, or midwinter, or equinox, or summer, or winter, or in whatever way the same quality of the season would remain like itself. There follows, therefore, a third motion, of declination, also with an annual revolution, but bending back against the order of the signs, that is, against the motion of the centre. And so, the two being nearly equal to each other and mutually opposed, it comes about that the axis of the Earth, and in it the equinoctial, the greatest of the parallels, look toward nearly the same part of the world, just as if they remained immobile. Meanwhile the Sun is seen to move along the obliquity of the zodiac with the same motion as the centre of the Earth, no differently than if it were itself the centre of the world — provided you remember that the distance of the Sun and the Earth has exceeded our sight in the sphere of the fixed stars:* that is, that it is imperceptible and of no sensible quantity, if viewed from the fixed stars. So that the axis and the terrestrial Equator should not, therefore, be moved along the course of the great orb, or Ecliptic, but should remain always parallel to itself, or equidistant from itself, in the situation in which it once was at the time of the first Equinox or Solstice, Copernicus

thinks it necessary that, while the centre of the Earth is carried round on the periphery of the annual orb, the axis of the terrestrial Equator should bend itself back little by little against the order of the signs, and so have a convertible inclination to the Ecliptic, or great orb, such that, if it were produced, it would strike sensibly the same parts of the world through the whole circuit of the year. Now, in order to set before the eyes, in a way, the difference of the solstitial days among themselves in the oblique sphere, and from the equinoctial days, which arises from this threefold motion, Copernicus drew, in the same chapter 11, the figure which we have described in number XVI, adding the Sun S at the centre and, beyond the annual orb EFBD, the celestial ecliptic ACLK. It is to be reproduced here.

Diagram. A woodcut in the left column: an outer circle, the celestial ecliptic ACLK, with a zodiac sign at each of its four lettered points; inside it the annual orb EFBD, with a small circle of the Earth at each of its four points F (top), B (left), E (right) and D (bottom); the rayed Sun S at the centre. Each small circle carries a diameter through the Earth's centre and the letters P, G, I, H.

- C (top of the outer circle), with the sign of Cancer above it
- L (left of the outer circle), with a zodiac sign [?] beside it
- A (right of the outer circle), with a zodiac sign [?] beside it
- K (bottom of the outer circle), with a zodiac sign [?] below it
- F (top small circle), with P above right, G left, I right, H below left
- B (left small circle), with P above, G left, I right, H below; then a small crescent-shaped sort [?] and N on the line toward the Sun
- E (right small circle), with P above, G left, I right, H below
- D (bottom small circle), with P above, G left, I right, H below
- S (centre): the Sun

[Margin: Representation of the winter-solstice day.] Let the centre of the Earth now first be at F, under the beginning of Cancer, and from it let the terrestrial Equinoctial PGHI be described, not in the plane of the Ecliptic, or of the great orb EFBD, yet so that the diameter GFI is the common section both of the terrestrial Equinoctial and of the great orb, or Ecliptic. Then draw the diameter HFP at right angles to GFI; and let the southern limit of the greatest declination of the two planes — that is, of the obliquity of the Ecliptic to the Equator, or rather of the Equator to the Ecliptic — be P, and the northern limit H. This obliquity is measured by the angle HFS on the one side and CFP on the other, of 23° 30′. With these things set out, the inhabitants of the Earth, while the Earth is at F, under C, will see the Sun S under Capricorn, making the winter turning, which the greatest northern declination H, turned toward the Sun, will display: *Since*, says Copernicus, *the declivity of the Equinoctial toward the line FS, through the diurnal revolution, turns out for itself the winter tropic, parallel to the terrestrial Equator, according to the distance which the angle of inclination contains under SFH*: or, perhaps more clearly, the line conceived immovably from the centre of the Sun to the centre of the Earth is understood, during one diurnal

revolution, to mark out on the terrestrial surface a circle parallel to the terrestrial Equator, with a declination as great as the angle SFH.

[Margin: Representation of the vernal Equinox.] Let the centre of the Earth now proceed in the order of the signs and, a quadrant completed, be at B; and meanwhile let P, the limit of greatest declination, recede by nearly as much against the order of the signs, having completed as it were a quadrant of its own little circle; and meanwhile, because of the equality of the revolutions, let the angle SFI remain equal to the angle FSB. For the diameters will remain parallel to each other — namely PFH to PBH, and GFI to GBI — and likewise the terrestrial Equinoctial will be kept parallel to itself; and these will likewise appear marked out in the highest heaven, because of its immense distance. Now, to the Earth at B, under the beginning of Libra, the Sun S will appear under the beginning of Aries, and the common section of the great orb and of the Equator will coincide in one and the same line GBIS, to which the diurnal revolution admits no obliquity of the already mentioned circles, or declination; for all the declination will be to the sides. Therefore the Sun will appear in the vernal Equinox. Still, the diameter GI will be in the common section of the already mentioned circles, and the plane of the terrestrial Equator will not for that reason coincide with the plane of the great orb.

[Margin: Representation of the summer-solstice day.] Let the centre of the Earth now depart from here, the already mentioned conditions being kept, and, a second quadrant completed, be at D, under the beginning of Capricorn; for to the Earth the Sun S will seem to enter Cancer, C. And because P, the southern limit of the declination by which the Equator is inclined to the Ecliptic, is turned toward the Sun, it will make the Sun seem to be in the opposite quarter, that is, the northern, and to run through the summer tropic, in accordance with the angle SDP, which measures the inclination.

[Margin: Representation of the Autumnal Equinox.] Finally, let the centre of the Earth go away from here and, a third quadrant likewise completed, come to E, under the beginning of Aries. For, because of the already mentioned parallelism of the axis with itself, and of the Equator with itself, the common section GEI of the terrestrial Equator and of the annual orb, or Ecliptic, will fall on the line SE, without any declination; and therefore the Sun S will seem, having entered Libra, L, to mark the Autumnal equinox.

[Margin: Another Representation of the solstitial days.] In another way, and by another diagram, Copernicus, in Book I, toward the end of chapter II, exhibits on the terrestrial globe the winter and summer tropic, parallel to its Equator on the surface of the terrestrial globe, as follows. In the underlying plane let there be the diameter AC of the great orb, and the common section of a circle erected perpendicular to this plane itself; in this circle, from the centre of the Earth placed at A, under Cancer, and at C, under Capricorn, let the circle DGFI be described through the poles. Let the equinoctial axis of the Earth be DF, the North Pole D and the South Pole F; and let GI be the diameter of the already mentioned Equinoctial.

Diagram. A woodcut in the right column: two circles of the Earth joined by a horizontal line, the diameter of the great orb; "Boreas" (North) above and "Auster" (South) below. Each circle carries diameters through its centre.

- Boreas (above the figure): North
- Auster (below the figure): South
- A (centre of the left circle), with D upper left, I right, L right on the line, G left, K lower left, F bottom
- C (centre of the right circle), with D top, N upper right (set as a reversed N), I right, F lower right
- M (on the joining line, at the left edge of the right circle), with G below it

These things posited, when the pole F turns itself toward the Sun, and the inclination of the Equinoctial circle to the great orb, or to the Ecliptic, is northern under the angle IAL, then the diurnal revolution about the axis will describe the tropic KL of Capricorn, that is, the southern parallel to the Equator, according to the distance IL, and will make the Sun appear at the beginning of Capricorn. Or, speaking more correctly, the diurnal motion about that axis will to the sight produce a conical surface, whose apex will be at A, the centre of the Earth, and whose base, KL, will be in a circle parallel to the Equinoctial itself. In the same way, but with the conditions reversed, everything will come about when the centre of the Earth is placed at C, under the beginning of Capricorn: the diurnal revolution, namely, will describe the northern parallel or tropic MN, and will make the Sun appear at the beginning of Cancer. From this it follows that, if nothing else happens to this reflexion of the axis, the axis describes on the terrestrial surface a little circle on either side, about the axis of the terrestrial Ecliptic, which is always equidistant from the great orb. Or certainly — because the terrestrial axis keeps the same situation with respect to the highest regions of the heaven — that the manner in which this retention comes about is more like rest than motion, or a certain containment and restraint, as Galileo and Gassendi have acutely noted.

[Margin: Galileo's diagram for the threefold motion of the Earth.] XXIII. The mention of Galileo opportunely suggests to us another way and another figure, by which he, in Dialogue 3 *On the System of the World*, brought this mixture of the Earth's three motions before the eye; we shall set out this figure more briefly. Let the great orb ♋, ♎, ♑, ♈ be understood as described from O, near the centre of the Sun; and let the Earth be understood as placed in turn at these four cardinal points by the annual revolution of the Earth's centre. Let its axis, produced in imagination beyond the Earth, be AB, parallel to itself in every place of the annual orb; and let the diameter of the great orb from Capricorn to Cancer be ♑ O ♋. Over this diameter, or over the plane of the great orb, let the axis AB be inclined by 23½°, that is, so that the angle O ♋ B, or O ♑ A, is 66½°. For you must conceive the axis AB, together with the line ♑O♋, in one and the same plane, which is right, or orthogonal, to the Ecliptic plane of the great orb ♋ ♎ ♑ ♈ — something Galileo represents only with

difficulty in this figure. Let the North pole of the terrestrial axis be A, above, and the South pole B, below; about this axis the Earth, by a revolution of 24 hours, describes innumerable parallels. But for clarity's sake let there now be marked on its surface the Equinoctial CD, and, parallel to it, the winter Tropic GN and the summer Tropic EF; then the Arctic circle IK and the Antarctic LM.

Diagram. A woodcut in the left column: the great orb drawn as a circle with the rayed Sun O at its centre and four straight lines running from O to four globes of the Earth standing on the orb — one at the top, one at the left (its left half shaded), one at the right (its right half shaded) and one at the bottom (shaded). Each globe has its axis A–B drawn through it and carries its parallels. The signs of the zodiac stand on the orb between the globes, with tick marks, and arrows on the upper left arc.

- Top globe: A (axis, upper right), I, E, C, K, F, G, D, N, L, B (axis, lower left), M (below, on the line from O); the sign of Libra ♎ on the globe
- Left globe: I, A, E, C, K, F, G, D, L, N, B, M; the sign of Capricorn ♑ on the globe
- Right globe: I, A, K, C, E, F, G, D (set reversed), L, N, M, B; the sign of Cancer ♋ on the globe
- Bottom globe: E, I, A, K, C, F, G, D, L, N, B, M; the sign of Aries ♈ on the globe
- O (centre): the Sun
- Zodiac signs on the orb: ♏ and ♐ (upper left), ♍ and ♌ (upper right), ♒ and ♓ (lower left), ♊ and ♉ (lower right)

[Margin: Representation of the day and Tropic of summer.] Let us now consider the Earth placed at the beginning of Capricorn. The axis AB declines from the perpendicular over the diameter of Capricorn and Cancer by 23½° toward the Sun O, and the arc IK is 47°, half of it 23½°. The Sun's illumination will therefore light up the terrestrial hemisphere exposed to the Sun, divided from the dark part by the boundary IM. This boundary will bisect the great circle CD, but will divide the other parallels into unequal parts, since IM does not pass through their poles AB. The parallel IK, moreover, with the circles enclosed within it, will remain wholly in the illuminated part, just as the parallel LM and those enclosed within it will remain wholly in the dark part. Further, since the arc IK is equal to the distance of the Tropics FDN, and its half to the half FD, and the arc from F to the Pole A is common to both, the arcs IKF and AFD will be equal, and each will make up a quadrant. And since the whole arc IDM is a semicircle, the arc FM will be a quadrant, equal to the arc FKI; and therefore the Sun O, in this place of the Earth, will be vertical to anyone standing at the point F. But because of the diurnal revolution about the stable axis AB, all the points of the parallel EF pass through the same point F. On this day, therefore, the Sun at the point of noon will be vertical in succession to all who live on the said parallel EF, and will seem to them to describe, by an apparent motion (which is really the Earth's), the Tropic that we call the

Tropic of Cancer. But for all inhabitants beyond the parallel EF, toward the North pole A, the Sun will decline from their zenith toward the south; whereas for those living on this side of EF, toward the Equinoctial CD and the South pole B, the noon Sun will decline from their zenith toward the North pole A. In this place it likewise appears that the semidiurnal arc ⋎ϐD is equal to the seminocturnal arc ⋎ϐC on the Equator CD, since the boundary of light IM bisects that circle alone; that the semidiurnal arcs on the parallels above CD, toward the North pole A, are greater than the seminocturnal; that below CD, toward B, they are smaller than the seminocturnal; and that those who are in the illuminated part — that is, in the arctic circle IK and within it toward A — enjoy perpetual day, whereas those in the Antarctic LM, and within it, are wrapped in perpetual night.

[Margin: Representation of the day and Tropic of winter.] Now let the Earth be in the diametrically opposite situation, that is, under the beginning of Cancer ♋. For it will be manifest that, since the axis AB has remained parallel to itself and has not sensibly changed its inclination, the aspect and situation of the Earth with respect to the Sun keep everything else they had in the earlier situation — in a similar way indeed, but with an opposite likeness. For the hemisphere that was dark before is lit by the Sun, toward which it is turned, and the Sun becomes vertical at noon for those living on the parallel GN; for the inhabitants of the parallel EF, on the other hand, it has declined toward the south, descending from their zenith through the whole arc ECG of 47°; and so the Sun has passed from one Tropic to the other, descending on the meridians through the said space of 47°. Further, those who dwell under the Arctic parallel IK and within it are overwhelmed by perpetual night, while those under the Antarctic LM, or within it, enjoy perpetual day; and the semidiurnal arcs on this side of the Equator CD, toward the South pole B, are greater than the seminocturnal, but beyond the Equator smaller than the seminocturnal; and to those living on the parallel EF the Sun O will seem to describe the Tropic that we call the Tropic of Capricorn.

[Margin: Representation of the equinoctial days.] Let us now look at the Earth in the figure above at the beginning of Libra ♎, from which it sees the Sun O under the beginning of Aries ♈. The axis of the Earth was understood in the earlier figures to be inclined over the diameter of Capricorn and Cancer, and therefore to lie in the same plane that, cutting the plane of the great orb along the line of Capricorn and Cancer, was erected perpendicular to it. If it is now transferred to the present figure, keeping its parallelism with itself, it will lie in a plane likewise erected to the surface of the great orb, and parallel to that plane which cuts the same surface orthogonally along the diameter of Capricorn and Cancer. Hence it comes about that the line O♎, drawn from the centre of the Sun to the centre of the Earth, turns out perpendicular to the axis AB. But the same line, drawn from the centre of the Sun to the centre of the Earth, is always perpendicular to the circle that terminates the light; therefore the circle terminating the light will pass through the poles A, B, and

the axis AB will lie in its plane. Now from the *Spherics* of Theodosius, a great circle passing through the poles of the parallels divides them all in two, that is, into equal portions; therefore the circle bounding the light, ACBD, will divide the parallels so that IK, EF, CD, GN and LM are each semicircles. The illuminated hemisphere will be the one that faces us in this diagram, and there will be the vernal Equinox for all the inhabitants of the Earth. From this we are given to understand the cause of the other equinox, the Autumnal, when the Earth is at the beginning of Aries ♈.

[Margin: Two notable points about the parallelism of the Earth's axis.] XXIV. Galileo, and also Kepler and Gassendi, add a further point. Since the terrestrial axis of the Equator remains parallel to itself through the whole circuit of the year, if it were produced at both ends beyond the Earth as far as the firmament, it would describe an enormous cylinder, or column. Each base of it would mark out on the firmament a circle as large indeed as the annual orb, but one which, seen from the Earth, would be like a point because of the immense distance of the fixed stars; and for this reason the axis of the Earth is said to look perpetually at the same part of the heaven, and to be turned toward the same point. Granted, strictly speaking, that this hypothesis, just as it does not recognise an Equator in the highest heaven, does not recognise a pole of the Equator either: the already mentioned bases could still be substituted for the poles at which the terrestrial axis would be said to look. But Galileo, using a subtler scrutiny, noticed in this same place a certain remarkable accident. For just as the axis of the Earth, keeping the same direction toward the highest sphere of the Fixed stars, brings it about that the Sun seems to us, within the circle of a single year, to be raised on the meridian through an arc of 47°, while meanwhile it exhibits no ascension on the meridian of any one and the same Fixed star, or sensible change of its declination; so, on the contrary, if the axis of the Earth persisted continuously in the same inclination toward the Sun — or, what comes to the same thing, toward the axis of the Zodiac — no change would appear in the Sun's meridian declination. The inhabitants of any one place would therefore always have the same variations of days and nights, and the same arrangement of the four seasons of the year; and so some would be forced to endure a perpetual winter, others a perpetual summer, while still others would enjoy a perpetual spring or autumn. But, on the contrary, the greatest change in declination would appear in the Fixed stars, so that on the Meridian within one year they would vary their declination through 47°, the full distance of the Tropics.

[Margin: Our explanation of the already mentioned parallelism.] XXV. And yet Copernicus explains the conservation of the axis of the terrestrial Equator, and of the Equator itself, each in its own parallelism, by a certain motion of the axis against the order of the signs; some, however, do not think this motion necessary, or necessarily to be distinguished from the other motions; and it might still seem obscure to someone what it is to carry the centre of the terrestrial globe along the track of the great Orb, or what it is for the axis and

the terrestrial Equator simply to follow the motion of the Earth's centre. So that both these doubts may be removed by us as far as possible, we say first: if the axis of the Equator, always inclined to the plane of the great orb, or Ecliptic, by 23½°, were carried round with the centre of the terrestrial globe about the periphery of the great orb in such a way that it always kept the same inclination to the axis of the great orb, it would keep neither its own parallelism with itself nor that of its Equator with its Equator, nor would it look toward the same regions of the highest heaven, but toward ever different ones. For by its annual motion it would describe either the frustum of a right cone or, if it is understood as produced, a right cone, whose axis would be at right angles to the circle that cuts the cone; and the lines of the terrestrial axis, produced further for its different situations, would cut one another at the vertex of the cone, making an angle of 23½° with the cone's axis. Therefore, if the pole of the terrestrial axis that is inclined toward the axis of the great orb were the north pole, a person who now lives under the north pole and sees above his zenith the Cynosure, that is, the pole star — or at least sees it not a whole 3° distant from his zenith — would after six months see it about 47° distant from his zenith. To make this evident, let ABCD be the great orb, whose diameter is AEC, and at right angles to it the axis of the great orb OER. Now let there be taken at A the angle EAK of 66½°, and

Diagram. A woodcut in the left column. At the left edge a vertical line carries the points F, H, P, O, T, V, X from top to bottom. The great orb is drawn as a circle with eight small globes of the Earth on it, lettered at their centres; E and O stand on its horizontal diameter between B and D. Straight lines run from the globes and from the points of the left line to the vertex K at the right, beyond which they open out to Y, R and Z. Around each globe stand the letters O and S, at the ends of its axis, and M and Q, at its equator.

- F, H, P, O, T, V, X (left vertical line, top to bottom)
- A (top globe), G, I, B (left globe), D, L, N, C (bottom globe): the eight places of the Earth's centre
- E, O (on the diameter of the great orb)
- K (vertex at the right)
- Y (upper right), R (right), Z (lower right)
- O, Q, M, S (around each globe: axis OS, equator MQ)

similarly at C the angle ECK equal to it. For when AK and CK are produced equally at both ends — on the one side beyond the point of concourse K as far as Z and Y, and on the other beyond the diameter of the great orb, KA to F and KC to X — and when the end-points F and X are joined by the straight line FOX, and the end-points Y and Z by the straight line YRZ, there will have been marked out the sides and the diameters of the bases of two right cones, namely the Cones FKX and YKZ, the former of which is cut orthogonally by the plane of the great orb. Next let eight situations of the centre of the terrestrial globe be chosen on the periphery of the annual orb: the beginning of Cancer at A, the middle of Leo at I, the beginning of Libra at B, the middle

of Scorpio at L, the beginning of Capricorn at C, the middle of Aquarius at N, the beginning of Aries at D, the middle of Taurus at G. From these points let the circle OMSQ be described, standing roughly for the globe of the Earth, in which let the Equator be MQ and its axis, at right angles to its Equator, OS. This is to be done in such a way that in every place of the Earth the terrestrial axis of the Equator keeps an inclination of 23½° to the plane of the Ecliptic, so that, produced in imagination, it meets on one side at the same point K of the axis of the great orb at which we said AK and CK meet, and is drawn from there as far as the line YZ, while on the other side it is likewise understood as produced as far as the line FX. With these conditions kept, the directing line of the axis, while the Earth is at I, will be PIK; while it is at B, OBK; at L, TLK; at N, VNK; at D, ODK; and when the Earth is at G, the directing line of the axis will be HGK; and so for the other places of the Earth. And so, because the same inclination to the axis of the great orb is kept — that is, because the lines of the produced axis meet at the same point K — the surface of a right cone will have been described at the end of the annual circuit; but neither will the terrestrial axis be parallel to itself, nor the terrestrial Equator to itself, since the axes of the Equators converge and meet at the point K. Let us now suppose that, while the Earth is at A, the eye is at the northern pole S of the terrestrial axis and sees at its zenith the Cynosure Z, set on the vertical line ASKZ, but at an immense distance. When after six months the Earth has come to C, the eye at S, remaining at the north pole, will have the line CSKY as its vertical line, and along it will see its vertical point, or Zenith, distant from the Cynosure by an angle as great as YKZ, namely 47°. For since by construction and hypothesis CAK is 66½°, and ACK as many, that is, 133° together, the angle AKC, by propositions 32 and 15 of the first book of the Elements, will be the supplement to two right angles, and so will the angle YKZ vertical to it, namely 47°; and its half, AKE, 23½°. KA or KC will therefore indeed be longer than CA, even if the figure draws it otherwise — yet not by so much that it cannot be disregarded beside the far greater KY or KZ; so that, if a straight line is drawn from A to the most remote point Y, it will be sensibly parallel to KY itself. And so, just as the whole of AEC vanishes because of the immense distance of the firmament and is like a point, so too does AK; and for the eye to be at S with the Earth placed at A, or at C, is the same as if it were at K. But first it would look toward its zenith along KZ, and afterwards along KY; therefore it would see the star Z, after 6 months, distant from its zenith by the whole angle YKZ, which we have shown to be 47°.

XXVI. Since, therefore, if the same inclination of the terrestrial axis is kept not only to the plane of the Ecliptic, or great orb, but also to the axis of the great orb itself, it is impossible for the parallelism of the terrestrial axis with itself, and of the terrestrial Equator with itself, to be kept: in order to obtain this parallelism we shall carry the centre of the terrestrial globe round through the circumference of the great orb in such a way that, with the axis produced at both ends in thought, we conceive its length on the surface of a cylinder

oblique to the great orb, and carry the axis round in the great orb along that surface. This can be seen in the following figure, in which the great orb, as above, is ABCD, and its diameter from Cancer A to Capricorn C is AC; and let this great orb cut the cylinder FAYZCV obliquely. With the Earth placed, then, at the eight already mentioned places A, I, B, L, C, N, D, G, lay its axis OAS on the side FAY of the cylinder. For if you carry the axis round, keeping it on the surface of this cylinder, you will see in every place of the Earth the axes OS parallel to one another, and therefore the Equators MQ parallel to one another as well, since their axis passes through them orthogonally. And conversely, if you keep that axis parallel to itself through the whole annual circuit, you will describe with the axis a certain cylinder, which, if it is produced further as far as the Fixed stars, will mark out at either end on the firmament a circle equal to the annual orb — namely FPV at one end and YKZ at the other — but one which, because of the immense distance, appears as a point; and therefore, whether the Earth is at A, or at B, or at C, etc., al- ways, by way of the axis OS, it will look at sensibly the same small part of the heaven.

[Margin: Whether the 3rd motion of the axis is superfluous in Copernicus's hypothesis.] This demonstration makes it plain that if someone, taking hold of the terrestrial axis by its poles, carries it round with each pole kept on the surface of the already mentioned cylinder, so that the centre of the terrestrial globe is always on the periphery of the annual orb, he will keep, by one and the same annual motion and without a third motion really distinct from it, the parallelism necessary to represent the inequalities and alternations of days and seasons. It is plain, too, that so far as this goes the reflexion of the terrestrial axis against the order of the signs is not necessary — granted that some other motion of the terrestrial axis may seem necessary to represent other phenomena, as we shall say shortly. It was perhaps for these phenomena, and not precisely for this parallelism, that Copernicus counted that motion as a third, or distinct, motion in Book I, chapter II.

Diagram. A woodcut in the left column: an oblique cylinder whose two ends are drawn as shaded lens-shaped ellipses. Eight small globes of the Earth stand along a curve through the middle of the cylinder, each crossed by its axis and its equator, with parallel lines running through them from one end of the cylinder to the other, and a vertical line through the middle.

- F, H, P, T, V (left-hand end, top to bottom)
- Y, V, K, B, Z (right-hand end, top to bottom)
- A, I, G, B, D, L, N, C (the eight globes, top to bottom)
- O, Q, M, S (around each globe: axis OS, equator MQ)

Explicatur Quartus Motus Terræ à Copernicanis adiunctus, qui tamen Copernico Tertius eſt.

The Fourth Motion of the Earth added by the Copernicans is explained, which for Copernicus, however, is the Third.

XXVII. We were saying just now that, for the parallelism of the terrestrial axis with itself — and so for representing those alternations of days and nights which the inhabitants of the oblique sphere experience — a third motion really distinct from the diurnal revolution of the terrestrial globe and from the annual revolution is not necessary, provided one carries the axis of the Earth round, while moving its centre, in such a way as to keep it on the surface of a Cylinder oblique to the great orb, on whose surface it has once been set along its length and with each of its poles. But the Fixed Stars seem to move in longitude by a certain slow progress, and Copernicus judged that this very motion, let alone the diurnal motion of the primum mobile, must be denied to the sphere of the Fixed stars. He therefore transferred the cause of this appearance to a motion of the Earth's axis, reflecting itself against the order of the signs in such a way that the annual revolution of this reflexion is somewhat slower than the revolution of the centre of the terrestrial globe. From this, he holds, it comes about that the equinoctial sections, that is, the points of the annual orb at which the equinoxes occur, become every year more and more westerly, though barely sensibly — that is, seem to regress against the order of the signs, contrary to their sequence — but that only after many years does this regression at last become known. Since, then, the apparent progress of the Fixed stars toward the East was detected through observations by Hipparchus, Ptolemy and the other Astronomers after them in no other way than by numbering their distance toward the East from the Equinoctial points on some parallel of the Ecliptic, the Fixed stars will seem to draw away from those points equally whether they themselves really move or, while they stay immovable in their sphere, the equinoctial points themselves move, but toward the West. For this reason Copernicus substituted this precession of the Equinoxes, or of the Equinoctial points, in place of the motion of the Fixed stars, referring its cause to the difference between the annual revolution of the centre of the terrestrial globe, going in the order of the signs, and the annual revolution of the declining axis, such that the axis departs insensibly from its parallelism. That is to imagine that the already mentioned cylinder, or its axis, moves little by little with its extremities P, K against the order of the signs, so that with them it describes in the sphere of the Fixed stars something like polar circles about the poles of the Ecliptic. From this you will understand those words of Copernicus, Book I, chapter II: *There follows, then, the third motion, of declination, also with an annual revolution* (understand: of the axis) *but against the order of the signs, that is, reflecting against the motion of the centre. And so, with both* (understand: motions) *nearly equal to each other and mutually opposed, it comes about that the axis of the Earth, and in it the greatest of the parallels, the equinoctial, look toward nearly the same part of the world, just as if they remained immovable.* Why he applied a limitation by using the particle *nearly*, he explains toward the end of the same chapter in these words: *We were saying, moreover, that the annual revolutions of the centre and of the declination are almost equal; for if this were exactly so, the equinoctial and solstitial points, and the whole obliquity of the zodiac, would by no means change*

under the sphere of the fixed stars. But since the difference is small, growing large only with time, it has been disclosed, from Ptolemy indeed down to us, as nearly 21°, by which those points now anticipate. For this reason some believed that the sphere of the fixed stars also moves, and accordingly they favoured a ninth higher sphere; and when that was not enough, more recent writers now add a tenth on top, yet without reaching the end, which we hope we shall attain from the motion of the Earth.

[Margin: What the Royal Road is in Kepler.] Therefore, to account both for the apparent motion of the Fixed stars in longitude, together with its anomaly, asserted [?] above all by the Alfonsines, and for the changes of the obliquity of the Ecliptic, Copernicus substituted a single motion of the terrestrial axis — but of an axis nodding with a twin libration — in place of the motion which others ascribed to the eighth, ninth and tenth sphere. How this libration of the terrestrial axis comes about, and in what way each anomaly results from it — namely that of the precession of the Equinoxes and that of the Obliquity of the Ecliptic — Copernicus sets out next, in Book 3, from chapter 3 to 6, as do Rheticus in the first *Narratio* and Kepler in Book 7 of the *Copernican Astronomy*; and we have already set it out sufficiently in Book 3, chapter 29. Kepler adds, however, that both the magnetic fibres and that animal faculty of which we spoke above concur in this motion as well; and that, in order that the variation of the obliquity of the Ecliptic and of the apparent latitude in some fixed stars may be discerned and brought back to a just measure, one must conceive in the heaven a certain mean Ecliptic, fixed in the plane of the great orb, which he himself calls the Royal Road [Via Regia]. From these things now gather the following.

[Margin: 1st Corollary.] *First*, that Rheticus, Galileo and Gassendi, who attribute four motions to the Earth, do not conflict — they do not conflict, I say — with Copernicus or Kepler, who in the places already mentioned ascribe only three motions to the Earth and call third the motion we set out from numbers 21 and 27 under the third and fourth. For the modification that the terrestrial axis receives from the twin libration, one in longitude and the other in latitude, to save the anomaly of the precession of the equinoxes and of the obliquity of the Ecliptic, they treat as a distinct motion — if not really, as they say in the schools, at least formally.

[Margin: 2nd Corollary.] *Second*, suppose a motion in longitude toward the East is granted to the Fixed stars themselves too, as even among the Copernicans Lansbergen and Boulliau grant; suppose this motion is uniform, so that there is no anomaly of the precession of the equinoxes, which I have taught to be more probable in Book 3, chapters 28 and 30, and Book 6, chapter 17; and suppose, finally, that there is no variation of the obliquity of the Ecliptic, let alone any anomaly of this variation itself, as I have likewise shown to be more likely in Book 3, chapter 27. Then no third motion will now be necessary to the Earth, and the Copernican hypothesis will be far easier both to understand and in its working. For we have already said that the diurnal and annual revolution of the Earth suffices to represent all the other phenomena, provided the annual

orb, with respect to the distance of the Fixed stars, has no sensible proportion, and the axis of the Earth's Equator is always kept in its parallelism — which, as we taught a little earlier, can come about without a distinct motion.

[Margin: 3rd Corollary.] [Margin: Praise of Copernicus.] *Third*. Posterity has never sufficiently admired, nor will it ever sufficiently admire, the loftiness of Copernicus's mind, the depth of that breast and the keenness of his genius. By the motion of one little globe (how small the Earth is with respect to the whole heaven!), and a threefold motion at that, he achieved what the greatest part of the Astronomers before him could barely sketch even with insane machines of spheres. For by the diurnal motion of the Earth he represented the primum mobile, and freed all the other Planets, and the Fixed stars too, from that motion which would otherwise have had to be carried out by vast orbs, or by circles or spirals, not without an apparent conflict with their own proper motion. In place of the three Eccentrics of the Sun, Venus and Mercury, and the three Epicycles of Saturn, Jupiter and Mars, and of their second anomaly in motion, he substituted a single orb and annual motion of the Earth; and by it he also eliminated the apparent imperfection of Stations and Retrogradations in those same planets, and the libration of their inconstant latitude and inclination to the Ecliptic. Finally, by the revolving libration of a single terrestrial axis he accounted for the proper motion of the fixed stars and the change of the obliquity of the Ecliptic, with the anomaly of each — that is, he cast three huge spheres down from the heaven and did away with them. And what so many Atlases before him could not do, this one Hercules dared to sustain. Would that he had kept himself within the limits of this hypothesis!

SCHOLIA.

Scholia.

I. Above, at number 7, we taught from Copernicus that the motion of heavy and light bodies descending naturally through the atmosphere — the part of the air full of watery and earthy vapours and exhalations — does not in reality take place in the space of the world along a straight line perpendicular to the Earth's surface, but along another line. That motion is compounded of the straight and the circular, and in these bodies the circular predominates, because the straight does not belong to them except when they are in a disordered state and carried out of their own place, whereas the circular belongs to them of itself and primarily, since it belongs to them perpetually and agrees with the motion of the whole Earth, of which they are parts. These things, I say, Copernicus taught in Book 1, chapter 8, and Galileo in Dialogues 1 and 2 on the system of the world. But of what figure and kind that line is, along which they so ascend or descend, Copernicus did not determine; Galileo, however, taught that it is circular, as I shall set out more fittingly in chapter 17. There I shall also explain the opinion of Boulliau, who holds that this motion is mixed of two circular ones, and I shall show their error in the same place, numbers 16, 17, 18.

[Margin: Error of an unnamed author about the serpentine line of heavy bodies descending.] II. What Kepler thought, he himself disclosed in a few words in Book 1 of the *Epitome of Copernican Astronomy*, page 132. There he first rebukes someone or other who affirmed, with too little consideration, that the fall of a stone toward the Earth fluctuates this side of the perpendicular or beyond it in serpentine windings, so that the number of windings answers to the gyrations of the Earth. Against this assertion he observes that the stone, left behind by the parts of the Earth over which it hung perpendicularly at the start, comes to be swept along by the neighbouring parts that follow, its fall always deflected toward the quarter into which the Earth turns — little indeed at the beginning, more and more at the end — because the sweep of a near carrier is the stronger. For he holds that heavy bodies tend to the Earth by a mutual magnetic force, so as to be united with it, and are seized or drawn by the Earth so that it may unite them to itself for the preservation of its whole globe.

[Margin: Figure of the descent of heavy bodies according to Kepler.] III. Having eliminated this error, Kepler at once adds in the same place that if some heavy body were to fall from a very remote place of the heaven, A, toward the Earth rotating in one fixed place, the figure of its motion would be roughly this one, which he admits he has drawn roughly in the first part of the attached diagram. In it the Earth's circle is divided into 24 parts, and the line of fall, AEF, into as many, but unequal ones, short above and longer below; and the parts of the circle, each in its own order having already discharged its office of drawing, have already returned to their places; but, for example, only three, overtaken by the fall of the heavy body, did not draw it perpendicularly at the end of the fall. So the figure of the already mentioned fall, as it approaches the Earth, seems roughly to imitate a parabolic line. But Kepler's error will become manifest from what is to be said in chapter 17, numbers 16 to 18.

IV. Pierre Gassendi, however, at the beginning of Letter 2 *De motu impresso a motore translato*, teaches that such motions, which seem to us perpendicular whether in descent or in ascent, are in reality parabolic in the space of the world. [Margin: Gassendi's opinion on the figure of the already mentioned[37] motion.] He uses, however, a heavy body thrown straight up, and the following figure — which we, however, have drawn more like a parabola. Let there be, then, a Tower AB, from whose foot A an iron ball is fired straight up from a musket, and let us suppose that, so far as our sense experiences it, the ball reaches the top B. Now if the Earth did not move at all, it would at once fall back from B to A along the perpendicular once its ascent was complete. Let the Earth now move from A, a western point, toward K, an eastern one, and the Tower together with it, and let the spaces the Tower occupies in its motion be marked at equal intervals: let these be CL, DN, EP, FR,

Diagram. A woodcut in the right column, in two parts side by side. On the left, Kepler's figure: a line of fall marked with dots runs from A at the top

[37]printed *pra-dicti*

through E down to F on a circle whose rim is ticked off into small divisions; C stands at the top inside the circle, B at its centre and D at its foot, with a short unlettered stroke inside the circle to the left of B. On the right, Gassendi's figure: a base line and a top line joined by nine verticals (the successive stations of the Tower), and a curve rising from A at the bottom right to R at the top and falling to K at the bottom left.

- A, E, F (the line of fall, top to bottom); C, B, D (the circle: top, centre, foot)
- K, I, H, G, F, E, D, C, A (the base line, left to right)
- β, Y, V, S, R, P, N, L, B (the top line, left to right)
- M, O, Q, R, T, X, Z (the curve, from A up to R and down to K)

GS, HV, IY, Kβ. When the ball reaches the top of the Tower, let the Tower itself already have completed half, AF, of the interval AK, and be at FR; and let it pass through the remaining half, FK, while the ball descends from the top of the Tower to its very foot. On these suppositions Gassendi affirms that, granted the Earth moves, the Tower carried from AB to CL would still leave the ball behind if the one who throws it did not cleave to the Earth itself at A — in the same way as, if someone standing outside a moving ship, on the nearby shore, were to fire a ball straight up from a musket while the ship's mast passes [?] opposite him, a ball of this kind would not follow the mast but would be left behind by it. Conversely, if the one who throws that ball along the height of the mast cleaves to the moving ship, the ball follows the mast and is not left behind by it, because he impresses a mixed impetus on the ball: mixed, that is, of the impetus that the fire impresses upward and of the one that the motion of the ship and of the sailor, as of a single mover, impresses on the ball crosswise. So too, since the one who throws the ball upward from A cleaves to the Earth running from A toward K, and is one mover with it, he impresses on the ball a motion mixed of the straight one upward and of the circular, or quasi-circular, one crosswise toward C; it follows that the ball is nowhere left behind by the Tower.

Now a ball thrown from a ship along the mast describes to the eyes of the sailors, in ascent as in descent, a straight perpendicular line, and the sailors in the same ship can notice no other figure of curvilinear motion, because that transverse motion is common to them and to the ball (when two Movables are moved with equal or similar speed toward the same quarter, neither seems to the other to change place, but to stay in the same place opposite the other); and yet in reality that ball is moved along a transverse and curvilinear path very like a parabola, as those notice who stand on the shore and are not moved with the motion of the ship. Just so, assuredly: the motion of the ball thrown upward from the foot of the Tower, A, by a mover standing on the Earth itself — which meanwhile runs from A toward K through C, D, etc. — is mixed of perpendicular and horizontal. By the proper force of the man throwing it, the ball is fired upward as far as the top of the tower; by the force of the motion of the Earth carrying the tower from A to F, it gets from A as far as R; and then,

by the force of the same motion, from R to K, where the Tower arrives once the other half of the interval is completed. And so that ball describes a parabolic line in the space of the world, namely the line AMOQRTXZK. For when the Tower is at CL, the ball has ascended to M; when the Tower has been carried to DH, the ball has ascended to O; and when the Tower has been advanced to EP, the ball has been carried up to Q; and when the tower has been carried over to FR, the ball reaches the top R. Soon it is to descend, and by a like descent it glides down from R through T, X, Z to K. Granted, neither the man who threw the ball up nor the bystanders clinging to the Earth would notice that transverse motion, since it is common to them and to the ball, equally swift, and made toward the same quarter of the world; and so the ball would seem to them to ascend and descend with a merely rectilinear and perpendicular motion.

[Margin: Anticipation of an objection, and its solution.] V. "You will say," Gassendi says in the same place, "what if someone were held in the air inside a chest carried off by a griffin? Would he notice the parabolic motion of the ball?" He answers: by no means, because according to the Copernicans he too, together with the griffin, the chest and this lowest air, is carried toward the East by a common motion — just as, when a quince is rolled round, its down is rolled round with it, and any little creature, say an ant, held within that down. He admits, though, that because of its fluidity the air does not cohere so firmly with the terraqueous vapours and exhalations, and so resists ever so slightly its being swept toward the East, and therefore seems to move gently toward the West; and that from this come those breezes of the surrounding air in open plains and on the Ocean within the torrid Zone, [Margin: The perpetual wind within the Tropics toward the West.] where the motion of the Earth toward the East is more vehement, while the resistance of the purer air toward the West appears greater and is felt far more evidently than in valleys or in the slower parts of the Earth near the poles. We said in the preceding chapter, number 6, that this very thing had been noticed earlier by Galileo; about this wind I remember reading in Ramusio's *Navigations* and in our own José de Acosta. Gassendi adds, as does Galileo in Dialogue 2, that just as, when there is inside a moving ship a vessel full of water with live fish swimming at will in that water, or the ship's cabin is full of birds, flies, etc., the motion of the ship in no way hinders the proper movements of the fish turning in every direction; nor, if you jump toward one side on the ship's deck, do you gain more or less space in the ship than if the same things were done with the ship standing still: so neither does the common motion of the atmosphere hinder the proper flights of birds, flies, etc., nor the particular blasts of the winds that drive clouds, straws and shavings. Since, however, the ball spoken of in scholium 4, when it begins to descend from the top R, has no impetus impressed on it by the Earth, nor perhaps by the air — or not as much as it had before from the Earth — but is moved crosswise only by the motion common to all terrestrial things, it is

therefore probable in this hypothesis that it will descend somewhat sooner from R to K, or rather between K and I; but by a difference imperceptible to us.

[Margin: A heavy body let fall from the mast of a running ship descends perpendicularly to the foot of the mast.] VI. Gassendi seems to suppose as certain that a stone thrown straight up by a sailor along the length of the mast, or let fall by a man at the top of the mast, descends perpendicularly to the same place in the ship. Lest you think he supposed this without grounds, consult his earlier letter on motion impressed by a transferred mover, written to Petrus Puteanus [Pierre Dupuy]. For there, from page 15, he relates as from a certain experiment that if someone goes out on the Seine in a skiff, with the current, the boat being driven by oarsmen as well so that it is carried with the greatest speed possible, and throws pebbles straight up from the prow, they fall back to the prow; if from the stern, to the stern; if from the middle of the boat, they likewise slide back to the middle. And although to those carried along in the same boat the ascent and descent of the pebble seems to be made perpendicularly, because they are borne by the same common horizontal motion, to those standing on the shore it was seen to describe a parabolic line, begun upward from the place of projection and ended at the place of the fall. Galileo asserts the very same of a heavy body let fall from the mast of a ship, whether moving or unmoving, which in either case slides down to the foot of the mast, in Dialogue 2 on the system of the world, Latin pages 104 and 111. He asserts it against the obstinate imagining of certain people who hold, without any experiment, that if the boat stands still the body falls perpendicularly to the foot of the mast, but if the boat moves swiftly toward the East, that heavy body will fall far beyond the foot of the mast toward the West; among these were Tycho, in the *Epistolae Astronomicae*, page 190, and Chiaramonti, in his defence of the *Antitycho*. Indeed he adds that if, with the boat at rest, a stone torn from the top of the mast, or let fall by a man clinging to the very top of the mast, comes down in the time of two pulses of the artery, and the boat is then moved so swiftly that within the same time it advances 20 cubits, that stone will still fall to the foot of the mast, and in no longer time than two pulses of the artery — although in the second case it has run through a much longer path, because the length of the space to be traversed was compensated by the greater speed, owing to the motion impressed by the mover, who is one with the moving ship.

[Margin: Notable points about motion impressed by a transferred mover, from Galileo and Gassendi.] VII. I now pass over many other experiments, though otherwise worth knowing, and admirable effects of mixed motion — that is, of motion impressed by a mover while he too is carried along, either by himself or by another — which are pursued at length by Galileo in the said Dialogue 2, from Latin page 113, by Cabeo in Book 1 of the *Meteorologica*, text 17, from question 2 to 10, and by Gassendi in the earlier of the already mentioned Letters. To our purpose I add only two, more worthy of note. The First concerns a ball thrown by a man who is being carried along in a chariot.

Tell the driver to urge on the horses and slacken the reins, and then you, who sit or stand in the chariot, throw a ball straight up with as great an impetus as you wish or can: it will fall back into your hand, if you want to recover it, for that ball follows you as you run along. The reason is that, besides the upward impetus put into the ball by the force of your arm considered by itself, you also impressed on it an impetus of horizontal motion crosswise, by the force of the motion of your whole body, with which you were being carried along together with the chariot. [Margin: Motion of a ball thrown upward by one who is running.] To you, indeed, who are swept crosswise with an equal motion, the ball will seem to describe a rectilinear and perpendicular motion as it rises and falls back; but to another, who is at rest on the road and not carried along by the same motion of the chariot, the ball will manifestly appear to describe a forward, curved and parabolic path, roughly like the line ARK marked above in the last diagram. The same thing, but downward, will happen if you let a ball drop from the speeding chariot: it will seem to you to fall onto the point over which your eye stands perpendicularly; but one who is outside the chariot will see it carried forward and falling to the ground along a curved path. [Margin: The art of those who strike birds while they fly.] The Second is the experiment, or art, of those who go after flying birds with a musket with such skill in aiming that they mostly hit them and bring them down. For, observing the bird's course and path and its speed, whether it flies toward them or away from them on an opposed or a following course, they swing the barrel, or iron tube, while firing the ball or the lead shot; and by that swing an impetus is impressed on the ball mixed of the straight one upward, by the force of the fire, and the transverse one, by the force of the swing of the arm and of the musket. So the ball either catches up with the fleeing bird or anticipates it as it flies toward them. And this, finally, is why one who leaps down from a speeding chariot falls, and why wine or water bursting from the pipe of a cask, and the ball of a bombard, fall to the ground along a parabolic line: namely, because of the motion impressed by a twofold mover or moving force — the perpendicular motion, downward, from their native gravity, and the transverse motion from the fire, or from the upper parts of the wine or water driving the lower ones out through the horizontal pipe. What I have said about the ball of a bombard and its parabolic fall, our Cabeo also observed in the *Meteorologica*, Book 1, text 17, question 4.

[Margin: The truth of the parabolic path, and the proportion in its speed.] VIII. That the line of heavy bodies thrown upward by a direct effort and sliding back downward is therefore parabolic, if the Earth moves with the diurnal motion, seems to have been sufficiently proved by Gassendi from a similar experiment of a shot or ball thrown up from a chariot or from a ship, but with a spectator outside the chariot and the ship observing the path of ascent and descent. As to why, in ascent, it completes in equal time a longer path near the beginning, such as AM, than near the end, such as QR, and conversely, in descent, RT is shorter than TX, etc.: the cause is that at the beginning of

the ascent the thrower's upward impetus is stronger and predominates over the transverse impetus; but when the ball has reached the top R, the impetus by force of gravity begins to move it downward, sluggishly at first, but more intensely at the end. More, however, is to be said of this figure below, in chapter 17, numbers 16, 17 and 18, where we shall inquire into the figure of this line from other, firmer arguments; for many things will have to be supposed there from our own experiments, not to mention Galileo's, on the descent of heavy bodies and the increase of their speed, which would be inserted here too inopportunely.

CAPVT V.

Chapter 5.

Proponuntur & Diſſoluuntur Decem Argumenta pro Reuolutione Diurna Terræ,deſumpta ex Terræ ipſius conditionibus Collatis cum Octauæ aut ſupremæ Sphæræ conditionibus.

Ten Arguments for the Diurnal Revolution of the Earth, drawn from the conditions of the Earth itself Compared with the conditions of the Eighth or supreme Sphere, are Proposed and Resolved.

[Margin: A very necessary admonition.] I. What I have resolved to observe, with the whole effort of my mind, in this so celebrated and arduous controversy, I would like to obtain from the Reader as well: namely, that all the arguments which are usually brought, or will seem capable of being brought, for the motion of the Earth or against it, be weighed with the fairest balance and a sincere disposition toward the truth, and that as much weight be given them as possible; that the arguments be unfolded one by one, since very many of them, often heaped together as if in a single sorites, confuse the mind; and that, with the prejudice of a presumed opinion set aside, the replies to the arguments be weighed with equal indifference, the final and decisive verdict being held back until the very end of this disputation. For this reason I shall not be reluctant to compress the arguments, once collected, into syllogistic form wherever it is needed.

I. Argumentum à Figura Terræ, vbi obiter de Figura Cæli.

1st Argument, from the Figure of the Earth; with, in passing, the Figure of the Heaven.

II. Copernicus led the way for the others in this argument, in Book 1, chapter 8, in these words: *Whether, then, the world is finite or infinite, let us leave it to the disputation of the natural philosophers [Physiologi], holding this as certain: that the Earth, enclosed by its vertices* — that is, enclosed by mountains — *is bounded by a globular surface. Why, then, do we still hesitate to grant it the mobility that by nature suits its form, rather than that the whole world, whose limit is unknown and cannot be known, should slide round?* It is as if he said: if the diurnal motion of the primum mobile must be granted to one of two bodies, namely either the

Earth or the supreme heaven, which contains the whole world in itself, it is to be attributed rather to the body which is certainly known to have a spherical figure — the figure that by its nature is the most apt for the motion of rotation [motus vertiginis], because it lacks angles — than to the one whose figure is unknown. But it is certainly established that the Earth, notwithstanding the very slight roughness and height of its peaks or mountains, is spherical on every side, as is plain from what we said in Book 2, chapter 1; whereas the supreme heaven was thought spherical by the Stoics, in Plutarch, Book 2 of the *De placitis*, chapter 1; by Pythagoras and Plato, in Laertius's *Lives*; [Margin: Whether the heaven is spherical or not is uncertain.] by Cleomedes, Book 1 of the *Cyclical Theory*; by Pliny the Elder, Book 2, chapter 2; by Aristotle, Book 2 of the *De caelo*, chapter 4; and by Ptolemy, Book 1, chapter 3. But the arguments of Aristotle, Cleomedes, Pliny, Ptolemy and others that are brought for its roundness are only probable arguments: such are those from the mobility, capacity and dignity of the spherical figure, or from a certain likeness to the divine Immensity, from empty space, etc. Those drawn from circular motion, moreover, fight only for concavity, not for convexity, and already involve the very thing we are now calling into controversy. And so the argument drawn from there for the finite extent of the outermost heaven and of the world has no force in the present question. Nor, again, is it plain from the sacred scriptures that the last heaven is round on every side as to its outermost surface. For those words of Ecclesiasticus 24, *I alone have compassed the circuit of heaven*, and *All things that are contained in the compass of heaven*, can be understood of the concave surface of the visible heaven, or of a circuit taken broadly for a boundary closing off some space on every side — in the way someone is said to have gone round a whole city even if it happens to have a square figure. [Margin: Opinions of the Fathers on the figure of the heaven.] For this reason the heaven was held to be only hemispherical, or spread out over the Earth like a vaulted chamber and arch, or a tent, by Saints Justin, questions 59 and 130; Basil, Homily 1 on the Hexaemeron; Ambrose, Book 1 of the *Hexaemeron*, chapter 6; Chrysostom, *On the Incomprehensible* [?] *Nature of God*, chapter 2; likewise Theophylact and Primasius on the Epistle to the Hebrews; Theodoret, in his book *On Matter and the World*; Lactantius, Book 3 *On False Wisdom*, chapter 24; Euthymios Zigabenos on Psalm 103; and Procopius, Severianus and Acacius on chapter 1 of Genesis. Indeed St Augustine, in Book 2 of the *De Genesi ad litteram*, chapter 9, doubted whether it was not rather a human fiction of those who say the heaven is spherical on every side, though he afterwards grants it — to say nothing of the profane authors who, in Plutarch, Book 2 of the *De placitis*, chapter 2, gave the World the figure of a top, or of an egg, or some other figure different from the spherical. Many other Fathers, however, favour the roundness of the heaven: above all St Nazianzen [Gregory of Nazianzus], Oration 4 against Julian, with Elias the interpreter there, and the oration on Holy Baptism, with Nicetas there; St Hilary on Psalm 135; St Ambrose, sermon 12 on Psalm 118; St Jerome, Book 11 on Isaiah; St Augustine on Psalm 103, and Book 1 of the *De Genesi ad litteram*,

chapter 20, and Book 2, chapter 9; Bede, in his book *On the Creation of the Six Days*; and with them Alensis, part 2, question 50, and the Scholastics on Book 2, distinction 14. Whatever may be said of the Empyrean, which some have thought round and some square or cubic, as I said in this Book 9, section 1, q. 6, chapter 1, number 31 — for there we taught that it is held to be immobile, whereas we are speaking of the heaven that is held to be mobile, at least as to appearance — even if the world is supposed finite, it is still uncertain whether it is spherical on every side; how much more so if it is uncertain whether it is finite or not. For that it is not infinite cannot be demonstrated by any natural arguments, since we once maintained in our Physical disputations that an actually Infinite magnitude can exist; and that figure is incompatible with an infinite body, as being what terminates and bounds it, is affirmed by so many that they cannot be convinced of the opposite. But we have digressed enough on the occasion of this argument, which — from the spherical figure of the Earth, though in very few words — was used by William Gilbert, Book 6 of *De magnete*, chapter 3; by Kepler, in the *Epitome of Copernican Astronomy*, page 110; by Origanus, in the dedication of his *Ephemerides*; and by Gassendi, Letter 2 *De motu impresso*, page 112. I am truly surprised that Gassendi counted this argument among the necessary ones, saying: *It seems worth considering that, if the Earth had been constituted by its own nature for rest, it would no doubt have been given either a pyramidal or a cubic form, not, as it has, a round one; for which reason, as this form is most fitted for motion, so the Earth seems to be moved wholly by its own nature.*

[Margin: Anticipation of an objection.] III. Nor should you say that this argument infers more than the Copernicans would wish: namely, that all the bodies of the world which the Copernicans would grant to be globular — such as the whole elementary sphere and the globes of the Planets and Fixed stars — must be moved with a motion of rotation about their own centre. For they will answer either by granting them this rotation, so that they likewise have in it a cause of the twinkling in the very distant Fixed stars, or by granting it for other causes, for which this rotation has been admitted by some in all the planets except the Moon, as we said in Book 3, chapter 4, and Book 7, section 1, chapter 3, scholium 3, and chapter 7, number 12. In the Moon, however, it is not admitted by those who move her by a simple Eccentric, because otherwise she would not always turn the same spots toward us in the same quarter; and conversely, because of this very appearance, the motion of rotation must be admitted by those who think she is moved by an Epicycle carried by an Eccentric. As for the whole elementary sphere, they will deny that the diurnal motion must necessarily be attributed to one of the two — namely either to the Earth or to the supreme air and fire — and deny that the sphere shares with the Earth in this hypothetical necessity that the motion be attributed at all events to one of them.

[Margin: The hypothetical form of the already mentioned Argument.] IV. The form of the already mentioned argument, then, can be either hypothetical

or categorical, that is, absolute. The hypothetical form is of this kind: *If the motion of diurnal rotation must necessarily be attributed either to the Heaven or to the Earth, it is to be attributed to the Earth rather than to the heaven. But the motion of diurnal rotation must necessarily be attributed either to the heaven or to the Earth; therefore etc.* The MAJOR premise has now been sufficiently established, because such a motion is to be attributed rather to the body that is certainly known to have a figure suited by its very nature to this rotation than to the body that either does not have such a figure or of which it is not certainly established whether it has it or not; and it is certainly established that the Earth has such a figure, whereas about the heaven there is doubt. The MINOR premise is sufficiently plain, because the world is divided exhaustively into the heaven and the Earth, some of the Elements being included under the name either of the heaven or of the Earth, and the celestial bodies under that of the heaven; nor does it appear to what other body the daily appearance of the diurnal rotation could be ascribed.

[Margin: 1st Reply to the Argument.] V. One can reply, first, by Conceding the Major and its proof on the supposition that this motion necessarily takes place through the rotation of some body — that is, through a motion of that body about its own centre, really distinct from its other motions — but by denying it if the motion does not necessarily take place in this way, and its appearance can instead be saved by a total translation of a body from one place to another; and by holding that the diurnal motion, which is called that of the primum mobile, can be attributed both to the individual globes of the Planets and to the individual fixed stars, carried round through a fluid heaven by their own movers one by one, without any rotation of the Heavens by whose motion they would be moved. But since it is more probable that the Planets are indeed moved in this way, but the Fixed stars by the motion and rotation of the eighth sphere, or at least together with the rotation of the Firmament, rather than each of the non-wandering stars individually in a liquid heaven, as we said in section 2, chapter 3 — for this reason:

[Margin: 2nd Reply.] VI. It is replied, *secondly*, by Conceding the major proposition and its confirmation if all other things are equal, but denying it if they are unequal, and if other reasons outweigh, or at least equal, this reason taken from the aptitude for the motion of rotation that comes from the roundness of the figure. Moreover, from all the arguments that we shall bring below against this diurnal motion of the Earth — at least taken together — there arises a reason that outweighs the one we were speaking of, or is equivalent to it.

[Margin: 3rd Reply.] VII. It is replied, *thirdly*, by Denying the Major; and, as to its proof, by conceding it if that aptitude arising from the figure were necessary for rotation, so that what lacks it would be wholly incapable of rotation, or were indeed useful, but ordained for that purpose per se or principally; otherwise, however, by denying the major of the proof. Now the spherical figure was not necessary for this rotation: for the diurnal motion did

not have to take place over some plane, nor within some solid body that would resist the enclosed moving thing if it were angular, but either in the air, or in the liquid aether, or in the ethereal breeze [aura aetherea], none of which resists. Nor, moreover, was this figure bestowed by God on the terraqueous globe for this as its principal end, since it was not granted for this end at all, as those who deny it this motion must say; but so that, its bulk and weight being balanced on every side, it would keep its equilibrium — and indeed the equilibrium of the Seas, lest otherwise the waters, not equally distant from the centre but some higher than others, should rush headlong to the lowest places; and so that living creatures, especially human beings, might enjoy the view of the whole celestial hemisphere, and the globe might lie open to the celestial influences equally on every side and in an orderly succession round its circuit; and for many other reasons which there is no need to list one by one. This is all the more so since I am quite certain that the Copernicans, even if the Earth were not round at all, would not on that account deny it the diurnal motion; and consequently the rotation is not rightly denied to the Eighth sphere even if it is not certain that it is round on the outside — granted that, if this rotation is conceded to it, its roundness may not improbably be confirmed from it. And from this the reply to the already mentioned Argument can be plain, if it is put forward without the hypothesis.

[Margin: The absolute form of the Argument, and the Reply.] VIII. For it could be said absolutely: *Of two bodies, the diurnal revolution is to be attributed rather to the one that is certainly spherical than to the one that is not spherical, or is not certainly established to be so, etc.* For any of the three replies already brought can be applied to this major proposition, but especially the second and the third.

II. Argumentum à Mobilitate Terræ raptilis ad motum Primi Mobilis.

2nd Argument, from the Mobility of the Earth, liable to be carried off [raptilis] by the motion of the Primum Mobile.

[Margin: The Argument of Mästlin and Kepler.] IX. "Not without reason" (these are Kepler's words in the *Epitome of Astronomy*, Book 1, page 106) "does Mästlin also ask this, namely in his preface to the *Narratio prima* of Rheticus: how can it happen that, when the whole system of the World is carried round — none of its orbs being excepted, not the sphere of fire (if there is one), not the upper Region of the air — this single little globe, whose diameter is less than one twenty-thousandth part of the diameter of the world, is not carried round together with it?" *How can it be*, says Mästlin, *that, when the whole system of the world is carried round, this single little point is not turned?* [Margin: And Galileo's [?].] But Galileo states this same point still more explicitly in his 7th objection [instantia], which he brings for the diurnal motion of the Earth in Dialogue 1 on the system of the world, page 86 of the Latin. For he says: *If we attribute the diurnal turning to the highest heaven, we must give it such great power that it carries with it the innumerable multitude of fixed stars, all of them very vast bodies and much larger than the Earth, and besides that all the spheres of the Planets, although both the*

latter and the former move by their own nature in the contrary direction. Moreover, we must also concede that the Element of Fire and the greater part of the air are carried round with them, and that only the small globe of the Earth resists such great power with stubborn rest. This in my judgement involves much difficulty; nor can I see how the Earth — a suspended body, balanced equally on its centre, indifferent to motion and rest, surrounded by an ambient liquid — should not itself also give way and be carried round together with the rest. But we find no such obstacles if we attribute the motion to the Earth, a very small body, insensible with respect to the Universe, and therefore unfit to do the Universe any violence. In these words, however, many things are involved and presupposed that belong to other arguments to be discussed later. In this place we weigh only the argument drawn from the mobility of the Earth, given the motion of the primum mobile. So that it may have its full edge, I put it into form as follows.

[Margin: The Argument reduced to form.] X. *If, of two movable bodies, one stands in such a relation to the motion of the other that, when the other is moved antecedently and about itself, it too must necessarily be moved, because it cannot resist that body's motion, while the other does not stand so, but can rest when the first is moved, then the motion is to be attributed rather to the movable body that does not necessarily draw the other's motion along with it. But if the heaven is moved with the diurnal rotation, the Earth must be carried off by the same rotation; whereas if the Earth is moved antecedently with the diurnal rotation, the heaven need not be carried off by the same rotation. Therefore the motion is to be attributed to the Earth rather than to the heaven.* The MAJOR premise of this Syllogism seems evident, lest motions be multiplied in nature in vain, and what can be done by fewer means be done without necessity by more. The MINOR premise seems already proved, both from the comparison of the heavens and the upper elements, which in the opinion of the Peripatetics cannot at all resist the motion of carrying-off [motus raptus], and, absolutely, from the smallness of the Earth, its indifference to motion and rest, and the absence of resistance from what surrounds it.

Nevertheless I reply very briefly: Granting the Major, I deny the minor proposition. For it is not necessary that, when the supreme heaven is moved, the Earth be moved and carried off; on the contrary, given a diurnal motion of the heaven such as appears to us, the Earth must rest, or not be moved with a sensible diurnal motion. For, *First*, it is more probable that the heaven of the Planets is liquid, which Kepler and Galileo themselves concede, as we taught in section 1, chapter 7; and consequently that the ethereal breeze adhering to the concavity of the Firmament is carried off by its motion in such a way that, through the breeze's own thinness, that motion at length dies away in the lower parts and is not sufficient to carry the globes of the Planets with it; and so the heavens of the Planets are not stirred by that motion, nor is the fire in the concave of the Moon carried off by it, much less the air or the Earth. Indeed, since we are dealing here with the controversy of a necessary argument — for Galileo supposes it to be such — it is not at all improbable, much less

impossible, that even the Fixed stars themselves are moved individually as in a liquid aether; and consequently far less danger of being carried off threatens the lower heavens and the elements. Besides, a Primum Mobile can be admitted in many other ways that communicates the impetus or pattern of its motion to the lower bodies without carrying them off, as is plain from the opinions brought in section 2, chapter 3. *Secondly*, even on the view of Aristotle, who admits solid heavens and the motion of carrying-off, that motion not improbably grows so faint from the fire to the air that it hardly reaches the middle region of the air, much less the lowest, because of the thinness and fluidity of these elements. For it is now well enough established, Galileo himself insisting on it, that the lightest feathers and straws are most unfit to receive impetus; how much more unfit, then, is the air? On the other side, the Earth is very dense and very heavy in all its parts, which strive towards the centre; and although heaviness of itself resists only a straight motion upward, yet, joined with density, it cannot be moved except by an enormous impetus extending through its whole depth — an impetus such as the air immediately surrounding it, affected by no sensible motion of carrying-off, cannot impress on it; nor is the Earth of itself indifferent to motion and to rest, as Galileo gratuitously supposes. Therefore it is not necessary that the Earth be carried off by the motion of the primum mobile.

I said, moreover, that it is rather necessary that the Earth not be carried off by the diurnal turning, given the phenomena that appear to us, because if the Earth were carried off by it we should always see the Sun, or the same star once seen, in the same circle of position — the Meridian, say — except in so far as they recede eastward by their slight proper motion; for we would follow their diurnal motion at an equal pace. Galileo, then, who in those dialogues so strictly demands the forms of consequences from others, ought to have been on his own guard, and not have asserted so readily as necessary what we have already shown to be not at all necessary.

III. Argumentum à Circulari Motu congruente per ſe potiùs quàm Recto Elementis, atque adeò Telluri.

3rd Argument, from Circular Motion as befitting the Elements per se rather than Straight motion, and so befitting the Earth as well.

XI. This argument too is *ad hominem*, as they say in the Schools, and is directed against the Peripatetics, who admit that Fire and the supreme region of the Air are moved perpetually by the concave of the Lunar heaven, through the force of the carrying-off of the primum Mobile — but per accidens, and not indeed against the nature of these elements, though still beyond nature [praeter naturam]; whereas straight motion upward from the centre is natural to them, as downward motion, or motion to the centre, is natural to Water and Earth. On these premises the Copernicans contend that circular motion should rather have been called natural, and per se, to these elements, and rectilinear motion preternatural and belonging to them per accidens; and so that perpetual

circular motion is naturally owed to the Earth as well. [Margin: Copernicus's doctrine of the motion of heavy and light bodies.] This was certainly the mind of Copernicus in Book 1, chapter 8, where he argues most speciously as follows: *Therefore what they say, that a simple body has a simple motion, is verified above all of circular motion, as long as the simple body remains in its natural place and in its unity. For in its place there is no motion other than circular, which remains wholly within itself, like a body at rest. Straight motion, however, comes upon things that wander from their natural place, or are thrust out of it, or are outside it in any way whatever. But nothing is so repugnant to the ordering of the whole and to the form of the world as for a thing to be outside its place. Straight motion, therefore, occurs only to things that are not rightly disposed, and are not perfect according to nature, while they are separated from their whole and desert its unity. Moreover, things carried upward and downward, even apart from circular motion, do not make a simple, uniform and equal motion; for they cannot be regulated by their lightness or by the impetus of their weight. And whatever falls, making a slow motion at the beginning, increases its speed as it falls; whereas, conversely, we see this earthly fire (for we see no other) languish at once when carried on high, as though the cause were acknowledged to be violence done to the matter. Circular motion, however, always revolves equally, for it has an unfailing cause; but the other has a cause that hastens to cease, through which, once bodies have reached their place, they cease to be heavy or light, and that motion ceases. Since, then, circular motion belongs to wholes, but straight motion also to parts, we can say that circular motion remains with straight, as an animal does with sickness. Indeed, Aristotle's dividing simple motion into three kinds — from the centre, to the centre, and about the centre — will be thought an act of reason only; just as we distinguish line, point and surface, although one cannot subsist without another, and none of them without body.*

[Margin: Galileo's propositions, full of various novelty.] XII. From this reasoning *Galileo* — but on the supposition of Aristotle's view, not his own, about the circular motion of Fire under the concave of the Moon — tries to teach, in Dialogue 2 on the System of the World, page 9 of the Latin, that straight motion cannot exist in a well-ordered World, but only circular, and that straight motion is of itself infinite, because a straight line is of itself indeterminate and infinite; and that straight motion perhaps existed in the first Chaos. He adds, however, that straight motion was instituted for bringing disordered bodies into order, and he brings in the authority of Plato, who thought that the bodies of the world were moved at first with straight motion and afterwards with circular, once they had acquired a certain degree of speed through straight motion. On page 17 he affirms that circular motion, and its determinate speed, cannot be acquired naturally, or without a miracle, without a preceding straight motion, but that once acquired it is from then on perpetual and uniform. On page 18 he says that straight motion cannot naturally be perpetual, but was given only to restore natural bodies to their place if they have fallen out of it, whereas circular motion can of itself be continued perpetually and is finite, because any point of the circumference is a beginning and an end; and that

therefore only circular motion and rest are suited to the conservation of order. On page 19 he denies that it is evident that heavy bodies descending are moved with a straight motion. On page 28 he says that rest, rather than straight motion downward, should have been called natural to the terrestrial globe, and on page 29 that straight motion is attributed with better reason to the parts than to the whole element. In the same place he affirms that the Peripatetics wrongly assign to the elements of Air and Fire circular motion as preternatural and straight motion as natural, since these elements always have been and always will be moved with circular motion, whereas as wholes they have never been moved with straight motion, nor will they ever be so moved; and that the Peripatetics have forgotten their own axiom that nothing violent is perpetual, and that the motion which is perpetual should rather have been called natural, and straight motion preternatural.

[Margin: The weakness of the Copernican and Galileist discourse against the Peripatetics.] XIII. And yet in the discourse of neither Copernicus nor Galileo do I see anything solidly put forward, either for themselves or against Aristotle, but only either an itch to pluck at Aristotle, or a beggary of very flimsy little reasons, scarcely consistent with themselves, scraped together for the diurnal motion of the Earth. For Aristotle does not affirm that a simple motion from the centre or to the centre — that is, straight motion — belongs to the Elements absolutely, and when motion is compared with rest; for absolutely he attributes to them, per se, rest in their natural place. But if they, as wholes or in their parts, are removed from it, he says that they have from the generator — that is, from their very birth — an intrinsic force of moving themselves with a simple motion, but the shortest possible, and so along a straight line to their natural place. Therefore, if Aristotle admitted that the World was made in time and that the elements were at first confused in Chaos, he would certainly assert that they afterwards acquired their place by straight motion; and that, if the whole Earth were placed outside its place, in the sphere of Fire, and the Fire in the middle of the elementary sphere, the Earth would descend to the place where it now is and the Fire would fly upward. Of this motion he had very probable, if not evident, arguments from the motion of the parts of Earth and Fire, which he supposed to move in this way on the testimony of the senses and by common consent. That the higher portion of the Fire and of the Air is carried off by the motion of the primum Mobile is not violent for them, since they are not on that account removed from their place; it is only beyond nature, and therefore it is not against that Peripatetic axiom, *Nothing violent is perpetual*. Now, granted that hypothesis — namely that they are removed from their place — it is natural for the elements to return to their place by the shortest and most determinate way, which is their distance from their natural place, rather than by a longer way; and the shortest is the straight line perpendicular to the earth, whereas that curvilinear line — mixed of straight and curved and fused into one — which the Copernicans are forced to admit in such a descent or ascent, is longer. Moreover, granted that same hypothesis, it is better for

the elements and their parts to be moved unequally, always accelerating their motion towards their native place, than with a uniform and equal motion: whether this happens because they are borne magnetically from one side and drawn from the other, and the more strongly the nearer they are; or because the impetus once impressed by heaviness or lightness ought not to be destroyed by a new impetus, while at every instant of time in which they are not in their natural place they have the force of producing in themselves impetus of a determinate degree, which, added to the earlier one or ones, makes the motion faster and faster towards the end. [Margin: Why does the motion of heavy and light bodies become faster?] Therefore, since an infinite impetus could not, or ought not, be given to them from the beginning — for it would have been in vain, if they were to be removed from their natural place only by a finite space, and that mostly a very short one — and yet so that they should not return slowly to their place, it was better that that speed should be finite but capable of increase to infinity [auctilis], that is, always growing until they reached the term of their natural rest. That straight line, then, is not infinite, but is determined here and now by whatever removes the parts of the elements from their natural place; nor does the inequality, difformly uni- formly increasing, stand in the way of the naturalness of such a motion; rather, it is most congruent with it. Nor does such a motion, in order to be natural, require perpetuity absolutely, but only in the way in which it is natural — namely conditionally, or on the hypothesis that the bodies must tend to their natural place, in which they are not; and if this hypothesis were in fact perpetual, the second act of that motion would also in fact be perpetual on either hypothesis. So much, then, for Aristotle, whose reasoning, though not set out by him as explicitly as by us, is far more solid, and more in conformity with the experience of the senses, on which physical science ought to build its reasonings.

[Margin: The arguments of Copernicus and Galileo [?] the motion of the Earth, very flimsy and inconsistent.] XIV. That the reasons of Copernicus and Galileo for the diurnal revolution of the Earth, begged from the comparison of rectilinear with circular motion, are flimsy and weak, and in places scarcely consistent with themselves, I show as follows. Copernicus prefers circular motion to straight for the reason that *circular motion remains wholly within itself, like a body at rest* — that is, because the whole movable body is not carried by its force to another place adequately distinct from the first. Therefore, when he carries the Earth round annually, this perfection is really taken away from circular motion, granted that it is not taken away formally by the force of the diurnal rotation. Next, if circular motion is more perfect because it is like rest, and is on that account owed to the Earth, then rest itself in its own place is more perfect, and consequently it is rest, rather than any motion, that is owed to the Earth. And so no sane Philosopher or Theologian would dare to ascribe to God a circular motion of rotation within himself, or within his own immensity: because local motion itself, even if it is not translational, when compared with rest and immobility, not only includes a change repugnant to God, but also

involves a perfection that is not simply simple; and absolutely it is better not to be moved, or better to rest, than to be moved and not to rest. Again, straight motion, as being enclosed in fact within two distinct termini that bound a finite interval, is finite, whereas circular motion is rather like the infinite, because it always begins to be repeated from the very point where it seemed about to end, since there is no greater reason from the nature of the circle why it should end at one point rather than another. Moreover, if straight motion is not natural to elements placed outside their place, because it is unequal and difform in speed, and therefore not simple, much less natural to them is a motion compounded of straight and circular, such as Copernicus attributes to them in their descent to their natural place; for besides the inequality it has in the space acquired in equal times, it also has an inequality of figure, which it is forced continually to change, approaching the circular or receding from it, always with a different course. For a straight line is much simpler than a parabolic or hyperbolic line, or any curvilinear line that is not circular — to say nothing of the length of this curvilinear path. Furthermore, when circular motion is said to be equal, this is already supposed gratuitously; for of itself it can be as unequal as rectilinear motion is. But Galileo too invents many such things for himself, of which kind are these: that circular motion, or its determinate speed, cannot be acquired naturally, or without a miracle, except through a preceding difform straight motion, but that once acquired it is perpetual and uniform. These are not only gratuitous but hardly coherent; for the means must fit the end and the end the means, but straight motion certainly does not fit circular motion, nor does difform motion fit uniform. Nor, when we spin a top, do we impress circular motion on it by means of straight motion, but by means of a circular rotation made with the cord; nor, conversely, when we impress straight motion on a stone with a sling, do we beg that straightness from circular motion: rather, we whirl the sling round so as to impress one impetus after another on the stone, which we afterwards convert into straight motion by means of a straight projection. Now, circular motion is said to be finite, and yet perpetual; but by the very fact that it is perpetual it is of itself infinite in duration. Nor is it terminated because any point can be taken as a beginning or an end; rather, by the very fact that none of them is determinately an end rather than a beginning, it has a certain indetermination, on account of which the circle is usually a symbol rather of infinity. The same Galileo, moreover, carps at Aristotle because he attributes per se to the elements straight motion, which as wholes they have never had, and says that circular motion, which fire and air always have, is per accidens. Let Aristotle, then, rebuke Galileo because he attributes per se to the Earth and Water a circular motion which they have never had, and says that straight motion — which, on Plato's view as adopted by Galileo, they had from the beginning — is intended by nature per accidens or secondarily. For he ought not to suppose what he is certain Aristotle will deny him, namely that the Earth is moved, or was at some time moved, circularly. But it is now tiresome to chase these flies, troublesome

though they are; and it is better to resolve the argument they are labouring at, once it has been compressed into form. It can, then, be formed thus.

[Margin: The 3rd Argument: its kernel, or form.] XV. *If Aristotle attributes to Fire and Air a diurnal circular motion, and that motion is more natural to them than straight motion, he must also concede to the Earth and Water a diurnal circular motion, as more natural to them than straight motion. But Aristotle attributes to Fire and Air a diurnal circular motion, and that motion is more natural to them than straight motion. Therefore Aristotle must concede to the Earth and Water a diurnal circular motion, as more natural to them than straight motion.* The MAJOR premise is plain from the parity of the elements, which Aristotle too acknowledges in this respect, that they agree in sharing a simple motion of the same kind, namely rectilinear. The former part of the MINOR is known from Aristotle; the latter part is proved as follows. That motion is more natural which is perpetual, uniform and simple, than one which either never was nor will be, or is not perpetual, nor uniform, nor simple. But the circular and diurnal motion of Fire and Air is perpetual, uniform and simple; whereas the straight motion either never was nor will be, if we are speaking of the whole element, or is not perpetual nor uniform, since towards the end it becomes faster and faster. Therefore, etc.

[Margin: 1st Reply, and the principal one.] It is replied, primarily, by conceding the Major of the first syllogism and the former part of the Minor, and denying the latter part of the Minor. To its confirmation the reply is: conceding its major if that motion is perpetual from an intrinsic principle, and the straight motion from an extrinsic one; denying it if the perpetual motion is from an extrinsic principle, and the straight motion, though not perpetual, is nevertheless from an intrinsic principle.

[Margin: 2nd Reply.] It is replied, secondarily, by Denying the Major and its proof drawn from parity. For even if it were natural for Fire to be moved circularly westward, and that from an intrinsic principle, so as to imitate the motion of the primum mobile, yet for the Earth, and for the Water enclosed in the Earth, this ought not to be natural; rest ought rather to be natural to them, so that the living creatures dwelling on them might experience in succession the alternations of risings and settings which they do in fact experience. Otherwise, if they were moved westward with the same motion, the creatures would not experience them, nor would the Phenomena be saved. In such a case, however, it would suffice that the four elements should agree in this: that if they were removed from their place, they would have a force of tending to it by straight motion. It is truly surprising that Galileo should wish to draw the Earth's circular motion out of the circular motion of Fire and Air asserted by Aristotle, and yet a contrary motion, namely eastward; for it ought rather to have been westward.

IV. Argumentum ab ingenti Magnitudine Octauæ ſphæræ & à ratione Totius collata cum Exilitate Telluris,tanquam particulæ Vniuerſi.

4th Argument, from the enormous Size of the Eighth sphere, and from the proportion [ratio] of the Whole compared with the Smallness of the Earth, as a tiny particle of the Universe.

XVI. We saw that this fourth argument was included in the second; but we thought it should be brought out separately and contested on its own, especially since the Copernicans try to arm it with certain specious similes. For Galileo, in Dialogue 2 on the system of the world, page 81 of the Latin, having said in the person of Salviati: *If we consider only the immense bulk of the Starry sphere in comparison with the smallness of the terrestrial globe, which is smaller by so many thousands of myriads, and consider besides the speed of the motion which the entire revolution must complete in the space of one day and night, I cannot persuade myself that anyone can be found to whom it would seem more reasonable and credible that this circumgyration falls to the celestial sphere and that the terrestrial globe stands unmoved* — at once brings in Sagredo, conceding that, if all else were equal on both hypotheses, it is more improbable to deny the diurnal motion to the Earth, and to the heaven ascribe it to the heaven, and that whoever does so is *not a little more absurd than a man who, climbing to the top of his watch-tower for no other purpose than to view the city and the surrounding countryside, should ask that the whole region be turned round for him, so that he need not undergo the labour of turning his head.* [Margin: Galileo's simile for this argument.] Kepler, however, preferred to come down from the watch-tower to the kitchen, so as to draw from there a simile suited to his palate. For in Book 1 of the *Epitome of Copernican Astronomy*, page 127, he writes: *But it is quite absurd that so great a mass* — he is speaking of the whole machine of the heaven — *which the Philosophers even contend is more perfect than the globe of the Earth, should travel so long a journey in going round, on account of the shape of this very narrow little ball (which is why the whole of it cannot enjoy the fire of the Sun at one time), when the Earth could relieve the heaven of this burden by a most economical turning of its own little body. For it would be just as if an inept cook, disdaining to turn the meat fixed on the spit, were instead to carry the fire round the meat in a circle.* [Margin: Kepler's simile.] For elsewhere he had said that the Earth roasts itself about its axis like a spit, by turning itself towards the Sun.

[Margin: Gassendi's similes.] Far more elegantly and urbanely, to be sure, Pierre Gassendi — not himself, indeed, absolutely a Copernican, but still taking on their persona, and inclined towards that Sect, were it not that the decrees of the sacred congregation stand in the way — argues as follows in Letter 2, *De motu impresso*, page 108: *Since reason persuades us that it is more likely that the ship moves towards the port than that the port and the ships, the houses and the adjacent lands move towards the ship itself; and, when someone views from a lofty tower the whole city set round about it, that it is more probable that the man has come up to the top of the tower and turns himself round in every direction than that the whole city sinks down and is turned round him; and, when an orator is delivering his*

peroration from the platform, that it is more fitting for his face to be directed in order to all the parts of the auditorium round about than for the whole auditorium to be turned round his motionless face — and similar things of that kind — *why, with the same reason persuading us, should it not become more likely, more probable and more fitting that the Earth moves towards the East and salutes in order the whole machine of the world and all the stars, than that so great a machine, and so many and such great stars, go round the Earth and salute it? Surely the ship is some sensible portion, or rather is of some magnitude, with respect to the port and the whole opposite shore; the man with respect to the city; the face with respect to the auditorium. But the Earth, compared with the amplitude of that so great machine, is not merely a point, but even, if anything more insensible can be imagined, something less than a point.* And so these men play the rhetorician. To this Lansbergen adds, in the *Progymnasmata astronomiae restitutae*, the character [ratio] of the Whole, saying that it is absurd that the Whole — namely the world, or the heaven — should be moved, and the part — namely the Earth — should rest. From this, then, there arises the following argument.

[Margin: The argument [?] in form.] XVII. *If, of two movable bodies, or bodies to be moved, one or the other must be moved in relation to the other, it is absurd that the one which is incomparably greater, and has the character of a whole, should be moved rather than the one which is incomparably smaller and has the character of a particle. But the whole heaven is incomparably greater than the Earth and has the character of a whole, while the Earth is like a particle. Therefore it is absurd that the whole Heaven should be moved rather than the Earth.*

[Margin: ıst Reply.] It is replied, first, by Conceding the Major if what is greater in quantity of mass is also greater than the other in quantity of power [virtus], or in perfection of substance and of its other properties; and by denying it if it is not greater in this second way, but smaller. And so the Minor proposition is conceded in the former sense and denied in the latter. For — I do not say the whole Earth with its inhabitants, but — a single man is incomparably greater than the whole machine of the heaven, setting apart the Intelligences, which are not parts of the heaven, nor forms of the celestial bodies. Nor, indeed, should one take here the bare globe of the Earth, without living creatures and without a creature that contemplates the divine works; for in that case perhaps neither the heaven nor the Earth would be moved — for what good, or to what end? We, therefore — unless we begrudge ourselves our own happiness, granted by the bounty of the Creator — we, I say, are those spectators for whom, as they sit in the theatre, that vast stage, lit with so many lights and revolving with its machines, displays its dances and catastrophes to be contemplated in turn. We are those kings and princes to whom the whole host of heaven — and, to borrow the words of the divine book, "that vessel of the camps on high" — repeating its review [lustrum] and running its rounds as if for a census, returns to be numbered. Unless perhaps we think it unworthy that God should have undertaken such great things for the sake of men, for whose instruction and redemption he expended even the Angels and — what is

inestimably greater — his only-begotten Son. As for the character of Whole and part, it is denied that the heaven, with respect to the Earth, has the character of a whole; for it is a co-part [compars], greater indeed than its fellow part in mass, but smaller in power. But someone could take the Intelligences together with the heaven — to which, as its movers, we conceded this motion in Section 2, chapter 1, numbers 4 and 9, in such a way that they go round together with it — and then the argument already mentioned would seem to have greater force, since in that way the heaven is greater than the Earth not only in mass but in power as well; although in reality the Intelligences are not an intrinsic part of the heaven, as the soul is of living creatures. For that reason a reply is given in another way.

[Margin: 2nd Reply.] It is replied, secondly, by Conceding the Major if what is greater is moved for the sake of the smaller as its ultimate end, and precisely in order to serve the smaller; but by denying it if it does not stop there, but orders its motion, or has it ordered, to its own nobler end, and so as to serve a worthier being, such as God, Best and Greatest. That the Intelligences, while they move the celestial bodies, move themselves — namely, to manifest His perfections and glory — need not be shown in this place, but is to be supposed as certain from sound Philosophy, let alone from Christian Theology.

[Margin: 3rd Reply, and retortion of the argument.] It is replied, thirdly, by Retorting the argument in part, in two ways. For if it is absurd that the heaven be moved for the sake of the Earth because it is of greater mass than the Earth, it would also be absurd that it be moved with any motion at all, let alone the diurnal one; and so the superior Planets would not be moved with their own motions either. For it cannot be said that they are moved per se about the Sun and per accidens about the Earth, since it is the Earth, not the Sun, that they perfect — or at least uncertain and disputed claims are being forced upon us. Likewise it would be absurd for the Earth to be moved with a diurnal motion for the sake of men, so as to offer them in succession the spectacles of the heaven to enjoy, since the whole Earth is of far greater mass than all men taken together.

V. Argumentum à Facilitate motus, ex Magnitudine Mobilis orta.

5th Argument, from the Ease of motion, arising from the Size of the Movable body.

XVIII. This is an offshoot no more fortunate than the preceding argument, from which it sprouted. Granted, say the Copernicans, that it is not absurd for what is greater to be moved for the sake of what is smaller; Nature nevertheless usually does what is easier, or less laborious and of less expense. But by the very fact that the Earth is of the smallest mass compared with the eighth sphere, let alone with the whole heaven, it is also easier to impart motion to it; and if motion is not exercised without some impressed quality, it is certainly of less expense to impress the impetus of rotation on this little sphere than on the most vast sphere of the Firmament. Let this argument too now be constructed according to the rule of the syllogism. *God and nature do what is easier and of*

less expense. But it is easier and of less expense to move the globe of the Earth with the diurnal rotation than the Machine of the Heaven. Therefore, etc.

[Margin: Syllogistic form of the argument, and 1st Reply.] It is replied, first, by Conceding the Major if other things are equal, and the same end that God and Nature intend can be obtained equally well; but by denying it otherwise. For God's purpose was to show the greatness of His Wisdom and Omnipotence: on the one hand by balancing the Earth immobile upon nothing — that is, upon its own centre — and upon air so fluid; on the other by joining so many motions, though contrary in appearance, in a single motion of the heaven or of the Planets, and in this way providing for living creatures, and especially for man. Nor can it be demonstrated by natural reason that this was not His end; and this end could not be obtained by moving the Earth. Granted, however, that by joining several motions in the single motion of the Earth He would also have shown great wisdom; still, as long as it is not certain, or more probable, that this rather than that was His purpose, we cannot conclude with certainty, or more probably, that He had regard to greater ease or to less expense. And indeed, if we follow our own estimation, weighing nothing but number or mass, it would have been of less expense to conserve the same light, once it had been produced in the air and the aether, than to produce new light over again at every moment; and less laborious to set the Planets going with a single and most simple motion, at once comprehensible in round numbers; to light the night with fewer fixed stars, and to guide sailors and farmers by fewer stars; to build the heaven and the Earth with less mass; and to create fewer species of plants, crops and animals. But God does not always look to what seems easier to us, but to what is better according to the end He has prescribed to Himself and to lower Nature.

[Margin: 2nd Reply.] One could reply, secondly, by Denying the minor proposition. For mass does not make its own motion more difficult insofar as it is mass, or precisely on account of its greater size, but only by reason of weight or of some similar quality; and I add that not even this holds absolutely, but only comparatively, with respect to the motive force, as also to the medium through which the body has to be moved. But neither for God nor for the intelligences can the machine of the heaven, under any of these heads, be more difficult to move than the Earth.

VI. Argumentum ab Infinitate Mundi Cæliq. ſupremi, vbi obiter quædam de Mundi Infinitate.

6th Argument, from the Infinity of the World and of the supreme Heaven, where, in passing, some things on the Infinity of the World.

XIX. Before we turn our arms elsewhere, away from the arguments drawn from Magnitude, which have been conquered and led in triumph, or at least invincibly repelled, we must dispose of the argument drawn from the Infinitude of the World. [Margin: Copernicus's discourse on the Infinity of the heaven.] Copernicus already touched on it in Book 1, chapter 8, arguing thus: *But*

according to that axiom of Physics, That which is infinite cannot be traversed, nor moved in any way, the heaven will necessarily stand still. But they say that outside the heaven there is no body, no place, no void, and absolutely nothing, and that therefore there is nowhere for the heaven to go out to. In that case it is surely a wonder if something can be held in by nothing. But if the heaven is infinite, and finite only at its inner concavity, it will perhaps be verified all the more that there is nothing outside the heaven, since everything, whatever size it occupies, will be within it; but the heaven will remain immobile. For the chief thing by which they strive to establish that the world is finite is motion. To these words he at once subjoins the conclusion that we recited above, but which must be repeated here: *Whether, then, the world is finite or infinite, let us leave to the disputation of the natural philosophers [Physiologi], holding this as certain: that the Earth, enclosed by its poles, is bounded by a globular surface. Why, then, do we still hesitate to grant it mobility, congruent by nature with its form, rather than have the whole world slide round, whose limit is not known and cannot be known? And why do we not admit that the appearance of the daily revolution belongs to the heaven, and its truth to the Earth?* To this argument, moreover, in the part where it rests on the spherical figure of the Earth, a sufficient reply has been given, unless I am mistaken, at number 5. But in the part where it touches on the uncertainty of infinity in the heaven and the certainty of finiteness in the Earth, it must be blunted in this place — though only after we have first heard what the Physico-Mathematicians, and not merely the Physicists, teach about the infinity of the world or of the supreme heaven.

[Margin: The opinion of Democritus, Epicurus and Metrodorus.] XX. If we believe Plutarch, in Book 2 of *De placitis philosophorum*, chapter 1, Democritus, Epicurus and their disciple Metrodorus posited innumerable worlds in the infinite, ranging without measure through its whole extent. Seleucus posited an unbounded world; [Margin: Diogenes's.] Diogenes an infinite Universe, but a finite World, for he distinguished the world from the Universe. [Margin: The Stoics'.] So did the Stoics, who, as Plutarch adds in the same place, said that the Universe consists of the World and the void, and that this whole made up of both is infinite, but that the World is distinct from the void, and is one, and is bounded by a globular figure. [Margin: Aristotle's.] Aristotle, however, in Book 3 of the *Physics*, tries to overturn every actual infinite; in Book 7 of the *Physics*, text 78, he denies that a finite thing can move in an infinite time; and in Book 1 of *De caelo*, from text 33, he contends that the World is not infinite, as very many of the ancient philosophers, he says, believed. His chief argument is that it is impossible for an infinite body to be moved circularly, let alone in a finite time: otherwise there would be a proportion between a finite and an infinite line comprehended by similar arcs and under an equal angle, and an infinite space could be traversed in a finite time, namely 24 hours. Besides this there are other, more obscure arguments, which can be seen up to text 75. In that place, therefore, he supposes as certain what is now in question, namely that the heaven turns circularly. [Margin: Pythagoras's.] [Margin: Plato's.] But

before him, too, Plato and Pythagoras, as Laertius reports in their lives, asserted that the World is finite and of spherical figure. [Margin: Cleomedes's [?].] Cleomedes too, in Book 1 of the τῆς κυκλικῆς θεωρίας [Greek: tēs kyklikēs theōrias, "of the circular theory" — the title of his work], relying on Aristotle's reasons, establishes that the World is both globular and finite. [Margin: Pliny.] Pliny the Elder, however, in Book 2, chapters 1, 2 and 7, speaks of the World in such a way as to make it God and Nature, or to make God himself the soul and nature of this World; and therefore he does not clearly express whether it is finite or infinite. For in chapter 1 he says: *The World, and this which it has pleased us to call by another name the heaven, by whose encircling sweep all things are covered, it is right to believe to be a divinity, eternal, immense, neither begotten nor ever to perish. What lies outside it is no concern of men to investigate, nor can the conjecture of the human mind grasp it. It is sacred, eternal, immense, whole in the whole, nay rather itself the whole; finite, yet like the infinite; certain of all things, and like the uncertain.* And a little later, against those who had posited infinite and innumerable Worlds, he adds: *Or, if this infinity can be assigned to the nature of the maker of all things, would not that same thing be more easily understood in one, especially in so great a work?* And soon after: *It is madness, sheer madness, to go out of it and, as though everything within it were plainly known, to scrutinize what lies outside — as if, indeed, one who does not know his own measure could take the measure of anything, or the mind of man could see what the world itself does not contain.* For Pliny is prone, in order to hammer out some clever and sublime sentence, to speak impiously and audaciously even of God himself. Thus in this place he makes this God of his uncertain and semi-corporeal, ignorant of his own immensity and incapable of it — although of this God he says in chapter 7: *Whoever God is (if indeed he is another)* — that is, distinct from the world — *and in whatever part, he is wholly sense, wholly sight, wholly hearing, wholly soul [animae], wholly mind [animi], wholly himself.* Since, however, he attributes a spherical figure to the world and acknowledges that it turns in the space of 24 hours, he seems to acknowledge it as bounded and finite. [Margin: The excellence [?] of the spherical figure [?].] For he says in chapter 2: *That its form is rounded into the shape of a perfect orb is shown, first, by the name and the consensus of mortals, who call it an orb, and also by the evidence of things: not only because such a figure in all its parts converges upon itself, must sustain itself, and encloses and holds itself together, needing no joints and sensing no end or beginning in any of its parts; nor only because it is most apt for the motion with which, as will soon appear, it must continually turn; but also by the proof of the eyes, because it is seen as convex and central from whatever side one looks, whereas this could not happen in any other figure.* He would have done better to say "concave", for the convexity of the heaven cannot be seen by us. Finally, in chapter 3, he writes of its motion as follows: *That this form, then, is carried round in an eternal and unresting circuit, with unspeakable speed, in the space of twenty-four hours, the risings and settings of the Sun have left no doubt. Whether the sound of so great a mass, rotating in ceaseless*

whirling, is immense and therefore beyond the sense of the ears, I could not easily say. So he again calls the immensity and infinity of the World into doubt.

XXI. Let it be enough to have sampled thus far these few, but rather select, opinions of the Ancients concerning the infinity of the world. What the more recent Scholastics and Physicists have thought I have reported more fully in the Physical questions, and this is not the place to repeat that controversy with all its arguments for and against. It need only be recalled that there are four opinions on this matter. [Margin: The opinions of the more recent on the Infinity of the World.] Some deny that the World is actually infinite, either because they think that every infinite which is, as they call it, categorematic is ἀδύνατον [Greek: adynaton, "impossible"], or because there is no sufficient indication of this infinity, and entities are not to be multiplied or extended without necessity. Others concede that the World is Infinite, but deny that it is bounded by any figure, or mobile as a whole within itself, let alone outside itself. Others again admit that it is Infinite, and yet bounded by a spherical figure, but deny that it can be moved in a finite time; for with them it is not the same thing to be finite and to be bounded or figured. [Margin: The acceptation of the Infinite.] For the Infinite, they say, is predicated analogically, and its first analogate is that which has all the perfections signified by the name of infinite: namely, that it embraces within itself all things possible in its own genus, and lacks a limit on every side, taken both negatively and privatively — that is, so that nothing can be designated in it to which the negation of further extension immediately belongs, even if that further extension were said to be impossible, and therefore the negation of it, as not connoting an aptitude for the thing denied, would be a mere negation and not a privation. For from these perfections, they say, it follows that it is untraversable. But among the secondary analogates participating in infinity, they also admit that which, although it is bounded by some figure and does not comprehend within itself everything possible in that genus, is nevertheless so great as to be untraversable, and not measurable by any finite and determinate measure of the kind that can be taken among the smallest and replicated, so that however many measures of this kind are replicated, there always remains something further in it to be taken. Finally, some agree that the World can be called infinite and configured externally with a spherical surface, and nevertheless movable and capable of being turned round in the finite time of 24 hours, but by an infinite power producing an infinite impetus; and they hold that infinity of time in the motion does not necessarily follow from the infinity of the space traversed. Or at least, if the World above the planetary heaven is fluid, or not wholly and continuously solid, they hold that the part which contains the stars visible to us and the Planets, being finite, can be moved within 24 hours, while the remaining part stands immobile. Among those who admit the infinity of the World, this opinion seems less absurd and less difficult to understand.

XXII. It remains, therefore, as I promised a little earlier, for us to indicate what certain Astronomers or Physico-Mathematicians have thought about

the infinity of the World and of the heaven. And indeed, of Copernicus's ambiguous opinion we have learned enough from himself, and also that, so far as it lies with him, the infinity of the World, if it were admitted, or even an infinite distance of the Fixed stars, would not hinder his hypothesis of the annual translation of the Earth at all, but would rather greatly help it. [Margin: William Gilbert's opinion on the Infinity of the World.] Moreover, William Gilbert, in Book 6 of *De magnete*, chapter 3, calls the sphere of the Fixed stars Immobile, and thinks that if the primum mobile were in it or above it, it would be infinite; and so he adds: *But of an infinity and of an infinite body there can be no motion, and therefore no diurnal motion of that most vast primum Mobile.* For this reason, on account of this argument among others, he concludes that the diurnal revolution belongs to the Earth rather than to the supreme heaven. [Margin: Marsilius Ficinus's opinion.] On the contrary, Marsilius Ficinus, on Plotinus's book *De caelo*, chapter 3, teaches that the World is not infinite: either because it would lack order and figure; or because, if it were imagined to be moved circularly, there would be slowness towards the centre and infinite speed from the centre upwards — for the parts nearer the centre, designable finitely to infinity, would be slower and slower, but those from the centre upwards faster and faster; and so on the one side there would be no motion of it at all, on account of the infinite slowness, while on the other it would take place in an instant, on account of the infinite speed; or else infinite slowness and speed would combine into a state, that is, rest. Or else because, since there would be an immense space between any lines drawn out from the centre, the effort of the mover would be in vain, and consequently there would be no motion. [Margin: Descartes's view.] Descartes, however, in the *Principles of Philosophy*, Part 1, from number 25 to 27, declares that we ought not to weary ourselves with disputations about the infinite, since, as we are finite, it seems absurd to him that we should try to determine anything about it, and so, as it were, to limit and comprehend it. All those things, however, to which no definite limit can be assigned beyond which God could not make more, or greater, or better things, he calls not Infinite but Indefinite; and he adds: *And these we shall call indefinite rather than infinite: both so that we may reserve the name of infinite for God alone, because in him alone, in every respect, we not only acknowledge no limits, but also understand positively that there are none; and also because we do not in the same way understand positively that other things lack limits in some respect, but only negatively confess that their limits, if they have any, cannot be found by us.* [Margin: Giordano Bruno's opinion. And Galileo's.] On the contrary, Giordano Bruno of Nola, in the book *De maximo et immenso* and in the book *De infinito et innumerabilibus*, does not hesitate to assert that this World is infinite upwards. Galileo, however, in Dialogue 3 on the system of the World, page 239 of the Latin, calls into doubt whether the World is finite, figured and has a centre, or is rather infinite, unbounded, and lacking a centre and circumference; and he says that Aristotle's reasons, which ultimately rest on the mobility of the diurnal motion, are as uncertain as the motion that Aristotle

assumed and attributed to the heaven rather than to the earth. Granted, he concedes for the time being to Simplicio — with whom he brings in Salviati disputing — for the sake of argument, that the World is finite and spherical and has a centre, so that, even with this liberally allowed, he may show that it does not follow that the Earth is at the centre of the universe, or that heavy bodies tending to the centre of the earth tend to the centre of the Universe. [Margin: Kepler's.] Among the Copernicans, however, Kepler — both in the book *De stella nova*, on the new star seen in Serpentarius, chapter 17, and in the *Epitome of Copernican Astronomy*, Book 1, page 39 — denies that any of the visible stars is distant from us by an actually infinite interval. For, he says, to be infinite and to be bounded are mutually repugnant; and likewise to be infinite and to have a finite proportion to another infinite. Both of these inconveniences would follow if some visible star were distant from us by an infinite interval; for the apparent diameter of the star would be increased by an infinite increment, as would the distance of the lines under whose angle it would be seen; and yet, so increased, it would still be enclosed within the termini, or points, bounding the diameter, and under a definite visual angle. Moreover, in the same place he denies that, even if some stars were not visible because of their huge distance, the space upwards is infinite, because their number would be finite, and from things finite in magnitude and number an infinite cannot be compounded. Some things in this discourse would need the file, if this were the place to range at large over this controversy. On it I wished only to indicate the bare opinions, so that it might be clear that, although among the Copernicans and some others there has been some suspicion of the infinity of the World, yet it has not been among all of them, let alone among everyone; and by what arguments, chiefly, its infinity is usually attacked. To these add the conjecture about finiteness taken from the Refractions of the Fixed stars, which I set out in Book 6, chapter 7, in the last Scholium. Now to the argument contained in Copernicus's words, which we form as follows.

[Margin: The 6th Argument reduced to form.] XXIII. *If the motion of diurnal revolution must be attributed to one or the other of the bodies — namely either the supreme heaven or the Earth — it must be attributed rather to the one that is certainly established to be movable than to the one that is not movable, or of which it is not certainly established whether it is movable or not. But it is certainly established that the Earth is movable, whereas the supreme heaven either is not movable, or it is not certainly established whether it is movable. Therefore, etc.* [Margin: Proof of the Major.] The MAJOR premise seems most certain. For although what we are speaking of could not have been uncertain to God, who imparted this motion to one or the other, yet for us, to whom it is not certain, prudence dictates — so that we do not expose ourselves to the danger of erring — that we ascribe the motion to the body to which we are certain it is not repugnant, rather than to the one whose repugnance to it we may legitimately suspect. [Margin: Proof of the Minor.] The MINOR premise, too, is already certain as regards the Earth, from its figure and its finite measure, found by Geometrical method, according

to what was said in Book 2, chapter 7. As regards the supreme heaven, however, it has not yet been demonstrated by anyone that it is not infinite; and if it is infinite, it is not certain that it is movable; or, if this has been demonstrated, it has been demonstrated by no other argument than from the diurnal motion, which is here called into controversy.

[Margin: Reply to the 6th Argument.] One can reply by Denying the Minor, if by the name of the supreme heaven is understood either the Firmament, or at least the aggregate of the visible Fixed stars. For it is physically evident to us that they are moved with the diurnal revolution, and therefore we are certain of their mobility from the motion itself, since Copernicus himself and the Copernicans admit that the appearance of this motion is in the heaven, if we will abide by sense — just as it is evident, in dependence on sensible experiments, that the Earth is of finite measure. Granted that for the evidence of the former a simple experiment suffices — by which we see the Fixed stars rise above the Horizon and roll down to the West, and after 24 hours return to the East, and traverse in a finite time arcs similar and proportional to arcs of the terrestrial circumference — whereas to measure the Earth and determine its figure more is needed. But if the Copernicans say that only one of these two is evident to us disjunctively, namely that either the Fixed stars or the Earth move, I say that this disjunctive is evident Mathematically, or even Metaphysically, supposing (as indeed our opponents suppose) that the local variation appearing between the stars and us takes place through local motion, and not through illusions of the eyes, or the reproduction of bodies, etc. But it does not follow from this that the other part is not physically evident, and the burden of proof lies on the one who denies it. I say nothing, meanwhile, of Sacred authority, for at present we are examining bare reasons. What happens, however, with the part of the heaven above the Fixed stars — whether it is moved and is finite, or is infinite and immobile — has nothing to do with this argument.

It is replied, secondly, by conceding the Major if other things are equal, and denying it if they are unequal, as in fact they are. For many other considerations are given that outweigh this comparison of certainty regarding the finite circumference of the Fixed stars and of the Earth, on account of which the diurnal motion is to be attributed rather to the Earth.

VII. Argumentum à ratione Loci & Continentis deductum.

7th Argument, drawn from the nature of Place and of the Container.

XXIV. Copernicus has not yet ceased to press the Earth's rest towards motion. For he says in Book 1, chapter 5: *Since the heaven is what contains and covers all things, the common place of all things, it is not at once apparent why motion should not be attributed rather to the contained than to the container, to the located rather than to the locating.* And again in chapter 8: *I add also that it would seem quite absurd for motion to be ascribed to the container or locating thing, and not rather to the contained and located, which is the earth.* This reasoning seems to have greater force against the Peripatetics with their Aristotle, who in Book 4 of the *Physics*, chapter 5,

text 42, define place as *the first immobile boundary of the containing body*, and who concede to the heaven the character [rationem] of universal place. [Margin: Form of the 7th Argument.] One may therefore argue thus: *Local motion is to be ascribed to the located rather than to the place, since place requires immobility; but the Heaven is the place with respect to the Earth, and the Earth the located. Therefore motion is to be ascribed to the Earth rather than to the heaven.*

[Margin: 1st Reply.] The Peripatetics would reply, from the same text 42 of the fourth book of the *Physics*, on the doctrine handed down by Aristotle from text 35, by distinguishing the major premise. They would concede it if it is understood either of a motion by which the whole place is carried, together with the located, to another place — for in that way the place would lose the character of place and take on the character of a vessel — or at least of a motion by which, in carrying off the located, the place ceases to surround it; and if it is understood of the immediate place. They would deny it if the proposition is understood of any local motion whatever, or of the remote place. The supreme Heaven, however, is not carried as a whole to another place, but remains in the same place as regards its whole being; or, if it is not strictly in a place, because it has no other body outside it by which it is contained, it is not carried to another place either. Much less does it leave the Earth outside itself, as the parts of a river leave the fish behind, even when they are at rest; rather it behaves like air stirred about a Tower, which does not on that account cease to be the place of the Tower. Nor is the supreme heaven the immediate or first surface containing the earth.

[Margin: 2nd Reply.] Someone might, secondly, reply by turning the argument back against the motion of the earth asserted by Copernicus. For the Earth too, together with the neighbouring air, is the place of plants and animals; therefore either it ought not to be moved, or, if it suffices for its mobility that it does not leave behind the things located in it, this suffices much more for the heaven, which neither leaves behind the things located in it nor carries them elsewhere with it like a vessel — as happens in the hypothesis of Copernicus, who adds an annual motion of translation of the earth as well. So for him the Earth turns out to be a vessel rather than a place.

VIII. Argumentum ab Incorruptibilitate Cæli deſumptum.

8th Argument, taken from the Incorruptibility of the Heaven.

XXV. Copernicus uses this argument in one way, Kepler, following Origanus, in another. [Margin: Copernicus's Argument.] The former argues thus in Book 1 of the *Revolutions*, chapter 8: *To these is added also that the condition of immobility is thought nobler and more divine than that of change and instability, which for this reason befits the earth rather than the world.* [Margin: Kepler's argument from Origanus.] But Kepler, in Book 1 of the *Epitome of Copernican Astronomy*, page 128, says that motion is of the genus of things that are not, but come to be; and that it therefore does not belong to those things that hold the degree of their perfection perpetually in themselves, but to those which, unless they

are moved, grow torpid with sloth and are corrupted — such as, in the Earths themselves, Water and Air, and many bodies living in these elements.

[Margin: Reply to Copernicus.] To Copernicus, however, the reply is that the heaven is not intrinsically incorruptible, as we said in Section 1, chapter 6, number 6; indeed Kepler admits that it is in fact capable of corruption. Next, if it is contended that it is incorruptible, then only that motion is repugnant or unbefitting to it which tends to corruption, and the diurnal revolution is certainly not of that kind: otherwise it ought long ago to have corrupted the whole Earth, to which he ascribes it. Further, this [?] motion is not incongruous even to the Intelligences themselves, let alone to the heaven or the stars. Finally, by this argument all motion would have had to be banished from the heavens; and so, since it proves too much, it proves nothing. [Margin: Reply to Kepler.] Kepler, for his part, will be abundantly answered if the Air is blown through by winds from time to time, and the Waters are preserved from putrefaction by the course of rivers and the ebb and flow of the Seas, besides the remedy of their saltiness; and since he does not fear corruption for the Earth, there is no reason for him to attribute motion to it on that account. If he grants that motion can belong to it on other grounds, let him grant also that it can belong to the heaven, even if the heaven does not need it for its own preservation. And finally, let him acknowledge, from this or from the motion of the Planets, that local motion was not instituted primarily of itself, or universally, to preserve from corruption by agitation and ventilation.

IX. Argumentum à Fluiditate Cæli, & Conſtanti Diſtantia Fixarum inter ſe.

9th Argument, from the Fluidity of the Heaven and the Constant Distance of the Fixed stars from one another.

XXVI. The Copernicans would perhaps[38] seem to have deserved very well of the heaven, being so solicitous for its integrity and health — especially those who, fearing for its fluid substance, or at any rate certainly feigning fear, thought that all motion was to be driven away from the bounds of the heaven, at least of the supreme heaven, lest either its substance or the stars themselves be scattered and flow apart through the great speed of the whirling. [Margin: Longomontanus's argument, supposing a fluid heaven.] Thus Longomontanus, who asserts not indeed the annual but nevertheless the diurnal motion of the Earth, confesses in Book 1 of the *Theorica*, towards the end of chapter 1, that he was confirmed by this argument too in transferring this motion to the Earth, when he says: *I add also that, since the fixed stars — which it would be reasonable to hold are removed at no equal distance from the centre or the earth, and hang in the most limpid heaven, and, like the globe of the earth, are balanced on their own centres with equal weights — have suffered no separation in their distances from one another since the founding of the world: who then, with such assertions, will attribute real motions to them, unless it be one who would stubbornly defend Sacred Scripture in a*

[38] printed *fotſitan*

few places, and in places which (as they seem) are accommodated to human capacity only in appearance? And in the same place, chapter 4, he repeats that among other things he was led to bring back both the diurnal motion and the motion of the precession of the Equinoxes from the heaven to the earth, *lest the stars fixed in the most limpid heaven be pulled apart from one another, if you presumed that each one was set going separately by some motion of conveyance: although perhaps this too might in some way be conceded in a celestial expansive stereoma* — that is, even if someone were to admit that solid heaven; for στερεάμα[39] [Greek: stereōma, "firmament"] signifies a solid structure [solidamentum], or a solid body.

[Margin: Gilbert's argumentation, supposing a fluid heaven.] XXVII. William Gilbert argues in a different way, from the liquid thinness of the Firmament, in Book 6 of *De magnete*, chapter 3. For he philosophizes thus: *Moreover, what craftsman ever discovered that the stars which we call fixed are in one and the same sphere, or established by reason that there are any real and, as it were, adamantine spheres? No one has ever demonstrated this very thing. Nor is there any doubt that, just as the planets stand off from the earth at unequal intervals, so those huge and most numerous lights are separated from the earth by various and most remote heights, and do not adhere to some spherical framework or firmament (as they imagine) and vaulted body. Thus the intervals of some are conceived by a kind of opinion rather than in reality, on account of their inscrutable distance; others far surpass them, and are by far the most remote; and since these are placed in the heaven at various distances, either in the thinnest aether, or in that most subtle fifth substance, or in a void, how will they remain in so great a whirling of a vast orb, a body most uncertain?* Therefore, since he holds that the non-wandering stars are in a fluid aether, and indeed separated from one another upwards and downwards at distances various beyond measure, he thinks it is to be feared that, if they were turned round with the motion of so vast a heaven, they would toss this way and that with a turbulent and many-wandering agitation, like planks in a shipwreck. And further on he again lays it down thus: *Not, therefore, are those confused crowds of stars carried or moved by the firmament, nor do they have their position from it; much less from the primum mobile; nor are they torn apart and disturbed by a contrary and most rapid impulse. Ptolemy of Alexandria seems to me too timid and faint-hearted, who shudders at the dissolution of this lower world if the earth were moved circularly. Why does he not fear the ruin, dissolution, disturbance and conflagration of the universe, and celestial and supercelestial calamities, from a motion beyond all thoughts, dreams, fables and poetic licences, insuperable, ineffable and incomprehensible? Therefore we are carried by the diurnal rotation of the earth — a motion, that is, more fitting; and as a skiff is moved upon the waters, so we, with the ear-* th are turned round, and yet we seem to ourselves to stand still and be at rest.* He also presses the argument from incredible speed, about which much more will have to be said separately in the following chapter.

[39]so printed, for στερέωμα

[Margin: Galileo's reasoning, supposing a fluid Heaven.] XXVIII. And this, to be sure, is the sixth confirmation of the diurnal rolling of the Earth, which Galileo counts in Dialogue 2 on the system of the world, page 85 of the Latin, when he says: *There is added an absurdity (and this is the sixth inconvenience) for those who discuss the matter soundly: that no thought can comprehend what the solidity of that most vast sphere must be, in whose depth so many stars are so tenaciously fixed that, without any variation of position among themselves, they are swept round in concord with so great a disparity of motions. Or if indeed the heaven is fluid, as it is more reasonable to believe, so that each star wanders through it by itself, by what law are their motions governed, and to what end does it come about that, seen from the earth, they appear as if made by one single sphere? It seems to me that, for obtaining this, it is a course as much easier and more convenient to make them immobile, not wandering, as it is readier to keep at one's beck a courtyard paved with stones than the crowd of children running about in it.*

[Margin: Gassendi's opinion on this.] Gassendi, however, although he does not state clearly enough whether he supposes the sphere of the Fixed stars solid or fluid, and makes no mention of a Crystalline heaven above it and below the primum mobile except as the opinion of others, nevertheless suggests, in Letter 2, *De motu impresso* etc., page 111, that a scattering of the stars by the diurnal rapidity is to be feared. *But how much*, he says, *do you think the absurdity that confronts us will be increased, once a point has been taken in the circuit of the primum mobile, especially with one or another crystalline heaven interposed? And let them object besides that the parts of the earth would be torn apart by that so great whirling: how much more will a tearing apart be to be feared for the parts of the primum mobile, snatched away as they are by an incomparably more rapid whirling?* But before we take this panic terror from their minds, let us bring the whole force of this argumentation to the rule of the syllogism, which in the end is as follows.

[Margin: Argument 9 in Form.] XXIX. *If the non-wandering Stars were moved with the motion of the Firmament, by the diurnal whirling, they would not keep perpetually the same distance from one another. But the non-wandering stars do keep perpetually the same distance from one another. Therefore the non-wandering stars are not moved by the diurnal whirling with the motion of the Firmament; it remains, therefore, that the Earth itself rather is moved with such a motion.* The MINOR premise is certain from what was said in Book 6, chapter 10. The MAJOR is proved, on the one hand, from the incredible speed of that whirling; on the other, from the fluidity of the Firmament, which is unfit to hold any fixed stars whatever always at the same distance from all the other stars; or else from its solidity, unless that solidity were supposed to be so great that human belief and estimation cannot reach it.

[Margin: Reply.] The reply is to deny the Major of the proposed syllogism, and its proof. For if the Firmament is supposed fluid, it is neither necessary nor congruous with the more probable reasoning that the non-wandering stars be moved with the motion of the Firmament; rather, they would more probably be supposed to be moved by Intelligences, either singly, or enclosed together

by some other bond, as it were a net, as we said in Section 1, chapter 7. For the chief reason for asserting a solid Firmament, after the authority of many Fathers and Doctors, is that very perpetuity of the equal distance of the Fixed stars, kept by those same stars and observed since the most ancient times of Astronomy. But if the Firmament is so solid that it neither rubs against another body apt to concur in kindling through the extrusion of hotter spirits or of latent little fires, nor strikes against another body, nor is forced by a sudden shock to stop or recoil, but is carried round with an even tenor of motion always within the same space, then it is feared in vain that it will break or burst apart, and so, its framework dissolved, disturb the intervals of the Fixed stars. Nor can Gassendi, of all people, reject this reply, since he replies in a similar way about the Earth in the same place, saying: *Although there is no danger for the parts of the Earth themselves, which, because they all cohere among themselves and are always carried with a natural and even motion, behave just as if they were at rest; and the only accident to be feared would be if the earth struck against a resisting body, or otherwise came to a sudden halt.* Could not GOD infuse into the Firmament as great a coherence and solidity as he put into the Earth, and much greater — even if that solidity had to be reckoned so great as to display an admirable work of God?

XXX. Therefore, if the Copernicans aim at this — to take from the divine works their wonderfulness, or to take from human minds either wonder at it or belief in it — they do not act consistently, since they press upon us the divine Omnipotence and Majesty of the Creator from the huge vastness of the interval between Saturn and the Fixed stars and from the immensity of that supreme heaven, as we shall see later; nor do they agree with the sayings of very many Wise men, and above all with those of the divine letters. [Margin: The glory of God shines out most of all in the Firmament and its motion.] For what else do those most celebrated verses of Psalm 18 proclaim, *The heavens tell the glory of God, and the Firmament announces the works of his hands*? How does it announce them, except chiefly by the harmony of the diurnal revolution, by which the alternating turns of days and nights, repeated by so constant a law ever since the founding of the world, rouse mortals with a kind of secret sound to the knowledge of God and the recognition of his power? For there follows: *Day pours out the word to day, and night shows knowledge to night*, and *Their sound has gone out into all the earth, and their words to the ends of the world.* What else, likewise, do those words of Ecclesiasticus, chapter 43, suggest to us: *A vessel of the camps on high, shining gloriously in the firmament of heaven*? Or what else does the Prophet Baruch signify in chapter 3, when, from the light of the stars rising in succession upon the world, he commends GOD with sublimity of speech, saying: *He who sends forth the light, and it goes; and he called it, and it obeys him with trembling. And the Stars gave light in their watches, and rejoiced; they were called and said, Here we are; and they shone with gladness for him who made them. This is our GOD, and no other shall be accounted against him.* What else, I say, do these passages intimate to us, but that it was and is as easy for our GOD

to direct the host of stars in order through the Intelligences at his nod, without any confusion of the place once assigned to them as their post, as it is for a most experienced Field-commander, or a most skilled Commander of an army, to move his forces in column, travelling however great and rapid marches, and to govern them through his centurions and tribunes, as if the camp, enclosed by its very rampart as by a kind of vessel, were being carried along? Finally, let us hear GOD himself questioning, in Job 38: *Will you be able to join the twinkling stars, the Pleiades, or can you scatter the circuit of Arcturus?* As if he said: Can you, or any manikin like you, bind together, in so tiny a space of the heaven as that which the asterism of the Pleiades occupies, so great a multitude of stars, with so perpetual and constant an order and an unchangeable constancy of intervals? (For the Telescope has revealed not seven only, but far more stars in that tract.) Or can it come about by your effort or anyone's that Arcturus, although it repeats its circuit every day in so ample a round and with such speed, should be scattered and otherwise thrust out, and forced to run off wandering along one path after another? By no means — truly, by no means.

X. Argumentum ab Indigentia Telluris & Independentia Stellarum à Terra, quæ non eſt finis, ad quem ordinata ſint ſidera.

10th Argument, from the Need of the Earth and the Independence of the Stars from the Earth, which is not the end to which the stars are ordered.

XXXI. This is the last of the arguments — of those, at least, that belong to this chapter. *It is ridiculous*, says Gilbert in Book 6 of *De magnete*, chapter 3, *that the Heaven should be moved for our sake, and for the sake of the earth, so small a globe.* For it seems unworthy of the majesty of the Heavens that they be moved with the diurnal motion for the Earth's benefit, when they themselves do not need that motion, while the Earth itself remains unmoved, which alone, or most of all, needs the good that follows from that motion. For whatever is moved locally is moved to this end: either to acquire some good which it never had — and thus animals are moved, which for this reason are called living things by the Philosopher, because they acquire for themselves something besides what they have from birth, and perfect themselves chiefly through motion, which plants accomplish without motion — or certainly it is moved to recover a lost good, or to restore itself to the place in which it can best be preserved; and in this way the bodies of the elements and of inanimate mixed bodies are moved. Lansbergen presses this argument in the *Progymnasmata*, adding that the earth is moved with less expenditure [?] than the stars, just as it is easier for a mother to bring her children up to the fire to warm them than to carry round the fire that warms them. Kepler too presses it, in Book 4 of the *Epitome of Copernican Astronomy*, page 500, saying: *The other argument destroys all motion whatever of the sphere of the Fixed stars* — indeed, I say, *of all the Planets. For it is not apparent to what good, since there is nothing outside from which or towards which that translation might vary both position and appearances; and it obtains by rest whatever it could acquire by any motion whatever. For it is from its rest that the motions of*

all bodies are understood, and unless it provided place — which it does most rightly by resting — nothing could be moved. The same author had said, moreover, in the *Dissertatio cum Nuncio sidereo*: *Let man therefore acknowledge that he is not the source and origin of the world's adornment, but depends on the true source and origin. Add this too, which I said in the Optics: that for the sake of contemplation — for which man was made, and adorned and equipped with eyes — man could not rest at the centre, but must travel round with an annual motion in this ship of the Earth, for the sake of surveying; just as surveyors of inaccessible things exchange one station for another, so as to obtain from the intervals between the stations a proper base for the surveying triangle.* For what he says here of the annual motion applies also, if it applies at all, to the diurnal. Let the argument now be gathered into a summary.

[Margin: Form of the 10th Argument.] XXXII. *Motion is to be attributed to that which needs that motion for its own good, and not to that which does not need it. But it is the Earth that needs the diurnal motion — indeed the annual too — for its own good, not the heaven. Therefore the diurnal motion, and indeed the annual too, is to be attributed to the Earth, and not to the heaven.*

The reply is to deny the Major if what is to be moved either is moved not from within but from outside, by something ordering it to the good of another; or if the movable, whether moved from within or from outside, is by its nature ordered to perfecting other things by means of that motion, so that its own good is to be perfective of others, while it meanwhile suffers no detriment itself — such are the heavens and the stars; and to concede it if none of these considerations applies [militet] in such a movable. As to what is added — that we are not the end to which the stars and the heavens and their motions are ordered — we deny that proposition. For we have already shown the opposite clearly enough in Section 2, chapter 4, number 10, as consonant with Sacred Scripture, let alone with the Fathers.

CAPVT VI.

Chapter 6.

Proponuntur & Diſſoluuntur Septem Argumenta ex conditionibus ipſius Motus Diurni, pro Telluris potiùs quàm Cæli Diurno motu.

Seven Arguments drawn from the conditions of the Diurnal Motion itself, for the Diurnal motion of the Earth rather than of the Heaven, are Proposed and Resolved.

I. The FIRST and chief argument of the Copernican sect — and, as the proverb says, its Achilles — is the one derived from the incredible speed of the Fixed stars, if they rather than the Earth were moved. To present this speed as incredible, various authors propose it according to the various distances of the Fixed stars from the Earth. [Margin: Mästlin's calculation for the speed of the Fixed stars.] For Mästlin, in his preface to Rheticus's *Narratio prima*,

first assumes from Reinhold, who computes the Ptolemaic dimensions of the orbs, a distance of the Fixed stars of 25,000 semidiameters of the earth; from this, he says, it follows that a fixed star placed on the Equator of the heaven is swept, in a single second of an hour — a little space of time in which you could scarcely utter three or four words in headlong speech — beyond fifteen hundred German miles. Next, assuming from Alphraganus [al-Farghānī] a distance of the Fixed stars from the earth of 20,110 semidiameters of the earth, and so a diameter of 40,220 terrestrial semidiameters, to each of which he assigns 860 German miles, he obtains a diameter of the sphere of the Fixed stars of 34,589,200 German miles; and, by the proportion of diameter to periphery, namely 7 to 22, he gathers the circumference of the sphere of the Fixed stars as 108,708,914 German miles. Dividing these by 24 hours, it follows that any one of the Fixed stars on the Equator traverses in each hour 4,529,538 — that is, more than forty-five hundred thousand German miles; in each minute of an hour 75,492, that is, more than seventy-five thousand; and in each second 1258, that is, more than twelve hundred German miles, of which 15 make one degree of the greatest circle on the earth. Or, if we believe Cardano, Book 5 of *De proportionibus*, propositions 58 and 218, and experiments often confirmed, says Mästlin — Cardano, who says that the artery of a man of temperate nature beats about four thousand times in one hour — it follows, he says, that in one beat of the human artery the said star traverses 1132, that is, more than eleven hundred German miles, and the primum mobile still more, if it is placed higher than the eighth sphere; *which surely exceeds all belief*, Mästlin concludes. Finally, in the Additions, or notes, to the same *Narratio prima* of Rheticus, page 119 (it should be 115, but the printers erred), he assumes a distance of the Fixed stars not of 14,000, Tycho's, but one still a little smaller, namely 13,500 terrestrial semidiameters, and so a diameter of the sphere of the Fixed stars of 27,000 semidiameters of the earth, that is, 23,220,000 German miles; from this he gathers the periphery of the celestial Equinoctial as 72,977,143. Dividing these by 24, he computes 3,040,714 German miles for each hour; and when these are divided again by 4000 beats of the human artery, there result for each beat 760 German miles, to be accomplished by any one star placed on the Equator of the eighth sphere. *But this is very absurd to believe*, Mästlin concludes there; [Margin: Mästlin's headlong verdict.] indeed, a little later he does not hesitate to say: *Whether this is to be believed of a natural celestial body, and not to be accepted as absurd and simply impossible, anyone will judge, if only he enjoys some use of reason. These and all absurdities of this kind are banished far from the hypotheses of Copernicus.* So if we listen to this man, either we must be mad and devoid of the use of reason, or we must certainly judge that it was impossible for God, as author of nature, to grant naturally to the Fixed stars so great a speed that they traverse 760 German miles in each beat of the human artery.

[Margin: Kepler's calculation for the speed of the Fixed stars, or of the Earth.] II. Let Mästlin be followed by his pupil Kepler. First, in the *Epitome of Copernican Astronomy*, Book 1, page 107, he affirms that it is more likely that

nature expresses a proportioned and small motion in the small Earth than, in the most vast heaven, a motion incredibly swift, and one that would surpass the vastness by its speed in an immense ratio. For motion, no less than magnitude, is a Geometrical thing: *To great things, therefore, slowness corresponds, and to small things speed, not the reverse, as we learn in the Harmonics*, he says. And he goes on: if the machine of the World were turned round the Earth, it would accomplish in one minute of an hour about fifty thousand German miles, supposing Tycho's distance of the Fixed stars. But if the Earth turns, those parts of it that rotate in the greatest circle of all accomplish in one minute of an hour only 3 German miles and three-quarters: a space that is neither the thirteen-thousandth nor the twenty-thousandth part of the space to be traversed by the Fixed stars. Again, in the same *Epitome*, Book 4, page 500, he tries to establish the diurnal motion of the Earth from the same speed, saying: *For if the outermost sphere of the Fixed stars has at least 4,000,000 diameters of the Sun in its diameter, its circumference will be 12,566,370 long. If the whole of it is turned within 24 hours, then in one hour 523,600 will be turned; in one minute 8727; in one second, which about equals a man's pulse, 145 diameters of the Sun will pass by, of which each is not less than 13 thousand German miles. So in the space of time in which the artery is once dilated and contracted again, a doubled pulse, about seventy-five hundred thousand miles of the greatest circle would be turned; and Saturn, in an orbit two thousand times narrower, will still cross almost 4000 miles.* But here he uses the distance of the Fixed stars that is owed to the Copernican hypothesis, with the annual motion of the Earth supposed — which has no force against the hypothesis of the standing Earth, in which the distance and the speed of the Fixed stars are much smaller. Moreover, the same Kepler had reasoned thus in the book *De stella nova*, on the new star seen in Serpentarius, chapter 16: if the Earth is moved with a diurnal motion, the parts placed on its Equator will accomplish 240 German miles in one hour; but since its centre accomplishes 740 German miles in each hour by the annual motion, if these are added to those, they make 980, or nearly a thousand German miles, which some city placed under the Equator would have to complete in one hour at midnight; and, a few things that are not to our purpose being interposed, he continues: *Nor indeed should this passage through a thousand miles in one hour still seem incredible to you. I bid you consider the proportion of the density of the air to the density of the aether, which I demonstrated in the Optics; from it, it follows that the passage of a thousand miles in one hour through the aether is calmer by many parts — an incredible number of them — than is the passage of one mile in one hour through our air.* Having laid this down, he passes to the Ptolemaic hypothesis and affirms that everything in it is found more incredible. For he says: *In that hypothesis the semidiameter of the sphere of the fixed stars holds twenty thousand semidiameters of the earth; the circumference therefore will be of sixty-three thousand.* [Margin: Kepler's slip.] This slip is manifest, but pardonable: he ought to have taken the diameter as 40,000 semidiameters of the earth, and, as 7 is to 22, so to make it to a circumference of about 126,000. From his own calculation, however, he gathers that in one hour a Fixed star must pass over

2625 terrestrial semidiameters, each of which contains 860 German miles. After this he repeats his own saying: *It is more credible that a subject be large without motion than that there be a large motion in a small subject*; and further below: *It is more difficult to intensify an accident beyond the measure of its subject than to have a subject without an accident* (and speed is an accident): *Copernicus therefore acts more plausibly, who enlarges the orb of the fixed stars without motion, than Ptolemy, who enlarges the motion of the fixed stars with an immense speed.* Throughout that whole discussion, however, he uses the proper motions of the Planets towards the East, showing that they are proportionally slower the farther they stand from the Earth; whereas, if the diurnal motion were attributed to the Fixed stars, the passage would run from Saturn, the slowest, to the Fixed stars, the swiftest. In this there is a foul — not to say a crafty — equivocation: for the Fixed stars and the Planets ought to be compared in the same genus of motion, not in a different one, as we shall teach below.

[Margin: Galileo's defence of Kepler.] III. Galileo, however, siding with Kepler in Dialogue 2 on the system of the World, page 198 of the Latin version, brings forward, intending to resolve it, an objection [instantiam] of Chiaramonti against Kepler, which runs as follows. Kepler is mistaken when he says that in the sphere of the Fixed stars the accident — that is, the speed — is increased beyond the measure of the subject; for in whatever proportion the circumference of the sphere is increased, in the same proportion the motion, or the space to be traversed by the moving body, is increased too. Galileo tries to resolve this by saying that Kepler was not so dull of wit as not to see it, but that by "the measure of the subject" he understood the norm and example of the other celestial bodies, which are slower the farther they are removed from the centre, as appears in the Planets and in the Medicean stars. But this resolution is frivolous, since the comparison is not made within the same species of motions, as will be said in the resolution of the second argument.

[Margin: Lansbergen's calculation for the speed of the Earth and of the Fixed stars.] IV. Longomontanus too, in Book 1 of the *Theorics*, chapter 1, calls the speed of the Fixed stars incredible. Lansbergen, for his part, in the 6th argument for the diurnal revolution of the Earth, says that any terrestrial point on the Equator accomplishes 225 German miles in one hour, and in one minute (that is, a second) the sixteenth part of a mile — which bears a fitting proportion to the motions we experience in nature, since a cannon ball traverses the sixteenth part of a German mile in one minute. But if the eighth sphere were moved, it would have to run through 643,848 miles in one minute [?]; and Saturn, if it were turned by the diurnal revolution, would complete 930 miles in one minute — which he thinks utterly absurd. [Margin: The speed of a cannon ball.] Lansbergen admits, however, on the strength of the Landgrave of Hesse's experiment, that a cannon ball runs through half a German mile in one minute of an hour before it strikes the ground; for Kepler affirms, at page 108 of the *Epitome*, that the Landgrave and Tycho tested this.

[Margin: Mulerius's calculation for the speed of the Fixed stars.] Nicolaus Mulerius, by contrast, at page 322 of the *Frisian Tables* [tabularum Friſicarum], supposing Tycho's distance of 14,000 semidiameters of the earth, or 12,040,000 miles, says that the great circle in the Firmament extends for 88,000 terrestrial semidiameters, or 75,680,000 miles, and that the hourly motion of a Fixed star on the Equator is therefore 3,153,333 miles. Yet from this speed he rightly rises to praise the power of God, exclaiming with the Psalmist, *The heavens tell the glory of God, etc.*, and His goodness, which for our sake moves the Firmament with so great a speed, repeating that verse of Psalm 8, *What is man, that you are mindful of him?* So far is he from gathering any absurdity from it.

[Margin: Gassendi's calculation for the speed of the Earth and of the Fixed stars.] V. Lastly, Gassendi, in Letter 2, *De motu impresso a motore translato*, page 111, rightly points out that there is no reason why it should be objected to the Copernicans as an absurdity that we, in some part of the earth — namely under the Equator — would be moved with so great a speed that a cannon ball would scarcely attain it. But he adds that the Copernicans can throw back at their opponents an incomparably more absurd speed of the Fixed stars. For a point of the Earth under the Equator, where the motion is swiftest, would run through about two hundred fathoms [Orgyas], or toises, in one second of an hour, that is, in the interval from one beat of the human artery to the next. *But since the circumference of the Firmament*, says Gassendi, *is, by the common opinion, more than fifty thousand times greater than the circumference of the Earth, a point on the circumference of the Firmament must in one second of an hour run through fifty thousand times as many fathoms.*

[Margin: Our calculation for the speed of the Fixed stars.] VI. So much for the measures of the Copernicans, if you except Mulerius. For these, however, the Reader may consult our Table of the speed of the Fixed stars, which we gave in Book 6, chapter 7, number 18. There we gathered that speed from other men's distances of the Fixed stars from the earth, and a much greater one from our own hypothesis. For in one second of an hour, or the interval of one pulse of the human artery, any Fixed star on the celestial Equator traverses, on the Ptolemaic reckoning, at most three semidiameters of the Earth; but on our hypothesis 152⅔ terrestrial semidiameters, that is, 629,128 Bolognese Italian miles, which are 157,282 German miles, according to what was said in Book 2, chapter 7, about our measures of the Terrestrial globe. So far, then, are we from wishing that speed diminished that, through our hypotheses, we rather make it — and admit it to be — about seventy times greater than it is in Tycho's hypothesis, and fifty times greater than in the Ptolemaic. And yet, this notwithstanding, we shall shortly show that the Achilles of the Copernicans has feet of clay, and weak ones. Let the argument first be put in form.

[Margin: Syllogistic form of the argument.] VII. *The diurnal motion is to be attributed rather to that body whose speed is more credible and more proportioned to the movable subject. But if the diurnal motion is attributed to the Earth, its speed is more credible, and more proportioned to the terrestrial mass, than if it is attributed to*

the sphere of the Fixed stars. Therefore the diurnal motion is to be attributed to the Earth rather than to the sphere of the Fixed stars. The MAJOR is evident of itself, and is clear also from the fact that Nature and God do what is easier in itself and more intelligible and credible to an intellectual creature. The MINOR is proved from the fact that the speed of the Earth turns out smaller by very many times than the speed of the Fixed stars, as was shown from number 1 to 6; and that it is more congruent with the measure of its subject, the mass of the Earth, than the speed of the Fixed stars is with the immensity of their sphere, as was shown in numbers 2 and 3.

[Margin: 1st Reply.] The reply is, first — the Major being granted, but not yet conceded (for God and Nature do not always do what is more credible to us because it is easier) — to deny the Minor as to both its parts. Thus the consequence of the argument adduced for the first part of the Minor is denied: for it does not follow that a speed is more credible precisely because it is smaller, and more incredible because it is in reality greater, unless the excess of the greater over the smaller is also contrary to sense, or out of proportion to the capacity of the subject and to its motive power, and the shortfall of the smaller is in conformity with sense. That is not what happens in our case. For the motion of the Fixed stars is indeed evident to sense, and their speed is very slight to sense, so much so that in one second of an hour — indeed in one minute of an hour — they scarcely seem to move. But the motion of the Earth is nothing whatever to sense; hence its speed is not a smaller speed but the utmost slowness — indeed no motion at all, if we stand by the physical evidence we have from sense — and so any speed of the Earth whatever is contrary to sense. If the comparison were made between two moving bodies of whose motion we had physical evidence, then indeed that speed would seem more credible which was smaller, or at least so great as not to offend sense, than one which offended sense by excessive rapidity, if the eye tried to follow it. For to the intellect no speed can be excessive, since the intellect comprehends a much greater one.

[Margin: The speed of the Fixed stars is proportioned to the subject.] As for the other part of the Minor, concerning speed proportioned to the subject, this too is denied, as I said, together with its proof. For if the comparison is made, as it must be, in the same genus or the same species of motion — namely in the diurnal motion — then however much the semidiameter of any sphere grows, the circumference, which is the space to be traversed by the diurnal motion, grows by just as much. Therefore, just as there would be nothing incongruous in the speed of the par- ts of the terrestrial surface, even if it were four thousand times greater than the speed of the parts that stand a single pace from the centre of the earth, so neither is the speed of the parts of the sphere of the Fixed stars incongruous, even if it is many thousands of times greater than the speed of the parts of the terrestrial surface. For it happens in this case just as if one pace of some giant were compared with one pace of an ant: each is proportioned to the legs of its own subject. Granted that a thousand paces of the ant are contained in a single pace of the giant, and that on account of this

number the pace which for the giant is a single and very easy one would seem excessively swift to the ant, through the multiplicity of its small parts. How sophistically, moreover, the comparison of the diurnal motion of the Fixed stars is transferred to the motion of the Planets towards the East will be clear from the resolution of the second argument, to be proposed shortly. So much for the comparison of the spheres as to their surfaces. For if they are compared as to their masses, the excess of the mass, or solidity, of the greater sphere over the smaller is greater than that of surface over surface, since the former is in the triplicate ratio of the diameters. But this will be clearer from what is to be said in chapter 30, number 8, where it is treated expressly.

[Margin: 2nd Reply.] The reply is, 2ndly, to concede the Major if that speed is more credible to the intellect with the evidence of the senses kept intact, and the authority that binds us; and to deny it if it is more credible to the intellect taken precisely — an intellect not only abstracting from sense and from binding authority, but even judging against both. With this distinction supposed, the Minor is then denied. For the Copernicans, against the evidence of sense, deny motion to the Fixed stars and grant it to the Earth[40], and are wholly bent on following mere intelligible reasons, with sense neglected or corrected, and with sacred authority dragged without necessity from the literal sense to a figurative and foreign one. We, indeed, are not yet disputing about Authority; yet we ought not always to forget it among our replies to their arguments. [Margin: A witty saying of Athanasius Kircher.] And not without wit did Athanasius Kircher say, in the *Magnes*, page 539: *God commends himself by the speed of the Sun, and you cannot believe it?* — namely in those words: *He has exulted like a giant to run his course.*

[Margin: 3rd Reply.] The reply is, 3rdly, to turn the argument back — not indeed against the Semi-Copernicans, who admit only the diurnal revolution of the Earth, but against the Copernicans, who add the annual one as well. For from the annual revolution there follows an immense distance of the Fixed stars, such that the diameter of the great orb becomes a point with respect to it, and indeed a most vast gap between Saturn and the Fixed stars, idle and destined for no stars. Just as they, then, to soften its incredibility, have recourse to the wonderfulness of the Power and Majesty of GOD the Maker, so let them allow the Omnipotence and Majesty of the same GOD to be commended by the remarkable speed of the Fixed stars — especially since the divine Power and Wisdom shine out much more in assigning to a sphere of moderate magnitude a remarkable speed, yet one serving some use, than in extending a mass into immensity without any motion or action equivalent to motion, and in such a way that it cannot appear to what use that inert mass, empty of sensible bodies, serves. To me, certainly, he would seem a man of greater genius and skill who built some great ship, full of sailors, soldiers and merchants, so that by the art of either sails or wheels it flew across a thousand miles of the Ocean in the space

[40]printed *Ter-tæ*

of one hour, than one who raised a crag right up beyond the clouds, but bare, and, its summit excepted, furnished with no grasses, plants, animals, gems or minerals.

II. Argumentum à Proportione motuum & Interuallorum, Subiectorumque Mobilium.

II. The Argument from the Proportion of the motions and of the Intervals, and of the Movable Subjects.

VIII. I would certainly be ashamed of this argument, since it is sophistical and rests on a manifest equivocation, if they did not press it so seriously on the occasion of the first argument related in this chapter — above all Kepler, in chapter 16 of *De stella nova* in Serpentarius, where he reckons up his reasons thus. [Margin: Kepler's calculation for the proportion of the celestial motions to their intervals.] *In the hypothesis of Copernicus*, he says, *the globe of Saturn passes over about 300 German miles in one hour; that of Jupiter 400; of Mars 600; of the Earth 740; of Venus 800; of Mercury 1000* — he is speaking of the proper motions towards the East — and he adds: *A beautiful proportion indeed, where the swifter is always the one nearer to the Sun, which is at rest and is the dispenser of all motion.* Then, having said that in Ptolemy's opinion everything is found more incredible, he adds that the Fixed stars, if they are moved, must pass over 2625 semidiameters of the earth in one hour, each of which contains 860 miles. *See here*, he says, *what an immense difference: Saturn, which in Ptolemy is next to the Fixed stars, so that it all but touches them, passes over 300 miles in one hour for Copernicus; for Ptolemy, 2,257,500 miles. It is therefore to be believed swifter in Ptolemy than in Copernicus by 7525 times. Whoever tries to grasp this incredible speed with his mind will be just as wearied — and even more severely — as one who tries to grasp the Copernican immensity. Meanwhile, Philosopher, weigh this: that a proportion of the accident to its subject is much more wanting here than anyone can rightly find wanting in Copernicus a proportion of one part of the World to another.*

[Margin: Galileo's discourse for the same proportion.] IX. On exactly the same reasoning Galileo, in Dialogue 2 on the system of the World, page 84 of the Latin, judges Ptolemy's hypothesis improbable, because, with all proportion removed, the order of motions manifest in nature is disturbed. *The order*, he says, *is this: the greater an orb is, the longer the time in which it finishes its revolution, and the smaller ones in a shorter. Thus Saturn, etc. No less perceptibly do we see in the Medicean stars that the one nearer to Jupiter completes its revolution in the shortest time, that is, in about forty-two hours; the next in three and a half days; the third in seven days; and the most remote in seventeen days. And this concordant tenor will not change at all, so long as we assign to the terrestrial globe a motion of 24 hours about itself. But if we establish the Earth as immobile, then it will be necessary to pass from the slowest sphere, Saturn's, to the incomparably greater sphere of the Fixed stars, and to attribute to it a revolution of 24 hours, etc. But if we assign mobility to the Earth, the order of the periods is perfectly consistent, and the passage*

runs from the most sluggish sphere, Saturn's, to the Fixed stars, wholly immobile. He repeats similar things in the same dialogue, page 199, where, refuting the replies of the *Antitycho* — that is, of Chiaramonti — on behalf of the Ptolemaic system, he says: *But this conflicts with the architectonic rules of nature herself, which observes this in the measure of the smaller spheres (as we see in the Planets, and most perceptibly in the Medicean stars): that she turns the smaller orbs round in smaller times too. Hence the time of Saturn's revolution is longer than all the times of the other, smaller spheres, being completed in 30 years. Now to pass from this sphere to another, very much greater one, and to fix its revolution at only 24 hours — this, one might not unreasonably say, is to wander outside the rules of measure.* [Margin: Lansbergen's similar discourse.] Here Galileo plainly Keplerizes, as does Lansbergen too in his commentaries on the diurnal and annual motion of the earth, who argues in the same way about the order already mentioned. But we shall soon uncover the equivocation, once we have bound the argument to the laws of the syllogism.

[Margin: Argument 2 reduced to Form.] X. *The diurnal motion is to be attributed rather to the sphere that is the smallest of all than to the one that is the greatest of all. But the sphere of the Earth is the smallest of all the spheres of the world, and the sphere of the Fixed stars the greatest. Therefore the diurnal motion is to be attributed to the sphere of the Earth rather than to the sphere of the Fixed stars.* The MINOR is called into doubt by no one. The MAJOR is proved by induction from the manifest motions of the spheres: for the greater each sphere is, whether of the Planets or of the satellites of Jupiter, the slower, or the longer in time, is its motion; and the smaller the sphere, the swifter, or the shorter in time, is its motion. On the other side, the shortest revolution among the manifest revolutions of the stars is that of 24 hours, and so it is the smallest.

[Margin: 1st Reply.] I reply, 1st, by denying the Major, and by distinguishing the first part of its proof. That part we grant rather than concede, if the discussion is about the proper motion of the Planets and of the Satellites of Jupiter towards the East — yet in such a way that not Saturn's sphere is admitted to be the most sluggish of all, but the sphere of the Fixed stars, since it cannot naturally complete its revolution towards the East except in about 26,000 years, or, according to Ptolemy, 36000. Nor is that induction to be conceded except of the celestial motions, as being manifest to sense, and so with the comparison made between those spheres of the world that are, beyond controversy, apparently moved — not, however, of the sphere of the elements, much less of the terrestrial one, whose immobility, rather than its motion, is manifest to sense. But that same first part of the proof we flatly deny, if the discussion is about the apparent motion common to all the stars, namely towards the West, which is called the motion of the primum mobile and could be called the motion of the common movable [communis mobilis], if not by being carried along, at least by imitation. For the greater the sphere, or the higher the star, that is moved by such a motion, the swifter is its diurnal revolution; and the smaller or lower, the slower — speaking here too of the spheres whose motion appears and is manifest to sense.

Hence the sphere of the Fixed stars revolves to the same Meridian in 24 hours, Saturn a very little more slowly, and so on for the rest, so that the Moon, the lowest of all, revolves towards the West from a Meridian back to the same Meridian only in 24 hours and about 48 minutes besides. The second part of the proof is perfectly true, namely that the revolution of 24 hours is the shortest of the manifest revolutions.

[Margin: 2nd Reply.] I reply, 2nd, by denying the Major, and by likewise denying the first part of its proof, without any distinction. For there are not in reality two motions in the Fixed stars or in the Planets, one towards the East and the other towards the West. Rather, as we laid down and set out, with many others, as the more probable view in Section 2, chapter 3, numbers 6, 7, 8 and 14, there is a single motion, along spiral or helicoid — or, as it were, screw-shaped — lines. Because it does not perfectly attain the idea of the primum mobile, but falls short of it in the time of revolution to the same Meridian by so much the more as it stands farther from it in place and nearer to the earth, it therefore seems, through that very slowness in attaining it, to be a motion towards the East. Nor, to save the appearance of this motion and to satisfy the senses themselves, is there any need to acknowledge that slowness as a distinct species of motion; it is enough to imagine a motion towards the West, slower and slower, so that, when the slownesses of many days are added together, the lower spheres, or the Planets, seem to be abandoned more and more by the higher ones, especially by the Fixed stars, and to be left behind towards the East. This common motion towards the West, moreover, is more manifest than its modification, or retardation, towards the East, since it is observable every day, whereas that retardation, if you except the Moon, requires several days to become manifest from observation. Therefore, in the order and proportion of motions, the induction must be made either solely, or at least preferably, according to this motion towards the Setting; and in it the motion is swifter in the greater spheres or the higher stars. With every right, then, is the first part of that proposition denied which was adduced for the proof of the Major.

[Margin: 3rd Reply.] I reply, therefore, 3rd, by turning the argument back thus. *The diurnal motion is to be attributed rather to the sphere which is the greatest of all and the most remote from the centre of the World than to the one which is the smallest and nearest the centre. But the sphere of the Fixed stars, or some sphere loftier than it, is the greatest and highest; the Earth, however, is the smallest and nearest the centre of the World. Therefore, etc.* The MINOR is evident. The MAJOR is proved both from the nature of the sphere and of the circle — in which, if there is a common motion of all the parts, those parts move most swiftly that are most remote from the centre — and chiefly by an induction made from the motions that are the most manifest of all in the stars. These are the daily motions towards the West and the revolutions to the same Meridian; for it is established that motions of this kind are more vigorous, and are completed in a shorter time, by the greater spheres or by the higher stars, as was said in the 2nd reply. Therefore, etc. And indeed, unless this is so, the manifest

order in the apparent motions of the bodies of the world is disturbed, and that most ancient Harmony of the heavens is taken away which, following Plato and Hesiod, Cicero, Macrobius, Ficinus, Glareanus, Zorlinus and very many of the Harmonists admired and received. For they assigned to the supreme sphere either the Mese or the Nete — that is, the highest-pitched of voices and strings — on account of its swiftest motion, and to the Moon the Hypate, the lowest-pitched, on account of its slowest motion. Granted, fewer others, looking to that slowness of the apparent revolution towards the East, attributed the Nete to the Moon and the Hypate to Saturn — namely Nicomachus and Boethius, as I said in Book 7, Section 1, chapter 1; but these had fewer followers among the Musicians. Yet if both opinions are to be received, and the one motion is to be distinguished into two, what more admirable order of divine Providence could shine out in these things than that a compensation should be made from the highest to the lowest and from the lowest to the highest, in running up and down through this harmonic system, and that the sphere whose motion towards the East was swifter should have the slower motion towards the West? This compensation, however, and this most beautiful antithesis, is taken away if the common motion, which evidently appears to sense in the stars, is attributed to the Earth, whose immobility is evident to sense.

Away with that sophism by which Kepler, Galileo and Lansbergen have tried to blind the eyes of the unskilled — considering only the appearance of the celestial motion towards the East, as though this were the single or chief motion among those manifest to sense, while meanwhile concealing the evidence of the common motion towards the West.

III. Argumentum à Repugnantia Motus proprij cum Motu Primi Mobilis in eodem ſubiecto.

III. The Argument from the Incompatibility of the Proper Motion with the Motion of the primum mobile in the same subject.

[Margin: Kepler's argumentation, ineffective.] XI. *If the first motion*, says Kepler in the *Epitome of Copernican Astronomy*, Book 1, page 104, *is in the celestial orbs, then two motions are in the same subject: one common to all the spheres, the other proper to each sphere. But it is much more probable that the first motion and the second motions are distinct in their subjects, so that the second motions, which are many, are each in its own sphere, while the first, which is single, is also in a single body, the Earth.* An ineffective argumentation indeed — as if the concept [ratio] of animal, because it is generic and common to every species of animate beings, had to exist separately, in subject and in reality, in a single Idea or in something else; or — to give an example in a sensible accident, of which kind apparent motion is — as if the common concept of colour had to be in a single subject, and whiteness, blackness and the other differences in the rest. But, you will say, if the two motions are in the same subject, they will fight each other. [Margin: Galileo's argumentation.] And so Galileo, in Dialogue 2 on the system of the

World, page 83 of the Latin, said: *If that great motion is attributed to the heaven, it must necessarily be made contrary to the particular motions of all the Planets, each of which, beyond controversy, has its own proper motion from West towards East, which we must then make to be carried off in the contrary direction, that is, from East to West, by that most rapid diurnal motion; whereas, if the rotation of the Earth about itself is asserted, the contrariety of motions is removed, and the single motion from West to East is accommodated to all the appearances and satisfies them all exactly.* [Margin: And Lansbergen's.] Lansbergen drives home the very same argument in his commentaries on the motion of the Earth, affirming that it is impossible for the eighth sphere to be moved by two contrary motions; and so he grants the proper motion towards the East to the Fixed stars (in which he differs from Copernicus), but attributes the common motion to the Earth.

[Margin: Gassendi's opinion.] XII. Gassendi strives to carry this point with much weightier formulas, in Letter 2, *De motu impresso a motore translato*, page 109, where he speaks thus of the motion of the primum mobile: *Although the matter might perhaps seem tolerable in some way, if that whole machine were carried round towards the Setting by a single and simple motion; yet since, on the other hand, so many Planets are also carried round towards the Rising, what could be more absurd than to feign, above the Planets and above all the Fixed stars and even the crystalline spheres, a most vast sphere of the primum mobile, which itself strains against the lower spheres as they strive towards the Rising with their own slow motions, and snatches them all away towards the Setting with incredible speed? Can anything be more violent, and yet be thought to be perpetual in the nature of things? Can anything be more tortuous, and yet be attributed to nature, which always proceeds to where it tends by the shortest and most level path?* And after inserting a few things in favour of the simplicity of the terrestrial motion, he teaches that the rapt motion of the primum mobile cannot be communicated to the lower orbs, which are not solid; and that even if they were solid it could not be, because they lack little handles by which they might be gripped by the higher orb. But this does not tell against the many others who, as I taught in Section 2, chapter 3, do not posit the primum mobile as distinct from the sphere of the Fixed stars and from the motions of the Planets, and yet attribute the diurnal motion to them. Let the argument now be put in form, as follows.

[Margin: 3rd Argument in form.] XIII. *It is impossible for the same movable body to be moved by two contrary motions; or, if they are not really contrary motions, *the appearance of contrariety is nevertheless to be avoided, as far as possible. But if the diurnal motion were attributed to the Fixed stars and the Planets rather than to the Earth, the same movable body would be moved by two contrary motions, or at least the appearance of contrariety would not be avoided as far as possible. Therefore the diurnal motion is not to be attributed to the Fixed stars and the Planets rather than to the Earth.* The MAJOR seems clear enough, and is to be denied by no one. The MINOR was proved in number 11: for one motion of the same movable body would either be, or would appear to take place, towards the East, the other towards the West; these are contrary termini, and therefore specify contrary

motions; and this appearance of contrariety would be very easily avoided by attributing the diurnal motion to the Earth.

[Margin: 1st Reply.] I reply, 1st, granting the Major, by denying the Minor and both parts of its proof. For there is no apparent contrariety in the motions already mentioned, except in the imagination of those who distinguish this motion, in reality, into two motions towards contrary quarters. Those, by contrast, who conceive it without that fiction — as a single motion towards the West, distinguished only by slowness or speed from another motion made towards the same quarter — remove from motion of this kind not only real but even apparent contrariety. If you nevertheless press that, by the very slowness by which they are left behind by the swifter movable, they seem to recede towards the East, and that the appearance of contrariety cannot be removed altogether, I reply that not even by granting the diurnal motion to the Earth is that appearance avoided; for the same things would appear as now. Therefore, if some cannot correct their fancy [phantasiam] so as not to see two contrary motions, they will not overcome it in this way either, if they choose to cling to the appearance. But what deception, I beg you, or what perversion is this: that of two movables running round in a circle towards the same part, the one that moves more slowly should be said, or seen, to run towards the opposite part? Read again, I beg you, what we have said about this apparent contrariety in Section 2, chapter 3, from number 6 to 8 and from 14, and in the scholia of the same chapter.

[Margin: 2nd Reply.] I reply, 2nd, by turning the argument back against the contrariety which, in Copernicus's hypothesis, has to be posited among the motions of heavy bodies as they are carried downwards, if they have been thrown obliquely towards the West, and in many other movables of this kind. If that contrariety is not absurd, neither will the apparent contrariety in the motions of the stars be absurd.

IV. Argumentum à Simplicitate Motuum.

IV. The Argument from the Simplicity of Motions.

XIV. But, they say, it cannot be denied that the simplicity of motions is greater if a single motion towards the East is attributed to the Planets, or even to the Fixed stars, without any admixture of a diurnal motion to be represented towards the West by spirals or in some other way, and if the appearance of the other motion is transferred to a single motion of the Earth. [Margin: Copernicus's opinion in favour of economy of motions.] Hence that saying of Copernicus in Book 1, chapter 10: *But the sagacity of nature is rather to be followed, which, just as it has most of all taken care not to produce anything superfluous or useless, so has rather often endowed one thing with many effects.* This, moreover, is the third reason by which Galileo, in Dialogue 2 on the system of the World, page 84, confirmed the diurnal rotation [volutationem] of the Earth. For he grants that whether the celestial motions towards the East and the West are contrary

or not is a dispute about words; but he adds: *I know that in reality it is much simpler and more agreeable to nature if we can save everything by one single motion than if we introduce two motions, which, if you do not want to call them contrary, you may call opposite. Nor do I condemn that introduction of opposite motions as impossible; nor do I affirm that a necessary demonstration is constructed from its negation, but only that something at least of weight is added towards probability.* [Margin: Gassendi's opinion in favour of the same economy.] With this agree those words of Gassendi in Letter 2, *De motu impresso a motore translato*: *With the Earth indeed turned towards the Rising — and nothing is more economical than this — their own slow motions towards the Rising will be left to the Planets themselves; and the motion which appears to be in them, as well as in the Fixed stars, towards the Setting will appear from the single turning of the Earth towards the Rising; so that this sole and simple turning is equivalent to that primum mobile, and meanwhile frees the lower orbs from that so great rapidity, violence and entanglement.* But, to say no more of these things, let the argument be framed in the following way.

[Margin: 4th Argument [?], enclosed in its form [?].] XV. *Of two motions, that one is to be chosen which is simpler and single, rather than the one which is multiple and composite. But the diurnal motion of the Earth is simpler and single, that of the Fixed stars and the Planets multiple and composite. Therefore the diurnal motion of the Earth is to be chosen rather than the diurnal motion of the Fixed stars and the Planets.* The MAJOR rests on that most widely received axiom of the Peripatetics: *What can be done by fewer things is done in vain by more.* Galileo, in Dialogue 2 on the system of the World, page 88, with excessive subtlety denies that the limitation "equally well" is to be understood in this axiom, because what is not done well, or not with full goodness, is not the same; whereas the axiom supposes that what is done by more things or by fewer is the same. The MINOR is easily proved: for once the diurnal motion of 24 hours is assigned to the Earth, there is no need to compound another motion towards the West with the proper motions of the Fixed stars and the Planets; and so the single and simple motion of the Earth will represent the appearance of all the diurnal motions.

[Margin: 1st Reply.] Someone might reply, 1st, by distinguishing the Major: granting it if other things stand equally on both sides — or if evidence gained through the senses, or even authority, which is superior to all reason, is equally satisfied — but otherwise denying it. In the case proposed, however, there stand against it both the Authority of Sacred Scripture, which attributes the diurnal motion to the stars and immobility to the Earth, and (what belongs to this place) the evidence of the senses concerning the diurnal revolution of the stars and the rest of the Earth.

[Margin: 2nd Reply.] I reply, however, 2nd, granting the Major, by denying the Minor. For more motions, both in number and in species, are multiplied in Copernicus's hypothesis, with the diurnal motion attributed to the Earth, than if it is attributed to the heaven and the stars — provided that all the conditions of motions which the diurnal motion given to the Earth necessarily draws with it

are considered, and that, on the other side, that hypothesis of the resting Earth is chosen which, by a single spiral line, represents all the varieties of motions in each of the Planets and in the sphere of the Fixed stars, according to what we have said so often, but above all in Section 2 of this Book, chapter 3, from number 14. For in this hypothesis there is a single motion of the Eighth sphere towards the West, but one that falls short little by little of the most perfect Idea of the first motion, which shines in the mind of the Intelligence or Intelligences that move that sphere along a spiral line; and this motion, perhaps, on the first day or the first three days before the Sun and the stars were made, was completed by a most perfect revolution of the light to the same Meridian. In the seven Planets, likewise, there is a single motion, also towards the West, but slower in the lower ones, and carried out by the Intelligences obliquely along spirals, looser in some cases and tighter in others. And so in all there are eight motions, with the little appendages of motions by which the satellites revolve about Saturn and Jupiter, which are not under discussion in this place; in the motion of heavy and light bodies, moreover, a simple motion along the shortest and simplest straight line suffices. But in Copernicus's hypothesis, besides the motions of the Seven Planets and the eighth — which he attributes to the terrestrial axis, slowly inclining itself against the order of the signs, while the semi-Copernicans attribute it to the Fixed stars — it is necessary to posit a ninth motion, really distinct, and to attribute it to the earth. [Margin: How mixed the motion of Heavy and Light bodies is, if the Earth is moved.] Once this is posited, in order that the countless experiments of the motions by which both inanimate and animate things are moved on the Earth, in the Water, and in the Air nearer to the lands may be saved, it is necessary — besides the straight motion of heavy and light bodies and the peculiar motions of animals, winds, clouds, etc. — to posit a circular motion included in those motions, though unknown to us or not seen by us. From these there arises a motion mixed of straight and circular: whether it be parabolic, according to what was said in the scholia of chapter 4 — which is certainly not the shortest, but must be completed along a longer path — or of some other figure, on which matter see below, chapter 17. Nor is this the end of the perplexities. For one must imagine that a clod of earth, torn from the earth's surface near the Poles and thrown up there along the perpendicular, moves with that slowness with which the circles of the earth that are parallel to the terrestrial Equator, but near its poles, are moved; but that, if the same clod is carried by someone in a box to another parallel, or is thrown upwards from the North towards the South with an oblique cast, it ascends and descends with a continually changing proportion of speed, so as to move more swiftly over the parallels nearer the Equator; and this although it is not joined to the Earth in the way that the Fixed stars are joined to the eighth sphere; and so that it changes innumerably its property of speed, and that line of its motion turns out to be composite in many ways. But if this is not a real composition, but a diverse affection and modification of a

single motion, it is much truer that in each Planet there is no real composition of motions, but only diverse affections of one motion.

[Margin: 3rd Reply.] I reply, therefore, 3rd, by turning the argument back thus: *Of two motions, that one is to be chosen which is simpler and single, rather than the one which is multiple and composite. But the diurnal motion attributed to the Fixed stars and the Planets, together with rectilinear motion for heavy and light bodies, is simple and single in each of the simple bodies; it is not such, however, if the proper motions are attributed to the Fixed stars and the Planets and the diurnal motion to the Earth. Therefore, etc.* On the MAJOR we both agree. The MINOR has already been set out and proved in the 2nd reply: for in place of eight motions in the heaven, and two in the Elements — one upwards, the other downwards — a ninth, of the Earth, is posited; to the rectilinear motion of the elements a mixture of curvilinear motion is added; and their ascent and descent takes place along a longer path than if they were carried by their own nature along a rectilinear and perpendicular course. The Copernicans, therefore, ought to have looked round at these things more attentively, or else not to have dissembled the multiplicity of motions, and of accidents in motions, which lies hidden in their own hypothesis.

V. Argumentum ab Inæquali participatione Motus Diurni, ſeu ab Inæquali Velocitate eorum, quæ motu diurno mouentur.

V. The Argument from the Unequal Participation in the Diurnal Motion, or from the Unequal Speed of the Things that are Moved by the Diurnal Motion.

XVI. Self-love [Philautia] is, to be sure, a flattering thing: each of us has long been affected by it towards his own opinion, and has grown so used to it that he does not recognize in his own hypothesis the very same spots or blemishes by which he thinks the hypotheses of others disfigured. Look, indeed, at Galileo in Dialogue 2 on the system of the World, page 85 of the Latin version, speaking thus against the common hypothesis, which grants the diurnal revolution to the Fixed stars: *The fourth difficulty is the immense disparity between the motions of the same stars, of which some will be moved most swiftly, in the vastest circles, and others most slowly, in the smallest circles, according as these or those are nearer to the poles or less near. This is certainly very inconvenient: both because we see that those whose motion is not doubted all move in the greatest circles, and also because it seems absurd to place bodies that move circularly at immense distances from the centre and to attribute to their motions the smallest circles.* [Margin: Galileo's discourse, but an utterly fallacious one.] These are the very words of Galileo, uttered in earnest, as appears; [Margin: Refutation of the Galilean Argument.] and yet for these words, I well know, any little beginner lightly grounded in the elements of the doctrine of the sphere would get a sound beating from Galileo himself. So there is no need to frame the argument in order to reject its fallacy. For it is no inconvenience that, in the same sphere turned about its own poles by a single rotation, the parts that lie on the circumference of great circles should move

more swiftly than those on the circumference of smaller circles, nearer the poles; rather, it is required by Geometrical necessity, since in the same time the former run through greater arcs and the latter lesser ones, whether that sphere then be large or small. Otherwise, if it were an inconvenience, it would be no less one in the diurnal revolution of the Earth, whose parts placed on the Equator would certainly have to move much more swiftly than the parts in the polar circles or situated near the poles. It is false, moreover, that the stars of whose motion we have no doubt at all all move in the greatest circles. For neither do the Planets, in their proper motion, always run in the Ecliptic — having acquired from it a latitude of several degrees, they describe smaller circles, especially if they are then also at perigee or perihelion — nor does their common motion, which is no less evident to sense in them than in the Fixed stars, since it is both daily and observable every day, take place along the greatest circles, but along circles parallel to the Equator, one after another. Lastly, the circles that are called smallest in the heaven, because they are nearest the Poles, are not called smallest in comparison with the circles of the Earth, but in comparison with the greatest circles of the same heaven or sphere; and in the respect in which they are, and are called, smallest, they are not the swiftest but the slowest — namely in comparison with the motion that takes place in the same time along the greatest circles. There is nothing absurd, then, if, at whatever distance from the earth they stand, they nevertheless describe circles which are the smallest in comparison with the greatest circles of the same sphere, and are not the smallest in comparison with the circles of the Earth. Indeed, if this were something absurd, it would be multiplied more by moving the Earth: for there would be an appearance of unequal speed in the heaven, and a real inequality in the parts of the Earth more or less remote from the poles.

VI. Argumentum ab Inconſtanti Declinatione, ac Velocitate Fixarum.

VI. The Argument from the Inconstant Declination and Speed of the Fixed Stars.

[Margin: Galileo's other argumentation, but an ineffective one.] XVII. Galileo sets out, but gets nowhere. In the same Dialogue 2 on the system of the World, and on the same page 85, he passes to another inconvenience, as he himself calls it, which follows from the diurnal motion attributed to the Fixed stars. *For*, to use his own words, *the stars themselves will vary their circles and their speed (which is the fifth inconvenience): since those which two thousand years ago were on the Equinoctial, and consequently described the greatest circles by their motion, decline in our time many degrees from the Equinoctial, it is necessary that they too have become slower in motion, and have been reduced to describing smaller circles. And it is not far from possible that a time should come in which one of them, always moved before, on being united with the Pole, should stand still, deprived of all motion, and after a rest of some time should begin to move again; whereas the other stars, which without doubt are moved, all describe, as has been said, the greatest*

circle of their orb and turn in it without variation. Are these your inconveniences, Galileo? And are these things said by you in earnest, or in pretence? But let the argument be framed, though there is scarcely need.

[Margin: Form of the 6th Argument.] XVIII. *If the Fixed stars are moved by a diurnal motion, it is necessary that the same Fixed stars at one time describe the greatest circles and be the swiftest, and at another describe smaller and the smallest circles, or even none, and so turn out less swift or least swift, or even not to be moved at all. But this is an inconvenience. Therefore the Fixed stars are not moved by a diurnal motion.* It remains, therefore, that such a motion belongs to the Earth.

[Margin: 1st Reply.] I reply, 1st, granting the Major if a proper motion in longitude is granted besides the diurnal motion of the Fixed stars, but denying the Minor. For this is no more an inconvenience than that the Planets at different times vary either their declinations or their latitudes, and are now swifter, now slower. Or, to draw an example from Copernicus's own storehouse, this is no more an inconvenience than that, in Copernicus's hypothesis, men, fish and birds — who daily, by walking, sailing, swimming and flying, straight or obliquely from the South towards the North or conversely from the North towards the South, vary their declination and their parallels of the Equator — should be moved now more slowly, now more swiftly, with the Earth on which they stand or whose diurnal motion they follow; indeed, that those who live under the poles of the terrestrial Equator should be moved by this motion not at all, or almost not at all. And yet variation of this kind is far more frequent, is entangled in very many perplexities of animate motions, and — what is the chief point — is feigned against all the evidence of the senses; whereas that variation in the Fixed stars as to declination and speed has been confirmed through many centuries by the evident experience of observations, and from it follow many things most worth knowing, and theorems and problems that lead us by the hand to the Glory of GOD the Creator. Of these we have brought forth some that are new and hitherto unheard of, in Book 6, chapter 11, but above all in chapter 19, where in problem 5 we taught the conditions required for a star to be able to reach the poles of the world, and that the star we now call the Polar star never has been, nor will be, exactly at the very Pole of the World; and in Problems 10 and 11, that there are some Fixed stars whose Right Ascension, although their Longitude increases, nevertheless increases in some centuries or periods and decreases in others, by a certain retrograde motion of declination, and that 94 such stars are found in the heaven among those hitherto counted with the naked eye; teaching, moreover, the way of discerning Geometrically the situation of these stars, and of determining the times in which their Right Ascension must increase and in which it must decrease. If Galileo had known this, how specious an inconvenience would he have brought forward against the hypothesis of the diurnal motion of the Fixed stars?

[Margin: 2nd Reply.] I reply, 2nd, by turning the argument back, and by throwing the inconvenience already mentioned back upon the Copernican hypothesis — not absolutely but ad hominem, as they say — in this way: *If*

the Earth were moved by a diurnal motion, it would be necessary that very many animals, following the common motion of the Earth, should very frequently vary their declination and their speed in this motion. But this, by your own account, is an inconvenience. Therefore, etc. The MAJOR has already been shown, from the change of the parallels through which animals pass. Moreover, it would be necessary for the axis of the Earth to nod and swing this way and that with so great an inequality and anomaly that the appearance of the declination of the Fixed stars might be saved through it; but this inconstancy is no less an inconvenience than the variation of declination in the Fixed stars. Therefore not on this account either is the diurnal motion to be attributed to the Earth.

VII. Argumentum ab Inæqualitate Dierum Naturalium, orta ex motuum Diurnorum Inæqualitate.

VII. The Argument from the Inequality of the Natural Days, Arising from the Inequality of the Diurnal Motions.

XIX. Kepler hones a subtler argument in the *Epitome of Copernican Astronomy*, page 108, which I once pointed out in Book 3, chapter 32, numbers 2 and 3. In it, it is supposed — as in fact appears — that the individual diurnal motions are indeed equal to sense among themselves and in their parts, but, compared with other diurnal motions, are unequal: slower, namely, when the Sun is at apogee, and swifter when it is at perigee. With this supposed, Kepler argues thus: [Margin: Kepler's argument, acute but weak.] *If, with the Earth alone excepted, the whole remaining machine of the World sustained its own motion, no cause could be devised why the laws of this inequality, tempered to the motion of the single Solar sphere alone, should overflow into the whole machine. But if, with the machine of the world at rest, the earth rotates, the cause of this inequality is easily given: from the change of the interval between the Sun and the Earth in the same proportion, since the Sun is the minister and helper of all motion.* He points to the same cause at page 550 of the same *Epitome*. But we deny that no cause of the effect already mentioned can be given. For the cause is that Solar days are the measure commonly used in counting the motions of the other Planets, and therefore their inequality overflows into the motions of the others. And if the cause of the inequality already mentioned is the change of the interval, or distance, between the Sun and the Earth, it makes no difference in this respect whether the Earth itself is moved, approaching and receding towards the Sun — even if it is said to be stirred up by the Sun magnetically, or by a force spread through its light, when it is nearer, and not so stirred up when it is farther away — or whether the Sun approaches and recedes towards the Earth, and in approaching is objectively [obiectiuè] drawn by the Earth, for whose sake it is moved, so that it is then somewhat swifter, while in receding it is not so drawn.

CAPVT VII.

Chapter VII.

Proponuntur, & Diſſoluuntur Tria reliqua Argumenta pro Motu Diurno Terræ.

The Three Remaining Arguments for the Diurnal Motion of the Earth are Proposed and Resolved.

I. Argumentum à Cometis.

I. The Argument from Comets.

I. Seneca once raised the doubt, in Book 7 of the *Natural Questions*, chapter 2, whether Comets were carried along by the diurnal turning of the heaven and yet were not dissolved; or whether, rather, they were not dispersed so quickly for this reason, that it was not the heaven but the Earth that revolved by a diurnal rotation. [Margin: Seneca's uncertainty about the diurnal motion of Comets.] Let us repeat his words from there: *It will be useful to inquire whether Comets are of the same condition as the higher bodies. For they seem to have certain things in common with them: risings and settings, and even their very appearance, though they spread out and extend farther; for they are equally fiery and bright. Therefore, if all the stars are earthy, the same lot will be theirs too. But if they are nothing else than pure fire, and last for six months at a time, and neither the turning of the world nor its speed dissolves them, these too can consist even of thin matter, and not be shattered on that account by the constant circling of the heaven. To this question it will also belong to have examined whether, so that we may know it, the world goes round while the earth stands still, or the earth is turned while the world stands still. For there have been those who said that it is we whom the nature of things carries along without our knowing it, and that risings and settings come about not by the motion of the heaven, but that we ourselves rise and set. The matter is worthy of contemplation, so that we may know in what state of things we are: whether we have been allotted the most sluggish seat or the swiftest; whether God drives everything round us, or drives us.* So far that noble Philosopher. But it was long ago laid down by Aristotle and the other Peripatetics that the diurnal motion of Comets is to be attributed to the turning of the fire, or of the highest region of the air, carried off towards the West by the heaven. [Margin: Copernicus's opinion on the diurnal motion of Comets.] Copernicus, by contrast, in Book 1, chapter 8, ascribes to the Earth their apparent motion, by which they are seen after 24 hours to revolve to almost the same place; yet he denies that the air in which Comets are generated is carried along by the Earth's rotation. For let us repeat his words too: *Conversely, they say with no less wonder that the highest region of the air follows the celestial motion — the Peripatetics, that is — as those sudden stars indicate, the Comets, I mean, and those called Pogoniae [bearded stars] by the Greeks, for whose generation they assign this very place, and which, like the other stars too, rise and set.* So much from the opinion of others; then he adds from his own: *We can say that, because of its great distance from the earth, that part of the air is deprived of that terrestrial motion. Hence the air which is nearest the earth will appear tranquil,*

as will the things suspended in it, unless they are tossed to and fro, as happens, by wind or some other impulse.

[Margin: Gassendi's opinion on the diurnal motion of Comets, before he had heeded the sacred edicts.] II. From this, then, Gassendi hammered out an argument for the diurnal rotation of the Earth in Letter 2, *De motu impresso a motore translato*, judging it much more probable that Comets have no motion other than their own proper one, and that a wandering one, while the diurnal motion which appears in them comes from the turning of the earth. For he argues thus there, at page 110: *I note meanwhile, on account of the Comets: since, besides their proper trajections [traiectiones], which are now to the north, now to the south, now elsewhere, they also appear to be carried by the diurnal turning from rising to setting, what could be thought more probable than that they are moved by a single motion, that is, by their proper trajection, and that that carrying-round towards the setting is only apparent, from the diurnal turning of the Earth towards the rising? This turning, certainly, is of such a kind that, whatever is above or beyond the Earth — whether it exists, or is born, or passes through the spaces of the world — it ascribes to it, as also appears, a turning towards the Setting.* The whole argument of Gassendi, therefore, consists in greater verisimilitude, as he himself judges, and so it can be gathered into form.

[Margin: The argument reduced to form.] III. *If Comets have in reality no motion other than their proper trajection, the diurnal Motion which appears in them besides is to be attributed to the Earth. But it is more probable that Comets have in reality no motion other than their proper trajection. Therefore the diurnal motion which appears in them besides is to be attributed to the Earth.* The MAJOR is proved because that diurnal motion can no longer be attributed to the turning of the fire or of the air, since the heavens of the Planets are not solid, and therefore the sphere of fire, still less that of the air, cannot be carried round by the concave of the Moon. The MINOR is proved because there is no small difficulty in assigning the cause of so wandering a trajection; therefore we ought not to increase it by adding another real motion, which would render the agitation of Comets more entangled and of more uncertain cause.

[Margin: Reply to the 1st Argument.] The reply, however, is to grant the Major and deny the Minor and the consequence of its proof. For that is not more probable which is against the physical evidence acquired through the senses — of which kind is the diurnal motion of the Comets, just as of the other stars; and the cause that moves the Comets by the trajectory motion can at the same time move them by the diurnal motion, especially since it is most probably some Intelligence, as I taught in Book 8, Section 1, chapter 6, number 12. Just as, therefore, we do not take the diurnal motion away from the Planets, although they have a proper motion in longitude, latitude and altitude so varied and anomalous, because Intelligences lead them round towards the West along a spiral line, so neither ought we to withdraw it from the Comets; for in this respect everything appears in Comets just as in the other stars, as we saw a little earlier rightly asserted by Seneca. Nor do I see why an argument for the

motion of the Earth should be taken from the Comets more than from the Planets; for the greater difficulty in determining the moving cause of Comets arises not from the diurnal motion towards the Setting, but from the admirable variety of their proper trajection, and whatever cause of it is assigned, that same cause will be able, much more easily, to move them by the common, that is, the diurnal, motion.

II. Argumentum ab Auræ perpetuis intra Tropicos verſùs Occidentem flabris.

II. The Argument from the Perpetual Blasts of the Breeze within the Tropics towards the West.

IV. It is reported by ship-masters — as the Dutch relate, and Ramusius [Giovanni Battista Ramusio] in his history of voyages, but far more eloquently our own Iosephus Acosta [José de Acosta] in Book 2 of *De natura novi orbis*, chapter 10 and chapters 13 to 16 — that the Torrid Zone is highly habitable. This is not only because of the vast quantity of sea water, which does not reflect the Sun's rays as the land does from its plains, valleys and hills; nor only because of the shortness of the summer days and of the winter nights; but also, since God provides so gently, because of the frequency of summer vapours and rains, and above all because of the singular benefit of a most gentle breeze, whose coolness is so pleasant that some, with Ludovicus Vives [Juan Luis Vives] on Book 13 of *De civitate Dei*, chapter 21, have not unfittingly placed the earthly Paradise, or the land of Eden, under the Equinoctial. [Margin: For what reasons is the Torrid Zone habitable [?]] I shall note here the choicer points, so that the force of the argument proposed, and its solution, may shine out more clearly. In chapter 13, then, Father Acosta relates that in those regions within the Torrid Zone which are blown upon by a sea wind rather than a land wind, there is in a manner a perpetual spring, especially on the Peruvian sea: [Margin: The variety of the winds within the Tropics, and their admirable effects.] *on which we remember being cold when we first sailed it, and that at the time when the Sun bore down most directly, that is, near the Equinoctial, in the month of March,* says the same Author. Yet he admits that on the plains of great Ethiopia, of Brazil, of Paytiti and of Paraguay the winds are so hot that you would think them lit furnaces, and that certain seas within the Tropics are also oppressive with heat — namely the sea of Hormuz, that of Mozambique, the Brazilian and the Panamanian — but that in the same Clime many other seas have a most temperate air. So from the abundance and variety of these winds he accounts for certain admirable effects in that tract of the world: why the summer Sun, even at midday, is kept off there better by the lightest shade — a mat, say, or straw — than in Spain by tiled roofs; why the summer nights there are not hot; and why in the province of Collao there is cold even under a thin shade, although those living in the Sun close by suffer an intolerable heat. The Peruvian region, though full of sand and gravel, is nevertheless wonderfully temperate; again, the city of La Plata is

most temperate, most pleasant and extremely fruitful, whereas that of Potosí, although it too is mountainous, lies in the same climate and stands a very short distance away, is nevertheless squalid, very cold and barren. *All these varieties, therefore, are produced chiefly by the nature of the wind,* says Father José; *for where the supply of the gentler breeze is withdrawn, the heat of the Sun is so great that it burns amid the snows; where it is restored, all heat easily departs; and where this cold wind is familiar and, as it were, a resident, it in no way allows earthy exhalations to gather as from burnt matter — the one cause which in Europe makes the summer nights almost more oppressive than the days themselves, because the earth, scorched by the force of the Sun, gives off thick and smoky breaths, as if coming out of a furnace.* He adds that in Europe the pleasantest time in summer is the morning, and the most oppressive the afternoon; whereas in the Torrid Zone, especially in the Barlovento Islands, the chief heat is felt from earliest morning until about the tenth hour, sometimes to the point of sweating; then, around midday and afterwards, with the sea breathing on them, the rest of the day passes pleasantly, and towards midday the Euroaquilo, called there the Brisa, cools the sailors and the inhabitants.

[Margin: Iosephus Acosta's praise of the temperateness of the Torrid Zone.] V. Acosta goes on, in chapter 14, about the temperateness of the air found here and there within the Torrid Zone, and concludes: *As I contemplate this kindliness of what is, as it were, a perpetual spring — since neither does winter press with cold nor summer bake with heat, so that all harms are kept off well enough by a little turf, and there is scarcely any need to change one's clothes — it has truly often come to my mind that, if men were freed from the chains of greed and of perverse opinions, they would easily have lived a pleasant and blessed life here. For what the Poet either invents or relates of the Elysian Fields, what of Tempe, and what Plato of that Atlantis of his, they would certainly have found in these regions, had they resolved, with a free-born liberty, to rule rather than to be slaves to money.* Next, in chapter 15, he digresses into the causes of the winds; and although he touches on them from the teachings of Peripatetic philosophy, he nevertheless refers the chief one to God, who, as we have it from Psalm 134, brings forth the winds out of his treasuries, so that the Truth spoke most truly in John 3: *you hear its voice* — that is, of the wind and of the spirit — *and you do not know where it comes from, or where it goes.* For it is hard to explain why along the whole sea-coast of Peru, over very long stretches, only one wind blows; and why the same South wind, which is most pleasant with them on this side of the Equator, becomes heavy and murky as soon as the Equator is crossed; why the North wind there gathers clouds and wrings them out into rain, and is held to be sluggish; why the Barlovento islands are cooled by perpetual North winds that bring abundant rain, while the South wind or the Africus can scarcely gain access there; why the region of Brazil before the promontory of St Augustine is perpetually beset by land winds, while that of Peru, in the same climate, is always blown upon by sea winds, and most pleasant ones. At the end of the chapter he notes what might be the cause of these varieties: namely that the torrid Zone is either full

of very high mountains, or for the most part interspersed with or surrounded by the Ocean, from which the winds are somewhat cold and rather subtle.

VI. Finally, in chapter 16, from the record of nautical observation, he affirms that within the Tropics, on either side, and especially on the high Ocean, winds blowing from the East and North prevail perpetually, while those from the West and South are rare and almost none; from which he concludes, rightly and no less elegantly, that this is the cause of so different a navigation. *For it is not the case,* says Acosta, *that, as the road from Thebes to Athens and from Athens to Thebes is the same, so is that from Spain to the Indies and from the Indies to Spain. The coming is indeed downhill, and — if I may play the poet — easy is the descent to the Indies; but to retrace one's step and escape back to the old world, that is no easy labour.* The cause, he says, is that fleets sailing from Spain to the West Indies enjoy almost perpetual favourable Euro-Aquilo winds, but those returning from there have the same winds against them; to escape these, they sail out beyond the Tropics to the degrees of a higher heaven, so as to find, on the way back, South or South-west winds that serve them — but by a longer course than before. [Margin: Routes to the Indies and from the Indies.] For the usual route to the West Indies and New Spain from the Fortunate Isles is almost straight; but the return is from New Carthage by way of Havana, the port of Cuba, and from there to within sight of Florida, past the ill-fortune of Bermuda, Terceira, and near the sacred promontory of St Vincent. Likewise, from New Spain and Peru across the Pacific to the Solomon islands the passage is easy and the return difficult, unless a West wind is sought beyond the Equinoctial and the Tropic of Cancer towards the shores of New Spain, or unless, once the Tropic of Capricorn is passed, you turn towards the region of Chile. In the same way, from Mexico to China and the Philippine islands the voyage is short and quick, and the return voyage slow and arduous. He admits, however, that this general law has its exceptions in places; for in the maritime region of the Southern Ocean, although it lies within the Tropics, the South wind prevails for almost the whole year. [Margin: Robert Dudley's testimony on the Winds within the Tropics.] Robert Dudley, Duke of Northumberland, says no differently in his *Arcana maritima*, recently published in Italian: for in Book 2, chapter 20, he says that within the Tropics there are fixed, that is, perpetual, winds from the East — as much from Africa towards the Americas, and from the Americas towards the Moluccas and the Philippines, as from there towards Africa — and that they usually blow with a uniform tenor and are healthy; so much so that, on the return from the East Indies towards the island of St Helena, the return is almost twice as short as the voyage from there to those Indies had been; and finally that it is a sure rule that those who sail according to the diurnal motion mostly enjoy favourable, healthy and fair winds, etc. These things have perhaps been told at too great length; but they were told so that, from the very narrative of so religious and experienced a man as we know Acosta to have been, the fiction of Galileo and Gassendi may at once collapse.

[Margin: Galileo's opinion on the apparent motion of the air towards the West.] VII. For Galileo, in Dialogue 4 on the System of the World, page 327 of the Latin edition, saw it to follow that within the Tropics, and on the open plains of the Ocean, where there is a smaller supply of thicker vapours, the air would not obey so readily the carrying-off by the terrestrial rotation, and therefore, by resisting that rotation as it revolves towards the East, would make a perpetual wind towards the West, or the appearance of a wind, which would be more perceptible where the rotation of the terrestrial globe is swifter, namely around the Equator. Seeing this, he does not hesitate to confirm the diurnal motion of the Earth from the perpetual winds which those sailing from the Mexican port across the Pacific to the East Indies, etc., experience. To this experiment Sagredo subscribes in the same place, affirming that in the Mediterranean too, unless accidental and transverse winds stand in the way, one sails back from Syria to Italy more quickly than from here to Syria — indeed almost four times as quickly. Salviati, however, not undeservedly refers this also to the waters running towards the Strait of Hercules, granted that in the Strait of Sicily the water runs eastward along one shore and westward along the other. [Margin: Gassendi's support for the same motion.] Gassendi too, in Letter 2, *De motu impresso*, page 92, having asserted that the air near the earth is carried along by the diurnal rolling of the Earth, nevertheless added that, because of its fluidity, it does not follow the earth so closely as not to be retarded somewhat towards the western parts: *They say,* he states there, *that this is evident not so much in valleys, within which the air is confined on either side and is not free, as on plains and open fields, where little vanes are observed, whenever calm prevails, to be turned towards the West; but that it is evident most of all at sea, and above all under the Equator itself, where the sails of ships are filled steadily and uniformly from the East; and that for this reason sailors find that they cover the same course both far more easily and in a shorter time towards the West than towards the East.* From this, then, Galileo and Gassendi confirm the rotation of the Earth, because, once it is asserted, a readier cause is to hand for that perpetual and uniform wind within the Tropics; and from these points one may argue thus.

[Margin: Argument 2 in Form.] VIII. *If a perpetual and uniform Wind blew towards the West within the Tropics, it would be a very probable sign of the diurnal revolution by which the Earth was turned towards the East. But it is established by experience that a Wind of this kind blows within the Tropics. Therefore it is a sign of the diurnal revolution by which the Earth is turned towards the East.* The MINOR seems already proved from the historical testimony of the ship-masters. The MAJOR, too, is firmly enough established by Galileo and Gassendi; for no other, more probable, cause can be adduced why such a wind should be felt perpetually within the Tropics rather than elsewhere, and towards the West rather than in another direction, except that the air in that open tract, being very little entangled with watery and earthy vapours, neither moves of itself with the motion of the earth and the water, since it is not of their nature, nor is carried at all towards the East by their rotation, because its thinness and fluidity

make it unfit to take on so great an impetus; and therefore, by forsaking, or not following, that motion, it behaves just as if it were moved towards the West.

[Margin: 1st Reply.] The reply, first, is to deny the Major. For it would not be a convertible sign of such a motion — granted that, if such a motion of the Earth were posited, it could very probably be adduced as at least a partial cause of such a wind, provided the wind were perceptible universally within the Tropics, and in the neighbouring Climes as well, where plains and seas spread out more widely. For it is no more a sign of the Earth going towards the East than of the Sun, which by its diurnal motion, with the whole chorus of the stars, towards the West, leads round with itself the vapours and exhalations it has raised, or successively raises one after another towards the quarter into which it is carried; or, again, of the sea waters, which are known to run towards the West, as a cause assisting the generation and motion of that breeze. For it is evident that all the waters of rivers, torrents and canals draw with them some breeze or other, a little wind, in the direction in which they themselves run down. And so the ship-masters, as we saw from Dudley, credit the benefit of that wind to the diurnal motion of the Sun. And it is certainly admitted among the Meteorologists that winds are raised mostly, and stronger, when the Sun is rising or about to rise, and more often from the Eastern quarter than from the Western; and that on the broad plains of the Ocean they are more uniform, for this reason: that there is no inequality there of mountains, valleys, woods and cities, which usually supplies the matter of unequal winds — as Descartes in particular teaches, briefly but lucidly, in his *Meteors*, chapter 4, marginal numbers 3, 4, 12 and 13.

[Margin: 2nd Reply.] I reply, second, by denying the Minor taken universally, or with those conditions which are required for such a wind to be an effect and sign of the terrestrial rotation. For neither does a wind of this kind blow in all the seas or plains within the Torrid Zone, however widely open, nor, where it does blow, does it blow directly from the eastern quarter, but mostly from a lateral quarter lying between the Solanus and the Aquilo, or else between the Solanus and the Auster; and the Euroaquilos are the more frequent; nor do they keep the same tenor all day, but vary perceptibly with the rising and the afternoon Sun, as we have already seen from Father Acosta's history.

III. Argumentum à Terræ Chymicæ & Terrellæ Magneticæ ſimilitudine. Vbi obiter quædam inſignia experimenta magnetica ope Terrellæ facta.

III. The Argument from the Likeness of the Chymical Earth and the Magnetic Terrella; where, in passing, certain notable magnetic experiments made with the help of the Terrella.

[Margin: The Chymical Earth.] IX. Iacobus Lansbergius [Jacob van Lansbergen], in the *Apologia* for Philips van Lansbergen on the motion of the Earth, chapter 5, adduces this argument as his eighth, and says: "The Physical Earth

of the Chymists (which is not a magnet, as Froidmont thought in his *Vesta*) is moved from West to East; therefore so too is the whole Earth, of which it is the image." Georgius Polaccus replies in his *Anticopernicus*, assertion 176, that this Hermetic Earth is to be left to the Paracelsians and the Fluddists, and that from its motion the motion of the whole Earth is not adequately argued. For my part, I have not yet been able to see this experiment. But granting it, we deny the consequence, in the same way as from the rectilinear downward motion of a ball of clay, or of a clod of earth let fall, one may not infer the downward motion of the whole Earth. Setting this aside, therefore, let us come to the argument taken from the Magnetic Terrella.

[Margin: What a Terrella is.] X. A Terrella — that is, a Magnetic one — is nothing other than a small sphere fashioned on the lathe from the Magnet stone; or it is a globular Magnet, customarily formed so by art, because this figure is the fittest of all for undertaking various magnetic experiments. So teach William Gilbert, in Book 1 of *De magnete*, chapter 3, and Book 6, chapter 1 — he gives experiments of this kind in Book 1, chapter 4; Book 2, chapter 27; Book 3, chapters 4, 7, 8, 9, 17; Book 4, chapter 2; Book 6, chapter 4 — and Father Niccolò Cabeo, in the *Philosophia Magnetica*, Book 1, chapters 5, 17, 18, 22, 24; Book 2, chapters 10, 11, 15; Book 3, chapters 4, 12, 16, 17, 19, 21, 34, 37; and Book 4, chapters 9, 11, 17, 36, 37, 39, 40, 41, 45. For in all these places experiments made with the help of the Terrella are explained. From them we shall select three or four, so that the reasoning of this argument may become clearer, first setting out the following diagram, in which let AZBN be the Meridian, and in it A the South Pole of the World and B the North, Z the Zenith and N the Nadir. Then let there be made, from the best Magnet obtainable — an Arabian one, say — a sphere as round as possible, CbDa, which from now on is to be called the Terrella.

Diagram. A large circle, the meridian, encloses a smaller hatched circle, the Terrella, whose centre lies a little below that of the large circle. A thread hangs from the top of the large circle to the top of the Terrella. Small arrows, each marked with a cross and a dot, stand at various points between the two circles and at the top and foot of the Terrella. Inside the Terrella a dotted vertical line carries R above and P below, and a dotted horizontal line runs from a to the point opposite it.

- [?] (head of the thread, top of the large circle): a curled sort, unread
- A (left, outside the large circle): South Pole of the World
- B (right, outside the large circle): North Pole of the World
- N (foot of the large circle): Nadir
- E (upper left, between the circles)
- F (upper right, between the circles)
- K (left, between the circles)
- M (left, between the circles)
- I (lower left, between the circles)
- H (foot left, between the circles)

- G (lower right, between the circles)
- S (right, between the circles)
- c or e [?] (top of the Terrella)
- R (inside the Terrella, upper)
- P (inside the Terrella, lower)
- D (foot of the Terrella)
- a (left of the Terrella)
- b or 6 [?] (right of the Terrella)

[Margin: Investigation of the Poles of the Terrella.] Since every Magnet has two faces, one southern and the other northern, one must first investigate in this Terrella each of these faces, that is, its poles. This is not done by suspending it from the thread ZC and noting which part it turns, by its own turning, towards the South and which towards the North, as if the points a and b were to be taken for the Poles of the Terrella. [Margin: 1st Experiment.] For, as Cabeo observes in Book 5, chapter 5, from this one concludes only that the south pole of the Terrella is in the hemisphere CaD, and the north pole in CbD; and it can happen that the former is at F or at G, and the latter at H or at E, because the point C, from which the Terrella was suspended, is not a full quadrant distant from its Poles, but less than a quadrant — 45 degrees, say. [Margin: 2nd Experiment.] A surer method, therefore, is if small bits of a rather fine iron wire, scarcely longer than a barley grain, you bring them up to various points of the Terrella. For where they adhere to the Terrella most straightly and perpendicularly, know that its poles are there; and where they touch the Terrella with equal balance and inclination on both sides, those points will be a full quadrant of a circle distant from the poles. [Margin: 3rd Experiment.] Another method, which Cabeo suggests in the same chapter 5, is of this kind. Apply a versorium, that is, a magnetic needle, on a pivot that can be carried round over the Terrella, to different parts of the Terrella's surface, until at some point it obtains a most exact equilibrium, as at C — that is, it inclines towards the Terrella no more with its point than with its little cross, but makes equal angles on both sides with its periphery. By the same device search out several other points, say R, P, D; for when a circumference is drawn through them, you will have the Equator of the Terrella, which is equidistant from either Pole. So, with one leg of a pair of compasses fixed at any one point of the circle CRPD, you will be able to describe with the other leg, bent, some meridian of the Terrella — say aCbD; for in it the points a and b, marked off at a quadrant's distance from the Equator, will be the Poles of the Terrella. Then, if you suspend the Terrella from C, a point of the Equator, by a thread twisted as little as possible, it will turn itself towards the poles of the World in such a way that with its north pole b it faces the North pole of the World, B, and with its south pole a it looks towards the South Pole of the World, A; and the axis of magnetic direction will be ba — keeping, however, as to its situation with respect to the Meridian, the declination from the Meridian which the place where you live requires, and which magnetic needles keep in any case.

And so, if the experiment is made at Bologna, the axis ba of the Terrella will decline towards the East by 3½°, and elsewhere otherwise, according to what was said in Book 2, chapter 18. When versoria, or needles, are applied to other points, say to E, F, G, H, you will see them incline more towards the Terrella with the point or with the little cross, as the magnetic face and the poles of the Terrella require; and at I and K the points will converge towards the pole a, while the little crosses, turning away from each other, will diverge. [Margin: 4th Experiment.] But what bears most on our subject is this: if you enclose the Terrella in a wooden box or a little cork net, and set the box on the water of a broad vessel so that it can float and turn freely; and close the room or chamber so that no wind can get in, and keep iron tools far away; you will see, not very long afterwards, the Terrella, by its own turning, come to rest at last in the situation befitting it, so that it faces with the pole b the North pole B, and with its pole a the South pole A, and rests there, with the axis ab raised towards the pole visible to you by about as much as the altitude of your pole requires. If you lived under the Equator, however, that axis would be parallel to the horizontal plane AB.

From this, then, Gilbert concludes in Book 6 of *De magnete*, chapter 4, that just as the Terrella has a natural faculty for circular motion about its own centre, by which it turns itself in the plane of the horizon with its poles to the poles of the World, so the whole earth, which in the same book he contends is a great Magnet, has a natural power for circular motion about its own centre; so much so that if it were pulled away from its situation, and its north pole were twisted elsewhere by some force, it would run back to its former situation by a natural rotation. He adds in chapter 6 that all terrestrial things, although separated from the earth, are nevertheless drawn round by it through magnetic effluvia, just as he thinks that the parts of each primary globe — the Sun, say, the Moon and the other Planets — would be drawn by them if they were separated from them. [Margin: The argument in form.] With these things posited, he seems to argue in this form: *The whole Earth stands with regard to the motion of rotation about its own centre as the Terrella stands with regard to the motion of rotation about its own centre. But the Terrella is moved circularly about its own centre; therefore the Earth too is moved about its own centre; therefore also by the diurnal motion.*

Someone might reply by denying the Major, on the ground that the Earth is not (as Gilbert contends it is, in Book 6 and elsewhere) a great magnet, or of the same lowest species as the magnet. For our Cabeo refuted this excellently in Book 1 of the *Philosophia Magnetica*, chapter 19, with many arguments indeed, but above all with this one: that if the Earth were a perfect magnet, it would snatch iron to itself from a far greater distance, in proportion to its bulk, than the tiny magnetic Terrella snatches a needle. But let us grant the Major for the time being.

[Margin: 1st Reply.] The reply, however, first, is this: granting the Major, we distinguish the Minor and the first Consequent, conceding the Minor and the Consequence if the Terrella and the Earth are moved out of their situation,

and denying them if they remain in their natural situation, which the axis requires; but we deny the second Consequence, for more is inferred than can be inferred from the premised propositions. For the Terrella has no rotation at all about its poles towards the East, even if it is suspended on its poles in the Meridian, whatever Petrus Peregrinus [Pierre de Maricourt] may have invented — whom Gilbert himself, in Book 6, chapter 4, not to mention Cabeo in Book 3, chapter 4, rejects, as we shall shortly see. Nor could it be proved to Father Athanasius Kircher or to Father Nicolaus Zucchius [Niccolò Zucchi], most learned writers on the Magnet, since they rather confirm the immobility of the Earth from the Earth's magnetism.

[Margin: 2nd Reply.] The reply, second, therefore, is to retort the argument: *The whole Earth stands with regard to motion and rest as the Terrella does. But the Terrella, as long as it is in its natural situation, is not moved; and when it is in an unnatural situation, it is not moved towards the East about its axis, but by another motion, one that turns the axis to the poles of the world. Therefore the whole Earth too, as long as it is in its natural situation, is not moved; and if it were in an unnatural situation, it would not be moved towards the East, but by another motion, etc.* Now it is established from the tradition of all the centuries in which observations of polar altitudes have been held that the Earth has been in its natural situation, and has always faced the poles of the world directly with the axis of its Equator. Therefore the Earth has always rested immobile, as it rests now too. The Major is already assumed by Gilbert, and urged by certain experiments in Book 1, the last chapter: namely, that the Earth has a magnetic virtue, by force of which it directs itself to the poles of the World, or keeps that direction acquired once from the beginning of the world. [Margin: Niccolò Cabeo's and Zucchi's arguments for the magnetic force of the Earth.] And it is further proved by Cabeo, in Book 1 of the *Philosophia Magnetica*, chapters 17 and 18, and by Father Niccolò Zucchi in his *Nova de machinis philosophia*, part 5, section 3. First, because it has been observed that the veins of earth and of stone which are seen on the sides of mountains after floods run out parallel to the axis of the world — which Kepler testifies was observed in Germany too — and meet the horizon, to sense, at an angle as great as the altitude of the pole. Next, the Earth communicates magnetic force to iron tools, as Gilbert tries to show in Book 3, chapter 12, but Cabeo shows by firmer experiments in Book 1, chapter 17. Now no one gives what he does not have; therefore the Earth has a magnetic virtue, which it communicates to iron tools — so much so that the bars of iron gratings drawn out of very old windows attract with their upper part the north point of a versorium, and with their lower part the southern little cross, in our regions, where the North pole is elevated; and they attract from a distance of four ells and more, which no magnet, however strong, can achieve. Again, if you bring a dagger or a sword, held by the hilt perpendicular to the horizon, up to a versorium or magnetic needle, the part that hangs over the versorium attracts the southern part of the versorium; and if you bring the same sword up to it from below, under the versorium, in a perpendicular ascent, the point of

the sword attracts the northern part of the versorium. Then, in chapter 18, he teaches that this magnetic force was given to the whole terrestrial globe so that it should keep itself in its situation towards the poles of the world, and could not be moved about its centre either by the force of winds or by a change of its centre of gravity. From this, too, the immobility of the Earth is confirmed.

[Margin: Petrus Peregrinus's fiction about the diurnal rotation of the Terrella.] XI. But since Petrus Peregrinus, as Gilbert reports in Book 6, chapter 4, steadfastly affirms that the Terrella, suspended on its poles in the Meridian, is moved circularly in a complete revolution of 24 hours, let us hear what those most experienced in magnetic matters think of this. And first, Gilbert adds in the same place: *This, however, it has not yet been our lot to see; and we doubt this motion too, on account of the weight of the stone itself; and also because the whole earth, as it is moved by itself, so is it also moved on by the other stars — which does not happen proportionally in any part, as in the Terrella. The earth is moved by its own primary form and by natural desire, for the conservation, perfection and adornment of its parts.*

[Margin: Cabeo's teaching on the motion of the Terrella and of the Magnet.] But Cabeo, in Book 3, chapter 4, teaches that a magnet, and therefore a Terrella, has no power of directing itself to any parts other than the polar ones, and has no eastern or western face; and this both a posteriori and a priori. A posteriori, indeed: because if you set a magnetic globe, exquisitely round, on its poles, found as described above, so that it can be turned very easily, and place its axis parallel to the axis of the earth; or insert it in two circles, in almost the same way as a nautical compass box is usually enclosed in bronze circles, so that, whatever the motion of the ship, the box stays parallel to its plane of the horizon; and set it, so inserted, magnetically, that is, by directing its axis to the poles of the Earth, and parallel to the axis of the Earth, it will never be moved towards any other quarter of the sky, except as far as the magnetic declination in that place requires; which is a sign that the Terrella neither regards nor loves any but the polar points. [Margin: The magnet regards not the heavens but the poles of the Earth.] A priori, moreover: because the Magnet does not, properly speaking, regard the heavens or points of the heavens — neither Ursa Minor, as Marsilius Ficinus thought; nor other stars, as Paracelsus did; nor the rising of a star in the tail of Ursa Major, as Gauricus [Luca Gaurico] and Cardano did; nor the Pole of the Zodiac, as Bessardus the Frenchman [Toussaint de Bessard] did; nor a point near the pole of the world, as the Conimbricenses and Martinus Cortesius [Martín Cortés] did — but the poles of the Earth, as Robert Norman, a most skilled sailor and an ingenious craftsman, first observed. Cabeo, too, in Book 1, chapter 15, Zucchi in the place cited above, and Athanasius Kircher in Book 1 of his *Magnes* confirm this by experiments. For if you hang by an untwisted silk thread a very thin little plate of steel, perfectly straight, eight or ten fingers in length and pierced in the middle, and excite it magnetically — that is, touch one end of the little plate with one face of a magnet and the other end with the other — you will see that the part which faces the northern hemisphere (if

you live in some northern climate) is not raised upwards so as to regard the North pole, or any part near this pole, but is depressed downwards towards the earth, and the more so the higher the pole stands in that climate; and that the other end is raised, so that it cuts the axis of the world crosswise. Lest you think that this comes from a less than exact equilibrium, destroy the magnetic virtue by a contrary touch — that is, touch with the north face the end of the little plate that you had touched with the south face, etc. — and you will see the end which was depressed before raised, and the other, which was raised, depressed. The same is proved by another experiment, from Gilbert's Book 5, chapter 9, and Cabeo's Book 3, chapter 4: namely, if you pass a straight iron wire through a little cork sphere, equally on either side, and set it on still water, sheltered from every breath of air, so that it sinks below the surface of the water but not to the bottom, and can turn itself freely — but before you put the wire in the water, its ends must be rubbed with a magnet. When this is done, you will see the wire decline towards the earth below the plane of the horizon with its north end, and indeed by almost as much as the altitude of the Pole is in that place. Again, magnetic needles brought up to a Terrella direct themselves to the poles of the Terrella; therefore they do not look to the poles of the world, or to the stars. Lastly, see the instrument which, invented by Gilbert in Book 5, Cabeo perfected in Book 1, chapter 20, and Kircher reported in Book 1 of the *Magnes*: for it teaches a way of suspending a very long magnetic needle in most exact equilibrium, so that, inside a vessel covered with a glass hemisphere, one may observe its motion, and by how many degrees, marked on the semicircle of the bowl, or lower hemisphere, it descends with its north point below the horizon. Granted that the measure of these inclinations which Gilbert assessed does not answer to the observations made by experts beyond the Equator, as Kircher testifies in his *Magnes*, Book 1, part 2, and Zucchi in part 5 of the *philosophia de machinis*, section 8. Since these things are so, and it is sufficiently plain from this that magnetic bodies do not regard or love the poles of the heavens of themselves and immediately, but rather the poles of the Earth, while the Earth itself so regards the poles of the world that, being spherical and homogeneous, it has no part of itself which is absolutely eastern or western — for the same part, say the land of Rome, which is eastern to the Mexicans, is nevertheless western to the people of Goa (which cannot be said of parts situated under the same Meridian, for some of them are absolutely northern and on this side of the Equator) — it follows, says Cabeo in Book 3, chapter 4, that the whole earth has no power of directing or moving itself to other points of the heavens than to the poles of the world.

[Margin: Athanasius Kircher's opinion on the Earth's motion impeded by magnetism.] XII. Further, Athanasius Kircher too, in Book 1 of his *Magnes*, part 2, from page 53, teaches that circular motion is more repugnant to the Earth than straight motion; for gravity alone impedes straight motion upwards or downwards, but by magnetism the Earth is directed along the axis of the world — so much so that even if there were a mountain as high as the Earth's

diameter, it still could not be moved from its situation by a driving wind. And in Book 2, part 5, he teaches that the itch to establish the Copernican motion is intolerable, for by force of it occasions for urging that motion are begged from every quarter out of uncertain and slippery experiments. However, Father Niccolò Zucchi, [Margin: Zucchi's opinion on the magnetism of the Earth.] in his *Nova de machinis philosophia*, part 5, section 10, does indeed reject the motion of the Earth, but nevertheless asserts that the magnetic force in the globe of the Earth is not there to impede the diurnal rotation about its axis, because the direction of the axis to the poles of the world can stand and that motion nevertheless be carried out. Next he mentions a certain new experiment, made by some investigator of nature or other, by which it is proved that a Magnet turns and settles itself, according to its own determinate plane, in the Meridian of any region whatever, if it is set up through its axis in a vertical line so that the one pole of the magnet which usually turns to the North is raised upwards; for then certainly those parts of it turn to the West which, when that same pole was turned downwards, always stood towards the East. He gives the reason for this there and in section 6: from the refraction of the magnetic rays diffused from the poles of the Earth to the mass from which the Magnet was formed — a refraction that was then made in the plane of the stone congruent with the meridian of the Earth, precisely along up and down, and not of itself from the magnetic force. But consult the Author yourself. Meanwhile it is sufficiently clear that this very motion, too, is not made from West to East upon the axis of the Equator, in the Equator of the Magnet itself. Therefore the diurnal conversion of the earth has not yet been proved by any magnetic experiments.

XIII. So much for the arguments which it seems can be devised for the diurnal revolution of the Earth about its axis and centre. For what Galileo sought out from the flux and reflux of the sea pertains not only to establishing the diurnal motion but to establishing other motions of the Earth as well, and will therefore have to be examined below in a separate chapter.

CAPVT VIII.

Chapter 8.

Propon[?]ntur & Diſſoluuntur Argumenta Decem pro Tellure extra centrum Mundi per Annuum motum collocanda, ex Conditionibus varijs Telluris & Planetarum, motum non inuoluentibus.

Ten Arguments for placing the Earth outside the centre of the World by an Annual motion, taken from various Conditions of the Earth and the Planets that do not involve motion, are proposed [?] and resolved.

[Margin: The order of examining the arguments for the annual motion of the Earth.] I. All the arguments which have been devised with most subtle

inquiry by the Philolaics and the Copernicans for removing the Earth, if it could be done, from the centre of the universe, and for carrying it about that centre in an annual orb, seem reducible to four general heads: namely, either to certain conditions of the Earth compared with the conditions of the Planets, but not yet involving motion; or to the motion of the Planets itself; or to other motions observed in the heavens; or, finally, to motions or changes observed in the sphere of the elements. We too, therefore, shall keep this order in weighing them.

I. Argumentum à Nobilitate Solis maiori quàm Telluris.

1. The argument from the nobility of the Sun, greater than that of the Earth.

II. There seems to be no doubt that the centre of the Universe is the noblest place in the world, since it is equally distant on every side from the extremes and holds the middle situation. Hence it came about that certain Pythagoreans, as Aristotle reports in *De caelo* 2, texts 72 and 73, placed in the middle of the Universe not the Earth but Fire, in order to assign the most honourable seat to the most worthy of bodies; and this Fire they called the guard, or prison, of Jupiter. That the chief author of this opinion and argument was Philolaus we have already learned from Plutarch, *De placitis* Book 3, chapters 11 and 13, and we have already reported his words in chapter 3 of this section, where, from number 3 on, we also treated of Vesta, placed by the ancients in the centre of the World as a terrestrial Fire. [Margin: The argument of Philolaus and Kepler.] This Fire, however, the Copernicans assert to be the Sun — above all Kepler, in Book 4 of the *Epitome of Copernican Astronomy*, page

444: for, they say, the Pythagoreans were accustomed in this way to conceal their doctrines under other words, and by Fire to have meant the Sun, as the most excellent of all bodies and the one to which the most fitting place is owed for guarding and preserving it; and this place, they say, is the centre of the Universe. Aristotle had tried to blunt this argument of the Pythagoreans, saying that the middle of magnitude and the middle of the nature of the thing are not the same in a cosmic body — just as in animals the principle of life, which is the heart, is not precisely in the middle of the animate body, if you look to the middle of magnitude, but in another, more fitting place — and that one must therefore seek elsewhere than from the centre of the bulk of the universe itself which place is more fitting for that principle on which the chief operations of the universe depend. To this Kepler says, in that place, that the case of animals, whose bodies are not round, is unlike that of the World, which is spherical and needs no operation outside itself; so that there is no reason why its heart should not be in the centre of the universe. He adds that Aristotle, in looking for baseness in the centre, does not do rightly, since the centre is the principle of a likeness that foreshadows the mystery of the Trinity: for by the centre the first person, the origin of the others, is designated. Oh, how many mysteries to

dress up this most impoverished argument! Reduced to the form of a syllogism, it is as follows.

[Margin: 1st Argument in form.] III. *To the most excellent of the cosmic bodies the most excellent place is owed. But the Sun, and not the Earth, is the most excellent body of the world, and the most excellent place of the Universe is the centre; therefore the centre of the Universe is owed to the Sun, not to the Earth.*

[Margin: Reply.] The reply is made by conceding the Major, of a place most excellent not by mere Geometric excellence but by the physical end and good which such a body ought to receive from others, or to communicate to others. But in this sense the latter part of the minor proposition is conceded, with regard to the natural end, of which we are speaking here; but the former part is denied. For the Earth ought not to be taken barely as a mere element, one of four or of three elements, but together with the living plants and the animals, and above all with men, for whose sake all the stars were made and are moved, as God attests in Deuteronomy; and taken thus it is the most excellent of all the bodies of the world, if, as is fitting, we value the greatness of its virtue and the dignity of its end rather than the greatness of its bulk. I said "speaking of the natural order and end": for if we speak of the supernatural, the centre of the Earth is the lowest and most wretched place, and the place of the prison of Hell, which is no longer the guard of Jupiter, nor of a saving Fire, but of the Fire destined for the eternal burning of the damned; whereas, on the contrary, the Empyrean — that is, the earth translated into heaven, or the Land of the living — is the most excellent place.

II. Argumentum à Centro Syſtematis Planetarij.

2. The argument from the centre of the Planetary System.

[Margin: 1st Reason of Copernicus for the centre of the Planetary System.] IV. The Copernicans have affirmed, for two causes chiefly, that the centre of the system of the Planets is not the Earth but the Sun. The first Copernicus touched on in Book 1, chapter 5, in these words: *For if anyone should deny that the earth holds the middle or centre of the world, and yet not admit that the distance is so great as to be comparable to the sphere of the non-wandering stars, but notable and evident with respect to the orbs of the Sun and the other stars; and should think that for this reason their motion appears varied, as though they were regulated about another centre than the centre of the earth — he could perhaps offer a not inept reason for the varied apparent motion. For the fact that the wandering stars are seen nearer to the earth, and the same ones farther away, necessarily argues that the centre of the earth is not the centre of their circles. Still less is it established whether the earth nods towards them and away from them, or they towards the earth and away from it.* Hence Galileo, in Dialogue 3 of the *System of the World*, page 241 of the Latin, when Simplicio asks how it is established that the Sun is in the centre of the revolution of the Planets, answers in the person of Salviati that this is made out by most evident and necessary observations. The first of these is that all the

Planets are found now nearer to the earth, now farther from it, by differences so enormous that Venus, at its most remote in its apogee, is found to be six times farther from us than when it is nearest and perigee; and Mars, when apogee, has a distance from the earth eight times greater than when perigee. This argument, however, he says, does not hold of the Moon, because it is established on other grounds that the Moon cannot be separated from the earth as the centre of its motion.

V. The same Copernicus brought forward the second reason in Book 1, chapter 10: namely, that the five lesser Planets complete their circuits about the Sun. Let us hear him, so that we may draw more clearly from the very source of this doctrine. [Margin: 2nd Reason of Copernicus.] *Therefore,* he says, *I think that what Martianus Capella, who wrote an Encyclopaedia, and certain other Latins knew well is by no means to be despised. For they think that Venus and Mercury run round the Sun, which exists in the middle, etc.* And shortly after: *What else, then, do they mean to signify than that the centre of those orbs is about the Sun? Thus indeed the orb of Mercury will be enclosed within that of Venus, which must be twice as large and more, and will obtain within that amplitude a place sufficient for itself. If anyone, taking occasion from this, should refer Saturn too, and Jupiter and Mars, to that very centre — provided he understands the size of their orbs to be so great that, together with them, it contains and surrounds even the earth abiding within — he will not err, as the canonical reckoning of their motions declares.* And he at once confirms this, saying: *For it is established that they are always nearer to the earth about their evening rising, that is, when they are opposed to the Sun, with the earth between them and the Sun; but most remote from the earth at their evening setting, when they are hidden about the Sun, namely when we have the Sun between them and the earth. These things indicate sufficiently that their centre belongs rather to the Sun, and is the same as that to which Venus and Mercury too refer their revolutions.* Galileo pursues exactly the same reasoning in Dialogue 3 of the *System of the World*, likewise page 241, and Kepler in Book 4 of the *Epitome of Copernican Astronomy*, page 536. But what Copernicus had assumed about Venus and Mercury on the mere authority of Capella and of certain Latins, Galileo himself affirms much more certainly by the experiment of the Telescope, by which it is now established that these Planets are illuminated by the Sun just as Moons going round it are. By these conjectures, then, partly, and partly by certain observations, both these men and the other Copernicans — above all Rheticus in his *Narratio prima*, and Kepler in the introduction to the *Commentaries on Mars* and in the *Epitome of Astronomy*, page 450 (who had nevertheless rightly advised, on page 449, that Copernicus's hypothesis is not disturbed if the Sun, rather than the centre of the great orb, is placed as the centre of the Universe) — try to persuade, and as it were to demonstrate, that the Sun is in the centre of the world; that it was therefore deservedly called by the Ancients [Greek: χορηγὸν (chorēgon, "leader of the chorus")], that is, the Choragus of the Planets and the moderator of Nature; and that those praises of the Sun are no more Poetic than Physical. Now, then, to the argument.

[Margin: 2nd Argument in Form.] VI. *That body ought to be in the centre of the Universe which is the centre of the System of the Planets; but the centre of the Planetary System is not the Earth but the Sun: therefore not the Earth but the Sun ought to be in the centre of the Universe.* The MAJOR is not indeed proved by the Copernicans, but rather supposed, as though it were certain that the other Planets are moved about the centre of the world, on the ground that the system of the Planets seems the chief part of the system of the world. The MINOR has already been proved at numbers 4 and 5, from the notable variety in the distance of the lesser Planets from the Earth, and from the circuit of Venus and Mercury about the Sun, from which the same becomes very probable of the three remaining Planets.

[Margin: 1st Reply.] I reply by distinguishing the Major, and denying it if that body is not the centre of the whole Planetary System taken adequately, nor of the Fixed stars, nor of the new Phenomena, nor of the elementary motions and of the animals living in them; for all these, just as they are to be considered for the integrity of the Universe, so too are they to be considered for constituting its centre, inasmuch as the centre includes a relation to all, and not only to some, of the great parts of the world. But if it is such a body, I concede the Major; and in this sense I deny the Minor. For granted that it is now certain, against Ptolemy, that the Sun is the centre about which Venus and Mercury revolve, and that, according to Tycho, the Sun is the Choragus of the five lesser Planets, which describe their orbs around it; it is nevertheless not the centre of the Lunar motion, nor of the motion of the Fixed stars, nor of the new stars, nor of the Comets, nor of the Elementary motions. Indeed, according to our system, which can probably be maintained with the phenomena saved, it is the centre only of the circulations of Mercury, Venus and Mars, as of its satellites; for the reasons indica- ted in Section 3, chapter 9. Indeed, if we wish to pursue every nicety, not even in the hypothesis of Kepler and Boulliau is the Sun the centre of the Planetary orbit, since they determine that orbit to be not a circle but an Ellipse, and place the Sun not in the centre of the Ellipse but only in one of the umbilici, or foci, of the Ellipse. But we do not lean on this last subtlety, because in every hypothesis of a Planetary orbit, on account of the Eccentricity required to excuse the anomaly, its centre is distant from the centre of the Universe — except the orbit of the Earth, whose centre, in the opinion of Copernicus, is in the very centre of the Universe, and is distant from the Sun only by as much as the Eccentricity of the Solar orb is in the hypothesis of the others.

[Margin: 2nd Reply.] I reply, secondly, by retorting the argument thus: *That body ought to be set in the centre of the Universe which is the centre of the motion of both the non-wandering and the wandering stars, of both the old and the new phenomena, and of both the elementary, or subcelestial, and the celestial bodies. But such a body is not the Sun but the Earth. Therefore not the Sun but the Earth ought to be set in the centre of the Universe.* The MAJOR certainly carries far greater probability than if the centre of the Universe is referred only to the

circulations of some Planets, all the other parts of the World being neglected. The MINOR has no difficulty except as regards the Planets, for it seems that not the Earth but the Sun is the centre of the Planetary system. But by the very fact that the Earth is the centre of the motion of the great Luminaries, namely the Moon and the Sun, it is also, through them, the centre of the other Planets, if these are determined to be carried about the Sun, as Tycho will have it. In our system, however, the difficulty is less: for the Sun is the centre only of Mercury, Venus and Mars, these being the satellites of the Sun and belonging to the same heaven, while the Earth is the centre of the orbs of Jupiter and Saturn and, through them, of the Jovian and Saturnian satellites.

III. Argumentum à Fonte Luminis & Caloris.

3. The argument from the source of Light and Heat.

[Margin: Kepler's opinion on the source and place of light.] VII. This argument seemed splendid indeed to Kepler, in the *Astronomiae Pars Optica*, chapter 1, page 7. There, having brought forward many praises of light — as the most excellent of visible things in the world, the matrix of all the faculties of animals, and the bond of the corporeal and the spiritual world, as he himself thinks, and therefore created by God on the very first day of the world — he added: *The Sun, then, is a certain body; in it is this faculty of communicating itself to all things, which we call light; to which, even for this reason alone, the middle place in the whole world is owed, so that it might diffuse itself equably and perpetually into the whole orb.* [Margin: Praises and place of the Sun, from Kepler.] But in the same work, chapter 6, from page 221 to 226, he pours himself out wholly in praises of the Sun, and attributes to it a density and quantity of matter as great as there is in the whole aethereal air, and yet such diaphaneity that our eye sees its depth and centre; and a soul, or vital faculty; and many other things. But to our purpose he says *that the office of the Sun in the world is, by the admission of all, that of the Heart in an animal, and that the motions of the Planets are dispensed from the Sun, etc.* Then in the *Epitome of Copernican Astronomy*, Book 4, page 448, he says that if the whole world, which is spherical, needs the light and heat of the Sun equally, the Sun is best placed in the middle, from where these may be distributed equally to all quarters of the world; that this happens more equably and more rightly if it rests in the centre of the World than if it goes round about the centre and, by receding from some and approaching others, illuminates and heats these more and those less; and that the comparison of a lantern, in whose middle we place the light, is not to be laughed at. But quite new are the things he adds on this subject in the same place, saying: *Of light, moreover, a particular demonstration is woven, supposing fitness, not necessity. Imagine the sphere of the Fixed stars to be a concave mirror: it is known that an eye placed in the centre of such a mirror sees itself on every side; and that if a light is in the centre, it is reflected on every side from the concave surface at right angles, and the reflected rays meet again in the centre. But this can happen at no other point of the*

concave except the centre. Therefore, since the Sun is the source of light, the eye of the world, the centre will be owed to it, so that (being itself, in the divine symbolism, the Father) it may contemplate itself in the whole concave surface (which bears the symbol of the Son of God), [Margin: Kepler's symbol of the Most Holy Trinity.] and may be well pleased with its own image in it, and by illuminating may make itself illustrious, and by heating may kindle itself.

VIII. But before Kepler, Copernicus, in Book 1, chapter 10, using another comparison, had assigned this very seat to the Sun in these words: [Margin: The praises and place of the Sun, from Copernicus.] *In the middle of all, moreover, resides the Sun. For who in this most beautiful temple would place this lamp in another or a better place than that from which it can illuminate the whole at once? For not ineptly do some call it the lantern of the world, others its mind, others its ruler; Trimegistus [Hermes Trismegistus] calls it the visible God, the Electra of Sophocles the all-seeing. Thus indeed the Sun, as though residing on a royal throne, governs the family of the stars that circles about it.* [Margin: And Pliny the Elder's.] To this may be drawn what Pliny the Elder utters most magnificently about the Sun in Book 2, chapter 6, and what we have recited from him in Book 3, chapter 1. From this, then, the following argument is built.

[Margin: 3rd Argument in Form.] IX. *That body ought to be set in the centre of the Universe which is the cause and source of the light and heat diffused through the whole world. But such is the Sun; therefore the Sun ought to be set in the centre of the universe.* The MINOR is certain with everyone. The MAJOR is proved both from the equal communication of heat and light, which ought to happen and cannot unless the source of light and heat is equally distant from all parts of the world; and from the comparison of the mirror, the lamp, etc., of which at numbers 7 and 8.

The reply is made by distinguishing the Major: denying it if the body is not the universal cause of light, and the very things which receive light from it do not receive light equally; but conceding, or granting, it if the body is the universal cause and illuminates all things in the same way. But in this sense the Minor is denied. For the Sun neither is the cause of light in the Fixed stars, as is plain from what was said in Book 6, chapter 2, even with Kepler himself agreeing; nor does it illuminate the Earth and the Planets equally — both on account of its unequal and from time to time variable distance from the Earth, and that of the Planets from it, and on account of its manner of illuminating them. For we discern some phases in the Moon; others in Mercury and Venus, which are sometimes seen sickle-shaped; others in Mars, which sometimes appears gibbous but never sickle-shaped; others in Jupiter and Saturn, which to sense always appear round — granted that these too receive their primary light from the Sun. That equality, therefore, is feigned, and put forward with little consideration. Nor is the comparison from the mirror valid either: since the sphere of the Fixed stars is not a mirror of the Sun, for otherwise the whole of it would shine together; nor are the Fixed stars images of the Sun, for otherwise they would not have their light from themselves, and all those equally distant

from the centre of the world would appear of the same magnitude, and at any point whatever of the celestial concavity from which rays could reach our eyes by reflection. It suffices, then, if the Sun is in such a place that it can illuminate the Earth and the Planets all round — but in that manner and measure which the end of divine Providence requires.

IV. Argumentum à Fonte Motus.

4. The argument from the source of Motion.

X. We have already heard this argument too hinted at by Copernicus in Book 1, chapter 10, when he calls the Sun the ruler and governor of the Planets driven round it. But Rheticus added force to it in his *Narratio prima*. [Margin: Rheticus's opinion on the source of motion and on the place of this source.] For when he had said: *In the common principles of Astronomy one could indeed see that all the celestial appearances direct themselves to the mean motion of the Sun, and that the whole harmony of the celestial motions is constituted and preserved by its governance. Hence the Sun was also called by the ancients* [Greek: χορηγὸς (chorēgos, "leader of the chorus")]*, governor of nature, and King. But how it should carry on this administration — whether in the way that God governs this whole universe, as Aristotle most beautifully depicts in* [Greek: περὶ κόσμου (peri kosmou, "On the World")]*, or whether it would act as God's administrator in nature by itself traversing the whole heaven and resting in no place — does not yet seem to be wholly explained and settled.* Having set these things down beforehand, I say, he adds shortly after: *The former mode of governance has been rejected, the latter received. But my Lord Doctor and Teacher* — he means Copernicus — *has decided that the condemned account of the governance of the Sun in the nature of things is to be recalled, yet so that the one received and approved too may keep its place. For he saw that in human affairs, too, there is no need for the Emperor himself to run [?] through the cities one by one in order to discharge, in short, the office laid on him by God; nor for the heart to migrate into the head, or the feet, and the other parts of the body for the conservation of the living creature, but for it to preside over its office through other* [Greek: ὄργανα (organa, "organs" or "instruments")] *destined by God for this.* After this he says, lest the mean motion of the Sun in the other planets should serve in imagination alone rather than have a true efficient cause, this was brought about by the annual motion of the Earth in the great orb; for from this the cause is ready to hand why the mean motion of the Sun flows into the theory of all the planets.

[Margin: Kepler's teaching on the source of motion and the place it holds.] XI. But Kepler strives to advance this very doctrine with other reasons, in the introduction to the *Commentaries on Mars*, page 4. There, from the fact that the Planets are spurred on by their approach to the Sun and checked, or slowed, by their recession from it, and that this same thing happens to the Earth if the Earth moves annually, he gathers by a physical conjecture that the Sun is the source of the motion of the Planets and of the Earth, and says: *It is therefore very*

likely that the source of the Earth's motion is where the source of the motion of the other five Planets is: namely, likewise in the Sun. It is therefore likely that the Earth moves, since a likely cause of its motion appears. So, with no delay interposed, he argues conversely thus: *On the contrary, that the Sun stays in its own place in the centre of the world is likely both for other reasons and above all for this one: that in it lies the source of motion of at least five Planets. For whether you follow Copernicus or Brahe, for both of them the source of the motion of the five Planets is in the Sun; in Copernicus also of a sixth, namely the Earth. But it is more likely that the source of all motion stays in its own place than that it moves.* But because he saw that the phenomena of the Sun are saved in the Tychonic hypothesis — that the Sun is slow in its recession from the Earth and swift in its approach, and that the Sun can be the source of light and yet move about the Earth — a little later he added these words: *Upon this most certain conclusion, then, by the physical conjecture used above, this Physical philosopheme would have to be built: that the SUN, together with that whole very great burden of the five Eccentrics (to speak crudely), is moved by the Earth; or that the source of the motion of the Sun and of the five Eccentrics attached to the Sun lies in the Earth. But let the two bodies, of the Sun and of the Earth, be inspected, and let judgement be made of both: to which of them does the source of the motion of the other body more fittingly belong? Does the Sun, which moves the other Planets, move the earth; or does the Earth move the Sun, the mover of the others, so many times larger than itself? Therefore, so that we are not compelled to concede that the Sun is moved by the Earth, which is absurd, immobility must be conceded to the Sun and motion to the Earth.* Kepler has, moreover, another reason peculiar to this hypothesis: that, according to his own fictions, the Sun moves the Planets by a magnetic force diffused by means of light, by which it seizes them and leads them round, as I have already explained in Book 3, chapter 2, number 7. Gathering this argumentation into a few words, then, one may reason thus for the Copernicans.

[Margin: 4th Argument in form.] XII. *That body ought to be at rest in the centre of the Universe which is the source of the motion of the Planets. But the Sun, and not the Earth, is the source of the motion of the Planets; therefore the Sun, and not the Earth, ought to be at rest in the centre of the Universe.* The MAJOR rests on likelihood, which could be strengthened from the analogy of a spring: for a spring does not move, but the river or stream flowing from the spring does; and in mechanics the movers usually stand in a fixed place, so that from there they may move other bodies more firmly. The MINOR is proved, first, from the fact that the motion of the Planets in their second anomaly is adjusted to the mean motion of the Sun — or, as the more recent writers will have it, to its true motion; next, from the magnetic faculty of the Sun, by which it leads the Planets round; and finally because it is more likely that the Sun is the source of the motion both of the Planets and of the Earth than that the Earth is the source of the motion of the Sun and, by means of the Sun, of all the Planets.

The reply, nevertheless, is made by distinguishing the Major. It is denied if that body is not the universal source of motions — that is, of all the Planets,

the Fixed stars, the new Phenomena, and the Elements and elementary bodies — or if it is not a source but an objective exemplar or rule of motions. It is conceded, or granted, if the body is of this kind; and in that sense the Minor is denied. For the Sun is the source neither of the motion of the Elements or of the elementary bodies, nor of the Fixed stars, nor of the satellites of Jupiter and Saturn. Nor is it even the source — that is, the efficient cause — of the motion of the Planets, as Kepler gratuitously feigns, but only an exemplar, which they in part follow or imitate by their motion, as a standard-bearer moves a column of soldiers, and a chorus-leader a chorus of dancers. Therefore the burden of carrying the Eccentrics of the Planets does not fall on the Sun, as though it were a porter, and Kepler's comparison is too crude, as he himself admits. Nor does the Earth move the Sun effectively, or by sustaining it, but objectively and finally: inasmuch as the Sun is moved about the Earth, and for the sake of those who dwell on it, by its own Intelligence — and in a metaphorical motion and mode of moving of this kind we deny that there is any absurdity. Indeed, taking the occasion of the question raised by Rheticus, it is likely that among the other ends intended by God this was one: that it should be evident, from the Sun's very agitation and restless course, that the Sun is not God but the minister of God, ordered to rise upon the good and the bad, and to carry the torch before all the inhabitants of the Earth. For I remember having read in the history of East India that the King of Peru, called Iinga, a worshipper of the Sun even on this very account — that the Sun runs round with such diligence as to help everyone — was refuted by a Spaniard: from this, he said, it should rather be gathered that the Sun is not God, since immobility befits God. What Kepler added about the magnitude of the Sun with respect to the Earth will be dissolved in the following argument.

V. Argumentum à Mole Solis ac Telluris.

5. The argument from the Bulk of the Sun and of the Earth.

[Margin: Kepler's argument for the annual motion of the Earth.] XIII. What is our fifth argument is Kepler's sixth, in Book 4 of the *Epitome of Copernican Astronomy*, page 544, beginning from these words: *For it is more credible that the body around which smaller ones go round is large.* He supposes it certain that Saturn, Jupiter, Mars, Venus and Mercury are smaller than the Sun, and likewise that the Moon is smaller than the Earth, which it goes round, and the Jovian satellites smaller than Jupiter. Then he concludes: *Now if the Sun moves, the Sun, the greatest body, and the three superior planets, each of them larger than the earth, will go round the smaller earth. It is therefore more credible that the Earth, a small body, goes round the great body of the Sun.*

[Margin: Reply.] The reply, however, is that this is not true unless other things are equal. For if what is lesser in bulk is greater in power and in the excellence of its end — and of this kind is the Earth, taken not bare but together with its inhabitants — it is more credible that it rests, and that the other bodies,

which are to serve it by their motion, move about it. Besides, the sphere of the elements, about which the Sun moves, is greater in bulk than the Sun; and yet the Copernicans move it, together with the whole Lunar heaven, about the Sun, which is so much smaller. So the argument, if it were valid, could be turned back against them.

VI. Argumentum ab Indigentia, & à Fine Motus.

6. The argument from Need, and from the End of Motion.

[Margin: Another argument of Kepler's for the annual motion of the Earth.] XIV. The Sun in no way needs the Earth, but the Earth needs the Sun and the other stars: it is fitting, therefore, that the Earth itself should move, and by its motion seek for itself the things it needs. But this argument has already been answered in chapter 5, numbers 31 and 32. As for the end of motion, Kepler, in the *Epitome of Copernican Astronomy*, Book 4, page 549, philosophizes thus. The Earth, he says, in almost these very words, is the dwelling of the contemplative creature; and it was not fitting that man, the inhabitant and observer of this world, should reside in one place of it as though in a closed chamber, for in that way he would never have arrived at the measurement and contemplation of stars so remote. It was necessary, therefore, that just as surveyors usually take different stations, so man, making use of the annual translation of the Earth, should walk around as though by navigation, so that he might more correctly view and measure the several members of this great house. [Margin: Reply to the argument.] But Kepler does not see that this was not necessary either. For it is abundantly sufficient for a man to come out into the open and contemplate the stars; or, if some motion is needed, to walk about the resting earth himself. Nor does the motion of the Earth contribute anything to this for him, since through it he comes no nearer to the Planets than in the hypothesis of a resting Earth; and to the fixed stars he is brought by no sensible approach through the motion of the Earth, since the whole annual orb is like a point with respect to the sphere of the Fixed stars, in the opinion of Copernicus. Conversely, if approach to the stars had been needed in order to measure the motions, diameters and intervals of the Planets and the Fixed stars — as though one had to measure them with ten-foot rods or ropes — this argument would prove too much: namely, that it would also have been needed for man to approach as near as possible to each of the stars, which is plainly quite false. Let Kepler cease, then, to assail the rest of the Earth with such light arguments, or to try with these little machines to move it from its seat; and let him remember that men need the stars not as masters, but as servants serving at God's command.

VII. Argumentum ab Analogia Microcoſmi.

7. The argument from the Analogy of the Microcosm.

[Margin: Another, very light argument, and its solution.] XV. Just as in the Microcosm, that is, the small world — namely man — the feet are moved with a progressive motion but the head is not, so in the Macrocosm, or great world, the Earth has the role of the feet, and accordingly it, rather than the Sun, ought to be moved. But both sides of the parity are denied. For the Earth is not the foot of the world but rather, according to the sacred Scriptures, the immobile footstool of the feet of God; nor, according to the Copernicans themselves, is it the lowest part. And even if it were, it would not have to be moved with that progressive motion with which the feet of an animal are moved, or for the same end, nor carry the head round with it; otherwise the Sun, or the whole heaven, as the head, would also be moved along with its motion. And so this argument too would prove too much, if it carried any weight amid so great a lightness of reasoning.

VIII. Argumentum à Congruenti Craſſitie Cælorum.

8. The argument from the Fitting Thickness of the Heavens.

XVI. Among the other things that seem to favour the Copernican system is, above all, the distinction of the heavens, on account of which no Planet enters the domain of another, and the fitting filling of the space set between them. [Margin: Copernicus's reason for placing the Earth between Mars and Venus.] Copernicus pointed to this in Book 1, chapter 10, when he said: *But with all these resting on one middle, that is, on the Sun, it is necessary that the space left between the convex orb of Venus and the concave of Mars be distinguished as an orb, or sphere, too, homocentric with them in both its surfaces, which may receive the Earth with its attendant Moon and whatever is contained beneath the Lunar globe. For we can in no way separate the Moon from the Earth, etc., especially since in that space we find a place for it that is fitting and amply sufficient.* [Margin: And Baranzano's.] Taking occasion from this, Dom Redento Baranzano, in Part 1 of the *Uranoscopia*, question 3, doubt 10, member 2, argues thus for Copernicus from the thickness of the spheres. Since the thickness of the Lunar heaven is twenty-one terrestrial semidiameters, the thickness of the heaven of Mercury 103, of Venus 933, of the Sun 44 and of Mars 7640, it would seem unfitting and far from all proportion that the Sun, the chief among the stars and of such great power, should have so tiny a heaven unless it were in the centre of the world, while Venus, which is smaller than it, and Mars had so great a depth in their heavens. This disproportion is certainly removed if the Sun is placed in the centre of the World, and the Earth with the Lunar heaven between Mars and Venus; for in this way the thicknesses of the spheres increase step by step: the sphere of Mercury comes out larger than the Sun, the sphere of Venus larger than Mercury's, and the thickness of the elementary sphere together with the heaven of the Moon larger than the sphere of Venus. One could also reason thus against the old hypotheses with the *Philolaus* published at Paris, chapter 7. The concave of the Sun and the convex of Venus are distant from

the centre of the earth by 1070 terrestrial semidiameters, according to certain Alphonsines; the concave of Venus and the convex of Mercury by 162⅔; and the concave of Mercury by 50⅔ semidiameters; but the Moon at the apogee of its Eccentric and Epicycle is distant 64 semidiameters 10′. There are lacking, therefore, 13 semidiameters of the earth and 46′ in the space assigned below the concave of the Sun, for it to be able to contain the whole orbs of Venus, Mercury and the Moon. This absurdity, however, is not found on this head in the Ptolemaic hypothesis, which assigns to the concave of the Sun a distance of 1146 semidiameters of the earth, as Chiaramonti noted in his *Antiphilolaus*, Part 1, chapter 7. Setting this aside, then, there remains the former argument of Copernicus and Baranzano, reduced to the following formula.

[Margin: 7th Argument in form.] XVII. *Unless the Sun is set in the centre of the Universe, and the Earth, with the Elements and the Lunar heaven, between the convex of Venus and the concave of Mars, the due proportion between the thickness of one sphere assigned to the Planets and that of another is not preserved. But it is absurd that the said proportion should not be preserved. Therefore the Sun must be set in the centre of the Universe, and the Earth, with the Elements and the heaven of the Moon, between the convex of Venus and the concave of Mars.* The MINOR seems certain from the orderly manner of the supreme Maker in determining the thickness of the heavens. The MAJOR has been proved, because to the Sun, the greatest of the planets, the smallest heaven, or the least thick possible [?], would belong, while to Venus and Mars, which are far smaller Planets, a far greater heaven would have been given, and the space between the convex of the Sun and the concave of Mars would be excessive and idle.

[Margin: 1st Reply.] The reply is made by denying the Major: first, because it would be a greater disproportion and absurdity for the Sun, which is the greatest of the Planets, to have no heaven than to have a small one, and for the Earth with the other elements — which are commonly reckoned subcelestial bodies, and of the most diverse properties — to be placed in the heaven. Moreover, for the Sun set immovably in the centre of the universe there would be no heaven at all; the whole space between it and Mercury would belong to the heaven of Mercury, since a heaven is ascribed to mobile Planets so that they may go round through it. Next, if Mercury and Venus are assigned to the Sun as satellites, there is no longer any reason why a distinct heaven should be ascribed to them, just as none is assigned to the companions of Jupiter and Saturn. Hence by the heaven of the Sun must be understood that whole heaven in which Venus and Mercury too move; and so it will be a very thick heaven. It will be far ampler still if, according to our system, Mars too belongs to the heaven of the Sun as a Satellite; for in this way a heaven of very great thickness will befit the greatest of the Planets, attended by the greatest satellites. On this thickness see what was said in Section 3, at the end of chapter 9. The remaining objections brought against Ptolemy and Alfonso, as men who always place Venus and Mercury below the Sun, are not valid against the Tychonic system or ours.

IX. Argumentum à Diſtinctione Cælorum.

9. The argument from the Distinction of the Heavens.

XVIII. Now, on the testimony of the Telescope, we are compelled to admit that Venus and Mercury move now below and now above the Sun; and the observations of Mars and of the Sun, and of the parallaxes owed to them, in such a way that fitting equations of the motions follow from them, prove well enough that Mars in perigee comes nearer to the Earth than the Sun. On this see what we said in Book 7, Section 2, chapter 3, scholium 4, and Section 6, chapter 4, scholia 3 and 4, and chapter 10, scholium 1. Now if we attribute to the Sun motion, and a place among the Planets already mentioned, it follows that those Planets, or at least Mars, penetrate the heaven of the Sun — which was objected as an absurdity against the Tychonic system by Rothmann, Hérigone, Tanner, Malapert, Amico and Argoli, as we said in Section 3, chapter 4, number 5, and chapter 8, number 5. [Margin: Kepler's argument against Tycho's system as regards the place of the Sun.] Indeed Kepler himself, in the *Epitome of Copernican Astronomy*, Book 4, page 544, puts it thus: *If Brahe's account holds, so that the orbit of Mars is one and a half times the orbit of the Sun, the body of Mars will at certain times move into that point of the world's space in which at other times the Sun was — which is quite incredible of primary Planets — and so their regions, which they pass through, are confounded; whereas in Copernicus they are not only distinct but separated by very wide empty intervals.* Anyone who wants a specimen of this difference should consult the figures of the systems set out by us: the Tychonic and ours in Section 3, chapters 8 and 9, and the Copernican in this Section 4, chapter 4, number 15. For in the Tychonic system and in ours the circle of Mars intersects the circle of the Sun's orbit when Mars approaches perigee; but in the Copernican the whole circle of Mars stays apart from the Sun among the orbits of the other planets, and does not intersect the great orb of the Earth; and the great orb itself performs what would otherwise be performed in Ptolemy by the Epicycle of Mars, and in Tycho by the orb of the Sun; and it is the Earth that approaches Mars when Mars stands opposite the Sun, and that is why Mars then appears so much larger than when it is far from perigee, and why, for the same reason, it undergoes a greater parallax than the Sun. This objection, moreover, has greater force with those who retain solid orbs; for they will not admit this penetration of Mars into the heaven of the Sun, for two reasons: namely on account of solidity, and on account of the distinction of the heavens. [Margin: Galileo's words on the same argument.] And so Galileo too, in Dialogue 3 on the system of the world, page 253, says: *Would you not call it, Simplicio, a very great absurdity if in the system of Ptolemy Mars should lower itself so far that, having broken through the orbit of the Sun, it descended below it and came nearer to the Earth than the solar body itself, and a little later was raised above that same body by an enormous interval?* Now to the form of the argument.

[Margin: 9th Argument in form.] XIX. *It is absurd that the distinction of the heavens should not be preserved, and that the heaven of one primary Planet should be per- meated by another primary. But unless the Sun rests in the centre of the world, and the Earth in its place performs an annual motion through the great orb, Mars, which is a primary Planet, sometimes penetrates the heaven of the Sun, which is itself also a primary Planet. Therefore, unless the Sun rests, etc., and the Earth moves, etc., an absurdity follows.* The MAJOR seems evident from the order owed to the more important parts of the world, such as the elements, which God set apart, and the heavens — especially since these are a kind of image of that hierarchical order which is found in the intelligible world, that is, among the nine choirs of Angels. The MINOR is proved because Mars, when opposite the Sun, appears at either of its perigees larger than itself at apogee: sixty times larger as regards the disc, and seven or eight times as regards the diameter, if it is viewed through a Telescope. Indeed, from our observations, its diameter near apogee ought to appear of only 10″ 6‴, but at either perigee of 92″ — that is, a diameter nine times larger; on this we said more in Book 7, Section 6, chapter 10, scholium 1. Moreover, if the apparent place of Mars, often observed in the acronychal position — that is, when it rises in the evening and is opposite the Sun — is to be reconciled by correct prosthaphaereses with the mean place of the tables, one must apply greater parallaxes to Mars than to the Sun, that is, a smaller distance of Mars from the earth in that position than of the Sun, as was said in Book 7, Section 2, chapter 3, scholium 4, and Section 6, chapter 4, scholia 3 and 4, and chapter 7, number 3. But these appearances cannot be saved, and brought into agreement with the due measure of the orbs, unless Mars comes very near the Earth and passes from a greater distance than the distance of the Sun to a smaller one, through the middle, or equal, distance; and unless, being sometimes without latitude at the moment when it is as far away as the Sun, it passes through that point of the Ecliptic through which the Sun passed that year or is about to pass, if the Sun is supposed to move through its annual orb. So it comes about that the Sun invades the heaven of Mars, and Mars the heaven of the Sun, in turn, with their rights and domains thrown into confusion.

[Margin: 1st Reply, but of the unskilled.] Someone might perhaps reply, first, by denying the Minor and its proofs, calling into suspicion of fallacy the observations of Tycho and others — as I regret has been done by some, whom I named in Book 7, Section 6, chapter 4, scholium 4. But in this matter Kepler, Maginus, Boulliau, Lansbergen and all the most skilled practitioners of the more recent Astronomy accepted those observations as valid. And that appearance of Mars as sometimes larger than Jupiter appears, of which Copernicus speaks in Book 1, chapter 10, and other similar ones reported by us in Book 7, Section 6, chapter 10, scholium 1, cannot be reconciled while preserving the commensuration of the orbs of Mars and the Sun deduced from the motions of both, unless Mars can sometimes come nearer to the Earth than the Sun does. Therefore the proof of the Minor, as regards this part, can be denied only by those less skilled in Astronomical subtleties.

[Margin: 2nd Reply, but a forced one.] Others perhaps might reply, second, by likewise denying the Minor and its proof. They would say that the heaven of the Sun is not an entire sphere but only a segment, round in the manner of a wheel, occupying as much aethereal space as the body of the Sun takes up when carried by its centre along the Ecliptic; that the heaven of Mars is likewise a round segment in the manner of a kind of wheel, occupying as much space as the bulk of Mars itself and, besides, as much as both its greatest elevation upward and its declination in latitude require; and that it can happen that the smaller wheel of the Sun is inserted so obliquely into the wheel of Mars that Mars at perigee is nearer to the earth than the Sun and yet never passes through the wheel of the Sun, the latitude of Mars so requiring. But these things are hard to maintain, and they would be said with little consistency, considering all the cases which the Theory of the Sun and of Mars admits by its nature, if the brief duration of the world does not by accident extend to these cases. Nor is there any need, for the solution of the argument proposed, of fictions of this kind, violently forced into the hypothesis of these Planets.

[Margin: 3rd Reply.] The reply, therefore, is made, third, by distinguishing the Major: denying it if this absurdity cannot be avoided without a greater absurdity; conceding it if it can. But in this sense I deny the Minor — that is, that with the Sun placed in the centre of the World and the Earth moved through the annual orb, the distinction of the heavens is preserved in such a way that a greater absurdity is avoided. For two far greater inconveniences are incurred. One is that the Sun is thrust down from the heaven into the centre of the world, so that in fact it has no heaven of its own, which is worse than sometimes admitting the path of another Planet into its own heaven. The other is that the Earth, with the whole elementary sphere, is transferred into the heaven, against common sense and against the order owed to the World, which requires rather that the Heavens be distinguished from the Elements by separate regions than that the Heavens be distinguished among themselves, or the Elements among themselves. But if the Copernicans have scorned this common sense of very many wise men, they cannot fairly take it ill if someone says, against their sense, that it is not absurd for the heaven of one Planet to be mixed, or interwoven, with the heaven of another.

[Margin: 4th Reply.] I reply, fourth: granting the Major, and conceding that part of the Minor in which it is said that Mars penetrates the heaven of the Sun, I deny that part in which Mars and the Sun are said each to be a primary Planet. For among the Copernicans the Sun is not even a Planet, since it is in no way moved by a motion of translation, much less by an errant and wandering motion. Among the others it is indeed a Primary Planet; but Mars, in our system, is not primary if compared with the Sun, for it is the Sun's largest satellite, and stands to the Sun as the outermost Satellite of Jupiter stands to Jupiter. Accordingly it is no wonder if, together with Venus and Mercury, it has been admitted within the same heaven of the Sun, as into the very spacious hall of its King. But if this likewise seems absurd to the Copernicans — namely

that Mars, one of the three superior Planets, is made by us from a primary into a secondary Planet and a satellite of Jupiter [?] — we say that it is much more absurd to make the Sun from a Planet into a non-Planet, to make the Earth with the sphere of the elements from a non-Planet into a Planet, and to make the Moon, which is one of the great luminaries, a secondary Planet and companion of the Earth. Nor will this dispute be ended unless one party gratuitously grants the other what that party supposes, although it is the very point in question.

[Margin: 5th Reply.] A fifth reply could be made by denying the Major. For since it is established that the heaven of the Planets is fluid, and that the Planets complete their courses freely in the liquid aether, there is no greater need to assign them distinct regions one by one than there is to assign distinct regions to the birds of the air and the fishes of the ocean; and a single Heaven suffices for all the Planets. If, however, someone still holds fast to the distinction of the heavens, at least by designation, let him use the 3rd or the 4th reply.

X. Argumentum ab Interuallis, & Numero Planetarum, ex quinque Corporum Regularium dimenſione deſumptis, ſeu ex Kepleri Myſterio Coſmographico.

10. The argument from the Intervals and the Number of the Planets, taken from the dimensions of the five Regular Bodies, or from Kepler's Mysterium Cosmographicum.

XX. This is that offspring of Kepler's genius, conceived by him not long after the year 1589, first published in the year 1596, and finally polished and reprinted in the year 1621. He was so pleased with it that he applied almost all the machinery of Astronomy to preserving its structure, and from it ordered almost the whole economy of this Art. It is, in short, the thing for love of which, above all, he adhered to the sect of the Copernicans, and he never ceased to establish it with all his effort. Here, then, the points necessary for grasping the force of the argument are to be gathered into a few words, reserving more for another place. [Margin: Kepler's discovery and the *Mysterium Cosmographicum*.] While he was inquiring more deeply into the proportion of the celestial orbs and into the causes of the number of the heavens, Kepler recognized that, if the Earth were received into the heaven between Mars and Venus and the Sun rested in the centre of the Universe, while the Copernican intervals of the Planets, and so also of the earth, were corrected somewhat, those intervals could be distributed according to the proportion that the five Regular bodies — otherwise called Platonic — have when inscribed in order one within another, with orbs between them. Since these bodies are only five, and there cannot be more or fewer, six Planets are required so that their five intervals may be filled. From this he judged that a necessary cause could be given for the number six of the primary Planets; for in place of the Sun he counts the Earth, together with the Moon as a kind of secondary Planet. [Margin: What are the Regular Bodies?] Now the Regular bodies are *Solid figures which are contained by planes*

that are equilateral and equiangular and equal among themselves. Of this kind is the *Cube*, or *Hexahedron*, that is, a figure bounded by six equal square faces; with 12 sides, and with angles solid, 8 in number. Next the *Tetrahedron*, or equilateral Pyramid, bounded by four equal equilateral triangles, with six sides and 4 angles; the *Octahedron*, bounded by eight equal equilateral triangles, with 12 sides and 6 angles; the *Dodecahedron*, bounded by twelve equal Pentagons, with 30 sides and 20 angles; and finally the *Icosahedron*, bounded by twenty equilateral and equal triangles, with 30 sides and 12 angles. That besides these five there can be no other bodies contained or bounded by plane faces equal among themselves and equilateral is demonstrated by the interpreters of Euclid — especially Clavius, in Book 13 of the *Elements*, in the scholia to proposition 18 — and by Kepler himself in Book 4 of the *Epitome of Copernican Astronomy*, p. 467. And in Book 2 of the *Harmonics* [Kepler's *Harmonice Mundi*], proposition 25, he rightly notes that the Cube, the Tetrahedron and the Dodecahedron are simple, and therefore primary, while the rest, the Octahedron and the Icosahedron, are secondary, as being composite. For the three former have the simplest angle, that is, a trilinear one, and a plane face of their own, distinct from the others; whereas the rest have an angle comprehended by more lines than three, and share their plane faces with the Tetrahedron.

XXI. Moreover, Kepler himself hands down this whole doctrine, compressed as it were into a compendium, in the preface of the *Mysterium Cosmographicum* and in chapters 2 and 14; in the preface of the *Harmonics*, and in Book 2 from proposition 25, and Book 5, chapter 9; and in Book 4 of the *Epitome of Copernican Astronomy* from p. 455. First, then, it is to be noted, on the authority of Aristotle and Plutarch, that the five Regular bodies were long ago fitted by the Platonists and Pythagoreans to the bodies of the world. Proclus did the same, on Book 13 of Euclid, asserting that Euclid set the five Regular bodies before himself as his ultimate end, and wrote everything he handed down in the *Elements* for their sake, except what leads to perfect Number. Petrus Ramus [Pierre de la Ramée], by contrast, contends in his *Scholae Mathematicae* that these five bodies should be removed from the end of Euclid's *Elements*, and Lazarus Schonerus [Lazarus Schöner] thought them of no use; against these, see Kepler in the preface of the *Harmonics*. Indeed, I read the following in Plutarch, Book 2 of the *De placitis*, chapter 6: *Since there are five solid figures, which are called mathematical, Pythagoras said that the Earth was made from the cubic one; Fire from the cone, or Pyramid; Air from the Octahedron, that is, the body of eight seats; Water from the Icosahedron, that is, the body of twenty seats; and the globe of the universe from the Dodecahedron, that is, the body of twelve seats. Plato Pythagorizes in all these things.* Kepler investigates the correspondences of this distribution, without any mention of Plutarch, in Book 2 of the *Harmonics*, prop. 25, and in the *Mysterium*, chapter 2. But there, and in that preface, he suspects that the Pythagoreans symbolically hid the names of the Planets under the names of the Elements: under the name of Earth, Saturn, nearest to immobility through its slowness; under the name of Water, Venus, who is feigned to have been

born from the foam of the Sea on account of the humours subject to her, and is therefore called Αφροδίτη [Greek: Aphroditē, the foam-born]; under the name of Air, Mercury, for the mobility of his speed; under the name of Fire, Mars, who is for that reason called Pyrois, from fire. But Proclus too said that, among other things, this is the end of Geometry: to teach how the heaven received figures fitting its several parts — namely from that great Demiurge who disposed all things in number, weight and measure. Kepler therefore judged more truly that the five intervals of the celestial orbs enclosing one another arose from those five figures; and so, if these are five, the primary Planets had to be six, just as five fingers are required to bound four intervals on one hand. The order and distribution of these bodies among the heavens of the Planets is handed down by Kepler, in the form in which he first conceived it, in this formula in the preface of the *Mysterium*: *The Earth is the circle that is the measure of all. Around it circumscribe a Dodecahedron; the circle comprehending this will be Mars. Around Mars circumscribe a Tetrahedron; the circle comprehending this will be Jupiter. Around Jupiter circumscribe a Cube; the circle comprehending this will be Saturn. Now inscribe an Icosahedron in the Earth; the circle inscribed in it will be Venus. Inscribe an Octahedron in Venus; the circle inscribed in it will be Mercury. There you have the reason for the number of the Planets.* Accordingly he counts the Moon as a secondary Planet and Companion of the Earth; the Sun, however, he does not reckon among the Planets, but places it in the centre of the Planetary system, as the source of light, heat and motion. Having examined the intervals at which the Planets stand from the Sun according to the Copernican measures — though somewhat emended by himself and by Mästlin — he concludes that those intervals are capable of the order and distribution already mentioned, as he teaches from chapter 2 of the *Mysterium Cosmographicum* to chapter 20, and in the *Epitome* from p. 468. Kepler grants, however, that the Copernican intervals come so near the proportions of those figures that they fall short only in some very small degree; but he says that this arises because the archetype of this mobile world consists not only of the beauty of the regular bodies but also of the Harmonic proportions and the concord of six voices, to whose Idea the fastest and the slowest motions of the Planets had to be tempered; and this tempering could not have been achieved unless the Harmost had varied those figural intervals a little.

XXII. Someone might perhaps wish us to represent this order and symmetry here in some diagram. But we cannot do so exactly, since those solid figures cannot be drawn in a plane in such a way as to keep the number of all their sides, planes and angles together with their magnitude; and if we used plane figures in their place, we would give the Reader occasion for manifold error. Let the diagram below suffice him, then. In it, from A — the centre of the annual orb, near the Sun according to Copernicus, but the Sun itself according to Kepler — let there be described, each within two circles, the orb of Saturn ♄ with its thickness, that of Jupiter ♃ with its own, and that of Mars ♂ with its own; and the Great Orb of the Earth, with the sphere of the Elements and the patio [?] of

the Moon's excursion, the whole of which we have also marked off above and below with a small black globe. Then, likewise within two circles, let there be described from the same centre A the orb of Venus ♀ with its thickness and the orb of Mercury ☿ with its own. This done, between the orbs of Saturn and Jupiter let the letter C be inscribed on either side, C, C, signifying the Cube to be inscribed in the Orb of Saturn but circumscribed about the orb of Jupiter. Between the orbs of Jupiter and Mars let the letters T, T be inscribed, indicating the Tetrahedron to be inscribed in the orb of Jupiter but circumscribed about the orb of Mars. In the third space, that is, between the orb of Mars and the annual orb of the Earth, let D, D be marked, to signify that in that space the Dodecahedron is to be inscribed in the orb of Mars but circumscribed about the annual orb of the Earth. In the fourth interval, namely between the annual orb and the orb of Venus, let the letter I be set in twice, indicating the Icosahedron to be inscribed in the annual orb but circumscribed about the orb of Venus. Finally, in the fifth space, that is, between the orbs of Venus and Mercury, let the letter O be inscribed twice, indicating the Octahedron to be inscribed in the orb of Venus but circumscribed about the orb of Mercury. Or else let the names be written out in full: in the first interval, of the Cube; in the second, of the Tetrahedron; in the 3rd, of the Dodecahedron; in the 4th, of the Icosahedron; in the 5th, of the Octahedron. And to aid the memory, the following distich will not be useless:

Cappa surrounds Jupiter; Tau, Mars; Delta, the Elements; Iota surrounds Venus; and Omicron, Mercury.

Diagram. In the right column, within the sentence that introduces it: Kepler's scheme of the planetary orbs and the five regular solids. A rayed disc at the centre carries the letter A; around it stand seven pairs of concentric circles, the orbs with their thicknesses. The planetary signs stand on the vertical axis above the centre, and the letters of the solids along the horizontal axis on either side, as the text prescribes; the names of the solids stand on the vertical axis below the centre.

- A (centre, in the rayed disc): the centre of the annual orb
- ♄, ♃, ♂ (upper axis, outermost first): Saturn, Jupiter, Mars
- ● (upper axis, between the circles of Mars and Venus): the small black globe marking the Great Orb of the Earth
- ♀, ☿ (upper axis, next to the centre): Venus, Mercury
- C, T, D, I, O (horizontal axis left of the centre, outermost first) and O, I, D, T, C (right of the centre, innermost first): Cube, Tetrahedron, Dodecahedron, Icosahedron, Octahedron
- oct. (lower axis, innermost): Octahedron
- icof. (lower axis): Icosahedron
- ● (lower axis, below "icof."): the small black globe marking the Great Orb of the Earth
- Dode[[c/i]]. (lower axis): Dodecahedron [?]
- Tetraëdr. (lower axis): Tetrahedron

• Cubus. (lower axis, outermost): Cube provided you remember that *Cappa*, that is C, signifies the Cube; *Tau*, that is T, the Tetrahedron; *Delta*, that is D, the Dodecahedron; *Iota*, namely I, the Icosahedron; and *Omicron*, that is O, the Octahedron.

XXIII. Now Kepler supposes in the *Epitome*, p. 455, on the strength of Astronomical observations and the demonstrations of Copernicus, that, relative to the distance of the Earth from the Sun, or to the semidiameter of the great orb, the distance of Saturn is about tenfold; that of Jupiter fivefold; that of Mars one and a half times; that of Venus subsesquitertial [that is, three-quarters]; and that of Mercury about one-third. Thus the diameter of the orb of Saturn is less than double that of its neighbour, the orb of Jupiter; Jupiter's is triple that of Mars; and Mars's is one and a half times that of the terrestrial orb described about the Sun. The terrestrial orb, in turn, is more than four-thirds of Venus's; and Venus's is five-thirds, or eight-fifths, of Mercury's — though he adds that the proportions of the distances are not the same in every part of the orbit but differ from part to part, especially in Mars and in Mercury. From p. 468 of the same *Epitome*, together with chapter 14 of the *Mysterium Cosmographicum*, the following numbers are gathered. The former signify the proportion of the semidiameter of the orb inscribed in a given Regular body, the semidiameter of the orb circumscribed about the same Regular body being set at 100,000 parts; the latter signify the proportion of the same semidiameters as they come out of the Copernican hypothesis. So from setting them side by side, or comparing them, it may appear how nearly the Copernican measures approach the measures of the intervals required for the regular bodies to be inscribed, in the order already mentioned, in the concave of the orb of the higher Planet, or circumscribed about the convex of the lower.

Of which parts the semidiameter of the circumscribed Orb is 100000, of such parts the semidiameter of the Orb inscribed in the	ought to be	Of which parts the semidiameter of the Concave is	Of such parts the semidiameter of the Convex is, by Copernicus	From Copernicus, Book 5, chapter
Cube	57735	♄ 100000	♃ 63500	9
Tetrahedron	33333	♃ 100000	♂ 33333	14
Dodecahedron	79465	♂ 100000	Earth 75700	19
Icosahedron	79465	Earth 100000	♀ 79500	21 22
Octahedron	57735	♀ 100000	☿ 72300	27
Square in the Octahedron	70711			

I explain the small table. If a circle, or rather an orb, is made whose semidiameter is 100,000 parts, and a Cube is inscribed in it, and in turn another orb is inscribed in this cube, the semidiameter of this orb will be 57,735 of those parts. Again, if the semidiameter of this second orb is divided into 100,000 equal parts, and a Tetrahedron is inscribed in it, and a third orb is inscribed in this Tetrahedron, the semidiameter of this orb will be 33,333 of those parts; and so for the rest.

[Margin: Kepler's philosopheme on the intervals of the Planets and the Regular bodies.] XXIV. With these things premised, Kepler philosophizes as follows. In the Ptolemaic hypothesis the distances of the Planets from the centre of the Earth and of the universe are so arranged that nothing lies between the lowest, or least, distance of Saturn and the highest, or greatest, of Jupiter; rather, the concavity of the heaven of Saturn touches the convex surface of the heaven of Jupiter immediately, and so for the rest. This is surely for no other reason than that there may be no void between the heavens that does not belong to the dominion and jurisdiction of some Planet, being occupied at least by its Epicycle, if not by the body of the star itself; but no Geometrical beauty shines out in this, to which God might be said to have looked. In the hypothesis of Copernicus, by contrast, between the concave of the higher Planet and the convex of the lower there lies a space that is not necessarily occupied by the body of the Planet, nor by any Epicycle of it, nor by any other orb. And these spaces are so interposed that three simple, or primary, Regular bodies can be inscribed between the heavens of the three superior Planets — namely the Cube in the concave of Saturn, the Tetrahedron in the concave of Jupiter, and the Dodecahedron in the concave of Mars — while the two remaining, secondary, figures can be inscribed: the Icosahedron in the concave of the Annual orb, and in the concave of Venus the Octahedron, which can be circumscribed about the convex of Mercury. These things, I say, can be done exactly as regards Mars and Venus, and almost exactly as regards the Earth: for the number required is 79,465, and Copernicus has 79,500. Nor is Mercury enormously out, if in the middle of the Octahedron you inscribe a square formed by its four middle sides: the circle inscribed in this square will have a semidiameter of 70,711 parts, of which the radius of the orb inscribed in the Octahedron is 100,000, while the Copernican measures supply 72,300 parts. Jupiter alone departs much from what is required: for the radius of the orb inscribed in the cube ought to be 57,735, but from Copernicus it is 63,500. Yet although it departs, it is nearer to the measure of this body than to that of any other. Such great analogy and nearness of the numbers, therefore, cannot be said to have been fortuitous in the mind and Idea of God the Creator; and if those numbers are corrected somewhat according to these ideal and archetypal ratios (preserving, however, the ratios which the harmonic proportions of the motions require according to the six musical voices), the most beautiful analogies arise, and a reason is given a priori why six primary Planets were created by God, why there is so great an interval between the concave of the higher heaven and the convex of the lower, and why the three primary Regular bodies correspond in order to the three superior Planets. But such analogies and causes cannot be given a priori in any hypothesis of the Earth at rest in the middle of the universe, and much less in the Ptolemaic. Therefore the Copernican hypothesis is to be preferred (although Copernicus was unaware of his own riches and did not see these analogies); and accordingly the Sun is to be placed in the centre of the Universe, never moved from it, and the Earth, mobile yearly, between Mars

and Venus. And here now is the argument coming into its proper form of its own accord, which I am pleased to set out again distinctly.

[Margin: Argument 10 reduced to Form.] XXV. *That Hypothesis is to be preferred to the others in which alone, or most of all, a reason is given a priori, through Geometrical and Harmonic proportions — and those the most beautiful — both of the number of the Planets and of the interval between the Planetary heavens. But such is the Hypothesis of Copernicus, and it requires the Sun to be immobile near the centre of the world, and the Earth to be mobile with an annual motion between the orbs of Mars and Venus. Therefore the Hypothesis of Copernicus is to be preferred to the others, and the Sun must be set immobile near the centre of the world, and the Earth, mobile with an annual motion, between the orbs of Mars and Venus.* The MAJOR does not seem open to doubt, except from someone who would deny what very many wise men have confessed — that God, in the structure of this visible world, looked to the most excellent Ideas of the intelligible world, which He had from eternity in His divine intellect and expressed by speaking His Word, according to that line of Boethius:

…. He, most beautiful himself, bearing the beautiful World in his mind, and forming it in a like image:

or who would deny what was said by Plato in the *Timaeus*, in the *Epinomis* and in the books of the *Republic*, and indeed before him by Pythagoras, and received with such applause by all the centuries — namely, that God, in founding and preserving the World, was a Geometer and a Harmost; or who would dare to deny what the sacred scriptures testify of God, Wisdom 11: *You have disposed all things in measure, and number, and weight*. As for the concord and harmony of the heavens, that will be clear from the same divine scriptures, Job 38, and from what is to be said below in its proper places. The MINOR seems to have been sufficiently proved by Kepler from what was said from number 20 to 25.

[Margin: 1st Reply.] I reply, however, 1. by distinguishing the Major and denying it: if the number of the Planets commonly received by the wise, and evident to sense itself, is not retained; if those proportions do not agree, again and again, with certain observations of the celestial motions; if they are not suited to the end and effects which Divine providence set before itself in the structure and administration of the World; and finally if, in such a hypothesis, not even those proportions are adequately fo- und. I concede the Major, however, if the four conditions already mentioned are found in that hypothesis; but in that sense I deny the Minor. For, *first*, in the Copernican hypothesis the Sun — which was reckoned by all the wise (apart from a few followers of the Copernican opinion) to hold the chief place among the Planets and to govern the rest by its motion — is thrown down from the heaven into the centre of the Universe: it has no heaven at all, and is no longer a Planet, but stands outside the number of the Planets. The Moon, too, which was likewise always held to be among the primary Planets, is not counted among the primary Planets in this hypothesis, but is the Earth's attendant; while the Earth itself — indeed the whole sphere of the elements — is carried up into the heaven by a

truly monstrous Uranomorphosis, and becomes a primary Planet. So it comes about that the two great Luminaries are not counted in the number of the Planets, whose cause is being sought here; and so the two Lights of the world are plucked out of that number like a pair of eyes; and that most celebrated septenary of the Planets and of the Strings, made illustrious by so many other analogies and conditions, as I indicated in Book 7, Section 1, chapter 1, perishes in our hands. So if Kepler had so robust a stomach that he could digest the striking of the Moon and the Sun from the roll of the seven Planets, let him not take it ill if someone says that the analogy of a few regular bodies with the intervals of some of the Planets was neglected, in the structure of the world, by the primary order of the divine Ideas. For it is much more evident that the Moon and the Sun are primary Planets, both in the measuring of Times and in the effects of nature, and that the Earth does not move, is not a Planet, and deserves the name neither of a star nor of a celestial body, than that the intervals of the Planetary heavens are taken from the regular bodies.

[Margin: 2nd Reply.] *Secondly*: neither the Prutenic tables, built on the Copernican hypothesis, nor the Copernico-Tychonic Rudolphine tables are such as to agree perpetually with the celestial observations; on the contrary, they stray from them again and again, and in the chief Phenomena at that, namely Eclipses, as is clear from what was said in Book 5 and is by now well established among the Astronomers of this age. And as for what concerns the apparent magnitude of Mars and the proportions between its bulk and its intervals, we showed sufficiently in Book 7, Section 6, chapter 10, scholia 1 and 3 taken together, that Kepler erred and conflicts with manifest observations of the diameter of Mars.

[Margin: 3rd Reply.] *Thirdly*: those purely Geometrical proportions derived from the five regular bodies do not seem of themselves suited to the ends and effects which God willed, in nature, to depend on the intervals and motions of the Planets; and I say the same of the Harmonic ones. For the excellence of the regular bodies does not consist in the proportions of the diameters of the inscribed or circumscribed circle, but in the very character of bodies bounded by such faces. The heavens of the Planets, however, do not act on these lower things by reason of their corpulence so bounded — especially since, once the fluidity of the heavens is granted, the heaven of the Planets is in fact a single one, and so that real distinction of heavens and of regular bodies perishes — but by means of the bodies of the Planets, their light, and the qualities flowing from that light. These bodies are contained within the Zodiac; they do not act on the celestial spheres, which we mark off by reason alone, nor are they acted upon by them, nor, insofar as they are bodies, do they receive from them their force or the governance of their power — much less from bodies that can be inscribed between those spheres. And it is far more probable that the Geometry and Harmony of Divine wisdom in the structure of the World was much higher and nobler than to have followed that material beauty of dimensions, and of a harmony accommodated to our ears. But just as we say metaphorically that a

Prince geometrizes well, or that a Commonwealth is administered Musically, if he chooses and applies means suited to the end of public happiness, distributes offices, rewards, punishments and burdens according to the endowments and merits of each, and so tempers himself to the characters and customs of his subjects as to preserve peace and concord among them and with himself, as well as the due subordination of the lower to the higher: so too, and far more worthily, must we think of God in the government of this world, even as regards natural effects. [Margin: How the Geometry of God is to be understood.] Thus in Wisdom 11, where the discourse concerned the governance by which God had so inflicted punishments on certain impious men that by the very things by which one had sinned he was punished, and so they were corrected — when otherwise He could have exterminated them at once in many ways by exercising His Omnipotence, as anyone who reads that chapter will see — for that reason, after the words *But you have disposed all things in measure, and number, and weight*, there follows at once: *For great power always remains with you alone, and who will resist the strength of your arm? For as the small weight that tips the balance, so is the world before you, and as a drop of the morning dew that comes down upon the earth. But you have mercy on all, O Lord, etc.* So too in chapter 32 of Ecclesiasticus it is said, *They have made you ruler*, and shortly after, *do not hinder the music*; and what music the discourse there concerns is made plain, namely the prudent and fitting government of subjects. But if it nonetheless pleases anyone to bind God to the compass and square of material Geometry, I should gladly ask Kepler why He did not keep the laws of any regular figures in the distribution of the Fixed stars, in the spots of the Moon, and in the surfaces of the Seas and of the great continents? Why, in organizing the body of an animal — especially of man, in whom we so greatly admire the depth of divine Wisdom — did He not conform the magnitudes, figures and intervals of the brain, the heart, the liver and the other parts to the properties of the regular bodies? Is it because these are not spherical bodies, nor destined for circular motion? But neither are the orbits or paths of the Planets spheres, or a great part of spheres, except among those who have introduced solid orbs (which Kepler rejects) without any necessity, and indeed against the protests of the observers of the heavens. It will doubtless be answered more truly that figures of this kind are not of themselves suited to the functions and effects which God and nature preordained in these works.

[Margin: 4th Reply.] *Fourthly* and finally: the proportions taken from the Regular bodies are not found adequately in the Copernican or the Keplerian hypothesis. For neither the size of the interval between Saturn and the Fixed stars, nor that between Mercury and the Sun, nor the thickness of the heaven of Saturn, is determined from them; and of the remaining five intervals there are only two that come very near, or near, to the proportions owed to the bodies to be inscribed, namely the intervals of Mars and of Venus. The rest differ greatly — not only between Jupiter and Saturn, and between Venus and Mercury, as Kepler admits in the places cited, granted that he begs a remedy for them from

far off, but also that of the Earth, if the space of the Lunar heaven carried round with it must be confined within the orbit of the Earth. And so in the *Mysterium Cosmographicum*, chapter 16, he grants that the little orb of the Moon stands out above the orb of the Earth like a gem above a ring, or like the orbs of the Satellites of Jupiter above Jupiter's orbit, and is not hindered by the spaces of the regular bodies, because the heavens are not adamantine or solid. Of Mercury I have already said above that, if one is to come near the measures of the Copernican hypothesis, its interval must be determined not by the Octahedron but by the square inscribed in that octahedron. Therefore, unless corrections of the measures are scraped together by a violent search through various windings, the analogy of the Planetary intervals with the capacity of the regular bodies shines out only in two, or at most in three, intervals.

[Margin: 2nd Reply.] XXVI. I reply, therefore, 2. by granting the Major and denying the Minor as to both its parts. That the proportions of the said bodies are not found exactly and adequately in the Copernican hypothesis, and that no reason for the true number of the Planets is given in it, is clear from what was said in the first reply, in the paragraphs *First* and *Fourthly*. And Kepler admitted, in Book 5 of the *Harmonics*, chapter 3, that the Harmonic proportions can be found in the Tychonic hypothesis too, or in others like it, and in the same place that the Tychonic observations conflict with that exact configuration of the regular bodies. For when he had said *that the Earth is one of the Planets and is carried among the stars around an immobile Sun*, he nevertheless at once added: *Let those, therefore, who are offended by the novelty of this science know that these harmonic speculations hold their place in the hypotheses of Tycho Brahe as well, because that Author has in common with Copernicus everything else that concerns the disposition of the bodies and the tempering of the motions.* And on the following page: *As regards the proportion of the Planetary orbits, it is indeed always such between each two neighbouring orbits that it readily appears that each of them approaches the single proportion of the orbs of one of the five solid figures, namely of the circumscribed orb of the figure to the inscribed orb; and yet it is not wholly equal, as I once ventured, concerning a finally perfected Astrono- my, to promise. For after completing the demonstration of the intervals from Brahe's observations, I found this, etc.* He then goes on to explain at greater length the shortfall or excess of the intervals that come out of the Tychonic observations, below or above the measures owed to the inscription of the regular bodies, and concludes: *In short, the Cube and the Octahedron, a pair, penetrate their planetary orbs somewhat; the Dodecahedron and the Icosahedron, a pair, do not quite reach theirs. The Tetrahedron touches both precisely: there it is less, there more, here equal, in the intervals of the Planets. From this it is clear that the very proportions of the intervals of the Planets from the Sun are not taken from the regular figures alone.* Read the rest in his own text. In the same Book 5 of the *Harmonics*, chapter 9, proposition 49, he bursts out into this exclamation: *It was good that the solid figures should yield, in the generation of the intervals, to the Harmonic ratios.* Since, therefore, in neither hypothesis — that of Copernicus, namely, or that of Tycho — are the intervals

of the Planets taken from the regular bodies taken exactly; and since it is the Harmonic proportions that should chiefly be considered (as Kepler at any rate holds), and these find a place in the Tychonic hypothesis as well (and, I add, in ours too, which in these respects is sufficiently equivalent to the Tychonic): we are not compelled by these proportions to prefer the annual motion of the Earth to the annual motion of the Sun, or the rest of the Sun to the immobility of the Earth.

CAPVT IX.

Chapter IX.

Proponuntur, & Diſſoluuntur Quinque Argumenta pro Annuo Terræ Motu, ex Planetarum Motibus & inſtrumentis motuum deſumpta.

Five Arguments for the Annual Motion of the Earth, taken from the Motions of the Planets and from the instruments of their motions, are Proposed and Resolved.

I. Although in the preceding Chapter mention was made of the motion of the Planets in certain arguments, chiefly the 2nd and the 4th, the arguments there were nevertheless not drawn from their motion itself, taken immediately, as will be done in this chapter. Let this be said to keep the titles sufficiently distinct.

I. Argumentum à Superfluitate motuum trium Superiorum Planetarum per Epicyclos, & duum Inferiorum per proprios Eccentricos, Sublata per vnicum ac ſimplicem Telluris Motum Annuum.

I. The Argument from the Superfluity of the motions — of the three Superior Planets through Epicycles, and of the two Inferior ones through their own Eccentrics — a superfluity removed by a single and simple Annual Motion of the Earth.

II. Here surely, if anywhere, the Copernicans seem with good reason to triumph, and with an irresistible battering-ram to shake the Ptolemaic system, or others akin to it, and all but bring it down. And to this point likewise tend those words of Copernicus in Book 1, chapter 10. For after he had said, *Whatever appears concerning the motion of the Sun is verified rather by the mobility of the earth*, he added shortly afterwards: *Which I think easier to concede than to have the understanding distracted into an almost infinite multitude of orbs, as those who have kept the earth in the middle of the world have been forced to do. But the sagacity of nature is rather to be followed, which, just as it has taken the greatest care not to have produced anything superfluous or useless, so has rather often endowed one thing with many effects.* [Margin: The annual orb of the Earth commended, from Copernicus.] Adhering to this, Rheticus in his *Narratio prima* warns

Astronomers that, just as Galen insists in Book 10 of *De usu partium* [*On the Usefulness of the Parts*] — namely that nature does nothing superfluous, and does not trouble to search out through many means what it can obtain by one or two — so they too, in framing hypotheses, should avoid superfluities. [Margin: And from Rheticus.] Then he infers: *Therefore, since we saw that by this single motion of the earth almost infinite appearances are satisfied, should we not attribute to God, the founder of nature, that skill which we see ordinary clockmakers possess? For they take the greatest care not to insert into the instrument any little wheel that is either superfluous or whose function another could supply more conveniently with its position slightly changed.* And explaining in particular the advantages that arise from the single motion of the Earth through its annual orb, he says: *To this is added that the motion of the earth in its orb produces the arguments of all the Planets except the Moon — that is, the Anomaly of the orb, or second inequality, which the Alphonsines call the Argument — and that it alone seems to be the cause of all the variation of motion, which appears, namely, in the three superior ones from the Sun, but in Venus and Mercury around the Sun.* And again he says that from this motion of the earth it follows *that all the variation of the apparent motion of the Planets, which seems to occur in them* [Greek: παρὰ τοὺς πρὸς τὸν ἥλιον σχηματισμοὺς (para tous pros ton hēlion schēmatismous, "according to their configurations with respect to the Sun")], *comes about through the annual motion of the earth in the great orb; and that the Planets in truth proceed with only the one remaining inequality, which is observed with respect to the parts of the Zodiac.* Ptolemy, then, and almost all the others down to Tycho, set out the first inequality of the motions in the Zodiac, in the five lesser Planets, by means of Eccentrics, and the second — which arises from their varying recession, or approach, and configuration with respect to the Sun — by Epicycles, or by orbs equivalent to Epicycles, but multiplied. Copernicus, by contrast, sets it out and saves it by a single annual orb of the Earth approaching the Planets and receding from them, and so achieves what in the Ptolemaic system three Epicycles of the superior Planets and two distinct Eccentrics of the Inferior Planets must achieve.

III. This, moreover, is what Kepler so insistently urges: first in the *Mysterium Cosmographicum*, chapter 1, where he says: [Margin: The same orb and motion commended, from Kepler.] *This orb and motion of the Earth, then, provides these advantages: that we do not need three eccentrics in the customary hypotheses, namely those of the Sun, of Venus and of Mercury. For because the Earth is carried round about these three Planets, the Earth-dwellers think that those three are carried round about themselves, who are immobile. And if there were more stars within the orb of the Earth, they would ascribe this motion to more as well. With this orb posited, the three great epicycles of Saturn, Jupiter and Mars also fall, together with their motions.* And then more explicitly in the *Epitome of Copernican Astronomy*, Book 4, page 542, where he admits indeed that those great Ptolemaic Epicycles were removed by Tycho too, yet says that Tycho left in the world the same thing that was effected by them: *namely, that any Planet, besides the motion that is truly to be granted to it, is still moved in addition by the very motion of the Sun, mixing the two into one; from*

which measure, since there are no solid orbs, most intricate spirals are produced in the space of the world. (Of this intricacy I displayed a diagram and a specimen, from Kepler himself, in Book 7, Section 1, at the end of the last chapter.) *Copernicus, by contrast, entirely strips the five planets of this extraneous motion of the Sun by a single simple motion of the centre of the earth, and brings it about that the centres of the six primary planets — of the Earth, namely, and of the other five — each describe in the space of the world a single orbit, simple and perpetually like itself, or a line very close to a circle.* So he says on page 543, alluding to the Elliptical orbit he himself introduced into Astronomy. Further, Lansbergen, in tract 4 of the *Apologia* for the motion of the earth, considering the five already-mentioned extraneous motions of the Planets removed by the annual motion of the Earth, and the eight diurnal motions of the eight spheres erased by the diurnal motion of the Earth, says: *By the Earth's motion is done what would have had to be accomplished by thirteen motions of the stars.* And he presses those physical axioms: *Nature proceeds in order, abhors superfluities, does nothing in vain.*

IV. But since Kepler knew that the already-mentioned motions of the Planets, which seem to us multiple, could be attributed to Intelligences leading the Planets round along a single spiral line, [Margin: A reply anticipated, from Kepler.] he met this reply in Book 4 of the *Epitome of Copernican Astronomy*, page 546, he said that the condition of the moving souls would be very hard, and harder still that of the Intelligences, when they are ordered to attend to very many things, so as to carry the planet in order with two motions distinct in every respect and mixed with each other; but if the Sun rests and the earth moves, the motion of each Planet is single, and it can be effected by corporeal magnetic powers, by a single revolution of the body of the Sun about its own centre, *and there is no need at all, anywhere, of the safeguard of a mind*, he says. And he refers the Reader to the *Commentaries on Mars*, where he teaches by what means the Sun, through a magnetic power diffused and emitted from itself through its light in every direction, grasps the Planets and whirls them round in a circle. But let us now come to the formula of the argument.

V. [Margin: The argument in form.] *That hypothesis is to be preferred to the others which accomplishes with fewer and simpler motions what the others cannot accomplish except with more, and more composite, motions. But the Hypothesis in which the Earth is moved through an annual orb is of this kind. Therefore it is to be preferred to the others.* The MAJOR is clear from the axioms already adduced so many times; for it is most widely accepted that God and Nature, and indeed art too, are not accustomed to bring about through more what can be done through fewer, and that means and instruments — indeed, generally, any entities at all — are not to be multiplied without necessity, or without greater utility. The MINOR has already been proved, since it has already been shown that from the annual motion of the Earth there follows that apparent second motion in the lesser Planets which represents the second inequality, called the annual, or the one tempered to the apparent motion of the Sun; and so that

what would otherwise have to be done by Epicycles, or by distinct Eccentrics, is done by a single annual orb.

[Margin: 1st Reply.] I reply, 1.: if it accomplishes the phenomena that are already evident to sense, and in a way no more absurd than is the composition of motions, I concede the Major; if it does otherwise, I deny the Major, and accordingly I deny the Minor. For the hypothesis in which the Earth is moved through an annual orb does not accomplish the phenomena evident to sense. Rather, it destroys and inverts those that are evident to common sense — such as the motion of the Sun, and the harmony of the Planets tempered to the Sun — and destroys them, moreover, without any necessity, and introduces a motion from which the intellect, relying on sense (as is fitting in Physics), recoils more than from a multiplicity of motions. [Margin: How far the fallibility of the senses is to be admitted.] Nor is there any reason to thrust at us that oft-repeated refrain, that the senses are deceptive and that this dispute is not to be decided from their estimation. For the answer is that they are deceptive neither always nor for the most part, and that their deception is detected when reasoning is applied — but reasoning that itself relies on more certain sensations. Otherwise, for as long as they are not caught erring in this way in some particular object, we must abide by them, and their estimation stands in possession as regards what our assent requires. For because some eyes are dim-sighted, or ours sometimes run with rheum, or an animal is sometimes sick, must we say that all eyes are dim-sighted? or that ours always suffer from rheum, or that an animal is never healthy? Moreover, does not the intellect itself err in many matters that do not depend immediately on sense, in such a way that the error cannot be attributed to the deception of the senses? You will certainly say so; for about the same sensible thing, and the same experience had through the senses — about the Rainbow, say, or the tide of the Sea — different people hold different and contradictory opinions, some one of which must be false. Is the intellect, then, not to be trusted either? Let those who are of this kind see to it, then, that they do not undermine the foundations of all the sciences.

To the proof of the Major I reply that those axioms are not to be taken absolutely, or in a purely arithmetical sense — as though God and Nature, or even Art, always aimed at unity, or at the number that comes as close as possible to unity — but relatively to the end set before them, and taken not only as regards the substance of the thing to be made, but also as regards the accidents befitting it for its being well, or better, in its own order. Nature could indeed have made and preserved Plants and Animals with fewer fibres, little veins, little nerves, little skins, little membranes, hairs, leaves, little roots, etc., but not so well and so fittingly. God too could have filled this world with fewer stars, with narrower tracts of Seas and Lands, and with a smaller number of insects, reptiles, fishes, birds and quadrupeds; but he would not have attained the end he attains as things are, nor in the way he attains it. And since Rheticus chose the comparison of Clocks: if someone were to add more wheels to some clock, so that the same hour was struck not once but twice, or so that, with

two distinct bells, the hours were struck on the larger and more sonorous one and the quarters of the hours on the smaller, he would be using wheels that, considering his end, are not superfluous but necessary. So in the case proposed: if God willed to display not the mere appearance of the motions of the Sun, and of the Planets that look to the Sun in their motion, but the real motion itself of the Sun and of the Planets — that motion now so quickened that they become ever more easterly, now slow, so that they seem neither to advance nor to go back, now finally retrograde against the order of the signs — and willed a variety of many effects to depend on this real variety, he is not to be thought to have introduced superfluous motions. I will give another example. A musical concert could certainly be so arranged that, with fewer voices and instruments but with a polyphonous Echo applied, there would seem to be more voices and more instruments; but if someone sets before himself not this mere appearance but a real multiplicity of voices and instruments, so as to produce the concert in that way, he will in no way be blameworthy, even if he could have used that Echo with equal ease.

[Margin: 2nd Reply.] I reply, 2.: if that hypothesis multiplies, in another kind, more motions, and more composite ones, than the other hypotheses do, I deny the Major; if it does not multiply them, let the Major pass. But in this sense the Minor is denied. For although the hypothesis that moves the Earth with an annual motion seems not to multiply motions, inasmuch as by this single motion it accomplishes five motions of the lesser Planets, yet in truth, looking more deeply, it multiplies other motions no less — indeed, more composite ones. For it multiplies the motion of the Moon: it makes the Moon, besides its monthly motion around the Earth, also move with an annual motion together with the Earth's centre, so that in this respect the Moon must perform not only the Moon's office but the Sun's. Next, it multiplies the same annual motion in the Earth, in the Water, and in the whole Elementary sphere; so that if the elements are four, the motions of these and of the Moon are five, and distinct, indeed, with the same distinction as the mobile bodies themselves. Besides, since not only the animals standing on the Earth but also the birds flying in the air, and all the meteorological bodies that hang or move in the air or in the fire (if there is such a thing), must follow the annual motion of the Earth, or of the Earth's centre, this number of motions is multiplied as many times as there are subjects moved by such a motion. Finally, since it is certain that the physical and astronomical phenomena cannot be saved unless a diurnal motion is added to the annual motion of the Earth — as the Copernicans freely admit — it is necessary to add the diurnal motion not only to the Earth but to all terrestrial and watery bodies, however far separated from the water and the earth, and, besides the rectilinear motion of the Elementary bodies, to introduce a curvilinear one made along a longer path. [Margin: Note the true economy of motions in the hypothesis of the Earth at rest.] All things considered, therefore, this annual motion of the Earth brings with it far more motions, both in number and in kind, than that hypothesis in which, by a wonderful economy of nature,

the phenomena and the real effects of these vicissitudes are saved: through the immobility of the Earth and of the Elements as regards their whole bodies, and a most simple rectilinear motion of the parts — and that not perpetual, but only in the cases in which the parts of the Elements are outside their natural place; and in the heaven through a single spiral line in each Planet, always towards the West, but with spirals now higher, now lower, now widened out, now narrower, led round by the Intelligences sometimes faster, sometimes slower.

[Margin: 3rd Reply.] One could therefore reply, 3., by turning back the argument drawn from the multiplicity of motions; it is enough to have pointed this out. I am pleased to transcribe here the words of Tycho from volume 1 of the *Progymnasmata*, page 661, where he says: *I admit indeed that the circuits of the five Planets which the ancients attributed to Epicycles are in this way dispatched very neatly, and with less trouble, in the Earth alone; that many mathematical absurdities committed by the Ancients, and an enormous incoherence, are guarded against by Copernicus; and that the celestial appearances are satisfied somewhat more accurately. Nevertheless, since this can also be accomplished in another way, as we have done in our hypothesis, equally well, if not more correctly, to what end is there need to fashion without good cause, out of the Earth, an opaque, gross and sluggish body, a kind of Star revolved even more multiply than the others — subject, that is, to a triple motion — not only against all Physical truth, but also with the authority of Sacred scripture opposing it, which ought to be paramount?*

II. Argumentum ab Imperfectione motuum cæleſtium, ſi reipſa tales ſint, quales apparent, Sublata, & meræ apparentiæ attributa, Redditaque à priori cauſa per annuum motum Terræ.

II. The Argument from the Imperfection of the celestial motions, if they are in reality such as they appear — an imperfection removed and attributed to mere appearance, and its cause given a priori through the annual motion of the Earth.

VI. The greater the variety and inconstancy in the celestial motions, the greater the imperfection they seem to suffer from. Now in the five lesser Planets there appears a manifold variety and inconstancy of motions. For, *first*, it is established that they sometimes cling to the same place of the sphere of the Fixed stars in such a way that they seem to stand still, their course checked; and sometimes, as if led by repentance of their direct motion, to Recede against the order of the signs and, like Penelope's web unwoven, to cancel or obliterate the motion by which they had previously advanced in the order of the signs. But Copernicus removed all this imperfection in Book 5, chapters 3, 35 and 36. For once an annual motion of the Earth is posited below the orbs of Saturn, Jupiter and Mars but above the orbs of Venus and Mercury, and in such a way that in their periodic motion beneath the Zodiac the three superior Planets are slower than the Earth, while Venus and Mercury are swifter than the Earth, the Earth must so approach these planets and recede from them that they seem

sometimes to advance, sometimes to stand still, and sometimes to go backward. The cause of these imperfections, therefore, is not physically in their motion — for they always advance — but is a mere appearance, and a purely optical cause, owing to the motion of commutation by which the Earth is carried round in a circle every year. For where the additive [adiectiuus] motion of the Planet is equal to the subtractive [ablatiuus] motion of commutation, the Planet will seem to stand still; where the subtractive motion of the Planet is greater than the additive motion of commutation, it will seem to go back; and where, finally, the subtractive motion of the Planet is less than the motion of commutation, it will seem to advance directly. We too explained this in Book 7, Section 5, chapter 4, with diagrams appended. [Margin: Kepler's doctrine of the Retrogradation of the Planets.] Comparing this hypothesis with the Ptolemaic, Kepler says in chapter 1 of the *Mysterium Cosmographicum*: *For it could have been asked of Ptolemy why the five Planets become Retrograde, but not the Luminaries? The answer, first concerning the Sun, is: because it is at rest; from which it comes about that the motion of the Earth, which is always direct, seems to be in the Sun itself, purely and undisturbed, only through the opposite part of the heaven. Concerning the Moon: because the annual motion of the Earth is truly common to its heaven together with the Earth; and two things that have the same motion in every respect seem to be at rest relative to each other. Hence the motion of the Earth is not perceived in the Moon, as it is in the other Planets. Concerning the Superior ones, Saturn, Jupiter and Mars, the answer is: because they themselves are slower than the Earth, and because that circle and motion of the earth is thought to be in them. Therefore, just as to those who looked out from the globe of Saturn the Earth would seem sometimes to advance, while it went through the half of the circle above the Sun; sometimes to go back, while it went through the other half, nearly; and to stand still at two points, namely those that bound the Direct and the Retrograde course: so it is necessary that to us, looking out from the Earth, Saturn should seem to be rolled towards the opposite parts, and so to go back at the time when, to one watching from Saturn, the earth would seem to go back; and Saturn to advance when the Earth advances, etc.* And he continues, saying: *The Inferior ones, Venus and Mercury, seem to go back for this reason, that they are swifter; so that, just as if the Earth stood unmoved, Venus, running in the more remote part of its circle, describes a path entirely contrary to the one it completes in the part of its circle near the Earth.* [Margin: Galileo's opinion on the cause of Retrogradations.] To this is added Salviati, or rather Galileo, who in dialogue 3 on the System of the World, page 252 of the Latin version, after saying, *In the Ptolemaic system there are diseases, and in the Copernican the remedies*, added a little later: *But what shall we say of the apparent motions of the Planets, so unequal that not only are they now swift and presently slower, but sometimes they even become entirely stationary, and even retrograde over a long space? To save this appearance Ptolemy introduced very great Epicycles, fitted to the individual Planets, with some rules of anomalies, or of incongruous motions, all of which are removed by a single, most simple motion of the earth.* Then, after some intervening words, Sagredo replies in the same place that these retrograde

and stationary motions have always seemed improbable to him, and that he therefore wishes it to be explained how the system of Copernicus excuses them; which Salviati shortly does, and so do we, as I said, in Book 7, Section 5, chapter 4, number 3. [Margin: Gassendi's concurrence.] The same cause — why the Luminaries do not seem to go back, while the other Planets do — is elegantly pursued by Gassendi in Letter 2, *De motu impresso*, page 145.

VII. [Margin: The 2nd imperfection removed by the annual motion of the Earth.] The *second* imperfection, or inequality, which seems to be in the motion of the five lesser Planets, and which is removed by transferring the true cause of such a phenomenon to the annual motion of the Earth, is the one Copernicus hinted at towards the end of Book 1, chapter 10, thus: *For from this one who contemplates not idly may notice why a greater progression and a lesser regression appears in Jupiter than in Saturn, and a lesser one than in Mars; and again a greater in Venus than in Mercury. And why such a reciprocation appears more frequently in Saturn than in Jupiter; more rarely still in Mars and in Venus than in Mercury.* For we have already shown in Book 7, Section 5, chapter 2, that the arc of direction of Saturn is less than that of Jupiter, and Jupiter's less than that of Mars, and greater in Venus than in Mercury, adding a twin table of this diversity; and in the same place, number 5, proposition 2, we added the cause why the Stations and Retrogradations of Saturn recur more often than those of Jupiter, and Jupiter's than those of Mars, and Mercury's than those of Venus. Now since Ptolemy tried to set these out by assigning a greater Epicycle to Mars than to Jupiter, to Jupiter than to Saturn, and to Venus than to Mercury, he seems to have multiplied motions and instruments, and yet not to have given the true cause of these inequalities: for the greater Eccentric of Saturn would seem to demand a greater Epicycle too. [Margin: The 3rd imperfection removed by the annual motion.] From this arises the *Third* imperfection in the Ptolemaic hypothesis, one removed by Copernicus. Hence that reproach of Kepler's in chapter 1 of the *Mysterium*: *So it could have been asked (but with Ptolemy answering nothing) why in great orbs there are such small Epicycles, and in small orbs such huge ones: that is, why the* [Greek: προσθαφαίρεσις (prosthaphairesis, "prosthaphaeresis", the equation)] *of Mars is greater than Jupiter's, and Jupiter's greater than Saturn's? And why does Mercury not also have a greater one than Venus, since it is lower than Venus, given that of the other four the lower always has the greater? Here the answer is easy*: namely, in the hypothesis of Copernicus, from which he gives the cause of these vicissitudes. [Margin: Kepler's second commendation of the annual motion of the Earth.] For, of course, as each of the Superior planets is nearer to the annual orb of the Earth, so the annual orb has a greater proportion to it, and appears greater. Mars, therefore, as nearest to the Earth's orb, has in its Radius more parts — of which the Radius of the annual orb is 100,000 — than Jupiter has, and Jupiter more than Saturn; and therefore the prosthaphaeresis arising from this is greatest in Mars, moderate in Jupiter, least in Saturn. Moreover, what the ancients thought were Epicycles in Mercury and Venus are in truth, in the hypothesis of Copernicus, Eccentric Orbs; and to Mercury, as the swiftest,

belongs the smallest orb, smaller than Venus's. So he says. [Margin: The 4th imperfection removed by the motion of the Earth.] And he soon adds a *Fourth* imperfection, or necessity, in these Retrogradations, whose true cause only the hypothesis of Copernicus seems to provide. *Likewise*, he says, *the ancients wondered, not without reason, why the three superior ones are always lowest in their Epicycle at opposition with the Sun* (understand: and Retrograde) *and highest at conjunction: so that, if the Earth and the Sun and Mars are in the same line, why Mars cannot then be in any other place of its Epicycle than at the Perigee. In Copernicus the cause is easily given. For it is not Mars in an Epicycle but the Earth in its orb that causes this variation, etc.* [Margin: Kepler's 3rd commendation of the annual motion.] For Mars could have been moved in its Epicycle against the order of the signs, and so have turned out direct at opposition and retrograde at conjunction with the Sun; nor can the cause why this motion was not made in that way be given through the Epicycle: one merely falls back on the effect and on experience, a posteriori. But since Gassendi treated these arguments not inelegantly, and it is useful to press this argument home in various ways so that its force may be set out more clearly, I am pleased to repeat from him, in other words, what has been said so far.

VIII. [Margin: Gassendi's confirmation of the causes already mentioned.] And so Gassendi, in Letter 2 of *De motu impresso a motore translato*, page 145, argues thus for the annual motion of the Earth: *Next, to speak also of the three more remote ones, Mars, Jupiter and Saturn: what is the reason that they never appear retrograde except when they have the Earth interposed between themselves and the Sun? or why are they always retrograde when they have it interposed? Why is the Epicycle of Mars greater than the Epicycle of Jupiter, and this one greater than the Epicycle of Saturn?* And a little later: *Why is Mars so much swifter than Jupiter on its Eccentric, and yet so much slower on its Epicycle, and Jupiter, in the same proportion, than Saturn, and so on for the rest?* And he answers at once that this must necessarily appear so, as the Earth proceeds between the Sun and them; for although both they and the Earth always advance towards the East, *it nevertheless comes about that, because the Earth moves faster than they do, when, approaching from far off, it is about to pass close by them, or to interpose itself between them and the Sun, they appear now to proceed more slowly than before; and when it approaches as if in a straight line, they appear as if to stand still; and thereafter also to go backward, since the Earth, pressing on faster, gradually leaves them behind, so that the retrogression appears greatest when the Earth is nearest and has them directly opposite; and afterwards it appears gradually to decrease, until, as the Earth departs from them as if along a straight line, they seem once more as if to stand still. After the second station, however, they appear to proceed directly and ever more swiftly, because the Earth thereafter, as it is about to pass beyond the Sun, observes them advancing according to the series of the Signs — they who, in the opposite position, seemed to go against that same series. Hence, then, it is that they cannot appear retrograde except in opposition with the Sun, etc. Hence the reason why the Epicycle of Mars appears greater than the Epicycle of Jupiter: because, namely, on account of its greater nearness,*

the retrogradation in it begins to appear nearer and ends farther, etc. Hence the reason why Mars is retrograde only once in two years: since the Earth, proceeding only twice as fast, cannot overtake it except in double the time in which it completes its own period. Hence the reason why Jupiter runs its course every year and becomes retrograde, and that only one month later; because the Earth is twelve times swifter and, having once left it behind, always overtakes it in the thirteenth month. Hence the reason why Saturn too becomes retrograde every year, and that scarcely half a month later, because the Earth is thirty times swifter and, having departed from it, overtakes it again in the twelfth month and scarcely half a month besides. With these foundations laid, then, let us build the following argument upon them.

IX. [Margin: The 2nd argument in form.] *That hypothesis is to be preferred to the others in which alone the imperfections of the Stations and Retrogradations are not real but merely apparent, and through which alone a reason is given a priori both of the Stations and Retrogradations already mentioned and of the other diversities attached to them. But such is the hypothesis of the Earth proceeding annually between the three superior and the two inferior Planets. Therefore this hypothesis is to be preferred to the others.* [Margin: Proof of the Major.] The MAJOR is evident to any well-disposed intellect: for who would not grant that it is better that, if any imperfection occurs in celestial motion, it should be apparent rather than true and real? Who, likewise, would be so ἄλογος [Greek: ἄλογος (alogos, "irrational")] as to prefer always to stumble along a blind path in the darkness of that hypothesis which cannot give the causes of such appearances, rather than in the light of the one that can show them? [Margin: Proof of the Minor.] The MINOR has already been shown above by an induction made both from the Stations and Retrogradations themselves, taken in themselves, and from the diversities connected with them — of which kind are these: that these vicissitudes are more frequent in Saturn than in Jupiter, and in Jupiter than in Mars; likewise, that the arc of Direction of Saturn is of more degrees than that of Jupiter, and Jupiter's than that of Mars; likewise, that Saturn, whose Eccentric is greatest, has the smallest Epicycle, or proportion to the annual orb; and Mars, whose Eccentric is the smallest among the three superior ones, the greatest Epicycle, and Jupiter a middling one; and that the Equation of the second inequality is greatest in Mars, the lowest of the superior ones, and yet least in Mercury, the lowest, or innermost, of the inferior ones; again, why they are always retrograde in the position nearest the Earth and direct in the most remote, and why this could not have happened otherwise; finally, why the Luminaries are incapable of Station and Retrogradation. For of all these a simple and plain reason is given by the annual motion of the Earth, compared with the proper motion of each Planet beneath the Zodiac.

[Margin: 1st Reply.] I reply, 1.: by distinguishing the Major, and denying and conceding it under precisely the distinction with which I distinguished the Major of argument 1, set out in number 5 of this chapter; and in the same way to the Minor, just as if that reply had been inserted here word for word.

[Margin: 2nd Reply.] I reply, 2.: by conceding the first part of the Major, if apparent Station and Retrogradation in the Planets is an imperfection; and by denying it, if it is not an imperfection but a great perfection in their motion, considering the end to which that motion is ordered and the vicissitude of the effects that depend on them in sublunary nature. For who would say that in harmonic concords and in rhythmic dances there are imperfections in what they call *Pauses*, and in the retreats or returns of vocal phrases, of sounds, of dance-steps, etc.? Or, in Dialectical reasoning, in the progression from causes to effects and the regression from effects to the causes to be demonstrated?

[Margin: 3rd Reply.] I reply, 3.: granting the Major for the present, by denying the Minor. For it is one thing to give a cause that is, at first sight, single and simple — and so to repeat the first argument of this chapter, which has been answered sufficiently; it is another to give the cause of such appearances in this hypothesis alone. But all the hypotheses celebrated so far — whether Ptolemy's, or Magini's, or Tycho's, or Our own — give the cause of these Phenomena a priori, though not in the same way, nor in the way the Copernicans want. For what these accomplish by the motion of the Earth compared with the motion of the five lesser Planets beneath the Zodiac, others accomplish by the motion of the Sun compared with the motion of the same Planets (the Earth standing still), but through an Epicycle carried by an Eccentric, or through a Spiral, or through some other equivalent orb. And in the Ptolemaic system, indeed, the Planets in reality never stand still or go back in their orbs, if you place the eye at the centre of their Epicycles, but always advance in the order of the Signs: just like the Copernican Earth viewed from the Sun as from the centre of the annual orb — granted that, if the Earth too were viewed from elsewhere, it would seem beneath the Fixed stars sometimes to stand still, sometimes to go back. In the Tychonic hypothesis, moreover, as Kepler admits in Book 4 of the *Epitome*, pages 535 and 537, the absurdity of the Retrogradations is avoided if the Sun is laid down to be the centre of the Planetary system and to carry with it the centres of the orbs of the five lesser Planets; and in this way, likewise, Astronomy is freed from two superfluous Eccentrics, those of Mercury and Venus, and from three great Epicycles, those of Mars, Jupiter and Saturn. Therefore, if the cause of these appearances is the concurrence of motions — the comparison, that is, of the lesser Planets on the one side with the Sun or the Earth on the other — the annual motion of the Sun can be received among the a priori causes just as well as that of the Earth; for either one is sufficient to represent them, with this difference, however: that the motion of the Sun, being evident to sense, has already taken possession of the judgements of the greater number of the Wise. The Copernican sect, however, seems to be of too delicate a taste, since for it no motion — neither the Sun's nor any other, except the Earth's — has the savour of the genuine a priori cause of such phenomena; a savour sharpened by the simplicity of the annual motion of the Earth, which is greater at first sight, but in depth far less, on account of the many other motions that it brings with it, as I said already in number 5, in Reply 2. But in reality the

motion of the Sun is much simpler, whether it overtakes the superior Planets or recedes from them, or carries the Epicycles of Venus and Mercury along with it by one and the same motion (for in this the hypothesis of Ptolemy is to be corrected). [Margin: Why the Luminaries are not Retrograde.] The cause, however, why the Luminaries are not retrograde has been sufficiently assigned by us in Book 7, Section 5, chapter 3, Theorems 3 and 4.

[Margin: 4th Reply, very noteworthy.] I reply, 4.: by turning the argument back. For from the annual motion of the Earth mixed with the diurnal it follows that all the points of the Terrestrial globe that lie outside its centre, and all bodies — terrestrial, watery, or semi-aerial — that follow the motion of the earth, whether fixed to the earth or detached from it or hanging, really become stationary twice every day as regards the diurnal motion, direct once and retrograde once, and yet that this is not perceived; and that the Moon, every month, as regards its monthly motion mixed with the annual motion of the earth's centre, becomes Stationary at the quadratures, Direct between the 1st and 2nd quadratures, Retrograde between the 2nd and the 1st of the following Lunation, Swiftest at the Full Moons and Slowest at the New Moons — as regards this motion, that is — and yet that not even this appears to us. So I said already in chapter 6, at the end of number 18, and shall say in chapter 19, number 12; and Galileo himself admits it, as will be said in chapter 14, number 4. Now if it is absurd to posit a real Retrogradation and Station in the celestial bodies, then, since the Earth with its kindred bodies is placed in the number of the Planets and is said to move in its own heaven with respect to the centre of the Universe, it is surely a greater absurdity to posit, in innumerable bodies diverse in number and species, a retrogradatio- n and Station — and one, indeed, so frequent that it is daily, and yet one that can be perceived by no observations, nor proved a posteriori — and to assert the same in the Moon every month, than to posit in a few other Planets a rarer Retrogradation and Station, and one, moreover, founded on observation. Or, if the former is not absurd, neither ought the latter to be absurd. There is no reason, therefore, why the Copernicans should press this argument so often.

III. Argumentum à Cauſa quæ aliarum quarundam rerum Aſtronomicarum redditur à priori per ſolum Annuum Telluris motum.

III. The Argument from the Cause of certain other Astronomical matters, which is given a priori through the annual motion of the Earth alone.

X. With this single motion of the earth substituted, says Kepler in chapter 1 of the *Mysterium Cosmographicum*, the causes of very many things are given which Ptolemy could not give from so many motions of the heaven. [Margin: Why do Venus and Mercury have the same motion as the Sun?] *For, first, Ptolemy could have been asked how it comes about that the three Eccentrics of the Sun, Venus and Mercury have equal revolutions? For the answer is that they are not truly*

revolved themselves, but the one Earth in their place. Moreover, as the same author notes in Book 4 of the *Epitome of Astronomy*, page 535, what Martianus Capella, Campanus [Campanus of Novara] and others — namely Vitruvius, Macrobius and Bede — had an inkling of, Galileo has now demonstrated in his *Sidereus Nuncius* from observations made with the Telescope on Venus, and Simon Marius [Simon Mayr] on Mercury, and we on both: that Venus and Mercury, when they proceed direct and come nearer to their morning Conjunction with the Sun, appear small and round, but when they become retrograde and are near their evening conjunction, appear very large and falcate, or horned. We gave diagrams of these phases, and the arguments, in Book 7, Section 1, chapter 2. From this, moreover, it is evidently gathered that they are illuminated by the Sun in such a way that they go round it. [Margin: Why do Venus and Mercury appear now below, now above the Sun?] Now it seems incongruous that these Planets should be carried now above, now below the Sun; and this incongruity is removed if an annual motion of the Earth is posited, for then their ascent and descent is merely apparent, according as the Earth becomes more remote from them or nearer to them as they go around the Sun.

XI. Likewise it seems unbefitting to the constancy of the celestial motions that the three superior Planets should sometimes scarcely equal little stars of the second magnitude — and this when they come to conjunction with the Sun — and sometimes equal or surpass stars of the first magnitude, and this when they come to opposition to the Sun. The cause of this diversity cannot be given by the ancient hypotheses, but it is given at once by the single annual motion of the Earth: for when the Earth interposes itself between them and the Sun, it approaches them, and therefore sees them much larger than when it recedes from them, so that the Sun is interposed between it and the Planets. [Margin: Copernicus's view on why Saturn, Jupiter and Mars are larger at opposition to the Sun.] *For from this it is possible to notice*, says *Copernicus* in Book 5, chapter 10, *that Saturn, Jupiter and Mars, when acronychal, are nearer to the earth than around their occultation and appearance. But Mars especially, when it shines all night, seems to equal Jupiter in magnitude, distinguished only by its ruddy colour; whereas there it is found scarcely among the stars of the second magnitude, etc. All of which proceed from the same cause, which lies in the motion of the earth.* [Margin: And Gemma Phrysius's.] Gemma Phrysius pursues this very point in a certain Letter to Ioannes Stadius [Johannes Stadius], dated at Louvain on 1555 February 28 [pridie Calendas Martias], asserting that the hypotheses of Ptolemy do not have such evident causes τῶν Φαινομένων [Greek: τῶν Φαινομένων (tōn phainomenōn, "of the phenomena")] as the Copernican ones have. *For that the three superior Planets*, so he goes on, *when* ἀχρόνυχοι [Greek: ἀχρόνυχοι (achronychoi, "acronychal", rising at nightfall)], *that is, placed diametrically opposite the Sun, are always at the perigee of their Epicycle, Ptolemy assumes, and this is* τὸ ὅτι [Greek: τὸ ὅτι (to hoti, "the that" — the bare fact)]. *But the hypotheses of Copernicus necessarily infer it, and demonstrate it* δί ὅτι [Greek: δί ὅτι (di hoti, "through what" — the reasoned why)]. Thus Ptolemy teaches the mere *That*

[Quod], but Copernicus the very *Why* [Propter quid]. Finally, *Kepler* strikes this very chord in his *Mysterium Cosmographicum*, chapter 1, when he says: *Likewise the ancients wondered, not without reason, why the three superior ones are always lowest in their Epicycle at opposition with the Sun, and highest at conjunction, etc. In Copernicus the cause is easily given. For it is not Mars in an Epicycle but the Earth in its orb that causes this variation.* Let the form of this argument, then, be as follows.

XII. [Margin: The argument in form.] *That hypothesis is to be put before the others which, either alone or more evidently, gives the reason a priori why Venus and Mercury have the same motion as the Sun, and seem now round above the Sun, now falcate below the Sun; and why Saturn, Jupiter and Mars always appear nearest the Earth at opposition with the Sun, and therefore largest, but at conjunction with the Sun most remote and smallest. But such is the hypothesis in which the Earth is carried around the Sun by an annual motion. Therefore, etc.* The MAJOR is clear of itself. The MINOR, too, remains sufficiently proved from what was said in numbers 10 and 11.

[Margin: 1st Reply.] I reply, 1.: with precisely the distinction I used in the first reply to the first argument of this chapter; reread it from number 5, and take it as though it were transcribed here.

[Margin: 2nd Reply.] I reply, 2.: granting the Major, by denying the Minor. For both in the Ptolemaic hypothesis — but as reformed by us — and in the Tychonic, the reason of the Phenomena already mentioned is given no less evidently. For it is certain, in the Copernican hypothesis as much as in our own and in the Tychonic, that Venus and Mercury go around the Sun and follow it as the centre of their motions; and from this, as from a true a priori cause, it follows that to the Earth — whether at rest, with the Sun moved annually around the Earth, or carried round the Sun at rest — they appear now round and slender, now large but falcate; and why they are moved by one and the same eccentric motion. Nor is the Earth, with an annual motion, any more the cause of this phenomenon than the Sun with an annual motion; indeed, if all else were equal, the motion of the Sun has the advantage in this, that it is more evident to our sense, so that, as long as we are not convinced of the opposite, we ought rather to assert it. Likewise it is false, what is imputed to Ptolemy, namely that he assumes that the three Superior Planets in the acronychal position, or at opposition with the Sun, are always perigean, or nearest the earth, and at conjunction apogean, or most remote, and gives no cause of this antithesis: for he refers the cause to their motion on the Epicycle in the order of the signs, tempered to the annual motion of the Sun. Tycho, however, refers the cause to Eccentric orbs, but ones carried round by the Sun; and this I wish to be confirmed by no other words and admission than those of Kepler himself. [Margin: Kepler's admission in favour of the Tychonic hypothesis.] For he has this in the *Epitome of Copernican Astronomy*, Book 4, page 537: *Now as regards the three superior ones, Aristarchus, Copernicus and Tycho Brahe demonstrate that, if we also place them around the Sun, and the Sun as the common centre, as*

it were, of the five Planets, so that the motion of the Sun, whether true or apparent, affects the whole orbs of the five Planets, then we are freed — as before, in Venus and Mercury, from two superfluous Eccentrics — so now, in the three superior ones, from Epicycles. As for what he adds — that the agreement which the planets already mentioned have with the motion of the Sun is blind and incredible — this is said gratuitously; and that agreement is much more credible than the annual motion of the Earth, or that magnetic attraction by which he would have them led round by the Sun itself. And if Ptolemy carried Venus and Mercury round perpetually beneath the Sun, this has now, after the use of the Telescope, been removed from his hypothesis.

IV. Argumentum à Latitudine & Inclinatione Planetarum minorum ad Eclipticam.

IV. The Argument from the Latitude and Inclination of the lesser Planets to the Ecliptic.

XIII. I once pointed to this argument in Book 7, Section 4, chapter 5, scholium 3. But Rheticus develops it at far greater length in his *Narratio prima*, on roughly the third page before the end, where, giving the reason why the annual orb of the Earth was called GREAT by Copernicus, he says: [Margin: Rheticus's encomium on behalf of the Great Orb.] *It is called the Great Orb because it has a notable magnitude in relation both to the orbs of the superior Planets and to those of the inferior, such as to be the occasion of the principal appearances*; then he adds: *Further, it is in the latitudes of the Planets above all that one may see how rightly the name Great is given to the circle carrying the centre, etc. The motions of the Planets in longitude do indeed bear remarkable testimony that the centre of the Earth describes the Orb we call Great; but in the latitudes of the Planets its uses, as if set in some conspicuous place, are more evident, since, though it never departs from the plane of the Ecliptic, it nevertheless stands as the principal cause of all the diversity of the appearances in latitude.* He then pursues the doctrine of the latitudes, and over the whole of the last two pages shows how, through the approach of the Earth to the Planets, the angles of inclination are optically increased — angles which the Ancients thou- ght to be increased on account of the Planet's position at the perigee of its Epicycle; and many other things connected with these, which there is no need to transcribe here.

XIV. [Margin: Kepler's view in favour of the latitude of the Planets in the hypothesis of the moving Earth.] Kepler urges this same point in a briefer discourse, but more than once. For in chapter 1 of the *Mysterium Cosmographicum* he said: *Likewise, because of this same approach of the earth to the planets, and its recession, in its orb, the five latitudes of the Planets themselves seem to us to receive some variation; and to save this libration Ptolemy had to establish five other motions, all of which fall once the single motion of the earth is posited.* Again, in the *Epitome of Copernican Astronomy*, Book 4, page 537, he affirmed that through the annual motion of the earth a great many complications in the motion of latitude are

removed from the Theoretical doctrine; and on page 543 he did not hesitate to say: *If the Epicycles go round while the earth stands still, whether according to Ptolemy or according to Brahe, those Epicycles, especially those of the inferior planets, will have to shake their sides in various ways as much as their head and feet — that is, to librate in a double manner; but if the Earth goes, all the orbs are inclined to the Ecliptic most constantly.* In Book 6 of the same *Epitome*, moreover, page 771, he inquires how it comes about that Ptolemy made the Epicycles of the Planets librating with a double reciprocation, when nevertheless, from the more recent observations he had adduced for Mars in chapter 14, their inclination has been found to be fixed and constant. And he answers that this happened through ignorance of the annual motion of the earth. For Ptolemy was looking at the limits of the Planetary orbit, stretched out towards certain parts of the fixed stars; and although in reality these limits decline constantly from the plane of the Ecliptic, nevertheless, as the Earth goes round, the Planetary orbit seems to stretch out towards the Earth now its northern limit, now its southern, now its nodes. But Ptolemy transferred the circuit of the Earth to the centre of the Epicycles of Venus and Mercury running through the Zodiac annually; and in this Epicycle he called perigee that point which at any given time is stretched out towards the earth, as if it were a single point, when in reality all the parts of this Epicycle (which Copernicus calls the Eccentric) become perigees in order, successively, by accident. So it came about that what Ptolemy named the perigee seemed, by a kind of libration, now to be in the North, now in the Ecliptic itself, now in the South. Having set out this doctrine, he adds this Epiphonema: *And behold a most evident argument for the annual motion of the Earth around the Sun. For since the Eccentrics of the Superior planets have fixed inclinations to the Ecliptic, why should the Eccentrics of the Inferior ones alone be laid down as librating, and with a double libration, when every libration of orbits is absurd in itself, because it produces a tortuous path of the planet instead of a circular one? As much probability, then, as the fixed inclination has, so much does the motion of the earth gain from it; and as much absurdity as the double libration has, so much does the immobility of the earth totter.* Gathering the argument into its formula, then, it runs thus.

XV. [Margin: Form of the 4th argument.] *That hypothesis is to be preferred to the others through which the latitudes of the five lesser Planets keep a constant angle of inclination to the Ecliptic, without libration of the orbits and without tortuousness of the planetary path. But the hypothesis of the Earth carried annually through the great orb is the only one of this kind. Therefore.* The MAJOR, at least other things being equal, is evident: for motions are not to be multiplied in vain, much less librations and windings of motions, if the celestial phenomena can be saved otherwise and without those complications. The proof of the MINOR has been sufficiently suggested by what was said in numbers 13 and 14: for it is by the approach and recession of the Earth to and from the Planets alone that the variation occurs in the angle by which the orbits of the Planets are inclined to the Plane of the Ecliptic; nor is it apparent how, without this motion of

the earth, that variation could come about and be explained without other complications.

[Margin: 1st Reply.] I reply, 1.: to the Major, with exactly the distinction I brought forward in the first reply to the first argument of this chapter, from number 5; since it is of great moment, I am neither ashamed nor weary to use and repeat it so often.

[Margin: 2nd Reply.] I reply, 2.: granting the Major[41], by denying the Minor. For both through a single spiral motion, made by an Intelligence towards the sides of the world with looser and tighter spirals, and in the Tychonic or Danish hypothesis, that constancy of Inclination can be saved by means of the parallelism of the planes, as is clear from the Theory of latitudes of Longomontanus, which we set out in Book 7, Section 4, chapter 4. That Ptolemy erred in the measure of the inclinations and of the latitudes, lacking suitable observations, does nothing against the hypothesis of the Earth at rest considered in itself. For the Earth, carried annually through the plane of the Ecliptic, by the very fact that its centre is always in the plane of the Ecliptic, cannot vary the latitudes of the Planets except by optically increasing the angle of inclination by its approach and diminishing it by its recession; the rest, which concerns the latitudes, the regions of the latitudes and the motion of the Nodes, as deduced from observations, must be saved through some theory other than the annual motion of the Earth in the Ecliptic. Now if the Planets, too, so temper their motion of latitude to the annual motion of the Sun that they obtain through this motion a latitude equal to, and of the same character as, the one they obtain through the motion of the Earth, and in such a way that the inclination of the planetary orbit to the Ecliptic, derived geometrically from such a latitude, is perpetually of the same measure — that is, that the plane of the Epicycle is always parallel to the plane of the Ecliptic — it will not be necessary for this to be accomplished through the annual motion of the Earth. For Copernicus already admits that all the Phenomena can stand saved both ways, as to the substance of the truth, in either hypothesis. And so that you may see that it is not from the hypothesis of the Earth at rest, in itself, that errors, or inconstancy of the latitudes and inclinations, arise, nor from the hypothesis of the moving earth that constancy and truth arise, I am pleased to show from Kepler himself what not only Ptolemy but also Copernicus did wrong in this, through what he himself sets out in the *Commentaries on Mars*, chapter 14, whose title is: *The planes of the Eccentrics are* ΑΤΑΛΑΝΤΑ [Greek: ΑΤΑΛΑΝΤΑ (atalanta, "not swaying", that is, not librating)], that is, non-librating. He begins, then, thus: *The perplexity of his hypothesis deceived Ptolemy, so that he heaped many monsters into the doctrine of latitudes. For when he considered that the plane of the Epicycle is twisted in all directions, and did not see at once, through those fogs of his hypothesis, that the plane of the Epicycle is parallel to the plane of the Ecliptic, he contrived a triple latitude; and so that contraries might be propped up by contraries, he wholly dislocated*

[41]printed *Maiote*, for *Maiore*

his Epicycle from its parallel position. And it was neither from the testimony of the observations, which he did not have so frequently, nor from their measure where he did have them, that he chose middle values (being distrustful of certainty), placing the extremes in error. Hence you may see that in the usual calculation — in Magini's Ephemerides, say — no conjunction at all of Mars and the Sun occurs that is not (as they say) bodily. If that were true, nature would have contrived the tempering of the latitudes in vain — contrived so that the agitations of the sublunary powers should not be excessive, through bodily conjunctions occurring frequently. So much concerning Ptolemy, and indeed concerning Magini too, who, although he uses Copernican numbers, nevertheless adheres to the Ptolemaic hypothesis. Then, concerning Copernicus, he goes on thus: *Copernicus, himself ignorant of his own riches, took upon himself the task of expressing Ptolemy entirely, not the nature of things — to which, nevertheless, he had come nearest of all. On which matter read Rheticus in his Narratio. For, delighted that through his approaches of the earth to the stars the appearances of the latitudes are increased, he nevertheless did not dare to reject the residual Ptolemaic increases of the latitudes (which this approach of the earth would not achieve); but, so as to express them too, he contrived librations of the planes of the Eccentrics, by which the angle of inclination (constant and fixed for Ptolemy) should be varied; and this (which is like a monster) not according to the laws of the motions of its own Eccentric, but of the earth's orb, which is entirely foreign to it: see Copernicus, Book 6, chapter 1.* See, then: from the bare hypothesis of the moving Earth, librations of these planes are not avoided unless the quantity of the latitudes is first corrected from more certain observations. Now let us see whether, in the Tychonic hypothesis too, and in the Ptolemaic though corrected, that constancy of the inclinations can remain saved. Kepler therefore continues in the same chapter; and when he had said that he had demonstrated, from the Tychonic observations made of Mars and in the preceding chapters — and would prove below, in part 5 of the Commentaries, with several arguments — that the declination of the orbit of Mars from the Ecliptic is everywhere constant, and that there is no libration of the inclinations of the Eccentric of Mars, he concludes universally, from the observations of nearly Mars alone, thus: *Let us therefore conclude this most firmly: that the inclination of the planes of the Eccentrics to the Ecliptic (for why should I not conclude it in general, when there is no cause why it should belong to one planet alone? although I have the same demonstrated from observations in Venus and in Mercury too) does not vary at all. And let him who follows Ptolemy learn from this that the plane of the Epicycle is perpetually parallel to the plane of the Ecliptic.* It suffices, therefore, for removing the complications of the librations and preserving the constancy of the inclinations, if the planes of the Ptolemaic Epicycles, or of the orbs substituted for them, are always parallel to the plane of the Ecliptic, whether this comes about with the Earth moving or with it at rest; nor is the perigee of the Planet, in which the greater latitude appears, produced more by the approach of the Earth to the Planet than by that of the Planet to the Earth.

V. Argumentum ab Aequante ſublato per Annuum Terræ motum.

V. Argument from the Equant, removed by the annual motion of the Earth.

XVI. [Margin: Rheticus's view against the Ptolemaic Equant.] This argument is very slight, as its solution will show; and yet Rheticus too invoked it in his *Narratio prima*, where, speaking of the annual motion of the Earth introduced by Copernicus, he says: *By this one means the Lord Teacher saw that it could conveniently come about that — what is most proper to circular motion — all the revolutions of the circles in the world should be moved uniformly and regularly about their own centres and not about alien ones.* [Margin: And Copernicus's.] Indeed Copernicus himself, discussing the hypotheses of the Ancients in Book 5, chapter 2, said: *They concede, therefore, that here too the uniformity of circular motion can come about about an alien centre and not its own, etc. These and similar things gave us the occasion to think about the mobility of the earth and about other ways by which uniformity and the principles of the art might remain, and the account of the apparent inequality be rendered more constant.* [Margin: And Galileo's.] Finally, Galileo, in Dialogue 3 on the system of the world, page 252 of the Latin version, brings in Salviati speaking thus: *Will not all the sects of Philosophers say that this is a great inconvenience — that some body movable in a circle should be moved irregularly about its own centre and regularly about some other point? Yet such deformed motions exist in the Ptolemaic fabric of the world; but in Copernicus all the motions are uniform about their own centre.* Let the argument now be formed in this way.

XVII. [Margin: 5th argument in form.] *That hypothesis is to be preferred to the others through which alone the absurdity of the Equant circle is removed — that is, of the circle in which neither the Planet nor the centre of the circle carrying the Planet moves, and yet about whose centre the Planet is supposed to be moved uniformly. But the only hypothesis of this kind is that of the Earth moved annually. Therefore, etc.* The MAJOR is evident from the uniformity owed to the celestial motions in their own circles, not in alien ones; otherwise, if they are moved uniformly in alien circles, they must be moved non-uniformly about their own centre, and so have not only an Optical but also a Physical, or real, inequality, which is absurd. The MINOR is established at least by the authority of Copernicus, Rheticus and Galileo, adduced a little earlier.

[Margin: 1st Reply.] I reply, 1.: if the Equant circle — that is, some real inequality in the motion of the Planets — is absurd, I concede the Major; if it is not absurd, I deny the Major. But I have taught, in Section 2, chapter 4 of this Book, that it is not absurd, but very fitting for them in relation to their end.

[Margin: 2nd Reply.] I reply, 2.: granting the Major, I deny the Minor. For the inconvenience of the Equant circle, or of the bisected Eccentricity, if there is any, is removed also in the hypothesis of the Earth at rest, as in the Tychonic, the Danish and our own; and conversely it is not removed by the annual motion of the Earth in itself. This is manifest from the fact that Kepler and Boulliau,

who defend the annual motion of the Earth as strongly as anyone, nevertheless retained, by bisecting the Eccentricity of the Planets, the equivalence of the Ptolemaic Equant; and, by moving the Planets in an Ellipse about the two foci, or umbilici, of the Ellipse, they conceded that one inequality is Optical with respect to one focus and the other Physical with respect to the other — as is clearly established from what was said in Book 3, chapters 22 and 23, and in Book 7, Section 2, chapter 5, numbers 12 and 13, where, and also in Book 7, Section 2, chapter 1, scholium 2, I have set out many things for the explanation and use of the Equant. Therefore the imperfection of the Equant, if there is any, is not of itself connected with the hypothesis of the Earth at rest, nor of itself torn away by the hypothesis of the Earth going round.

CAPVT X. Proponuntur, & Diſſoluuntur Quinque Argumenta pro Annuo Terræ motu, deſumpta ex quarũdam aliarum rerum cæleſtium Motibus, aut Variationibus.

CHAPTER X. Five Arguments for the Annual Motion of the Earth, taken from the Motions or Variations of certain other celestial things, are set out and refuted.

I. Argumentum à Fixarum diametro, motu, & diſtantia apparenti variatis.

I. Argument from the diameter, motion and apparent distance of the Fixed stars, as varied.

I. I remember having read in someone — or at any rate having received in letters sent to me from Germany — that a certain man had dared to assert that the same Fixed stars were seen by him larger in the northern quarter when the Earth saw the Sun under the beginning of Capricorn than when it saw it under the beginning of Cancer. From this he gathered that the effect came about through the Earth's approach to the northern quarter, by force of which Arcturus would then appear larger than itself when the Earth had receded to the southern Quarter. Io. Pena and Io. Stadius too, as I reported in chapter 3, scholium 2, thought that by the Earth's approach the motion of the Fixed stars is seen faster, and by its recession slower; we refuted them there. Finally, Rothmann, in Tycho, tome 1, page 684, said that the distances of the Fixed stars from one another are increased by 2′ through the approach of the Earth and diminished through its recession, and that he had observed this. But either this is a wholly voluntary Fiction — since nothing of the kind has appeared to us, who have been observing the stars for many years now — or that phenomenon arose from the differing thickness of the vapours on summer and winter nights. Otherwise, it is clear enough from Copernicus himself, *De Revolutionibus* Book 1, chapters 5, 6 and 10, that the whole annual orb, through which the Earth is supposed to move, has no sensible ratio to the sphere of the eighth Orb, or of the Fixed stars, but is like a point; and so, whether Arcturus or any other

Fixed star is viewed from the Earth placed under Cancer or from the same placed under Capricorn, no variation can appear of itself either in its apparent magnitude or in its meridian altitude. [Margin: The variation of the Fixed stars in magnitude is a fiction.] And yet if one did appear, it ought to appear greatest in the stars placed on the Ecliptic, smaller in those distant from the Ecliptic, and none in those near the poles of the Ecliptic, as Galileo demonstrates in Dialogue 3 on the System of the World, from page 281 of the Latin.

II. Argumentum à Satellitibus Iouis.

II. Argument from the Satellites of Jupiter.

II. [Margin: How the revolutions of the Jovian Satellites are regular.] Go to Kepler, in the *Epitome of Copernican Astronomy*, Book 4, part 2, page 537, and you will read in it this argument: *Marius,* he says, *in his Jovian world, discovered that the restitutions of the Jovian satellites about Jupiter, which we project from the centre of the earth onto Jupiter, are by no means regular, but that they are regular if they are referred to lines drawn from the centre of the Sun through Jupiter. This, of course, stands in the place of a very great argument that the orbit of Jupiter is ordered about the Sun, and that the distance of the Sun from the centre of Jupiter's orbit is certain and in a certain way fixed, whereas the Earth varies its distances from this centre through the year.* But it can be answered that this has not been demonstrated by Simon Marius, but deduced only from conjectures that are not very strong; and that, this granted, it does not follow from it, necessarily or even very probably, that the Earth is moved with an annual motion rather than the Sun, but at most that the Sun is the centre of the Jovian system; or that, just as the Sun is the first exemplar of one carrying its satellites Mercury, Venus and Mars round itself, so it is for Jupiter, leading its satellites round.

III. Argumentum à Lunæ Variatione, & Libratione.

III. Argument from the Variation and Libration of the Moon.

III. [Margin: The cause of the lunar Variation.] Tycho observed that, besides the first and second inequality, or anomaly, which the Moon undergoes, it has a third inequality as well, more manifest in the Octants but ceasing in the Syzygies and quadratures, as we set out in Book 4, chapter 18, number 8, and chapter 26, number 9. Tycho and Longomontanus refer its cause to a libration of the lunar Epicycle, as we showed in the same place; Kepler, by contrast, to the illumination of the Sun and its species reflected from the earth onto the Moon, as we set out in the same Book 4, chapter 27, from number 5, and in scholium 6. But Ismaël Boulliau, in Book 3 of the *Astronomia Philolaica*, chapter 11, thinks that the truer cause of this Variation is the Reflection of the line of the lunar apogee, arising from the annual motion of the Earth, by which the whole lunar system is carried along. For when the apogee is advanced by the reflection from the Syzygies towards the dichotomies, the Moon thereby acquires a new acceleration; and when it is drawn back from the quadratures

towards the Syzygies, the Moon undergoes a certain slowness in the Zodiac. This he thinks is the genuine, proximate and simple cause of this Variation, whose measure he investigates afterwards in chapter 12. See more on these matters, if you please, in our Book 4, chapter 27, numbers 13 and 14.

IV. [Margin: The cause of the lunar Libration.] Besides this Variation, a certain Libration has been observed in the Moon, by force of which the lunar spots appear now nearer to the eastern and western limb, now farther from them; so that when the Caspian Spot is most remote from the western edge of the Moon, then the Spot called by Langrenus [Michael Florent van Langren] and by us the Lake of Posidonius — which Galileo, however, calls the Corican and Hevelius the Mareotic — is nearest to the eastern limb of the Moon, etc. More on this libration below, in the Appendix to Book 4, on the Moon. Galileo, in the dialogues, said that this libration was instituted so that the Moon might keep a certain direction towards the axis of the moving earth; but Boulliau, in Book 3 of the *Astronomia Philolaica*, chapter 13, rejects this and concludes that the axis of the Moon about which that libration is performed is either perpendicular to the plane of the Ecliptic or insensibly inclined to it — by as great an angle, perhaps, as that by which the planes of the Ecliptic and of the lunar orbit are inclined to each other, namely about 5⅓°: see the diagram of this matter in our Book 4, chapter 9, number 4. With these premised, let the Argument be:

V. [Margin: The argument in the form of an Enthymeme, and a twofold Reply.] *The Variation of the lunar motion and the Libration of the lunar body are better explained on the supposition of the Earth's annual motion than on the supposition of its rest. Therefore the annual motion of the Earth is to be supposed rather than its rest.* It is answered, 1., by denying the antecedent: for they are explained equally well by a libration of the lunar Epicycle, or of the lunar body itself, according to what is to be said in the Appendix to Book 4. I reply, 2.: granting the antecedent, by denying the Consequence, because other things are not equal. Indeed there are other, greater absurdities in the annual motion of the Earth, and the exclusion of these outweighs this very slight convenience, by which a partial cause of the lunar Variation and Libration could be brought forward through the annual motion of the Earth; and that lunar Libration is completed wholly within a month, and has no analogy with the annual motion of the earth. Besides, what Boulliau and Hevelius have so far published about that libration agrees neither with themselves nor with our observations, so that this matter must be deliberated on more maturely, and a more uncertain cause must not be sought from uncertain phenomena.

IV. Argumentum à Nouis Stellis.

IV. Argument from the New Stars.

VI. Thomas Diggesseus [Thomas Digges], an Englishman of Kent, on the occasion of the new star which was seen in Cassiopeia in 1572, thought — to

use his own words as reported in Tycho, tome 1, page 658 — *that an opportunity had been offered of testing whether the motion of the Earth supposed in Copernicus's theories will be the sole cause why this star diminishes in apparent magnitude. For if it is so, at the vernal Equinox, always decreasing, it would be seen at its least magnitude. But afterwards, if it lasts, growing little by little, in the following July it will be of almost the same brightness as in its first appearance. But at the autumnal Equinox it will be seen of unusual magnitude and also of unusual splendour. No other cause could be assigned to a variation of such apparent quantities than its elongations from the Earth.* So Digges, but with a wholly empty hope. [Margin: Thomas Digges's empty conjecture about the motion of the Earth from the magnitude of the new star.] For that star, in 1572, about the beginning of November, was seen to exceed Lyra, Sirius, and Jupiter itself when acronychal; in January of 1573 it was a little less than Jupiter but a little greater than Lyra; in February and March it was seen equal to Lyra; in April and May it resembled stars of the second magnitude, and in June, July and August stars of the third rank; but in September, October and November, more attenuated, it scarcely equalled stars of the fourth class; in December of 1573 and January of 1574 it was equal to stars of the fifth brightness, and in February to those of the sixth; and at last, in March, it became invisible, as Tycho narrates in tome 1, page 301. Therefore it did not change its magnitude in the way that the approach and recession of the Earth would require: for it ought to have been seen equal to Jupiter again in November of 1573. Moreover, as Tycho notes against Thomas on page 660, it ought to have appeared greatest when the Sun was about the beginning of Scorpio and the Earth about the beginning of Taurus, and least when the Sun was placed about the beginning of Taurus and the Earth about the beginning of Scorpio, if that variation had come from the annual motion of the Earth; for the longitude of the star was about the 7th degree of Taurus. "But let us grant," says Tycho in the same place, from page 658, "that the star grew and decreased according to its approach to the earth, and that it was, as Digges would have it, about the region of Mars, with a parallax of 2′; it would certainly have undergone the parallax of the annual orb, and at the autumnal Equinox it would not have been seen under the same place of the Fixed stars, and equally distant from the Fixed stars of Cassiopeia, as that under which it was seen at the vernal Equinox. And yet Digges admits, in his treatise on this star, that it stood motionless for the whole time of its appearance and was seen under the same place of the Fixed stars." But if he had located it in the eighth sphere, its distance from the Earth according to Copernicus is so great that the diameter of the annual orb is insensible in relation to it, and no variation in the apparent magnitude of the star could have been detected from the mere approach and recession of the Earth. Since, therefore, that star decreased continuously, Digges must acknowledge that it either withdrew into the depth of the aether or perished little by little.

V. Argumentum à Traiectione Cometarum.

V. Argument from the Trajectory of Comets.

VII. [Margin: Kepler's argument for the annual motion of the Earth from the motion of Comets.] Here indeed Kepler triumphs, in his history of the three Comets which appeared in 1618. For there, in Epichireme 8, which is found on page 91, he argues thus: *With the trajectory of the Comet supposed rectilinear, and slackening little by little from a uniform motion by ordered decrements, the Comet of 1618 offers a twofold testimony to the motion of the Earth: one by its motion in longitude, the other by its motion in latitude.* As to longitude, he strives to prove this from the fact that the orbit of the Comet, divided into daily motions, appeared divided disorderly: *But no straight line,* he says, *divided into parts by ordered decrements can appear divided disorderly while the Earth is at rest; it can, however, if the Earth goes round the Sun and makes progressions that are sensible in comparison with the Comet's speed.* See his Theorem 30 of the same little work. As to latitude, the same author busies himself to suggest that, from the observed latitudes of the Comet, the Comet deflected in two places from the direct path of a great circle towards the Ecliptic — first in Libra and Virgo, and again in Cancer — as the Earth deflected towards the Sun, and that it declined little from a straight line parallel to the straight line of the trajectory along which he supposes the Comet to have proceeded. For by this supposition he concludes, on page 92: *Therefore the apparent path of the Comet, if the true trajectory keeps its own proper order of decrements, necessarily becomes tortuous; and conversely, because the path of the Comet was tortuous, then, given a rectilinear and ordered trajectory, it follows that the Earth moves, etc.* And on page 93 he says that he was led to assert the rectilinear motion of Comets not by a desire for novelty, but because he found in rectilinear motion evident causes of many appearances which it is not easy even to dream of in a circular motion of the Comet. And on page 97, asking why the Comets of 1607 and 1618, and many others, having become slow, began to curve their paths, he answers that it was a mere deception of sight, as are also the stations and regressions of the Planets, and that this arises from the recession of the Earth: for a Comet much elongated from the Earth along a straight line much also diminishes the appearance of its latitude and its progressions in longitude. Finally, on page 98, he ends the first little book with this clause: *In short, as many Comets as there are in the heavens, so many arguments are there (besides those drawn from the motions of the Planets) that the Earth is moved with an annual motion about the Sun. Farewell, Ptolemy; I return to Aristarchus, with Copernicus as my guide.* Which appearances of Comets they are that seem to be better explained by a rectilinear motion of the Comets mixed with the annual motion of the Earth, I have already taught sufficiently from Kepler in Book 8, Section 1, chapter 26, where I treat this argument somewhat more fully. [Margin: Solution of the argument.] There, from number 5, I refuted this argument — which is not, of itself, without some probability — by four objections brought against Kepler; let the Reader have recourse to them,

so that we are not forced to repeat the same things here. Let him remember, besides, that the absurdities in the annual motion of the Earth, already reviewed in chapter 9 of this section after number 5, outweigh this spark of probability struck violently out of the Comets in favour of the annual motion, as if out of flint — especially since Kepler's argument does not proceed absolutely, but from a feigned hypothesis, both of the rectilinear motion of the Comets and of its decreasing in order, and he has demonstrated neither hypothesis. Tycho, moreover, would answer with the words found in the letter to Magini written in 1590 on the Kalends of December [1590 December 1], which is on page 81 of Magini's own *Directions of the primum mobile*, where he says this: *Indeed even the Comets themselves of which I spoke, insofar as some of them proceeded about the region opposite the Sun, denied the annual motion of the earth asserted by Copernicus: for it took nothing away from their destined course, much less forced them, as we watched, to be driven backwards or held back with a slower gait.*

CAPVT XI. Proponuntur & Diſſoluuntur Duo Argumenta pro Motu Terræ Annuo, vnum ex Mutatione Lineæ Meridianæ; alterum ex Mutatione Altitudinis Poli, de quibus multa ſcitu digna referuntur.

CHAPTER XI. Two Arguments for the Annual Motion of the Earth are set out and refuted: one from the Change of the Meridian Line, the other from the Change of the Altitude of the Pole; on which many things worth knowing are reported.

I. So far we have treated those Arguments which, relying on true phenomena, seemed to win some probability for the annual motion of the Earth — except, however, arguments 1 and 5 of the preceding chapter, which, lacking the truth of the phenomenon they assumed, collapsed at once. In this chapter, by contrast, we must fight with mere shadows and dreams, and so the victory will be easier, once the very falsity of the assumptions is detected. On this occasion, however, two very celebrated questions will be dealt with, pertaining partly to the doctrine of the primum mobile and partly to Cosmography: namely, whether the Meridian Line and the Altitude of the Pole change from time to time.

I. Argumentum à Mutatione Lineæ Meridianæ.

I. Argument from the Change of the Meridian Line.

II. Iosephus Scaliger [Joseph Justus Scaliger] judged that the Meridian Line changes, not so much from observation as from the Theory of the equinoctial precessions. For in that *Diatriba* of his on the anticipation of the Equinoxes he seemed to himself to have demonstrated that the poles of the Equinoctial are different from the poles of the World; that the poles of the Equinoctial circle are indeed movable; and that from this comes the motion of the equinoctial points against the order of the signs, on account of which the Fixed stars appear to

move in the order of the signs. His chief argument was that the star they call the Pole star, which is at the tip of the tail of Ursa Minor, was, as it is now, so also formerly, in the time of Eratosthenes and Hipparchus, and indeed of Eudoxus, the most northerly of all the stars of this constellation, and yet it was formerly distant from the Pole of the celestial Equator by 18°, [Margin: Scaliger's fiction about the change of the Meridian line.] in the time of Hipparchus by 12° 24′, while in the year of CHRIST 1600 it would be distant only 3° 24′. From this fiction, among the other corollaries, he likewise deduces this one, which is fifth in his order and is expressed on page 70 in these words: *Meridian Lines drawn on a plane change their position with the progress of time.* "For all Meridian lines," he says there, "pass through the poles of the Equator; but the Poles of the Equator are movable; therefore the Meridian lines too are movable" — namely, against the order of the signs. And at the end of the Corollary he brings this inference: *Therefore, as many equinoctial circles as there are, so many will be their poles; and as often as the poles change, so often will the Meridian lines passing through those poles manifestly change. For the Meridians do not pass through the poles of the World, but through the poles of the Equinoctial.*

III. But just as it is false that the star at the tip of the tail of Ursa Minor was formerly the Pole star and the most northerly of all, so it is false that the poles of the World are different from the poles of the Equator; and consequently what is deduced from this is false, namely that the poles of the Equator, the Meridian circles, and the Meridian lines that indicate them, all change. From what source that false assumption crept into Scaliger's mind, and the confutation of this fiction, we have already sufficiently indicated in Book 3, chapter 28, number 10, and in Book 6, chapter 4, in the Appendix on the pole star, from number 3. Scaliger has, moreover, been abundantly refuted by most learned men of our Society: by Dionysius Petavius [Denis Pétau], in Book 3 of the dissertations of the *Uranologion*, and by Paulus Guldinus [Paul Guldin], in the Refutation of the *Elenchus* of Calvisius, written against the Gregorian Calendar, Book 2, chapters 9 and 10. Further, this change of the Meridian line, of the kind that ought to exist on Scaliger's view — namely, that the northern part of this line should deflect continually towards the West — has not been confirmed by any experiment. Nor, if it were granted, could anything be derived from this change to confirm the motion of the Earth; for a change of this kind does not follow from the motion of the Earth, but from the motion of the poles of the Equator. Otherwise, if it came from the annual motion of the Earth, the whole change of the Meridian lines would have to be completed within any one year, and the same change would have to return whenever the Earth was in the same place of the Ecliptic.

IV. Let us now hear Galileo, at the end of Dialogue 4 on the System of the World, on the second page from the end, narrating thus in the person of Salviati: [Margin: Cesare Marsili, asserter of a change in the Meridian line.] *There arises at this time a certain fifth novelty, from which the mobility of the terrestrial globe may be argued, through what has been very subtly uncovered by the Most Illustrious*

Lord Caesar [Cesare Marsili], sprung from the most noble family of the Marsili of Bologna, and himself enrolled in the College of the Lincean Academicians, who relates in a certain most learned writing that he has observed a certain continual, though very slow, change in the Meridian line. How great that change was, however, and towards what quarter it was noticed, and by what means the observation was made, Salviati does not add there. I, for my part, have not been able to find this writing, although in the papers of the most excellent Lord Carolus Antonius Manzinus [Carlo Antonio Manzini] and of Bartholomaeus Provalius I have read certain observations made in the presence of the already mentioned Lord Cesare Marsili on that marble line which Master Egnatius Dantes [Egnazio Danti] had constructed here in Bologna in the very great Church of San Petronio, as I reported when dealing with the Bologna Gnomon in Book 3, chapter 6, from number 3, where I displayed a diagram of this Gnomon and of the already mentioned marble line. From this line, perhaps, the occasion was taken for suspecting this change. For some Bolognese think that that marble line, or path, laid by Egnatius Dantes in the pavement of San Petronio, was the Meridian line, when in fact it is not, but deflects from the exact Meridian line towards the East in its northern part by 9° 6′ 20″, as I discovered by a most accurate observation together with Fathers Franciscus Maria Grimaldus [Francesco Maria Grimaldi] and Franciscus Zenus. The reason why that marble path could not be laid upon the meridian line itself was the obstacle of the wall and of the front column, which is near the steps of the high altar: for that line would have turned out very short, and would not have shown the people clearly the days of the Solstices and Equinoxes, as Egnatius Dantes wished, at the time when the question of correcting the Roman Calendar was being most actively debated, because the vernal Equinox had regressed from the 21st of March to the 11th. But Egnatius Dantes himself, too, in the description of that Gnomon which is annexed to his *Anemographia*, published in 1578, teaches clearly enough that that marble path is not the Meridian line itself, but declines from the Meridian towards the East in its northern part, since he says that the image, or ray, of the Sun descends a little after noon to that marble line. His own words are these: *Since, however, the Sun does not always reach the points of the equinoxes at noon or near it, the altitude of the Sun must for that reason be determined while it is on the gnomonic line; we shall have it from the proportion of the line AB, that is, of the height of the gnomon, to the line drawn from A, the beginning of the gnomon, and terminated at the ellipse of the Sun; and if to this is added what the Sun has travelled after noon in descending toward the gnomon, and with the altitude of the equator of Bologna, then the Sun will be said, etc.*

V. But wherever this suspicion finally arose for Lord Marsili, so long as no change in the meridian line is demonstrated, we can by no means be persuaded of it — we, I say, who have long since found out by manifold experiment how deceptive the observation of this line is, and who have therefore been forced to resort to new and subtler methods of verifying it. I indicated some of them in Book 1, chapter 10, in the scholia, and will set them out more fully in Book

10, among the Cosmographical Problems. Meanwhile one must guard against refractions, the variation of the solar declinations, and many other things that hinder the exact perfection of this procedure. I am therefore not surprised that some, as Father Iacobus Viua once wrote to me from Germany, found a difference between the meridian line taken from the fixed stars and the one taken from the Sun, and again between one taken from the Sun before noon and one taken after noon. But if it is once exactly obtained, and many years later is investigated again at the same place on the Earth by the same method and with the same care, I have not the least doubt that it will be found exactly the same. I have proof of this from the orientation of the most holy House of the most blessed VIRGIN at Loreto. [Margin: The orientation of the House of Loreto.] For the Angels placed it so that, as then, so now it faces exactly the four cardinal points of the world: one straight line drawn at right angles from one wall to the opposite wall marks the meridian, and another, cutting this at right angles, designates the parallel of the equator. This was so that it might be understood to be a most safe refuge, to which mortals could flee from the four quarters of the world. [Margin: Three arguments: the meridian line is unchangeable.]

Moreover, the unchangeability of the meridian lines follows rather from Copernicus's own hypothesis. For in order to safeguard intact, through the motion of the Earth, all the phenomena that the usual hypothesis explains with the Earth at rest, he laid down that the axis of the terrestrial equator should always move parallel to itself and towards the same parts of the heaven, as we set out in chapter 4, numbers 16, 22, 23 and 25. From this it follows that the meridians are always the same on the terrestrial globe and perpetually face the same parts of the world, as far as sense can tell, and that no sensible difference can ever be discerned between the meridian lines of one and the same place, provided they have been laid out with equal care. A change of the meridian line would therefore rather stand in the way of the Copernican position. Galileo did not consider this; out of an excessive itch to prop up the Earth's annual motion from any quarter whatever, as soon as he heard that Marsili asserted a change in the meridian line, he conceived the hope of hammering out from this too some clever argument for the Earth's annual motion. [Margin: Galileo's itch to argue for the annual motion of the Earth.] Lastly, the magnetism of the Earth itself, which Galileo did not reject, requires that its axis keep its position most constantly, turned immovably and perpetually towards the same points of the heaven, as William Gilbert clearly teaches in *De magnete* book 6, chapter 2, Cabeo in book 3, chapter 4, and Zucchius [Niccolò Zucchi] in the *Nova de machinis philosophia*, part 5, section 10. We therefore do not yet believe what Athanasius Kircher, in book 2 of the *Magnes*, p. 489, promises to demonstrate: that, from static principles, the terrestrial globe has changed its position in every age. [Margin: The immobility of the terrestrial globe.] For he has not yet demonstrated it; nor are the changes of heavy bodies that occur on the surface of the Earth sufficient to cause a motion of trepidation, since, in respect of the

whole terrestrial globe and of the air to be driven round it, they are so light that they would not equal the finest atom added to an enormous balance. And yet this motion of trepidation would do nothing towards a change of the meridian line, or towards establishing the Earth's annual motion.

II. Argumentum à Mutatione Altitudinis Poli.

II. The argument from the change in the altitude of the pole.

VI. Two questions of no small importance present themselves for investigation here: first, whether the altitudes of the pole at the same places on the Earth change by some annual or secular variation; second, whether, if we were certain of such a variation, the Earth's annual motion would follow from it. As to the first, Pliny the Elder seems to have given occasion to both these questions in book 36, chapter 10, with this account: [Margin: Pliny's account of the change of the equinoctial shadow. [?]] *To the obelisk which stands in the Campus Martius the Deified Augustus gave a remarkable use: to catch the shadows of the Sun, and so the lengths of the days and nights. A pavement of stone was laid out to the measure of the obelisk's height, so that the shadow at Rome at the sixth hour of the closing day should be equal to it, and that day by day it should decrease little by little along rulers of bronze set into the stone, and then grow again. A thing worth knowing, and of fertile ingenuity. Manlius the mathematician added a gilded ball to the apex, so that at its top the shadow would be gathered into itself, whereas the apex would throw out lengthenings now one way and now another — a principle, they say, learned from the human head. This observation has not agreed for nearly thirty years now: whether because the Sun's own course has fallen out of tune and the heaven has changed in some way; or because the whole Earth has been shifted somewhat from its centre, as I hear has been detected in other places too; or because the gnomon alone was twisted there by earthquakes; or because the mass has settled through the floods of the Tiber — although its foundations are said to have been laid as deep in the earth as the height of the thing set upon them.* From these words nothing certain or even probable can be gathered for the motion of the whole Earth, only the signs of Pliny's wavering mind. It is more credible that the error arose partly from the day of the true equinox, which was wrongly noted in the calendar and treated as immovable, so that the equinoctial shadow did not appear as long as it was thought it ought to be; and partly from the settling of the stone pavement, the ground having been trodden down for almost a whole century, so that in Pliny's time the marble path no longer made a right angle with the perpendicular line of the obelisk, as it had done in Augustus's time. But Scaliger, in his *Diatriba* on the equinoxes, p. 72, misusing this passage of Pliny and asserting that no variation occurred in the obelisk or through the settling of the stone, attributes the whole discrepancy to a change of the meridian line. Leaving Pliny and Scaliger aside, then, let us come down to more recent authors.

VII. Towards the end of the fourteenth century Dominicus Maria Ferrariensis [Domenico Maria Novara], a man endowed with the highest gifts and the teacher of Nicolaus Copernicus, was the first, so far as I know, to raise this opinion about a change in the altitude of the pole, in a certain treatise or prognostication published at Bologna in the year 1489. From it Maginus, in Canon 8 of the *Secunda mobilia*, and William Gilbert, *De magnete* book 6, chapter 2, selected these words: [Margin: Domenico Maria's opinion on the altered altitudes of the pole.] *In past years,* says Domenico Maria, *while studying Ptolemy's Geography, I found that the elevations of the north pole he gives for the several regions fall short of those of our time by one degree and ten minutes; and this difference can by no means be ascribed to a fault of the table, for it is not credible that the whole series of the book should be equally corrupted in the numbers of the tables. It is therefore necessary to grant that the north pole has moved towards the vertical point. So observation over a long time has now begun to reveal to us what lay hidden from our forebears — not through their idleness, but because they lacked the long-period observations of predecessors. For very few places before Ptolemy had been observed for their elevations of the pole, as he himself testifies at the beginning of his Geography, where he says: Hipparchus alone handed down to us the latitudes of a few places; but very many of the distances, especially those lying towards the rising or setting of the Sun, were conceived from a certain general tradition — not through the idleness of those authors, but because the use of a more careful mathematics did not yet exist. It is no wonder, then, if earlier men did not perceive this very slow motion; for in one thousand and seventy years it shows itself to have moved about one degree towards the zenith of the inhabitants. The strait of Cadiz shows this: there, in Ptolemy's time, the north pole appeared elevated above the horizon by 36 degrees and a quarter, but now by 37 and two fifths. Leucopetra in Calabria likewise shows a similar difference, as do the individual places of Italy — those, that is, which have not changed down to our own times. From this motion, therefore, the places now inhabited will at last become deserted, and others, which are now scorched by the torrid zone, will be brought, though over a long space of time, to our temperate climate; so that this very slow motion is completed in the course of three hundred and ninety-five thousand years.* [Margin: The period of this fictitious change. [?]]

Moreover, Giordano Bruno of Nola, a devotee of a great many novelties, subscribed to this invention of Domenico Maria in his books *De maximo et immenso* and *De infinito ac innumerabilibus*, page 306. [Margin: The opinion of Bruno and Maginus on the changed altitude of the pole. [?]] And, what may surprise you more, so did Io. Antonius Maginus, in his Tables of the *Secunda mobilia*, Canon 8, where he says that he increased the latitudes of places in his catalogue because of the more recent observations of Petrus Pitatus [Pietro Pitati] and others, who found them increased and greater than in Ptolemy's time. And he adds: *Indeed we hold that the latitudes of other places of Ptolemy too ought to be increased, both on this ground and on the authority of Domenico Maria of Ferrara, a man endowed with a divine genius, who was the teacher of Nicolaus Copernicus. It pleases me to share his opinion on this matter with students, especially*

as I know that his writings cannot easily come into everyone's hands: for in a certain old prognostication for the year 1489, printed at Bologna, he puts these words at the head: In past years, etc. — the words we reported above.

VIII. Io. Stadius asserted a plainly contrary account of this change in the History of Astronomy prefixed to his Bergen Tables, following Io. Penna, though with his name suppressed (enough has been said of him in chapter 3 of this section, from scholium 2). [Margin: Io. Stadius's different opinion on this change.] He affirms that the centre of the Earth rises while the motion of the fixed stars is accelerating, and descends while it is slowing, so that it necessarily follows that the Earth moves up and down and is not the centre of the universe. To confirm this invention he calls on Pliny, already cited by us at number 6, and says: *In Pliny's age the whole Earth was detected to have been moved somewhat from its centre; and the same is shown by the latitude of cities and regions, which has been decreasing in various places since Ptolemy's age and plainly reveals itself. And to confirm the matter with an example: Ptolemy, in the Geography, records the latitude of Rome as 41⅔ degrees; and lest you object that some error crept in with Ptolemy — in the city of Rome, on the day of the equinox, the shadow falls short of the gnomon by a ninth part, as Pliny reports and Vitruvius testifies in book 9. Now since, when two sides about the earth of a right-angled triangle are given, each of the angles is given, here too (because 8 and 9 are given) one of the acute angles is given as 48⅓ degrees, which subtends the Sun's altitude above the horizon, and the other as 41⅔ degrees, by which the equator declined from the vertical point of Rome — that is, the latitude of the city of Rome, which Ptolemy, more recently, defined as 41° 40′. But the observation of more recent men, as Erasmus Reinhold reports, shows the same latitude in this our age to be 41 degrees and a sixth, so that you may be in doubt whether you would prove that half a degree has been lost through the centre of the world or through a tilting of the Earth.* Stadius thus holds that the altitudes of the pole have diminished from Ptolemy to these times, whereas Domenico Maria and Maginus thought they had increased. In so far as his opinion rests on the unequal motion of the fixed stars, we have already refuted it, and Penna's, in the scholia of chapter 3; the rest will be refuted shortly, once we have reviewed the opinions of others.

IX. After this, Christoph Rothmann, mathematician to the Landgrave of Hesse, reported to Tycho, as Tycho himself relates in volume 1 of the *Progymnasmata*, p. 684, [Margin: Rothmann's opinion on the changed altitude of the pole.] *that he had sometimes noticed that the altitude of the pole shows itself different by one or two minutes in summer from what it is in winter; and that the intervals between certain fixed stars, especially those separated by a great distance, come out different at one time from another, so that they too undergo a variation of about two minutes. And he thought it was made probable from this that the Earth does not perpetually and immovably occupy the centre of the universe.* Finally, Pierre Gassendi, at the end of his *Judicium de novem stellis circa Iovem visis* [Judgement on the nine stars seen around Jupiter], relates a certain observation made by Alexander Calignonus, which, if it were true, would be an argument that the

Earth librates in some meridian and by its libration varies the altitudes of the pole. [Margin: Alexander Calignonus's observation on the change of the plumb line.] For he says that Calignonus hung a plumb line from a height of thirty feet, enclosed in a tube so that it would be safe from movements of the air; that he fixed a point beneath the plumb-bob, and beneath that downward-pointing point, when the plumb line was at rest, set another point standing vertically and fixed in a cube. With these prepared, he observed that the point of the plumb line moved for six hours from north to south, but curving towards the east, and for six hours from south to north, by a path curved towards the west; and that it reached the limit of its excursion towards the south at noon itself, and again at midnight. From this observation, kept up for a month, Calignonus concluded that the Earth has a certain motion from south-west to north-east [ex Africo in Cęciam], and from north-east to south-west, and that the ebb and flow of the sea's tide is to be ascribed to this motion rather than to the Galilean motion of the Earth. Gassendi, though he suspends his assent, does not oppose this observation so far as not to wonder whether observers of the heavens should be warned that, in taking the altitude of the pole from the meridian altitudes of the Pole Star, they should beware of those hours at which the altitude of the culminating star is not the same. For if the Earth nods in this way, the altitude of the stars is not the same at midnight and at the sixth hour; but in this century the greatest altitude of the Pole Star is to be observed in Europe around the sixth hour of the evening, and the least around the sixth hour of the morning.

X. We have thus found three opinions so far on the change of the altitudes of the pole. [Margin: Summary of the three opinions brought forward.] The first is of a change that is secular, that is, not perceptible except over the lapse of centuries — whether it increases these altitudes, as Domenico Maria, Giordano Bruno and Maginus held, or decreases them, as Io. Stadius held. The second is of an annual change, owing to the Earth's annual approach to and withdrawal from the fixed stars; this was Christoph Rothmann's. The third is of a diurnal change, owing to a libration of the Earth from south-west to north-east and back, returning to the same position twice a day; Alexander Calignonus affirmed it, and Gassendi did not deny it. We now undertake to refute these in order, and much more accurately than was done against Domenico Maria, Stadius, Maginus and Rothmann by William Gilbert in *De magnete* book 6, chapter 2; by Tycho in volume 1 of the *Progymnasmata*, p. 684, and in the *Astronomiae instauratae mechanica*, page 8 before the end, and also in a letter to Io. Antonius Maginus dated 1590 December 1, by the old calendar [calendis Decembris anni veteris], which we have at the front of Maginus's own Tables of Directions of the primum mobile, p. 82; by Kepler in the *Astronomiae Pars Optica*, p. 148; and by Villebrordus Snellius [Willebrord Snel], *Eratosthenes Batavus* book 1, chapter 8, where, speaking of Domenico Maria contriving great things out of a very few slight observations, he says: *Here he makes us an elephant out of a gnat; for if he had examined Ptolemy a little more carefully, he could have*

done with this prognostication without any effort. [Margin: Authors asserting the unchangeability of the altitudes of the pole.]

XI. First, then, against Domenico Maria and his followers there stands the magnetism of the Earth, which, as Gilbert, Cabeo and Zucchi, already cited above, contend, requires that its axis remain immovably in the same position towards the same points of the heaven, and therefore that the altitudes of the pole do not vary. [Margin: 1st argument for the unchangeability of the altitudes of the pole.] Next, for a just comparison to be made between the altitudes of the pole of one and the same place taken in antiquity and taken recently, both would have to be most exact. [Margin: 2nd argument.] But it is well enough established that those taken in antiquity were not so precise, for many reasons. First, because the parallax and refraction of the Sun were neglected, not only in the Sun's solstitial altitudes but also, and chiefly, in measuring the longest and the shortest day of the year — the measure from which, as is established, Ptolemy in the *Geography* determined very many altitudes of the pole, that is, latitudes of places. For the horizontal refractions apparently increase the length of the day: by raising the Sun they make it seem to rise earlier and set later than the obliquity of the horizon and the Sun's declination would require, and this more in winter than in summer. It could therefore happen that differences in the refractions prevailing at different times on the same horizon produced a difference in the altitude of the pole — a matter indicated by Gilbert, book 6, chapter 2, illustrated by Kepler in the *Optics*, p. 148, and not omitted by Tycho. Moreover, when the ancients deduced the altitude of the pole from the ratio of the equinoctial shadow to the gnomon, then, even granting that the end of the shadow and the right angle of the gnomon had been applied most exactly, the equinox still did not fall at noon, and the variation of the declination on the equinoctial days was large; nor was even the day due to the astronomical equinox certain to the observers. At any rate, in all these points there lurks some suspicion of error, which makes those altitudes uncertain. But the principal fault in those observations was that the altitude of the Sun's upper limb was used in place of the altitude of the Sun's centre — the upper limb, of course, being what bounds the end of the shadow with its ray — when the Sun's apparent semidiameter ought to have been subtracted from that altitude. We have shown this by many examples in our Geographical book, and will make it plain with some a little below; and Snel noted this very fault in those observations in his *Eratosthenes Batavus*, book 1. Suppose, then (as could easily happen), that the moment of the equinox was mistaken by a whole day, and that the observation of the equinoctial shadow was made on March 25, which the civil calendar marked as the day of the equinox, but that the equinox fell at noon on the 24th or the 26th. Here, then, is an error of 24 minutes, since it is established from the table of solar declinations that on the days nearest the equinox the Sun's declination varies by a whole minute every hour. Add to this error the error of about 16 minutes contained in the Sun's apparent semidiameter: do you not see that an error of 40′ could arise from this in estimating the altitude of the

pole — or of a whole degree and more, if the observation was made two days before or after the true equinox? And what if Ptolemy determined some of these by conjecture from the position of the regions, as Gilbert suspects in *De magnete* book 6, chapter 2, saying: *Since Ptolemy received only certain latitudes from Hipparchus, and did not observe them himself at most places, it is likely that, knowing the position of the regions, he estimated the latitude of the cities by probable conjecture only, and then committed it to his tables. So one may see in our own Britain that the latitudes of cities are wrong by two or three degrees, as experience teaches. Therefore no new motion should be introduced on account of these errors, nor should the Earth's renowned magnetic nature be disfigured by an opinion so lightly conceived.* [Margin: The Ptolemaic altitudes of the pole are very erroneous.] And this was precisely the reason why Tycho, in the *Astronomiae instauratae mechanica*, page 8 before the end, earnestly wished that the Republic of Venice would send skilled men to Alexandria in Egypt to take the polar altitude of that city exactly — both for other purposes, and so that, by comparing it with the altitude once taken by Ptolemy accurately enough, as it is fair to believe, it could be determined whether or not the altitudes of the pole change with the lapse of time. For he judged that the other altitudes, which Ptolemy did not observe but handed down on others' authority or from his own conjecture, were by no means to be trusted. And our Clavius, too, in the *Sphaera*, page 284 in my copy, declares that the latitudes of places recorded by Ptolemy not rarely depart from the truth by one or two degrees. Yet not even the more recent astronomers have always been so exact in this business as to incur no error even of a minute, as we shall soon see concerning the altitude of the pole at Rome. The altitude at Paris is also worth noting: in observing it, Orontius found 48° 40′, Vieta [François Viète] 48° 49′, Gassendi and Hortensius [Martinus Hortensius] 48° 42′ or 43′, Hérigone 48° 55′, Father Georgius Furnerius [Georges Fournier] 48° 50′ — whether this arose from the differing size, placement and use of the instruments, or from parallax, or from some other source. [Margin: The altitude of the pole at Paris varies.] It is no wonder, then, if Rothmann could obtain an altitude of the pole at Kassel differing by two minutes at different times of the year; and Tycho, in volume 1, p. 684, rightly reproved him for not hesitating to build a motion of the Earth on this difference, which could easily have arisen from error. [Margin: Rothmann reproved by Tycho over the altitude of the pole.] This is all the more so since it is established from the Hessian observations published by Villebrordus Snellius that, because the instrument was badly placed, the altitude of the pole at Kassel was found to differ from itself by several minutes. The same thing happened to Tycho with the altitude at Prague, as the same Snel reports in the Bohemian observations: at first he found it 50° 6′, and Longomontanus, Kepler and Lansbergen set it at that figure in their catalogues; but afterwards he found 50° 4′ 30″. [Margin: The altitude of the pole at Prague.]

XII. But grant that the observations of neither the ancients nor the moderns were erroneous in observing the altitude of the pole. Even so, for Domenico

Maria's divination to be true, all, or very many, of the altitudes of the pole observed by the ancients would have to be found increased when compared with the more recent ones — and by 70 minutes at that, from Ptolemy to the year of the Lord 1490. [Margin: 3rd argument against Domenico Maria.] But this is false. Some have rather diminished, some are found to be of the same quantity as of old, and some have indeed increased, but by many fewer, or by more, minutes than 70′. This must be shown by several examples, beginning, as is fitting, with Rome.

At Rome, then, around the times of Augustus, the equinoctial shadow was found to stand to the gnomon in the ratio of 8 to 9, as Vitruvius, book 9, chapter 8, and Pliny, book 2, chapter 72, expressly affirm. From these two sides, by the rules of rectilinear right-angled triangles, the meridian altitude of the Sun's upper limb at the equinoxes is found to have been 48° 22′; but when the Sun's semidiameter, about 16′, is subtracted, 48° 6′ remains. [Margin: The altitude of the pole at Rome examined.] Such, then, will have been the Sun's meridian altitude at Rome at that time, if we neglect the parallax, which for us does not exceed 30″, and suppose that the observation chosen out of many for that ratio of 8 to 9 was the one in which the equinox took place at noon itself or very close to it. Consequently the complement of the altitude to the quadrant, that is, the altitude of the pole, will have been 41° 54′. Ptolemy, however, puts it at 41° 40′. The reason is that, as is clear from the *Almagest* book 2, chapters 5 and 6, he deduced the meridian equinoctial and solstitial shadows from the altitude of the pole, and conversely the altitude of the pole from these shadows, without taking any account of the Sun's semidiameter or making any mention of it. He therefore used the equinoctial shadow cast by the upper limb, which gives an altitude of the limb of 48° 22′, or in round numbers 20′, and took as the altitude of the pole its complement, 41° 38′, or in round numbers 40′. If, then, the Sun's semidiameter of 16′ is added to this altitude of the pole, the altitude of the pole comes out, as above, at 41° 54′ or 56′; and Ptolemy cannot be reconciled with Vitruvius and Pliny in any other way. Now Clavius, in the *Sphaera*, puts it at 41° 56′ from more recent observations, and Latinus Ursinus [Latino Orsini], in part 3 of the *Radius Latinus*, chapter 5, puts it at 41° 54′ from his own observation — the same figure that Bellarmatus [Girolamo Bellarmato] and Maginus also give in their geographical tables. Granted, Tycho, in the letter to Maginus found on folio 81 of Maginus's own *Directions*, infers it to be 42° from the observation of Regiomontanus made when he was at Rome, which is the figure the Alphonsine and Rudolphine Tables give. It is therefore as great now as it was of old, and it has not increased, as Domenico Maria would have it, by a sensible and evident difference, if we follow Ptolemy corrected and reconciled with Vitruvius and Pliny; much less has it increased by 70′. [Margin: Ptolemy's errors in the altitudes of the pole.]

But if Domenico Maria contends that Ptolemy is not to be corrected and that his numbers are to be taken as they stand, I in turn shall show either that Ptolemy contradicts himself or that the altitude of the pole at Rome was not

smaller in the past than it is now. For in the 6th table of Europe Ptolemy says that the longest day observed at Rome was $15^h\ 5^m$. The semidiurnal arc was therefore $7^h\ 32½^m$, or 113° 7′, and its supplement to the semicircle gives the lower angle at the pole of the world, 66° 53′. With this, and with the complement of the Sun's greatest declination, which in truth was then too 66° 30′, as I showed in Book 3, chapter 27, there follows, by the second rule of right-angled spherical triangles, an altitude of the pole at Rome of 42° 5′. But if, on the strength of our own observation, we reduce the semidiurnal time by about 4 minutes on account of the refractions that increase it — that is, increase the lower angle at the pole of the world by one degree — the altitude of the pole comes out at 40° 53′, which contradicts Ptolemy, who puts it at 41° 40′ in his Tables. Indeed, if we used the obliquity of the ecliptic that he asserts, 23° 51⅓′, the altitude of the pole at Rome would come out at 40° 25′, which contradicts Ptolemy much more. So it does not serve Domenico Maria to cling to an uncorrected Ptolemy.

But here is another argument for applying a correction to Ptolemy because he neglected the Sun's semidiameter, and this in the altitude of the pole at Alexandria in Egypt. There, on the testimony of Vitruvius, book 9, chapter 8, the equinoctial shadow was found to stand to the gnomon as 3 to 5. Now let it be: as 3 is to 5, so is the whole sine of seven ciphers to 16666666, which is the tangent of 59° 2′; and this gives the altitude of the Sun's upper limb, namely 59° 2′. If you subtract from it the Sun's semidiameter of 16′, there remains 58° 46′, and so the altitude of the pole at Alexandria is 31° 14′. [Margin: The altitude of the pole at Alexandria examined.] But if the Sun's semidiameter is not subtracted, the altitude of the pole would be 30° 58′, exactly what Ptolemy gives in book 5 of the *Almagest*, chapter 13, and in book 4 of the *Geography*, table 13. It is clear, then, that he erred by 16 minutes through neglecting the Sun's semidiameter. Into this error — what is more surprising — fell not only Gemma Phrysius, in chapter 21 of the *Radius Astronomicus*, but also Snel, in book 1 of the *Eratosthenes Batavus*, chapter 8: from the ratio of 3 to 5 they inferred an altitude of the Sun of 59° 2′ and of the pole of 30° 58′, when that was the altitude of the limb, not of the Sun's centre. [Margin: The slip of Gemma Phrysius and Snel.] I found that a man here in Bologna, otherwise very learned in geometry and astronomy, had fallen into an error similar to this: measuring the altitude of the ☉ from the turned shadow, and subtracting the semidiameter of the ☉ from the altitude of the lower limb when he ought to have added it, he differed from us by a full 32′.

XIII. But so that the inconstant difference between the altitudes of the pole handed down by Ptolemy and those observed more accurately in these latest times may appear, I shall select the following from my Geographical book, and so make the falsity of Domenico Maria's opinion manifest.

Ptolemy's Altitudes of the Pole compared with the Observations of the Moderns.

Places	Ptolemy	Others	From the observations of
Aix [Aquae Sextiae]	43° 45′	43° 33′	Petrus Gassendus [Pierre Gassendi]
Avignon	44° 0′	43° 53′	Anton. Franc. Payen
Augsburg [Augusta Vindelicorum]	46° 20′	48° 22′	Paul Hainzel
Barcelona	41° 0′	41° 26′	Father Io. Bapt. Cysatus [Johann Baptist Cysat]
Bologna, Bo[?]onia Felsina	43° 30′	44° 29½′	Our observations
Florence	43° 0′	43° 40′	Egnatius Dantes
Genoa	42° 50′	44° 27′	D. Vincentius Reinerius [Vincenzo Renieri]
London	54° 0′	51° 32′	Vrichtus and Brichtus
Leiden [Lugdunum Batavorum]	53° 20′	52° 10′	Villebrordus Snellius [Snel]
Majorca	39° 15′	39° 35′	D. Vincentius Mutus [Vicenç Mut]
Marseille	43° 6′	43° 20′, or 17′	Gassendi or others, from Raynaudus's letters
Messina	38° 30′	38° 11′	D. Carolus Vintimilius [Carlo Ventimiglia]
Modena	43° 40′	44° 40′	Father Franc. M. Grimaldus [Francesco Maria Grimaldi]
Palermo	37° 0′	38° 10′	D. Carlo Ventimiglia
Paris	48° 30′	48° 50′	Father Georges Fournier
Parma	43° 30′	44° 51′	From our observation
Pisa	42° 45′	43° 28′	D. Vincenzo Renieri
Ravenna	44° 0′	44° 24′	From ours
Reggio [Regium Lepidi]	43° 30′	44° 44′	D. Antonius Rocca [Antonio Rocca]
Toulouse	44° 15′	43° 30′	Io. Fernelius [Jean Fernel]
Valencia in Spain	39° 0′	39° 30′	Hieronymus Munosius [Jerónimo Muñoz]
Lisbon [Ulyssipo]	40° 10′	38° 40′, or 38′	Nonius [Pedro Nunes]; ours

XIV. So much against the first opinion, which held that the altitudes of the pole vary, but only after many years, by a kind of secular change — and even if that were granted, nothing would follow for the annual motion of the Earth. Next came the second opinion, Rothmann's, which asserts that the altitudes of the pole vary every year because of the Earth's annual motion, so that the highest value taken in winter is one thing and that taken in summer another. Since, however, this does not exceed 2′, it is very easy to put that variation down to the instruments, or to the use of the instruments, as we have already said. Yet Tycho, in volume 1 of the *Progymnasmata*, p. 685, judged that if it were admitted, the annual motion of the Earth could probably be inferred from it. Galileo, by contrast, in Dialogue 3 on the system of the world, from p. 276 of the Latin edition, teaches that no change in the pole would be detected from this, because the Earth's axis does not look to the fixed pole itself in the heavens but to one point after another; but that a change would appear in the apparent altitude of some fixed star, and in its distance from the pole, if Copernicus admitted any sensible parallax in the fixed stars on account of the diameter of the annual orb. On p. 282, however, he tries to show that such a change, caused by the annual motion, will not appear in any fixed stars that lie in the ecliptic — granted that they will appear larger than themselves as the Earth approaches them; whereas in stars placed outside the plane of the ecliptic a

change in meridian altitude will appear, greater the farther they stand from the ecliptic, but a change in apparent magnitude correspondingly smaller. Thus if some star is very close to the pole of the world, it will appear of the same magnitude all year round, but to someone observing that same star once and then again after six months, the difference in its elevation above the horizon will appear the greater, the greater the ratio of the annual orb to the eighth sphere. [Margin: Galileo's doctrine of the change in the altitude of the fixed stars due to the annual motion of the Earth.] "But Galileo errs here," says Father Francesco Maria Grimaldi, "for, to take an example, for an inhabitant of the right sphere, a fixed star at the beginning of ♋, even though it lies in the ecliptic itself, will have a much smaller meridian altitude when the Earth is placed at the beginning of ♋ than when it is placed in ♎ or ♈, if the annual orb is supposed very large. Indeed, while the Earth is under ♈, that fixed star will rise for this inhabitant at midnight itself, and after 6 hours will be on the meridian, if the annual orb bears no ratio to the sphere of the fixed stars; but quite otherwise if it bears a large one — as will be readily clear to anyone who examines a Copernican sphere." [Margin: Galileo's slip.] But since Copernicus, as Tycho also noted above, knew that these differences do not appear, and that, granted the Earth's annual motion, the unchangeableness of these altitudes and magnitudes could not hold if the ratio of the diameter of the annual orb to the diameter of the fixed stars were sensible, he therefore laid down that this ratio is insensible, and that the diameter of the annual orb is like a point compared with the diameter of the sphere of the fixed stars.

XV. To Alexander Calignonus's experiment we reply with an experiment of our own that came out quite the contrary. We hung a plumb line from a similar height in a room closed on every side, and drew a small line, perpendicular and very fine, on a small white board near the bob, against the silk thread. Within half an hour the plumb line came so completely to rest that the thread coincided exactly with the line and did not nod this way or that with any sensible wobble — even when it was watched from below, with a mirror placed underneath and a point fixed into the bob, to see whether any motion could be detected in it. I tested this more than once, formerly at Ferrara and afterwards at Bologna, with Father Francesco Maria Grimaldi as witness. So I cannot but believe my own eyes; and perhaps the opposite happened to Calignonus because the plumb line's thread was twisted, or because the place he had hung the plumb line from trembled, or for some other cause. But see what I said about these matters in Book 2, chap. 21, scholia 2 and 3. Yet even if we granted him this, what could be inferred from it would be at most a diurnal libration, not an annual motion of the Earth. Let Argument 2 now run as follows, with its reply.

[Margin: Argument 2 and its reply.]

XVI. *The altitudes of the pole change, either over a long course of centuries, or yearly, or daily; therefore the Earth moves with an annual motion.*

The reply is, first, to deny the antecedent: as regards the secular change, this is clear from what was said from number 11 to 13 inclusive; as regards the

annual, from the end of number 11 and from number 14; and as regards the daily, from what was said in number 15.

I reply, second: granting the antecedent as to its first and last part, I deny the consequence. For those changes would not establish an annual motion of the Earth in the ecliptic, but some libration, requiring for its period either many centuries, or a day, or only half a day.

CAPVT XII. Proponitur & Diſſoluitur Argumentum pro Motu Annuo Terræ à motu Macularum Solis deſumptum; eaq. occaſione Doctrina de his Maculis à Galilæo & Scheinero tradita, in Epitomem redigitur; quo luculentiùs Argumentum hoc eiuſq. ſolutio declarentur.

CHAPTER XII. The Argument for the Annual Motion of the Earth taken from the motion of the Spots of the Sun is set out and refuted; and on this occasion the Doctrine of these Spots handed down by Galileo and Scheiner is reduced to an Epitome, so that this Argument and its solution may be made clearer.

I. Galileo, in Dialogue 3 on the Ptolemaic and Copernican system of the world, tried to use the very spots of the Sun to besmirch and destroy the Sun's annual motion and to fasten an annual motion on the Earth. Indeed, he counted this among the three strongest arguments for the Earth's motion, in Dialogue 4, on the penultimate page, where he says: *From the conversations held over these four days we therefore have notable testimonies for the Copernican system, of which these three — taken first from the stations and retrogradations of the planets and their approaches to and elongations from the Earth, second from the revolution of the Sun upon itself and from what is observed in its spots, third from the ebb and flow of the sea — seem to have a most evident force of proof.* Since, I say, he attributes so much to this argument, we must, lest we seem to have failed in a task once undertaken, search this ford more diligently; and, as the opportunity offers, explain the motion of these spots here, where it is easier to understand for readers now practised, than it would perhaps have been in Book 3, chap. 3.

I. First, then, Galileo, as he himself relates in the history of the Solar Spots, in three letters to Marcus Velserus [Markus Welser] written, in which that history is comprised. Velserus had forewarned him of these observations, published by our Apelles, who was then hiding behind the picture — that is, by Christoph Scheiner, who for the time being suppressed his name at the order of Father Theodorus Busaeus, the Provincial, who did not want these novelties, not yet fully explored, to be made public by our men. In that history, I say, in letter 1, Galileo approves Apelles's opinion on the nearness of the spots to the Sun and on their real existence, because of the imperceptible parallax between them and the Sun. [Margin: Galileo's opinions on the spots of the Sun, from his letters.] But Apelles had said that they move around the Sun from east to west — considering, that is, the motion visible to us. [Margin: From letter 1.] Galileo grants this, if their whole revolution around the Sun is not considered;

but if it is considered, he says they move in the way that Venus and Mercury move around the Sun, namely from west to east — which is true if the motion is regarded above, that is, in the Sun's hidden hemisphere. He adds that the evening spots, observed near sunset, change their position daily and descend from the upper parts of the Sun towards the lower, whereas the morning spots ascend from the lower parts of the Sun towards the upper. He denies that their substance is very dense, and says instead that it is like smoke or clouds; he likewise denies that they are fixed or wandering stars; finally, he denies that they move in circles separate from the Sun, since otherwise they would have to pass more quickly than Mercury does beneath the Sun — and yet Mercury crosses the disc of the Sun in about six hours, whereas the spots stay beneath it for many days.

III. In letter 2, to the same M. Velserus, he confirms some things said in letter 1, and affirms the following propositions about the solar spots. [Margin: From letter 2.] Besides the disordered and irregular motion by which several spots flow together into one, or one disperses into several, they have another motion, ordered, common and greatest, by which they are carried round uniformly, in lines parallel to one another, by the globe of the Sun to which they are contiguous; so that this rotation of the Sun about its centre, and of the spots about the globe of the Sun, completes its period or revolution in about one lunar month; and this rotation is like the turning of the planets in their orbs, inasmuch as they revolve from west to east. He confirms their contiguity with the Sun, and their rotation with it, from the fact that when they appear or disappear near the edges of the Sun they look of small length, though keeping the same breadth, but in the middle of the solar disc they look longer. Further, he says that the multitude of spots does not appear over the whole disc of the Sun, but only in a certain middle zone of the disc, which does not exceed 56° or 58° of the 360° that make up the circumference of the great circle on the Sun's globe. Thus, just as the greatest declination of the planets from the equator does not exceed 23½° towards north or south in the Sun, but in the other planets, which have latitude, does not exceed 28° or 29°, so the greatest declination of the solar spots from the middle of the Sun's disc does not exceed 28° or 29° of the solar circumference towards south or north. From this he deduced that there are two poles on the surface of the solar globe, about which that rotation takes place, while the other points describe the circumferences of parallel circles, of which the greatest is the one standing equally distant on either side from those poles. After demonstrating by several arguments, in the same letter 2, that these spots are contiguous to the Sun, he teaches that many of them return and travel again along the same parallel. Towards the end of the letter he teaches a method of observing the spots; and from their sudden generation and destruction on the disc of the Sun, before they complete their turning from limb to limb, he confirms both the fluidity and the corruptibility of the parts of the heavens.

IV. Finally, the third letter teaches that hardly one spot in a thousand observed is found to cross the zone already mentioned, and that they do not decline from the middle of the Sun beyond 29° or 30°. [Margin: From letter 3.] Their rectilinear motion, he says, is an argument that they move around an axis perpendicular to the ecliptic. And because Apelles had said that a certain spot passing through the Sun's great circle appeared on the Sun's disc for 16 days, another in a smaller parallel for 14 days, and some for only 9 days, Galileo denies that this diversity is possible. He says that all the spots that complete their course, and are not extinguished before their apparent setting, stay an equal time on the disc of the Sun, a time he defines as a little longer than 14 days in one hemisphere of the Sun; and he proves this from his own observations. He also teaches that if one of the spots crossed the disc of the Sun in 16 days, and another, standing as far as possible from the middle, namely at 30°, crossed it in 14 or 13 days and 20½ hours, the semidiameter of the aetherial sphere in which those spots revolved would be twice as great as the semidiameter of the solar body. He confirms besides that the spots are on the very surface of the solar globe and are carried round by its rotation, from the faculae, brighter than the rest of the Sun itself, that appear on it and move with the same motion as the spots; for it is not credible that any body brighter than the Sun should be found outside the Sun. He adds that we can imitate the production and destruction of the solar spots if we throw wax, pitch or bitumen on a red-hot iron: for a black spot then forms on the iron, and smoke rises from it, spreading out into various figures. Many spots, he says, are suddenly born and die away in the very middle of the Sun; yet they are not within the Sun's substance, as if they were its lakes or caverns. This, in summary, is Galileo's doctrine of the spots of the Sun, reduced to an epitome from those three letters, with many other things deliberately left out that have no bearing on our argument but concern Venus or Mercury seen below the Sun, or the spots of the Moon, or the satellites of Jupiter, or the reflection of the Sun's light from the Earth. Let us now see what the same author taught, under the persona of Salviati, about the motion of these spots in Dialogue 3 on the system of the world.

V. In that Dialogue, then, on p. 337 of the Italian but p. 255 of the Latin version, he affirms that the Sun itself, by its spots, is a most splendid witness, beyond all exception, for the Earth's annual motion. [Margin: Galileo's doctrine of the motion of the spots of the Sun, from Dialogue 3.] He affirms also that he was the first discoverer and observer of the solar spots, as of all the other celestial novelties, when he was professing mathematics at the university of Padua in the year 1610; that he discussed them with certain persons still living at Padua and Venice; and that he was the first to assert, from the sudden eruption and dissolution of these spots, the alterability of the heavens. He relates further that he had once thought that these spots, being contiguous to the Sun or adhering to its surface, were carried round by its roughly monthly rotation in such a way that this rotation took place about an axis of the solar globe erect to the plane of the ecliptic: for the arcs that the spots described on the Sun's disc appeared

to the observer's eye as straight lines, parallel to the plane of the ecliptic. If any deviation from that straightness appeared, he judged that it did not lie in their essential and primary motion, but came from an accidental concourse of small spots into one larger one, or from the tumultuous breaking-up of one spot into several very small ones. He says, however, that having interrupted his observations of the spots and afterwards resumed them, he came upon a very large spot, dense and solitary, and observed its passage around midday over very many successive days. When he noticed that its path was curved, he at once conceived a great hope of deciding about the motion of the Sun or of the Earth from this motion, and, turning to his host Filippo Salviati, he expressed that hope in these words: *Ah, Filippo, a way seems to open to me from here to consequences of great moment. For if the axis about which the Sun revolves is not erect perpendicular to the plane of the ecliptic, but is inclined over it, as the curved path of the spot lately observed suggests, we shall take from it a conjecture about the state of the Sun and the Earth than which no other accident has ever supplied us with one firmer or more probable.* [Margin: Galileo now calls probable the conjecture that, at the end of Dialogue 4, he calls a most evident testimony.]

At Salviati's request, then, he sets out this conjecture in almost these words. If the Earth, he says, is carried with an annual motion along the ecliptic around the Sun, so that the Sun, remaining at the centre of the ecliptic, rotates upon itself about that centre — not about the axis of the ecliptic itself (which would be the axis of the Earth's annual motion) but about an inclined axis — then wonderful changes must be presented to us in the apparent motions of the spots, even if the said axis of the Sun is supposed to persist perpetually and unchangeably in the same inclination and to be directed always towards the same point of the universe. For if the terrestrial globe goes round the Sun with an annual motion, it will be necessary, *first*, that to us, who are carried round with it, the passages of the spots should sometimes — but only twice a year — appear to be made along a straight line; but at all other times along a curve. *Second*, during one half of the year the curvature of these arcs will appear to us inclined the opposite way from the other half: that is, for six months the convexity of the arcs will face the upper part of the Sun's disc, and for the other six months the lower. [Margin: The variation of the motion of the spots according to Galileo.] *Third*, since the spots begin to appear and rise to our eye on the left side of the Sun's disc, and cease or are hidden on the right side, the eastern terms, that is, the terms of first appearance, will for six months be lower than the terms of hiding, and for the other six months higher: that is, spots rising from higher points and descending from them will be hidden, as they proceed, at lower points. Only on two days of the year will these terms stand in equilibrium; after that balance the inclination of the spots' passage will begin to grow greater and greater, until after 3 months it becomes greatest, and then, decreasing over another 3 months, it returns to equilibrium. *Fourth*, on the very days on which the obliquity or inclination of the spots' path is

greatest, the paths will look rectilinear; but on the days of the equilibrium already mentioned, as curved as possible.

VI. To illustrate this doctrine with Galileo by diagrams, as far as can be done on a plane (globes would be more suitable), let O, in the following figures, be the centre of the ecliptic, or great orb, and also of the solar globe, whose whole hemisphere we may suppose is seen by us earth-dwellers because of its immense distance. About this centre let the circle ABCD be described, representing the horizon of the solar hemisphere, that is, the great circle that bounds the hemisphere of the solar globe visible to us and divides it from the invisible one.

Diagram. A woodcut of three circles set side by side across the left column, uncaptioned; the text calls them the first, second and third figures. The first circle carries a horizontal and a vertical diameter through the centre O, an inclined diameter, a solid line studded with dots (the path of a spot) and two dashed lines parallel to it; the second and third circles each carry a horizontal diameter, an inclined axis, and a dotted arc above the centre (the hidden or visible path of a spot), with a further inclined diameter in the third.

- First circle: E, A (top), L, G, M (upper right), D (inside, right of the horizontal diameter), O (centre), B, H, F, K (left, from top to bottom), C, I (bottom)
- Second circle: A (top), E, G (above the centre), O (centre), F, I (below the centre), C (bottom), B (left), D (right)
- Third circle: A (top), E, S (upper left of the centre), G, D (right), O (centre), N, I (lower right), F (left), B (left, beside the second circle's D)

And since our eye, no less than the centre both of the Earth and of the Sun, lies in the plane of the ecliptic, if we imagine the body of the Sun cut by the plane of the ecliptic, that section will appear to our eye as a straight line; let it be BOD. The perpendicular AOC drawn to it will be the axis of the ecliptic, about which the Earth's annual motion takes place. Let us now think of the body of the Sun, immobile about its own centre, as revolving upon itself not about the axis AOC, perpendicular to the plane of the ecliptic, but about the axis EOI, somewhat inclined to the ecliptic; and let this axis be fixed and unchangeable, that is, always keeping the same inclination towards the same points of the firmament and of the universe. In the solar horizon let a point F be taken, equally distant on either side from the poles E and I, and from it let the diameter FOG be drawn through O, perpendicular to the axis EOI: it will be the diameter of the great circle described about the poles E, I. Now if the solar horizon ABCD, just as it passes through the axis of the ecliptic AOC, also passes, with respect to us who move with the Earth, through the poles E and I, then the great circle whose diameter is FOG will stand erect at right angles to the circle, or solar horizon, ABCD, on which the ray from our eye falls perpendicularly at O. The same ray will therefore fall in the plane of the circle whose diameter is FOG, and its circumference will look to us like a straight line, and so will the motion of the spots along it, as is marked in the first figure by the line FOG; while the paths of other spots will appear to describe

other straight lines parallel to FOG, such as HL and KM. [Margin: Case 1: the rectilinear path, explained by figure 1.] The same will happen after the Earth, six months later, has come to the point of seeing the other hemisphere of the Sun, which was previously hidden from it, provided it is so placed that the solar horizon ABCD passes through the poles E and I. But since, because of the Earth's annual motion, the solar horizon changes continually and passes through the poles E and I only at the two times or moments already mentioned, it follows that only at those two moments of the year will the spots' path appear to us exactly rectilinear. Hence it also appears that, since the appearance and beginning of the motion from the side F proceeds towards G, for us observing at midday the path of the spots ascends from the left and lower part towards the right and upper part of the solar horizon. But if the Earth has now reached the diametrically opposite part, the passage will be from the left but upper part G, descending to the lower right, F; for in this second case you must imagine your eye placed not in front of this first figure ABCD but behind it.

VII. Now suppose that the Earth has left that position and after three months has completed a quadrant of the annual orb (look at the second figure), and that the plane of our meridian passes through the axis of the ecliptic AOC. [Margin: Case 2: the curvilinear path, set out by figure 2.] The axis of the Sun's revolution, with its poles, will lie in that same plane, one pole visible to us, call it I, and the other, E, invisible, that is, lying hidden in the surface of the solar hemisphere that is concealed from us. For since the axis EI is inclined towards us at its lower part, the great circle described about it will also be raised upwards for us, and will be such as is marked by BGDF. So if a spot rises at B, its path visible to us will appear curved, running along BGD, so that its convexity bends upwards towards the northern limb of the Sun — and with the greatest curvature, and in the hemisphere of the Sun visible to us; while the rest of the spots' path, which will be hidden, runs along DFB with its convexity turned downward. Conversely, if the Earth, moving on from here, completes a semicircle after six months, the pole of the solar axis visible to us will be E, and the path of a spot risen at B will be visible along the arc BFD, its convexity swelling downward towards the southern limb of the Sun, while the rest of the path, hidden from us, will run along the arc DGB. Finally, for us observing at midday, the terms of appearance B and of hiding D will be in equilibrium, that is, equally distant from C or A. And what I have said of a spot risen at B holds for all the others risen elsewhere, for they will describe arcs parallel to the arc BGD, or, after six months, to BFD. But suppose the Earth is outside the positions already mentioned, so that neither the solar horizon ABCD nor the meridian AOC passes through the poles E, I of the solar axis, as you see in the third figure, but the axis is EOI. Then the diameter of the great circle described about it will be FOG, and the circumference of the circle FNGS. So if the pole visible to us is E, the visible path of the spots will be the curve FNG, though not with the greatest curvature, and its convex side will bend downwards; nor will the terms of appearance F and of hiding G be in equilibrium, but F will

be below and G above. [Margin: 3. Figure, and explanation of the case [?].] Conversely, with the Earth diametrically opposite this position, and the pole I visible to it, the curvilinear path will run along the upper arc GSF, with the appearance at the upper point G and the hiding at the lower part F, but from left to right; for here too, in the second case of this third figure, you must imagine the observer's eye to be behind the disc ABCD.

VIII. These are the wonderful vicissitudes that Galileo divined, by his conjecture, would come about — provided, that is, that three suppositions are admitted: namely, that the Sun by its rotation carries the spots round with it; that its axis is inclined to the plane of the ecliptic, but with the same inclination remaining perpetually; and that the Earth is carried round with an annual motion about the Sun, which remains at the centre of the world. When Giovanni Francesco Sagredo asks whether such vicissitudes were afterwards actually observed, Salviati (who plays Galileo's part in those dialogues) answers that the observations were continued at different times of the year and that the outcome answered the prediction most exactly. Then Sagredo, turning to Simplicio, concludes (p. 260 of the Latin, but p. 344 of the Italian): *Sir Simplicio, if these things that Salviati has set out are true (and it is not right to doubt his assertions), the Ptolemaics and Aristotelians will need solid arguments, strong conjectures and very firm experiments, if they are to counterbalance an objection of such weight and not incur the final ruin of their opinion.*

But Simplicio replies, acutely and soundly, that even if the phenomena of the solar spots are saved once the Earth's annual motion and the other conditions mentioned above are granted, the argument still does not hold when converted — namely, that if such are the phenomena of the spots, the Earth moves and the inclination of the solar axis is fixed — unless it is demonstrated that those phenomena cannot be defended otherwise. For why could they not be saved with an annual motion of the Sun posited about an immobile Earth? Salviati then concedes that this can indeed be done, but says that one would have to admit many and continuous motions, by which the Sun's axis perpetually changes its inclination to the axis of the ecliptic, and on account of which that axis is bent from moment to moment towards different points of the universe; and that this axis must therefore have its own monthly revolution, distinct from the annual revolution — since once every year the whole variety of the spots' motion is completed exactly and recurs again — and that the axis must have a convertible inclination, so that it lies sometimes in the plane of the solar horizon, and the path of the spots is therefore rectilinear, sometimes in the observer's meridian but outside the horizon of the solar globe, and sometimes in yet other planes. But it is far more likely, says Salviati, that the axis of the Sun's rotation is single and has a fixed inclination to the plane of the ecliptic, that the Sun is moved by no other motion than a simple revolution of about 27 or 28 days about its own immobile centre, and that all the remaining vicissitudes of the spots come about through the single and simple annual motion of the Earth — rather than that, with the Earth standing still, the Sun and its axis should be

entangled in multiple motions. Therefore it is also more likely that the Earth, rather than the Sun, moves annually. [Margin: The Earth's annual motion is inferred from what has been said.]

IX. Struck by this argument among others, Pierre Gassendi, in Letter 2 of *De motu impresso a motore translato*, p. 148, did not hesitate to break out in these words: *It is remarkable that the paths of the spots, when the Sun is at the equinoctial points, describe as it were straight lines on its disc, and at the other points, especially at the tropics, curved ones; and that this cannot be saved more conveniently than on the ground that, while the Sun always revolves about the same poles, its own or the ecliptic's, and carries the spots along with it, the Earth places itself towards it — now, as it were, to the right or left at the equinoxes, now, as it were, above or below at the solstices — in such a way that, because of the constancy of its own axis already set out, it sees the spots proceeding across the Sun's disc not in one manner only, but with the variety of straightness and curvature already mentioned: so Galileo demonstrates.* [Margin: Gassendi's opinion on the Earth's annual motion, but his errors about the motions of the spots.] In these words a twofold error lies hidden. First, neither Galileo nor Scheiner, in their later work, admits that the Sun's rotation takes place about the poles of the ecliptic; otherwise the path of the spots would always appear rectilinear to us. Second, it is not true that their path is seen rectilinear at the equinoxes and most curvilinear at the solstices; on the contrary, it is seen rectilinear rather about the beginning of December and of June, that is, nearer the solstices, and most curvilinear about the beginning of March and of September, that is, nearer the equinoxes, as we shall presently relate from Scheiner's most certain observations. Now we must reply to the argument drawn from this motion, set out in form.

X. *It is more probable that the apparent motions of the solar spots come about through simpler and fewer motions than through more numerous and more composite ones. But if the Earth moves annually about the Sun, rather than the Sun about an immobile Earth, the apparent motions of the solar spots come about through simpler and fewer motions than if the Sun is supposed to move annually about an immobile Earth. Therefore it is more probable that the Earth, rather than the Sun, moves annually.* [Margin: The argument set out in form.] The MAJOR is clear from those most widely received propositions, rehearsed so many times in this section: namely, that GOD and Nature do not strive to do through more what can be done through fewer, and that consequently not even we are permitted to introduce into the nature of things a multiplicity of any entities whatever, still less of motions, without necessity. The proof of the MINOR is sufficiently indicated by comparing what was said in numbers 5, 6 and 7 with what was said at the end of number 8; and it will be confirmed from the change of the solar axis asserted by Scheiner and by others who explain these phenomena on the hypothesis of a resting Earth.

I reply, 1 and 2, in the same way as I replied to the major of Argument 1, which I set out in chapter 9, number 5: namely, by conceding the major if the simplicity of these motions, greater at first sight, neither involves other,

greater absurdities, nor saves other phenomena more evident to the senses than these, nor, when examined from within, involves many other motions along with it; but otherwise by denying the major of the syllogism, and its proof. [Margin: Replies 1 and 2.] And in the same sense as I denied it there, so I deny the minor here. For if the Earth is supposed to move and the Sun to remain at the centre of the world, certain phenomena more evident to the senses than the motion of the solar spots are destroyed; and in the Earth's motion there lie hidden very many other motions, multiple in kind and in number, most intricate, and no easier for the intellect to grasp than the motion of a solar axis that has a convertible inclination — since the same convertibility of inclination of the Earth's axis to the equator is involved in at least a twofold libration, one because of the precession of the equinoxes, the other because of the obliquity of the ecliptic, if we listen to Copernicus.

I reply, 3: granting the major, by denying the minor. [Margin: Reply 3 [?].] For if the Earth is supposed to move annually, it must also be supposed to move with a diurnal revolution about its own axis; and the phenomena of the spots further require a rotation of the Sun about its own axis, completed in, as it were, a monthly revolution; and these are three motions really distinct. But if the Earth is supposed to be at rest, a threefold motion of the Sun must likewise be posited: one, of the Sun's centre, which is at once annual and diurnal, along a spiral line, by which it therefore appears to move towards the east because, in its motion towards the west, it is slower than the fixed stars and the ideal motion of the primum mobile; the second, an annual motion of the poles of the solar axis about its own fixed axis, from west to east; and the third, that of the quasi-monthly rotation about another, mobile axis, from east to west, as we shall make plain from Scheiner in numbers 22 and 23. In either hypothesis, therefore, the multiplicity of real motions is equal; but in other respects there is a great disparity, on account of which the verdict seems rather to go for the Earth's rest.

Chiaramonti would reply, 4, in his *Defence of the Antitycho* [Defenſione Antitychonis], part 4, chapter 18, that the phenomena of the spots can be defended without any motion of the Sun's axis or annual revolution of the Earth, if the spots are said to move in their own epicycles. [Margin: Reply 4, Chiaramonti's, rejected.] But this reply supposes that the sunspots do not inhere in the very body of the Sun, which is false; and it moreover multiplies the motions to as many as there are spots, which favours Galileo's argument rather than destroying it.

XI. It is now time, towards the solution of the argument — for which I pledged my word in Book 3, chapter 3, at the end of number 8, and in the title of this chapter — for me to set out the phenomena of the solar spots, especially those that concern motion, more clearly than I did there, and to reduce to an epitome, for my reader who may perhaps lack books of this kind, what Scheiner first wrote about them in his *Apelles post tabulam*, and what he afterwards handed down to posterity in the *Rosa Ursina*, correcting himself,

and having by then become greater and more illustrious than himself, not to say than the Sun. For just as it is not to be counted a discredit to Galileo, so neither is it to Scheiner, if in their first observations, because of the novelty and intricacy of the phenomenon and the lofty and arduous contemplation that it otherwise demanded, they said some things less consonant with the truth; for afterwards, by correcting themselves, they earned the praise of astronomical sincerity. [Margin: Scheiner's encomium.] And Scheiner discoursed on this subject in the *Rosa Ursina* with so many and such exact observations, theorems and problems that he earned from Reinardus, or Renatus, Descartes [René Descartes] and from Johannes Hevelius the eulogy that nothing more accurate could have been done in this kind, or can be hoped for in future; and Father Nicolaus Zucchius [Niccolò Zucchi] of our Society, who was a constant witness of Scheiner's observations, and his encourager and helper, began, in his new and noble *Philosophy of Machines* [Philoſophia de Machinis], part 5, section 12, with this eulogy: *Let there first be presupposed here the canons of motion detected in the Sun in this our age, which Father Christoph Scheiner of our Society established by the most constant labour of many years, consulting the sky more often day by day, taking the rules of a better astronomy into counsel and wisely adding new ones; and later inspectors, not of books only but of the stars as well, have so confirmed them that from them even those who had formerly been his censors have corrected their errors and supplied their omissions, daring to contradict him in nothing; and most recently a man of the highest merit in astronomy has published the ordered courses of the sunspots over three years, whom I shall presently name, in his book* Selenographia. [Margin: Father Nicolaus Zucchius's eulogy for Scheiner.]

Doctrina Chriſtophori Scheineri de Maculis Solaribus, ex Apelle poſt tabulam latente, ſeu Epiſtolis ad Marcum Velſerum ſcriptis.

The doctrine of Christoph Scheiner on the solar spots, from his Apelles hiding behind the picture, that is, from his letters written to Marcus Velserus [Markus Welser].

[Margin: From letter 1.] XII. In the first letter, dated 1611 November 12, he relates that about eight months earlier, namely in March of the year 1611, he had turned a telescope, which enlarged an object almost eight hundred times in surface, on the Su- n, in order to explore its optical diameter in relation to the Moon, and on that occasion had seen certain blackish spots on the Sun, but had not then been concerned about them. Returning to this observation in the month of October, however, he at first suspected that those spots arose from some hidden defect in the air, or in the eye, or in the glasses of the telescope. But when, with different eyes called in as witnesses, and with the glasses exchanged and eight tubes used, the same spots appeared in the same order, position and number, he concluded that they did not arise from a defect of the eyes or of the glasses — nor from the air either: because they were moved with the motion of the primum mobile together with the Sun the whole day, whereas nothing in

the air is moved so constantly with such a motion beneath so tiny a portion of the sky as the Sun occupies; because they admitted no parallax the whole day, as they would have had to if they had been in the air near us, but appeared on the Sun's disc the whole day; because by a proper motion they turned, together with the Sun, from the Sun's eastern limb to its western over several days, until they vanished from sight; and finally because their phenomena were seen unchanged through mists and clouds. But since he then thought it unworthy to admit, in the Sun, a most luminous body, spots darker than some of those of the Moon, and since the same spots had not yet been seen to return, but always other and new ones, he judged that they are not turned round by the Sun and are not in the Sun, but go like stars, either below or around the Sun, with their own peculiar motions, and eclipse it for us. He admits, however, that not all the observations he had given Welser were at that time most exact, and that some, examined without the tube, appear simple which through the tube appear aggregated from many.

[Margin: From letters 2 and 3.] XIII. In the second letter, dated 1611 December 19, he reports that he had been intent on the conjunction of Venus with the Sun which, according to Magini's *Ephemerides*, was to begin on December 11 at the 11th hour of the night and last at least 40 hours; but Venus was not seen beneath the Sun, and from this he took the argument that in its superior conjunction it is carried above the Sun. In the third letter, written on December 26, he contends that the spots are not in the very body of the Sun. For since they appear beneath the Sun for less than fifteen whole days from their first appearance to their hiding, they ought, if they turned round with the Sun, to return after another fifteen days or a little later; yet in an experiment of two months none was seen to return. On the other hand, from the absence of any parallax from the Sun; from the fact that at their appearance and hiding near the Sun's limbs they grow thin; from the fact that several seen in the middle of the Sun appear to merge into one at the edges; and from their being fast in the middle of the Sun's disc and slow at the limbs, he gathers that they are both near the Sun and move around the Sun with their own motions; that they are certain heliacal bodies [ſidera quędam Heliaca], very dense, opaque and deep, inasmuch as they block the Sun's light; that they are wandering stars rather than fixed; and that just as there are satellites around Saturn and Jupiter, so there are around the Sun, etc.

[Margin: From letter 4.] XIV. In the fourth letter, written to the same man on 1612 January 16, from 30 observations made from 1611 December 10 to 1612 January 12, he affirms that the spots at their entry onto the Sun and exit from it are seen small, slower and blacker, and also joined together, but in the middle of the solar disc large, faster, less black and more separated from one another; that some, and very large ones, arise unexpectedly not at the limb but almost in the middle of the Sun's disc, or suddenly fail there before they reach the Sun's edge; and that some divide from one into two, and that almost in the middle of the Sun, or two merge into one, and so continue divided or joined until their

exit. He adds that their path appears parallel to the ecliptic, but he suspends judgement. He thinks it certain, however, that those which cross the middle of the Sun stay longer beneath the Sun than those far from the middle; and from this he takes a new argument that they are not fixed to the Sun as it revolves about its own centre: for one spot was seen beneath the Sun for at least 16 days, another, farther from the middle of the Sun, stayed beneath the Sun at most 14 days, and another 13.

[Margin: From letter 5.] XV. Finally, in the fifth letter, dated to Welser 1612 July 25, he first confirms that these spots are not an illusion or a mere spectre appearing through a defect of the eye, of the glass or of the air, since the same spots are seen by anyone's eye, in any situation under the open air, without glasses, in the same order, position and number, if a ray of the Sun, passing through a bare hole no bigger than a pea, is received in a darkened room on a paper parallel to the Sun's disc; and moreover because the same spots were seen and observed in different places — namely at Rome by Fathers Christophorus Gruembergerus [Christoph Grienberger] and Paul Guldin, distinguished mathematicians of the Society of Jesus, and by Galileo and others. From these and from his own observations he concludes, together with a certain Venetian who concealed his name under that of Protogenes, that these spots are real bodies, not in the Sun but near the Sun; that they are not spherical but of an irregular and variable figure, and therefore are not stars; yet that their interposition, unknown to the old astrology, does great harm, since in varying sublunary effects so many bodies, persisting so long beneath the Sun, have more power than Mercury alone, conjoined with the Sun and accustomed to stay beneath it for only a few hours. [Margin: A wound to judicial astrology from the spots of the Sun.]

Doctrina Chriſtophori Scheineri de Maculis Solis ex Roſa Vrſina in Epitomen Redacta.

The doctrine of Christoph Scheiner on the spots of the Sun, from the Rosa Ursina, reduced to an epitome.

XVI. In Book 1. Galileo, in the *Discourse on the Comet of the Year 1618*, p. 3, in the *Trutinator*, pp. 2 and 209, and in the dialogues on the system of the world, p. 337 of the Italian, sells himself as the first discoverer of the solar spots and of all the novelties of the sky, and accuses Apelles, that is, Scheiner, of theft, as if he had begun to observe the spots after seeing Galileo's writings about them and had then boasted himself their first observer. Scheiner therefore clears himself of this imposture. For he says that, with no prior rumour or indication, he began to observe them in March of the year 1611, while he was exploring the size of the Sun with a telescope; that afterwards, in October, he resumed the observations and informed Father Adam Tanner and Father Jacob Gretser of them, who passed this on to M. Welser, the Maecenas of all men of letters; and that Welser at last, by many letters, extorted from him the novelty

of this phenomenon and urged that it be published in print. This was done in the three earlier letters, dated in the year 1611, but under the feigned name of Apelles, because Father Theodorus Busaeus, the Provincial, did not wish these novelties, not yet sufficiently explored, to be published under Scheiner's own name. Welser, moreover, at once communicated Scheiner's letters to Galileo, on 1612 January 6. [Margin: Scheiner vindicated from the imposture of the theft of Galileo's observations.] But Galileo, replying to him on May 4 of the same year, affirms that he had observed the sunspots eighteen months before; and yet he produces no observation of the year 1611 with a diagram, but only those of the year 1612, from April 5. He ought, however, to have produced diagrams of spots observed in the year 1611 and at the beginning of the year 1612, as Scheiner did, so as to make it at least probable in this way that he had not been later than Scheiner in observing them. How, then, could Scheiner have stolen his own observations, earlier in time and made before April of the year 1612, from the observations of Galileo, which were published neither then nor afterwards? Galileo, it is true, excuses himself in that letter for not having published his observations earlier, because of suspense of mind and a fear of erring; but Scheiner had meanwhile published many of his propositions outright, and indeed ones contrary to what Galileo said. For Scheiner had already then said that the apparent motions of the spots are unequal, that their stays beneath the Sun differ, that some are blacker than the spots of the Moon, and things like these, the opposite of which Galileo affirms in his letters to Welser. Therefore, if Galileo wrote nothing about the spots of the Sun before he saw Scheiner's *Apelles post tabulam*, communicated to him by Welser — indeed settled nothing certain about them — whereas Scheiner had already then published in print many things, and things contrary to those which Galileo afterwards decided, how could Scheiner have stolen his observations from Galileo's writings? Next, it is false that Scheiner put himself forward in his letters as the first observer. For he did not deny there that others had observed the spots, nor did he arrogate to himself the invidious title of first discoverer; he only related what had befallen him unexpectedly, without any prior rumour or indication — which could have been true even if, a thousand years before, someone else had observed the same thing without Scheiner knowing it. Granted that he really was so, as the facts themselves, which say more than words, and the chronological comparison of other observations with Scheiner's, attest. To this I add that the *Rosa Ursina*, in which among other things the curvilinear motion of the spots is demonstrated, except at two times of the year, began to be printed in the year 1626 and was published in the year 1630, whereas Galileo's dialogues were published in the ye- ar 1632; and that, since in the third of them Galileo himself corrects himself and has admitted the curvilinear motion of the spots, which he had always previously thought rectilinear, some may suspect that he did this after seeing the *Rosa Ursina*, or after hearing what Father Scheiner had communicated at Rome to many outsiders, whom he admitted in good faith as witnesses and observers of his observations — especially since Galileo says in

that dialogue that the path of the spots must be rectilinear only twice a year, and for six months curved upwards and for the other six curved downwards, and affirms that the outcome answered this prediction of his, and yet does not dare to say in which months of the year these vicissitudes occur; which he certainly ought to have said, and would have said, if he had been able to determine these things from his own observations. It is therefore no slight indication that he inserted this correction of himself into the Dialogues not so much from his own observations as from seeing or hearing of other people's — which, however, I would not dare to affirm absolutely. Further, in this book Scheiner sets out the diagrams of observations he formerly made at Ingolstadt.

[Margin: From Book 2.] XVII. In Book 2 he teaches the use of the helioscope, which has coloured glasses, and of the telioscope, which has glasses or lenses that are not coloured; and he demonstrates from the foundations of optics the choice, testing and combination of lenses, and also the admission of the species through a bare hole and their transmission through the telioscope; the difficulties in observing spots and faculae, and the precautions to be taken; likewise the practice of drawing in the vertical line, or plumb line, and the ecliptic; and the times suitable for observing — although at the end of Book 3 he exhibits a new machine with which the solar spots are observed most easily.

[Margin: From Book 3.] In Book 3 he exhibits and explains the observations of the spots themselves, both his own and those of others. He warns in advance, on page 161, that the paths of the spots proceed obliquely to the ecliptic, from the north-east to the south-west, from the spring equilibrium, which occurs at the end of February and the beginning of March, up to the autumn equilibrium, which occurs at the end of August and the beginning of September; that in the first three months, March, April and May, the arcs of the paths are concave towards the south-east and convex towards the north-west; but that in the other three, later months — after the rectilinear station held at the end of May and the beginning of June, that is, in June, July and August — the concavities of the paths turn towards the north-west and the convexities towards the south-east. [Margin: When the motion of the spots is oblique to the ecliptic, and towards which quarters.] Everything happens the contrary way from the autumn equilibrium, which occurs at the end of August and the beginning of September, up to the spring equilibrium, which occurs at the end of February and the beginning of March. For in its first three months, September, October and November, the paths of the spots are concave towards the north-east and convex towards the south-west; but after the winter rectilinear station has been held at the end of November, the bends of the spots in December, January and February are concave towards the south-west and convex towards the north-east. Now this book contains observations made from 1618 to 1627, which are comprised in 71 images; and the number of observations shown in them amounts, by my reckoning, to 1425, the greatest part of which were made by Scheiner himself at Rome, and a few by others of our Society at Ingolstadt, Freiburg and Douai. Finally, he ends this book with a new telescopic and

heliotropic instrument by Father Grienberger, with which, without the angle of the ecliptic and the vertical, without drawing in a plumb line, and without the altitude of the Sun, the course of the spots, the position of the ecliptic and the other things necessary for this business are taken most expeditiously. To it are added demonstrations proving the instrument, devised by Grienberger himself, and a table of right ascensions and declinations for one quadrant of the ecliptic, at every 5′, constructed by the same man.

[Margin: From part 1 of Book 4.] XVIII. In part 1 of the fourth book Scheiner demonstrates the following propositions from the observations of Book 3. Chapter 1: "All the spots," he says, "that belong to the Sun never appear outside the Sun as long as they are visible, but rise with it and, when the diurnal motion is completed, set with it, from whatever part of the world they are viewed." And chapter 2: All the spots always appear to move from the eastern quarter of the Sun to its western. And chapters 3, 4, 5 and 6: No part of the heaven of the Moon, of Mercury or of Venus can cause any spot on the Sun by its interposition. And chapter 7: No spot or facula appears outside the visible disc of the Sun; the light of the solar faculae is stronger than the rest of the Sun's light and than the daylight spread through the aether; the faculae of the Sun are parts of the Sun, and since they move with the same motion as the spots, they prove that the spots are in the Sun itself, and therefore that the Sun turns about its own centre — otherwise, if the faculae were outside the Sun, those placed at the edges would shine outside the Sun. Hence epicycles and eccentrics proper to the spots and faculae are excluded, and from this alone it is gathered that they are not stars distinct from the Sun. And chapter 8: No spot has so far been seen to stand still on the Sun, or to go back, or to rush ahead, after so many years of observations, that is, from 1611 to 1627; from which it is confirmed that they are carried round by the Sun, and not by eccentrics or epicycles of their own. And chapter 9: The paths of all spots appearing at the same time appear, other things being equal, of the same kind and position — that is, all rectilinear, or all curvilinear and swelling with their convexity towards the same quarter; from which it is gathered that there is a certain motion universal to them all, from some common subject that moves them.

And chapter 10: All paths of the spots described yearly on the same parallels are sensibly similar to one another, and for the most part equal in time. That is, if, for example, on 1627 August 22 certain motions are described by a spot in a certain parallel of the Sun, then in 1628 too, on August 22, any spot that is in that parallel will describe a motion similar to the one already mentioned — curvilinear upwards, that is, if that one was curvilinear upwards — and so for past and future years. And chapter 11: All spots in parallels equally distant from the greatest parallel of the solar globe, and separated from each other by an interval of six months, have similar motions, but subcontrary in position and direction; and from this uniformity it is confirmed that they are moved with the motion of one universal subject. And chapter 12: The paths of all spots and faculae become sensibly rectilinear twice a year, namely once about the end of

November and the beginning of December, and again about the end of May and the beginning of June; and yet their paths are oblique to the ecliptic, and the time of the rectilinear stays beneath the Sun is about two weeks, or nearly 14 days. [Margin: When the path is rectilinear [?].] But the direction of the rectilinear paths near midwinter is from the south-east to the north-west, and of those near midsummer from the north-east to the south-west. And chapter 13: that the sphere in which the spots are thought to turn about the Sun is not larger than the Sun and homocentric with it; and chapter 14: nor heterocentric either; and chapter 15: nor larger than the Sun and of mixed eccentricity.

And chapter 16: All the paths of the spots turn from rectilinear to curvilinear, so that from about the beginning of December, through the three months December, January and February, they become daily more curvilinear, the arc swelling upwards towards the northern part of the Sun, and at the beginning of March they become most curved. From then on the curvature decreases more and more through March, April and May, until at the end of May they become sensibly rectilinear; from here they pass again into curvilinear paths, so that from the beginning of June their curvature grows continually, but downwards towards the south, through the three months June, July and August, at the end of which this curvature is greatest; and from the beginning of September the curvature decreases through three further months, September, October and November, until at the end of November the paths become rectilinear. [Margin: When the paths of the spots are curvilinear.] And in chapter 17 he gathers from the motions described that the motion of the spots is sensibly regular and ordered, uniformly difform, and about a mobile axis of the Sun, since the spots have appeared so constantly fastest in the middle and proportionally slower near the limbs, always from east to west, and with a reciprocal passage from rectilinear to curvilinear paths. The motion of this axis and of its poles from west to east — such that in one half-year one pole is in the part of the Sun visible to us and the other in the invisible part, and in turn, in the other half-year, the one that was visible is hidden — became known from the half-yearly contrariety of the course; and he gathers that from the ascent and descent of the axis and the poles comes the whole vicissitude apparent in the motion of the spots. And chapter 18: The paths of all the spots are rectilinear about the beginning of June, and most curvilinear towards the south about the beginning of September; but rectilinear about the beginning of December, and most curved towards the north about the beginning of March. And when they are most curved, they are then also in equilibrium, so that the terms of rising and of hiding, if the Sun is, at its centre, equally distant [Margin: When the curvature is greatest, and the equilibrium [?].]

from the horizon, or in the nonagesimal degree of the ecliptic, appear equally raised. And in Chapter 19: all the courses of the spots, if the annual and the monthly are considered each separately, are regular — that is, both the one produced by the Sun's poles as they describe small circles in their annual circling, and the one completed in about the space of a month; but if they

are compounded together, a certain irregularity arises from them, though it is imperceptible in a single passage. This he shows here for the rectilinear motion, and in Chapter 20 for the curvilinear. From Chapter 21 to 33, however, he shows that this irregularity comes from the refraction of the Sun's rays transmitted through glass lenses into the image of the Sun as it is received; and that this makes the spots appear to traverse greater arcs near the Sun's margins in less time, and smaller arcs near the middle of the Sun in more time; and that consequently no parallax of the spots, and no distance from the Sun, can be inferred from this irregularity. And in Chapter 34 he infers the true regularity of the spots' motion from corrected experiments, and that if they are in the Sun it must be regular; and that they are in the Sun he confirms by the arguments indicated above and by others still to be indicated. So much for the motion.

XIX. Now, in the same first part of Book 4, Chapter 35 deals with the increase and diminution of the spots and faculae — not only the optical or apparent, but also the physical and real. He says that the spots are on the physical surface of the Sun, as hills are on the surface of the Earth, which is not geometrically spherical; and that none of them moves about its own centre. Moreover, although all spots, other things being equal, appear optically smaller near the Sun's margins and larger towards the middle of the solar disc, yet sometimes, by physical increase, they appear larger near the margins, and, by physical diminution, near the middle, a compensation having taken place. [Margin: Increase and diminution of the spots.] And Chapter 36 shows that round solar spots, turned about the Sun, show to the sight, from the aspect, the same configuration on the Sun's hemisphere as they would display if they were similarly rotated about their own centre joined with the centre of the Sun. Chapter 37 asserts that physical increase and decrease have belonged to all the spots observed so far, but varied and successive; that the whole time of increase of one and the same spot is almost equal to its time of decrease, granted that the parts of this time do not have an equal or uniform quantity of magnitude, and that the times of increase of one spot are not equal to the times of increase of another; that the diameter of the spots is sometimes physically increased twofold, threefold and more; and that the increases of some quite often come about through the aggregation of several, just as decreases come about through disaggregation. Chapter 38 reviews the various ways in which physical augmentation can take place, and chooses as the most probable the one that occurs not by alteration or rarefaction alone, but by generation from the solar matter. Chapter 39 deals with the variable obscurity and clarity of the spots; and Chapter 40 with the nucleus of the spots, that is, the darker part in the middle. Chapter 41: that the spots cannot be shadows cast onto the Sun. Chapter 42: nor can they be a merely apparent opacity; but, Chapter 43, a true opacity is in them, and, Chapter 44, a true colour; and, Chapter 45, that this colour is probably blackness; Chapter 46, that it is physically intensified in the nuclei — about which, in Chapter 47, he disposes of many small questions, as in Chapter 48 of many about the light and clarity of the spots. [Margin: Obscurity, opacity, colour of the spots.] Next,

in Chapter 49, he deals with the shapes of the spots: that they are irregular and slender, and changeable from time to time. And in Chapter 50: that the physical surface of the Sun is alterable in many ways, by so great a variety of spots, faculae and shadows. And Chapter 51 teaches by many arguments that spots and faculae are generated and perish physically on the surface of the Sun. [Margin: Rising [?] and destruction of the spots, physical and astronomical.]

To understand these arguments, know that the astronomical rising of the spots is the name given when, because of the Sun's rotation, they begin to appear at its eastern margin, which for us northerners facing south is on the left; and astronomical setting is the name given when, because of the same rotation, they begin to be hidden at the Sun's western margin, on our right, and to traverse the upper hemisphere of the Sun, which is hidden from us. Physical rising, by contrast, is spoken of when they actually come into being anew, and therefore appear indiscriminately in any part of the solar disc, provided that part is visible to us; and physical setting, or destruction, is when they are extinguished and actually cease to be. With these premises laid down: the greatest part of the spots, says Scheiner, is born directly on the face of the Sun, far from the eastern margin, grows, and advances to the western limb until it sets astronomically. Likewise a great part of them rises astronomically, but before astronomical setting wholly fails physically. There are many, too, that have neither astronomical rising nor setting, but begin and cease to be on the face of the Sun between the two. Some, however, last from astronomical rising to astronomical setting, whether they are among those that arose physically in the hemisphere of the Sun hidden from us, or are those whose duration equals an entire rotation of the Sun of about 27 days. Rarely, moreover, does a large spot appear that is not born physically and does not fail without astronomical rising and setting; and spots of this kind grow with an escort of many, which nevertheless diminish after about two or three days, as the retinue of spots withdraws. Of the faculae, many perish after astronomical rising, before setting; many flash out anew on the face of the Sun without astronomical rising and set astronomically; some, though more rarely, are kindled and extinguished in the middle of the Sun without astronomical rising or setting. [Margin: Rising and destruction of the faculae.] Moreover, not all return after a rotation of 27 days, nor have any lasted so far that, in one whole year, not to say in several, they have returned through repeated rotations; therefore most perished before the end of the year. Besides, the secondary spots and faculae, which lie outside the middle zone of the Sun, are in perpetual change, and the Sun's surface is full of them. Finally, there have been some months in which no large spot appeared in the Sun. Are these not physically evident arguments of a real rising and destruction?

Again, in Chapter 52 he confirms the same generation and corruption from the fact that the greater number of spots are kindled into faculae. For this can happen neither by rarefaction alone, nor by their dispersal from the Sun brought about by rarefaction, as though that part of the Sun which previously suffered

eclipse from the spots now shines through; for in that way they would not appear at the margin of the Sun, if we suppose it, as the Peripatetics posit, both perfectly round and of uniform density and opacity. In that place he refutes at length certain people who say that the Sun has a thin crust, transparent in some places and opaque in others, in which it is enclosed, or who defend these phenomena rather forcibly in some other way, through translucency. Then Chapter 53 teaches that solar spots cannot be produced by the mere concourse of opaque parts, if hard orbs are supposed to rotate perpetually about the Sun. And that both they and the faculae are generated and corrupted, he again confirms in Chapter 54 from the manner of their beginning and ending. [Margin: Various modes of destruction and rising of the spots.] For some spots in the middle of the Sun's disc decrease in bulk and are attenuated into a shadow, so that they can scarcely be distinguished from the rest of the Sun's surface except by agitating the instrument; this shadow sometimes turns into a facula, sometimes vanishes without a facula, and sometimes splits into minute spots like black points, and these points, having advanced together in order over several days, pass off into faculae. Not rarely they leave behind them a trail and track, facular or shadowy; or they spread into one continuous shadow; or, as others vanish, they regain strength and grow into new spots, which again pass off into shadows or torches [faces] in a kind of reciprocal struggle. Often some spots seem to be overwhelmed on all sides by brighter matter until they become torches; and faculae are sometimes pure, sometimes surrounded by cloudy traces. But all the spots end at last either in faculae, or in shadows, or in the ordinary surface of the Sun. At their rising, moreover, many, from the thinnest shadows like spiders' webs, become thicker, blacken and grow; many bubble up from very small and separate ones and, having spread, make one continuous spot; many, from faculae joined with a shadow, degenerate into spots. Yet the beginnings are always smaller in intensity and extension, and the increases are completed sometimes after a few hours, sometimes only after whole days. Again, faculae are less stable than spots, harder to observe, and mostly larger than the spots from which they arose. Yet it is not in a continuous chain, as it were, of reciprocal rising and destruction that a facula comes from a spot, and from this facula a second spot, and from the second spot a second facula, and so on to infinity; this succession is often interrupted, and lasts only as far as the condition of the underlying matter allows. Who, having weighed all this, would not grant their physical rising and destruction? For what greater sign of such rising and corruption do we have even in the sublunary region?

XX. With these things established, he now shows in Chapter 55 that the true place of the spots and faculae is in the Sun itself or very near it, from the absence of any parallax. For they were observed at the same time by different observers in different places on the Earth and found in the same part of the Sun, as appears from the diagrams; and if any were observed at a different time, they were nevertheless found by the later observers to have advanced towards the western part of the solar disc by just as much as the proportion of the intervening time

to the rotation of about 27 days required. From this, at least, it follows that they are not so far from the Sun as to admit a parallax perceptible from us. [Margin: Place of the spots and faculae.] Scheiner affirms, moreover, that many observations were sent to himself and to Father Paul Guldin from the West Indies by Father Gaspar Ruess, whom he had once instructed at Ingolstadt, and from the East Indies by others; and that when these were compared with his own and with those of others made in Europe, no parallax whatever was detected, but the same spots were seen in the same place on the Sun in both. Father Carolus Malapercius [Charles Malapert] also noted this, comparing his own observations with those made at Coimbra by Father Gulielmus Vuely from the year 1620 to 1623, and with those received at Kalisz in Poland. Assuming Tycho's distances, moreover, Scheiner shows that the spots, from the absence of any sensible parallax, are nearer to the Sun than the Sun itself at apogee is to itself placed at its mean distance. And he concludes the chapter by asserting that when the Sun is without any spot, this is so over the whole globe of the Earth; that when a new spot rises, this happens everywhere at the same time and at the same place on the Sun; and that when some spot occupies the centre of the solar disc or the ecliptic, this happens everywhere on Earth at the same time; and that no deviation of any spot from the Sun's disc has ever been noted.

But in Chapter 56 he teaches that the spots are in the very body of the Sun, from the apparent inequality of refraction introduced into the motion of the spots by the observing lenses; [Margin: The spots[42] are shown to be in the Sun itself.] and in Chapter 57 from the faculae, which were shown above to be in the Sun — therefore the spots too, which are moved with equal motion and in the same order; and in Chapter 58 from the slenderness the spots usually show near the Sun's horizon, or limb; and in Chapter 59 from the intervals of two spots, or of one advancing day by day; and in Chapter 60 from the proportional intervals of the times and motions with which the spots are moved when they proceed with a rectilinear course along parallel lines on the Sun's disc; and in Chapter 61 from the quasi-elliptical path of the spots when the course is curvilinear; and in Chapter 62 from the properties of the curvilinear paths, which recur at half-yearly intervals; and in Chapter 63 from the intervals of a single curvilinear course; and in Chapter 64 from the polar small circle which the axis of the spots' rotation describes; and in Chapter 65 from the periodic return of many spots which are not corrupted before 28 days, their return being argued from the same place, time, motion and parallel. Such was the one that in the year 1625, from March 22 at $8\frac{1}{8}^{h}$ in the morning to April 4 at $1\frac{1}{9}^{h}$ in the evening, was in the Sun's southern parallel; and another of the same year, from May 11 to June 18, in the northern parallel; and a third which, in the same year, rose on June 8 but was completed on the 19th, and lasted until July 15, from north to south. [Margin: Examples of returning spots.] And from a fourth spot which, born in the same year on July 12 but full-grown on the 24th, set

[42] printed *Macula*, singular, with the plural verb

astronomically, and, having traversed the hidden hemisphere of the Sun, rose astronomically on August 6 at $7\frac{1}{2}^{h}$ in the morning, and set astronomically on the 19th at $4\frac{1}{3}^{h}$ in the afternoon; and so with the rest. And in Chapter 66 from secondary spots going about the poles and small circles of this motion with a motion proportional to the others. I would add that it is possible that the spots are on the physical surface of the Sun itself and are carried round with the motion of the Sun's rotation; and if this possible case were actually brought into being, we would have no other indications and arguments, or none greater, than those we have now; therefore these arguments are entirely sufficient to conclude that the matter stands so, rather than that the spots are moved by many motions of their own.

XXI. In part 2 of book 2 he explains the theory of the solar phenomenon, especially as it bears on the motion of the spots, and in Chapter 1, reduced to a short summary, he teaches the following. [Margin: From part 2 of book 4.] The regular apparent motion of the spots about the Sun's centre is twofold: one annual, the other almost monthly. Since this motion is rectilinear twice a year, at intervals of six months, namely at the beginning of June and of December, its axis must lie in the apparent circle of the Sun, or horizon. [Margin: Preparation for the theory of this phenomenon.] And since for six months it is curvilinear towards the south and for six towards the north, the curvature on each side gradually swelling up to the maximum and gradually subsiding, in an even proportion of times and curvatures, the said axis must for six months raise its one half above the plane of the solar circle, the other half being depressed by as much. And since the curvature of the spots' motion grows from the beginning of December for three months up to the greatest curvature, and decreases over another three, and the same then happens in the opposite region of the Sun, the axis already mentioned must be carried round annually, and small circles must be described by its poles on the Sun's surface, one towards the south and the other towards the north, one half of each appearing in the hemisphere of the Sun manifest to us, the other lying hidden in the concealed one. Further, he calls the solar ecliptic the common section of the plane of the celestial ecliptic and of the open solar circle, or solar horizon. He says that the poles from which the small circles already mentioned are described about the fixed axis of the Sun lie on the perimeter of the solar circle, or in the solar horizon, a quadrant distant on either side from the solar ecliptic; and that the fixed axis, about which the small circles are described, is the diameter of the Sun that cuts the line of the solar ecliptic orthogonally at the centre of the Sun. [Margin: Position, magnitude and period [?] of the polar small circles.] And although the semidiameter of the polar small circles already mentioned has been found to be sometimes 6 degrees and sometimes 8, of which the Sun's perimeter is 360, yet more frequently and more carefully it has been found to be 7, which he also chooses. The periodic time is annual, and its beginning is most probably taken from the contact by which the lower pole reaches the south-western part of the solar horizon about the end of November and the beginning of

December, so that the root of this motion can be fixed at about the first day of December, at the contact of the small circle with the south-western part of the solar horizon. This annual motion takes place from west to east, and contrary to the monthly motion of the spots, which takes place about this mobile axis in such a way that they tend from the eastern region of the Sun to the western in the part visible to us. Finally, the duration of a rotation of the spots, when it is complete, sometimes scarcely exceeds 25 days, often reaches 27, and sometimes 28. [Margin: Monthly period of the spots.] Chapter 2 follows, in which is the theory of this motion.

Syſtema Generale ac Theorica Motus Macularum Solarium.

The general system and theory of the motion of the solar spots.

Diagram. A large circle, the solar disc, with its centre at E, crossed by the horizontal diameter A–B and by four further chords, R–L and K–S above the diameter and T–N and M–Q below it. At the top stands a small circle about C, cut by the chord G–O, and at the bottom a small circle about D, cut by the chord P–F; straight lines drawn from the arcs of both small circles converge through E. "Boreos." (north) is set above the figure, "Auster." (south) below it, "Oriens." (east) vertically on the left and "Occidens." (west) vertically on the right.

- Boreos. (top): north
- Z (top of the upper small circle)
- l, m, n, o (points on the upper arc of the upper small circle, left to right with Z between m and n)
- G, O (ends of the chord through the upper small circle, on the large circle)
- C (centre of the upper small circle)
- t, r, p, s, q (points within the lower part of the upper small circle)
- R, K, A, T, M (left ends of the chords, top to bottom)
- L, S, B, N, Q (right ends of the chords, top to bottom)
- E (centre of the large circle)
- Oriens. (left margin of the figure): east
- Occidens. (right margin of the figure): west
- c, b [?], e, d, a (points within the upper part of the lower small circle)
- P, F (ends of the chord through the lower small circle, on the large circle)
- D (centre of the lower small circle)
- f, g [?], Y, h, i (points on the lower arc of the lower small circle, left to right)
- Auster. (bottom): south

[Margin: From book 4, part 2, chapter 2.] XXII. In the preceding figure, let the solar horizon BCAD be described from the centre E of the solar globe; it separates the hemisphere of the Sun visible to us from the hid- den one (although the hemisphere is not seen by us as such, because of its immense distance, but as a flat disc); and for convenience let the Sun be imagined to

be in the meridian or the nonagesimal, etc.; and let the four cardinal points in the solar horizon be A, the eastern; B, the western; C, the northern; and D, the southern. Let the line of the solar ecliptic be AEB, and let its fixed axis, parallel to the axis of the celestial ecliptic, be CD, whose northern pole on the Sun's perimeter is C and southern pole D. For because of the Sun's very great distance we can take ADBC as the perimeter of the greatest circle in the Sun, and the line CD as its axis, granted that, strictly, we do not see the greatest circle of the Sun. Now let the small circle GZor be described from the pole C, and the small circle FYPc from the pole D, whose semidiameter CZ, or DY, subtends 7 degrees of the solar perimeter, of which the whole perimeter ACBD is 360. Let the small circle GO be called the Arctic, or Northern, and the small circle PF the Antarctic, or Southern. These small circles are to be thought of not as lying in the same plane as the solar horizon, as this page shows them, but as erected orthogonally to it; and they are to be divided into twelve equal segments, for twelve signs, as it were, and the twelve months of the year. These small circles are described by an annual motion of the poles of that axis about which the monthly motion of the spots takes place; and this axis we shall call the mobile axis.

At the beginning of December, then, let the southern pole of the mobile axis be at the contact F, and the northern at G, so that the mobile axis is FEG. Then let the pole F be moved on the surface of the Sun visible to us, but towards the east, so that when December is over, at the beginning of January, it is at a, and the pole G, by a contrary motion, has reached l, but on the surface of the Sun hidden from us; and let the mobile axis be aEl. At the beginning of February, let the southern pole be at b and the northern at m, and the mobile axis then be bEm. And finally, when the quadrant Fc has been completed, let the southern pole be at c at the beginning of March, and the northern at Z, and the mobile axis be cEZ. And let these poles continue in this way, so that at the beginning of April the southern is at d and the northern at n; at the beginning of May the southern at e and the northern at o; and at the beginning of June let the southern be at the contact P of the solar horizon, setting heliacally for us, or hiding itself behind the back of the Sun, while the northern rises for us at the contact O, about to traverse over six months the semicircle OrG visible to us, just as the southern will traverse the hidden semicircle PYF. So at the beginning of July the southern pole will be at f, and the northern at p; at the beginning of August the southern at g and the northern at q; at the beginning of September the southern at Y and the northern at r; at the beginning of October the southern at h and the northern at s; and at the beginning of November the southern at i and the northern at t. Finally, when the annual revolution has been completed, at the beginning of December the southern pole will again be at the contact F, rising optically for us, and the southern [?] at the contact G, setting optically for us.

[Margin: At the beginning of December.] XXIII. When the southern pole is at the contact F, and the southern [?] at G, the paths of the spots are rectilinear

to sense, but not parallel to the solar ecliptic AEB; rather, the greatest parallel described by the path of the spots is TES (which, however, to avoid confusion, I have not drawn in the figure), whose greatest southern declination from the solar ecliptic, namely the arc TA, and greatest northern declination, BS, are equal to the arc GC, that is, of 7 degrees; and the motion of the spots is from the eastern point T towards S, or at any rate along parallels to the line TES.

[Margin: At the beginning of June.] When, however, the southern pole is at the contact P and the northern at O, the motion of the spots from east to west is rectilinear to sense, but the greatest parallel described by them is KEN, declining towards the north by the arc AK and towards the south by the arc BN, of 7 degrees.

[Margin: At the beginning of March [?].] When the southern pole is in the middle of the visible semicircle, that is, at c, and the northern at Z, the curvature of the spots' paths is greatest, with the convexity swelling towards the north; but the diameter of the greatest parallel described by the spots in their monthly motion coincides with the line AEB of the solar ecliptic and does not decline from it at all; and this station is called the vernal equilibrium. For there, the Sun being, as we have supposed, in the meridian or in the nonagesimal degree of the ecliptic, the ends A and B of the greatest parallel appear equally raised above our horizon.

[Margin: At the beginning of September.] When the southern pole is in the middle of the hidden semicircle, at Y, and the northern in the middle of the visible one, at r, the curvature of the paths is greatest, but with the convexity swelling towards the south; and then too the diameter of the greatest parallel coincides with AEB without any declination, and there is the autumnal equilibrium, the ends A and B being equally raised, etc.

At the remaining times of the year the convexity of the paths behaves as follows. For the greater part of December, and almost the whole of January and February, it swells towards the arc ARC of the solar perimeter, intercepted between the eastern and northern points; for the greater part of March, in April, and for almost the whole of May, they swell towards CLB, intercepted between the northern and western cardinal points of the Sun; but for almost the whole of June, the whole of July and almost the whole of August they swell towards AMD, the arc intercepted between the rising and the southern cardinal points; and finally, for almost the whole of September, the whole of October and almost the whole of November they swell towards BQD, the arc intercepted between the western and southern cardinal points of the solar perimeter.

So you see that by a single annual motion of the solar axis in the order of the signs (if you look at its visible pole), and by a quasi-monthly motion of the spots against the order of the signs, that is, from rising to setting (if you look at their path visible to us), made about that mobile axis, the phenomena of the motions of the spots are saved; and that this annual motion of the solar axis is

posited more safely, and without absurdity, than an annual motion of the Earth. [Margin: A corollary in favour of the Earth's rest.]

XXIV. The Torrid Zone of the Sun, so to speak, or Royal Road, within which the greatest part of the faculae and spots, and of the ordinary ones, proceeds, is contained by the arc MAR on the rising side and the arc QBL on the setting side. Its bounds are parallels of the solar ecliptic, but within it the paths and bends are wonderfully inclined and alternating. The whole breadth of this zone does not exceed 60 degrees, of which 30 are to be understood in the arc AM or BQ, and the remaining 30 in the arc AR or BL, granted that in a small figure we have not represented that quantity. Therefore, since the zones that are, so to speak, frigid, because of the scarcity of faculae, are of 7 degrees, as large as CG and DP, there remain 53 degrees each for the remaining, temperate zones, in which rare and not very conspicuous spots or faculae occur — namely the arcs RG and OL, or MP and FQ.

XXV. This done, Scheiner treats in Chapter 3 of the size of the circle of the Sun visible to us and its difference from the greatest circle of the Sun; in Chapters 4 and 5 of the Sun's apparent diameter and the way of observing it; and in Chapter 6 of the size of the Sun and of the spots, both visual and true, supposing Tycho's distance of the Sun and the size of the Earth's diameter, which he supposes as can be seen in the table below. [Margin: From book 4, part 2.]

According to Tycho and Scheiner

	German miles	Roman miles
Semidiameter of the Earth	860	3440
Semidiameter of the Sun	4602 1619/4000	18411 6/10
Circumference of the Earth	5403 2152/4000	21614 152/1000
Circumference of the Sun	28920 3612/4000	155683 612/1000

With these assumptions, he sets out various classes of spots with their size, beginning from the smallest, which are like a point, and proceeding up to the greatest, which I have reduced to a synopsis in the following table. [Margin: Size of the spots.]

Order of the spots	Proportion of the visual diameter of the spots to the visual diameter of the Sun	Size of the diameter of the spots in Roman miles
Simple: A	1 to 2000	18 4/10
B	10 to 2000	187 2/5
C	As 50 to 2000	964 0
D	100 to 2000	1928 0
E	143 to 2000	2640 0
Composite: F	285 to 2000	4419 0
G	As 333 to 2000	6292 0
H	500 to 2000	9640 0

He gives an example of spot E in 1625 April, and says it was equal to Europe; of spot F, on 1626 May 26, greater than Africa; and of spot G, in the year

1625, March 25, equal to Asia; and finally of spot H he gives an example on 1625 April 19, which he says was nearly equal to the disc of the Earth, or to half its circumference. But since the Sun is about seven times higher for us than for Tycho, and its diameter is to the diameter of the Earth not as 5 ½ to 1 but as about 34 to 1, as we said in Book 3, chapter 11, the spots just mentioned therefore turn out about seven times larger than Scheiner set them. The remaining matters he treats up to the end of the book concern the qualities and substance of the Sun and of the heavens, about which enough has been said elsewhere.

[Margin: Chiaramonti's blunder about the shadows of the solar spots.] XXVI. When I had written this, I came upon Book 9 of Chiaramonti's *De Universo*, in chapter 1 of which he reports the opinions of several people on the spots of the Sun, especially that of Ioannes Iarde, who thinks they are stars and calls them the Bourbon stars [sidera Borbonia]. In chapter 5, however, he contends that they lie far below the Moon. His reason is that if they were adjacent to the Sun, and one of them were so large as to equal the terrestrial globe, its shadow would still be no longer than two semidiameters of the Earth; whereas, if it had to shade a small part of the Sun for us, the apex of its shadow would have to reach the Earth and be very nearly 1105 terrestrial semidiameters long, which is the least distance of the Sun set by Copernicus. But the good old man is mistaken. For an opaque body to cover part of a luminous body for us, it is not necessary that its shadow reach our eye, as is clear from central but annular eclipses of the Sun, which we treated in Book 5, chapter 8, number 4, and chapter 9, problem 7; it is enough that it intercept some rays of the luminous body from us. The privation of these rays, in the air and in our eye, cannot be called a shadow simply, but at most a shadow in a certain respect [secundum quid].

CAPVT XIII. Diſſoluuntur quinque Argumenta pro motu Terræ vel Annuo vel Diurno, ex motu multarum rerum versùs Orientem deſumpta.

CHAPTER XIII. Five Arguments for the motion of the Earth, either Annual or Diurnal, taken from the motion of many things towards the East, are refuted.

[Margin: 1st argument: from the perpetual wind of the Torrid Zone.] I. Some take the first argument from the breeze or wind that blows almost perpetually from the east within the Tropics, which Galileo and Gassendi ascribe to the motion of the Earth towards the east. But this argument remains sufficiently answered by what was said in chapter 7, number 4. Mastrius and Bellutus, however (disputation 4 on the heavens, question 4, number 121), add that if the Earth moved, this perpetual wind would be felt better on land than at sea, inasmuch as the air is enclosed within valleys, mountains and trees.

[Margin: 2nd argument: from the veins of metals.] II. Others draw the second argument from the fibres and veins in the mines of metals, which are turned towards the east; this, they say, seems to come from nowhere else than from the Earth's perpetual motion towards the eastern quarter. But this, if it is true, rather indicates that it comes about from the illumination and heat of the Sun, which begin from the east. Otherwise, if it proceeded from the motion of the Earth, why do rivers, which are more fluid, not bend their course towards the east? So say Mastrius and Bellutus in the place indicated a little earlier. I said "if it is true", because the veins of stones, and of trees, are found to be turned rather towards the poles of the world.

[Margin: 3rd argument: from an iron plate turned towards the poles.] III. Thirdly, some say: let an iron plate be hung in balance by a bronze wire in a small, well-closed room, so that one of its ends points to the east and the other to the west; a little later it will bend so as to point to the Poles. They therefore think this an argument for the motion of the Earth towards the east, when they ought rather to recognise the magnetic force of iron, which turns itself towards the motionless axis of the Earth.

[Margin: 4th argument: from the danger of corruption.] IV. Fourthly, others say that things which are not agitated perpetually or frequently tend little by little towards corruption; therefore, lest the Earth, the nurse of all things, should be corrupted, it had to be agitated perpetually towards the east. They also add a soul to the Earth, as a salt, so that it should not putrefy. But if the talk is of partial corruption at the surface, that cannot be avoided by the motion of the Earth; if of total corruption, there is no danger, since it is not easily alterable, except around the moister parts of its surface.

V. The fifth argument, from the motions of the sea and from its flux and reflux, needs a longer examination; it is therefore to be set out in a separate chapter, the following one, and brought to the balance.

CAPVT XIV. Ad Argumentum pro diurno & annuo Telluris motu ex Maris Aeſtu deriuatum, intelligendum ac diſſoluendum, tractantur accuratè tum hiſtoria Aeſtus Marini, & aliorum Maris motuum, tum Opiniones celebres de Cauſa Aeſtus Maris.

CHAPTER XIV. For understanding and refuting the Argument for the diurnal and annual motion of the Earth derived from the Tide of the Sea, there are treated accurately both the history of the Sea Tide and of the other motions of the Sea, and the celebrated Opinions on the Cause of the Tide of the Sea.

I. Here, surely, if anywhere, Galileo exulted, in Dialogue 4 on the twofold system of the world. For the first page of that dialogue, which in the Italian is 409 but in the Latin 309, begins thus: *After I had examined again and more often by myself the effects and accidents, partly seen by me myself, partly learned from others, which are observed in the motions of water; and had moreover read*

and heard the notable vanities that many have brought forward to give the reason of these accidents, I felt myself not lightly drawn to admit these two conclusions, the necessary presuppositions being made, however. [Margin: Galileo's two conclusions on the tide of the sea.] The conclusions, which he adds at once, are these: *If the terrestrial globe is immobile, then the flux and reflux of the sea cannot happen naturally. On the contrary, if the motions already assigned are conferred on the same globe, it is necessary that the sea be subject to flux and reflux, according to all those things that are observed in it.* A few lines later, however, he admits that he does not claim so much for himself as to be able to give an adequate reason for all the effects that may perhaps be found in seas situated outside the Mediterranean, *but nevertheless to propose a key that may open the door of a road never trodden by others*; and he adds: *And although in other seas remote from us accidents unknown to our Mediterranean may be found, the reason and cause that I shall produce will not for that reason cease to be true, provided it is confirmed by the accidents of our sea and fully satisfies them: for in the end there must be a single, true and primary cause of effects of the same kind.* From this it appears that his reasoning and judgement stumble at once, on the very first threshold. For if the accidents of the sea tide outside the Mediterranean are not of the same nature as the accidents seen in the Mediterranean, but very different, and in some places in a way opposite — and that in seas much vaster than the Mediterranean — how can he affirm with so confident a mind that a cause satisfying the tide of the Mediterranean will also satisfy all the tides of all seas? He does just as if someone who had discovered a cause of quartan fever that satisfied all its properties were to assert that the same cause is the cause of all fevers, although he had not known their accidents beforehand. But let us hear from him, partly in the person of Salviati, partly in that of Sagredo, the history of the sea tide observed in the Mediterranean, so that we may come to know thoroughly the foundation of his opinion.

[Margin: The threefold period of the sea tide.] II. He says, then, that three periods are observed in the fluxes and refluxes of sea water. The first, chief and greatest is diurnal, by which the waters are alternately raised for about 6 hours and lowered for another 6 hours. The second is monthly, which seems not indeed to be produced by the Moon, but to vary the first by alteration, with a notable difference according as the Moon is new, or full, or halved. The third is annual, which seems to depend on the Sun, inasmuch as the Sun makes the diurnal fluxes greater or lesser, and different at the solstices from those that occur at the equinoxes. [Margin: In the diurnal motion, a threefold variety.] In the diurnal tide, moreover, he affirms that three diversities are noticed. In some places the waters swell and subside without progressive motion; in others, without rising and falling, they now advance towards the east, now run back towards the west; in some places, finally, both the rising or swelling and the progressive and regressive course vary, as happens at Venice, where the waters swell as they approach and subside as they recede. They do this at the far end of the length of gulfs that extend from west to east and are bounded by such shores as give the water room to spread itself when, raised, it swells. But if its

course were cut off by mountains or by higher embankments, there it would swell and subside without progressive motion. Further, the waters of the sea run and run back without swelling or change of height in the middle parts of the Mediterranean, as happens most evidently in the Faro of Messina, or Sicilian Strait, between Scylla and Charybdis, where the running waters are very swift because of the narrowness of the channel. But in the more open seas, and around the Balearic islands, Corsica, Sardinia, Elba [Ilua], the African coast of Sicily, Malta, Crete, etc., the changes of height are very small, but the currents of the waters are notable, especially where the sea is confined between two islands, or between some island and the mainland.

[Margin: A cause of the sea tide refuted by Galileo.] III. Having narrated these things first, he refutes two causes of these effects brought forward by some. The first comes from some Peripatetic or other, who, clinging to the text of Aristotle, had said that the deeper waters by the weight of their mass expel the less deep, which, thrust upwards, strive to descend, and that from this continual struggle the flux and reflux arise. For only waters that have a higher surface drive away the lower and humbler ones, whereas deeper waters cannot do this; and then, when the higher have settled on the lower, they soon come to rest and are brought into equilibrium with them. The second cause is the one that not a few refer to the Moon: that it predominates over moist bodies, and entices to itself and raises a heap of waters, which, continually following the Moon, make a swelling in the part to which the Moon is vertical; and that when the Moon has gone below the horizon, that swelling returns after six hours because of the tractive power which the Moon has communicated to the opposite degree of the zodiac; or else that the Moon by its temperate heat rarefies the waters, and that they therefore swell. This one, I say, he rejects on the ground that the Moon passes over the whole Mediterranean every day, and yet the waters are not raised except at its more easterly extremities and near Venice; that a moderate heat does not suffice to raise water by force of rarefaction, as anyone can test in water that is tepid and not yet boiling; and that no reason for the disparity can be given from it, why the Moon should rarefy the waters at Venice and not rarefy them at Ancona, Naples and Genoa. So he counts these and similar causes among poetic fictions.

And when Simplicio in that dialogue had said that he would rather refer this effect to a miracle, as being supernatural, since Aristotle too, at the beginning of the *Mechanical Questions*, seems to ascribe to a miracle the cause of certain things whose causes are hidden, or to count them among marvels, Salviati replies that if a miracle is to be introduced for the sea tide, it is rather the motion of the Earth that should be introduced, made by miracle and supernaturally; upon its motion the tide of the sea would then follow naturally and by a simpler way, and there would be no need of more miracles. [Margin: Another cause of the tide rejected [?].] For with the Earth motionless no reason can be given why the water at Venice swells by ten palms and more within 5 or 6 hours, with no change of weight, of density, of coldness or of saltiness (lest anyone

have recourse to rarefaction), but by a manifest inflow onto the shores, and yet is not raised perceptibly at Ancona, at Dyrrachium or at Corfu [Corcyra]. For if anyone has recourse to the waters of the Ocean entering through the Strait of Gibraltar [Gaditanum fretum], that is, a channel no more than 8 miles wide: if that water had to reach the farthest shores of the Mediterranean in six hours, and cover two or three thousand miles, it would certainly travel more than 400 miles in one hour, which is an incredible velocity; nor could ships in that strait, or near it, make headway against it, which they nevertheless do. Since, then, this cannot be explained if the Earth, as a vessel containing the seas, rests motionless, it must be investigated whether the causes of these vicissitudes can be explained by the motion of this vessel. This he does

[Margin: The likeness of the tide, taken from the water of a moved vessel.] by considering what would happen to a vessel or ship full of water if the vessel or ship were moved with a gentle progressive course towards one side, but with unequal velocity. For the water, being fluid and not firmly cohering to the vessel, would, while the vessel was retarded, keep part of the impetus it had conceived, run forward, and be raised at the vessel's leading end. On the contrary, if the vessel's motion were quickened, the water, keeping part of its slowness and remaining somewhat behind, would, before it had wholly conceived the new impetus, recede to the following part of the vessel and be raised there; in the middle of the vessel, however, hardly any swelling of the water would appear, but rather a more notable current than at the extreme edges. Now effects of this kind may be seen in the tides of the Mediterranean. It is therefore consistent that its vessel, that is, the Earth, is moved by such motions as now retard it, now quicken it; and that the Earth, further, because of the mixture of the annual and the diurnal motion, is moved now more slowly, now more swiftly, he teaches by the following diagram.

[Margin: The motion of the Earth made unequal by the diurnal and the annual.] IV. From A as centre let the great or annual orb BCDE be described; and on its periphery, taking any point, say B, let the terrestrial globe FGHK be described, which is to be understood as moved, with the motion of the centre B, towards the east, C, so that in one solar year it runs through the whole periphery of the great orb.

Diagram. In the right column: a large circle with centre A, the great or annual orb, with C at the left, D at the foot and E at the right; at its top, on the circumference, the point B. About B are drawn a small circle, the terrestrial globe, and a middle-sized circle passing through O at its top and L at its foot, with a small crescent figure drawn on it at O and another at L. A long arc runs from P at the left, through O, to a point at the right. M and N stand inside the large circle on either side of A. No caption.

- O (top of the middle circle, beside a crescent figure)
- H (top of the small circle)
- K (left of the small circle)
- B (centre of the small circle, on the great circle)

- G (right of the small circle)
- F (foot of the small circle)
- L (foot of the middle circle, beside a crescent figure)
- P (left end of the long arc)
- [?] (right end of the long arc: a swash sort, Q or an ampersand)
- M (inside the great circle, left of A)
- A (centre of the great circle)
- N (inside the great circle, right of A)
- C (left on the great circle)
- E (right on the great circle)
- D (foot of the great circle)

Further, let the same globe be understood to revolve daily about its centre B, likewise towards the east, from H through K into F and G, until it has completed its diurnal revolution when 24 hours have passed. For from this it comes about that any part of the circumference of this globe is moved, though at different times, by contrary motions: while the parts near H are moved towards the left side, K, the parts placed at F are moved towards G, the right quarter; and while the parts go up from G into H, the parts from K go down into F. With this contrariety of motions posited, therefore, when the motion of the diurnal turning is joined with the annual motion of the centre B, there must result at the surface of the Earth a motion now quite accelerated, now retarded by just as much. For the parts around H will be moved most swiftly, since the annual and the diurnal motion conspire in the same direction, that is, towards the left, the one tending into C, the other into K: *Therefore in such a case the diurnal motion increases and accelerates the annual motion* (these are Galileo's own words). By contrast, in the part F the motion is retarded, because although by the force of the annual motion it is carried towards the left, yet by the force of the diurnal motion it is carried towards the right: *Therefore the diurnal motion will subtract from the annual motion.* Around the points G and K, however, the annual motion is nearly simple and uniform, since the diurnal adds nothing or very little to the annual motion, or subtracts nothing or very little from it, inasmuch as there it is carried neither to the left nor to the right, neither up nor down. This inequality of the motions of the Earth, then, is the chief and primary cause of the sea tide, especially of the tide made lengthwise, towards east and west.

[Margin: 1st accident.] V. After this Galileo considers five notable accidents of the sea tide. The first is that water raised at the end of some vessel by the force of acceleration or retardation does not stay in that elevation, but not only brings itself back to equilibrium by flowing back, but also, by its own weight, with the impetus so conceived, springs back to the opposite side, raising itself towards the other edge of the vessel, just as the bob of a pendulum after its descent rises through the opposite arc. [Margin: 2nd accident.] The second is that the reciprocations of the rising water already mentioned, from one side to the other, are more frequent in shorter vessels and rarer in longer ones, just

as the vibrations of a pendulum hung from a shorter cord are more frequent than those of one hung from a longer. [Margin: 3rd accident.] The third is that, among waters enclosed in channels of equal length, those that are deeper complete their reciprocations in shorter times, while those that are less deep do so less frequently. [Margin: 4th accident.] The fourth is that waters enclosed in a vessel, or in the channels of the Earth, are alternately raised and lowered around the ends of the vessel without progressive motion, unless perhaps in swelling they overflow beyond the embankments; but the parts placed in the middle run out forwards and backwards without perceptible swelling. [Margin: 5th accident.] The fifth is that in small vessels the acceleration and retardation are shared in the same way by the whole vessel and by every part of it; but not so in the immense channels of the sea. For when one end is retarded because of the mixture of the diurnal and annual motion of the Earth, the other end is still carried along by a very swift motion: as in the preceding figure, when the parts of the sea at H are moved swiftly, the parts at G are nevertheless moved less swiftly, and the parts at F most slowly, if the stretch of sea is so long as to equal the quadrant HG or the semicircle FGH.

[Margin: Cause of the 1st effect of the sea tide.] VI. When these things have been well considered, it is not difficult to give the reason of several effects that are conspicuous in the tide of the sea. First, in lakes, pools and small seas no perceptible tides appear, because of the frequency of the reciprocating vibrations, before they can complete their effect from the retardation and acceleration of the Earth, which take place alternately every 12 hours. For before that, the waters, somewhat raised at the ends because of the narrowness of the vessel, slip back by their own weight to equilibrium within one, two or three hours, and so a notable tide is prevented. [Margin: Cause of the 2nd effect.] Secondly, although the period of velocity takes 12 hours through the semicircle GHK, and the period of slowness 12 hours through the semicircle HFG, yet from the different length and depth of the seas it comes about that those alternations take place every 6 hours, or at least that these are observed more frequently in our seas, though not in all. For in the Hellespont, in the Aegean, and in the Euripus at Euboea the periods are quite short and very different from one another. [Margin: Cause of the 3rd effect.] Thirdly, therefore, in seas and gulfs that are long indeed, but extended in length not towards east and west but towards the poles, as the Red Sea nearly is, almost no tide appears, because the motion of the Earth is lengthwise, not towards the poles of the world, and so the impulses of the waters strike the meridians, not the parallels of the equator. [Margin: Cause of the 4th effect.] Fourthly, from what we experience in a vessel full of water and agitated towards one side, it is easily established why at the extremities of the Mediterranean the water is raised notably, so that at Venice it rises to 5 or 6 feet, whereas elsewhere the elevation is very small, so much so that around Corsica and Sardinia, and in the estuaries of Rome and of the port of Livorno [Ligurni portus], it does not exceed half a foot. [Margin: Cause of the 5th effect.] Fifthly, therefore, the same quantity of water, although

otherwise it runs gently, must nevertheless run with great impetus through the narrower channel if the whole of it has to be poured through a narrow channel out of a wider one, as happens in the Sicilian Strait, in the Strait of Magellan, and in the sea enclosed between Africa and the island of St Lawrence. Sixthly, however, these vicissitudes can be varied not a little both by the blowing of winds and by rivers breaking into the sea, especially as regards the course of the waters, which are carried in the direction in which the rivers drive them. Thus in the Thracian Bosporus the water always runs from the Euxine into the Propontis, and in the Sicilian Strait it runs from the Ionian into the Tyrrhenian, more swiftly, however, in the Sicilian, because it extends lengthwise.

VII. Thus far Galileo on the cause of the diurnal period of the sea tide. [Margin: Cause of the monthly variation of the sea tide.] After this, having interposed a few things on the perpetual breeze within the Tropics, on the easier navigation towards the west, and on the falsity of the motion of fire from the concave of the Moon, he passes on to investigating the cause of the monthly variation, and affirms, on page 333 of the Latin, that if the annual motion of the Earth were not altered, the monthly period of the sea tide would cease. He teaches, further, that the cause of this monthly alteration is that in the annual orb not the Earth alone is carried round, but the whole lunar system LO. When the Moon is at new moon, at L, and is near the Sun, A, or the centre of the great orb, the great orb with respect to it is the arc MLN etc., describable from A as centre; but when it is at full moon, at O, the great orb with respect to the Moon is POQ etc., any arc of which is greater than any similar arc, that is, an arc of as many degrees, of the smaller circle MLN etc. Now it is certain that, other things remaining equal, the same mobile moved by the same power is moved in a shorter time through similar arcs of a smaller circle than through those of a greater. We see this not only in celestial motions but also in terrestrial ones: for from this it comes about that the vibrations of a shorter pendulum, nearer the centre, are completed in a shorter time than those of a longer pendulum, because they traverse arcs of a smaller circle. From this likewise it comes about that the iron rod which the makers of horizontal clocks place parallel to the horizon and free to swing to the sides, to regulate the clock's time, makes the hours longer if the leaden weights hung at the heads of the rod are hung far from the rod's centre, but quickens them and makes them shorter if the weights are placed nearer the centre, because in the one case they must traverse arcs of a smaller circle, in the other of a greater. Likewise, therefore, the Moon at new moon must traverse in the annual orb similar arcs in the smaller circle MLN, and hence in a shorter time; but at full moons arcs of the greater circle POQ, and in a longer time. That force, then, which moves towards the east not the Earth FGHK alone but the whole lunar system OL, must, because of the Moon, share those monthly varieties with the terrestrial globe, make the annual motion non-uniform, and pour that variety back into the tide of the sea, which is founded on the annual motion of the Earth together with the diurnal. That astronomers have not observed these monthly variations in the annual

motion of the Earth, or, as others will have it, of the Sun, is no wonder, since many other subtleties remain up to now unobserved by astronomers, or only roughly noticed.

[Margin: Cause of the annual variation in the tide of the sea.] VIII. There remained the cause of the annual variation in the tide of the sea to be adduced, which Galileo begins to set out from page 340 of the Latin, 451 of the Italian, having first said, however, that it is rather difficult to understand and requires a great abstraction of mind. He says, then, that the inequality of the additions and subtractions which the diurnal turning of the Earth communicates to the annual motion depends on the inclination of the axis of the diurnal motion to the plane of the great orb, or ecliptic; because of this inclination the Earth's equator cuts the ecliptic, always keeping the same angle of inclination. He says that the addition is as great as the whole diameter of the equator when the centre of the Earth is at the solstitial points, but that outside these it is smaller and smaller, according as the centre of the Earth is less and less distant from the equinoctial points, where those additions become least. But to make the matter clearer: in the following figure let the great orb AIP be drawn, at whose circumference's point A let there be one of the solstitial points, where the centre of the Earth CFBG is. Let its axis CAB be inclined to the plane of the ecliptic, or great orb, and fall in the plane of the solstitial colure, which colure passes through the axis both of the equator and of the ecliptic, since the diameter AP is the common section of the solstitial colure and of the plane of the ecliptic.

Diagram. In the right column: a large ellipse, the great orb, with A at its left end and P at its right end, joined by the line AP; I is its lowest point. At A a small globe figure: an inclined axis from C (lower left) through A to B (upper right), a vertical line through A, and an inclined ellipse, with G and D above, E, M and F below. At I a second globe figure built the same way, with G, D and B above, H and S at the left, L at the right, and C, E and F below. No caption.

- G (upper left of the globe at A)
- D (above the globe at A)
- B (upper right of the globe at A)
- A (left end of the great orb, centre of the first globe)
- P (right end of the great orb)
- C (lower left of the globe at A)
- E (below the globe at A)
- M (below the globe at A, right of E)
- F (lower right of the globe at A)
- G (upper left of the globe at I)
- D (above the globe at I)
- B (upper right of the globe at I)
- I (lowest point of the great orb, centre of the second globe)
- V (a small capital just right of I, on the line to L)
- L (right of the globe at I)
- H (far left of the globe at I)

- S (left of the globe at I)
- C (lower left of the globe at I)
- E (below the globe at I)
- F (lower right of the globe at I)

To avoid confusion, however, let only the equinoctial circle DGEF be drawn, whose common section with the plane of the great orb is the line DE, so that the half DFE of the equinoctial itself remains inclined below the plane of the great orb, and the other half, DGE, is raised above it. Now let the daily revolution of this equinoctial take place in the order of the points DGEF, and let the motion of the centre be from A towards I. [Margin: Variation at the solstices.] While the centre of the terrestrial globe is at A, the axis CAB, erected on the diameter DE of the equator, falls in the solstitial colure, and the common section of this colure and the great orb is AP. It follows that the line AP is perpendicular to the line DE, since the colure is erected on the great orb; and consequently the line DE touches the great orb at the point A. In this position, therefore, the motion of the centre along the arc AM, which by virtue of the annual motion amounts to about one degree a day, behaves just as if it were made along the tangent DAE. And since the point D, carried by the daily rotation through G to E, adds to the motion of the centre, as if moving along the line DE, as much as the whole diameter DE, and conversely subtracts just as much while it moves through the other, lower semicircle EFD, the result is that in this place the additions and subtractions are measured by the whole diameter DE.

[Margin: Variation at the equinoxes.] IX. Now let the centre of the terrestrial globe be moved to I, one of the equinoctial points, and let the equinoctial there be GEFD, with its common section with the great orb DE, and the axis CIB, with the same inclination. But the line touching the great orb at the point I is no longer DE: it is another line that cuts DE at right angles, namely the line HIL, along which the motion of the centre I, advancing through the circumference of the great orb, is to be understood as laid down. With this posited, the diameter DE no longer measures the additions and subtractions that the daily motion brings to the annual motion, since DE does not extend along the line of the annual motion HIL; on the contrary, it cuts it orthogonally. The end-points D, E therefore neither advance nor subtract anything. The additions and subtractions must instead be sought from the diameter that falls in the plane erected on the great orb and cuts that plane along the line HIL; this diameter will be the line GF. The additive motion, so to speak, will be the one made from the point G through the semicircle GEF, and the ablative motion the one made through the semicircle FDG. Further, since this diameter GF does not lie in the line HIL of the annual motion but cuts it at the point I, with the end-point G staying raised above the plane of the great orb and F depressed below it, it does not determine the additions and subtractions by its whole length. Their quantity is to be taken instead from the parts of the line HL intercepted between the perpendiculars raised upon it from the end-points G, F; let these be understood to be GS, FV. The measure of the additions is thus

the line SV, smaller than GF or DE, which was the measure of the additions at the solstices. And from this, finally, comes that third variety of the annual period which appears in the tide of the sea, when one compares the tides that occur at the solstices and at the equinoxes.

[Margin: Galileo's conclusion.] X. Galileo therefore concludes, on p. 342 of the Latin version, that the three fixed and perpetual causes of the tide of the sea are these: the inequality of the daily motion communicated to the annual motion; that inequality varied monthly by the annual motion, by which the whole lunar system is carried faster at new moons and slower at full moons; and finally that inequality varied annually, at the solstices and equinoxes, by the inclination of the axis of the daily motion to the plane of the great orb. The remaining causes are accidental and without rule, partly unobserved and partly unobservable: namely the blowing of the winds, and the depths of the seas and their various lengths, positions and inclinations. He wonders, moreover, that Kepler, to whom those three motions of the Earth were known, nevertheless referred the tide of the sea to the predominance of the Moon over water and to other occult properties. And on p. 344 of the Latin, but p. 456 of the Italian, he holds that this argument, drawn from the tide of the sea, is one of the chief arguments for the motion of the Earth, saying: *"From the conversations of these four days, then, we have notable testimonies in favour of the Copernican system, of which these three seem quite conclusive: the first taken from the stations, retrogradations, and approaches to and recessions from the Earth of the planets; the second from the revolution of the Sun upon itself and from what is observed in its spots; and the third from the flux and reflux of the sea."* [Margin: Gassendi's vote for Galileo.] Pierre Gassendi subscribed to this so far that, in Letter 2 of *De motu impresso a motore translato*, from p. 149 to 155, he reduced Galileo's reasons to an epitome with the example of a small boat full of water, and after the argument from the solar spots, which he likewise thought demonstrative of the Earth's motion, he said: *"Thus Galileo demonstrates; to him is owed, moreover, that from the motions he assigned to the Earth he has explained the tide of the sea in such a way that he seems at last to have found its genuine cause."* [Margin: Authors who attack Galileo.] But the opposite view was held by Chiaramonti, in his Italian defence of the *Antitycho*, part 4, chapters 24, 25, 26, 27 and 28, where he refutes Galileo; by Father Cabeo, on Book 2 of the *Meteorologica*, text 6, from question 7 to 10; and by Father Georgius Furnerius [Georges Fournier] of the Society of Jesus, in Book 9 of the *Hydrographia*, from chapter 14 to 20. These two Fathers refute Galileo's invention far more fully and solidly. But neither their arguments nor ours against Galileo can be properly understood unless a history of the tide of the sea is set out first, and a more accurate one than was given in Book 2, chapter 15, where we were instructing the beginner and there was no occasion to descend so deeply into this controversy.

Hiſtoria Motuum Maris ex diuerſis Scriptoribus Collecta.

A History of the Motions of the Sea, Collected from Various Writers.

XI. Besides those who have described the Indian voyages for us, this history is touched on above all by not a few authors named by us in Book 2, chapter 15, Scholium 1. To these must be added Bartholomaeus Crescentius [Bartolomeo Crescenzio], Book 3 of the *Nautica Mediterranea*, from chapter 2 to 4; Bartholomaeus Mastrius, disputation 4 of *De caelo et meteoris*, question 4, from number 148; Hieronymus Trimarchus, Book 2 of the *Meteora*, disputation 1, from sections 6 and 7; Federicus Delphinus [Federico Delfino], in his opuscule *De maris aestu*; Albert the Great, *De proprietatibus elementorum*, treatise 2, chapter 6; Franciscus Resta [Francesco Resta], Book 3 of *De meteoris aqueis*, in the treatise on the sea, from chapter 13 to 16; Georges Fournier, Book 9 of the *Hydrographia*; Lucas Aurigarius [Lucas Janszoon Waghenaer] of Holland, in the *Thesaurus marinus*; and Claudius Duretus [Claude Duret]. Most of these grant the sea three ordinary motions: one in longitude, another in latitude, a third in altitude. The extraordinary motions produced by the winds we do not consider here.

Primus Maris motus à Septentrione verſus Auſtrum.

The First Motion of the Sea, from North towards South.

XII. The motion of the sea from north towards south was recognized long ago, together with Aristotle (*Meteorologica* 2, chapter 1, text 6), by Alexander of Aphrodisias, St Thomas [Aquinas], Vicomercatus [Francesco Vimercati], Nyphus and most of the interpreters at that place, and also by Albert the Great (treatise 3 of the *Meteora*, chapter 6). For the Maeotic marsh [Sea of Azov] flows into the Euxine Sea [Black Sea] through the Cimmerian Bosporus, the Pontus into the Propontis, the Propontis into the Aegean through the Thracian Bosporus, and the Aegean into the Mediterranean. [Margin: 1. Cause of this motion.] Aristotle gives a twofold cause of this motion. One is the multitude and size of the rivers bursting into the Maeotis and the Euxine, such as the Tanais, the Danube and the Borysthenes, compared with the narrowness and slight depth of the Maeotis and the Euxine, which cannot contain so great a mass of water within their beds but are forced to discharge it at once into wider and deeper seas. For the Maeotic marsh, if we believe the geographers, has scarcely six or seven ells of depth; and Aristotle adds that the Euxine is deeper than the Maeotis, the Aegean sea deeper than the Euxine, and the Sicilian deeper than the Aegean, while the Sardinian and Tyrrhenian are the deepest of the whole Mediterranean. From the same abundance of waters bursting into the Euxine, Strabo (Book 1 of the *Geography*) reports that the Pontus forced itself a way into the Propontis, which it did not have from the beginning, and by continual battering at the Strait of Hercules opened itself an entrance into the Mediterranean. The other cause the Philosopher refers to the height of the land, which is more elevated around the Arctic pole; from this, he says, it follows that

a very great force of waters runs down from the Arctic pole towards the south. As for the Caspian Sea, which does not visibly run down into the Euxine or any other sea, although great rivers discharge into it and it is not very large, the reason is that it does not communicate with the Euxine through any strait or channel. It is therefore forced, as not a few geographers hold, to send its waters through underground passages to the Euxine, and from there to the Mediterranean. [Margin: 1. Confirmation of this motion.] This motion from north to south is further confirmed, first, by the easier and faster navigation from the north to the south: for, as Fromondus [Libert Froidmont] asserts, one sails more quickly and readily in the Atlantic from Denmark and Hollan- d into Spain than from Spain into Holland or Denmark. Our Fournier, too, in Book 9, chapter 22 of the *Hydrographia*, affirms that ships setting out from the Azores towards the equator complete their voyage much more quickly than those that try to return to the Azores from Brazil or from Havana, which are southern places. For these, so as to be less hindered in their oblique course by the waters running towards the south, must coast the shores of America between Florida and the island of Bahama and Virginia, and cover a route ten times longer than the one they had made from the Azores to Brazil. This greater speed is seen in its own way also in navigation from the Euxine to the Aegean, and from the Aegean into the rest of the Mediterranean, even when the winds are still; so that Franciscus Vallesius [Francisco Vallés], in chapter 50 of the *Sacra philosophia*, was wrong to refer it to the north winds. [Margin: Vallés's error about the cause of this motion.] Fournier adds, in Book 9, chapter 31, that in the year 1630 a ship which had sailed from Dieppe and had reached, in February, the latitude of 2° near the equinoctial, after running for 24 hours towards the Libycus [the south-west], although it reckoned it had moved further south, was nevertheless still at 2°, where it had been before, because of the currents towards the Graecus [the north-east]. [Margin: 2. Confirmation.] Secondly, it is confirmed most evidently by the masses of ice which, on the testimony of Fournier at the place cited above, roll down from the north towards the south, as those have found who have sailed across the Atlantic to North America, or from there to Norway. For those going towards Bergen in Norway along Vaëgas or Nieulandia must always steer their course beyond Hunem, unless they want to perish from the ice rushing down from the north. And in the year 1635 a French fleet sailing towards Canada came upon some huge masses of ice like bells, one of them so vast that they could scarcely get clear of it over a space of 40 leagues. Nor does anyone ever sail to Canada without coming upon such floating islands of ice from time to time, rolling down from the north. Since, therefore, such masses of ice always rush from the northern region towards the equator, it is manifest that the sea runs perpetually from there towards the south. [Margin: 3. Confirmation.] Thirdly, it is confirmed by those who have made the voyage to the Moluccas or the Philippine islands: they have experienced an incredible speed of the sea, now from north towards south for 12 hours, now from south towards north for another 12, as Fournier relates at the same place and in chapter

31. Fromondus also notes, from the *Enchiridion* of Bartholinus the Dane, Book 4, chapter 8, that the Baltic Sea not only flows out from the north but also flows back into the north; Bartholinus attributes this to rivers flowing on the one side from the north and on the other into the north, and driving the Baltic with different motions. Many geographers too, especially Mercator Plantinus, Maginus and Bercius, relate that under the Arctic pole there is an immense whirlpool into which the sea is carried very rapidly through four Euripi; how this could have been known, however, Pifferus wonders, on chapter 3 of the *Sphere*. [Margin: The flux of the sea is to be distinguished from this motion.] This motion, however, does not properly include the well-known flux and reflux that runs from the Gothic sea onto the shores of France, and back again from the shores of France towards the Gothic sea; for this is the tide of the sea, reciprocating to and fro every day. It is also seen daily at Spitsbergen, at an elevation of 24°, from north to south and back again from south to north. Further, the cause of the sea's course from north to south in the Gothic sea and the Atlantic, or, at the Moluccas and Philippines, now to the north and now to the south, cannot be referred either to rivers or to the height of mountains from which waters slide down into the sea. In that tract there are no such rivers flowing towards the south, nor is that part of the land higher than the Mountains of the Moon or than America; on the contrary, the Holland and Zeeland sea appears higher than many islands, and the Rhine and many other rivers flow from the Alps towards the north. [Margin: Cause of this motion.] It seems rather to our Fournier, in Book 9, chapter 22 of the *Hydrographia*, that the cause should be referred to the Sun, which continually diminishes the waters within the tropics by its heat and its attraction of vapours. The result is that, as the waters at the surface fail, the waters run by their own weight, on one side from the north and on the other from the south, towards the equator, to level out their surface. This is especially so if it is true, as the same author affirms at that place, that when the Sun is in Capricorn the waters within the tropics cross the equator and rush towards the Tropic of Capricorn, and conversely, with the Sun placed in Cancer, the waters rush from the Tropic of Capricorn towards the Tropic of Cancer. Franciscus Resta is of the same opinion, in Book 3 of the *Meteorologia*, treatise 1, *De mari*, chapter 16, affirming that the Sun's action makes the waters flow now towards one tropic, now towards the other. Nor does he think contemptible what Aristotle had said on another occasion (section 23, problem 20): namely, that salt waters are more stable because of a certain viscosity like oil, whereas fresh waters flow more easily; and the waters near the poles, it is agreed, are less salt, but they could not flow towards the salt waters enclosed within the tropics unless these, by their own depression, gave way to them, leaving a place to be levelled up by the polar waters. Mastrius too, in disputation 4 of *De caelo ac meteoris*, question 4, number 148, thinks that the waters of the sea flow from the south and from the north towards the equator, because, through the excess of heat, much of the water contained in the torrid zone is turned into air, while, on the contrary, near the poles a great

quantity of water rains down into the sea from snows and mists. Bartolomeo Crescenzio, however, in Book 3 of the *Nautica Mediterranea*, chapter 2, ascribes this motion to the action of the Sun in such a way that, while he admits that the water flows from the poles to the torrid zone, he does not admit that the waters within the torrid zone become lower because of vapours consumed. Rather, because waters are drawn up by the Sun and turned first into vapours and then into air, the remaining waters of the seas rush by force of continuity to fill the place of those that were under the torrid zone. Finally, our Cabeo too, on Book 2 of the *Meteorologica*, text 6, question 2, approves the same cause: namely, that there is great consumption of water in the hotter places about the equator and a great abundance of rains and snows about the poles, and that therefore, to preserve the equilibrium of the sea, the waters flow together from both poles towards the equator. But he adds that, just as in very cold weather the colder the surrounding air, the more copious the water that distils through an alembic, so in the polar parts, where the surface of the earth is colder, the vapours rising from the bowels of the earth into the hollows of the polar mountains condense more easily into water and distil into more copious water; and therefore larger rivers or more copious springs of water burst forth from there. [Margin: Are rivers more numerous and larger from the poles, or under the equator?] But Fournier and Crescenzio teach the opposite: namely, that more and larger rivers spring up under the equator, such as the Nile, the Manicongus, the Cuana, the Zaire, the Coanza, the Niger and others in Africa; and in America the Maragnon, the Aureliana, the Silver River [Río de la Plata], the river of St Dominic, the river of St John, and many other huge rivers, with which the rivers that roll down from the northern mountains into the sea can in no way be compared. They think the cause is that the ground under the torrid zone is filled with more and larger pores, because of the exhalations and vapours perpetually drawn out by the Sun, and that more waters rush from underground reservoirs of water to fill these pores. But this is not the place to debate the matter.

Secundus Maris Motus ab Oriente in Occidentem.

The Second Motion of the Sea, from East to West.

XIII. This motion in the sea is no less evident than the preceding one. For it is established that one sails much more quickly, even when the wind drops or drives the sails with an even force, from Palestine to Spain than from Spain to Palestine, and from England to Ireland than from Ireland to England. One also sails more quickly from Spain to the West Indies, so that one reaches Mexico or the islands on this side of it in about one month, or at most two months; whereas one cannot sail back from there in turn except in four, five or even six months, and to escape the waters running towards the west one must catch stronger winds blowing towards the east, and therefore an oblique course towards the north must be set. And the Portuguese too, when they have rounded the

Cape of Good Hope and make for the East Indies, reach Goa or Malacca much more slowly, even with favourable winds, than when sailing back from there westward. Many ship-masters testify to this, and our Father Antonius Magallianus, a Portuguese, confirmed it in the year 1633, when he was appointed Procurator of the Province of Goa to the General Congregation. For the sea runs so violently towards the Cape of Good Hope, especially between Ethiopia and the island of St Lawrence [Madagascar], that the danger of shipwreck there is far greater than in the Sicilian Strait or in the sandy places of Flanders. And our Fathers returning to Europe from the kingdom of China or from the East Indies have affirmed that at times that eastward course cannot be resisted even with favourable winds, especially under the equator. Hence, from this motion or westward course of the waters, the Vallis Iosaphat at the island of St Lawrence is notorious for shipwrecks, as are, in the Gulf of Mexico, the island called Os Draconis [the Dragon's Mouth] and the waters near Guadeloupe and the harbour of Cartagena and of Santa Marta. This goes so far that ships cannot make headway against it even with the wind helping, and can scarcely cover one mile in one day, or 40 miles in 75 days, the sounding-lead being very often swept away towards the west. So Trimarchus testifies, in Book 2 of the *Meteora*, disputation 1, section 6, from the account of ship-masters who were Spanish knights, when he was in Madrid; and so does Willebrord Snellius [Villebrordus Snellius] in his *Tiphys Batavus*, Book 2, proposition 8, adding that between Brazil and Angola this motion of the sea is carried towards the north-west from April 20 to July 20, and towards the south-west from October 20 to January 20. Adrianus Metius also relates, and Fournier after him in Book 9 of the *Hydrographia*, chapter 23, that a certain ship's captain sailing from Brazil to the island of St Helena, which lies east of Brazil, after sailing for several days with favourable winds on the same sixth parallel of the equator, and reckoning himself very close to that island, found that he was still near the coast of Brazil. Moreover, those who sail from France to Brazil head straight for Guinea, so that, having caught the sea's westward course there, they can sail more successfully; for this course is much more perceptible within the torrid zone than far from it. But in the Pacific Sea this motion is most evident. Those sailing from Acapulco head south to the 10th degree of south latitude and, keeping their course on that parallel, reach the Islands of Thieves [Ladrones] in 85 days, having covered 3000 leagues without any use of their sails; and from there, leaving those islands and climbing to the 13th degree of latitude, they reach with the same good fortune Manila, which is in Luzon, one of the Philippines. From the nautical ephemeris of Thomas Roës [Thomas Roe], an English nobleman who was sent by his king as ambassador to the princes of India in the year 1614, and especially from the diary of Gualterus Peyton [Walter Peyton], it is established that, when the English reached the island of Mohelina [Mohéli] on June 22, the inhabitants of that region showed them a stream that flowed eastward for 15 days and westward for another 15 days; and the English confirmed this by their own trial. That island lies between the Cape of Good Hope and the island of St

Lawrence, or Madagascar. No less marvellous is what the lord de Hayes reports, as cited by Fournier, in his itinerary from France to Constantinople: namely, that in the Danube the water runs down faster in the morning and evening than around midday, and that near Buda and Belgrade this is manifest from the wheels of the mills. [Margin: Cause of this motion.] From this Fournier gathers that the cause of the sea's motion towards the west is the Sun, which perpetually draws up vapours and exhalations, especially within the torrid zone, and does so in its own course towards the west; the waters must consequently run in from the east, so as to preserve the equilibrium of the sea by a continuous course. Granted, Kepler, in the introduction to the *Commentaries on Mars*, attributes this to the magnetic power of the Moon drawing out the waters while it moves westward by the daily motion; and he says this power is so great that, if the Earth did not in turn attract the waters of the sea to itself, all the waters of the sea would be raised up and would flow into the body of the Moon. Fournier thinks this was said gratuitously [?]. Others, with Vicomercatus, attribute it to the daily motion of the heavens, which sweeps air and sea towards the west; Franciscus Resta, in the chapter 16 already named, does not oppose this, nor does Mastrius, cited above. Tyennensis, however, in Book 2 of *De elementis*, attributes it partly to the heat of the Sun raising vapours, partly to an occult and as it were magnetic power, by which it carries the waters of the sea along with it, just as it carries the clouds, to wherever it is itself borne by the daily motion; and Trimarchus, at the place already cited, does not disagree. But Scaliger, in exercise 52, Contarenus [Gasparo Contarini], the Conimbricenses and Aversa [Raffaele Aversa] ascribe this to occult qualities of the heavens, with which Mastrius too tacitly agrees. Further, Cabeo, on Book 2 of the *Meteorologica*, text 6, question 3, on the supposition that there is such a motion, places its cause partly in the perpetual or frequent easterly winds, especially within the torrid zone, and partly in the Sun drying up the waters, etc. [Margin: By whom this motion has been called into doubt.] I said "on the supposition", because he himself has doubts about this motion and rather thinks it false; and Fromondus, in Book 5, chapter 1, article 6, denies it, and says that the whole of that ease of navigation westward comes from the easterly winds blowing continually, especially within the tropics. These winds were recognized and attested by Iosephus Acosta [José de Acosta], in Book 3, chapters 4 and 8, and by the nautical diary of the Dutch for the year 1595. From these sources we learn that a ship set out from Lima in the year 1580 and in February and March made 2700 leagues westward on the same parallel, with the winds in the Pacific Sea perpetually seconding it, and that Alvarus Mandana [Álvaro de Mendaña], setting out from Peru, reached the Solomon Islands in a short time by the same favour. Hence it is (says Fromondus, and Cabeo nearly subscribes) that the Spaniards head straight for the Canaries, so as to fill their sails there with the winds blowing from the east and reach the island of San Domingo by a very short route; but on the return they are forced to leave the torrid zone as far as the 37th or even the 40th degree of elevation, so as to enjoy the west winds,

with which they return to Spain as best they can by a long and oblique course, granted that from the 40th degree the winds are already inconstant, on the testimony of Acosta, Book 3, chapter 7.

[Margin: Confirmation of this motion.] But navigation eastward is very difficult even with the winds equal, and indeed with favourable winds blowing towards the east, as we related above. Add to this, from Scaliger, exercise 52, who learned from ship-masters that from the island of Maidegascar, under the eighth southern parallel, one reaches Moabar in India in 20 days, whereas one does not return from there, even with an equal force of winds, except in three months. Since, I say, these things are so, and very many ship-masters, or those most expert in navigation since Columbus, recognize, besides the winds, also and above all this course of the waters, it must not be said that they have been deceived in a matter of such moment.

Motus Maris ex prædictis duobus ortus.

A Motion of the Sea Arising from the Two Already Mentioned.

XIV. Therefore, from the motion of the sea from the pole, especially the Arctic, towards the equator, and from its motion towards the west, there follows a certain other motion in a circle, by which it seems to return eastward through certain straits and along the shores; for while it strikes against the opposite shores it is forced to flow back. Thus Contarenus, in Book 2 of *De elementis*, Scaliger, in exercise 52, and others have noted that the Mediterranean, striking on the shores of Africa and Spain, since it cannot flow out wholly through the narrows of the Strait of Gibraltar as quickly as would be needed, returns by a winding course towards the shores of Africa as far as the coasts of Asia Minor; by this bend it flows along the shore of Dalmatia towards Venice, and from Venice is carried along the shores of Italy as far as the Sicilian Strait. In the Sicilian Strait, moreover, it is very well known that one part of the waters is carried upward and the other downward at the same time, very rapidly. A reciprocal motion up and down, however, is observed in most Euripi or straits, and in the Sicilian Strait indeed the water moves in a circle twice each day towards opposite parts; but this belongs to the tide, of which more below. Scaliger adds that in the Scythian Ocean the waters run eastward, and in the Atlantic towards the Land of Labrador [Terra Laboratoris]. Remarkable, too, is what Fournier says in Book 9, chapter 6: that the Mediterranean flows out through the Strait of Gibraltar for six hours, but the Ocean comes in as victor for 18 hours. But if anyone wants other notable examples of sea waters running towards different regions of the world, let him consult Fournier, Book 9 of the *Hydrographia*, chapter 31.

De Tertio motu Maris, qui Aeſtus dicitur, ſeu Fluxu ac Refluxu in communi, vbi quædam Admiranda.

On the Third Motion of the Sea, which is called the Tide, or Flux and Reflux, in general; with some remarkable things.

XV. Although the motion of the sea, especially the one we pointed out in number 14, can be called flux and reflux, nevertheless, when this word is used more strictly by ship-masters and the more skilled hydrographers, the name "flux and reflux of the sea" means the motion that is not merely boreal but takes place with a certain alteration, on account of which the sea swells and subsides as if by boiling up and cooling down. For this reason it is also called the tide [aestus] of the sea, and the bays and harbours in which the approach of the swelling sea and the retreat of the subsiding sea are manifest are called estuaries [aestuaria]. Some, however, think that *aestus*, like *aestas* (summer), is named from burnt air, or from the abundance of heat. Now the Phoenicians of old, and afterwards the other ship-masters, observed three variations of this tide: one that occurs twice daily, a second that occurs twice every month, at the new moon and the full moon, and a third that occurs every year, in March and September. We shall discuss these in order. Meanwhile some general points must be set out first.

[Margin: The boiling-up of the sea from the bottom.] First, then, the tide occurs not only near the shores, as Bodinus [Jean Bodin] thought — moved perhaps by Pliny the Elder's statement in Book 2, chapter 97: "But these motions are detected more near the shores than in the open sea" — but also around the middle and in the depths; from there the sea, boiling up, drives sea beasts upward to its surface, as Albert the Great (*De proprietatibus elementorum*, treatise 2, chapter 6), Vallesius (*De sacra philosophia*, chapter 52) and Franciscus Resta (treatise 1, *De mari*, chapter 13) affirm. Nor does Pliny deny this absolutely; he speaks comparatively.

[Margin: Wells and springs that ebb and flow with the sea.] Not only the sea, moreover, but certain springs ebb and flow like seas. Such was the spring at Gades [Cádiz], enclosed in the manner of a well, near the temple of Hercules, which swelled and subsided with the Ocean, but at other times did so at different times, as Pliny reports (Book 2, chapter 97). Ortellius [Abraham Ortelius] also relates that in Connacht in Ireland a spring is found on the summit of a mountain which, following the rule of the nearby sea, swells and subsides twice daily. Pliny adds, in the place cited, that on the bank of the Baetis [Guadalquivir] there was a town whose wells sank as the tide rose and rose as it fell, and were motionless at the times between, and that a single well at Seville had the same nature. Finally Albert the Great (*De proprietatibus elementorum*, treatise 2, chapter 5) reports that in certain cities of Arabia beside the sea, as in the city of Azim and in Albastrach, the fresh waters used to ebb and flow with the sea.

[Margin: Whether men die only when the tide is ebbing.] But remarkable, if true, is what Pliny relates from Aristotle in Book 2, chapter 98, saying: "To these things Aristotle adds that no animal dies except when the tide is ebbing. This has been much observed in the Gallic Ocean, and established only in the case of man." Bodin thinks this false. But Levinus Lemnius [Lieven Lemmens] (*De naturae miraculis*, Book 4, chapter 1), although he does not dare to reject Pliny, nevertheless adds that he has found that some people died at the Ocean's approach, and that in the coastal tract of the Low Countries the fat are in danger when the tide is coming in, the thin when it is going out. Fournier, moreover, affirms (Book 9, chapter 1) that mixed bodies never abound more in humours than when the sea surges and swells, and are never more dried out than when the sea subsides and falls; therefore, since the Moon rules over the humours, this is a sign that the tide of the sea has a great connection with the Moon.

Further, it is agreed that if any shore feels the tide, the tide is greatest where the mouths of great rivers are, and that coasts which stretch out into the sea experience it sooner than those that have receded inland. By contrast, rivers themselves, lakes and pools do not undergo a tide of themselves, just as seas that resemble pools, such as the Caspian, do not, unless it is communicated to them from the sea through their mouths. [Margin: Pliny's general statement on the tide.] Lastly, we should recall Pliny's statement in Book 2, chapter 97: "But all tides cover and flood greater spaces in the Ocean than in the rest of the sea," and so on; "neither lakes nor rivers are moved in the same way."

Hiſtoria Aeſtus Maris accurata.

An Accurate History of the Tide of the Sea.

XVI. The tides of Europe stand as follows. In the Mediterranean the greater part of the shores lacks a perceptible tide. Certainly, just as there is none in the Caspian (although that belongs rather to Asia), so there is none in the Euxine Sea either. [Margin: The tide of the Mediterranean.] Of the Euxine there is Pliny's statement in Book 2, chapter 97: "The Pontus always flows out into the Propontis; the sea never flows back inward into the Pontus." In that place he adds: "On the third day one reaches Utica from Italy, when the tide is running high." For the African shores at Utica, Tunis, Hippo and certain other coasts feel some tide, but the island of Malta a greater one. In the Adriatic gulf, around Ravenna and Aquileia, the tide was once so great, on the testimony of Procopius (Book 1 of the *Gothic War*), that it covered many thousands of paces, and sailors waited for the tide in the morning so as to bring in their goods more quickly; in the evening it usually subsided and flowed back. Now it covers about two miles of the Venetian shore and, boiling up from the bottom, as Contarenus noted, rises 3, 5 or 6 feet; for this reason the city has from time to time to protect itself with barriers and embankments, so as not to be overwhelmed by the tide. But around Corsica and Sardinia, in the estuaries of Rome, and around Elba, Malta and Crete it rises scarcely half a foot. On the coast of Provence, at Marseille, it

rises a whole foot. In the rest of the Tyrrhenian Sea no tide is felt, either on the western coasts of Italy or on the eastern shores of France and Spain; nor is any ascribed to the shores of Greece.

[Margin: Euripi.] A greater tide, or flux and reflux of the waves, is seen in Euripi and straits. Pliny (Book 2, chapter 97) testifies that the Euripus of Tauromenium [Taormina] flows and ebbs more often in a day; and Patritius, in his *Pancosmia*, says that in Liburnia, at the town of Assuerum, there is a very narrow Euripus of 10 paces, where the sea surges more than twenty times a day. [Margin: The Chalcidic Euripus.] But nothing is more memorable than the Chalcidic Euripus between Boeotia and the island of Euboea. There the sea flows and ebbs, not at uncertain times or turns, as Livy held (Decade 3, Book 8), but seven times by day and seven times by night, as Cicero (*De natura deorum*, Book 1), Strabo (Book 1), Pomponius Mela (Book 2, chapter 4) and Pliny (Book 2, chapter 97) affirm. St Basil describes this Euripus elegantly in Homily 6 of the *Hexaemeron*; it sometimes rushes with such force that it seems to be pouring down from the highest mountains. But Gillius [Pierre Gilles], who investigated those places, affirms, as Franciscus Resta relates (treatise 1, *De mari*), on the report of the inhabitants, that this Euripus now ebbs and flows only four times, not seven, and that sometimes it does not keep even these turns. Scaliger (exercise 52) refers the cause to waters shut in by rocks lying beneath, which are poured back in alternation.

[Margin: Did Aristotle throw himself into the Euripus?] Some, moreover, report — from Saints Justin and Nazianzen [Gregory of Nazianzus], and from Procopius, Caelius, Vimercatus (commentary 4), Maginus in his description of Euboea, and others — that Aristotle, despairing of grasping the cause of this Euripus, threw himself headlong into it, saying: "Since I do not grasp you, Euripus, you grasp Aristotle." But Laertius says nothing of the kind about Aristotle in his life of him.

[Margin: The tide of the Sicilian Strait.] Before we leave the Mediterranean, however, we must consider the tide of the Sicilian, or Messina, Strait. There, although the water at Scylla and Charybdis rises only a little more than one palm, it nevertheless flows and ebbs up and down very rapidly, so much so that Aristotle counted it among the wonders of the world. Bartholomaeus Crescentius relates (*Nautica Mediterranea*, Book 3) that on 1594 August 3, as he was entering this strait near Scylla and Charybdis, the waters drove on his stern; but when he reached the middle of the channel, another current of water rushed in from Reggio, which, meeting the first, produced a surge of water as if boiling, with swelling and eddies, just as a cauldron full of water usually boils when a fire is set beneath it. This swelling of the tide lasted for a quarter of an hour, until the water rolling out from Reggio overcame the other, which had come from Scyllaeum, or from the tower of the Pharos of Messina. He adds further that certain knights of Messina assured him that this meeting of the waters, and the tide, occurs every six hours, which they attributed to waters shut up in caves and poured back out of them; but Crescentius attributes it

to the Moon, whose 17th day it then was, and it was the 15th Italian hour. In the year 1595, too, when the same Crescentius wanted to leave the mouth of the strait with galleys, he experienced no small fear from that struggle of the waters: the water running from Reggio towards the Pharos drove the ships so that one passed the next very swiftly, but when they came nearer the shore, the other water, meeting them from the opposite side, drove them back — certainly because of the circular motion of the waters. I have often heard similar things from Fathers of our Society who sailed to Sicily to preach or to govern.

[Margin: Is there any tide in the Baltic?] Outside the Mediterranean, the European shores of the Scythian and Baltic Seas have no perceptible tide, on the testimony of Bartholinus the Dane. Cardano, however (*De rerum varietate*, Book 1, chapter 4), reports that beyond Sweden there is a promontory called Nasus, so named from its likeness to a nose, beneath which is a whirlpool that boils every six hours with so rapid a tide that, as it flows back, it swallows ships; Olaus Magnus affirms this too. [Margin: The tide of Norway, Denmark, England, Holland, Flanders.] The remaining shores of Norway, especially the western ones, and also the English, Irish, Danish, Dutch and Flemish shores, undergo huge tides, so much so that in Britain the shores are laid bare for 9 miles, and the Thames feels the tide up to London, as far as 50 or 60 miles, the water rising about 20 cubits. Indeed Pliny says, in Book 2, chapter 97: "Pytheas of Massilia is the authority that the tides swell 80 cubits above Britain." Georgius Bruin [Georg Braun] nearly agrees with him in his *Bristolia*, granting that this swelling rises to 66 feet; and so does Fournier (Book 9, chapter 20), who says he observed himself that in the gulf of the Manche [the English Channel] the tide sometimes rises 70 feet. Also worthy of note is what Bede relates in his life of St Cuthbert, chapter 17, as given in Surius, volume 2: namely, that in the region of Lindisfarne, as the tide of the British Ocean comes in twice daily, an island is formed, and when the shores are twice laid bare again, the place that had been an island, joined to the mainland, appears. [Margin: A remarkable spring.] I said above, from Ortellius, that in Connacht in Ireland there is a mountain on whose summit a spring swells twice daily and subsides twice, following the tide of the nearby sea. They report, too, that terrible tides occur daily on the shores of Iceland. [Margin: The tide of the Low Countries and of France.] In the Low Countries and at Calais the tide rises at new and full moon to 15 or even 18 and 20 feet. The French shores facing the west also experience a very great tide, but two things in them are most worthy of note. The first is that the mouth of the Garonne fills in seven hours and diminishes in five, as Scaliger noted in exercise 52 — although the Lord of Candal, as reported by Fournier (Book 9 of the *Hydrographia*, chapter 1), observed that it fills in 6⅖ equinoctial hours, but empties and subsides in 6 hours. The second is what Franciscus Resta relates (treatise 2, *De mari*, chapter 15), and Fournier (Book 9, chapter 21), and what I seem to have heard myself from Fathers of our Society: namely, that at Mont-Saint-Michel and Saint-Malo the tide rises to 70 feet, and sometimes 90, and recedes over three leagues; and that on the coast of Normandy an island is

formed every day, to which, when the sea has flowed back, one returns across the sand, since it has now become not an island but mainland.

[Margin: The tide of Portugal and Spain.] It is well known that all the rivers of Portugal undergo a tide at their mouths, but above all the Tagus, which therefore flows back towards its source for up to 4 miles. The Baetis, however, runs back as far as 50 miles. On this occasion I repeat Pliny's statement in Book 2, chapter 97: [Margin: Remarkable wells.] "On the bank of the Baetis there is a town whose wells diminish as the tide rises and grow as it recedes, and are motionless at the times between. The same nature belongs to a single well in the town of Hispalis [Seville], the others being ordinary." A little earlier in the same chapter he had said: "At Gades the spring nearest the temple of Hercules, enclosed in the manner of a well, sometimes rises and falls together with the Ocean, but at other times does both at contrary times." But now that we have passed Gades [?], let us move on to the African coasts.

[Margin: The tides of Africa.] The shores of Africa washed by the Mediterranean experience their own tide, as we have said. Those washed by the Atlantic Ocean, however, experience none that is perceptible until one reaches the Tropic of Cancer; for within the Tropics the sea ebbs and flows, especially beyond the equator, near the river Niger. Indeed, at 11 degrees a flux like the Portuguese one begins and extends as far as the island of São Tomé, which lies under the equator; there, however, it is not greater than the Venetian tide, as Trimarchus heard from ship-masters and reported in Book 2 of the *Meteora*, disputation 1, section 7, and Ramusius [Giovanni Battista Ramusio] says the same (volume 1 of the *Navigationi*, page 126). At Guinea, moreover, the sea boils with the tide with such force that ships can scarcely hold fast with three anchors so as not to be dashed against the shore. The mouths of the river Niger, as Fromondus relates in his *Meteora*, swell for 4 hours and subside for 8; or, to use Scaliger's words (exercise 52): "At Ginega in Ethiopia it comes in for four hours and goes out for eight." Those mouths are 20 miles wide; and Ramusius, too, in volume 1 of the *Navigationi*, page 118, confirms a flux of 4 hours and a reflux of 8.

[Margin: The tide of the Red Sea.] Finally, in the Erythraean or Red Sea (whatever Galileo may deny, since it harms the cause he put forward) the daily tide is so great that, as Scaliger says (exercise 52), the haters of Holy Scripture have dared rashly to lie that Moses crossed over on dry ground, having seized the chance of the ebbing sea. This, however, could not have happened, the same author says, because as far as Suez, a town that lies in the innermost recess, the sea so covers that shore that it never uncovers the sea-bed where the Hebrews crossed. [Margin: Moses made no use of the opportunity of this tide.] Moreover, as Trimarchus points out (Book 2 of the *Meteora*, disputation 1, section 7), between the place the sea reaches when it recedes and the place where Moses crossed dry-shod with the Hebrew people there lie thirty miles and more. Nor could this have been hidden from the Egyptians, who, had they feared a fresh tide coming so suddenly, would not have committed themselves

to it so fearlessly. Now, since we too have crossed with Moses through the Red Sea into Asia, let us sample the more notable features of the Asian tide.

[Margin: The tides of Asia.] The tides of Asia are unknown towards the north, but manifest towards the south and the east. How anxious they made Alexander the Great and his admirals at the mouths of the Indus is related at length by Quintus Curtius in Book 9, [Margin: The tide of the East Indies.] and by Nearchus in Ramusius, volume 1 of the *Navigationi*: for the sea receded with such force that it left three ships stranded on the dry shore, and the tide, returning later, restored them to their course. But pleasant is what Pliny reports in Book 13, chapter 25: "Alexander's soldiers who sailed to the Indians," he says, "reported that the foliage of the sea trees was green in the water, and when taken out it at once dried in the Sun into salt," and so on; and a little after: "In the same tract the tide covers forests of islands, although they are taller than plane trees and the tallest poplars; their leaves are those of the laurel, their flower that of the violet in scent and in colour; their berries, like olives, are themselves of pleasant scent, growing in autumn, the leaves never falling [?]. The smaller of these the sea covers entirely; the tops of the largest stand out, and ships are tied to them, and, when the tide has receded, to their roots."

Of a gulf 60 miles distant from the river Indus, Linscosanus [Jan Huygen van Linschoten] relates (*East India*, part 2, chapter 9) that the ebb and flow of the sea there is the most violent of all [?] that has been seen anywhere, so much so that it was a terror to Alexander the Great, let alone an astonishment. Further, at Calicut, as the Fathers of our Society relate, especially Nicolaus Godignus [Nicolau Godinho] (*De rebus Abyssinorum*, Book 1, chapter 11), and as Trimarchus attests in the section already cited, the risings of the sea are greater at the full moons, whereas at the mouths of the Indus they occur at the new moons. Ramusius adds (volume 1 of the *Navigationi*, page 131) that every six hours the sea there ebbs so far that it leaves ships on dry ground. At the coast of the Ganges the tide is worthy of the mouth of so great a river, [Margin: The remarkable tide of Cambay.] but it is greater on the shores of Cambay. There, as both the Fathers of our Society and most writers on this subject report, the sea pours onto the shore in 2 hours and covers about 30 leagues — elsewhere 15 miles, as Trimarchus has it, and Ramusius (volume 1, page 330) — and with such speed that unless people, warned by cymbals, flee at a run, they are overwhelmed by the tide; and when the sea likewise ebbs in two hours, very many ships are left stuck on dry ground. The inhabitants of that place, however, have in some spots very large pits in which the ships settle until the tide calms, so as not to be swept dangerously out to sea by it; but for this skilled ship-masters are needed, so that the vessels can stay in those pits when the sea ebbs, as Odoardus Barbosa [Duarte Barbosa] relates in Ramusius, in the place cited.

Of the coast of Persia Scaliger (exercise 52) says: "Near the shore of Diuoban-del Rumi an island is sometimes formed; at times it is part of the mainland. That name means Holy Port of the Turks. But neither that return nor that

occupation, with the shores laid free, occurs in all places." In the kingdom of China the river Yanna (a name meaning "son of the sea") undergoes a tide every day as far as Nanchinum [Nanjing], or as far as the lake nearest Nanquinum, although at Pachinum [Beijing] it is felt only at new and full moon. So says our Nicolaus Trigautius [Nicolas Trigault], *De Christiana expeditione apud Sinas*, Book 3, chapter 9. At the Moluccas and the Philippines the sea does not rise more than 3 feet, says Fournier (Book 9, chapter 21).

[Margin: The tides of America.] Finally, the tides of America stand as follows. The northern shores of North America have no tide, but the southern ones have a manifest tide, and at Veragua a strong one. In some places, on the testimony of Ouetanus (chapter 9) and Petrus Hispalensis (part 5, chapter 1), it runs out as far as eight thousand paces; and on the Mexican coast (except, however, the Gulf, which is immune) it is so swift in ebbing that it seems to vanish at once from sight. At the islands of Martinique and the Caribs the sea rises scarcely one foot. Near Cuba and Hispaniola there is no perceptible tide except at the full and new moons, especially in March and September, for then the waters are higher and more agitated. The river of the Amazons, or Marañón, is said to feel the tide and the saltness of the sea for as much as a hundred leagues. On the shores of Peru and of Paria the tide is no small one, and near Panama it covers two leagues. But nowhere is the tide more wonderful than in the Magellanic Strait, where two tides, one from the Atlantic and the other from the Pacific Sea, meeting each other, raise the waters into a heap with a manifest swelling, as Fathers Iosephus Acosta (Book 3 of the history of the New World, chapters 13 and 14) and Alphonsus Ovalius [Alonso de Ovalle] of our Society, in his *History of Chile*, testify. Bartholomaeus Crescentius drew a diagram of this tide (*Nautica Mediterranea*, Book 3, chapter 2), stating that this strait is 100 leagues long and very winding. Sailing on from here to the Archipelago of St Lazarus, one finds a tide which every 12 hours is carried towards the south with such force that it has broken a cable 27 inches thick, as Fournier relates (Book 9 of the *Hydrographia*, chapter 16).

De Diurna Periodo & Tempore Aeſtus.

On the Daily Period and Time of the Tide.

XVII. [Margin: 1. Observation concerning the daily time of the tide.] First. From the common opinion and observation of experts, from the ancient Phoenicians right up to our own times, it is established, generally spea- king, that in the places where there is a tide, as soon as the Moon has risen about one sign above the horizon, the increase of the sea begins. It flows in so that it rises above the banks and, unless their height prevents it, spreads beyond its usual limits towards the land, while at the mouths of rivers it drives their waters back towards their source; and this swelling grows continuously until the Moon has reached the meridian of that place — or rather, as Simon Steuinus [Simon Stevin] thinks, the nonagesimal degree of the ecliptic reckoned from

the horizon. For then the greatest increase occurs, which is usually called *aqua viva*, "living water", or *vivum aquarum*, "the living of the waters". When the Moon, having left the meridian, has begun to descend towards the horizon, the waters of the sea begin to subside and flow back, until the Moon is about one sign from the horizon; for then the greatest decrease occurs. After it the waters stand still for a little while, until, the Moon being one sign below the horizon, they begin to grow again, up to its arrival at the nadir of the meridian — that is, the part of the meridian lying below the horizon in the hidden hemisphere — or rather at the nonagesimal degree of the ecliptic counted below the horizon; at that time there are again living waters and the highest growths of the sea. Next, when the Moon has begun to rise towards the eastern horizon, the waters of the sea begin to subside; and when, before its cosmical rising, the Moon is one sign from the horizon, they sink entirely, and stay in that state until the Moon is raised one sign above the horizon. Thus in the period of one lunar day, which consists of 24 solar hours and ⅘, that is, 48 minutes of an hour, there occur twice daily in the places mentioned the flux and the tide, and twice the reflux and the subsiding of the tide; and in most places the sea rises for six hours and falls for six.

Granted, this rule is not kept in some places. For, as has already been said, the mouths of the Garonne fill in seven hours and subside in five, if we believe Scaliger; or else the sea there rises in 6⅖ hours and falls in 6 hours, if we stand by the Lord of Candal and Fournier. Around the coast of Hippo in Africa the *malinae* and *ledones*, that is, the fluxes and refluxes, behave as follows, on the testimony of St Augustine (*De miraculis Sacrae Scripturae*, Book 1, chapter 7): "The great *malina* boils up for five hours, and for seven hours uncovers the backs of the shores." At the mouths of the river Niger the increases take four hours and the decreases eight; at the coasts of Cambay the tide comes in very rapidly in two hours, and the reflux in two; and in the Chalcidic Euripus the tide once occurred seven times a day, but now four. Finally, in some places it follows no fixed rule, which is regarded as a prodigy, as between Staten and Bergen on the coasts of Norway.

[Margin: A notable passage of Pliny on the tide of the sea.] With this qualification we must understand the words of Pliny, Book 2, chapter 97, which I am pleased to set down here: "Much has been said about the nature of waters; but the flowing-in and flowing-back of the tides of the sea is the most wonderful. It happens in many ways, but the cause lies in the Sun and the Moon. Twice between two risings of the Moon they flow in, and twice they flow back, always in twenty-four hours. First, as the heavens lift themselves up with the Moon, they swell; then, as it declines from the meridian summit of the sky towards its setting, they subside; again, as it approaches from its setting beneath the lowest parts of the sky and the regions opposite the meridian, they flood; and from then until it rises again they draw themselves back." The rest of his words concern the second observation.

[Margin: 2. Observation.] Second. It has been observed that the tide does not begin at the same hour, even in the same place, but later each day by 48^m, because the Moon rises later each day by as many minutes. This is owing to its own proper motion, by which it advances each day towards the east by about 12 degrees, and these require 48 minutes of the diurnal motion towards the west. Therefore, since at Antwerp, at new and full moon, the highest tide occurs at 6^h after midnight, on the following day it occurs only at 6^h 48^m, and on the third day after new and full moon only at 7^h 36^m; and so on, on each of the 15 days that lie between the syzygies of the Moon, the tides are put off by 48 minutes. And although the tides of the sea depend so greatly on the Moon, their turns nevertheless return after hours that are not temporal but equinoctial. [Margin: Pliny's remaining words.] Pliny himself recognised and taught all this, in Book 2, chapter 97, saying: "Nor does it ever flow back at the same time as on the day before, as though the star were pumping [?], drawing the seas with it in a greedy draught, and rising continually from a different place than the day before; yet they reciprocate at equal intervals, and always in six hours — not hours of any given day and night or place, but equinoctial hours. And therefore they are unequal in terms of the common hours, according as more of the measures of day or of night fall into them, and they are equal everywhere only at the equinox. A huge argument, full of light, of the daily alternation, that those are dull who deny that the stars pass beneath the earth and rise again the same, and that from then on a similar face is presented to the lands, indeed to the whole of nature, in the same workings of rising and setting," and so on. Thus the daily period of the tides has as its measure the lunar day of 24^h 48^m.

[Margin: 3. Observation.] Third. It has been observed, as Fournier has it (Book 9 of the *Hydrographia*, chapters 1 and 7), that although some places lie under the same meridian, or very near it, they nevertheless do not have the tide at the same hour. This happens partly because of the situation of the ports — for ports that stretch further out into the sea feel the tide sooner, and are freed from it sooner, than those that have receded inland, especially those set back in oblique bays; partly because of reefs or shallows, sandbanks and similar obstacles; and partly, finally, because of the nature of the soil and of the air, which in neighbouring places is sometimes very different, so much so that one place has the greatest serenity and another the most turbulent air. From the first of the three causes mentioned, the waters surging from south to north fill Bessunda, a port of Spitsbergen, at 3^h European time, and yet in a port a little further up the water is at its highest only at 9^h, although the one is not more than 20 German miles from the other. Thus on the coast of Frisia, on the day of new and of full moon, the living water, or highest tide, occurs at 9^h before noon, and yet at Enkhuizen it occurs at noon itself, and at Amsterdam at 3^h after noon, although these places are very near one another; for Amsterdam lies further within the gulf of the sea than Enkhuizen. So, if the coasts of Britain, Normandy and Picardy are considered, it will easily appear why the tide occurs

an hour sooner on the British coast, which stretches further out towards the west, than in Normandy. From the second cause it comes about that in some places the sea does not grow gradually but, once the obstacles are overcome, suddenly takes its whole increase, and the waves of the surging Ocean roll very swiftly one after another, like barrels, into the ports and onto the shores, so that they outrun the speed of any horse; this is seen at Mont-Saint-Michel near Avranches, on the bank of the Seine, and at the mouth of the Garonne.

[Margin: The error of Petrus Medinensis.] From what has been said it is clear that the rule of Petrus Medinensis [Pedro de Medina], which Crescentius refutes (*Nautica Mediterranea*, Book 3, chapter 3), is false: namely, that the sea fills when the Moon is at Graecus, that is, between north and east, or at Libycus, that is, between south and west, and that, by contrast, it falls when the Moon is at Magister, that is, between north and west, or when it is between east and south. For although this can happen in some places and at some times, it does not happen always or everywhere. [Margin: 4. Observation.] Rather, what the Lord of Candal observed, and Fournier reports from him (Book 9, chapter 1), is true: that the tide of the sea occurs in the same place each day at the same time at which the Moon cuts the same hour circle, that is, the circle drawn through the poles of the world.

De Menſtruis Periodis ac varietatibus Aeſtuum.

On the Monthly Periods and Variations of the Tides.

XVIII. Besides the increases which the sea undergoes every day as the Moon comes to the meridian above or below the horizon, it also at times undergoes greater ones at full moons than at new moons, and at new moons than far from them, and the least at the quadratures or a little after the quadratures; elsewhere, however, it undergoes more at new moons than at full moons. This is so true that even on those shores on which no perceptible tide appears during the whole rest of the month, some swelling nevertheless appears at new or full moon, as happens in the Gulf of Mexico and near Cuba and Hispaniola, and also on some coasts of the Mediterranean, and in that branch of the river Yanson which reaches as far as the city of Pachinum. In some places, therefore, the increases are greater at new moons, as at Calicut; elsewhere, by contrast, they are greater at full moons, as at the mouths of the Indus. In some places, too, the highest increases occur not on the very day of new or full moon, but two or three days after, as at Dieppe and elsewhere, where the tides are greater on the 17th day of the Moon than on the 15th; just as the standing, or middle state, of the waters occurs not at the quadratures themselves but about two days after. Hence that proverb among the Venetians: "From the eighth to the ninth the water does not move; from the twenty-first to the

[Margin: The Venetian proverb on the sea tide [?] around □ ☽.] twenty-second it goes neither up nor down." Rendered into Latin, this runs: "From the eighth to the ninth day the water does not move; from the twenty-first to

the twenty-second it neither rises nor falls." And at that time, as Crescentius says in Book 3 of the *Nautica Mediterranea*, chapter 2, the waters of the sea are called "waters of gall" [aquae fellis], whereas at the time of conjunction ☌ and opposition ☍ they are called "living waters" [aquae vivae]. There he adds, from his own experiments, that the greatest increases of the waters come at the new moon; that from then on they become smaller up to the eighth day of the Moon, when they are at their lowest, and so continue until the 11th day, within which days the smallest tides occur; that from the 11th to the 17th they rise, so that on the 17th day they are at their highest, and from there they fall until the 22nd and so persist until the 25th, within which days again the smallest flows occur.

[Margin: St Augustine's view on the sea tide.] But how the monthly variation of the tides behaves along the coasts of Africa, at Hippo and the neighbouring shores, or even along the shores of the Ocean, we shall learn better from St Augustine himself, in Book 1 [?] of *On the Miracles of Holy Scripture*, chapter 7, whose words are: "Concerning the daily floodings and ebbings of the Ocean the question always arises anew. For this daily flooding is accomplished twice a day, from time to time, always within twenty-four hours, and the alternation of Ledo and Malina accompanies it in alternate weeks. Ledo has six hours of flooding and as many of ebbing. But the great Malina boils up for five hours and for seven hours uncovers the backs of the shores. Yet it shows an agreement with the Moon: it always begins three days and seven hours before the Moon is born, and after the beginnings of the nascent Moon it usually keeps another three days and seven hours. Likewise it begins three days and twelve hours before the full moon, and after as much time again it completes the term of its course. Every common year has six Malinas of each season — of spring, that is, and of summer, autumn and winter — by lunar reckoning, that is, twenty-four in all; except, of course, the embolismic years, which hold twenty-six Malinas." On his view, therefore, a common year of twelve lunations has 24 great tides, twelve at the new moons and as many at the full moons. These begin to rise above the ordinary 3^{d} 7^{h} before the new moon and grow until the Moon is 3^{d} 7^{h} old; but the Malinas, or great tides of the full moons, begin to grow 3^{d} 12^{h} before the full Moon and increase until 3^{d} 12^{h} after the full moon.

[Margin: Scaliger's, [?] and Trimarchus's.] Here I should like to add the words of Scaliger, *Exercise* 52: "Indeed in the quarters there is a calm [malacia], commonly called *calma*; at the full moons the seas are more swollen, so that they seem to lift themselves up out of longing for the star." Now if we believe Iosephus Scaliger, Book 2 of *De emendatione temporum*, page 162, the first Franco-Saxons, who lived on the coasts of Gaul, distinguished the beginnings and ends of the lunar months in no other way than by the tides that occurred at the new moon, so exact and regular were they. Fournier subscribes to this in Book 9, chapter 21, adding that on the coast of Gaul the new-moon tide lasts for two or three days, and then falls until the eighth day, when the water is at its lowest, and stays so until the 11th day; then it rises until the 14th day

and stays so until the 17th; from there it diminishes until the 22nd and stays so until the 25th; then it rises towards the new moon. Trimarchus (*Meteora* 2, disputation 1, section 7) speaks a little differently of the tide of the sea. He says that on the 8th and 21st days of the Moon there is no tide, or the least possible, and that sailors therefore say the water is then tired and languid; but that at the new and the full moon it is greatest, and that sailors therefore call the first the "head of the water" [caput aquae] and the second "living water" [aqua viva]; yet he admits that these things are commonly observed in the Adriatic gulf. Chiaramonti, moreover, says in Book 13 of *De universo*, chapter 22, that at the harbour of Cesenatico sailors call the water that occurs at the quadratures of the Moon "water of gall" and "dead water".

[Margin: Resta's [?] opinion [?].] Franciscus Resta, treatise 1, *De mari*, in his *Meteorologia*, chapter 15, says: "At the interlunium and the full moon the tides are vehement, at the quadratures slack; somewhat fuller at full moon than at new moon."

[Margin: Albert the Great's [?] observation [?] of a single tide [?] in each month [?].] What I said above about the Chinese sea at the mouths of the Yanson also seems to be true of some other sea; for Albert the Great says, in treatise 2 of *De proprietatibus elementorum*, chapter 6: "A sea has been found which comes in from the beginning of the Moon until full moon, and goes out from full moon until the complete failing of the Moon, making in one month only one flow and one ebb."

[Margin: Pliny's [?] authority [?].] Nor is Pliny's opinion on this matter obscure, in Book 2, chapter 97, where he says: "For the lunar difference is manifold, and first in periods of seven days: the tides are moderate from the new moon to the half moon; from that point it overflows more fully, and at the full they seethe most. Then they grow milder, equal to the first ones by the seventh day; and again, on the other side, they increase at the half moon and are equal at the conjunction with the Sun. Plainly, when the same Moon is northern and withdrawing further from the lands, they are milder than when, having digressed to the south, it exerts its force with a nearer effort." He holds, then, that at the new moon the tides are great and at the full moon greater; that from the new moon and from the full moon they diminish for the first seven days of the Moon, down to the first and second quadratures; and that after each of these they take on increases. But he adds: "Yet not at the very moments of time that I have named, but a few days after; just as not at the full or the last moon either, but afterwards."

[Margin: Borro's singular view on the tide at ☌.] Although this is so, Hieronymus Borrus Aretinus [Girolamo Borro of Arezzo], in his dialogue on the flux and reflux of the sea, page 132, nevertheless says that at the new moons the tide is almost imperceptible, and that the Venetians therefore say then that the sea is "in oil" [Mare esse in Oleo], because it stands still just as if it were oil; and yet he concedes that the waters of the sea are then raised so high that they cannot be sailed without danger. But this, he says, comes not from the

tide properly so called but from the storms and winds and similar changes that occur at the new moons; and he adds that on those same days all the humours are greatly altered and give signs of a notable change. To this could be referred what Galen handed down in Book 3 of *De diebus criticis*, referring their changes chiefly to the Moon — although Io. Picus [Giovanni Pico della Mirandola], in Book 3 of *Against the Astrologers*, chapter 16, tries to refute him at length. But it does not seem likely to me that so many other ship-masters and writers would have asserted the greatness of the tides at the new moons unless they had noted it with the sea also calm. And surely the Venetians would be very unskilled to call the sea oil if it were heaved up and tossed by the most turbulent tempests and storms. Unless I am mistaken, then, they mean that the sea is at that time at its highest tide, but in such a way that it keeps that swelling calmly for some time; or else that the tide is perhaps not greatest on the very day of the new moon, but some days after. Yet the same Borro relates on page 175, from observation, that in the first 15 days of the Moon the flows are greater and longer-lasting than the ebbs, but in the remaining 15 days the former are smaller than the latter. So much for the monthly vicissitudes of the sea's tide.

De Periodis & varietatibus Annuis Aeſtuum, alijſque ad annuas reductis.

On the Annual Periods and Variations of the Tides, and Others Reduced to Annual Ones.

[Margin: The period of 8 years of the tides, from Pliny, and on the equinoctial tides.] XIX. Let us begin with Pliny, Book 2, chapter 97, so as to bring some light from his own words. Speaking of the differences of the tides that come from the Moon, after he had indicated the monthly ones, he added: "Over periods of eight years, too, they are brought back to the beginnings of their motion and to equal increases at the hundredth circuit of the Moon, the annual causes of the Sun augmenting all of these: swelling most at the two equinoxes, and more at the autumnal than at the vernal; but empty at midwinter, and more so at the solstice." And before him Tacitus, in Book 1 of the *Annals*, had said that "under the equinoctial star the ocean swells most". But why do these turns not rather return in 19 years, since the Moon and the Sun then return very nearly to the same turns — or at any rate in other lunar periods? We have said more about these in Book 4, chapter 19. But for now we are not inquiring into the cause, being devoted only to the history of the tides.

[Margin: St Augustine's authority on the equinoctial tides.] Let us hear, then, St Augustine, Book 1 of *On the Miracles of Holy Scripture*, chapter 7, speaking of the Malinas, or tides: "Of each Malina of the times already mentioned, the two middle ones — namely the equinoctial ones — and another when either the day or the night completes the term of its course, usually become stronger than usual and higher in flooding; but in the intervening spaces the Ledo is always interrupted again by the same amount." Indeed, if we listen to Trimarchus

(Book 2, *Meteora*, disputation 1, section 7), sailors speaking at least of the Adriatic tides think them greater in winter than in summer, and by day than by night. Franciscus Resta, in his *Meteorologia*, treatise 1, *De mari*, chapter 15, adds from Albert the Great and others that the tide varies with the Moon's nearness to the sea (as we have also learned from Pliny, Book 2, chapter 97, who says that when the Moon has digressed to the south the tides become greater because of the nearer force of the star); with the Moon's nearness to the zenith of those who live by the sea, that is, according as it is nearer to their vertex; and next with its varying relation to the Sun. On longer days, he says, the daytime tides are stronger, and conversely the night tides when the nights are longer than the days; yet in such a way that from the vernal equinox to the summer solstice, and from the summer solstice to the autumnal equinox, the flows are greater than in the rest of the year, but stronger in the lengthening of the days than in their shortening. He affirms, moreover, that the tides are greater when the Moon is in Pisces and at the end of Aquarius, and when the Pleiades assist the Moon. Delphinus, for his part, says in his treatise on this matter that Venus and Jupiter, when near the Sun and the Moon, help to stir up the waters. And Hieronymus Borrus, in his dialogue on the tide of the sea, page 175, says it has been observed that the flows are greater and longer-lasting than the ebbs in proportion as the Sun and Moon stay more hours above the horizon, but that in proportion as the luminaries are more hours below the horizon, the flows are smaller and shorter than the ebbs. I fear, however, that some uncertain things are still being reported about these matters, and from the presumption of conjecture rather than from trustworthy history. But this deserves full credence, which Fournier reports from the observation of the Lord of Candal in Book 9 of the *Hydrographia*, chapter 21: namely, that at the mouths of the Garonne and on the Medoacensian coast the tides are never greater than at the new and full moons that occur in September and then in March, and that the waters are lowest at the winter solstices, and next at the summer ones.

De Regulis Nauticis diſcernendi Horam aquæ viuæ ſeu ſummi æſtus, dato quouis die Lunæ in dato Portu Europæ.

On the Nautical Rules for Determining the Hour of Living Water, or of the Highest Tide, on Any Given Day of the Moon in a Given Port of Europe.

XX. The highest increase of the tide, wherever it occurs daily, depends on the Moon to such an extent, and follows its ascent to the meridian above the horizon, or its descent to the meridian below the horizon, with such fixed steps, that ship-masters long ago drew up for themselves very certain rules by which one can know at what hour the highest tide will come in this or that port, starting from the day of the new and the full Moon and expecting the highest tide each day 48 minutes later than the day before. We shall give a specimen of this, partly from Bartholomaeus Crescentius, Book 3 of the *Nautica Mediterranea*, chapter 3, and partly from our Fournier, Book 9 of the *Hydrographia*, chapters

2, 3 and 4. Crescentius admits that he drew rules of this kind from a certain distinguished Cantabrian ship-master, whom Nicolaus Sagrus of Ragusa, or Epidaurus [Nikola Sagroević], introduces in his dialogues on the tide of the sea. From him, then, we shall reduce into a shorter table the hours at which the highest tide occurs on the day of the new and the full Moon at the places listed below. For once this is known, the hour of the highest tide on the following days, counted from the new or the full moon, will easily be known by adding each day four fifths of one hour, that is, 48 minutes. For example: in the Strait of Cádiz [freto Gaditano], on the day of the new and the full Moon, the living water, or highest tide, comes at $1\frac{1}{2}^{h}$ after midnight and again at $1\frac{1}{2}^{h}$ after noon. So if you want to know the hour of living water for the second day of the Moon, or the 16th, which is the second after full moon, add 48^{m} to $1\frac{1}{2}^{h}$, that is, to $1^{h}\ 30^{m}$, and you will have $2^{h}\ 18^{m}$ after midnight, and likewise after noon. Thus on the third day of the Moon and on the 17th the living water will be there at $3^{h}\ 6^{m}$, both after noon and after midnight, as is clear by adding 48 minutes to $2^{h}\ 18^{m}$; and so on for the rest. We shall give the vernacular names, as he also gives them, since the Latin ones are often lacking or do not correspond exactly, and since on maps it is the vernacular ones that are usually written; and so that no one should take them for Latin, we shall mark them with a distinctive type.

Prima Tabula Horarum Aquæ Viuæ ſeu ſummi Aeſtus die Nouæ ac Plenæ Lunæ, in horis tum à meridie tum à media nocte numeratis, ex Nauclero Cantabro & Creſcentio.

First Table of the Hours of Living Water, or of the Highest Tide, on the Day of the New and the Full Moon, in Hours Counted Both from Noon and from Midnight, from the Cantabrian Ship-master and Crescentius.

Place	Hour of living water on the day of ☌ & ☍ [?], from midnight and from noon
In the Strait of Cádiz, commonly Stretto di Gibilterra, and also in the port of Calesi, and along the whole shore from the Cape of Carteia, commonly Capo di Tariffa, to the cape commonly called Capo di Rutta, the water is full and living at	$1^{h}\ 30^{m}$
From Capo di Rutta to the Cape of S. Maria	$2^{h}\ 15^{m}$
From the Cape of S. Maria to the Cape of S. Vincentius, and from there to the Celtic or Artabrian promontory, commonly Capo de Finis terrae,	
… and over a great part of the Cantabrian coast, commonly Costa di Biscaglia	$3^{h}\ 0^{m}$
On the coasts called de Caccioppi near Bordeaux, commonly Bordeos, and at the places called Las Bollienas and Ollona	$2^{h}\ 15^{m}$
In the straight run between La Rochelle, commonly Roccella, and the Isle of the King [Insulam Regis], which lies by the promontory del Plomo, up to the cape called Raso di Hontenaut	$3^{h}\ 0^{m}$

Place	Hour of living water on the day of ☌ & ☍ [?], from midnight and from noon
From the place called il Forno to the island di Hebas, in the run of the water	3^h 45^m
But in the ports beside these runs of the waters it comes later, namely at	4^h 30^m
From the island Hebas to Carnesule and at the place called Caschetti	5^h 15^m
But at the places on the mainland of France that face the three places already mentioned	6^h 15^m
From the place Carnesul through all the ports, shores and capes of Normandy and Picardy, as also at the places Blancarnes, Cales and Neuport	9^h 0^m
But in the middle of the Channel between France and England, off the places already mentioned, from Carnesul to Blancarnes	12^h 0^m
But from Cales to Neuport, in the run of the water ten miles out from land, the living water comes at	0^h 45^m
From Cales to Gravelines, in the run of the water	1^h 30^m
From Gravelines along the whole coast of Flanders, and in the island of Zeeland	3^h 0^m
But in the run of the water beside the coast already mentioned	3^h 45^m
On leaving Flanders towards the Channel of England: although the water is full at hour 3 on the shore and at 3^h 45^m in the run of the water, yet the further you withdraw from the shore towards the cape called de Dobla, the water will be full at hour 3, but in the run of the water at 4^h 30^m; and this course of the water, until it tires, lasts from Zeeland to the cape called di Monge, or as far as las Motricas	
At the place called Godoina and at Duna on Cape Dobla, up to Capo di Romaneus, on the shore, that is, and in the ports	9^h 0^m
But in the course of the waters	12^h 0^m
On Capo di Romaneus, and in the port Cameracensis, or de Cambray, and along the whole neighbouring coast	9^h 45^m
But in the run of the waters off the places already mentioned	11^h 15^m
At Erlaga, Beuceppe and the bay called di Soran, and along the whole coast that extends from the cape of Erlaga to the island called di Vuicche	10^h 30^m
But in the run of the water beside the coast already mentioned	12^h 0^m
Before you enter by the mouth of S. Lena to the island di Vuicche, you will find two syrtes, or sandy shallows, one called Almages and the other Ostriar, where, as also in the port Porceinua, the water is living at	11^h 15^m
In the Channel itself, between the island Vuicche and England, there are three syrtes, one called Zenciuiglia, another Ambra and the third Calzesores, where, as also in the port of Antona, the water is full at noon itself and at midnight, or	12^h 0^m
At Giaramua and at the point, or Aguglia, di Vuicche	9^h 15^m
But the water runs further still, up to hour 11¼	11^h 15^m
From the Aguglia already mentioned to Cape Porlan, and on its coast and in its port	9^h 30^m
But in the run	11^h 15^m
From Cape Porlan to Cape God-Esterch, in the run	10^h 30^m
But in the port Artemua and on the neighbouring shore	8^h 15^m
From Cape God-Esterch to Cape Lisuarte, in the run, that is	9^h 0^m
But in the ports di Plemua, Faiut, Falamua	7^h 30^m
From Cape Lisuarte to the Cape of Cornoualia, so cal-	

Place	Hour of living water
led Lungosneus, which is the chief cape of England, in the run of the water	$7^h\ 30^m$
At the place called Pesanz and Musol	$6^h\ 15^m$
In the ports of all the Surlingan islands	$4^h\ 30^m$
But outside their ports, at the capes of the islands	$6^h\ 0^m$
On the coast of Ireland, at the place called Manga di S. Giorgio, at the entrance di Duolin	$10^h\ 30^m$
In the port di Mirafurda	$5^h\ 15^m$
In the island dell'Hondectenebi, and at the place called Los Olmos	$6^h\ 0^m$
In the port di Patristo	$5^h\ 30^m$
At the place called Quinque Roda, and at another called Ancia Pristol	$6^h\ 45^m$
In the city called Galuei, as far as Gafurda, and in their ports, as also in Semerich, in … Tingle, in S. Michaele, in Baiam, Balensemor, Guinzala, Corca, Iiocla and Gattafurda, the water is full and living on the days of the new and the full Moon at	$4^h\ 30^m$

So much from Crescentius and that distinguished Cantabrian ship-master. There now follows another table, from Father Georgius Furnerius of our Society, Book 9 of the *Hydrographia*, chapter 2. We shall reduce it to another form, a little clearer unless I am mistaken, adding capital letters at the head of the former table, so that by means of them we may indicate, in the other part of the table, which places belong to such a position and time of full water. We shall teach the use of the table in numbers XXI and XXII.

Second Table of the Hours of Living Water, or of the Highest Tide, on Any Day of the Moon in the Ports Listed Below, from Father Georgius Furnerius of the Society of Jesus.

Part I of the table

Day of the Moon	Day of the Moon	A	B	C	D	E	F	G	H
		S. & N.	S. ¼ S.O / N. ¼ NE	S. S. O. / N. N E.	SO. ¼ S. / N E. ¼ N.	S. O. / N. E.	SO. ¼ O. / NE ¼ E.	O. S. O. / E. N. E.	O. ¼ SO. / E. ¼ N.E.
☌ 0	☍ 15	$12^h\ 0^m$	$12^h\ 45^m$	$1^h\ 30^m$	$2^h\ 15^m$	$3^h\ 0^m$	$3^h\ 45^m$	$4^h\ 30^m$	$5^h\ 15^m$
1	16	$12^h\ 48^m$	$1^h\ 33^m$	$2^h\ 18^m$	$3^h\ 3^m$	$3^h\ 48^m$	$4^h\ 33^m$	$5^h\ 18^m$	$6^h\ 3^m$
2	17	$1^h\ 36^m$	$2^h\ 21^m$	$3^h\ 6^m$	$3^h\ 51^m$	$4^h\ 36^m$	$5^h\ 21^m$	$6^h\ 6^m$	$6^h\ 51^m$
3	18	$2^h\ 36^m$	$3^h\ 9^m$	$3^h\ 54^m$	$4^h\ 39^m$	$5^h\ 24^m$	$6^h\ 9^m$	$6^h\ 54^m$	$7^h\ 39^m$
4	19	$3^h\ 12^m$	$3^h\ 57^m$	$4^h\ 42^m$	$5^h\ 27^m$	$6^h\ 12^m$	$6^h\ 57^m$	$7^h\ 42^m$	$8^h\ 27^m$
5	20	$4^h\ 0^m$	$4^h\ 45^m$	$5^h\ 30^m$	$6^h\ 15^m$	$7^h\ 0^m$	$7^h\ 45^m$	$8^h\ 30^m$	$9^h\ 15^m$
6	21	$4^h\ 48^m$	$5^h\ 33^m$	$6^h\ 18^m$	$7^h\ 3^m$	$7^h\ 48^m$	$8^h\ 33^m$	$9^h\ 18^m$	$10^h\ 3^m$
7	22	$5^h\ 36^m$	$6^h\ 21^m$	$7^h\ 6^m$	$7^h\ 51^m$	$8^h\ 36^m$	$9^h\ 21^m$	$10^h\ 6^m$	$10^h\ 51^m$
8	23	$6^h\ 24^m$	$7^h\ 9^m$	$7^h\ 54^m$	$8^h\ 39^m$	$9^h\ 24^m$	$10^h\ 9^m$	$10^h\ 54^m$	$11^h\ 39^m$
9	24	$7^h\ 12^m$	$7^h\ 57^m$	$8^h\ 42^m$	$9^h\ 29^m$	$10^h\ 12^m$	$10^h\ 57^m$	$11^h\ 42^m$	$12^h\ 27^m$
10	25	$8^h\ 0^m$	$8^h\ 45^m$	$9^h\ 30^m$	$10^h\ 27^m$	$11^h\ 0^m$	$11^h\ 45^m$	$12^h\ 30^m$	$1^h\ 15^m$
11	26	$8^h\ 48^m$	$9^h\ 33^m$	$10^h\ 18^m$	$11^h\ 3^m$	$11^h\ 48^m$	$12^h\ 33^m$	$1^h\ 18^m$	$2^h\ 3^m$
12	27	$9^h\ 36^m$	$10^h\ 21^m$	$11^h\ 6^m$	$11^h\ 51^m$	$12^h\ 36^m$	$1^h\ 21^m$	$2^h\ 6^m$	$2^h\ 51^m$
13	28	$10^h\ 24^m$	$11^h\ 9^m$	$11^h\ 54^m$	$12^h\ 39^m$	$1^h\ 24^m$	$2^h\ 9^m$	$2^h\ 54^m$	$3^h\ 39^m$

Day of the Moon	Day of the Moon	A	B	C	D	E	F	G	H
14	29	$11^h\ 12^m$	$11^h\ 57^m$	$12^h\ 42^m$	$1^h\ 27^m$	$2^h\ 12^m$	$2^h\ 57^m$	$3^h\ 42^m$	$4^h\ 27^m$
☍ 15	♂ 30	$12^h\ 0^m$	$12^h\ 45^m$	$1^h\ 30^m$	$2^h\ 15^m$	$3^h\ 0^m$	$3^h\ 45^m$	$4^h\ 30^m$	$5^h\ 15^m$

Part II of the table

Day	of the Moon	K	L	M	N	O	P	Q	R
		E & O	O ¼ NO / E ¼ SE	O. N.O. / E SE	NO. ¼ O / S.E. ¼ E	N. O. / S. E.	NO ¼ N / SE ¼ S	N. N.O. / S. S. E.	N ¼ NO / S ¼ SE
♂ 0	☍ 15	$6^h\ 0^m$	$6^h\ 46^m$	$7^h\ 30^m$	$8^h\ 15^m$	$9^h\ 0^m$	$9^h\ 45^m$	$10^h\ 30^m$	$11^h\ 15^m$
1	16	$6^h\ 48^m$	$7^h\ 33^m$	$8^h\ 18^m$	$9^h\ 3^m$	$9^h\ 48^m$	$10^h\ 39^m$	$11^h\ 18^m$	$12^h\ 3^m$
2	17	$7^h\ 36^m$	$8^h\ 21^m$	$9^h\ 6^m$	$9^h\ 51^m$	$10^h\ 36^m$	$11^h\ 21^m$	$12^h\ 6^m$	$12^h\ 51^m$
3	18	$8^h\ 24^m$	$9^h\ 9^m$	$9^h\ 54^m$	$10^h\ 39^m$	$11^h\ 24^m$	$12^h\ 9^m$	$12^h\ 54^m$	$1^h\ 39^m$
4	19	$9^h\ 12^m$	$9^h\ 57^m$	$10^h\ 42^m$	$11^h\ 27^m$	$12^h\ 12^m$	$12^h\ 57^m$	$1^h\ 42^m$	$2^h\ 27^m$
5	20	$10^h\ 0^m$	$10^h\ 45^m$	$11^h\ 30^m$	$12^h\ 15^m$	$1^h\ 0^m$	$1^h\ 45^m$	$2^h\ 30^m$	$3^h\ 15^m$
6	21	$10^h\ 48^m$	$11^h\ 33^m$	$12^h\ 18^m$	$1^h\ 3^m$	$1^h\ 48^m$	$2^h\ 33^m$	$3^h\ 18^m$	$4^h\ 3^m$
7	22	$11^h\ 36^m$	$12^h\ 21^m$	$1^h\ 6^m$	$1^h\ 51^m$	$2^h\ 36^m$	$3^h\ 21^m$	$4^h\ 6^m$	$4^h\ 51^m$
8	23	$12^h\ 24^m$	$1^h\ 9^m$	$1^h\ 54^m$	$2^h\ 39^m$	$3^h\ 24^m$	$4^h\ 9^m$	$4^h\ 54^m$	$5^h\ 39^m$
9	24	$1^h\ 12^m$	$1^h\ 57^m$	$2^h\ 42^m$	$3^h\ 27^m$	$4^h\ 12^m$	$4^h\ 57^m$	$5^h\ 42^m$	$6^h\ 27^m$
10	25	$2^h\ 0^m$	$2^h\ 45^m$	$3^h\ 30^m$	$4^h\ 15^m$	$5^h\ 0^m$	$5^h\ 45^m$	$6^h\ 30^m$	$7^h\ 15^m$
11	26	$2^h\ 48^m$	$3^h\ 33^m$	$4^h\ 18^m$	$5^h\ 3^m$	$5^h\ 48^m$	$6^h\ 33^m$	$7^h\ 18^m$	$8^h\ 3^m$
12	27	$3^h\ 36^m$	$4^h\ 21^m$	$5^h\ 6^m$	$5^h\ 51^m$	$6^h\ 36^m$	$7^h\ 21^m$	$8^h\ 6^m$	$8^h\ 51^m$
13	28	$4^h\ 24^m$	$5^h\ 9^m$	$5^h\ 54^m$	$6^h\ 39^m$	$7^h\ 24^m$	$8^h\ 9^m$	$8^h\ 54^m$	$9^h\ 39^m$
14	29	$5^h\ 12^m$	$6^h\ 57^m$	$6^h\ 42^m$	$7^h\ 27^m$	$8^h\ 12^m$	$8^h\ 57^m$	$9^h\ 42^m$	$10^h\ 27^m$
☍ 15	♂ 30	$6^h\ 0^m$	$7^h\ 45^m$	$7^h\ 30^m$	$8^h\ 15^m$	$9^h\ 0^m$	$9^h\ 45^m$	$10^h\ 30^m$	$11^h\ 15^m$

Loca Reſpondentia literis in fronte tabulæ poſitis.

The Places Corresponding to the Letters Set at the Head of the Table.

Let-ter	Places	Hr	Min
		On the day of the ♂ and ☍ of the ☾, high water is after noon and after midnight:	
		Hr	Min
A	At the island of Jutland; off the coasts of Heuer, Eyder and the Elbe at Emden. Off Enkhuizen and Horn. Along the whole coast of Flanders, in England. At the river de Beuechier at Porshmout; at Hampton and Baruich. In France at Honfleur and at Caen. In Barbary, beyond Cape Quintin as far as Boiador. Above these stretches the tidal streams come down from Nesse as far as Boulogne.	12	0

Letter	Places	Hr	Min
B	Within the Meue at Camp Vere, at Vlissingen and Middelburg, at the coasts of Beuesier in the sea at Vinckelzee, in the sea of Garcenzey. Above these stretches the streams of the surging or ebbing sea come down from Boulogne, into the Somme.	12	45
C	Off the Meuse, that is, the Mosa, and Goeree; off Vere on the coast of Zeeland, off the Thames, and off Iarmuyen towards the western side of Vuict, at Morbian, off Blauet, at the Fair Island, and beyond these stretches as far as Cape Quentin. The tidal streams, moreover, flow down from Gravelines to Boulogne.	1	30
D	At Houge, outside the Ras de Fontenas; above the Fair Island; off Vuielingue; off the Meuse, that is, the Mosa. The tidal streams flow down between, through the middle, to Calais, and beyond Gravelines as far as Duncherquen, and from Caen as far as Estaples.	2	15
E	At Amsterdam, Rotterdam and Dort, outside the Shoals of Flanders, at the point off Conquet; at the mouths of the Garonne; on the southern coast of Britannia among the Pictones, that is, Poitou; in Aquitaine, commonly Guascogna; in Cantabria, that is, Biscaglia, in Galicia and Lusitania, and on the southern shore of Ireland. The tidal streams come down beyond Cape de Hague as far as the island of Ornay, and from Garnzeu to the Quasquettes.	3	0
F	Within Caletum, or Calais, and the Mosa, or la Meuse, at Rouen, at S. Mahe, at Rupella, commonly Rocchelle, at the Cassiterides islands, now the Sorlinges, at Venta Belgarum, now Bristou. In the harbours and on the shores of Aquitaine, that is, Gascony, in Galicia and Lusitania, and on the western coast of Ireland. The tidal streams, moreover, rush from Stuissart to Diepen.	3	45
G	From Thessel as far as Caletum, or Calais, at S. Paulus, or S. Pol, outside the harbour; between Garnezer and the Seven Islands; outside Four, and on the southern coast of Ireland. The tidal streams run and come down from Barfleur to Stuissart.	4	30
H	At Plemuda; in the sea of Galles, at Dormuda and Valmuda, at Mulfortium in the southern harbours of Ireland. The tidal streams roll down from the island of Basau, Cape de Four and the Cassiterides, or Sorlingues, to Cape Lysart, and from Portland to Vuicht.	5	15
K	At S. Malo, at Cancalen, at Antwerp, or Anuers, and off it at Bremen, at Hamburg, commonly Hamborg; at Thessel, at Goësan, or Goës, at Gouda. The tidal streams flow down from the headland, or promontory, of England to Lezordia.	6	0
L	At Bristol, or Bristou, at Cay between Foye and Valmuda, off S. Nicolaus. The tidal streams come down from the island of Bas to Matuanem along the length of the land.	6	46
M	At Thessel above Raden, at Lezarda near land by Pleymouth, in the sea at Passage. The tidal streams arrive descending from Barfleur to the Cape of the Seine.	7	30
N	Outside the Quasquettes near Vuicht, and from Vuicht to Beuesier near land, outside Vlie. The tidal streams come down from behind Garnozey and within the Seven Islands.	8	15
O	Within the Seine, or La Seyne, at the Ras of Portland; on all the shores of Friesland, between GarneZaij and les Quasquettes; and at Cherebourg. The tidal streams, having slipped down, reach between Morlaix and Octobers.	9	0
P	Near Garnezey, at the channel next to Lesartia, at Iarmouth, at the point or tip of Vuicht. The tidal streams flow down off Cancale.	9	45

Let-ter	Places	Hr	Min
Q	At Diepen and S. Valerius, at Fesqua, and the greater part of the shores of Picardy and Normandy, and at Senegal. The tidal streams come down from Barfleur to Alborga.	10	30
R	Off the Thames, at Hansonna and at Estreham. The tidal streams come down beyond Fontenay as far as Cape de Four.	11	15

Vſus Præcedentis Tabulæ.

The Use of the Preceding Table.

XXI. Across the head of the preceding table run three bands. In the first are large characters in alphabetical order; they denote the places belonging to that column, which are to be indicated after the tables by the same characters. In the second are the points of the winds, following the more recent ship-masters, by which these men have been accustomed to indicate not only the situation of places but also the hour of the tide on the day of the new and the full Moon: they assign three hours to each of the eight rhumbs, 1½ hours to half a rhumb, and three quarters of an hour to a quarter of a rhumb. So when they want to express that full and high water occurs at Amsterdam or Rotterdam on the day of new and full Moon at 3 o'clock after noon and at 3 after midnight, as you see in Column E, they say that Amsterdam and Rotterdam lie Sud Ouest and Nord Est; this is indicated by the initial letters S O and N E set in the second band of the head. If you wish to know the winds and points indicated by these letters, consult our tables 2 and 3 set out in Book 2, Chapter 17. Where this fraction ¼ stands, it signifies one quarter of the next rhumb. If anyone does not wish to use this mode of indication, he may omit it, since it is not at all necessary to the reader. Finally, in the third band of the head stand the letters H. M., signifying Hours and Minutes, that is, sixtieths of an hour.

Moreover, in the 1st and 2nd parts of the table there are ten columns. The first of these contains the first 15 days of the lunation, counted from the ☌, that is, from the day of new Moon; the second the remaining 15 days, counted from the ☍, that is, from the full Moon. The rest contain the hours at which the highest tides, or high waters, occur in the places indicated by the capital letters of the 1st or 2nd head, corresponding to the age of the Moon, that is, to the days counted from new Moon or from full Moon. With these laid down, we shall teach, following Fournier, how to solve the principal hydrographic problem by means of the table already mentioned, or also by Crescentius's table.

Problema. Dato Die Lunæ, ſcire Horam Aquæ viuæ ſeu ſummi æſtus, in dato Portu aut littore.

Problem. Given the Day of the Moon, to Find the Hour of High Water, or of the Highest Tide, in a Given Harbour or on a Given Shore.

XXII. Look up in the catalogue of places the place you are in doubt about, and see which of the initial letters it belongs to. Then look for that initial letter in the topmost head of the first or second part of Fournier's table: in its column, opposite the given day of the Moon, which is marked in the first or second column, you will find the hour, both after midnight and after noon, at which the highest tide occurs in that place. [Margin: Example of the problem.] EXAMPLE: We wish to know at what hour the highest tide occurs at the mouths of the Garonne on the day of new and full Moon, or also on the 4th day.

If you run through the catalogue of places, you will find that the mouths of the Garonne belong to the letter E; this letter is found above the seventh column at the head of the first part of the table. Going down that column, then, you will see opposite day ☌ 0, that is, the day of new Moon, and opposite day 15 ☍, that is, the day of full Moon, the hour 3 0; but opposite day 4 after new Moon, the hour 6 12′. Conclude, therefore, that at the mouths of the Garonne the highest tide occurs on the day of both new and full Moon at 3 o'clock after midnight and again at 3 after noon; but on the 4th day of the Moon (to which day 19 also corresponds) it occurs at 6 hours and 12 minutes, that is, one fifth of an hour, after midnight, and again at 6 12″ after noon.

[Margin: The way to check the figures of the table.] If you use Crescentius's table, in which only the hours of the highest tide on the day of new and full Moon are marked, and you are concerned about other days, multiply 48′ by the number of days, that is, of the age of the ☾, and divide the sum by 60. Add the quotient of the division, with its fractions if there are any, to the hours of the highest tide at new or full Moon, and subtract 12 hours from the sum if it exceeds them; the remainder will give the hour of the highest tide for the given day of the Moon. In the example adduced a little earlier, the 4th day of the Moon is given, and the hour of the highest tide at the mouths of the Garonne on the day of new Moon is 3 o'clock. Multiplying 48′ by 4, then, gives 192 minutes; dividing these by 60 gives a quotient of 3 hours 12′; adding this quotient to 3 hours gives 6 hours 12′, as above. And this is the way to check the figures of Fournier's table, in case some error has crept in through the fault of the printers — as we found more than once that one had crept into Fournier's table, and corrected it.

Indicantur Opiniones de Cauſa Aeſtus Maris.

The Opinions on the Cause of the Tide of the Sea Are Set Out.

XXIII. Our Causinus, in his *Dogma de Resurrectione*, commending the divine works of the first week, said very sharply that the flow and ebb of the sea is *the grave of human curiosity*. For no schools, says Simon Maiolus [Simone Maioli] in Colloquy 10 of the *Dies caniculares*, have yet untied this question sufficiently. Not without reason does Julius Caesar Scaliger begin this treatment in Exercise 52 with these words: *Perhaps silence would have done more for the opinion of a*

good reputation than boldness. For what I am accustomed to proclaim everywhere, that we know nothing, suits above all this inquiry, which treats the motion of the sea; and further down: *Now some god from the machine* [Greek: θεὸς … ἀπὸ μηχανῆς (theos … apo mēchanēs, "a god … from the machine")] *must be called up by us. For who would not say that in this matter philosophy itself stammers?*

[Margin: The difficulty of knowing the cause of the tide, but more so of the ebb of the sea.] The ebb of the sea seems much more difficult to explain than its flow. For this reason St Augustine — having said, partly in Book 7 of the *City of God*, chapter 22, and partly in Book 1 of *De mirabilibus Sacrae Scripturae*, chapter 7, that the Romans consecrated two goddesses to this matter, namely Venilia, or Malina, for the flow of the sea, which the Italians call *Marea*, and Salacia, or Ledona, or Lidona, for the ebb — said: *But the flooding of this rational persistence, the one that recedes, is hidden from our minds; human knowledge prepares no less progress for those who do not know.* And a little later: *We are permitted to observe the swellings of the sea as it floods, but we are deprived of understanding of it as it recedes.* To these add the verses of Lucan adduced in Book 2, chapter 15, at the end of Scholium 5.

And truly the features of this motion are so varied and so wonderful that they will always seem to defeat every effort of the human mind, whoever wishes to investigate their sufficient cause and reason and, having investigated it, to explain it. [Margin: The heads of the difficulty, and a summary of the wonders in the tide of the sea.] For what mortal could explain why a large part of the sea lacks a tide, when a large part is likewise subject to one? Why it displays such dependence on the Moon, and yet, when the Moon is silent at new Moon or lies hidden below the horizon, the same tides return, or greater ones? Why in many places the flow is completed in six hours and the ebb in as many, and yet in some places the rise takes about seven hours and the fall five, as roughly at the mouths of the Garonne; elsewhere the waters flow in for four hours and flow back for eight, as at the mouths of the Niger and the coasts of Guinea; elsewhere, as on the coast of Cambay, they burst in very rapidly in two hours and in two hours flee back and are swallowed up again? Why, again, in other places fifteen whole days are required — namely from new Moon to full Moon — for the fullness of the waters, growing continuously, to reach its highest increase, and as many days for them to subside, namely from full Moon to the following new Moon, which happens in the Chinese sea at the mouths of the river Pachinensis, and also in some seas around Cuba and Hispaniola? Why, moreover, at the mouths of the Indus the tides are greater at full Moon than at new Moon, while near Calicut, on the contrary, they are greater at new Moons than at full Moons, but universally they are greater at new and full Moons than at the quadratures? Further, why, in harbours or river-mouths lying close together and almost on the same meridian, the highest tide nevertheless occurs some hours sooner at one than at the other? Besides, why in some places the Ocean swells so proudly that it rises with its tide by 80 cubits, as in Britain, while elsewhere, as at Marseille, Ancona and Rimini, they

scarcely rise to a foot or a foot and a half? Finally, who could relate why, of the wells at Gades and at Seville, some rose with the tide of the sea and sank with its ebb, some did the reverse, and some behaved in both ways at different times? We have chosen to gather all these truly wonderful features of the sea's tide into a single summary, so that we need not repeat them so often when we review the individual opinions to show their insufficiency.

[Margin: 1. The opinion of Timaeus and Chiaramonti.] XXIV. The first opinion, that of Timaeus in Plato and in Plutarch, Book 3 of *De placitis philosophorum*, chapter 17, assigns the cause to rivers. *Timaeus,* says Plutarch, *alleges as the cause the rivers that burst through the mountains of the Celts into the Atlantic: by thrusting the sea forward with their impetus they stir up the tide, and conversely, when the impetus slackens, they drive the sea back again and make the ebbing tide.* This very opinion of Timaeus was quite recently revived by Scipione Chiaramonti in Book 13 of *De universo*, chapter 22, but embracing all rivers, not only those of the Celts; and he adds that by the dashing of the waves against the shores the water is thinned into exhalations and vapours. Since, therefore, the waters of the rivers continually flowing in produce so great an increase that the motion that occurs either in the ebb or in the interval between flow and ebb is not enough to evaporate them, the waves of the sea must swell and cause the tide; but from this very swelling they collide with one another with greater motion, and the evaporation increases from that, until the water that was in excess, thinned into exhalations, decreases and appears to flow back. For since he believed he had shown earlier that the tide of the sea arises from an increase of the waters, and yet not from rarefaction, he argued that it can arise from no other source than the water of the rivers flowing in, since water naturally increases in no other way than either by rarefaction or by the addition of water to water, through a new generation of waters or through their inflow and concourse into one place. And in chapter 26 he accordingly asserts that on the coasts of the Ligurian and the rest of the Tyrrhenian sea no perceptible flow occurs, because few and small rivers enter that sea. As for why the tide is smaller at the Moon's quadratures than on the days nearest to new and full Moon, so that the waters are then called "waters of gall" [aquae fellis], he gives the cause in these words, supposing, from Ptolemy, that the Moon is then nearer to the Earth: *The motion of the water of the sea, as it glides onto the shore and glides back, is due to evaporation, as I said before. In the waters of gall the approach is greater than in the flow, smaller than in the ebb; and as it glides back, the falling back is greater than in the flow, smaller than in the ebb. The collision for evaporation is therefore smaller than in the flow, but greater than in the ebb; for this reason, that the Moon is nearer the Earth, it helps the water towards evaporation more than when it is more remote, and therefore a smaller collision of the waves suffices to dissolve the same amount of water into vapours.*

[Margin: Refutation of the 1st opinion.] XXV. But against both opinions there are many and most evident arguments. For the tide would have to occur on the opposite banks towards which the rivers would thrust the water; and

so the Celtic, or Gallic, rivers would raise a tide at the islands and mainlands lying opposite, towards the west of Gaul. And where the waters of two great rivers meet each other head-on — say, the river Niger flowing from Africa towards America, and the river of the Amazons flowing from America — there, that is, in the intervening sea, a greater tide and heaping-up of waters would occur than at the shores. Moreover, since very great rivers enter the Caspian Sea, a very powerful tide would have to occur on its shores or in its middle. Further, the tide would have to increase continuously, just as the rivers run down into the sea continuously, and indeed more in spring than in autumn, because of the abundance of melting snows. All of this is false. In addition, the very mouths of the rivers, rather, suffer the tide from the sea, and are driven back by it towards their springs: when the sea flows in, the rivers flow back, and when it flows back, they flow. So Franciscus Resta [Francesco Resta] sharply noted against Timaeus in Book 3 of the *Meteorologia*, tractate 1, chapter 16, adding that there are many seas which receive great rivers and yet have no tide, or do not have a tide proportionate to the size of the rivers. Vicomercatus [Francesco Vimercati], for his part, used to object to Timaeus the scantiness of the water the rivers carry into the sea, which is unfit to raise so vast and wide a surface by a perceptible increase. Chiaramonti, on the contrary, in Book 13 of *De universo*, chapters 17 and 19, undertakes to demonstrate that the water brought in by the rivers bears a perceptible proportion to the water of the sea. In chapter 17 he supposes, for good measure, that the Earth is covered with waters on every side, and that their depth everywhere is one Italian mile; that the semidiameter of the Earth alone is 3034 Italian miles, but with the water 3035, and so the diameter is 6070. He makes its ratio to the circumference 7 to 22, and therefore the circumference 19,077 miles. From this, by Archimedes' rules, he gathers that the solidity of the terrestrial sphere together with the waters is 117,142,218,733 cubic miles. [Margin: The quantity of the water of the sea, by Chiaramonti's calculation.] Likewise, taking the diameter of the Earth alone as 6068, he gathers a circumference of 19,070 and a solidity of 117,033,475,546 cubic miles. Subtracting these cubes from the former, therefore, there remain for the water alone 108,743,187 cubic miles, so that the Earth alone contains the water 1076 times. But because only half of the Earth is covered with waters, and even this half is not everywhere one mile deep but at most half a mile, only a fourth part of the preceding sum actually falls to the water of the sea, namely 27,185,797 cubic miles. In chapter 19, to investigate the quantity of water that the rivers carry into the sea, he uses the river Arno, on which, he says, boats travel from Florence towards Pisa as far as Callo, 40 miles distant, in 8 hours when the river is moderately fast, and that without oars or sails. So that water covers 40 miles in 8 hours, that is, 13,333 perches (for he gives a mile 333⅓ perches), and therefore completes a single perch of length in at most two seconds of an hour. Now let the Maragnon have the same velocity; its width at the mouth is 70 leagues, that is (says Chiaramonti), 105 miles, or 35,000 perches; and let its depth likewise be one perch everywhere at the mouth. Then the

Maragnon will pour out 35,000 cubic perches in two seconds of an hour, and in a whole year 3,030,480. What of so many other rivers of the whole world?

In the calculation just described, however, considering the water the rivers bring in over a whole year is superfluous, since the tide of the sea occurs in many places daily, and every twelve hours. One ought therefore to have considered the water brought in in 12 hours, or at most in one day. Then, to establish what proportion it bears to the water of the whole sea, one ought to have gathered a more probable sum from all the rivers of the world, at least those described in geographical maps and histories, and compared it with the water of the sea. We have done this elsewhere, in a complete little work shared with judicious men, which we shall perhaps publish in Volume 2 of the *Almagest*, Book 4, which will be the geographical book; and we showed, with the greatest probability, that all the known rivers taken together do not pour more than 150 cubic miles into the sea in 6 hours. [Margin: The quantity of the sea, by our calculation.] Distributed into paces, or cubic feet, these would make 150,000,000,000 paces, and 18,750,000,000,000 feet. Now, from what was said in Book 2, chapter 14, the surface of the terraqueous globe is 336,434,176 square Italian miles. Half of it, therefore, as we said in the same place, falls to the surface of all the seas taken together, namely 168,217,088 square miles, [Margin: The tribute of the rivers carried into the sea.] which make 168,217,088,000,000 square paces, that is, 4205,427,200,000,000 square feet. Dividing these by the whole tribute of the rivers, which is 18750,000,000,000 cubic feet, the quotient comes out as 224, neglecting fractions. If, therefore, the water of the rivers were to be distributed over each foot of the sea's surface, added at an equal height everywhere, the height of water falling to each foot would be a 224th part of a foot — scarcely the thickness of a grain of barley. So far is that water from being able to produce the tide of the sea; and much less still, if the sea transmits a great part of the water it receives from the rivers through subterranean passages to another neighbouring sea, and that sea to other abysses, as Fournier learnedly discusses with regard to the Caspian and the Mediterranean in Book 9 of the *Hydrographia*, chapter 6.

[Margin: 2. The opinion of Plato.] XXVI. The second opinion was that of Plato himself, who attributed the tide of the sea to waters alternately bursting out of Tartarus, or the abyss, and being swallowed back. Plutarch says this of him in Book 3 of *De placitis*, chapter 17: *Plato ascribes the cause to a certain lifting: for the waters are raised up by the opening of a certain cave and flow out widely, and then flow back into the same place, and from this the surging seas are tossed.* [Margin: Refutation of the opinion [?].] But from this the cause can be given of almost none of the features we reviewed at number XXIII, and especially not of those connected with the phases of the Moon. Granted — let us not deny it — that there are deep abysses of waters in the bowels of the Earth, and that their whirlpools contribute something to the variety of the tides; and Philippus, as cited by Resta, refers to this the words of Job, chapter 38, in the reading of the Seventy translators: *Have you entered into the springs of the sea?*

[Margin: 3. Opinion, and that of Pandulfus Sfondratus.] XXVII. The third opinion was that of Pandulfus Sfondratus [Pandolfo Sfondrati], in a treatise on this subject. He attributes the tide to the motions of the sea from east to west and from north to south, because so much water cannot pass at once through the narrows of the Strait of Gades, of the Strait of Magellan and the like; and he says that these motions of the sea occur so that it may run to make up what the Sun has consumed into vapours, and to keep the sea in its equilibrium. [Margin: Refutation of the 3rd opinion.] But the tide of the sea is far different from those two motions, and has far more wonderful and different features, of which no reason can be given through the motions just mentioned alone. And truly, since the Pacific does not run eastward before the tide, it cannot be the cause of the tide in the Strait of Magellan that comes from the Pacific Sea. Nor can the sea's course towards the north be the cause of those huge flows that occur at the coasts of Britain, since the sea rather runs from the north towards the equator — and not in alternating spells of six hours, but perpetually.

[Margin: 4. The opinion ascribed to Io. Picus and Buccaferreus.] XXVIII. The fourth opinion is attributed to Io. Picus and Buccaferreus [Ludovico Boccadiferro], on the ground that they refer this effect to the diversity of the shores, which resist and repel the waters, and to the differing slope of the basins in which the sea is contained — just as in a shaken vessel the water floats this way and that towards the sides unevenly, according to the vessel's bottom and its differing rims. But of Io. Picus's opinion shortly. Meanwhile, whoever's it may be, it is shown to be insufficient by the connection of the sea's tide with the Moon and by the features enumerated at number 23. [Margin: Refutation of the 4th opinion.] Add that, while the tide occurs on one shore, an ebb would have to occur from the opposite shore, which, however, no history confirms.

[Margin: 5. The opinion of Adelandus, approved by Picus.] XXIX. The fifth opinion was that of Adelandus, speaking from the opinion of the Saracens. Io. Picus writes of him thus in Book 3 *Against the Astrologers*, chapter 15: *Since,* he says, *the arms of the sea themselves — those, namely, that the interposed mass of land divides — hasten with a hurried impetus to meet and flow together, it comes about, both from the interposed mountains and from the very situation of the land, that while they fall short of that same course they are carried back, and that from there they are recalled, by the very situation of the place, to where both a paternal and a natural motion impels them.* And a little later Picus says: *This opinion of his is not greatly at variance with likelihood, since the parts of each element are borne by a natural propensity towards their own wholeness. What wonder, then, if, shut off in place by the lands lying between, and present, they strike the shores of the Ocean as they come up while they eagerly seek to run together, and, repelled by the obstacle, likewise draw back? With this cause removed (as happens in the southern ocean), we see that these tides do not appear.* Besides the differing situation of the shores, therefore, these authors suppose a natural propensity of water towards mutual concourse and, as it were, embrace through the arms of the sea, so that the parts may be preserved with their whole. [Margin: Refutation of the 5th opinion.]

But against this opinion, besides the connection of the sea's tide with the Moon and the effects reviewed at number 23, there are the arguments of Franciscus Resta, Book 3 of the *Meteorologia*, tractate 1, chapter 14, and of Cabeo, Book 2 of the *Meteorologica*, text 6, question 6. For if water seeks union with other water, and the tide occurs for that reason, it ought to occur also in lakes, pools and the Caspian Sea. Next, since, apart from the Caspian, the other seas are already continuous, what need was there of a greater union? And in what part of the sea, after all, must this union take place? Besides, there are no further seas on this side of those shores at which the sea nevertheless makes a tide by approaching them: for either the tide stops at the banks, if they are low, or, overflowing above them, it covers the shores, so that the sea seems rather to seek union with the land than with water. For what sea is there on this side of the Venetian, Gallic, British or Indian coast? None, certainly. And yet to these coasts there flows in, with so great a tide, the sea. It seems, then, rather to flee the waters that are in the middle of the ocean, and to be separated from them by a kind of sedition, than to be joined to them — unless perhaps it repents of that sedition and, by flowing back, rolls back to the place it had withdrawn from. But if you say that near the shores there is a friendly and sympathetic water with which it wishes to be joined, then let it cling to that water once it has joined itself to it, and not be parted from it by flowing back. But enough has been said against this fiction.

[Margin: 6. The opinion of the Stoics and of Apollonius of Tyana.] XXX. The sixth opinion was that of the Stoics, as reported by Solinus, chapter 36. They held that the world is an animal, and that through chasms of the earth close to the floor of the sea, as though through bodily nostrils of breath, it alternately sucks the waters back and blows them out, by a reciprocal inhaling and exhaling, and so stirs up the tide as if panting. So too Apollonius of Tyana attributes the same to spirits panting beneath the sea; and that animal faculty which Kepler attributes to the whole globe, compacted of earth and water, favours them not a little. Cabeo rises against this fiction with several arguments, Book 2 of the *Meteorologica*, text 6, question 6; [Margin: Refutation of the 6th opinion.] but it is enough to have enumerated the features of the sea's tide, connected chiefly with the Moon, as we did at number 23: for it is at once apparent that those effects cannot be produced by this panting, however great the soul. Rather, someone might attribute this to the inhaling and exhaling of the Moon, an opinion St Basil mentions in Homily 6 on the *Hexaemeron*, saying: *The sea, as if by the exhalations of the Moon, is drawn backward, and by her breathing-in again is driven up to its proper measure.*

[Margin: 7. The opinion of Alpetragius and of certain Arabs.] XXXI. The seventh opinion attributes the tide of the sea to the diurnal motion, which sweeps all things along with it. So certain Arabs, as reported by Albert the Great, *De proprietatibus elementorum*, tractate 2, chapter 6, and among them Alpetragius, if we believe Io. Picus, Book 3 *Against the Astrologers*, chapter 15. But if this opinion were true, the flow and ebb would occur only in the

direction of longitude, [Margin: Refutation of the 7th opinion.] and would be greatest under the Equator; nor would the vicissitudes of the lunar motions produce so great a variety of tides. Besides, it is false that the sea is swept along by the diurnal turning of the heaven, especially once the fluidity of the heavens is posited, for which enough was argued in Section 1 of this Book, chapter 7.

[Margin: 8. Opinion, likewise of Io. Picus.] XXXII. The eighth opinion, which Io. Picus adduces in second place in Book 3 *Against the Astrologers*, chapter 15, refers the cause to vapours and winds stirred up from the denser water: from these come turmoil and violent motion, and from that comes heat seeking more room, and from that the tide; and when this has cooled, the seas are laid flat and subside. [Margin: Refutation of the 8th opinion.] But since Picus is wholly occupied there in excluding the Moon, and yet so great a relation of the tides to the Moon is confirmed by history, he ought to have shown how, without the Moon, the vicissitudes of the tides are nevertheless adjusted to the Moon's motions. Add to this that even a calm sea, stirred by no winds, is nonetheless subject to the tide; indeed, when a violent storm comes on, the tide properly so called is diminished, as Trimarchus affirms, Book 2 of the *Meteorologica*, disputation 1, section 7.

[Margin: 9. The opinion of Thomas Lydianus.] XXXIII. The ninth opinion attributes the tide of the sea to ebullition, but ebullition caused by heat and subterranean fires — to which I, as a younger man, had ascribed a large part of this cause. So Thomas Lydianus [Thomas Lydiat], as reported by Fromondus, says that a fire lies hidden beneath the floor of the sea and kindles the abundant bitumen present there, so that the sea swells as the fire exhales. And although Franciscus Resta, Book 3 of the *Meteorologia*, tractate 1, chapter 14, does not think it improbable that the tide is a kind of ebullition of the sea, like that of boiling water, produced by spirits and breaths rising from the sea floor — for if a spirit shut up in the earth stirs up earthquakes and moves the earth, how much more easily can it move the waters and raise them — he nevertheless afterwards drew back from this view. [Margin: Refutation of the 9th opinion.] For the necessity always remains of giving the reason why the tides occur with those vicissitudes preserved which we reviewed at number 23, and why they receive their laws from the Moon in so many effects. Cabeo adds, Book 2 of the *Meteorologica*, text 6, question 6, that motions produced by fire are lawless and tumultuous, as appears in earthquakes, in Vesuvius, in Etna; but tides are periodic and return at fixed times. Finally, against this opinion stand the arguments against rarefaction to be adduced at number 37.

[Margin: 10. The opinion of Lessius.] XXXIV. The tenth opinion has recourse to Intelligences, which, just as they are the movers of the heavens, are also the movers of the seas. This opinion is attributed by Fromondus to Leonardus Lessius [Leonard Leys], who asserts that the Intelligence presiding over the waters is called in the Apocalypse "the angel of the waters". On this occasion Franciscus Resta, in the passage just cited, says that it could also be attributed to evil spirits, since of Behemoth, that is, an evil demon, it is

said in Job 41: *He will make the deep sea boil like a pot, and will make it as when ointments boil.* But this opinion has not been accepted, for three reasons. [Margin: Refutation of the 10th opinion.] First, because it has recourse to a machine without necessity, or out of weariness with inquiring into the proper cause in nature. Second, because the tides of the sea are irregular in many places, and so do not seem to be the work of an Intelligence. Third, because the sea's tide is altogether lunatic [Lunaticus], that is, adjusted to the vicissitudes of the Moon; so it seems that recourse should be had to the Moon rather than to an Intelligence — and to the Moon, in the end, most authors have recourse, though without excluding the Sun, as we shall soon see.

[Margin: 11. The opinion of Pliny and Pytheas.] XXXV. The eleventh opinion, then, throws the tide of the sea back onto the Luminaries, but chiefly onto the Moon. Pliny the Elder, indeed, asserted this without determining it, in Book 2, chapter 97, saying: *That the tides of the sea come in and go back is most wonderful: in many ways indeed, but the cause lies in the Sun and the Moon*; nor does he explain the manner. But in Plutarch, Book 3 of *De placitis*, chapter 17: *Pytheas of Massilia: the approach of the sea comes about with the waxing of the Moon, and its recession with her waning.* This opinion has by far the most authors on its side; but the whole difficulty lies in the manner in which the Luminaries produce the varieties of the tide that we adduced at number 23 — the Moon especially, even when she lies hidden below the horizon, or is new and does not reflect the Sun's rays to the earth and the sea, or is within the Torrid zone and does not strike the waters of the temperate or frigid zones with perpendicular rays. Meanwhile, to the common opinion belongs that passage of Caesar, Book 4 of the *Commentaries*: [Margin: Caesar's opinion.] *On the same night it happened that there was a full Moon, which usually makes the greatest tides in the Ocean.*

[Margin: 12. The opinion of Contarenus and Faber.] XXXVI. The twelfth opinion, accordingly, has recourse to occult influences of the Luminaries, as do Contarenus [Gasparo Contarini] and Faber, and certain astrologers whom Picus censures in Book 3 *Against the Astrologers*, chapter 15 — on the ground that the Moon could not move the waters in this way when she is beneath the lowest part of the earth, since the light of the Moon, which is the vehicle of influences, cannot then reach them. But this opinion does not solve the question; it is just as if one said that the cause of the tide is unknown and hidden from us. [Margin: Refutation of the 12th opinion.] Cardinal Contarenus, moreover, adds that by this occult power of the Moon the waters of the sea are rarefied, and that, once they have been rarefied to the highest degree, they produce the tide. But it does not appear how the Moon, placed beneath the lowest part of the earth, could rarefy them — besides the other objections we shall shortly raise against this rarefaction.

[Margin: 13. Opinion.] XXXVII. The thirteenth opinion, closely akin to the ninth, holds that the Luminaries, and above all the Moon, produce the tide by heating and rarefying the waters and thinning them into vapours, but

not consuming them. [Margin: Of Contarenus, Aversa, Crescentius, Borrus, Rheita.] So hold Contarenus, Book 2 of *De elementis*; Raphael Auersa [Raffaele Aversa], tome 2 of the *Philosophia*, question 42, section 7; Crescentius, Book 3 of the *Nautica*; Hieronymus Borrus [Girolamo Borro], in his treatise on this subject, page 125; and most recently Antonius Maria de Rheita [Anton Maria Schyrleus de Rheita], in his *Radius Sidereomysticus*, Book 4, chapter 3. For they say that the Moon, as she ascends, gradually rarefies the waters until, as she comes to the meridian, they swell, rarefied to the highest degree; and that as she descends from the meridian, striking the waters with more oblique rays, she diminishes the rarefaction, and the waters, condensed, flow back more and more and return to their former state, or are resolved into vapours. But Galileo, in the fourth dialogue *On the System of the World*, Chiaramonti, Book 13 of *De universo*, chapter 22, and Franciscus Resta, tractate 1 *De mari*, chapter 14, prove that the waters of the sea are not rarefied by the Moon's heat during the tide. For if the water were rarer, it would also be lighter in the same bulk; some heat would be felt in it, and some smoke would appear, as in boiling water. But if an equal quantity of the same sea water, drawn before the tide and during the tide, is weighed, it is found to be of the same weight, and no sign of rarefaction or heat is detected in it, whether from density or from smoke. Nor do ships sink deeper during the tide than at other times, which would necessarily happen if the water were rarefied and became lighter. [Margin: Refutation of the 13th opinion.] Galileo adds that, if the tide were produced in this way, a violent heat would be necessary (which cannot come from the Moon), and so great that it would make the sea water unbearable from its heat to a finger dipped in it. Against this view Cabeo, Book 2 of the *Meteorologica*, text 6, question 6, adduces must, which, swelling violently, boils without unbearable heat; and, conversely, water heated long before, which nevertheless boils no more, because its subtler parts, rarefied in the first boiling, into bubbles have vanished, and these into the air. But granted that rarefaction is not ruled out on this head, it still remains to give the reason why the Moon, placed beneath the earth, can rarefy the waters more as she approaches the lowest meridian than when she is above the horizon and receding from the meridian; and why she can do more on more remote waters, such as those around Ireland, England, Holland and Belgium, where the tides are strongest, than on nearer ones, such as those from Gades as far as the Tropic of Cancer, where there is no perceptible tide; and why, likewise, she does more at new Moon than at the quadratures, in which she both pours more light onto the waters and comes nearer to them; and so of the other diversities adduced at number 23. To the first difficulty, however, Borrus and others answer, with Albumasar, Book 2 of the *Introduction*, that the rays of the Moon, when she is below the horizon, are reflected back onto the earth and water of our hemisphere by the opposite heaven. But these men suppose that the heaven is hard and fit for reflecting light; and besides, that light would be much fainter, and the flow would accordingly be almost three times smaller.

[Margin: 14. The opinion of Scotus, Io. a S. Thoma, Resta, Mastrius, and nearly that of Pliny.] XXXVIII. The fourteenth view has recourse to that power of the Moon by which she predominates over humours. For just as the humours of plants, of shellfish and of the other animals are stirred, and abound especially at full Moon, so the waters of the sea seem able to be stirred and to swell by the action of the Moon, and to grow not only through rarefaction but also through a new increase. So Scotus, on Book 2, distinction 14, question 3; Ioannes a S. Thoma [João Poinsot], as reported by Mastrius, disputation 4 *De caelo ac meteoris*, question 4, number 154; nor does it displease Mastrius himself; Resta acquiesces in it, tractate 1 *De mari*, chapter 14; and Pliny favours it, Book 2, chapter 99, where, after setting out the tide of the sea in chapters 97 and 98, he says: *From which a true conjecture arises, that the Moon is not without reason thought to be the star of breath; that it is this which saturates the earth, and as it approaches fills bodies, and as it departs empties them; that therefore shellfish grow with its increase, and that those creatures which have no blood feel its breath most*, etc.; and in chapter 101: *The Moon, a feminine and soft star, loosens moisture and draws it, does not carry it away.* Just as shellfish, then, abound in moisture — not because they feed more copiously at full Moon, as Vallesius [Francisco Vallés] would have it, chapter 71 of the *De sacra philosophia*, but because of the warmer nights and the heat that loosens congealed humours, as Aristotle teaches, Book 1 of *De partibus animalium*, chapter 5 — so tides occur when the thick humours are loosened. [Margin: The 14th opinion is refuted.] But against this account there militate the things we brought, toward the end of number 37, against opinion 13; and besides, all waters — of the sea, of rivers and of lakes — ought to undergo a tide, at least at full Moon, if indeed the Moon, by predominating over all moist bodies, can produce the swelling of moist things.

[Margin: 15. The opinion of Aristotle, Heraclitus, Thyaneus, Vallesius.] XXXIX. The fifteenth opinion [?] refers the cause of the tide to breaths and blasts beneath the waters and around the waters of the sea, stirred up by the Sun and by the Moon according to their varying disposition. Plutarch says, Book 3 of *De placitis*, chapter 17: *Aristotle and Heraclitus hold that this is produced by the Sun; for by its turning it carries round with it a great many blasts, and when these rush in and push the Atlantic forward, the Atlantic swells and makes the tide; and when they are at last lulled, it draws back, gathering in its tide, and this is the reciprocation.* Perhaps Thyaneus [Apollonius of Tyana] meant the same when he said that it is produced by spirits bursting out of the sea and stirred up around it; and so too Vallesius, chapter 52 of the *De sacra philosophia*, who refers the proximate cause of the tide to blasts shut up within the caverns of the earth, but the remote cause to the Moon and the Sun, which generate and stir those blasts. [Margin: The opinion of Trimarchus.] Trimarchus too, Book 2 of the *Meteorologica*, disputation 1, section 7, numbers 86 and 87, refers this motion to exhalations under the sea, like those under the earth at the time of earthquakes, which, once they have been rarefied by the heat of the Sun and the Moon, seek an exit and burst out with great force; sailors attest that when the tide comes

on, ships tremble, even in calm weather, as in an earthquake. These exhalations bursting out into blasts, he thinks, were infused into the sea by God from the beginning of the world, yet are preserved by the action of the stars; and, pressed by the mass of the sea, they are driven back into the caverns, and from this the ebb results. [Margin: The opinion of Bacon.] Indeed, before him Rogerius Baccon [Roger Bacon], in the *Letter to Clement V*, Supreme Pontiff, said that the light of the Moon stirs up vapours under the waters, which engorge the waters with bubbling swellings, vapours that it cannot consume; that fewer are stirred up by weaker rays, when she shines obliquely on the seas, and more by stronger and more direct rays, when she strikes them on the meridian; and that the same rays, while the Moon is beneath the lowest part of the earth, are reflected from the highest heaven, or from the eighth or ninth sphere, as being dense and bounding the sight. But he adds that the tide becomes more sluggish as the Moon nears the meridian, because of the vapours dissolved by the stronger rays; and for this he is censured by Io. Picus, Book 3 *Against the Astrologers*, chapter 15, on the ground that he speaks against the experience of a greater tide on the meridian.

[Margin: Cabeo's opinion.] Our own Cabeo, Book 2 of the *Meteorologica*, text 6, question 9, having taught that this motion does not arise from rarefaction or from a real increase of the water, nor from a local motion alone impressed by the Moon — since then the tide would occur everywhere and always toward the west — adds that in the Moon, besides heat and light, there are other active qualities, manifest to us in their effects though unnamed, from which come the catarrhs and very troublesome fluxes in lunatics at the time of the lunar syzygies, even if they live shut up in their rooms. Similarly, then, the tide arises not from a thinning of the water itself, but from a thinning, or ebullition, of spirits rising from the floor of the sea — not as though they boil from heat, but because they are moved by the Moon by a peculiar force, just as a swelling occurs in must without heat, and just as catarrhs are moved by the Moon. The spirits on the floor of the sea, moreover, are analogous to the spirits of saltpetre and sulphur, and so the tide occurs only in those waters in which there are spirits of this kind; nor do clouds, or the earth interposed between the Moon and our sea, stand in the way, just as the walls of houses do not prevent the Moon from stirring, by her peculiar power, the spirits or humours in our bodies and provoking fluxes or catarrhs. [Margin: Fournier's opinion.] Finally, our Fournier likewise, Book 9 of the *Hydrographia*, chapter 8, professes that the tide of the sea is a kind of secret and a very great mystery to be observed in the sea, but that its probable cause is a stirring, by the Moon, of the vapours and breaths enclosed in the sea water. For just as the causes of fevers can be reduced to two heads — namely, to the disposition of the humours of the human body and to the active power of an external agent — and the mass of humours, like a kind of leaven, is cooked by the force of the external agent, is inflated, putrefies and, percolated through the blood, is inflamed, so that from the diversity of the inflammable humours there arises a diverse species of fevers, and from that same

diversity, and also from the diverse power of acting of the external agent, the motions and periods of fevers are slowed or quickened: so the material cause of the sea's tide is hot vapours and breaths, which are in the soil and ground lying beneath the sea and are probably of the same nature as the rest of the earth from which metals are dug. For it is established that in digging these out, when the diggers have gone beyond 80 or 100 paces, though they always felt cold before, they soon meet so great a heat that they cannot bear it beyond three hours without danger and must withdraw from there; this has been reported above all by those who explored the mines of Hungary, whose depth is greater than 300 cubits. And although hot and viscous breaths and vapours of this kind come from the Sun, the Moon nevertheless has the power to stir them by her influxes; and, swelling them further, partly by her light and partly by her other influxes, she raises the seas and causes the tide, according to the disposition both of the vapours and breaths already mentioned and of the harbours, shores and coasts. From this, too, comes what Pliny said in Book 2, chapter 98: *All seas are purged at full Moon, some also at a fixed time. Around Messina and Mylae refuse like dung is cast out on the shore, from which comes the fable that the oxen of the Sun are stabled there.* Strabo had noted this earlier, in Book 1 of the *Geography*, saying: *When the sea belches, the shores give back a hoarse voice, so that the flood has a certain force such that it repels what is foreign. And indeed they call this a kind of purging of the sea, by which both corpses and the fragments of shipwrecks are carried out onto the land.* Finally, Fournier says that if anyone attentively considers the history of the sea's tide, he will find in it all the symptoms of fevers, and that the only difference lies in two points. [Margin: The tide of the sea [?] is its fever.] The first is that the Moon applies herself to stirring the sea more regularly than any external agent applies itself to the human body in provoking fevers; the second, that the human body, because of the greater compactness of its parts, cannot swell from fevers and alter its former shape. Had Pliny heard this, he would far more willingly have pronounced what he said in Book 2, chapter 97: *But these motions are detected about the shores more than in the deep, since in the body too it is the extremities that feel the pulse of the veins, that is, the breath, more.*

XL. The sixteenth opinion prefers to recognise in the Moon a certain magnetic power, by which, through loca- l motion, she attracts the waters to herself and entices them, until, by their own weight and because of the excessive heap, they fall back again and subside. [Margin: 16. Opinion: of the Conimbricenses, Zanardus, Scaliger.] Of this opinion seem to be the Conimbricenses, on Book 2 of the *Meteorologica*, tractate 8; Zanardus, question 30; and Scaliger, Exercise 52. Scaliger, having said of the tide of the sea, on the authority of many authors, *Since it had been observed to follow the course of the Moon, they judged the Moon to be its author*, and having at once objected, *But the Moon does not touch the waters*, at once solves the objection, saying: *This gave trouble to some of the Peripatetics, to whom the magnet too ought to have given it. If that motion is produced in iron by the stone without contact, why will the sea not follow the body of the noblest star?* And towards the end of that exercise: *I would dare to affirm that there is then no*

new generation of waters — namely, when the tide occurs; *with what purpose I should profess rarefaction, I truly do not know. But that they are lifted up at that time, like iron by the magnet, every month, in all places except that one shore alone, has been found by continual observations.* He speaks chiefly of the full Moons, at which, however, the increases of the sea at the coasts of the Indus are smaller than at the new Moons. [Margin: Gilbert's.] William Gilbert tacitly insinuates this very thing, Book 6 of *De magnete*, chapter 6, when he attributes to both the Moon and the Earth a magnetic power for attracting and leading round kindred bodies. [Margin: Kepler's.] But most lucidly of all Kepler, in the Introduction to the *Commentaries on Mars*, whose opinion we shall set out only in his own very words, distributed into several propositions, which are as follows.

[Margin: Kepler's theses [?] on the tide of the sea.] 1. *If the Moon and the Earth were not held, each in its own circuit, by an animal force or some other equivalent one, the Earth would ascend to the Moon by a fifty-fourth part of the interval, and the Moon would descend to the Earth by about fifty-three parts of the interval, and there they would be joined — supposing, however, that the substance of both is of one and the same density.*

2. *If the Earth ceased to attract its waters to itself, all the waters of the sea would be lifted up and would flow into the body of the Moon.*
3. *The orb of the attractive power that is in the Moon is stretched out as far as the earth, and entices the waters under the Torrid zone — to meet it, that is, wherever it falls on the zenith of a place: imperceptibly in enclosed seas, but perceptibly where the beds of the Ocean are widest and the waters have ample freedom of reciprocation. When this happens, the shores of the lateral zones and climates are laid bare, and any shores, even under the Torrid zone, formed by the more recessed bays of the neighbouring Ocean. And so, as the waters rise in the wider bed of the Ocean, it can happen that in its narrower bays — provided they are not too tightly enclosed — the waters even seem to flee from the Moon when she is present: for they subside, their supply of waters having been drawn off outside.*
4. *But since the Moon flies swiftly across the zenith, and the waters cannot follow so swiftly, a flow of the Ocean does indeed occur under the Torrid zone towards the west, until it strikes the opposite shores and is bent back by them; but at the Moon's departure the gathering of the waters, or the army that is on the march towards the Torrid zone, is dissolved, as being deserted by the traction that had called it forth; and, having gained an impetus, it returns, as in vessels of water, leaps against its own shores and covers them; and this impetus, during the Moon's absence, begets another impetus, until the Moon, returning, takes up the reins of this impetus, moderates it, and leads it round together with her own motion. Thus all shores equally open are filled at the same hours; the more recessed ones later, and some in different ways because of the differing approaches of the Ocean.*

Yet the same Kepler, in the *Epitome of Copernican Astronomy*, Book 1, page 128, adds as an auxiliary cause of the tide the natural inertia of the waters, which

hang back in the west while the earth withdraws itself towards the east; and from this, he says, comes a flow of the waters that makes navigation towards the west easier, which, however, is not felt on the shores of America towards the east, because of the vast length of the Ocean, which strikes no obstacle.

[Margin: Refutation of the [?]6th opinion.] But how many things are asserted here, how many concealed, against the testimony of history? For how false it is that the tide occurs primarily and of itself from east to west, when several tides occur from north to south and the reverse. Moreover, if the Moon enticed the waters by a magnetic force and by local motion alone, she would certainly entice those that are nearer to her, or would entice all of them most when she herself came nearest to them. But the opposite happens: for the greater tides occur at the English, Dutch and Belgian shores, whose waters are farther from the Moon than in many parts of the Torrid zone; and when the Moon, at the quadratures, descends towards the earth, the tides are nevertheless the smallest. Again, if in more recessed seas the tides are smaller than in more open ones, why is the tide not greater in the Tyrrhenian than in the Adriatic, especially in places set under the same meridian? Finally, there are many other features, recounted by us at number 23, whose cause does not appear in this magnetic power.

[Margin: 17. Opinion. Of Seleucus. And of Galileo.] XLI. The seventeenth opinion ascribes this motion of the sea to the motion of the Earth, diurnal and annual together. This was devised long ago by one Seleucus, whoever he was, of whom Plutarch speaks thus expressly, Book 3 of *De placitis philosophorum*, chapter 17: *Seleucus the mathematician, who himself also moves the Earth, says that the turning of the Moon runs counter to the Earth's revolution and motion; and that, as by this impetus the breath gathers itself between these two bodies and rushes into the Atlantic, the sea, with probable reason, is thrown into disturbance.* But Galileo, though passing over Seleucus's name in silence through forgetfulness, criticises his opinion in the fourth dialogue *On the System of the World*, Latin page 343, on the ground that the Moon's motion is not contrary to the Earth's motion, since it is directed towards the same quarter. And so Galileo himself explained the tide of the sea in another way, through the unequal motion of the Earth, as we have already related from number 1 to number 10. We shall shortly show this to be most evidently false, from number 43 on.

[Margin: 18. Our opinion.] XLII. The eighteenth — our own opinion, gathered and selected from many of the preceding — is that the tide of the sea is likewise produced by the Moon, which stirs the humours, vapours and breaths on the floor of the sea, and moves them without perceptible rarefaction at the surface of the sea, according to their diversity, as opinions 14 and 15 hold; yet in such a way that to its variety there concur not only the Sun and the other positions of the stars, but also the inrush of rivers nearer to the tidal places, as opinion 1 holds; and the twofold course of the sea, of which we spoke from number 11 to 15, as opinion 3 holds; and the whirlpools of waters vomiting out the waters from subterranean chasms and sucking them back again, as opinion

2 holds; and the differing situations of the basins and shores, as opinion 4 holds; and at times winds, as opinion 8 holds; and subterranean fires, as opinion 9 holds. Yet even when all these are fused into one, the effects we enumerated at number 23 will always be worthy of wonder, and especially the one seen in the daily period of the tide that returns when the Moon, below the horizon, approaches the lowest part of the meridian. For I am not yet persuaded that the Moon transmits her power through the whole terrestrial globe, and that it is not at all blunted or weakened; or that it reaches as far as the opposite part of the highest heaven, from which it would be reflected to us after twice traversing an immense space (since we hold that the other heavens are by no means solid). Nor can we explain this daily vicissitude through the Moon's monthly libration and the varying emergences or concealments of her spots at the edges. Nor have I so far found anyone who satisfies me on this point; and I am not ashamed not to know it, since I do not doubt that I am ignorant of many other things easier to know. [Margin: The final cause of the sea's tide.] The final cause of these motions Franciscus Resta touched on briefly and readily, following Bodinus [Jean Bodin], Book 3 of the *Meteorologia*, tractate 1, chapter 15. For the tide of the sea serves now for washing away and keeping off its filth, now for fisheries, now for the more convenient arrival of ships for bringing in and carrying out goods, now for salt-pans, the natural workshops for making salt, now for stirring up many animals; and I add, for supplying abundant matter for the praises of God the Creator and for acknowledging the weakness of our wit. Now to Galileo, who imposed on us the necessity of so lengthy a treatment, lest we should seem to wish to demolish with a light arm an argument that he himself judged to be of such weight for the motion of the Earth.

Refutatio Cauſæ Diurni Aeſtus à Galilæo adductæ.

Refutation of the cause of the daily tide adduced by Galileo.

XLIII. Here we must go over again the doctrine of Galileo set out from number 4 to 7, together with the diagram given in the same place. For he said that the cause of the daily flow and ebb, which returns every twelve hours, is the unequal diurnal motion of the Earth HKFG about its own centre B, towards the eastern quarter, from H to K; that this motion is faster in the upper semicircle GHK, because it assists the motion of the centre B, carried by the force of the annual motion towards the same quarter, from B to C; but that in the lower semicircle KFG the Earth's motion is slower, since it strives against the annual motion of the centre B; and that therefore there occurs alterna- tely the flow and the ebb, and the other things we described there. But against this cause there are very many arguments, partly of others, partly our own; and the first, that of Chiaramonti, which strikes directly at Galileo's foundation, must be set out, although in truth it is invalid.

Diagram. Galileo's figure for the tide, set in the left column. A small circle about the centre B carries the points H (top), K (left), G (right) and F (bottom). A larger circle passes above it through O, and a long arc runs from P on the left through O to Q at the right. Below, a large circle about A carries C (left), E (right) and D (bottom), with M and N inside it on either side of A and an inner arc running from M up towards L and down to N. Two small crescent Moon signs are drawn, one on the arc just below O and one beside L.

- O (top, on the upper arc): O
- H, K, G, F (on the small circle about B): H, K, G, F
- B (centre of the small circle): B
- P (left end of the upper arc): P
- sign at the right end of the upper arc: a loop with a curled tail resembling the sign of Leo [?]
- L (below F): L
- A (centre of the large circle): A
- M, N (inside the large circle): M, N
- C, E, D (on the large circle, left, right, bottom): C, E, D

[Margin: 1. Argument against Galileo, from Chiaramonti and Cabeo, but invalid.] *First.* Scipione Chiaramonti, in his Italian defence of the *Antitycho*, part 4, chapter 25, charges Galileo with a truly shameful equivocation: Galileo transfers properties that presuppose a true and real inequality of motion to an inequality that is merely apparent. For the inequality of the Earth's diurnal motion (this motion being granted meanwhile for the sake of argument) is not real, says Chiaramonti, but optical, and merely apparent to an eye imagined to be at A. The parts GHK will appear faster beneath the Firmament, because the centre B of the circle GHKF is carried forward in the same direction, as if it were the centre of an epicycle; but the parts KFG will appear retrograde a little below the points K and G, because their motion is contrary to the motion of the centre B. It is by this reasoning that Galileo himself tried to show that the retrogradation of the planets is not real but apparent. Add that for us, who are not at A but on the surface GHKF, on which we watch the motion of the sea, the motion of the Earth not only is not unequal — since Galileo admits that in reality it is equal in itself — but cannot even appear unequal: both because we are turned in the same way with the Earth, and because we cannot observe by any sense even any motion of it at all, let alone its inequality. But if we were at the centre B, the motion of the Earth would appear equal to us.

[Margin: Galileo's argument in form.] Now Galileo's argument, reduced to form, ran like this: *In every vessel full of water and moved now more slowly, now more quickly, the water flows now to one edge, now to the other edge. But the Earth is a vessel of this kind; therefore, etc.* To this argument Chiaramonti grants the major premise as regards real speed and slowness, and denies it as regards merely apparent; the minor premise, by which the Earth is supposed to move more slowly and more quickly, he denies as regards real speed and slowness, and grants as regards apparent speed, for an eye imagined at A. Cabeo touched

on something similar, somewhat obscurely, in Book 2 of the *Meteorologica*, text 6, at the end of question 8. But both are mistaken. For not only for an eye at A, but for one placed anywhere, the point H of the Earth, in the space of the world, through the motion compounded of the diurnal motion about the centre B and the annual motion of the centre B about the centre A, would have completed a much longer path than if it had been moved by the diurnal motion alone about the centre B. So much so that, according to what was said with Kepler in chapter 6, number 2, such a point on the Equator would complete in one hour 240 German miles by the diurnal motion, but 740 by the force of the annual motion, that is, 980 miles by the compounded motions; whereas the point F would complete 500 miles in one hour.

[Margin: 2. Argument against Galileo.] *Second.* Galileo contradicts both himself and Copernicus, since he attributes to the water a motion different from, and sometimes contrary to, the motion of the Earth, and yet attributes its origin to the diurnal and annual turning of the Earth itself, and not to anything else. I too noted this inconsistency long ago, and I have seen it remarked by Chiaramonti in the defence of the *Antitycho*, part 4, chapter 25, and by our Fournier, Book 9 of the *Hydrographia*, chapter 15. For Copernicus, raising against himself in Book 1 of the *De revolutionibus*, chapter 7, the objection why heavy bodies thrown straight upward fall back onto the same point of the Earth, although that point has meanwhile been carried away from under them with such speed, and why the clouds themselves, and whatever else hangs in the air, do not appear always to be carried to the west if we are rolled with the Earth towards the east, replies in chapter 8 with these words: *What then should we say of the clouds and the other things hanging in the air in any way, or sinking down, or again tending upward? Only that not only the Earth with the watery element joined to it moves in this way, but also no small part of the air, and whatever things have a kinship with the Earth in the same way. Either because the neighbouring air, mixed with earthy or watery matter, follows the same nature as the Earth; or because the motion of the air is acquired, a motion it shares from the Earth through contiguity, in perpetual revolution and without resistance.* Galileo accepts this doctrine in Dialogue 2 on the System of the World. Therefore, if not only watery bodies but also the clouds and things like them, which are near the Earth and yet cohere with it less, are moved with the same speed and kind of motion as the Earth towards the east, then the whole water of any sea must likewise be moved equally with the Earth's globe in the same direction, if this common motion is considered; nor could it be moved elsewhere, or unequally, except by some particular motion from outside, for instance by winds. Therefore, when Galileo attributes to the sea a motion of flow and ebb different from the diurnal turning of the Earth, on the ground that water is fluid and does not wholly obey that motion — which the air near the Earth, though much more fluid, does obey — he contradicts both Copernicus and himself.

[Margin: 3. Argument.] *Third.* If the diurnal motion of the Earth together with the annual could produce the flow and ebb of the sea, it would have to

produce it through a sudden change of motion and an abrupt passage from speed to slowness, or from slowness to speed, as is evident in a vessel full of water: for unless the water is shaken by a sudden change, it does not perceptibly surge, nor is it raised now to one edge, now to the other. But even if the Earth is admitted to move unequally, the change from speed to slowness still does not happen all at once, in a single impulse, nor immediately, but gradually and by degrees, without being felt, about the points K and G, exactly as in planets moved on an epicycle. I afterwards found this argument in Cabeo, Book 2 of the *Meteorologica*, text 6, question 8. Indeed, this inequality is itself imperceptible, especially far from the points F and H, as we shall show in chapter 19, from numbers 9 and 11.

[Margin: 4. Argument.] *Fourth.* From the account of the daily tide of the sea given from number 17, and from the nautical rules for determining the hour of the highest tide, or high water, given from numbers 20, 21 and 22, it is evident that the tide of the sea has a very great connection with the Moon's motion, both daily and monthly. It is evident, too, that in the same places where it occurs, the tide is postponed each day by about 48 minutes — that is, by as many minutes as the Moon rises later each day because of its proper motion — and that the highest tide occurs as the Moon approaches the meridian above or below the horizon, even outside midday or midnight. Sailors gathered these rules from long observation and very certain experiments, since ignorance of them would have been very dangerous to them. But if the tide of the sea depended on the motion of the Earth, and took its vicissitudes and laws from it, the rules already mentioned would be vain and most deceptive: for no cause can be drawn from the motion of the Earth why the highest tide of the whole day or night should occur as the Moon approaches the meridian, or why it should be delayed each day by 48^{m} of time [minuta horaria]. And so Galileo, in that Dialogue 4, as if in jest, not to say contempt, belittles the opinion of those who ascribe the tide of the sea to the Moon; and he either shows himself ignorant of nautical matters, or cunningly conceals their rules. But he could not deceive the experts, nor will he ever be able to persuade sailors to take the rules of the tides from this feigned motion of the Earth, uncertain and imperceptible as it is, and to abandon those which they obtained from the motion of the Moon, with such agreement among practitioners and such success in navigation.

[Margin: 5. Argument.] *Fifth.* If the flow and ebb of the sea came from the diurnal whirling of the Earth towards the east, but unequal in the way Galileo imagines, it would have to be universal for every sea extended along the longitude of the Earth from west to east, and be felt alternately on every eastern and western shore. And thus the coast of Africa, which runs from the western mouth of the Strait of Cádiz [Strait of Gibraltar] as far as the Tropic of Cancer, and is free of tide, would have to experience a tide when the waters returned from the eastern coast of America; or the western coast of Spain would feel a tide as the waters came back from the coast of Syria, at least during those 8 hours in which the people of Cádiz have observed the waters of the Mediterranean

flowing out through that strait into the Ocean. I say this because Galileo says that no tide occurs on the eastern coast of Spain, on the ground that the waters of the Ocean, entering through the Strait of Cádiz, resist the waters of the Mediterranean carried towards Spain; which is not true of the whole day, but at most of 15 or 16 hours.

[Margin: 6. Argument.] *Sixth.* If the tide of the sea came from the diurnal turning of the Earth, as Galileo holds, then on those shores which face towards the eastern or the western quarter, and at the same time lie under very nearly the same meridian, the tide would occur in the same way, at the same hour and with the same measure, without perceptible difference. But this conflicts with daily experience and with the account of the tide already given: for if anyone considers the tides that occur on the coasts of England, the Low Countries, France, Portugal and Guinea, he will find a very marked difference both in the time and in the measure of the tides. Moreover, it would have to be that at the time when the sea is high on one eastern shore, it is low on the western shore opposite it, which is likewise against the account of the sea's tide; and much less could the waters of two seas, meeting each other in the same stretch of sea, cause a tide; rather, at the same time both would run towards the east, or both towards the west. But it happens otherwise in the Strait of Magellan: there the waters rushing in from the Atlantic Ocean on one side and from the Pacific Sea on the other, and meeting each other, produce a notable tide in the middle of that strait, and afterwards, subsiding, flow back, the one into the Atlantic, the other into the Pacific.

[Margin: [?]. Argument.] *Seventh.* If the sole cause of the sea's tide were the inequality of the Earth's motion towards the east, then certainly no perceptible or great tide could occur on shores and in ports situated towards the north or the south; and that is why Galileo, from either ignorance or cunning, denied that a tide occurs in the Red Sea. But the account of the sea's tide shows, by very many plain experiments, that this is false. For a tide occurs not only in the Red Sea, but also on the southern coasts of America beyond the Gulf of Mexico, and likewise on the southern coast of the East Indies from Calicut to Cambodia, where it is so rapid and violent that in two hours it covers at least 15 miles of shore, and in as many hours, flowing back, lays the same shore bare; it drives the waters of the river Indus back upstream towards their source, and then, flowing back, leaves ships high and dry. And if the English, the Dutch and others who sail every year to Spitsbergen to take whales, as far as the 80th degree of polar altitude, are asked, they will testify that the waves of the tidal Ocean roll down and rush from south to north, and indeed more violently than happens anywhere from east to west, or from west to east. So too in the Archipelago of St Lazarus the tide is carried every day towards the south with such force that it breaks ropes 27 fingers thick, as Fournier relates in Book 9 of the *Hydrographia*, chapter 16; he uses this very argument against Galileo, as does Cabeo, Book 2 of the *Meteorologica*, text 6, question 8.

[Margin: 8. Argument.] *Eighth.* The continuous motion of the Earth towards the east, so rapid that in 24 hours it would turn the whole globe of the Earth with the waters enclosed in it, would hinder not only the tides strictly so called, which occur through the motion of the sea towards the poles or the Equator, but also the other flows of the sea running towards the north or towards the south: for it would at length overcome their motion and roll them along together with itself. But it is established that the waters of the sea run from the north or from the south towards the Equator, as is clear from the account given at number 12. Moreover, there would be hindered both the tide, or flow and ebb, of the Euripus of Chalcis, and that of the currents between Africa and the island of St Lawrence, and the course of the ice rushing down from the north, of which we spoke at the same number 12.

[Margin: [?]. Argument.] *Ninth.* In the preceding figure Galileo supposes the motion of the Earth to be slower in the lower semicircle KFG, which is lit by the Sun, and faster in the nocturnal semicircle GHK. So if the tide arose from the motion of the Earth, the sea would have to flow everywhere for about 12 hours each day and ebb for as many; and of the two effects of this motion, one would be felt in places moving from K through F to G, the other in places moving from G through H to K, in perpetual succession. Yet observation is against this motion.

[Margin: 10. Argument.] *Tenth.* If the tide of the sea, properly and strictly so called, is rightly considered, it is not a mere local motion, but a kind of swelling upward from the very floor of the sea — granted that the waters, swelling in this way, then run towards the lower parts of the sea or of the shores. But if it came from the motion of the Earth, it would be a mere flowing in and flowing back, by local motion, of waters going and returning this way and that.

[Margin: 11. Argument.] *Eleventh.* If the motion of the Earth towards the east, by its inequality, were the sole cause of the sea's tide, then everywhere the variation arising from the passage from speed to slowness would always be observed at sunrise, and that arising from slowness to speed at sunset; and the variation arising from the greatest slowness would be observed at midday, while that arising from the greatest speed would be observed at midnight. For in the preceding figure set out by Galileo, the passage from the fast motion of the Earth to the slow occurs at the point K of the terrestrial Equator, and similarly on its parallels, where the Sun A begins to be seen; and the passage from the slower motion to the faster occurs about the point G, where the Sun ceases to be seen and appears to set; while the greatest slowness occurs at the point F, for which the Sun A makes midday, and the greatest speed at the point H, where it is midnight. But no experiment supports this observation; rather, almost the whole account of the sea's tide opposes it. For its variations, as to the beginning and end of the tide, or as to its greatest or least measure, do not depend on the four cardinal points of the day, but on the Moon's elevation above the horizon and her descent, and on her approach to the top or the bottom of the meridian.

[Margin: 12. Argument against Galileo.] *Twelfth.* Suppose the flow and ebb of the sea arose from the Earth's diurnal whirling in 24 hours. Since in each hour 15 degrees of the terrestrial Equator and of each of its parallels revolve towards the east, then, for this motion to be proportioned to the impetus of the Earth driving the waters, the waters running from west to east would have to reach from the eastern shore of a western region to the opposite shore in about as much time as the degrees of longitude intercepted between the western and the eastern shore require: namely after about one hour if they were 15 degrees apart, after two if 30 degrees, and so on. And so from the coast of Spain the waves would at some time have to reach the coast of Syria in 3⅔ hours, since the two are about 55 degrees apart in longitude; and from the coast of North America, in the land of Florida, to the western coast of Spain — just as from the coast of New France to the western coast of France — they would at some time take the same amount of time, since these too are about 55 degrees apart in longitude: at some time, I say, namely when, because of the inequality of the diurnal motion that Galileo asserts, they struck the shores of the western land and from there turned back eastward to the opposite shores on the same parallel. And from the eastern coast of England to the western coast of Denmark or Norway, which are six or seven degrees of longitude apart, the waves would flow within 24^{m} or 28^{m} of time. But three absurdities would follow from this, that is, three effects that conflict with nautical experience. First, ships sailing eastward on parallels of this kind would experience an incredible speed of their own accord, even when the winds did not oppose them, and such as they have never experienced. For on the parallel of latitude 30°, along which one generally sails from Spain to Syria, each degree contains 61⅕ Italian miles, of which, according to what was said in Book 2, chapter 7, one degree on the Equator contains 72¼. Therefore, if in each hour the water of the sea carrying a ship completed 15 degrees, that ship would certainly run through 918 Italian miles in one hour, [Margin: The greatest speed of ships.] and in latitude 45° it would complete 756 miles in one hour; whereas, even on the fastest course, it barely completes 500 miles in a whole day in the Mediterranean, and 200 in the Ocean, as I said in Book 2, chapter 16, number 8. *Second*, ships meeting on a contrary course the waves flowing back from west to east would meanwhile experience an incredible and insuperable resistance; and yet it is certain that those sailing westward are perpetually helped by the course of the sea towards the west, as we showed at number 13. *Third*, it would follow that in that stretch of sea which lies between England and Denmark or Norway the waves would flow within 24^{m} or 28^{m} of time, and flow back as often to the opposite shores; and so there would be 24 tides a day and as many ebbs — which has not so far been detected, not only there, but not even in any Euripus.

[Margin: 13. Argument.] *Thirteenth.* From Galileo's opinion — admitted, though with little consistency as regards the motion of the Earth, yet as a hypothesis — it would follow that no perceptible tide occurs in seas or gulfs that are rather narrow along their length and enclosed within neighbouring

shores, especially those facing north and south. Of this kind are not only the Red Sea, of which we spoke above, but also the sea between England, Norway, Denmark and the Low Countries; our Adriatic Sea, which is narrower than the Tyrrhenian, or Gulf of Venice; the Persian Gulf; the Bay of Bengal; and the Gulf of China. And yet notable tides occur in them, so much so that between England and the Low Countries they rise 50 and 70 feet on the one side and 15 and 20 on the other, as our Fournier often observed and related in Book 9 of the *Hydrographia*, chapter 20. But reread the account of the sea's tide, where the tides of gulfs and straits are discussed.

Refutatur Cauſa varietatis Menſtruæ & Annuæ marini Aeſtus à Galilæo ficta.

The cause of the monthly and annual variation of the sea's tide devised by Galileo is refuted.

XLIV. Recall what I said at number 7. Galileo holds that the diurnal motion of the Earth and of the lunar heaven, taken together, is slower at new moons, because the Moon must then describe in the same time a greater arc of the annual orb, but faster at full moons, because it describes a smaller arc. And from what was said at numbers 8 and 9, on Galileo's hypothesis the diurnal motion adds more to the annual motion at the solstices than at the equinoxes. In this way he tries to defend the monthly and annual variations that occur in the sea's tide, which, however, he did not set out definitely — I believe lest a candid exposition of them should harm his hypothesis. But against Galileo's hypothesis there are the following arguments, taken in great part from Cabeo, Book 2 of the *Meteorologica*, text 6, questions 9 and 10, and from Fournier, Book 9 of the *Hydrographia*, chapter 19.

[Margin: 1. Argument against Galileo.] *First.* If at new moons the diurnal motion of the Earth turns out faster than at full moons, and this difference is so great that it produces a perceptible difference in the sea's tide, then this difference would long ago have been observed in the diurnal motion of the Sun, for which Galileo and all the Copernicans substitute the motion of the Earth — and the more easily, the more often this happens within the year, that is, at least twelve times at new moons and twelve times at full moons. But neither Copernicus nor Tycho nor any other astronomer observed this difference, though they observed other differences of the celestial motions that are subtler, and that grow large only with time.

[Margin: 2. Argument.] *Second.* For the Moon to be faster at new moon and slower at full moon, it would not be necessary for the Earth itself to move faster or more slowly at that time. The Earth's customary motion would suffice, together with the varying distance of the Moon from the centre of the annual orb: brought nearer to it, the Moon would be faster at ☌ [conjunction]; brought further away, it would appear slower at ☍ [opposition].

[Margin: 3. Principal argument.] *Third, and principally.* From Galileo's hypothesis it would follow that greater tides occur at new moons, since the Earth is then quickened in its diurnal motion and the waters consequently surge more; and greater also at the quadratures than at full moons, and indeed on the very days of new moon. But we have already shown at number 18, from the account of the sea's tide, that the contrary happens. For in most places the tide is greater, or not less, at full moons than at new moons; nowhere, however, is it greater at the quadratures than at full moons; and in many places the highest increases of the tides do not occur on the very day of new or full moon, but one or two or even three days after, as we said there.

[Margin: 4. Argument.] *Fourth.* Suppose it follows — because the diurnal motion of the Earth is fastest at the solstices, as Cabeo, though rather timidly, gathers from Galileo's doctrine set out at numbers 8 and 9, since the diurnal motion conspires most with the annual and adds as much as the whole diameter of the Earth requires — that the tide is greater at the solstices than at the equinoxes, and greatest of all at solstitial new moons, as the same Cabeo gathers. But this is contrary to experience and to the account given at number 19. If, on the other hand, as Fournier gathers from it, the tide is smaller at the solstices than at the equinoxes, it would follow that the tide occurring at a solstitial full moon grows each day towards the quadrature, the further one moves from the solstitial day; and conversely that tides occurring at equinoctial quadratures are greater than tides occurring at solstitial new or full moons. But experience establishes that both are false, and that tides occurring at the solstices, whether the Moon is new or full, are far greater than those occurring at the quadratures, as was said at number 18.

To anyone, then, who rightly considers this whole hypothesis of Galileo, it will be clear that for the most part it not only does not explain the cause of the sea's tide, but even destroys the effect itself, and utterly overturns almost the whole account of the tide handed down by the most expert sailors. Now, therefore, to sail out of this tide, let us dissolve Galileo's argument, reduced to form.

[Margin: Galileo's argument in syllogistic form.] XLV. *That hypothesis is to be rejected by which no account of the tide of the sea can be given, and that one, on the contrary, is to be embraced by which it can. But by the hypothesis of the immobile Earth no account of the tide of the sea can be given; and by the hypothesis of the Earth moved with a diurnal and an annual motion it can. Therefore the hypothesis of the immobile Earth is to be rejected, and the hypothesis of the Earth moved with a diurnal and an annual motion is to be embraced.* [Margin: Proof of the major premise.] The MAJOR premise does not seem to need calling into question. For even if other things were unequal, and by the hypothesis of the Earth at rest an account were given equally well or better of some motions, at least the elemental ones, still the variety of the sea's tide is so great, and the question of the sea's tide is so celebrated and of such moment, that if it can at last be settled by some hypothesis, and a sufficient and highly probable cause can be adduced of all, or

of the principal, effects and accidents that appear in that motion, there is no reason why that hypothesis should not be preferred to one which can exhibit no sufficient cause of them. The first part of the MINOR premise, which is negative, is proved from Galileo's statements scattered here and there, indicated from number 3 to 10; and from the refutations and difficulties against all the opinions on the cause of the sea's tide that suppose the Earth unmoved, or do not count its motion among the causes, which we ourselves adduced from number 23 to 42. The second part is clear from Galileo's doctrine, explained from number 2 to 10.

[Margin: 1. Reply [?].] I reply, *first*, by granting the major premise and denying the minor as to both its parts. As to the first part: because without the motion of the Earth, through the 18th opinion presented at number 42, an account is given — not indeed evident as to all the effects of the sea's tide, but evident as to some and probable as to others. And where that account falls short, it falls short at the point on which Galileo, by the hypothesis of the moving Earth, has given no account at all: namely, why the tide occurs in just the same way when the Moon is placed below the horizon as when the Moon is above the horizon. For Galileo recognizes no necessary connection between the daily tide and the motion and position of the Moon, nor does he adduce from the motion of the Earth a particular cause for this particular effect, just as he does not for many other effects either. Therefore, although by the hypothesis of the Earth at rest the account given of the sea's tides is not absolute and wholly satisfying to our mind, yet, compared with the accounts which Galileo tried to beg from the hypothesis of the moving Earth, it is rendered much more probable, agreeing with very many effects and conflicting with none.

The second part of the minor premise, however, is denied because of the twelve arguments made against Galileo at number 43 and the four at number 44. From these it is clearly established that Galileo, in very many respects, conflicts with the most manifest experiments and observations about the sea's tide, made from the time of the Phoenicians to our own times; and, as our Fournier rightly said in Book 9, chapter 16 of his most learned *Hydrographia*, that he investigated the cause of a thing that does not exist at all in the nature of things — that is, the cause of a certain chimerical tide, devised by himself, but not of the tide such as it really is, and such as it has long been observed to be by those skilled in maritime matters.

[Margin: 2. Reply [?].] It is replied, *second*, by turning very many of the arguments set out at numbers 43 and 44 back against Galileo, saying: If the Earth moved, and by its motion were the efficient cause of the sea's tide in the way Galileo explains, the sea's tide would not be such as it really is; therefore either the Earth does not move, or by its motion it is not the cause of the sea's tide in the way Galileo explains.

CAPVT XV.

Chapter 15.

The system of Signor Giovanni Battista Baliani is set out, for explaining the cause of the sea's tides through the motion of the Earth better than Galileo managed — not asserted, but devised as a mere hypothesis.

I. I am indeed convinced that the Most Illustrious Signor Giovanni Battista Baliani, a patrician of Genoa, will not be unwilling if I publish the letter he wrote to me. It contains nothing secret that may not be revealed, and it takes nothing away from the esteem he long ago won among prudent judges of these matters by his most learned books on the natural motion of heavy bodies, solid and liquid. Indeed, the letter that follows will show his singular piety and prudence joined to keenness of mind, since he pays the Sacred Congregation of Cardinals such reverence, as is fitting, in this matter of the Earth's motion. His letter to me, faithfully rendered into Latin, runs as follows.

[Margin: Signor Baliani's letter to the author of this Almagest.]

Some time ago Signor Hieronymus Bardius showed me a letter of Your Paternity's, in which you express a wish to see my system, in which the Earth would revolve around the Moon. I replied that I do not deny that this conception once came into my mind, or my fancy. I came close to writing to Your Paternity about it, but refrained, because I gave this figment no assent and did not think it true, for many reasons, but chiefly because I believed it to be contrary to the decrees of those who hold the authority to judge matters pertaining to the Faith. But when I was very recently in conversation with Father Niccolò Cabeo (who, to my great sorrow, departed this life not long ago), and he warmly recommended Your Paternity's Almagest to me, the occasion arose for me to write these few lines to Your Paternity: to ask that you kindly let me know when the New Almagest may be hoped to come to light, and at the same time to give the reason for which I once conceived the system already mentioned, of the Earth around the Moon. It arose from my having seen that Galileo thought it could be demonstrated that the Sun is the centre of the flux and reflux of the sea — in which he plainly erred, as I wrote to him at once, after he had sent me his book. On that occasion I found that a true cause of the sea's tide would emerge if the Earth revolved around the Moon. I did not, however, assent absolutely to this hypothesis, for the reason already mentioned, and also because I do not yet see how the Moon could be placed at the centre of the universe, or how, with the Sun placed at this centre and the Moon set at the centre of the Earth's epicycle (which are the two ways of defending the sea's tide), all the celestial phenomena could be saved. So I wished to tell Your Paternity this slight foundation for the report made to you that I had it in mind to publish such a system. And finally I reverently kiss your hands. Given at Genoa, 1650 July 9.

Your Very Reverend Paternity's
Most devoted servant
Giovanni Battista Baliani.

II. When I had answered this most courteous letter that it seemed to me very difficult to uphold all the celestial phenomena by the system already mentioned without multiplying the motions in the Moon, he did not hesitate to write back to me this second letter.

[Margin: Another letter of the same man to the same.]

What Your Paternity told me in your most courteous letter about the publication of the Almagest would have been far more welcome had it offered me hope of a much earlier edition of that work. I pray nonetheless for Your Paternity's life and the very best of health, so that you may complete so great a work for the use and learning of men of letters. I then considered what Your Paternity wrote to me about that system of mine. And although I persist in believing that the Earth is in fact at the centre of the universe, I am still not persuaded that, if the Sun were placed there and the Earth turned around the Moon, it would be necessary to ascribe to the Moon other motions besides those that Copernicus conceived and those that he himself followed. For at first sight it seems to me that the appearances can be defended without new motions of the Moon around the Earth — at least none worth considering; but that one would nevertheless have to suppose the Great Orb much larger, and assume the Sun's distance far greater, and so also the distance of the fixed stars much longer, than even the Copernicans themselves reckon. I do not doubt that this would be a great paradox. Notwithstanding this, it seems to me that if GOD had willed to found such a system, it would answer to the experiments both of the celestial motions and of the sea's tide. May Your Paternity hold me excused if I importunately press these tiresome propositions, full of tedium, upon you, and may you deign to command me in something on your own account. Genoa, 1650 July 30.

Your Very Reverend Paternity's
Most obliged servant
Giovanni Battista Baliani.

III. This is what I had to point out to my reader about the system already mentioned — not so much to add the judgement of so great a man against Galileo, as so that others may learn from his example: if the conceptions of their own minds happen to be by no means in agreement with the authority of Holy Scripture or of the sanctions of the Church, then either not to give birth to them, but to suppress them by a pious and prudent, and therefore fruitful, miscarriage; or, if they happen to have given birth, to confine them within the swaddling-bands and cradle of mere hypothesis.

CAPVT XVI.

Chapter 16.

Toward a better understanding and solution of the argument for the diurnal and annual motion of the Earth together, drawn from the increase in the speed of heavy bodies, select theorems and problems on the speed of heavy and light bodies and on the increase of speed are put forward, drawn from our own experiments both in air and in water.

CAPVT XVI.

I. For twenty centuries and more there has flourished in the academies of the natural philosophers, especially the Peripatetics, together with Aristotle (*De caelo* 1, chap. 88), a principle and axiom gathered by induction from experiments and well enough known to the senses of itself: *heavy bodies descending by natural motion through a lighter medium move continually faster and faster the nearer they approach the end toward which they tend* — that is, the nearer they approach the centre of the Earth, whether this happens because this centre is the centre of gravity, or because it is the centre of the whole terrestrial element, to which whole all earthly bodies seek to be joined. This increase of speed, plain to the attentive eye itself, has shown itself far more evidently in the much more violent sound and the much more powerful impact that follow when some heavy, hard body is let fall from a higher place rather than from a lower one. Aeschylus, had he survived, could have borne witness to this. An eagle, hovering very high above in lofty flight directly over him, let fall a tortoise it had earlier seized in its talons onto the bald scalp of his head, which it took for a stone; and seeing the tortoise broken, it suddenly swooped down to devour the creature's flesh, like a kernel taken out of its natural shell. For they say Aeschylus died of that blow, which would by no means have happened to him from it, if that body had been let fall from close by. So the tortoise, a brute animal, when it struck and was struck back, broke the shell of a rational soul — a skull holding no ignoble brain — in such a way that, shattered in the wound of the other, who was soon to die, it perished itself. And while it left the corpse of Aeschylus in that field exposed to the crows, as far as lay in it, it went as food — itself the meaner fare, and the slowest of creatures — to the nobler and swiftest of birds. Yet this increase of impetus, inseparably connected with the increase of speed and manifest to the three principal senses, sight, hearing and touch — indeed even to the birds themselves — has not lacked people who dared to call it into doubt, or even flatly to deny it.

[Margin: Five errors concerning the descent of heavy bodies.]

II. In this matter, however, I have found five different errors. [Margin: 1st error.] First, some deny that heavy bodies descending naturally move uniformly difformly, that is, with an unequal but ever greater speed toward the end; and they dare to affirm that these bodies move equally, considering not only their real motion, but also the motion that appears to us to take place along a straight perpendicular line. Among them is Simplicius, on *De caelo* 1, text 88, commentary 86, toward the end, where he calls this into doubt. [Margin: Of Simplicius.] But plainly in this class is our Rodericus Arriaga [Rodrigo de Arriaga], disputation 4 *De generatione*, section 5, subsection 3, affirming that the same mobile, whether it is heavy and moves downward or light and moves upward, always moves with equal speed. [Margin: Of Arriaga.] Niccolò Cabeo, too, on book 1 of the *Meteorologica*, text 17, at the end of question 3, professes himself undecided on this matter, when he said of a descending heavy body: *Nor is this very thing certain to me, whether the speed grows uniformly difformly in equal parts of time and of space, and consequently whether it can make a triangle of*

speed; therefore I can establish nothing certain. [Margin: Of Cabeo.] He is alluding to Galileo's triangle, of which more below.

[Margin: 2nd error.] Secondly, Galileo, in dialogue 2 on the twofold system of the world, Latin page 120, Italian page 159, concedes an apparent inequality in the descent of heavy bodies along a perpendicular line. [Margin: Of Galileo.] But he says that this motion, because of the annual motion of the Earth, in reality takes place along a circular line, and in such a way that equal arcs are traversed in equal times. The author of *Philolaus* too — that is, Boulliau (for in book 1, chap. 7 of the *Astronomia Philolaica* he professes himself the author of that book) — teaches in book 1 of *De vero Mundi Systemate*, chap. 4, that this motion, which appears to us to be in a straight line and irregular, in reality takes place, or is composed, from two circular motions, each of which is equal and uniform, granted that one of them is faster than the other. [Margin: And of Philolaus revived, or of Boulliau.] We shall explain all of this in the next chapter, with figures attached.

[Margin: 3rd error.] Thirdly, some grant that the motion already mentioned is uniformly difform, at least as it appears to take place in a straight line, if any heavy body is considered by itself. But if two heavy bodies of different gravity are let fall from the same height to a ground or floor equally distant, they deny that the heavier moves faster and reaches the floor sooner than the lighter. So Galileo, whose person Salviati bears in dialogue 2 on the two systems of the world, Latin page 164, Italian page 218. [Margin: Of Galileo.] When he wished to investigate the time in which an iron ball would reach the Earth from the concave of the Moon, and Sagredo had said: *Let us take, then, a ball of determinate weight, and indeed that very one whose time of descent from the Moon we wish to measure*, Salviati replies: *That makes no difference at all, for balls that weigh one, ten, a hundred, indeed a thousand pounds will measure those same hundred cubits in exactly the same time*; and he adds that he has certain and firm experiments of this, to be brought out in due time in a treatise or dialogues on motion. Since, then, he wants the calculation he presently sets up from the experiment with a height of a hundred cubits to hold good for investigating the time in which an iron ball would descend from the Moon to the Earth, he certainly holds that if two balls were dropped from the very concave of the Moon, one of one pound, the other of a thousand or any number of pounds, each would traverse that space in the same time and strike the Earth at the same moment — at least if they are of the same material or the same species. This same thing, finally, Giovanni Battista Baliani, patrician of Genoa, affirms in book 1 of his short but very pithy treatise on the natural motion of heavy bodies, solid and liquid. [Margin: Of Baliani.] There he says that when he was serving as prefect of the fortress of Savona he found by repeated experiments *that two cannonballs — either both of iron, or one of stone and the other of lead — let fall by him at exactly the same moment of time through a space of fifty feet, even if one weighed only a single pound and the other fifty, struck the ground in an indivisible moment of time, so that only one blow of the two was perceived by the senses.* He then adds that he used two

bodies of almost the same bulk, one of wax, the other of lead: *I found*, he says, *in the wax one a somewhat longer delay in the descent, yet far below the proportion of the weights. For that wax ball, descending through the given distance of fifty feet, was about one foot from the ground when the lead one touched the plane beneath, the intervening air, unless I am mistaken, resisting appreciably and impeding the motion.* From these and other experiments he admits that he came round to the opinion that perhaps gravity behaves as the agent and the material body as the patient, and therefore that heavy bodies move according to the proportion of gravity to matter; and that where they are borne naturally by perpendicular motion without impediment, they move equally, because where there is more gravity there is equally more matter, or material quantity. But if some resistance is added, the motion is regulated according to the excess of the agent's power over the resistance of the patient, or over the impediments to the motions.

[Margin: Of Cabeo.] Niccolò Cabeo agrees with Baliani, ever since he came to know him at Genoa while he was preaching there, and I could never draw him away from that opinion. So much so that on *Meteorologica* 1, text 17, questions 5 and 6, he most emphatically affirms, from his own often repeated experiments, that if two balls are dropped together from the same height, one of one ounce, the other of ten pounds or of any greater weight — whether both are of lead, or one is of lead and the other of stone or wood — then, provided that the air is still and the lighter one is not of such slight weight that, unable to overcome the resistance of the air or breeze, it floats in the air (as a feather or paper does), both will reach the ground at the same moment, and no appreciable difference can be noted in the fall. From this he infers that the speed of all falling bodies is of itself equal. I do not know from what height he dropped those balls. But this I remember for certain: when we were together at Ferrara in the year 1634 and were dropping stones of different weight, and also wooden balls, from the tower of our church of the Society of JESUS, I placed a bronze basin under one and a wooden board under the other, so as to distinguish better by the difference of sound which reached the ground sooner; and I noticed that the heavier one arrived a little sooner. But because that difference was slight — for the place in the tower from which they were dropped did not exceed 80 feet — he could never be brought to admit that inequality, or any, or a difference in the fall.

[Margin: Of Arriaga.] Finally, Arriaga too, disputation 4 *De generatione*, section 5, subsection 3, and, subscribing to him, Bartholomaeus Mastrius and Bonaventura Bellutus, disputation 3 *De caelo*, question 3, from no. 52 to no. 57, assert that any two heavy bodies whatever, whether of the same or of different species, or bulk, or shape, however great their difference in gravity, descend from the same height in the same time and glide down with a speed equal of itself. The foundation of so great an assertion is Arriaga's experiment, in which he says that he often dropped together from the same table a dry crust of bread of two fingers, the pen with which he was writing, and a huge stone which he could scarcely hold up in his hands, and noticed that they struck the floor

together at the same moment and moved equally fast. From this he exclaims and complains that in so easy a matter none of the philosophers ever confirmed this descent by experience, but that nearly all remained in the faith of their forebears.

[Margin: 4th error.] Fourthly, others asserted that heavier bodies descend sooner and move faster, but affirmed too great a difference in the descent and speed of two bodies that are not equally heavy. For they said that speed is to speed as gravity is to gravity, so that if, of two balls, one is of one pound and the other of ten pounds, and both are dropped together from the same height, the one of ten pounds reaches the ground ten times sooner than the other, or in the same time traverses a space ten times longer. So Aristotle taught. [Margin: Of Aristotle.] For he taught not only in *De caelo* 2, text 46, *that what is greater is borne faster*, but also in texts 77 and 79 *that a small particle of earth, if raised and released, is borne downward, and the greater it is, the faster it mo-* ves; but also in *De caelo* 3, text 27, that as body is to body of the same species, or as gravity is to gravity in two bodies of the same species, so speed is to speed, and space traversed in an equal time to space. Many of the Peripatetics later lent their votes to this.

[Margin: 5th error.] Fifthly and lastly, some have strayed from the truth in establishing the increase of speed of heavy bodies and its proportion, though in different ways. Galileo, in dialogue 2 on the two systems of the world, Latin page 163, Italian page 217, had asserted from his experiments that an increase of this kind takes place according to the odd numbers counted from unity — or, what comes to the same thing, that the spaces traversed by a heavy body in natural descent are to one another as the squares of the times, that is, have to one another the doubled proportion of the proportion that the times in which that passage was made have to one another. So if a heavy body covers one palm in the first interval of time, in two intervals it covers 4, in three 9, in four 16, and so on; and therefore, taking the intervals separately, 1 palm answers to the first, 3 palms to the second, 5 to the third, 7 to the fourth. When, I say, Galileo had asserted this from his experiments, and Gassendi had subscribed to it in his second letter *De motu impresso a motore translato*, Cabeo nevertheless, on *Meteorologica* 1, text 17, question 3, not only calls it into doubt, but says that this increase (if it be granted at all that heavy bodies move with unequal speed, uniformly difformly) more probably takes place in arithmetic progression: so that if in the first interval of time the speed was as 1, in the second it is 2, in the third 3, and so on, according to arithmetic progression. [Margin: Of Cabeo.] Baliani, too, in book 4 on the motion of heavy bodies, page 110, thought this much truer. [Margin: Of Baliani.]

[Margin: Of Chiaramonti.] Scipione Chiaramonti, however, in book 12 of *De Universo*, chap. 28, in order to show that the increase of this speed does not grow without limit, but at length decreases so that the speed becomes equal, says that he divided a line AB, about a perch and a half long, into three equal parts, namely AC, CD and DB. To measure the time he used both a beating

clock and water flowing gently from the small spout of a vessel, always with the same weight. The weight, he says, reached B from A in a time of 514 small parts; from C to B in a time of 451; and from D to B in a time of 320.

Diagram. A vertical line set in the left column beside Chiaramonti's account, divided into three parts by two short cross-ticks; the labels are read off the block.

- A (top end of the line)
- C (upper cross-tick)
- D (lower cross-tick)
- B (bottom end of the line)

Since, then, says Chiaramonti, the time from C to D is equal to the time from D to B (understand: if the weight is dropped now from C, now from D), if you subtract DB, 320, from the time CB, 451, there remains DB, of 131 parts. Again, since the whole time from A to D is as much as from C to B, that is, 451 parts, and AB is 514, if you subtract AD from AB, DB remains, a time of 63. Finally, since the time AC equals the time DB if the weight is dropped from A to C or from D to B, and AC, or DB, is 320, then to run through the equal parts the following times are required in order: for the 1st, AC, a time as 320; for the 2nd, CD, as 131; for the 3rd, DB, a time of 63. But the ratio of 320 to 131 is greater than that of 131 to 63. Therefore the ratio of the increase of speed diminishes as the descent proceeds.

So far I have spoken of the — I do not say opinions, but errors — of others, because their opposite is now, for us and for others who were present as witnesses at our observations (which will be reported shortly), not merely probable but evident and certain. And so we shall proceed not from likely conjectures but from infallible physico-mathematical science, by way of the following experiments, to certain conclusions.

I. Classis Experimentorum pro Leuitate distincta à Grauitate.

First class of experiments, for lightness as distinct from gravity.

[Margin: 1st experiment: of air piercing water so as to tend upward.]

III. Take a glass tube or small pipe AB, as we sketch it in the first figure following, at least three feet long and as straight as possible, closed at one end, A, by continuous glass. Fill it almost wholly with water, leaving only a small space AM, as large as the breadth of the smallest finger, the ear finger. Then close the mouth of the tube with a wooden lid, but smear round the vents with bitumen, or with a cerate made of virgin wax, pitch, and powder of Armenian bole or of flower of red stone, so that no more air can enter, much less water escape even drop by drop. When this is done, turn downward the part of the tube whose top, AM, the enclosed air will now have occupied, setting the tube up perpendicular to the horizon. You will see that the air which at B, the bottom of the tube, had had the form of a cylinder occupying the whole cavity of the tube in breadth, soon rises, and rises in such a way that, narrowing itself,

it stretches out lengthwise; and that the upper surface of that little cylinder, which was flat like a little disc, has now become conoidal, as you see at C — so that, namely, it may more easily pass through the water, and as it were perforate it by that shape, and by narrowing itself give way on all sides to the water tending downward. You will notice moreover that this blunt cone, or rather cone of air, becomes more pointed and narrower in its ascent, and hence that more of the descending water passes between its convexity and the cavity of the glass than before; and that finally, when it has flown up to the top of the water, it restores itself to its former cylindrical shape, if the tube's cavity is such. These points being posited, which are most true and which I have often shown to many people, it follows (philosophizing, however, sincerely and prudently) that although the ascent of the air and the yielding of the water begin at the same moment, yet by nature and causality the air moves upward before the water flows in to fill the place the air vacates. For it is the air that, swelling upward, enters the water and forces it to yield to it, as if a wedge were driven into its middle. Otherwise, if the air tended upward for this reason, that it is thrust out upward by the water tending downward, the water would rather pass wedgewise through the air (just as water falling outside a tube breaks through the air beneath it, and does not surround it and receive it into itself) — and that all the more easily, the less fit the air is, by its rarity, to resist what drives it; or at least the water would blunt from above the swelling with which the air swells upward, and, compressing it from below as with forceps, would constrict its lowest part into a conoidal shape. Now if the air is prior in this motion and behaves actively, it certainly does so by some force moving it upward; and philosophers and non-philosophers alike commonly call this lightness. There is therefore a lightness distinct from gravity, and lightness does not consist in the privation of greater gravity.

[Margin: 2nd experiment.]

IV. But take another experiment, perhaps more evident to you, or try it yourself. Fill the tube already mentioned, or another, DE, in the 2nd figure, almost wholly with water, but pour three or four drops of oil on top, still leaving a very small space for a little portion of air, DL. Then close it well, as above, and with the end D turned downward set it upright, perpendicular to the horizon. You will see the air borne upward very quickly and the oil very slowly, so that the air which was at E, penetrating the oil F, is already at H when the oil is still a little above F; and when the air has reached the top of the tube, D, the oil will not yet have got through half the way. So, while the oil is struggling upward around the middle of the tube, turn the lower part of the tube upward again: at once the air H will fly upward, passing wedgewise through the oil G, and will fly through to K. Sometimes, however, it will pass upward through the oil, perforated in this way, in such a manner that a little portion of air, formed into a small globe I, clings to the oil at its top; and then the oil G will tend upward far faster than it would otherwise in the same position, because it will enjoy a twofold motion: one from its intrinsic lightness, the other of traction, by

which it is drawn by the air I. It is therefore a sign that the air is borne upward by its own lightness; otherwise the water, pressing on the oil G, would more easily compress the air I or scatter it to the sides. Nor can any physical and solid reason be given why that compound of oil and air moves faster than the oil alone. For it cannot be said that the air is thrust upward by the oil G as it descends, as the less heavy by the heavier, since the eye plainly sees the oil G borne continuously upward, and not falling back down intermittently so as to drive the air. Nor can it be said that the water, by descending faster, also expels the oil upward faster, since there is no greater mass of water above the oil than before, and the water would very easily compress the bubble of air I. Nor, finally, can it be said that the compound of G and I is something lighter than an equal quantity of water in the same bulk, and that the water therefore exceeds it in gravity more than it exceeds the oil taken by itself, and therefore seeks to occupy its place sooner. For if there is no lightness, and the little portion of air I has some gravity, then rather a heavier body has been made from it and the oil than the oil alone would be.

[Margin: 3rd experiment.]

V. Finally, make a cylindrical vessel ONPQ whose bottom OP is parallel to the horizon, and set on the bottom a wooden cylinder TOPX — but one such that it floats in water, that is, it is not heavier than water but specifically lighter. Holding a rod SR in your hand, press the cylinder down with it, so that it is not lifted upward while someone else fills the vessel with water. Once the agitation of the water has settled, remove the rod SR: the cylinder will then be carried upward with such impetus that it leaps out above the water, occupying in that leap the space ZNQV, and afterwards, settling back, it will float at rest with no small part of it above the water.

That leap, therefore, is a sign of an impetus produced by an intrinsic lightness. For the water cannot thrust the cylinder further than it needs to in order to occupy the lower place, unless the water itself is first carried upward; and that is against its own gravity, and the water does not usually do it except to fill a vacuum and to succeed into the place of another body moved upward by some other force. Moreover, when the cylinder was on the bottom OP, no part of the water can be found that thrusts it upward. Not the part on the bottom: for I suppose a physically perfect cylinder and a bottom of the vessel exactly flat, so that no sensible part of the water can slip in underneath while the cylinder is held there by force by the hand S, by means of the stick RS. Nor the part of the water NTXQ, equal to the breadth of the cylinder: for if that part weighed, it would rather hold the cylinder down in proportion to its own measure. Nor the parts TO on one side and PX on the other, pressing on the sides of the cylinder. For either they press so that the pressure acts along a line parallel to the horizon, or downward, and so they would hold it in the same position; or upward with an oblique effort — and not even this can they do unless they themselves first tend upward, which, as I said, is against the gravity of water, which naturally tends downward or is at rest, and is never carried

upward except to prevent a vacuum, such as would arise from a body raised upward by another force distinct from the power of the water. And this very motion is held by many to be rather one of traction, exerted by a body tending back. Nor, finally, the parts enclosed between NT or QX and the sides of the vessel: for neither do they actually weigh downward, or not more than the parts enclosed within NTXQ, which are of greater bulk and would prevail by their weight in holding the cylinder down.

But what will the adversaries say if in the bottom of the vessel there were a wide hole Y, narrower than the cylinder, however, and closed, which was opened at the same moment that the hand lifts the rod RS? For the water would certainly flow out downward, and yet that cylindrical wood would tend upward. Let them acknowledge, therefore, some lightness in that wood which, by producing an impetus upward, first by nature moves and drives the water, and is the cause that the water, being a fluid body, yields to it so as to slip into its place, lest a vacuum be given. Let them acknowledge too that the water does not actually exert gravity, but rather that the upper parts of the water are pushed by the wooden cylinder and yield place to it by moving aside to the sides, so as to fill the place of those parts which slip in below into the place of the cylinder.

Diagram. Four numbered experiment figures stand in the left column between the opening lines of § V and the rest of its text.

- Figure 1: a tall stippled tube (water), A at the top, M near the top (the small air space), C at the base of a white bell-shaped bubble of air in the lower half, B at the foot.
- Figure 2: a tube with hatched bands at the top (L) and the foot (F), D at the top, E at the foot; three bell-shaped bodies — K beside the upper white one, G beside the dark middle one (the oil), H beside the lower white one; I stands between the upper and middle ones, with a small round mark [?] just below it.
- Figure 3: a cylindrical vessel N Q / O P, with a block T X (R inside it) on its bottom; a hand S presses a rod down onto the block; a dotted rectangle Z V stands above the rim (the space of the leap); Y marks the foot below the vessel (the hole of the closing question).
- Figure 4: a tall tube, A D at the top, B C at the foot, crossed by dotted lines E F, I K, L M and G H, each with a dot on its right-hand dotted line (and a dot at E).

II. Classis Experimentorum Pro Inæquali motu Grauium in Aere velociùs ac velociùs descendentium, quò plus accedunt ad terminum, ad quem tendunt.

Second class of experiments: for the unequal motion of heavy bodies in the air, descending faster and faster the nearer they come to the end towards which they tend.

[Margin: 1st experiment, from sound and hearing.]

VI. The first experiment is taken from sound. Drop a ball of wood, bone or metal from a height of 10 feet into a basin set beneath it, and attend to the ringing that arises from the impact. Then drop that same ball from a height of 20 feet, or two perches: you will hear a much greater and more widely diffused sound. Next raise the basin to a height of 10 feet and drop the same ball into it from a height of 10 feet: you will hear a ringing like the first. In the second fall, therefore, the ball acquired a greater impetus from falling from a greater height than from the lesser one in the first and third falls; and in the second half of its path downward it acquired more impetus than in the first half. So it moved downward with an unequal velocity, and so unequal that in the second fall it descended faster through the last 10 feet than through the first 10. For it is clear from constant experiments that a faster motion of the moving body follows a greater impetus.

If you want a more homely experiment, pour water into a cup from a vessel two or three finger-breadths away from the cup, and you will hear no noise; raise the vessel to two or three feet, and you will hear a noise from the falling water. Hence those who lived at the Catadupa, or cataracts of the Nile, are said by Cicero in the *Dream of Scipio* to have gone deaf from the crash of the waters rushing down from on high in a headlong fall.

[Margin: 2nd experiment, which can be estimated by touch itself.]

The second is taken from an impact perceptible to touch itself. Put your hand under a playing ball while someone else drops it from a height of 10 feet, and you will experience a very light impact; but if it is dropped from a height of 50 feet or more, you will feel your hand struck, not without some pain. The ball, therefore, acquired a greater impetus from the higher fall. This greater impetus, as I was saying, the wretched Aeschylus felt from the tortoise dropped by an eagle from on high onto his head; and the brute bird itself foresaw it, having no doubt of the outcome to come. Elpenor felt it when he fell from a tower; hence Ovid's well-known comparison in Book 1 of the *Tristia*:

> He who falls on level ground (though even this scarcely happens)
> falls so that he can rise from the earth he has touched; but wretched
> Elpenor, fallen from a high roof, met his king as a tearful shade.

And hence that adage, and the very well-known warning of the Poet:

> ..... They are raised on high that they may fall with a heavier crash.....

Finally, do not those who run down some slope conceive so much impetus that, even if they wish, they cannot afterwards check the course which they could very easily have checked at the beginning?

[Margin: 3rd experiment, from an impact that can be estimated by the eyes.]

VII. The third experiment is taken from the greater impact of bodies falling from a height, an impact that the eyes can estimate. A ball of clay which, dropped from a small height, is not itself broken, or cannot break an eggshell or a nutshell placed perpendicularly beneath it, or raise a weight placed in a wooden balance, or penetrate a palm's depth of water, at length accomplishes all

these things if it is dropped from a higher place: for it breaks and is broken, and it raises that weight. So too a wooden ball, or a playing ball, falling from a low height into a cistern or a large vessel full of water, sinks a few finger-breadths below the water; but if it falls from a very high place, it penetrates many feet below the water, and sometimes right to the bottom. And by countless other experiments of this kind it is made evident that a heavy body falling naturally from a higher place has always acquired a greater and greater impetus at the end of its motion.

[Margin: 4th experiment, from the rebound of a playing ball.]

The fourth is taken from the repercussion and rebound of a playing ball upward. We had a leather ball of this kind, very hard and no larger than the yolk of an egg, so that it would strike the ground at a more acute angle, dropped from a height of 37 feet onto a stone-paved floor, and it rebounded up to 7½ feet; but when it was dropped from a height of 73 feet, it rebounded to 11¼ feet. Another leather ball, however, larger and striking the floor at a more obtuse angle, rebounded to about 6 feet when dropped from a height of 37 feet, and to 7½ feet when dropped from a height of 73 feet. But I seem to myself to be playing, so long as I do not proceed to nobler experiments, which demonstrate most evidently not only the inequality in the motion of heavy bodies, but also that the increase of their speed grows uniformly difformly toward the end of the motion.

[Margin: 5th experiment.]

VIII. The fifth experiment, then, which we made very often, was the measurement of the space that a heavy body covers in natural descent in equal times. I had indeed tried this at Ferrara with Father Cabeo in the year 1634, but from the tower of our church, which does not exceed a hundred feet, and with a pendulum [Perpendiculo] whose exact times in the primum mobile I did not yet know. But when I was at Bologna in the year 1640, and had by then examined pendulums of different lengths by the transits of the fixed stars through mid-heaven, I chose for this experiment the smallest one, which has a length, from the vertex of the motion to the centre of the little ball, of one inch and fifteen hundredths more of an inch of the ancient Roman foot.

[Margin: A pendulum applied to the most precise measurement of times.]

As I showed and set out in Book 2, chap. 21, a single simple vibration of this pendulum is equal to 10 thirds of hourly time of the primum mobile, and six vibrations equal one second most exactly. So a single simple vibration of it is very nearly equal to the time that musicians usually mark by the note of the semiquaver [ſemichromatis], if the choirmaster or conductor regulating the voices raises and lowers his hand in the ordinary way. But because the oscillations, or vibrations, of so short a pendulum are very fast and very frequent, and I judged that not even an error of one should be admitted, we used to begin the count again from one after each ten vibrations, each ten being marked by raising a finger of a hand already closed, so that no confusion or deception should creep into the count before the eye. And we used to pronounce the numbers

below ten very rapidly one after another with a shortened nomenclature, such as these Italian words, but pronounced more briefly, as is commonly done here at Bologna: *Vn, dù, tri, quatr, cinq, sei, sett, ott, nou, dies.* If, as I said, you set musical semiquavers under these, and follow the ordinary measure of the musical beat, you will have a very close specimen of the times that a single simple vibration of this pendulum of ours measures.

We also accustomed others to this way of counting, especially Fathers Francesco Maria Grimaldi and Georgius Cassianus, whose help I used a great deal in the experiment about to be set out. And it is indeed wonderful to tell: when those two and I used such pendulums — they standing on the cornice [coronis] of the Asinelli Tower, I on the floor of the platform below — and each of us separately had noted on a sheet the vibrations in which the heavy body descended from there to the floor, there was never, though the experiments were repeated, a difference between us of a single whole little vibration. I know that some will scarcely believe this, and yet I testify that it was most truly so, and the Fathers of our Society already mentioned will always attest it. So much for the pendulum and the measurement of times.

After this we prepared a very large basket full of clay balls, each of which weighed eight ounces. For measuring the shorter intervals we used the windows of our College; for the higher ones, various towers — not the whole towers, but their windows or small windows: namely the tower of San Francesco, which is 150 Roman feet high, that of San Giacomo, 189 feet, that of San Petronio, 200 feet, and that of San Pietro, 208 feet; but chiefly and most frequently the Asinelli Tower, which is 312 feet high in all, but from the cornice to the base, or parapegma, only 280 feet. Since this tower is most convenient for experiments of this kind, just as if it had been built for this purpose, I wish to set it before the eyes.

[Margin: The height of the towers of Bologna.]

[Margin: Description of the Asinelli Tower.]

IX. In the following diagram, let IBCD be the shaft of the tower, the thicker part, standing on a nearly cubical base VYZX much thicker than the shaft of the tower. From this base rises the parapegma YZH, enclosed by rather broad stone railings [cancellis], so that at least six men can walk safely round the tower on it at once, in a single row, between the railings and the tower. At the top, moreover, the cornice BC projects, enclosed by beaked stone railings, so that from the openings G, Φ, O, as from windows, any man of ordinary stature can look down safely and, letting a plumb line down from there to the floor of the parapegma ID, measure the height GI, as we did more than once; and this, as I said, we found to be 280 ancient Roman feet. The other heights we measured partly with rods, and the whole height also by altimetry, together with Father Grimaldi.

[Margin: The suitability of the Asinelli Tower for these experiments.]

The tower, therefore, has very great advantages for this business. For balls dropped from the windows G, Φ, O fall perpendicularly onto the floor ID,

neither striking the foot of the tower nor falling outside the railings YZ. Next, there is no need to stop anyone from crossing the square laid round its base while the balls are being dropped from the cornice; they can be dropped again and again without danger to anyone. Besides, the tower has certain iron bands around F and T, with iron clasps binding the opposite walls together, which we used as limits for measuring the remaining interval to be covered by the ball when it had reached T or F. The line NH, moreover, stands in for the other towers and heights that we used.

Diagram. A woodcut of the Asinelli Tower, standing in the right column beside the text of § X: a tall square brick shaft with a cupola and weathervane on top, set on a broad base; to its right a vertical measuring line.

- G, Φ, O (crowning parapet; Φ in the central opening, O at the right): the openings from which the balls are dropped
- B (left), C (right): the projecting cornice at the top of the shaft
- Q, R, S (down the right side of the shaft), F (left, low on the shaft), T (low on the shaft): points on the tower; F and T are the iron bands of the text
- I, D (foot of the shaft): the floor of the parapegma
- Y (left), Z (right): the railings of the parapegma on the base
- V (left), X (right): the foot of the base
- N (top), M, L, K, β, H (foot), dots on the vertical line: the line NH, standing for the other towers and heights

X. In the year 1640, then, in the month of May, and at other times afterwards, we sought the height Hβ that would be such that a clay ball of eight ounces, let go to itself, would strike the floor after exactly five vibrations of the pendulum already mentioned, that is, in a time of 50 thirds; and we found it to be 10 Roman feet, the experiment being often repeated. Next we sought the height needed for the descent of another ball of the same species and weight in double the time, that is, in ten vibrations, and we found it to be 40 feet, which the interval KH marks. Climbing higher from here, we sought the height due to a time three times longer, that is, to exactly 15 vibrations, for the descent of such a ball, and we found LH to be 90 feet. So for a time of 20 vibrations we found the height MH to be 160 feet; and for 25 vibrations NH to be 250 feet. Finally, because we could not climb to as great a height as 30 vibrations require, a ball was dropped nevertheless, and often, from the cornice of the Asinelli Tower, namely from G to I, an interval of 280 feet; and it struck the floor when we — I standing at I, and Fathers Grimaldi and Cassianus at G — had counted 26 vibrations, as was clear when we exchanged our sheets with one another.

Now let us imagine the intervals of the line NH transferred to intervals of the line OT, and let the interval traversed by the ball in the first five vibrations, that is, in the first time, be OC: for it will be 10 feet, as much, namely, as βH was. The interval of the second time, OQ, will be equal to KH, 40 feet; the interval of the third time, OR, will be equal to LH, 90 feet; and the interval of the fourth time, OS, will be equal to the space MH, 160 feet. Finally, the whole

interval OT, comprised in the 25 vibrations of the fifth time, will be equal to the whole NH, 250 feet. Note, however, that here we take for T some mark on the face of the tower between S and the floor.

[Margin: The increase of the speed of heavy bodies.]

The ball already mentioned, therefore, descended faster and faster the further it receded from O and the nearer it approached D. Taking the intervals separately, each answering to the measures of equal times, it covered OC, 10 feet, in the first time; CQ, 30 feet, in the second; QR, 50 feet, in the third; RS, 70 feet, in the fourth; and ST, 90 feet, in the fifth; and these numbers added together make 250. This increase is remarkable and worth noting, and it is to be confirmed by the following experiment. For here we are inquiring only whether heavy bodies falling naturally through the air along a straight line perpendicular to the horizon descend uniformly or difformly uniformly, and whether with a decrease or with an increase of speed.

III. Classis Experimentorum pro Proportione Incrementi velocitatis Grauium per Aerem descendentium.

Third class of experiments: for the proportion of the increase of speed of heavy bodies descending through the air.

XI. It is true that in the year 1629, when, with Father Daniele Bartolo [Daniello Bartoli] and Signor Alphonsus Iseus, a distinguished geometer, I first began on another occasion to examine two pendulums of the same length and weight let go together from the same point — whether each would always proceed at an equal pace along its arc — I noticed that the oscillations of one and the same pendulum are all, to sense, equal to one another in time, or synchronous; and afterwards, at Ferrara in the year 1634, I established this very thing most certainly with Father Cabeo. Yet I had not yet understood or learned the proportion of the increase of speed of heavy bodies handed down by Galileo in the second dialogue on the system of the world, and asserted to follow the odd numbers [pariter impares] beginning from unity. Indeed, at that time I had suspected from my cruder experiments that it was continually triple, that is, according to these numbers: 1, 3, 9, 27.

[Margin: The proportion of the increase of speed asserted by Galileo.]

Afterwards, however, when permission was granted me to read those dialogues, which the Sacred Congregation of the Index had prohibited and marked with censures, I found in them, at Italian page 217, Latin page 163, that the increase already mentioned was found by him through experiments to follow the odd numbers [pariter impares] counted from unity, namely as 1, 3, 5, 7, 9, 11, etc. I suspected, however, that some deception lay hidden in his experiments, because in the same second dialogue, Italian page 219, Latin 164, he asserts that an iron ball of a hundred pounds, dropped from a height of a hundred cubits, reached the earth in five seconds of time; whereas my clay ball of 8 ounces descended from a much greater height, namely from GI, 280 feet, which make

187 cubits, in exactly 26 vibrations of my pendulum, which make a time of the primum mobile of 4 seconds and 20 thirds. And I was certain that there had been no sensible error in my counting of the time; and Galileo's error, which lay hidden in a time not adjusted exactly to the time of the primum mobile and the transit of the fixed stars through mid-heaven, I transferred to the intervals covered in the descent of that ball. Moreover, I found it hard to believe that he could have used an iron ball of so great a weight, especially since he had not named the tower from which he had had it dropped. So, full of this suspicion, I began, as I said, already from the year 1640, the measurement of this increase with all precision, hoping that I might find some other proportion, perhaps closer to my own. But in fact I found that the one Galileo had asserted is rather the true one.

[Margin: 1st experiment.]

For from what was said in the preceding experiment, set out in number 10, I recognised that the increase followed the proportion of the feet 10, 30, 50, 70, 90, which is just as if it were expressed in the smallest numbers 1, 3, 5, 7, 9.

[Margin: 2nd experiment.]

But not yet fully satisfied with that, I changed the time and took 6 vibrations of the pendulum, which make one whole second of the primum mobile, and with Father Grimaldi I sought the height due to them, such that a clay ball of eight ounces, let go to itself, would reach the floor; and I found βH, 15 feet. Then at the end of another second, that is, of exactly 12 vibrations, I obtained the height KH, 60 feet; at the end of 18 vibrations, that is, of the third second, it was LH, 135 feet; and at the end of 24 vibrations, that is, of the fourth second, it was MH, 240 feet. But a similar ball traversed the 280 feet in the Asinelli Tower, again and again, in 26 vibrations, that is, in 4 seconds and 20 thirds. For although in the greater distances a foot or two taken beyond or short does not introduce a difference of one whole vibration, I nevertheless found that the number of vibrations already mentioned answered most exactly to the intervals already mentioned.

[Margin: The same proportion confirmed by us.]

Hence by this experiment likewise, separating the intervals due severally to equal times, I was certain that this proportion was kept in the feet 15, 45, 75, 105, which is exactly such as holds between the numbers 1, 3, 5, 7: for as 1 is to 3, so is 15 to 45, etc. I found the same by some other experiments, which I pass over here for brevity's sake and because they are entangled with fractions; a third one, however, I shall set out in the following small table.

[Margin: 3rd experiment.]

So I went with Father Grimaldi to Father Bonaventura Cavalerius [Bonaventura Cavalieri], principal Professor of Mathematics in the University of Bologna and formerly a pupil of Galileo, and told him of the agreement of my experiments with Galileo's, at least as regards this proportion; for, confined to his bed or his chair by gout in the hands and feet together, he could not be present at

them. It is incredible to tell how much he was cheered by this joint testimony of ours.

[Margin: The proportion of the spaces covered by heavy bodies in descent.]

XII. Those not ignorant of geometry now recognise that the spaces covered in equal compounded times by heavy bodies of this kind descending naturally are to one another in the doubled ratio of their times, that is, are to one another as the squares of the compounded times. Galileo gathers this very thing in that second dialogue, on the same page, 217, or 163. For in the first experiment the vibrations were, in order, these: 5, 10, 15, 20, 25, whose square numbers (that is, those produced by multiplying the same number into itself) are 25, 100, 225, 400, 625; and the spaces covered were, in order, 10, 40, 90, 160, 250 feet. Now as 25 is to 100, so is 10 to 40; and as 100 to 225, so 40 to 90; and as 225 to 400, so 90 to 160; and finally as 400 to 625, so 160 to 250. So too in the 2nd experiment the order of the vibrations was 6, 12, 18, 24, 26, whose squares are 36, 144, 324, 576, 676; and the intervals traversed and compounded were, in order, 15, 60, 135, 240, 280 feet. But as 36 is to 144, so is 15 to 60; and as 144 to 324, so 60 to 135; and again as 324 to 576, so 135 to 240; and finally as 576 to 676, so 240 to 280. The spaces already mentioned, therefore, are to one another as the squares of the vibrations, or times.

For ease, however, those equal times can be reduced to the smallest numbers, so that the first time counts as 1, the second as 2, the third as 3, the fourth as 4 and the fifth as 5; and so these squares, 1, 4, 9, 16, 25, can be applied to examine the proportion just stated. In the third experiment, however (which you have set out in the small table about to be shown), there ought to have been 288 feet for the proportion already mentioned to be kept exactly; but we could not conveniently test this except from a height of 280 feet. I wish, then, to gather what we have said so far about so beautiful a proportion, and its foundations, into one small table, and to offer it to the Reader to taste in a brief synopsis.

I add, however, that this proportion was not found by us, as I once hoped, in the weight that a falling ball can raise, granted that it tends toward this proportion. For if a ball falling from a height of one foot raises a weight of one ounce, it will not raise a weight twice as great from a height of 4 feet, or a quarter-pound weight from a height of 9 feet, etc. See below, the 12th class of experiments.

Order of experiments	Simple vibrations of a pendulum 1 15/100 inches high	Time of the primum mobile answering to the vibrations (seconds, thirds)	Square numbers of the vibrations	Spaces covered by a clay ball of 8 ounces at the end of the times (Roman feet)	Spaces covered separately in the single times (Roman feet)	Proportion of the increase of speed of heavy bodies in our air (smallest numbers)
	5	0″ 50‴	25	10	10	1
	10	1″ 40‴	100	40	30	3
I.	15	2″ 30‴	225	90	50	5
	20	3″ 20‴	400	160	70	7

Order of experiments	Simple vibrations of a pendulum 1 15/100 inches high	Time of the primum mobile answering to the vibrations (seconds, thirds)	Square numbers of the vibrations	Spaces covered by a clay ball of 8 ounces at the end of the times (Roman feet)	Spaces covered separately in the single times (Roman feet)	Proportion of the increase of speed of heavy bodies in our air (smallest numbers)
	25	4″ 10‴	625	250	90	9
	6	1″ 0‴	36	15	15	1
	12	2″ 0‴	144	60	45	3
II.	18	3″ 0‴	324	135	75	5
	24	4″ 0‴	576	240	105	7
	26	4″ 20‴	676	280	40	8 1/6
	6 1/2	1″ 5‴	42	18	18	1
III.	13 0	2″ 10‴	169	72	54	3
	19 1/2	3″ 15‴	381	162	90	5
	26 0	4″ 20‴	676	280	118	6 7/128 or 6 7/126 [?]

IV. Classis Experimentorum pro Duorum Grauium diuersi ponderis Deseensu [recte: Descensu] Inaequali ex eadem altitudine per Aerem.

IV. Class of experiments: for the unequal descent through the air of two heavy bodies of different weight from the same height.

XIII. This experiment is very deceptive unless it is carried out with great diligence and circumspection, and unless conclusions are drawn from it only after prudent regard to everything that can concur to quicken or retard a motion. For the best comparison of all, and one that would remove every doubt, would be to let go, from the same height and at the same moment of time, two balls of the same size and the same species and yet of different weight: if, that is, out of two balls of clay or of metal — one with a diameter of one palm and weighing one pound, the other with a diameter of two palms and weighing eight pounds — we could so rarefy the one-palm ball that, without any other body entering it, it would become two palms across and still keep its solidity. For then, if the heavier descended faster, it would descend by the force of its greater heaviness alone, and the slowness of the other could not be put down either to a difference of species or to a difference of size, which, because of a blunter and more spread-out figure, would break with more difficulty through the larger body of air beneath it; since in everything else there would be parity, except in heaviness.

Nor, on the other hand, is it useful to drop two cylinders or two prisms of equal breadth but of unequal height and weight, made of the same material; for they waver about their centre of gravity in the air and leave the cause of the faster or slower descent in doubt. But since this cannot be done, Father Grimaldi and I prepared what comes next to it and is the best means among the rest: twelve clay balls of solid clay, each weighing 20 ounces, and another

twelve of clay equal in size to the first but hollow inside, each of ten ounces, and so half as heavy as the first. Then we collected many other balls, differing in species, or in size, or in weight, and set up various comparisons.

In the year 1640 in the month of May, in 1645 in August, in 1648 in October, and most recently in the year 1650, we dropped various pairs from the cornice of the Asinelli Tower, having called together many witnesses from our Society. Though not all of them were always present at the same experiments, those present were, namely, Fathers Stephanus Ghisonus, Camillus Rodengus, Iacobus Maria Pallavicinus, Francesco Maria Grimaldi, Vincentius Maria Grimaldus, Franciscus Zenus [?], Paulus Casatus with his pupils, Franciscus Adurnus and Octavius Rubeus — all distinguished for intelligence, judgement and religious integrity.

[Margin: Witnesses and helpers [?] of the experiment.]

And indeed among them were three or four Masters of Philosophy or Theology who, with Galileo, or with Cabeo and Arriaga, had held that any two heavy bodies whatever, let go together from the same height, however great, descend to the earth in the same physical moment of time. But they at once laid this opinion aside.

[Margin: 1st experiment.]

For the lighter clay ball, that is, the one of 10 ounces, let go from G at the same moment as the other clay ball, of the same size but of 20 ounces, was let go from O, appeared still at F, at least 15 feet distant from the floor I, at the moment when the heavier had struck the same floor at D and had already burst into countless fragments. Even before either of them reached the floor, in fact, the lighter — which, to tell them apart, Father Grimaldi by agreement always let go with his left hand — was already plainly seen, well separated from the heavier, a little below the middle of the tower. And when the time was counted by means of the pendulum, the heavier descended in exactly 23 vibrations, that is, in 3″ 30‴ [?], and the lighter in 26 vibrations, namely in 4″ 20‴ of the primum mobile. This experiment, repeated twelve times, always had the same outcome.

Since, then, the difference of the interval FI was easier to observe and made a greater discrepancy than the difference of time, it seemed advisable, in the descent of the other balls, to attend to the interval still remaining to the ball descending more slowly rather than to the difference of time. What we obtained the following table will show more briefly and more clearly; in it the earlier experiments concern pairs of balls of the same size, the later ones balls of different size.

Order of experiments	Balls of the same size	Weight of the balls: ounces	Weight of the balls: drachms	Ball that was faster	Distance of the slower from the floor already struck by the faster
1	Clay, hollow	10	0	The heavier	15 feet
	Clay, solid	20	0		

Order of experiments	Balls of the same size	Weight of the balls: ounces	Weight of the balls: drachms	Ball that was faster	Distance of the slower from the floor already struck by the faster
2	Clay, hollow	20	0	The heavier	16
	Clay, solid	45	0		
3	Clay	9	0	The heavier	20
	Wood	2	4		
4	Clay	20	0	The heavier	12
	Wax	15	0		
5	Wood	4	6	The heavier	15
	Wax	6	7		
6	Wax	1	5	The heavier	30
	Iron	11	7		

[Margin: The 1st experiment was repeated twelve times, the 2nd twice.]
Second part of the preceding table.

Order	Balls of unequal size	Weight of the balls: ounces	Weight of the balls: drachms	Ball that was faster	Distance etc.
7	Clay	5	0	The heavier	5
	Clay	4	0		
8	Clay	21	4	The heavier	12
	Clay	11	4		
9	Clay	27	3	The heavier	15
	Clay	14	1		
10	Clay	18	7	The heavier	35
	Wax	1	0		
11	Clay	2	0	The heavier	25
	Lead	2	4		
12	Lead	2	4	The lead one	40
	Wood	2	4		
13	Clay	7	0	The heavier	10
	Clay	62	0		
14	Clay	10	4	The heavier	13
	Clay	23	0		
15	Clay	53	0	The heavier	8
	Clay	6	4		
16	Clay	53	0	The heavier	9
	Clay	7	1		
17	Walnut wood	2	1	The walnut one	2
	Beech wood	4	7		
18	Clay	11	0	The lead one	1
	Lead	1	7		
19	Clay	33	0	The clay one	2

Order	Balls of unequal size	Weight of the balls: ounces	Weight of the balls: drachms	Ball that was faster	Distance etc.
	Lead	1	0		
20	Clay	38	0	The clay one	3
	Lead	1	0		
21	Lead	1	0	The heavier	5
	Lead	0	4		

Corollaria ex praecedenti Tabula.

Corollaries from the preceding table.

XIV. I here take it that a thing is heavier in species which, in an equal size naturally proper to it, is of greater weight: as lead is said to be heavier than wax, because if two balls one palm across were weighed, one of lead and the other of wax, the lead one would weigh more pounds, or ounces, or drachms, etc., than the wax one. And that a thing is heavier in the individual which is of greater weight absolutely, whether it is of the same species as the other, or of a different one, in respect of which it is called heavier: as a wax ball of a hundred pounds is called heavier than a lead ball of one ounce. Hence arise five useful combinations.

[Margin: 1st combination: of things equally heavy.]

For some things are found that are equally heavy in species and in the individual, as are two lead spheres of one pound each; and these are naturally of the same size — naturally, for nothing prevents one of them, rarefied supernaturally or preternaturally, from becoming of greater size.

[Margin: 2nd combination: of things equally heavy.]

Others are equally heavy in species but not in the individual, as are two lead balls of which one weighs one pound and the other two. Of these, if they are solid, the one that is heavier in the individual, that is, the two-pound one, is also larger — granted that by supernatural or preternatural condensation it could become equal to the other or smaller. But if one of them is hollow inside, or filled with air or with a lighter body, it can, though lighter in the individual, equal the size of the other.

[Margin: 3rd combination [?]: of things equally [?] heavy.]

Others again are equally heavy in the individual but not in species, such as a lead ball and a wax ball each weighing one pound; and naturally the one that is lighter in species, the wax one, is larger — granted that supernaturally or preternaturally it could be condensed so as not to be larger than the other.

[Margin: 4th combination [?]: of heavier things.]

Again, some things are heavier than others both in species and in the individual, as if a lead ball weighs 10 pounds and a wax ball one pound; and these can be equal to the others in size, or smaller, or larger.

[Margin: 5th combination [?]: of heavier things.]

Some, finally, are heavier in species but lighter in the individual, as a small lead ball of one ounce is heavier than a wax ball of a hundred pounds; and these are

naturally smaller than the others. In this combination, through the correlative term, is contained the combination of what is heavier in the individual but lighter in species. For if lead of one ounce, heavier as regards species, is lighter in the individual than wax of a hundred pounds, then conversely wax of a hundred pounds is heavier in the individual but lighter in species than lead of one ounce. With these things laid down, the corollaries written below follow from the preceding table.

[Margin: 1st corollary.]

First. Two spheres equally heavy in species and in the individual naturally descend equally fast through the same medium — understand always: if they are let go together from the same height; for they are of equal size, and so everything is equal. If, however, one of them, supernaturally or preternaturally, became either larger by rarefaction or smaller by condensation, the smaller would descend faster, because the angle of contact — the interior one, with which it would strike the plane of the air beneath it — would be more acute.

[Margin: 2nd corollary.]

Second. Of two spheres equally heavy in species but not in the individual, the one heavier in the individual naturally descends faster through the air: whether they are of equal size, because one of them is hollow inside, as is clear from experiment 1, repeated twelve times, and from experiment 2; or of different size, as is clear from experiments 7, 8, 9, 13, 14, 15, 16, 21. Among these there are some in which I would believe that the speed did not keep, against the excess of individual heaviness, a proportion like the one that appeared in the others — either because of too great an excess of size, which would make an obtuse angle, or because the balls were not perfectly rounded all over. But it is better to report experiments faithfully than to dress them up by trimming them.

[Margin: 3rd corollary.]

Third. Of two spheres equally heavy in the individual but not in species, the one heavier in species naturally descends faster through the air. This is established by experiment 12. And the reason is that, naturally, the one heavier in species is of smaller size than the other, which is equally heavy with it in the individual; therefore it surpasses that other not only in specific heaviness but also in the sharpness of its figure, or of its angle of contact, which arises from the smallness of the sphere — besides the fact that in our case wood is more porous than lead and, because of the lightness of the air or vapours hidden within its pores, less suited to gravitate.

[Margin: 4th corollary.]

Fourth. Of two spheres of which one is heavier both in species and in the individual, the heavier naturally descends faster through the air: whether it is larger than the other, as appeared in experiment 10; or equal to it, as was established by experiments 3, 4, 5, 6; or even smaller, as we saw in experiment 11.

[Margin: 5th corollary.]

Fifth. Of two spheres of which one is heavier in species but not in the individual, the one heavier in species can (speaking naturally) descend faster through the air; it can also descend equally fast; and it can descend more slowly. An example of the first case we had in experiments 12, 17 and 18; an example of the third case in experiments 19 and 20. For the second and third we have a sufficient argument in comparing experiment 8 with 18. In both, the clay ball was of 11 ounces, the drachms being neglected for the moment. Now the clay ball of 11 ounces, let go with the clay ball of 21 ounces in experiment 8, was so much slower that it was 12 feet from the floor when the clay ball of 21 ounces struck it; whereas the clay ball, likewise of 11 ounces, let go in experiment 18 with a lead ball of about 2 ounces, was slower only by as much as the interval of one foot marks. Therefore, if the clay ball of 21 ounces had been let go with the lead ball of about 2 ounces, it would have left the lead ball behind by an interval of 11 feet, or at least of more than one, and would have turned out faster in descent. Since, then, by an increase of individual heaviness it went from slower to faster, the individual heaviness of the clay could certainly have been increased by so little that it would descend equally fast with the lead. I said *at least of more than one* because I am moved by experiments 19 and 20, in which a small lead ball of one ounce gained more in speed through its smallness than it fell short of the heavier clay ball by reason of weight — although that weight was far greater — so that it was indeed slower than the clay ball, but only very slightly slower. The more perfect roundness of the lead balls, compared with the clay ones, also contributed to this.

[Margin: 6th corollary.]

Sixth. Of two spheres let go together from the same height through the same air, the one that is lighter on every count never naturally descends faster or equally fast; for the other is heavier either in species and in the individual, or in the individual, or in species. Therefore, speaking simply, heavier things descend faster naturally — at least in our air. But it rarely happens that they are of different weight and the defect of individual heaviness is compensated by the sharpness that comes of smallness in size, so that they descend equally fast.

V. Classis Experimentorum pro Inaequali descensu Grauium per Aquam.

V. Class of experiments: for the unequal descent of heavy bodies through water.

XV. Although it is by no means necessary, it is nevertheless very useful — both for the argument now in hand and for the completeness of physical speculations — to explore whether, and in what proportion, the speed of heavy bodies descending naturally through water continually increases.

[Margin: 1st experiment.]

First, then, we used two glass tubes, one of which was 3 1/3 Roman feet high and the other 4 feet and 2 inches. We divided them in two by a mark, filled them with water, set them upright to the perpendicular, and let various small

balls down through them. A lead ball of 4 drachms and 10 grains completed the first half of the first tube in about 5 small vibrations of our smallest pendulum, that is, in 50‴, and the remaining half in two vibrations, that is, in 20‴; so it completed the whole height of the tube in 7 vibrations, or in one second of the primum mobile and 10‴ besides. It was therefore far faster at the end of its motion than at the beginning. A glass ball too, whose weight was 15 grains of wheat, measured out the whole tube in its descent in exactly 2″, that is, in 12 vibrations of the smallest pendulum, but the first half in 8 or 8 1/2 vibrations, and the second in about 4. Finally, a bone ball weighing 2 drachms and 6 grains descended through the whole tube in 10 vibrations, namely 1″ 40‴; through the first half of the tube in 7 vibrations, namely 1″ 10‴; and through the second in 3 vibrations, or 30‴. Altogether, then, heavy bodies of these three species descended much faster at the end of their motion than at the beginning. For the rest, see the following small table; these things will be confirmed further by the small table to be shown at number 17.

Length of the glass tube	Order of experi-ments	Balls let down through the glass tube full of water	Weight: ounces	Weight: drachms	Weight: grains	Time of descent through the first half of the tube	Time of descent through the second half of the tube
	1	Lead	0	4	10	0″ 50‴	0″ 20‴
	2	Lead	0	2	6	1″ 10‴	0″ 30‴
3 1/3 feet	3	Glass	0	0	15	1″ 25‴	0″ 40‴
	4	Glass	0	0	7 1/2	1″ 40‴	0″ 50‴
	5	Bone	0	2	8	1″ 10‴	0″ 30‴
	6	Bone	0	0	10	1″ 30‴	0″ 40‴
	7	Lead	0	4	20	0″ 50‴	0″ 20‴
4 1/6 feet	8	Lead	0	1	0	1″ 20‴	0″ 35‴
	9	Of ebony	0	2	14	4″ 50‴	2″ 0‴
	10	Of ebony	0	1	10	5″ 0‴	2″ 0‴

VI. Classis Experimentorum Pro Incremento Velocitatis Grauium descendentium per Aquam.

VI. Class of experiments: for the increase of speed of heavy bodies descending through water.

[Margin: Examination of experiment 1.]

XVI. *First.* From the experiments of the table just preceding one may get wind of the proportion we are investigating; for it comes very close to the one we found in air and explained at numbers 11 and 12 — namely, to the proportion of the spaces with the squares of the times. For in experiment 1 the lead ball completed the first half of the tube in 50‴ and the second in 20‴, and so the whole tube in 70‴. But as space 1 is to one twice as long, that is, to 2, or as the half is to the whole, so, as nearly as can be, is the square of the first time, 50‴, namely 2500, to the square of the whole time, 70‴, namely 4900; for 2500 is a little more than half of 4900.

[Margin: Examination of experiment 2.]

Again, in experiment 2 the lead ball covered the first half in 70‴ and the second in 30‴, so that the whole time of passing through the tube was 100‴. But as 1 to 2, so very nearly is the square of the number 70, 4900, to the square of the number 100, 10,000; for 4900 is a little under 5000, which is half the number 10,000.

[Margin: Examination of experiment 3.]

In experiment 3 the whole time of descent of the larger glass ball was 125‴, and the time of the first half of the space was 85‴ (for we resolved the second into 60 thirds). Now as 1 to 2, so very nearly is the square of the number 85, 7225, to the square of the number 125, 15,625, whose half is 7812 1/2; and 7225 comes close to that number.

[Margin: Examination of experiment 4.]

Now in experiment 4 the time in which the smaller glass ball descended to the bottom of the tube was 150‴, but the time of the first half of the space was 100‴. And as 1 to 2, so very nearly is the square of the number 100, 10,000, to the square of the number 150, 22,500.

[Margin: Examination of experiment 5.]

In experiment 5 the larger bone ball passed through the whole tube in 100‴ and through the first half in 70‴. And as 1 to 2, so is the square of the number 70, 4900, to the square of the number 100, 10,000, very nearly.

[Margin: Examination of experiment 6.]

In experiment 6 the other bone ball ran through the tube in 130‴ and through the first half in 90‴. Now as 1 to 2, so very nearly is the square of the number 90, 8100, to the square of the number 130, 16,900.

[Margin: Examination of experiment 7 and of the rest.]

Let us now examine the experiments made in the longer tube, of 4 1/6 feet, and very nearly the same proportion will be found. For in experiment 7 the time of the first half was 50‴, whose square is 2500, while the time of the whole space was 70‴, whose square is 4900 — that is, very nearly twice as large, just as the whole space is double its half. It is well known, moreover, that to these differences of the squares there answer much smaller differences still in their roots.

When the rest are examined in this way, then, the squares of the two times — of the first time for the half space, and of the whole time for the whole space — are found to have stood very nearly as 1 is to 2, which the following small table sets before the eyes.

Order of experiments	Square of the first time	Its root	Square of the whole time	Its root
1	2500	50‴	4900	70‴
2	4900	70	10000	100
3	7225	85	15625	125
4	10000	100	22500	150
5	4900	70	10000	100

Order of experiments	Square of the first time	Its root	Square of the whole time	Its root
6	8100	90	16900	130
7	2500	50	4900	70
8	6400	80	13225	115
9	84100	290	168100	410
10	90000	300	176400	420

XVII. But to investigate by more certain experiments whether the proportion just mentioned was true and continuous, we used three tubes whose heights kept the order of the squares, so that the first was as 1, the second as 4 and the third as 9: that is, the first was of 18 inches, the second of 72 inches and the third of 162 inches — which is the same as saying that the first was of 1 ½ feet, the second of 6 feet and the third of 13 ½ feet. Thus, if the time of the second space completed were double the time of the first, and the time of the third space, or tube, triple the time of the first tube, each traversed by the same ball let down through these tubes filled with water and set perpendicularly upright, we would be certain of the proportion just mentioned. Moreover, since the tubes were unsuited to balls of larger bulk because of their narrowness, and since, as the water fluctuated in the descent and passed through the narrow space, the balls struck alternately against the sides of the tubes, we used a well whose water was 12 feet deep and a cistern whose water was exactly 3 feet deep. The depth of the water in the cistern was therefore to the depth of the well as 1 to 4, that is, as the square of unity to the square of two. In this way too it would be established for us whether the time of descent through the well water of 12 feet was double the time of descent of the same ball through the cistern water of 3 feet, as it ought to be if the increase of speed went according to the squares of the times. What we obtained will be clear from the selected experiments faithfully set out in the following table.

Order of experiments	Balls let down through water	Weight: drachms	Weight: scrupples	Weight: grains	Time of descent, depth 1 ½ feet	Time of descent, depth 6 feet	Time of descent, depth 13 ½ feet
Through water enclosed in tubes							
1	Ebony	2	0	14	3″ 0‴	6″ 10‴	9″ 20‴
2	Ebony	1	0	10	4″ 20‴	8″ 30‴	12″ 50‴
3	Holm-oak	2	1	6	1″ 10‴	2″ 20‴	4″ 0‴
4	Holm-oak	1	1	0	1″ 20‴	2″ 40‴	4″ 10‴
5	Horn	2	0	18	1″ 10‴	2″ 20‴	4″ 10‴
6	Horn	0	2	16	1″ 30‴	2″ 40‴	4″ 20‴
7	Clay	8	0	0	1″ 0‴	2″ 0‴	3″ 10‴
8	Clay	2	1	6	1″ 40‴	3″ 20‴	5″ 0‴
9	Lead	4	0	10	0″ 40‴	1″ 20‴	2″ 10‴
10	Lead	1	1	10	1″ 10‴	2″ 20‴	3″ 40‴

Order of experiments	Balls let down through water	Weight: drachms	Weight: scru-ples	Weight: grains	Time of descent, depth 1 ½ feet	Time of descent, depth 6 feet	Time of descent, depth 13 ½ feet
11	Ivory	24	0	0	1″ 0‴	2″ 0‴	✠ ✠
12	Bone	0	2	10	1″ 20‴	2″ 40‴	4″ 20‴
13	Boxwood	3	1	18	4″ 40‴	9″ 30‴	15″ 10‴

Order of experiments	Larger balls	Weight: pounds	Weight: ounces	Time of descent, depth 3 feet	Time of descent, depth 12 feet
Through well water					
14	Iron	2	1	0″ 50‴	1″ 40‴
15	Iron	1	0	1″ 10‴	2″ 20‴
16	Ivory	0	3	1″ 40‴	3″ 30‴

XVIII. Of the experiments just mentioned, those that agree exactly with the proportion just mentioned are 8, 11, 14 and 15; for the spaces covered in equal times compounded stood to one another as the squares of the compounded times. Experiments 1, 2, 4, 6, 7, 9, 10, 12 and 16 nevertheless support it very nearly, since they exceed or fall short of the desired measure of time by one or at most two vibrations of the pendulum, that is, by 10‴ or 20‴. The rest exceed it by a little more. We, however, set more store by experiment 14 and 15, because the width of the well, the entirely free descent of the balls and the great force of the iron for overcoming the resistance of the water gave us a more evident observation, and one repeated more often. It therefore seems more certain to us that heavy bodies descending naturally through water also have the same increase of speed that they have in air — provided corrected and selected observations are considered. This is said of bodies whose heaviness in species stands to the heaviness of water as it does in iron and similar bodies, and is greater than the heaviness of ivory.

VII. Classis Experimentorum pro Inaequali Descensu per Aquam Grauium Duorum diuersi ponderis.

VII. Class of experiments: for the unequal descent through water of two heavy bodies of different weight.

XIX. Now from the tables set out at numbers 15, 16 and 17 it is apparent that, of two balls of the same species, the one that was heavier in the individual and greater in bulk than the other always descended more quickly from the same height to the same bottom of the water. For the lead ball of 4 drachms and 10 grains ran through the first tube in 70‴, which the lead ball lighter by almost half, that is, of 2 drachms and 6 grains, completed only in 100‴; and so with the other comparisons in the table of number 15. So too in the table of number 17 you will see that the pairs of balls of the same species, or nearly the

same, always descended in such a way that the heavier descended more quickly. This was so evident in water that several of our people who were present — setting aside even a more exact count of the time, and judging the time of the fall by common esti- mation — discerned with certainty that the lighter ball descended somewhat more slowly. I said *nearly of the same species*, because the ivory and the bone balls were not of the same lowest species; and I did not have two boxwood balls that would descend in water, for boxwood is rarely found that sinks to the bottom of the water unless it is knotty.

But since up to this point we had compared balls of different bulk with one another, we wished to compare balls of the same bulk as well. Since we could not attend to the difference of space — either because of the opacity of the tube, or because their narrowness made it hard to let two balls go together — we used the difference of time, which, because of the slowness of motion through a denser medium, namely through water, was more evidently observable for us. Among the balls of the same bulk we made two pairs of nearly the same species, namely one clay ball solid and the other hollow inside; the rest were of different species. The difference of time, then, was as you see in the small table immediately following.

Time of descent of balls of the same bulk.

Order of experiments	Balls let down through water	Weight: ounces	Weight: drachms	Weight: scruples	Weight: grains	Time of descent	Depth of water, feet
1	Clay	1	0	0	0	2″ 0‴	6
	Boxwood	0	3	1	18	9″ 30‴	
2	Lead	0	4	0	10	1″ 20‴	6
	Holm-oak	0	1	1	0	2″ 40‴	
3	Iron	12	0	0	0	2″ 20‴	12
	Ivory	3	0	0	0	3″ 40‴	
4	Stone	37	0	0	0	3″ 20‴	12
	Stone	12	0	0	0	4″ 0‴	
5	Stone	12	0	0	0	4″ 0‴	12
	Iron	25	0	0	0	1″ 40‴	

The stone balls, moreover, were made of the dust of baked brick bound together, and so were of far greater bulk than all the others.

VIII. Classis experimentorum pro Ascensu Inaequali Leuium per Aquam, seorsim sumptorum.

VIII. Class of experiments: for the unequal ascent of light bodies through water, taken separately.

[Margin: The ascent of air and oil through water.]

XX. At numbers 3 and 4 we have already established positive lightness by experiments taken from air and oil enclosed with water inside glass pipes; and we used the same pipes described there to test the inequality of the ascent of

air and of oil through water. Look, then, at the first and second figures set out at number 5. In the first, the air B reached C, the middle of the pipe, in 30 vibrations of the somewhat larger pendulum, but, ascending from C to A, it completed the remaining half in 24 vibrations.

[Margin: 1st experiment.]

The oil, on the other hand, in the second pipe ascended from F to G, the middle of the pipe, in 14 vibrations, and from G to L in 12 vibrations. The reason why the air, which ought to have been faster, was nevertheless slower than the oil, was this: the air alone, keeping its rarity, rose through the narrow channel of the pipe in such a way that it left only the thinnest of paths around itself for the water succeeding into its place; and since the water had to be thinned out, it needed brief delays of time. The oil F, by contrast, pushed by the air E as it penetrated upward, lost some of its original bulk during the ascent, a part of it adhering to the underside of the air itself and flying upward with the air. Hence it came about that around the oil the path for the water succeeding into its place was looser, and so great a delay was not needed, since the water required almost no thinning. But setting aside these rudiments of experiments begun in pipes that were too narrow, let us come to others that are more certain.

[Margin: 2nd experiment.]

Second, then, we used tubes full of water with heights of 4 or 6 feet; letting various balls lighter than water go from the bottom, and then letting the same balls go upward from the middle of the tube, we always found that light bodies move faster towards the end of their motion. For example, a small glass sphere full of air inside, let go from the middle of the tube, ascended upward through a height of 3 feet in a time of one second and 20‴; let go from the bottom of the tube, it ascended through a height of 6 feet in a time of 2″ and 10‴. But if it moved uniformly, it ought to have completed double the space in double the time, namely in 2″ and 40‴. What I say of this one we found by very many other experiments with various small balls lighter than water. But since the narrowness of the tubes did not allow so convenient an operation, we decided to go back to the well, whose water was 14 feet deep. We therefore let down to its bottom a pole made up of partial shafts fastened together; at its lower end was a cylindrical vessel with a lid that could be tipped open in an instant by a drawing-cord, and enclosed in the vessel itself was a sphere lighter than water, which the weight of the lid held inside the vessel. Then, once the pole had been let into the well until the vessel was at the bottom of the well, and the vibrations of the pendulum had begun to be counted at the same moment as the lid was raised, we counted the time in which the sphere rose from the vessel and the bottom of the well to the surface of the water. The same operation was repeated three times, four times or more, so that I and Father Grimaldi, and the others who were present, might be made more and more certain of the same number of vibrations. Further, 7 feet having been marked on the pole just mentioned, the vessel with the same sphere enclosed in it was let down

to the middle of the well water; and the sphere being freed by the raising of the lid, turned back or tipped up so that it could at once make its way upward, we began at the same moment to count the vibrations of the pendulum, until the sphere burst out at the surface of the water. The more select experiments, and those repeated more often, we shall set out in the following table with the utmost good faith; we do not doubt that it will be borne out for those who are willing to try the same things with equal diligence. The pendulum used to count the times equals 10‴ of the hour with each simple vibration, so that 6 vibrations equal one second of the hour. Note, however, that the wax sphere and the bladder were not exactly round on every side, as the other spheres were.

Species of spheres	Diameter: inches of the Roman foot	Diameter: fraction of an inch	Weight: ounces	Weight: drachms	Weight: scruples	Ascent through 14 Roman feet: vibrations	Ascent through 14 Roman feet: time	Ascent through 7 Roman feet: vibrations	Ascent through 7 Roman feet: time
Beech	3	0	5	0	0	28	4″ 40‴	16	2″ 40‴
Beech	2	1/5	2	6	0	37	6″ 10‴	22	3″ 40‴
Wax	2	1/5	3	0	0	88	14″ 40‴	49	8″ 10‴
Glass full of air	3	0	1	4	0	26	4″ 20‴	15	2″ 30‴
Glass full of air	2	1/5	0	7	0	33	5″ 30‴	19	3″ 10‴
Glass full of air	1	1/2	0	1	2 1/2	38	6″ 20‴	20	3″ 20‴
Bladder	2	0	0	0	0 9/24	30	5″ 0‴	16	2″ 40‴
Glass full of water	3	0	9	1	0				
Glass full of water	2	1/5	3	5	1				

From all these and very many other experiments it is evident to us that light bodies in their natural ascent through water move unequally, in such a way that they move faster and pass through the first half of the space in a longer time than the second. For example, ascending through the first half of the water, that is, the seven-foot depth, the beech sphere tra- versed it in 16 vibrations, and the whole water, twice as high, that is, of 14 feet, in 28 vibrations. It therefore completed the later half in 12 vibrations, when it ought to have taken 16 had it moved evenly; and so with the others. The reason why the wax sphere ascends so slowly is both its lack of the most perfect roundness and the slight difference of specific heaviness between water and wax. For the wax sphere and the glass sphere filled with air each had the same diameter, namely 2 1/5 inches, and the wax sphere weighed 3 ounces, the glass one 7 drachms. We filled that same

glass sphere with water, and its weight was 3 ounces, 5 drachms and one scruple. Taking away, then, the weight of the glass alone, which was about 7 drachms, there remains a weight of water of 2 ounces, 6 drachms and one scruple — but of water whose diameter is smaller by twice the thickness of the glass sphere, which was certainly small. From this we gather that, if a sphere of water were equal to the wax sphere, the water sphere would be slightly heavier. This is also clear from the immersion itself, for the wax floats in water in such a way that, although it does not descend to the bottom, it is nevertheless almost wholly submerged below the surface.

IX. Classis Experimentorum pro Incremento Velocitatis Leuium per Aquam Ascendentium.

IX. Class of experiments: for the increase of speed of light bodies ascending through water.

XXI. From the experiments of the eighth class it is quite clear that the light bodies we used do not increase in their ascent through water in such a way that the spaces covered stand to one another as the squares of the times; rather, the square of the whole time in which 14 feet were traversed is much more than double the square of the time in which 7 feet were traversed. For example, the time in which the larger beech sphere traversed 14 feet was 28 vibrations, whose square is 784; but the time in which the same sphere traversed 7 feet was 16 vibrations, whose square is 256. Yet 784 to 256 is more than as 2 to 1, since it is as 3 to 1 and more. And so you will find with the other times corresponding to the whole space and to the half; you may see the squares of these times in the following small table, in the same order as we kept in the preceding one.

Vibrations	Squares	Vibrations	Squares
28	784	16	256
37	1369	22	484
88	7744	49	2401
26	626	15	225
33	1089	19	361
38	1444	20	400
30	900	16	256

Therefore the proportion that bodies of this kind keep is inconstant. In some, the time of the first equal space seems to be to the time of the second equal space as 4 to 3; in some, as 5 to 4; in some, nearer to equality, as 10 to 9. For in the first experiment 28 vibrations belong to the whole space of 14 feet, but 16 vibrations to its first half; when these are taken from 28, 12 remain. And as 16 is to 12, so is 4 to 3. Likewise, when 22 is taken from 37, 15 remain; and as 22 is to 15, so very nearly is 4 to 3. But it is not so in the other experiments, as you will learn if you subtract the time assigned to the first half from the time of the whole space.

[Margin: Why is the increase of speed less in light bodies than in heavy ones?]

The reason why bodies of this kind do not have so great an increase of speed in water is not heterogeneity alone — as in the glass spheres made up of glass, which is heavier in species than water, and of air; or in the wooden ones, which soak water into their pores the more, the more often the experiment is repeated within a short time. The sole or chief reason is that the excess of the heaviness of water over the heaviness of the bodies just mentioned is far smaller than the excess of the heaviness of a metal or stone sphere over the heaviness of water, or over the slight heaviness of our air. But if bodies were found as much lighter than water as metal or stone bodies are heavier than water — let alone than air — then it is very probable that the increase of speed would go according to the squares of the times. Thus, if a lighter sphere ascended through a space of 7 feet in a time of 5 vibrations, whose square is 25, it would complete a space twice as great in 7 vibrations, whose square, 49, is very nearly double 25.

X. Classis Experimentorum pro Inaequali Ascensu binorum Leuium per Aquam, sed differentis Leuitatis.

X. Class of experiments: for the unequal ascent through water of two light bodies, but of different lightness.

XXII. From the same small table of number 20 it is quite apparent that the bodies which are lighter either in species and in the individual together, or at least in species, ascend more quickly through water. For when three spheres of different species but of the same bulk are compared with one another — namely the smaller beech sphere, the wax sphere and the glass sphere filled with air (so that the inequality may not be put down to shape or bulk) — it is plain that the lightest, namely the glass-and-air sphere of 7 drachms, ascended in 33 vibrations, the beech sphere of 22 drachms in 37 vibrations, and the wax sphere of 24 drachms in 88 vibrations. Again, comparing the larger beech sphere with the larger glass-and-air sphere of the same bulk, that is, of three inches in diameter, the glass-and-air sphere of 12 drachms ascended in 26 vibrations, but the beech sphere of 5 ounces, or 40 drachms, in 28 vibrations. These comparisons are for bodies lighter in species and in the individual together. For bodies lighter in species only, compare the larger beech sphere with the wax sphere. The bladder, for its part, was not perfectly spherical, as we have already noted; moreover, it was soaked through in the water so quickly and took up the moisture so much that, having become heavier, it exceeded the weight assigned to it in the table.

XI. Classis Experimentorum, pro lapsu globorum diuersorum per planum inclinatum ad Horizontem ad diuersos inclinationum gradus.

XI. Class of experiments: for the fall of various balls along a plane inclined to the horizon, at various degrees of inclination.

XXIII. We prepared a wooden tube, or canal, of 35 feet. Raising it from the plane of the horizon, we counted the time in which both water and various balls descended along the whole slope of the canal, and we repeated this at various times in the presence of some Fathers of our Society. We shall represent the experiments in the following diagram and in the table belonging to it. In the diagram, AB is the perpendicular line and AC is parallel to the horizon; above AC, on AB but moved forward towards C, the various heights of the right-hand end of the canal were measured. Further, the angles ACL, ACK, etc. are the angles at which the canal was raised from the horizon; the rest you will learn from the table.

Diagram. A right-angled fan of lines in the right column: a vertical line AB stands at the right, with points marked on it from top to bottom B, L, K, I, H, G, F, E, D, A; from the point C at the lower left straight lines run to each of these points, the lowest lying along the base CA, so that the lines rise at successively steeper angles. The vertical line runs on a little above B. A letter M stands above the upper lines, left of centre; the text does not cite it here.

- C (lower left): the vertex at which the inclined lines meet
- M (above the upper lines, left of centre): label, not cited in the text on this page
- B, L, K, I, H, G, F, E, D (on the vertical line, top to bottom): the heights to which the right-hand end of the canal was raised
- A (foot of the vertical line): the right angle, end of the horizontal CA

Do not be surprised that in this experiment we did not use numbers of feet or of degrees that were always round: because of the length of the wooden canal we needed the steps of a certain stone staircase, on which the other end of the canal rested, and their height varied.

EXPERIMENTA PER CANALEM PEDVM 35.

Experiments with the canal of 35 feet.

Height of the canal	Feet	Inches	An-gle	Angle of elevation	Water: time of fall	Wooden ball: time of fall	Clay ball: time of fall	Line of travel
AD	0	11 ⅕	ACD	1° 32′	15″ 40‴	18″ 0‴	19″ 0‴	CD
AE	1	10 ½	ACE	3° 4′	9″ 10‴	11″ 0‴	12″ 0‴	CE
AF	3	5 0	ACF	5° 36′	6″ 40‴	8″ 10‴	8″ 50‴	CF
AG	4	7 ⅔	ACG	7° 37′	6″ 0‴	6″ 30‴	7″ 20‴	CG
AH	6	2 ½	ACH	10° 14′	5″ 20‴	5″ 10‴	5″ 40‴	CH
AI	7	5 0	ACI	12° 15′	4″ 40‴	4″ 30‴	5″ 0‴	CI
AK	8	8 ⅓	ACK	14° 21′	4″ 10‴	4″ 0‴	4″ 10‴	CK
AL	10	0 ⅓	ACL	16° 32′	4″ 0‴	3″ 40‴	3″ 50‴	CL

Height of the canal	Feet	Inches	An-gle	Angle of elevation	Water: time of fall	Wooden ball: time of fall	Clay ball: time of fall	Line of travel
AB	12	6 ⅓	ACB	20° 56′	3″ 50‴	2″ 40‴	3″ 0‴	CB

Do not be surprised that the clay ball, though heavier than the wooden one, was slower. It descended through the canal with more continuous friction, whereas the wooden ball, hopping along by its lightness, was carried further on. Now, if water raised one foot from the horizon runs down 35 feet, or 7 paces, in a time of 15″, then, were it to run down with uniform speed, it would seem to run 28 paces in one minute and 1680 paces in one hour. But from a fall of 5 feet, or one pace, it covered 7 paces in about 6″, and therefore 4200 paces in an hour. Yet because of the great force of the waters, a river whose fall is one pace to the mile usually covers about 2000 paces in an hour, notwithstanding the obliquity and winding of its waters, as we said in Book 2, chapter 13, number 8.

XII. Classis Experimentorum pro Eleuatione maioris & maioris ponderis à Graui naturaliter descendente ex maiori ac maiori Altitudine.

XII. Class of experiments: for the raising of a greater and greater weight by a heavy body descending naturally from a greater and greater height.

[Margin: A balance of a construction suited to this.]

To test this I made a balance whose pans were not hung from the beam, or its arms, but fixed on top of it, so that no cords or little chains should hinder the perpendicular descent of the ball onto the very centre of the pans. The beam itself was supported at its middle by a well-steadied prop with a large foot. Once an exact equilibrium of the pans had been found, a second prop was set under the right pan, to keep it in equilibrium when loaded with any weight placed on it — that is, to hold the beam parallel to the horizon. On the other pan, the left one, I gently placed a scruple at its centre, that is, the twenty-fourth part of an ounce; by its force the right pan was raised to a height of 4 digits measured across, and no more, as I often found by trial. Having chosen this height, I fitted a small bell above the right pan, 4 digits away from it, so that the raised pan would make its rising known by the ring, and its failure to rise by the absence of a ring — to the ears more surely than to the eye. Further, I chose a leather playing ball, quite hard, brought to the weight of one ounce by lead inserted among its stuffing, so that no other kind of ball, in falling and striking the left pan, should either be bruised by the pan or bruise or break it.

With these things prepared, I went to a certain square column having the shape of a parallelogram — we have two such in our Solarium — and placed a weight of two ounces on the right pan. I let the ball fall perpendicularly onto the centre of the left pan along the side of the column, with the help of a gnomon that showed this position, while Father Francesco Maria Grimaldi watched whether the ball fell on the very centre of the pan. In this way, by

experiments repeated many times, I determined what height was required for the ball already mentioned to raise a weight of two ounces; then what height, to raise a weight of three ounces; and finally what height, to raise a weight of four ounces. To raise it, I say, in such a way that from the ring — slight, indeed almost the least possible — on the one hand, and from the absence of a ring, if the height were a little less, on the other, the height required to raise the right pan up to the height of 4 digits was made certain. Now, to raise two ounces a height was required of 2 Roman feet and almost a dodrans [three-quarters of a foot], that is, of almost 33 or 32 inches. For the height held at about this limit: from a height of 31 inches the right pan was not raised as far as striking the bell, though it came very near, while from a height of 34 inches the stroke was already strong, and so on. To raise a weight of 3 ounces a height of 84 inches was required, and for a weight of 4 ounces a height of 116 inches, or a little more, as you see in the following small table.

To raise a weight of (ounces)	Let it fall perpendicularly from a height of (inches of the Roman foot)	Difference
2	32	
		52
3	84	
		32
4	116	

A leather ball of one ounce: to raise the weights written below to a height of exactly 4 digits, it must be let fall from the height written below.

[Margin: 1st Corollary.]

Given this, it is now plain that the higher the place from which a heavy body is let fall, the greater the impetus it acquires. For the ounce ball, which by itself cannot raise the pan loaded with a two-ounce weight, or *dextans* [?], can nevertheless do so if it is let fall from a height of 32 inches, etc.; and it raises a weight of 3 ounces from a height of 84 inches, and a weight of 4 ounces from a height of 116 inches.

[Margin: 2nd Corollary.]

It is clear, further, that it is not improbable that the acquired heaviness, or impetus, which a heavy body descending naturally gains at the end of equal times stands to the impetus gained at the end of the following equal time as space stands to space traversed in times that are equal but taken separately; that is, that it has the proportion which the odd numbers counted from unity have. This would certainly be a most beautiful thing, and most worth knowing, if it could be confirmed by continued experiments. For the leather ball by itself already has a gravitation, or impetus, as 1, that is, of an ounce, since by its force it balances one ounce; and with one scruple added it raises an ounce up to 4 digits. From a height of 32 inches, by itself, it raises a weight of 2 ounces to the same height of 4 digits; it has therefore acquired an impetus of one ounce and one scruple. But if it had had to raise the pan only to the least of sensible

heights, a gain of one ounce, that is, of an ounce's impetus, would have been enough; and for this gain a height of very nearly 30 inches would have been enough. When, however, it descends from a height of 116 inches, which is almost four times the height of 30 inches, the time completed was twice as long as that in which it descends from a height of 30 inches. For it is plain from the earlier experiments of Class 3 that if a heavy body has traversed a space as 1 at the end of the first time, it has traversed a space as 4 at the end of the second equal time; and therefore, arguing conversely, if the first space is to the second as 1 to 4, the first time is equal to the second time taken separately. Now at the end of this second time the ball raised a weight of 4 ounces; it therefore acquired an impetus of a quadrans, that is, of ounc- es 3; but in the second time, taken separately, it acquired an impetus of 2 ounces. Now, if the space traversed in the first time is taken as 1, the space traversed in the second time, taken separately, was as 3.

[Margin: How great is the impetus acquired [?] in the motion of heavy bodies?]

Therefore the impetuses acquired at the end of equal times stand to one another as the spaces traversed in times that are equal, but taken separately. I wish we had been able to establish or examine this proportion by experiments continued to greater heights. But we could not: from a height of 270 inches, which was what was needed for the ball to descend in a time triple the first, it could not be let fall so as to land exactly on the centre of the empty pan again and again, as is required to make an experiment certain; for that is too difficult, and is not done without uncertainty. But what I have suspected from this first trace of the proportion I preferred to share with the reader rather than keep entirely silent.

XIII. Classis Experimentorum pro Perfossione Aquae facta à globo decidente ex maiori & maiori altitudine.

XIII. Class of experiments: for the boring through water made by a ball falling from a greater and greater height.

We used a wooden ball of 2 ounces. Let fall perpendicularly over a bucket of water 7 inches of the Roman foot deep, it did not reach the bottom of the bucket from a distance above the water's surface of less than 7 inches; but from a distance of 7 inches it now reached the bottom with a very slight blow, and from a greater distance with a greater blow. We then had a wooden vessel of two congii [bicongium] filled with so much water that its depth was 14 inches. And yet the same ball did not penetrate that whole depth, nor strike the bottom, except from a height of 10 feet measured from the bottom of the water — but from the surface, 8 feet and 10 inches, that is, 106 inches. This might certainly seem surprising. Perhaps it happens because of the resistance of the deeper water and the agitation already set up in it; or rather because the impetus acquired in the air is diminished more and more until it is overcome,

and the wooden ball flies back upward, since it is lighter than water. Since this return happens at once, it is a sign of a very swift extinction of that impetus — namely by the lightness, which resists more and more by producing an impetus upward, until this overcomes the earlier impetus.

Theoremata selecta ex praemissis Experimentis.

Selected theorems from the experiments set out above.

[Margin: 1st Theorem.]

XXIV. *Lightness does not consist in the privation of greater heaviness, but is a quality positively distinct from heaviness.* This is against Epicurus and Strato, and indeed against Plato as reported by Simplicius on Book 1 of *De caelo*, text 89, comment 87; and against Galileo in his treatise on the things that float on water, p. 22. Aristotle refutes the first of these in the same place: if air or fire were thrust upward by an extrinsic force, by water and earth as heavier bodies, they would not move faster at the end but slower, as happens to every body moved by an external force; and besides, a greater fire would move upward more slowly than a smaller fire. But since these assumptions of Aristotle — about the greater speed of the natural motion of the elements at the end, and of a greater fire than of a smaller — could be denied by someone, we decided to confirm them by experiments with air and oil, which are bodies lighter than water, as is clear from what was said in numbers 20, 21 and 22; and, further, to establish this theorem with physical evidence by new arguments and experiments, as we think was done in numbers 3, 4 and 5. As for Galileo's claim that fiery exhalations rise faster through water than through air — whereas, if they were positively light, they ought to rise faster through air, as being the thinner medium — this arises from the comparison, and from the greater excess that the lightness of fire has over the partial lightness of water than over the lightness of air.

[Margin: 2nd Theorem.]

Heavy bodies descending, and light bodies ascending, by natural motion move with a uniformly difform speed, one that grows greater and greater towards the end of the motion. This is, first, against Arriaga, *De generatione*, disputation 4, section 5, subsection 3, who asserts that this motion is equal even in appearance; and against Simplicius on Book 1 of *De caelo*, text 88, and Cabeo on Book 1 of the *Meteorologica*, text 17, question 3, who doubted its inequality, although it is so commonly asserted. Next, it is against Galileo, in the second dialogue on the two Systems of the World, and Boulliau in his *Philolaus*, Book 1, chapter 4. They grant that the motion of heavy bodies appears unequal to us, as it seems to take place along a straight perpendicular line, but hold that in reality it takes place along one or more circular lines, so that it is equal, although because of the motion of the Earth it appears rectilinear and unequal. But against both views the experiments reported from number 6 to 13 for heavy bodies, and from number 20 for light ones, plainly militate: all of them against

the former opinion, or suspicion; and against the latter, those which rest not on the measurement alone of the unequal space traversed in equal times, but on the greater sound, stroke and raising produced by a greater impetus. For if bodies moved equally, from however great a distance they had begun their motion, they would acquire no greater force by which they could produce a greater sound or stroke.

[Margin: 3rd Theorem.]

The increase of speed of heavy bodies descending naturally through the air is according to the odd numbers counted from unity. Or: the spaces covered in equal times by the motion already mentioned have to one another the duplicate ratio of the compounded times at whose end they have been traversed. Or: the spaces already mentioned are to one another as the squares of the compounded times.

I said "naturally" not only to exclude violent or supernatural motions, but also any preternatural ones, such as the motion that comes from the agitation of our air; for the moving bodies must be such as to overcome this ordinary agitation.

This is against Chiaramonti, who in Book 12 of *De Universo*, chapter 28, held that this increment is not increased but diminished, fearing an infinite speed, as I said in number 2. It is also against Cabeo, who on Book 1 of the *Meteorologica*, text 17, question 3, held it more probable that this increment takes place according to the arithmetic progression: so that if in the first time a heavy body covered 1 foot, in the second equal time it would cover 2 taken separately, in the third time 3, and in the fourth 4; and so, if the motion stopped at the end of the fourth time, the feet added together would sum to 10. Baliani judged this same progression the more probable in Book 4 of *De motu naturali gravium*, pp. 110 to 113, even though he had agreed with us in Book 1, proposition 6.

[Margin: Baliani's later opinion is worse than his earlier one.]

But why did he then depart from himself? He answers himself: *the sixth proposition rests on experiments subject to the deception of the senses, by which an insensible error cannot be detected. This happens here because the portions of time equal to that first one — in which the first portion of the motion, independent of impetus, is completed — cannot be perceived, being insensible, just as that first portion of the motion is insensible; if they were perceived, we would see the motion increase according to the natural progression.* Next, on p. 113, he gives this rule for finding the spaces covered after equal times: *Multiply the number of times, if it is even, by its half, and add the half; if odd, by the portion greater than the half; and the sum of the spaces covered in the given time will result. Let 4 times be given: multiply by 2; to the product 8 add the half, 2; it makes 10, the sum of the spaces. Let 9 times be given: multiply by 5; the product 45 is the sum of the spaces. The motion is therefore increased, unless I am mistaken, according to an arithmetic progression — not the one of the odd numbers from unity believed until now, but the natural one. Nevertheless, since nearly the same effects follow because the discrepancy is insensible, it is no wonder that it was believed that the spaces are in the duplicate ratio of the times; for even if*

this is perhaps not precisely true, it is nevertheless so close to the truth that the senses could not perceive the truth in the experiments employed.

But Baliani acted prudently in not affirming this opinion of his as certain, and in qualifying it with expressions of doubt. For not only Galileo, as he himself did in the second dialogue on the system of the world (to which Gassendi subscribed in his letters *De motu impresso*), but we too, by most certain experiments over sensible intervals of time, have found the proportion already mentioned of the odd numbers from unity. How false the rule given here by Baliani and Cabeo is, and how different the effects that follow, is clear from the experiment we spoke of in number 12. In the first equal time of one second the clay ball covered 15 feet, and at the end of the fourth second it had covered exactly 240 feet. But according to the rule of Baliani and Cabeo it would have covered only 150 feet, or only ten times the first space. Let Baliani therefore restore the ear- lier opinion asserted in his Book 1, proposition 6, as being better than this later one and proved by experiments that are least fallacious.

[Margin: 4th Theorem.]

The increase of speed of some heavy bodies descending naturally through water is in itself as great as that of the same bodies descending through air, although the descent itself is slower through water than through air. That is, their speed grows according to the odd numbers; or, in covering space, it keeps the triplicate proportion of the times; or, the spaces traversed in water in equal times are to one another as the squares of the aggregated times. This is clear from the experiments reported in numbers 16, 17 and 18. I said "some", namely metal, stone and similar bodies; for bodies of boxwood, or of ebony or holm-oak, although they are heavier than water, nevertheless descend more slowly and do not acquire so great an increase of speed in water.

[Margin: [?] Theorem.]

In most light bodies ascending naturally through water, the increase of speed is, per accidens, not as great as that of metal or stone heavy bodies descending through air or through water. This is clear from the experiments reported in number 20 and examined in number 21, where the reason for the disparity is also indicated; and what would happen of itself, with the hindrances that intrude into the experiments removed, I said in the same place.

[Margin: 6th Theorem.]

Of two heavy bodies of the same species, bulk and figure, but of different weight, let fall through the air from the same height together at the same moment, the one that is heavier in the individual descends more quickly to the same pavement or ground. This theorem, and the ones that follow, are against Niccolò Cabeo, Rodericus Arriaga, Bartholomaeus Mastrius and Bonaventura Bellutus, in the places cited in number 2 under the 5th error. From crude and almost laughable little experiments these men did not hesitate to assert universally that any two heavy bodies let fall together descend equally fast and strike the ground at the same moment, without any sensible difference. Now this sixth theorem

is established by the 21 experiments that have their place in tables 1 and 2 in number 13, and on which see number 14, corollary 2.

[Margin: 7th Theorem.]

Of two heavy bodies of the same bulk and figure, but of different species, let fall through the air from the same height at the same moment, the one that is heavier in species, and therefore in the individual, descends more quickly. This is against Cabeo, Arriaga, Mastrius and Bellutus, cited above. But it is not against Baliani, since he admits that of two balls of the same bulk, one of wax and the other of lead, let fall from the height of the fortress of Savona, fifty feet, the wax one was somewhat slower, so that it was about one foot from the ground when the lead one touched the plane beneath. Now our theorem is clear from the experiments reported in the table of number 13, namely experiments 3, 4, 5 and 6, repeated many times. And reasoning a priori plainly teaches it. All the other conditions are equal, and one cannot appeal either to a resistance arising from a more obtuse angle, by which a greater mass of air meets the surfaces, or to a figure less suited to penetrating the medium, since both bodies are of the same bulk and figure. On the other hand, what is heavier in species is heavier in the individual as well, inasmuch as it is compared with a body of equal bulk but of a different species, and therefore lighter in species. It is therefore necessary that the greater heaviness, in species and in the individual, should produce an effect proportioned to itself — that is, should gravitate more and move with a greater impetus the body in which it is naturally implanted. Nor can it be said that the defect of specific heaviness in the one is compensated by an excess of individual heaviness; for then (speaking naturally) what is of the lighter species would have to be of greater bulk, which is against the hypothesis.

[Margin: 8th Theorem.]

Of two heavy bodies of the same species and figure, but of different bulk, and so of different individual heaviness, let fall from the same height at the same moment, the one that is heavier in the individual descends more quickly. This is against not only Cabeo, Arriaga, Mastrius and Bellutus, but also Galileo, in the second dialogue on the System of the World, and Giovanni Battista Baliani, Book 1 of *De motu gravium solidorum ac liquidorum*. Of these, Galileo affirms that two iron balls let fall together from the same height, even were it the concave of the lunar heaven, descend to the earth in the same time; and Baliani, that heavy bodies, whether of the same or of a different species, very unequal in bulk and heaviness, descend naturally through the same space in an equal time and strike the ground beneath at the same indivisible moment of time.

[Margin: The occasion of the errors of Galileo and Baliani.]

The occasion of this erroneous opinion was the too short space that they used: in the places listed above, Galileo admits to having used a height of only a hundred cubits, that is, 150 feet, and Baliani a much smaller height, namely 50 feet. Now when two heavy bodies of the same species, but of different bulk and so of different individual heaviness, are used, a smaller difference appears in their fall. Therefore a greater height is required in these than in the others for

the difference to be evident to the senses, and far greater attention is required to distinguish the sound and the blow of the one from those of the other. Perhaps, too, Galileo denied this inequality so that it should not harm the motion of the Earth, as we shall say in chapter 17, number 12. The theorem proposed, however, is quite clear from the experiments shown in the table in number 13 and gathered there in corollary 2, namely experiments 7, 8, 9, 13, 14, 15, 16 and 21 of that table.

[Margin: 9th Theorem.]

Of two heavy bodies of the same figure and of the same individual heaviness, let fall together through the air from the same height, the one that is heavier in species descends more quickly. This is against Cabeo, Arriaga, Mastrius, Bellutus and Baliani, cited above. What Galileo thought about these is not yet certainly known, for he seems only to have compared balls of the same species together. But on the side of this theorem stand experiment 12 in the table of number 13, and corollary 3 set out in number 14.

[Margin: 10th Theorem.]

Of two heavy bodies of the same figure, but of different species and of different heaviness, both specific and individual, let fall together through the air from the same height, the one that is heavier descends more quickly, whether it is of greater, smaller or equal size. This is against Cabeo, Arriaga, Mastrius and Bellutus; and, as regards the last part of the theorem, about equal size, against Baliani. But the whole theorem is sufficiently proved by the experiments cited in number 13 and in number 14 for corollary 4.

[Margin: 11th Theorem.]

Of two heavy bodies of the same figure, one of which is heavier in species and the other heavier in the individual, let fall together through the air from the same height, sometimes both descend equally fast, sometimes the one heavier in species descends faster, and sometimes the one heavier in the individual. See what we said in number 14, corollary 5, where we also indicated the causes of this variety: above all, the air enclosed within the pores of whichever is less dense or compact; then the size of the sphere, which strikes the air with a more obtuse angle; then the roughness of the surface. As for Baliani's claim in Book 1 *De motu gravium*, that if all hindrances are removed heavy bodies descend equally — because, although one is lighter and the other heavier, yet in the heavier one the greater heaviness must also move a greater matter — this is neither true nor consistent. It is not true, because it conflicts with our experiments. It is not consistent, because if the body is heavier it will also be greater in bulk (for he is speaking of absolute heaviness, heaviness in the individual), and so it will meet hindrance from the quantity of air to be penetrated and from the more obtuse angle with which it tries to strike that air.

[Margin: 12th Theorem.]

Of two heavy bodies of different weight let fall together from the same height, generally one descends faster than the other; and the one that descends faster is always heavier in some respect, namely either in species and in the individual, or in the

individual, or in species. Hence, if one must pronounce absolutely, it must be asserted that the heavier descends faster. This is, as it were, a summary of all the preceding theorems gathered into one, and it is sufficiently established from what was said in number 14, corollary 6. The same is to be said of light bodies in ascent, according to what was said in number 22.

[Margin: 13th Theorem.]

Of two heavy bodies descending unequally from the same height, the difference in speed is not proportional to the difference of heaviness alone, but is much smaller. This is against Aristotle and some Peripatetics on Book 3 of *De caelo*, text 27, where among other things he says of two bodies of unequal bulk: *For the speed of the smaller will stand to that of the larger as the larger body to the smaller body.* Hence, comparing two bodies of the same species, he thought that the one heavier than the other in the individual moves naturally faster in the measure that it is heavier, and so, if one is twice as heavy as the other, it also covers twice as great a space in an equal time. But this, rightly, is refuted by Galileo in Dialogue 2 of the *System of the World*, p. 164 of the Latin edition, and by Baliani in Book 1 of *De motu naturali gravium*, p. 5; and, what is chief, by the most evident experiments that we performed and gathered into a table in number 13. For whether you look at the difference in time or in space, it was never as great as the difference in weight. Thus a clay ball, hollow inside and of 10 ounces, covered the height from the cornice to the lowest base of the Asinelli tower in 4″ 20‴ of the primum mobile, while a clay ball of the same size but of 20 ounces travelled the same height in 3″ 30‴[43]; and when this one was already striking the pavement, the lighter was no more than about 15 feet from the pavement, out of the 280 that made up the height already mentioned. But if the speed of the heavier had been greater in the same measure as its heaviness, the ball of 20 ounces would have descended in half the time of the ball of 10 ounces, that is, in 2″ 10‴[44], and the lighter would have been 140 feet from the pavement when the heavier struck it. So in all the other experiments the excess of the space covered by the heavier body over the space covered by the lighter was found to be far smaller than the excess of heaviness over heaviness. Hence that proportion, which at first sight seemed probable to Aristotle and his followers, vanished entirely once it was brought back to experiment.

[Margin: Why speed does not stand to speed as heaviness to heaviness.]

Since it is difficult to give an a priori reason why the effect, speed to speed, does not keep the proportion that the cause has to the cause, namely heaviness to heaviness, it has come about that not a few of those just named have thought that any two heavy bodies, however much they differ in weight, descend equally of themselves, if the things that retard one of them by accident are removed. But we say that no case can naturally be given in which two bodies are of different weight and yet all the hindrances retarding one of them are removed from them. For the heavier body will either be greater in bulk, so

[43]printed "Tertijs 30‴"; the word fixes the order as thirds
[44]printed "Tertijs 10‴"; the word fixes the order as thirds

that it will bore through the air with a more obtuse angle; or it will be lighter in species, and enclose small portions of air or fire within its pores; or it will have a rougher surface, or be allotted a figure less suited to penetrating the air — and by these the force of the greater heaviness in moving is blunted. But suppose two bodies are given of the same species, bulk, figure and smoothness, and one is nevertheless twice as light as the other in the individual, but, on account of a greater rarity acquired preternaturally or supernaturally, has been allotted an extension and size equal to the other's. Even so, the one that is twice as heavy will not on that account descend twice as fast. Perhaps this is because the principal mover is the very substance of the movable body, which uses heaviness as a merely instrumental cause; and the substance of heavy bodies of the same species requires that they reach their natural place as soon as possible, with a determinate increment of speed indeed, but a very similar one. So the body that is lighter in the individual is moved faster by the force of its substantial principle than it otherwise could be, if bare heaviness alone were considered.

I now come to some selected problems. Their practice is not only desirable in itself, but will also be of use to us for the arguments to be adduced in chapter 17 against the motion of the Earth, or for the motion of the Earth.

Problemata selecta ex praemissis Theorematibus.

Selected problems from the preceding theorems.

[Margin: 1st Problem.]

XXV. *Given the time in which some heavy body, descending naturally, has passed through a given height: to find the time in which it passed through a smaller height, or would have passed through a greater one.*

[Margin: 1st Example.]

The problem will be explained by examples, and its practice taught at the same time. We said in number 12 that a clay ball of 8 ounces descended from the cornice of the Asinelli tower through a space of 280 ancient Roman feet in a time of 4″ 20‴; that is, if you reduce the time to thirds alone, it passed through that space in a time of 260 thirds. Now let someone ask in what time it passed through half of that space, namely 140 feet. To solve this problem from the foundations of theorem 3, let the greater space be to the smaller space sought as the square of the known time is to the square of the time sought; for the root of this square will be the time sought. Now the number of thirds in the given time, 260‴, squared, is 67,600. Let it be, then, as the space 280 to the space 140, so 67,600 to 33,800, the nearest square root of which last number is 184. Therefore such a ball passed through 140 feet in 184‴, and the remaining 140 in 76‴.

[Margin: 2nd Example, for a mile.]

Again, let someone else ask in what time such a ball would pass through a whole mile, which contains 5000 feet. Let it be, then, as the smaller space, which was of 280 feet, to the greater space, that is of 5000 feet, so the square of

the time of 260‴, 67,600, to the square 1207107‴, whose nearest root is 1098‴, that is, 18″ 18‴.

[Margin: 3rd Example, for the semidiameter of the Earth.]

By the same device you will find that such a heavy body, if it descended through a medium such as the air nearest us, would pass through one semidiameter of the Earth, which for us contains 4139 Italian miles, in a time of 70685‴. For as 1 mile is to 4139, so is the square 1207107‴ to the square 4,996,015,873, whose nearest root is 70685‴, which make 19′ 38″ 5‴. In so much time, therefore, the heavy body already mentioned would reach the centre of the Earth, if a well bored all the way there were full of air.

[Margin: 4th Example, for the distance of the Moon.]

But to go further, at least as far as the Moon: since the upper region of the air, and the fire, if there is one, is a less resisting medium, you will readily grant that the ball already mentioned, or at least a larger one, namely of one pound, will pass through the first semidiameter of the Earth, the one nearest the lunar heaven, in 19′, whose square is 361. To someone who asks, then, in what time a clay ball of one pound would descend from the top of the lunar heaven, assumed to be 70 semidiameters distant (granted that no one assigns it so great a distance), I answer confidently that it will descend in 159 minutes of the hour, that is, in 2 hours and 39 minutes. For as 1 semidiameter of the Earth is to 70 semidiameters, so is the square 361, corresponding to the root of 19 minutes, to the square 25,270, whose nearest root gives 159′, that is, 2 hours 39′. Elsewhere, however, in Book 2, chapter 21, and Book 7, section 6, chapter 5, proceeding somewhat differently, we concluded that such a ball would cover the lunar distance in about 3 hours. For the distance of the Sun, of Saturn and of the fixed stars, see the table set out in Book 7, section 6, chapter 5, in which, however, we neglected the small fractions, for the reasons given there.

[Margin: 2nd Problem.]

XXVI. *Given the time in which some heavy body, descending naturally, has passed through a given height: to find another height, smaller or greater, which it would pass through in another given time.* Let the square of the greater time be to the square of the smaller time as the given greater space is to the smaller space sought. Or, if the space sought is to be greater, let the square of the shorter given time be to the square of the longer time as the given smaller space is to the greater space sought.

[Margin: Example.]

For instance: since the clay ball of 8 ounces runs through 280 feet in 260‴, if you ask how great a space it will run through in one hour, that is, in 216000‴, let it be as the square of the number 260, 67,600, is to the square of the number 216,000, the number 46,656,000,000, so 280 feet to the feet sought. They will be 153 308 875 feet, that is, 30 661 775 paces, that is, 30,662 Italian miles. In 24 hours, therefore, whose square is 576, it would pass through 30,662 multiplied 576 times, that is, 17 661 212 miles.

[Margin: 3rd Problem.]

XXVII. *Given the space that the same heavy body has covered in air and in water, and the time corresponding to each space: to find the probable difference between the heaviness, or at least the density, of water and of air.* First, by Problem 1, find in what time that heavy body would cover a space of air equal to the space of water it passed through; then compare the time found with the time in which the same heavy body covered the same amount of space in water. For the more time it used up in the water than in the air, the greater, probably, is the resistance of water than of air, whether that resistance is said to arise from heaviness or rather from greater density — unless the wetness, which together with the density makes water viscous, also helps the resistance that the moving body has to overcome.

[Margin: Example.]

Example. A small lead ball of 4 drachms, or half an ounce, covered for me 280 feet in air in 24 vibrations of the pendulum, but in water it covered 4 feet in 7 vibrations of the same pendulum — that is, there in 4″, here in 1″ 10‴. I ask in what time it would have passed through

280 feet of water? Let it be as 4 feet to 280 feet, so 49, the square of 7 vibrations, to 3430, which is the square of approximately 59 vibrations. Therefore, as 59 is to 24, so, probably, is the resistance and density of well water to the resistance and density of our air, the air nearest the Earth — unless someone would have density stand to density as the square of the time to the square of the time. In that case, in our instance, the density of water would stand to that of our air as 576, which is the square of 24 vibrations, to 3430, which is the square of 59 vibrations.

[Margin: 4th Problem, on investigating the depth of the sea.]

XXVIII. *To explore the depth of the sea, or of any water, with hindrances by accident set aside. First*, take a glass tube of, say, 10 feet, filled with water of the same species as the water whose depth you wish to measure, and set up perpendicular to the horizon. Into it let down a ball of cork, or a gall, D, into whose base a lead hook ABF has been inserted through a loop or rather loose ring, the greater part of the hook being BF; and with the pendulum measure precisely the time in which the whole aggregate of gall and hook reaches the bottom. Then, as soon as the hook has tipped over and freed the ball D to go upward, measure likewise the time in which the ball D rises to the top of the water; and keep each time separately, and also their sum. *Second*, let the same aggregate of ball and hook down into the sea, and measure the time that passes between the letting-down and the return of the ball to the surface of the water. Distribute this sum, like a common profit, to the aggregate of ball and hook, or to the descent alone, by the rule of partnership or by the rule of three, and you will have, distinctly, the time in which the ball with the hook descended to the bottom of the sea. *Third*, then, by the second problem given a little earlier, find the desired depth of the sea. I said *with hindrances by accident removed*: such would be the case if that weight struck some point or hidden rock, or the back

of a fish then swimming beneath, or if the sea were stirred from the bottom by the force of the winds.

Diagram. A small figure set in the left column beside the description of the sounding device. On the left, a dark ball D with a small hook beneath it; on the right, a horizontal bar on a vertical post, the post running down to a small pointed vessel.

- D (above the ball): the ball of cork or gall
- A (left of the hook), c (small letter between A and B), B (right of the hook), F (lower end of the hook): the lead hook ABF, whose greater part is BF
- M (left end of the bar), E (at the post), R (right end of the bar)
- 3 (beside the post)
- G (mouth of the vessel), K (point of the vessel)
- 2½ (beside the vessel)

[Margin: Example.]

Example. Suppose the aggregate of ball and hook has passed through 10 feet of sea water in the time of six vibrations of our smallest pendulum, or 60 thirds of the primum mobile, while the ball rose to the same height in 10 vibrations, or one second and 40 thirds[45]. The sum of ascent and descent, then, is 2″ 40‴, or, resolved into thirds, 160‴. Next, suppose that the ball with the hook, let down into the sea, reached the bottom of the sea and returned to the surface in a time of one minute, or 60″, which are 3600‴. For if 3600‴ are owed to the whole time of 160‴, then to the time alone in which the hook with the ball descended through 10 feet of water, which was 60‴, there are owed 1350‴ — the thirds, of course, in which that hook descended to the bottom of the sea. Finally, as 3600, the square of the time 60‴, is to 1,822,500, the square of the time 1350‴, so 10 feet to 5062 feet, that is, to one Italian mile and 62 feet besides, since a pace consists of five feet. So great, then, will be the depth of that sea. Thus you see the problem of Leo Baptista Albertus [Leon Battista Alberti], which we discussed in Book 2, chapter 5, number 11, corrected and restored in a far nobler way.

[Margin: 5th Problem.]

XXIX. *Given the time in which one heavy body, in natural descent, measures a given height, and the difference of the interval which the other heavy body has covered more slowly than the first: to find the difference of interval that will arise in a given greater time.* Find by problem 2 the interval due to each of the two heavy bodies for the second time, given or assumed; for the difference of these intervals will be the very one you are seeking.

[Margin: 1st Example.]

Example. We said in number 13, in the table at experiment 21, that a small lead sphere of one ounce, let fall from the cornice of the Asinelli tower, touched the pavement of the base, which is 280 feet distant from there, when another

[45]printed "40″. Tertijs"; the word fixes the order as thirds

small sphere of 4 drachms, let fall at the same time from the same place, was 5 feet from the pavement. Further, the one-ounce sphere, while I was then measuring the time, descended in 24 vibrations, or 4″. It is asked how far the half-ounce sphere would have been from the pavement if such motion had continued for three hours, that is, 10800″. First, then, let it be as 16, the square of the time 4″, to the square of 10,800 seconds, which is 116,640,000, so 280 feet to 2,041,200,000 feet, which are 408,240,000 paces, or 408,240 Italian miles. Again, let it be as 16 to 116,640,000, so the 275 feet that the half-ounce sphere covered to 2,006,625,000 feet. Subtracted from 2,041,200,000 feet, these leave a difference of 34,585,000 feet, namely 6,917,000 paces, which make 6917 Italian miles. Further, the semidiameter of the Earth (which for us is 4139 miles) is contained 91 times in 408,240 Italian miles. Therefore, if two of the balls already mentioned, or similar ones of the same species, one of them twice as heavy as the other, were let fall together from a height of 91 terrestrial semidiameters, which is about one and a half times the distance of the Moon, and the medium were such as our air, then when at the end of three hours the heavier struck the Earth, the lighter would still be 6917 Italian miles from the Earth. But if you make the calculation for six hours, that is, 21600″, whose square is 266,560,000, you will find that the one-ounce sphere would pass through 929,210 miles, and the half-ounce one 916,300; their difference at the end of six hours would be 12,910 miles, that is, more than three semidiameters of the Earth.

[Margin: 2nd Example.]

But I should like to weigh another remarkable case, which we discussed in number 13, experiment 12. A lead ball of 2½ ounces covered 56 paces, or 280 feet, in about 4″; and at the moment it struck the pavement, a wooden ball of greater bulk but equal weight, that is, of 2½ ounces, was clearly 40 feet, that is, 8 paces, from the pavement; so the wooden ball had then covered 48 paces. I ask how great the difference of interval would be if each motion had lasted 6 hours. The square of 4 seconds is 16, and the square of the 21,600 seconds that make up 6 hours is 266,560,000. Let it be, then, as 16 to 266,560,000, so 56 paces to 929,210,000 paces, that is, 929,210 Italian miles. Again, let it be as 16 to 266,560,000, so 48 paces to 799,680,000, that is, 799,680 miles. The difference of these miles is 129,530. Therefore, if the balls already mentioned had been let fall together from a height of 929,210 miles, that is, of 224½ terrestrial semidiameters, and the air or medium had been such as our air, then when the lead ball of 2½ ounces had struck the Earth, the wooden one of 2½ ounces would have been 129,530 miles from the Earth, that is, about 31¼ terrestrial semidiameters.

[Margin: 6th Problem.]

XXX. *Given a heavy body that would reach the centre of the Earth from the surface of the Earth in six hours, through a well full of air or water: to find its slowness, that is, in what time it would have to pass through its first pace.* I suppose that the semidiameter of the Earth contains 4,139,000 paces. Therefore, according to problem 1, let it be as 4,139,000 paces to 1 pace, so 466,560,000, the square [?]

of 21,600 seconds, which make up 6 hours, to the square 112¾, whose nearest square root is 10½. Such a heavy body, therefore, would have to use up about 10″ in running naturally through its first single pace, or five feet, that is, 60 vibrations of our smallest pendulum. But such a body will hardly be found; and among many experiments made in water, the body that came, barely, close to being suitable for this was the ebony ball discussed in number 15, experiment 9 or 10, in the small table; for it covered 4⅙ feet in a time of 7″. But this is much more difficult in air.

CAPVT XVII.

Chapter 17.

An & Quomodo per Diurnum simul & Annuum Terrae motum reddatur vnicè aut meliùs ratio Incrementi velocitatis Grauium & Leuium; ideòque hinc talis motus vehementer confirmetur. Qua occasione disseritur de Figura, quam in hypothesi Terrae motae, Grauia & Leuia motu suo naturali describunt, & Argumentum Galilaei pro motu Terrae inde sumptum tandem dissoluitur.

Whether, and how, the diurnal and annual motion of the Earth together gives the one true, or a better, reason for the increment of the speed of heavy and light bodies; and whether such a motion is therefore strongly confirmed from this. On this occasion the figure is discussed that heavy and light bodies describe by their natural motion on the hypothesis of the moving Earth, and Galileo's argument for the motion of the Earth drawn from it is at last dissolved.

I. The Copernicans saw that, with the Earth moving daily and yearly, the line that heavy bodies describe naturally in descending, and light bodies in ascending, cannot really be a straight line perpendicular to the horizon, such as it appears to us. Some of that sect therefore laboured hard to determine the figure of this line. Kepler, adding no proof, and Gassendi, drawing a likeness from the case of a ball let fall from a mover that is meanwhile carried along, judged it to be parabolic, or very close to parabolic, as we have already set out in chapter 4 of this section, in scholia 2 and 3. But Galileo and Boulliau taught that it is circular, or composed of circular lines. Nor did they stop there: from the motion made along such a line they seemed to themselves to adduce a solid and true cause why the motion of heavy bodies appears faster and faster towards the end; and from this they drew an argument, at first sight very strong, for confirming the motion of the Earth. We shall sharpen their argument more than they did themselves, and yet we shall turn the wound back from it upon the Copernican hypothesis. But it is best first to dispose of what Boulliau devises.

[Margin: Boulliau's opinion.]

II. First, then, Boulliau, in chapter 4 of his *Philolaus*, censures the various opinions of those who have tried to bring forward an a priori cause of the ever greater speed of heavy bodies towards the end, but above all of those who take

refuge in a greater gravitation, that is, a second act of heaviness. He says: *Others hold that a descending body acquires greater heaviness*; and he soon carps at this, saying: *A tasteless and insipid reason, labouring under manifest falsity. For heaviness, whatever it may finally be, follows the bulk of the matter, and while that remains the same, the same heaviness remains. From the view of these men, then, it would follow that something of substance could be produced by local motion, which is false.* But, as *Scipione Chiaramonti* rightly answers him in Part 1 of the *Antiphilolaus*, chapter 4, those authors do not mean a new heaviness in first act, acquired through the motion itself, but in second act, which not a few therefore call gravitation, or a downward translative impetus. Having rejected this and the remaining opinions, then, Boulliau concludes: *The reason for the acceleration of the motion of falling bodies therefore depends wholly on the circular motion, which the part must have in common with the whole, and which it would without doubt keep in falling, unless the violence of its state moved it with a violent motion.* He holds, further, that this motion comes about through two opposed circular motions, by which the movable body is kept, to appearance, in the same straight line, and appears unequal in it, although in reality each of the circular motions is equal. To show this he uses Copernicus's demonstration in Book 3, chapter 4, made for the motion of libration and the anomaly of the equinoxes, which we too have set out in Book 3, chapter 29, number 10. But here it must be repeated in summary and applied to this case.

[Margin: A rectilinear motion from two circular ones.]

III. In the following diagram, from the centre A describe the circle BKE and a smaller one, CIDL, whose semidiameter AC is half the semidiameter AB. Let another circle, HGA, be equal to the smaller circle, and let its centre F be carried in the order of the signs along the circumference of the circle CIDL, from C to I. Meanwhile, let some movable body on the circumference of the circle AGH be moved with a motion twice as fast against the order of the signs, the motion of the centre F beginning from C, and that of the other movable body, H, from B. With these posited, Copernicus shows that, as these motions meet each other in opposite directions, the movable body is held in the straight line AB. When the centre F has completed the half-quadrant CF, the movable body H, which was at B, has completed the quadrant HG and is at G; and when the centre F is at I, the quadrant CFI of its circle now being complete, it follows that the movable body, having completed the semicircle HGA, is at A. Hence, if these motions continue in this way, the movable body will be kept in the straight line BAE; and although its motion along that line will appear unequal, in reality it will be equal in the circles already mentioned. So the motion of heavy bodies, which appears to us to take place in a straight line and to be faster and faster, is in truth compounded of two circular and equal motions.

Diagram. Set in the right column beside the construction of paragraph III. A large circle with a vertical and a horizontal diameter; a smaller circle about the same centre; and a third circle, equal to the smaller, passing through the centre and through a point on the great circle at the upper left.

- B (top of the great circle), E (bottom of the great circle)
- K (left end of the horizontal diameter), L (right end of the horizontal diameter)
- A (centre)
- C (on the vertical diameter above A, on the smaller circle), I (left on the smaller circle), D (below A, on the smaller circle)
- H (on the great circle at upper left), G (on the vertical diameter below B), F (near the centre of the third circle)

[Margin: Chiaramonti's arguments against Boulliau.]

IV. Chiaramonti, in that same chapter 4, rejects this doctrine as badly applied to the increment of speed we are dealing with. *First*, because Boulliau wrongly calls the downward motion of heavy bodies towards the centre of the Earth violent, since it is most natural, supposing that they are outside their place. *Second*, because he supposes that KAL is the surface of the Earth, and that a heavy body let fall from the top of the tower, B, has an inclination to the circular motion BHK, but that the points of the arc HG resist it and are carried towards B with a speed double that with which the heavy body would wish to be carried from B towards K; and that from this contrariety, therefore, it comes about that the heavy body falls onto the Earth at A along a straight line, with a motion perpendicular to sense. But the diurnal and annual motions of the Earth are not contrary, since they are in the same direction. Nor can any motion be imagined that opposes the motion of the heavy body: for neither is the air near the Earth moved in a different direction — it moves in the same one — nor does a heaviness resisting the circular motion move through a circle that strives against the first circle. *Third*, he objects against Boulliau the excessive speed that would follow from such a motion. For let the heavy body be at B, and the tower BA be of 10 perches, of which BG is imagined to be the tenth part, that is, one perch, that is, 10,000 of the parts of which AB, the whole sine, is 100,000. For to the versed sine BG, of 10,000 parts, there corresponds the arc BH, of 25° 51′. And from Archimedes' *De dimensione circuli*, the circumference is to the diameter as 22 to 7. Therefore, if the circumference BKE etc. is 360°, and the diameter BE 200,000 parts, the arc BH, which is 25° 51′, will be 45,134 such parts, and its proportion to BG, set at 10,000 parts, will be greater than four and a half times. Now a stone let fall from B passes through the perch BG in a shorter time than two beats of a human artery. But let there be two beats in which the arc BH, namely an arc of 25° 51′, is turned with an equal motion (as Philolaus would have it). The whole circle of 360°, therefore, will be turned in a time of 27 1323/1551 beats, or about 28. And the circumference whose diameter BE is 20 perches is about 63 perches. But a mile contains 333⅓ perches of Cesena; hence, if 63 perches are run through in 28 beats, a mile will certainly be run through by such a movable body in 148 beats, that is, in about two minutes of an hour. Therefore in one hour, by the force of the circular motion, it would cover 30 miles, and in one day 820 miles — a portentous speed, as Chiaramonti calls it, which he denies would suit any bird.

[Margin: Chiaramonti's double error on the speed of heavy bodies.]

But this third attack is frivolous, and it errs in two ways. First, in the Copernican hypothesis: for on that hypothesis, in the motion of birds, besides the motion that appears to us to be their own and only motion, there must also be included the common and circular motion by which they accompany the Earth, if they fly towards the east; and so, on such a hypothesis, they fly past a far greater space than the one that appears to us. Granted, let it be enough for Boulliau if, as far as the apparent motion goes, no more than 10 perches seem to be passed through. Next, he errs outright when he says that a passage through 820 miles made in one day is incredible and portentous. For it is far smaller than the speed that would in reality suit a stone if it were let fall from such a height that one day would be needed to run through it. For from what was shown in chapter 16, number 24, Theorem 3, and number 26, Problem 2, a clay ball of about one pound would pass through 30,662 Italian miles in one hour, and 17 661 212 miles in 24 hours. This is far more wonderful, and yet truer.

[Margin: Our arguments against Boulliau.]

V. Boulliau must therefore be attacked otherwise than by that third argument. First, if any heavy body really covered equal spaces on its circle in equal times, it would strike the Earth with no greater impetus when let fall from a height than when let fall from a low place; and this is against the very many and most manifest experiments set out in chapter 16 from number 6. Second, from a motion ordered through the two circles already mentioned there does not follow the increment of speed that is manifestly apparent in the natural motion of heavy bodies — an increment which we showed in chapter 16, numbers 11 and 12, to follow the odd numbers counted from unity, and which we drove home in number 24, Theorem 3. That it does not follow is easily demonstrated. For according to Boulliau the heavy body B reaches A when the circle CFD, proceeding with equal motion in the order of the signs together with the circle BKE, has completed a quadrant; for then the movable body on the circle HGA has completed a semicircle against the order of the signs. Now let the whole time of descent be divided into four equal parts. In the first time, H, the outermost point of the diameter AFH, will complete a quarter of a quadrant, namely 22° 30′; imagine these degrees contained in the arc BH. Its versed sine BG, therefore, will be 76 of the parts of which the whole sine AB is 1000. At the end of the second time it will complete 45°, whose versed sine is 293; at the end of the third time it will complete 67° 30′, whose versed sine is 617; and finally, at the end of the fourth time, it will complete the quadrant BHK, whose versed sine AB is the same as the whole sine, of 1000 parts. Now let the first versed sine be subtracted from the second, the second from the third, and the third from the fourth: there will remain the spaces covered in each of the four times, as you see in the first column of the following small table.

Spaces covered, separately	Spaces to be covered, separately	Odd numbers
76	76	1
217	218	3
324	380	5
383	532	7

But according to the increment of the odd numbers, which is owed to the speed of heavy bodies, if the space of the first time is 76, the spaces of the other times ought separately to be as you see in the second part of the table. So although the space of the second time, covered by force of Boulliau's circles, is nearly equal to the space to be covered, yet the third falls short of the space owed to the increment of speed, and the fourth much more so. Hence, since the clay ball in our experiments, as I said in chapter 16, number 12, covered 240 Roman feet in four seconds of time, and 15 feet in the first second, then if Boulliau's reasoning held it would have covered no more than 76 feet at the end of the fourth second. This is absurd, and conflicts with the most evident experiments. Third, Boulliau cannot, through the circles already mentioned, give a reason for the increment of speed of light bodies upwards, such as purer air and fire, because these are not carried to the centre of their whole but to its circumference. Let us hear, then, Galileo, who is believed to have come nearer to the truth with a single circular motion.

Galilæi Demonstrationes, quibus conatus est in hypothesi Telluris motae Rationem afferre Incrementi velocitatis Grauium.

Galileo's demonstrations, by which he tried, on the hypothesis of the moving Earth, to give the reason for the increment of the speed of heavy bodies.

[Margin: Galileo's doctrine on the line described by the motion of heavy bodies.]

VI. In the second dialogue of *On the Twofold System of the World*, from page 157 of the Italian, but page 119 of the Latin version, Galileo pronounces the following propositions in the person of Filippo Salviati.

[Margin: Galileo's first proposition.]

The first is this. If the straight motion of heavy bodies towards the centre of the Earth appeared uniform, then, since the circular motion of the Earth towards the east is also posited as uniform, the line compounded of the two motions would be a spiral — namely one of those that Archimedes defined in his book *On Spirals*. And he says that such a line arises when a point moves uniformly along a straight line that is carried round uniformly about one of the line's extremities, fixed as the centre of the circumduction. But since the apparent motion of heavy bodies is uniformly difform, and continually accelerates, it is necessary that the line described, compounded of this motion and the diurnal motion of the Earth, should recede successively, in an ever greater proportion, from the circumference of the circle that the centre of gravity would have described had the stone stayed at the top of the tower. It is necessary, moreover,

that this recession should be slight and very small at the beginning, because the heavy body passes from rest — that is, from the privation of downward motion — and enters upon downward motion, and so necessarily passes through all the degrees of slowness lying between rest and any speed whatever, degrees which he had earlier concluded to be infinite.

[Margin: Galileo's second proposition.]

Second: since the natural descent of heavy bodies from the top of the tower to the centre of the Earth is carried on of itself, as to its own term, he says it necessarily follows that the line of the compound motion recedes from the circumference of the circle described by the top of the tower in such a way, in an ever greater proportion, that it nevertheless ends at the centre of the Earth.

[Margin: Galileo's third proposition.]

VII. Third, on page 160 of the Italian, 121 of the Latin, he asserts that the line already mentioned is circular, or extremely close to circular, because in it the conditions prerequired in the first and second propositions are preserved. On the preceding page he teaches how this circular line is to be drawn. Let A be the centre of the Earth; from it, with the semidiameter BA as interval, let the quadrant BIM etc. of the Earth's circumference be described. Further, let there be the tower BC, whose vertex C describes about the centre A a quadrant of a circle towards the same side, namely the east, towards which the Earth moves by its rotation. Now let the whole interval AC be bisected at E, and from the centre E, with the interval AE, let the semicircle AIVC be described. *Through this*, says Galileo, *I now assert that it can be believed with fair probability that a stone falling from the top of the tower C is moved with a motion compounded of the common circular motion and its own straight one.* For if on the circumference CD some equal arcs are marked, namely CF, FG, GH, HL, LD, and from the centre A straight lines AF, AG, AH, AL, AD are drawn to the ends of those arcs, then the portions of these lines intercepted between the two circumferences CD and BI, namely OF, PG, QH, RL, ID, will always represent for us the same tower CB, carried round by the rotation of the terrestrial globe towards D. Along these the stone, in its fall, will seem to descend with a rectilinear motion to us who, standing at the foot of the tower B, are carried through the arc BI in the same direction with a common and equal motion. Further, the portions of the said lines intercepted between the circumference CD and CI, namely FS, GT, HV, LX, DI, will be the spaces covered apparently downward by the stone at the end of each of the times; and the places in which the stone will be found at the end of

Diagram. Set in the right column beside the construction of paragraph VII. A vertical line C B E A with a right angle at its ends; an outer arc from C leftwards to D (the path of the top of the tower), an inner arc from B leftwards through I to M (the Earth's circumference); straight lines from the centre A to points on the outer arc; and a semicircle from A up through I to C, crossing those lines.

- C (top right, top of the tower), B (right, foot of the tower), E (on the vertical line between B and A), A (bottom right, centre of the Earth)
- F, G, H, L, D (on the outer arc, from right to left)
- O, P, Q, R, I (on the inner arc, from right to left), M (lower end of the inner arc, at the left)
- S, T, V, X (where the semicircle crosses the lines from A, from right to left) those times will be S, T, V, X, I. These points recede from the circumference CD in an ever-increasing proportion, and so the motion of the stone seems to us to be accelerated more and more; and at last this motion, if nothing hindered it, is understood as due to end at the centre of the Earth, A, once the semicircle CIA has been completed.

[Margin: Galileo's fourth proposition, in three members.]

VIII. Fourth, on page 159 of the Italian, 120 of the Latin, Galileo asserts that the natural motion of the stone is in reality circular, although it appears straight to us, and indeed that it runs along the simple circumference of a single circle. Further, that this movable body in falling completes no greater space than if it had stayed at the top of the tower C. For the whole arc CD, which it would have described by staying at the top of the tower, is precisely equal to the arc CI, which it completes in falling; just as, one by one, the arc CF is equal to the arc CS, the arc FG to the arc ST, the arc GH to the arc TV, the arc HL to the arc VX, and the arc DL to the arc XI. From this follows the third member of this proposition, which he himself calls admirable: namely, that the true and real motion of this stone is not accelerated, but is in reality equal and uniform, granted that it appears unequal to us; since to the equal times in which the top of the tower C moves, marked by the equal arcs CF, FG, GH, HL, there correspond real equal spaces CS, ST, TV, VX, XI. *This*, Galileo adds, *frees us from the labour of investigating the causes of the acceleration of this motion, since the movable body, both while standing on the tower and in falling, is always moved in the same way, that is, circularly, with the same speed and a uniform tenor.*

That the arcs already mentioned are equal to one another is, moreover, very readily demonstrated by drawing the line EI from the centre E. For the semidiameter AC of the circle CD is by construction double the semidiameter EC of the circle CIA. Therefore the whole circumference of the first to the whole of the second, a quadrant to a quadrant, and any arc of the greater circumference to any similar arc of the smaller, are in double proportion; for the circumferences of circles are to one another as their diameters, by theorem 5 of Book 11 of the *Collections* of Pappus of Alexandria. Hence any half arc of the circumference CD is equal to a similar arc of the smaller circumference CI. Now the angle CEI, standing at the centre E, is double the angle CAD, which stands at the circumference of the smaller circle but at the centre of the greater, by Proposition 20 of the third book of the *Elements*. Therefore the arc CI, on which the angle CEI stands, is double the arc CD, on which the angle CAD stands; so CD is half of the arc of the greater circle similar to the arc CI, and accordingly the arc CD is equal to the arc CI. By the same reasoning, drawing

straight lines from E to the points S, T, V, it will be proved that the arc CF is equal to the arc CS, the arc CG to the arc CT, and the arc CH to the arc CV. Hence, since the arc CD is divided into the equal arcs already mentioned, FG will be equal to the arc ST, the arc GH to the arc TV, the arc HL to the arc VX, and the arc LD to the arc XI: which was to be shown. This done, Galileo concludes, on page 160, but 121 of the Latin, in these words: *Although I would not now say that the matter stands precisely so as regards the motion of descending heavy bodies, this I do certainly affirm: if the line described by the falling body is not exactly this one, it is nevertheless extremely close to it.* Now let us look at the objections against Galileo: one of them Chiaramonti's, the rest our own.

[Margin: Chiaramonti's objection against Galileo.]

[Margin: But fallacious.]

IX. For Scipione Chiaramonti, in the Italian defence of his *Antitycho*, Part 4, chapter 10, objects to Galileo that it is impossible for a stone in falling to be moved along the circumference CI and yet to represent to us a motion made along a perpendicular straight line; and that whoever says otherwise commits a very great error in mathematics. For, taking the arc CI as one Italian mile and the height of the tower BC as 50 paces, he shows that this arc will have a declination from the horizontal line six times smaller than the channels suited to turning mill-wheels commonly have. But Chiaramonti was certainly deceived. For Galileo never said that the arc CI itself, or its parts, by itself or through its own portions however small, represents to us a perpendicular straight line; had he said so, Chiaramonti's objection would be valid. What he said was that, on the hypothesis of the diurnal rotation, a stone falling from a tower is at every instant of its motion at such a point of the circumference CI that the straight line drawn from the top of the tower, the term from which it fell, through the stone falls on the same point of the Earth over which the stone stood perpendicularly at the beginning of its fall, because the top of the tower and that point of the Earth are moved with a like circular motion towards the east; and so to our eye, remaining at that place on the Earth and consequently carried along with the Earth, the motion seems to be made along a perpendicular straight line. Setting Chiaramonti aside, then, let us come to the more solid objections, devised partly by us and partly by Father Francesco Maria Grimaldi, with whom I have often been accustomed to discuss this hypothesis.

[Margin: Objection[s?] ours and Father Grimaldi's against Galileo.]

X. First, then, I say what I brought against Boulliau. If the motion of heavy bodies falling naturally were in reality equal, and only seemed unequal through a deception of the eye, a body would fall with no greater impetus from a higher place than from a lower one, and accordingly those innumerable effects would not follow which do follow from the more vehement percussion of bodies falling from a higher place, some of which we reviewed in chapter 16 from number 6.

[Margin: First objection.]

[Margin: Inequality of impetus shows an inequality of the motion really made.]

Since, then, touch itself and hearing evidently discern that greater percussion, it cannot be said to be a mere appearance and a mockery of the senses. But that greater percussion necessarily and naturally presupposes a greater impetus, and a greater impetus a greater real increment of speed. Therefore they are not moved equally, but with an ever greater real speed.

[Margin: Second objection.]

[Margin: Heavy bodies descending unequally along the same circle would not appear in the same li[?] … straight.]

XI. Second: by most evident experiments, exhibited before many witnesses most worthy of credit and set out in chapter 16, number 13, and drawn in number 14 into various corollaries, it is established that if two heavy bodies of different weight are let fall together from the same height, the heavier descends faster: if it is heavier both in the individual and in species, as there in corollary 4; indeed, even if it is heavier in the individual but equally heavy with the other in species, as in corollary 2; or if it is heavier in species but equally heavy in the individual. But so that no room is left for any evasion, I shall choose this last case. Of it, in the same chapter, number 13, experiment 12, I related that a wooden ball of 2½ ounces, let fall from the cornice of the Asinelli Tower at Bologna together with a lead ball likewise of 2½ ounces, stood 40 feet, or 8 paces, from the pavement at the base of the same tower when the lead ball, striking the pavement, had already traversed a height of 280 feet, or 56 paces; and so the wooden ball had covered no more than 48 paces in the same time. From this, by example 2 of Problem 5, which is in the same chapter 16, number 29, I deduced that if such balls had been let fall from a height of 929,210 Italian miles, the outcome would have been that at the end of 6 hours the lead ball would strike the Earth, while the wooden ball would still be 129,530 miles from the Earth, that is, very nearly 31¼ semidiameters of the Earth. Such experiments, and similar ones, cannot be saved by the motion of heavy bodies along the circle Galileo determined, or along one insensibly different from it. For, repeating the figure of number 7: if both balls are let fall together from the top of the tower C, and each must reach the centre of the Earth, A, along the semicircular circumference CIA, then at every same moment of time for which the motion lasts each ball must appear — indeed must really be — at the same point of the same circumference. So when the lead ball is at S, the wooden one too is at S, and appears there to our eye remaining at O; when the lead ball is at T, the wooden one too is there; and when the lead ball has struck the surface of the Earth at I, the wooden one too is there and strikes the Earth. Otherwise, if the wooden ball were not at the same points of the same circumference, but at those of another, greater one, drawn nevertheless through the vertex C, this circular line would not end at the centre of the Earth but at some other point below that centre; and this is against the nature of heavy bodies, and against Galileo's second proposition, of which I spoke in number 6. But if the wooden

ball had been, for the whole time of the motion, at the same points of the same circumference as the lead one, it would have descended with precisely equal speed, and would not have stood a full 40 feet from the Earth — which is against a most evident experiment. Hence Father Grimaldi suspected that Galileo, in that same second dialogue on the system of the world, denied for this reason that of two heavy bodies, of however different weight, let fall together from the same term, the heavier reaches the Earth sooner: namely, lest this should harm this motion of his of heavy bodies along the circle already mentioned. But perhaps his occasion for denying it was that he used two balls of the same species but of different weight and different bulk, in which the difference of descent and of percussion is far smaller in appearance than in other comparisons; nor does it show itself evidently unless the bodies are let fall from a notable height, whereas Galileo used no height greater than 100 cubits, as he testifies in the same dialogue.

[Margin: A way of escape is closed to Galileo.]

Nor should you say that that inequality can be defended if the wooden ball is carried round along another circumference, insensibly greater than the circumference CI. For if that happened, both balls would have struck the point I at a sensibly identical moment, and the difference of interval would not have been so sensible as to come to 40 feet. Moreover, if both balls had been let fall from a height of 929,210 miles, and the medium had been of the same density as the air near us, and neither ball had been consumed by heat conceived from the motion, or dissipated before its arrival at the Earth, then when the lead ball had reached the centre of the Earth, A, after 6 hours had elapsed, the wooden ball would have stood 31¼ semidiameters of the Earth from the centre A, as I showed in chapter 16, number 29, at the end. This difference is very notable.

[Margin: Third objection.]

[Margin: The descent of heavy bodies is not of the same rationale everywhere [?].]

XII. Third: the motion of heavy bodies along the circle that Galileo assigned is unfit to save the greatest part of the motions by which heavy bodies naturally descend. This is so both because it saves only the motion of heavy bodies descending through the plane of the equator, and because it does not save even the hour of all motions, but only of those which, because of their lightness or lack of greater heaviness, are so slow that they cannot reach the centre of the Earth except in a time of 6 hours. Scarcely any of these can be found if the descent has to take place through air; and if through water, heavy bodies that descend so slowly are very rare. Such, however, is a ball of the rarer kind of ebony let fall through water, as appears from the problem I gave for this purpose in chapter 16, number 30. But very many heavy bodies, in number and in species, neither descend through the plane of the equator nor are so slow as to need 6 hours to pass through a semidiameter of the Earth, or 4139 miles, as appears from what was said in chapter 16, number 25, example 3, and number 30.

I said, first, that the motion of heavy bodies is not saved unless they descend through the plane of the equator, and so unless the tower from which the stone falls stands on the equator itself. For Galileo's demonstration, brought forward in numbers 7 and 8, requires that the circle along whose circumference CIA the stone is carried round lie in the same plane in which lie both the centre of the Earth, A, and the top of the tower, C, and so too the circle CD described by that same top through the force of the diurnal rotation — as is manifest from the construction of the figure and from the hypothesis on which that demonstration rests. And this cannot happen unless that tower stands on the equator. For if it stands off the equator, under the poles, the descent of the stone will in reality be along a straight line, and so not along a circle; and if it stands on some parallel to the equator, one parallel will be described by the foot of the tower, B, and another by the vertex of the tower, C, and moreover the plane of neither will lie in the plane in which the centre of the Earth is, but in a far different plane. Moreover, Galileo himself, in that second dialogue, page 237 of the Italian, 179 of the Latin, when answering the objections of our Scheiner, acknowledged that this line is circular only on the equator, and that on the other parallels it describes a conical surface. He was prompted, namely, by that question of Scheiner's which stands in the *Disquisitiones*, page 31, and runs as follows: *Why does the centre of a sphere that has fallen under the equator describe a spiral in its plane; under other parallels describe a spiral on a cone; and under the pole descend on the axis, running down a gyral line traced on a cylindrical surface?* To which he answered in these words: *Because, of the lines drawn from the centre to the circumference of the sphere (which are, namely, those along which heavy bodies descend), the one that ends at the equinoctial traces a circle, while those that end at the other parallels describe conical surfaces; the axis, finally, describes nothing else, but remains in its being.* Galileo was therefore wrong, on page 121 of the Latin, but 160 of the Italian, to define universally or indefinitely that this line is circular; nor did he correct this statement of his, as he ought to have, having forgotten, I believe, what he had said.

[Margin: Galileo does not correct[?] his statements, where he ought to from what he had conceded [?].]

I said, second, that not even under the equator is the motion of heavy bodies saved, unless they are so slow that their arrival at the centre of the Earth (if access were given there through a well full of air or of water) needs precisely 6 hours. For if Galileo's demonstration, of which numbers 7 and 8 speak, is to be right, the semicircumference CIA of the circle along which the heavy body is carried round must be equal to the quadrant CD etc. that the top of the tower, C, describes by the force of the diurnal rotation; and so, when the heavy body has reached the centre of the Earth, A, from C through I, having completed the semicircle CIA, the top of the tower, C, must meanwhile have completed a quadrant of the diurnal rotation, that is, in 6 hours. Otherwise, if the heavy body reaches A along the arc CIA either faster or slower than in 6 hours, Galileo's whole demonstration collapses. Since, however, these two

conditions can in reality be joined together — namely, that the place from which the heavy body is let fall be on the equator, and that the body be so slow as to need 6 hours to pass through the semidiameter of the Earth — from this we shall draw a very strong argument against the diurnal motion of the Earth, and so against the annual motion too, which does not stand without the diurnal; and it is the following.

[Margin: Fourth objection.]

[Margin: Nowhere does a heavy body descend along a circle, if the annual motion is admitted.]

XIII. Fourth: not even on the equator, even if the heavy body is such as would reach the centre of the Earth in 6 hours, is it true that the line it describes is circular, or insensibly different from a circular one, unless the motion of the Earth is simply diurnal, about its own unmoved centre. But if the centre of the Earth moves with an annual motion, as Galileo would have it together with Copernicus, the line of the movable body already mentioned will not be circular, but will describe in the space of the world a line very different from a circular one. This is evident. For while in one hour it would have to describe 15° of the circular line by the force of the diurnal motion, the centre of the Earth is meanwhile carried forward in the Great Orb 2′ and 3″. These are nevertheless far greater than 15° of the terrestrial circumference, even if the Great Orb is not as large as the distance of the Sun found by us in Book 3, chapter 7, requires, but only as large as the Copernican distance — that is, if its diameter is only 2284 semidiameters of the Earth, or 1142 diameters of the Earth. For it follows that the circumference of the Great Orb is to the circumference of the terrestrial globe as 1142 to 1, since circumferences are to one another as their diameters, by theorem 5 of Pappus of Alexandria, Book 11 of the *Mathematical Collections*. Therefore one degree of the Great Orb is to one degree of the terrestrial circumference as 1142 to 1. But 2′ 30″ are the 24th part of one degree; hence, dividing 1142 by 24, 2′ 30″ of the Great Orb will be to one degree of the terrestrial circumference as 47 7/12 to 1, and to 15 terrestrial degrees as 47 7/12 to 15, that is, in a greater proportion than triple. Hence in one hour, by the force of the annual motion alone, the heavy body already mentioned will cover in the space of the world a space more than three times greater than it would cover on the terrestrial circumference by the force of the diurnal motion. But if the diameter of the Great Orb is, according to us, 7300 diameters of the Earth, dividing these by 24 gives 304¼; and so the space covered in one hour by the force of the annual motion will be to the space covered by the force of the diurnal motion as 304¼ to 15, that is, more than twenty times greater. Since, then, the circumference of the Great Orb is not parallel to the terrestrial circumference, but passes through its centre in such a way that, because of the inclination of the equator to the ecliptic and the parallelism of the equator with itself, a heavy body following the motion of the centre and of the circumference of the Earth is perpetually in different planes; and since the portion of the Great Orb corresponding to one hour is three times,

indeed twenty times, greater than the portion of the terrestrial circumference corresponding to one hour; it is necessary that the line compounded of both motions should differ notably from a circular one — and much more so if the motion lasts for 6 hours.

[Margin: Fifth objection.]

[Margin: No heavy body descends along a circle, even granting the diurnal motion alone.]

XIV. Fifth: I say that even if some heavy body descended on the equator from the surface to the centre of the Earth in precisely 6 hours, and the Earth moved only with the diurnal motion, the line it describes would still not be circular, but would fall within the circumference of the semicircle. Otherwise the apparent increment of speed would not proceed according to the squares of the times — and yet it does so, not only by our experiments but also by Galileo's experiments and his own admission. To make this manifest, let the figure set out in number 7 be repeated here, and let the quantities of the spaces FS, GT, HV, etc., covered by the heavy body in equal times, be investigated until the vertex of the tower, C, has completed a whole quadrant in 6 hours, and the heavy body the whole semicircle CIA. For the spaces already mentioned will be found to fall short more and more of the quantity necessary for them to keep the proportion of the squares of the times.

[Margin: The way of finding the spaces covered by a heavy body, if it descended along a circular line.]

The way of inquiring into the quantity of the spaces already mentioned will, moreover, be declared very briefly by an example. Let the first arc taken, FC, be of one degree, which the Earth completes by its diurnal motion in 4^{m} of time; for the arc CS, intercepted between the same lines AC and AF, will be double in number of degrees, from what was demonstrated in number 8, that is, of 2°. Therefore the complement to the semicircle, namely the arc SIA, will be of 178°, and its half of 89°, whose sine is 99,984½ of the parts of which the radius EC is 100,000, while the diameter, or greatest of the chords, AC, is 200,000. That sine, doubled, will give the chord AS as 199,969 parts; and this, subtracted from the radius of the quadrant, AC — that is, from AF — of 200,000 parts, will leave the segment FS as 31 parts. In the same way, if the arc CG is of 2°, CT will be 4° and TIA 176°, and its half 88°, whose sine is 99,939; doubled, this will give the chord AT as 199,878, and this, subtracted from the whole radius AG, will leave the space or segment GT as 122 parts; and so for the rest.

Now if the space FS, of 31 such parts, is taken as the first, and as 1, and as the space passed through by the heavy body in the first time, or in the first 4 minutes of time, then the space GT, passed through at the end of the second of the equal times, must stand to the space FS as 4 to 1. For the square of unity, assigned to the first time, is 1, but the square of two, assigned to the second time, is 4. Let it be, then, as 1 to 4, so 31 to another number: one finds not the 122 parts that we said lie in the space GT, but 124 parts. Therefore, if GT, in

order that the proportion due to the squares of the times be kept, is greater than what arises from the circular line, it is necessary that the line which the heavy body describes in descending and keeping the proportion already mentioned should, if the Earth moves with the diurnal motion alone, draw back inward from the line CIA, ever more towards the diameter AC, though it will at last pass through the centre A.

But lest anyone should think that this difference, which is moderate at the beginning, is also moderate and negligible as it goes on, and that the line is therefore insensibly different from a circular one, let him inspect the following table; its explanation, to be sought from what has just been said, will be given beneath it.

Arc of the semi-circle (°)	Chords of the arcs, of which the greatest is 200,000	Complements of the chords to the greatest chord	Square numbers, to be multiplied by 31	Spaces due by the proportion of the square numbers	Arc of the semi-circle (°)	Chords of the arcs, etc.	Complements of the chords to the greatest chord	Square numbers, to be multiplied by 31	Spaces due by the proportion of the square numbers
178	199969	31	1	31	88	138931	61069	2116	65596
176	199878	122	4	124	86	136399	63601	2209	68479
174	199725	275	9	279	84	133826	66174	2304	71424
172	199512	488	16	496	82	131211	68789	2401	74431
170	199238	762	25	775	80	128557	71443	2500	77500
168	198904	1096	36	1116	78	125864	74136	2601	80631
166	198509	1491	49	1519	76	123132	76868	2704	83824
164	198053	1946	64	1984	74	120363	79637	2809	87079
162	197537	2462	81	2511	72	117557	82443	2916	90396
160	196962	3038	100	3010	70	114715	85285	3025	93775
158	196305	3695	121	3751	68	111838	88162	3136	97216
156	195629	4371	144	4464	66	108827	91173	3249	100719
154	194874	5126	169	5239	64	105984	95016	3364	104284
152	194059	5941	196	6076	62	103007	96993	3481	107911
150	193185	6815	225	6975	60	100000	100000	3600	111600
148	192252	7748	256	7936	58	96962	103038	3721	115351
146	191260	8740	289	8959	56	93894	106106	3844	119164
144	190211	9789	324	10064	54	90798	109202	3969	123039
142	189103	10896	361	11191	52	87674	112326	4096	126976
140	187938	12062	400	12400	50	84524	115476	4225	130975
138	186716	13284	441	13671	48	81348	118652	4356	135036
136	185436	14564	484	15004	46	78146	121854	4489	139159
134	184100	15900	529	16399	44	74922	125078	4624	143344
132	182709	17291	576	17856	42	71674	128326	4761	147591
130	181261	18739	625	19375	40	68404	131596	4900	151900
128	179758	20242	676	20956	38	65114	134886	5041	156271
126	178201	21799	729	22599	36	61804	138196	5184	160704
124	176589	23411	784	24304	34	58474	141526	5329	165199
122	174923	25076	841	26071	32	55128	144872	5476	169756
120	173205	26795	900	27900	30	51764	148236	5625	174375

Arc of the semi-circle (°)	Chords of the arcs, of which the greatest is 200,000	Comple-ments of the chords to the greatest chord	Square num-bers, to be mul-tiplied by 31	Spaces due by the propor-tion of the square numbers	Arc of the semi-circle (°)	Chords of the arcs, etc.	Comple-ments of the chords to the greatest chord	Square num-bers, to be mul-tiplied by 31	Spaces due by the propor-tion of the square numbers
118	171433	28567	961	29791	28	48384	151616	5776	179056
116	169609	30391	1024	31744	26	44990	155010	5929	183799
114	167734	32266	1089	33759	24	41582	158418	6084	188604
112	165807	34193	1156	35836	22	38162	161838	6241	193461
110	163830	36170	1225	37925	20	34730	165370	6400	194000
108	161803	38196	1296	40176	18	31286	168714	6561	203391
106	159727	40273	1369	42439	16	27834	172166	6724	208444
104	157602	42398	1444	44764	14	24374	175626	6889	213559
102	155429	44571	1521	47151	12	20906	179094	7056	218736
100	153208	46792	1600	49600	10	17431	182569	7225	223975
98	150942	49058	1681	52111	8	13951	186049	7396	229276
96	148628	51372	1764	54684	6	10468	189532	7569	234639
94	146270	53730	1849	57319	4	6980	193020	7744	240064
92	143877	56123	1936	60016	2	3490	196510	7921	245551
90	141421	58579	2025	65775	0	0	200000	8100	251100

[Margin: See the preceding figure.]

Now, to explain the preceding table: note how in it, having divided the quadrant CD etc. into 90°, and the semicircle CIA likewise into 90 pairs of degrees, I traced out the chords of the supplements to the semicircle, which are contained in the second column, and the segments intercepted between the semicircle CIA and the quadrant CD etc., which are given in the 3rd column. In the fourth column there is a continuous series of square numbers beginning from unity; by multiplying the first space, FS, of 31 parts, by these, one obtains the spaces or segments that are due if the proportion of the squares is to be kept. For you will see that at the end of the quadrant from the vertex C, completed in 6 hours, the space covered by the heavy body ought to be 251,100 parts, and yet, if the heavy body has to descend along the semicircle CIA, it cannot be more than 200,000 — namely, as much as AC; and that at the end of the other times too that proportion falls short more and more, whereas it ought either to be kept or rather to increase.

[Margin: Sixth objection.]

[Margin: The motion of heavy bodies is not in reality equal [?], and so not circular either.]

XV. Sixth: the demonstration of the equal motion of the heavy body already mentioned — namely, of one descending on the equator so slowly that it needs 6 hours to reach the centre of the Earth — which we brought forward from Galileo in number 8, does not hold unless the foot of the tower, B, and its vertex, C — that is, the circumference of the terrestrial globe — are supposed to move equally, and A, the centre of the Earth, to be immobile. But if, besides

the diurnal motion, an annual motion of the centre is also given to the Earth, Galileo holds that the compounded motion of any heavy body following the motion of the Earth is unequal in reality, now slow, now fast. He thinks this so true, in this hypothesis, that Galileo founded the cause of the tide of the sea on the inequality of the Earth's motion compounded of the diurnal and the annual, as we have already seen in chapter 14, from number 4 onward.

[Margin: Galileo's inconsistency with himself.]

Galileo therefore contradicts himself when he attributes a real equality to the motion of such heavy bodies, which he does not grant even to the Earth. And granted that this inequality is not the one we experience in the apparent motion of heavy bodies, still another inequality, unknown to sense, is substituted in its place, so that the one manifest to sense is removed — which is a greater absurdity.

[Margin: Objection [?].]

[Margin: It is falsely asserted, universally, that heavy bodies descend along a circular line.]

XVI. Seventh: Galileo errs when he pronounces universally or indefinitely that, in the Copernican hypothesis, the line that heavy bodies describe in their natural descent is circular, or as close to circular as possible. For this is not true even in the plane of the equator, where it ought most of all to be verified, as will be evident from what has been said so far and what is still to be said; and it is much less true outside the equator. To show this to the reader, we shall set out a threefold figure, by which we shall outline this line as best we can. For we cannot represent, in the one plane of this page, the diversity of the planes through which heavy bodies are carried in descending in this hypothesis, nor keep the due proportions, because of the narrowness of the leaves. From this, however, you will better learn what Scheiner indicated very lightly about the line of this motion on a cylindrical or conical surface in the *Disquisitiones Mathematicae*, page 31, and what Galileo conceded at his prompting, as we said in number 12.

[Margin: The first figure, for a heavy body descending in the parallel sphere.]

Let the first figure be for a heavy body let fall perpendicularly above one of the poles of the terrestrial equator, with an imagined spectator of this descent, in the parallel sphere. Let ABCD be the ecliptic, or Great Orb, carrying the centre of the terrestrial globe round with the annual motion from A through B towards the eastern quarter, C, and from C through D to A, the western quarter. For just as the axis of the Earth's ecliptic describes its own cylinder in its annual circuit, so too the axis of the equator describes its own, but one inclined to the cylinder of the ecliptic.

Diagram. The first figure, set in the left column: a cylinder seen obliquely. An upper arc E–M–N–F–S–a–G stands over a lower arc A–D–Z–C, the two joined by verticals E–A and G–C; vertical lines drop from M, N, F, S and a. The line from M carries the marks 1, 4 and 9; dots O, Q, T and Z stand on the verticals from N, F, S and a; H stands above the upper arc. Along the lower

edge run five small circles, the Earth at its successive positions, with centres K, V, B, X and Y, each with a horizontal diameter and lettered I at the left, R at the right, L below and P above (the circle B shows no P).

- E, H, M, N, F, S, a, G: points on the upper arc (H above it)
- 1, 4, 9: marks on the vertical from M
- O, Q, T, Z: dots on the verticals from N, F, S and a
- A, D, C: points on the lower arc
- K, V, B, X, Y: centres of the five circles
- P, I, R, L: the pole, left, right and lower letters of the circles (P on K, V and X; I, R, L on all five)

Now let the Earth be understood as ILRP, and its centre carried by the Great Orb from the point K to the points V, B, X, Y, over an interval of four equal times, to which the equal arcs of the Great Orb KV, VB, BX, XY correspond. Let the terrestrial equator be IR, its axis LP, and its north pole P; and let the axis be produced through the pole P outward, towards the north, by as much as the height of the tower or place from which the heavy body is let fall — say, by as much as KPM. For this line too, carried round with the axis of the equator, will describe the cylinder ACGE, one of whose bases is ABCD and the other EFGH; and at the end of the first of the equal times the tower, which was in the position PM, will be in PN; at the end of the second in PF; at the end of the third in PS; and at the end of the fourth in Za.

Heavy bodies, however, descend naturally in such a way that, if the whole time of the descent is divided into 4 equal parts (as we have assumed here) and the first space is taken as 1, the space covered at the end of the second time is as 4, at the end of the third as 9, and at the end of the fourth as 16 — as is established by our experiments in chapter 16, number 12, and by Galileo's own admission in the second dialogue *On the System of the World*. Let the height PM, therefore, be divided into 16 equal parts, and let there be marked on it the part M1, of one such part, the part M4, of four, and the part M9, of nine parts. Now if a ball heavier than the medium through which it descends were let fall from the vertex M, above the pole P, and the Earth moved with no motion other than the diurnal, about its own centre K, from I to R, and its centre always remained at K, then this ball would certainly descend not only apparently but also really along the same straight line MP, falling perpendicularly on the pole P. At the end of the first time it would have covered the space M1, at the end of the second the space M4, at the end of the third the space M9, and at the end of the fourth the whole space MP, which is 16 of the parts of which M1 is one.

But the centre is carried from K to V, etc., by the force of the annual motion, and the tower or height MP is carried round over the surface of the cylinder, which is to be conceived in the space of the world, on Copernicus's hypothesis. Therefore, when at the end of the first time the tower is at PN, the ball will be seen to have covered the space NO, which is equal to M1, or as 1; at the end of the second the space FQ, or as 4; at the end of the third the space ST, as 9; and at the end of the fourth the space aZ, which is as 16. However, since the eye of

the spectator, supposed to be at the pole P, is likewise carried along with that tower, the ball will appear to him always to be in the same straight line, even though in reality, on this hypothesis, the ball has described on the cylindrical surface the curved line MOQTZ, far longer than the straight line MP and not lying in one plane.

This line will therefore not be a cylindrical section that is a circle — as is the section whose plane cuts the cylinder parallel to the base — nor will it be an ellipse, such as is the section whose cutting plane is not parallel to the base of the cylinder, according to what was demonstrated by Serenus of Antinoë, who is usually published with Apollonius, in Book 1 of the *Sections of the Cylinder*, Propositions 5, 6 and 17. The reason why the points O, Q, T, Z are not in one plane is the proportion of the square numbers. Otherwise, if they were in one plane, the section would be elliptical; but then it would be cut in half at Q by the straight line FP, which is the middle one among five lines standing at equal intervals from one another, and the parts FQ would be 8 of the 16 parts; and the second line, NP, and the fourth, SP, would be cut so that NO would be equal to PT, that is, 7 parts each, as could easily be demonstrated if there were need and we were not hastening on to other matters.

[Margin: The spin owed to a ball descending above the poles.]

It should be noted here, however, that a ball let fall perpendicularly above the pole of the terrestrial equator would, while it descended along the line MOQTZ, at the same time be carried round about its own centre with a spin imitating the diurnal spin of the Earth. I wish it could be observed by an inhabitant under the poles of the world! For from it either the truth of the Earth's diurnal spin would become manifest, if such a spin were discerned on the surface of that ball, or its falsity, if it were not discerned.

[Margin: The 2nd figure, for a heavy body descending in the plane of the equator.]

XVII. Now let the second figure be for a ball let fall from the vertex of a tower standing in the plane of the equator, or in the right sphere. Let A be the centre of the Great Orb; from it describe the arc BDC, divided into four equal parts, Bρ, ρD, DP, PC, corresponding to the four equal times in which the ball let fall from the vertex of the tower is supposed to slide down to the surface of the Earth; and let it be understood that at the beginning of the motion the centre of the Earth is at C, at the end of the first time at P, at the end of the second at D, at the end of the third at ρ, and at the end of the fourth at B. Let the height of the tower be FH, and the terrestrial equator EFG, which, in the four times already mentioned, revolves about the centre C towards the east, from F to E, so that the foot of the tower, F, covers the arc FE, and its vertex the arc HK. Here, if there were no motion other than the diurnal, the ball let fall from the vertex H would have covered at the end of the first time the space LM, as 1 of the parts of which FH is 16, and the tower would be on the line CL. At the end of the second, the tower would be on the line CN, and the space covered by the ball would be NO, namely 4 such parts; at the end of the third,

the tower would be on the line Cm, and the space covered would be mI, or 9 parts; and finally, at the end of the fourth time, with the tower carried to the line CK, the whole space KE, of 16 parts, would have been covered.

But at the end of the first time the centre of the Earth is supposed to have been carried from C to P by the force of the annual motion, and meanwhile the tower, by the force of the diurnal motion, has completed the arc XQ of the equator towards the same side; therefore the tower is on the line QR, and the space apparently covered by the ball is RS, of 1 part. At the end of the second time, with the centre of the Earth brought to D, and the tower completing by the force of the diurnal spin the arc TV, double the arc xQ, the tower will be on the line XV, and the space covered by the ball will appear to be XZ, that is, 4 parts. Then, with the centre of the Earth carried by the annual motion to ρ, and the tower to the line αβ because of the arc Yα completed by the force of the diurnal rotation, triple the arc xQ, the ball will appear to have covered the space βγ, of 9 of the parts of which RS is one. Finally, with the centre of the Earth transported to B, and the tower to the line εθ because of the arc δθ completed by the force of the diurnal spin — indeed quadruple the arc xQ — the ball will appear to have covered the whole space εθ, of 16 parts.

Diagram. The second figure, set in the left column: from A at the foot, radii rise to an arc of the Great Orb carrying five small circles, the Earth at its successive positions, with centres B, ρ [?], D, P and C. Above them an outer arc through ε, β, X, R to H, and inside it a curve through θ, δ, γ, Z and S; from each circle a tower line rises to the outer arc. At the right, around the circle C, lines fan out to K, m, N, L, H, with O, M, I, E, F marked on them and G below.

- A: the centre of the Great Orb, at the foot
- B, ρ [?], D, P, C: centres of the five circles
- R, X, β, ε, H: on the outer arc
- S, Z, γ, δ, θ: on the inner curve
- Y, a, V, T, Q, x: on the circles and tower lines (x stands right of Q)
- K, m, N, L, O, M, I, E, F, G: on the fan of lines about the circle C

Hence the line described by the ball in the space of the world will be the curved line HSZγθ, although to an eye carried likewise along with the foot of the tower from F to the points Q, V, a, θ the ball will appear always to descend along the same straight line, parallel to the tower. Meanwhile the vertex of the tower, H, will describe the curved line HLRXβε. Neither of these will be circular, because it will always come closer to the centre A, from which it is certainly farther when the ball is at H — for it is then distant by the whole aggregate of the semidiameter of the Great Orb, AC, the semidiameter of the Earth, CF, and the height of the tower, FH — than in any of the other positions already mentioned. Indeed, since the plane of the equator, because of the parallelism it keeps with itself, is always carried by the Great Orb, which is inclined to it, to one plane after another, the line HSZγθ cannot lie in one plane, and therefore it cannot be parabolic, hyperbolic or elliptical: for these

lines require that they lie in one and the same plane cutting a cone or a cylinder, actually or potentially. The same thing happens proportionally in the other positions of the tower, granted that we have chosen a single case, namely when it is midnight for that tower.

[Margin: Figure for the oblique sphere.]

XVIII. Now let the third figure be for a heavy body let fall from the vertex of a tower standing in the oblique sphere (or on the circumference of some terrestrial parallel declining from the equator), but standing perpendicularly on the Earth's surface, and so placed that, if it were produced, it would run into the centre of the Earth. Let there be marked, then, the arc ACB, a portion of the Great Orb, on which the centre of the Earth FKEG is first at B; and let the circle of the terrestrial equator, inclined to the Great Orb, be DBE, whose axis KBQ etc. is understood to be produced as far as H. Let the parallel be FQG, on whose circumference the tower GL stands erected perpendicularly to the globe of the Earth; and let the terrestrial equator turn with the diurnal spin from E through B to D, the eastern point, until, a revolution of 24 hours being completed, the point E returns to the original beginning of the revolution. For it is certain in the Copernican hypothesis that the whole globe of the Earth, with all the parallels to the equator, turns towards the same quarter. And just as the semidiameter of the Earth, BG, will describe the cone BGF, whose base is the parallel circle indicated by GQF and described by the point G, which is at the foot of the tower, so the tower GL, joined with the Earth's semidiameter BG, will describe a larger cone, BLM, whose vertex, common with the former, will be the centre of the Earth, B, and its base the circle ML. For the semidiameter of this circle makes a right angle with the axis BH; and therefore, if, while this axis BH stays fixed as a side, the other sides are carried round it until they complete one whole revolution, a cone will be described by them, by definition 18 of Book 11 of Euclid's *Elements*.

Diagram. The third figure, set full width: five hatched globes in a row, each with a cone rising from its centre, and a dotted curve running from the first cone over the tops of the others to L at the far right. The fifth globe, centre B, carries the inclined circle D–B–E, the parallel F–R–Q–P–G, the tower GL and the cone BLM, with the axis dotted down to K.

- Φ, π, [?]: on the first cone and globe (an unresolved sort beside a dot on the globe's rim)
- A, I, C, X, B: centres of the five globes
- x, [?], ω, θ: on the second cone and globe (the unresolved sort beside a dot on the dotted curve)
- ε, η, ζ, ξ: on the third cone (ε at its top, η and ζ within it, ξ at its right edge)
- [?], [?], β, a: on the fourth cone and globe (two unresolved sorts at its top)
- D, E, K: on the fifth globe (inclined circle and foot of the axis)
- M, N, H, O, L: on the top of the fifth cone

- F, V, T, S, R, Q, P, G: on the parallel and the tower lines of the fifth globe

Let us now imagine that the ball let fall from the vertex L reaches the surface of the Earth in four equal times, and that the Earth meanwhile moves with the diurnal motion alone; and so, the arc FG of the parallel being divided into four equal arcs corresponding to the times already mentioned, namely GP, PQ, QR, RF, that the tower is likewise carried first to the line PO, then to the line QH, third to the line RN, and fourth to the line FM. Therefore, since heavy bodies descending naturally pass through spaces that stand to one another in order as the squares of the times, if the height GL is divided into 16 equal parts, of which the one passed through at the end of the first time is OS, that is, 1; the second, at the end of the second time, HT, of 4 parts; the third, at the end of the third time, NV, of 9 parts; and the fourth, at the end of the fourth, MF, of 16 parts, then the tower will indeed describe part of a conical surface, in such a way that the ball, which always appears to be in the same straight perpendicular line, will describe about the surface of the cone MBL the curved line LSTVF; and the proportion of the square numbers does not allow this line to lie in one and the same plane. So even in this case that line will be neither circular, nor elliptical, nor parabolic, nor hyperbolic — even if B, the centre of the Earth, were not meanwhile moving towards A by the annual motion.

Now let the centre B move towards A, the eastern point, into X, by the annual motion, and so carry the cone described by the tower and the Earth's semidiameter to another position. Meanwhile, through the diurnal spin, let the tower have completed the arc aβ and stand on the line βγ: for the ball will appear, at the end of the first of four equal times, to have completed the space γδ, which is taken as 1. Next, at the end of the second time, let the Earth's centre be at C, and the whole cone carried along with it, so that the tower, by the force of the diurnal spin, has completed the arc ξζ, double the arc aβ, and stands on the line εζ: for the ball will appear to have completed the space εη, which will be as 4. Third, let the centre of the Earth be carried by the force of the annual motion to Y, with the whole cone standing on it, and let the tower, through the diurnal rotation, have completed the arc ωθ, triple the arc aβ, and stand on the line xθ: for the ball will appear at λ, having now completed the space xλ, which will be as 9. Finally, at the end of the fourth time, let the centre of the Earth have passed to A by the force of the annual motion, carrying the Earth with it together with the cone already mentioned; and let the tower now have run through, by the diurnal motion, the arc πμ, four times the arc aβ, so that the tower stands on the line μΦ: for then the ball will be on the Earth's surface, that is, at the foot of the tower, μ.

So although the ball will always appear in the same straight line in which the eye stationed at the foot of the tower stands — since the eye is carried by the force of both motions from G to the points β, ζ, θ, μ — in reality it will make the curved line Lδηλμ. This line can be neither circular, since it always approaches more and more closely to its own centre, which is the centre of the

Great Orb; nor elliptical, nor parabolic, nor hyperbolic, since it does not lie in one plane. For it is described about the cone in such a way that it is nevertheless carried into one plane after another, both because of the proportion of the square numbers that it keeps in approaching the Great Orb, and because of the inclination of the terrestrial equator to the Great Orb, the equator remaining always parallel to itself.

[Margin: A warning on correcting the figures.] In the figures already mentioned we drew the centre of the terrestrial globe at its several positions with arcs of the Great Orb between them equal to the arc that the Earth's diameter subtends, so that the figures would not confuse the viewer. Those intervals, however, may be greater or smaller, according to the time of the whole descent of the heavy bodies. And what we have said about the position of the tower that has midnight holds equally for any other position of the tower.

[Margin: Corollaries for the figure of the motion of heavy bodies.] From what has been said it is clear that in no case is the line already mentioned circular, elliptical, hyperbolic or parabolic, since it is not drawn in one plane — which is nevertheless required, as is clear from the sections of Apollonius, Book 1, proposition 4, on the circle cutting a cone; prop. 11, on the parabolic section; prop. 12, on the hyperbolic section; and prop. 13, on the elliptical section; and also from the cylindrical sections of Serenus, Book 1, props. 5, 6 and 17, on the circle and the ellipse arising from the section of a cylinder.

[Margin: The errors of Kepler, Gassendi, Boulliau and others about the figure of the motion of heavy bodies, but above all of Galileo.] Accordingly, these erred: Kepler, who, as stated in chapter 4, scholium 2, described this line as if the Earth's motion were diurnal only, and as approaching too closely, at the beginning of the motion, to the straight perpendicular line; Gassendi, who, as stated in the same place, scholium 4 — namely in the second letter *De motu impresso* — held it to be universally parabolic; Boulliau, in Book 1 of his *Philolaus*, chapter 4, who asserted that it was composed of two circular lines in one plane; and those who, as reported in Chiaramonti, Book 12 of *De Universo*, chapter 21, said that it is like the quadratrix of Nicomedes. But most of all Galileo erred, who in the second Dialogue on the system of the world asserted that it is circular, or as near to circular as can be, and yet ends at the centre of the Earth. Against him, besides these errors, there is also the following objection. Granted, as I said in number 12, he recognised, as if through a lattice, that this line is a spiral about a cone in the parallels of the equator, and in the case above the poles a spiral about a cylinder — but at the prompting and warning of Scheiner, who of all came closest to the truth on this point. We, however, had already drawn the figures already mentioned by our own wit before we had read that passage of Scheiner.

[Margin: 8th objection.] XIX. In the second Dialogue on the twofold system of the world, page 223 of the Italian but 168 of the Latin version, Galileo teaches the following about the pendulum or plumb-line: *"I put that observation of the pendulum before you for this purpose: that you should understand that the*

impetus acquired in the descending arc, where the motion is natural, is of itself able to drive the same ball of the pendulum, by a violent motion, through an equal space in a similar ascending arc — of itself, I say, once all external impediments are removed. I also believe it is understood beyond doubt that, just as in the descending arc the speed continually grows as far as the lowest point of the plumb-line, so too from that same point along the other, ascending arc the speed continually diminishes as far as the extreme and highest point; and indeed diminishes in the same proportion in which it grew at the start, so that the degrees of speed at points equally distant from the lowest point are equal to one another." [Margin: Once inequality of impetus is granted, inequality of motion is wrongly denied.] [Margin: Galileo's doctrine of the speed of descent and ascent of the pendulum along an arc.] *"On this occasion I say that the reason this motion is not perpetual is that in the ascent the heaviness takes away something from the upward-driving impetus; and so the arc of ascent is not precisely as great as the arc of descent, and the heaviness prevails more and more by a contrary impetus, until it destroys that impetus entirely and the plumb-line comes to rest."*

[Margin: And on the motion of heavy bodies through a well as long as the diameter of the Earth.] But Salviati, playing the part of Galileo, goes on and says: *"And from this (reasoning with probability) I think I can believe that, if the terrestrial globe were bored through to the centre, and indeed beyond it as far as the nadir, a cannon-ball would acquire in its descent so much impetus of speed that, once past the centre, it would drive the ball upward through as great a space as the space of its descent; in such a way, that is, that beyond the centre its speed would continually lessen by degrees of decrement corresponding exactly to the degrees of increment acquired in the descent."* So far Galileo, as concerns our subject.

Galileo therefore admits that a heavy body acquires a real impetus, ever greater, and so great that it is able to drive that same ball upward. But if it really moves uniformly along a circle, as follows from his own doctrine given in number 8 and from his own most explicit statement, and the inequality of speed is a mere appearance to the eyes, or to the intervals as viewed, which it seems to pass through in a straight perpendicular line, then it cannot continually acquire an increment of greater impetus, an impetus able to drive upward; or if it does acquire it, the inequality of speed that follows from it by natural necessity is real, not apparent. [Margin: Galileo at variance with himself.] Galileo therefore contradicts himself, and his propositions on page 120, on real uniformity, and on page 168 of the Latin, on the increment of impetus, cannot be reconciled with each other.

Having set out these not untimely remarks on this motion, it is now time to put forward the argument for the Earth's motion that Galileo and Boulliau have built from the motion of heavy bodies, and to dissolve it. Cast into syllogistic form, the argument runs as follows.

[Margin: The argument in form for the motion of the Earth.] XX. *Heavy bodies, descending naturally, acquire by their apparent motion a continual increase of speed. But the reason for this increase is given most readily if the Earth moves*

at least with the diurnal motion; if, however, it is immobile, no reason can be given in which the intellect can rest. Therefore the Earth moves at least with the diurnal motion. [Margin: Proof of the major and the minor.] The MAJOR is clear from many experiments, but chiefly from those set out at length in chapter 16 from number 6. The first, affirmative part of the MINOR is proved thus: if the Earth moves at least with the diurnal spin, the real descent of heavy bodies turns out in fact to be circular and uniform, and the whole inequality of their speed is thrown back onto mere appearance, the eyes falsely judging that they move along the same straight perpendicular line and pass through ever greater spaces in it in equal times — whether they are then said to move along two circles, according to Boulliau's opinion set out in numbers 2 and 3, or along a simple circle, according to Galileo's opinion explained in numbers 7 and 8. The second, negative part of the MINOR is proved from the insufficiency of the reasons that the Peripatetics, or others, have so far adduced for such an increase, which Simplicius in particular reports, in Book 1 *De caelo*, text 88, and others too. For the chief reason usually brought forward is the acquisition of greater and greater gravitation, or impetus, intensifying the earlier impetus. But if this were the cause, heavier bodies would also have to descend faster, and the faster the heavier they are than the others; and so a ball of iron weighing two pounds would have to descend from the same height to the ground in half the time taken by an iron ball of one pound. This is established to be utterly false, as is clear from the experiments set out in chapter 16, number 13, and from theorem 13 of the same chapter.

[Margin: 1st reply.] I reply, first: granting the Major, I deny the first part of the Minor, and its proof. For once the diurnal motion of the Earth is posited, some appearance of an increase of speed is indeed saved by a motion that is in fact uniform; but the other effects are not saved, which presuppose a real increase of a continually greater impetus, and so a speed of motion that is in fact greater and greater. Such are the stronger blow, the louder sound, and the raising of a greater weight, the higher the place from which the heavy bodies fall, according to what was said against Boulliau in number 5 and against Galileo in number 10. Next, Boulliau's two circles do not save the increase of speed according to the odd numbers counted from unity, as we showed in number 5. [Margin: Galileo's many errors gathered into one.] And Galileo's single circle does not save the real increase of the impetus of heavy bodies, which nevertheless had to be saved, even on what Galileo grants elsewhere, according to what was said in numbers 10 and 19. Nor, again, is the unequal descent of two heavy bodies of different weight let fall together from the same height saved, as I showed in number 11. Moreover, that descent along a semicircle is not saved unless the heavy body is at the equator and so slow that it requires exactly 6 hours to reach the centre of the Earth; for once these two conditions are removed — and they are verified in very few cases — the line of the motion already mentioned is not circular, as I showed in numbers 12, 14, 16, 17. Indeed, not even at the equator is that semicircle produced by the motion of a heavy

body, even one so slow that it would reach the centre of the Earth in 6 hours, unless the Earth's motion is diurnal only, with the annual motion of the centre excluded. Since Galileo does not exclude that motion, it makes the fall unequal in reality, and along a line different from a circle; so Galileo again contradicts himself, as I showed in numbers 13 and 15. Indeed, even if the heavy body is at the equator and descends to the centre of the Earth in 6 hours, and the Earth moves with the diurnal motion alone, the line of descent is not circular but notably different from a circle towards the end, as I showed in number 14. Finally, by this hypothesis Galileo cannot save the increase of speed of light bodies that ascend naturally, even above the air bordering the Earth: for these are neither carried to one determinate point as to a centre, nor do they follow the motion of the Earth. Perhaps for this reason he preferred to deny positive lightness, against the experiments adduced in chapter 16, number 3.

[Margin: The objection rejected.] Nor may you say that, granted Boulliau and Galileo have not saved all the properties of heavy bodies descending naturally, they can nevertheless be saved in the hypothesis of the moving Earth, since one can assign the line they describe, and in it the inequality of the parts passed through in equal time, together with the inequality of the blow, the sound and so on, and the inequality of speed of heavy bodies of different weight. For the reply is that this real inequality cannot be saved at the equator, if the heavy body is such that it requires 6 hours to arrive at the centre of the Earth, as I shall demonstrate below. Outside the equator it can indeed be saved by a line such as we described in number 17; but by that line the difficulty of the real increase of the speed of heavy bodies is not removed, but rather increased. For along such a line that unequal motion turns out to be produced along curved lines, of different kinds, about a cone, by a longer path than if, with the Earth standing still, the heavy bodies moved along the simplest straight line, as happens with heavy bodies falling above the poles of the terrestrial axis. And, what is absurd, the same heavy body, its heaviness unchanged and let fall from the same height, is compelled without any necessity to travel in reality a much longer path than if it fell above the poles of the terrestrial axis; and a longer one in one parallel than in another; and, finally, a longer one by night than by day.

[Margin: 2nd reply.] I reply, second: I deny the second part of the Minor, which was the negative one. For a sufficient reason for the increase already mentioned can be given in the hypothesis of the immobile Earth, as will appear in the next chapter, number 10. Nor is it necessary, if the reason is to be sufficient, that the motion downward be the faster the heavier the body is; just as it is not necessary, if the cause of refraction is the inclination of a ray upon a medium of different density, and the inclination of one ray is twice the inclination of another, or the density it falls on is twice as great, that its refraction too be twice the refraction of the other. Or, if the greater distance of a star from the Earth is the cause of smaller refraction and parallax, other things being equal, it is not necessary that the refraction or parallax therefore become half as great because its distance has become twice as great.

[Margin: Corollary against Galileo's boasting.] From what has been said so far, then, it is clear into what shameful errors in geometry and physico-mathematics Galileo fell, who boasted with such great but empty glory that he had found the way by which heavy bodies, descending naturally, are seen to acquire an increase of speed.

CAPVT XVIII.

Chapter 18.

AN & QUOMODO IN HYPOTHESI TERRÆ QUIESCENTIS REDDATUR RATIO À PRIORI, CONTINUI INCREMENTI VELOCITATIS GRAUIUM, & LEUIUM NATURALITER DESCENDENTIUM, VEL ASCENDENTIUM.

Whether, and how, in the hypothesis of the resting Earth, a reason a priori can be given for the continual increase of speed of heavy bodies and light bodies descending, or ascending, naturally.

I. Various opinions on this subject are reported, in the first place by Simplicius on Book 1 *De caelo*, text 88, comment 86; by Franciscus Bonamicus [Francesco Buonamici], Book 4 *De motu*, chapter 37; by the Conimbricenses, Book 2 *De caelo*, chapter 6, question 1; by Zabarella, Book 1 *De motu gravium et levium*, chapter 16; by Chiaramonti, Book 12 of *De Universo*, chapter 30; by Contarenus, Book 1 *De elementis*; and by Pererius on Book 4 of the *Physics*, chapter 3. We must touch on them briefly here, to show that the hypothesis of the immobile Earth does not lack a way of bringing forward an a priori cause for which heavy and light bodies move faster and faster towards the end. Giving that cause is now harder than it was before our experiments and Galileo's. For until then it was not known what proportion they continually keep in such an increase, and so it was enough to bring forward a cause of greater and greater speed. Now, however, it is known — at least by those who have made the experiments, or are willing to make them carefully — that this increase (speaking at least of heavy bodies) follows the progression of the odd numbers counted from unity; so a reason must be given for this proportion too.

[Margin: 1st opinion, Hipparchus's, on the cause of the increase of speed.] II. The *first* opinion was that of *Hipparchus*. He said that if heavy bodies are thrown upward by force, they begin their downward motion when their heaviness begins to prevail over the power impressed by the thrower; but because something of that impressed impetus still lasts, though it is overcome more and more by the innate heaviness, the descent is therefore slower at the beginning and faster at the end. And if heavy bodies have not been thrown upward but let down, there still remains in them something of the force that held them back, which is used up only little by little; and so the motion of heavy bodies is slower at the beginning and faster towards the end. Alexander, in Simplicius, rejects this cause on the ground that heavy bodies are not rarely generated outside their own place, where they were held back by nothing, and yet are still faster towards the end of the motion. I add that if Hipparchus's

reasoning held, heavy bodies thrown downward would always move faster at the beginning than at the end, because of the double motive power downward joined in them — one from the intrinsic power, the other from the extrinsic thrower — the second of which power gradually weakens. But this is not universally true: for sometimes the impetus with which they are sent down is so slack that the increase of speed towards the end of the motion is far greater than near the start. Finally, one who does not simply let a heavy body down, but lets it go downward, entirely ceases to hold it back, and leaves nothing of a retentive quality in that heavy body.

[Margin: 2nd opinion, attributed to Alexander.] III. The *second* opinion, which some attribute to Alexander, held that the cause is that heavy bodies are altered less by contrary qualities the nearer they come to their own place. But an iron ball descending through the air is certainly not altered at all by the air; and if it is altered, it is rather some moisture that it acquires, which grows towards the end; and a glass ball is altered much less in descending through water.

[Margin: [?] opinion, of Themistius and Kepler.] IV. The *third* opinion was that of *Themistius*, on Book 8 of the *Physics*, text 76, who says that bodies move faster at the end for this reason: they are attracted by the places akin to them, to which they are carried. Or that of *Kepler*, in the introduction to the *Commentaries on Mars*, who says that they are drawn by the globe of earth and water through a certain magnetic power. But whether the place attracts only objectively or effectively, it would certainly attract and draw a body akin to it more strongly when the body is nearer than when it is further away, even if it had not been brought nearer by motion from far off. And so, if one ball were let fall from a window 50 feet high at the moment when another ball of the same kind, shape and weight, let fall from a height of 100 feet, had reached that window, they would be attracted equally, being now at an equal distance; and yet the one let fall from the height of 100 feet reaches the ground faster and strikes it with a greater blow.

[Margin: 4th opinion, of Iamblichus, Sirenius and Durandus.] V. The *fourth* opinion, of *Iamblichus* and *Sirenius*, was that this happens from the smaller resistance of the medium, since, that is, a smaller depth of air remains to be passed through; or rather, as *Durandus* says (on Book 2, distinction 14, question 1), because the lower air is the thicker the nearer it is to the Earth, and so, levitating upward less, it resists the downward motion less. But against these stands the case just brought forward, of two heavy bodies equal in kind, bulk, shape and weight, one of which is let fall from a tower 100 feet high, and the other let fall when the first reaches a window 50 feet high: for then an equal depth of air remains to be overcome by each, and yet the one let fall from the hundred-foot height reaches the ground much sooner. Against Durandus in particular there tells the very thickness of the air, which is less apt to give way and to move upward or to the sides. And when a ball heavier than water is let

fall through water of uniform thickness, as we have often done, it still descends faster at the end.

[Margin: 5th opinion, of Contarenus and Burlaeus.] VI. The *fifth* opinion is that of *Contarenus*, Book 1 *De elementis*, and of *Burlaeus* [Walter Burley], on Book 8 of the *Physics*, text 76, who put the cause in the air rushing in from behind to fill the vacuum, rarefied more and more by the motion, and so driving the heavy body downward with greater effort. But if a half-burnt and still smoking material is let fall, or a heavy body with a piece of a lighted candle fixed on top of it, the smoke or the flame is seen for a long time standing upright; so the air does not press on that heavy body from behind. So too with light bodies rising through water: if you hang a thread below one, the thread stays stretched downward and is not driven upward by the water. Moreover, the lower air is not more rarefied on account of the motion completed in the upper parts.

[Margin: 6th opinion, of Zabarella, Pererius and Dandinus.] VII. The *sixth* opinion is that of *Zabarella*, Book 1 *De motu gravium et levium*, chapter 16; of *Pererius*, Book 4 of the *Physics*, chapter 3; and of *Dandinus*, Book 2 *De anima*, in a digression on this motion. Although they grant that the increase of speed is helped by the air rushing in from behind, they refer the chief cause to the air in front, already moved and dispersed in the direction in which the heavy or light body is carried. For just as a moving body moves faster through air already moved in the same direction, and a ship going downstream on a river moves faster, than through unmoved air or wandering water, so it moves much faster through air moved twice, and faster still through air moved three times, and so on.

But against the air from behind enough was said in number 6. Against the air in front there tells the wind, which drives that air crosswise and yet cannot move a heavy body of great weight from its perpendicular motion. Next, it is not numerically the same part of the air that is moved again and again by the descending heavy body, since the air runs in at the sides and around the body to fill the place the body successively leaves; so the air is not rightly said to be moved twice, three times, four times. Universally, against those who fall back on the motion of the air or water in front, there tells the priority of the motion of the heavy body itself. For the air or water beneath is moved because it is driven by the heavy body tending downward, and the air or water is moved faster because that heavy body moves faster. Its increased speed is therefore presupposed before the air or water moves faster, at least by priority of nature and of cause; so the commotion of the air or water cannot be the cause of the greater speed in the descent of heavy bodies. The comparison with the river does not hold here, because the motion of the river is presupposed antecedently to the motion of the ship. Finally, against this opinion tells the slower descent of a lighter ball let fall together with a heavier one.

[Margin: 7th opinion, of Aristotle, Alexander, St Thomas, Albertus de Saxonia, the Conimbricenses, Cabeo, Chiaramonti, Bonamicus.] VIII. The

seventh opinion was that of *Aristotle* and *Alexander*, in *Simplicius* on Book 1 *De caelo*, text 88 — an opinion that Simplicius himself also prefers to the others, on the supposition that such an increase is granted (for in that same place he himself doubts it); likewise that of *St Thomas Aquinas* and *Albertus de Saxonia*, in *Bonamicus*; of Bonamicus himself, in his book *De motu*, chapter 37; of the *Conimbricenses*, on Book 2 *De caelo*, chapter 6, question 1; of *Cabeo*, on Book 1 of the *Meteorologica*, text 17, question 3; and of *Chiaramonti*, Book 12 of *De Universo*, chapter 30. They put the cause in greater and greater gravitation, or in greater accidental gravity, or in the second act of gravity, which by the motion itself becomes more and more perfect in second act — what Aristotle called the apposition of gravity or levity, understand: in second act. For in Book 1 *De caelo*, text 88, he says that if the speed of the elements were increased to infinity, gravity or levity would be increased to infinity; and in the *Mechanics*, question 19, he teaches that if a heavy weight is placed on an axe, it often does not split the body, which the axe nevertheless does split if it is swung from on high by itself, without any other weight, because it takes on some gravity. From this it appears that by the name of gravity he does not mean gravity in first act — for with another weight placed on top, that aggregate already had greater gravity, and yet it did not split the body — but means impetus, and gravity in second act, or a translative quality. Chiaramonti, however, calls the second act of gravity itself motion, which, he admits, can produce another motion by the mediation of gravity; others understand impetus as distinct from motion taken formally.

Since, then, at every moment in which a heavy or light body is outside its place and is prevented by no impediment or contrary mover, it produces some determinate degree of impetus suited to its own gravity or levity, and there is no reason why the earlier impetus should cease, it comes about that, as new and still newer degrees of impetus of the same kind are added, the impetus in the same part of the subject becomes more and more intense, and so the motion becomes faster and faster. But Bonamicus, besides this principal cause, also adds the ever smaller resistance of the medium. This opinion would no doubt satisfy us if a cause had to be assigned only for the continual increase of speed. For what is more consonant with the nature of heavy or light bodies than that, at any instant in which they are away from their natural place, gravity or levity should produce an impetus of the same kind and amount as it produced in the first instant of the motion, and that this should remain so long as it is not destroyed by a contrary impetus, or the body has not reached its natural place?

But from this it would follow that the proportion of this increase goes according to the simple and natural arithmetic progression. For let us suppose, for ease of explanation, that the impetus produced in the first instant of the motion is so great in a given heavy body that by its force the body passes through a single palm in the first and smallest of the equal times. Then, with a second degree of impetus added, equal to the first, it ought in the 2nd of the equal times to pass through no more than two palms, which together with

the first would make 3 palms at the end of the second time. And with a third impetus added, it ought in the 3rd of the equal times to pass through 3 palms separately, which together with the 3 preceding would make six palms; and so on. So the spaces completed in each of the equal times separately would go according to the increase of the arithmetic progression 1, 2, 3, 4, 5, 6; and from these the spaces passed through at the end of the aggregated times would arise in this proportion: 1, 3, 6, 10, 15, 21. And Cabeo, in the place just cited, and Baliani, Book 4 *De motu gravium*, page 110, judged it more probable that the increase goes in this way. But it is now certain to Galileo, to us, and to anyone willing to make the experiment, that this increase is much greater, and indeed follows the progression of the odd numbers counted from unity: so that if in the first of the equal times a heavy body passes through 1 palm, in the second it passes through 3 palms separately, in the third 5, in the fourth 7, in the fifth 9, and so on. From this the spaces aggregated at the end of the times arise according to the order of the square numbers 1, 4, 9, 16, 25. The reason for this increase, therefore, is what is sought.

[Margin: 8th opinion, of a recent philosopher.] IX. The *eighth* opinion, then, was that of a certain recent professor of philosophy, who had been present at some of our experiments and had no doubt at all about the proportion already mentioned. It held that in the first minimum of the duration of this motion gravity produces 1 degree of impetus, by whose force the heavy body passes through one determinate part of the space; that in the second instant, or rather minimum of duration, gravity produces another degree, and the earlier impetus produces another impetus like itself, so that there are three degrees of impetus, and therefore in the second of the equal times a space three times greater is passed through than in the first time; but that meanwhile the older impetus perishes, and so two degrees remain, which produce two more degrees, while gravity always produces its accustomed degree, so that there are five degrees, from which follows a passage through a space five times greater than the first space; and that in the fourth moment the two oldest of these 5 degrees of impetus perish, and the three remaining produce three more, to which gravity, adding its own new degree of impetus, completes 7 degrees, from which follows motion through a space seven times greater than the first; and so on. But this opinion is little in keeping with the natures of things. For why should impetus perish if it has no contrary and the moving body has not reached its place? Or, since all the degrees of this impetus are of the same species, why should so many degrees perish in the very instant in which others, and indeed more of them, are to be produced in their place? For example, why should two of the 5 degrees perish in the fourth instant and three remain, if in that same instant 4 degrees must be produced, which with the three remaining make 7? Is it not less wasteful that only 2 be produced anew, which, added to the five earlier ones, make 7? Certainly it is.

[Margin: 9th opinion, our own, in three parts.] X. The *ninth* opinion, then, and our own, to which we hold until something better occurs to us, is this. At

any instant in which a heavy body is outside its place and the obstacle has been removed, gravity produces one degree of impetus; and that very impetus which was produced by gravity produces another like itself in the instant immediately following, so that all those produced remain and make the earlier impetus more intense. Not all of them, however, produce degrees like themselves at each and every moment, but only once, as soon as they have been produced. If this is so, it follows that in the first instant of the descent one degree is produced by gravity alone; in the second instant (to speak this way for ease) both gravity produces its own degree and the impetus produced by gravity immediately before produces a degree of impetus like itself, and so there are three; in the third instant gravity produces its own and the more recent impetus its own, which, added to the earlier ones, make five degrees of impetus; in the fourth instant gravity begets its own degree and the newest impetus its own, which, joined to the earlier ones, make seven degrees; and so on, two degrees always being produced, one by gravity, the other by the most recently produced impetus. Otherwise, if all the existing degrees each produced their own degrees in every instant, they would exceed their nearest radical principle, which is gravity.

[Margin: 2nd part of our opinion.] If this does not please, let us say that in the first instant, or minimum of time, gravity produces only half of the impetus which it can produce whole in the remaining instants, on account of the difficulty of beginning motion, whatever that difficulty may finally be. Or rather let it be said that in the first instant the heavy body descends by force of an impetus which, before the descent, it had always kept produced in itself, but which was impeded — by a supporting body, for example — from effecting the descent; whereas in descent that heavy body produces for itself, at each instant, a degree of impetus twice as great as that first impetus which it was producing while not descending. Accordingly, if in the first instant it descends, say, a space of one palm by force of the first impetus, in the second instant it will descend three palms — one by force of the same first impetus, which persists, and two by force of the impetus produced in the first instant of the descent — and in the third instant it will descend five palms, because another impetus has now been added, effecting descent through two palms. Proceeding in this way, the progression of the odd numbers beginning from unity will likewise be kept.

[Margin: 3rd part of our opinion.] But suppose someone will not have impetus stand to impetus as speed stands to speed, but holds that an impetus merely doubled, through its union with another impetus equal to it, can have as much force as if it were itself doubled, so that the first two impetuses produce motion as 4 when the first had produced it as 1. Then, surely, three impetuses at the end of the third equal time, each behaving as if it were tripled, would together produce motion as 9, four as 16, and 5 as 25, and so on. If, however, someone says that the cause a priori was the will of the Author of Nature, who could have established another proportion of this increase for heavy and light bodies, but determined on this one as being both beautiful in itself — on

account of the order of the square numbers, which the increments of circles also keep when their diameters grow equally — and suited to the end intended, namely that these bodies should reach their place as soon as possible, yet within certain limits of speed; and that GOD willed to leave traces of his providence in these effects, so that, not finding in nature itself a sufficient cause of them, we should necessarily acknowledge the first cause: such a person would neither be bringing a new mode into philosophy nor summoning God from the machine without any necessity. Certainly he would philosophize much less absurdly than one who had recourse to the motion of the Earth, since that motion is unfit to give the cause of this increase and brings with it many other tangles of spiral motions, as we showed in chapter 17.

Hactenus De Argumentis Pro motu Terræ Excogitatis. Sequuntur Argumenta pro Terræ Immobilitate.

So far, the arguments devised for the motion of the Earth. There follow the arguments for the immobility of the Earth.

CAPVT XIX.

Chapter 19.

Proponuntur Quinque Argumenta ex Incremento velocitatis Grauium ac Leuium contra Terræ motum Diurnum, aut Diurnum ſimul & Annuum.

Five arguments are put forward, from the increase of speed of heavy and light bodies, against the diurnal motion of the Earth, or the diurnal and annual together.

I. It was with the greatest deliberation, and on purpose, that in chapter 16 we set out beforehand so many experiments on the natural descent of heavy bodies and ascent of light ones: we wished to lay down in advance firmer foundations for the arguments against the motion of the Earth that we shall presently bring forward. These arguments will surely have as much physical evidence for those who are willing to try the same experiments, or as much credibility for those who trust our experiments, as the experiments themselves have evidence. But first one axiom, or quasi-axiom, must be set down, which no physicist will deny. It is as follows.

Axiom. The increase of speed of the same heavy body descending naturally from the same height through the same medium, or of the same light body ascending naturally to the same height through the same medium, is such and so great at any place on the terrestrial globe as it is at another place on the terrestrial globe, if all its intrinsic conditions are the same in both.

[Margin: Exposition of the axiom.] To explain the axiom, take as an example the clay ball of 8 ounces which, as was said in chapter 16, number 12, we have often used. By repeated and most certain experiments we found that it descended from a height of 240 feet in a time of 4^s (seconds of time), but in such a way that at the end of the first second it had already passed through 15 feet, at the end of the second 60 feet, at the end of the third 135 feet, and at

the end of the fourth 240 feet — and this in our parallel of Bologna. Galileo testifies that he found results like these, in Dia- logue 2 on the System of the World, at some other Italian parallel of the Equator, I do not know which. I say, then, that if this same ball, keeping all its intrinsic conditions — its gravity, shape, bulk and so on — were let fall from the top of a tower standing at any other place on the Earth's surface, whether at the Equator, or at the poles of the terrestrial globe, or at any other parallel; if the height from which it was let fall were 240 Roman feet, such feet as made up the height we used; and if the medium were the same, namely air of the same rarity and calm: then the increase of speed would be such and so great as we found it in the parallel of Bologna. *Such*, I say — that is, really unequal, or with a real and sensible increase of impetus and speed, if in our parallel it went with a real inequality, or a real and sensible increase of speed. *So great*, moreover — that is, following the order of the square numbers beginning from unity in the spaces completed after equal times — if it has been found by certain experiments to be so great in our parallel.

[Margin: The reason for the axiom.] The *reason* for the axiom is this. Since everything that can concur in determining the increase of speed is the same in both places, and no miracle is supposed to occur (for we are speaking of natural descent), no cause can be soundly assigned why that increase should be real in one place and only apparent, not real, in another, or should follow the proportion already mentioned in one place and not in another. Otherwise no one could gather for himself, from an induction made in some places about the motion of natural bodies or its other sensible effects, first principles that are universal and true in every place — such as this one: every natural body is mobile; every fire is naturally combustive; and so with very many that are commonly held as axioms in physics. This being laid down, now to the arguments.

I. Argumentum à Reali Velocitatis Incremento Grauium contra Motum Diurnum Terræ.

First argument, from the real increase of speed of heavy bodies, against the diurnal motion of the Earth.

II. *Many heavy bodies let fall through air lying in the plane of the Equator would descend to the earth with a real and notable increase of speed, and not merely an apparent one. But if the Earth moved with the diurnal motion only, about its own centre, no heavy bodies let fall through air lying in the plane of the Equator would descend to the earth with a real and notable increase of speed, but only with an apparent one. Therefore the Earth either does not move, or does not move with the diurnal motion only.*

[Margin: Proof of the major.] The MAJOR is proved as follows. Any ball let fall by us through the air in the parallel of Bologna descends to the earth with an increase of speed that is not only apparent, growing according to the order of the square numbers, as we showed in chapter 16, number 12, but also real. This was established from the more violent impact, the louder sound, the

deeper penetration of water, the greater rebound and recoil, the raising of a greater weight, and other similar effects, the higher the place from which the balls were let fall — according to what was said in that same chapter 16 from number 6 on, and, for the raising of a weight and the penetration of water, at number 23, experiments 12 and 13 — and as anyone can easily try for himself. These effects, unless one chooses to be perverse, are a natural sign of an ever greater impetus acquired in the descent, and hence of a real increase of speed following naturally from the greater impetus, so long as there is no impediment retarding its second act. They are, I say, a sign of this as of their cause, just as the same projectile, thrown by an arm that drives it more strongly and therefore making a greater wound or a greater blow on the same subject, is a physically evident sign of a greater impressed impetus — or of a greater commotion of the air, as certain haters of impetus and impressed quality say. Therefore, by the axiom set down above, the same would happen to these balls at the Equator, let fall in like manner through air of the same character; and so the increase of speed there too would be real, and not merely apparent.

III. The MINOR, in which the whole difficulty lies and which requires some mathematics, is proved by first setting out the scheme which we already expounded in chapter 17, number 7. [Margin: Exposition of the figure.] Let there be described, then, from the Earth's centre A the arcs CD and BM, the first passing through the top C of the tower BC, the second through the tower's foot B, the tower standing in the plane of the Equator. Then, the Earth's semidiameter AC being bisected at E, let another arc be described from C towards the same eastern quarter M, which is either the semicircle CIA or at least so great a portion of it that straight lines drawn from A to the arc CD fall upon it, or even cut it. Since the tower BC is supposed to move towards DM by the motion of the diurnal rotation alone, and uniformly — that is, describing with its top and its foot equal arcs in the same circumference in equal times — let the arc CD be divided into any number of equal arcs corresponding to the equal times already mentioned, say into the arcs CF, FG, GH, HL, LD; and to the points of division let the straight lines AF, AG, AH, AL, AD be drawn from the Earth's centre A. Finally, from E, the centre of the smaller circumference, let a straight line be drawn to some point where the circumference CIA intersects one of the straight lines already mentioned, say to the point I.

Diagram. A woodcut set in the right column beside the construction. From the Earth's centre A (foot) the vertical AC rises to C (top right), with E on it about halfway; B stands just below C at the tower's foot. Two concentric arcs run leftward from the top: the outer arc from C through F, G, H, L to D, and the inner arc from B through O, P, Q, R towards M. From C a second curve runs down through S, T, V, X to I, continuing as the semicircle CIA that bulges to the left and returns to A. Straight lines fan from A up to F, G, H, L and D on the outer arc, and a straight line joins E to I.

- C (top right): top of the tower, end of the outer arc

- F, G, H, L, D (outer arc, right to left): the points of division of the arc CD
- S, T, V, X (on the curve from C): the points cut on the small circle by AF, AG, AH, AL
- B (right, below C): foot of the tower
- O, P, Q, R (inner arc, right to left): points on the arc BM
- I (left, where the curve meets the inner arc): the point to which the line from E is drawn
- M (far left): the eastern end of the arc BM
- E (on AC): centre of the smaller circumference
- A (foot): the Earth's centre

[Margin: 1st proposition, for the proof of the minor.] These things marked out, *first* I say that if the Earth moved with the diurnal motion alone, a heavy body let fall from the top C of a tower standing in the plane of the Equator would describe by its natural motion a portion of the line CTI which would be, to all sense, circular, or a portion of a circular circumference. I show it thus. A clay ball let fall by us from the top of the tower C descends to the earth in 4^{s} (seconds of time), and in such a way that at the end of the first second it has passed through 15 ancient Roman feet, at the end of the second 60 feet, at the end of the third 135 feet, at the end of the fourth 240 feet. Therefore, by the axiom set down above, it would do the same at the Equator; and, the same proportion of the square numbers being kept, it would describe in that time a line circular to the utmost fineness of sense. For the spaces completed in the four seconds, namely FS, GT, HV and LX, would be as great as they must be if the line CSTVX drawn through their ends is to be circular; and conversely, if the circular line CSTVX is drawn from the centre E and the size of the spaces FS, GT, HV, LX is investigated, FS is found as 1, GT as 4, HV as 9 and LX as 16, which is the proportion of the squares beginning from unity; or, if the first space FS is taken as 15 feet, GT will be found to be 60 feet, HV 135 feet and LX 240 feet.

The spaces already mentioned are investigated by the method already given in chapter 17, number 14, which is as follows. In one second of time a point on the terrestrial Equator traverses, by force of the diurnal rotation, the arc CF of 15″, and the arcs FG, GH, HL are each equal to it; but the arc CS, since it is described from E, which bisects AC, and is intercepted by the same lines AC and AF, is of twice as many seconds, as we showed in chapter 17, number 8. The arc CS will therefore be of 30″. And since the arc CG is of 30″, the arc CT will for the same reason be of 60″, or one minute; and since the arc CH is of 45″, the arc CV will be of 90″, that is, of 1′ 30″; and finally, since the arc CL is of one minute, CX will be of 2′. The complements will therefore be: AIS, 179° 59′ 30″; AIT, 179° 59′ 0″; AIV, 179° 58′ 30″; and AIX, 179° 58′ 0″. The chords of these, taken in order from the doubled sines, will be: AS, 19 999 999 947 parts, of which the diameter or greatest chord is 20 000 000 000; the chord AT, 19 999 999 788 such parts; the chord AV, 19 999 999 524 parts; and the chord AX,

19 999 999 154. If these chords are subtracted from the greatest chord AC, to which the radii AF, AG, AH, AL are equal, there remain the spaces FS of 53 parts, GT of 212, HV of 476, LX of 846. But the proportion between these is exactly the same as between the squares 1, 4, 9, 16. For if one makes: as the first space FS, 1, is to 4, so is 53 to another, GT will come out at 212 parts. Again, if one makes: as FS, 1, is to 9, so is 53 to another, HV will come out at 476 parts; and finally, if one makes: as FS, 1, is to 16, so is 53 to another, LX will come out at 846 parts. Therefore, if a curved line is described continuously through the ends of the lines completed by that ball, keeping the proportion of the squares in the spaces, that line will be circular; and conversely, if it is circular, those spaces or segments intercepted between that circular line and the arc CD will have the proportion already mentioned. Since, then, that heavy body travels by a single line and path, and by no other than the one that the ends of the completed spaces describe, that line will be circular. The same will happen if the time of descent does not exceed 4^{m} (minutes of time), and so if the whole arc CL does not exceed one degree, as will be plain to anyone who examines the chords and the complements of the chords to the greatest chord. We have chosen, moreover, to set the numbers already mentioned before the eyes in the synopsis of a single small table; for it, look at the preceding figure, which is also placed on page 399.

Arcs	Arcs	Arcs, complements	Chords, of which AC is 20 000 000 000	Residual spaces	Proportion of the spaces
CF 0′ 15″	CS 0′ 30″	AIS 179° 59′ 30″	19999999947	FS 53	1
CG 0′ 30″	CT 1′ 0″	AIT 179° 59′ 0″	19999999788	GT 212	4
CH 0′ 45″	CV 1′ 30″	AIV 179° 58′ 30″	19999999524	HV 476	9
CL 1′ 0″	CX 2′ 0″	AIX 179° 58′ 0″	19999999154	LX 846	16

[Margin: 2nd proposition, for the proof of the minor.] IV. *Second*, I say that the arcs CS, ST, TV, VX are each equal to the arcs CF, FG, GH, HL, and so are equal among themselves, just as the latter are equal among themselves and correspond to equal times. For the circumference CD has as its semidiameter the straight line AC, which is twice as great as AE, the semidiameter of the smaller circumference CI, since by construction its centre E bisects AC. Now by theorem 5 of Book 11 of the *Mathematical Collections* of *Pappus of Alexandria*, the circumferences of circles are to one another as their diameters, or as their semidiameters. Therefore, just as a quadrant whose semidiameter is AC equals two quadrants of a circumference having as semidiameter AE, which is half of AC, so the arc CF, which is of 15″, is equal to the arc CS, which is of 30″; and for the same reason the arc FG is equal to the arc ST, the arc GH to the arc TV, and the arc HL to the arc VX, in true and absolute length, because the former are each of 15″ and the latter of 30″. That these arcs are of 30″, and

of twice as many units of arc as the arcs CF, FG, GH, HL, has already been shown in chapter 17, number 8; and it is easily proved by drawing straight lines from E to the points S, T, V, X, as EI was then drawn to the point I. For by the 20th of the third book of the *Elements*, the angles at the centre E turn out double the angles formed at the circumference at A by the same lines; but the angles at A are at the centre of the circumference CD and measure it, just as the angles at E measure the arcs of the circumference CI. Hence, if the angles at E, made by the same straight lines, are double the angles at A — the angle BEI, for instance, is double the angle BAI, that is CAD — the arc CI too is of twice as many units of arc as the arc CD, and so with the rest. These things having been shown first, the minor is now to be proved.

[Margin: Proof of the minor.] The MINOR, then, is proved. For the ball spoken of in the proof of the major would describe at the Equator, by its descent, a line circular to the utmost fineness of sense, as we showed in number 3; and this granted, it would pass through equal arcs in equal times, as was shown here in number 4. Therefore it would not descend with a real inequality, or with a real increase of speed — if, that is, the Earth moved with the diurnal motion alone. This is what was to be demonstrated in the minor. The MAJOR having thus been demonstrated physically and the MINOR physico-mathematically, it follows that either the Earth does not move at all, or it does not move with the diurnal motion only.

[Margin: 1st reply of the Copernicans, but a trifling one.] V. The Copernicans might perhaps reply that the impetus by force of which a heavy body descending from a higher place makes a greater impact does indeed grow continually insofar as it comes from the principle moving it downward, namely from gravity; but that it does not grow insofar as it comes from the principle moving it forward towards the east, being instead equal, or produced equally and uniformly, as the uniformity of the diurnal motion requires, to which the heavy body must be attuned in its forward motion; and so from the greater impact one cannot argue a real increase of impetus with respect to that other effect, namely a greater speed in the real descent along the circular line already mentioned. But this reply, though necessarily devised from the nature of the Copernican hypothesis that is to be defended, is nonetheless, taken absolutely, trifling. For it is sufficiently established by induction that a mobile body, by force of an impetus coming either from within or from without, is moved, by priority of nature, before it strikes another body towards which it is moved, and that impact necessarily presupposes the local motion of the striker; and that if a greater impact arises as the impetus grows, a greater or swifter motion also precedes it by some priority, so that the same impetus, of itself, naturally and with impediments removed, effects these two effects, namely motion and impact; and that just as there cannot be impact without previous motion, so there cannot be greater impact without greater motion arising from the increase of impetus. Indeed, a greater impetus is ordered of itself and primarily to swifter motion, and secondarily, or as it were incidentally, to the impact

on a body that impedes further motion. Grant, then, that the impetus of a heavy body descending along a circular line would not grow by force of the common principle moving it forward towards the east: still, it would have to grow by force of gravity, and by its increase cause a swifter motion along the path by which it really descends before it caused a greater and greater impact. Nor can any rational physical cause be assigned that would allow this impetus to grow insofar as it is percussive while it tends downward, and yet would impede its increase insofar as it is really motive downward, and so impede the increase of speed. And conversely, if anything can physically impede the increase of impetus as motive, by inhibiting it, or by tempering it and holding it in perpetual uniformity, by that very fact it would also impede the secondary effect that depends on the primary by physical necessity, namely the greater impact. Since, then, the impact is manifestly greater the higher the point from which the heavy body falls, it is also manifest and physically evident that a greater and greater speed preceded, not only apparently but really; but this could not precede if the uniformity of the Earth's diurnal motion is supposed; therefore the supposition of this motion is false and absurd.

[Margin: 2nd reply for the Copernicans.] Secondly, someone might perhaps reply that the motion of descending heavy bodies, insofar as it is forward on account of imitating the diurnal and the annual terrestrial motion, is not uniform, because it does not take place at the same distance from the centre of the Earth and of the annual orb, but at a smaller and smaller distance; and so it completes arcs of circles that are indeed similar to one another in equal times, but smaller, that is, of fewer paces or feet. Hence, by force of the principle that of itself moves circularly, the motion of descending heavy bodies is retarded insofar as it is motion, granted that it is not diminished insofar as it is percussive. [Margin: But useless.] But this reply is of no help either, for the dilemma already stated always holds. Either it retards the real motion so as to reduce it to perpetual uniformity — and then, by the necessity by which impact follows the vehemence of the motion it comes from, the impact would be rendered uniform from whatever height it took place — or it does not reduce it, but still leaves it, if not in its former inequality, at least in a very notable one; and then the motion too, though not with so great an inequality as otherwise, would in fact have a notable one. In reality, however, it would not notably diminish the inequality of heavy bodies, because in the figure to be set out at number 9 the semidiameter of the Earth's orb CF is 25 870 000 Roman feet, nor much greater or smaller on the Copernican reckoning; while the semidiameter of the annual orb, according to Copernicus, is 29 543 740 000 Roman feet, from what is to be said in the table at the end of number 13. Hence the whole of AF is 29 569 610 000. [Margin: See the figure to be set below at number 9.] Now the Asinelli tower, from whose height we let heavy bodies fall, namely FH, is 240 feet; therefore the whole of AH is 29 569 610 240. Now in one second of time, by force of the circular diurnal motion, 15″ of the Equator are passed through, to which 1875 ¼ feet correspond; by force of the annual motion, 2‴

28′′′′ are made (as many, that is, as the mean motion of the Sun would make), which is, according to Copetnico [?], 5965 feet, as will be clear shortly from the same table. Therefore the motion compounded of the diurnal and the annual, corresponding to one second of time, is 7840 ¼ feet; by this, namely, a point on the surface of the terrestrial Equator, say the point F, is carried in one second of time to X. Now let it be done: as the semidiameter AF, of 29 569 610 000 feet, is to FX, of 7840 ¼ feet, so is the semidiameter AH, of 29 569 610 240 feet, to another; and the arc HE will be found to be 7840 and 7 394 284 160/29 569 610 000 feet, which is insensibly greater than the arc FX. Much less sensibly, then, do the arcs HE and FX differ from each other; and yet the inequality of the motion of heavy bodies in their descent from the tower already mentioned is as great as that between these numbers, 15, 60, 135, 240, from what was said in chapter 16, number 12. Therefore, since the diurnal motion together with the annual is retarded, on account of the variation of the circles, by not even a quarter of a foot, the whole of that inequality which comes from the principle moving downward is left intact as far as whole feet are concerned, and still goes according to the numbers 15, 60, 135, 240. From this one might rather argue that, if the path along which the heavy body actually descends is rendered equal by the principle moving it forward circularly, retarding the principle that of itself moves it downward unequally, then this circular principle must be called very unequal of itself, and must intend a motion very different from the diurnal and annual motions, which are feigned to be equal of themselves.

[Margin: 3rd reply for the Copernicans.] Thirdly, the Copernicans will reply by taking refuge in the familiar example of the ship, as in the last little boat left from the shipwreck of their hypothesis. They will say that it can happen, on the hypothesis of a standing Earth, that while a ship is carried towards the East, a ball let fall downward from the top of the ship's mast, having at the same time received the ship's impetus from the mast, really falls along a curved line to the foot of the mast in such a way that it makes a greater impact the higher the mast is; and yet its motion, really, and as it would be observed by someone standing on the shore, would always be equal. [Margin: But insufficient.] But this reply can neither be established by any experiment a posteriori nor be made persuasive by any reason a priori. For if the ball's motion were blunted in such a way that, though it would of itself have been unequal, it were rendered always equal, then the impact made from any interval whatever would also be equal. But because this experiment is not easy in practice, we ourselves will suggest to the Copernicans an easier way of testing the effects arising from the motion of one and the same body from two principles, one of which moves it along a straight line, unequally of itself, the other along a circle, and equally.

Let the figure set out at number 3 be repeated here. In it let AC be a beam hollowed out lengthwise in the manner of a channel, or troughed, and floating on still water in such a way that the end A is held immobile, as though at the centre of the motion, while the other end, C, can be carried round with an equal and uniform motion from C through G to D. Let there be, besides, some

ball at C, connected by a small cord to a large weight hanging from A under the water, so that it can be drawn along the trough of the beam; and let the depth of the pond be so great that the weight does not reach the bottom of the water when the ball has come from C to A — that is, let the depth of the water be greater than the length of the beam AC. First, then, with the beam at rest, let the weight be released downward from A, and let the time in which the ball moves from C to A be measured with the pendulum, which we have often mentioned in measuring seconds of time; at the same time let the impact the ball makes on some body placed at A be carefully observed; and let the same be done when the ball comes again from E to A, which is a path shorter by half. Then, the ball having been brought back again to C, at the same instant at which the weight is released from A and the ball begins to move from C, let the end C of the beam begin to be carried round equally towards D; and let the time in which the ball C reaches A be measured, and the impact noted. For it will appear that the impact is greater when the ball comes from C than when it comes from E while the beam was at rest, and that the time from E to A exceeds half the time spent from C to A. When the beam is carried round equally, however, an inequality will indeed be observed in the impacts and times already mentioned, but not so great a one, because the transverse motion blunts the motion along the straight line CA that would otherwise follow of itself — and the more so, the swifter the transverse motion of the beam. But if the speed of the circular motion is precisely so great that the motion of the ball released both from C and from E turns out really equal, at least to sense — whether its real path runs along the circle CSTI or not — then the impact too will be as great, to sense, for the ball released from C as for the one released from E, and the time spent from E to A will be half the time spent from C to A. All this will be brought about still more easily if the beam AC is so inclined to the plane of the horizon that the ball runs freely down to A, the immobile end, while meanwhile the other end, C, raised up, is carried along a circle parallel to the horizon. And from this the Copernicans will learn that an impetus which of itself would grow unequally cannot be restrained to equality without the inequality of the impact being impeded too. Finally, it is manifest from other, more certain experiments that the same heavy body, if let fall straight down, moves more swiftly than if let fall obliquely — as in the experiments of Class II adduced above in chapter 16, number 22, compared with others in which the same ball was let fall perpendicularly downward — and in rivers, one branch of which flows down through a straight channel and the other through meanders and a less steep channel.

Applicatio eiusdem Argumenti ad plurima Corpora Grauia.

Application of the same argument to very many heavy bodies.

VI. We have used the example of the clay ball of 8 ounces because we employed that ball in particular most frequently, it being more easily obtained in view of the number of such balls needed for repeating the experiments rather

often, since for each of them a ball of its own was required, to be substituted in place of the previous one once that was broken or bruised. Yet although we have used it, the argument made holds nonetheless for the greatest part of those heavy bodies which are so much heavier than our air that they can overcome its fluctuations, made by an ordinary breeze, without disturbance of the proportion already mentioned. For such bodies are either heavier in species than the clay ball or not much lighter, so that they descend from a height of 240 feet either in a shorter time than 4^{s} or in one not much longer. Nor would you find any that requires one whole minute of time to measure out so small a height by natural descent. For a leather playing-ball and an orange are not fit to overcome the customary commotion of our air without fluctuating somewhat in their descent, as we have found more than once, and I say the same of certain wooden balls of a lighter kind; and yet these very balls reached the ground from a height of 280 feet, namely from the crown of the Asinelli tower, in a shorter time than 8^{s}. Therefore all bodies (and they are surely very many) that are fit to overcome the fluctuation of the air without disturbance of the proportion due of itself to the increment of speed descend from a height of 240 feet in a shorter time than 8^{s}. And yet, even if they required a whole minute of time for it — the time in which a point of the Earth's surface, by force of the diurnal turning, passes through 15′ of the Equator or of a parallel of the Equator — the argument would still hold. For within the first minute of the first degree of this motion, the line described by the heavy body on account of the diurnal motion alone is circular to the utmost fineness of sense, from which it would follow that its motion is in fact equal, as we showed in number 4. That the line is circular will be plain to anyone who investigates the chords, and from them the spaces that are the complements of the chords, in the way we taught in chapter 17, number 14, and in this chapter at number 3. So that this may be more evident, we shall investigate it at this place. Let the whole time of descent of the ball, then, est [?] a height of 240 feet, be 60^{s}, which make one minute. This minute, divided accordingly into four equal times, namely into periods of 15^{s} of time each, would require for each of them 3′ 45″ of the Equator; but for the sake of abundance and ease let there be 4′ in each of the arcs CF, FG, GH, HL of the figure set out earlier, for there will be twice as many minu- tes, namely 8′, in the arcs CS, ST, TV, VX; hence the arc CT will be of 16′, CV of 24′ and CX of 32′. Their complements, then, will be AIS, 179° 52′; AIT, 179° 44′; AIV, 179° 36′; and AIX, 179° 28′. From their halves the right sines will be found; these, doubled, will give the chords of the arcs already mentioned, and the chords, subtracted from the greatest chord AC, will leave the spaces passed through at the end of the equal times, in parts of which AC is taken, as above, as 20 000 000 000 — as can be seen in the following small table.

Arcs: arc	Arcs: ′	Arcs: arc	Arcs: value	Complements to the semicircle: arc	Complements to the semicircle: value	Halves of the complements
CF	8′	CS	0° 16′	AIS	179° 52′	89° 56′

Arcs: arc	Arcs: ′	Arcs: arc	Arcs: value	Complements to the semicircle: arc	Complements to the semicircle: value	Halves of the complements
CG	16′	CF [?]	0° 32′	AIT	179° 44′	89° 52′
CH	24′	CV	0° 48′	AIV	179° 36′	89° 48′
CL	32′	CX	1° 4′	AIX	179° 28′	89° 44′

Sines of the halves	Chords: chord	Chords: value	Segments or spaces: segment	Segments or spaces: value	Proportion of the spaces
9999993231	AS	19999986462	FS	13538	1
9999972923	AT	19999945846	GT	54154	4
9999939076	AV	19999878152	HV	121848	9
9999891692	AX	19999783384	LX	216616	16

Now let it be done: as the first space FS, 1, is to the second, GT, 4, so 13,538 to another, and you will find 54,156. Again, as FS, 1, is to HV, 9, so 13,538 to another, and you will find 121,842. Finally, as FS, 1, is to LX, 16, so 13,538 to another, and you will find 216,608. The spaces found from the chords subtracted from the greatest chord, then, fall short of those found by the proportion due to the square numbers by 2 parts in GT, but exceed them by 6 parts in HV and by 8 parts in LX — parts of which the whole AC, that is, the Earth's semidiameter, is 20 000 000 000. Further, the Earth's semidiameter, as we showed in Book 2, chapter 7, contains 4139 modern Italian and Bolognese miles, that is, 4 139 000 paces, or 20 695 000 feet, or 248 340 000 inches. Let it be done, then: as 20 000 000 000 is to 8 parts, so 248 340 000 inches to another; and you will find not one whole inch, or a finger's breadth, but this fraction of a single inch, 198,632/2000000, that is, nearly a tenth part of an inch. Hence LX would differ at most by one tenth part of a finger taken in breadth from the space it ought to have if the line CSTVX were geometrically circular, and HV and GT by much less. Therefore, if a heavy body did not require a whole minute of time for its descent from a height of 240 feet, it would still describe in its descent a line circular to sense at the Equator, according to what was said in number 3, and would consequently move in fact equally, according to what was said in number 4. The argument already mentioned therefore holds for all bodies so heavy that they do not require a time longer than one minute of time for natural descent through a height of 240 feet. Of this kind is the greatest part of heavy bodies, namely those suited to descending through our air without sensible fluctuation, or without such fluctuation as would move them off a straight line perpendicular to sense.

Applicatio Argumenti eiusdem ad Grauia extra Aequatorem posita.

Application of the same argument to heavy bodies placed outside the Equator.

VII. We have used the argument made so far supposing that the heavy body descends in the plane of the Equator, because the demonstration of the real

equality of motion, which would necessarily follow from the circular motion, has greater evidence if the line of motion lies in one and the same plane, as the nature of a perfectly circular line requires. Yet it has its own physical evidence, sufficient for inferring the same equality, in places outside the Equator too — and the more truly, the less those places are distant from the Equator. For outside the Equator heavy bodies, in descending, do describe (once the diurnal motion of the Earth is admitted) the curved line of their motion on the surface of a right cone that has its vertex at the centre of the Earth and its axis coinciding with the axis of the Equator, and that motion begins from the periphery of the conical base, as we showed in chapter 17, number 18. Even so, within the time of 4^{s} of time in which, in the preceding argument, we suppose the ball to have descended from a height of 240 feet, the revolution of that cone is so small with respect to its transverse magnitude, or the thickness of the cone taken near its base, that to sense it is just as if the motion took place on a plane surface; nor can that slight deflection from one plane to another be so great as to produce a real inequality of motion that is sensible, and as great as the impact proves, together with the other similar effects that indicate a real increase of impetus. But lest anyone think this was said out of a certain careless complacency, let us undertake the demonstration of what has been said — and indeed one adapted to a parallel declining 45 degrees from the Equator, such as is very nearly the one in which we made our experiments and Galileo made his, so that from this, as from a middle case, judgement may be made of the others.

In the following diagram let there be the cone ABC, whose vertex is at the Earth's centre B, and let BM be the axis coinciding with the axis of the Equator. Let the base of the cone be the circle AECD, turned towards the viewer of this page in an upright position for ease of demonstration, although by the laws of perspective it ought to be represented in oblique view, as an ellipse. Let the parallel of the Equator in which the foot of the tower stands be FHG, and the tower itself GC. From its top, or a window, C, let a ball be let fall and descend to the point H of the Earth's surface along the curved line LH in a time of 4^{s} of time, in which the point C, by force of the diurnal turning, completes the arc CK, which is of one minute, since it lies in a circle parallel to the Equator; for the arc of the Equator corresponding to 4^{s} of time is that great.

Diagram. A circle (the base AECD of the cone) drawn upright, with D at its top, A at its left and C at its right, and its centre M; from A and C two straight lines converge downward to the vertex B at the Earth's centre. A vertical line runs from D through M to E at the foot of the circle. A line from M rises to the upper right, with P marked on it; lines run from M towards C. O and K stand near the lower right of the circle, with L on the circle's lower right arc. Below the circle a horizontal line F–H–G crosses the cone; a dotted line drops from K to H, and straight lines run from C and from L down to H.

- D (top of circle): D
- P (upper right, inside circle): P
- M (centre of circle): M

- A (left end of circle): A
- C (right end of circle): C
- O (inside circle, lower right): O
- K (inside circle, below O): K
- L (on lower right arc): L
- E (foot of circle on the vertical): E
- F (left end of horizontal line): F
- H (middle of horizontal line): H
- G (right end of horizontal line): G
- B (vertex at foot): B

Its half, namely the arc CL, is of 30″. The second sine [Sinus Secundus] of this arc, PC, equal to the straight line MO, is 9 999 999 894 of the parts of which the radius ML is 10 000 000 000. If you subtract the sine PC from this, there remains the versed sine, or sagitta, OL, of 106 such parts. But so that the quantity of the arc CLK, and of the line OL by which the arc CLK departs from the straight line COK, may be known in parts taken determinately, it must be recalled that the Earth's semidiameter contains 20 695 000 Bolognese feet and its diameter 41 390 000, as I said in number 6, towards the end. The result is that the circumference of a great circle is 129 964 600 feet, for from what was said in Book 1, chapter 4, the diameter is to the circumference as 100 to 314; and therefore one minute of the Equator contains 6020 feet. But because we suppose the circle AECD to decline 45° from the Equator, the arc CK, by the problem of Io. Antonius Maginus [Giovanni Antonio Magini] and Iosephus Moletius [Giuseppe Moleti] on Book 1, chapter 20, of Ptolemy's *Geography*, and the ta- ble given there in Moletius, will be 42″ 25‴ of the Equator. Now the periphery AECD is to the periphery of the terrestrial Equator as 42″ 25‴ is to 60″; and since circumferences are to one another as their diameters, or semidiameters, and conversely, therefore as 60″ is to 42″ 25‴, so is the Earth's semidiameter to ML, the semidiameter of the parallel. Let it be done, then: as 1 minute, or 60″, is to 42″ 25‴, so 6020 feet to another; and the arc CLK will come out at 4219 feet. Again, let it be done: as 60″ is to 42″ 25‴, so the 20 695 000 feet of the terrestrial semidiameter to the feet of the semidiameter ML, 14 441 500. Finally, if one makes it: as the radius ML, of 10 000 000 000 parts, is to the versed sine OL, of 106 parts, so 14 441 500 feet to another, the sagitta OL will be found to be 0 153/1000 feet, that is, about a 6½th part of a foot. Hence the arc CLK, which is 4219 feet, does not deflect from the straight and plane line COK, where it deflects most, by more than two inches of a foot; and these, in an arc of 4219 feet, or 844 paces, or in the circular periphery of nearly one mile, are not a sensible portion. Therefore, if a heavy body descending from C went along the arc CLK in a time of 4^{s} of time, it would not decline from the plane surface by more than 2 inches of a foot, or 3 fingers' breadth. But it goes along LH, which departs less from the plane surface and comes closer to the straight line CLG lying in the same plane; it therefore deflects still less from the plane surface, although in geometric rigour it goes along a conical surface.

Nor does this conflict with what was said in chapter 17, number 18. For there we argued against Galileo, who pronounced universally that the line of motion of heavy bodies is circular, without restricting his proposition to a heavy body that completes its descent in 4 seconds, and who, besides the diurnal motion, supposed the annual motion of the Earth and spoke of the whole semicircle continued down to the centre of the Earth.

II. Argumentum contra motum Terræ Diurnum simul & Annuum, à Reali Incremento velocitatis Grauium desumptum.

II. An argument against the diurnal and annual motion of the Earth taken together, drawn from the real increase of the speed of heavy bodies.

VIII. I see that the argument made so far tells only against the Semi-Copernicans, who attribute only the diurnal turning to the Earth, and whom I reviewed in chapter 2. But the Copernicans could ascribe the real inequality of the descent of heavy bodies to the annual motion of the Earth combined with the diurnal — granted that Galileo, who asserts both motions, denies this inequality, as we said in chapter 17, number 8. Our weapons must therefore be aimed against both motions together, namely by the following argument.

Many heavy bodies, let fall through the air that lies in the plane of the Equator, would descend to the earth with a real and notable increase of speed, growing equally, at whatever time of day they were let fall. But if the Earth moved with the diurnal and the annual motion together, some of those many heavy bodies, let fall through the air that lies in the plane of the Equator, would not descend with a real and notable increase of speed, growing equally, when let fall at any time of day whatever. Therefore the Earth does not move with the diurnal and the annual motion together.

[Margin: Proof of the major.] The MAJOR was proved in number 2. For here nothing else is added to the major of that argument except that the increment of speed grows in the same way at whatever time of day the heavy body is let fall downward. This is sufficiently established from our experiments, especially those with clay balls, made in the morning, in the evening and around midday; for we always obtained the same proportion of the square numbers, and the same impact, the same raising of a weight and the same penetration into the depth of water, with the same ball let fall from the same height. And the same will be established for anyone who does not neglect these experiments.

[Margin: 1st part of the minor is proved.] IX. The MINOR is proved as regards the qualifiers *real* and *notable* increase. For by force of the precisely diurnal motion, the descent of the clay ball of 8 ounces through 240 Roman feet in a time of 4^s of time would be really equal, granted that it is apparently unequal, and the increment of speed would turn out to be in fact none, or negligible, as was shown from number 2 to number 5; and this demonstration was applied in number 6 to very many heavy bodies. By force of the annual motion too, the motion of that ball would be equal, without any difference worth attending to, because the annual motion of the Earth's centre within 4^s of time is entirely equal, and in each second makes $2''' 27'''' 51^v$, just as the Sun

does on the hypothesis of a moving Sun; [Margin: The annual motion of the Earth, in a time of 4 seconds, would be entirely equal to sense.] and the top of the tower from which the ball is let fall describes in this time a line differing insensibly from a circular one, while the ball already mentioned meanwhile describes arcs equal to the arcs described by the top of the tower, without any notable difference, or any worth attending to, as we shall show shortly.

[Margin: Explanation of the figure.] In the following diagram, describe from A, the centre of the annual orb, the arc BC, on which let the centre of the Earth, carried from C to B, the eastern point, be at C at the beginning of the ball's descent; at the end of the first second of time at P; at the end of the second at D; at the end of the third at Q; at the end of the fourth at B. And on the straight line AH, produced from A, let there be the tower, or height, HF, of 240 feet, so that its top H, at the interval AH, describes in a time of 4^{s} of time the arc HG, similar to the arc CB — such an arc as it would describe by force of the annual motion of the Earth alone.

Diagram. An upper arc running from K at the left through G, M, L and E to H at the right; below it a second arc through B, Q, D, P and C, along which five overlapping circles are strung. Straight lines rise from the vertex A at the foot of the figure through B, Q, D, P and C to the upper arc, with V marked on the right-hand line. A curve through marked points runs from H down through R, S and T towards O near K; I and O stand at the left near K, and F at the upper right below H. Below T, between the first two circles, a sort is printed turned on its side; it reads as Z or N according to the direction of turning.

- K (upper left end of the upper arc): K
- G (upper arc): G
- M (upper arc): M
- L (upper arc): L
- E (upper arc): E
- H (upper right end of the upper arc): H
- S (below L): S
- R (below E): R
- T (below M): T
- Y (below S): Y
- X (below R): X
- I (left, near K): I
- O (beside I): O
- Z [?] (turned sort below T): Z or N [?]
- F (upper right, below H): F
- B (left end of the lower arc): B
- Q (lower arc): Q
- D (lower arc): D
- P (lower arc): P
- C (right end of the lower arc): C

- V (on the right-hand line): V
- A (vertex at the foot): A

If this happened, the ball let fall from H would appear at R at the end of the first second of time, having completed the space ER, which is as 1, and the tower would be at EX; at the end of the second second the ball would be at S, having passed through the space LS, as 4, and the tower at LY; at the end of the third the ball would be at T, having passed through the space MT, as 9, and the tower would be at MZ; finally, at the end of the fourth second the ball would be at O, having completed the space GO, of 16 parts of which ER is one; and the tower would be at GO, and the path of the ball would be HRSTO. In reality, however, in geometric and astronomical rigour, according to the hypothesis of Copernicus, the tower FH also moves in the Equator towards P with the diurnal motion, completing in a time of 4^s of time one whole minute of the Equator. Hence at the end of the fourth second it is not at GO but a little further east and more inclined, say at KI, having completed, namely, the arc IO of one minute; and therefore the top H has described not the arc HG but the arc HK. [Margin: 1st step of the demonstration.] I say, however, that it has described, to sense, or without any difference worth attending to, an arc similar, and indeed equal, to the arc HG and coinciding with it; and hence that the tower KI can be taken as standing in the position GO, and in the preceding times at MZ, LY, EX, just as if it had not been inclined at all by force of the diurnal turning. To show this, I produce the straight line KI to B, and join K to the centre A by the straight line AK. For in the triangle KAB, besides the angle ABK of 179° 59′, as being the complement of the angle KBG, which measures the arc OI of one minute passed through by the tower in a time of 4^s, the side BK is given, made up of the height of 240 Roman feet and the Earth's semidiameter BI; and BI, from what was said in Book 2, chapter 7, is 5174 ancient Roman miles, that is, 5 174 000 Roman paces, namely 25 870 000 geometric feet. Hence the whole of BK is 25 870 240 feet. Further, from what was said in Book 3, chapter 7, the mean distance of the Sun from the Earth is 7300 terrestrial semidiameters, that is, in Roma- n feet 188 851 000 000. Joining, then, BG (which is as great as BK) and AB, the whole of AG comes to 188 876 870 240 Roman feet, and each of AH, AE, AL and AM is as great. Now if, following the laws of triangles, you use a radius, or whole sine, of 12 figures, as in Pitiscus's great *Canon*, and you make: as the sum of the sides AB and BK is to their difference, so the tangent of the half-sum of the angles adjacent to the base AK (a half-sum of 30″) is to another number, you will find the tangent of the angle of 0° 0′ 29″ 56‴ 40⁗. Added to the half-sum already mentioned, that is, to 30″, this angle gives the angle AKB, 59″ 56‴ 40⁗; subtracted from the same half-sum, it leaves the angle BAK, 3‴ 20⁗.

[Margin: 2nd step.] With these angles obtained, if again by the rules of trigonometry one makes: as the sine of the angle AKB is to the given opposite side AB, so the sine of the angle ABK is to the opposite side AK, the side AK will be found to be 188 876 870 238 7/10 Roman feet. Therefore the line AK,

by which the top of the tower stands distant, at the end of the ball's descent, from A, the centre of the Great Orb, is very nearly equal to the line AG and to the others, AM, AL, AE and AH, each of which is 188 876 870 240 Roman or geometric feet. Nor does it differ from them except by 1 3/10 feet, a tiny difference entirely negligible in respect of the radius AG, which is of more than 180 000 000 000 feet. Indeed, even if AK differed from AG by the whole height of the tower IK, that is, by 240 feet (which is nevertheless impossible, since the tower has described only the arc OI, which is of one minute), that difference could still be disregarded. Therefore, if AK is equal to sense to AG, much more are the remaining straight lines drawn from A to the arc HK equal to the same straight lines produced to the arc HG. So the arc HG can be taken for the arc HK without any appreciable difference, and the arc HK as an arc perfectly circular, so far as our purpose goes; and the position of the tower at the end of the descent can be taken just as if it were at OG, or on the line AG, and much more, at the preceding times, as if it were on the lines AE, AL and AM. Further, in a time of 4 seconds of time the centre of the Earth (whose motion is as great as the mean motion of the Sun) runs through the arc CB, $9''' \; 51'''' \; 24^{v}$; and the top of the tower runs through very nearly as much from H to G; and HK, described from its own centre, is of the same number of sexagesimal parts.

[Margin: 3rd step.] Now let AH be bisected at V, and from the centre V, at the interval VH, let a circular arc HO be described through the top of the tower H. It will be of twice as many sexagesimal parts as the arc HG, namely $19''' \; 42'''' \; 48^{v}$; and any arc of it will be of twice as many sexagesimal parts as any arc of the periphery HG cut off by the same straight lines drawn from the centre A. Hence, as regards absolute length, the whole arc HG will be equal to the whole arc HO, and the arc HE to the arc HR, EL to RS, LM to ST and MG to TO. All these proportions are evident from what was demonstrated with a similar figure in number 4 of this chapter. I chose, moreover, a position of the tower opposite the Sun, in the night hemisphere, because in such a position there is a danger of a greater inequality in the increase of speed [?], as I shall say in number 12.

[Margin: 4th step.] XI. Finally I say that the ball mentioned above, let fall from H, describes in a time of 4 seconds the circular line HRSTO, to sense, equal to HG or HK; and hence, from what has just been said, that in equal times it completes arcs as great as those the point H travels over on the arc HG, that is, equal to sense. For at the end of each of the four seconds the ball appears in the perpendicular of the tower, that is, at EX, LY, MZ and GO; and, as to appearance, from what was proved in chapter 16, number 12, the first space completed by the ball, namely ER, is as 1, the second, LS, as 4, the third, MT, as 9, and the fourth, GO, as 16. And these same spaces, keeping this proportion, lie between the circular line HG and HO, if the former is described with a semidiameter twice as great as the latter (as we supposed in the construction), so long as the first arc HK does not exceed one minute, as is evident from what was demonstrated a posteriori by the tables of sines and chords in number 3.

Since, I say, these things are so, and the ball already mentioned describes only one line in its descent, it follows that it describes the circular line HRSTO, and therefore moves equally to sense, without any notable real increase of speed.

XII. But some scruple might still remain in someone's mind on account of the appendage added to the annual motion by the diurnal motion of the Earth; and from it one might suspect some notable inequality in the motion of heavy bodies, and a great one, which, although it would be imperceptible with astronomical instruments, would nevertheless in absolute terms amount to many miles, or paces, or feet; and one might suspect that from it arises that more vehement impact, and the real increase of impetus made manifest by that very impact. [Margin: The second [?] part of the minor [?] is proved [?].] We shall show that not even in this way is that inequality notable, or so great that the more vehement impact could be ascribed to it: indeed, that sometimes it will be none at all, sometimes even with a decrease, and for the most part negligible and insufficient to defend the experiments of a real increase of the impetus of heavy bodies — an increase that comes from the more vehement impact, the higher the place from which they descend. We shall use the figure we used in Book 7, section 5, chapter 3, for the stations and retrogressions of the planets. For, from Galileo himself in Dialogue 4 on the System of the World, according to Copernicus the orb of the terrestrial globe stands to the Great Orb as an epicycle to an eccentric; and the points of the Earth's surface, and the bodies that complete the diurnal turning along with it, become as it were stationary twice every day, are in a manner retrograde for almost the whole day, and as it were direct for the whole night, so far as concerns that addition of motion which the diurnal motion makes to the annual. Let there be understood, then, described from the centre E, the arc LDK of the Great Orb, carrying D, the centre of the terrestrial globe ALCK, towards the eastern quarter L; and let the point A on the periphery of the terrestrial Equator, or a terrestrial body placed there, move towards L, completing the whole turning ALCKA in 24 hours.

Diagram. A circle set in the right column, with R and A at its top, F and B on its upper left and H on its upper right. A nearly horizontal arc crosses it from L at the left through P, a second lettered point, S and D near the centre, and O, to K at the right; G stands at the lower left, V, T, I and C along the lower edge, M at the lower right. Straight lines from points on the circle converge downward in a long cone to E at its foot. The letter just right of P is a capital with a bowl and a tail that reads as R or Q; the I beside T stands hard against T and may be a stroke of the drawing.

- R (top of the circle): R
- A (top of the circle): A
- F (upper left): F
- B (upper left, below F): B
- H (upper right): H
- S (centre, left of D): S

- D (centre): D
- L (left end of the horizontal arc): L
- P (on the arc, right of L): P
- R [?] (right of P): R or Q [?]
- O (on the arc, right of centre): O
- K (right end of the horizontal arc): K
- G (lower left): G
- V (lower edge): V
- T (lower edge): T
- I [?] (against T): I [?]
- C (lowest point of the circle): C
- M (lower right): M
- E (vertex of the cone, at the foot): E

[Margin: 1st case.] First, it is evident that around the point L, at which the Sun, placed at E, rises, and around K, at which it sets, the diurnal motion neither adds anything to nor takes anything from the annual motion of the centre D. So if the descending ball, completing along with the Earth a small part of the diurnal turning within 4 seconds of time, that is, running through one minute of the Equator ALCK, is very near the morning place L or the evening place K, its motion is as equal as the annual motion itself, and is not varied at all by the diurnal.

[Margin: 2nd case.] Second, since over the whole arc KAL the motion is compounded of the annual motion of the centre D and the diurnal, which drives towards the same quarter as the centre, the diurnal adds to the mean annual motion in such a way that it adds more and more from K through H to A, and most of all at A, but less and less from A to L. Conversely, because over the whole arc LCK the diurnal motion tends towards the contrary quarter, it subtracts from the annual motion, yet in such a way that it subtracts more and more from L to C, but less from C to K. Hence it comes about that, although any terrestrial body always moves forward in cosmic space, it nevertheless really moves unequally (if we wish astronomical rigour to be kept): it is swifter than the mean motion of the centre on the arc KAL, and swiftest at A, the point of midnight, but slower on the arc LCK, and slowest around C, the point of midnight [?]; and in this way it imitates the direct motion and retrogradation of the planets, and the inequality of the prosthaphaereses that the epicycle imposes on the eccentric.

[Margin: 3rd case.] Third, from what has just been said it follows that over the whole left arc ALC the inequality of the annual motion produced by the diurnal is one of decrease, because on the arc AL the diurnal motion adds continually less and less to the annual, while from L to C it subtracts more and more. Conversely, on the right arc CKA, the inequality is one of continual increase of the mo- tion, because from C to K the mean annual motion is diminished less and less, and from K to A it is increased more and more. And so the first two parts of the proposition asserted at the beginning of number 12

stand established: namely, that sometimes the diurnal motion introduces no inequality into the annual motion, as at the points L and K; and that sometimes it introduces one in such a way that it is rather with a decrease of speed, as on the arc ALC. There remains to be proved the third part, concerning the very slight inequality that the diurnal motion adds to the annual, even if it is considered in paces or feet.

XIII. Before we show this, however, we need to know in advance how many paces or feet a point placed on the terrestrial Equator is carried forward in each second of time by force of the diurnal motion, and the centre of the Earth by force of the annual motion. Both of these depend on the paces and feet contained in the semidiameter of the Earth and in the semidiameter of the Great Orb. For when these are doubled, the diameter is had; and if one makes: as 100 is to 314, so the diameter to another number, the circumference is had. Divided by 360, this gives the paces or feet contained in one degree; these, divided by 60, give the paces or feet of one minute; and so, by continued division by sixties, the number of feet contained in one second, third or fourth becomes known. Therefore, if it is known through how many sexagesimal parts of a degree the centre of the Earth is carried forward by force of the annual motion in each second of time, and likewise a point of the terrestrial Equator by force of the diurnal motion, the number of feet passed through by force of each motion will at once be known. It must therefore be supposed, from what was said in Book 2, chapter 7, that the semidiameter of the Earth contains 5 174 000 ancient Roman paces, that is, 25 870 000 feet; and that the semidiameter of the Great Orb, from what was said about the mean distance of the Sun in Book 3, chapter 7, contains 1142 semidiameters of the Earth for Copernicus, 3381 for Kepler, and 7300 for us. Further, from the tables of the primum mobile, any point of the Equator passes through 15″ of the Equator in one second of time; and from the tables of the mean motion of the Sun, according to what was said in Book 3, chapter 17, the centre of the Earth passes through 2‴ 27⁗ 51ᵛ in each second of time. With these suppositions made, you will learn the rest from the following small tables, in which, as regards terrestrial measures, we follow only our own opinion.

Of the Earth	Ancient Roman miles	Roman paces	Roman feet
Diameter	10348	10 348 000	51 740 000
Circumference	32512	32 512 000	162 560 000
1 degree of the Equator	90 1/3	9333 1/3[46]	451 665
1 minute	1 505/1000	1505 1/2	7527 1/2
15 seconds	✠	376 1/4	1875 1/4
1 second	✠	25 1/12	125 5/12
Of the Great Orb			
Diameter — Copernicus	11 817 496	11 817 496 000	59 087 480 000
Diameter — Kepler	35 006 588	35 006 588 000	175 032 940 000

[46]so printed (the figures are confirmed); 90 1/3 miles and 451 665 feet imply 90 333 1/3 paces

Of the Earth	Ancient Roman miles	Roman paces	Roman feet
Diameter — us	75 540 400	75 540 400 000	377 702 000 000
Circumference — Copernicus	37 106 937 1/2	37 106 937 500	185 534 687 500
Circumference — Kepler	109 854 698 1/3	109 854 698 320	549 273 491 600
Circumference — us	237 196 856	237 196 856 000	1 185 984 280 000
1 degree — Copernicus	103 074 5/6	103 074 833	515 374 165
1 degree — Kepler	305 152	305 152 000	1 525 760 000
1 degree — us	658 880 56/360	658 880 020	3 294 400 100
1 minute — Copernicus	1718	1 718 000	8 590 000
1 minute — Kepler	5085 52/60	5 085 750	25 428 750
1 minute — us	10981 1/3	10 981 333	54 906 665
1 second — Copernicus	28 38/60	28 633	143 165
1 second — Kepler	84 3/4	84 766	423 830
1 second — us	171 1/3	171 333	856 665
1 third — Copernicus	✠	477 13/60	2386 1/12
1 third — Kepler	1 5/12	1412 46/60	7063 5/6
1 third — us	2 51/60	2855	14 277 3/4
2 28/60 thirds — Copernicus	1 193/1000	1193	5965
2 28/60 thirds — Kepler	3 532/1000	3532	17 660
2 28/60 thirds — us	7 138/1000	7138	35 690

We shall, however, apply these measures to modern Italian miles in another way in chapter 22, from number 1; but here we chose them so as to use the geometric foot, which is the ancient Roman foot, just as we used it in observing the descent of heavy bodies.

XIV. Taking up again now the figure of number 12, let us suppose that the plane of the terrestrial Equator (although it is in reality inclined to the plane of the Great Orb) is nevertheless in the same plane; for it will not be difficult to judge what happens to it when it is inclined to that plane. And since the clay ball mentioned above descends from a height of 240 feet in 4 seconds of time, let this time be divided into four parts, and, with the ball supposed at G at the beginning of its motion, let there be for the single seconds four equal arcs GV, VT, TI, IC, each of which, from what was said a little before, is of 15″, but of 1875 1/4 feet, from the first part of the preceding small table. Let the points of these divisions be joined to the centres D and E by the straight lines DG, GE; DV, VE; DT, TE; DI, IE. The task now is to find the four angles at E. This will not be difficult. For in the triangle EDI there are given the angle EDI, measuring the arc CI, of 15″; the semidiameter of the Earth, DI, as 1; and the semidiameter of the Great Orb, ED — for me 7300 of the parts of which DI is 1, but for Copernicus 1142. Therefore the angle DEI will not remain hidden. Similarly, in the triangle EDT there are given DT, 1, and DE, as above, and the angle EDT, of 30″. Hence the angle DET will become known; and when DEI is subtracted from it, IET will remain. In a similar way the remaining angles at E will be found, since the angles EDV, 45″, and EDG, 60″ or one minute, are known. So let one make: as the sum of the two given sides is to their difference, so the tangent of the half-sum of the angles at the base; and

the tangent of the angle to be subtracted from the half-sum already mentioned will come out, so that the angle sought at E remains. [Margin: Summary of the rule.] Or, if you are used to the shortcuts of logarithms, join into one sum the residue of the logarithm of the sum of the given sides, the logarithm of the difference of the same sides, and the mesologarithm of the half-sum of the angles at the base; for the result will be the mesologarithm of the angle to be subtracted from the half-sum already mentioned, so that the angle at E is left known. By this method we found the angles written below.

[Margin: 1st small table.]

Angles	Copernicus ″″	Copernicus ᵛ	Us ″″	Us ᵛ
CEI	59	0	56	0
IET	55	0	52	0
TEV	51	30	48	30
VEG	48	30	45	30

The arc CI, however, which is viewed from E in a direct position, is 1875 1/4 feet, as I said in the first part of the small table of number 13; and these feet the diurnal motion takes away from the annual, which for me is 28,555 1/4 feet, from the last part of the small table of number 13, when the point I runs through the arc IC. The remaining arcs, IT, TV and VG, are viewed more and more obliquely, and they subtract fewer feet from the annual motion the smaller the angles they subtend. Therefore, if one makes: as the angle 59 is to 55, so 1875 1/4 feet to another number, we shall have very nearly the feet that the diurnal motion subtracts from the annual on the arc TI; and so for the rest. Let there be, then, for these four arcs a second small table, both in the Copernican measures and in ours, between which the Keplerian lie midway.

[Margin: 2nd small table.]

Arc	Copernicus: feet	Copernicus: difference	Us: feet	Us: difference
CI	1875 1/4		1875 1/4	
IT	1748	127	1741	134
TV	1637	111	1624	117
VG	1542	95	1523	101

Therefore, if we subtract these arcs from the annual motion corresponding to one second of time, which is, as I said, 28,555 1/4 feet, we shall have the equated annual motion of the terrestrial points when they are not more than one minute distant from the point C, in which position the differences are greatest and the inequality is greatest, just as also at the point A. But, extending the trigonometric calculation from the point C up to the arc six minutes distant from the point C, I find that the differences now come out so equal that they do not differ by one whole foot, as is evident from the following small table, compiled from our measures.

[Margin: 3rd small table.]

Arc of distance from the point C	Differences of the arcs in feet	Arc of distance from the point C	Differences of the arcs in feet
0′ 15″	✠	3′ 15″	26
0′ 30″	134	3′ 30″	25
0′ 45″	117	3′ 45″	25
1′ 0″	101	4′ 0″	24
1′ 15″	87	4′ 15″	24
1′ 30″	75	4′ 30″	24
1′ 45″	64	4′ 45″	23
2′ 0″	54	5′ 0″	23
2′ 15″	45	5′ 15″	23
2′ 30″	37	5′ 30″	23
2′ 45″	31	5′ 45″	22
3′ 0″	27	6′ 0″	22

It is evident, then, as I said, that over almost the whole diurnal circle the diurnal motion of 4 seconds of time introduces into the annual no sensible or notable inequality. For, apart from the arcs of 6 minutes before or after the points A and C, the difference of the additions or subtractions does not reach one whole foot; and yet the difference of the impacts is proportioned to the difference and inequality of the apparent increase, which amounts to many feet. For in the first second the ball already mentioned completes 15 feet, in the second 60 feet, in the third 135 feet, and in the fourth 240, as has often been said — proportioned, I say, after its own kind, so that the impact from a height of 60 feet is much greater than from 15, and from 135 than from 60, and so on. Of this distinction we gave a sufficient specimen in chapter 16, number 23, in class 12 of the experiments.

Applicatio eiusdem 2. Argumenti ad Plurima corpora Grauia, etiam extra Aequatorem.

Application of the same 2nd argument to very many heavy bodies, outside the Equator as well.

XV. Argument 2, which rests on the clay ball of 8 ounces, holds for most heavy bodies of the same bulk that are heavier than it, and so descend from a height of 240 feet in a time no greater than 4 seconds of time, according to what was said in number 6. It holds, moreover, in very many, indeed in almost all, places outside the Equator. For neither does the diurnal motion, in so small a time, carry the ball sensibly round outside the same plane, as I showed in number 7; nor does the annual motion, likewise in so small a time — 4 seconds of time — transfer the same ball into planes so different that, by reason of them, the ball already mentioned could acquire as notable a real increase of speed as the impact it makes demonstrates when it is let fall from a higher place. For although in that much time the centre of the Earth runs through 28 2/5 ancient Roman miles, according to our measures set out at the end of number 13, at the foot of the last table, since in each second it runs through 7 1/10 miles,

nevertheless the plane through which the ball began to descend, following the parallelism of the Equator, declines uniformly from its former position.

III. Argumentum Contra Motum Diurnum Terræ.

III. An argument against the diurnal motion of the Earth.

XVI. *A heavy body the same in species and in number, retaining the same gravity, bulk, density, shape and other intrinsic conditions, and let fall from the same height through the same medium, descends naturally with an equal increase of real speed, over whatever part of the Earth it is let fall. But if the Earth moved with the diurnal motion only, a heavy body of this kind would not descend with an equal increase of real speed when let fall over whatever part of the Earth. Therefore the Earth either does not move, or does not move with the diurnal motion only.*

[Margin: Proof of the major.] The MAJOR is evident from the axiom set out at the beginning of the chapter, and it cannot be denied except by someone who now wishes to defend the presumed hypothesis of the Earth's motion, which is in dispute here. For he has no other reason for denying this axiom; nor has he proved from better-known things that his hypothesis is in fact so, but only that it is possible, with the astronomical experiments saved.

Diagram. Set in the right column. An upper arc runs from E at the left through M, N, F, S and a small italic a to G at the right, with H above it; a lower arc runs from A through D to C, and vertical lines join the two arcs. On the line falling from M, the small figures 1, 4 and 9 stand beside dots; O, Q and T stand beside heavy dots on the next three verticals, and Z beside a dot near C. Along the foot, five circles are strung from A to C, each lettered: the first with P above, I to the left, K at the centre, R to the right and L below; the second with P, V at the centre, R, I and L; the third with B at the centre, R, I and L; the fourth with P, I, X at the centre, R and L; the fifth with Y at the centre, I, R and L. Letters standing between two circles are assigned to the nearer one.

- E (left end of the upper arc): E
- H (above the upper arc): H
- G (right end of the upper arc): G
- M (upper arc): M
- N (upper arc): N
- F (upper arc): F
- S (upper arc): S
- a (upper arc, small italic): a
- 1 (on the vertical from M): 1
- 4 (on the vertical from M): 4
- 9 (on the vertical from M): 9
- O (beside a dot): O
- Q (beside a dot): Q
- T (beside a dot): T
- Z (beside a dot near C): Z
- D (lower arc): D

- A (left end of the lower arc): A
- C (right end of the lower arc): C
- P, I, K, R, L (first circle): P, I, K, R, L
- P, V, R, I, L (second circle): P, V, R, I, L
- B, R, I, L (third circle): B, R, I, L
- P, X, R, I, L (fourth circle): P, X, R, I, L
- Y, I, R, L (fifth circle): Y, I, R, L

[Margin: Proof of the minor.] The MINOR is proved by repeating figure 1, which we set out in chapter 17, number 16. For in it a ball let fall from the top M of a tower, standing perpendicularly above either pole of the terrestrial Equator, would in fact descend along the perpendicular line MP, and would imitate the turning of the Earth by no other motion than the turning of its parts about its own centre of gravity; but the centre itself, along with the heavy body, would descen- d straight down; for it would not be carried anywhere else by force of the annual motion, which we do not suppose in this argument. Therefore the increase of speed, and of the spaces completed in equal times, would in reality be such as it appears to those who test and observe it: namely, following the series of the square numbers beginning from unity. In the first second the space completed would be M1, as one; at the end of the second, M4, as four; at the end of the third, M9, as nine; and at the end of the fourth, MP, as 16.

By contrast, the same heavy body let fall at the Equator would indeed move unequally as to appearance by force of the diurnal motion alone, but equally as to the real space traversed in four seconds of time, as we showed in Argument 1, number 3, and also applied to the other parallels of the Equator in number 7. Therefore, if the Earth moved with the diurnal motion only, the same heavy body, its intrinsic conditions remaining the same, let fall from the same height — say of 240 feet — through air of the same rarity and stillness, would not descend with an equal increase of real speed above every place of the Earth whatever.

[Margin: A threefold reply to argument 3.] XVII. The opponents may perhaps reply, first, that air as rare cannot be found under the poles of the world as far away from that region; next, that the Earth attracts heavy bodies along a straight line, because the magnetic virtue pours itself out along the length of the terrestrial axis; and lastly, that in heavy bodies there are two virtues: one of moving themselves downward through a lighter medium, and this they are left to exercise in every place; the other of following the diurnal motion of the Earth, on the hypothesis that they have been let fall above a place of the Earth that is turned by the diurnal motion, and that this second virtue is not exercised in act from the centre of gravity of the bodies if they are let fall above the terrestrial poles, because the poles do not share in the diurnal turning. Hence, they would say, other things are not equal: on account of the first of these disparities the Minor can be denied, and on account of the other two the Major can be denied or distinguished — conceded, indeed, if heavy bodies are either

drawn extrinsically by the Earth in the same way in every place, or exert the same intrinsic motive virtue of their own in second act, but denied otherwise.

[Margin: The reply is rejected.] But as to the first: the air under the poles of the world can never naturally be so thick as to retard notably the motion of descending heavy bodies, and to impede that real increase of the spaces traversed in equal time according to the proportion that the squares of the times have. Or, if you prefer, transfer this argument to a heavy body descending through water, which is of the same density in every place, and so on. As to the magnetic force, we already rejected it in chapter 18, number 4, from partnership among the motive causes in this case, because otherwise it would snatch nearer bodies faster and more vehemently, with greater collision and impact, than more remote ones, which is contrary to manifest experiments. The third part of the reply, however, has no foundation from elsewhere or from better-known things, but only from the mere fiction of this hypothesis, namely of the diurnal motion of the Earth. It is just as if one said that outside the poles bodies move with the Earth because the Earth moves, but above the poles they do not move with the Earth because those points of the Earth do not move. That would be rightly said if it were already known from elsewhere that the Earth moves in reality, and not merely that it can move with most of the celestial phenomena saved. Therefore, to assert this motion as a fact, that other virtue would have to have been known beforehand by some physical experiment — just as in these bodies we have recognised an active virtue of tending either upward or obliquely, or at least a passive virtue of not resisting such motions if they are drawn towards the part where there would be danger of a vacuum. Physicists acknowledge that virtue not without reason, because of the common good of the universe, which prevails over the other virtue that moves towards the body's own good. But in our case, whose good does this virtue of going round and following the motion of the Earth serve? Not that of the heavy body itself, for it is better for it to arrive straight, and by the shortest possible route, at the place owed to its nature. Not that of the universe, or of another, better body, for that circuit of heavy bodies confers no benefit on anything. Nor can any other good be imagined here except the agreement of the parts in motion with their whole; [Margin: A notable objection [?] against [?] those who assert [?] the motion of the Earth.] and on account of that same agreement we likewise say that heavy bodies ought not to move in a circle, since neither does the Earth move. Since, then, the force of each argument taken a priori is equal, each for its own hypothesis, it must be discerned from elsewhere which of the two agrees in reality with experiments and reasons a posteriori; nor should a reason be thrust forward that is no better known than the reason for the contradictory hypothesis — a point very much to be noted. Here, however, we suppose that the other arguments for the motion of the Earth have already been solidly resolved, as in truth they were.

IV. Argumentum contra motum Diurnum simul & Annuum Terræ.

IV. Argument against the diurnal and annual motion of the Earth together.

XVIII. This argument is exactly the same as Argument 3, if to the Minor you add the hypothesis of the annual motion as well, and in the conclusion exclude the annual motion as well. For outside the poles of the world heavy bodies, by force of both motions, would move without a real and notable increase of speed along a curved line whose nearly equal parts would be traversed in equal times — or at least parts not so unequal as are the parts of the straight line along which they would move above the poles of the Earth, as is clear from what was said for Argument 2. For here that portion of the path which would have to be traversed outside the poles by force of the diurnal motion would cease, and only that portion would have to be traversed which is supposed to be run through by force of the annual motion; for at the poles the diurnal motion is nothing.

V. Argumentum contra motum Terræ Diurnum simul & Annuum, à motu Leuium desumptum.

V. Argument against the diurnal and annual motion of the Earth together, taken from the motion of light bodies.

XIX. All the arguments made so far hold in their own way for light bodies ascending upward by natural motion, perpendicularly as to appearance; for they too acquire a real increase of impetus, as has already been proved by our own experiments in chapter 16. But the argument would be more evident if we had some body so light that it did not share the nature of earth and water — as glass spheres full of air do share it, and artificial fires, and fiery rockets thrown upward, and smoke, vapours, little clouds and the like, in which some earthy or watery matter is always mixed. But if we had a purely aerial or fiery body that ascended perpendicularly upward, so that such an ascent could be observed with the eyes, we should at once be certain that the Earth does not move with the diurnal motion — at least if it still persevered in perpendicular ascent beyond our own air. For a body of this kind ought not to follow the diurnal or annual motion of the Earth, as not being akin to it; so if the Earth moved, the body would certainly be left behind by it in aerial space, and its ascent could not appear perpendicular to us, but would appear oblique towards the West. Conversely, if it appeared perpendicular, that would be a sign that the whole Earth, and the part of it in which the observer stood, remained unmoved. Yet the greater the probability that such an ascent has, from reason a priori, in purely aerial corpuscles and in exhalations — even if it cannot be noticed by the eyes, except perhaps in some comets — the more probable is the immobility of the Earth than its mobility. But the remaining arguments brought forward above give us, for whom the experiments of the real increase of impetus are established, physical evidence of the immobility of the Earth.

CAPVT XX.

Chapter 20.

Proponuntur Argumenta sex à motu Rectilineo & Perpendiculari Grauium Leuiumque contra Diurnum simul & Annuum motum Terræ. Quorum pleraq. Physicè sunt euidentia.

Six arguments are proposed, taken from the rectilinear and perpendicular motion of heavy and light bodies, against the diurnal and annual motion of the Earth together. Most of them are physically evident.

I. Very ancient and very celebrated is the subdivision of simple local motion into motion towards the centre of the universe, motion from the centre, and motion around the centre; or into circu- lar motion, which belongs to the celestial bodies; straight motion downward, which suits heavy bodies; and straight motion upward, which suits light bodies — whether they are elementary or mixed, but inanimate, or have not been allotted any other motion by a soul. [Margin: The three species of simple local motion, from Aristotle.] Aristotle gives this division at the very beginning of Book 1 of *On the Heavens*, chapter 2, or texts 5 and 6, in these words: "Every motion that takes place with respect to place, which we call locomotion [latio], is either straight, or circular, or mixed from these; for these two alone are simple. The cause is that among magnitudes too these alone are simple, the straight and the circular. Circular motion, then, is that which takes place about the centre; straight, that which is up and down. By 'up' I mean motion from the centre, and by 'down' motion towards the centre. Hence every simple locomotion must be either from the centre, or towards the centre, or circular." Then in chapter 3, or text 17: "Let heavy, then, be that which is naturally borne towards the centre, and light that which is borne from the centre; heaviest, that which underlies all things that are borne downward, and lightest, that which stands above all things that are borne upward." He repeats this in Book 4, text 26. With this foundation laid, among the other arguments by which he establishes the immobility of the Earth in Book 2 of *On the Heavens*, chapter 13, text 101, he has this one: "It is manifest, therefore, that the Earth must be at the centre of the universe and immobile, both for the reasons stated and because weights thrown violently upward are borne back perpendicularly to the same place, even if that force should throw them to an infinite distance."

[Margin: Copernicus's teaching on the distinction of straight and circular motion.] II. But Copernicus, in Book 1 of the *Revolutions*, chapter 8, tried to overturn this Peripatetic teaching as contrary to his hypotheses. He holds that the motion of heavy and light bodies is compounded of the straight and the circular, and so is not a simple motion; that in it the circular predominates per se, as perpetual and as suited to a body already sound, that is, set in its own place, while the straight belongs to it per accidens, as to a sick body and one in disorder, being placed outside its own place; and that straight motion does

not exist in reality, but is distinguished from the circular only in thought. Let us pick out his own words. First he says: "We must admit that the motion of falling and rising bodies is twofold in comparison with the world, and altogether compounded of straight and circular." Then, having said that not only earthy things but fiery ones too, inasmuch as they are kindled from earthy matter, follow the nature of their whole, that is, are carried round with the Earth, he adds: "Therefore what they say, that the motion of a simple body is simple, is verified above all of the circular, so long as the simple body remains in its natural place and in its unity; for in its place there is no motion other than circular, which remains wholly within itself, like a body at rest. Straight motion, however, comes upon those things that wander from their natural place or are thrust out of it, etc. Nothing is so repugnant to the order of the whole and to the form of the world as to be outside its own place. Straight motion therefore happens only to things that are not in a right condition, and are not perfect according to nature, while they are separated from their whole and *diserunt* [?] its unity." At last, after a few remarks on the unequal motion of heavy and light bodies, slow at the beginning but swifter as it proceeds, and on the uniformity of circular motion, he concludes: "Since, then, circular motion belongs to wholes, but to parts straight motion as well, we can say that the circular remains with the straight as 'animal' remains with 'sick'. Indeed, the fact that Aristotle distributed simple motion into three kinds — from the centre, to the centre, and about the centre — will be thought merely an act of reason, just as we separate the line, the point and the surface, although one cannot subsist without another, and none of them without body." So far Copernicus.

[Margin: Galileo's teaching on straight and circular motion, against Aristotle.] III. Galileo, however, with far more tunnels, from the very beginning of Dialogue 1 on the two systems of the world, has laboured to undermine and overturn the foundations of Aristotelian motion already mentioned. First he says that the helical line drawn around a cylinder seems to be counted among the simple lines. He reproaches Aristotle for taking the simplicity of motion merely from the simplicity of the line, and so handing down a defective teaching, since among simple motions he counts as straight only the motion that takes place up and down — whereas motions can be straight, and therefore simple, even if they take place not up or down but forward, backward, to the right or to the left, provided only they come from a moving principle — and for acknowledging the motion that takes place in a straight line by a heavy or light body as compounded in a qualified sense. Then he says that straight motion cannot exist in a well-ordered world: both because it befits only things that are disordered and placed outside their own place, and because straight motion is infinite by its nature, since a straight line is of itself infinite and indeterminate; and nature, as Aristotle agrees, does not of itself undertake things that cannot be completed, such as infinite motion is. Besides this, he approves Plato's speculation about straight motion at the beginning of the world: that through it the bodies that were in Chaos might acquire a determinate degree of speed,

the motion being afterwards converted into circular motion, intended per se. In circular motion, moreover, each point of the circumference is a beginning and an end. Further, he denies that Aristotle has sufficiently proved that heavy bodies move to the centre of the universe from the fact that they move to the centre of the Earth, or downward; for, he says, those two centres are not one and the same, and the centre of the universe is an imaginary point, destitute of all virtue. Again, he carps at the Peripatetics because they attribute to the whole Earth, as natural to it, a straight motion downward to the centre of the universe — a motion it never had, but is supposed it would have if it had been placed outside that centre — whereas they call præternatural the circular motion of fire in its orb, which they themselves grant it in reality.

In Dialogue 2, moreover — pages 119 and 132 of the Italian, but 90 and 99 of the Latin version — he sets out Aristotle's argument for the rest of the Earth, taken from the perpendicular descent of heavy bodies, or from the falling back of projectiles to the same place from which they were thrown upward perpendicularly along a straight line. He denies that this argument holds unless the Earth is supposed to be at rest. For unless this is supposed, so long as we are uncertain whether the Earth moves or not, we are also uncertain whether the descent of heavy bodies takes place along a line that is really straight and perpendicular, or along a curved and oblique one; since it is certain that, if the Earth moves, the descent must take place along a curved line, or one mixed of straight and circular. Aristotle, he says, ought not to have denied that motion to heavy bodies falling downward, since he granted it to fire, which tends straight upward of itself but is carried round circularly by the motion of the heaven. He also adduces the example, repeated so often, of the stone that falls from the top of a ship's mast to its foot whether the ship moves or is at rest — but if it is at rest the descent is rectilinear, while if the ship moves the descent becomes in reality curvilinear and transverse. We said more about these matters in the scholia of chapter 4. Notwithstanding these points, we shall show that the argument of Aristotle, Ptolemy and others for the rest of the Earth holds good. It can be framed as follows.

[Margin: 1st form of the argument.] IV. Heavy bodies left to themselves, through air or through water, descend of themselves to the Earth along a straight line falling perpendicularly on the Earth; and heavy bodies thrown upward along a straight line perpendicular to the Earth fall back of themselves to the same place from which they were thrown, along the same straight line, likewise falling perpendicularly on the Earth. But if the Earth moved with the diurnal turning of 24 hours alone, or with the annual motion as well, they would not descend, nor fall back to the same place, along a straight line perpendicular to the Earth. Therefore the Earth moves neither with the diurnal turning of 24 hours nor with the annual motion.

I chose to frame in this way the argument that Aristotle indicated to us briefly above, in *On the Heavens* 2, chapter 13, text 10 [?]. Not only do his interpreters accept it at that place, but so do Petrus Alliacensis, question 3 on the *Sphere*,

conclusion 3; Clavius on chapter 1 of the *Sphere*, page 96 in my copy; Tycho in the *Epistolae Astronomicae*, page 167; Scheiner in the *Disquisitiones Mathematicae*, from page 30; Chiaramonti in the Italian defence of the *Antitycho*, part 2, chapter 3; and most of the physicists and cosmographers. In the Major I added the qualifier "of themselves" [per se], to exclude the impediment that might arise per accidens from the wind or another mover knocking the descending body sideways. In the Minor I expressed the turning of the Earth as the one the Copernicans posit, namely the one completed wholly in 24 hours; for otherwise, if the revolution were much slower, it could be imagined so slow that it would not physically alter the straightness of the descent of heavy bodies falling in a very short time. With these things set out, the Major and the Minor must be proved. We shall show the Minor first, because we shall dispatch it more briefly; and almost the whole difficulty lies in the Major.

[Margin: Proof of the Minor.] V. The MINOR, then, is proved as follows: in the time in which heavy bodies usually descend, or fall back to the same place, the Earth, by its diurnal turning towards the East, and much mo- re by the annual motion, would withdraw the place on the Earth's surface above which the heavy body had been let fall, or from which it had been thrown perpendicularly upward, from the place in cosmic space where it was when the heavy body was first released or thrown — and that far and notably. Therefore, for that heavy body to appear always on the same perpendicular straight line, and to fall back apparently to the same place, it would have to depart from its former straight line and descend in such a way as to move transversely, following the former place of the Earth — and indeed along a curved line, striking the Earth obliquely in reality. This is evident from the figures set out in chapter 17, numbers 16, 17 and 18; and from the admission both of Copernicus and Galileo, in the passages cited in numbers 2 and 3, and of Kepler, Gassendi and Boulliau, who teach that this motion is in reality curvilinear, as was said in chapter 4, scholia 2 and 4, and chapter 17, number 3. Hence that descent, or falling back, would indeed seem in appearance rectilinear and perpendicular to the Earth, but in cosmic space it would in reality be curvilinear, and would strike the Earth obliquely.

[Margin: Proof of the Major [?].] VI. The MAJOR is proved as follows. A thing must be asserted to be such when it is physically evident, and is not established to be false from a higher light, or from more evident, or at least more certain, principles. For to deny it, or to assert its opposite, it is not enough that the opposite is possible, or has certain congruences, or perfections desirable under some aspect. But that heavy bodies left to themselves, or thrown perpendicularly upward, descend or fall back along one and the same straight line perpendicular to the Earth is in itself so physically evident that it is not established to be false from a higher light — that is, neither from more evident nor even from more certain principles. Therefore such motion must be asserted to be straight, and consequently it is straight. We have, moreover, included in this proof two propositions which seem themselves to need further proof.

[Margin: Proof of the proposition [?].] The first is that a thing must be asserted to be such when it is so physically evident that it is not established to be false from principles of a higher light, which produce greater evidence or certainty; and that, to deny it, it is not enough that the opposite is possible or has certain congruences. This proposition is manifest from the whole of physical science, which would otherwise perish if it were not true, since the whole of that science rests on first principles drawn from sensations duly had. But sensations are not duly had, nor fit to generate physical evidence, if they are found by other sensations, equally or more manifest, to be deceptive and in need of correction. So it is when, having drawn a stick out of water, we see and confirm by touch that it is straight, though it had previously appeared bent; or when, while the ship is running, we see from the bank or the shore that a stone let fall perpendicularly from the top of the mast describes in the air a curved line striking the bottom of the ship obliquely, though to us, or to others carried along together in that ship, it seemed to descend along a straight and perpendicular line; or when we prove that the Sun, which looks a foot across to us, is not a foot across, from the distance of the Sun found through eclipses or lunar dichotomies, and so through other, more certain sensations. Therefore, since the senses per se — that is, usually, indeed always, if they are applied in the due manner — present objects to us such as they are in reality, or reveal other things through which we can correct the deceptiveness of other sensations, possession stands on the side of the sensations, and the presumption [?] must be made in their favour, so long as the opposite is not proved from the principles of a superior light: whether of the Catholic Faith, which has certainty without evidence, or of Metaphysics, Mathematics, or a science subalternate to Mathematics and Physics, but one in which the principles of mathematics predominate.

[Margin: The 4 degrees of natural evidence.] At this point I think it should be recalled to memory — or even brought to the notice of those who perhaps never learned it — that there are four degrees of evidence in the natural sciences, corresponding to the four total natural sciences (taking "natural" as opposed to "supernatural"): namely Metaphysics, Mathematics, Physics and Moral science. These are nevertheless reduced to three, because metaphysical and mathematical evidence have the same force. Metaphysical and mathematical evidence is that which makes a proposition so evident that it is altogether impossible for the matter to be otherwise, the first principles of Metaphysics and Mathematics being kept. These principles are most absolutely true, and rest on this axiom, the most manifest of all: "It is impossible for the same thing to be and not to be"; or on axioms arising from it next, or very near, such as are very many divisions of beings — as when we say, "Every being is either corporeal or incorporeal; either substance or accident; either finite or infinite," etc.; or when we say, "Every whole is greater than its part," or, "If from equals you take away equals, the remainders are equal." For these are so true that not even by divine power could they fail to be true.

The second evidence is physical: that which makes a proposition so evident that it is impossible for it not to be so, the first principles of Physics being kept — that is, the mode of being and operating of natural things and causes — even though it is not repugnant to the principles of Metaphysics or Mathematics, or to the absolute power of God, for it to be otherwise. In this way it is evident that every heavy body left to itself tends downward; that all fire burns; that daily, in the torrid and temperate zones, the Sun rises above the horizon and always moves in a circle; that every animal is mortal, indeed will die; that an eclipse of the Sun happens only at new moons; that some substance proper to it underlies every accident; and so of similar things — even though, if God wills, as at times he has willed, these things can be otherwise. For the axe-head floated on the water at the command of Elisha, and the body of St Peter on the sea at the bidding of Christ; the fire of the Babylonian furnace did not burn the three youths; the Sun stood still at the command of Joshua; at the Saviour's death the Sun suffered an eclipse by the Moon while the Moon was at the full; and daily, in the most holy Sacrament of the Eucharist, physical accidents are found without the proper substance of the bread and wine. Evidence of this kind, therefore, is not altogether absolute, but has implicitly annexed to it a certain hypothesis and a tacit limitation: namely, if Nature exists and operates in its accustomed manner, the manner owed to it; or if the mode of being and operating of natural causes as such is what is considered.

Moral evidence, finally, is when, the principles of prudence being kept, the matter cannot be otherwise, even though it is repugnant neither to the principles of Metaphysics nor to those of Physics for it to be otherwise. In this way it is evident that Rome, or the Ocean, exists to someone who has never seen them, but has heard from the constant report of honest men that both are to be found. Moreover, physical and moral evidence persists, and has the force of determining the intellect to assent, so long as it has not been established from divine revelation, or from the principles of Metaphysics or Mathematics, that the opposite is true. In the same way the probability of an opinion also remains for as long as its opposite has not been evidently known.

With these things laid down, I said that what is so physically evident that it is not established to be false from the principles of a higher light must be asserted; otherwise that evidence ceases, because the tacit condition on which it rested ceases. I said, however, that to deny that something is, which has been manifest by such evidence, it is not enough that the opposite is possible, or has some congruences in favour of its real existence. Otherwise we could assert the real existence of all things that are possible and that, if they existed, would certainly have some goodness and perfection — which is erroneous, even if we had a sign of the existence of neither of two contradictories. How much more erroneous, then, is it to deny that something is such, when we have sufficient signs of it, and to deny it because its opposite has certain congruences and desirabilities and is not otherwise impossible? Are not the equality of the lunar month, the uniformity of the motion of the Moon and of all the planets, and configurations

of the fixed stars according to regular figures possible — indeed, do they not have great and desirable congruences? Certainly they do. Would anyone therefore dare to deny that some lunations are shorter and others longer? Or, if you refer this to an apparent inequality, what of the figures of the stars? Must they too be called regular in reality, and irregular only optically? If anyone dares to say so, let him go on, by all means, and rub his brow still harder, and affirm that in reality there is no summer, no winter, no pains, no diseases, no death, but that these only appear — on the ground that a temperate and equable state of the weather and of the humours, and an immortal life, are possible and more desirable entities. For it is no more evident that there is summer and winter, etc., than that the natural descent of heavy bodies is rectilinear and perpendicular to the Earth. But if you say [?] that, in the examples listed, the appearance of those things cannot be joined with the privation of their real existence, or at least not without a miracle, I too shall say that the appearance of the straight motion of heavy bodies cannot be joined with the real privation of straightness without a miracle. And in the controversy over the notion and definition of a miracle I shall have as my supporters all the best theologians, who recognise so great a miracle in the motion of the whole Earth and in the rest of the Sun that they deny this motion could be produced naturally even by the whole of angelic power. We ought to adhere to them rather than to the Galileists and the rest of the Copernicans, for whom the Sun stands still and the Earth is turned round without any miracle at all. Finally, it must be noted that the proposition "Heavy bodies descend either along a straight line or along a line that is not straight" (which alone the adversaries concede to be evident) is evident metaphysically — understand: on the supposition that they descend. Here, however, we require and defend only physical evidence.

[Margin: Proof of the 2nd proposition.] VII. Secondly, in the proof of the Major we said that it is physically evident that heavy bodies left to themselves, or thrown upward to the perpendicular, descend and fall back along one and the same straight line falling perpendicularly on the Earth; and that it is evident in such a way that the falsity of this straightness cannot be proved from any higher light, or from more certain principles. For it is established by the experiments of all ages and places that this motion appears straight — not to some people or at some times straight and to others or at other times not straight, but to all people and always straight. Nor does any authority of God oppose this appearance; rather, it supports it in the highest degree by the very fact that it very often affirms to us that the Earth is immobile. Finally, no reason drawn either from Metaphysics or from Mathematics certainly and evidently resists the straightness of this motion. This is clear both from the solution of the arguments adduced for the motion of the Earth from chapter 5 to chapter 19, and from the admission of the Copernicans, who openly concede that on either hypothesis — that is, of the Earth unmoved or moved — all the phenomena can be saved. And indeed Galileo, in Dialogue 2 on the system of the world, page 81 of the Latin, says most expressly: "And this motion of 24 hours, as regards

this first appearance, in no way prevents its belonging as well to the Earth alone as to the whole rest of the world except the Earth. For the same appearances would be seen in the one position as in the other." From no effect, therefore, can it be proved that the Earth moves, and hence that the descent of heavy bodies is not, as it appears, rectilinear and perpendicular with respect to the Earth. And let this suffice for the Aristotelian argument. Let us hear, however, the reply of the Copernicans.

Responsio Copernicanorum ad 1. Argumentum, eiusq. Inualiditas.

The reply of the Copernicans to the 1st argument, and its invalidity.

VIII. The Copernicans reply, however — Rothmann in Tycho's letters, page 185; Galileo in Dialogue 2 on the system of the world; and Gassendi in Letter 2 of *De motu impresso a motore translato*, pages 80 and 106 — by distinguishing the Major. They concede it if it is certain from elsewhere that the Earth does not move and that our senses are not deceived in judging the rectilinear motion of heavy bodies, or if the motion of the Earth is not common both to us who observe the motion of the heavy bodies and to the heavy bodies themselves; otherwise they deny the Major. And they press the example of a man who did not know of the motion of the ship in which he is carried, and who would therefore judge rectilinear the fall of a stone let fall from the top of the mast, or the descent of a ball let fall through the water of a vessel enclosed in that ship — a fall that, because of the ship's motion, would in reality be curvilinear in the space of the world, even though it would not be recognised as such, because the eye observing that motion is carried along together with the ship. And Gassendi, although he indeed suspends his assent, nevertheless adds certain things there which, to use Chiaramonti's words, would take away altogether the criterion of sense — that is, the power of judging that rests on sensations — however rightly the sensitive power were disposed, the object duly presented, and the medium such as not to disturb the sensation. [Margin: Gassendi's dissertation against the deceptiveness of the senses.] For Gassendi says: "Now the globe of the Earth is the one into which we were transported as if asleep, or rather in which we were born without thinking of it. But from the time we opened our eyes, and contemplated besides this globe another, namely the Sun, in the same expanse, as it were, or space of the world as the Earth, the question has been to which of these globes rest belongs, and to which motion. We, for our part, unhesitatingly pronounced that rest undoubtedly belongs to the Earth and motion to the Sun. Pythagoras, Plato, Aristarchus and other ancients, and likewise Copernicus, Galileo, Kepler and many more recent men, warned that perhaps we are deceived, and that the Sun is at rest and the Earth moves; we nevertheless held out against them. And we became the more obstinate the more attentively we observed that the parts of the Earth are not drawn away from one another, but that that mountain is always to the north, this city to the south, that river to the east, this island to the west; and that we meanwhile run about upon the Earth just as we run about through the house or city in

which we were born — so that, just as neither the house runs about through the city, nor the city through the province or the Earth, so the Earth itself also does not seem to run about through the spaces of the world. Such indeed is our imagination from our birth onward; nor do we notice that the Earth is perhaps like a ship, and that it may happen that, being on the Earth, we are deceived just as that inland man is while he is on a ship. For what will be the difference? Or what the reason for a difference? And what the way of distinguishing truth from error? I do not, indeed, say that the Earth therefore moves and the Sun is at rest; but I do say that better reasons must be sought than those drawn from sense itself. And if we had never had experience of the deceptiveness of our sight, we would seem excusable, and could object that nothing is more manifest or more certain than sight, and that, once faith is withdrawn from the eyes, nothing is worthy of belief. But since experience has taught us that sight is deceived so greatly, and that faith is not always to be placed in the eyes — and this above all when motion is in question — what else can be answered than that both sight and the other senses must be corrected by reason?" So Gassendi, with a declamatory elegance of style indeed, but one that strikes nothing, and does not even prick Aristotle.

[Margin: The reply offered is rejected.] IX. For the reply and the distinction already mentioned are worth nothing. Physical evidence does not require that we be certain from elsewhere that the Earth is at rest and that our sense is in no way deceived; it requires that sense is not deceived once every application of the power, and every examination of the medium and of the object naturally possible to us, has been made. Otherwise one would have to proceed to infinity from one sensation to another and yet another, with a perpetual fear of error; or, if this certainty had to be sought from other principles that presuppose no sensation, it would no longer be physical certainty but metaphysical, mathematical or theological — which would be demanded here unseasonably and unskilfully. The example of the man observing the fall of a heavy body in a ship, moreover, is no parallel. For neither all men nor always are ignorant that the ship moves, and natural means are given of detecting that ignorance, and so of correcting the deceptiveness of sense. By contrast, no natural means is given by which we could certainly and evidently detect that the Earth moves, and we with the Earth, and hence that sense is deceived in judging the rectilinear motion of heavy bodies. On the contrary: from the more vehement percussion when they fall from a higher place, we have an argument from the sensations themselves that the increase of speed is real, and hence that the Earth does not move, as is clear from the preceding chapter. The a priori reasons for the motion of the Earth drawn from certain congruences, moreover, have already been abundantly resolved from chapter 5 to chapter 19; and in the motion of the Earth there is no lack of many other absurdities, just as in its rest there is no lack of very many utilities, and of congruences with the end intended by God and desirable to us. Therefore, even if the weights of the a priori reasons were equal on both sides, there will always be this disparity: for the rest of the Earth

and the rectilinear motion of heavy bodies there stands the universal sensation of all men at every time and place, whereas for the curvilinear motion of heavy bodies and the motion of the Earth there is no sensation, nor can there be. And on him who asserts that sense is deceived lies the burden of proof — and indeed of a very strong one; otherwise all tribunals will award the suit to so many eyewitnesses.

II. Argumentum à Breuitate & Compendio itineris Grauium Leuiumq. naturaliter ad suum locum properantium.

II. Argument from the brevity and shortness of the path of heavy and light bodies hastening naturally to their own place.

[Margin: Form of the 2nd argument.] X. The nature of heavy and light bodies requires that, if they are outside the place owed to them, they reach it, once impediments are removed, by tending to it along the shortest possible path. But if the Earth moved with the diurnal motion alone, or with the annual as well, neither heavy nor light bodies placed outside the place owed to them would reach it by tending along the shortest path, however completely the impediments were removed. Therefore the Earth moves with neither the diurnal nor the annual motion.

[Margin: Proof of the MINOR.] The MINOR is proved first, because there is no controversy about its truth, nor can the adversaries deny it, if the path a body would have to traverse on the hypothesis of the moving Earth is compared with its path on the hypothesis of the Earth at rest. For if the Earth does not move at all, heavy and light bodies left to themselves tend to and reach their own place along a straight line perpendicular to the place they tend to, and no path can be shorter than that. But if the Earth moves with the diurnal motion alone, and much more if it moves with the annual motion too, the heavy bodies must turn aside from the straight path downward, following the Earth and its motion: both the diurnal motion, so that they fall on the point above which they were let fall perpendicularly or thrown upward — a point already withdrawn from the former perpendicular by the turning of the Earth — and the annual motion, so that they do not fall outside the whole Earth. They must therefore describe by their motion a transverse and curvilinear path, such as we sketched in chapter 17 from number 16. And the same must be said of light bodies tending upward, for these too must appear, to an eye carried along with the Earth, to lie on the same straight line drawn perpendicularly from the Earth's surface. Thus, for example, take the clay ball which, as stated in chapter 16, number 12, we observed to descend from an apparent height of 240 feet in a time of 4^s. If it followed only the diurnal motion of the Earth, so as to fall on the same place above which it was let fall, it would have had to run through at least 7527 Roman feet, had it been at the Equator — namely as many as are contained in one minute of the Equator, according to what was said in chapter 19, number 13, in the first part of the table; for one minute of the Equator turns in every 4 hour-seconds. And if it followed besides the annual motion of the

Earth, then, since the centre of the Earth runs through 2‴ 28⁗ of the annual orb in each hour-second — that is, 9‴ 52⁗ in every 4 seconds — it would have had to pass, by the force of this motion alone, through at least 23,860 feet, or 4772 Roman paces, that is, 4 4/5 miles, if we follow the Copernican diameter of the Great Orb; but if we follow ours, it would have had to traverse at least 142,760 Roman feet, or 28,552 paces, that is, 28½ ancient Roman miles, as may be gathered from the table of chapter 19, number 13. These distances exceed the shortest, perpendicular path of 240 feet by a very long interval.

[Margin: Proof of the MAJOR.] The MAJOR is proved because in inanimate bodies, whose motion is not determined by a soul, Nature takes the short way, and does not do through more what it can do through fewer. The Copernicans indeed use this axiom — or rather abuse it — to attribute the simplicity and unity of the diurnal motion to the Earth alone, and to deny it to the fixed stars and the planets. Moreover, that very increase of impetus by which heavy and light bodies move faster and faster towards the end of their motion is an argument of a nature that aims at the shortest and most expeditious path, so that they may reach their own place. We must, however, hear the reply the Copernicans could devise.

[Margin: The 1st reply to Argument 2, of little strength.] For one can reply, first, by distinguishing the Major: conceding it comparatively — with respect to the place of their whole, to which they seek to be joined, that place being meanwhile carried along, and supposing that they imitate the motion by which their place is moved; for under these conditions they cannot be carried by a shorter path — but denying it absolutely, or when the comparison is made with the place over which they hung perpendicularly, or on which they stood, when they were first let fall. But who does not see that this has been invented out of the necessity of the hypothesis of the Earth's motion? Since that hypothesis is disputed, it ought not to be presumed as certain; rather, the controversy must be settled from other, better-known things. And it is better known that the nature of these bodies requires a priori — indeed, if we rest on the observation and testimony of the senses, it is also better known a posteriori that it pursues — the joining of the parts with their whole, and their restoration to their own place by the shortest path. Surely it would be better for them to be united with it at once along the straight line, and afterwards, if it moves circularly, to move with it.

[Margin: 2nd reply.] The second reply is to concede the Major if the brevity of the path of elementary bodies does not presuppose multiple motions and paths of the celestial bodies, and to deny it if it does. Now if the brevity of the path of heavy and light bodies along the line perpendicular to the Earth is kept, the diurnal and annual motion of the Earth must be transferred from the Earth to the stars; and it seems that the simplicity of the celestial bodies, as being simple, should be favoured rather than that of the elementary bodies. On the hypothesis of the immobile Earth, however, the foundation drawn from the sensations always prevails, on account of which both the rest of the Earth

and the diurnal motion of the stars are reasonably asserted; and the hypothesis of the moving Earth lacks this foundation. Besides, we have already shown, in chapter 6 from number 11 to 15 and in chapter 9 from number 1 to 5, that more motions are multiplied if the Earth moves, even counting the necessary motions of the stars, than if it is at rest.

III. Argumentum à Simplicitate motus Grauium & Leuium.

III. Argument from the simplicity of the motion of heavy and light bodies.

[Margin: Form of the 3rd argument.] XI. Species of motions, or diverse inequalities and diverse degrees of speed in the same mobile body, are not to be multiplied without necessity. But if the Earth moved with the diurnal or also the annual motion, species of motions, and diverse inequalities and degrees of speed in the same mobile body, would be multiplied in heavy and light bodies without necessity. Therefore the Earth moves with neither motion. [Margin: Proof of the MAJOR.] The MAJOR is used by the Copernicans to substitute one single diurnal and one single annual motion of the Earth for the many other motions of the fixed stars and planets that would have to be admitted if the Earth were immobile, as is clear from chapter 6, number 14, and chapter 9, number 2; and it rests on these axioms: "God and nature do nothing in vain," and likewise, "Entities are not to be multiplied without necessity." The MINOR is easily shown from the different figures of motion that heavy bodies would describe in descending if the Earth were turned with both motions, or at least with one of them — figures we sketched after a fashion in chapter 17, numbers 16, 17 and 18. [Margin: How varied, unequal and nowhere constant the motion of heavy bodies would be, if the Earth moved.] For if a heavy body were let fall above the poles of the terrestrial Equator, and the Earth moved with the diurnal motion alone, the descent would take place along a straight line perpendicular to the Earth; but if it moved with the annual motion as well, the body would describe a curved and oblique line on a cylindrical surface. If it were let fall on the parallels of the Equator, it would describe a curved line about a conical surface — and indeed the surface of a cone that varies, or is carried by the annual motion to different places; but if at the Equator, the line would be described in nearly the same plane, as we showed in the same place. Added to this, in following the diurnal motion of the Earth, heavy bodies would have to traverse more path in the same time at the Equator than on the parallels, since on the parallels the motion of the parts of the Earth's surface is slower. So the same heavy body, let fall above different places of the Earth, would descend by different paths and with very different degrees of speed. Besides this, it would acquire another inequality from the different times of the same day, at which the diurnal motion would sometimes add to the annual, sometimes subtract from it, and sometimes neither add nor subtract [?] anything, according to what was said in chapter 19, number 12. The same must be understood, proportionally, of the ascent of light bodies. Moreover, no necessity appears for multiplying so many varieties of motion, since with the

Earth at rest all the celestial phenomena and all the experiments on elementary motions are saved.

[Margin: A reply of no great strength.] The Copernicans will reply by denying the Minor. For there is a necessity — not indeed a posteriori, from sensations or phenomena, but a priori — that motions not be multiplied in the heavens without necessity; and in an equal contest between celestial and elementary bodies, multiplicity of motions must be allowed to the elementary rather than to the celestial bodies. For through it, since it presupposes the motion of the Earth, very many other motions are dispensed with, which would otherwise have to be acknow- ledged in the heavens, if they are excluded from the Earth. But in reality there are more motions in the world if the Earth moves than if it is at rest. For in the heavens motions are not really multiplied; rather, there is in each body a single diurnal motion, though it is slower in some and swifter in others. Besides, the disparity always prevails, on account of the physical evidence of the motion of the celestial bodies and of the rest of the Earth.

IV. Argumentum à Termino motus Grauium ac Leuium, ad quem tendunt.

IV. Argument from the term of the motion of heavy and light bodies, to which they tend.

[Margin: 4th argument, an enthymeme [?].] XII. If the Earth moves with the diurnal motion, no reason can be given, antecedent to the hypothesis of the motion or rest of the Earth, why heavy and light bodies tend to the term they tend to, rather than to another of the same or even of better condition for terminating that motion. But such a reason can be given if the Earth does not move. Therefore the hypothesis of the immobile Earth is to be preferred to the hypothesis of the mobile Earth. About the consequence there seems to be no doubt, at least if other things are equal. [Margin: Proof of the 1st part of the antecedent.] The first part of the antecedent is proved by taking a single example, so that a judgement can be made about the others. Above some rock placed at the Equator, let there be a glass sphere, which is thrown from there straight up to the perpendicular, or at any rate is let fall from a tower above that rock. It is certain from observations that this sphere will fall on that rock, unless per accidens it is violently pushed elsewhere, or pulled back, or impeded. Let there be, moreover, at the terrestrial Equator, a lake or pond, distant from the rock already mentioned towards the west by so many paces that, if the glass sphere in reality descended by a straight path and did not follow the turning of the terrestrial rotation, it would fall into the lake or pond. Granting the diurnal turning of the Earth, I ask why that glass falls on the rock. Suppose you say this happens because that glass shares in the motion that the whole Earth has, since it is an earthly body, and therefore must move towards the same part as the Earth and about the same centre about which the Earth moves. Then this reason is not antecedent to this disputed hypothesis about the motion of the Earth, but presupposes it — and presupposes it in such a way that, even if its

contradictory were supposed, namely the privation of motion in the Earth, the glass would nonetheless fall on that rock. So the motion of the Earth can no more be adduced as the cause of the descent upon the rock than its rest can. If you say this happens so that the sphere may be united with its whole, or be better preserved there, or on account of the kinship the glass sphere has with the rock, or because it is drawn magnetically to it by the Earth, you give no satisfaction. For it has no greater kinship with the rock than with the water of the lake or pond; nor is it better preserved there than here, since it is rather broken by the rock; nor is there in the rock a mass of glass having the character of the whole. But if the character of the whole lies in the whole terraqueous globe, the sphere will be united with its whole, or drawn by it, no less if it falls into the water than if it falls on the rock. Besides, as Scheiner rightly argues in the *Disquisitiones Mathematicae*, page 30, if there were a globe of water about the centre of the Earth, and the land about this globe, a clod of earth placed at that centre would nevertheless not rise to its whole, but would remain there: therefore heavy bodies do not descend so as to be united with their whole, but so as to lie beneath bodies lighter than themselves. The objection has greater force, however, if the clod is at a distance from the centre, lest they say that it does not move because the centre does not move either. If you say that the sphere falls on the rock so that its motion may always appear to us on the same straight line, you invent a cause arbitrarily; for our sight is not the end of that fall. And then what will you say of a drop of rain falling on sand or on burning coals? So, whichever way you turn, you can devise no solid cause of the fall already mentioned that is antecedent to the hypothesis of the motion of the Earth.

[Margin: Proof of the 2nd part of the antecedent.] XIII. The second part of the antecedent is proved. The cause why a heavy body falls back to the place over which it hangs perpendicularly is not an appetite for uniting itself to its whole (otherwise a stone, for example, let fall down a well would have to unite itself to the side of the well dug in the earth), but that the parts of the universe in the elementary system may return as soon as possible to their own place, and their order with respect to position be restored — namely, that the heavier lie beneath the lighter and the lighter stand above the heavier. This cannot happen as soon as possible, nor by a shorter [?] path, than along the line perpendicular to the Earth, that is, the line tending straight to the centre of all heavy bodies. For although this centre is a point, and is not of itself either the mover or the terminator of the motion of all heavy bodies, it is nevertheless directive of it, as that towards which, if they tend, it is certain they will return most quickly to the place owed to them in the order of the universe. In the same way a standard is not a sign under which all the soldiers can gather, as in a place or covering able to hold them all, but is directive of their motion; and the Cynosure is not the harbour of those who sail, but directive of their navigation. For even if the whole Earth had burst apart into various parts, those parts would flow together again to the centre, not so as to be at the centre,

but so as to lie beneath lighter bodies; and for this they needed some common point that would objectively determine their shortest and surest path. Similarly, lighter bodies must recede from the centre of the heavier, on the supposition that about this centre, especially on a straight line, there are heavier bodies not impeded from descending beneath the lighter ones; and for this reason they are said to move from the middle. Hence it is no obstacle that Kepler, in the Introduction to the *Commentaries on Mars*, insists that the centre of the Earth is a mathematical point and nothing, and so can move heavy bodies to itself neither effectively nor objectively. The words of this author are these: "A mathematical point, whether it is the centre of the world or not, cannot move heavy bodies either effectively or objectively, so that they approach it. Let the physicists prove that this power belongs to a point, which is neither a body nor is understood otherwise than through relation alone. It is impossible that the form of a stone, in moving its body, should seek a mathematical point, or the middle of the world, without regard to the body in which that point is. Let the physicists prove that natural things have a sympathy for what is nothing." But we do not say that they tend to the centre without regard to the body whose centre it is; on the contrary, we suppose that heavier bodies are placed all around it, and that heavy bodies tend to these so as to lie beneath lighter ones, and so to be co-ordinated with them in the universe. Nature, and God the Geometer, have used this centre as a term towards which the heavier bodies, and from which the lighter, are to be carried — not, however, as a term to which, and in which, they are to be placed; just as a centre is used by one who describes a circumference on which certain things are to be placed. It is false, therefore, that the centre does not move objectively and in the manner of a sign, or even of a term of relation and order. As for what Kepler adds in the same place — "Heavy bodies (even if we place the Earth at the centre of the world) are not borne to the centre of the world as to the centre of the world, but as to the centre of a round kindred body, namely the Earth" — this we admit so far as it goes. But it follows from this that this centre is the term from which their distance in a round body is to be measured, and that their direction is to be taken along a line that is a radius or semidiameter of the round body; for this line alone is what, by its revolution about the centre, constitutes a round body. And hence the motion of heavy and light bodies naturally descending or ascending must take place along a line perpendicular to the round surface of the terrestrial globe; and this cannot happen, with the phenomena and the falling back to the same place saved, unless the Earth is at rest.

The Copernicans could reply, however, that the consequence of the enthymeme is true if other things are equal, or if a suitable reason can be given for the many phenomena that appear in the heavens on the supposition of the Earth's immobility; and that otherwise it is not true. And so they seem in some fashion able to escape the force of this argument, and to be held by it only within the bounds of probability. Yet the foundation drawn from the sensations always prevails, and the hypothesis of the moving Earth lacks it.

V. Argumentum à sphæra catenulis appensa & deorsum ab Angelo dimissa.

V. Argument from a sphere hung on little chains and let fall downward by an angel.

XIV. If an angel hovering over a point of the Earth let fall downward a metal sphere, but one tied to a chain, holding back the other end of the chain, that sphere would tend straight towards the Earth and would stretch the chain along the perpendicular above the former point of the Earth. But this could not be if the Earth moved. The Copernicans will reply by denying the Major: for the sphere would tend towards the east with an oblique motion, and so would drag the chain with it. Or, if the angel's force in holding back the chain were greater, it would indeed come to rest at last, but inclined towards the east.

VI. Argumentum à Linearum & Figurarum, Mensurarumq. confusione.

VI. Argument from the confusion of lines, figures and measures.

XV. I bring this argument under the title of this chapter because it has a great affinity with what was said about the straight line described by the motion of heavy and light bodies. For just as it is physically evident that these bodies describe a straight line in their descent or ascent, so it is evident to architects that they describe a straight line with a plumb line stretched out and let down; and to practical geometers that they draw a straight line with the help of a ruler or a stretched thread, a circular periphery with the help of compasses, and a right angle with the help of a square or of the problems of practical geometry, and from there construct various rectangular figures. But if the Earth moved, these things would not be such in reality in the space of the world, but only optically, or apparently, on the board, the paper, the wall, and so on; nor would vision be continued along one and the same straight line. Nor could someone who had run towards the East, or had drawn furrows with a plough, truly say that he had run through three miles in one hour, or had finished ploughing one iugerum in one day. For he would have measured out much more space in the world, especially if the Earth moved with the annual motion as well: besides the apparent motion, he would in addition have completed the motion that the Earth meanwhile completed by its own motion. But all these things, however foreign they are to common sense and however paradoxical to the common view, the Copernicans will deny are absurd to an intellect that knows the reasons for which they assert the motion of the Earth.

CAPVT XXI.

Chapter 21.

CAPVT XXI.

PROPONUNTUR ARGUMENTA QUATUORDECIM EX MOTU CORPORUM ELEMENTARIUM VERSÙS QUATUOR MUNDI CARDINES, SEU PLAGAS QUATUOR VENTORUM PRINCIPALIUM, CONTRA MOTUM DIURNUM TELLURIS. QUÆ MULTÒ MAGIS MILITANT CONTRA ANNUUM DIURNO CONIUNCTUM: EX QUIBUS QUINQUE SUNT PHYSICÈ INSOLUBILIA.

Fourteen arguments are proposed, taken from the motion of elementary bodies towards the four cardinal points of the world, or the regions of the four principal winds, against the diurnal motion of the Earth. They militate much more against the annual motion joined to the diurnal. Of these, five are physically insoluble.

I. The arguments that are commonly made, or could be made, against the diurnal motion of the Earth, taken separately or joined with the annual, are almost innumerable, if the motion of bodies, whether animate or lacking a soul, towards the East is compared with their motion towards the poles or towards the West. But they are reduced to fewer, because most of them differ only by reason of their subject, while the form and the ground of the difficulty are the same: namely, that from the turning of the Earth towards the East it seems to follow that the motion of terrestrial, aquatic and flying bodies turns out swifter towards the East, slower towards the West, or even at times impossible, while towards the poles it behaves in a middle way; and since these consequences do not answer to the experiments, this motion must therefore be counted among false hypotheses. [Margin: Buchanan's poem against the turning of the Earth.] Buchanan touched elegantly on many of these points in book 1 of his *Sphere*, composed in verse, in these lines:

> Even the birds, swimming through the air with the gentle oarage of their wings, would lament their woods snatched away by the swift turning of the Earth, and their nests with their tender brood, and perhaps with their dear mate; nor would the turtle-dove dare to entrust itself alone to the West Wind, lest, as the Earth fled, it should weep for its marriage carried far away and mourn its widowed love in long grief. What of when the light-armed Persians first join battle with the Medes, and the army stands against them with equal arms — the pole standing still and the ground fleeing — while the missile iron flies suspended in the empty air? The one side, running against [?] the weapons, would put itself upon them; the other would never suffer wounds, for the weapons, carried away from the enemy by the turning of the Earth, would fall in vain beside their master's footsteps. What of when the Earth, driven on, twists itself always towards the West, if the blue levels of the sea lie sluggish and unmoved in their lazy pools? Meanwhile, must not either a great part of the Earth float upon the waters, the compact of nature broken — so that the light wave, carrying heavy rocks on its flood, does not give way, and the water does not bore through the high mountains, though before it could not be taught to carry even the smallest pebbles? Or, if the Earth, solid and unbroken, should resist

the waves, the watery part of the sea would daily overwhelm a part of it, and night would always look upon the land that the Sun had just seen, sunk in the strait and squalid with shapeless sand.

But let us treat this matter seriously, with arguments selected from the motions of these things.

I. Argumentum à pendentibus vel meantibus in Aere versùs Occidentem.

I. Argument from things hanging or travelling in the air towards the West.

[Margin: Form of the 1st argument.] II. If the Earth moved with the diurnal turning, or also with the annual translation, clouds hanging in the air, and rising smoke, and birds, however much they hold themselves suspended or fly towards the East, would appear always to be carried towards the West. But this is against manifest experience. Therefore the Earth does not move with the diurnal turning, and much less with the annual translation. The MINOR is clear to those who observe; the MAJOR is proved. For the part of the Earth on which we stand while observing the things already mentioned would snatch us away towards the East with a far swifter motion than any wind could drive the clouds, or any impetus from within could drive the birds, towards the East. For, from what was said in chapter 19, number 13, a point of the terrestrial Equator would pass in one hour-second — that is, in about one beat of the pulse — through 15″ of the terrestrial circumference, namely 376 Roman paces; and in one hour through 15 degrees, that is, 1355 ancient Roman miles. This is roughly the argument of Ptolemy in book 1 of the *Great Construction*, chapter 7; of Petrus Aliacensis, question 3 on Sacrobosco's *Sphere*, conclusion 3; of Clavius on chapter 1 of the *Sphere*, page 196 in my copy; and of Scheiner in the *Disquisitiones Mathematicae* from page 30. But it is worth transcribing Ptolemy's words here: "They think that nothing can resist them if they suppose the heaven immobile and the Earth revolving daily from west to east about the same axis." Against them he adds: "For it escapes them that, as far as the appearances in the stars are concerned, nothing perhaps prevents someone from thinking that things are so; but from the things that happen around us and in the air this will seem quite ridiculous. Why? Because they cannot deny that the revolution of the Earth, if it turned in this way, would be simply the swiftest of all the motions about it, since it would complete so great a revolution in so short a time; so that the things that are not in it would always appear to move with a single motion contrary to the Earth. Thus neither clouds nor anything else among things flying or thrown would ever appear to be carried towards the east; but the Earth itself would outrun them all, and would so resist their motion towards the east that all the rest, left behind, would appear to advance towards the west. For even if they say that the air is likewise carried round with the Earth at equal speed, the things that form in that air would nonetheless always be left behind by the motion of both, etc."

[Margin: Copernicus's reply to Argument 1.] Copernicus, however, replies very briefly, implicitly denying the Major, in book 1, chapter 8, in these words:

"What then shall we say of the clouds and the other things hanging in the air in whatever way, or settling, or tending upward to the heights? Only that not merely the Earth with the watery element joined to it moves in this way, but also no small part of the air, and whatever things have kinship with the Earth in the same way. Either the neighbouring air, mixed with earthy or watery matter, follows the same nature as the Earth; or the motion of the air is acquired, and it shares it from the Earth through contiguity, with perpetual revolution and without resistance." And a little below: "We must admit that the motion of falling and rising things is double, by comparison with the world, and altogether compounded of straight and circular. For since the things that are pressed down by their own weight are especially earthy, there is no doubt that the parts preserve the same nature as their whole." That is, they move with a double motion: one common to them and to all things akin to water or earth, about the centre of the Earth, about which the Earth and the water turn — a motion we cannot perceive, because we are carried along with the Earth; the other their own, and this alone we can notice. Therefore, if clouds, smoke, birds and so on do not use their own motion, they will seem to us, who are carried round together with them, to stand still and hang in the air, because they are nevertheless carried round towards the east by the common motion; but if they move with their own motion towards the west or the east, this motion alone will appear, separated from the motion common to them and to us. This reply also holds for things hanging or travelling in the very thin air above the Peruvian and other mountains, where air fit for breathing is said scarcely to be found: for those hanging bodies too, being of an earthy or watery nature, share the common circular motion towards the east. Their own motion towards the west, moreover, is nothing other than a certain subtraction and diminution of the common motion towards the east, as Galileo rightly explains in Dialogue 2 on the system of the world, page 136 of the Latin. He adds that if flies, butterflies and similar flying creatures are shut up in the larger cabin of a decked ship, and there is a vessel full of water there with fishes in it, then whether the ship stands still or runs, the same proper motion of the flying and swimming creatures will appear in either case; and that the same therefore happens with the proper motions of bodies, whether the Earth stands still or moves.

II. Argumentum à motu difficiliori versùs Occasum, & faciliori versùs Ortum futuro.

II. Argument from the motion that would be harder towards the west and easier towards the east.

[Margin: Form of the 2nd argument.] III. If the Earth moved with the diurnal motion, it would be harder to walk, swim or fly towards the West than towards the East. The consequent is false, and against common experience. Therefore the antecedent is false too. So argue Petrus Aliacensis on the *Sphere*, question 3, conclusion 3, and Scheiner in the *Disquisitiones*, page 31. They prove

the Major because the air too would have to move towards the East together with the Earth: therefore things walking, swimming or flying towards the West would feel the air coming against them, and that most rapidly, and would be driven back by it towards the East; so that to make any progress towards the West they would need far greater force and effort. About the Minor there is no controversy, for in itself, and as a rule, this difficulty is not felt.

[Margin: Reply to Argument 2.] The reply, however, is to distinguish the Major: it is conceded if the bodies walking, swimming or flying had, besides their own motion, no motion common to them, to the Earth and to our air, circular towards the East — a motion by which they would be carried towards the East prior in nature to being driven there by the air. But if they had this motion, as they are supposed to have it, the Major is denied.

III. Argumentum à rebus præsertim fluidis in Ortum obuertendis.

III. Argument from things, especially fluid ones, that would be turned towards the east.

[Margin: Form of the 3rd argument.] IV. Granting the diurnal turning of the Earth, the vanes of towers, the hair of women who have left it undressed, the flames and smoke of fiery bolides (or their radiant tails, and those of comets), threads tied to balls and thrown into the air, the sails of ships, the leaves of trees, the ears of grain in open fields, and smoke rising from chimneys and furnaces, would all be turned perpetually towards the East. The consequent is false; therefore so is that from which it would follow, namely the turning of the Earth. And it is much more false that the Earth moves with the annual motion on top of the diurnal, because the effects already mentioned would follow much more.

[Margin: Reply to Argument 3.] The reply is, as above, to concede the Major if the bodies already mentioned did not move towards the East with a motion common with the Earth, circular, and of equal speed; otherwise the Major is denied.

IV. Argumentum à Dissipatione multarum rerum in Aere.

IV. Argument from the dispersal of many things in the air.

[Margin: 4th argument.] V. If the Earth and the water, and with them all earthy and watery bodies, moved with the diurnal turning towards the same part of the world at equal speed, lead bullets would melt in the air from the excessive heat conceived from so rapid an impetus; water rising from the pipes of fountains would be scattered into imperceptible drops, and any cloud into mists; flames in the open air would be extinguished; the sound of bells would either be scattered or be heard more easily and more quickly in the western part; no fragrance of odours could be perceived by those to the east, since they are carried away very swiftly towards the East; and finally, the atoms we see flitting in a ray of the Sun would float off, scattered elsewhere. But we see none of these things follow. Therefore neither the Earth, nor the water, nor these bodies with the Earth or the water, move with the diurnal turning.

[Margin: Reply to Argument 4.] The reply is to deny the Major, if the air moves with equal speed and towards the same region of the world as the bodies already mentioned, as the asserters of the Earth's motion suppose it to move; for then the friction and resistance of the air, on account of which those effects would otherwise follow, cease.

V. Argumentum à Bombardæ globo exploso in Occidentem & in Orientem.

V. Argument from a cannon ball fired towards the west and towards the east.

VI. This argument is very celebrated, and is supported by the experiments of William, Landgrave of Hesse [William IV of Hesse-Kassel], and of Tycho. On these we must hear first of all Tycho himself, in the *Letters*, book 1, page 189: "For, that these things may be understood more clearly," he says, "an iron ball fired obliquely from the largest cannon, which they call a cartaun, scarcely reaches the ground, spent by its motion, within two minutes of time; and in that time it would have to be rolled round twenty thousand greater paces by the diurnal motion in the parallel of Germany, if the Earth were subject to the diurnal motion. For so Rothmann himself told me: that his Prince had once made a trial in firing a ball from a very large cannon (though not for this reason, but only to test the space and time of the ball's advance)." [Margin: The Landgrave's experiment on the time and space of the fall of a cannon ball.] Kepler adds, in book 1 of the *Epitome of Copernican Astronomy*, page 108: "The Landgrave and Brahe measured the time that elapses while a great cannon ball, driven out by the force of the fires, flies across through the air before it strikes the ground: they found it to be two minutes, and the space of its passage a great German mile." And from this Hérigone took it, when he says in tome 5, page 625, that the ball already mentioned had travelled four Italian miles in the air in that time before it struck the ground. But to return to Tycho: in that place he suggests to us his argument against the motion of the Earth, beginning with these words: "And what, I ask, will happen if an iron ball, or a lead one, or even a stone one, is fired from a larger cannon directed towards the east, and from that same cannon, placed in the same spot towards the west, and raised on both sides at equal angles with the horizon with respect to the former inclination? Is it to be thought possible that the ball, fired on both sides with the same quantity and force of powder, would cover just as much space on the ground, on account of the natural knowledge of motion by which any ball formed of earthly things would accompany the whole Earth? Where, then, will that most violent motion remain, aroused against nature by the gunpowder — a motion which is surely in some way a rival to that other natural motion, however very swift, by which the Earth would have to be turned round in a circle?"

[Margin: Tycho's argument from a cannon ball against the motion of the Earth.] After this he adds that there are three motions in that ball: one by which, by reason of its gravity, it would tend to the centre of the Earth along a

straight line, if it were not impeded; a second by which, through its consent with the Earth, it would imitate exactly the circulation of the whole Earth, if it were not impeded by its gravity; and a third, violent, from nitre mixed with sulphur and set alight with charcoal, by which it is driven to where it would not otherwise go naturally. With these laid down, he argues thus. If that third motion, produced by the gunpowder, has so much force that it impedes for a long time the natural descent of the ball intended by gravity — and impedes it so far that the ball is carried down towards the Earth only after a long space has been crossed, and only after two minutes of time — by what privilege will the second motion, by which that ball is said to imitate the circulation of the Earth, not be impeded by that violent third motion? Is it because the second motion is natural? But the first, intended by gravity, is natural too. Is it because the medium is different? By no means, for it is the same thin air, and it resists the violent motion no more. If, then, the violent motion impeded the second motion as well, some difference would appear in the motion and fall of the ball fired towards the East, compared with one fired towards the West; and yet none appears. "For experience testifies," Tycho says there, "that a ball of the same size and weight, fired in the way we have described, in turn, by the force of gunpowder of the same quantity and strength, leaves behind it very nearly the same space of the Earth's surface whether it is shot towards the east, with the same inclination of the same cannon, as I said, or towards the west — especially when the air is fairly calm and nothing per accidens furthers or retards this impulse or that. And yet, on account of the Earth's diurnal motion (if there were one), most rapid as it would be, the ball fired towards the east could by no means measure out so much space of the Earth's surface, since the Earth would somewhat outrun it by its own motion; while for the one fired in like manner towards the west, the Earth would then withdraw some of its surface by its own motion, and so increase the space intercepted." After this he narrates the Landgrave's experiment, which I reported at the beginning, and finally concludes: "By this single argument I think it sufficiently shown that the Earth has no proper motion from west to east, until he" — namely Rothmann, with whom he was disputing about this — "or someone else shows clearly, by invincible reasons, how it can happen that that excessively violent motion of which I spoke is not impeded at all by those two natural motions that he assumes, or even does not disturb them by any trace." Tycho's argument, therefore, reduced to the laws of the syllogism, runs as follows.

[Margin: Form of Argument 5.] VII. If the Earth were subject to the diurnal turning, the same ball from the same cannon, fired in the same way, would pass through less space when sent towards the East than when sent towards the West. But the same ball from the same cannon, fired in the same way, does not pass through less space towards the East than towards the West. Therefore the Earth is not subject to the diurnal turning. The MINOR is certain from the experiments asserted by Tycho. The MAJOR has been proved by the discourse of Tycho quoted above.

[Margin: 1st Reply, Gilbert's, to Argument 5.] William Gilbert replies, in book 6 of *De magnete*, chapter 5, by having recourse to the magnetic effluvia of the Earth, which draw the cannon ball round in the same way both towards the east and towards the west. For after saying that some people doubt how the balls of a larger culverin, shot with a similar quantity and vigour of gunpowder, and also with equal direction and altitude through the same air, could be shot an equal interval from one fixed place towards Eurus and towards the west, if the Earth moves towards Eurus, he adds: "But those who bring forward arguments of this kind are deceived: they do not notice the nature of the primary globes, such as the globe of the Earth is, and the combination with their own globes, even if the parts are not joined to them by solid parts. The Earth, however, does not move with its diurnal revolution by a separation of its more solid circumference from the bodies poured round it; rather, all the effluvia poured round it, and the heavy bodies in them, driven by force in whatever way, proceed uniformly together with the Earth by a general coherence." He adds, moreover, that gravity is an appetite by which the parts desire to cohere with their primary globes, and that they move with them; and that for this reason bodies are neither accelerated nor retarded on account of the diurnal revolution of the Earth. And he finally concludes that a projectile thrown with equal force towards Eurus and towards the west covers an equal distance on either side, even if the diurnal turning proceeds, just as a man's twenty paces make an equal interval both towards the east and towards the west. "In no way, therefore," he says, "is the diurnal motion of the Earth refuted by the illustrious Tycho Brahe with such arguments."

[Margin: 2nd Reply, Kepler's, to Argument 5.] Kepler, himself also an asserter of magnetic attractions, replies in clearer words, in the *Epitome of Copernican Astronomy*, book 1, page 134, beginning with these words: "Let the same judgement, then, mutatis mutandis, be passed on cannon balls as well. A great ball, lasting two minutes of one hour in its flight through the air, crosses towards the west through one German mile on the ground; meanwhile the ground beneath the Equator comes to meet it through eight miles. So, with respect to the space of the world, the ball is still carried away towards the region opposite to the violent motion, namely towards the east, by seven miles; and the firing in the opposite direction does it no other good than to use up the eighth mile and make the ball follow towards the east more slowly. The powder cannot shake the ball entirely out of the hands, as it were, of the Earth; it always stays fast, entangled in the drawing power." He could not have pictured the case of a ball fired towards the west in clearer formulas. He goes on, then, about the same ball fired towards the east: "Conversely, the ball sent towards the east is advanced in the same interval of time, by the rapt motion of the Earth itself, through 8 miles, and itself adds a ninth, since it is fired violently likewise towards the east. Thus, whether it is fired towards the east or towards the west, it is always carried towards the east; only this one a little more than that one. But this compounded space of the world has nothing to do with the

space on the ground that men can measure. That space is nearly the same on either side, because the force is the same, and because the magnetic bonds from which the ball is, as it were, snatched away and carried further are the same on either side." He adds, however, that if some slight difference were to exist, the opportunity of testing it by experiment would be lacking. "For who," he says, "will make me certain of the same force of powder in each firing, and of the other circumstances being the same on either side?"

[Margin: 3rd Reply, Hérigone's.] The reply of Pierre Hérigone in tome 5, page 625, however, is not so clear, and assumes other measures: namely, that the cannon ball already mentioned, fired at the Equator towards the west, moves towards the east in two hour-minutes through 26 Italian miles, and strikes and overturns walls because the motion of the walls, by the force of the diurnal turning, is faster than the cannon ball's motion towards the east; yet in such a way, he says, that if the ball stood altogether immobile, it would strike them more violently.

But replies of this kind are worthless, since they do not even touch the force of Tycho's argument. For Tycho does not hold that both motive powers of the ball are compounded in such a way that the elongation of the ball from the term of its motion turns out in reality such as one would have to understand it to result if each power produced its whole motion separately, or even successively. Rather, he contends that the powers already mentioned must be joined in such a way that one of them is nonetheless broken and tempered: namely, if the motive power towards the east would by itself have carried the cannon ball through 8 miles, then, when the impetus of the kindled powder, motive towards the west through 1 mile, comes upon it, it is weakened, and is not understood in fact to carry the ball, for its part, except through, for example, 7 ½ miles.

[Margin: 4th Reply.] The reply, however, is to deny the Major, because a greater space would rather appear to be traversed towards the East than towards the West, since towards the East the natural motion of the diurnal turning carried out by the ball would conspire with the motion from the kindled powder, which would then be by no means violent. Towards the West, on the other hand, the violent motion from the kindled powder, although it would blunt the motion produced by the diurnal turning, would nonetheless be blunted by it, and indeed more so, as by the stronger.

VI. Argumentum ab eiusdem Bombardæ globo prope polos, & in diuersis Parallelis explosæ.

VI. Argument from a ball of the same cannon fired near the poles and in different parallels.

VIII. A second argument of Tycho's is contained in the same *Letters*, at the end of page 189 and the beginning of page 190, in this discourse: "I add this too: that if, about the poles of the Earth, where the diurnal motion (if there were one) ends in rest, the same testing were made with a musket towards any part of the horizon in the manner described before, the outcome would be in

every way the same as if it were made midway between the two poles, at the Equator, where the motion of the Earth's circumference would have to be most rapid; just as also, in any horizon, if a ball is fired towards the east and the west in like manner, it covers the same space as one fired towards the south and the north with a similar impulse — whereas, if the Earth had any diurnal motion, that motion would regard the west and the east, but not likewise the south and the north. Since, then, these things happen uniformly everywhere, etc." Let this now be the form of the argument.

[Margin: Form of the 6th argument.] If the Earth moved with the diurnal motion, a ball fired in like manner from a cannon or a musket would pass through a notably different space when it was sent near or towards the poles than when it was sent in the parallels nearer the Equator, or towards the south or the north. But this is against the experiments. Therefore the Earth does not move with the diurnal motion. The MINOR seems to be established by the experiments themselves, if faith is placed in Tycho. The MAJOR is proved because, if the ball were fired towards the poles along the plane of the same meridian, less variation would be brought upon it by the diurnal motion than if it were fired now towards the east, now towards the west; and if in the parallels nearer the pole, the ball would be carried more slowly with the Earth, and if in those nearer the Equator, more quickly with the Earth, other things being equal, as is supposed.

The usual reply is to deny the Major, or to distinguish it: conceding it of a difference of real space in the space of the world, and denying it of a difference of apparent space, that is, of space that falls under our measurement. But in fact the argument holds, and this reply is worthless, according to what was said towards the end in connection with the preceding argument.

VII. Argumentum à globo item Bombardæ sumptum, & à Scopo, quem versus Orientem & versùs Occidentem feriret.

VII. Argument taken likewise from a cannon ball, and from the target that it would strike towards the east and towards the west.

[Margin: Argument against the diurnal motion of the Earth, falsely attributed to Tycho by Galileo.] IX. I am surprised that Galileo, in the second Dialogue on the System of the World, mentions Tycho's cannon more than once and yet does not set out Tycho's own argument, but one different from Tycho's, which he expounds as follows on page 131 of the Latin edition. If cannon shots were fired at an eastern target and the Earth moved, they would strike above the target; if at a western one, they would strike below the target. For the eastern parts of the Earth are continually depressed by the diurnal motion below the tangent parallel to the horizon, and conversely the western parts are raised; this is why the western stars are seen to become lower. Consequently, shots aimed along that tangent at an eastern target, which is meanwhile depressed while the ball runs along the tangent, ought to come out higher than they should, and western shots lower than they should, since the target has meanwhile been

raised. But this variation of aim is not confirmed by experiments; therefore it is a sign that the Earth does not move. To this argument, however, I draw from Galileo a twofold reply, which I will bring forward after the argument has been reduced to form.

[Margin: Formula of the 7th argument.] If the Earth moved with the diurnal motion, or even with the annual, a ball from the same cannon, fired either towards the east or towards the west [?], would be cheated of striking the target at which it was aimed. But it is not cheated. Therefore the Earth does not move with the diurnal motion. The MINOR is assumed as certain from observation. The MAJOR seems established by the reasoning of Galileo just quoted. Therefore, etc.

[Margin: 1st Reply, from Galileo.] Galileo's first reply, roughly, is to deny the Major. Even if the eastern target were continually depressed below the tangent by the force of the diurnal motion — if we speak of the first tangent, immobile in the space of the world — the cannon ball nevertheless, before it bends downward by the force of gravity, runs along one moving tangent after another, a tangent that is continually inclined, just as the whole length of the cannon is inclined by the force of the diurnal motion; and it does not cease to look towards the same target. Conversely, the axis of a cannon pointed towards the west is raised by the force of the diurnal motion, and along its length one must conceive one raised tangent after another, no less than for the western target. So in neither case would an aim that was otherwise duly taken be cheated of its effect.

[Margin: 2nd Reply.] Galileo's second reply does not indeed deny the Minor, but calls it into doubt. There he corrects certain Copernicans who, with excessive liberality, indulge their opponents and grant them certain experiments as true and certain, although they have never been performed. Among these he suspects this one can be counted: namely, whether shots towards the east come out higher than they should, and shots towards the west lower than they should. Indeed, he is convinced that this has never been tested by anyone, and that the experiment cannot easily be made: for no gunner will be found so skilled that he would claim to be able to hit the target itself with every shot, or not to miss the target by more than one cubit, if he has to fire from a distance of about 500 cubits or more. Further, if the Earth moved, the error due to that motion could not be greater than one cubit; and so in this experiment, however often repeated, the motion of the Earth could not be convicted of falsehood, since an error just as large can occur while the Earth is at rest. [Margin: The time in which a cannon ball strikes the target.] To prove this by a rougher calculation, he supposes that a shot from a larger culverin is made on the equinoctial line itself, where the greatest variation will arise because of the very swift motion of the equinoctial parts of the Earth, at a target 500 cubits distant towards the west. And he says it is certain that the culverin's ball reaches the target very quickly, in no longer time than a traveller takes to complete two paces, which he affirms to be less than one second of time. For if a traveller covers three Italian miles,

that is, 9000 cubits, in one hour, or 3600^{s}, then in one second he covers two and a half paces; so a single second of time is more than enough for the cannon ball's motion as far as the target. In so small a time, by the force of the diurnal revolution, 15″ of the Equator rise; and consequently the western target and horizon, in that circle whose semidiameter is supposed to be 500 cubits, will be raised by 15″. Now, from the tables of sines, the chord of an arc of one minute is less than 30 parts of which the radius is 100,000; therefore the chord of 15″ is less than 15 parts of which the semidiameter is 200,000. Therefore it will be less than four hundredths of one of those parts of which the semidiameter is 500 cubits. So the raising of the target, and the error of the ball due to the diurnal motion of the Earth, would not exceed four hundredths of one cubit, or about one finger's breadth; and it cannot be established by such an experiment whether the Earth is at rest or moves. Here, moreover, I have neither the leisure, nor is it useful, to correct Galileo's reasoning and calculation; I note only that he confused the chord with the sine of an arc of one minute. For the sine of this arc is 29 parts of which the radius is 100,000, so the chord is not 30 such parts but 58.

VIII. Argumentum à Bombardæ globo nunc in Septentrionem, nunc in Orientem Occidentemue exploso, per P. Franciscum Mariam Grimaldum excogitatum contra Motum Diurnum & Annuum Terræ.

VIII. Argument from a cannon ball fired now towards the north, now towards the east or west, devised by Father Francesco Maria Grimaldi against the diurnal and annual motion of the Earth.

[Margin: Father Grimaldi's theorem against the motion of the Earth.] X. So long as we look at the apparent spaces that bodies usually traverse, the arguments drawn from them against the Earth's motion do not have so manifest a force. But if we consider the increase of real percussion and impetus, somewhat stronger weapons against its motion are supplied to us from that source, as is clear from what was said in chapter 19. From this, then, it seemed possible to take another argument, resting on the impacts of cannon balls. Let it be supposed first — for the sake of explanation, not of necessity — that the ball of a very large cannon, that is, one weighing 60 or 80 pounds, reaches straight to a target 250 paces distant in the time of two beats of the human artery, or about two seconds of time, which those skilled in this art would readily grant. And in the following figure let the mouth of the cannon be A, aimed at the eastern target B, which is distant by the interval AB of 250 paces. Now if the Earth, with its kindred bodies, were not moved with a common motion towards the east, the ball from A would reach the target B in a time of 2^{s}. But because both are carried with equal motion by the diurnal turning, and in two seconds complete 30″ of the Equator — that is, 752 Roman geometric paces, from the first small table of chapter 19, no. [?] 13, set out above — let the chord of this arc be AC, taken as a nearly horizontal line, along which the mouth of the cannon A is understood to be carried over to C, and the target from B to D,

each having completed 752 paces. For the target D will be distant from the mouth C by the interval CD of 250 paces, as before, and the ball I, fired from A, will strike it. Now let the cannon PA be aimed towards the northern target E, so that the cannon is in the position AQ, and let the interval AE be 250 paces,

Diagram. A woodcut in the right column illustrating Grimaldi's cannon argument. A horizontal cannon lies with its breech at P and its mouth at A; a second cannon hangs vertically from A down to Q. To the right of A a ball stands at B, on a dotted line running to a second horizontal cannon whose mouth is at C; from C a further cannon hangs vertically down to R. A dotted vertical rises from A to a ball at E; a solid curve rises from A through K towards G, and a dotted straight line runs from A through H towards G; a dotted vertical rises from C to L, just below G. Above G stands N over a circled ball F, and a dotted line runs from G out to M. To the right of C stands a pair of balls labelled I and D.

- P (left, breech of the horizontal cannon): P
- A (mouth of the left horizontal cannon): A
- Q (foot of the cannon hanging from A): Q
- B (ball to the right of A): B
- C (mouth of the right horizontal cannon): C
- R (foot of the cannon hanging from C): R
- E (ball above A): E
- K (on the curve from A): K
- H (on the dotted line from A): H
- L (top of the dotted vertical from C): L
- G (upper right): G
- N (above F): N
- F (circled ball beside G): F
- M (end of the dotted line from G): M
- I, D (pair of balls right of C): I, D

that is, exactly equal to the interval AB; and let everything else be equal as well — namely, the cannon cooled, the same ball, the quantity and quality of the gunpowder of the same measure and kind, the inclination or elevation of the cannon, the temper of the air, and so on — so that in the same time of 2^s the ball from A could reach E along the straight line AE, if the Earth stood still. But since the Earth is posited to be carried at least by the diurnal motion, and with it every heavy body akin to it, then certainly, when the cannon has been carried over into the position CR, having completed the 752 paces of the interval AC, the target too will have been carried in its parallel to N, and the cannon ball to F, where it will strike the target, having completed, as far as appearance goes, only the interval CF of 250 paces. In reality, however, it will have completed in the space of the world a much greater path, namely AKF, whose chord AHL will be 825 paces. This will be clear from the laws of triangles if, in the triangle ACF, right-angled at C, in which AC of 752 paces and CF of 250 paces are given, the base AHF

is sought; and in the same operation the angle AFC will be found to be 70° 35′, to which the angle NFM is equal.

With these suppositions, it seems that the cannon ball would strike the northern target N with a far weaker blow and a feebler percussion than the eastern target D, so that the breaking of a wall, or the pushing away of another ball set up as the target, would be notably less; and this difference would easily have been observed, or could be observed. For two causes would concur to weaken the impetus of a ball sent towards the north. [Margin: 1st cause of the weaker impetus towards the north.] *First*, the impetus of the ball on its exit from the mouth of the cannon A would be impressed all at once, and would be of such a kind in itself, and so great, that if the Earth were immobile it would strike the target B just as well as E, along the straight line AB or AE, at an equal interval of 250 feet. But both the Earth and the ball are posited to be carried towards the east; and while the ball strives towards E, the diurnal motion prevails, so that it deviates and is twisted away from the straight line AE and drawn along the curve AKF towards F (because at the beginning this motion is swifter, and the ball is carried beyond the straight line AHF which it would describe if the motion were uniform). It is therefore necessary that its impetus be broken and greatly weakened, since it is resisted by the diurnal motion and is forced to go by the longer and oblique path AKF, when otherwise it would go by the straight and shorter path, along AE, to E. This happens in the same proportional way as when someone drops a lead ball down a tube standing perpendicularly on his hand and very swiftly carries the tube sideways: the impetus and descent of the ball would certainly be greatly retarded and weakened, and in its fall it would strike the hand beneath with a much smaller blow than if it had been allowed to fall from the same height through a tube that was not moved.

[Margin: 2nd cause weakening it towards the north.] *Second*, although to the eye at C the cannon ball would seem to strike the target N at the point F along the line FN, in reality it would strike it with a very oblique blow along the track LM. The line of direction along which the impetus, twisted by the force of the diurnal motion away from the straight lines AE and FN, would be spread would be AHL — indeed it would be a portion of the curve AKF, declining from the straight line CF still more than by the angle AFC, which we found above to be 70° 35′. Hence it would come about that the ball would strike the target not with its point G, which is foremost in the motion and is carried, like the head of a battering ram, along the straight line LG, but with its lateral point F, which declines from the real path of the impetus LGM — the path it would run through in the space of the world, and along which it would strike every obstacle standing in that path. So, striking not straight at F but with a fleeting, as it were, and very oblique contact, which would rather deserve the name of rubbing or grazing than of a blow and a wound, it would scarcely strike the target N. And if beside N there were another target at G, placed to the right towards the Greek wind, the ball would strike that one with

a much greater impetus, even if neither the cannon nor the ball were aimed at it, since it would be driven against it according to the real direction of its impetus; and there it would break through a wall with greater ruin. And if the target N and the target G, equal in all other respects, could be pushed away, the target G would be driven much further — say, through the space GM — than the other, say through a space equal to FN. For we commonly see in the game of billiards, or with leather balls struck with rackets, how much weaker the blow and the push are if one ball is struck by the other with a contact that slips away as if along a tangent, or if the ball is caught on a slicing, slanted racket and sent back, than if the diameter of one ball strikes straight against the diameter of the other with its full impetus, or the leather ball is caught and sent back by a perpendicular meeting. For these two causes, then, which would weaken a northward blow compared with an eastward or westward one, and which nevertheless do not in reality weaken it, it seems clear that the Earth does not move with the diurnal motion, and much less with the annual, which would weaken that blow more. Let the argument now be put in form.

[Margin: Form of the 8th argument.] If the Earth moved with the diurnal motion, or even with the annual, the blow of a cannon ball fired towards the north or towards the south would be much weaker than towards the west or towards the east. The consequent is false; therefore so is the antecedent from which it follows. What is said of the cannon ball, moreover, holds for many other projectiles. The MAJOR seems sufficiently proved by the preceding reasoning. The MINOR is assumed as certain from an experiment that very skilled gunners can perform, who aim so surely that they are able to stop up the mouths of enemy cannons with their balls; for they would at some time have observed this difference, or could observe it.

[Margin: Reply of the Copernicans.] The Copernicans will perhaps reply by denying the Major. For, they will say, the motion of the cannon ball, insofar as it comes from the kindled powder, has of itself its direction towards the target, which from the very beginning, by the kind of motion impressed on it, it intends to strike straight; and although it is carried along an oblique path, nevertheless, because the target is carried along with it with equal motion and velocity (even in the very brief, almost momentary, time in which it is struck), it comes about that the percussion is finally made directly. Just so, if an eye looking through a narrow tube at the flame of a candle placed towards the north is carried by a ship towards the east together with the candle, it will see the flame with a direct gaze; a ball on a gaming table, likewise carried by a ship towards the east and pushed against a ball to the north of it, will strike that ball directly; and finally a hammer, driven by the arm of a sailor carried towards the east against the ship's mast standing to the north, will strike through the mast directly towards the north — however much the ball and the hammer are in reality snatched sideways by the ship's motion and run through an oblique path aimed by two principles of motion mixed together. [Margin: Weakness of the reply.] The reply, however, is good as to the straightness of the percussion,

but weak as to its vehemence, which would be diminished and enfeebled all the more, the more the motion snatching the ball or the hammer sideways resists the impetus produced at the beginning. For we are certain from very many experiments that a motion once impressed, and moving towards one part, is weakened and diminished by an impetus that moves not only towards the contrary part but also towards a different part, that is, sideways. It is true indeed that in the example of billiards and of the hammer, because of the brevity of the path, the difference between percussions made once with the ship unmoved and again with it in motion is not so notable; and much less in the case of the hammer, whose impetus is restored by the arm as by a conjoined principle, even though this restoration is not noticed.

IX. Argumentum Nostrum contra motum Terræ Diurnum simul & Annuum, ab ictu & percussione maiore globi versus Orientem, quàm versus Occidentem proiecti.

IX. Our own argument against the diurnal and annual motion of the Earth together, from the greater blow and percussion of a ball thrown towards the east than of one thrown towards the west.

XI. This argument proceeds in a way altogether contrary to Tycho's, adduced in number 7; for it runs as follows. [Margin: Form of the 9th argument.] If the Earth moved with the diurnal or even the annual motion, the same ball, pushed with the same force over the same distance, once at an eastern target and again at a western one, would strike the eastern target with a stronger blow than the western. The consequent is false; therefore so is the antecedent. The MINOR is most certain from experiments: both with billiards, in which we push ivory balls on a gaming table against another ball that is unmoved and equally distant, now towards the east, now towards the west; and with many other similar projectiles. The MAJOR is proved because the impetus of a ball thrown towards the east would not be retarded by the diurnal or annual motion of the Earth, but would rather be helped by it, and would in turn assist it, since both strive towards the same region. Conversely, the impetus of a ball sent towards the west would retard or blunt the impetus moving towards the east, and con- versely would be blunted by it, since they tend in contrary directions (granting that the impetus of the diurnal and annual motion together is much stronger), and so the one resists the other — not absolutely, but in a certain respect. So, if a clay ball of an ounce, shot by a crossbow downward from the top of a tower at a target of clay 30 feet distant, strikes and pierces the clay with a far greater blow, and penetrates it further, than if the same ball were driven by the same crossbow upward against a clay target of the same softness, also 30 feet distant, the reason is plainly that in the former case, besides the impetus impressed by the crossbow, there is also an impetus produced by gravity, which by itself would strike the clay in a headlong fall, and each helps the other; in the latter case, by contrast, the impetus produced by gravity resists the impetus impressed by the crossbow and blunts it more and more. [Margin:

No suitable reply.] To this argument I certainly find no solid reply, nor one that is persuasive while the natures of things are preserved. For the examples that could be brought forward, on the hypothesis of a standing Earth, of projectiles sent now towards the east, now towards the west, over the deck or planks of ships while the ship is always carried towards the east, either have an uncertain or unobservable difference of percussion, because of the brevity of the path, or fall short of perfect likeness and do not include two intrinsic impetuses which in one case help each other and in the other blunt each other by mutual resistance.

X. Argumentum à Contrarietate Motuum.

X. Argument from the contrariety of motions.

[Margin: Form of the 10th argument.] XII. If the Earth moved with the diurnal motion, or even with the annual, the same mobile body would be moved at the same time with two contrary motions. But it is physically repugnant for the same mobile body to be moved at the same time with two contrary motions. Therefore the Earth moves with neither the diurnal nor the annual motion. The MAJOR is proved because it is manifest that many bodies really move towards the west; but if the Earth moved towards the east, then, for many other experiments to be saved, those same bodies would have to be moved meanwhile towards the east by a certain motion common to all terrestrial and watery bodies — and this is exactly what Copernicus and his followers assert. Therefore they would be moved at the same time towards different regions and termini: by their proper motion they would approach that very western terminus from which they would meanwhile be receding by the common motion; or, while approaching the eastern terminus by the force of the common motion, they would recede from it by the force of their proper motion. The MINOR is proved because it is impossible for the same thing at the same time to approach and recede with respect to the same terminus, since recession has annexed to it the privation of approach to the terminus from which the recession is made, and a privation and a form cannot be physically exercised together.

[Margin: 1st Reply to Argument 10.] The first reply, however, is to deny the Major in the sense in which the Minor is true, that is, of two motions truly and really contrary [?] in relation to termini fixed and immobile in the space of the world. For even if, comparing the terminus a quo on the surface of the earth or the water with the terminus ad quem which bodies tending westward would apparently approach, they would seem to recede from the eastern terminus a quo, yet in reality they would always approach the eastern terminus fixed in the space of the world. For the common motion of the Earth's diurnal revolution, and much more of its annual revolution, is swifter than the proper motion of any body tending towards the west — granted that, because of this proper motion, the approach towards the east would be diminished, an approach which would otherwise be greater if there were no motion at all towards the west. The reply will become clearer [?] by an example. Let a traveller on the Equator go forward for one hour towards the west, departing

from some column and covering three thousand Roman paces on the surface of the earth. In the meantime the diurnal turning of the Earth will snatch him towards the east, just as it does the column; but it will snatch the column in such a way that in the space of the world it rolls it towards the east through 15°, that is, through 1,355,000 paces, whereas it will snatch the traveller in such a way that it rolls him through only 1,352,000 paces, or 1352 Italian miles — the 3 miles that the traveller has meanwhile covered being subtracted. He covered them not by receding towards the west from the fixed place of the world in which the column stood, but by receding towards the east from that fixed place less than he would have receded had he let himself be snatched by the common motion alone, without any proper motion of his own. See the figure and its exposition in Section 2 of this Book, chapter 3, scholium 1.

[Margin: 2nd Reply [?].] Galileo replies, second, in the second Dialogue on the System of the World, page 200 of the Latin edition, by turning the argument back. For if these two motions are denied to the Earth, two contrary motions must be attributed to the celestial bodies; in whatever way, then, they can be reconciled in the heaven, in the same way they can be reconciled on the Earth. He adds, moreover, that if any fatigue were to be feared, it should rather be feared for the starry sphere than for the Earth, since the sphere would have to carry much greater bodies than the Earth does.

XI. Argumentum à Multiplicatione motuum eiusdem speciei minimè necessaria.

XI. Argument from the wholly unnecessary multiplication of motions of the same kind.

[Margin: Form of the 11th argument.] XIII. Motions are not to be multiplied without necessity. But if the Earth moved with the diurnal, or even the annual, motion, innumerable motions would be multiplied without necessity. Therefore the Earth moves with neither the diurnal nor the annual motion. The MAJOR is established by an axiom received by all, and so insisted upon by the Copernicans that on this very ground they try to take the diurnal motion away from the fixed stars and the planets and transfer it to the Earth alone, and to accomplish the motions of the epicycles of the five lesser planets by the single motion of the Great Orb. The MINOR is proved because, to save the phenomena of sublunary motions, each and every terrestrial and aquatic body — among which there are innumerable mixed bodies, some animate, some inanimate — would have to have, besides its own proper motions, a common motion by which it would revolve with the Earth daily and yearly towards the east. But all these motions can be avoided, with every phenomenon whatever saved, if the Earth is at rest, as the Copernicans openly concede; and so they are multiplied without necessity.

[Margin: Reply.] The Copernicans, however, could reply either by turning the argument back, or by distinguishing the Major: conceding it if, by not multiplying motions in sublunary bodies, a multiplication of motions of the

same kind in the celestial bodies could be avoided, and otherwise denying it. But in the sense in which they will concede the Major, they will deny the Minor. For if the diurnal motion is not conceded to the Earth and its kindred bodies, it will have to be conceded to the innumerable fixed stars and to all the planets. Likewise, if the annual motion through the Great Orb is not attributed to the centre of the Earth and to the whole elementary system together with the lunar heaven, it will have to be attributed to five distinct epicycles of the lesser planets, or to other circles or ellipses equivalent to them. Since, therefore, these two motions must be multiplied in one set of bodies or the other, and no necessity appears for multiplying them in the celestial rather than in the subcelestial bodies, the reason for attributing them to the one rather than to the other must be sought from elsewhere. Let this, then, be the twelfth argument.

XII. Argumentum à Multiplicatione motuum in ijs corporibus, in quibus maius est fundamentum eos multiplicandi, & minor multiplicatio fit.

XII. Argument from the multiplication of motions in those bodies in which there is a greater ground for multiplying them and the multiplication is smaller.

[Margin: Form of the 12th argument.] XIV. If the diurnal and annual motion is to be attributed to one set of bodies or the other, it is rather to be attributed to those in which a greater ground appears and the multiplication is smaller, than to those in which a lesser ground appears and the multiplication is greater. But a greater ground appears for attributing the diurnal and annual motion to the celestial bodies, and in them the multiplication would be smaller than in the Earth and the remaining bodies. Therefore it is rather to be attributed to the celestial bodies. The MAJOR is evident. The MINOR is easily proved, because we have a sufficient ground for the diurnal and annual motion from phenomena and observations manifest to the senses; nor is there any repugnance to the contrary, nor any argument for asserting that the senses are deceived in these matters. Next, if the diurnal motion is attributed to the celestial bodies, it is attributed only to the eighth sphere, by whose single motion the fixed stars can be moved, and to the seven planets with the 4 companions of Jupiter and the 2 of Saturn — that is, to 14 bodies in all. And if the annual motion is attributed to the Sun, and the motion proportional to it to the 5 planets in their epicycles, it is attributed to only six bodies; or, if here too you add the companions of Saturn and Jupiter, it is not attributed except to 13 bodies. But if the diurnal and annual motion is attributed to sublunary bodies, a motion is attributed to them that rests on no observation perceptible to the senses, that is, on no a posteriori ground obtained through the senses. The diurnal motion would have to be attributed to the Earth and to absolutely all the sublunary bodies situated below the upper region of the air, which are far more than 14 both in number and in kind; and the annual motion not only to all sublunary bodies but also to the Moon itself and to the lunar heaven. [Margin: No reply to Argument 12.] To this argument I see not even the shadow of a solid reply — unless the Copernicans, abandoning and scorning the ground taken from the senses, have

recourse to congruences a priori. But since the greatest of these congruences consists in avoiding a multiplicity of motions, they cut their own throats with their own sword: they fall into a far more numerous multitude of motions, and meanwhile undermine the foundations of physics, that is, sensations.

XIII. Argumentum à Destructione motuum, qui apparent, & Substitutione motuum, qui non apparent.

XIII. Argument from the destruction of the motions that appear, and the substitution of motions that do not appear.

[Margin: Form of the 13th argument.] XV. If the diurnal motion, or the annual as well, is attributed to the Earth, the motions that appear are destroyed without necessary reason, so that none of them really exist; and in their place motions are substituted that never appear. But that is absurd. Therefore, etc. The MINOR is self-evident. The MAJOR is proved because both the diurnal and the annual motion of the Earth, and of the other bodies that imitate it in these motions, is greater than the proper motion of any of them. From this it follows that the proper motion which appears in these bodies towards the west is not really a motion, since by it the body does not approach the same fixed point towards the west; it is only a diminution of the greater motion towards the east, as was said in the reply to argument 10. The diurnal and annual motion of the Earth and its kindred bodies, on the other hand, which is substituted as the real one, in no way appears.

XIV. Argumentum à Multiplici Inæqualitate ac difformitate motus in eodem mobili absq. necessitate.

XIV. Argument from the manifold inequality and nonuniformity of motion in the same mobile body without necessity.

[Margin: Form of the 14th argument.] XVI. If at least the diurnal motion is conceded to the Earth, a motion unequal and nonuniform with innumerable variety must be attributed to the same mobile body, without any necessity. But this is unfitting. Therefore, etc. The MINOR is certain in itself. The MAJOR is evident, because the same terrestrial body, even when separated from the earth — say a cannon ball shot towards the east, or a cloud, or a bird — must move faster at the Equator than in the next parallel, faster in that one than in the following one, and so on for all the others without limit, as many as can be described up to the poles. Hence, if a bird flies from the Equator towards the poles, straight or obliquely, then even though its flight is uniform to appearance, or even faster and faster, it must in reality move continually slower and slower, so as to adapt itself to the slower and slower motion of the parallels. This would certainly happen more often, and in more bodies, than if the diurnal motion were attributed to the fixed stars. The Copernicans do not deny that this happens without necessity; [Margin: No reply.] and they cannot reply to this argument except by saying that this variety follows from their hypothesis, whose foundations they have not in sensations but in ideal reasons

and in certain congruences. But among the greatest of these congruences is the one they so often inculcate, that simplicity and uniformity of motions be established — and this they violate far more numerously, by attributing it to so many sublunary bodies. And so the reply comes to nothing and destroys itself. If you wish to know the inequality of the motion of terrestrial bodies to a definite measure — as they stand either at the Equator or in other parallels, and as they move about noon or about midnight — run through the small tables to be given in the following chapter at the end of number 1, especially 5, 6 and 7.

†

CAPVT XXII.

Chapter 22.

Proponuntur Argumenta Quinque sed inualida ab Impetu & Velocitate nimia Telluris & Cognatorum Corporum, si mouerentur motu siue Diurno solùm, siue etiam Annuo, & vtriusque motus Mensuræ hac occasione traduntur alia ratione quàm factum sit capite 19. numero 13. vt hac fortasse aliquibus magis satisfiat.

Five arguments are proposed, but invalid ones, from the impetus and excessive velocity of the Earth and its kindred bodies, if they moved either with the diurnal motion alone or with the annual as well. On this occasion the measures of both motions are given by another method than was done in chapter 19, number 13, so that this one may perhaps satisfy some readers more.

I. Before we discuss the effects that the velocity of the moving Earth would produce, that velocity itself must be set out in determinate parts of space known to us, which the Earth would traverse in one hour, or in one minute of an hour, or in one second. For this we need four elements, or foundations, for constructing the following Table. [Margin: 1st foundation of the following tables.] The first is how many miles are contained in the diameter and in the greatest circumference of the Earth, such as the Equator is; on this question we reported a great many opinions in Book 2, chapter 7, at the end of scholium 9. Now the authors who dispute about the motion of the Earth, most of them, together with the Germans and some seamen, assign 60 Italian miles, or 15 German, to one degree of the terrestrial Equator, and 1720 German miles to the diameter of the Earth — so Tycho, Longomontanus, William Gilbert, Mästlin, Metius [Adriaan Metius], Hérigone, Scheiner and, for the most part, Kepler. We, on the other hand, found by exact measurement 72 ¼ Italian miles of this Bologna measure in one degree of the Equator, and 8278 for the diameter of the Earth; close to this come Gassendi, who assigns 73, very many seamen, who assign 70, most of the Arabs, and also Fernel and Snel, who assign about 68 Italian miles to it, as we said in the same place. Since that is so, we shall construct the table according to only these two opinions, namely that of 60 Italian or 15 German miles, and that of 72 ¼ Italian. [Margin: 2nd foundation of the table.] The second thing we need is how many minutes on the Equator

correspond to one degree of the parallel equally distant on either side from the Equator and from the poles of the Equator, that is, declining 45 degrees; and likewise how many correspond to one degree of an almost smallest parallel, that is, distant by a single degree from the pole and declining 89 degrees from the Equator — so that a judgement can be made of the rest from the middle and the extremes. Now, by the problem given by Giuseppe Moletti and Giovanni Antonio Magini on chapter 23 of Book 1 of Ptolemy's *Geography*, and by the table constructed by the same Moletti, to one degree of the parallel declining 45° there correspond on the Equator 42′ 24″ 35″ [?]; but to one degree of the parallel declining 89° there corresponds on the Equator 1′ 2″ 51‴. [Margin: 3rd foundation of the table.] The third thing we need is the semidiameter of the Great Orb, which doubled gives the diameter. Hence, if one makes, as 100 is to 314, so the given diameter to another number, the circumference of the Great Orb is obtained, according to the rules given in Book 1, chapter 4; the circumference divided by 360 gives the size of one degree, a degree divided by 60 gives that of one minute, and a minute subdivided by 60 gives the size of one second. Among the opinions on the semidiameter of the Great Orb — that is, on the mean distance between the Sun and the Earth — there are several, reported by us in Book 3, chapter 7, at the end; but it will be enough to follow two that are, as it were, the extremes, and one in the middle. The first is that of Copernicus, who puts that distance at 1142 terrestrial semidiameters, and many use this figure in the present controversy; the second is ours, by which we assign 7300 semidiameters of the Earth to this distance; the third, as it were midway between these, was Kepler's, who finally assigned 3381 terrestrial semidiameters to it. Now if, with the Germans and many others, we assign 1720 German miles, or 6880 Italian, to the diameter of the Earth, then in the diameter of the Great Orb, which for Copernicus is 2284 semidia- meters, it will be 3928 480 German miles, or 15 713 920 Italian. For Kepler, for whom that diameter is 6762 terrestrial semidiameters, it will be 11 630 640 German miles, or 46 522 560 Italian. For us, for whom that diameter is 14,600 terrestrial semidiameters, it will be, according to our opinion on the diameter of the Earth, 120 858 800 Italian miles of Bologna. From these the rest will be derived, as can be seen in the table. [Margin: 4th foundation.] The fourth and last thing we need is how many degrees or minutes some one point of the Equator traverses on its own in one hour, one minute, or one second of an hour by force of the diurnal revolution, and likewise how many minutes of the circumference of the Great Orb the centre of the Earth traverses in one hour, or one minute, or one second of an hour, so that we can apply to them the corresponding number of miles. For if a point of the Equator, or the point corresponding to it in another parallel, is in such a position that the Sun apparently rises or sets for it, the diurnal motion will add nothing to the annual motion and take nothing from it; but if it is at the point of midnight, the diurnal motion must be added to the annual; and if at the point of noon, subtracted — so that in this way the

velocity of the parts of the earth becomes known from the composite motion. On these foundations, then, the following Tables rest.

I. TABVLA.

Table I.

Table I	Copernicus, Tycho, Mästlin, Kepler, Longomontanus, Gilbert, Scheiner, Hérigone, etc.: German miles	Italian miles	Paces	Ours: Italian miles	Paces
Diameter of the Earth	1720	6880	✠	8278	
Circumference of the Earth	5400	21600	✠	26010	
Equator: degree 1	15	60		72 ¼	
Equator: minute 1	0 ¼	1		1 204/1000	
Equator: second 1	0	0	16 ⅔	0	20 1/15
Parallel declining 45°: degree 1	10 ½	42 5/12		51 1/6	
Parallel declining 45°: minute 1	0	0	706 11/12	0	852 ¾
Parallel declining 45°: second 1	0	0	11 47/60	0	14 ¼
Parallel declining 89°: degree 1	0	1 1/20		1 ¼	
Parallel declining 89°: minute 1	0	0	17 ½	0	20 5/6
Parallel declining 89°: second 1	0	0	0 ⅕	0	about ⅖

II. TABVLA.

Table II. A terrestrial point traverses:

Table II	Copernicus, Tycho, Mästlin, Kepler, etc.: German miles	Italian miles	Paces	Ours: Italian miles	Paces
In 1 hour, 15 degrees; therefore, at the Equator	225	900		1087 ½	
In the parallel declining 45°	157 ½	636 ¼		767 ½	
In the parallel declining 89°	about 4	15 ¾		18 ¾	
In 1 minute of an hour, 15 minutes; therefore, at the Equator	3 ¾	15		18 6/100	
In the parallel declining 45°	2 65/100	10 604/1000		12 791/1000	

Table II	Copernicus, Tycho, Mästlin, Kepler, etc.: German miles	Italian miles	Paces	Ours: Italian miles	Paces
In the parallel declining 89°	0	0	262 ½	0	312 ½
In 1 second of an hour, 15 seconds; therefore, at the Equator	0	0	250	0	301
In the parallel declining 45°	0	0	175 ¾	0	213 ¾
In the parallel declining 89°	0	0	3	0	5

III. TABVLA.
Table III.

Table III — Great Orb	Copernicus: German miles	Copernicus: Italian miles	Kepler: German miles	Kepler: Italian miles	Ours: Italian miles
Diameter	3928 480	15 713 920	11 630 640	46 522 560	120 858 800
Circumference	12 235 427 ⅕	48 941 708 ⅘	36 520 209 ⅗	146 080 838 ⅖	379 496 632
1 degree	33987 11/36	135 949 2/9	101445 1/36	405 780 1/9	1054 157 14/45
1 minute	566 27/60	2265 13/15	1690 ¾	6763	17569 3/10
1 second	9 13/30	37 11/15	28 1/6	112 ⅔	292 ⅚

IV. TABVLA.
Table IV. The centre of the Earth traverses in the Great Orb:

Table IV	Arc traversed	Copernicus: German miles	Copernicus: Italian miles	Kepler: German miles	Kepler: Italian miles	Ours: Italian miles
In 1 hour	2′ 28″	1397 ¼	5589	3607 1/6	14428 ⅔	43338
In 1 minute	2″ 28‴[47]	23 3/10	93 2/15	60 7/60	240 7/15	722 3/10
In 1 second	2‴ 28 [?]	0 23/60	1 33/60	1	4	12 1/30 [?]

Compounding, therefore, the diurnal motion of some point on the surface of the Earth with the annual motion of the centre, the space and the velocity come out as below.

V. TABVLA.
Table V. If the Sun rises or sets for a point of the Earth, that point of the Earth traverses the miles written below:

Table V	Copernicus: German miles	Copernicus: Italian miles	Kepler: German miles	Kepler: Italian miles	Ours: Italian miles
1 hour	1397 ¼	5589	3607 1/6	14428 ⅔	43338

[47]so printed (the figure is confirmed); the series wants 28‴

Table V	Copernicus: German miles	Copernicus: Italian miles	Kepler: German miles	Kepler: Italian miles	Ours: Italian miles
1 minute	23 3/10	93 2/15	60 7/60	240 7/15	722 3/10
1 second	0 23/60	1 33/60	1	4	12 1/30

But if it is midnight at some point of the Earth, that point traverses a space compounded of the diurnal and the annual space, that is:

VI. TABVLA.

Table VI.

Table VI	Copernicus: German miles	Copernicus: Italian miles	Kepler: German miles	Kepler: Italian miles	Ours: Italian miles
1 hour, at the Equator	1622	6488	3832	15328	44425 ⅕
In the parallel declining 45°	1555	6220	3765	15060	44105 ½
In the parallel declining 89°	1401	5604	3611	14444	43356 ¾
1 minute, at the Equator	27	108	75 ⅛	300 ½	740
In the parallel declining 45°	26	104	70 ⅗	182 ⅖	735
In the parallel declining 89°	23 ⅗	93 ⅗	49 ⅖	197 ⅗	723
1 second, at the Equator	0 23/93	1 33/60	1 1/16	4 ¼	12 301/1000
In the parallel declining 45°	0 25/93	1 32/60	1 1/20	4 ⅕	12 214/1000
In the parallel declining 89°	0 22/93	1 31/60	1 3/4000	4 3/1000	12 5/1000

Conversely, if it is noon at some point of the Earth, the diurnal space must be subtracted from the annual, and so that point traverses the miles written below.

VII. TABVLA.

Table VII.

Table VII	Copernicus: German miles	Copernicus: Italian miles	Kepler: German miles	Kepler: Italian miles	Ours: Italian miles
1 hour, at the Equator	1172	4688	3382	13528	42250 ½
In the parallel declining 45°	1240	4960	3450	13800	42570 ½
In the parallel declining 89°	1393	5572	3603	14412	43319 ¼

Table VII	Copernicus: German miles	Copernicus: Italian miles	Kepler: German miles	Kepler: Italian miles	Ours: Italian miles
1 minute, at the Equator	19 ¼	77	45 7/60	180 28/60	704
In the parallel declining 45°	20 65/100	82 ⅗	49 ⅖	197 ⅗	709 ⅕
In the parallel declining 89°	23 ⅕	92 ⅘	59 ¾	239	721 ⅔
1 second, at the Equator	0 23/93	1 31/60	0 15/16	3 ¾	11 699/1000
In the parallel declining 45°	0 23/93	1 32/60	0 19/20	3 ⅘	11 786/1000
In the parallel declining 89°	0 24/93	1 33/60	0 3997/4000	3 997/1000	11 995/1000

I. Argumentum ab Inertia Telluris ad omnem motum, præsertim, circularem, & tam celerem.

I. Argument from the inertness of the Earth to all motion, especially circular motion, and motion so swift.

[Margin: Form of the 1st argument.] II. The Earth is the heaviest and densest of all bodies; therefore it is naturally incapable of any motion, especially circular. The antecedent is denied by no one of sound judgement. The consequence is proved because heaviness was given to bodies so that, in proportion to their heaviness, they should tend towards the centre of the universe along a straight line, and rest there once they have reached their place. Therefore, if the earth is the heaviest of all bodies, it must occupy the centre of the universe with its own centre, and rest there; and if it were removed from it, it would return to it naturally by no other motion than a rectilinear one. This is the sum of the argument and of its proof, which the Peripatetics hold implicitly, in other words, together with Aristotle, *De caelo* 2, chapter 13, and Tycho in his *Epistolae Astronomicae*, page 147. But the Copernicans, and Galileo first of all, will easily deny the consequence of this argument, as is clear from their foundations set out above in chapter 20, numbers 2 and 3. For they will say that heaviness is not a force of tending towards the centre of the universe, but an inclination which parts have to be united with their wholes and their primary globes; that rectilinear motion does not belong to those parts except apparently, whereas in reality circular motion alone belongs to them, and per se; that if they are outside their place, a curved motion belongs to them, mixed of the circular and of another striving towards the straight, but not straight; and that to the primary globes of the world themselves, of which kind the Earth is, circular motion about the centre of the world belongs per se. Hence, if the Earth were placed at the centre of the world, it would withdraw from there along some curve to the periphery of the Great Orb, and thereafter continue its circuit along that periphery. Kepler, moreover, in Book 1 of the *Epitome of Copernican Astronomy*, page 108, denies that the whole Earth can be called heavy insofar as

it is whole, since there is nothing outside it that attracts it — granted that he attributes to the Sun a force of attracting the Earth and the other planets, or of driving them round; and he adds that the motion which must be attributed to the fixed stars, if it is denied to the Earth, is much faster.

II. Argumentum à nimia Velocitate Terræ, quàm nullum mobile versùs Orientem assequi posset.

II. Argument from the excessive velocity of the Earth, which no mobile body moving towards the east could overtake.

[Margin: Buchanan's argument against the motion of the Earth.] III. George Buchanan painted this popular argument in poetic colours, in Book 1 of his *Sphaera* set in verse, in these lines:

Picture in your mind the Pole standing still, its body unmoved, its fires sluggish; and the Earth rolling itself round in a circle, and through thrice eight hours of shadow and of light closing its perpetual round along its own tracks. This force of its course swift arrows could not equal, nor the wings of birds, nor the blasts of the wind, nor the stones that the violence of sulphurous flame drives out whenever the heat shut up in the hollow bronze rages.

[Margin: Form of the 2nd argument.] The argument can therefore be framed thus. If the Earth revolved towards the east with the diurnal motion, or the annual as well, no elementary body moving towards the east could reach the places of the earth it was heading for — at least near the Equator. But this is contrary to manifest experiments. Therefore the Earth moves with neither the diurnal nor the annual motion. [Margin: Proof of the Minor.] The MINOR is evident: for birds flying to their nest, ships sailing to an eastern harbour, and arrows and the balls of muskets and of cannon aimed at an eastern target, all get there.

[Margin: Proof of the Major.] The MAJOR is proved by comparing the velocity of the Earth set out in the tables given above, in number 1, with the natural velocity of any bodies whatever, especially near the Equator. For by the diurnal motion alone the Earth — that is, a point of the earth at the Equator — traverses 301 paces in one second of an hour, that is, in about one beat of the human pulse; whereas not only no ship or bird, but not even the ball of a larger culverin, covers more than about 125 paces in one second. And if a portion of the annual motion is joined to the diurnal, there is no need to restrict the argument to a body at the Equator: for even when the motion is least, that is, about noon, a point of the earth traverses very nearly 12 Italian miles in one second, whether it is at the Equator or near the Pole, as is clear from the last cells of the 7th Table, set opposite one second of an hour.

The reply, however, is to deny the Major, because birds, ships, arrows, cannon balls and all bodies of this kind are moved not by their proper motion alone, as appears to us, but also by the common motion with which the earth moves, even though this does not appear to us; and this motion is as great as the Earth's. Therefore, carried along by an equal motion, they can reach an

eastern place and target by their own proper motion, which they add over and above the common one. [Margin: Reply to Argument 2.] So, at any rate, the Copernicans.

III. Argumentum ab ingenti Sonitu, Verbere Aeris, & Vento perpetuo.

III. Argument from the enormous sound, the beating of the air, and a perpetual wind.

[Margin: Buchanan's argument against the motion of the Earth.] IV. George Buchanan likewise did not pass over this argument, when he sang in Book 1 of the *Sphaera*:

Do you not see, when boys shake their little rattles in the right hand, or when an arrow, loosed from the whistling string, flies; or when the wind, let out from the throat of the bellows, fans the panting fire in the furnaces? Do you not see with what a murmur the air hisses and groans to be split? If, then, at a slight impulse the air, struck, roars with a sound and bellows back, what sound shall we imagine there would be, what murmurs, while the Earth, driven headlong, twists itself round in a circle, and so many forests at once, and the steep summits of mountains, cleave the indignant air, and tear it, and [?] it?

Simplicius raises exactly this objection against Salviati, that is, against Galileo, in Galileo's Dialogue 2 on the system of the world, Latin page 186, in the person of a certain author: "The author goes on to show that in the doctrine of Copernicus one has to deny the senses and the most powerful sensations. That would be the case if we, who feel the breath of even the lightest breeze, could not feel the onslaught of a perpetual Wind striking us with such velocity that it flies across more than 2529 miles an hour — for that is the space the centre of the earth traverses by its annual motion in one hour along the circumference of the Great Orb, as he himself carefully computes. But, says the same author, in Copernicus's opinion the surrounding air is moved with the earth, and yet the motion of the air, swifter than any wind, is not felt; instead, unless something else intervenes, there is the greatest calm. What, then, is a true deception of the sense, if this is not a deception?" Finally, Kepler raises similar objections against himself in Book 1 of the *Epitome of Copernican Astronomy*, page 137, with this little question: "If the Earth were revolved with this very swift motion, would a perpetual wind be felt from the direction opposite to the motion?" The replies to these will be brought forward shortly, once the argument has been put into form, in this way:

[Margin: Form of the 3rd argument.] If the Earth revolved with the diurnal motion, or with the annual as well, a perpetual wind blowing towards the west would be felt by us, and the air, dashed against mountains, towers and the leaves of trees, would give out an enormous sound and whistling. But neither is felt by us or by any mortal. Therefore the Earth revolves neither with the diurnal motion nor with the annual. The MINOR seems certain from daily experience, when the air is calm and no winds blow from elsewhere. [Margin: Proof of the Major.] The MAJOR is proved thus. Either the air around us is not moved with the earth at equal velocity — and then, as we and the mountains

and forests are carried eastward and go to meet the unmoving air, the result would be exactly as if the air ran against us, the mountains and the forests, all at rest, with equal velocity. Or the air too is moved with the earth at equal velocity — and then in one hour, by force of the annual motion, it flies through 5589 Italian miles for Copernicus, 14,428 ⅔ for Kepler, and 43,338 for us, as is clear from number 1, table 4. How, then, can so rapid a motion of the air — by day towards the east, by night towards the west, by force of the annual motion — not be felt by us?

[Margin: 1st reply to Argument 3.] The Copernicans' first reply is to deny the second part of the Minor, namely that no mortal feels a perpetual wind or breeze blowing towards the west. For Galileo, Dialogue 4, page 327, and with him Gassendi in his second letter *De motu impresso*, holds that within the Tropics a perpetual breeze and wind are felt, and that for this reason sailing westward is quicker than sailing eastward — as we said elsewhere, in chapter 4 at the end of number 6 and in its scholium 5, and in chapter 7, number 4. But, they hold, it is not felt to be as vehement as the velocity of the terrestrial motion would otherwise require, because the air moves together with the earth towards the east, but somewhat more slowly, since by its fluidity [fluxibilitas] it does not entirely comply with the earth's motion. This reply, however, supposes that the rapture of the air towards the east comes from contact with the earth; otherwise, if it came from an intrinsic principle, its fluidity would be no obstacle to its moving with the earth at equal velocity. On page 138 of the *Epitome of Copernican Astronomy*, Kepler first puts forward a similar reply drawn from other effects, answering that question about the perception of a perpetual wind thus: "This could be conceded of the highest summits of the mountains; and in this cooling a cause could be sought for the perennial snows even under the torrid Zone, and likewise for the coldness of the east winds, the pleasantness of the morning, and the like — provided that both the lower mountain country and the valleys hidden among their summits, which animals inhabit, were safe, and the air in them were undisturbed and quiet through this exterior friction, and there were freedom in it for the bubbling vapours to rush in every direction. But it is not necessary for us to concede what is objected." He therefore brings forward another reply, which we shall add at once.

[Margin: 2nd reply to Argument 3.] The second reply, with the Copernicans, is to deny the Major. For the whole of the air that is in the lowest and middle region of the air, being full of terrestrial and watery vapours and exhalations, is moved eastward at equal velocity with the earth by the intrinsic force of the motion common to them and to the terraqueous globe; and we likewise, and the mountains, and the forests with their leaves, are moved the same way. Therefore, "just as we do not feel the blow of a spear pursuing us if we avoid it by fleeing at equal velocity," says Galileo, page 187, Dialogue 2, "so neither do we feel the blow of the air, since we are carried in the same direction with it at equal velocity." Kepler, moreover, in Book 1 of the *Epitome of Copernican Astronomy*, page 38, says that the ethereal aura, which is not moved with our air,

is so many times thinner than our air, which animals breathe, that the friction against the ethereal aura over a thousand miles in one hour is more tranquil, and so less perceptible, than the friction of a man's face against the air as he walks half a mile through it in one hour. So, on page 108, he had said: "The ethereal aura, through which the highest ridges of the mountains are carried, is thinner not merely eightfold but almost infinitely many times than the air through which a leaden ball is carried. Therefore the passage of the ridges through the aether is very many times more tranquil than that of an iron ball through the air." Kepler therefore holds that the air enclosed within the ordinary mountains is moved with the earth, being watery, and that the ethereal aura begins at the highest ridges of the mountains; and that, although it is not moved with the motion of the earth, it cannot, because of its thinness, produce blasts or winds that are very perceptible, much less a great sound and crash.

IV. Argumentum à Concussione ac ruina ædificiorum, & Proiectione corporum infirmiter Terræ adhærentium.

IV. Argument from the shaking and collapse of buildings, and from the flinging off of bodies weakly attached to the Earth.

[Margin: Buchanan's argument against the motion of the Earth.] V. This argument, such as it is, belongs to a great many writers, against both motions of the earth but especially the diurnal; among them Buchanan, in Book 1 of the *Sphaera*, sings thus:

So if the Earth, driven with so swift a motion, went into the West, and came back again into the East, it would shake all things together with it, and with a vast crash the unforeseen slaughter of a sudden collapse would overwhelm temples, houses, and cities, even with their wretched citizens.

[Margin: And of Clavius.] Clavius, too, says it explicitly, in Book 1 on the *Sphere*, page 196 in my copy: "If the Earth," he says, "were turned about the axis of the world with such speed that it completed its circuit in the space of 24 hours, as some people fable, all buildings would collapse and could by no means stand for long; and there is no one who does not see that this is entirely false. Nor is the reply of some people valid, who say that buildings do not collapse because of the excessive speed of the motion, just as water contained in a vessel does not flow out if the vessel is swung round very swiftly. It is not valid, I say, because the whole impetus of the water is impressed towards the lower parts of the vessel, not towards its mouth; but the impetus is impressed on buildings towards the outermost parts of the earth, etc." The true cause, however, why the water does not fall in that case is that the impetus impressed on the water and the vessel together, which moves them in a circle, is greater than the one that the heaviness of the water, moving it downward, produces; and this impetus, continued very swiftly in a circle, carries the upper parts of the water to another place, in which they are no longer above the air.

Galileo and Kepler raise this same argument against themselves and solve it: Galileo in Dialogue 2 on the system of the world, from Latin page 139 to 148,

but in the Italian from page 184 to 213; Kepler in the *Epitome of Astro-* nomy*, pages 156[48] and 137; *Philolaus*, part 1, chapter 6; and Redemptus Baranzanus [Redento Baranzano], part 1 of the *Uranoscopia*, page 111. But before all of these, Copernicus, Book 1, chapter 7, where he too attributes this argument to Ptolemy. [Margin: Copernicus's [?] against himself.] Copernicus's words are: "For the motion would have to be most violent, and its speed unsurpassable, that carried the whole circuit of the earth round in 24 hours. But things set going by a sudden whirling seem altogether unfit for coming together, and rather, if united, to be scattered, unless they are held together by some firmness of cohesion. And long ago," he says — that is, Ptolemy, whom he had named a little before — "the earth, scattered, would have fallen out of the very heaven, which is quite ridiculous; and all the more would animals, and any other loose weights, by no means remain unshaken." He replies to this objection in chapter 8. [Margin: Ptolemy's argument against the motion of the Earth.] Ptolemy's words, in Book 1 of the *Almagest*, chapter 7, had been: "If one single motion, common to the other heavy bodies, also belonged to the Earth itself, it is clear that because of so great an excess in its turning, proportionate to its magnitude, it would outrun them; and, leaving behind in the air the animals and the other weights, it would itself very swiftly fall out even beyond the heaven itself." Chiaramonti subscribes to this in the *Antiphilolaus*, parts 1 and 6. Let the argument now be gathered into form, as below.

[Margin: Form of the 4th argument.] VI. If the Earth went round with the diurnal motion, or with the annual as well, all buildings would be shaken and would fall, and the other things that stand on the earth's surface but cohere with it not very firmly would be shaken off by its whirling and flung away. But neither happens. Therefore the Earth goes round neither with the diurnal nor with the annual motion. The MINOR is evident: for we do not experience perpetual landslides or earthquakes, but only rarely, and those from some cause other than the whirling of the whole earth. The MAJOR is proved from the very rapid whirling of wheels. For things placed on their circumference that do not cohere very firmly with it are thrust off and flung far away. So mill wheels standing half out of the water wet the places nearby with a spray of flung drops; so the wheels of carts driven fast push off the mud sticking to them; and boys, setting a stone in the notch of a reed as in a pair of tongs and swinging the reed round, throw the stone a long distance. Much more, then, would the Earth eventually shake buildings and bring them down, and not only shake fruit and leaves from the trees but thrust off the animals creeping over it, and ships, and the stones lying in the beds of torrents left without water, and sands and the like — by as much as its whirling is faster than the whirling of any wheel.

[Margin: Reply to Argument 4.] The Copernicans already named, however, reply by denying the Major, and against its proof, drawn from a similar case, they deny the parity. For bodies placed on wheels are not moved of themselves,

[48]so printed; the same work is cited below at page 136

or with a velocity equal to the wheel's; and the impetus impressed on them by the wheel follows the straight line tangent to the wheel at the point where those bodies are and from which the impetus leaves, even if, by reason of their own heaviness, they then decline downward from the straightness of that tangent. The same is to be said of the stone thrust out of the notch of the reed. But all heavy bodies, whether fixed firmly in the earth, like buildings, or weakly, like sands, ships, the stones of torrents, and the leaves and fruit of trees, are moved with the common motion and at equal speed with the earth, in an entirely uniform manner; and they neither strike against anything nor are shaken by a reciprocating agitation to and fro. In the same way, not even cups full of wine are overturned if a boat is carried down a river very rapidly but at uniform velocity; nor do the Antipodes fall down, because they strive towards the centre of the earth with an equal concert. If anyone wants more about the cause for which bodies placed on wheels are thrust off, let him consult Galileo in the places cited a little earlier, and Niccolò Cabeo, Book 1 on the *Meteorologica*, text 17, question 8, where he denies that bodies thrust off in this way move along the tangent of the circle at the point from which those projectiles are torn away, because heaviness inclines those projectiles downward and twists them away from the straightness of the tangent.

V. Argumentum ab impetus Sensatione futura in nobis, si moueremur cum Terra.

V. Argument from the sensation of impetus that would arise in us if we were moved with the Earth.

VII. Before I had read the seeds of this argument in Kepler, Book 1 of the *Epitome of Copernican Astronomy*, page 136, it had long seemed to me impossible that any animal, and we above all, should not feel in ourselves, and in all the organs and vessels of our body, an impetus impressed either by the earth or by our common motive faculty, if we were carried round together with the earth at so great a velocity as we set out in the tables given in number 1; yet the comparison with those who are carried in a ship seemed to remove this suspicion. Father Francesco Maria Grimaldi had not read Kepler in that place either; and yet he too, while we were discussing this matter, asserted to me that, just as someone falling from a window, even if he falls straight along the perpendicular and has his eyes closed, feels himself slipping downward, so we too would feel our motion and impetus if we were turned with the earth in so headlong or so very rapid a whirling. Not much later, rereading the already mentioned passage of Kepler on another occasion related to this one, I came across this very objection, framed in this question: [Margin: Kepler's argument against the motion of the earth, and the reply.] "But surely we would at least feel this motion in our bodies, even with our eyes closed?" To which he replies at once: "Not at all. For not even in ships is the motion always felt when they run on uniformly, although there natural bodies are carried by their heaviness not to the ships as ships, but beyond the ships, outward and downward to the

lands beneath, which are at rest with respect to the river, while the ship carries them across this line. How much less shall we feel the motion of the Earth, we who are attracted by the heaviness of our bodies nowhere else than to the moving earth on which we stand; and therefore we are not carried outside the line of natural attraction, since that line accompanies us as we are carried along together." So much from Kepler, and it is almost enough; but for the sake of the natural philosophers and the logicians I must solve this very argument reduced to the rules of the syllogism.

[Margin: Form of the 5th argument.] If the Earth, and we with the Earth, were carried eastward at equal velocity, it could not be that we should not at some time feel this motion in ourselves. But we never feel such a motion. Therefore neither the Earth, nor we with the Earth, are carried eastward at equal velocity. About the MINOR I do not think anyone has any doubt. The MAJOR is proved because, in falling downward, or in leaping upward and falling back downward, we nevertheless feel, even if we close our eyes meanwhile, the impetus impressed on our bodies, either upward by the soul or downward by heaviness. Therefore, likewise, whether the Earth and the Air, of a terrestrial or watery nature, carry us eastward, or whether there is in our bodies a common faculty and force of going round in a circle and following the Earth, we should nevertheless at length feel this very impetus with some sense, at least with the common sense, since it is received in us subjectively and perhaps emanates from us effectively.

[Margin: Reply to the 5th argument.] The reply, however, is to deny the Major, on the ground and by the comparison Kepler has just brought forward. What is brought forward to confirm the Major is weak and involves a disparity. For in the cases cited we feel the motion and the impetus precisely because neither the air nor the earth moves with us with the same motion upward or downward. So it happens that, if not with our eyes, at least by touch, we feel ourselves leaving one place and acquiring another; and in a leap upward there is a struggle between the impetus upward from the soul and the impetus downward from heaviness. But in the common motion eastward nothing of the kind happens, and we have no indication of this common motion, or of a new place acquired through it, whether we compare the place we occupy with the fixed Stars or with other things — understand, as long as we are not moving with a proper motion, for that one does appear to us. Nor should it seem surprising that there is some faculty in us which exercises its act without our feeling it; for the vital faculty does this perpetually in concocting and digesting food.

CAPVT XXIII.

Chapter 23.

CAPVT XXIII.

Proponuntur ac Dissoluuntur Quatuor Argumenta sumpta ex Stellarum Ortu & Occasu, siue in aperto, siue ex puteo visis, & ex Solis Vmbris Solstitialibus, aut Eclipsi in morte CHRISTI.

Four arguments are proposed and refuted, taken from the rising and setting of the stars, whether seen in the open or from a well, and from the solstitial shadows of the Sun, or from the eclipse at the death of CHRIST.

I. Up to this point the arguments against the motion of the Earth have been sought from what appears in elementary bodies; now they are to be drawn from the celestial phenomena — but these, in the end, we shall acknowledge to be weak. They are not to be despised, however, lest they should seem to be rejected unheard and without cause.

I. Argumentum ab Occasu apparenti Siderum.

I. Argument from the apparent setting of the stars.

[Margin: Form of Argument 1.] If the Earth moved with the diurnal motion, the western mountains would seem to us to rise rather than the stars to descend below the horizon, and the eastern mountains to descend rather than the stars to rise above the horizon. But observation shows otherwise: the mountains in both quarters appear motionless, and the stars appear to rise from the East and to descend into the West; indeed, it is from this that the one part of the heaven is called the East [Oriens] and the other the West [Occidens]. Therefore the Earth does not move with the diurnal motion. [Margin: Kepler's reply to Argument 1.] So Kepler objects to himself in Book 1 of the *Epitome of Copernican Astronomy*, page 129. But he replies at once by denying the consequence of the Major. For ascent, he says, is judged by sight from the leaving behind of the plane on which we stand, and from the approach of a thing to the vertex, towards which a man's stature is raised upright. Now it is the mountains themselves that form the visible plain for us, and they do not approach the vertex, because as far as they advance, so far the line on which the spectator's stature stands erect advances with them, and the vertex too lies on that line. So the vertex, as far as sight goes, is always equally distant from the extremities of the ground on which the spectator stands, that is, from the outermost mountains. The mountains therefore cannot be seen to rise, even though in the hypothesis of the moving Earth they truly do rise.

[Margin: Empty corroboration of Argument 1.] If you press the argument and say that the mountains must at least seem to move, if not to rise or descend, the reply is to deny this consequence too. For if they seemed to move, it would certainly be by no other motion than one of ascent or descent; so, for the very reason that they do not seem to rise or descend, they will not seem even to move. Virgil's line holds good here:

We sail forth from the harbour, and lands and cities recede,

although it is the ship that really recedes. So those who sail downstream, if they are not already aware of their own motion and the ship's, will think that

the shores and mountains nearby are coming to meet them against the current; and so in similar cases.

II. Argumentum à Fixis Stellis ex Puteo spectatis.

II. Argument from the fixed stars viewed from a well.

[Margin: Form of Argument 2.] II. If the Earth, rather than the Firmament, moved with the diurnal whirling, then we, looking from the bottom of a well at some fixed star standing over our vertex, could not see it for as long as we actually do see it; instead, carried eastward together with the well, we would lose sight of it in a single beat of the artery. The consequent is false; therefore so is the antecedent from which it follows. This, namely, is how Bartholomaeus Mastrius and Bonaventura Bellutus argue, together with Alexander Tassonus [Alessandro Tassoni], in disputation 4 *On the Heavens*, question 4, article 3, number 117. They try to prove the consequence of the MAJOR from the fact that the mouth of a well is no wider than one ell — as small a space as the Earth traverses with its diurnal whirling in the twinkling of an eye — while from the bottom of a well we can see no more of the heaven than what is comprised under the angle made at the centre of the eye by two lines drawn from the eye through the outermost edges of the well into the heaven. Since, then, the space of the heaven comprised by the said angle is far greater than the mouth of the well, they infer from this that the same star can be seen from the bottom of the well for so long a time, and that the Fixed stars, rather than the Earth, move with the diurnal motion. But these authors are to be forgiven, as men little devoted to Geometry, since even some friend of ours or other, by no means lightly versed in Geometry, thought that this argument had demonstrative force against the motion of the Earth, and tried to add weight to it with his calculations. For they say that in one second of an hour, or roughly one pulse of the human artery, the Earth, by force of the diurnal whirling, traverses under the Equator a great many paces, let alone a single ell, which is the width of a well; and that therefore the whole width of the well would at once withdraw from the space lying beneath the celestial space that contains the fixed star, etc. Nor, however, is it necessary to go down into a well; it is enough to watch the stars passing by from a very long tube, or from the far end of some portico closed on all sides, through a window.

[Margin: Reply to Argument 2.] The reply, nevertheless, is to distinguish the consequence of the Major. It is conceded if the eye and the bottom of the well were at the centre of the Earth and the mouth of the well were not much wider than three hundred of our paces; or if the eye and the bottom of the well were near the surface of the earth, but the visual angle made by straight lines drawn from the eye through the opposite edges of the well did not comprise more seconds in the heaven than fifteen of the circumference described from the eye itself. Otherwise, the consequence of the Major is denied. As to its proof, it is conceded that a point of the earth's surface traverses more paces in one beat of the human artery, that is, in very nearly one second of an hour —

especially in the parallels near the Equator — than the usual width of a well; but the consequence is denied. For the measure of the time for which a Star stays in view of an eye placed at the bottom of the well is not the physical quantity of the well's width, determined in paces or other measures known to us, nor its proportion to the physical quantity of the whole terrestrial circumference. It is an optical quantity, namely that of the visual angle, relative to the arc of the circumference described from the centre of the Earth; for that circumference is the one whose entire revolution measures the diurnal time of 24 hours. Hence, if the eye is at the centre of the earth, the measure of that time, or of the star's duration in view of an eye placed at the bottom of so deep a well, is the arc of the circumference, terrestrial as well as celestial, comprised by two straight lines drawn from the eye through the opposite edges of the well; and in this case I say that, if the star is to remain in view for a longer time than one second of an hour, or one beat of the human artery, the mouth of the well must be wider than three hundred of our paces. But if the eye is near the surface of the earth, the measure of the said time is not the arc of the circumference of the terrestrial globe comprised by the edges of the well, because that arc is not described from the eye as its centre; it is a celestial or terrestrial arc similar to the arc comprised by the visual angle, that is, by the lines drawn from the eye through the opposite edges, if the doubled parallax of the star is subtracted from it. And in this case I say that, if the star is not to remain in view of an eye placed at the bottom of the well for more than one second of an hour, such an arc and angle must not comprise more than 15″. Although these things can easily be understood by those skilled in Geometry, it is nevertheless worth the effort to help both them and others with a diagram and with some problems — very opportune ones — taking the occasion from this argument.

III. From the centre of the Earth, A, describe BCD, the semiperiphery of the celestial Equator, and beneath it ELF, the semicircumference of the terrestrial Equator, in whose plane let there be the well aSOb. Through its bottom describe from A the semicircumference GIK, which the eye I, placed at the bottom of the well, is supposed to describe while it is moved with the Earth by the diurnal whirling about the centre A, towards B, the eastern point; and let the width of the well be the straight line SO, subtending the arc SLO. Now since the walls of wells and of all buildings built to the perpendicular, and the perpendicular lines of the walls, meet at A, the centre of the Earth, then if a well is imagined dug right down to the centre of the earth, the well will be ASO. Let there be such a well, then, and the eye placed at A. For, the straight lines ASP and AOQ having been produced from A through the edges of the well, S and O, as far as the heaven: as soon as the eastern edge of the well has come to S, if a star is at P on the line ASP, the eye will begin to see it, and will be able to see it until the edge O of the well has come to S. I said, moreover, that a star cannot be seen by an eye placed at the centre of the Earth for longer than one second of an hour if the mouth of the well, OS, is not wider than

three hundred paces. This I shall demonstrate with the help of the following problem, so that the benefit may be more general.

Diagram. A semicircle on the diameter BD with centre A, the Earth's centre, and two smaller concentric semicircles within it. Straight lines run from A and from a point I above A out to the outer arc; small star marks stand beside M and P on that arc. A short arc above I carries the letters of the well's mouth; the italic letters a and b stand either side of I.

- M, P, C, Q [?], N (along the top of the outer arc, left to right): points of the celestial Equator; star marks beside M and P
- V (left end of the outer arc)
- R, [?], S, L, O, X (along the small arc above I, left to right)
- a, b (italic, either side of I)
- I (the eye at the bottom of the well)
- T, H (left, near the inner arcs)
- B, E, G, A, K, F, D (along the diameter, left to right)

I. Problema. Data temporis mora, quo Stella ab oculo in fundo putei existente in centro Terrae sub Aequatore videri poterit, & data insuper semidiametro Terrae, Inuestigare Latitudinem oris putei, ad talem moram necessariam.

Problem I. Given the stay of time for which a star can be seen by an eye at the bottom of a well at the centre of the Earth, under the Equator, and given also the Earth's semidiameter: to find the width of the mouth of the well needed for such a stay.

IV. From A, the centre of the Earth, let the perpendicular ALC be drawn, bisecting the arc SLO, the chord SO and the angle SAO, by propositions 9, 10 and 11 of Euclid's first book. For there arises the rectilinear triangle ALO, right-angled at L, in which are given the base AO, the semidiameter of the earth, and the angle OAL. This angle is half the angle OAS, which subtends the arc OS, known from the given stay of time converted into degrees of the Equator. Therefore, by the laws of Trigonometry, the side OL will not remain hidden, in the units in which AO is known; and OL doubled will then give the chord OS, that is, the required width of the well.

EXEMPLVM.

Example.

[Margin: Example for a stay of one second.] Let the stay of the time already mentioned be one second of an hour. From the tables of the primum mobile the arc OS corresponding to it is 15″; its half, the arc OL, is 7½″, and the angle OAL is just as large. Now since, from what was said in Book 2, chapter 7, the Earth's semidiameter AO is 4139 Bologna miles, AO will be 4,139,000 Bologna paces. From these data the side OL is found to be 150 paces. Therefore the required width of the well, OS, is 300 paces.

Now imagine the eye at the bottom of the well, at I, equally distant from the sides Ob and Sa, and let the straight lines ISM and ION be produced from I

through the edges O and S of the well as far as the heaven. They will intercept the arc MCN, much greater than the arc PCQ intercepted by the visual angle SAO. Therefore the eye I will see the star — which rises for it earlier, from M — with a stay longer than that with which the eye A sees the star rising at P, by as much as the arc MCN of the Equator is greater than the arc PCQ, each arc being converted into time. But if this time, in this position of the eye too, is to be no greater than one beat of the human artery, or one second of an hour, then, as I said, the depth IL of the well and the narrowness of its mouth OS must be such that the angle MIN does not comprise more than 15″ of the Equator — at least if the star is a fixed star, or lacks sensible parallax. Let us suppose, then, that the depth of the well is at least 100 paces, or 500 feet, although hardly any well is usually found that exceeds 100 cubits, that is, 150 feet; and let the stay of the star in view of the eye I be one second of an hour; and let it be proposed to find the width of the well beyond which it cannot go, that stay being given. This will be found by the following Problem, with an example added as well, as you see.

II. Problema. Data mora stellae sub aspectu oculi in fundo putei eam spectantis, & in Aequatore existentis; data insuper putei profunditate: Inuestigare Oris putealis latitudinem, illi morae congruentem.

Problem II. Given the stay of a star in view of an eye that watches it from the bottom of a well and is situated on the Equator, and given also the depth of the well: to find the width of the well's mouth that agrees with that stay.

V. In the preceding figure, since the arc MCN is given from the time of the star's stay in view of the eye I, converted into degrees of the Equator, the angle MAN, from which that arc has been described, is also given. And when AIC is produced, bisecting the angle MAN just as it bisects SIO, the angle CAN will be known, since it is half the angle MAN. Now let the straight lines AM and AN be produced from A. For in the triangle AIN, the angle ANI, which is the parallax of the star, and the angle NAI, taken together, are equal to the external angle CIN, by proposition 32 of the first book of Euclid's *Elements*. So if the parallax N is known and is added to the angle NAI, the angle CIN, that is LIO, will be known, in the small rectilinear triangle LIO, right-angled at L, in which the depth of the well, IL, is given. Therefore, by the canons of triangles, the side LO will become evident, and doubled it will give the required width of the well, OS.

EXEMPLVM.

Example.

Let the given stay of the star be 1 second of an hour, to which on the Equator there corresponds the arc MCN, that is, the angle MAN, of 15″. So half of this angle, CAN, is 7½″; and the angle CIN, that is LIO, is just as large, to every subtlety of sense, if the angle N, or the parallax, is supposed insensible, as it is supposed in the fixed stars. Now, in the triangle LIO, let the depth of the well,

or the side LI, be given as 100 paces, or 500 feet, that is, 6000 inches. From this, and from the angle LIO of 7½″, the side OL will come out at 21⅘ inches. So the whole width of the well, OS, will be 43⅗ inches, that is, 3 feet and 7⅗ inches. But if it is greater, the star will stay in view of the eye I for a longer time than one second of an hour.

III. Problema. Data Latitudine & Profunditate Putei sub Aequatore, Inuestigare temporis moram, qua stella videri poterit ab oculo in fundo putei collocato.

Problem III. Given the width and depth of a well under the Equator: to find the stay of time for which a star can be seen by an eye placed at the bottom of the well.

VI. In the triangle LIO of the preceding figure, right-angled at L, the depth of the well, LI, is given as one side about the right angle, and LO as the other, since it is half the well's width OS. Therefore the angle LIO will become known by Trigonometry. When the parallax of the star, if there is any — that is, the angle ANI — is subtracted from it, the angle IAN will be known, by proposition 32 of Euclid's first book; this is the angle CAN, which measures the arc CN of the Equator. Doubled, it will give the arc MCN, which, converted at last into time, will show the stay of the star in view of the eye I.

EXEMPLVM.

Example.

Let the depth of the well, LI, be given as 100 feet, and the width SO as 4 feet, and so the half-width LO as 2 feet. Then, by the laws of right-angled triangles, the angle LIO will be 8′ 50″; and CAN will be just as large, if N, the parallactic angle, is insensible, as it is in the fixed stars. So the whole angle MAN, that is, the arc MCN, will be 17′ 40″, to which there corresponds a time of one minute and very nearly 11 seconds, that is, about 71 pulses of the human artery. When these have elapsed, the star M, which began to be seen by the eye I under the edge S, will set for it under the edge O, having by then been carried to N by the force of the diurnal celestial revolution. But if it is the Earth that turns, the edge O of the well will in that much time have been carried with the well towards T, the eastern point, through so great an arc of the terrestrial circumference that through it a straight line, namely IM, can be drawn from the eye to the fixed star at M, now supposed motionless. For the size of this arc, consult the following Problem.

IV. Problema. Data Putei Profunditate ac Latitudine, Inuestigare arcum circumferentiae terrestris Aequatoris, qui vi diurnae vertiginis transire debet ab ortu ad occasum stellae, respectu oculi in centro putei spectatum. Et si nota fuerit quantitas vnius Gradus Aequatoris terrestris in pedibus aut passibus &c. Determinare in his mensuris quantitatem praedicti arcus.

Problem IV. Given the depth and width of a well: to find the arc of the circumference of the terrestrial Equator which, by force of the diurnal whirling, must pass over between the rising and the setting of a star, considered with respect to an eye at the centre of the well. And, if the quantity of one degree of the terrestrial Equator in feet or paces etc. is known: to determine the quantity of the already mentioned arc in these measures.

VII. Let the time of the star's stay in view of an eye placed at the bottom of the given well be found by Problem 3, and from it the arc of the Equator corresponding to that time; for the required arc of the terrestrial circumference will be similar to it. Thus if, of the well SabO, the depth LI and the half-width OL are given, and from them the angle LIO, then the angle CAN is given, provided that the parallax N, if there is any, has been subtracted from the angle LIO. This doubled will give the angle MAN, which measures both the arc MCN and the arc ZLX, because each is described from the common centre A and intercepted by the same two straight lines AZM and AXN, drawn from that same centre A. Now that the arc ZLX is known, if an arc equal to it is taken from the edge S towards T, you will have the arc ST; and the point T will be the one to which the eastern edge S will have been carried after the said time. Taking, then, towards the right the arc TR, determined by the chord TR equal to the well's width OS, R will be the western edge; and the line drawn through R from the eye H, now carried here, will fall on the star M as it sets for the eye H. Finally, if one makes: as 60′ is to the paces or feet contained in one degree of the terrestrial Equator, so are the minutes of the arc found, ZLX or TRS, to another number, the said arc will be known in these measures too.

EXEMPLVM.

Example.

Let the width of the well be 4 feet and its depth 100 feet, as in the example of Problem 3. Then, from what was said there, the arc MCN will be 17′ 40″. Therefore ZLX and TRS will each be 17′ 40″ as well. Now, from what was said in Book 2, chapter 7, one degree of the terrestrial Equator holds 72,500 Bologna paces, that is, 362 500 feet. Therefore, as 60′ is to 362 500 feet, so is 17⅔′ to 46 319 feet. Hence, although the said well is 4 feet wide, the same star will nevertheless stay in view of an eye placed at its bottom until an arc of the terrestrial circumference of 46,319 feet has passed, if the well is 100 feet deep. [Margin: Error in geometry of the authors of Argument 2.] From this it is gathered how badly those men sinned against Geometry who thought that the star must vanish from the eyes as soon as as much of the terrestrial circumference has passed as the width of the well.

III. Argumentum ex Vmbris Gnomonum sub Tropicis in diebus Solstitialibus.

III. Argument from the shadows of gnomons under the Tropics on the solstitial days.

[Margin: Form of the 3rd argument, and the reply.] VIII. If the Earth moved with the diurnal motion, gnomons erected under the Tropics on the solstitial days would cast sensible shadows, except at the single moment of noon. But this is contrary to the experiments and observations made at Syene and other places under the Tropic of Cancer, and reported by Strabo, Pliny the Elder and others. Therefore etc. The reply, as above to Argument 2, is to deny the Major, for the same reason. For since gnomons erected perpendicularly on the surface of the earth converge on the centre of the earth, the two outermost ones, which, if produced into the heaven, would touch the edges of the Sun, subtend a similar arc both on the surface of the earth and in the heaven of the Sun, namely about 30′, which require two minutes of time of the diurnal revolution. For that much time, therefore, throughout the whole space contained between the two outermost gnomons already mentioned, no sensible shadow of them appears, whether the Sun moves with the diurnal motion while the Earth with its gnomons stands still, or the Earth moves while the Sun stands still. For, as I said, the arc is similar, and either arc is the measure of a time that has the same proportion to the whole day as that arc has to its whole circumference. Further, on Syene in particular and other places under the Tropic of Cancer, and on no midday shadows being seen on the day of the solstice, Cleomedes can be consulted, Book 1 of the *Cyclical Theory*, chapter 10; Strabo, Book 17; Pliny the Elder, Book 2, chapter 75; Ptolemy, Book 4 of the *Geography*, chapter 5, in the 3rd table of Africa; and Macrobius, *On the Dream of Scipio*, Book 2, chapter 8.

IV. Argumentum ab Eclipsi Solis visa in morte CHRISTI.

IV. Argument from the eclipse of the Sun seen at the death of CHRIST.

[Margin: Form of the 4th argument.] IX. The eclipse of the Sun at the death of CHRIST was total, and lasted as such for three hours. But if the Earth turned with the diurnal whirling, it would not have lasted total for three hours. Therefore the Earth does not turn with the diurnal whirling. The MAJOR is certain from the Gospel, where it is said that darkness came over the whole earth from the sixth hour until the ninth hour. The MINOR is proved because the Earth's very rapid whirling would have completed 45° in those three hours, and would have withdrawn the whole of Palestine under another quarter of the heaven, between which and the Sun the Moon would not have been interposed. And it is confirmed from the 5th letter of Dionysius the Areopagite to Polycarp, which we reported in Book 5, chapter 18, number 3, where he describes the motion of the Luminaries in that eclipse from East towards West. From this Tannerus, *De caelo*, question 9, and Inchofer [Melchior Inchofer], chapter 7 of the *Tractatus Syllepticus*, infer that the diurnal motion belongs to the Luminaries; but if the Earth too had been endowed with a diurnal motion, it would have carried Palestine away from a position suitable for seeing the Eclipse twice as

fast, since the Luminaries would be meeting it with their own motion and going of their own accord to meet the terrestrial revolution.

[Margin: Reply to Argument 4.] The reply is to deny the Minor. That miraculous eclipse was not total, and of three hours, by force of the diurnal and common motion (although, had it been by force of that motion, a miracle could have been worked in that motion too), but by force of the Moon's proper motion compared with the Sun's proper motion. This motion was varied supernaturally in the Moon: the Moon was drawn back from the Full Moon, and from diametral opposition to the Sun, to Conjunction with the Sun, and was kept beneath it, between it and Palestine, for three hours — which could have happened in just the same way whether the Earth moved or not. But the rest that concerns this miracle we have pursued diligently in that chapter 18 of Book 5.

CAPVT XXIV.

Chapter 24.

Proponuntur ac Dissoluuntur Quatuor Argumenta contra Terrae motum Diurnum & Annuum ex Principio & simplicitate huius motus sumpta.

Four arguments against the Earth's diurnal and annual motion, taken from the principle and the simplicity of this motion, are proposed and refuted.

I. Aristotle, in Book 2 of *De caelo*, chapter 12, had censured certain of the Ancients, especially Anaximander, who, though concerned with why the whole Earth does not move downward but rests at the centre of the World, had given no reason from the Earth's own nature, nor any reason why its parts are carried to the middle of the world; and he himself laid down that one must first inquire whether it moves, and then where it rests. Accordingly, in chapter 13, text 96, he tries to show that the earth by its own nature moves with neither motion. For, repeating what he had related in chapter 13, he says: "For, as we have said, some make it one of the stars, while others, placing it in the middle, say that it is rolled and moved about the middle and its own poles." And he adds at once: [Margin: Aristotle's 1st argument against the motion of the Earth.] "But that this is impossible is evident, taking as a principle that, if it is carried, whether it exists in the middle or outside the middle, it must be moved by this motion violently; for this motion is not the Earth's own, since then each of its particles would have this motion; but in fact they are all carried in a straight line to the middle. Therefore, since this motion is violent, or beyond nature, it cannot be everlasting; but the order of the World is everlasting." His argument, then, reduced to the rules of the syllogism, is as follows.

I. Argumentum Aristotelis ex Violentia motus.

I. Aristotle's argument from the violence of the motion.

[Margin: Form of the 1st argument.] II. If the whole Earth moved, either with the annual and diurnal motion together or with the diurnal motion alone,

its motion would be violent or preternatural; but a violent or preternatural motion ought not to be attributed to the whole Earth. Therefore etc.

The MAJOR is proved because neither motion would come from a natural principle of the Earth itself. For if the circular motion, whether of translation or of the whirling alone, were natural to the whole Earth, it would be natural to its parts as well. But it is not natural to the parts of the Earth; therefore not to the whole Earth either. That it is not natural to the parts is proved because none of them moves in a circle, either by whirling or by translation; rather, when let go, they are carried downward along a straight line perpendicular to the middle. The MINOR is proved because a motion that is not everlasting ought not to be attributed to the Earth, which is part of the World, since the order of the World in its other parts and motions — namely in the heaven of the world, in generations, etc. — is everlasting. But if a violent or preternatural motion were attributed to the Earth, it would not be everlasting; for nothing violent is perpetual.

[Margin: 1st reply to Argument 1.] The Copernicans will reply, first, by denying the Major. To its proof they will concede that circular motion is natural both to the whole Earth and to its parts, as long as the parts are joined to their whole; but if they are separated, they will say that the motion natural to them is one mixed of circular and straight, or some other like it, according to the shape of motion required, so that by tending downward they fall back on the point of the Earth above which they were let go perpendicularly — on which enough was said in chapter 17. And where it is said in the proof of the Major that circular or quasi-circular motion does not belong to the parts, they will deny it. To the proof of this statement, taken from the apparent straightness of the downward motion of heavy bodies, they will concede that their motion as it appears to us is rectilinear, but will deny this of the real motion, which takes place in the space of the world. [Margin: Objection against the reply.] Against this reply, however, the Aristotelians will object that it is absurd to abandon sense and, in Physics, to destroy the motion that appears so as to substitute a motion that neither appears nor can appear — and this for certain fitting reasons, but by no means necessary ones.

[Margin: 2nd reply to the argument.] Second, the Copernicans will reply to the Minor by denying it as to its second part, concerning preternatural motion; or they will turn the argument back against Aristotle, who attributes to fire a circular motion from outside, owing to the diurnal revolution of the heaven, and so a preternatural one, and yet a perpetual one. Therefore a motion in a circle can also be attributed to the parts of the earth, if they are drawn magnetically by the whole globe of the Earth, as Gilbert, Kepler, Galileo and Gassendi hold; and indeed to the whole Earth as well, if it too is drawn round by the Sun's whirling about its own centre, as Kepler holds.

II. Argumentum à Simplicitate motus debita Telluri.

II. Argument from the simplicity of motion owed to the Earth.

III. Our Scheiner uses a very long sorites and heaped-up questions in the *Disquisitiones Mathematicae*, from page 31 to 35, to which Galileo tries to reply in Dialogue 2, from page 175 to 182 of the Latin, but from page 232 to 240 of the Italian. But since Scheiner, at last, on page 35, gathers what he had said before into three Enthymemes, I shall report them in order with the replies that Galileo in part, and other Copernicans in part, have suggested from their hypotheses. His first argument is of this kind:

[Margin: Scheiner's 1st enthymeme against the motion of the Earth.] "The Copernican motion of the Earth removes from the Universe the simple motion of things upward and downward; therefore it is not to be admitted." The Antecedent is plain, because on the Copernican motion of the Earth heavy and light bodies must naturally descend or ascend to their place in such a way that they describe a line that is not, indeed, perfectly circular, but quasi-circular, and varying according to the different parallels, as is plain from the figures brought forward in chapter 17, numbers 16, 17 and 18. The Consequence is proved because it is evident from the perpetual experience of the senses that a simple motion along a straight line, upward and downward and falling perpendicularly on the Earth, belongs to heavy and light bodies.

The Copernicans reply by denying the Antecedent as regards the motion that is apparent and evident to sense; for the hypothesis of Copernicus too represents this motion as rectilinear. In this sense they concede the Consequence; but they concede the Antecedent of the real line of this motion, against which there is no experience of sense, since sense stands towards it merely negatively and is directed only to the apparent motion.

III. Argumentum ab Incommodis Principij huius motus, quodcumque illud assignetur.

III. Argument from the difficulties of the principle of this motion, whatever principle is assigned.

[Margin: Scheiner's 2nd enthymeme.] IV. Scheiner's second argument, on the same page 35, which for us here is the 3rd, is this: "The Copernican motion of the Earth necessarily introduces into all things, those carried from the centre as well as those carried to the centre, some circular motion, whose principle, however — whether natural or not natural, whether internal or external — cannot be assigned without innumerable absurdities. Therefore the motion of the Earth is not to be admitted." Yet the very many absurdities he listed earlier militate only against an external and non-natural principle, which the better Copernicans do not assign; and not even Kepler, or the others who acknowledge magnetic tractions by the earth, think that parts of the earth placed outside their place are drawn by the terrestrial globe unnaturally, but naturally, with an inclination concurring in them to receive the traction passively. Let us hear, then, what he brings against those who assign this principle as natural and intrinsic. He says that, if circular motion is natural to the Earth, the straight motion, which certainly appears in those bodies, cannot be natural, since these

motions differ from each other in species. Moreover, if the whole earth together with the water were annihilated, no hail or rain would fall from a cloud along an apparent straight line, but would only be carried round naturally — which he affirms is contrary to experiments and to reason. Finally, if beneath the earth, about the Earth's centre, there were a sphere of air, and a stone were placed at the centre of the earth, the stone would have to rise naturally from that centre to the rest of the earth, and afterwards turn with it, so as to be joined in this way to its whole and to imitate its circular motion — which likewise seems absurd, since that stone ought to remain beneath the air and at the centre. Therefore the motion of the Earth or of its parts is not from an intrinsic principle, nor natural.

[Margin: Reply to Argument 3.] Galileo, nevertheless, would reply by denying the Antecedent. To its proof he will say that the principle of this motion is natural and intrinsic, and that nothing prevents the same thing being the principle of a motion really circular and apparently rectilinear, because by it the eye too is carried round in a circle, and so cannot see the motion except always on the same straight line. Next, if the terraqueous globe were annihilated, it is uncertain what miracle would happen in the rest of the elementary sphere, granted this miracle; perhaps, however, the hail and rain would either descend with the rarefied air to fill the vacuum, or would really go round in a circle about the place where the globe of the Earth had been, or, because they would not know that the Earth had been annihilated, would descend just as they do now as far as that centre which had formerly been the Earth's centre. Lastly, the stone would remain at the centre of the Earth, because that is the centre of heavy bodies, about which, or towards which, they tend so as to be united in this way to their whole, yet in such a way that, if they are not impeded, they tend to the place where all the parts of the globe conspire to arrange themselves equally around it. That stone, however, would turn about its own centre and the Earth's.

IV. Argumentum à Peruersione motus recti terrestris pro quasi violento habiti, & circularis pro naturali.

IV. Argument from the perversion by which the Earth's straight motion is held to be quasi-violent, and the circular motion natural.

[Margin: 4th argument, from Scheiner.] V. Scheiner's third argument, on that page 35, is of this kind: "The Copernican motion of the Earth makes straight motion accidental to heavy bodies, and quasi-violent, and on the contrary makes circular motion necessary and natural; to things most diverse in nature it gives one nature of moving; it calls a stone up from the centre of the earth of its own accord; the same stone lest it fall downward from the vault, it holds suspended by the mere love with which it longs to dwell with its whole, etc. All of which are contrary to reason and experience. Therefore the motion of the earth is not to be admitted."

[Margin: Reply.] The reply is to deny the Major as regards the claim of violence in the apparent straight motion. For this motion too is natural, though secondarily, because it belongs to a body that is out of order and tending to its own place, whereas circular motion belongs to a body already set in its own place. As for the stone held suspended outside the centre of the earth, in a terrestrial vault above air spread round the centre of the earth, so that no other terrestrial body were at the centre: the Major holds, according to what was said about the terminus ad quem of the motion of heavy bodies in Argument 4 of chapter 20, number 12. For the stone could not be torn away from its whole except violently, since the character of the whole [ratio totius] would not be at the centre. What the Copernicans nevertheless try to reply was stated in the same place.

INTERROGATIONIBUS, & QUAESTIUNCULIS MULTIS SCHEINERI DE PRINCIPIO HUIUS MOTUS, SATISFIT HAC OCCASIONE EX COPERNICANORUM HYPOTHESI PRAECIPUÈ GALILAEI.

On this occasion the many questions and petty queries of Scheiner about the principle of this motion are answered from the hypothesis of the Copernicans, chiefly of Galileo.

[Margin: 1st question.] VI. This circular motion of heavy and light bodies — from what principle does it come? Internal or external? If external, does God excite it by a continuous miracle? Or an Angel? Or the Air? And this last, indeed, many assign. But against this is, etc.

[Margin: Reply.] The reply is that it comes from an internal principle, and much less from the Air, since the Air itself is carried round together with the globe of the Earth and Water.

[Margin: 2nd question.] If the highest part of the Air is not moved with the diurnal motion of the Earth but is exempt from it, how then would heavy bodies let fall through it be carried round in a circle?

[Margin: Reply.] The reply is, 1, that no one has yet made this experiment — whether heavy bodies let fall through the highest region of the air move in the same way as those let fall through our own air. Secondly, that heavy bodies are carried round not from outside but by an intrinsic force of following and imitating the motion of their primary globe.

[Margin: 3rd question.] If this circular motion comes from an internal Principle, this principle is either an accident or a substance. If the first, of what kind is it? For no quality of local motion around [qualitas locomotiva circum] seems to have been recognized so far. And even if there were one, how, after all, would it be found in things so contrary? In fire as in water? In air as in earth? In living things as in things lacking a soul?

[Margin: Reply.] The reply is that it is an accident, whose effect we cannot recognize through the senses, because we too are moved together with the Earth; but if we were outside the elementary world, we should see this circular motion. Further, contrary motions do not belong to things diverse and contrary: that is, downward motion does not belong to light things, nor upward motion

to heavy ones, just as the motions that depend on a soul do not belong to inanimate things. But motions that are not contrary but common — such as circular motion, which even on Aristotle's testimony can be mixed with straight motion — can belong to diverse and contrary natures, just as downward motion belongs to innumerable bodies differing in species.

[Margin: 4th question.] If the Earth stood still by the will of God, would the other things be rotated or not? If the latter, it is false that they are whirled round by nature; if the former, the earlier questions return. And it would surely be strange that the gull could not hover over its little fish, the lark over its little nest, the crow over a snail and a rock, even when it wished to.

[Margin: Reply.] The reply is, 1, that it is uncertain what God would will to be the case with these things if he performed that miracle. Secondly, that if no other miracle took place than the stopping of the earth, the remaining bodies akin to the earth — which are accustomed to be led by a blind impulse, and therefore neither understand nor sense the privation of motion brought about in the Earth — would continue in their motions. Nor could those animals then hover over the bodies already mentioned, because these bodies are supposed to be checked together with the Earth, while the animals themselves would be carried round by natural necessity, unimpeded by God. Thirdly: suppose that the Earth, which Scheiner sets standing still, were suddenly snatched elsewhere by a divine miracle together with the bodies adhering to it, while the gull, the lark, etc. hovered over the bodies already mentioned and were moved by no motion of their own; then they certainly could not hover over them. Just so, in the hypothesis of Copernicus, they could not do so if the Earth were stopped by a miracle and this miracle did not take place in them, but their natural principle were allowed to move them round.

[Margin: 5th question.] How does it come about that these things, so various, move only from west to east, parallel [?] to the Equator? That they always move and never rest?

[Margin: Reply.] The reply is that this happens exactly as, in the opposite and Ptolemaic hypothesis, the fixed Stars move from east to west, describing lines parallel to the Equator as regards the diurnal motion, and that without cessation.

[Margin: 6th question.] Why are the higher ones swifter and the lower ones slower? [Margin: Reply.] The reply is: because, by the common law of circles described from the same centre, if all revolve in the same time, the parts more remote from the centre are carried through greater circles and are therefore swifter, since they complete a greater space in the same time. But this seems to assign a geometrical rather than a physical reason.

[Margin: 7th question.] [Margin: Reply.] Why are those nearer the Equinoctial carried in a greater circle, and those more remote in a smaller one? The reply is that this happens in the same way as, in the contrary hypothesis, the Stars nearer the Equinoctial move in greater circles than the more remote ones; and they would do so even if they moved of themselves and not

with the motion of the eighth sphere, so as to keep the laws and proportion of the diurnal revolution. But the stars have this from an Intelligence, or from their own nature, granted a natural motion by which they transfer themselves to such a position in the sphere; heavy bodies, however, do not have it from their own nature that they should demand to be now above one parallel, now above another. And so the reply does not satisfy equally.

[Margin: 8th question.] Why would the same ball, under the Equinoctial, revolve as a whole about the centre of the earth in the greatest circuit and with incredible speed, but under the pole about its own centre, in no circuit and with the utmost slowness? For in its descent above the poles along a straight line it would cover a far smaller space in the same time than if it descended in the plane of the Equator.

[Margin: Reply.] Galileo replies, on page 236 of the Italian and 178 of the Latin, that this happens in imitation of the Fixed stars, which would do the same if the diurnal motion were attributed to them and one star at the Equator were compared with a star fixed at the pole of the Equator. So the ball, because it imitates the motion of its primary globe, is carried round, if it is placed at the Equator, just as the terrestrial Equator is; if it is above the poles, it is not carried round, just as the pole is not, which is supposed immobile as regards the diurnal motion.

[Margin: 9th question.] Why does the same thing — a lead ball, for example — if it has once gone round the earth describing a great circle at the Equator, not travel round the same earth along a great circle wherever it is placed, but, when moved outside the Equinoctial, is driven in smaller circles?

[Margin: Reply.] The reply is that the same thing happens to it, as regards the diurnal circuit, as would happen — indeed, according to Ptolemy, already does happen — to the fixed stars, which were once at the Equator and afterwards lay on parallels of the Equator. But see the reply to Question 7.

[Margin: 10th question.] If circular motion [latio circularis] is natural to heavy and light bodies, what kind is the motion that takes place along a straight line? For if it is natural, how is the motion that goes round also natural, since it differs in species from the straight? If it is violent, how does it come about that a fiery missile flying upward lifts its sparkling head away from the earth and does not instead whirl round, since the upward motion is the weaker and violent one, and the circular motion the stronger and natural one? Why do falling thunderbolts not trail their tail according to the rule of circular motion, which is natural to them, but rather according to the course of the violent one, so that they stretch it upward?

[Margin: Reply.] Galileo replies that it has been said countless times that this circular motion is natural and intended primarily and of itself, whereas the straight motion — or rather the one mixed of straight and circular — is natural on the supposition that the bodies are out of order and outside their own place. Further, that we do not notice motion of this kind because it is common to us.

[Margin: 11th question.] Why does the centre of a sphere let fall under the Equator describe a spiral in the Equator's plane? Under other parallels describe a spiral on a cone? Under the pole describe a straight line coinciding with the axis, while the sphere itself, with its outermost point, runs along a spiral line traced on a cylindrical surface?

[Margin: Reply.] Galileo replies that this happens because heavy bodies descend along lines drawn from the centre to the circumference of the sphere; and lines of this kind, as the earth is moved by the force of the diurnal whirling, describe a circle at the Equator, while those that end at parallels describe conical surfaces; the axis itself, however, describes nothing, but remains within itself. Galileo therefore concludes, on page 179 of the Latin and 237 of the Italian, that from all these petty queries of Scheiner he can gather nothing that carries any weight against the motion of the Earth; and rightly. For had Scheiner looked more deeply into the Copernican hypothesis, he could have answered himself, and so have refrained from many, if not all, petty queries of this kind.

CAPVT XXV.

Chapter 25.

Proponuntur & Soluuntur Vndecim Argumenta contra Solum Annuum Terrae motum; & primò illa, quae Aristoteles & eius in hoc sectatores sumpsere ex loco Telluri in Mundo debito, spectata ipsius natura, & loco ac motu Grauium ac Leuium ipsi debito. Et Colliguntur selecti Aristotelis Textus ad id spectantes, cum explicatiunculis, vbi opus est.

Eleven arguments against the annual motion of the Earth alone are proposed and solved; and first those that Aristotle and his followers in this matter took from the place owed to the Earth in the World, having regard to its nature and to the place and motion of Heavy and Light bodies owed to it. And the selected Texts of Aristotle bearing on this are collected, with brief explanations where needed.

I. So far we have dealt with the arguments that militate against the Earth's diurnal motion only, or against the diurnal and annual motions together; from here on we deal with those that could be devised, either by others or by us, against the annual motion alone. First there present themselves the arguments of the Peripatetics and of many others, derived from the nature of heavy and light bodies and from that motion of theirs by which they seek their right seat in the universe. These Aristotle first of all hurled against certain Pythagoreans in Book 2 of *De caelo*, chapters 12 and 13; Ptolemy, *Almagest* Book 1, chapter 7; Alfraganus, differentia 4; Cleomedes, *Cyclical Theory* Book 1, chapter 9; Albert the Great, *De caelo et mundo* Book 2, tract 1, chapter 7, and *De natura loci* tract 1, chapter 3; Johannes de Sacrobosco in the *Sphere*, chapter 1, and Clavius there, page 135; Petrus Aliacensis, question 3 on the *Sphere*; Maurolycus [Francesco Maurolico], *Cosmographia* dialogue 1, page 10; Barocius, *Cosmographia* Book 1, page 35; Blancanus in the *Sphere*, Book 4, chapter 1; Scheiner in the *Disquisitiones Mathematicae*, page 35; Fortunius Licetus [Fortunio Liceti] in his disputations on

the single centre of motion; Io. Elephantutius on the structure of the universal orb; Chiaramonti, *De Universo* Book 12, passim; and, lest we neglect the Poets, Georgius Bucanhanus in Book 1 of the *Sphere*. There he first describes the opinion of the Pythagoreans, who lived at Tarentum and Croton and said that Fire is at the middle of the universe and that the Earth, with another, opposite Earth, turns about it, in these verses:

[Margin: Buchanan's verses on the position of Fire and Earth in the Universe.]

> Nor lighter was the error, on the place of earth and wave, of those same sages who dwelt within the lofty walls of Croton and in Spartan Tarentum: for nature encloses nothing in the whole of its body purer or better than the living seed of flame, etc. To this, then, by far the worthiest seat seemed the secret inner shrine of the orb, the recess of the middle, from where, as from a sacred sanctuary, it might pour itself out into every part — as, if it is lawful to compare great things with small, the HEART, fount of blood and of life, which quickens the sluggish limbs in all living creatures, spreads from the cavern of the breast its vital heat through the other members. Next after fire, for them, the Earth is set; and yet they did not establish this one alone, but also an Antichthon, an earth opposite to it, rolling its eternal circuits round the fire.

Having set this out, he at once rejects them, saying:

> Perceive how far these things depart from true reason: for since heavy bodies have one force of moving of their own accord, so as to flee the vaults of heaven and flow into the middle centre of the world, it must be that the Earth, lowest, settles of its own accord, and, poised in the empty air at the middle station of the world, shapes itself into a sphere.

II. These arguments are taken chiefly from Aristotle, Book 2 of *De caelo*, chapters 12 and 13, as we shall soon see. Further: Aristotle's arguments are touched on lightly by some, and by certain people are even cited unfaithfully, or enfeebled; their force must not be dissembled by us, but brought out in its full extent; and the text is neither always so clear in itself, nor translated so clearly from Greek into the Latin tongue, that it could not seem obscure to some. It will therefore be worth the effort to transcribe here his selected texts that bear on this matter, with a very brief paraphrase where needed, so that from them we may extract the thread of Peripatetic doctrine and the force of the arguments, like a kernel from its shell.

[Margin: Texts of Aristotle selected for this controversy.] Accordingly Aristotle, in Book 2 of *De caelo*, chapter 12, text 72, proposes the questions to be treated in that and the following chapters, saying: "It remains to speak of the Earth: where it is placed, and whether it is among the things that are at rest or among those that move; and of its shape." And shortly afterwards he says: [Margin: *De caelo* 2, text 72.] "While most of those who say that the whole

heaven is finite say that it lies at the middle, those who live around Italy and are called Pythagoreans say the contrary: for they say that Fire is at the middle, and that the Earth is one of the stars and, being carried in a circle about the middle, produces night and day." That is, it is carried round the centre of the universe by an annual motion in such a way that it nevertheless also produces the alternations of night and day by a diurnal whirling. He goes on in the same text and relates of the same men: "Moreover, they invent another earth opposite to this one, which they call by the name Antichthon, not seeking the cause from the things that appear to sense, but rather striving to drag the appearances towards certain opinions and reasons of their own, and trying to adorn these" — that is, with the colours of certain congruences. [Margin: Text 73.] For he adds in text 73, as if divining what was afterwards to be invented by the Copernicans: "To many others too it will seem that the middle ought not to be assigned to the Earth, since they take their argument not from the things that appear, but rather from reasons. For they think that the most honourable region is owed to the most honourable thing; and that Fire is more honourable than Earth, and the Boundary than the intermediate parts; and that the Extremity and the centre have the character of a boundary. Reasoning from this, therefore, they hold that it is not the Earth that is set at the middle of the sphere, but rather Fire. Further, the Pythagoreans add this reason too: that it is most fitting that what is most principal in the universe be guarded, and that the middle of the world is most apt for this (which they accordingly call the prison of Jupiter), and that this region therefore belongs to Fire." The Copernicans, however, understood by this most honourable and most principal Fire the Sun itself.

But the Philosopher, at once, in text 74, criticizes the Pythagoreans and the others who differ in this way on two counts. First, that they use an equivocation and confuse the middle of magnitude, such as the centre of a sphere and of the world, with the middle of the thing and of its nature, which is not always the same, as is plain in animals: for in them the middle of the animal and the middle of the body are not the same. For whether the brain is the principal part of the animal, in which the soul exercises the common sensations, or the heart, in which lies the principle of the vital operations, neither is at the middle of the body's magnitude, that is, of the organic body; and in us, indeed, the heart is much nearer the head than the feet — an example Aristotle does not indeed state, but it is enough for him to have pointed to this. Secondly, between the two extremes of a sphere, namely the centre and the surface, he says that the surface is the more honourable, because it has the character of the end and of the container, the centre that of the finished, or of what is bounded and contained by another. With this explanation set out first, you will better understand the words of text 74, which run thus: [Margin: Text 74.] "As if the middle were said simply, that is, in only one way, and the middle of magnitude were the same as the middle of the thing and of its nature. And yet, just as in animals the middle of the animal and that of the body are not the same, so too, and much more, must it be judged of the whole heaven and world. For this reason, then,

they ought not to have been perturbed about the places of the universe, nor to have brought that guard, namely Jupiter's, to the centre, but to have sought what sort of thing the middle of the thing is, and where by its nature it is apt to be. For a middle of this kind is a principle, and more honourable. But the middle of place resembles the extreme more than the principle: for the middle of place is that which is bounded, while what bounds is the end; and the end and the container are more honourable than what is bounded, for the latter is matter, whereas the former is the substance of its constitution." After this, in text 75, he hints at the opi- nions of certain people about the motion of the earth, and among other things these words are worth noting: [Margin: Text 75.] "Those who deny that the earth is placed at the middle say that it moves in a circle about the middle, and not only this earth but also the Antichthon." And shortly afterwards he says that some, from the fact that in the celestial phenomena no notable variation appears to us who observe them from the surface of the earth, although we are distant from the centre of the Earth by a whole semidiameter, thought that everything could appear to us just the same whether the Earth is at the middle of the Universe or not. For this is the sense of those words: "Since the Earth is not the centre, but is distant from the centre by the whole of its hemisphere, they think nothing prevents the Phenomena from turning out in the same way if we did not live at the centre of the world as if the Earth were at the middle of the world; for not even now does anything notable happen to us, who are distant from the centre by half the diameter." Finally, he closes text 75 by relating the opinion of Plato, or of Timaeus, about the Earth at the centre of the world, but turned round by a whirling about its own centre and an immobile axis: "Some say that, while lying at the centre, it is rolled and moved about the pole that is always immobile, as is written in the *Timaeus*." That is, about the pole of the axis of the terrestrial Equator.

[Margin: Texts 76 and 77.] Further, in text 76 he sets out the opinions on the shape of the Earth, and in text 77 he brings forward a most weighty question, one worthy of our understanding, saying: "For perhaps it would be the mark of a rather irrational mind not to wonder how a small particle of earth, if it is raised and released, is carried downward and will not rest, and the larger part always moves faster; whereas the whole earth, if one imagined it raised and then released, would not be carried downward but, great as its weight is, would rest? And again, if someone took hold of its particles and withdrew the whole earth before they fell, they would nonetheless be carried downward with nothing resisting." From these doubts, he says in text 78, various solutions arose. Xenophanes of Colophon said that the whole Earth is not carried downward because it has an infinite root below, by which it is supported; Thales of Miletus said that it floats on the waters. [Margin: Text 79.] Him Aristotle criticizes in text 79: as if the water itself did not need another body to support it, and as if the whole earth, being heavier, ought not to descend through the water, seeing that any part of earth released descends through water. [Margin: Text 80.] Hence in text 80 he observes that some philosophize only so far as satisfies themselves,

and rest content with the reasons that first occur to them, as long as they have nothing to say against them. But he warns that one who philosophizes rightly must behave otherwise, and adds that golden admonition: [Margin: Aristotle's admonition for philosophizing rightly.] "Therefore one who is to inquire well must be fit to raise objections by means of the objections proper to each genus; and this comes from having considered all the differences."

[Margin: Texts 81, 82, etc.] Next, in text 81, he relates the opinion of Anaximenes, Anaxagoras and Democritus, who attributed a broad shape to the Earth so that it could not be carried downward; these he attacks from text 82 to 85. In that text he reports Empedocles as saying that the Earth is not carried downward because it is prevented by the most rapid revolution of the heaven, in the way that water in a cup swung swiftly round cannot fall, up or down. [Margin: From text 86 to 89.] Against him he presses several objections from text 86 to 89, where this is to be noted: "Or is it that heavy and light are not determined by the whirling either, but that, heavy and light things existing beforehand, the former come to the middle, while the latter rise above because of the motion?"

[Margin: Text 90.] Then, in text 90, he relates the opinion of Anaximander, who said that the whole Earth is not moved because, since it is at the middle, there is no reason why it should be carried to one side rather than another, since all regions of the world are, for the whole earth, downward in one respect and upward in another, and so none is more upward than downward. [Margin: Text 91.] This, he says in text 91, is said wittily indeed, but not truly; otherwise any body whatever, once placed at the middle of the world — whether it were earth, fire, air or water — would rest there perpetually, since the reason Anaximander adduced is not proper to the Earth itself; and moreover a reason should have been given why the parts of the earth are carried to the middle. [Margin: Text 92.] So in text 92 he says: "For it seems not only to remain at the middle but also to be carried to the middle: for where any particle of it is carried, there the whole must also be carried; and where something is carried according to nature, there it must also remain according to nature." [Margin: Text 93.] Further, in text 93, he says that it is absurd to ask why the Earth remains at the middle and not to ask why fire remains at the extremity: "For if the extreme place belongs to that by its own nature, it is plain that some place must belong to the Earth too by nature." The remaining points with which he presses Anaximander sharply can be read in texts 94 and 95, with which chapter 12 is completed.

III. In chapter 13, however, Aristotle, in texts 96 and 97, first determines whether the Earth moves, and tries to show that it does not move — a priori, indeed, in text 96, from the nature of the whole Earth, which, however, he investigates from the motion of the parts of the earth. He begins text 96 thus: [Margin: Text 96.] "Let us say first whether it has motion or remains still. For, as we said, some make it one of the stars, while others, placing it at the middle, say that it is rolled and moved about its own middle and pole. That this is

impossible we shall try to make plain, beginning from this first argument: that if the earth, whether it exists at the middle or outside the middle, is moved, it must be moved violently with a motion of this kind; for it does not belong to the earth itself by nature — otherwise each one of its particles would have this motion; but in fact they are all carried along a straight line to the middle. Therefore, since this motion is violent and beyond nature, it is not possible for it to be everlasting; but the order of the World is everlasting." [Margin: Text 97.] Afterwards, in text 97, he adds an argument a posteriori from the double motion of the Planets, which he says would have to belong to the Earth too if it moved, and yet could not be in it with the phenomena saved; but this argument belongs to chapter 26. He then goes on to other arguments from the motion of heavy bodies to the centre of the Universe, and in text 98 says: [Margin: Text 98.] "Moreover, the motion of the parts and of the whole itself according to nature is to the middle of the universe; for this is why it is placed, and even now exists, at its centre."

To prove this he needed the solution of two questions: the first is whether heavy bodies are carried to the centre of the Earth; the second, whether they are carried to the centre of the Earth insofar as it is the centre of the Earth, or insofar as it is the same, in subject and in reality, as the centre of the Universe. He solves the latter first, saying in text 99: [Margin: Text 99.] "But someone might doubt, since the middle of both is the same, to which of the two the things that have weight, and the parts of the earth, are carried according to nature: to the centre or middle of the Earth as it is the middle of the Earth, or as it is the middle of the Universe? I say, accordingly, that they must be carried to what is the middle of the Universe; for fire and whatever light things are carried to the parts contrary to those to which heavy things tend are carried to the middle of the extreme containing place." That is, towards the highest surface of the heaven, which is the universal place. "But it happens that the same middle belongs to the Earth and to the universe together; for they are carried to the middle of the Earth too, but accidentally, insofar as the Earth has its own middle at the middle of the Universe." In these two [?] texts Aristotle seemed to Galileo and Kepler to assume what is in question, namely that the centre of the Earth and that of the universe are the same; on this, more later.

Meanwhile the Philosopher solves the second question easily in text 100, by saying: [Margin: Text 100.] "That they are also carried to the middle of the Earth is shown by this sign: weights carried towards it move along lines that are not parallel but at similar angles; therefore they are all carried to one middle, which is indeed the middle of the Earth." Or, as Ptolemy argues in Book 1, chapter 7, all weights fall perpendicularly on the Earth's surface, and everywhere make right angles on both sides with the Tangent to the globe of the Earth at the point above which they are released; and so it comes about that the lines of all plumb-lines, and of walls raised to the perpendicular, converge at the centre of the Earth.

From what has now been said in texts 98, 99 and 100, Aristotle concludes in text 101 in this way: [Margin: Text 101.] "It is plain, therefore, that the Earth must be placed at the middle of the Universe, and be immobile." But someone could object that from the propositions already mentioned, if they are granted, it follows only that the earth does not move with a motion of translation, and does not follow that it does not move with a motion of whirling about the immobile centre of itself and of the universe. So he adds in the same text 101: "And for the causes stated, and because weights violently thrown upward are carried back along the perpendicular to the same place, even if that force should throw them to infinity. That it therefore neither moves nor lies outside the middle is plain from these things." Then, going back, he gathers from the natural rest of the earth that the motion of heavy bodies and of parts of the Earth receding from the middle is violent, and concludes from this that the motion of the whole earth receding from the middle is much more violent. Accordingly, in text 102 he argues thus: [Margin: Text 102.] "Moreover, the cause of rest is plain from what has been said. For indeed if the earth is by its own nature disposed to be carried from every side to the middle, as fire too appears to be carried from the middle to the extremity" — that is, carried by its own nature — "it is impossible that the motion of any particle of it by which it moves away from the middle should not be violent; for one motion belongs to one body, and a simple motion to a simple body, and not contrary motions." That is, two contrary motions do not belong naturally to one simple body, such as the Earth is. "But the motion away from the middle is contrary to the motion toward the middle. Therefore, if it is impossible for any particle of earth to move away from the middle" — understand: naturally — "it is evident that it is still more impossible for the whole Earth too" — namely, to move away from the middle of the universe; "for to the place to which the part is disposed by nature to be carried, the whole too is disposed by nature to be carried. Hence, if it is impossible for it to be moved except by a stronger force, it must necessarily move [?] in the very middle."

[Margin: Text 103.]

Lastly, in text 103 he confirms this position of the Earth by another argument, from the celestial Phenomena, which turn out just as if the Earth were in the middle; but these, like text 97 too, belong to the arguments to be proposed in chapter 26. And he concludes chapter 13 with his customary recapitulation: "So much, then, has been said about the Earth itself: about its place, its rest and its motion, and how they stand." For in chapter 14 he deals with the Figure of the Earth, and with that finished he completes Book 2 of *De caelo*. In Book 3 he treats of the Generation, number and difference of the Elements. And in Book 4 he deals with Heaviness and Lightness, and with which of them belong to the individual elements, and in what degree. Since this doctrine is very useful for understanding better Aristotle's arguments for the place of the Earth, we shall reduce it to the briefest compendium, selecting those points that bear most on this question.

SELECTA QUASI IN EPITOMEN, EX LIBRO 4. DE CÆLO.

Selections, as it were in an Epitome, from Book 4 of De caelo.

[Margin: Texts 1 and 2.]

IV. In text 1 of Book 4 of *De caelo*, then, he says that he will treat of the Heavy and the Light: what each is, what their nature is, and why they have these powers. In text 2 he supposes that the Heavy and the Light can in some way be moved naturally, but he observes that no name has been given to their actuality, unless one thinks that propensity or inclination to motion is a name suited to it. He adds that Heavy and Light things have within themselves something like incitements to motion, and that all authors use their powers, namely heaviness and lightness, yet few have given any determination about them.

[Margin: Text 3.]

In text 3 he says that the Ancients discussed only what is lighter or heavier than another, as bronze is heavier than wood — that is, what is light and heavy comparatively, and not what is simply and absolutely heavy or light.

[Margin: Text 4.]

Now note the words of text 4: "For the former" — that is, Light things — "are always disposed by nature to be carried from the middle, and the latter always toward the middle" — that is, heavy things. "Of these, that which is carried from the middle we say is carried upward, and that which is carried toward the middle, downward. For it is absurd not to think that in the heaven" — that is, in the World, which he often names in this way from its more important part — "there is something that is up and something else that is down, as certain people hold. For they say that there is not one thing up and another down, if it is alike on every side and anyone can walk everywhere with his feet set opposite to his own." So Timaeus held, as Simplicius reports on this text. "We, however, call up the very extremity of the Universe, which is up both in position and first in nature."

[Margin: The distinction of up and down established.]

[Margin: Text 5 explained.]

In text 5, from the fact that the heaven has not only an extremity but also a middle, he concludes that there is an up and a down, provided that the heaven is spherical on every side. Otherwise, if only a single hemisphere is supposed, whatever is its middle will not have the character of down. For then one of the two extremes will be the earth itself, and the middle will be something lying between the heaven and the earth; and so the notion of a middle in the universe is not enough to establish the difference of position called down, unless the universe is spherical. With this paraphrase you will now understand this text, which is otherwise obscure to many. It runs: "Since there is an extremity of the heaven and a middle, it is clear that there will be both an up and a down, as many also say, but not adequately. The cause of this is that they think the heaven is not alike on every side, but that there is only a single hemisphere, the

one above us. But those who think that it is such all round, and that the middle stands alike toward the universe, will call the one up and the middle down."

So far, then, is it from following that the position of up and down is abolished, because the earth can be walked about all round, and feet can stand at different times on a quarter opposite to themselves, or because each person can become his own Antipode, that Aristotle rather establishes that position from this very fact.

[Margin: Text 6.]

Having established it, he repeats in text 6 his definition or description, saying: "We call simply Light, then, what is carried upward and to the extremity; and simply Heavy what is carried downward and to the middle. But light relative to another, or Lighter, is that which, of two heavy things equal in bulk, is carried upward by nature more swiftly than the other." Here corrupt texts have "downward" in place of "upward."

[Margin: Text 8.]

Having determined these points, in texts 8 and 9 he presses the Platonists, who in the *Timaeus* defined the heavier as "that which consists of more of the same parts, and the lighter as that which consists of fewer, as a larger piece of lead is heavier than a smaller one, and bronze than bronze," and likewise for two other individuals of the same species. In this they would not have erred had they defined only what is heavier in the individual; but in this way, he says, nothing is determined about heavy and light taken simply. He adds: "In the same way they say that lead is heavier than wood too."

[Margin: Text 9.]

He shows in text 9 that this is absurd: first, because fire would be now lighter, now heavier, according as it consisted of more or fewer parts, and so the simply Light would be abolished; second, because a larger fire, being heavier, would be carried upward more slowly than a smaller one. And yet Fire is always light, and simply light, and the larger it is, the more swiftly it is carried upward.

[Margin: Text 10.]

When in text 10 he denies that what consists of fewer triangles is light and that what consists of more is heavier, he alludes to Plato in the *Timaeus*, who assigns the Cube to Earth, the Icosahedron to Water, the Octahedron to Air and the Pyramid to Fire, as Simplicius notes there.

[Margin: Text 11.]

Next, in text 11 he censures those who defined Heavier things by the multitude of their planes or by greater bulk, as though bulk consisted of indivisible planes. But many things are heavier than others and yet smaller in bulk, as bronze is heavier than wool.

[Margin: Texts 12 to 20.]

Leucippus and Democritus, as Simplicius reports here in commentary 4, attributed the cause of heaviness to the solidity of the atoms and the cause of lightness to the interposition of the void; from text 12 to text 20 he therefore harries them with many objections. The chief of these is that there could be

gold and earth containing more void within them than some fire, and yet they would not for that reason turn out lighter; and that the void is not moved and does not need a place upward, since it is itself rather a place, if its existence is admitted.

[Margin: Text 22.]

In text 22 he takes it as an axiom that each natural body is carried naturally to its own place and flees from the contrary place, insofar as the place is such. For as in other changes there is a passage from one contrary to the other that is intended of itself — in Generation, for example, to the form; in alteration to the quality; in augmentation to the quantity of the substance — so in natural locomotion the mobile body must tend to its own place, insofar as it is its place. He therefore censures certain of the Ancients who said that Heavy and Light things are carried down and up because they are carried toward what is like them. For although it is true that in the place to which they are carried there are things of a like nature — namely, that in the place to which earthy things are carried there is earth, and in the place to which airy things are carried there is air — nonetheless the formal terminus of the local motion of Heavy and Light things is the place, insofar as it is such a place in the universe as befits them. This he confirms as follows: "For if someone were to move the Earth to where the Moon now is, each of its parts would be carried not to it, but to the place where it now also is." Let the Copernicans note this, who say that heavy and light things are carried to their wholes not by reason of place but because of kinship in nature.

[Margin: Text 23.]

Yet Aristotle grants in text 23 that the intermediate elements are also carried toward things like themselves, for another reason: water, he says, is like air, and air like fire. But this he denies of the extreme elements.

[Margin: Text 24.]

And in text 24 he teaches again that this likeness is not the formal terminus of the local motion of heavy and light things, but the place itself, up and down, insofar as it is such: just as what is curable, if it is changed as curable, is moved toward health and not toward whiteness, even supposing whiteness were found joined with health. "To ask," says the Philosopher, "why fire is carried upward and earth downward is the same as to ask why the curable, if it is moved and changed insofar as it is curable, tends to health and not to whiteness." This doctrine he confir- [Margin: Text 25.]

ms in text 25: when air comes from water, three changes intervene — Generation, terminating at the substantial form; alteration, terminating at the qualities; and at once locomotion to the place above water. Granted, then, that air is more like air than water, it is still not carried to air because it is like it; for this likeness consists in a quality, which it has already acquired by force of the alteration. It therefore ascends upward to such a place insofar as this place is owed to the lighter body. He therefore concludes that each of these is carried to its own place insofar as such a place is owed to them in the universe.

[Margin: Text 26.]

With these points subtly argued, in texts 26 and 27 he defines the Heavy and the Light, both simply and relatively, with this new formula: "First, then, it has been defined, as seems right to everyone, that the simply heavy is what sinks beneath all things, and the Light what rises above all things."

[Margin: Text 27.]

And in text 27: "Heavy and light relative to another are those in which both are present: for they rise above some things and sink beneath others, as air and water do; for neither of these is simply light or heavy," etc.

[Margin: Texts 28 and 29.]

And in text 28 he teaches that the cause of heaviness and lightness in composite or mixed bodies is a greater or lesser participation in the qualities and differences of the elements. At once, in text 29, he gives his opinion on the things that participate in heaviness, in these words: "It happens, therefore, that the same things do not appear heavy and light everywhere, because of the difference of the primary bodies. For example, in air a piece of wood weighing one talent will be heavier than lead weighing one pound, but in water it will be lighter. The cause is that all things have heaviness except fire, and all things have lightness except earth," etc.

[Margin: Text 30.]

And in text 30: "For in its own region everything has heaviness except fire, even air itself. A sign of this is that an inflated skin pulls more" — namely, it pulls down the scale-pan, or the hand that holds it up — "than an empty one." And he concludes this text thus: "Hence, if something has more air than earth and water, it may be lighter in water but heavier in air; for it does not rise above air, but it does rise above water."

[Margin: Text 31.]

Next comes text 31, in which, defining once more, he says: "I call simply light what is always naturally disposed to be carried upward, and heavy what is always naturally disposed to be carried downward, when not hindered."

[Margin: Text 32.]

Against certain people who say that all things have some heaviness, he affirms that there are some things that are simply light and have no heaviness; and in text 32 he proves this from the fact that fire rises above all things and sinks beneath none of the elementary bodies — yet it would sink beneath them, he says, if it had any heaviness. But his opponents would say that it sinks beneath none because all things are heavier than fire.

[Margin: Text 33.]

And in text 33 he confirms that there is a middle in the Universe, to which heavy things are carried and from which light things are carried, because there is no motion downward to infinity; and he adds this confirmation: "Further, fire appears to be carried upward at similar angles, and earth and everything that has heaviness downward. Hence they must be carried to the middle. Whether

this happens accidentally toward the middle of the Earth or toward that of the Universe, since their centre is the same, is another discussion."

[Margin: Text 34.]

Aristotle goes on, and from text 34 onward proves the same proposition — namely, that there are a simply Heavy and a simply Light — by two most fitting reasons. For if there is both something that sinks beneath all things and something that rises above all things, and there is a middle and an extremity in the universe to which certain bodies are carried simply, is it not reasonable that there should be a simply Heavy and a simply Light?

[Margin: Text 39.]

I pass over the following texts for now, since they do not bear much on our matter, and come to text 39, in which I would have these propositions noted above all others: "If it is withdrawn, there is motion into what comes next below: air into the region of water, and water into that of earth. But upward, into the region that belongs to fire, air will not be carried when the fire is removed, except by force — just as water is drawn upward by the contiguous upper surface if someone draws it upward faster than the water is carried downward; nor will water be carried into the region of air, except by force." And a little later: "As earth is not carried upward, so neither is fire carried downward when the air is removed" — understand: except by force, by something drawing it in order to fill a void — "for it has no heaviness in its own region, nor has earth lightness." Texts 40 and 41 do not bear on our matter, nor do the remaining four, that is, 42, 43, 44 and 45. In the last of these, however, he teaches that the shapes of bodies do not produce the Heavy and the Light, and so are not motive downward or upward; but that, motion from Heaviness or from Lightness being supposed, they contribute to a greater or lesser speed. With these premises laid down, we must come to the arguments.

I. Argumentum pro Telluris loco in centro Vniuersi.

Argument 1, for the Earth's place at the centre of the Universe.

V. "The centre of the Earth is where the terminus is to which all Heavy things naturally tend of themselves. But a terminus of this kind is the centre of the Universe. Therefore the Earth is at the centre of the Universe."

The MAJOR is proved as follows. On the one hand, the centre of the Earth is where the terminus is to which all Heavy things are carried. For, released all round and from any place whatever, they are carried along straight lines that are not parallel to one another but converge in one point, and so fall perpendicularly on the convex surface of the terraqueous globe — that is, they make right angles on both sides with the straight line touching the terrestrial globe at the point of contact; and this happens everywhere on earth. Hence Aristotle, *De caelo* Book 2, text 100, and Ptolemy, *Almagest* Book 1, chapter 7, rightly conclude from this that all Heavy things are carried to one point, which lies within the Earth equally distant from the convex surface of the terrestrial globe; and such a point is the centre of the Earth. On the other hand, if not

some heavy things only, nor only at some times, but all heavy things and always are carried to one and the same terminus, and that naturally and from within, it follows that this is the terminus to which they are carried not accidentally but of themselves. Therefore the centre of the Earth, in reality and in fact, is that terminus to which all heavy things naturally tend of themselves.

[Margin: First proof of the Minor.]

The MINOR is proved, first, by an exclusive proof, one that excludes the centre of the Earth. For even if the Earth were not where it is, but were annihilated or moved elsewhere, Water would nonetheless, with the Earth removed, descend into the place where the Earth had been; and if the Water in turn were removed, Air would take the place of the water by natural descent. So teach reason, supported by experiments — not these indeed, but similar ones — and Aristotle, *De caelo* Book 4, texts 22 and 39, already cited above. For we see that when a part of the earth on which water lay is removed, the water descends into the place where that part of the earth was, and air descends into the place where the water was. And the account is the same for the whole element and for the part, and for a motion of the same species, from the same principle and to the same terminus, as the Philosopher taught more than once, especially in *De caelo* Book 2, texts 96 and 102, already cited. Nor should you say that the air and water would then descend to fill a void, and so not naturally but by force and by attraction. For both water and air itself have some heaviness, as Aristotle proved by the experiment of the skin, *De caelo* Book 4, text 30; they would therefore rush down by the force of their innate heaviness, and not by attraction, to fill the void and at the same time to occupy the place owed to the heavier body. Therefore in that case the centre of heavy things would not be the centre of the Earth, since the Earth would either not exist or exist elsewhere. Hence, if the centre of Heavy things can be separated from the centre of the Earth, the centre of the Earth is not of itself the terminus to which Heavy things are carried of themselves insofar as it is the centre of the Earth, but accidentally. Aristotle does not indeed state this proof, but he conveys it implicitly in the texts already mentioned.

[Margin: Second proof of the Minor.]

Second, the Minor is proved by an inclusive proof, which is Aristotle's, *De caelo* Book 2, text 99. For the terminus to which heavy things are carried of themselves is the terminus from which light things are carried to the opposite terminus, since along the same line, making similar angles everywhere, the former descend and the latter ascend, as is said in the same place and in *De caelo* Book 4, text 33, cited above, and as is evident by experiment. But Light things are carried toward the outermost surface of the universal place that contains the middle of the world, that is, toward the outermost surface of the heaven; and indeed by a perpendicular ascent, and at angles, with respect to the celestial surface, like those that heavy things make in descending at the surface of the earth (if we abide by the judgement of the senses, as we must so long as nothing is established to the contrary, whether by evidence or by higher authority).

Therefore the terminus from which light things recede is the middle or centre of the world; and so the same is the terminus to which heavy things are carried of themselves. I have chosen to set out Aristotle's argument in this way so that its force might show itself as fully as possible. Yet two distinguished Copernicans, Galileo and Kepler, report it in other, almost equivalent words, though they answer it differently.

[Margin: The same argument propounded by Galileo and solved.]

VI. For Galileo, in Dialogue 1 on the *System of the World*, Italian page 26, Latin page 20, brings in Simplicio arguing thus with Salviati: "In the second place you call into doubt whether the parts of the Earth move, as Aristotle affirms, toward the centre of the World — as though he had not demonstrated this conclusively by contrary motions, when he argues in this way: The motion of heavy things is contrary to the motion of light things. But the motion of light things, as is apparent, tends directly upward, that is, toward the circumference of the World; therefore the motion of heavy things tends straight toward the centre of the World; and it happens accidentally that it tends toward the centre of the Earth, since these centres coincide by chance." From Galileo, in the same place, I gather a threefold reply to this argument.

[Margin: Galileo's first solution.]

The first is the one by which he implicitly denies the Minor of the argument proposed by Simplicio. For although fire tends straight upward to a circumference greater than the terrestrial circumference — such as the concave of the Lunar heaven, to which Aristotle more than once grants that Fire tends — it does not follow from this that this circumference is the circumference of the Universe or of the highest heaven, or concentric with the World. Nor can this be affirmed unless it is presupposed that the centre from which we see light things ascend is the same as the centre of the World, and so that the centre of the terrestrial globe is the same as the centre of the World. But this is the very thing we are in doubt about, and about which Aristotle himself raised a doubt in text 99 of the second book of *De caelo*. Hence Galileo asserts that Aristotle, by supposing the very point in question, manifestly commits a paralogism.

[Margin: Aristotle accused of paralogism by Galileo, and defended by Chiaramonti, but not sufficiently.]

But Chiaramonti, *De Universo* Book 12, chapter 22, defends Aristotle. He says that in that text 99 Aristotle did not intend to prove that heavy things are carried downward — for this he supposed from common sense and from a most widely received axiom, that heavy things are carried downward to the lowest place of the world and that this is the centre of the universe — but intended to prove that heavy things are carried to the centre of the Earth not of themselves and insofar as it is the centre of the Earth, but insofar as it is the centre of the World. Yet granted that he did not want to prove that heavy things tend downward, he did want to prove that heavy things tend to the centre of the universe, from the fact that they tend of themselves to the point from which light things recede by ascending straight upward. And he supposed this point

to be the middle of the universe because he supposed the circumference to which light things are carried to be the circumference of the World. That he should suppose it to be this circumference rather than that of some other sphere eccentric to the world arose from nowhere else than from a preconceived opinion that the circumference of the World was described from the centre from which light things recede and to which heavy things tend — that is, from the centre of the Earth. Hence it is that, without any further demonstration, he asserted in *De caelo* Book 2, text 98, and *De caelo* Book 4, text 33, already cited, that the centre of the Earth and the centre of the Universe are the same. Therefore, although Aristotle in *De caelo* Book 2, text 101, concluded that the Earth is in the middle of the World, yet since he proved it only by supposing that its centre is the same as the centre of the World, he seems to have supposed what was in dispute — and what he himself, a little before, in text 73, had related was denied by the Italian Pythagoreans.

[Margin: Galileo's second solution.]

Second, Galileo, in the same place and in the person of Sagredo, holds that, even if it were granted that the circumference which fire seeks in a straight line is the circumference of the World, it still does not follow from this that heavy things descending along the same line tend to the centre of the World, unless that line is supposed to be produced as far as the centre of the World and to be traversed in its whole length by their descent. For from any point within a circle, even one far from the centre, a straight line can be drawn toward the circumference, indeed produced as far as the circumference itself; and along that line ascent and descent can take place without passing, in the descent, the point already mentioned, and so without reaching the centre.

[Margin: Third solution.]

Third, Galileo, on the following page and in the person of Salviati, denies that heavy things are carried of themselves to the centre not of the Earth but of the Universe. For it is uncertain whether the Universe has a centre, and where it is, because of the infinity, or at least the vastness, of the world beyond what we see, and because the centre is a nothing, devoid of every faculty; whereas the centre of the Earth is certain, and heavy things are carried to it in order to be united with their whole and to take refuge with their universal mother.

[Margin: The centre of the World unfit to terminate the motion of heavy things of itself.]

This objection — that the centre of the World is unfit to move and attract by way of an end, or to terminate the motion of heavy things of itself — is pressed also by Kepler, *Epitome of Copernican Astronomy* Book 1, page 95, and by Gassendi, Letter 2, *De motu impresso*.

VII. Kepler, moreover, in the *Epitome of Copernican Astronomy* Book 1, page 99, sets out Aristotle's argument thus: "Aristotle proved that the motion of heavy things tends to the centre of the World from the fact that the motion of light things is toward the surface of the World, since the motions of heavy and light things tend toward contrary quarters."

[Margin: The same argument propounded by Kepler and solved, but insufficiently.]

To this argument he appends a reply that suffers from a greater fault than Aristotle's argument seems to suffer from. For he says that heavy and light are not said absolutely but only comparatively. Otherwise, if fire and smoke were absolutely light, they would fly off from the Earth as far as the outermost heaven; yet clouds of smoke, once they have passed the lowest region of the air, rest suspended. This, he says, is a proof that they do not of themselves, by their own nature, seek the outermost parts of the World, nor even flee from the centre of the earth, but are pushed by heavier things and give way to them — as one of the two pans of a balance, though heavy, seeks the higher place because it is drawn by the other, heavier one. With this said, he concludes: "Therefore it is false that light things seek higher places of themselves; false that they seek the very surface of the World."

[Margin: Five faults in Kepler's solution.]

But this reply errs in many ways. First, it is false that there is nothing absolutely heavy or light. For what of itself lies beneath all things in the elemental sphere is absolutely heavy, and what rises above all things is absolutely light; and there is in the elemental sphere some body that of itself sinks beneath all things, and a body that rises above all things. Second, it is false that, if there were something absolutely light, it would reach as far as the outermost heaven; for it would be enough for it to reach above all the other elemental bodies that are heavier than it, or less light. Third, he wrongly takes clouds of smoke as an example of simply light things, since they retain something of watery heaviness, and so do not reach even the highest region of the air, and are heavier than pure air. Fourth, from the suspension of clouds he wrongly infers that neither these nor light things as a whole move upward of themselves and from within, but that they are thrust out by the heavier parts of denser air. For we have arguments from elsewhere that show physically a positive lightness and an upward motion of light things of themselves and from within, as we have taught elsewhere; and at any rate that thrusting-out ought not to have been taken as certain in order to exclude the earth from the centre of the World, since it is less certain than the point at issue. Fifth and last, he wrongly attributes to Aristotle the saying that light things seek the outermost surface of the World; for Aristotle said only that they tend toward the outermost surface of the World — and if someone says that he will sail toward Africa, it does not follow that he wants to arrive in Africa. Therefore the first and second solutions of the Aristotelian argument brought forward by Galileo (for the third will be questioned below) are more solid than Kepler's. Now we must answer the same argument as proposed by us.

[Margin: Our reply to Argument 1.]

VIII. I reply, then, to Argument 1, proposed at number 5, by granting the Major and denying the Minor. And to its First proof it can be replied on behalf of the Copernicans that, if the whole Earth were moved elsewhere, all heavy

things too would be carried there — indeed, that the whole elemental sphere would be moved there with the Earth, unless some new miracle were worked in the matter. But if the Earth were removed entirely, or annihilated, then the heaviest of the remaining elements would succeed into its place, and its centre would then become of itself the centre of heavy things — not insofar as it is the centre of Water, but insofar as it is the centre of the body that would then be the heaviest in the World, or at least insofar as it is the centre at which the heaviest body of all, namely the Earth, ought naturally to exist. But it does not follow from this that the centre of the Earth, at least when the Earth exists, is not of itself the terminus toward which all heavy things move and from which all light things move. So too the outermost surface of the eighth sphere, as the highest of the celestial spheres, is for Aristotle of itself the universal place of all visible things; granted that, if it were destroyed and Saturn's Sphere remained, this sphere would then become the most universal place, containing all things.

[Margin: Reply to the second proof of the Minor.]

To the Second proof of the Minor I deny the Consequence. For granted that light things, from anywhere on earth, are allowed to be carried along a line falling perpendicularly on the outermost surface of the World, it does not follow from this that they are carried to that surface of themselves, but accidentally, insofar as it is exterior to and concentric with the sphere of the elements, to whose outermost surface light things tend of themselves. Hence, if light things tend of themselves to the outermost surface of the elemental sphere, they also recede of themselves from its centre; and therefore heavy things are carried of themselves to the centre of the elemental sphere, not insofar as it is the same as the centre of the World, but insofar as it is the centre of the elemental sphere. Moreover, the order of the Elements requires that the centre of the elemental sphere be the centre of the Earth first, by a certain priority of foundation and of nature, rather than of the other bodies. Therefore the terminus per se to which heavy things are carried, and from which light things are carried, is with better reason the centre of the Earth as Earth than the centre of the universe as it is the centre of the universe. Next, neither the Pythagoreans nor the Copernicans would grant that light things ascend, from anywhere on earth, at right angles formed on the very surface of the outermost heaven in geometrical rigour, but only to sense: both because they say that the whole Great Orb, along which they carry the Earth's centre around, is like a point relative to the vast sphere of the Fixed stars; and because the curved line along which they say light things really ascend is not perceived by us, carried along as we are with the Earth.

II. Argumentum pro Identitate centri Terræ cum centro Mundi ex Perpendiculari Ascensu Leuium versùs Cælum vltimum.

Argument 2, for the identity of the centre of the Earth with the centre of the World, from the perpendicular ascent of light things toward the outermost heaven.

[Margin: Form of Argument 2.]

IX. Light things, released from any point whatever of the convex surface of the terrestrial globe, ascend along a line that is not only straight but also falls perpendicularly both on the convexity of the earth and on the concave of the celestial hemisphere seen by us, that is, of the visible World. Therefore they ascend from the middle of the World. But the same middle from which they ascend is the middle of the Earth. Therefore the middle, or centre, of the Earth and of the World are the same. On the MINOR the Pythagoreans and Copernicans agree with the Peripatetics, nor do experiments allow it to be doubted.

[Margin: Proof of the MAJOR.]

The Antecedent of the MAJOR is proved both by the authority of Aristotle, already adduced, *De caelo* Book 2, text 99, and *De caelo* Book 4, text 33, taken together, and by the experience of the eyes. For, placed anywhere on earth, we see a whole hemisphere of the highest heaven, that is, of the visible World, and we see light things ascend straight toward it in such a way that the line of ascent, produced in imagination, makes equal angles on both sides: mixtilinear angles with the circumference passing through our zenith, but rectilinear — indeed right — angles with another straight line, conceived as touching the convex surface of the outermost heaven at the point on which the straight line of the ascent of light things falls. The Consequence of the Major is proved because it is impossible for a smaller sphere to be inscribed within a larger in such a way that, from any point whatever of the convexity of the smaller sphere from which perpendiculars are drawn to the surface of the larger, a hemisphere of that larger sphere can be seen — unless they are concentric; indeed, unless the smaller is so small that the whole of it can be reckoned as the physical centre of the larger. This is easily evident from Geometry.

[Margin: Reply to Argument 2.]

I reply. Grant that the World is finite and spherical, and that what we see of it is reckoned by everyone as a hemisphere (for the common people, ignorant of the fallacies of sense and of Optics, reckon the upper parts of the World to be a vault, or the arched roof of an oven, depressed in the middle and nearer to us than its extremities resting on the horizon). These things being granted, I say, I concede the whole Major physically and optically, that is, as to the appearance to the senses. First, because the distance of the highest heaven is so great that not only the globe of the earth but also the great orb, in which either the Sun or the Earth is carried, is like a point, so that no inequality of the angles already mentioned can be discerned. Second, because, if our senses themselves are supposed to be carried along with the Earth, they cannot discern this common motion of heavy things made along a line that is in fact curved, but are compelled to see them always appearing on the same straight line. But I deny the Major as to astronomical rigour, according to which the question now is whether the Earth is at the centre of the World, or is as far from it as the Sun is from the Earth; for it can be that far away with the phenomena already mentioned preserved.

III. Argumentum ex Termino formali motus Grauium & Leuium.

Argument 3, from the formal terminus of the motion of heavy and light things.

[Margin: Form of Argument 3.]

X. Heaviness and Lightness were given to bodies by God and by Nature so that, if they are outside their own place, they may return to it — not so that they may be joined to their own wholes similar to themselves, but so that they may tend [rendant] [recte: tendant] to their proper place in the World insofar as it is a place: heavy things downward, light things upward. Therefore they have their determinate place in the World. Therefore the Earth and all Heavy things are carried to the centre of the World, and Light things from the centre of the World. The Antecedent was proved most acutely by Aristotle in *De caelo* Book 4, texts 22, 23 and 24, reported above. The first consequence is proved because the motion of heavy and light things as such is not spontaneous, as in animals, nor free, as in men, but is determined by nature; and so they are said to be moved by what generates them, because whatever generates them co-generates in them a principle that determines them to one mode of acting and of moving themselves. Therefore, if their local motion is determinate, their proper terminus, namely a place in the World, is determinate too. The second consequence is proved because every other place outside the centre of the World is, by its own nature, indeterminate and indifferent with respect to heavy and light things; for no reason appears why heavy things ought to tend toward that place rather than toward another place in the World.

[Margin: Reply to Argument 3.]

I reply by granting the antecedent (which, however, Copernicus, Gilbert, Kepler and Galileo deny, maintaining that heaviness and lightness are nothing other than an inclination of the parts to tend to their primary globes, or to kindred bodies similar to themselves), and by denying both the first Consequence, if by a determinate place is understood a fixed and immovable place, and the second. For if the centre of the Earth is naturally carried along the circumference of the Great Orb, any points whatever of that circumference are determined for that office at their own moments of the year; and from this is taken the reason why, at such a time of the year, heavy things ought to tend to such a point in the World rather than another.

IV. Argumentum ex Negatione Descensus totius Telluris deorsum.

Argument 4, from the denial of a descent of the whole Earth downward.

[Margin: Formula of Argument 4.]

XI. If the centre of the Earth is not at the centre of the World, no adequate reason can be brought why the whole Earth is not carried either downward or toward that region of the world toward which the parts of the earth placed outside it are carried; but a reason can be brought if it is at the centre of the World. Therefore the centre of the Earth is at the centre of the World. The Antecedent is proved because all the other reasons that others have tried to adduce are sufficiently refuted by Aristotle, *De caelo* Book 2, chapter 12, from

text 77 on. Otherwise there is no reason why the Earth, placed outside the centre of the World, should rest in one part of the World rather than in another, and not be carried where its parts are carried; for, from *De caelo* Book 2, texts 96 and 102: "Where the parts are carried by their nature, there the whole too is naturally apt to be carried." On the other hand, if it is at the centre of the World, it will be equidistant from every region of the World, and so there will be no reason why it should be carried toward one region rather than another; and accordingly it will remain there unmoved.

[Margin: Reply of Kepler and Gassendi.]

Kepler, in the *Epitome of Copernican Astronomy* Book 1, page 102, and Pierre Gassendi, Letter 2, *De motu impresso*, page 113, reply by denying the antecedent. The cause, they say, is that the whole Earth, insofar as it is whole and compacted of parts already united to itself, is not heavy: because there is nothing outside the earth that attracts it magnetically, says Kepler; or because, as Gassendi says, heaviness is an affection not of the whole Earth but of the parts torn away from it. For the globe of the earth, considered in itself as a whole, does not need a motion by which to betake itself into its own place, from which it never goes out, since it is itself its own place. For this motion, he says, unites the parts and gathers them into one; and so, both on the side inhabited by us and on the side of the Antipodes, and on every side, they conspire toward the same point; but it is not a motion that gathers the Earth together with some other body. But this reply does not satisfy. For if by the name of heaviness is understood the act of gravitating itself, the question is about this very thing: why it does not gravitate toward a place that can be designated outside itself. If, on the other hand, heaviness in first act is understood, the parts do not acquire it from the separation itself, or from what separates them from their whole; for often such a separating agent is unfit for this — as fire hurling the rocks of Vesuvius upward, or a cannon hurling its ball. Therefore they are presupposed to have had it even when they adhered to the Earth; and so the whole Earth, consisting of similar parts, has heaviness. And rightly does Aristotle say, *De caelo* Book 2, text 77: "Perhaps it would be the mark of a more irrational mind not to wonder in what way a small particle of earth, if it is raised and released, is carried and does not want to rest, and a larger one always faster; whereas if someone were to release the whole Earth after raising it, it would not be carried at all; but now so great a heaviness is at rest." And in *Meteorologica* Book 1, chapter 4, he said: "In the middle, then, and about the middle is what is heaviest and coldest."

[Margin: The weight of the whole Earth wrongly investigated by Chiaramonti.]

Admittedly, I do not approve the attempt of Chiaramonti, in *De Universo* Book 12, in the appendix to chapter 13. By weighing a cube of our own pounded earth, he thought that from its proportion to the cubes contained in the globe of the earth he could come close to the weight of the whole Earth. I do not approve it, not only because within the Earth there are many waters and many caverns, as he himself admits, but because the cube of earth

he used, however much it was pounded and compressed, was much lighter than a cube of pure Earth, such as is found in the bowels of the earth. This is proved from the following. He says there that the cube was one inch of the foot of Cesena high — he shows the figure of that foot there in a linear diagram, and it is two inches of the ancient Roman foot — and that the earthen cube weighed 7½ ounces. Yet a lead cube of this kind, which, by his own concession in the same chapter 13, must be lighter than an equal cube of pure Earth, weighs two pounds and more. Therefore, by multiplying by 7½, as he does, the inch-cubes that are in the mass of the earth, we do not come close to the true weight of the whole Earth — although he says that the whole Earth weighs 2711,787,896,857,350,340,085,118 pounds.

[Margin: Second reply to Argument 4.]

A better reply, then, can be made with the hypothesis of the Copernicans preserved: by denying the Antecedent, and saying that the cause why the whole Earth, though very heavy, is not apparently carried along a straight line toward the centre of the World is this — that it is determined by its nature to a circular motion about the centre of the World, so that it must always stay equidistant from it, while its parts gravitate only toward the centre of the Earth itself.

V. Argumentum ex Translatione, aut Dissipatione Telluris in alienum locum.

Argument 5, from the transfer or the dispersal of the Earth into a foreign place.

XII. If the whole Earth were transferred to where the Moon now is, heavy things would not be carried to it, but to the centre of the World. And if the whole Earth were divided into particles that were carried off elsewhere, they would return nowhere else than to the centre of the World. Therefore the centre of the Earth and that of the World are the same. The Antecedent is gathered from Aristotle, *De caelo* Book 4, text 22; therefore, etc.

[Margin: Lunar waters adhering to the Moon, from Kepler.]

Galileo will reply, in Dialogue 2 on the *System of the World*, Latin page 181, by denying the Antecedent; for he says that, if the aggregate of heavy things were transposed to any place whatever, the parts separated from the whole would flow together there. But Kepler would deny the same antecedent much more strongly, since in the *Epitome of Copernican Astronomy* Book 1, page 100, he affirms that there are waters on the Moon, and heavy ones at that, and yet they do not fall to the earth, much less to the centre of the World, because they are kindred to the Lunar globe and cohere with it — by the Magnetic force of the Lunar globe, to be sure; and so he denies that all heavy things flow together to the centre of the Earth.

VI. Argumentum ex Loco debito Grauissimo & Infimo Corporum.

Argument 6, from the place due to the heaviest and lowest of bodies.

[Margin: Form of Argument 6.]

XIII. To the heaviest and densest of bodies the lowest place in the World is due. But the Earth is the heaviest, densest and lowest of Bodies, and the lowest place of the World is its centre. Therefore the Earth's centre must be at the centre of the World.

[Margin: Reply to Argument 6.]

The reply is by conceding the Major concerning the lowest place in relation to the elemental system, that is, in that part of the World which consists of heavy and light things; in this sense the Copernicans deny the second part of the Minor. But absolutely, and in relation to the system of the universe, they will deny the Major, and in this sense they will concede the Minor.

VII. Argumentum ex Definitione Grauium & Leuium.

Argument 7, from the definition of heavy and light things.

[Margin: Form of Argument 7.]

XIV. If the Earth were not at the centre of the Universe, the commonly received definition of heavy and light things would not be good. But the consequent is absurd; therefore so is that from which it follows; and so the Earth is at the centre of the Universe. The MINOR is evident. The MAJOR is proved: for the definitions of these handed down by Aristotle in *De caelo* Book 4, and commonly received, are three, but because the terms are equivalent they conspire into one. For in text 6 it is said: "Simply, then, we call light what is carried upward and to the extremity, and heavy simply what is carried downward and to the middle." Next, in text 26: "Let the first thing be defined as it seems to everyone: that heavy simply is what lies beneath all things, and light, what rises above all things." Finally, in text 31: "I call simply light what is always naturally apt to be carried upward, and heavy, what is always naturally apt to be carried downward, when not prevented." Now from the first definition, and from *De caelo* Book 4, text 4, to be carried downward and to be carried to the middle of the World are the same; and nothing can lie beneath all things unless it is at the centre of the Universe. Therefore, if the Earth, which is simply heavy, is not at the centre of the Universe, the definition of things simply heavy and light will not stand.

[Margin: Reply to Argument 7.]

The reply is by denying the Major. As to its proof: if by the middle is understood the middle of the Elemental system, that is, of the aggregate of heavy and light things as such, it is conceded that Aristotle's definitions can be commonly received. Yet even so they are not received by the Platonists, who, on the testimony of Aristotle himself in *De caelo* Book 4, texts 8 and 9, defined heavy as what consists of more of the same things and light as what consists of fewer, or heavy as what consists of more triangles and light as what consists of fewer; nor by those who distinguished heavy things by the full and light things by the void, or by more voids — namely Democritus and Leucippus, whom Aristotle nevertheless presses from text 12 to text 20 of *De caelo* Book 4; nor, finally, by the Copernicans, who hold that heaviness is an inclination to

tend to its own whole, and that lightness consists in a lesser heaviness. But if by the middle is understood the centre of the Universe and of the whole World system, it is denied that those definitions must necessarily be received, except by those who presuppose from elsewhere that the centre of the elemental system is the same as the centre of the World; for by those who do not suppose this, but either deny it or call it into dispute, they need not be admitted. And for the essential coordination of heavy and light things it is enough that the Heaviest lies beneath all things, the Lightest rises above all things, and the heavier lies beneath the lighter, along a straight line drawn from the centre of the elemental sphere.

VIII. Argumentum à Destructione Leuitatis positiuæ.

Argument 8, from the destruction of positive lightness.

[Margin: Form of Argument 8.]

XV. If the centre of the Earth and of the elemental system is not at the centre of the World, positive lightness is taken away and is resolved into a privation of greater heaviness. But this is absurd; therefore, etc. The MINOR is evident from the experiments and reasons adduced in chapter 16, numbers 3, 4 and 5.

[Margin: Proof of the Major.]

The MAJOR is proved, first, by the consent of the Copernicans, who with Copernicus, Book 1, chapter 8, deny that there is any positive lightness — as Kepler denies it in the *Epitome of Copernican Astronomy* Book 1, page 99, Galileo in the treatise on the things that float on water, and Gassendi in the Letter *De motu impresso*, pages 118 and 119. It is proved, second, by reason. For granted that the globe of Earth, Water and Air can be the place to which heavy things tend by their heaviness, even if this globe is transferred — since they have, after all, a centre toward which to direct their motion and descent, namely the point that is on the circumference of the annual orb — yet no place can be assigned to which light things may gather by ascent as to their own, since they are forced to leave it at once; nor is there a place in the space of the world toward which they ought by their nature to strive through an intrinsic principle of motion.

[Margin: First reply to Argument 8.]

I reply, first, by denying the Major. For the place of light things — of elemental fire, say, if there is such a thing — is a portion of the elemental sphere situated between the concave of the Moon and the highest region of the air; and particles of fire carried up there adhere to it no less than particles of earth adhere to the earth. And just as particles of earth, although through the transfer of the elemental system they lose, in the space of the World, the place that the whole earth also loses — that is, the remote place, or, as others call it, the mathematical place — do not, however, lose the pro- ximate and physical place, so neither do the particles of fire, although the elemental system is carried around elsewhere together with the Lunar heaven.

[Margin: Second reply, but a frivolous one.]

Second, not a few will reply by denying the Minor, with Epicurus, Democritus and Leucippus, whom Copernicus followed implicitly, Book 1, chapter 8, but explicitly Galileo in the treatise on the things that are borne on water, Kepler in the *Epitome of Copernican Astronomy* Book 1, page 99, and Gassendi, Letter 2, *De motu impresso*, page 118. For they said that lightness consists in the mere privation of greater heaviness; that Fire is called lightest because it is the least heavy of all — that is, Gassendi adds, because it is drawn by the Earth least of all bodies; and that the things we call light do not ascend upward by the force of an intrinsic principle, but are thrust upward by heavier things striving to occupy the lower place due to them. To confirm this he adduces certain things that suggest there is a twofold lightness: one, which he attacks with such light arguments; the other, the very lightness of the arguments.

[Margin: Gassendi's objections against positive lightness.]

First, he says, fire does not burn in a fireplace if the room is so closed that no air can enter through any little chink. But if the fireplace is open, why does the air that is in the room not thrust the fire upward? (For if the fire is carried upward in this case, the experiment would be void.) If you say it is because so great a rarefaction of the air cannot take place so suddenly as to fill as much space as the flame occupied, then the cause why the fire does not ascend is not a lack of thrusting-out, but the difficulty of filling the void. Second, he says that the feet of someone sitting at a fire are warmed in front, while the heels are cooled, as colder air runs to the heels on its way toward the fire. But grant that the experiment is true: from where do you prove that the air runs up in order to thrust out the fire, and not rather in order to succeed into the place of the flame flying upward? Third, he says that fire burns more vehemently if the logs lie on an iron grate rather than on the ash itself. Yet this happens not because the air underneath pushes the fire upward — for thrusting-out cannot of itself be the cause of a greater fire, since it supposes that the whole fire that is to be thrust out already exists — but because the proximate material of the ignition, and the flame itself, require rarefaction all around, and on that side this is impeded by the ash. Fourth and finally, he says that the furnace of the Chymists which they call the blast furnace [flatuoſam], without other bellows, by the air alone coming in from below, fans and inflames the coals standing above it. But who would doubt that the wind can drive the subtler parts and spirits of the fire, and force them into the pores of the coals in such a way that, united, they act more strongly and inflame the coals? Yet they will do the same even if they come in not from below but from above, as when we kindle coals lying underneath with a fan. So let these arguments go, and any lighter than these that have been devised in vain against the lightness of bodies.

IX. Argumentum à Confusione Grauium cum Leuibus, & vicissim.

Argument 9, from the confusion of heavy things with light things, and the reverse.

[Margin: Form of Argument 9.]

XVI. If the Earth is outside the centre of the World, nothing will be absolutely heavy or light; the same thing that is called heavy will at the same time have to be called light, and what is light, heavy. The consequent is unfitting; therefore so is the Antecedent from which it follows.

The MAJOR is proved because, while what is called heavy descends toward the earth, it will nevertheless at the same time recede from the middle, that is, from the centre of the universe; and what is called light, while ascending from the Earth, will approach the middle of the world. But what is carried from the middle is light, and what is carried to the middle is heavy; therefore the same thing will be at once heavy and light.

[Margin: Reply.]

I reply by denying the Major and its proof. For in the opinion that places the Earth outside the middle of the World, those definitions that distinguish heavy things from light things by approach to the middle of the World are not valid.

X. Argumentum à Fine motus Grauium & Leuium.

Argument 10, from the end of the motion of heavy and light things.

[Margin: Form of Argument 10.]

XVII. Heaviness and Lightness are implanted in inanimate bodies so that they may rest in their own place and not easily be moved away from it; or, if they have been removed from it, may return to those places in such a way as to rest there. Therefore the centre of the Earth, and of the whole elemental sphere, is not outside the centre of the World. The Antecedent is proved, first, from the Philosopher, *De caelo* Book 2, text 92, where it is said: "For where any particle of it — that is, of the Earth — is carried, there the whole too must be carried; and where something is carried according to nature, there it must also remain according to nature." Next, by reason. For by this the end of the motions that are in animate things or in celestial bodies is distinguished from the end of the motion that is in the elements and in inanimate mixed bodies: motion is given to animate things so that by spontaneous motion they may acquire for themselves something beyond what they have from birth; to celestial things, so that they may be moved not by acquiring something for themselves but for the sake of others; but to the Elements and to inanimate things motion is given for neither purpose, but precisely so that they may retain and recover the place due to them in the Universe. The Consequence is proved because, if the Earth is placed outside the centre of the world for some cause — above all so that through its motion many phenomena of the Sun and of the five lesser Planets may be more easily explained — then, if it moves, neither it nor the remaining elements retain their place in the World any longer, but change it perpetually.

[Margin: Reply to Argument 10.]

The reply is by conceding the Antecedent concerning the motion of heavy and light things as such; and in this sense the Consequence is denied. For when the Earth is carried around along the annual orb, they do not change place

by gravitation and levitation, that is, insofar as they are Heavy or Light; for once they have arrived at their own Physical place, they remain in it — granted that they change their Mathematical place through the circular motion, which belongs to the Earth not as a heavy thing, but insofar as it is supposed to be a kind of Planet.

XI. Argumentum à non necessaria Multiplicatione Centrorum, & Situs eorum.

Argument 11, from the unnecessary multiplication of centres and of their positions.

[Margin: Form of Argument 11.]

XVIII. Centres of systems in the World are not to be multiplied without necessity, and much less are the finite positions of centres to be multiplied to infinity. But if the Earth's centre is not the same as the centre of the World, then without necessity not only are the centres of systems multiplied, but also the finite positions of centres, to infinity. Therefore, etc. The MAJOR is certain from the axiom repeated so many times above, and admitted by the Copernicans themselves. The MINOR is proved because it follows from this that the centre of the system of the whole World is really different from the centre of the system of the Elements, which is the same as the centre of the Earth; and because the centre of the Earth is supposed to move perpetually, it also perpetually acquires one position after another, to infinity. But these things happen without necessity, since, once the identity of the centre of the elements with that of the World is posited, all the phenomena can be saved.

[Margin: First reply to Argument 11.]

The reply is, first, by denying the Minor. For in the opinion of the Copernicans the necessity is not indeed a posteriori, nor drawn from what appears to sense, but a priori, on account of that very axiom which is used in the Major — namely, that the centres of the Planetary system should not be multiplied; that is, that the Sun's centre, or the centre of the annual orb, should not be posited as different from the centre of the World.

[Margin: Second reply.]

And so the reply is, second, by retorting the argument, and putting this Minor under the Major of Argument 11: But if the Sun's centre, or the centre of the Great Orb, is not the same as the centre of the World, the centres of the systems of the World and of the Planetary system are multiplied without necessity. If, however, this necessity is proved from elsewhere, then this argument will be better framed against the Copernican hypothesis than against the opposite one.

CAPVT XXVI.

Chapter 26.

CAPVT XXVI.

Proponuntur ac Dissoluuntur Quinq. Argumenta contra Annuum Terrae motum & situm extra Vniuersi centrum, ex Situ deorsum & Peruersione systematis Mundani.

Five arguments against the annual motion of the Earth and its position outside the centre of the Universe, drawn from its downward position and from the overturning of the system of the World, are proposed and resolved.

I. Argumentum à Differentia situs Sursum ac Deorsum.

Argument 1, from the difference between the upward and the downward position.

[Margin: Form of Argument 1.]

I. From whatever part of the Earth's circuit the heavens are viewed, everyone judges that the Heavens are up and the Earth down. But this judgement cannot stand unless they judge that the Earth is at the centre of the World. Therefore, by the common judgement of mankind, the Earth is at the centre of the World. The MAJOR is proved not only by the authority of Sacred Scripture — which we are not yet dealing with here — which says in Proverbs 25: "The heaven above and the earth beneath; therefore let thy words be few"; and in Deuteronomy 4: "Know therefore this day, and consider it in thy heart, that the Lord himself is God in heaven above and in the earth beneath, and there is no other"; and in Isaiah 6: "Look upon the earth beneath." It is proved also by the common consent of very many wise men, so much so that Aristotle said in *De caelo* 4, text 4: "For it is absurd not to think that there is something in the World that is up, and something else that is down." The MINOR is proved because, in whatever part of the earthly circuit they are, people judge that what is simply down is the point about which the circumference of the heavens appears to be described, and which seems to be equally distant from the highest heaven. Therefore they judge that point to be the centre of the World.

[Margin: Reply to Argument 1.]

The reply is by conceding the Minor of the centre of the World taken Physically and as to appearance, and denying it of the centre taken Mathematically, or according to Astronomical rigour. For the distance of the highest heaven is so great in comparison with the Earth's distance from the centre of the World that, in the Copernican hypothesis, the latter vanishes against the former, and the whole Great Orb, along which the Earth is carried around, is like a point relative to the highest sphere.

II. Argumentum ex ratione Infimi.

Argument 2, from the nature of what is lowest.

[Margin: Form of Argument 2, and reply.]

II. The lowest place is owed to the lowest of the bodies of the World. But the Earth is the lowest not only of the elements but of all bodies, and the centre of the World is the lowest place. Therefore, etc.

Kepler, in the *Epitome of Copernican Astronomy*, Book 1, page 101, and Gassendi, in Letter 2, *De motu impresso*, page 113, reply by denying both the

Major and the Minor: for places need not correspond to bodies in nobility, nor is the Earth less noble than the Moon and perhaps than certain other Planets. And indeed the Sun is nobler than Saturn, Jupiter and Mars, yet not higher in place.

It might, however, be replied that the Earth, taken together with men and the other living creatures, is not the lowest, and so the Minor would be denied. Gassendi, for his part, says that nothing in the World is absolutely lowest or highest, but only relatively to the position of a living creature, and above all of man, whose highest part is the head and lowest the feet. Hence the part that hangs over our crown is called highest, and what lies beneath our feet lowest; so that what is for us the Nadir and the bottom of the heavens is for our Antipodes the Zenith and the top of the heavens. Nor does what is in the middle at once have the character of lowest, as is plain in the navel of the human body.

III. Argumentum à nimia Licentia collocandi Tellurem vbilibet.

Argument 3, from the excessive licence of placing the Earth anywhere one likes.

[Margin: Form of Argument 3.]

III. If the Earth is not at the centre of the Universe, everyone will be free to place it wherever he wishes in the Universe. The consequent is absurd. Therefore, etc.

[Margin: Reply.]

The reply is by denying the Major. For no one will be free to place the Earth in a position from which the whole hemisphere of the heavens does not appear to us, to sense, on every side. With this phenomenon preserved, however, it will be permitted. For, as Gassendi notes in Letter 2, *De motu impresso*, page 115, the distance of the Fixed stars is so great that we judge them all to lie in a single surface with the Planets and with one another. From this he infers that if we were on the Moon or on the Sun, the heavens would nonetheless appear spherical to us, and their hemisphere on every side. He adds that, just as the Ptolemaics admit that the Planets revolve around the Earth and yet do not say that their orbs are concentric with the Earth, so it does not follow that, because we see the heavens turn around the Earth, we therefore know where the centre of the heavens is; and that centre may be many semidiameters of the Earth away from the Earth.

IV. Argumentum à situ Inferni loci seu Inferorum.

Argument 4, from the position of Hell, the place of those below.

[Margin: Form of Argument 4.]

VI. If the Earth is not at the centre of the World, the place of the damned would not be Hell [Infernus], nor would all who are banished there descend into Hell; rather, the place of the damned would be in the heavens. But these things are absurd. Therefore, etc. The MAJOR is proved because those who were carried towards the centre of the Earth, at about noon, would recede from

the middle of the world — that is, from the Sun, or from the centre of the Great Orb, which is close to the Sun — and so would ascend rather than descend.

[Margin: Reply.]

The Copernicans will reply by flatly denying the Major. For Hell, the place of the damned, is understood relatively — both to the surface of the Earth, on which men live as wayfarers [Viatores], and to the highest region of the Empyrean, in which the blessed [comprehenſores] dwell; the damned recede from both in going down to Gehenna. Nor, they will say, is it absurd that the damned are in the Planetary heaven — but not in the Empyrean heaven; nor must that saying of CHRIST, "I saw Satan like lightning falling from heaven," necessarily be understood of the Planetary heaven, but of the Empyrean heaven. But as for the sacred authorities concerning the place of the Earth, they must be treated seriously below.

5. Argumentum à Peruersione Systematis Elementaris ac Planetarÿ, numerique Planetarum.

Argument 5, from the overturning of the Elementary and Planetary system, and of the number of the Planets.

[Margin: Form of Argument 5.]

V. If the Earth moves along the Great Orb around the Sun, as the Copernicans imagine, the whole system — not only the elementary but also the Planetary — is overturned and thrown into disorder, against the common sense not so much of the crowd as of the wise; and this without any necessity. Therefore the Earth does not move along the Great Orb, etc. The Antecedent is proved because it is the common sense of the wise that all the Elements and elementary bodies lie below all the heavens and are distinct from them no less in place than in nature and condition; that the Moon is the lowest of all the Planets; that the Sun and Moon are the two chief Luminaries and the primary Planets; and finally that the Sun is higher than the Moon but lower than Saturn, Jupiter and Mars. But the Copernicans place in the heavens, and among the Planets, not only the Earth but also the whole Elementary sphere, and with them all mixed bodies, animate and inanimate, so that by their account even beetles and ants are now, as it were, secondary little planets. The Moon they insert between the heaven of Mars and that of Venus, and make it a secondary Planet, the Earth's companion. Finally the Sun, cast out of every heaven and struck off the whole number of the Planets, they set motionless at or near the centre of the World. All these things they invent compelled by no necessity of the celestial or elementary Phenomena, but merely in pursuit of certain ideas and intelligible congruences.

[Margin: Reply to the Argument.]

The Copernicans reply by conceding the Antecedent in all its parts, except the denial of necessity. That denial is proved only of a necessity taken from the experience of the senses; but they will say that they were compelled to it by a necessity a priori, from archetypal and Ideal causes, and so as not to

admit other absurdities into the system of the World — such as they consider Retrogradation and Station, the double inequality of the motion of the Planets, and the superfluity of epicy- cles, and of Librations. Or they will say that this very necessity is not indeed absolute, but only from the supposition of what is better in itself and what seems more worthy of God, the Maker and Founder. Therefore, although the argument as framed has very great force with those accustomed to the Peripatetic manner of philosophising, with minds of the Platonic school, which contemplate intelligible Ideas rather than the images of things received through the senses, it is not of such weight.

CAPVT XXVII.

Chapter 27.

PROPONUNTUR AC DISSOLUUNTUR NOUEM ARGUMENTA SUMPTA EX CAELESTIBUS PHAENOMENIS, & PRAECIPUÈ EX ECLIPSIBUS, & AB EUIDENTIA MOTUS SOLIS, CONTRA MOTUM TELLURIS ANNUUM.

Nine arguments against the annual motion of the Earth, taken from the Celestial Phenomena, and chiefly from Eclipses, and from the evidence of the motion of the Sun, are proposed and resolved.

I. To show that the globe of the Earth is at the centre of the heavens and of this visible World, many arguments are commonly brought forward, drawn from what appears to us of the heavens and the celestial bodies. On these see Aristotle, *De caelo* 2, chapter 13, texts 97 and 103, and his interpreters there; Ptolemy, *Almagest* Book 1, chapters 5 and 6; Cleomedes, *Cyclical Theory* Book 1, chapter 9; Alfraganus, difference 4; Sacrobosco in the *Sphere*, chapter 1, and Clavius on it, from page 135 to 143; Maurolycus [Francesco Maurolico], dialogue 1 of the *Cosmographia*, page 9; Barocius, *Cosmographia* Book 1, page 33; Blancanus in the *Sphaera*, Book 4, chapter 1; Liceti in the disputations *De Terra unico centro motus*; and Georgius Bucanhanus, Book 1 of the *Sphere* — with whose verses, it seems, we should make our prelude:

[Margin: Bucanhanus's verses for the Earth's position in the middle of the World.]

> Nor is it hard to know this by sure signs. If the earth stood less far from the eastern gate than from the Tartessian bound that receives the weary axles, then Phoebus, when he put forth his head bright with flashing rays, would show a larger orb as he rose in the morning, and a smaller one again as night came on. Why should I tell of the stars, both those that glide along the same perpetual track and those that, with unequal steps, wander constantly through the liquid fields of heaven? When they travel nearer, they will seem to us of greater bodily size; while they are farther off, they will show a face smaller in proportion to the distance. Nay, observe the Sun, when from the summit of heaven, straight down, he strikes the Ethiopians with no point of his rays, when the vine-leaf shields the

tender fruit with its shade: if the ruler of light then drove his chariot nearer the eastern lands, the part of daylight still to be travelled would be greater than the part travelled, and in the morning the Sun would climb the heights of heaven up the opposing slope in a few hours, and late, over a greater span, would hide his team in the western ocean. Conversely, if the earth, near the western goal, stood far from the eastern, the Titan, long climbing in the morning the lofty summit of the aether, would quickly plunge into the western waves; nor would he, midway in the high aether, divide the times of daylight from his throne into equal spans, as he now does. Moreover, if the Sun showed himself nearer to the Indians than he sets to the western peoples, he would cast shorter shadows in equal hours than when, declining, shaking his reins, he makes for night along the shores of Calpe. For if an opaque body is set before a light, it stretches its dark shadows out the longer the farther you remove the light from the dense body. Now, since the Sun casts equal shadows in equal spans, he faces the earth with equal spans on every side. Equally near the western gate and the eastern, the orb of the earth neither leans down, inclined towards the hot South, nor rises towards the Maenalian Bear; but, hanging at the midpoint between the two Poles, it marks off the surrounding heaven with an equal bound. Do you not see when the swallow, messenger of the gentle spring, gives warning that the darkness is equal to the daylight? Or when Libra tempers days and nights in a just balance and a milder heat warms the mellow grapes, when Phoebus, rising in the early morning from the rosy east, pours the darts of his rays upon the moist lands, towered houses, walls armed with battlements, and the tree of the field — the boundary that shuts out quarrels between neighbours — cast the points of their Shadows straight towards the late couch of Phoebus; again, when in the evening, departing late, the panting Phoebus stables his horses in the waves, the point of the shadows is stretched towards the rosy rising; and when the Sun, declining towards the rainy South, holds Capricorn [Aegocerota], or when on high towards the Bears he flashes out and kindles the arms of the watery Crab, the shadow, slanting, makes now for the South wind, now for the North. But if the earth, leaving its station in the middle of the world, inclined itself more towards the Parrhasian Bear, every shadow would stretch towards the freezing Bear; if it leaned more towards the moist South, every shadow would stretch towards the moist South, and the evening shadow, flowing crosswise, would then cut the shadow stretched out far from the eastern mountains. Nor, if the earth were not at the middle of the axis between the two poles, would Delia, her brother's light withdrawn, grow pale from

the sudden shadow of the earth as often as, with heaven divided by an equal bound, it sets its orb against the rays of the Sun.

Most of the arguments already mentioned, however, which the other Authors indicated above also bring forward, avail only to show that the Earth cannot be outside the plane of the celestial Ecliptic, or, running off towards the South or the North, outside the centre of the Equator — which not even the Copernicans say. Therefore, passing over them, we shall select the remaining arguments.

I. Argumentum Generale ex Phaenomenis congruentibus situi Terrae in Mundi medio.

Argument 1, the general one, from the Phenomena that agree with the Earth's position in the middle of the World.

[Margin: Form of Argument 1.]

II. If the Earth is supposed to be at the centre of the World, all the Phenomena of the heavens are saved, and this supposition corresponds to the observations made by Astronomers. Therefore the Earth is at the centre of the World. This is, namely, the argument of Aristotle, *De caelo* 2, text 103, where he says: "The things said by the Mathematicians about Astrology also bear witness. For the appearances come about as the figures by which the order of the stars is determined change, just as if the Earth were placed in the middle."

[Margin: Aristotle's argument for the Earth's position.]

[Margin: Reply to Argument 1.]

The Copernicans reply by conceding the Antecedent, denying the Consequence, and retorting the argument: For the same Phenomena are saved if the Earth is supposed to turn about its own axis with the motion of rotation [motu vertiginis], and around the Sun with the annual motion of translation; therefore the Earth moves in this way. Since, then, the said Phenomena are saved in either way, the consequence is necessary for neither hypothesis; and the argument, reduced to form, rests on two particular affirmatives, from which nothing follows necessarily.

II. Argumentum à Duobus motibus apparentibus in Planetis.

Argument 2, from the two motions that appear in the Planets.

[Margin: Argument 2.]

III. If the Earth moved circularly, it would have to be moved by more than one motion. But it cannot be moved by more than one motion. Therefore the Earth does not move circularly. The MAJOR is plain by induction over all the spheres that move circularly; so if the Earth were placed among the Planets, it too would have to be moved by more than one motion. The MINOR is proved because great changes would then appear in the fixed stars and in their risings and settings, and these do not appear. This is Aristotle's argument, *De caelo* Book 2, text 97, which it is worth giving in his own words:

[Margin: Aristotle's other argument.]

"Moreover," he says, "all the things that are carried in a circular course seem to lag behind and to be moved by more than one course, except the first sphere; so that the Earth too, whether it is carried about the middle or placed in the middle, must be moved by two courses. But if this were to happen, there would necessarily come about chan- ges and turnings of the fixed stars. But this is not seen to happen; rather, the same stars always rise and set at the same places on it." The Philosopher, however, excepts the first sphere, that is, the primum mobile, to which he assigns a single motion, namely the diurnal one.

[Margin: Reply to the argument[49].]

The reply is by conceding the Major of real motions and denying it of apparent ones; and conversely, by denying the Minor of real motions and conceding it of apparent ones, of which alone the proof holds. For since, in this hypothesis, the eyes of the observers are moved together with the Earth by both motions, the Earth's own motions cannot appear to us; yet from its real motions it follows that the stars appear to us of the same size, according to what will be said shortly.

III. Argumentum ab Hemisphaerio ac medietate Caeli semper apparente.

Argument 3, from the hemisphere, or half, of the heavens that always appears.

[Margin: Form of Argument 3.]

IV. If the Earth were not in the middle of the World, the whole hemisphere of the heavens — that is, half of any heaven — could not appear to us wherever on earth we are placed, nor would six signs of the Zodiac always stand above the horizon. But the observation of all places on earth and of all ages teaches the contrary. Therefore, etc.

[Margin: Reply.]

The reply is by distinguishing the Major. It is conceded if the Earth were neither moved by the diurnal motion of 24 hours, nor the semidiameter of the annual orb of insensible proportion to the semidiameter of the sphere of the fixed stars, nor, finally, the semidiameter of the terrestrial globe small, to sense, with respect to the semidiameter of the heavens; otherwise the Major is denied. For the diurnal motion of 24 hours brings it about that things stand with the rising and setting of the stars just as if the stars themselves turned around the Earth as the centre of their revolution. The immense distance of the Fixed stars and of the visible signs of the Zodiac, in comparison with the distance of the Earth from the centre of the World and of the Fixed stars — which to sense is nothing — brings it about that six signs always appear, and the whole hemisphere of the sphere of the Fixed stars. Finally, the smallness of the Earth's semidiameter, compared with the semidiameter of the other spheres belonging to the Planets, brings it about that only a very small and barely sensible portion of the planetary heaven is hidden from us; and that the Earth's semidiameter, when we face one quadrant of the heavens, does not take from us an arc greater than the horizontal parallax of each Planet. That parallax is indeed a little more

[49]printed *argam.*, for *argum.*

than one degree in the Moon, but less than one degree in the other Planets; how great it is appears from what was said in Book 3, chapter 8, on the parallaxes of the Sun; in Book 4, chapter 14, on those of the Moon; and in Book 7, section 6, chapter 7, on those of the other Planets. When the horizontal parallax is taken from the quadrant, that is, from 90 degrees, there therefore remain 89 degrees and more — if you except the Moon — which we see when turned towards the East, and as many which we see when turned towards the West. So it is no wonder that we see almost the whole half of the Planetary heaven as well. And hence it also comes about that the Earth seems a point with respect to the heavens, especially the highest, as Ptolemy (*Almagest* Book 1, chapter 6), Alfraganus (difference 4), Proclus in the *Sphere*, Clavius in the *Sphere* (page 143) and others contend.

IV. Argumentum à Magnitudine apparenti Stellarum.

Argument 4, from the apparent size of the Stars.

[Margin: Form of Argument 4.]

V. If the Earth were not in the middle of the visible World, the same Stars would not be seen equal to themselves at rising, at midnight and at setting. But they are seen equal to themselves. Therefore, etc. The MAJOR is proved because they would be seen larger from the part of the Earth that was nearer to the heavens. But if you say that the vapours of the horizon hinder this view, take stars seen near the Meridian in different horizons, says Clavius on page 136 of the *Sphere*: for then everyone sees Arcturus of the same size, and so with the others — even at different times of the year.

[Margin: Reply.]

The reply is by conceding the Major if the distance of the Earth from the centre of the World were sensible, or of notable proportion, relative to the distance of the Stars, or if the Earth were not in the middle of the World Physically and to sense; but by denying it if the proportion between the said distances is insensible, or if the Earth is in the middle Physically, though not Mathematically.

V. Argumentum ab Vmbris Gnomonum.

Argument 5, from the shadows of gnomons.

[Margin: Form of Argument 5.]

VI. If the Earth were not in the middle of the World, the eastern shadows of gnomons would not be equal to the western ones at an equal altitude of the Sun, nor would they describe a straight line on a horizontal plane on the Equinoctial days. The consequent is false and against observations; therefore so is the antecedent from which it follows. The MAJOR is proved by Cleomedes, Book 1 of the *Cyclical Theory*, chapter 9. For if the Earth came nearer to the East, the shadows of gnomons would be shorter with the Sun rising than with it setting, since the nearer the luminous body is, other things being equal, the more the shadows contract; conversely, if the Earth came nearer to the West,

the shadows of the setting Sun would be shorter than those of the rising Sun. And if the Earth came nearer to one of the poles, the equinoctial shadows would describe a curved line, as they do on days away from the Equinoxes.

[Margin: Reply.]

The reply is by conceding the Major if, because of its position on the annual orb, the Earth came notably nearer to the rising Sun than to the setting Sun; otherwise, by denying it. Now the Earth is no farther from the Sun, whether rising or setting, than the Sun is from the Earth in the other hypothesis, since the Sun is supposed to be at the centre of the Earth's Great Orb, or very close to that centre. And the argument taken from the antemeridian semidiurnal arc, which, it seems, would not be equal to the postmeridian one, is answered in the same way.

VI. Argumentum à Vicissitudine dierum & noctium.

Argument 6, from the alternation of days and nights.

[Margin: Form of Argument 6, and reply.]

VII. If the Earth were outside the centre of the world, there would not occur each year two Equinoxes and two Solstices, nor the other alternations of days that appear in the threefold sphere — namely the Right, the Parallel and the Oblique. The consequent is false. Therefore, etc.

The reply is by conceding the Major if the Earth were not carried round yearly in the Plane of the Ecliptic about the Sun, which stands motionless at or near the centre of the world; or not in such a way that the terrestrial Equator is inclined to the Ecliptic by 23½° and always parallel to itself. Otherwise the Major is denied.

VII. Argumentum ab Eclipsibus Lunae.

Argument 7, from the Eclipses of the Moon.

[Margin: Form of Argument 7.]

VIII. If the Earth were not in the middle of the World, Eclipses of the Moon would not always occur with the Moon standing under the place of the Zodiac opposite to the one under which the Sun is. But Eclipses of the Moon always occur with the Moon and the Sun standing under opposite places of the Zodiac. Therefore the Earth is in the middle of the World. The MINOR is admitted among the Astronomers of every age. The MAJOR seems to be shown by Ptolemy, *Almagest* Book 1, chapter 5, in these words: "And, to touch on it briefly, the whole order that appears in the increase and decrease of days and nights would be thrown into utter confusion unless the earth held the middle position. Moreover, eclipses of the Moon too would happen in any parts of the heavens whatever, and could not take place at the place opposite the Sun, since the Earth would often set itself against it not at opposition but at intervals smaller than a semicircle." This argument is pressed by Averroes, *De caelo* Book 2, on text 103; Regiomontanus, *Epitome of the Almagest* Book 1; Maurolycus, dialogue 1 of the *Cosmographia*, page 9; and our Clavius on the *Sphere*, page

142. But Maurolycus and Clavius apply themselves to this proof more earnestly: each of them thinks the matter can be settled from only two Eclipses, using this figure, in which two Lunar Eclipses made at different places of the Zodiac are represented, but in such a way that the place of the Sun under the Zodiac is opposite to the Moon's place under the Zodiac — that is, when the centres of the Luminaries are opposed along a diameter. From these, they say, it follows that the Earth, whose shadow C eclipses the Moon, is on both diameters, and so at their common intersection. Therefore, since all the diameters of the World intersect one another at the centre of the World, it is neces-

Diagram. A small woodcut set into the right column beside the description of the two lunar eclipses: a circle (the Zodiac) carrying, at its upper left and upper right, two rayed discs (the Sun) and, at its lower left and lower right, two dark round sorts shaded on their lower half (the eclipsed Moon). Two diameters cross at the centre, each joining one Sun to the Moon diagonally opposite it. One letter stands on the figure.

- C (just above the crossing of the two diameters): the Earth's shadow, "whose shadow C eclipses the Moon".

sary, they think, that the Earth is at the centre of the World; and Maurolycus thinks this so manifest that he concludes that *nothing ambiguous remains*.

[Margin: Reply to Argument 7.]

The reply, however, is by distinguishing the Major. It is conceded if the Earth were outside the centre of the World in such a way that not the Earth itself, but the Sun as well as the Moon revolved around that centre — the Sun yearly, the Moon monthly. But the same Major is denied if the Earth's centre, carrying the lunar heaven with it, revolves yearly around the Sun, which is very close to the centre of the world, while the Moon completes its monthly revolutions around the Earth. For then the Sun can be seen from the Earth under any place of the Zodiac and the Moon under the opposite place, and so any lunar eclipse whatever can occur with the Luminaries in diametral opposition. The argument of the astronomers already mentioned therefore does not strike at the Copernicans, but at those who might perhaps imagine that the Sun moves yearly around the centre of the World and that the Earth, having no annual revolution, is outside the centre of the World. For the Copernicans, however, look at the Scheme set out in chapter 4, number 17.

VIII. Argumentum ab Eclipſibus Lunæ Horizontalibus.

Argument 8, from horizontal Eclipses of the Moon.

[Margin: Form of Argument 8.]

IX. *If the Earth were outside the centre of the World, Eclipses of the Moon could never occur with the luminaries appearing on the Horizon on either side. But such Eclipses do sometimes occur. Therefore the Earth is not outside the centre of the World.* This, indeed, is the argument in which Andrea Argoli, out of inexperience, sang a triumph over Copernicus in his Ephemeris for the year 1668, on the occasion of the lunar eclipse due on May 25. With respect to the Roman Horizon that

eclipse will fall so that in the morning, as the Sun rises, the Moon, diametrically opposite it and already about to set below the Horizon, will appear eclipsed. This author's words are:

[Margin: Argoli's argument, but an empty one, against the Copernicans.]

"The second Eclipse of the Moon, May 25 at 16^h 26^m. We have deliberately set out its calculation and figure, since it occurs almost at the rising of the Sun, and for this reason: that people may see whether, at the opposition of the Luminaries, both are seen from the Horizon — a sign that the earth is set in the middle of the Firmament, against the Copernicans. Yet this can be seen constantly from the rising or setting of Palilicium [Aldebaran] or Antares, stars diametrically opposite to each other: when one of them is placed in the East, the other offers itself to be seen in the West, and conversely."

The Major of the proposed argument is therefore proved as follows. If a lunar Eclipse must occur in such a way that the Moon is not only diametrically opposed to the Sun but both Luminaries also appear on the Horizon at the same time, then a whole hemisphere, that is, a semicircle of the Zodiac, must be seen, since the Luminaries are then a whole semicircle apart from each other. But a whole semicircle of the Zodiac cannot be seen by us unless we and our Earth are at the centre of the celestial Zodiac, and so at the centre of the Firmament. The MINOR is proved by examples of horizontal Eclipses, and by the witness of Pliny the Elder (Book 2, chapter 13) and of Mästlin in his work on Eclipses; we say more about these in Book 5, chapter 5, proposition 5. Indeed, there is no Eclipse of the Moon that is not of this kind with respect to some Horizon, since there is no moment at which the Sun is not rising for some Horizon.

[Margin: Reply to Argument 8.]

The reply, however, is by distinguishing the Major. It is conceded if the Earth were outside the centre of the World but motionless, and the Sun turned in the annual orb around the centre of the World, as Argoli still imagines. But the Major is denied if the Earth's centre revolves with an annual period around the Sun, which is set near the centre of the World, and the Moon revolves around the Earth in a monthly circuit. Nor is it necessary, for the phenomenon of such Eclipses, that we see a semicircle of the celestial Zodiac. It is enough that the Earth be in the Ecliptic which it describes with its centre, placed so that it sees the Sun under one point of this Ecliptic and the Moon under the opposite point, as far as the Moon's Latitude allows, and so that the Earth is on a straight line set between the Luminaries; everything else is joined to these Eclipses only accidentally. For Aldebaran in the East and Antares in the West to be visible at the same time, however — and indeed by every one of the Earth's inhabitants — the Earth must be in the middle of the World Physically, and in such a way that its distance from the centre of the World bears no sensible ratio to the distance of the Fixed stars, as has been said more than once.

IX. Argumentum ab Euidentia motus Solis.

Argument 9, from the Evidence of the Sun's motion.

[Margin: Form of Argument 9.]

X. *If it is Physically evident that the Sun moves yearly around the centre of the World, it is evident that the Earth does not so move but is at the centre of the World. But it is Physically evident that the Sun moves yearly around the centre of the World. Therefore it is evident that the Earth does not so move but is at the centre of the World.* The MAJOR is certain, at least on the Copernican hypothesis, in which the Earth is led round in the annual orb about the motionless Sun solely so that it may perform what the Sun is supposed by others to perform in the annual orb about the resting earth — granted that, in Astronomical rigour, it is not inconsistent for both globes to be carried round yearly at an equal pace under the Zodiac. The MINOR is proved because it is no less, indeed more, Physically evident that the Sun moves with a proper motion than that the other Planets likewise move with a proper motion towards the East. For the motions of the others too are adjusted to the Sun's motion; and the daily elongation of the Planets from some one of the Fixed stars towards the East, which is the evidence of their proper motion, appears no less, indeed more, in the Sun. Now it is so Physically evident that the other Planets move with a proper motion that their motion cannot be converted into a motion of the Earth; therefore not even the apparent motion of the Sun can be converted into a motion of the Earth without injury to Physical evidence.

[Margin: Reply to Argument 9, but an invalid one.]

They reply: let the Major pass; and they deny the Minor. For it is evident only as a disjunction that either the Sun moves yearly around the Earth or the Earth around the Sun; since it is certain that, if we were on the Sun, the Earth would seem to us to move under the Zodiac exactly as the Sun now seems to move to us who are placed on the Earth. And the reason why we cannot perceive the Earth's annual motion, if there is one, is that the motion is common to us and to our eyes, which are carried along if the Earth is carried. For the same reason, if we were placed on the Sun, we would not perceive the Sun's motion either, even if the Sun really moved and the Earth stood still. This being so, they say, those who philosophize circumspectly ought not to deduce from this appearance the real motion of the one rather than of the other, but only the apparent motion of the Sun. To the proof of the Minor the reply is by denying the parity. The Phenomena of all the Planets cannot be saved if the Earth is posited to move around them while they rest, and itself to perform their periods; but the phenomena both of the Planets and of the Sun can stand if the Earth is posited to move yearly around the Sun, resting near the centre of the World, in such a way that its motion lies between the orb of Mars and that of Venus, etc., according to the system set out in chapter 4, number 15, and chapter 8, number 22. Nevertheless, that disjunctive proposition is evident not merely Physically but Mathematically; but Physical evidence alone, taken absolutely, stands for the motion of the Sun, and it suffices so long as the opposite is not demonstrated.

CAPVT XXVIII. Proponuntur ac Diſſoluuntur Tria Argumenta Contra motum Annuum Terræ; ſumpta ex Parallaxi Orbis Annui.

Chapter 28. Three Arguments against the annual motion of the Earth, taken from the Parallax of the annual orb, are proposed and resolved.

[Margin: Definition of the Parallax of the annual orb.]

I. The PARALLAX of the annual orb is a variation of view arising from the different position of the observer, who is carried with the Earth through the annual orb. This variation can be either in the apparent size of the Stars, or in their altitude or distance from the vertex, or in their distance from some visible and motionless point or object — such as the Sun is in the Copernican hypothesis. Accordingly the asserters of the Earth's immobility contend: if the Earth were carried through the annual orb, now to one place and now to another, the variations already mentioned would appear in some stars; since they do not appear, the argu-

ment is that the Earth does not move through the annual orb.

[Margin: Explanation of the scheme for the Parallax of the annual orb.]

Now, so that the force of the Arguments and their Solutions may be grasped, let there be, in the following scheme, at A the centre of the Sun and of the World, around which the semicircle of the Ecliptic, or of the annual orb, is BDC, so that the diameter of the annual orb is BAC and its semidiameter AB. Let the centre of the Earth be at one time at B, where the night hemisphere of the earth is BK and the day hemisphere — the one lit by the Sun — is BS; and at another time, 6 months later, at C, where the night hemisphere is CN and the day hemisphere CR. This happens, for example, when the Earth is carried from one Solstitial point to the other, or from one Equinoctial point to the other. From the same centre A let a great semicircle EFG be described in the sphere of the Fixed stars, whose diameter EAG coincides with the diameter of the annual orb. Let this semicircle be understood as erected at right angles to the plane of the Ecliptic; let it be, for example, the Solstitial Colure, and at the same time serve as the Meridian; and in it let F be the Pole of the Ecliptic, where some star is imagined to be, to which the straight lines BF and CF are drawn from the centres B and C. Let the eye on the surface of the earth be once at I and once at O, and through these points let the straight lines AIM and AOP be drawn, touching the earth at I and at O and serving as the Physical Horizon. Through it the eye at I sees the Sun A setting for it (since the diurnal motion goes from S to I, towards K, etc.), but at O sees the Sun rising for it. Again, let there be another star L, outside the Ecliptic and the poles of the Ecliptic, to which the straight lines BL and CL are drawn from the Earth's centres B and C. Neglecting now the parallax of the terrestrial globe, let the eye at B be imagined to see L along the straight line BL: it will see it raised by the angle GBL, much greater than when, carried to C, it sees it along CL raised by the angle GCL. In this case, therefore, the parallax of the annual orb will be the

angle BLC, arising from the diameter BAC, just as in the former case it would be the angle BFC. We shall call this Parallax parallax in itself, whether it is then observable or, accidentally, not observable, because the star L or F, once seen when the earth is at B, cannot be seen six months later, when the earth is at C, the day or the brightness of the Sun preventing it. For we shall later separate these aspects, and teach what can be observed when the Earth has been carried along.

Diagram. A woodcut in the left column illustrating the parallax of the annual orb. A vertical diameter G–K–B–S–A–R–C–N–E runs down the left side; a large arc (part of the great semicircle EFG in the sphere of the fixed stars) sweeps from G at the top through L, F and V to E at the bottom. Two small circles, the Earth at its two positions, are centred at B and C, each with one half hatched dark (the night hemisphere). A horizontal line runs from H through K and M to L at the top. Straight lines from B and C run to L, F and V, and lines from A pass through I and O towards M and P; a dotted line runs from B towards L.

- G (top of the vertical diameter): end of the diameter EAG of the great semicircle
- H (top left, on the horizontal line): end of the tangent HKL
- K (top of the upper Earth): the eye in the tangent HKL; end of the night hemisphere BK
- M (on the horizontal line, between K and L): end of the line AIM
- L (top right, on the arc): the star L outside the Ecliptic and its poles
- B (centre of the upper Earth): the Earth's centre at one time
- I (on the upper Earth's edge): the eye at I, where AIM touches the Earth
- S (below B): the end of the day hemisphere BS; the Antipode of K
- A (on the diameter, at the level of F): the centre of the Sun and of the World
- D (right of A): a point on the semicircle of the Ecliptic BDC
- F (right, on the arc, level with A): the Pole of the Ecliptic, where a star is imagined
- R (between A and C): end of the day hemisphere CR
- O (on the lower Earth's edge): the eye at O, where AOP touches the Earth
- V (right, on the arc below F): no referent in the text on this page
- C (centre of the lower Earth): the Earth's centre six months later
- P (below O, right): end of the line AOP
- N (below C): end of the night hemisphere CN
- E (bottom of the vertical diameter): end of the diameter EAG

[Margin: Galileo's [?] first proposition.]

II. With these premises established, Galileo, in Dialogue 3 of the *System of the World* (page 376 of the Italian, 385 of the Latin), demonstrates that *the greatest parallax of the annual orb that stars can undergo in altitude, or in distance from the vertex, is that of the stars at the Poles of the Ecliptic; there is none for those in the*

Ecliptic; and it is the smaller, the nearer they are to the Ecliptic and the farther from the poles of the Ecliptic. Thus in the preceding figure the greatest parallax is that of star F, and there is none for star G or E; and that of star L is smaller than that of star F, because the parallactic angle BFC is greater than the parallactic angle BLC — indeed it is the greatest of all the angles contained on the same base BAC within the same periphery EFG — as he shows in the same place. It can also be gathered from the fact that the angle diminishes from F towards G, so that at G it finally vanishes. Hence Fixed stars placed in the Ecliptic are never raised or depressed more or less because of the motion of the earth, granted that they become now nearer to the eye, now farther from it.

[Margin: Galileo's [?] second proposition.]

And for that reason, on page 373 of the Italian (382 of the Latin) of the same dialogue, he demonstrated this other proposition, which for him comes first: *the greatest Parallax of the annual orb that stars can undergo in apparent size is that of the stars in the Ecliptic; there is none for those at the poles of the Ecliptic; and it is the smaller, the nearer they are to the poles of the Ecliptic and the farther from the Ecliptic.* The first proposition, however, holds in the case supposed in the description of the figure, with the distance of the Fixed stars G and E taken as immense; otherwise a case can be given in which even the stars G and E acquire some parallax from the annual orb, according to what was said in chapter 11, number 14.

III. It is now worth inquiring how great the greatest parallax of the annual orb would be, supposing both the moderate distance of the Sun and the Fixed stars from the Earth that some asserters of the Earth's immobility posit, and the distance that the more eminent Copernicans posit. We shall obtain this easily in the triangle ABF, right-angled at A. In it, from the hypotheses of the authors already mentioned, the side AB is given — the semidiameter of the annual orb, that is, the mean distance of the Sun from the Earth — and the side AF, which is the distance of the Fixed stars from the centre of the World. Therefore, by the Fifth [proposition] of Plane Right-angled Triangles, treated in Book 10, section 1, chapter 2, the angle AFB will become known; and by doubling it we shall have the whole parallactic angle BFC, standing on the diameter BC.

[Margin: The slip of the Copernicans over the Parallax of the annual orb.]

Granted, all the Copernicans until now have pressed the angle AFB on us, absolutely, as the parallax of the annual orb, as we showed in Book 6, chapter 7, numbers 10 to 15 — whether they used dissimulation in order to diminish this parallax, or rather through an inadvertence that arose from their habit with the parallax of the terrestrial globe, which the Earth's semidiameter produces. For just as in the preceding figure — with the star placed at L, rising for the eye at K and seen along the tangent HKL, and with the straight line BL drawn from the centre B — the horizontal parallax of the star is the angle BLK, standing on the Earth's semidiameter BK, so they reckoned the parallax of the annual orb from the semidiameter AB of the annual orb alone, and no greater than the angle AFB. But just as, if someone, with the earth standing still, were carried from K

to S, the place of his Antipode, and could see the star L from both places, the whole parallax would be the angle SLK, arising from the diameter KS; so, with the Earth carried from B to C, the Parallax of the annual orb, taken adequately, turns out to be BFC, because of the whole diameter BC. To show the difference between the two views, however, we shall set out in the following Table both the angle AFB and the angle CFB, with its quantity derived from the given sides AB and AF that follow. We took the first of these from Book 3, chapter 7, and the second from Book 6, chapter 7, where we also cited the authors of the distances and the places where they give them.

TABVLA PARALLAXIS MAXIMÆ ORBIS ANNVI.

Table of the Greatest Parallax of the Annual Orb.

Authors of the distances	Distance in Earth semidiameters: of the Sun from the Earth, or side AB	Distance of the Fixed stars from the centre of the World, or side AF	Semiparallax, or angle AFB	Parallax, or angle CFB
Ptolemy	1126	20 220	3° 11′ 34″	6° 23′ 8″
al-Battānī	1068	19 000	3° 13′ 20″	6° 26′ 40″
Tycho	1120	14 000	4° 35′ 30″	9° 11′ 0″
We	7300	100 000	4° 10′ 30″	8° 21′ 0″
Kepler	3469	60 000 000	0° 0′ 12″	0° 0′ 24″
Lansbergen	1498½	41 958 000	0° 0′ 7⅓″	0° 0′ 15″
Galileo	1208	13 046 400	0° 0′ 20″	0° 0′ 40″
Hortensius	1498½	10 312 227	0° 0′ 30″	0° 1′ 0″
Hérigone	1200	1 440 000	0° 3′ 0″	0° 6′ 0″

[Margin: Copernicans.]

It should be noted, however, that Lansbergen, together with Hortensius, once admitted a Parallax of the annual orb in the Fixed stars of 30″ (in fact a semiparallax), as though it could not be detected by any observations; but that later, in the *Uranometria*, Book 3, element 7, he reduced it to 7″ 22‴. Doubling it, therefore, as is proper, the whole parallax comes to 14″ 44‴, or nearly 15″.

[Margin: The distance of the Fixed stars that must be asserted if the Parallax of the annual orb is not to be greater than 10 Seconds.]

IV. Besides knowing the parallax already mentioned, since we judge that with our Instruments properly applied a difference in the altitude of the stars can be detected that is not less than ten Seconds, it is worth investigating what distance of the Fixed stars from the centre of the World the Authors must establish if they want the parallax of the annual orb, at its greatest possible, not to exceed 10″ — that is, if the angle CFB already mentioned is to be 10″ and AFB 5″ — with the side AB, the distance between the Sun and the Earth, taken as great as each of the Authors listed below supposes. We shall settle this easily by analysing the triangle ABF already mentioned, right-angled at A, in which AB is given and the opposite angle AFB is assumed to be 5″. Then by the 7th [proposition] of Plane Right-angled Triangles we shall find how great the distance AF of the Fixed stars must be. Here, then, is the small table below.

With the angle AFB 5″ and CFB 10″, and AB in Earth semidiameters		AF, in Earth semidiameters, is
Copernicus	1150	47 439 800
Kepler	3469	142 746 428
Galileo	1208	49 832 416
Lansbergen	1498½	61 616 122
Hortensius	1498½	61 616 122
Boulliau	1460	60 227 920
Hérigone	1200	49 502 400
Wendelen	14656	604 589 312
Us	7300	301 146 419

[Margin: There is no sensible variation in the apparent size of the Fixed stars because of the Parallax of the annual orb.]

V. It remains to examine how much larger a Fixed star of the first magnitude, such as Sirius, would appear if the Earth approached it by the whole diameter of the annual orb, and if such a star were imagined to be in the Ecliptic, taking the least distance of the Fixed stars from the centre of the World that some Copernicans posit — namely 1 440 000 terrestrial semidiameters, the figure Hérigone posits. Now the apparent diameter of Sirius is only 18 Seconds, as we showed in Book 7, section 6, chapter 11, in the first table. In the following figure, then, let AC be the distance of Sirius from the centre of the world C, and let the apparent semidiameter of Sirius stand under the angle ACB, of 9″. For when the tangent CB and the semidiameter AB are drawn, there are given in the triangle ABC, right-angled at the point of contact B, the Base AC, 1 440 000, and the adjacent angle ACB, 9″. From these, by the first [proposition] of these Triangles, the true semidiameter of Sirius is found to be 62 and 784/1000 terrestrial semidiameters. Now suppose the eye has approached the star, so that Sirius is distant from the eye at C by the interval CP, namely 1 437 600 semidiameters — subtracting, that is, from the semidiameter of 1 440 000 the 2400 that Hérigone gives to the diameter of the annual orb (although in the Copernican hypothesis only the semidiameter would have to be subtracted, unless distances from the centre of the earth are taken instead of distances from the centre of the World) — and let DP be 62 and 784/1000. Then in the triangle PDC, right-angled at D, the angle PCD will be found, by the Second [proposition] of plane right-angled triangles, to be 9″ 2‴. The whole apparent diameter of Sirius will therefore be 18″ 4‴, and so, because of the approach already mentioned, no greater by more than 4‴ — a difference that is entirely unobservable. How much less observable, then, will it be in other, smaller stars, outside the Ecliptic and more distant from the centre of the world? With these preparations made, let us come to the Arguments.

Diagram. A small woodcut set into the left column beside the computation for Sirius: two circles, one above the other, representing the star seen from two distances. In the upper circle a short radius runs from the centre A to the point B on its left edge; in the lower circle a radius runs from the centre P to

the point D on its left edge. From C at the foot two long lines run upward, touching the circles at B and D, and a line runs from C up through P to A.

- A (centre of the upper circle): Sirius, at the distance AC from the centre of the world C
- B (left edge of the upper circle): point of contact of the tangent CB
- D (left edge of the lower circle): point of contact, right angle of the triangle PDC
- P (centre of the lower circle): Sirius at the nearer distance CP
- C (foot of the figure): the centre of the world, and the eye

I. Argumentum à Parallaxi Orbis Annui in Altitudine Fixarum.

Argument 1, from the Parallax of the annual orb in the Altitude of the Fixed stars.

VI. *If the Earth moved in the Annual Orb, some sensible parallax would be observed in the meridian altitude of the Fixed stars, taken after three or six months at the same Horizon with large instruments, exactly made and applied. But no such parallax has appeared so far. Therefore the Earth does not move in the Annual Orb.*

[Margin: Proof of the Major.]

The MAJOR appears to be proved by the Trigonometric calculation and the table at the end of number 3, at least for stars placed at the pole of the Ecliptic or not far from it, such as the star in the bend of Draco and, in the other hemisphere, the star in the breast of the fish Dorado. It can also be proved from Sirius, which we have observed on the Meridian more than once: shortly after the autumnal Equinox in the morning, with the Sun rising, and shortly after the vernal Equinox in the evening, with the Sun setting. For the brightness of this star is so great that, with a clear horizon, it can be seen even for about a quarter of an hour before sunset or after sunrise, and both its distance from the Sun as seen and its meridian altitude can be taken. Now let Sirius be at L in the figure set out in number 1, and let its Southern latitude GL, that is, the angle GAL, be 39° 32′ 58″, from what was said in Book 6, chapter 24. Let AB, following Hérigone, be 1200 terrestrial semidiameters, and AL 1 440 000. From these and the included angle BAL, of 39° 32′ 58″, the angle ALB comes out, by the 3rd [proposition] of Plane Oblique-angled Triangles, as 1′ 50″; therefore the whole parallactic angle CLB is very nearly 3′ 40″. Now let the Polar star be at V, whose latitude EV, that is, the angle EAV, is 66° 2′; and in the triangle CAV, besides the angle already mentioned, let there be given, as above, AC, 1200, and AV, 1 440 000 semidiameters of the earth. The angle AVC will be found to be 2′ 38″, and BVC nearly 5′ 16″. So great, then, would the parallax in the altitude of these stars appear if they could be viewed on the Meridian from the earth carried from B to C through the whole diameter BAC. But if Kepler's distances are taken, AB 3469 and AL or AV 60 000 000, ALB comes out as 8″ and CLB nearly 16″; but AVC as 12″ and CVB nearly 24″.

[Margin: Proof of the Minor.]

The MINOR is proved because neither have we detected so great a parallax in the altitude of Sirius, nor have either we or Tycho detected it in the altitude

of the Polar star. Indeed, in the Year 1586 Tycho observed the greatest meridian altitude of that star at midnight after the autumnal Equinox; and in the Years 1577, 1581, 1586 and 1589, after the winter Solstice, he observed its least altitude towards morning and its greatest after evening; and from all these observations he found its distance from the pole of the world always the same. In particular, in the Year 1586 he found it, from the Equinoctial and the winter observation, to be 2° 56′ 10″, when he ought to have found a different one because of the Earth's translation over the space of three months; and he detected no parallax at all, not even of ten Seconds, as Longomontanus also relates in Book 1 of the *Theorics*, chapter 1, page 159. Kepler too confirms this in Book 4 of the *Epitome of Copernican Astronomy*, page 493, saying of Tycho: *"He observed the greatest altitude of the polar star, which at this time is in 7° of Aries, in the year 1586, at midnight after the autumnal equinox, and it was 58° 51′. He observed the same also about the winter Solstice, on December 26, at about hour 6 in the evening, and again found 58° 51′; so there was no difference at all. Yet in the month of September the horizon cut the sphere of the Fixed stars lower, by almost the whole semidiameter of the orb in which the earth is carried round, than on December 26, since then the Sun appeared in Libra, now in Capricorn. The same happened also when the least altitude was observed at midnight after the vernal Equinox, and after the winter Solstice in the morning at hour 6: on both occasions 52° 59′ 30″ was found, although in the month of March the Horizon cut the Fixed stars higher, by almost the whole semidiameter of the orb in which the Earth is carried, than in December. Therefore this diameter of the orb in which the earth is carried round is not sensible with the Brahean instruments."*

[Margin: Reply 1, Kepler's, a faulty one.]

VII. 1. Kepler would reply, on the same page 493, by denying the Major, because from his distances there does not follow a parallax of the annual orb greater than 12 Seconds, which he denies can be perceived by any diligence of craftsmen, especially *since the diameter of the polar star appears to equal at least one minute, and the diligence of craftsmen is not to be trusted about a fifth part of one minute.* But this reply suffers from four faults. *First*, it estimates the parallax at 12″ from the semidiameter of the annual orb alone, whereas the whole parallax, which must be estimated from the diameter, is 24″, as is clear from the table set out in number 3. *Second*, the apparent diameter of the Polar star is

only 7″ 54‴, or at most 8″, according to our corrected observations given in Book 7, section 6, chapter 11, in the first table. *Third*, even if this diameter were one minute, threads so fine can nevertheless be stretched at the centre of an astronomical quadrant that one can sight along them at the very centre of the star, and its disc is cut in two by them. *Fourth*, he does an injustice to his own Tycho, who often asserts in his *Mechanica* that he observed the altitudes of the Sun and of the Fixed stars down to a distinction of ten Seconds, that is, of the sixth part of one minute.

[Margin: Reply 2, Hortensius's, a faulty one.]

VIII. 2. Hortensius, in the dissertation with Gassendi on Mercury seen under the ☉ [Sun], replies by denying the Minor: for he admits a parallax of 30″, but

denies that it can be clearly perceived with any instruments. But his reply too errs in two ways. *First*, that parallax was derived by him from the semidiameter of the annual orb alone; from the whole diameter it comes to 60″, that is, a whole minute. *Second*, he too does an injustice to Tycho and to other more diligent Astronomers of this century, who in observing the altitude of the stars descend at least to tens of Seconds.

[Margin: Reply 3, Mästlin's and Hérigone's, insufficient.]

IX. 3. Mästlin, in his addition to the *Narratio prima* of Rheticus, page 114, and Pierre Hérigone, in volume 5 of his *Cursus Mathematicus*, page 615, reply by denying the Major. For although the parallax arising from the annual orb is in itself so great that, if the same star could be seen on the Meridian or on the same vertical by the same observer, remaining at the same Horizon, in two observations taken six months apart, it would be 3′ near the pole of the Ecliptic, as Hérigone grants — or 2′ or 3′ in stars rising or setting about midnight, as Mästlin admits — yet it is unobservable per accidens. This is either because of the horizontal refractions of stars rising or setting, or because, if a star appears on the Meridian once by night, it cannot appear on it six months later, the daylight of the Sun preventing it; and after three months that parallax would be much less than 2′ or 3′, and so, because of its smallness, not clearly sensible. But this reply too, unless something else is added, is insufficient, at least as regards stars observable on the Meridian. *First*, Sirius, the Sun notwithstanding, can be observed six months later on the same Meridian on which it was observed before, as I said in the proof of the Major. *Second*, the very parallax that Hérigone admits will occur after 6 months is, from his own distances, nearly three Minutes, since it is estimated from the semidiameter; for the parallax from the whole diameter would be 6 minutes, as is clear from the table of number 3. And who would deny that a parallax of 3 Minutes can be perceived, on account of its smallness, unless he were utterly unskilled in astronomical observations?

[Margin: Reply 4, ours.]

X. 4. The reply, therefore, is to deny the Major. *First*, because Copernicus wants so great a distance of the Fixed stars to be assumed that all parallax of the annual orb vanishes, even if this interval had to be increased finitely to infinity. And although the Major tells against Hérigone, and almost against Mästlin and Hortensius — especially in Sirius, which some of them thought could not be observed on the Meridian once and again six months later — it does not tell in the same way against the other Copernicans in the table of number 3, and much less against Kepler. For that parallax, as great as it comes out when derived from the whole diameter of the annual orb according to their distances, can be fitted only to stars placed at the pole of the Ecliptic, and none of these can be seen on the Meridian once and again six months later, since the brilliance of the Sun prevents one of the two observations. Sirius, on the other hand, which can be seen twice, at the two ends of roughly six months, is so far from the pole of the Ecliptic that the parallax of the annual orb on Kepler's distances does not come out greater than 16″. The parallax that would reach

24″ in the polar star is not observable, since the light of the Sun prevents one of the observations. And the distance of these and of all the Fixed stars from the pole of the Ecliptic, like their latitude, is invariable. *Second*, because the Parallax which in itself would be observable after an interval of three months in other Fixed stars visible on the Meridian by night, if the centre of the Earth ran from N to A along the very diameter of the annual orb, cannot in fact be observed, since the centre of the Earth is carried along the circumference of the annual orb from C to D. In that position neither the diameter nor the semidiameter of the annual orb can fall in the plane of the Meridian, and so no triangle can be formed in which a parallactic angle arises. It is no wonder, then, if after three months no parallax of the orb, or of the semi-orb, can be observed, in this hypothesis of the Earth's motion as well. *Third*, because not even the Parallax of Sirius already mentioned is observable, for another reason: namely, when Sirius appears on the Meridian at sunrise or sunset, the Sun appears to the Earth under the Equinoctial points, or near them, and not under the Solstitial points; and in the Copernican hypothesis the Earth is at or near the Equinoctial points, in which position the diameter of the annual orb through the centre of the earth lies in a plane entirely different from the plane of the Meridian — for it lies in the plane of the Horizon, and cuts the plane of the Meridian nearly at right angles. Hence no triangle can be formed on it in which the meridian altitude and its parallax arise.

II. Argumentum à Parallaxi Orbis Annui in Distantia Sirij à Sole.

Argument 2, from the Parallax of the annual orb in the Distance of Sirius from the Sun.

XI. *If the Earth moved through the annual Orb, some Parallax of the annual Orb would appear in the distance of Sirius, taken from the Sun once when it is rising and again, 6 months later, when it is setting. But none appears. Therefore, etc.*

The MAJOR is proved by taking up again the figure set out in number 1, in which, however, let EFG be a great circle drawn through the Equinoctial points (which are here to be imagined as G and E) and through the centre L of Sirius. For since Sirius is seen not only when the Sun rises or sets at or near the equinoctial points, but also somewhat after sunrise and before sunset, its distance from the Sun can be taken. Let AF, then, be the solstitial colure, and Sirius at L. When the Sun A rises to the eye at O, near the autumnal Equinox, Sirius will appear distant from the Sun by the acute angle AOL; but when, near the vernal Equinox, the Sun sets to the eye at I, Sirius L will appear distant from the Sun by the obtuse angle AIL (supply what is missing in the figure). Therefore the difference between the angles already mentioned will be the parallax arising from the annual orb.

The MINOR is proved by experience, because no such parallax in fact appears in the distance of Sirius from the Sun.

Father Francesco Maria Grimaldi proves the same Major in another way. If the earth is at A, the distance of Sirius from the Sun at B, the point of the

Autumnal Equinox, when it is rising, observed through the angle BAL, together with the same distance of Sirius from the Sun when it is setting at C, the point of the Vernal Equinox, observed through the angle CAL, will make up two right angles, or 180°, by [proposition] 13 of the first book of Euclid. But if, with the Sun motionless at A, the earth, placed now at B, observes the distance of Sirius from the Sun through the angle LBA, and now at C observes the same through the angle LCA, these two distances will not make up 180°; the quantity of the parallactic angle BLC will be lacking.

[Margin: Reply to Argument 2.]

XII. I reply by denying both the Minor and the consequence of the Major. For that difference between the obtuse and the acute angle does not arise from the motion of the Earth through the annual orb, but from the distance of Sirius from the solstitial colure, since Sirius in this century is nearer to the autumnal Equinox than to the vernal. So it comes about that, whether the Earth moves through its annual orb or the Sun through its annual one, Sirius must appear distant from the rising Sun at the autumnal Equinox by an acute angle, and from the setting Sun at the vernal Equinox by an obtuse angle. But if Sirius were at F, the point of the solstitial colure, it would appear, in the hypothesis of the Earth moving as much as in that of the Earth at rest, distant from the Sun at either Equinox by a right angle, AIF and AOF, formed by one line running straight from the eye to the centre of Sirius and by another which would be tangent to the earth at the point of the eye; and that same line would be the physical Horizon, in which the centre of the Sun would rise or set for the eye at O and at I. I say nothing, meanwhile, of the variation of the Solar refractions at the horizon, which could disturb the clarity of this parallax even if it were otherwise sensible in itself — although Copernicus would avoid this sensibility by increasing the distance of the Fixed stars to infinity. And from this, likewise, the reply is given to the other proof of the Major: namely, that because of the immense and unconstrained distance of the Fixed stars the lines AL, BL, CL would all turn out parallel, and therefore the angles at A, B, C would appear equal; besides which, that observation is too slippery, and it can scarcely happen that the Equinox falls at the rising or setting of the Sun.

III. Argumentum à Parallaxi in Magnitudine apparenti Fixarum.

Argument 3, from the Parallax in the apparent Size of the Fixed stars.

XIII. *If the Earth moved through the annual orb, a great variation would appear in the apparent diameter of the Fixed stars. But none appears. Therefore, etc.*

[Margin: Reply to Argument 3.]

The reply is to deny the Major, because not even in Sirius would the variation be greater than 4‴, even if Sirius were supposed to be in the Ecliptic, as was shown in number 5, and if the stars were no less distant from the centre of the World than 1,440,000 terrestrial semidiameters, as Hérigone posits; and so slight a difference as 4‴ cannot be clearly observed with any instruments. How much less, if the star were far from the Ecliptic, or if Kepler's distance

of the Fixed stars were assumed, or as great a distance as Copernicus allows, who allows so great a one in his hypothesis that every difference of this kind becomes entirely insensible. As for the fact that in the Planets, especially in Mars and Venus, a notable variation of apparent size is seen, this is no more an argument for these Planets approaching an immobile Earth than for a moving Earth approaching them.

CAPVT XXIX. Proponuntur Tria Argumenta Contra motum Telluris Annuum, sumpta ex nimia Distantia Fixarum, & magnitudine Octauæ sphæræ. Vbi opiniones de illarum Distantia ex Copernicanorum varijs hypothesibus recensentur cum fundamentis earum.

Chapter 29. Three Arguments against the annual motion of the Earth are proposed, taken from the excessive Distance of the Fixed stars and the size of the eighth sphere. Here the opinions on their Distance drawn from the various hypotheses of the Copernicans are reviewed, together with their grounds.

I. From number 4 of the preceding chapter it is established that, if the parallax of the Fixed stars arising from the annual Orb of the Earth must be unobservable, or at least not exceed 10″, the distance of the Fixed stars must be greater than 47,000,000 terrestrial semidiameters, as is gathered from the table placed there. But from so great a distance it follows that between Saturn and the Fixed stars there is an incredible space, and that the sphere of the Fixed stars is of an immense bulk, surpassing almost all belief. But since different authors suppose different values in computing each of these magnitudes, I think it right to rehearse the principal opinions here — those of the Copernicans themselves, those of others, and our own.

Opiniones Copernicanorum.

Opinions of the Copernicans.

[Margin: Opinion 1, that of Aristarchus of Samos.]

II. The *first* and most ancient Opinion was that of Aristarchus of Samos. Although he said that the annual orb, through which he held the Earth to be carried round, is to the sphere of the Fixed stars as a point or centre, he must be understood in the sense in which Cosmographers usually say that the Earth is like a point with respect to the heavens, namely to the senses. And this is how Archimedes kindly interpreted him at the beginning of the *Sand-Reckoner*: having said that it is impossible for the Earth's annual orb to be to the sphere of the Fixed stars as a centre, because a centre is nothing, or has no ratio at all to a sphere, he added: *"But it must be granted that Aristarchus meant this: since we suppose the Earth to be placed about the centre of the world, he thought it must be laid down (taking his demonstrations from the Phenomena) that the ratio which the Earth has to the World as we have described it is the same ratio that the sphere of the circle carrying the Earth round has to the sphere of the Fixed stars."* Now the

World which Archimedes had described a little before is the annual orb of the Sun itself, for he had said: *"You are not unaware that most astronomers call the World the sphere whose centre is the centre of the Earth, and whose Radius is equal to the straight line lying between the centre of the Sun and the centre of the Earth."* So, on Archimedes' interpretation, Aristarchus thought that the ratio between the annual orb and the orb of the Fixed stars is the same as that between the Earth and the annual orb. We must therefore find this ratio according to the view of Aristarchus.

[Margin: Aristarchus's view on the distance of the Sun.]

Now Aristarchus, in the book *On the Sizes and Distances of the Sun and Moon*, proposition 7, demonstrated this Theorem: *"The distance by which the Sun is distant from the Earth is greater than eighteen times the Moon's distance from the Earth, but less than twenty times."* That is to say, it is to the Moon's distance as 19, approximately, to 1. Plutarch alludes to this in Book 2 of *De placitis Philosophorum*, chapter 31, and Pliny the Elder in Book 2, chapter 21, saying: *"Many have also tried to investigate the distances of the stars from the Earth, and they have reported that the Sun is nineteen parts as far from the Moon as the Moon itself is from the Earth."* Now according to the Ancients the Moon is distant from the earth by 64 terrestrial semidiameters, which multiplied by 19 make the Sun's distance, that is, the semidiameter of the Annual Orb, 1216 semidiameters of the earth; and there are as many diameters of the earth in the diameter of the annual orb. This number 1216 multiplied by itself gives the Square number 1478 656, which multiplied again by 1216 makes the Cubic number 1788 045 696; while the cube of one diameter of the Earth is unity, or 1. Aristarchus therefore holds that the sphere of the Fixed stars contains the annual orb nearly 1800 000 000 times, and that the semidiameter of the annual orb is to the semidiameter of the sphere of the Fixed stars as the semidiameter of the Earth is to the semidiameter of the annual orb. Let it be, then, as 1 to 1216, so 1216 to a fourth term, and the semidiameter of the orb of the Fixed stars will come out as 1478 656 terrestrial semidiameters. Mästlin too, among others, admits this proportion, as we shall see shortly.

[Margin: Opinion 2, Mästlin's.]

III. The *second* Opinion is that of Michael Mästlin, in his additions to the *Narratio prima* of Georg Joachim Rheticus, page 114. There he does not hesitate to admit, in Fixed stars rising or setting about midnight, a horizontal parallax of two or three Minutes, which he nevertheless denies can be observed in that position, because of the horizontal refractions that absorb it. He adds that the Copernican hypothesis does not require the distance of the Fixed stars to vanish entirely when compared with the distance of the Sun from the Earth, but only that it vanish to the senses and the judgement of the eyes, that is, that the parallax cannot be observed; and that it neither contradicts Copernicus nor leads to any absurdity if the distance of the Fixed stars from the centre of the universe contains the greatest distance of Saturn from the same centre seven hundred times and more. Taking Saturn's distance from Tycho as 12 900 terrestrial

semidiameters, the distance of the Fixed stars then contains 9 030 000 terrestrial semidiameters, 12,900 being multiplied, that is, by 700. But a little later he says: *"He therefore cannot be convicted of any absurdity who does not hesitate to assert that the ratio of the distance of the fixed stars from the centre of the World is at the least equal to the ratio of the distance of the Sun from the Earth, that is, to the semidiameter of the Great Orb, and that on this account the Orb of the Fixed stars can also, in this way, be called like the infinite."* This is exactly what we showed in number 2 that Aristarchus held. Now the mean distance of the Sun from the Earth, from what was said in Book 3, chapter 7, is for Mästlin 1160 semidiameters of the earth. If, then, it is made as 1 to 1160, so 1160 to a fourth term, the distance of the Fixed stars will come out as 1 345 600 terrestrial semidiameters. Lower down on the same page he writes as follows: *"It is therefore beyond doubt that Copernicus had regard to the judgement of the eyes, not to the universal exclusion of all parallax; for this remoteness of the starry orb is sufficient for the demonstrations of the motions, a remoteness because of which, and in comparison with which, the size of the Great Orb cannot be discerned by the eyes. And that this remoteness, indicated in the way described and weighed according to all the circumstances, should, even if it does not exceed the greatest altitude of Saturn seven hundred times, yet equal it a hundred or two hundred times, is altogether most probable."* Since, then, he says in the same place that the greatest distance of Saturn from the centre of the world rises to 12 900 semidiameters of the earth, and that a distance that great, or no less, is established by Astronomical demonstrations, it follows that the most probable distance of the Fixed stars from the centre of the World, in his opinion, is either 1,290,000 or even 2 580 000 semidiameters of the Earth. Finally, elsewhere he sets almost the same distance as Galileo, namely 13 046 400 terrestrial semidiameters. So inconsistent was he on this matter, as the figures below show; among these, however, he seems to have settled on the middle one,

which contains Saturn's distance two hundred times, that is, the one that has 2 580 000 terrestrial semidiameters.

Distance of the Fixed stars from the centre of the World admitted by Mästlin, in semidiameters of the Earth
1 290 000
1 345 600
2 580 000
9030 000
13 046 400

[Margin: Opinion 3, Kepler's.]

IV. The *third* Opinion was Kepler's, in his book on the New Star in Serpentarius (*De stella nova*), chapter 16. There he gives the Sun a semidiameter of 6 semidiameters of the earth, the Earth's distance from the Sun 1432 terrestrial semidiameters, and Saturn's distance ten times this, namely 14 320; and he says it is probable that Saturn's distance is the mean proportional between the two

immobile bodies, that is, between the Sun, the mover of the Planets, and the sphere of the Fixed stars. He therefore makes it: as 6 to 14 320, so 14,320 to 34 077 066⅔ terrestrial semidiameters; and that is the distance of the Fixed stars he sets there. But in the *Epitome of Copernican Astronomy*, Book 4, pages 479, 485 and 492, he chooses a semidiameter of the Sun of 15 terrestrial semidiameters, a distance of the Sun from the Earth of 3469 semidiameters of the earth, and a distance of Saturn of 30 000 semidiameters of the earth. Keeping, then, the same proportion mentioned above, he makes it: as 15 to 30 000, so 30 000 to 60 000 000 semidiameters of the earth; and he rests content with this distance of the Fixed stars.

[Margin: Opinion 4, Lansbergen's and Hortensius's.]

V. The *fourth* Opinion was that of Lansbergen, formerly, in his commentary on the motion of the Earth, and of Martinus Hortensius, in the dissertation with Gassendi on Mercury seen in the Sun and Venus unseen. They judged that the parallax of the Fixed stars produced by the annual orb can reach 30″ without evident inconvenience to observation; and from this, and from a semidiameter of the annual orb of 1498½ terrestrial semidiameters, they gathered by the rules of right-angled triangles a distance of the Fixed stars of 10 312 227 terrestrial semidiameters. But later that parallax seemed too great to Lansbergen. In Book 3 of the *Uranometria*, element 7, he therefore makes it: as the periodic time of one revolution of the Earth through the annual orb, that is, one year, is to the radius of its orb, 10000, so the periodic time of the revolution of the Fixed stars, which he sets at 28 000 years, is to the radius of the sphere of the Fixed stars, 280 000 000. Since, then, the radius of the annual orb is for Lansbergen 1498½ terrestrial semidiameters, if it is made as 10 000 to 1498½, so 280 000 000 to a fourth term, the distance of the Fixed stars comes out as 41 958 000 terrestrial semidiameters. From this he concludes that the ancient and the more recent Astronomers were utterly mistaken in making one heaven out of two, since between the heaven of the Planets and the heaven of the Fixed stars there is a vast gap; and that, as the Sun is the centre of the Planetary sphere, so the whole sphere of the Planets is, as it were, the centre of the sphere of the Fixed stars.

[Margin: Opinion 5, Galileo's.]

VI. The *fifth* Opinion was Galileo's, in the third dialogue on the System of the World. There, on page 244 of the Latin and 323 of the Italian, he does not disapprove of Simplicio saying that it is more probable that not all the Fixed stars are equally distant from the centre of the World, but some higher than others, and that their innumerable multitude is contained between two surfaces, one convex and the other concave, separated from each other by a great interval; and he confirms this on page 284 of the Latin. On page 270 of the Latin and 357 of the Italian he determines the distance of the Fixed stars from the ratio of the periodic revolution of the Fixed stars, which he takes from Ptolemy as 36 000 years. Since, then, the revolution of the Sun, or of the Earth, is completed in one year and that of Saturn in 30 years, while Saturn's distance, as he thinks, is nine times higher than the Earth's, he argues thus: *"If the orb of*

Saturn, being nine times greater than the orb of the Sun, revolves in a time thirty times greater, then by the ratio reversed [per rationem euersam] the orb, or distance, of the Fixed stars, since it revolves 36 000 times more slowly, must be 10 800 semidiameters of the Great Orb." In this way this distance comes out five times greater than the one he had computed a little before from the hypothesis that a Fixed star of the sixth magnitude equals the Sun, namely on page 266 of the Latin and 351 of the Italian, where from that supposition he had deduced a distance of the Fixed stars of not more than 2160 semidiameters of the Great Orb; and in the same place he had said that the distance of the Sun from the earth is, by the agreement of all, 1208 terrestrial semidiameters. Multiplying, then, 10,800 by 1208 gives 13 046 400 for the distance of the Fixed stars from the centre of the World.

[Margin: Opinion 6, Hérigone's.]

VII. The *sixth* Opinion is that of Pierre Hérigone, who in volume 5 of the *Cursus Mathematicus*, page 618, assumes a distance of the Sun from the Earth of 1200 terrestrial semidiameters, and on page 615 supposes that the ratio of the semidiameter of the Earth to the semidiameter of the annual orb is the same as that of the semidiameter of the annual orb to the semidiameter of the Firmament. Hence, if it is made as 1 to 1200, so 1200 to a fourth term, the distance of the Fixed stars comes out as 1 440 000, from which he gathers a parallax of 2′ or 3′.

[Margin: Opinion 7, Gassendi's.]

VIII. The *seventh* Opinion is that of Pierre Gassendi, in the second letter *De motu impresso a motore translato*. He asserts that in the Copernican hypothesis the size of the World must be conceived as so great that the whole sphere of the heaven of the Planets, seen from the Fixed stars, would appear like a single star — in the way that the cluster of Jupiter and the Jovian stars appears to us as a single Planet — or at least so that the whole annual orb, seen from the Fixed stars, would be [?] as it were a point, or as it were the least of the tiny stars. Taking, then, a semidiameter of the annual orb of 1150 semidiameters of the earth, which he does not reject in Tycho in Book 2 of the *Institutio Astronomica*, and an apparent semidiameter of the smallest Fixed stars visible to the naked eye which we established, in Book 6, chapter 11, is not two Seconds, since the little star Alcor has an apparent diameter of only 4″ 24‴ — from this angle, I say, and from the side opposite it, of 1150 semidiameters, by the problem given in the preceding chapter, number 5, it follows that the distance of the Fixed stars is 118 602 274 terrestrial semidiameters.

[Margin: Gassendi, once a Copernican but not afterwards.]

I have counted Gassendi among the supporters of the Copernican sect because he did in fact favour it for a long time, before he had seen the decrees of the sacred Congregation; once he had seen them, or heard of them, he acquiesced piously and prudently, as is clear from the end of the same letter.

Opiniones non Copernicanorum, sed ex Hypothesi Copernicæa disserentium.

Opinions of those who are not Copernicans, but who reason from the Copernican Hypothesis.

[Margin: Opinion 8, Magini's.]

IX. The *eighth* Opinion was that of Giovanni Antonio Magini, who in Book II of the *Primum Mobile*, Problem 29, has these words: *"To leave aside the opinion of Copernicus, who asserts that between the sphere of Saturn and the Eighth there is so immense a vastness that the eighth sphere is removed from the Earth by at least 7 850 000 semidiameters."* This number, however, is nowhere found in Copernicus. But whether Tycho took the number from here, or Magini took this number from Tycho, I do not know.

[Margin: And Tycho's.]

Certainly Tycho, in the letter to Rothmann dated 1589 November 24, old style, page 167, has these words: *"As for the other motion of the Earth, the Annual, which would remove the eighth sphere so far that the Orb this motion would describe vanishes with respect to it: tell me, does it seem probable to you that the space from the Sun, then the centre of the universe, to Saturn should be contained more than 700 times between Saturn and the sphere of the fixed stars, and that all of it should be filled with no stars at all? Yet this must be so, if the Annual Orb of the Earth is to appear even as large as one Minute. Indeed, then the Fixed stars of the third magnitude too, which have one minute in diameter, will necessarily be equal to this whole Annual Orb, that is, they will comprise in their diameter 2284 semidiameters of the Earth; for they will be distant 7 850 000 of the same semidiameters, approximately."* So too, in volume I of the *Progymnasmata*, page 481, he affirms that it follows from the Copernican hypothesis that, if the semidiameter of the annual orb must vanish against the distance of the Fixed stars, this distance is 700 times greater than the distance of Saturn from the centre of the World.

[Margin: Opinion 9, Longomontanus's.]

X. The *ninth* Opinion, the one closest to the Tychonic, is that of Longomontanus, in the *Astronomia Danica*, Book I of the *Theorics*, chapter I, folio 159 in my copy. He takes from Tycho a semidiameter of the Annual Orb of 1150 terrestrial semidiameters, and holds that the annual Orb, in order to vanish against the sphere of the Fixed stars, must be of one minute, and so the semidiameter of the annual Orb must subtend an angle of 30″. Let the figure set out in the preceding chapter, number 5, now be repeated here, and in it let DP be the semidiameter of the annual orb, subtending the angle DCP of 30″; and so let the other acute angle, CPD, in the triangle CDP, right-angled at D, be 89° 59′ 30″; and let the distance of the Fixed stars, CD, be sought by his Method, which is this. As 1 454 441, the right Sine of the angle DCP of 30″, is to the side DP, of 1150 semidia-

meters of the earth, so 9 999 999 894, the right Sine of the angle CPD, of 89° 59′ 30″, is to CD, 7 906 818 semidiameters of the earth, that is, to the distance of the Fixed stars; and this is somewhat greater than the one Tycho deduced.

[Margin: Opinion 10, Scheiner's.]

XI. The *tenth* Opinion is that of our Christoph Scheiner, in the *Disquisitiones Mathematicae*, pages 25, 26 and 27. To remove the Fixed stars from all suspicion of parallax, in keeping with Copernicus's mind, he uses this proportion: as the Earth's Semidiameter, 1, is to Saturn's greatest distance, 10 872 terrestrial semidiameters, so the semidiameter of the annual Orb, 1208 terrestrial semidiameters, is to the distance of the Fixed stars, 13 133 376 terrestrial semidiameters. He confirms that this is not excessive from Tycho, Book 1 of the *Progymnasmata*, pages 480 and 481, who says that the Copernican distance of Saturn from the Earth is 12 900 terrestrial semidiameters, and that this must be multiplied more than seven hundred times to obtain the Copernican distance of the Fixed stars. And so, if it is multiplied by 800, Scheiner obtains 10 320 000 terrestrial semidiameters, that is, not much less than the preceding.

[Margin: Opinion 11, Chiaramonti's.]

XII. The *eleventh* Opinion is that of Scipione Chiaramonti, in the Italian defence of his *Antitycho*, part 4, chapter 13. There he assumes from Copernicus that the semidiameter of the Annual Orb is like a point compared with the semidiameter of the Firmament, so that the same Fixed star can be seen through a sight [dioptra] parallel to the horizon whether the eye is at the centre of the annual orb, or on its circumference, or on the surface of the Earth; and that the chord subtending the diameter of a Fixed star is at least 1105 terrestrial semidiameters, which is the least distance of the Sun from the earth according to Copernicus. Cubing 1105, then, he obtains 1 349 232 625, namely the ratio of the solid area, or solidity, of such a star to the solidity of the terrestrial globe (though he ought to have said: to the eighth part of the Earth's solidity). "But," he says, "the Sun contains the Earth only 166 or 167 times; therefore such a star would contain the Sun 8 079 237 times." And since Galileo, in the dialogues, page 265 of the Italian, had said that the Fixed stars are no less bright than the Sun, he concludes that if a single Fixed star is greater than the Sun 8 079 237 times, and yet should give no more light than the Sun, then, for the distance to be proportional to the size, the distance of the Sun, which is 1105 semidiameters of the earth, must be multiplied by 8 079 237; that is, the distance is 8 927 556 885 terrestrial semidiameters.

[Margin: Opinion 12, ours.]

XIII. The *twelfth* Opinion is ours: namely, that the distance of the Fixed stars must be so great that, when the Earth is carried through the annual Orb from one point of the annual circumference to the opposite point, no sensible parallax at all — that is, one of 10″ — can be observed in any Fixed stars, not even in those placed at the poles of the Ecliptic. On this supposition, from the various semidiameters of the annual orb set by the Copernicans, or by quasi-Copernicans, we derived a different distance of the Fixed stars by the Problem given in the preceding chapter, number 4, where we also displayed a table for these distances. We shall nevertheless repeat it here, so that it can be

compared alongside the distances collected from others up to this point. Here, then, in the following Synopsis, are the opinions already mentioned.

Table I. Distance of the Fixed stars deduced from the Copernican Hypothesis by various authors, with respect to the centre of the Earth or of the Universe

Order of the opinions	**Authors**	**Semidiameters of the Earth**
1	Aristarchus	1 478 656
		1 290 000
		1 345 600
2	Mästlin	2 580 000
		9 030 000
		13 046 400
3	Kepler, in the New Star	34 077 066⅔
	But in the *Epitome of Astronomy*	60 000 000
4	Lansbergen formerly, and Hortensius	10 312 227
	Lansbergen afterwards	41 958 000

Remainder of the Preceding Table

Order of the opinions	**Authors**	**Semidiameters of the Earth**
5	Galileo	13 046 400
6	Hérigone	1 440 000
7	Gassendi	118 602 274
8	Magini and Tycho	7 850 000
9	Longomontanus	7 906 818
10	Scheiner	13 133 376
11	Chiaramonti	8 927 556 885
12	We, taking the Semidiameter of the Annual Orb from: Copernicus	47 439 800
	Hérigone	49 502 400
	Galileo	49 832 416
	Boulliau	60 227 920
	Lansbergen	61 616 122
	Kepler	142 746 428
	Ourselves	301 146 419

Table I. Distance of the Fixed stars deduced from the Copernican Hypothesis by various authors, with respect to the centre of the Earth or of the Universe

	Wendelen	604 589 312

De Mole & Distantia Sphæræ Fixarum comparata ad molem Terræ, Orbis annui, & Orbis Saturni, & ad Distantiam Solis ac Saturni.

On the Bulk and the Distance of the Sphere of the Fixed stars, compared with the bulk of the Earth, of the annual Orb and of the Orb of Saturn, and with the Distance of the Sun and of Saturn.

XIV. An argument against the Copernican hypothesis can be taken not only from the distance but also from the bulk of the sphere of the Fixed stars, and that bulk seems comparable above all with three bodies: with the globe of the Earth; with the Great, or annual, Orb, which the Ptolemaic hypothesis would call the sphere of the Solar heaven; and with the Orb of Saturn, that is, with the whole sphere enclosing the whole system of the elements and of the Planets. Since this is so, we must derive the bulk from the distance of the Fixed stars, that is, from the semidiameter of the sphere of the Fixed stars, by proposition 18 of book 12 of Euclid's *Elements*, where it is shown that spheres are to one another in the triplicate ratio of their diameters, or by proposition 12 of book 8 of the same *Elements*, by which it is shown that spheres are to one another as the cubes of their diameters; on these we have said more in Book 7, section 6, chapter 8, problem 8. It is not worth the trouble, however, to bring in all the distances of the Fixed stars reviewed a little earlier, or their exact numbers. It suffices to use some selected ones, the more notable — partly the extremes, partly the middle ones — and their numbers rounded in Millions of terrestrial semidiameters. Further, we shall always make the semidiameter of the orb of Saturn ten times the semidiameter of the annual orb; how great the semidiameter of the annual orb is, we have already said in the preceding chapter, in the table of number 4, and in Book 3, at the end of chapter 7. On these premises, let us display in the following table the diameters of the three spheres — namely of the annual orb, of the sphere of Saturn, or the Planetary sphere, and of the sphere of the Fixed stars — in parts of which the diameter of the terrestrial globe is one, so that from this we may afterwards derive the proportion of the spheres.

Table II. Diameters of the Three Notable spheres, expressed in diameters of the Earth

From the Authors	**Annual Orb: Diam. of the Earth**	**Sphere of Saturn (♄): Diam. of the Earth**	**Sphere of the Fixed stars: Diam. of the Earth**
Aristarchus	1216	12160	1 500 000
Copernicus	1142	11420	47 000000

Table II. Diameters of the Three Notable spheres, expressed in diameters of the Earth

Mästlin	1160	11600	2600 000
Hérigone	1200	12000	1400 000
Galileo	1208	12080	13 000 000
Scheiner	1208	12080	13 000 000
Tycho	1150	11500	8 000 000
Longomontanus	1288	12880	8 000 000
Boulliau	1460	14600	60 000 000
Lansbergen	1498½	14985	61 000 000
Kepler	3469	34690	60 000 000
Ourselves	7300	73000	300 000 000
Wendelen	14656	146560	600 000 000
Gassendi	1150	11500	120 000 000

[Margin: Notice for the following table.]

XV. Next, the cubic numbers of the numbers already mentioned must be found. This is done by multiplying them by themselves and multiplying the product again by the first number. Thus 1142 multiplied by 1142 makes this square number, 1 304 164, which, multiplied again by 1142, makes the cube, or cubic number, 1 489 355 288. In its place, however, we shall put in the following table the first two initial figures, namely 15, with 8 zeros [cyphrae], so that from these round numbers a sufficient judgement can be formed more quickly about the bulk of the Sphere of the Fixed stars compared with the bulk of the spheres of the annual Orb and of Saturn. Further, we shall not attach the zeros themselves, written out, to the initial figures, but the number of zeros to be added to the initial figures, so as not to give the Printers occasion to err. For the rest, when you see, for example, that 8 zeros are to be added to the initial figure 15, know that it is the same as if you read 1 500 000 000; and so for the others.

Table III. Cubes of the Numbers of the preceding Table, in round figures

Deduced from the Authors	**Cube of the Diameter of the Annual Orb, in cubic diameters of the Earth: Figures**	**with ze-ros**	**Cube of the Diameter of the sphere of Saturn, in cubic diameters of the Earth: Figures**	**with ze-ros**	**Cube of the Diameter of the sphere of the Fixed stars, in cubic diameters of the Earth: Figures**	**with ze-ros**
Aristarchus	18	with 8	18	with 11	23	with 16
Copernicus	15	8	15	11	10	22
Mästlin	17	8	17	11	18	18
Hérigone	17	8	17	11	27	17

Table III. Cubes of the Numbers of the preceding Table, in round figures						
Galileo	17	8	17	11	22	20
Scheiner	17	8	17	11	22	20
Tycho	16	8	16	11	51	18
Longomontanus	22	8	22	11	51	18
Boulliau	34	8	34	11	22	21
Lansbergen	34	8	34	11	23	22
Kepler	42	9	42	12	22	21
Ourselves	39	10	39	13	27	24
Wendelen	34	11	34	14	22	25
Gassendi	16	8	16	11	17	23

XVI. Dividing, then, the cube of the diameter of the sphere of the Fixed stars, set in the last column of the preceding table, by the cube of the diameter of the sphere of Saturn in the penultimate column, or by the cube of the diameter of the annual orb set in the 2nd column, you will find how many times, according to the hypotheses of the authors already mentioned, the sphere of the Fixed stars contains either the sphere of Saturn or that of the annual Orb. The cubic number set in the last column, moreover, shows by itself, without any further division, how many times the sphere of the Fixed stars contains the globe of the Earth. For example, according to Copernicus's hypotheses, the sphere of the Fixed stars contains the Earth 100 000 000 000 000 000 000 000 times, that is, the number 10 taken with 22 zeros; and so for the others. But dividing the same number by 15 with 11 zeros, the sphere of the Fixed stars contains the sphere of Saturn 66 666 666 666⅓ times. Finally, dividing the same number by 15 with eight zeros, the sphere of the Fixed stars contains the annual orb 66 666 666 666 666⅓ times; and so for the others. Let this, then, be Table IV, in which we have likewise chosen round numbers and their two initial figures, with zeros substituted in place of the remaining figures.

Table IV. The Sphere of the Fixed stars Contains the Planetary Sphere and the Annual Orb the Number of Times written below

Deduced from the Authors	**The Planetary Sphere**	**The Annual Orb**
Aristarchus	130 000	130 000 000
Copernicus	67 000 000 000	67 000 000 000 000
Mästlin	11 000 000	11 000 000 000
Hérigone	1600 000	1600 000 000

Remainder of Table IV

Table IV. The Sphere of the Fixed stars Contains the Planetary Sphere and the Annual Orb the Number of Times written below

Deduced from the Authors	**The Planetary Sphere**	**The Annual Orb**
Galileo	1900 000 000	1900 000 000 000
Scheiner	1900 000 000	1900 000 000 000
Tycho	32 000 000	32 000 000 000
Longomontanus	23 000 000	23 000 000 000
Boulliau	6500 000 000	6500 000 000 000
Lansbergen	65 000 000 000	65 000 000 000 000
Kepler	520 000 000	520 000 000 000
Ourselves	69 000 000 000	69 000 000 000 000
Wendelen	61 000 000 000	61 000 000 000 000
Gassendi	1100 000 000 000	1100 000 000 000 000

Let no one be surprised, however, if according to some Authors, who disagree greatly among themselves about the semidiameter of the Annual Orb, the total of the sphere of the Fixed stars compared with the annual orb, or with the Planetary sphere, nonetheless does not differ so much. For when the semidiameter of the annual orb is increased, the semidiameter of the orb of Saturn is also increased proportionally, and so are their orbs; and the result is that these orbs are contained fewer times in the sphere of the Fixed stars, since this sphere too must grow proportionally with the increase of the semidiameter of the annual orb, in order to avoid a sensible parallax — a parallax that would be the greater, the greater the semidiameter of the annual orb. It is no wonder, therefore, if among the measures of Copernicus, Lansbergen, Wendelen and our own, set in Table 4, there is so close an affinity, namely of the numbers 67, 65, 69, 61, with 9 zeros in the first column and 12 in the second. These matters having been prepared sufficiently, indeed abundantly, for the proof of the following Arguments, the arguments themselves must now be set out.

I. Argumentum ab Immensitate sphæræ Fixarum.

Argument 1, from the Immensity of the sphere of the Fixed stars.

[Margin: The Form of Argument 1.]

XVII. *If the Earth moved through the Annual Orb, the sphere of the Fixed stars would turn out Immense, and of an altogether incredible, or improbable, bulk. Therefore the Earth does not move through the Annual Orb.*

[Margin: Scheiner's argument from the vastness of the sphere of the Fixed stars.]

This is the substance of the Argument that Scheiner urges with such weight of words in the *Disquisitiones Mathematicae*, page 27, where he says: *"This so great immensity of the Firmament and vastness of the stars Copernicus himself noticed well enough, in Book 1 of the Revolutions, chapter 6; nor do his followers dare to hide it — so Rheticus, in the Narratio prima, page 118 — but from it they go on to magnify the majesty of the creator. Ridiculously, since they, that is, the stars, appear so tiny, and even the most learned man can scarcely grasp this enormity. But the wiser Astronomers, both of old and now, would rightly have taken this superfluous bulk ill."*

[Margin: And Archimedes's.]

At this point he also brings in Archimedes himself, as one who counts this bulk among impossibilities. The words of Archimedes, from the *Sand-Reckoner* [Arenarius], that is, from the book on the number of the sand, faithfully translated from the Greek, are these: *"Aristarchus of Samos, then, refuting and altering these things which are found written among the Astrologers, put out certain writings filled with certain hypotheses, from which it is evident that the World is much greater than the World described up to now. For he supposes that the fixed stars and the Sun remain motionless; that the Earth is carried around the Sun on the circumference of a circle which is situated in the middle of the course of the Planets; and that the Sphere of the Fixed stars is situated about the same centre as the Sun, and is of such a size that the circle about which the Earth is supposed to be carried has the same ratio to the distance of the Fixed stars as the centre of any sphere has to its surface. But this is manifestly impossible: since the centre of a sphere has no magnitude, it must not be supposed to have any ratio at all to the surface of the sphere. Yet it must be granted that Aristarchus understood this: namely, since we think the Earth placed about the centre of the world, he held that it must be established, by demonstrations drawn from the phenomena, that whatever ratio the Earth has to the World as described by us, that same*

proportion is had by the sphere of the circle that carries the Earth around, relative to the sphere of the fixed stars." Further, the World that Archimedes had spoken of before extends as far as the annual Orb of the Sun; for he had said at the beginning of the *Sand-Reckoner*: *"You are not unaware, however, that the World is called by many Astrologers the sphere whose centre is the centre of the Earth, and whose Radius is equal to the straight line that lies between the centre of the Sun and the centre of the Earth."* Although Archimedes, then, thought it absurd that the annual Orb should be like a point compared with the sphere of the Fixed stars, he nevertheless interprets Aristarchus more kindly, and gathers from his hypothesis that the proportion of the annual orb to the orb of the Fixed stars is that of the Earth to the annual Orb, or to the World as commonly understood. So, just as we are accustomed to say that the Earth is like a point compared

with the whole heaven, the annual orb would be like a point compared with the orb of the Fixed stars in Aristarchus's hypothesis — that is, not absolutely a point and nothing, but comparatively, or to sense. Whatever may be true of Aristarchus's mind, however, the Antecedent is now proved from Table 4, set out in number 16. For in it, if you take the smallest measure, the one dug out of Aristarchus, the sphere of the Fixed stars contains the annual Orb 130 000 000 times; but if you take the Copernican measure, 67 000 000 000 000 times; if Lansbergen's, 65 000 000 000 000 times; if ours, 69 000 000 000 000 times. Nor are the other measures, deduced from the hypotheses of others, so small that they do not contain an incredible vastness of the orb of the Fixed stars, as will be clear to anyone who surveys that Table.

[Margin: Reply to argument 1.]

XVIII. The Copernicans, however, *reply* by denying the Antecedent, namely that from the motion of the Earth through the annual orb there follows a Bulk of the sphere of the Fixed stars so great that it is incredible. The Reasons for which they deny that it is incredible or improbable are four.

[Margin: The 1st Reason of the Reply.]

The *first* is Mästlin's, who denies that, in Copernicus's mind, a distance of the Fixed stars, or a bulk of the sphere of the Fixed stars, is required so great that against it the distance of the Earth from the Sun, or the annual orb, vanishes entirely; it is enough, he holds, if it vanishes to sense. See his own words, already reported in number 3, in the 2nd opinion; and the 1st opinion, that of Aristarchus, which does not require so great a distance of the Fixed stars in the hypothesis of the moving Earth, supports him.

[Margin: The 2nd Reason of the Reply.]

The *second* reason, which Kepler and Galileo suggest, is the comparison of this distance and Magnitude of the sphere of the Fixed stars in the Copernican hypothesis with the speed of the same sphere in the Ptolemaic hypothesis. For they say that that Copernican Magnitude, though motionless, is far more probable than the Ptolemaic smallness moving with so rapid a speed. But this very comparison must be dealt with more calmly below, in chapter 30.

[Margin: The 3rd Reason of the Reply.]

The *third* reason is the comparison of the other spheres with one another, whose size must be admitted, and is admitted, in the hypothesis of the motionless Earth. Is not the heaven of the Fixed stars, says Gassendi in the second letter *De motu impresso*, page 124, by common opinion so vast that our Earth, however great, compared with it is nevertheless only like a point? Just as this, then, which is commonly reckoned a Paradox, is nevertheless most true in Astronomical rigour, why should it seem incredible that the whole annual Orb is like a point with respect to the sphere of the Fixed stars? Who would believe that the Sun, which is commonly reckoned a foot across, is greater than the Earth 38,600 times, as we showed in Book 3, chapter 11? Moreover, if you follow the opinion of many Copernicans, the excess of the Planetary sphere over the sphere of the terraqueous globe is greater than the excess of the sphere of the Fixed stars

over the Planetary sphere, or over the sphere of the annual orb. For, as above in Table 3, the Planetary sphere on Kepler's hypothesis contains the Earth approximately 42 000 000 000 000 times; but in Table 4 the sphere of the Fixed stars, on the same Kepler's hypothesis, contains the Planetary sphere only 520 000 000 times, and the annual Orb 520 000 000 000 times; and so for not a few others. Just as so great a vastness of the sphere of Saturn, and its excess with respect to our Earth, is not incredible to the wise, why then should a smaller excess of the sphere of the Fixed stars over the sphere of Saturn, or over that of the annual orb, seem incredible? For on this reasoning, says Galileo in the third dialogue, elephants will seem incredible to ants, and whales to little fish.

[Margin: The 4th Reason of the Reply.]

The *fourth* and chief reason is the commendation of the Divine Omnipotence and Magnificence, for which so great a bulk of the work wins esteem. And so *Copernicus*, having said in Book 1, chapter 10, that from the highest of the wandering stars, Saturn, to the sphere of the Fixed stars there is a very great distance, and that between the moved bodies, that is, the Planets, and the unmoved, that is, the Fixed stars, there lies the greatest interval, replaced incredulity with wonder, exclaiming: *"So great, indeed, is this divine Fabric of the Best and Greatest!"*

[Margin: Rothmann's opinion for the immensity of the heaven, in commendation of the Divine Majesty.]

So Christoph Rothmann, in the Letter to Tycho dated 1590 April 18, which is among Tycho's letters, page 186, replies to Tycho thus: *"Why should it not seem probable to me that the space from the Sun to Saturn is contained so many times within the remoteness between Saturn and the fixed stars? Or what absurdity follows if a star of the third magnitude equals the whole annual orb? Does this conflict with the divine will, or is it impossible to the divine Nature, or does it not befit an infinite Nature? You must demonstrate these things without fail, if you mean to gather some absurdity from this [?]. Things that commonly seem absurd at first sight cannot so easily be convicted of absurdity. But the Divine Wisdom and Majesty is far greater; and however great a vastness and size of the World you grant, it will still have no proportion to the infinite Creator. The greater the King, the greater and more ample the palace he thinks befits his Majesty. What will you think of GOD?"* And indeed Tycho himself, in the same Letters, page 191, conceded that the World is of almost infinite bulk, when he said: *"And although the heaven, the Universe, is indeed finite in place and in time, yet it is like the infinite in its incredible magnitude and perpetuity."* Why then, in the same place and on page 167, does he call incredible that Magnitude which the Copernicans ascribe to the sphere of the Fixed stars?

[Margin: The opinion of Rheticus and of Pliny, for the same.]

Rheticus, certainly, in the *Narratio prima*, commending this Copernican magnificence, more rightly confirmed it with that well-known sentence of Pliny's, Book 2, chapter 1: *"Of the world, or heaven, by whose curve all things are covered, etc., it is no concern of men to search out what lies outside, nor can the*

conjecture of the human mind grasp it. It is sacred, etc., immense, etc., finite and like the infinite."

[Margin: Mästlin's opinion, for the same.]

But Mästlin, in the additions to Rheticus's *Narratio prima*, page 114, judges it harsh, and almost injurious to the Divine Omnipotence and Wisdom, to call the sphere of the Fixed stars that Tycho gathered from the Copernican hypothesis too vast and useless.

[Margin: Lansbergen's opinion, for the same.]

So too *Lansbergen*, in Book 3 of the *Uranometria*, element 7, concludes: *"This is our opinion on the distance of the fixed stars from the Earth; and as it is not foreign to Geometry, so too it detracts nothing from the Majesty and the infinite Power of GOD, Best and Greatest, but rather magnifies and illustrates it much more. For it teaches that, since the sphere of the fixed stars is so vast, we must be astonished at the infinite Power of GOD the Architect, and worship and venerate it submissively."* So, finally, Gassendi, in the second letter *De motu impresso*, turning the argument back, says that just as the Ptolemaics argue that there is no reason to think God willed to make the World so vast and august, so the Copernicans argue that there is no reason to think God willed to make the World so narrow; and since neither side can demonstrate what pleased God, the latter opinion, which amplifies the majesty of things, seems the more to commend the Magnificence of the supreme Maker.

Finally, what if God willed to sketch, in so vast a bulk, some trace of infinity and immensity? What if, in proportion to the innumerable number of the Fixed stars, he granted them a region so much ampler than he granted the seven Planets? What if, for the sake of the Saints, he also willed to construct so great a proof of his works, so that they might see how much he has wrought for them and for the throne of Christ the Lord, and how much greater things are to be expected from him above the Empyrean?

[Margin: Baruch 3.]

Let the holy Prophet Baruch, indeed, close this reply with that Epiphonema: *"O Israel, how great is the house of God, and how vast the place of his possession? It is great and has no end; high and immeasurable."*

II. Argumentum ab Immenso Interuallo inter Saturnum & Fixas, eoq. Stellis vacuo & otioso.

Argument 2, from the Immense Interval between Saturn and the Fixed stars, an interval empty of Stars and idle.

[Margin: The Form of Argument 2.]

XIX. *If the Earth moved through the Annual Orb, there would be between Saturn and the Fixed stars an immense space, empty of stars and altogether idle. But this is absurd. Therefore, etc.* This argument, more than the preceding one, is made much of above all by *Tycho*, in the letters, page 167, whose words I have already reported in number 9, but chiefly on page 192, where

against Rothmann, who attributes so great a vastness to the divine Omnipotence, he says that God is indeed omnipotent, but does nothing in a disordered way; on the contrary, he disposes all things in weight, number and measure, and contrives nothing empty or useless, nothing that does not answer to itself in a fixed harmony; and that accordingly an interval of more than 7 000 000 semidiameters of the Earth, empty of stars and destined for no use of the Earth's inhabitants, ought not to be admitted between Saturn and the Fixed stars, since the Fixed stars themselves, which lie above it, were established for their use.

[Margin: Tycho's argument against [?] Copernicus.]

[Margin: Rothmann sings a palinode [?].]

And he adds: *"Away with this ageometry and asymmetry, this disordered way of philosophizing, utterly foreign to the divine Wisdom and Providence — which even ROTHMANN himself, when he was with me, no longer dared to defend; rather, in the course of our discussions, he at last admitted, not unwillingly, that a great absurdity, joined with an equal incredibility, lay beneath these things."* And a little later he confesses that the world is indeed finite and has no proportion to the infinite Creator, but that as regards its parts it is proportioned to the whole in the highest degree, just as one may see in the parts of animals, and above all in the Microcosm, that is, in the human body — the singular symmetry of which Microcosm *Durerus* [Albrecht Dürer] demonstrated. He presses the same absurdity in volume 1 of the *Progymnasmata*, page 481.

[Margin: Scheiner's argument against [?] Copernicus.]

No less sharply does *Scheiner* press it, on page 28 of the *Disquisitiones Mathematicae*, saying against the Copernicans: *"What good are such great machines — were they produced for the sake of so tiny a speck, the Earth? Why, then, are they so remote that they appear so small, and have almost no power over the Earth? What is that insane abyss between them and Saturn for? All things, it seems, are in vain."* Accordingly, at the end of the same page, he concludes thus: *"The Copernican motion of the Earth puts between Saturn and the Firmament an immense vastness of distance, the use of which they themselves do not know; therefore it does not seem admissible."* Finally *Longomontanus*, Book 1 of the *Theorics*, chapter 1: *"To what purpose is the empty gap between Saturn and the fixed stars, which is drawn from the premisses as* 7 894 818 *semidiameters of the earth?"*

[Margin: Proof of the Major.]

The MAJOR of our argument, moreover, is proved from Table II, set out in number 14: namely, if you subtract the numbers of the second column from the numbers of the third, or last, column. For example, if from Kepler's distance of the Fixed stars, which is 60 000 000, you subtract the distance of Saturn, which is about 35 000, there will remain between Saturn and the Fixed stars a gap of 59 965 000 terrestrial semidiameters; and so with the hypotheses of the others, from all of which that interval is always gathered to be huge.

[Margin: Proof of the Minor.]

The MINOR is proved from an axiom often employed by the Copernicans when they busy themselves with banishing the Ptolemaic Epicycles and the

like. For nothing is more often on their lips than that GOD and Nature do nothing in vain, and do not attempt through more means what can be done through fewer. Therefore they act absurdly when they put so vast a space, in vain and without any use, between the sphere of the Planets and that of the Fixed stars.

[Margin: The 1st Reply, Copernicus's, insufficient.]

XX. *Copernicus* would reply, 1st, by denying the Minor, in Book 1 of the *Revolutions*, chapter 10: because *"between the moved and the unmoved there had to be the greatest difference"*, that is, between the Planets, which are moved, and the Fixed stars, which he held to be motionless. But since for him the Sun too is motionless, and yet is not distant from Mercury — which is moved, and is innermost and nearest to it — by more than 561 terrestrial semidiameters, as we showed from Copernicus in Book 7, section 6, chapter 2, in Table 4, a few more semidiameters of the Earth would have sufficed between Saturn, the outermost of the moving bodies, and the Fixed stars.

[Margin: The 2nd Reply, the Copernicans', insufficient.]

Rheticus, Mästlin, Galileo, Lansbergen and *Gassendi* would reply, 2nd, by having recourse to the Divine Omnipotence and Magnificence, in accordance with what we reviewed in number 18, in the 4th reason of the reply; and so by denying the Minor. But granted that the divine Omnipotence and Majesty is commended by the huge bulk of the sphere of the Fixed stars, insofar as it is the sphere of the Fixed stars, or by that part of its bulk which the innumerable population of the Fixed stars inhabits, it is nevertheless not commended by that insane and useless space between the Fixed stars and Saturn — whether it is supposed empty of every other body, or filled only with ethereal air — unless its use is apparent. *Galileo*, however, persists, in the third dialogue, page 272, and says *that it is rash to call vain and superfluous everything in the Universe that does not serve our uses.*

[Margin: The 3rd Reply, that of René Descartes.]

Renatus de Cartes [René Descartes], then, would reply, 3rd, in the *Principles of Philosophy*, part 3, by denying rather the Major; for in number 20 he says: *"We are not yet certain how far the Fixed stars are distant from us, nor can we imagine them so remote that this conflicts with the Phenomena. Let us not be content to suppose them above Saturn, as all commonly admit, but let us take the liberty of judging them as much higher as we please. For if we wished to compare their height with the distances known to us here above the earth, the height already granted to them by everyone would be no less incredible than any greater one. But if we look to the Omnipotence of God the Creator, none can be conceived so great that it is for that reason less credible than any smaller one. And I shall show below that, to explain conveniently the Phenomena not only of the Planets but also of the Comets, a very great space must be set between them and the sphere of Saturn."* Yet from number 119, where he begins to treat of Comets, he nowhere shows this; rather, he supposes from his own principles immense vortices, through which the Fixed stars are moved and are changed sometimes into Comets, sometimes into new

Planets. Seneca nevertheless supports him, in Book 7 of the *Natural Questions*, chapter 24, where he judges that innumerable stars unknown to us run through the immense space of the heavens, and so that what seems empty to us is not empty of stars. But we have already shown in Book 8 that it has not yet been demonstrated by anyone that Comets or new stars were above Saturn; nor is it fitting, for the sake of one or another such phenomenon, to set so great a vastness between Saturn and the Fixed stars, since for these Phenomena there amply suffice, in part the region of air or of fire itself, in part the fluid heaven of the Planets, in part the heaven of the Fixed stars, especially if that too is admitted to be liquid; or at least some moderate space between the Fixed stars and Saturn would suffice — say, as much as there is from the centre of the World to Saturn.

[Margin: The 4th Reply, the Copernicans'.]

Not a few Copernicans reply, 4th, by implicitly denying the Major, when they indicate that the space above Saturn is not wholly empty of stars, granted that the stars which can be observed by us with the naked eye, and of which there can be no sensible parallax, are removed from the sphere of Saturn by a vast interval. They could therefore say that there are innumerable stars which can be distinguished only with the Telescope, and which cannot be observed in such a way that their parallax, if they have any a little smaller than Saturn's parallax, could be observed; and that very many stars of this kind are scattered within the interval already mentioned, and that it is not idle and empty, as Tycho, Scheiner and others thought. I had devised this reply for myself when I came upon another akin to it in *Galileo*, in the third dialogue *On the System of the World*, page 284 of the Latin edition, where he says that some of the fixed stars are two or three times more remote than others, and that it may happen that one day, by means of the Telescope, some variation like the variation of the Planets is observed in some of them. So too *Mästlin*, in the additions to Rheticus's *Narratio prima*, page 113, denies that the gap must be so vast, since not all the Fixed stars are equally distant from the centre — Tycho himself conceding this in volume 1 of the *Progymnasmata*, 470 and 481. For the rest, although this reply cannot be convicted of falsity by anyone, it is nevertheless apparent that it was devised arbitrarily to protect the hypothesis of Copernicus. For who would believe that God removed all the stars that appear larger to us, and whose parallax, if there were any, we could observe, so far from us that we could not observe that parallax; and that he made the others, which as regards their distance are otherwise subject to parallax, so small that they could not be observed by us with the naked eye? It would surely follow — if one may speak so — that God, in distributing the Fixed stars right from the beginning of the World, was a Copernican, that is, a supporter of this sect, which he nevertheless foresaw would one day be condemned by sacred decrees.

III. Argumentum à Solis exilitate non valentis Fixas illustrare ob ingentem distantiam.

Argument 3, from the smallness of the Sun, which is unable to illuminate the Fixed stars on account of their huge distance.

[Margin: The Form of Argument 3.]

XXI. *If the Earth moved through the Annual orb around the Sun, the Fixed stars would have to be removed so far from the Sun that the Sun, seen from them, would be like a point, and so could not illuminate the stars with its light. The consequent is false. Therefore so is that from which it follows.*

Gassendi replies, in the second letter *De motu impresso*, page 126, first by denying the Major: for it could spread its rays all the way there, even if it were like a point; secondly by denying the Minor: for the Fixed stars have their own light from themselves, not from the Sun — which we too taught in Book 6, chapter 2.

CAPVT XXX. Proponitur Argumentum ab Ingenti Mole Fixarum, Contra motum Terræ Annuum. Qua occasione disputatur An hæc sit incredibilior in Fixis immotis, quàm Celeritas illarum motui diurno in hypothesi Terræ quiescentis conueniens?

Chapter 30. The Argument from the Huge Bulk of the Fixed stars is put forward against the annual motion of the Earth. On this occasion it is disputed whether this bulk is more incredible in motionless Fixed stars than the Speed that befits them for the diurnal motion in the hypothesis of the resting Earth.

I. To most people the huge Magnitude of the Fixed stars that follows from their distance in the Copernican hypothesis has seemed much more absurd than the bulk of the whole sphere of the Fixed stars — whether the Fixed stars themselves are compared with the globe of the Earth, or with the Sun, or with the whole Annual Orb. The Copernicans believe, however, that this magnitude came out greater than it should for Tycho, Longomontanus, Scheiner, Chiaramonti and some others, because they assumed too large an apparent diameter for the Fixed stars.

[Margin: Tycho's opinion on the magnitude of the Fixed stars in the Copernican hypothesis.]

For Tycho, in the letter of the year 1589, November 24, written to Rothmann, page 167, says: *"Indeed, even then the fixed stars of the third magnitude, which have one Minute in diameter, will necessarily be equal to this whole Annual Orb, that is, they will comprise 2284 semidiameters of the Earth in diameter; for they will be distant about 7 850 000 of the same semidiameters. What shall we say of the stars of the first magnitude, some of which occupy two, and some three, Minutes in visible diameter?"* He repeats this on page 191. He takes from Copernicus, moreover,

a semidiameter of the annual orb of 1142 terrestrial semidiameters, and so a diameter of 2284.

II. *Longomontanus* likewise, in Book 1 of the *Theorics*, chapter 1, page 1, investigates the magnitude of the fixed stars with a diagram added, in the way that we too did in Book 6, chapter 9, number 9.

[Margin: The method of investigating the true magnitude of a Star.]

Diagram. A woodcut set in the left column. A circle, the star, with centre B; a vertical line runs from H at the top of the circle through B and F down to the apex G; from G two straight lines touch the circle at D on the left and at E on the right; a flatter arc, the portion of the circumference of the highest heaven, runs from A on the left across the circle near the level of B to C on the right; straight lines join B to D and B to E.

- H (top of the circle): a point of the star's circumference
- B (centre of the circle): the centre of the star
- A (left end of the arc): one end of the arc ABC of the highest heaven
- D (left, where the tangent meets the circle): point of tangency
- E (right, where the tangent meets the circle): point of tangency
- C (right end of the arc): the other end of the arc ABC
- F (on the vertical below B): a point of the star's circumference
- G (foot of the figure): the centre of the World, from which the arc is described

For let ABC be a portion of the circumference of the highest heaven, described from the centre G, and let DHEF be some Fixed star, whose centre is B; and from G let the straight lines GD and GE be drawn, touching the globe of the star at D and E. By the 18th of the third book of Euclid, they will make the angles at D and E right angles with the star's semidiameters BD on the one side and BE on the other; and the angle DGE will subtend and measure the apparent diameter of the star. When the angle G is bisected by the straight line BG, a double triangle arises, right-angled at D and at E. Therefore, if in either triangle, say DBG, the base BG is given — that is, the distance of the Fixed star from G, the centre of the World — and the angle DGB, which measures the apparent semidiameter of the star, then by the rules of right-angled rectilinear triangles the true size of the semidiameter DB will at once be found, in parts of which GB is known. When BD is doubled, the diameter will be known; multiplied by itself, it will give the square number, and this, multiplied again by the number of the diameter, will give the cubic number, which indicates how many times the solidity of the earth is contained in the Solidity of the star — provided that the true diameter of the star is known in diameters of the earth. Accordingly *Longomontanus*, having found from the Copernican hypothesis the distance of the Fixed stars, BG, to be 7 906 818 semidiameters of the earth, and having assumed the apparent semidiameter of a Fixed star of the first magnitude, that is, the angle BGD, to be 1 Minute, sets up the proportion: as the Radius BG, of 10 000 000 parts, is to the distance BG of 7 906 818 semidiameters of the earth,

[Margin: Longomontanus's opinion on the magnitude of the Fixed stars in the Copernican hypothesis.]

so is the Tangent of the angle BGD of 1 Minute, 2909, to BD, 2300 terrestrial semidiameters; and so he gathers the diameter of such a star to be 2300 terrestrial diameters. The square of this number multiplied into itself is 5 290 000; and this, multiplied again by 2300, makes the Cube 12 167 000 000. *"So many times, then,"* he says, *"does a star of the first magnitude contain the Earth in the Copernican hypothesis,"* since, when the terrestrial diameter is taken as 1, its cube is 1. And since the Sun contains the Earth 140 times, as Longomontanus thinks, if 12 167 000 000 is divided by 140, the same star will be found to be greater than the Sun 86 907 143 times. Finally, since the diameter of the annual Orb is 1142 diameters of the earth for Copernicus, while the diameter of the Fixed star already mentioned is 2300 diameters of the earth — that is, about twice as great — and stands to it as 2 to 1, and since the cube of the number 2 is 8 and the cube of unity is 1, it follows that a Fixed star of the first rank comes out eight times greater than the whole annual orb. *Longomontanus* therefore concludes in the same place: *"Since, then, both from the immense remoteness of the fixed stars from the earth, and from the gap between the last of the Planets and the orb of those same Fixed stars, and from the incredible magnitude of a fixed star of the first rank that follows from this, relative to the annual orb of the earth, likewise to the Sun, and much more to the earth, all the well-ordered symmetry of the parts of the world is easily taken away, besides the absurd, etc., I therefore think that the Copernican hypothesis — of the annual motion of the earth above all, and of its libration upon its poles — is rightly to be removed."*

[Margin: Scheiner's opinion on the magnitude of the Fixed stars in the Copernican hypothesis.]

III. But *Christoph Scheiner*, in the *Disquisitiones Mathematicae*, page 26, having derived from the Copernican hypothesis a distance of the Fixed stars of 13 133 376 semidiameters of the earth, assumes a distance of the Sun of 1208 semidiameters of the earth, and its true diameter of 11 terrestrial semidiameters. Multiplying 13 133 376 by these, he gathers the sum 144 467 136; dividing this by 1208, he finds 119 592 semidiameters of the earth, which the diameter of the Sun would contain if it were in the Firmament. Then, putting the visual diameter of the Sun at apogee at 30′, and the visual diameter of a fixed star of the first magnitude at 2′, he sets up the proportion: as 30 is to 119 592, so is 2 to 7972 semidiameters of the earth, comprised in the diameter of the fixed star; and by the same reasoning, for a star of 1 apparent minute, or of the third magnitude, he gathers 3986 semidiameters. From this he deduces that the diameter of the annual orb, which is 2416 semidiameters of the earth, is contained in the diameter of a star of the first rank 3 181/604 times, in the diameter of a star of the third rank 1 785/1208 times, and in a star of the sixth order about once. Finally, since the cube of the diameter of the annual orb, taken as 1, is 1, and the cube of the diameter of a Fixed star of the first rank, taken as 3 18/60, is about 32, he gathers that a star such as the Dog-star contains the annual orb within itself 32

times. Accordingly, on page 28, he concludes thus: *"The Copernican motion of the Earth makes any star of the first magnitude thirty times, and more, larger than the entire heaven of the Sun, and any little star seen in the starry sphere larger than that same heaven. Therefore it does not seem admissible."*

[Margin: Chiaramonti's opinion on the magnitude of the Fixed stars in the Copernican hypothesis.]

IV. By what method, further, *Chiaramonti*, in the defence of the *Antitycho*, part 3, chapter 13, gathers that a fixed star of the first magnitude in the Copernican hypothesis contains the globe of the earth 1 349 232 625 times, and the Sun 8 079 237 times, has already been stated in the preceding chapter, number 12.

[Margin: And Lansbergen's.]

Among the Copernicans, *Lansbergen*, in the *Uranometria*, Book 3, from element 20 on, attributing to a Fixed star of the first magnitude a diameter of 1 minute, and to the distance of the Fixed stars 41 958 000 terrestrial semidiameters, gathers by the method described in number 2 the true diameter of a Fixed star, and sets it at 40 712 terrestrial diameters; and he gathers that the Great Orb is contained in its solidity 20 053 times. For the diameter of the annual orb is, for him, 1498½ diameters of the earth, whose cube is 3 364 884 747; and the cube of the number 40 712 is 67 478 794 224 128, which contains the former cube 20 053 times. For a star of the sixth rank, however, he makes the apparent semidiameter 2½″ and the true one 3388 terrestrial semidiameters; from which he deduces that its diameter is to the diameter of the Annual Orb as 3388 to 1498½, and that its solidity, or cube, is greater than the solidity of the annual Orb 11½ times. For the remaining stars, further, see what I reported in Book 6, chapter 9, at the end of Scholium 4.

[Margin: Hortensius's opinion on the same.]

V. *Hortensius*, however, himself an Astronomer of the Copernican sect, in the dissertation with Gassendi on Mercury seen in the

Sun and Venus unseen, gives the stars of the first magnitude only 8″ for their apparent diameter, and those of the sixth magnitude 2″. From this, and from the distance of the Fixed stars, which for him is 10 312 227 semidiameters of the earth, and from the semidiameter of the Annual Orb, 1498½ semidiameters of the earth, he gathers that a star of the first magnitude is smaller than the Great Orb 422 times, and a star of the third magnitude 27 826 times — so far is it from being equal to it or greater.

[Margin: Galileo's opinion on the same magnitude of the Fixed stars.]

But *Galileo* makes them smaller still. For in the third dialogue *On the System of the World*, page 265 of the Latin and 351 of the Italian, he rebukes those who gather from the system of Copernicus that some Fixed star is greater than the whole Great Orb; and on page 267 he teaches a method of measuring the apparent diameter of the Fixed stars, from which he asserts, on page 268, that the apparent diameter of a star of the first magnitude does not exceed 5″ — from which there follows a bulk far smaller than that of the annual Orb. But I have reported his method in Book 6, chapter 9, number 3.

[Margin: Our opinion on the Magnitude of the Fixed stars.]

VI. What I, for my part, accomplished together with Father Francesco Maria Grimaldi in investigating the apparent diameters of the Fixed stars by a far more certain method, I have already related in Book 6, chapter 9, from number 6, and in Book 7, section 6, chapter 9; and I illustrated it with examples and demonstrations in Book 7, section 6, chapters 10 and 11, and showed that a star of the first magnitude, such as Sirius, has an apparent diameter of 18″, while the smallest of the stars visible to the naked eye, such as the little star in the tail of Ursa Major called Alcor, has a diameter of 4″ 24‴. From these, according to the various opinions on the distance of the Fixed stars, I deduced the various true Magnitudes of these stars and arranged them in three tables, set out in that chapter 11, of which the last two rest on Copernican distances; and I think the Reader should be referred to them for fuller instruction. Here, however, I shall repeat what is more necessary for setting out the force of this argument, which I have in hand. In the 1st and 3rd tables I shall give the cubic diameters of the earth contained in the bodies of the Fixed stars Sirius and Alcor, so that from these two, the greatest and the smallest of the Fixed stars visible to the naked eye, a judgement may be made about the rest; and in the 2nd and 4th tables I shall give how many times stars of this kind contain the Annual Orb, or are contained in it — dividing, that is, the number of cubic terrestrial diameters contained in the body of the star by the number of cubic diameters contained in the annual orb, taken from the preceding chapter, number 15, in the 1st column of table 3.

Table I. The True Magnitude of the Greatest of the Fixed stars, namely SIRIUS, and of the Smallest visible to the naked eye, namely ALCOR; the apparent diameter of Sirius being put at 18″ and of Alcor at 4″ 24‴, and the Distance of the Fixed stars being that asserted by the four Copernicans written below.

From the authors	Distance of the Fixed stars, in semidiameters of the Earth	Semidiameter of the Annual Orb, Earth semidiameters	Sirius therefore contains: diameter, Earth diameters	Sirius: body, the Earth so many times	Alcor contains: diameter, Earth diameters	Alcor: body, the Earth so many times
Hortensius	10 312 227	1498½	899	726 572 600	442	86 355 888
Galileo	13 046 400	1208	1138	1 473 760 072	558	173 741 112
Lansbergen	41 958 000	1498½	3658	48 947 466 312	1796	5 793 206 336
Kepler	60 000 000	3469	5232	143 219 897 228	2568	16 933 994 432

Table II.	Sirius, therefore	Times	And Alcor	Times
Hortensius	is contained in the Annual Orb	about 5	is contained in the Annual Orb	39½
Galileo	is contained in the Annual Orb	about 1 2/15	is contained in the Annual Orb	10
Lansbergen	contains the Annual Orb	about 145	contains the Annual Orb	1 12/17
Kepler	contains the Annual Orb	about 3⅓	is contained in the Annual Orb	2 9/17

Table III. The True Magnitude of SIRIUS and ALCOR, their Apparent Semidiameter being supposed 18″ for Sirius and 4″ 24‴ for Alcor, the Distance of the Fixed stars being so great that the Parallax of the Annual Orb does not exceed 10″, and the Semidiameter of the Annual Orb being supposed from the Authors written below.

From the authors	Distance of the Fixed stars, in terrestrial semidiameters	Semidiameter of the Annual Orb, Earth semidiameters	Sirius therefore contains: diameter, Earth diameters	Sirius: body, the Earth so many times	Alcor contains: diameter, Earth diameters	Alcor: body, the Earth so many times
Copernicus	47 439 800	1142	4170	71 677 713 000	1992	4 378 454 048
Hérigone	49 502 400	1200	4350	82 312 875 000	2068	8 844 058 432
Galileo	49 832 416	1208	4380	88 427 672 000	2092	9 155 562 688
Boulliau	60 227 920	1460	5300	148 877 000 000	2530	15 941 277 000
Lansbergen	61 616 122	1498½	5424	159 371 956 024	2588	17 333 761 472
Kepler	142 746 428	3469	12550	1 976 656 375 000	6000	216 000 000 000

Table IV.	The Annual Orb is therefore contained by Sirius, times	by Alcor, times
Copernicus	48	4
Hérigone	48	4⅕
Galileo	52	5⅓
Boulliau	44	4⅔
Lansbergen	47	5
Kepler	46	5

[Margin: The Argument in Form.]

VII. With these premisses, the Argument is Formed thus. *If the Earth moved through the Annual orb, the Fixed stars not only of the first but also of the lowest magnitude would have too great a bulk, that is, one greater than the Sun and the Annual Orb itself, or than the solar heaven. But this is absurd and incredible. Therefore, etc.* The MAJOR is proved from the tables set out a little before. The MINOR is proved from the comparison of the Fixed stars with the Sun and the other

Planets, which are much smaller; from the symmetry of the Universe; and from Rothmann himself, Copernican though he was, who at last yielded to Tycho on this point, as Tycho himself relates in the letters, page 192, saying of Rothmann:

[Margin: Rothmann's confession of the absurdity of the Copernican hypothesis.]

"For I remember that he once said to me, among other things, that if the Copernican assumption held in the truth of the matter, it would be necessary that most of the fixed stars should exceed the whole annual Orb, or the sphere of the Sun, in their true size, before so inexhaustible a distance as Copernicus's reasoning necessarily attributes to them would grant them the visible size that we discern from the Earth: just as

had been objected to him by me. Yet he did not deny that this surpassed all belief and seemed utterly absurd; nor did he try any longer to excuse it by the divine Omnipotence; and he admitted, not unwillingly, that a great asymmetry too was introduced by these things."

[Margin: 1. Reply, insufficient.]

First, *Galileo and Hortensius* reply, as we saw in number 5, by denying the Major: Tycho and others, they say, deduced from too large an apparent diameter of the Fixed stars a magnitude for them larger than is right. But granted that Tycho, Longomontanus, Scheiner and some others assumed too large a diameter for them, Galileo and Hortensius themselves nevertheless assumed too small a one. If, however, we assume the diameter that is to be chosen from our more certain observations, together with the Annual Orb asserted by the Copernicans, and with the distance of the Fixed stars either as asserted by the same men, as we did in the 1st table, or as it must be asserted if they wish to avoid a parallax of 10″ arising from the whole diameter of the Annual Orb, as we did in the 3rd Table — taking these things together, I say, Sirius, which is a Fixed star of the first magnitude, and Alcor, which is a star of the sixth and lowest magnitude (of those visible, that is, without the Telescope), turn out too large with respect to the annual Orb, as is plain from the 2nd Table; indeed both turn out larger than the annual Orb, Sirius at least 44 times, and Alcor 4 times. Therefore, if the matter is handled honestly, the Copernicans cannot evade the Major.

[Margin: 2. Reply, and its threefold reason, but insufficient.]

Secondly, the Copernicans reply by denying the Minor: first, in view of the Divine Magnificence, which grows the greater from the vastness of these stars; next, by comparing other bodies of the World with one another, which all Cosmographers must nevertheless admit. For it must be admitted that the Sun, to say nothing of the rest, although it looks a foot across, is nevertheless greater than the terrestrial globe not only 140 or 160 times but 38 600 times, if the distance of it that we elicited from the lunar dichotomies is true — indeed 262 144 times, if the distance of the Sun asserted by Wendelen were true, as I showed in Book 3, chapter 11, problem 1. Finally, by comparing the Copernican

bulk of the Fixed stars with their Ptolemaic speed: for the former, they say, is much more credible than the latter.

But none of these reasons satisfies. Not the *first*: for if God had wished His Magnificence to be recognized by mortals in this life from the vastness of the Fixed stars, there was no reason for Him to remove them from our eyes by so great an interval that they should appear so tiny to us, and should scarcely equal with their apparent diameter a hundredth part of the Sun's apparent diameter; or at least He would have allowed us to come, by some other sensible sign, to certain knowledge of this distance and magnitude. That He has by no means done so is plain from what has been said so far, since all the Astronomical phenomena are saved without the Copernican hypothesis, while physical experiments with heavy bodies and the impacts of projectiles plainly convict that hypothesis of falsity. Nor is the *second* reason valid. That the Sun should be of so great a bulk is not incredible, if its functions in the Universe are considered: for the Sun is the source of the Light of all the Planets, and the universal secondary cause of all natural sublunary effects. It is no wonder, then, if it is so much greater than the Earth. But it seems wonderful and hardly credible that any little fixed star whatever should be greater not only than the Earth and the Sun — and that far more so than the Sun is — but greater by so many times even than the whole heaven of the Sun itself, when no reason appears for so great a bulk. For if the reason had been that it should act on these lower things more abundantly and powerfully than the Sun, there was no reason why it should be removed so far from the Planetary system. And it is established from Sacred Scripture, namely from Deuteronomy, that all the stars were created for the service of men, and that on this ground too their worship is forbidden, since they serve us and do not rule us. Finally, how weak the *third* reason is — about the greater likelihood of a huge bulk of the Fixed stars without motion than of a moderate magnitude with remarkable speed — will become plain from what is about to be said.

Diſcutitur An credibilior ſit Moles Fixarum in Copernicæa hypotheſi, quàm earumdem Celeritas in Ptolemaica hypotheſi.

It is discussed whether the Bulk of the Fixed stars in the Copernican hypothesis is more credible than their Speed in the Ptolemaic hypothesis.

VIII. That the speed of the Fixed stars is not incredible in the Ptolemaic hypothesis, nor indeed in our own, we showed in chapter 6. There, from number 1 to 7, we enumerated the opinions and calculations of the Copernicans against this speed, and in numbers 7 and 8 we refuted their twofold argument, full of equivocation and sophistical fallacy. We therefore refer the Reader there, certain that, if he is willing to judge without prejudice, he will pronounce against the Copernicans on this point. And to gather in summary the chief reasons adduced there: I said, and I say, that the speed of the Fixed stars does

not increase beyond the measure of its subject, as Galileo, following Kepler, pretends; for it increases only as much as their distance does. For it is certain that in any wheel or movable sphere the parts of the circumference move the faster with the common motion (for it is in this that the comparison must be made) the farther they are from the centre. Next, the speed of the Fixed stars seems excessive because we express it in miles of our own paces, just as a Giant's speed would seem excessive if someone expressed a single one of his paces cut up into thousands of an ant's paces.

[Margin: 1. Reason against the Copernicans.]

Comparing, moreover, the Ptolemaic speed of the Fixed stars with their Copernican Magnitude: *first*, I say that this comparison does not tell against those semi-Copernicans who attribute only the diurnal motion to the Earth, and not to the Fixed stars, as Longomontanus, Origanus [David Origanus], Argoli, etc. do; for to them the Copernican magnitude of the Fixed stars seemed absurd. Nor can its absurdity be removed by turning the diurnal rapidity of the Fixed stars back upon them, since they transfer it from the Fixed stars to the Earth.

[Margin: 2. Reason against the Copernicans.]

Secondly, I said, and I say, that the speed of the Fixed stars arises from the magnitude of the circumference of the eighth sphere, as they call it — compared by the Copernicans with the circumference of the terrestrial globe — and from its having to turn round in the time of 24 hours; whereas the immensity of the Fixed stars arises from the solidity or corpulence of the Fixed stars, which, if the comparison is to be on equal terms, must certainly be compared with the globe of the Earth. But the excess of the corpulence of a single Fixed star over the globe of the Earth, in the Copernican hypothesis, is far greater than the excess of the circumference of the eighth sphere, turning round in 24 hours in the Ptolemaic and in our own hypothesis, over the circumference of the terrestrial globe. For since circumferences are to one another as their diameters, by Theorem 5 of Book 11 of the *Mathematical Collections* of Pappus of Alexandria, the diameter of the eighth sphere contains the diameter of the Earth only 20 220 times if you follow Ptolemy, or 40 440 times if Alfraganus, or 100 000 or 210 000 times if one of our own opinions, according to what was said in Book 6, chapter 7, number 6, and in the 1st Table. But not only Sirius, the greatest of the stars, but even Alcor, the smallest, turns out in the Copernican hypothesis to be greater than the Earth by far more times, as will at once be plain to anyone who runs through tables 1 and 3 shown in this chapter, in the columns that give how many times these stars contain the Earth. If the Copernicans wish to draw us back to the comparison of a Fixed star with the annual orb, so that its excess may appear smaller, we shall not refuse, provided that they too (as is fair and just) compare the circumference of the Fixed stars, taken from the Ptolemaic hypothesis or from ours, with the circumference of the annual orb, Ptolemaic or ours, and from both estimate the speed of the diurnal motion. Now Sirius must contain the annual orb at least 44 times, as is

plain from the 4th table of this chapter and its most solid foundations; whereas the Ptolemaic circumference of the Fixed stars is to the circumference of the annual orb as the diameter 20 220 to the diameter 1210, that is, as 16 86/121 to 1. And our circumference of the Fixed stars is to the annual orb as the diameter 100 000 to 7300, that is, as 13 51/73 to 1; or as 210 000 to 7300, that is, as 28 56/73 to 1. The excess of Sirius over the Earth — and indeed over the Copernican Annual Orb itself — is therefore much greater than the excess of the diurnal speed and of the circumference of the sphere of the Fixed stars, Ptolemaic or ours, over the diurnal speed and the circumference of the terrestrial globe or of the Annual Orb.

[Margin: 3. Reason against the Copernicans.]

Thirdly, as I said in the same place, other things being equal, a moderate magnitude of a body in the World, commensurate with its other parts, together with some swift motion, is more credible than an immoderate one that is inert and without motion. And God Himself, in the sacred writings, commends Himself by the speed of the Sun — which, He revealed to us through the Psalmist, exulted like a giant to run the way of its diurnal motion — rather than by the magnitude of the bulk of the fixed stars, of which He nowhere made mention, but rather of their innumerable multitude; and

among the stars He commended the Sun alone for its magnitude, calling it a *Giant* in Psalm 18, and *"an admirable vessel, the work of the most High"* in Ecclesiasticus — the Sun which, if the Copernicans were listened to, would turn out a pygmy, a contemptible little vessel beside the Fixed stars. Let the Conclusion, therefore, stand.

CONCLVSIO.

Conclusion.

The Diurnal Speed of the Fixed stars in the Hypothesis of the resting Earth is more credible than their insane Bulk in the Hypothesis of the Earth going round through the Annual Orb.

CAPVT XXXI.

Chapter 31.

Proponuntur Argumenta 6. Contra motum Terræ Annuum, ex Refractione Fixarum.

Six Arguments are proposed against the Annual motion of the Earth, from the Refraction of the Fixed stars.

I. From Parallax we pass conveniently to the Refraction of the Fixed stars, from which, it seems, the following arguments can be drawn against the excessive distance of the Fixed stars that follows from the annual motion of the Earth.

I. Argumentum à Deſtructione ſenſibilis Refractionis Fixarum in Aere noſtro.

Argument I, from the Destruction of the sensible Refraction of the Fixed stars in our Air.

If the Earth revolved in the annual orb, there would be no sensible Refractions of the Fixed stars in the air that surrounds the earth more closely. But the Consequent is false; therefore so is that from which it follows.

[Margin: Proof of the Major.]

The MAJOR is proved. If all sensible inclination of the ray above the surface of our air were taken away, all sensible refraction would be taken away, since refraction, by the common opinion of the writers on Dioptrics (to be reported in Book 10, section 6, chapter 2), is always less than the inclination of the ray in the same transparent medium. But if the Earth revolved in the annual orb, it would be necessary, in order to avoid a sensible parallax of the Fixed stars, to remove them so far from us that the whole diameter of the annual orb — let alone the orb of the Earth clothed with its air — would be insensible with respect to the distance of the Fixed stars; that is, it would be like a point escaping every visual angle, or incapable of grounding a sensible angle of inclination.

[Margin: Proof of the Minor.]

[Margin: Sign of the Refraction of the Fixed stars.]

The MINOR is plain from the observations of Tycho and of others, and from our own. In them, when the true altitude of the Fixed stars, known beforehand by calculation, is compared with the altitude seen or observed, especially near the horizon, the latter is found greater than the true — which can come about from no other source than Refraction, whose property it is to raise the stars in appearance. And *Kepler* himself, a most zealous follower of the Copernicans, in the *Epitome of Copernican Astronomy*, Book 1, page 61, adduces this from a frequent observation of Tycho's to confirm the Refractions of the Fixed stars: the star of the Lion's Tail and Spica, when they are near the Meridian, were found by instruments to be 35° and nearly 2′ apart; but when, in the eastern quarter, the Lion's Tail has reached an altitude of 34° 30′, Spica, in very nearly the same vertical, is already beginning to appear on the physical horizon — which could not happen unless a refraction of about 30′ diminished Spica's distance from the Lion's Tail and raised it above both horizons.

[Margin: Reply.]

The reply, however, is to deny the Major; and as to its proof, the rest being granted, it is denied that from the smallness of the orb of the earth, taken together with the refractive air and compared with the orb and the distance of the Fixed stars, it follows that there is no angle of inclination of the rays, or that the angle is insensible with respect to an eye on the surface of the Earth, or to an angle standing at the surface of the air — granted that the whole sphere of the elements would appear from the Fixed stars as a point, indeed would not even appear at all. For we measure this angle and its size from the given

refraction, the height of the refractive air and the semidiameter of the earth, as we shall teach in Book 10, section 6, problem 45. But see here the reply to the 3rd argument.

II. Argumentum ab Exceſſu Refractionis ſupra Inclinationem.

Argument II, from the Excess of Refraction over Inclination.

[Margin: Form of the 2nd Argument.]

II. *If the Earth moved in the Annual Orb, the Refraction of the ray of the Fixed stars would turn out greater than its Inclination above the surface of the refractive air. But this is inadmissible; therefore, etc.*

[Margin: Proof of the Major and of the Minor.]

The MAJOR is urged thus: because of the huge distance of the Fixed stars, the inclination would turn out insensible, or much less than 30′; and yet the horizontal Refraction of the Fixed stars, according to Tycho, volume 1 of the *Progymnasmata*, page 280, is 30′. The MINOR is proved from the laws of the Opticians in Dioptrics: refraction is always less than inclination, since the Inclination itself, or the angle equal to the inclination, is composed of the angle of Refraction and the refracted angle, as we shall say in Book 10, section 6, chapter 2, axiom 8.

The reply is to deny the Major, and as to its proof, the consequence is denied. For the horizontal refraction of the Fixed stars would still be much less than the inclination of the ray, as will presently be established from the reply to the 3rd argument, however great a distance of the Fixed stars the Copernicans might imagine.

III. Argumentum ab Exceſſu Refractionis Fixarum ſupra dimidium Inclinationis.

Argument III, from the Excess of the Refraction of the Fixed stars over half the Inclination.

III. *If the Earth moved in the Annual Orb, the Refraction of the Fixed stars would be greater than half the angle of Inclination of their ray. But this is inadmissible; therefore, etc.* The MAJOR is proved by the same reasoning by which the Major of the 1st and 2nd Argument was proved. The Minor is proved from the laws of the Opticians: for the refractions in glass themselves, which are greater than refractions in air, are very nearly a third part of the inclination up to the thirtieth degree of Inclination, as we shall teach in Book 10, section 6, chapter 3, proposition 3. But the sensible refractions of the Fixed stars cease before the thirtieth degree of inclination is reached; therefore, before that degree, they must always be less than half the inclination.

[Margin: Reply.]

I reply by denying the Major, since the horizontal refractions of the Fixed stars themselves, which are the greatest, can never equal half the angle of inclination, however great a distance of the Fixed stars the Copernicans may

posit. It will be worth the trouble to demonstrate this here; for I learned from a letter he sent me that a man otherwise most expert in Optics, and a very close friend of mine, had counted on a great argument against the Copernicans in this matter — but in vain.

[Margin: Explanation of the figure.]

From the centre of the Earth, A, let three arcs of vertical circles, each more than a quadrant, be described: one, BCD, in the Heaven of the Fixed stars; another, EFG, of the refractive air; and the third, HIK, of the terrestrial surface. Let the true horizon be AL and the physical horizon HC, the vertical line AB, and the eye at H. Now let a Fixed star be at

Diagram. A woodcut set within the right column. From A, at the lower left, a vertical line rises through H, E and B to a letter at the top. Three concentric arcs are drawn about A: the outermost, the heaven of the fixed stars, through B, P and C down to D at the foot; a middle arc, the refractive air, through E and F down to G; and an innermost arc, the Earth's surface, through H and I down to K. A horizontal line runs from H through F and C to O; a line from A runs to N, with L on it near the outer arc; a line runs from A to D. From F one line rises to P and another runs up to R; further lines run down to the right from F toward D and the lower-right corner.

- Z (top of the vertical): unexplained on this page
- B (vertical, on the outer arc)
- P (outer arc, upper right): unexplained on this page
- R (upper left, end of the line from F)
- [?] (left edge, above E; a small letter, Y or V): unexplained on this page
- E (vertical, on the middle arc)
- H (vertical, on the inner arc; the eye)
- F (middle arc; point of incidence)
- C (outer arc, on the horizontal from H)
- O (right, end of the horizontal through C): unexplained on this page
- I (inner arc)
- L (on the line from A to N)
- N (right, end of the line from A): unexplained on this page
- A (lower left; centre of the Earth)
- K, G (foot of the inner and middle arcs)
- D (foot of the outer arc; the star)
- [?] (lower right corner; a round letter with a tail, Q or O): unexplained on this page

D, distant from the centre A by the straight line AD, and sending its ray DF to the point F of the air, so that it enters the air and does not go straight on to R, but, from the point of incidence F, is refracted to the eye H along the horizontal line HF, and so appears to the eye H as if it were at C. And let there be drawn from A,

through F, the perpendicular AP. Then the angle of refraction will be HFR; the refracted angle HFA, equal to the angle PFC by the 15th of the first book of

the *Elements*; and the angle of inclination PFD, by which, namely, the incident ray DF declines from the perpendicular PF. By the definitions of the Opticians this angle is equal to the refracted angle PFC, that is AFH, and the angle of refraction CFD, that is HFR, taken together; and it is always, at least in air, more than double the angle of refraction.

[Margin: 1st step: for the Inclination.]

IV. With these things laid down, the inclination PFD is to be sought. This will easily be done, given, from Book 2, chapter 7, the semidiameter of the earth AH, 4139 Bolognese miles, and AF, made up of AI, which equals AH itself, and of the height of the refractive air IF, which I shall show in Book 10, section 6, problem 51, to be 20 miles in some places; so the whole AF is at times 4159 miles. Therefore in the triangle AHF, right-angled at H (for the horizon HC makes right angles with the vertical AB), the angle AFH is found by the rules of triangles to be 84° 22′, and PFC is just as great. If you add to it the horizontal refraction of the Fixed stars according to Tycho, RFH, that is CFD, 30′, the angle of Inclination PFD will come out 84° 52′.

[Margin: 2nd step: for the Parallax.]

Let us now suppose, first, that the distance of the Fixed stars AD is as great as Tycho assumed, namely 14,000 terrestrial semidiameters, that is, 5 794 600 Bolognese miles — the 4139 of AH, that is, being multiplied by 14,000. For in the triangle AFD, given AD, 5 794 600 of the parts of which AF is 4159, and the angle AFD, which is the complement of the inclination PFD already found and is therefore 95° 8′, it follows that the parallactic angle FDA is 15″ — a parallax not of the orb of the earth taken bare, but of that orb increased by the thickness of the refractive air.

[Margin: 3rd step: for the decrease of the Inclination.]

V. Now let the Heaven of the Fixed stars be not the Tychonic BCD but the Copernican ZOQ, which because of its huge distance is just as if it had been described from the centre of the annual orb. The star that was supposed at D is then at Q, and the one that appeared at C appears at O, through the refracted horizontal ray HF; the incident ray at F is QF, and the inclination is PFQ. The task now is to find how much smaller the inclination PFQ can become than PFD when the distance AD is increased and carried out as far as Q, as far as the Copernicans wish — that is, how great the angle DFQ is, which is the difference of the inclinations already mentioned. But I say briefly that even if AD is increased to infinity, the star Q remaining at the same true distance from the vertex Z, that is, the angle ZAQ remaining the same, the angle DFQ can never become greater than, nor even equal to, the parallactic angle ADF set down before. For by the 16th and 32nd of the first book of Euclid's *Elements*, the exterior angle ADF is greater than either of the interior angles DFQ and DQF, being equal to both of them taken together. Therefore the inclination PFD, found in number 4 to be 84° 52′, cannot become less than 84° 51′ 45″ even if you increase the distance AQ to infinity, since no more than 15″ can be taken from it. And I say the same of any other parallax ADF, which, it is

certain, cannot be imagined in the Fixed stars to be of one degree, nor even of 4′, for it would exceed the parallax of the Sun — just as it is not permitted to imagine the height of the refractive air greater than 100 Italian miles, as we shall teach in Book 10, section 6, problem 51. Thus, everything computed, the Inclination of the ray of the Fixed stars above the refractive air is greater than 84°, and it can never decrease so far as not always to be greater than double the horizontal refraction of the Fixed stars, if this is put one degree less. Nor, conversely, can the Refraction of the Fixed stars ever reach half this inclination, that is, 42°: for even if the air became as dense as water, the refraction from air to water corresponding to an inclination of 84° would not exceed 35°, as is plain from *Vitellio* [Witelo], Book 10, proposition 8, and from the anaclastic tables to be placed at the end of Book 10.

But someone will say that, as the distance of the Fixed stars increases, the angle OFQ would diminish, so that at last it would perish entirely, together with the refraction. This argument will indeed seem more plausible, since it is easier, when the star's distance is freely increased, for the inclination of the ray to decrease until it coincides with the horizontal line OF than for it to reach the perpendicular PF itself. The reply, however, is to deny the consequence, because, from what has been said, that angle could not decrease beyond 15 Seconds.

IV. Argumentum à Diſtantia Fixarum, ex Refractione Horizontali earumdem deducta.

Argument IV, from the Distance of the Fixed stars, deduced from their Horizontal Refraction.

[Margin: Form of the 4th Argument.]

VI. *If the horizontal Refraction of the Fixed stars is not less than 30′, it follows that their distance is much smaller than the hypothesis of the Copernicans, who revolve the Earth in the annual orb, requires. But it is not less than 30′; therefore the Earth is not revolved in the annual orb.* The MINOR is certain from the observations of Tycho, volume 1 of the *Progymnasmata*, page 280; indeed, from Kepler himself, who, although a Copernican, nevertheless admits a horizontal refraction of the Fixed stars of 30′ in the *Rudolphine Tables*, in the last precept and in the table of refractions that he places there on the last page of the tables; and so do most of the more recent Astronomers.

[Margin: 1st proof of the Major.]

The MAJOR is proved, first, geometrically, in the figure set out in number 3, by considering the triangles AHF and AFD. For AH, the semidiameter of the Earth, is 4139 Bolognese miles, from Book 2, chapter 7; and AF cannot be greater than 4239 miles nor less than 4141 miles, because the height of the refractive air IF cannot be imagined greater than 100 miles or less than 2 — indeed, from what is to be said in Book 10, section 6, problem 51, it is about 20, so that the whole AF is 4159 miles. From the side AH and the base AF, by

the laws of triangles, the angle AFH becomes manifest; and equal to it are both the vertically opposite angle PFC, by the 15th of the first book of *Euclid*, and the alternate angle FAL between the two parallel horizons, by the 29th of the same book. Now if you add the refraction of 30′, that is, the angle HFR, to which CFD is equal by the 15th of the first book of *Euclid*, to the angle PFC, the inclination PFD will be known; and by the 13th of the first book of *Euclid* the angle AFD will be manifest, as its supplement to two right angles. Finally, the angle of depression of the Fixed star that has undergone the horizontal refraction, that is LAD, is known: it is as great as the horizontal refraction, if Tycho, in gathering that refraction, neglected the parallax of the Fixed stars, deeming it nil; or, if he did compute it, LAD is a little less than 30′, yet greater than 29′. But if it is greater than 29′ and less than 30′, and is added to the angle FAL, already known as I said, the angle FAD will be known in the triangle AFD, in which the angle AFD and the side AF are manifest. Therefore, by the laws of triangles, the side AD will be known, that is, the distance of the Fixed stars from the Earth; and if LAD is greater than 29′ and less than 30′, or even exactly 30′, this distance will be much smaller than the Copernican hypothesis requires.

[Margin: 2nd proof.]

VII. It is proved, secondly, as follows. As the difference of the horizontal refractions of the Sun and the Moon is to the difference of their distances from the earth, so is the difference of the horizontal refractions of the Fixed stars and of either Luminary to the difference between the distance of the Fixed stars and the distance of either the Moon or the Sun. Now the horizontal refraction, according to *Tycho*, volume 1 of the *Progymnasmata*, pages 79, 124 and 280, and to *Kepler* at the end of the *Rudolphine Tables*, is 33′ for the Moon, 34′ for the Sun and 30′ for the Fixed stars. But because of the excessive parallax of the Sun, as *Kepler* notes in the last precept of the *Rudolphine Tables*, the solar refraction came out greater than the lunar, whereas, because of the smaller Inclination that follows from the greater distance, it ought to have been smaller, and at most 32′. The distance of the Sun to that of the Moon, moreover, is for Copernicus as 1 140 to about 60, or twentyfold, for Kepler sixtyfold, and for us as 140 to 1, from what was said in Book 3, chapter 7, and Book 4, chapter 14. Therefore, if the difference of two minutes between the refractions of the Sun and the Moon does not presuppose an excess of the solar distance over the lunar greater than twentyfold, or sixtyfold, or a hundred-and-fortyfold, then the difference of two minutes between the refraction of the Fixed stars and that of the Sun does not presuppose a greater excess of the distance of the Fixed stars from the earth over the solar distance than either twentyfold, or sixtyfold, or a hundred-and-fortyfold. But the distance of the Sun, from what was said in Book 3, chapter 7, is not greater than 1180 terrestrial semidiameters for *Copernicus*, 3469 for *Kepler*, and 7600 for us. Therefore the distance of the Fixed stars cannot be greater than 23 600 terrestrial semidiameters for Copernicus, 208 140 for Kepler, and 1 064 000 for us, as is plain to anyone who multiplies 1180 by 20, or 3469 by

60, or 7600 by 140. But these distances are much smaller than the Copernican hypothesis requires, since, from what was said in Book 6, chapter 7, number 17, no one among the Copernicans requires a distance of the Fixed stars smaller than 47 000 000 semidiameters of the earth.

[Margin: [?] Reply to the [?] proof of the Major.]

VIII. The reply, however, is to deny the Major. As to its first proof, the rest being granted, it is denied that from the angles already mentioned and the side AF it follows that the distance of the Fixed stars is smaller than the Copernican one. For if the angle LAD were precisely 30′, as great as the refraction CFD, then FD and AD would be parallel, by the 28th of the first book of Euclid, and so the triangle AFD would perish, and the distance AD could be infinite; or at any rate, if this triangle were retained, there would be more than two right angles in it, contrary to the 32nd of the first book of Euclid. This is easily shown. For let the angle PFC, and therefore FAL, be 84° 22′, from the calculation of number 4. When the refraction of 30′ is added to each, both PFD and FAD will come out 84° 52′; and by the 13th of the first book of Euclid, AFD will be 95° 8′, as the supplement to two right angles. Therefore AFD, 95° 8′, and FAD, 84° 52′, taken together, will make 180°, that is, two right angles; so the angle ADF, however small it is imagined, will together with them make more than two right angles. But if LAD is put less than 30′ and greater than 29′, the parallactic angle ADF will indeed not be greater than one minute; yet, with everything else laid down in the proof of the Major left intact, it can be of so few Seconds that the distance AD comes out infinitely greater than the Copernicans require, as is plain from what was said in Book 6, chapter 7, number 17. Therefore, unless a Parallax of the Fixed stars is posited distinct from the simple Refraction — the one raising the star from D to L — the integral Refraction cannot be known, which raises it from D to C, overcoming the horizontal parallax, which is as great as the arc CL. But the parallax of the Fixed stars itself cannot be known unless their distance is first supposed. Besides, the parallactic angle ADF is not taken from the semidiameter of the earth AH or AI, but from the semidiameter AF; and the horizontal Refraction makes the star standing at D appear higher by as much as the angle CFD, which is somewhat greater than the arc CD, since that arc is described not from F but from A. So the Refraction, strictly taken, is greater than the arc CD. But let this be said for good measure.

[Margin: [?] Reply to the 2nd proof of the Major.]

IX. To the 2nd Proof of the Major, the Major is denied — that is, that proportion of distances and refractions, which is framed in too topical a manner and cannot be proved either from the common principles of Optics or Dioptrics or from the proper principles of Anaclastics. Rather, the decrease of the Refraction follows the decrease of the Inclination, which is unknown as long as the integral Refraction, including the correction for parallax, is not known; and the parallax of the Fixed stars is unknown as long as their distance is unknown.

V. Argumentum à Deſtructione Refractionum Fixarum in lentibus Teleſcopij.

Argument V, from the Destruction of the Refractions of the Fixed stars in the lenses of the Telescope.

[Margin: Form of the 5th Argument.]

X. *If the Earth were moved in the Annual Orb, the Fixed stars would not be represented larger, by the force of refraction, through the lenses of the Telescope than without them; but they are so represented; therefore etc.* The Minor is plain. The MAJOR is proved, because that representation requires a sensible refraction in the glass lenses, made towards the perpendicular, as is known from Dioptrics. But if the Earth revolved in the annual orb, there would, because of the huge distance of the Fixed stars, be no sensible inclination of the ray of the Fixed stars upon the glass lenses, and therefore no sensible refraction either.

[Margin: Proof of the Major.]

[Margin: Reply to Argument 5.]

I reply by denying the Major, for the reason stated once already in the reply to the proof brought for the major of argument 1.

VI. Argumentum à Multiplicatione, vel incertitudine diſtantiæ Veræ à vertice, & Parallaxium.

Argument VI, from the Multiplication, or the uncertainty, of the True distance from the vertex, and of the Parallaxes.

[Margin: Form of the 6th Argument.]

XI. *If the Earth moves in the Annual Orb, either we are uncertain what the true distance of a star from the vertex is, or at any rate a twofold distance and a twofold parallax are multiplied without necessity. But this is absurd; therefore etc.* The Minor is plain. The MAJOR is proved, because if we are not certain whether the earth moves or not, we do not know from where we ought to estimate and measure the true distance of a Fixed star from the vertex — whether from the centre of the Earth or from the centre of the Annual Orb. And if from the centre of the annual orb, then both this distance and the parallax of the annual orb, over and above the parallax of the orb of the earth, are multiplied without necessity, since without them all the Phenomena are saved in the hypothesis of the moved earth.

[Margin: Reply to Argument 5.]

The reply is to deny both the first part of the Major, concerning uncertainty, and the second, concerning a multiplication made without necessity. For by the very fact that the Earth is supposed to move in the annual orb, it is certain that the true distance of a Fixed star from the vertex is to be estimated principally from the centre of the Annual Orb, and the apparent distance from the surface of the Earth — granted that, secondarily, a true distance can be estimated from the centre of the orb of the earth, provided that the one is not confused with

the other but they are kept distinct. As for the necessity: although it is not a posteriori, from the phenomena, the Copernicans will nevertheless say that it is a priori, from the most fitting reasons, so that other entities are not multiplied without necessity; and so they will evade the argument.

CAPVT XXXII.

Chapter XXXII.

Proponitur Argumentum contra Hypotheſim Copernicanam, à nimia licentia fingendi diuerſa ſyſtemata.

An Argument is proposed against the Copernican Hypothesis, from the excessive licence of devising different systems.

[Margin: Form of the Argument.]

I. *If, abandoning sense and following certain fitting reasons, one may devise a system such as Aristarchus and Copernicus devised, there will be no certain System in Astronomy; rather, anyone ingenious and imbued with Astronomical learning will be free to devise several different systems. But this seems inadmissible. Therefore etc.* The MAJOR is proved. For someone could place the Moon at the centre of the universe, as Signor Baliani once thought, from what was said in chapter 15; or any one of the Planets, if not all of them together; or he could move the Sun up and down in a librating circle around the Earth — the Earth set at the centre of the World but turned round by the diurnal motion alone — as Father Francesco Maria Grimaldi once thought, for the sake of exercise, but not with any intention of asserting it; and so on with similar things.

[Margin: Reply.]

The reply is to deny the Major's consequence. For it does not follow, from one system determined by Aristarchus or Copernicus, that others equally good can be devised and so render Astronomy uncertain; it follows only that one must either stand by the Copernican system or devise another of equal perfection — which the Copernicans [?] will deny can be done.

CAPVT XXXIII.

Chapter XXXIII.

Rediguntur in Epitomen Argumenta & Solutiones Argumentorum, hactenus facta pro Motu Diurno & Annuo Terræ.

The Arguments, and the Solutions of the Arguments, made so far for the Diurnal and Annual Motion of the Earth, are reduced to an Epitome.

I would not wish anyone, whether from weariness of the labour or through inexperience in Geometrical demonstrations, to fly to the summary in this chapter in such a way as to think that he can draw from it a knowledge

sufficient for passing a certain judgement on these hypotheses among those versed in these matters, without the danger of discoursing on them with little consistency and firmness, or at least with little learning, in ignorance of the Authors who supplied these Arguments. For no one will avoid a danger of this kind except one who has read through the preceding chapters of this section. For a knowledge of things neither small nor few, and indeed often remote from the senses of the common people, must first be possessed by anyone who, on a controversy that is the most celebrated among Astronomical controversies in this age and has been debated among the most learned men on both sides,

wishes to pass a decisive — indeed, as they call it, a peremptory — sentence, and to uphold it once passed. Nevertheless I could not but gather the chief heads of the arguments and replies into a single view, so that I might pluck [?] the conclusions to be set down below from nearer branches, whose long roots, driven higher and deeper, can be seen in what has gone before. Meanwhile let no one be surprised if, for the sake of brevity, we do not bring the Arguments in syllogistic form, and indeed sometimes vary the order of the propositions.

Argumenta 20. Pro Motu Diurno Terræ ac Solutiones eorum; quæ tradita ſunt à Capite 5. ad 7. incluſiuè

The 20 Arguments for the Diurnal Motion of the Earth, and their Solutions, which were given from Chapter 5 to 7 inclusive.

[Margin: See chapter 5, from number 1 to 8.]

I. *The First Argument.* The diurnal motion should be attributed to the Earth, which is certainly spherical and therefore suited to rotation about its own centre, rather than to the heaven of the Fixed stars, of whose roundness, and therefore of whose aptitude for this rotation, we are not certain.

[Margin: Threefold reply.]

The reply was to grant the argument, *first*, if the appearance of the diurnal motion must necessarily be saved by a rotation about a centre, really distinct from the other motions that appear in the heaven; *secondly*, if all else is equal; *thirdly*, if a figure suited to this rotation is required, or if the roundness of the earth is ordained of itself for this rotation. Otherwise, however, it was to deny it: for the appearances of all the celestial motions can be represented by a single real motion of the Fixed stars and a single one in each of the Planets; there are many other reasons against the diurnal motion of the Earth which outweigh this slight little reason taken from sphericity, so that all else is not equal; and the roundness of the Earth is ordained of itself for other ends than for motion.

[Margin: See chapter 5, from number 8 to 11.]

II. *The Second Argument.* If the whole heaven is moved westward by a diurnal rotation, all the elements must be moved by it in the same direction with the rapt motion [motus raptus], and so must the earth, since it hangs in the middle of the air and is not at all hindered from this carrying-along either by any resistance of the air, which is none, or by the force of its own heaviness, since

this does not hinder circular motion but straight motion upward. But if the Earth is moved by a diurnal rotation, the heaven need not be moved by it, and so the multiplication of motions is spared. Therefore the diurnal motion should be ascribed to the Earth rather than to the heaven.

[Margin: The threefold reason of the reply.]

The reply was to deny the consequence of the Major: first, because if the Earth were moved with the heaven by the rapt motion and the rotation of 24 hours, very many celestial Phenomena, which are discussed there, would be lost to us; second, because that rapt motion supposes the heaven solid as far as the Moon, whereas the fluidity of the Planetary heaven is more probable; and finally, because not even the wiser Peripatetics grant that the rapt motion reaches as far as the middle region of the Air, let alone the lowest; and even if it did reach it, it would still be powerless, because of its fluidity and rarity, to impress motion through the whole depth of the Earth, a body most gross and most dense. It was further denied to Galileo that the Earth is of itself as indifferent to motion as to stability.

[Margin: See chapter 5, from number 11 to 16.]

III. *The Third Argument.* Circular motion is more natural to the elements than straight motion, since it is uniform and finite, and belongs to them when they are already ordered and set in their own place; whereas straight motion — uniformly difform in the ascent of light bodies and the descent of heavy ones, and infinite — belongs only to things disordered and, as it were, sick, or faring ill outside their own place. Therefore, if Aristotle attributed circular motion to Fire and to the uppermost region of the air — from without, and violent or preternatural — much more ought he to have granted circular motion to them and to the other elements, and so to the Earth.

[Margin: 1st reply.]

The reply was, 1st, to deny the Major. For no circular motion belongs to the elements from within, but at most a rapt motion from without, which therefore ought not to be called absolutely natural. Indeed, no motion at all belongs absolutely to them when they are well ordered and set in their own place; to disordered ones, however, belongs the motion that takes place along the shortest line and as swiftly as their nature allows — such as is the motion of heavy and light bodies along a straight perpendicular line, uniformly difformly, always accelerated towards the end, and as finite as their removal from their proper place is finite. Therefore this motion, on the supposition of their removal from their place, befits them rather than any other. For these and other reasons we taught that the Peripatetic doctrine on this point is much more solid than the Copernican or the Galilean, and indeed that the latter is hardly consistent with itself: as when they say that circular motion is likened rather to the finite, and straight motion to the infinite, whereas straight motion has ends on both sides and circular motion has none, but always begins again where it seemed about to end. Or when they say that circular motion is more perfect and more congruent with the elements because it is more like rest, as being made wholly

within itself: rather, then, rest itself, being more perfect than circular motion, should be attributed to the elements; for, as in God immobility supposes less need than motion does, it is therefore to be attributed to God as a perfection simply simple. Besides, once the annual motion of translation is added to the Earth, that perfection really perishes, and the diurnal motion is no longer made wholly within itself. See many other points in their place.

[Margin: 2nd reply.]

The reply was, 2nd, to deny the Consequence regarding circular motion towards the West, such as Aristotle attributed to the element of Fire; for if he had attributed this motion to the earth too, rather than motion towards the East, as the Copernicans do, the celestial Phenomena would not be saved.

[Margin: See chapter 5, from number 16 to 18.]

IV. *The Fourth Argument.* It is absurd that the whole heaven and the sphere of the Fixed stars, which has the character of the whole and is incomparably greater than the Earth, should be moved for the sake of the Earth, which is so tiny a particle of the Universe, rather than the Earth itself. Therefore the Earth rather than the heaven is moved with the diurnal motion.

[Margin: Threefold reply.]

The reply was to deny the Antecedent: first, because the heaven, though greater in bulk, is nevertheless less in power and in perfection of substance than the Earth taken together with men and the other animals, for the moving Intelligences are not an intrinsic part of the heaven; second, because neither the heaven nor the moving Intelligences are moved for the sake of the Earth and its inhabitants as for their ultimate end, but so that they may tell the glory of God and serve the Creator — for which end it was not even absurd that Angels, and the very Son of God, should be destined to the salvation of men. Otherwise it would also be absurd for the Planets to be moved around the Earth, or for the Earth to be moved, for the sake of men, who are smaller than the Earth itself; and so the argument was turned back.

[Margin: See chapter 5, from number 18.]

V. *The Fifth Argument.* It is easier, and of less expense, to move the tiny globe of the Earth than the immense machine of the heaven; therefore God and Nature, which does what is easier, move the Earth rather than the heaven with the diurnal motion.

[Margin: Threefold reply.]

The reply was to deny both the Antecedent — because greatness of bulk, taken precisely, does not make its motion more difficult unless some other quality is added, resistive or obstructive in comparison with the motive force, and there is none such with respect to God, or to the Intelligences, in the heaven any more than on the Earth — and the Consequence, because all else is not equal, and there are other, stronger reasons for which God and Nature would not do this, which at first sight seems easier; just as in many other things too Nature has not pursued what seemed easier or of less expense.

[Margin: See chapter 5, from number 18 to 24.]

VI. *The Sixth Argument.* The diurnal motion should be attributed to the body that is certainly established to be movable in this way rather than to one of whose mobility we are not certain. But of the mobility of the Earth we are certain, because we are certain of its finiteness; whereas of the mobility of the highest heaven we are as uncertain as it is uncertain whether it is finite or infinite. For if it were infinite, it would not be movable by a diurnal revolution, or at least it is disputed among the Physicists whether it would be movable.

[Margin: Threefold reply.]

The reply was to grant the Major if all else is equal and deny it if unequal (as in truth they are unequal), and to deny the Minor. For it is physically evident to us that the Fixed stars move: the possession of this evidence lies with the senses, and the proof to the contrary is incumbent on the one who denies it; and from this we gather their mobility more immediately than from finiteness. The evidence of this disjunctive proposition, however — Either the Fixed stars move, or we with the Earth — is not Physical only, but Mathematical, or Metaphysical, which is not necessarily required in Physics; otherwise not even the finiteness of the Earth would be evident, since its measure depends on experiments that are only physically evident. Many things bearing on learning about the infinity of the World were said on that occasion.

[Margin: See chapter 5, number 24.]

VII. *The Seventh Argument.* Local motion should be attributed to the thing located rather than to the place, since a place of itself requires immobility. But the highest heaven is the place with respect to the Earth, and the Earth is the thing located; therefore motion should be attributed to the Earth rather than to the heaven.

[Margin: Twofold or threefold reply.]

The reply was to grant the Major concerning that local motion by which a place loses the character of a place, because it is carried elsewhere together with the thing located and so takes on the character of a vessel, or else deserts the thing located; but to deny the Major concerning motion that is not repugnant to the essence of a physical place — although the Minor is not true either, concerning the immediate place. The argument was moreover turned back: for neither ought the Earth to be moved together with the neighbouring air, since the Earth, together with that air, is the place of animals and plants; and much less ought it to be moved with a motion of translation, which destroys the immobility of the place — such as is the annual motion that the Copernicans add to the diurnal.

[Margin: See chapter 5, number 25.]

VIII. *The Eighth Argument.* Immobility is nobler than motion and more akin to incorruptibility, since local motion is instituted either to preserve from corruption what would otherwise wither in torpor, or else it tends towards corruption. Therefore immobility should be granted to the heaven, and motion to the Earth, as being more subject to corruption than the heaven.

[Margin: Reply.]

The reply was to deny that part of the Antecedent concerning the end of local motion. For local motion is not of itself instituted to preserve from corruption, and much less is the circular diurnal motion; nor, again, does it tend of itself towards corruption, but towards place as place. Further, neither is the heaven, as is supposed, incorruptible by its own nature and from within; nor is the diurnal revolution of the Earth together with the neighbouring air useful for preserving the Earth's surface from corruption, but rather the agitation of the air and the water by the winds, the rivers, the ebb and flow of the sea, etc.

[Margin: See chapter 5, from number 26 to 30.]

IX. *The Ninth Argument.* All the Fixed stars, from the observations of all ages compared with one another, keep perpetually among themselves the same distance that they once had. But if they were moved by a diurnal rotation, they would not keep it: for either they would be moved with the motion of a solid Firmament, and its parts would be scattered by excessive speed; or they would move of themselves in a fluid heaven, and would be scattered much more and drift with a wandering motion, both because of the rapidity of the rotation and because of the fluidity.

[Margin: Reply.]

The reply was to grant the Major and deny the Minor. For if the Firmament is solid, as is gathered from the authority of several Fathers, it is not for that reason worn away, or shaken by a reciprocal motion, nor does it strike against another hard body or grow hot; and therefore no scattering of its parts is to be feared for it. Indeed, from that very preservation of the distance of the Fixed stars among themselves, together with the supposition of their motion, its solidity and indissipable firmness is the more probably inferred. But if it is Fluid, the stars are nevertheless moved by Intelligences, so that they keep that equality of distance and are moved like *the vessel of the camps* [vas castrorum], as *Ecclesiasticus* chapter 43 speaks of these stars, and remain in their watches while they move, as *Baruch* 3 says. Finally, God commends himself in Job 38 on this ground: that the circuit of Arcturus and of the Pleiades, and the constellation, cannot be scattered. Nor indeed ought the wonder that shines forth in a host of stars advancing in so great and so perpetual an order to detract from belief in the divine works — and much less among the Copernicans, who, as will be said later, are accustomed to commend the divine Omnipotence and Magnificence from the immensity of the interval between Saturn and the Fixed stars, and from the vastness of the eighth sphere, and indeed of Fixed stars greater than the Annual Orb itself, in their own hypothesis.

[Margin: See chapter 5, numbers 31 and 32.]

X. *The Tenth Argument.* It seems absurd, indeed ridiculous, that the Fixed Stars and their sphere — so vast, needing nothing terrestrial, but wholly independent of the Earth — should be moved around so small an Earth, and for the favour and good of the Earth as the end of their motion, rather than the Earth itself, which depends on the heaven and needs its influences. Therefore the Earth rather than the heaven is moved with the diurnal motion.

[Margin: Reply.]

The reply was to deny the Antecedent. For the Earth, being full of animals, especially rational ones, is greater than the heaven as regards the magnitude of power and of nobler substance; and it does not need the heaven and the stars as masters but as servants, which, as is established from the sacred Scriptures, are moved for the good of men in such a way that the ultimate end is nevertheless the glory of God.

[Margin: See chapter 6, from number 1 to 7.]

XI. *The Eleventh Argument.* If the diurnal motion is ascribed to the Fixed stars rather than to the Earth, their speed becomes altogether incredible and far exceeds the measure of the subject. For from the Ptolemaic distance of the Fixed stars it follows that any one of the Fixed stars on the celestial Equator passes through 760 German miles in one beat of the human artery — indeed many more, if we follow the Keplerian distance. And if we follow Ours, then in each pulse of the artery each Fixed star passes through 152⅔ semidiameters of the Earth, that is, 629 128 Italian miles, or 157 282 German miles. Therefore the Earth rather than the Fixed stars is moved with the diurnal motion: for in one pulse of the human artery a point on the terrestrial Equator has to pass through no more than 376 geometric paces, from what was said in chapter 19, number 13.

[Margin: Reply.]

The reply was to deny both parts of the Antecedent. The first, because a speed does not become incredible merely because it is greater than the speed of something else, unless it exceeds the capacity or the motive power of the subject, or would offend the senses by its rapidity. None of these is found in the speed of the Fixed stars, because it is commensurate with their distance and with the magnitude of their circumference, and comes from the power of the Intelligences, and yet it is scarcely perceptible to the senses, since they scarcely seem to move within a short time. The second part, concerning the measure of the subject, is denied because, if any two circles differing in magnitude by however much are described about the same centre, and each is revolved whole in the same time, the motion of their parts, or of similar arcs, is as proportioned as those arcs are similar to each other; and the arcs of the circles already mentioned that are enclosed within the same lines drawn from the centre are similar to each other and of as many degrees. Therefore 629 128 Italian miles are as proportioned to one Second of the celestial Equator as 376 paces are to one Second of the terrestrial Equator, as the proportion of the distance from the same centre requires, with the straight lines drawn from it to each circumference serving as legs — of which the former would suit the heaven as a giant, the latter the Earth as an ant. Add that a speed which saves the evidence of the senses is more credible than one which destroys it; and the speed of the Fixed stars asserted by the Ptolemaics is much more credible than the bulk of the Fixed stars asserted by the Copernicans, as was shown in chapter 30, number 8.

[Margin: See chapter 6, from number 8 to 11.]

XII. *The Twelfth Argument.* The motion of the celestial bodies is the slower the farther they are from the centre of the universe: thus the motion of Jupiter is slower than the motion of Mars, and the motion of Saturn than the motion of Jupiter, etc. But if the diurnal motion were attributed to the Fixed stars, it would be far swifter than the motion of Saturn, although the Fixed stars are much farther from the centre of the Universe than Saturn is; and so it would exceed the measure of the subject and the proportion of the intervals. It is therefore to be attributed not to the Fixed stars but to the Earth.

[Margin: Reply.]

The reply was to grant the Major concerning the proper motion that appears towards the East, but to deny it concerning the common or diurnal motion towards the West — or, as they call it, the motion of the primum mobile. For if all the circles about the same centre are moved in the same direction in the same time, the arcs of the more remote circles must be greater, and therefore their motion swifter. But in the Minor the terms are changed, and the discussion concerns the diurnal and common motion, whereas in the Major it concerned the proper motion. And yet (who would believe it?) Galileo, Kepler and Lansbergen fell into this so shameful a fallacy and equivocation, as we showed in its place from their own words — as if they did not know what is known even to beginners in Astronomy: that the proper motion of the Fixed stars is so much slower than the motion of Saturn that, while Saturn requires barely 30 years for one revolution, the Fixed stars nevertheless require 26000 years, or indeed, according to Ptolemy, 36000. For the rest bearing on this learning, see chapter 6, from number 9 to 11.

[Margin: See chapter 6, from number 11.]

XIII. *The Thirteenth Argument.* If the Fixed stars and the other stars were moved towards the West by the diurnal motion, or the common motion of the primum mobile, while a proper motion towards the East cannot be denied them, the same movable body would be moved by two contrary motions at the same time — which is either impossible, or at least so hard to grasp that even the appearance and suspicion of it should be avoided. But it is avoided if the Earth alone, by a diurnal revolution towards the East,

is turned, while the stars individually perform their own simple motions towards the East. Therefore the diurnal motion should be ascribed to the Earth rather than to the Fixed stars and the other heavenly bodies.

[Margin: Twofold reply.]

The reply was, first, to deny the Major. For there is a single motion in the Fixed stars and a single one in Saturn (and so for the rest), towards the West, by spirals; but it is swifter in the Fixed stars than in Saturn, and swifter in Saturn than in Jupiter, etc., because each day the higher stars are revolved to the same meridian sooner than the lower ones. From this arose the illusion by which the lower stars, being left behind by the swifter ones moving towards the West, are commonly thought to recede towards the East — and so the occasion of

suspecting two contrary motions. But this contrariety is as false as it would be in any two bodies running round a circle in the same direction, one of which reaches the same goal more slowly than the other. Yet this very appearance of contrariety would not be avoided by the motion of the Earth, because, once that motion is supposed, everything would nonetheless appear just as it does now, as the Copernicans profess. Next, the argument was turned back against the contrariety of motions that would follow, on the Copernican hypothesis, in heavy bodies or in bodies projected towards the West.

[Margin: See chapter 6, numbers 14 and 15.]

XIV. *The Fourteenth Argument.* Fewer and simpler motions are posited if a single diurnal motion is attributed to the Earth, and to each planet its own motion towards the East, than if a diurnal motion is added to the Fixed stars and the Planets over and above their proper motion. Therefore etc.

[Margin: Twofold reply.]

The reply was to deny the Antecedent. For the diurnal motion of the Fixed stars and of the Planets is not in reality distinct from their proper motion, since the proper motion is not such except in appearance, on account of their falling behind and their later arrival at the meridian; and so no more than 8 motions are posited in the heaven, one in the Fixed stars and 7 in the Planets. (For there is no point here in counting the motions of the Satellites of Saturn and of Jupiter, because these too must be multiplied no less in the Copernican hypothesis than in the more common one.) But if a diurnal motion is given to the Earth, that motion is then really distinct from the proper motion of the Fixed stars (which, of course, Lansbergen, Longomontanus, Boulliau and many others among the Copernicans or Semi-Copernicans admit in the Fixed stars) and from the 7 motions of the Planets; and moreover, in the motion of heavy and light bodies — indeed in all terrestrial and aquatic mixed bodies — several motions mixed of straight and circular must be admitted. Therefore, when everything is reckoned as it really is, more motions are multiplied, and more compounded ones among terrestrial things; hence the argument was turned back against the Copernicans.

[Margin: See chapter 6, number 16.]

XV. *The Fifteenth Argument.* If the Fixed stars were moved by the diurnal motion rather than the Earth, then, not only in different times but in the same time as well, some would describe great circles — namely those on the Equator — and others the smallest circles — namely those near the poles; and so the former would be the swiftest, the latter the slowest. But this is absurd. Therefore the Earth rather than the Fixed stars is moved by the diurnal motion. I should be ashamed even to have put this argument forward, were not the whole shame to overflow onto Galileo, who urges it in earnest — and who would certainly be thrashed by mere beginners in Astronomy.

[Margin: Threefold reply.]

For the reply was, first, to deny the Minor, since this arises from geometrical necessity: namely, that of the parts of a spherical surface revolved by a single

motion of the sphere, those nearer the poles describe smaller circles and move more slowly than those farther from the poles of the sphere. Second, that comparative particle of the Major, *rather than the Earth*, was denied. For granting — without conceding — a diurnal motion of the Earth, neither would the appearance of slowness in the Fixed stars nearer the pole, and of speed in the Fixed stars near the Equator, be removed; and in the parts of the Earth itself there would really be that inequality of speed and of circles, according as they are nearer to or farther from the poles of the terrestrial revolution. Therefore, if this were absurd, the absurdity would be double, or in a double subject: one apparent, in the eighth sphere; the other real, in the Earth. Whereas it would be single, or real in a single subject, if the diurnal motion is attributed to the Fixed stars. So the argument already mentioned can accordingly be turned back *ad hominem*.

[Margin: See chapter 6, numbers 17 and 18.]

XVI. *The Sixteenth Argument.* If the Fixed stars were moved by the diurnal motion, they would perpetually vary their declination from the Equator, and therefore their speed as well, so that a star that had once been the swiftest, on the Equator, would in some age become the slowest because of its nearness to the Pole. But this is absurd. Therefore etc. So too Galileo, and in earnest — although this argument has no kernel at all.

[Margin: Threefold reply [?].]

The reply was, first, to deny the Major if the diurnal motion is considered precisely, and to concede it if an apparent proper motion is granted to the Fixed stars — that is, if that single motion towards the West is completed by spirals in latitude. *Secondly*, granting the Major, to deny the Minor: for this is no more absurd in the heaven than it would be on the Earth if it moved with the diurnal motion and besides with a motion of the axis and a libration accounting for the proper motion that appears in the Fixed stars. Indeed, it would follow more on the Earth, where animals, rivers, ships, clouds, etc. perpetually change their declination from the terrestrial Equator, more frequently and with greater variety. Accordingly the reply was, *Thirdly*, to turn the argument back. But what we indicated there about Fixed stars that will reach the pole, and about the retrogradation of some of them in right ascension, and set out in book 6, chapters 11 and 19, problems 5, 10, 11, cannot be explained in a few words in this place.

[Margin: See chapter 6, number 19.]

XVII. *The Seventeenth Argument.* The annual inequality of the natural days, which is manifest to Astronomers, has a more evident cause if the diurnal motion is attributed to the Earth together with the annual one, inasmuch as the Earth is quickened by its approach to the Sun and slowed by its recession. So *Kepler*.

[Margin: Reply.]

The reply to him was to deny the Antecedent. For if that inequality arises from the variation of the distance between the Sun and the Earth, then whether

the Earth really approaches the Sun or the Sun the Earth, we have an equal account of the inequality either way.

[Margin: See chapter 7, numbers 1, 2 and 3.]

XVIII. *The Eighteenth Argument.* It is more probable that the Comets do not have a diurnal motion in themselves, but only their own trajections, so that we do not increase the difficulties in so wandering and manifold a motion. Therefore the diurnal motion towards the West that appears in the Comets really comes from the turning of the Earth towards the East; for it cannot be attributed to a sweeping-along of the fire or the air, since the heavens of the Planets are not solid and do not sweep the fire along, etc.

[Margin: Reply.]

The reply was to deny the Antecedent. For just as the diurnal motion in the Planets is more evident than their proper motion, so it is in the Comets too; and therefore, just as we must not deny the diurnal motion to the Planets, so neither to the Comets. Otherwise we should rather be multiplying motions — namely a proper trajectory motion in the Comets and a common diurnal motion in the Earth — whereas the same Intelligence or force that moves the Comets with the diurnal motion can move them by spirals looser than in the Planets, so that by the looseness of the spirals a motion in latitude, and by the slowness of the revolution to the meridian or the horizon a motion in longitude towards the East, is represented, though by a single motion in reality.

[Margin: See chapter 7, from number 4 to 8.]

XIX. *The Nineteenth Argument.* The perpetual wind and breeze within the Tropics, which is at least a partial cause of the easier and shorter navigation towards the West than towards the East, more probably comes from the diurnal motion of the Earth towards the East, which leaves the air behind; for the air, because of its fluxibility, does not receive so great an impetus, and is not carried along by the Earth's rotation so as to follow it exactly. Therefore it is more probable that the Earth is moved by a diurnal motion than not. For the rest, to grasp the force of this argument better, see the place cited.

[Margin: Reply.]

The reply was to deny the Antecedent. For since the diurnal motion of the Sun and the other Planets towards the West is more evident to sense, the cause of that perpetual breeze should rather be attributed to the Sun, with the other Planets assisting it, carrying vapours and exhalations around or raising them successively towards the West — especially since the breeze is not so perpetual and continuous as it would have to be if it came from the Earth's rotation. On this variation consult the history of these Winds that we selected, in chapter 7, from number 4.

[Margin: See chapter 7, from number 9 to the end.]

XX. *The Twentieth Argument.* A small sphere made of the magnet-stone, which is called a terrella, if it is suspended on the poles of its own axis — the axis along which the magnetic virtue extends towards the Poles of the terrestrial Equator — revolves with a diurnal revolution towards the East. The

spagyrists also report this of the chymical or Hermetic Earth. Therefore the whole Earth too, which is a great Magnet, is turned towards the East with a diurnal revolution.

[Margin: Reply.]

The reply was to deny the Antecedent. For the experiment — which was not performed, but feigned in the imagination by Petrus Peregrinus from a conjecture that seemed probable to himself — has been neither found nor held to be true by Gilbert, nor by the Fathers of our Society Athanasius Kircher, Cabeo or Niccolò Zucchi, who have written best of all on this subject; indeed,

rather, they teach that a Magnetic virtue has been implanted in the Earth so that it keeps itself immovably in the position that GOD once established for it with respect to the Poles of the World. The terrella already mentioned is indeed moved towards the Poles under the meridian, if it is suspended from the poles of the meridian, until its axis coincides with the axis of the terrestrial Equator; but this motion is not the one we are disputing about. Yet the Earth is not a great magnet. For the rest of the learning needed here, see chapter 7, from number 9.

Argumenta 29. Pro ſitu Solis in centro Vniuerſi, & Motu Telluris Annuo circa Vniuerſi centrum; ſimul cum Diurno: ac Solutiones eorum, quæ traditæ ſunt à Capite 8. ad 18. incluſiuè.

29 Arguments for the position of the Sun at the centre of the Universe and for the annual motion of the Earth about the centre of the Universe, together with the diurnal motion; and their Solutions, as given from chapter 8 to 18 inclusive.

[Margin: See chapter 8, numbers 2 and 3.]

XXI. *The First Argument.* The centre of the World is the noblest place, and the Sun is nobler than the Earth; therefore that place is owed to the Sun rather than to the Earth.

[Margin: Reply.]

The reply was to concede the first part of the antecedent in the natural order — for in the supernatural order the nobler place is the Empyrean, and the lowest and worst is Hell, the place of the damned — but the latter part was denied. For the Earth, with its living things and animals, especially the rational ones, is nobler than the Sun.

[Margin: See chapter 8, numbers 4, 5, 6.]

XXII. *The Second Argument.* The Sun is the centre of the Planetary system, since it is now certain that Mercury and Venus move about it, from which a similar conjecture is made about the others. Therefore it must be the centre of the Universe.

[Margin: Twofold reply.]

The reply was, 1, to deny the Antecedent concerning the system of the Planets taken adequately: for the Sun is not the centre of the Moon's motion — indeed,

probably not even of Saturn's and Jupiter's, for the reasons given in their place. 2, to deny the consequence, because the centre of the Universe must be the centre of all the bodies of the World — that is, of the elements and of all the celestial bodies, and so also of the sphere of the Fixed stars and of the new Phenomena — and such a centre is the Earth, or the Earth's centre; and so the argument was turned back against the Copernicans.

[Margin: See chapter 8, numbers 7, 8, 9.]

XXIII. *The Third Argument.* The Sun is the source of the light and heat of the whole Universe. Therefore it must be placed in the middle of the Universe, so as to illuminate all things equally.

[Margin: Reply.]

The reply was to deny both the Antecedent, because the Sun is not the source of the light of the Fixed stars, and the Consequence, because it does not illuminate all the Planets equally, as is clear from the variety of their phases. It suffices, then, if it moves in such a heaven that it can illuminate the Earth and the Planets in the way it now illuminates them, according to the measure of divine Providence.

[Margin: See chapter 8, numbers 10, 11, 12.]

XXIV. *The Fourth Argument.* The Sun is the source of the motion of the Planets, since they are attuned to its motion. Therefore it must be at the centre of the World.

[Margin: Reply.]

The reply was to deny the Consequence, because the Sun is not the source of the motion of the Fixed stars, of the New Phenomena, of the Elements, etc. Indeed, it is not even the efficient cause of the motion of the Planets, as Kepler imagines, but an exemplary cause with respect to some aspect of the motion; whereas the Earth, with men, is the final and objective cause of the motion of the stars.

[Margin: See chapter 8, number 13.]

XXV. *The Fifth Argument.* It is more credible that the Planets and the Earth are moved about that which is greater than all of them, namely about the Sun, than that the Sun is moved about the Earth, which is smaller than many Planets.

[Margin: Twofold reply.]

The reply was to concede this likelihood if other things are equal, and if what is greater in bulk is also greater in power and in perfection of substance — in the way that the Earth, with men, is greater than all the Planets; but if magnitude of bulk alone is considered, the proposition is denied. Next, if the ground for rest is to be taken from bulk, then it is more credible that the Sun is moved about the elementary sphere than that this sphere, with the lunar heaven, is moved about the Sun — the Sun being, indeed, much smaller than that sphere.

[Margin: See chapter 8, number 14.]

XXVI. *The Sixth Argument.* It is the Earth that needs the Sun and the other Planets, and in it is the creature that measures out and contemplates the divine works — not in the Sun. Therefore the Earth rather ought, by an annual

translation of itself, to approach the various parts of the world in order to contemplate them, and to bring back from them the wares of their influences.

[Margin: Reply.]

The reply was that the Earth needs the stars not as masters but as servants, waiting on it by divine command and ordinance and serving man; and that for contemplating and measuring the intervals and magnitudes of the stars the motion of the Earth contributes nothing more than the motion of the Sun and the other stars about an immovable Earth, since the same things appear in either hypothesis; and that it suffices for man to go out into the open and, by observations duly made, to measure the motions, intervals and magnitudes of the stars.

[Margin: See chapter 8, number 15.]

XXVII. *The Seventh Argument.* In the Microcosm, that is, in man, the feet are moved with progressive motion, not the head. Therefore in the Macrocosm too, that is, in the World, the Earth must be moved, not the Sun.

[Margin: Reply.]

The reply was to deny the Consequence: both because the Earth is not the foot in the World — and much less so in the Copernican hypothesis, since in that hypothesis it is not the lowest — but is the immovable footstool of the feet of GOD, as the Royal psalmist says; and because, just as the head too is carried along with the motion of the feet, so with the motion of the Earth either the Sun or the sphere of the Fixed stars ought to be carried along, since one or the other would have to be called the head; but the Copernicans concede neither.

[Margin: See chapter 8, numbers 16 and 17.]

XXVIII. *The Eighth Argument.* Unless the Sun is at the centre of the World, and the Earth, with the Elements and the lunar heaven, moves between the convex of Venus and the concave of Mars, to the Sun, the greatest of the Planets, falls the least thick heaven of all, and to Venus and Mars by far the greatest; and too great a space would lie between the convex of the Sun and the concave of Mars.

[Margin: Twofold reply.]

The reply was that it is a greater absurdity that no heaven be given to the Sun, placed immovably at the centre, than that some heaven be given to it outside the centre; and to count the Earth, with the elements, among the celestial bodies. Next, if Mercury, Venus, or even Mars are supposed satellites of the Sun, all these belong to a single heaven; on this we said more in our new system, at the end of the third section.

[Margin: See chapter 8, numbers 18 and 19.]

XXIX. *The Ninth Argument.* Unless the Sun rests at the centre of the World, and the Earth moves through the Annual Orb, approaching Mars and receding from it, Mars at perigee must not infrequently be nearer to the Earth than the Sun is, and so must penetrate the heaven of the Sun, and thus the distinction of the heavens is removed. But this is unfitting. Therefore etc.

[Margin: Threefold reply.]

The reply was to deny the Minor. For if the Sun rests at the centre, it no longer has any need of a heaven; and if the Earth proceeds annually between Venus and Mars, it becomes, with its elements, one of the Planets. But it is a greater absurdity to cast the Sun out of every heaven and to raise the Earth with its elements into the heaven in its place than for the heaven of one Planet to be mingled with that of another. Next, if Mars, as we said in our system, is a satellite of the Sun together with Venus and Mercury, the same heaven serves the Sun and these three. Finally, since the heaven of the Planets is fluid, there is no greater necessity to assign the Planets a distinct heaven than to assign birds or fishes a distinct region of air or water.

[Margin: See chapter 8, from number 20 to 26.]

XXX. *The Tenth Argument.* If the Earth is moved by an annual motion about the Sun placed at the centre of the World, and the Earth's Annual Orb lies between the concave of Mars and the convex orb of Venus, an account is given of the number of the Planets, and of the intervals between the Planets, from most beautiful proportions worthy of God as their maker — namely from the commensurations of the five Regular bodies inscribed in or circumscribed about spheres. Therefore it is far more probable that the Earth is moved in this way than that it is not moved. The Antecedent is proved: for if you circumscribe a Dodecahedron about the annual orb of the Earth, the circle circumscribed about it will be the concave of the orb of Mars; if you circumscribe a Tetrahedron about the convex of the orb of Mars, the circle enclosing it will be the concave of the orb of Jupiter; and if you circumscribe a Cube, or Hexahedron, about the convex of the orb of Jupiter, the circle embracing it will be the concave of Saturn. Next, if you inscribe an Icosahedron in the concave of the Annual orb, the circle inscribed in this Icosahedron will be the convex of the orb of Venus; finally, if you inscribe an Octahedron in the concave of Venus, the circle inscribed in it will be the convex of the orb of Mercury. Kepler called this the *Mysterium Cosmographicum*, and fitted it, in imitation of the Pythagoreans and the Platonists, to the order and symmetry of the World. On this truly ingenious application see what was said in chapter 8, from number 20.

[Margin: Reply 1.]

The reply was fourfold. First, by denying the Antecedent: because that beauty in the distribution of the planetary intervals, apparent at first sight, contains many absurdities within it, and yet does not give an account of the number of the Planets. For it destroys the number of the primary Planets commonly received by the learned, since it turns the Sun from a Planet into a Fixed star; does not count the Moon, in this distribution of the bodies, among the primary Planets, but makes it the Earth's companion, while making the Earth, together with the elements, a primary Planet; and reckons Mercury and Venus among the Primary ones. Consequently, from that proportion no more than 6 Planets arise, namely Saturn, Jupiter, Mars, the Earth or sphere of the Elements, Venus and Mercury. But what is more absurd than to strike off the roll of the Planets the two great Luminaries, by whose motions, most evident to sense, the years

and months are distinguished, and to tear apart that Septenary number of the Planets — as it were the Heptachord of the World, far better known and more celebrated than the Pythagorean distribution of the bodies of the World?

[Margin: Reply 2.]

Secondly, the Antecedent was denied, because not even so is an account of the planetary intervals given adequately, since no account is given of the thickness of the heavens, especially of Saturn's and Mercury's, nor of the interval between Saturn and the Fixed stars. Indeed, of the remaining five intervals there are only two that correspond closely to the distances owed to the Copernican hypothesis and derived from observations — namely those of Mars and of Venus; the rest are far off, and cannot be brought into agreement with the proportions of the Regular bodies except by numbers violently begged, as I showed in chapter 8, number 25, reply 4.

[Margin: Reply 3.]

Thirdly, the Consequence is denied: since, if we wish to make the intervals of the Planets conform to the measures of the Regular bodies, the Phenomena are not saved, as I showed in the same place, in reply 2.

[Margin: Reply 4.]

Fourthly, the Consequence is likewise denied, because that proportion and beauty of the Regular bodies does not seem suited to the Physical effects that divine Providence willed to exist in the nature of things. For it consists in a determinate number of regular faces by which those five of the bodies are bounded; but the intervals of the Planets are not taken from bodies as bodies; nor in fluid heavens is there a real distinction of the heavens, much less of Regular bodies; nor do they exert influence on these lower things as such bodies, but by reason of light and of the influences flowing through light into the lower things. Besides, neither the Platonists nor the Pythagoreans distributed those bodies in this way; rather, they assigned the Cube to the Earth, the Tetrahedron to Fire, the Octahedron to Air, the Icosahedron to Water and the Dodecahedron to the globe of the Universe — and that not with regard to quantity and measure, as Kepler does, but speaking metaphorically and with regard to other properties: namely stability in the Cube, the mobility of Fire in the Tetrahedron or pyramid, etc., as may be seen in Plutarch, book 2 of *De placitis*, chapter 6. Moreover, Kepler himself confessed that harmonic proportions are found in the hypothesis of the immovable Earth as well, as I showed in chapter 8, number 26.

[Margin: See chapter 9, from number 1 to 5.]

XXXI. *The Eleventh Argument.* Motions, or the instruments of motions, are not to be multiplied without necessity. But if the Earth is at rest, the motions of the second Anomaly are multiplied in the five lesser Planets, and so are the instruments of these motions — namely three Epicycles in the three superior Planets and two Eccentrics in the two inferior Planets — and that without necessity, since all of them can be accomplished by the single Annual Orb of

the Earth. Therefore the Earth moves through the Annual orb rather than being at rest.

[Margin: Twofold reply.]

The reply was, 1, to deny the Minor, because the motions are not really multiplied if there is a single spiral motion in each Planet. Moreover, the multiplication of apparent motions comes from a twofold necessity. *The first* is that the motions evident to sense be saved — such as the motion of the Sun, and the harmony of the other Planets attuned to the motion of the Sun — and that in their place motions of the Earth and of the other elements not be introduced, from which the intellect recoils more than from a mere multiplicity of motions. *The other* necessity is that other motions, more manifold and differing in species, number and accidents, not be multiplied — namely an annual motion in the lunar heaven, in the Earth and all the Elements, and in the inhabitants of the elements. 2, *the reply was* by distinguishing the Major: denying it of mere arithmetical multiplication, and conceding it of a multiplication that is useless or superfluous for the end intended by God and by Nature — for the sake of which end, in many other things too, what could otherwise be done arithmetically by fewer means is accomplished by more, but not equally well, and without attaining the end.

[Margin: See chapter 8, from number 6 to 9.]

XXXII. *The Twelfth Argument.* In the apparent motion of the Planets there are many imperfections, if that motion were really such as it appears — chiefly the Stations and Retrogradations, and the excess of the Epicycle of Mars and of Jupiter over the Epicycles of the others, although their Eccentrics are the smallest; and the more frequent alternations of Stations and Retrogradations in the slower Planets; and the like. But if they are not real but merely apparent, they are not Physical imperfections but Optical ones, and more tolerable. Yet if the Earth does not move, they are real and Physical; if it moves, they are merely apparent and Optical, for they all occur because of the approach of the Earth, moving annually, to the Planets as they go towards the East on a mere and simple eccentric, or because of the Earth's recession from them. Therefore the Earth moves annually rather than not.

[Margin: Fourfold reply.]

The reply was: *First*, by denying that they are imperfections absolutely; for neither inequality in motion, nor pauses, nor an acceleration and retardation suited to the end of the motion, is an imperfection, but a perfection — just as in harmonic concords and rhythmic dances. *Secondly*, by denying that they are imperfections comparatively, that is, greater than those that must be admitted in the hypothesis of the Earth moved annually and daily. For every motion of the Sun must be extinguished, which is worse than a momentary station or retrogradation of some Planets; the Earth with its elements must be transferred into the heaven; innumerable motions must be attributed to all who inhabit the Earth, the Water and the Air near the Earth; and the lunar heaven must be inserted between Mars and Venus and moved with an annual motion together

with the Earth, etc. *Thirdly*, by denying that they are real imperfections in one hypothesis and apparent ones in the other; for in both they are apparent, because they come from the position of the eye: if the eye were at the centre of the Epicycle of the five Planets, they would be seen always to advance. *Fourthly*, the argument was turned back: because in the hypothesis of the Earth's annual motion mixed with the diurnal, one must admit a far more frequent appearance of Stations and Retrogradations, and an inequality of motion mixed of the diurnal and the annual — and that without any necessity, or foundation taken from the senses, which perceive no motion of the Earth, much less its inequality. But these things cannot be understood perfectly unless what was said in chapter 8, from number 6 to 9, and in chapter 14, from number 4, is read. Moreover, Kepler confessed that in the Tychonic hypothesis too, and in others equivalent to it, an account of the Stations and Retrogradations can be given, as I showed in the same place.

[Margin: See chapter 9, from number 10 to 12.]

XXXIII. *The Thirteenth Argument.* There are many other things in the heaven of which no sufficient account can be given by the hypotheses of others, but of which an account is given by the single annual motion of the Earth as it approaches the Planets and recedes from them: namely, why the Eccentrics of the Sun, Venus and Mercury have equal revolutions; why the Superior Planets appear now very large, now very small; and why at opposition with the Sun they are always low, and at conjunction high, etc. Therefore the Earth moves with an Annual motion.

[Margin: Twofold reply.]

The reply was, 1, to deny the Antecedent: for in the hypothesis of the others, too, a sufficient account is given. For whether the Earth itself approaches the Planets, which are moved by no other motion than a single one in longitude, or, with the Earth unmoved, the Planets themselves approach the Earth by an apparently twofold motion, the same Phenomena result; indeed, fewer motions arise if the individual Planets, as we suppose, are carried round by an Intelligence along a single spiral line representing several motions in length and breadth. 2, the Consequence was denied, because, granted that an account of the effects already mentioned were given by the annual motion of the Earth, many other more absurd things would nevertheless lie hidden in this motion, which we have indicated above more than once; and still others will be uncovered in the next chapter.

[Margin: See chapter 9, from number 13 to 15.]

XXXIV. *The Fourteenth Argument.* If the Earth moves with an annual motion, the inclination of the orbit of the five lesser Planets keeps a constant angle, and there is no need for a libration of the orbs or for a tortuousness of the Planet's path;

but it is otherwise if the Earth does not move in this way. Therefore the Earth does move in this way.

[Margin: Twofold reply [?].] *The reply was*: 1. By denying the Antecedent, since that constancy of the angle can be saved in other hypotheses of a resting Earth as well, as I showed in chapter 9. Next, the consequence is denied, because in the hypothesis of the moving Earth there lie hidden many other imperfections, far greater than an inconstant inclination of the orbit and a winding path of the Planets would be.

[Margin: See chapter 9, numbers 16 and 17.] XXXV. *The Fifteenth Argument.* The equant circle [Circulus Æquans] is out of keeping in astronomy, since the Planet is supposed to move uniformly with respect to it, even though it does not move on that circle but on another. But this imperfection is removed by the annual motion of the Earth, and not by its rest. Therefore etc.

[Margin: Reply.] *The reply was*: First, by denying the Major, if the equant seems absurd because it supposes a real inequality in the motion of the Planets; for we have shown sufficiently, in section 2, chapter 4, that this inequality must be admitted and is no imperfection. Second, we denied the Minor: for the imperfection of the equant, taken as the bisection of the eccentricity, is not removed in the hypothesis of Kepler and Boulliau, who nevertheless posit an annual motion of the Earth; and it is removed in the hypothesis of Tycho, of Longomontanus, and in our own.

[Margin: See chapter [?]0, number 1.] XXXVI. *The Sixteenth Argument.* The same Fixed stars appear swifter and larger than themselves (other things being equal as to the clearness of the air and the distance from the horizon) at one time of the year than at the opposite time, as Io. Stadius, Io. Pena and Rothmann have asserted. Therefore it is a sign that the Earth approaches them at one time and recedes from them at the other.

[Margin: Reply.] *The reply was*: by denying the Antecedent, which is contradicted both by our own experience and by the common experience of observers, and by the very hypothesis of Copernicus, in which it is supposed that the annual orb produces no sensible Parallax, or variation of view, in the Fixed stars.

[Margin: See chapter [?]0, number 2.] XXXVII. *The Seventeenth Argument.* The revolution of Jupiter's satellites around Jupiter is not regular if it is reckoned from lines drawn from the centre of the earth; but it is regular if reckoned from the centre of the Sun — so Simon Marius observed. Therefore the Earth, rather than the Sun, moves with the annual motion.

[Margin: Reply.] *The reply was*: granting the Antecedent, though it may be doubted, by denying the consequence. For from it there follows not the motion of the Earth, but at most that the Sun is the centre of the motions of the Planets, or that it serves Jupiter as a model for revolving its satellites in the same way as the Sun leads its own, Venus and Mercury, around itself.

[Margin: See chapter [?], number 3 [?] 5.] XXXVIII. *The Eighteenth Argument.* The variation of motion and the libration of the Moon's body are better explained by supposing an annual motion of the Earth. Therefore etc.

[Margin: Reply.] *The reply was*: by denying the Antecedent — both because they are explained equally well by a small librating circle [circellus libratorius]; and because that libration and variation are monthly, and therefore have no fitting account from the annual motion as such; and because many things concerning it have been falsely supposed by Hevelius and Boulliau, which we shall expose in the Appendix to book 4. Besides, there are many other absurdities in the Earth's motion which outweigh this very slight convenience, if convenience there were.

[Margin: See chapter [?], number 6.] XXXIX. *The Nineteenth Argument.* The new star which appeared in Cassiopeia in the year 1572 gave evidence, by its own increase, of the Earth's approach to it. [Margin: Reply.] *The reply was*: by denying the antecedent, since the star decreased continuously, even at the time when the Earth, had it moved annually, would have approached it; see what was said in chapter 10, number 6.

[Margin: See chapter [?], number 7.] XL. *The Twentieth Argument.* The path of the Comets, as to their motion both in longitude and in latitude, is explained much more readily in the hypothesis of an Earth carried round annually. Therefore the Earth moves annually.

[Margin: Twofold reply [?].] *The reply was*: 1. By denying the Antecedent, since Tycho rather confirms the Earth's rest from it: for if the Earth moved with this motion, some Comets ought to have appeared Retrograde and slower, and yet they did not appear so. 2. By denying the Consequence — both because of the other absurdities of the Earth's annual motion, pointed out in chapter 9 from number 5, which extinguish this little spark of probability violently struck out of the Comets; and because Kepler supposes a rectilinear and regularly decreasing motion in the Comets, although he has demonstrated neither. But we have said more on this matter both in chapter 10, number 7, and in book 8, section 1, chapter 26.

[Margin: See chapter 11, from number 1 to 5.] XLI. *The Twenty-first Argument.* The Meridian line gradually, after some centuries, changes its position and declines from the original plane of the Meridian. Therefore this arises from the motion of the Earth.

[Margin: Reply.] *The reply was*: 1. By denying the Antecedent, which Iosephus Scaliger deduced *a priori* from most fallacious principles, and Caesar Marsilius [Cesare Marsili] from slippery little observations. 2. By denying the Consequence, because the Copernican hypothesis rather requires a stable appearance of the meridian line; but see what was said in chapter 11, from number 1 to 5.

[Margin: See chapter 11, from number 6 to 16.] XLII. *The Twenty-second Argument.* The altitudes of the Pole at one and the same place have been found to vary: either after many centuries, as Dominicus Maria [Domenico Maria Novara], Iosephus Scaliger, Io. Stadius and Io. Antonius Maginus noticed; or at least twice a year, as Rothmann observed; or finally every day, as Alexander Calignonus observed from the nutation of plumb-lines towards the sides of

the world, and as Gassendi did not deny. Therefore the Earth moves annually. Someone could confirm this if a pendulum 40 Paris feet long and weighing 5 pounds performed, on the day of the summer Solstice, 20 or 30 fewer vibrations in one hour in the evening than in the morning — for Mersenne thought that the truth of one or the other hypothesis could be decided from this, and wrote to the most illustrious Count of Alais, Governor of Provence; Father Paulus Casatus [Paolo Casati] showed me a copy of his letter.

[Margin: Twofold reply.] *The reply was*: 1. By denying the Antecedent, and by exposing the fallacies, and indeed the inconsistency, in the observations. 2. By denying the Consequence drawn from the first and third kinds of change: for from these there would follow a secular motion, or a diurnal motion towards the poles, but not an annual one; and yet that motion is opposed by the magnetism of the Earth, which many Copernicans have attributed to it precisely so that the Earth may keep itself in the same position with respect to the Poles. But see chapter 11, from number 6 to 16.

[Margin: The argument from the vibrations of the pendulum for deciding between the Ptolemaic and the Copernican Hypothesis.] As for the number of evening vibrations of the pendulum, etc.: I replied that an experiment of this kind is tried in vain, since it would be better tried with a shorter pendulum, as one that exhibits a greater number of vibrations. And yet neither in the morning, nor in the evening, nor about Midday or Midnight (which would be better, since the difference which the diurnal motion would add to the annual would then be more evident than in the morning or evening) has any certain and sensible difference appeared to us between the vibrations of one equal hour, marked by the transits of the stars, and those of another. Nor can such a difference arise of itself from the motion of the earth, since the pendulum itself must follow both motions of the earth, according to Copernicus; and all the phenomena of natural motions must be saved equally in either hypothesis — whatever may be the case with violent or preternatural motions.

[Margin: See the whole of chapter 12.] XLIII. *The Twenty-third Argument.* The apparent path of the Solar Spots, and their truly wonderful and most certain alternations, are explained by simpler and fewer motions if an annual motion of the Earth is asserted than if one of the Sun. Therefore the Earth, rather than the Sun, moves annually.

[Margin: Twofold reply.] *The reply was*: 1. By denying the Antecedent: for in either hypothesis only three real motions are necessary to save the Phenomena of the Solar Spots. Namely, in the hypothesis of the moving Earth: the annual motion of the Earth, the diurnal motion of the Earth, and the motion of rotation [motus Vertiginis] of the Sun about its own centre. In the hypothesis of the motionless Earth, on the other hand, there are required: the spiral motion of the Sun's centre, but slower than the motion of the Fixed stars and of the primum mobile, and therefore representing in itself, though single, both the diurnal and the annual motion; and the annual motion of the poles of the Sun's mobile Axis about its fixed axis, performed from West to East; and a third

motion, an as it were monthly rotation about another mobile axis, from East to West. Therefore in the number of real motions there is parity. But there is this disparity: Galileo distributes these motions to two really distinct subjects — namely the diurnal and the annual to the Earth, and the monthly rotation to the Sun — and so it is no wonder if the Sun's motion seems simpler; whereas Scheiner gives all three to the same subject, namely the Sun; and these motions are more evident to the senses. 2. The Consequence was denied, because of the many other inconveniences lurking in the motions of the Earth, but already pointed out often, and especially in chapter 9 from number 5.

[Margin: See chapter 13, number 1.] XLIV. *The Twenty-fourth Argument* is taken from the perpetual wind which is felt blowing towards the West, especially in the Torrid Zone; for from this some people beg a little breath of support for the annual motion too. [Margin: Reply.] But this argument would require continuous, and over the whole tract of the Torrid Zone, win- ds, if the cause were the motion of the Earth — which, however, is false according to the historical record. And another, more probable cause shows itself in the Sun, which raises one vapour and exhalation after another in succession as it moves towards the West. Indeed, a wind of this kind would have to run towards the East by day and towards the West by night, which is very much to be noted.

[Margin: See chapter 13, number 2.] XLV. *The Twenty-fifth Argument.* The fibres and veins of metal in mines are seen to face the East: therefore this comes from the annual motion of the Earth towards the East. [Margin: Reply.] But the Consequence was denied, because, if this is true, it arises from the illumination of the Sun, which rises and takes its increase from the eastern quarter. For why would rivers, which are more fluid, not bend their course towards the East because of the Earth's annual motion, overcoming very many obstacles? It is truer, however, that those veins face the Poles of the world, as appears in quarries and in the cut trunks of trees.

[Margin: See chapter 13, number 3.] XLVI. *The Twenty-sixth Argument.* An iron plate hung in equilibrium from a bronze thread in a small, closed room, so that it faces the East with one end and the West with the other, nevertheless turns itself at last towards the poles. From this some have suspected that the cause is the motion of the Earth compounded of the annual and the diurnal, [Margin: Reply.] whereas they ought rather to recognize the magnetic force of the iron, which directs itself towards the poles of the world.

[Margin: See chapter 13, number 4.] XLVII. Some take the *Twenty-seventh Argument* from the danger of putrefaction, unless the Earth is either perpetually agitated or preserved by a soul, as if by salt. [Margin: Reply.] But corruption at the surface of the earth is not in fact avoided altogether, since it occurs in very many parts; and in the depths it is feared in vain.

[Margin: See the whole of chapter 14.] XLVIII. *The Twenty-eighth Argument.* The alternating changes in the Tide of the Sea, which all the Schools have admired up to now — whether you consider the daily ones, which mostly return every six hours; or the monthly ones, which vary notably at New Moons

and Full Moons; or the annual ones, which show a remarkable difference at the Equinoxes and Solstices — are explained in no other more convenient way, and with no more evident, or at least more probable, cause adduced, than by an unequal motion of the Earth, arising from the varied mixture of the diurnal motion with the annual. Therefore, even from this one advantage, each of the two motions of the Earth comes out more probable than rest. As to how Galileo, and Gassendi subscribing to him, explain the flux and reflux of the Sea by both motions of the Earth, see chapter 14, from number 1 to 10.

[Margin: Reply.] *The reply was*: by denying the Antecedent. First, because that inequality which would arise from the diurnal and annual motion of the earth would be insensible and without any sudden change in itself, and therefore unfit to shake the Earth and, by the shaking, to cause the tide of the sea. Second, because even if it were sensible, it would not suffice to produce the very many variations of the tides that are manifest. Indeed, Galileo's doctrine would overturn the most certain Rules of the most experienced pilots [Naucleri], which depend on the phases and motions of the Moon; and the greatest part of the historical record of the Tide of the Sea would be false. On the other hand, although no opinion on the cause of the tide of the Sea has yet stood out that removes all difficulties and satisfies an intellect contemplating all the differences of the tides, there are nevertheless some that are far more probable than Galileo's opinion — which Chiaramonti, Cabeo and Fournier have therefore deservedly hissed off the stage. But to check Galileo's boasting in this argument, we resolved in chapter 14 to gather, by a most diligent inquiry, the historical record of the motions of the Sea, and above all of the tide, which we did from number 11 to 20; the choicest Rules of the pilots, which was done from number 20 to 22; and finally the opinions on the cause of the tide of the Sea, namely from number 23 to 43. Once these are read through, Galileo's whole argument will without doubt vanish, just like a bubble or the thinnest foam of the Sea. Whatever may be the case with the system of Signor Giovanni Battista Baliani, in which he asserts that the tide of the Sea is better explained with the Moon placed at the centre and the Earth around it — a system which he devised not absolutely, but in order to beat Galileo's hypothesis with a better one — out of reverence for the decrees of the Sacred Congregation he did not publish it, as we said in chapter 15.

[Margin: See the whole of chapters 16 and 17.] XLIX. *The Twenty-ninth Argument.* By the diurnal motion of the Earth together with the annual alone, the difficulty is removed of giving the reason why heavy and light bodies, in their natural descent or ascent, acquire a continuous increase of speed, that is, move faster and faster towards the end. For it is shown that their motion is in reality circular and uniform in the spaces of the World, granted that to us, carried along together with them, it appears rectilinear and non-uniform. Therefore, even for this one reason, the motion of the Earth is more probable than rest.

[Margin: Twofold reply.] *The reply was*: 1. By denying the Antecedent, because by the figure of motion that Boulliau, or above all Galileo, ascribed to this motion, the real difference in the impact of heavy bodies is not saved — a difference which is the greater, and evident from the most manifest effects, the higher the place from which a heavy body descends; and it presupposes a real increase of impetus, which Galileo acknowledged elsewhere, though forgetting himself. Next, by that same circular figure the motion of heavy bodies is not saved unless they are in the plane of the Equator, and of such slowness that they would require exactly 6 hours for their arrival at the centre of the earth, and unless the motion of the Earth is diurnal only; although not even with these conditions laid down will that line be circular from the beginning to the end of the motion. Outside the Equator, on the other hand, the line of this motion would be a spiral about a Cone or a Cylinder, but always unfit to save the real difference of impacts, because the motion of heavy and light bodies would in reality be uniform, or without sensible inequality. [Margin: See chapter 18.] *Secondly*, the Antecedent was denied as regards the particle "alone" [solum]. For the reason of the increase already mentioned is given in another way, more Physical and better; for which reason, having collected the opinions of others in chapter 18, we chose to add our own view on the cause of this increase as well.

CAPVT XXXIV.

Chapter XXXIV.

The arguments of very many authors, put forward up to now for the Immobility of the Earth, are reduced to an Epitome, with their Solutions. And first, the 38 Arguments against the Diurnal and the Annual motion together, from Chapter 19 to 24 inclusive.

[Margin: See chapter 19, from number 1 to 7.] I. THE FIRST *Argument.* Many heavy bodies let fall through the air in the plane of the Equator would descend to the Earth with a real, and not only an apparent, increase of speed. But if the Earth moved with the motion of diurnal rotation, they would descend not with a real increase, but only with an apparent increase of speed. Therefore the Earth does not move with the motion of diurnal rotation. The Major was demonstrated from our own most certain experiments and those of others — made, indeed, outside the Equator, but shown to hold at the Equator as well. The Minor, too, was demonstrated from the real Uniformity of motion which the bodies would necessarily keep by force of the diurnal rotation; and the Argument is in form. [Margin: No solid reply.] And so no solid reply could be found to this argument, by me at least; rather, on account of its Physico-mathematical evidence, it has wrung necessary assent from me and from others to whom I put it. Some frivolous and insufficient replies, however, were brought forward and rejected in number 5.

The same Argument was applied, in chapter 19, number 7, to heavy bodies placed outside the Equator.

[Margin: See chapter 19, from number 8 to 15.] II. *The Second Argument* is altogether the same as the preceding one, except that in the Minor the annual motion was also added, and in the Conclusion the same was excluded. This argument was demonstrated in chapter 19, from number 8 to 15 inclusive. Those demonstrations, being subordinate to Geometry rather than to Physics, cannot be reduced to a summary; but they tend to this: [Margin: No solid reply.] it is demonstrated that the motion of heavy bodies, if the Earth moved with the diurnal and annual motion together, would still be really uniform, or without sensible inequality, and therefore physically incompatible with a real increase of impetus, which we know for certain from the stronger impact. And so not even for this argument could a solution be found up to now that was not sophistical and full of empty evasion. Likewise the same argument was applied, in chapter 19, number 15, to heavy bodies placed outside the Equator.

[Margin: See chapter 19, numbers 16 and [?]7.] III. *The Third Argument.* If the Earth moved with the diurnal motion, the same heavy body — keeping the same heaviness, shape, bulk, density and all its other intrinsic conditions, and let fall from an equal height through air of the same temperature — would nevertheless not descend with equal speed when let fall over any part of the Earth whatever. For one let fall at the Equator would descend without real inequality along a curved line, granted that it would apparently descend non-uniformly along a straight line; but one let fall over the poles of the terrestrial Equator would descend not only apparently but also really along a straight line, covering unequal spaces in equal times, as was shown in chapter 19, number 16. But this is absurd in itself, and contrary to the nature of heavy bodies that are in the same condition; nor is the absurdity avoided in any other way than by supposing the hypothesis of the Earth's diurnal motion, with circumstances invented by the Copernicans. But the motion of the earth is in dispute, and must not be supposed as known, but proved or disproved from better-known things; and it is better known that the same thing, remaining the same, if it is a natural agent, always does the same thing, than that the Earth moves.

[Margin: Three replies rejected [?].] *Three replies* by which the argument could be evaded were rejected as empty in chapter 19, number 17: one, which is sought from the thicker air beneath the poles; another, from the magnetic power of the earth; and a third, from a twofold power of heavy bodies — one moving downwards for the good of the parts of heavy bodies, the other moving circularly for the common good of the parts that follow their whole. It is clear that this was invented on the presumption of this hypothesis; otherwise it serves no good purpose, since it is better for heavy bodies to reach their place and their whole by a straight and as short as possible a way, etc., as discussed there. Therefore the Argument already mentioned has Physical evidence.

[Margin: See chapter 19, number 18.] IV. *The Fourth Argument.* It is the same as the 3rd, but applied to excluding the Annual motion. For with that motion posited, there would still be no diurnal motion at the poles of the Earth; so heavy bodies would not have to traverse that portion of the line which outside

the poles they would have to traverse by force of the diurnal motion, but only the portion they traverse by force of the annual motion, which carries the poles along; and so the motion would in reality be non-uniform, precisely because of the descent over different parts of the Earth.

[Margin: See chapter 19, number 19.] V. *The Fifth Argument.* It is the same as the 1st, 2nd, 3rd and 4th, but applied to light bodies tending upwards. On account of the real increase of impetus which we have observed in light bodies too, by certain experiments — though these are not so readily available to everyone — it compels us to retain the immobility of the Earth.

[Margin: See chapter 20, from number 1 to 9.] VI. *The Sixth Argument.* It is physically evident that heavy bodies naturally descend along a straight line perpendicular to the Earth, and that, if they have been thrown upwards along it, they fall back along the same line to the same place; and this is so evident that its evidence cannot be convicted of falsity either by other, more certain sensations, or by any necessary *a priori* reasons, or from things revealed by God. And so the sense, and the intellect relying on sensations of this kind, is in possession of this evidence, and can never be ousted from it by mere congruences in favour of the opposite. Now if the Earth moved with the diurnal or even the annual motion, heavy bodies would really descend along a curved and longer line, as has already been demonstrated in chapter 17, from number 16, and as the Copernicans themselves admit.

[Margin: A reply, but invalid.] *All the replies* conspire in the end to this: to teach that sense is deceptive, and that therefore only one of two things, under a disjunction, is evident — namely, that heavy bodies descend either along such a line as they appear to descend along, that is, a straight one, or along one different from the line along which they appear to descend; but which of these it is must be determined from something other than sense, since the senses are deceptive and cannot discern it. But that disjunctive proposition is evident not only Physically but also Mathematically and Metaphysically; and for us in this place Physical evidence suffices — evidence which the sensation, not of one man or a few men or times, but of all, repeated almost infinitely, has produced, and which keeps its force as long as the contrary is not proved. But I have said more on the degrees of this and of the other species of evidence in chapter 20, number 6. Certainly, if it is not evident to sense that heavy bodies descend along a straight line, nothing will be evident to it, and the whole of Physical science will perish: for nothing presents itself to sense in such a way that it could not be otherwise by the absolute power of God.

[Margin: See chapter 20, number 10.] VII. *The Seventh Argument.* The nature of heavy and light bodies requires that they return to their place and be united to their whole by the shortest possible way. But if the Earth moved with the diurnal or even the annual motion, they would not return to their place by the shortest way: not along a straight line perpendicular to the earth, but along a much longer curved one, as we showed in chapter 20, number 10. Therefore the Earth moves neither with the diurnal nor with the annual motion.

[Margin: A reply not very firm.] *The reply was* that the shortness of this return is to be taken not absolutely, but relative to the motions necessary to heavy bodies, among which is the curvilinear motion, so that they fall onto the place over which they hung perpendicularly at the moment they were let go. But who does not see that this has been scraped together by begging, not from the nature of heavy bodies, but to defend the hypothesis of the earth's motion? For it was more connatural to heavy bodies to move downwards along a straight perpendicular line, and after their union with the Earth, to rest with it if it rests, or to move circularly with it if it moves; but in order that they fall onto the same place, this longer motion along a curvilinear path had to be feigned. A better reply seemed to be that of those who concede the Major, unless, with the Earth at rest, the diurnal motion had to be multiplied in the stars, and the motion of the second inequality in the five lesser Planets. But we have already said in the preceding chapter, in the replies to Argument 13, 14 and 11, number 31, that more motions are multiplied in the World if the Earth moves than if it rests, even counting the motions of the stars; and so each Reply seems not very firm.

[Margin: See chapter 20, numbers 12 and 13.] VIII. *The Eighth Argument.* If the Earth does not move, a most natural reason can be given why heavy bodies, in descending, tend to the place to which we see them tend: namely, so that as soon as possible, and by the shortest possible way, they may lie beneath less heavy bodies, and so occupy the place due to them in the system of the elements. If, on the contrary, the Earth moves, no such reason can be given except by recourse to the very motion of the Earth, which is in dispute. For why does water let fall onto dust or sand or live coals fall into the dust or onto the coals, by which it is to be consumed? Certainly not so as to be joined to the sand or the coals, etc. But see the whole discussion conducted in chapter 20, from number 12. Certainly, from what we know, no one of sound judgement would say that a stone placed near the centre of the Earth would ascend through the water surrounding it on every side to the rest of the earth, so as to be united to its whole; since we do not see a stone let fall down a well cling to the side of the well, so as to be united at once in this way to its whole, namely the earth.

[Margin: A reply not very firm.] Any *reply* presupposes what is in question, and has no foundation prior to the motion of the Earth.

[Margin: See chapter 20, number 11.] IX. *The Ninth Argument.* Species of motions, or different inequalities and different degrees of speed in the same mobile, are not to be multiplied without necessity. But they are multiplied if the Earth moves with the diurnal or even the annual motion, as was shown in chapter 20, number 11.

[Margin: A reply not very firm.] *The reply was* by denying that this happens without necessity; for it happens so that the diurnal motion is not multiplied in the Fixed stars and in the Planets, and, besides, the Epicyclic motion in the Planets. But the Reply collapses, because in fact more motions are posited in the system of the World if the Earth moves than if it rests — and without a

foundation taken from the senses, indeed contrary to sense. For the apparent motion in the Planets and in the Fixed stars, even with the earth at rest, is a single one in each, namely the diurnal motion towards the West, but slower in those nearer the Earth; and on account of this slowness a second motion, towards the East, is believed to exist.

[Margin: See chapter 20, number 14.] X. *The Tenth Argument.* If an Angel let down a metal sphere of great weight hung from a chain, holding the other end of the chain motionless, that chain would be stretched by the force of the sphere perpendicularly towards the Earth; but according to the Copernicans it ought to be bent obliquely towards the East. [Margin: Reply.] But from this neither hypothesis can be convicted of falsity, because each person must imagine this experiment according to the properties of his own hypothesis.

[Margin: See chapter 20, number 15.] XI. *The Eleventh Argument.* Granted the motion of the Earth, no straight lines that we mark out with a cord, a thread or ink would really be straight. But before God and the Angels they would be different figures, etc. [Margin: Reply.] But the Copernicans will deny that the reasons for which they assert the motion of the Earth are absurd to those who understand them; and since they have once dared to deny evidence that is Physi-

cal — the evidence of the natural motion of heavy bodies along a straight line — what wonder is it if in other, similar cases too they have denied the real straightness of the lines we draw, and conceded only an apparent one?

[Margin: See chapter 21, from number 1.] XII. *The Twelfth Argument.* If the Earth moved with the diurnal or even the annual motion, clouds and birds hanging in the air, or trying to fly towards the East, and things like them, would nevertheless appear to fly towards the West, since they would be left behind by the Earth, and by us, who are carried towards the East together with the Earth much faster. But this is contrary to the experiments. Therefore etc.

[Margin: Reply.] *The Copernicans reply* by denying the Major: all bodies akin to the Earth and the Water, besides their own private motion, if they have one, also have the motion common to the whole earth and water, by which they are carried towards the East with equal speed, that is, through similar arcs; granted that this motion is not seen by us, because it is common to us as well.

[Margin: See chapter 21, number 3.] XIII. *The Thirteenth Argument* is taken from the motion being more difficult westward than eastward, because of the resistance of the air coming to meet it. But the Copernicans say that bodies akin to the Earth, no differently from the air near the earth, share a common motion towards the East.

[Margin: See chapter 21, number 4.] XIV. *The Fourteenth Argument.* If the Earth moved daily or even yearly, the vanes of towers, hair, flames and smoke would be turned eastward, or rather westward. But the Copernicans deny this, on account of the common motion towards the East that the terrestrial air and the other bodies akin to the Earth have.

[Margin: See chapter 21, number 5.] XV. *The Fifteenth Argument* is drawn from the scattering of many things that would arise from the motion of the earth; which, however, the Copernicans deny, on account of the eastward common motion, so often mentioned, that all bodies akin to the earth have.

[Margin: See chapter 21, numbers 6 and 7.] XVI. *The Sixteenth Argument.* If the Earth turned with a diurnal rotation, or even with the annual motion, the same cannon-ball, fired in the same way and with the same force, would cover less space when sent towards the East than towards the West. For the Earth, carrying the cannon off towards the East, would follow the eastern ball but abandon the western one, that is, the one sent at a western target; because the force impressed on the ball by the kindled powder would resist, and break, the force intrinsic to the ball that imitates the diurnal motion — just as that same force overcomes the heaviness of the same ball, not allowing it to descend for some distance from the straight line along which the cannon-ball is carried; granted that it is overcome in this, that it carries the ball more slowly. But the consequent is contrary to the experiments of Tycho and of the Landgrave of Hesse, on which more in chapter 21, number 6.

[Margin: Reply.] *The reply was* by denying the Major. Indeed, the case would rather be the opposite: the ball sent towards the East would land farther from the cannon than one sent towards the West, because towards the East both moving powers would agree; whereas in a ball fired towards the West, the force of the fiery exhalation would resist the power moving it eastward in such a way that the latter would nonetheless resist the former more, and blunt it more, being stronger, perpetual, natural, etc.

[Margin: See chapter 21, number 8.] XVII. *The Seventeenth Argument.* If the Earth moved with the diurnal or even the annual motion, the same cannon-ball, fired in the same way, would cover a notably different distance if it were fired along parallels near the poles than if along parallels near the Equator, or in the plane of the Meridian towards the poles: because of the very slow motion of the earth near the poles, and the smaller resistance arising from it to the motion of a ball sent towards the poles. But this is contrary to the experiments of Tycho, etc. Therefore etc. [Margin: No solid reply.] No valid reply to this argument appears, unless perhaps someone were to deny that experiments of this kind have been made. But they certainly can be made; nor will the difference between the intervals already mentioned be insensible, since each motion is so violent.

[Margin: See chapter 21, number 9.] XVIII. *The Eighteenth Argument.* If the Earth moved with the diurnal or even the annual motion, a cannon-ball fired either towards the East or towards the West would fail to strike the target: for if it were aimed towards the East it would strike above the target, and if towards the West, below it, because by force of the earth's motion the eastern target would be lowered and the western one raised. But this is contrary to experiment.

[Margin: Reply.] *Galileo replied*, 1. by calling these experiments into doubt, as inadequate for discerning so fine a difference of excess or defect from the target; but better, by denying the Major, because the ball would not run along the same tangent to the Earth, as the inexpert imagine, but along one after another, and at the end of its motion would be found on the one that can be drawn from the target to the mouth of the cannon. But you should see more in chapter 21, number 9.

[Margin: See chapter 21, number 10.] XIX. *The Nineteenth Argument.* If the Earth moved with the diurnal or even the annual motion, the impact of a cannon-ball, or of one driven by a rammer or a ballista, sent towards either pole of the world would be much weaker than the impact of the same sent towards the East or the West, even if everything else were equal. But this is contrary to the experiments: therefore the Earth would move with neither of those motions. The Minor is not in dispute. The Major was demonstrated Physically by Father Francesco Maria Grimaldi, chapter 21, number 10. [Margin: No reply.] Nor has any reply been offered that weakens the stronger part of the Argument made there.

[Margin: See chapter 21, number 11.] XX. *The Twentieth Argument.* If the Earth moved with the diurnal or even the annual motion, the same ball — from cannon [bombardibus], or from a ballista, or driven by a rammer — sent in the same way once towards the East and again towards the West, would strike the eastern target with a greater impact than the western one at an equal distance. But the consequent is utterly false, and conflicts with our experiments and with those open to anyone; therefore the Earth moves with neither the diurnal nor the annual motion. [Margin: No reply that is solid.] For this argument no solid reply has been found.

[Margin: See chapter 21, number 12.] XXI. *The Twenty-first Argument.* If the Earth moved with the diurnal or also the annual motion, the same mobile could be moved at the same time with two contrary motions; for whatever we now see moving towards the West would nevertheless also be moving towards the East, with a motion equal to the Earth's. But this is impossible. Therefore etc.

[Margin: A twofold reply.] *The reply was*, 1. by denying the Major in the sense in which the Minor is to be conceded. For the mobile would not really be approaching the same terminus, fixed in the space of the world, from which it was meanwhile receding; in fact it would always be approaching the eastern terminus — though less swiftly than if, by its striving towards the West, it did not diminish the speed towards the East that it would otherwise have. 2. The argument was turned back upon the two contrary motions of the heavens: if these can be reconciled in the heavens, they can also be reconciled on the Earth by an equivalent reasoning. But see chapter 21, number 12.

[Margin: See chapter 21, number 13.] XXII. *The Twenty-second Argument.* If the Earth moved with the diurnal or also the annual motion, innumerable motions of the same species would be multiplied without necessity; for it would

be necessary to attribute to each individual animal, besides its own proper motion, the motion common to the Earth, so that the apparent experiences might be saved. But this multiplication is absurd. Therefore.

[Margin: Reply.] *The reply was* by denying the Major: there is a necessity, namely, that motions should not be multiplied in the heavens. But there is less foundation for multiplying these motions in the heavens than on the Earth, and fewer are in fact multiplied if they are attributed to the heavens than if to the Earth; and from this arises the following, very strong argument.

[Margin: See chapter 21, number 14.] XXIII. *The Twenty-third Argument.* There is a greater foundation for attributing the diurnal motion to the stars and the annual to the Sun than the diurnal and the annual to the Earth; and if the former is done, the multiplication of real motions is smaller than if they are attributed to the Earth. Therefore the diurnal motion is to be attributed to the stars, and the annual to the Sun, rather than to the Earth. See the force of the argument in chapter 21, number 14, and you will find no solid reply. [Margin: No solid reply.]

[Margin: See chapter 21, number 15.] XXIV. *The Twenty-fourth Argument.* If the Earth moves with the diurnal or even the annual motion, motions that manifestly appear to us must be done away with without any necessary reason, and in their place motions must be substituted that do not appear: which is surely absurd in itself. See chapter 21, number 15.

[Margin: See chapter 21, number 16.] XXV. *The Twenty-fifth Argument.* If the Earth moved with the diurnal and annual motion, a motion variable by innumerable inequality would have to be attributed to the same mobile, and far more so than if the motion is attributed to the stars. Therefore etc. See chapter 21, number 16, where we said that no firm reply to this argument appears. [Margin: No solid reply.]

[Margin: See chapter 22, numbers 1 and 2.] XXVI. *The Twenty-sixth Argument.* The Earth is extremely dense and extremely heavy; therefore it is inert towards every motion. [Margin: Reply.] *The reply was* by denying the Consequence as regards circular motion, to which heaviness offers no resistance. But as to whether the Earth as a whole, insofar as it is a whole, is heavy, see chapter 21, number 2.

[Margin: See chapter 22, number 3.] XXVII. *The Twenty-seventh Argument.* The speed of the diurnal motion would be so great that it would overcome the flight of birds, let alone the course of rivers or of ships towards the East; therefore no birds, ships etc. could reach their eastern terminus, which is contrary to the experiments.

[Margin: Reply.] *The reply was* by denying the Consequence, because in Copernicus's hypothesis birds, ships etc. move not by their own motion alone but also by the common motion, equally with the Earth. How great the speed of the Earth would be, I have taught accurately in chapter 21, number 1.

[Margin: See chapter 22, number 4.] XXVIII. *The Twenty-eighth Argument.* If the Earth moved with the diurnal and annual motion, we would feel a

perpetual wind towards the West, and a noise and whistling from the air dashed against trees, mountains, towers etc. But we feel these not at all.

[Margin: Reply.] *The reply was* by denying the Minor as regards the almost perpetual winds within the Tropics, and the Major as regards the other effects, because the air near the Earth too, being full of exhalations and vapours, would move with the common motion together with the Earth.

[Margin: See chapter 22, numbers 5 and 6.] XXIX. *The Twenty-ninth Argument* is from the shaking and collapse of buildings, or the flinging off of bodies weakly attached to the Earth, which would seem bound to follow from the Earth's rapid rotation.

[Margin: Reply.] *The reply was* that there is no such danger in the Copernican hypothesis, as is clear from what was said in chapter 22, number 6.

[Margin: See chapter 22, number 7.] XXX. *The Thirtieth Argument.* We ought to feel this motion in ourselves, if it were common to us with the earth. [Margin: Reply.] But we have shown in chapter 22, number 7, that there is in truth no necessity for this sensation.

[Margin: See chapter 23, number 1.] XXXI. *The Thirty-first Argument.* If the Earth turned towards the East, the eastern mountains would appear to us to descend and the western ones to ascend. [Margin: Reply.] But this consequence was denied by the Copernicans, because the plane on which we stand with the mountains, and the mountains themselves, move with us with an equal motion; and so we do not notice their motion, which is common to us.

[Margin: See chapter 23, from number 1 to 8.] XXXII. *The Thirty-second Argument.* If the Earth moved with at least the diurnal motion, we, looking up from the bottom of a well at some Fixed star standing over our zenith, would lose sight of it once a blink of the eye had passed, because in the blink of an eye the whole breadth of the well's mouth would pass by, by force of the diurnal rotation. But this is contrary to the experiments. Therefore.

[Margin: Reply.] *The reply was* by denying the consequence of the Major, because our wells neither reach the centre of the Earth nor are much wider than four or five feet; so the visual angle at the bottom of such wells takes in much more of the circumference of the heavens than the terrestrial arc that the Earth traverses in one blink of the eye. But this reply will become clearer from the Problems, very well worth knowing, which we gave on this occasion in chapter 23, from number 2 to 8.

[Margin: See chapter 23, number 8.] XXXIII. *The Thirty-third Argument.* If the Earth moved with at least the diurnal motion, Gnomons erected in the Tropics on the solstitial days would cast sensible shadows at the time nearest to Noon. But Strabo and Pliny the Elder relate that it turns out otherwise. [Margin: Reply.] Yet the consequence of the Major is denied, for the cause adduced in chapter 23, number 8.

[Margin: See chapter 23, number 9.] XXXIV. *The Thirty-fourth Argument.* The eclipse of the Sun at the death of CHRIST was total for three hours; but if

the Earth had been turned with the diurnal motion, it would not have lasted total for three hours, for the Earth's rotation would at once have withdrawn Palestine into another position, from which it could have seen the Sun. Therefore.

[Margin: Reply.] *The reply was* by denying the Minor, because the Moon could have been kept between the Sun and Palestine for three hours, carried forward towards the East as far as the motion of the Earth would require. For the rest, see chapter 23, number 9.

[Margin: See chapter 24, numbers 1 and 2.] XXXV. *The Thirty-fifth Argument.* Circular motion is violent or preternatural to the particles of the earth, and therefore to the whole Earth too; therefore it does not belong to the Earth perpetually. So Aristotle.

[Margin: A twofold reply.] *The reply was*, 1. by denying the Antecedent: for in the Copernican hypothesis a really circular motion belongs naturally to the parts of the Earth when they are united to their whole, and a motion mixed of straight and circular if they are separated — granted that it does not appear to us insofar as it is circular, because we too are turned with such a motion together with the Earth. 2. The argument was turned back against Aristotle, who admits in the elementary Fire a perpetual circular motion, although a preternatural one.

[Margin: See chapter 24, number 3.] XXXVI. *The Thirty-sixth Argument.* The Copernican motion of the Earth removes from the Universe the simple motion of things up and down; therefore it is not to be admitted. [Margin: Reply.] *The Copernicans reply* by denying the antecedent as regards apparent motion.

[Margin: See chapter 24, number 4.] XXXVII. *The Thirty-seventh Argument.* No principle of the earth's motion can be assigned without great inconveniences. Therefore. *The Copernicans reply* by denying the antecedent, and they say that the principle is intrinsic and natural: see chapter 24, number 4. [Margin: Reply.]

[Margin: See chapter 24, from number 5 and 6.] XXXVIII. *The Thirty-eighth Argument.* The Copernican motion of the Earth makes the motion of Heavy and Light bodies unnatural, and the circular motion natural — and that in things most diverse in nature — and it is entangled in insoluble questions. Therefore.

[Margin: Reply.] *The Copernicans reply* by denying the 1st and the 3rd part of the antecedent; and the eleven principal Questions put by Scheiner were answered in chapter 24, number 6.

Argumenta 39. Contra Situm & Motum Annuum Terræ, ſeorſim à Diurno conſideratum, & Solutiones eorum traditæ à cap.25. ad 31. Incluſiuè.

39 Arguments against the Position and the Annual Motion of the Earth, considered separately from the Diurnal, and their Solutions, given from chapter 25 to 31 inclusive.

[Margin: See chapter 25, from number 1 to 5.] XXXIX. THE FIRST *Argument.* The centre of the Earth is where the terminus is to which all Heavy bodies tend naturally and of themselves; but this terminus is the centre of the

Universe; therefore the centre of the Earth is at the centre of the Universe. Aristotle proves the Minor: 1. Because, even if the Earth were taken away, heavy bodies would still tend to the centre of the Universe; therefore heavy bodies are not carried to the centre of the Earth of themselves and as the Earth's centre, but insofar as it is the centre of the Universe. 2. Because the terminus to which heavy bodies tend of themselves is the one that is the terminus from which light bodies move of themselves; but light bodies recede of themselves from the centre of the Earth insofar as it is the centre of the universe, since they tend towards the circumference of the highest heaven, that is, of the Universe, along perpendicular lines.

[Margin: A manifold reply.] *Galileo replied* in three ways, and not badly from his own hypothesis, as is clear in chapter 25, number 6; and Kepler too, but in Kepler's solution we noted five faults, on which see chapter 25, number 7. *Our own reply was*, granting the Major, by denying the Minor and the Antecedent of the proof: for if the Earth were taken away, that is, moved elsewhere, heavy bodies would tend to the centre of the Earth; but if it were destroyed, with the remaining elements and mixed bodies left, they would tend to the centre of the Water, which would then take the place of the earth — that is, to the centre of the heaviest body. To the 2nd proof the Minor was denied: for light bodies tend to the highest circumference of the elementary sphere, which Aristotle supposed, but did not prove, to be concentric with the Universe. Whether in this he begged the question or reasoned fallaciously, see what was said in chapter 25, number 6.

[Margin: See chapter 25, number 9.] XL. *The Second Argument.* Light bodies ascend from any point of the earth's surface towards the highest heaven along a straight line perpendicular both to the convexity of the terrestrial globe and to the concavity of the highest heaven; therefore they ascend from the middle of the Universe; but they ascend by receding from the middle of the earth; therefore the middle of the Earth and the middle, or centre, of the Universe are the same.

[Margin: Reply.] *The reply was* by denying the Major: for they tend not to the highest heaven but towards the highest surface of the elementary sphere, which is concentric with the Earth; but whether it is concentric with the Universe is doubtful, unless that is shown from elsewhere; and the Copernicans contend that it is eccentric.

[Margin: See chapter 25, number 10.] XLI. *The Third Argument.* Heaviness and Lightness were given to bodies not so that they should be united to their whole and to what is like them, but so that they should keep or recover a determinate place in the universe; for place, insofar as it is place, is the formal terminus of locomotion. Therefore they have an immovable place at or around the centre of the Universe; but they would not have this if the Earth moved through the annual orb.

[Margin: Reply.] *The reply was* by denying the Major; for they were given so that bodies should acquire and keep a place in the elementary system, wherever that system may be, abstracting from its mobility and immobility.

[Margin: See chapter 25, number 11.] XLII. *The Fourth Argument.* Unless the Earth occupies the World's cen- tre, no sufficient reason can be given why it does not, as a whole, move downwards, or to the place to which its parts move. Therefore it occupies the centre of the World.

[Margin: A twofold reply.] *The reply was given by Kepler*, by denying the Antecedent: for the reason is that the whole Earth, insofar as it is a whole, is not heavy — granted that Chiaramonti said both that it is heavy and that its weight can be known approximately; on which see chapter 25, number 11. *Other Copernicans, however, would reply* that the cause is that the whole Earth is naturally determined to circular motion around the centre of the world.

[Margin: See chapter 25, number 12.] XLIII *The Fifth Argument.* If the Earth were transferred to the Moon, heavy bodies would nevertheless not tend there, but to the centre of the Universe etc. [Margin: Reply.] *The Copernicans reply* by denying the antecedent, which Aristotle has not yet shown.

[Margin: See chapter 25, number 13.] XLIV. *The Sixth Argument.* The lowest place is owed to the heaviest and lowest of bodies: the Earth is the heaviest, and the centre of the World is the lowest place. Therefore. [Margin: Reply.] *They reply* by conceding this of the lowest place in the elementary system, and denying it of the absolutely lowest.

[Margin: See chapter 25, number 14.] XLV. *The Seventh Argument.* Those bodies are called heavy without qualification which are carried towards the Middle, and light those carried away from the Middle; but these definitions would perish if this Middle were not the Middle of the Universe. [Margin: Reply.] *They reply* by denying the Minor; for it is enough if it is the middle of the elementary system.

[Margin: See chapter 25, number 15.] XLVI. *The Eighth Argument.* Unless the centre of the Earth and of the elementary system is the centre of the Universe, positive lightness is removed and reduced to a privation of greater heaviness. But this is contrary to reasons resting on our experiments. Therefore. The Major is proved from the lack of a determinate place for light bodies.

[Margin: Reply.] *The reply was* by denying the Major: for the Physical place of light bodies — of fire, for instance — would be the space between the Moon and the heavier elements, even if the whole elementary sphere were transferred to another Mathematical place in the space of the World.

[Margin: See chapter 25, number 16.] XLVII. *The Ninth Argument.* Heavy bodies, in descending to the centre of the Earth, would sometimes recede from the centre of the World, if the Earth were outside it; and light bodies would approach it by receding from the earth. Therefore the definitions of Heavy and Light would be confounded.

[Margin: Reply.] *The reply was* by conceding the consequence as regards the definitions as Aristotle understood them, and denying it as regards definitions

rightly understood, that is, given in relation to the middle of the elementary system.

[Margin: See chapter 25, number 17.] XLVIII. *The Tenth Argument.* Heaviness and Lightness are implanted in bodies so that they may rest in their own place, or tend towards it in such a way as to rest in it. But they would never rest if the Earth were revolved with the elements through the annual orb.

[Margin: Reply.] *The reply was* by conceding the Major as regards the place of Heavy and Light bodies as such — that is, that heavy bodies should stand beneath light ones, and light ones stand above heavy ones, in the elementary system; and in this sense the Minor was denied.

[Margin: See chapter 25, number 18.] XLIX. *The Eleventh Argument.* Centres, and positions of centres, would be multiplied without necessity: for the centre of the Universe would be one thing, and the centre of the Earth and of the elementary system another, and always a different one. Therefore etc.

[Margin: A twofold reply.] *The reply was*, 1. by denying the Antecedent: for the necessity, *a priori*, lies in all the arguments, or all the best of them, on account of which the Copernicans posit the annual motion of the Earth. 2. By turning the argument back: for otherwise the centre of the Universe and the centre of the Planetary system — which is now held to be the Sun — would be multiplied, unless the Sun were at the centre of the World and the Earth went around it.

[Margin: See chapter 26, number 1.] L. *The Twelfth Argument.* All who look at the sky from any part of the Earth judge that the sky is up and the Earth down; but this judgement would be false if the Earth were outside the centre of the World. Therefore.

[Margin: Reply.] *The reply was* by conceding the Minor as regards the centre taken also Physically and to sense, and denying it as regards the centre taken Mathematically.

[Margin: See chapter 26, number 2.] LI. *The Thirteenth Argument.* The Earth is the lowest not only of the elements but of all the Bodies of the World; therefore the lowest place is owed to it not only in the elementary System but also in the World, and that is the centre of the World.

[Margin: Reply.] *The reply was* by denying the Antecedent, if the Earth is taken together with men and the other animals. For other replies, see chapter 26, number 2.

[Margin: See chapter 26, number 3.] LII. *The Fourteenth Argument.* Too much licence would be given for placing the Earth wherever one pleased.

[Margin: Reply.] *The reply was* by denying the antecedent: for the Earth must stand in such a position that half of the heavens is visible to sense on every side, and the remaining phenomena of the heavens are saved.

[Margin: See chapter 26, number 4.] LIII. *The Fifteenth Argument.* With the Earth placed outside the centre of the World, Hell [Infernus], the place of the damned, would not be the nether place; indeed some who tend towards it would ascend. Therefore.

[Margin: Reply.] *The reply was* by denying the Antecedent: for that place is called Hell in comparison with the surface of the earth, on which men were wayfarers, and with the Empyrean, the place of the comprehensors; and these two relations would remain, notwithstanding the motion of the Earth. For the rest, see chapter 26, number 4.

[Margin: See chapter 26, number 5.] LIV. *The Sixteenth Argument.* If the Earth is in the Annual Orb with the elements, the order of the Planetary and elementary system is overturned: the elementary sphere becomes one great Planet and is placed in the heavens; and the number of the primary Planets, 7, is overthrown, since the Sun ceases to be a Planet, and the Moon a primary Planet — for it becomes the earth's attendant — while the Earth with the elements becomes a primary Planet, and only 6 primary Planets are retained. But this is most absurd.

[Margin: Reply.] *The Copernicans reply*, granting the Major, by distinguishing the Minor: conceding it with respect to those who, accustomed to the Peripatos, judge everything from the senses; denying it with respect to those who, contemplating the Platonic Ideas and initiated into the Pythagorean mysteries, judge the order of things from archetypal reasons.

[Margin: See chapter 27, numbers 1 and 2.] LV. *The Seventeenth Argument.* With the Earth placed at the centre of the World, all the Phenomena of the heavens are saved. Therefore. *The reply is* by denying the Consequence, because they are saved even if the Earth moves in the annual orb, with the diurnal motion.

[Margin: Chapter 27, number 3.] LVI. *The Eighteenth Argument.* More motions would have to be attributed to the Earth, with more changes in the stars, etc. *Reply*: by conceding the first and denying the second.

[Margin: Chapter 27, number 4.] LVII. *The Nineteenth Argument.* A whole hemisphere of the sky could not be seen by us from any part of the earth. *Reply*: by denying the consequence, because no more of the sky is hidden than the horizontal parallax of the heavenly bodies, which, the Moon excepted, is less than a whole degree.

[Margin: Chapter 27, number 5.] LVIII. *The Twentieth Argument.* The Fixed Stars that the Earth approached would appear larger than themselves. *Reply*: by denying it, if the diameter of the annual orb were of insensible proportion to the distance of the Fixed stars, as Copernicus supposes.

[Margin: Chapter 27, number 6.] LIX. *The Twenty-first Argument.* The eastern shadows of gnomons, at an equal altitude of the Sun above the horizon, would not be equal to the western ones. *Reply*: by denying the consequence, because in both cases the Earth would be equally distant from the Sun, to sense.

[Margin: Chapter 27, number 7.] LX. *The Twenty-second Argument.* The alternations of days and nights in the threefold sphere would not occur in the way they now do. *Reply*: by denying the consequence, which arises from ignorance of the Copernican hypothesis set out in chapter 4.

[Margin: See chapter 27, number 8.] LXI. *The Twenty-third Argument.* Eclipses of the Moon would not always occur with the Moon set under the place of the Zodiac opposite to the one under which the Sun is. [Margin: Reply.] *Reply*: by denying the consequence: for the Sun would not be revolving around an Earth moved annually, but the Earth around the Sun, carrying the lunar heaven with it, etc.; therefore in an eclipse of the Moon the Earth would always be interposed between the Moon and the Sun on some diameter of the heavens drawn through the centre of the heavens, that is, through the centre of the Sun or of the annual orb.

[Margin: See chapter 27, number 9.] LXII. *The Twenty-fourth Argument.* But at least eclipses of the Moon would not happen with both Luminaries appearing on the horizon. [Margin: Reply.] *Reply*: by denying the consequence, for the reason indicated in the reply to the 23rd Argument. On the ignorance of Argoli, who makes much of this argument, see chapter 27, number 9.

[Margin: See chapter 27, number 10.] LXIII. *The Twenty-fifth Argument.* It is Physically evident that the Sun moves with the annual motion, that is, that it is slower in the diurnal motion in such a way that after 365¼ days it very nearly overtakes the same Fixed star again. Therefore it is Physically evident that the Earth does not move in this way; for only one of the two is possible in Astronomy.

[Margin: Reply invalid.] *They reply* by denying the Antecedent: for only this disjunction is evident — either that the Sun moves annually around an Earth that is not carried round, or that the Earth moves annually around a Sun that is not carried round. But, as has been said above more than once, this disjunction is evident not only Physically but also Mathe- matically. But here physical evidence suffices to determine one of the two hypotheses, provided it is of that kind and nothing contradicts it. Therefore no valid reply is brought against this Argument.

[Margin: See chapter 28, from number 1 to 6.] LXIV. *The Twenty-sixth Argument.* If the Earth moved through the annual orb, then, when it had been carried from one point of the annual orb to the opposite point after 6 months, a sensible variation would appear in the altitude of the same Fixed stars, that is, a notable parallax — at least in the stars nearer the pole. But astronomers have never noted any parallax in the Fixed stars within the same year. Therefore.

[Margin: Reply.] *The reply is* by conceding the Major against certain Copernicans who set too small a distance for the Fixed stars; but by denying it on Copernicus's Hypothesis, which requires so great a distance of the Fixed stars that the diameter of the annual orb vanishes, that is, makes no sensible parallax in the Fixed stars. Much on this parallax has been set out in chapter 28, from number 1 to 7, and from there onwards the various replies of the Copernicans; read them.

[Margin: See chapter 28, number 11.] LXV. *The Twenty-seventh Argument.* If the Earth moved through the annual orb, then, with the earth approaching the Fixed stars after 6 months at the Equinoctial points, a great parallax would be

felt in the apparent distance of Sirius from the Sun rising or setting. But none is felt. Therefore.

[Margin: Reply.] *The reply was* by denying the Major, because that variation arises from the distance of Sirius from the solstitial colure, whether the Earth stands still or moves. And if any other variation is said to be in prospect, it is removed at once by the distance of the Fixed stars [Eixarum] [recte: Fixarum], increased finitely to infinity by the Copernicans.

[Margin: See chapter 28, number 13.] LXVI. *The Twenty-eighth Argument.* But surely a notable parallax would be felt in the apparent magnitude of the Fixed stars. [Margin: Reply.] *The reply is* by denying it of the Fixed stars: for even if they were in the Ecliptic, that variation would not be greater than 4‴; indeed it would be much smaller, if their distance were increased as much as Copernicus permits.

[Margin: See chapter 29, from number 1 to 18.] LXVII. *The Twenty-ninth Argument.* Granted the annual motion of the Earth, then, so that no sensible Parallax should follow, one must set the distance of the Fixed stars from the centre of the Earth and of the Universe so great that their sphere would turn out to be of mad and wholly incredible bulk; as is clear from what was said in chapter 29 on the distance of the Fixed stars in Copernicus's hypothesis, from number 1 to 13 inclusive, where you have a selection of opinions, and on the bulk of the sphere of the Fixed stars, from number 14 to 16 inclusive.

[Margin: Reply.] *The Copernicans nevertheless reply* by denying that this bulk or distance is incredible — granted that it is admirable, and commends the Divine Omnipotence and Magnificence all the more. But see what was said in chapter 29, number 18.

[Margin: See chapter 29, numbers 19 and 20.] LXVIII. *The Thirtieth Argument.* If the Earth moved through the annual orb, then, to avoid a sensible Parallax of the Fixed stars arising from the diameter of the annual orb, one would have to set between Saturn and the Fixed stars an immense space, empty of stars and idle. But this is absurd and exceeds all belief, especially since the Copernicans themselves so often insist that God and Nature do nothing idly or in vain. Therefore.

[Margin: Reply.] I showed in chapter 29, number 20, that *the Replies of the Copernicans* are invalid and lean on begged, forced or improbable subtleties — granted that the last of them, which is the 4th there, cannot be convicted of falsity: namely, if someone says that in that space there are many of those Fixed stars which cannot be discerned without a Telescope, and whose place nevertheless cannot be observed so exactly with Astronomical Instruments that, if they have any parallax, it could be felt.

[Margin: See chapter 29, number 21.] LXIX. *The Thirty-first Argument.* But the stars would be so distant from the Sun that the Sun, viewed from them, would be like a point, and so unfit to illuminate them. [Margin: Reply.] *Reply*: by conceding the whole; for the Fixed stars do not have their light from the Sun.

[Margin: See chapter 30, the whole.] LXX. *The Thirty-second Argument.* If the Earth moved through the annual orb, the distance of the Fixed stars would be so great that their Magnitude would turn out greater than the whole annual orb; but this is incredible. Therefore. Moreover, we demonstrated in chapter 30, from number 1 to 6, that this Magnitude would be so great; for Galileo and Hortensius tried in vain to diminish it, from what was said in the same place, number 7. Reply 1.

[Margin: Reply.] *But other Copernicans reply* by denying the incredibility of so great a magnitude, again extolling the Magnificence of God on this account, and turning the argument back against the excessive speed of the Fixed stars that would follow from the Ptolemaic hypothesis, which they think more incredible than this Magnitude. On that occasion we showed, in number 8, that that speed is more credible than this bulk of the Fixed stars; and in number 7 we said that, if God had wished to be commended by this bulk, He would have given us some indication of it a posteriori, and would not have shown them to us so small, etc. But see that discussion, so that you may learn that the Reply, although it cannot be refuted as false, cannot satisfy more prudent men.

[Margin: See chapter 31, number 1.] LXXI. *The Thirty-third Argument.* If the Earth revolved in the annual orb, the distance of the Fixed stars would be so great that no sensible Refractions of the Fixed stars would occur, because the Inclination of the incident ray into our Air, in which the refraction would have to take place, would become insensible, as would the whole orb of the earth and of the surrounding air; whereas that Inclination is always greater than the refraction. But a sensible Refraction has been observed in the Fixed stars, indeed one of 30′ at the horizon, which the more recent Copernicans admit. Therefore.

[Margin: Reply.] *The reply was*: by denying the Major. For although the earth with its air, seen from the Fixed stars, would be like a point, nevertheless the angle of Inclination of the ray of the Fixed stars to us, who are close to it, is and would be very large; and we measure it from data of sensible and known magnitude, namely from the horizontal Refraction, the semidiameter of the earth, and the semidiameter of the refracting air.

[Margin: See chapter 31, number 2.] LXXII. *The Thirty-fourth Argument.* But at least the Refraction would not be greater than the Inclination of the incident Ray. But we likewise denied this consequence, in chapter 31, number 2, as will be clear a fortiori from the reply to the following argument.

[Margin: See chapter 31, from number 3 to 6.] LXXIII. *The Thirty-fifth Argument.* Yet the distance of the Fixed stars would be so great that the Refractions could not be less than half the Inclination; but this is against the canons of Dioptrics. Therefore.

[Margin: Reply.] *The reply was* by denying the Major, the opposite of which was clearly shown in chapter 31, from number 3 to 6, by calculations made most subtly; and it was shown that not even through a distance of the Fixed stars increased finitely to infinity would the refraction come to half the

Inclination; indeed, that the stars cannot be raised by any distance whatever in such a way that, while they appear on the physical horizon, they are really on it, all refraction having vanished.

[Margin: See chapter 31, from number 6 to 9.] LXXIV. *The Thirty-sixth Argument.* From a certain semidiameter of the earth, the altitude of the refracting air, the horizontal refraction of the Fixed stars and their true distance from the zenith; or at least from the proportion between the refractions of the Fixed stars derived from these, compared with the proportion of the refractions of the Sun and Moon; and then from the proportion of the distance of the Sun and Moon from the earth, it follows that the distance of the Fixed stars from the earth must be far less than the Copernican hypothesis requires for an annual motion of the earth. [Margin: Reply.] *The reply, however, was* by denying that this follows from the first of the positions already mentioned; and it was shown by a most certain proof that the distance of the Fixed stars can increase finitely to infinity, without prejudice to what is stated in the first position. As for the proportion already mentioned, which is touched on in the second supposition, we denied that it should be relied on.

[Margin: See chapter 31, number 10.] LXXV. *The Thirty-seventh Argument.* In the lenses of the Telescope all refractions of Fixed stars so remote would cease; and therefore the Fixed stars could not appear larger to us by means of the Telescope, because the inclinations of the rays upon the lenses would cease; which is contrary to experiments. [Margin: Reply.] *The reply was* by denying the consequence, for the same reason that we indicated in the reply to argument 33.

[Margin: See chapter 31, number 11.] LXXVI. *The Thirty-eighth Argument.* With the centres of the earth and of the universe separated by the semidiameter of the annual orb, we would be uncertain from which we ought to estimate the true altitude of the stars. [Margin: Reply.] *The reply is* by denying the consequence: for this measure would have to be estimated from both, though in different ways.

[Margin: See chapter 32, the whole.] LXXVII. *The Thirty-ninth Argument.* If the Copernican hypothesis were admitted, too much licence would be given to minds to invent one system after another, and to place at the centre of the World now this Planet, now that one, etc.

[Margin: Reply.] *The reply is* that some systems would be unfit for saving all the celestial Phenomena; but of the suitable ones, none is more apt than the Copernican.

CAPVT XXXV. In Quo ex meris Rationibus, & circumscripta omni Auctoritate Concluditur, Vtra Hypothesium absolutè asserenda sit, Illane quæ Motum, an illa quæ Immobilitatem Terræ supponit?

Chapter 35. In which it is concluded, from reasons alone and with all authority set aside, which of the hypotheses is to be asserted absolutely: the one that supposes the motion of the Earth, or the one that supposes its immobility?

I. CONCLVSIO.

Conclusion I.

I. *If the celestial Phenomena alone are considered, they are saved with Astronomical and Mathematical rigour in either Hypothesis, that is, as much in the one that supposes the immobility of the Earth as in the one that supposes the Diurnal and Annual motion of the Earth. Nor have any arguments drawn from the appearances in the heavens been devised so far that demonstrate the truth of the one and the falsity of the other.*

This is clear to anyone who surveys the Arguments and their solutions indicated in chapters 33 and 34, and discussed more fully from chapter 5 to 32 of this section, if one picks out those that are derived from the celestial motions and from other things that appear in the heavens. For all those arguments on either side have been answered — more or less, indeed, but still in such a way that the solutions have their own probability, or at least cannot be refuted as false. On this occasion, moreover, it will be evident that there are many arguments which, at first sight and when unforeseen, could alarm the defender of one or other hypothesis; and yet, examined closely, they have no force or very little, and not as much as the followers of either school have so far boasted. Indeed, not a few of those brought against the Immobility of the Earth are sophistical and full of fallacy, and yet are seriously pressed by the Copernicans — such are those in chapter 33, numbers 3, 12, 15, 48, 49 — or they are founded on doubtful experiments or slippery suppositions, like those we indicated in the same chapter at numbers 20, 36, 37, 39, 41, 42, 45. Just so, conversely, many arguments thrown out against the motion of the Earth fall in vain, because they arose from ignorance of the Copernican hypothesis, which was not examined with as much depth as was needed, not even by Tycho, Clavius, Scheiner and other great men. And when the Copernicans boast that the motion of the Earth and the rest of the Sun have been demonstrated, that is either a solemn lie, or it must be understood on the mere hypothesis — that is, that if the motion of the Earth etc. is supposed, the Phenomena of the heavens can be saved, and indeed sometimes with greater elegance in some respect, and with a more probable rationale. Indeed, Copernicus himself and Galileo, and not a few of the Copernicans, grant that all the phenomena of the heavens are saved equally in either hypothesis.

II. CONCLVSIO.

Conclusion II.

II. *If the certain and manifest Physical experiments in elementary bodies are considered, all are saved in either of the hypotheses already mentioned, according to Physicomathematical rigour, except those that concern the impact of heavy and light bodies naturally descending or ascending, or the impact [?] or velocity of projectiles directed now towards the poles of the world, now towards the East, now towards the West. For these cannot be saved in Physicomathematical rigour except in the hypothesis of the resting Earth alone.*

The first part is likewise evident to anyone who runs through the Arguments drawn from the motions of elementary things, with their solutions, indicated in chapters 33 and 34. The latter part is evident from Father Grimaldi's demonstrations and our own, indicated in chapter 34 at numbers 1, 2, 19 and 20, but set out clearly in their own places. Just as these, in so celebrated a controversy, are certainly not to be lightly regarded by fair judges of things, so it is right to credit them to God, who in His munificence has favoured our labours. Therefore, although Ptolemy had scented other experiments — or perhaps these, but confusedly — he declared skilfully, in book 1, chapter 7 of the *Magna constructio* [the *Almagest*], that the hypothesis of the moving Earth could be shaken by these rather than by Astronomical ones, when he said: *"For it escapes them"* — namely, the movers of the Earth — *"that as far as the appearances in the stars are concerned, nothing perhaps prevents things being so, if one considers it rather simply. But from the accidents that occur around us and in the air, this will seem very ridiculous."* [Margin: Ptolemy's [?] prudent judgement on the force of the arguments against the motion of the Earth.]

III. CONCLVSIO.

Conclusion III.

III. *If, moreover, Physical Evidence alone is considered (which indeed alone carries weight in Physical matters), there are some physically evident arguments for the Immobility of the Earth, but none for its motion.*

The first part will be clear to anyone who reads through the arguments in chapter 34 indicated at numbers 3, 4, 5, 6, 17, 19, 20, 23, 63, to which those we touched on in the same place at numbers 8 and 9 are also reduced. I said, moreover, that it carries weight in Physical science — to which, namely, Astronomy is subalternate — so long as mathematics does not resist those arguments; nor is it necessary that they have Mathematical evidence, when that cannot be had.

The negative latter part is evident from all the arguments for the motion of the Earth drawn from sensible experiment: so far are they from containing Physical evidence that they rather tend against the evidence of the senses, that is, against Physical evidence. The rest proceed from mere congruities, in a hypothesis about a thing that is not impossible and that has certain attractions.

IV. CONCLVSIO.

Conclusion IV.

IV. *If, setting aside the demonstrative and evident Arguments, one considers those which in themselves contain mere probability, there are not a few of this kind for each hypothesis, and so many and of such a kind that, according to each person's cast of mind, they can incline the intellect to either hypothesis — were it not that sacred Authority, that of the Divine Scriptures, stood in the way; and with that standing in the way, all that probability ceases.*

This conclusion is likewise clear enough when one considers the weightier arguments made for the motion of the Earth in chapter 33 — for instance those at numbers 9, 11, 29, 30, 32, 43, and any others of this kind — and those against the motion of the Earth indicated in chapter 34, for example at numbers 32, 54, 68, 70. I said "in themselves", because in the understanding of those who have assented to the Arguments we spoke of in conclusions 2 and 3, as demonstrative and evident against the motion of the Earth, no arguments for the motion of the earth will meanwhile be able actually to generate probability; and much less if the authority of divine Scripture stands in the way.

V. CONCLVSIO.

Conclusion V.

[Margin: The principal conclusion.] V. *Considering Reason alone and Intrinsic Arguments, with all Authority set aside, the Hypothesis that supposes the Immobility or Rest of the Earth is to be asserted absolutely as true; and that Hypothesis which attributes to the Earth either the Diurnal motion alone, or the Diurnal and Annual motion, is false and in conflict with Physical, indeed also with Physicomathematical, demonstrations.*

This conclusion follows from Conclusions 2 and 3.

VI. CONCLVSIO.

Conclusion VI.

VI. *For anyone who wishes either to champion or to attack either of the hypotheses already mentioned, with all the arguments devised so far drawn up in battle array, no amount of expertise in Physical or Theological matters suffices, unless he is also most expert in Astronomy, and not lightly versed in Geometry and Arithmetic and in not a few subalternate Mathematical disciplines.*

For it is clear from what was indicated in chapters 33 and 34 that, again and again, principles or conclusions of the more recondite Astronomy are required in the Arguments and their solutions; and I say the same of Geometry and Arithmetic, etc. Otherwise, we know that not only have purely Physical men failed to understand, or are going to fail to understand, even the terms of the arguments, but also that some who greeted Mathematics from the threshold have fallen into manifest errors. Let these things be taken as said for the sake of

counsel and admonition, not of indi- rect self-display. For if in this argument we have achieved anything not unworthy of learned men, we credit it to God alone, the Giver of Life, Talent, Judgement, Industry and of all other opportunities, and ascribe nothing, or very little, to our own weakness.

VII. CONCLVSIO.

Conclusion VII.

VII. *It is fair that the followers of the Copernican hypothesis in future, if there are any, should acknowledge that Theologians and Churchmen have sufficient expertise to pass judgement on these hypotheses; and that the Copernican hypothesis is rejected not from excessive or mere reverence towards Ecclesiastical decrees or the Sacred Scriptures, but also from a deeper examination of both hypotheses, and from reasons drawn from it with the utmost circumspection.*

Those who dispute with the Copernicans here and there know what this is aimed at — those who feel that, in speech or in writing, they are again and again charged with an ill-advised piety and a kind of religious timidity: as if, bound solely by this, and not philosophising with a free mind, they had clung immovably to the hypothesis of the Motionless Earth like polyps to a rock. For so Kepler, above all in the introduction to the *Commentaries on Mars*: *"But there are many more of those who are moved by piety not to assent to Copernicus, fearing that a lie may be imputed to the Holy Spirit speaking in the Scriptures, if we should say that the Earth moves and the Sun stands still."* We shall read similar things from him below, in chapter 38, number 3.

CAPVT XXXVI. Sacræ Scripturæ Authoritates pro Solis Motu & Immobilitate Terræ afferuntur.

Chapter 36. The authorities of Sacred Scripture for the motion of the Sun and the immobility of the Earth are brought forward.

I. This business is of greater moment than some of the Copernicans have thought. For on the occasion of this controversy about the motion and rest of the Earth — which in itself is purely Physical and Astronomical — both the literal sense of Sacred Scripture and the right to judge and define its true sense are now being turned by them into a controversy, if we accept their interpretation. Accordingly, both for the plan of the work I have undertaken, and for the duty of the Religious life and of Theology, which I professed by teaching publicly and privately for almost a whole decade, I must inquire with all the diligence and sincerity I can whether, and how far, the statements of the divine scriptures avail to settle this question — especially as weighed by the unanimous consensus of the Fathers and of the expositors. All the more since every year, according to the most praiseworthy custom of our Society, all the Masters of Theology and of Arts, together with our Scholastics, at the opening of the schools, in the profession of Faith prescribed by the Most Holy Council of Trent, are accustomed, by a most sacred oath upon the holy Gospel

of God, among other things to promise, vow and swear thus: *"Likewise I admit Sacred Scripture according to the sense which Holy Mother Church has held and holds, to whom it belongs to judge of the true sense and interpretation of the Sacred Scriptures; nor will I ever take and interpret it otherwise than according to the unanimous consensus of the Fathers."* [Margin: From the bull of Pius IV on the form of the oath of the Profession of Faith.] Similar things are found in Session 4, in the decree on the edition and use of the sacred books. And always to be kept before one's eyes, even in matters hard to understand, is that excellent saying of Saint Augustine in book 2 of *De Genesi ad litteram*, chapter 5: *"In whatever way, and whatever kind of waters are there, let us not in the least doubt that they are there. For the authority of this Scripture is greater than all the capacity of human ingenuity"*; and in chapter 9: *"For what divine authority says is true, rather than what human weakness conjectures."* [Margin: Saint Augustine's saying on the authority of Sacred Scripture [?].]

II. On the other hand, however, we must beware lest, when Sacred Scripture is capable of several interpretations, we cling to one that can be convicted of falsity on other grounds, and summon a defence for our errors from the divine scriptures. For the same Saint Augustine says most prudently, in book 1 of *De Genesi ad litteram*, chapter 18: *"And in obscure matters, very remote from our eyes, if we read anything written about them, even in the divine writings, that can, without prejudice to the Faith with which we are imbued, give rise to one opinion and another, let us not throw ourselves into any one of them with headlong affirmation in such a way that, if perhaps the truth, discussed more carefully, rightly overturns it, we fall — fighting not for the meaning of the divine Scriptures but for our own, so that we want ours to be the meaning of the Scriptures, when we ought rather to want the Scriptures' meaning to be ours."* [Margin: A saying of the same Saint Augustine on the use of Sacred Scripture.] He gives a similar warning at the end of book 2 of *De Genesi ad litteram*.

Moreover, we must take care not to hand the sacred scriptures over to ridicule by those to whom the error is manifest that we would wish to defend by misusing them: as if someone wished to maintain that a clod of earth placed on water can float, because of a wrongly understood Psalm 135. And so the same holy Doctor says, in book 2 of *De Genesi ad litteram*, chapter 1: *"Here it occurs to me to warn of the error to be avoided which we warned of in book 1 — lest perhaps, because it is written in the psalms, 'He founded the earth upon the waters', one of us should think that this testimony of the scriptures must be relied on against those who discourse subtly about the weights of the elements. For they, not being held by the authority of our writings, and not knowing in what sense it was said, will sooner ridicule the holy books than repudiate what they have either grasped by certain reasons or proved by the most manifest experiments."* Explaining that passage of the Psalm literally, he says it is to be understood of islands and continents that have a higher position above the waters, and of the roofs of caves made firm with a hanging solidity above the waters; and so he concludes: *"Therefore no one can understand what was said, 'He founded the earth upon the waters', even*

literally, in such a way as to think that the weight of the waters has been set by the natural order as if beneath the earthy weight to support it." Further, in book 1 of *De Genesi ad litteram*, chapter 19, having taught that it can happen that the author or writer of Sacred Scripture wished to be understood in a double sense, he insists that we must take care not to seek out from the divine utterances a defence for a falsehood easily made manifest by the rules of Astronomy, to the great detriment of the Catholic reputation. I shall not hesitate to copy his words here, because they serve our business very well: *"That he meant both"* — namely, the Sacred Writer — *"is not unfittingly believed, if a definite circumstance supports each opinion. For it very often happens that even a non-Christian knows something about the earth, about the heavens, about the other elements of this world, about the motion and revolution, or also the magnitude and intervals, of the stars, about the definite eclipses of the Sun and Moon, about the cycles of years and seasons, about the natures of animals, shrubs, stones and other things of this kind, in such a way that he holds it by most certain reasoning or experience. But it is exceedingly shameful and pernicious, and to be guarded against above all, that any unbeliever should hear a Christian, speaking about these things as if according to the Christian writings, talking such nonsense that, seeing him err by the whole sky (as the saying goes), he can scarcely hold back his laughter. And it is not so much that a man in error is laughed at that is troublesome, but that our authors are believed by those outside to have held such views, and, to the great ruin of those whose salvation we labour for, are censured and rejected as unlearned. For when they have caught someone from the number of Christians erring in a matter which they know very well, and asserting his vain opinion on the basis of our books, how are they going to believe those books about the resurrection of the dead, and about the hope of eternal life and the kingdom of heaven, when they have thought them written falsely about things which they have already been able to experience or to grasp by indubitable numbers? For what trouble and sorrow rash and presumptuous men bring upon prudent brethren cannot be adequately expressed: since, whenever they begin to be rebuked and refuted over their wrong and false opinion by those who are not bound by the authority of our books, they try to bring forward those same holy books, from which to prove what they have said with the lightest rashness and the most open falsity, etc."* With these preliminary warnings, let us approach the sacred scriptures.

Solis Motus ex Sacra Scriptura.

The Motion of the Sun from Sacred Scripture.

[Margin: The rising and setting of the Sun are frequent in the Scriptures.] III. First, Scripture often states expressly that the Sun rises and sets, and goes forth over the earth. So Genesis 15: *"And when the Sun was setting, a deep sleep fell upon Abraham"*; and chapter 19: *"The Sun went forth over the earth, and Lot entered Segor"*; and chapter 32: *"And at once the Sun rose upon him"*; and Judges 19: *"And the Sun set on them near Gabaa"*; and 3 Kings, chapter 22: *"Before the Sun set"*; and 2 Paralipomenon, chapter 18: *"He died as the Sun was setting"*; and Judith 14: *"And it shall be, when the Sun has gone forth"*; and Esther 11: *"Light and*

the Sun rose"; and in very many other places in which the rising or setting of the Sun is mentioned — but above all Matthew 5: *"Who makes his Sun rise upon the good and the bad"*; and Ephesians 4: *"Let not the Sun go down upon your anger."* Besides these, I have counted 40 and more other places on this point.

IV. Second, it is ascribed to a Miracle of divine Omnipotence that it can check the course of the Sun, and, conversely, shake the Earth, which stands unmoved. Job 9: *"Who shakes the earth out of its place, and its pillars tremble; who commands the Sun, and it does not rise"* — that is, if he commands it, it will not rise — [Margin: Joshua stops the Sun and the Moon.] as will happen at the end of the world, of which Habakkuk 3 says: *"The Sun and the Moon stood still"*; and as happened at the command of Joshua, chapter 10: *"Then Joshua spoke to the Lord on the day on which he delivered the Amorrhite into the sight of the children of Israel, and he said before them: Sun, do not move against Gabaon, and Moon, against the valley of Aialon. And the Sun and the Moon stood still, until the nation avenged itself on its enemies."* So the Vulgate. Granted that the Hebrew reading has *Sile* ["be silent"] for *ne movearis* ["do not move"], and *Siluit* ["was silent"] for *stetit* ["stood"], the Septuagint has [Greek: στήτω ὁ ἥλιος (stētō ho hēlios, "let the Sun stand")], "Stand, Sun." The Canonical writer continues in the same place, saying: *"Is this not written in the book of the Just? So the Sun stood still in the midst of heaven, and did not hasten to set for the space of one day. There was not, before or after, so long a day, God obeying the voice of a man and fighting for Israel."* In this place the Septuagint translators have: *"And the Sun in the midst of heaven did not proceed to the West"*; so it seems to have been checked at the time when it was at midday. To this refers that passage of 1 Paralipomenon, chapter 4: *"Who made the Sun stand"*; and of Ecclesiasticus 46: *"Was not the Sun hindered in his anger, and one day made as two?"*

Almost a twin to this portent, by far the most celebrated, was that other one in which the Sun, at God's command, turned back towards the East, so that the shadow of the gnomon on the dial of Ahaz went back 10 lines. [Margin: The Sun's going back on the dial of Ahaz.] For although 4 Kings, chapter 20, says: *"So Isaiah the Prophet called upon the Lord, and he brought the shadow back by the lines by which it had already gone down on the dial of Ahaz, ten steps backward"*, that bringing-back was nevertheless made in the motion of the Sun itself as it went back, as Isaiah himself, the agent and witness of this miracle, explains in chapter 38, saying expressly: *"Behold, I will make the shadow of the lines, by which it had gone down on the dial of Ahaz in the Sun, go back ten lines. E [?] the Sun went back ten lines, by the steps by which it had gone down."* And Ecclesiasticus confirms it in chapter 48, speaking of Hezekiah: *"In his days the Sun went back, and he added life to the King."* So the same portent appeared not only on the dial of Ahaz but on others too. This was the reason why the Babylonians, the most attentive observers of the stars and of the celestial motions, having noticed it and having heard its cause from the report of the miracle already spread abroad, sent envoys to Hezekiah, so as to be better informed of its circumstances. Book 2 of Paralipomenon, chapter 32, testifies to this embassy in these words: *"Yet in*

the embassy of the princes of Babylon, who had been sent to him to inquire about the portent that had happened upon the earth," etc.

[Margin: The motion of the Sun described in detail in the Sacred Scriptures.] V. Third, the motion of the Sun — chiefly the diurnal — and its vicissitudes, effects and speed are described in detail in the sacred writings. For in Psalm 18 the glory of God, appearing particularly in the Sun, is sung thus: *"In the Sun he has set his tabernacle; and he, like a bridegroom coming forth from his bridal chamber, has rejoiced like a giant to run his course; his going forth is from the highest heaven, and his circuit even to its highest, and there is none who hides himself from his heat."* Here, in the Astronomical manner, the beginning and end of the day are taken from the summit and at the summit of heaven, that is, from the culmination of the Meridian and the greatest altitude of the Sun. And what is clearer than those words of Ecclesiastes, chapter 1: *"A generation passes and a generation comes, but the Earth stands for ever. The Sun rises and sets, and returns to its place; and, reborn there, it circles through the South and bends to the North; surveying all things round about, the spirit goes on, and returns to its circles."* But passing from Ecclesiastes to Ecclesiasticus, it is truly worth noting how often the motion of the Sun is suggested by the latter. Chapter 26 says: *"As the Sun rising,"* etc.; and chapter 33: *"Why does day surpass day, and light light, and year year, by the Sun? They were separated by the knowledge of the Lord, the Sun having been made and keeping his command; they are divided by the wisdom of God."* It belongs, then, to the Sun and not to the Earth to obey God and to distinguish the days. And chapter 43: *"The Sun, at its appearing, announcing at its going forth, an admirable vessel, the work of the Most High,"* etc. *"Great is the Lord who made it, and at his words it has hastened its course."* Thus God himself commends his own greatness by the speed of the Sun's course. That speed is indeed as much greater than the speed of the Earth would be if it were turned by a diurnal rotation, as the circumference of the Ecliptic, along which the centre of the Sun advances, is greater than the circumference of the terrestrial globe; and in our view it is greater by 7300 times. The perpetual obedience of the same Sun towards God in its motion is commended in Baruch 6: *"The Sun indeed, and the Moon and the stars, being bright and sent forth for useful purposes, are obedient."* The rest we pass over, because for anyone whom these do not satisfy, no other testimonies would be enough.

Terræ Quies & Immobilitas ex Sacris Literis.

The Rest and Immobility of the Earth from the Sacred Writings.

VI. In book 1 of Paralipomenon, chapter 16, in the Canticle of David after the bringing back of the ark, it is said: *"Let all the earth be moved before his face, for he has founded the orb immovable"* — namely, the orb of the earth. And in Job 9 the power of God is praised because he could, if he wished, move the Earth from its place: *"Who shakes the earth out of its place, and its pillars tremble"* — and these are Job's own words; and chapter 26: *"Who stretches out the North over the void, and hangs the earth upon nothing"* — again Job's own words; and in chapter

38 God himself says: *"Upon what are its bases made firm?"* For that suspension and balancing signifies immobility; as also in Proverbs 8: *"When he was hanging up the foundations of the earth"*; and in Isaiah, chapter 40: *"Who has measured the waters in his fist, and weighed the heavens with his palm? Who has hung the mass of the earth on three fingers, and weighed the mountains in a balance, and the hills in a scale?"* Then in Psalm 74: *"The earth is melted, and all who dwell in it: I have made firm its pillars"*; that is, although the animals and inhabitants of the earth are melted like wax or ice and have vanished, the Earth itself nevertheless remains immovable, resting as if on the firmest pillars, by my command and power. On this point God is again commended in Psalm 92: *"For he has made firm the orb of the earth, which shall not be moved"*; and in Psalm 95: *"For he has set right the orb of the earth, which shall not be moved"*; and in Psalm 103: *"You who founded the earth upon its own stability: it shall not be bowed down for ever and ever"*; and in Psalm 118, verse 90: *"Your truth is from generation to generation; you founded the earth, and it abides."* Isaiah too, chapter 44: *"I am the Lord, who makes all things, who alone stretches out the heavens, who establishes the earth, and there is none with me."*

As for that passage of Ecclesiastes 1: *"A generation passes, and a generation comes; but the earth stands for ever"* — in Hebrew it reads thus: *"The Earth is fixed and steadfast for ever."* So the first and simple sense is that it stands, in the way that standing is opposed to transport, or local motion — especially since it is set against the local motion of the Sun, when it is said: *"The Sun rises and sets."* The secondary literal sense is that it does not pass away like the generations of men, that is, that it does not perish as a whole. For Saint Augustine teaches by examples, or in practice — book 12 of the *Confessions*, chapters 8, 19, 20, 24 and 31, and book 1 of *De Genesi ad litteram*, chapter 19 — and Saint Thomas, part 1, question 1, article 10, that Sacred Scripture can have many literal senses. But the same Saint Augustine, in book 1 of *De Genesi ad litteram*, chapter 19, and in book 3 of *De doctrina Christiana*, chapter 23, likewise teaches that when Scripture admits several senses that are not in conflict with one another, all of them are to be accepted together.

De Situ Terræ ex Sacra Scriptura.

The Position of the Earth from Sacred Scripture.

VII. The divine writings not rarely teach that Heaven is simply up and the Earth down:

namely, Deuteronomy 4, verse 39: *"Know therefore and consider in your heart that the Lord himself is God in heaven above and on earth below."* Joshua 2, verse 11, agrees with this: *"For the Lord your God, he is God in heaven above and on earth below."* And 3 Kings, chapter 8: *"Lord God of Israel, there is no God like you in heaven above and on earth below."* In Proverbs 25 too it is said: *"Heaven above and earth below."* And Jeremiah 31: *"Thus says the Lord: If the heavens above can be measured, and the foundations of the earth below searched out."* And Joel, chapter 2, which is quoted in chapter 2 of the Acts of the Apostles: *"I will give*

wonders in heaven above, and signs on earth below." Add to these many other places in which heaven is set against the Earth: Genesis 1: *"God created heaven and Earth"*; and Psalm 113: *"The heaven of heaven is the Lord's, but the Earth he has given to the children of men"*; and Colossians 1: *"In the Heavens and on Earth."* The Copernican system seems to agree little with these passages, since it does not place the Earth simply down [?], but sets the Sun at the centre of the universe as in the lowest place; and since it does not distinguish the Earth from heaven, but places it in heaven among the Planets.

CAPVT XXXVII. Quomodo Patres & Sacri Interpretes prædicta Sacræ Scripturæ loca intellexerint.

Chapter 37. How the Fathers and the Sacred Interpreters understood the passages of Sacred Scripture already mentioned.

De Motu Solis, ac Miraculis in Solis Motu diuinitus factis.

On the Motion of the Sun, and on the Miracles Divinely Worked in the Motion of the Sun.

I. I state very briefly that all the Holy Fathers, and all the Interpreters of Sacred Scripture not condemned by the Church, who have explained the passages adduced on the motion of the Sun, understood them first in the literal sense, of a true and real motion of the Sun — granted that some, on this foundation, afterwards built some senses that are not literal. To this belong all those whom we reviewed in section 1, chapter 1, questions 7, 8 and 9, and chapter 4, questions 2, 5 and 6, on the creation of Light and on the distinction of days and nights made by that light and afterwards by the Sun; for all of them speak in such a way that they suppose that the Sun, by a local motion of transport, brings about the alternations of days and nights, and also of years. To these Saint Augustine must be added, in book 1 of *De Genesi ad litteram*, chapter 10. There, explaining how day was made from evening and morning, he put forward an opinion that would explain this without local motion of the light and of the Sun, saying: *"Or must it be said that, since this work of God was quickly accomplished, the light stood for as long, with no night following, as it took for the space of a day to be completed; and that the night following the light remained for as long as it took for the space of nocturnal time to pass and for the morning of the following day to come, one and the first day being over?"* But he rejects it at once, adding: *"But if I say this, I fear I may be laughed at, both by those who know with the greatest certainty and by those who can very easily observe, that at the time when it is night with us, the presence of light illuminates those parts of the World through which the Sun returns from its Setting to its Rising; and that therefore through all 24 hours, around the circuit of the whole round, there is never lacking day in one place and night in another,"* etc. And in book 2 of *De Genesi ad litteram*, chapter 15, he has this: *"But the fixed hours and days and years, which we know by common usage, would not come about except by the motions of the stars. So if we understand times, days and years in this way —*

times as certain divisions which we reckon by clocks, or which are very well known in the sky, when the Sun rises from the East up to its Meridian altitude and from there declines again into the West, so that next either the Moon or some star can be noticed emerging from the East immediately after the setting of the Sun, which likewise, when it has come to the middle altitude of the sky, indicates the middle of the night, and is then about to set when, with the Sun's return, it becomes morning; whole days as the circuit of the Sun from East to West; and years either as these customary rounds of the Sun — not when it returns to the East, which it does daily, but when it returns to the same places among the stars, which it does only after three hundred and sixty-five days and six hours, that is, a quarter of a whole day, have been completed; which part, taken four times, compels one day to be inserted, which the Romans call the bissextile, so that the same circuit may be returned to — or else greater and more hidden years, for when the courses of other stars are completed, greater years are said to come about: if, then, we understand times, days and years in this way, no one doubts that these come about by the stars and Luminaries" — that is, by the motion of the stars and Luminaries, as he had said at the beginning.

[Margin: What kind of miracle Joshua's was in the course of the Sun.] II. As regards the Miracle of the Sun worked by Joshua, all likewise agree on a true and real checking of the motion of the solar body itself, as may be seen in Serarius [Nicolas Serarius] on chapter 10 of Joshua, from question 17 to 27; in Iacobus Salianus [Jacques Salian], volume 2 of the *Annals of the Old Testament*, at the year of the World 2584, from number 168, or page 303; in Cornelius à Lapide on chapter 46 of Ecclesiasticus, verse 5; in Oliverius Bonartius [Olivier Bonaert] in the same place; and in Cosmus Magalianus [Cosme de Magalhães] on chapter 10 of Joshua, section 1, from annotation 4. Nor did the Fathers understand this prodigy in any other sense — especially Saint Jerome, who says on chapter 28 of Isaiah: *"Trusting in God, who dwelt in him, Jesus [Joshua] said: Let the Sun stand"*; and Saint Ambrose, book 1 of the *De officiis*, chapter 40: *"Jesus Nave, in greatness of mind and of faith, cried out: Let the Sun stand; and it stood, until this victory was completed"*; and Saint Bernard, as found in the author of his life, book 2, chapter 7: *"Joshua fights, and, so that the day, prolonged, may suffice for the completion of the victory, he does not so much pray the Sun to stand as command it: his faith merits the obedience of the Sun as much as victory over the enemy laid low."* The Saints speak in the same way: Dionysius in the letter to Polycarp; Chrysostom, homily 27 on the Epistle to the Hebrews; Saint Augustine, book 21 of the *City of God*, chapter 8, and book 11 of the *Confessions*, chapter 23; Saint Ephrem, volume 3, in the sermon on the sinful woman; Saint Maximus, in the letter to Thalassius; the Author of the *Wonders of Sacred Scripture*, book 2, chapter 4; Tertullian, *Against the Psychics*, chapter 10; and others, of whom below.

But it is disputed whether the Sun was at midday or near the horizon, and how many hours that day had. That it was midday is held by the Author of the *Wonders of Sacred Scripture*, book 2, chapter 4, among the works of Saint Augustine: *"Jesus son of Nun commanded the Sun at midday not to move"*; by

Theodoret, question 13 on Joshua, from these words: *"For just as, when the Prophet was fighting, the Sun stood still"*; by Serarius, questions 21 and 22 on Joshua; by Lyranus, Arias [Benito Arias Montano], Radak [R. David Kimchi] and Vatablus [François Vatable] on chapter 10 of Joshua; and by Salianus, above. And the 70 Interpreters strongly favour them, saying: *"The Sun in the midst of heaven did not proceed to the West."* So too Sedulius sang:

The Sun stood still at Gabaon, and on the summit of mid-heaven fixed its panting light, the evening put off, unused to reining in the day.

Yet that the Sun was near its setting was held by Severus Sulpitius [Sulpicius Severus] in the *Ecclesiastical History*; by Comestor in the *Historia scholastica*, on chapter 9 of Joshua; by Dionysius Carthusianus; by Abulensis [Alonso Tostado], question 12; and by Hugo and Caietanus [Tommaso de Vio] on chapter 10 of Joshua — because for *ne moveare* ["do not move"] the Hebrews read *Expecta* ["wait"]. And so Caietanus says: *"For had he not seen that the setting of the Sun was imminent, he would not have said: Sun, wait."* Granted that other Hebrews read *Sol tace* ["Sun, be silent"], Magalianus and Cornelius à Lapide, above, subscribe to this 2nd opinion: both because those words of Joshua 10, *"the Sun did not hasten to set"*, indicate clearly enough that it was about to rush headlong below the horizon at once, had it not been restrained by Joshua's command; and because a miracle is not to be multiplied, nor brought about except under pressing necessity, and the phrase *"in the midst of heaven"* can be taken for any part of the visible hemisphere, which, with respect to some horizon, is nevertheless always on the meridian.

III. It is further asked to how many hours that day was prolonged; and whether those words of Joshua 10, *"It did not hasten to set for the space of one day"*, and those of Ecclesiasticus 46, *"And one day was made as two"*, ought to be understood of a whole natural day of 24 hours, or of an artificial day of 12 temporal hours, or — since the interpreters judge from the circumstances of the story that it was about the summer Solstice, and at Jerusalem, which has a pole altitude of 32° 10′, the longest day is of 14^{h} 5^{m} — are to be understood of a day of 14 equal hours. [Margin: How long was Joshua's day?] For if the Sun stood still for 24 hours, these, added to the temporal hours of the artificial day that had gone before, make 36 hours; but added to the equal hours, 14^{h} 5^{m}, they make 38^{h} 5^{m}. If, however, the Sun stood still for only 14 hours, these, added to the 14 hours likewise elapsed, make 28 hours. Indeed, if that standing were reckoned from midday, according to the opinion of those who judge that Joshua's command was given to the Sun when it held the meridian, there would be only 21 hours. That day lasted 36 hours, in the opinion of the Paraphrast — the Chaldean — of Dionysius Carthusianus and of Emmanuel Sà [Manuel de Sá]. The view is also attributed to Saint Justin, in the dialogue against Trypho, and it seems to be confirmed by the fact that Joshua 10 says there was not so long a day before or after — whereas the day prolonged by Isaiah's prayers as a favour to King Hezekiah is judged by some to have been of 32 hours. But even if that were granted, the writer of the book of Joshua spoke

of the days before Joshua and after him, up to his own age in which he was writing — whether he was Joshua himself, or Samuel, or someone else older than Isaiah. Within those 36 hours, however, some count a whole natural day of 24 hours together with the 12 hours of the artificial day that was prolonged, as Serarius does. But that day was composed of two artificial days, that is, of 24 temporal or 28 unequal hours, in the opinion of Caietanus, Vatablus, Rabbi Eliezer, Rabbi Kimhi, Magalianus on chapter 10 of Joshua, Serarius in the same place, question 23, and Cornelius à Lapide and Salianus, above; for this was enough to complete the victory. But these speak of a day not including the night. I am therefore surprised that Toletus [Francisco de Toledo], in book 4 of the *Physics*, chapter 14, question 16, favoured a stay of only three hours. Nor indeed does Saint Augustine say this, in book 11 of the *Confessions*, chapter 23; rather, he says: *"So that Joshua might finish the battle as victor, the sun stood, but time went on."* Saint Dionysius, in the letter to Polycarp: *"The Sun and Moon stand for one day"*; and Saints Jerome and Ephrem affirm one day — and so, indeed, does Tertullian.

[Margin: Whether the Sun alone stood still.] IV. It is also commonly asked whether the Sun alone stood still, or all the stars together with their leader and Choragus. Saint Dionysius the Areopagite, in the letter to Polycarp, adduces both miracles as an outstanding argument of divine power, and *trum* [?] determines; but he says that it would be more admirable if, while the Sun stood still, the other stars had completed their courses. It is more probable, however, that the whole heaven stood still with all the stars, as is held by the Author of the *Wonders of Sacred Scripture*, book 2, chapter 4; Saint Maximus, to Thalassius; Abulensis on chapter 10 of Joshua; Masius [Andreas Masius] and Caietanus in the same place; the Conimbricenses, book 2 of the *De caelo*, chapter 6, question 2, article 3; Serarius on chapter 10 of Joshua, question 26; and Salianus, Cornelius à Lapide and Bonartius, above. This is so both that the order of the celestial motions should not be disturbed, and because Joshua's command to the Moon seems to indicate it — the Moon, whose light was not necessary for completing the victory while the Sun was present. Rather, he seems to have said implicitly, and by Divine prompting: "Sun, and all you other lights of the world, do not move" — in brief, comprehending the whole host of heaven under the name of the Luminaries, as the king and queen of the stars. But if it was the Earth that had to stop, as the Copernicans would have it, there was no reason why God, who wished to work this prodigy at Joshua's prayer, should urge him by an inner impulse to stop the Moon as well; or certainly Saint Dionysius and the other sacred writers would have raised this question in vain — unless the Copernicans pretend that the miracle of the diurnal revolution was worked in the Earth, but that of the proper motion in the other Planets.

V. Finally, it is asked whether this prodigy was performed on any other occasion. For in 1 Paralipomenon, chapter 4, verse 22, someone among the sons of Sela is named *"Who made the Sun stand"*. The Rabbis say that this was

Elimelem, the husband of Noemi; but he was not of the sons of Sela, but of the sons of Phares. And in Hebrew, in place of that periphrasis *"Who made the Sun stand"*, there stands the name *Iocim*, or *Ioakim*, whose etymology is *"he made stand"*. But whether it was a fable invented by the Rabbis, as Caietanus on that passage of Paralipomenon and Serarius, question 25 on chapter 10 of Joshua, think, and the noun *Solem* ["the Sun"] was inserted into the sacred Bible by certain people; or whether he was rather a *stator Salis* [?], because like Elijah he brought drought upon the lands, as Magalianus holds on chapter 10 of Joshua, verse 22, annotation 7; or whether those words are to be read merely materially, as Salianus thinks in scholium 3 at the year of the World 2801, or as the etymology of a proper name, abstracting from the truth of its signification, as Gaspar Sanctius [Gaspar Sánchez] would have it on 4 Kings, chapter 20, and Bonartius, above; or whether, finally, he really made the Sun stand, as Saint Jerome reports from the tradition of the Hebrews, without rejecting it, in the book *De traditionibus Hebraeorum* on that passage, together with Abulensis in the same place, question 18, Lyranus, Hugo, Rabanus, Dionysius Carthusianus and many others — all this seems uncertain. Nor does Cornelius à Lapide, above, define anything, just as I do not either, out of reverence for the Vulgate edition, which I prefer to accept as it lies.

On this occasion, moreover, Cornelius à Lapide and Bonartius, on chapter 46 of Ecclesiasticus, add other prodigies, if not as great, still similar. For Saint Mutius the Abbot, as Ruffinus [Rufinus of Aquileia] relates in book 2 of the *Lives of the Fathers*, [Margin: Saint Mutius the Abbot and Bessarion stop the Sun.] chapter 9, going at God's prompting to visit one of the brethren who was sick: *"When he saw the Sun already sinking, he said to it: In the name of our Lord JESUS Christ, stand a little while on your way and wait for me, until I reach the village. And though it had already begun to sink in some part, it stopped, and did not set before the man of God had reached the village. And all who stood and watched the Sun's stays in its setting marvelled; and he said to them: Do you not remember the word of the Lord: If you have faith like a grain of mustard seed, you will do greater signs than these?"* So Ruffinus, and from him Maiolus in Colloquy 1 of the *Dies Caniculares*. And for a similar reason the Abbot Bessarion and another old man are said to have stopped the Sun, in book 6 of the *Lives of the Fathers*, chapter 2. [Margin: The Sun stands still under Charlemagne.] Turpin, Archbishop of Reims, who was an intimate of Charlemagne, relates in chapter 26 of the *Life of Charles* that the Sun also stood still under Charlemagne, to avenge the Saracens' killing of Roland and other princes of Charles in the defeat at Roncevaux; but Baronius [Cesare Baronio], Bellarminus [Roberto Bellarmino] and Possevinus [Antonio Possevino], in *De scriptoribus ecclesiasticis*, mark this author as suspect and fabulous. [Margin: And under Charles V.] Again, that the Sun stood still under Charles V, when he overcame the Saxon in battle, is affirmed by Ludovicus Avila [Luis de Ávila y Zúñiga] in book 2 of the *German War*, by Consalvus de Illescas [Gonzalo de Illescas] in part 2 of the *History of the Pontiffs*, and by others, whom Christoph Scheiner follows in the book *De refractionibus*

caelestibus, chapter 32. [Margin: And under Godefroy de Bouillon.] Likewise, Eucherius relates in his history, chapter 21, and Genebrardus from him in the *Chronicle*, book 4, that on the day on which Godefridus Bullionius [Godefroy de Bouillon] stormed Jerusalem, in the Year 1099 — on the sixth weekday, Friday, and at the hour at which Christ had been crucified there — the Sun went back some degrees. [Margin: And under Francisco Ximénez.] In the preceding century too, at the time when the Spaniards took Oran and defeated the Moors, the Sun stood still for 4 hours, as Alvarus Gomez [Álvar Gómez de Castro] testifies in book 4 of the *Deeds of Cardinal Francisco Ximénez*. Finally, Bonartius, in the place cited, relates from letters written from Madrid, and on the oath of Fathers of our Society, that the Sun, on the very point of setting, stood still for five hours and more, until our men, with others, could escape from the shoals that lie between Goa and Malacca, into which the ship's master had carelessly run, with the greatest danger. The danger was increased by a wind that suddenly arose and drove the ship onto the shoals. He says that this miracle was worked through the intercession of Saint Francis Xavier, to whom they had commended themselves, adding a vow to bring a silver lamp to his tomb. Cornelius à Lapide, however, explains portents of this kind as occurring not in the globe of the Sun itself but in its image, refracted through clouds or vapours of the horizon: "otherwise," he says, "if the Sun itself had stood still, the Astronomers of the whole world would have noted it." But I come to the other miracle, worked under Hezekiah.

[Margin: The miracle on the dial of Ahaz.] VI. For the Holy Fathers and Interpreters likewise treat of the Sun's going back under King Hezekiah, at the prayer and entreaties of Isaiah — on 4 Kings, chapter 20, or on chapter 38 of Isaiah, or on chapter 48 of Ecclesiasticus, or on book 2 of Paralipomenon, chapter 32, where this miracle is mentioned. But the most diligent of all are Gaspar Sanctius on 4 Kings, chapter 20; Salianus, volume 4 of the *Annals*, at the Year of the World 3322, from number 86; Cornelius à Lapide on chapter 38 of Isaiah and on chapter 48 of Ecclesiasticus; and Oliverius Bonartius on chapter 48 of Ecclesiasticus — all of our Society. The principal question about this miracle, then, is whether only the shadow went back on the dial of Ahaz, or the Sun itself. [Margin: Whether the Sun itself, or only the shadow, went back.] That only the shadow went back is held by Burgensis [Paul of Burgos] on 4 Kings, Vatablus, Arias Montanus on Isaiah, and Emmanuel Sà. Their reasons are these: the going back of the Sun is a greater miracle than its checking, and ought not to have been passed over in silence by Isaiah and by the writer of the book of Kings, who nevertheless mention only the shadow; nor was the choice offered to the King about anything but the shadow, since he, lying sick in bed, could not see the Sun, but only the shadow through the window; and finally, the Babylonians would not have been in doubt about the portent, since it would have been observed at Babylon too, and not merely carried to them by report. On the other hand, that the Sun itself went back is affirmed by the Siracid in Ecclesiasticus 48. Having said there: *"Isaiah, the great Prophet, and faithful in the*

sight of God", he at once added: *"In his days the Sun went back, and he added life to the King."* In Greek it is [Greek: ἀνεπόδισεν ὅ ἥλιος (anepodisen ho hēlios, "the Sun went back")], "the Sun stepped back"; and there the words [Greek: τὸ (to, the article, here "the words")] *"he added life to the King"* are referred to Isaiah rather than to the Sun — unless you understand that the Sun added it, that is, signified by its going back that it was to be added. Indeed, Isaiah himself says in chapter 38: *"And the Sun went back ten lines."* And in 2 Paralipomenon, chapter 32, that portent is said to have happened not at Jerusalem only but *"upon the earth"*, which signifies universality. Rightly, therefore, the opinion that the Sun itself stepped back was held by Saint Dionysius in the letter to Polycarp; Saints Jerome and Cyril on Isaiah; Procopius, Lyranus, Haymo, Hugo and Adamus in the same place; the Author of the *Wonders of Sacred Scripture*, book 2, chapter 48; Theodoret on Psalm 29; Elias Cretensis on the 19th oration of Saint Nazianzen [Gregory of Nazianzus]; Abulensis; Sanctius; Cornelius à Lapide on chapter 38 of Isaiah and on 4 Kings, chapter 20 — where Sanctius says that this is the constant opinion of almost all the Fathers and Interpreters; and finally Serarius on Joshua, and Salianus and Bonartius, above. Granted, Isaiah expressed to the King nothing but the going back of the shadow; but he did so as the effect from which the King could more conveniently and more evidently recognise the greatness of the miracle worked in the Sun. Nor did the Babylonian envoys doubt the substance of the prodigy, but only its manner and circumstances. There is the further point that dials can be made, by natural means, on which the shadow alone goes back, as Clavius demonstrates in the *Fabrica instrumenti ad horologia*, chapter 21. There he wisely refutes Petrus Nonius [Pedro Nunes], who says that this portent was produced by the sciateric art, and therefore naturally, when in fact it was worked miraculously and supernaturally.

[Margin: The time of the miracle, and the kind of dial.] VII. It is disputed, moreover, at what time of day, and on what kind of sundial, this miracle was worked. Scripture, in 4 Kings, chapter 20, brings in Isaiah speaking thus: *"Do you wish the shadow to go up ten lines, or to return as many steps?"*; and soon after: *"And Hezekiah said: It is easy for the shadow to grow ten lines; nor do I wish this to happen, but that it return backward ten steps. So Isaiah the Prophet called upon the Lord, and he brought the shadow back by the lines by which it had already gone down on the dial of Ahaz, ten steps backward."* And Isaiah 38: *"And the Sun went back ten lines, by the steps by which it had gone down."* From these words Cornelius à Lapide, on chapter 38 of Isaiah, gathers that the dial stood on a plane vertical to the Horizon, erected so that its face was turned toward the South. For since it is probable that it stood in full view on the wall of the royal palace, for common use, those conditions — the ascent and descent of the shadow, or rather the ascent of a growing shadow, and 10 hours from Rising to Setting — cannot be verified except of such a dial, since on it an ascent through 10 steps or lines could be marked, and a descent through as many lines. Concave hemispheres would have been less apt and convenient for this, and less exposed to observers. The same author further gathers that the time was midday, or close to it. For

as the Sun rose to the meridian the shadow had already gone down through 10 lines, and it would have gone up through as many if the King had wished; so the shadow seems to have stood at the point that ended the descent already made and began the ascent to come. [Margin: The antiquity of the sundial.] On this occasion note, in passing, that the inventor of the sundial was not Anaximenes, as Pliny the Elder thinks (book 2, chapter 76), who flourished about the fiftieth Olympiad; for Ahaz lived about the first Olympiad, that is, about 200 years before Anaximenes, and many more years before L. Papyrius Cursor, who was the first to set up a sundial for the Romans, at the temple of Quirinus, about the time of the first Punic War, as Pliny relates in book 7, chapter 60. But see Clavius, book 1 of the *Gnomonice*, page 7.

[Margin: Were the 10 lines of hours, or of half-hours?] VIII. Third, it is asked whether those ten lines marked ten whole hours, or halves or quarters of hours; on this depends the solution of the question about the length of that day. Another question, however, is also involved here: whether the going back of the Sun and of the shadow was made suddenly and in a moment, or gradually and successively. That those lines were lines of hours, not of half-hours, was indicated by Saint Dionysius in the letter to Polycarp; by the Author of the *Wonders of Sacred Scripture*, book 2, chapter 28; by Eucherius on 4 Kings, chapter 20; and by the Chaldean Paraphrast, Bede and Angelomus in the same place. And from these premises Saint Dionysius indicated that the day was 32 hours long: namely 12 temporal hours, and 10 which the Sun added by going back, and as many others which it added by going forward; and so the artificial day would have been nearly tripled. Others, however, say that the day was doubled: so the Author of the *Wonders of Sacred Scripture*; Procopius on chapter 38 of Isaiah; Rabbi Kimhi, as cited by Vatablus; Tornielus at the year of the world 3322, number 6; Abulensis and Caietanus; and Sanctius on Isaiah. On their view, either those lines were lines of half-hours, or else the Sun went back to the East in a moment of time, or at least very swiftly, and afterwards, advancing in its ordinary succession, added another 10 hours to the 12 hours, or 12 hours to the 10 already elapsed, so that the day was 22 hours long. For if the lines had been lines of hours, and the Sun had kept its accustomed slowness both in going back and in going forward, the Sun's shadow could not have grown through another 10 lines, as Isaiah had offered the King; otherwise the artificial day at Jerusalem would naturally have been 20 hours long — hours, that is, as marked on the dial. Cornelius à Lapide on Isaiah, however, thinks that it was then midday, and that only 10 hours were marked on that dial — five before midday, through 10 half-hour lines, and as many after midday; and so he likewise judges that the day was 15 hours long, a number certainly well suited to signify the 15 years which God granted Hezekiah over and above. From this it will likewise follow that the day lengthened at Joshua's command was longer than this day prolonged at Isaiah's prayers — although this is by no means necessary, since those words of Joshua 10, *"there was not before and after so long a day"*, refer only to the time at which the author of that book was writing. [Margin: Whether

the other stars went back.] On this occasion too, however, not the Sun alone but the other stars as well went back proportionally, so that the arrangement of the heavens should not be disturbed: so teach Saint Dionysius to Polycarp; the Author of the *Wonders of Sacred Scripture*, book 2, chapter 28; Abulensis on 4 Kings, question 33; Baradius, volume 1, book 5, chapter 14; Cornelius on chapter 38 of Isaiah; Salianus, above; and Bonartius.

[Margin: Our opinion on this miracle.] IX. For my part, I explain the miracle already mentioned as follows, always saving a better judgement. *First*, I say that Isaiah, when he offered the King a choice either of the ascent of the shadow or of its going back, offered some miracle in either case, and therefore an ascent of the shadow to be completed not after 10 or 5 hours, by the ordinary descent of the Sun to its Setting, but at once and almost in a single instant. The King, however, chose the going back of the shadow: either he had not understood that miracle, since the Prophet had not expressed it, or, when the choice was now being given, he was choosing a greater and more evident miracle, open to no ambiguity. For whether that miracle was to be completed very swiftly and almost in an instant, or with the succession customary to the Sun's motion, some miracle at any rate was to come, namely in the going back of the shadow through the same lines along which it had already advanced since the Sun's rising; and if this very thing was to be done in a moment, there was to be a double miracle within the one miracle. *Second*, I say that the going back of the Sun and of the shadow was made in a moment, or with the greatest possible speed, which is what those words signify: *"And he brought the shadow back by the lines by which it had already gone down."* For there was no reason to defer the completion of the miracle, and to keep the King in suspense and the Prophet occupied in the palace to no purpose for 10 or 5 hours. Finally, reason demands that the ascent of the shadow in going back should have been such as it would have been in going forward, if the King had chosen the first part of the offer; and that was without doubt going to be miraculous, and so without the customary delay of the Sun's course. Nor was it in keeping with the divine liberality, when the King was choosing a greater and surer miracle, to restrict it to the going back alone. *Third*, I say that, although it is gathered from the sacred text and from the property of that dial that the time was not yet past midday — for the shadow had gone down through ten lines, owing to the ascent of the Sun, in such a way that it could still have gone up through ten other lines as the Sun descended to its Setting, and could have grown, or advanced, to as many remaining lines, if the King had wished it to advance so suddenly at God's command; and yet it had already gone down through all the lines through which it went back, and had not gone down through some of them and begun to go up through any — it does not follow, nevertheless, that it was the very moment of midday. It could have been a time before midday, and the Prophet could still have made the shadow (had the King chosen the bare ascent) pass the meridian line at once and hurry to the last of the eastern lines, the Sun rushing headlong to its setting in an almost momentary leap.

Fourth, I say that the ten lines through which the shadow had already gone down cannot have been lines marking or bounding whole hours. For the hours before midday at Jerusalem, even on the artificial solstitial day, could not have been 10, but only 7^{h} $2½^{m}$, since the longest artificial day there is only 14^{h} 5^{m}. It is more probable, therefore, that they were lines of half-hours, and that on that dial not only the hours but also the half-hours were marked, as is done by the more careful makers of dials, in either the same or different colours. The ten lines, therefore, took in five whole hours. *Fifth*, I say that it is not probable that the craftsman, who was so diligent that on the wall of the royal palace not only the hours but also the di- stinctions of the half-hours were marked by him, should have drawn a dial so maimed and imperfect that for the whole day it marked only 10 hours, namely five before midday and as many after midday. For if those hours were unequal, or temporal, the first hour after sunrise and the last before sunset would have been missing, since the artificial day is of 12 temporal hours. If, on the other hand, the hours had been equal, then on the days nearest the summer solstice two hours would have been missing after sunrise and two before sunset, since the artificial day was, as I said, of 14 equal hours; and at the time of the equinox one hour would have been missing at each end. Nor would the dial have served accurately except on the days of the winter solstice, on which the day there was of 10 equal hours. Let it be granted, then, that at least 12 unequal hours were marked on that dial, by 24 half-hour lines; for it is more probable that the Hebrews, like the Romans and the Greeks, used temporal hours, as we have shown in our Chronological work. Therefore, since Isaiah brought the shadow back through all the lines through which it had already gone down, and these were 10, it is very probable that it was then the 5th hour from sunrise, and that one hour still remained until midday; and so, since the Sun covered these hours twice that day by its motion, the whole day was 17 temporal hours long. If, however, anyone would have those hours be equal, and the day a winter one, then it would have been midday, and the 10 hours of the artificial day, with 5 added over and above, would make up 15 hours — suitable for signifying the 15 years granted to the King by God over and above, granted that this signification is not necessary.

There are some, however, who think that the dial consisted of steps, like those of a staircase, rather than of lines, as we shall see below from Saints Hilary, Cyril and Jerome, and from Elias Cretensis. [Margin: Whether the dial of Ahaz was a kind of staircase.] But a dial of this kind — one that would serve all year round, be open at once to everyone, could be seen by the King lying in bed, and show at once the bringing back or going back of the shadow through 10 lines — would have been extremely difficult. Nor does it follow, from Scripture's saying *Steps*, that they were the real steps of some staircase. Scripture called them so because the lines on vertical dials are arranged about the gnomon in such a way that they imitate the steps of spiral staircases, whose plan an architect may lay out on the remaining plane; and in the same way the lines of the parallels on globes or geographical maps imitate the steps of a

staircase — the origin of the name *Climates* — and yet they are not in reality steps by which one climbs.

X. So much, up to this point, for the substance and circumstances of each miracle, whose exposition was not to be interrupted by the opinions of the Fathers. Here, however, I wish to set those opinions out, beginning with Saint Dionysius the Areopagite. [Margin: The opinion of Saint Dionysius the Areopagite on the standing and the going back of the Sun under Joshua and Hezekiah.] In the Letter to Polycarp, speaking of Apollophanes the Sophist, who wickedly used divine testimonies against divine things, he says: *"Apollophanes, being a wise man, ought to have understood that nothing in the order and motion of the heavens could ever have been changed from what it is, unless that order had had him who both keeps it in being and brought it forth, by whose motion and nod it was led to that point — he who, as it is in the divine writings, makes and changes all things. Why, then, does he not worship him whom from this too we have come to know as truly the God of all, admiring his power, which is the cause of all things and cannot be expressed in words? For by him the Sun and the Moon, by a force and a standing above the nature of the universe of things, are fixed wholly immovable; and for a whole day all things stand at the same points. Or, what is greater: while all the higher bodies were moved in a circle, the bodies contained in their embrace were not carried round in a circle together with them. And when another day was made, by continuation, almost three times longer, by a full twenty hours, either the universe took up again a contrary course for so long a time, and turned back with contrary revolutions so admirable; or the Sun, its varied motion contracted in its course into ten hours, completed the same whole again, going back, in another ten hours. This indeed terrified the Babylonians, rightly and deservedly, and subdued them to Hezekiah without a battle, as though he were equal and like to God, and excelled men."*

[Margin: Saint Hilary's opinion on the dial of Ahaz.] As for the dial of Ahaz, Saint Hilary, in the preface to Psalm 119, seems to hold that it distinguished the hours by certain steps of a staircase. For he says: *"In the book of Isaiah, when Hezekiah prayed, the sign given him of his prolonged well-being lies in the steps. Now there were in his house ten steps, which the Sun, beyond the course of its time, climbed with its light advancing, and again, on the day of his death, descended with its light descending."* And on Psalm 135 he says: *"That too belonged to a great miracle, when, at the word of Jesus Nave, the Sun stood still over against Gabaon, and the Moon over against the valley of Haylon. I do not know what more wonderful than this could be done: when, at the word of a man, the Sun and the Moon were held back from the law of their course for the stay of one day."* In the same place he describes the immobility of the Earth, as we shall see below.

[Margin: The opinion of Saint Cyril of Alexandria on the same dial and miracle.] Saint Cyril of Alexandria also seems to have held the same view as Saint Hilary, when in book 3 on Isaiah, at chapter 38, he says: *"They say that Ahaz, the father of Hezekiah, had certain steps made in his house, as if by a kind of machine and art, which would number the hours, as it were, and would measure the course of the Sun by the passage of the shadow made on them. That Hezekiah was to*

return to life God made plain, by the going back of the Sun's shadow and by the day being prolonged to an unaccustomed measure of hours." And on chapter 39 of Isaiah he teaches that the miracle was worked in the Sun itself, so that this was one of the principal reasons for which Merodach, son of Ladan, King of Babylon, sent envoys to Hezekiah with gifts and letters: *"But it was not only this that struck the Babylonian with wonder — that Hezekiah, I say, had been sick and had returned to good health again — but that altogether great and truly immense sign. By an ineffable and divine nod the Sun was ordered to run back and go backward, so that the shadow too went back 10 steps. The Babylonians and the Chaldeans, called by name the Sophi, that is, the wise, since they boast of being subtly learned in knowing the course of the Sun and the rising and setting of the stars, were not, it seems, ignorant of this reversion and of the unaccustomed length of that day,"* etc.

[Margin: Saint Gregory Nazianzen's view.] Let Saint Gregory Nazianzen be added to these. In oration 18, which is in praise of Saint Cyprian the Martyr, he lists these among the ancient miracles: *"The sea is divided, bread is poured down like rain, the Sun's course is checked,"* etc.; and in oration 20: *"Who divided the sea, held back the course of the Sun, checked the river,"* etc. Indeed, earlier, in oration 3, he had said: *"The Sun standing, and the Moon held back, and the course of the Jordan cut off,"* etc. In these places he points to the miracle of Joshua. But in oration 19 he touches on the other miracle, saying of Hezekiah: *"Whom God presented with an increase of life, and showed it by the shadow of the steps driven back, honouring the king, that is, at once with a favour and with a sign, and confirming it by the increase of the day."* [Margin: The opinion of Elias Cretensis on the dial of Ahaz, etc.] Elias Cretensis expounds this passage as follows: *"In the commentaries of the ancients we find that that day was of thirty-two hours. For there were twelve steps of the house, by which Ahaz, his father, reckoning the hours and giving his attention to this art, perceived the course of the Sun exactly; and at that time the Sun had run its course and reached the tenth hour, and in the same way the shadow had gone down ten steps. The Sun went back again ten steps, and so there came to be twenty hours; after which, when the Sun had again traversed its course in its own order, it went off to its setting. Thus the length of that day came to thirty-two hours, and this miracle spread through the whole world; for all saw the Sun go back."*

Theodoret agrees with these on Psalm 20, which begins (*"Lord, in your strength the King shall rejoice"*), and which he thinks was composed by David, in a prophetic spirit, about Hezekiah. So, explaining the verse (*"He asked life of you, and you gave it to him"*) and the following one (*"his glory is great in your salvation; glory and great splendour,"* etc.), [Margin: Theodoret's testimony on the miracle of Hezekiah.] he adds: *"You not only granted him life, but made him illustrious and famous, both by the wonderful destruction of his enemies and also because the Sun ran in reversed order; and this became so well known by land and sea that the king of the Babylonians sent gifts to those who had formerly lavished gifts on the Assyrians."*

Saint Augustine also mentions both miracles, in book 21 of the *City of God*, chapter 8, where he first relates the following about the star of Venus, from

M. Varro: [Margin: Venus changes its course.] *"In the heavens there occurred a wonderful portent. For in the most noble star of Venus, which Plautus calls Vesperugo and Homer Hesperos, calling it most beautiful, Castor writes that so great a portent occurred that it changed its colour, size, shape and course — which neither before nor since has happened so. Adrastus of Cyzicus and Dion of Naples, noble Mathematicians, said that this happened in the reign of King Ogyges."* And, a few words later: *"It certainly disturbed then the Canons of the Astrologers, if there were any by then,"* etc. But, to our point: [Margin: Saint Augustine's view.] *"We, however, read in the divine books that the Sun itself also stood still, when the holy man Jesus Nave had asked this of the Lord God, until victory should end the battle he had begun; and that it went back, so that the fifteen years added to King Hezekiah's life might be signified by this prodigy too, joined to God's promise."* Let us now hear Saint Jerome on chapter 7 of Isaiah, where, explaining the words (*"Ask for himself [?] a sign from the Lord God of yours, whether in the depth of hell or in the height above"*), and teaching by many examples that some signs were given on earth and others in heaven, he adds: *"I think that Hezekiah too, when the Sun went back ten lines, received a sign from heaven, and so did Jesus son of Nave at Gabaon and Haialon, when the Sun and the Moon stood still."* [Margin: Saint Jerome's opinion on the dial of Ahaz.] And on chapter 38 of Isaiah he indicates two opinions on the form of the dial, in these words: *"The sign is given that the Sun should go back ten steps, which we, following Symmachus, have translated as lines and dial; he understood the steps as lines, so as to make the sense plainer to readers. Or else the steps had been built by mechanical art in such a way that the shadow, descending through each of them, marked off the spaces of the hours; and this sign was a figure both of the present time and of the future, how the Sun would return to its beginning,"* etc. And on chapter 39: *"And because among the Chaldeans, whose King was Nebuchadnezzar, there is observation of the heavenly bodies, and the courses of the stars are known by long use and practice — as is shown too at the Lord's nativity — they understood that the Sun had gone back, and the spaces of the day been doubled, in the service of him whom they held to be the only God. And when they inquired into the causes and the reason of this miracle, as the report flew through all nations, they learned that the course of the brightest sign too had been changed on account of the illness of the king of Judah."* Saint Gregory, most briefly, on 1 Kings, chapter 2: *"Joshua fixed the Sun by his prayers."*

XI. That verse of Psalm 18, *"In the Sun he has set his tabernacle,"* etc., is taken by Saint Augustine, Saint Bernard, Cassiodorus and others in a mystical sense, of CHRIST and the Church, or of CHRIST in the womb of the Virgin MARY, etc.; its literal sense, nevertheless, concerns the Sun, the most illustrious of all the Planets, and its very swift motion. The words *"His going forth is from the highest heaven, and his circuit even to its highest"* are interpreted by many, as our Lorinus says on this passage, of the diurnal motion of the Sun from Rising to Setting: [Margin: The opinion of Caesarius.] especially by Caesarius, dialogue 1, Saint Chrysostom, homily 6 on Genesis, and Theodoret on Psalm 18; and by Saint Thomas, of the point of midday and of midnight, or of the same point of

midday, from which the Sun would begin the day astronomically and, meeting itself again at the same point, would end it. [Margin: The opinion of Arias and Caietanus.] Arias Montanus, on the other hand, understands it of the annual motion from one Equinoctial or Solstitial point to the other; and Caietanus of both motions, so that the diurnal is indicated by the going forth from the highest heaven, and the annual by the circuit, or return. We need not linger here over that passage of Esdras, book 3, chapter 34, *"The swift course of the Sun turns the heaven round in a circle to its place in one day"*, for the book is apocryphal. [Margin: What does Saint Nazianzen say of the Sun?] Let us rather hear Saint Gregory Nazianzen in oration 20, in which he compares Saint Basil to the Sun, saying: *"In David, Psalm 18, the Sun is praised for its beauty and size, its course and speed, its force and power: for it resembles a bridegroom in splendour and a giant in size, and, advancing far and wide, it has such force that it lights the furthest regions equally from the furthest, nor is its heat diminished in any way by distances of place. For Basil, beauty was virtue; size, Theology; course, the perpetual activity of virtue, reaching toward God by daily ascents; power, the sowing and distribution of doctrine. So I shall not shrink from saying even this: that his sound went out into all the earth, and the force of his words to the ends of the world."* He thus understands David to the letter, just as Saint John Chrysostom does in homily 6 on chapter 1 of Genesis. [Margin: Saint Chrysostom.] For Chrysostom says: *"Declaring its beauty, blessed David himself also said, 'as a bridegroom coming forth,' etc. Do you see how he declared its beauty and its speed in working? For by saying 'its going forth is from the highest heaven, and its circuit even to the highest of heaven', he signified to us how in one moment of time it runs through the whole world and sends its rays from end to end,"* etc. [Margin: Theodoret.] Let Theodoret follow him, on Psalm 18, who reads thus with three interpreters: *"He set the tabernacle of the Sun in them"*; and he adds: *"namely, in the heavens"*; and a little later: *"He set a tabernacle for the Sun in the heavens, so that it should be carried by them; in beauty it was to imitate a bridegroom coming forth from his chamber with much splendour, and in speed a kind of giant running with great strength, with nothing to hinder it from advancing further. For so it rises from the East and, running across in the space of one day, reaches the West."* And since David had said first, *"The heavens tell the glory of God"*, the Holy Fathers teach from this that the beauty and speed of the Sun, and its most ordered course, constant since the beginning of the world, are a most splendid argument of divine power and providence. [Margin: Saints Athanasius, Cyril of Jerusalem, Basil, Ambrose, Nazianzen, Nyssen.] So, above all, Saint Athanasius, in the oration *Against the Idols*, page 9; Saint Cyril of Jerusalem, Catechesis 9; Saint Basil, homily 6 on the *Hexaemeron*; Saint Ambrose, book 4 of the *Hexaemeron*, chapter 6; and Saint Gregory Nazianzen, oration 34, last page — who, weaving together outstanding encomia of the Sun, extol above all its course and motion. Saint Gregory of Nyssa, in oration 10 on the Canticle, proclaims the motion of the Sun about the Earth thus: *"For who does not know the motion of the Sun — that from the East, through the southern region of the heaven, it inclines by its appointed course toward the Setting? And since*

the figure of the Earth is globular, as those skilled in these things say, from whichever part it has been lit by the Sun, darkness must be produced on the other, opposite part," etc.

XII. Finally there remains that most celebrated testimony for the motion of the Sun, Ecclesiastes 1: *"The Sun rises and sets,"* etc., *"and returns to its place, and, reborn there,"* etc. [Margin: Saint Jerome on the motion of the Sun.] Saint Jerome, on chapter 1 of Ecclesiastes, interprets this passage of both motions of the Sun, first the diurnal and afterwards the annual — granted that he begins thus, in a florid and almost poetic style: *"The Sun itself, which was given to mortals for light, shows daily, by its rising and its setting, the passing away of the world. After it has dipped its burning wheel in the Ocean, it returns by ways unknown to us to the place from which it had set out, and, the course of the night completed, it bursts forth again in haste from its chamber."* Read the rest in his work. With Saint Jerome, moreover, the whole of this passage is understood of both courses of the Sun, the diurnal and the annual, by Saints Bonaventure and Thaumaturgus; by Theophilus of Alexandria in the 3rd Paschal Letter; and likewise by Olympiodorus, Hugo Victorinus [Hugh of Saint Victor], Hugo Cardinalis [Hugh of Saint-Cher], Lyranus, Albinus, Carthusianus, Caietanus and Io. Arboreus. But it is understood either solely or chiefly of the diurnal course by Saint John Chrysostom, homily 12 to the people of Antioch; by Saint Thomas on chapter 37 of Job; and by Lorinus on chapter 1 of Ecclesiastes. Yet certainly, on the one and the same day on which it circles through the South, it is not bent toward the North. Nor, even if it were moved by the force of two orbs, namely the Eccentric and the Epicycle, would it for that reason return into its many circles; and much less on the same day from the aux to the opposite of the aux, as Lorinus seems to want. For the revolution of these orbs is not completed in full except by the annual motion; and it is properly of that motion that both the bending now to the South, now to the North, and the plurality of circuits into which it returns, are verified. It is better, therefore, with Saint Jerome and the many others above, that those earlier words, up to *"and reborn there"*, are understood of the one diurnal circuit, and the rest of the remaining diurnal circuits that make up the annual course.

Moreover, to signify the acceleration of the Sun's motion, and, as it were, its longing to return to begin the new day, the words *"returns to its place"* are rendered by Aquila [Greek: προσέπνευσε (prosepneuse, "breathed toward")], or [Greek: ειασπνεῖ (eiaspnei, glossed on the page with the following Latin)] — that is, *aspirat*, "breathes toward" — and by the Chaldean, *"pants toward its place"*. The following words, *"and, reborn there, it circles through the South and bends to the North"*, the Chaldean paraphrast translates thus: *"It walks through the way of the abyss along the whole side of the South, and returns to the side of the North."* Finally, the words *"Surveying all things round about, the spirit goes on"* are separated by punctuation by Hugo Victorinus, who begins a new period from *"The spirit goes on,"* etc., understanding by the name of spirit the air and wind, which is carried round in a circle by the rapt motion [motus raptus].

This view did not displease Thaumaturgus either, in his paraphrase, nor Saint Bonaventure and Caietanus in their commentaries. [Margin: Whether, and why, the Sun is called Spirit.] Saint Nyssenus, however, interprets it of man, who is named spirit from his better part. But Saint Jerome understands the Sun, because it animates, breathes, etc., and he uses the testimony of Virgil, who sang in book 1 of the *Georgics*:

> *In the beginning the heaven and the earth and the liquid plains, The shining globe of the Moon and the Titanian stars, A spirit within nourishes,*
> etc.

But there Virgil signifies, from a false opinion, that the world is animated, not merely the Sun. Nor did Saint Jerome think the Sun animated, at least not by an intellective soul, since in the letter to Avitus, 59, and on Deuteronomy, chapter 32, and on chapter 45 of Isaiah, he counts it among the errors of Origen that he attributed a rational soul to the heavens. For my part, as I have already said in section 2, chapter 1, number 4, I am very well pleased with the opinion of Saint Thomas in opusculum 10, at article 6, who understands by the name of spirit an Angel, who is said figuratively to go on in a circuit because he makes the Sun go on while he moves it — in the way in which, in 1. Romans 8 [?], it is said, *"The Spirit intercedes for us"*, that is, causes intercession. And this opinion, in the end, seems to be subscribed to by

Lorinus, on chapter 1 of Ecclesiastes, toward the end of verse 6. [Margin: The unanimous consensus of the Fathers on the motion of the Sun.] Besides so many Fathers and interpreters who taught that the Sun moves locally, as we have so far shown abundantly, there concur also Origen, homily 3 on Matthew; Isidore of Pelusium, letter 100; Isidore of Seville, book 3 of the *Origins*, chapter 49; Saint Chrysostom on Psalm 134; Saint Justin, question 59; Glycas, part 2 of the *Annals*, page 271; Claudius Mamertus, book 2 of *De statu animae*, chapter 13; and, in the end, who does not? With good reason, therefore, we conclude that there is a unanimous consensus of the Fathers that the heaven moves round in a circle with local motion, and that Sacred Scripture was so understood by them.

Sententia Patrum de Stabilitate Loco ac Libramento Terrae ex Sacra Scriptura.

The Opinion of the Fathers on the Stability, Place and Poise of the Earth, from Sacred Scripture.

[Margin: Saint Basil on the place and immobility of the Earth.] XIII. In the first place there presents itself Saint Basil, homily 1 of the *Hexaemeron*, where he approves Aristotle's opinion and says: *"Setting these things aside, then, I advise that we do not even ask on what subject the earth is founded and rests; for in this way the mind will be thrown into a kind of dizziness, while its reasoning finds no certain way out"*; and again: *"This answer is indeed the safest for us for acquiring an understanding of Scripture, and useful to the hearers."* Then: *"But our discourse has already shown that the middle of the universe is the lowest place. Do not wonder, then, if the earth falls away in no part of itself, since it occupies the very middle place of the*

universe, the place most suited to it by nature." [Margin: Saint Ambrose.] Saint Ambrose imitates him, as is his custom, in book 1 of the *Hexaemeron*, chapter 6: *"Let it suffice for knowledge, what the series of the divine Scriptures contains: that he hangs the earth upon nothing. What is it to us to discuss whether it hangs in the air or upon water, so that a controversy should arise from it as to how the thin and softer nature of air could sustain the earthly mass?"* Afterwards he concludes that it has its stability not from being suspended in the middle as if in an equal balance, *"but because the majesty of God binds it by law through his will, so that it remains stable above an unstable and empty nature."* And again: *"The earth, moreover, is the foundation on which we stand; although both the discourse from outside celebrates it as being in the very centre of the heaven, and Scripture seems to signify this, when Job says: He hangs the earth upon nothing."* [Margin: Job, chapter 26.] [Margin: Saint Chrysostom.] Saint Chrysostom concurs, homily 2 on Psalm 103, at the words (*"you who founded the earth upon your stability; it shall not be inclined,"* etc.): *"But what,"* he says, *"is this stability? The effecting power, of course; for in his hand are all the ends of the earth."* And in homily 12 to the people of Antioch: *"The earth is fixed, but the waters are always moving."* [Margin: Saint Nazianzen.] [Margin: Saint Hilary.] Saint Nazianzen, oration 34, and Saint Hilary on Psalm 135 have recourse to the same power of God, Hilary explaining the words (*"who made firm the earth above the waters"*): *"Let it suffice to have learned from the authority of the Prophet that the earth stands above the waters with a hanging firmness."* [Margin: Saint Justin.] So too Saint Justin Martyr, question 130, where it is concluded most elegantly: *"The waters, then, sustain the heaven; the Earth sustains the waters; and the divine nod sustains the Earth — the nod which,"* he says, *"hangs the Earth upon nothing."* [Margin: Saint Damascene.] And so too Saint John Damascene, book 2 of *On the Orthodox Faith*, chapter 10: *"The earth is one of the four elements, endowed with dryness, coldness and heaviness, and without motion,"* etc. *"But on what thing it sits, or on what foundation it rests, no mortal has been able to say."* [Margin: Saint Nazianzen again.] There is also extant, in the *Catena of the Fathers* on Job, a saying of Saint Gregory Nazianzen about the Earth: *"By what reason, then, does it remain stable and firm? What is it that sustains it, and what makes firm that very thing which sustains it? Reason indeed finds nothing on which it may rest, except the will of God, which hung it and made it firm; nor is there anything else by which the earth is propped up or held together."* [Margin: Saint Jerome.] Saint Jerome was plainly of the same mind on chapter 26 of Job, expounding the saying (*"who hangs the earth upon nothing"*): *"This is to be understood thus: either that the earth was made from nothing, or that there is nothing beneath the earth by which the earth is sustained, since it itself sustains all things, and is itself balanced immovably by the power of God; for in him are all things, and by him all things are held together; of whom the Apostle says: upholding all things by the word of his power."* On Psalm 12, moreover, he affirms that the Earth is the lowest of all things, *"because the Earth is the end of creatures"*; and on chapter 13 of Jeremiah: *"Just as the heaven cannot be higher than it is, so the earth cannot be lower than it is."* [Margin: Saint Prosper.] Add to these Saint Prosper on Psalm 103, who in this tossing takes refuge in the

divine power as in a sacred anchor: *"Of this Earth it is laboriously asserted that it is not to be inclined for ever and ever, although it has been said: heaven and earth shall pass away. For that it was founded upon its firmness we can understand thus: that the force by which it is sustained and held together is called its firmness; and although this is hidden from us, it is nevertheless rightly believed to lie in the wisdom and power of the Creator, who made all things firm in himself, in whom he also founded all things."* [Margin: Saint Athanasius.] Saint Athanasius, however, seems to have held that the immediate cause is heaviness, in the oration *Against the Idols*, when he says: *"that the Earth, remaining immovable, bears fruit, and, being by nature the heaviest of all things, does not sink, but stands stable and immovable."* [Margin: Procopius.] So too Procopius, on chapter 1 of Genesis and on chapter 13 of Isaiah, explaining the words (*"Upon this I will trouble the heaven, and I will move the earth from its place"*), adds: *"For, suspended in an equal balance, since it stands at an equal distance on every side, it must be unmoved."* Theodoret hints at the same intrinsic force on Psalm 103: *"When he had built it upon itself, he granted it never to be moved"*; and Clement of Alexandria, in the oration of exhortation to the Gentiles: *"This has adorned the Universe also for you, in measure and harmony, and has brought the discord of the elements into order, so that the whole world might become a harmony for it; and it made the Earth stable and solid."* As for the other interpreters, they may be seen on chapters 26 and 38 of Job, on Psalm 103, and on chapter 1 of Ecclesiastes.

[Margin: Twofold cause of the stability of the Earth.] XIV. From what has been said so far we gather two causes of the Earth's stability. [Margin: 1. Extrinsic.] One is, as it were, extrinsic, which most of the Fathers acknowledge: namely the divine will and omnipotence. Of these, the will decreed from eternity that the Earth should be placed and should stand in this virtual part of the divine immensity itself rather than another — or, as others put it, in this region of imaginary space rather than another; the omnipotence, carrying out the divine decree, created the Earth from the beginning of the world and now preserves it in the same place, so that there is no created power that could move it from its place. For most of the Scholastics, with Saint Thomas in opusculum 10, article 16, and on book 2 of *De caelo*, lecture 2, teach that not even the Angels can do this — not so much because they could not overcome the whole weight of the earth, as lest the order of the universe established by God be disturbed. And Saint Thomas adds: *"But if it is asked about circular motion, by which the said order is not varied, it seems that the earth rests naturally."* [Margin: 2. Intrinsic cause.] The other cause is some intrinsic virtue conferred on the Earth by God, by which it can hold itself in the place once assigned to it: whether that is magnetism, as our Fathers Cabeo, Athanasius Kircher and Nicolaus Zucchius hold; or the weight and heaviness of all its parts toward that centre which God willed to be at once the centre of the Earth and of the Universe, from which it comes about that the Earth recedes equally from the extremes. [Margin: Ancient profane authors for the stability of the Earth.] And this cause, besides Saint Athanasius, Procopius and Theodoret, is acknowledged by most of the

philosophers, with Aristotle, book 2 of *De caelo*, from text 77, and by many other profane writers: namely Cicero in the *Dream of Scipio*, and Macrobius on it, book 1, chapter 22; Vitruvius, book 9, chapter 4; Pliny the Elder, book 2, chapter 5; Cleomedes, book 1 of the *Cyclical Theory*, chapter 9; Martianus Capella, page 192; Manilius, book 1 of the *Astronomicon*, chapter 2; and Ovid, book 1 of the *Metamorphoses*, in this verse:

Nor did the earth hang in the encircling air, poised by its own weights.

To him belongs this also:

The earth stands by its own force; from standing by force it is called Vesta.

On account of this very force, too, the Pythagoreans and Platonists attributed the Cube to the Earth as the symbol of stability, on the testimony of Plutarch, book 2 of *De placitis*, chapter 6; and the Roman priests used to sacrifice seated to Ops at rest — who is the Earth, so named because she brings help [opem] to all — according to the same Plutarch in the *Roman Questions*. And finally Cleanthes brought a charge before the Areopagites against Aristarchus of Samos, who carried the Earth round through the annual orb, on the ground that he had moved the sacred things of Vesta from their place, as the same Plutarch relates in the little work *On the Face of the Moon*. It should seem the less surprising, then, if we too bring a charge before the Sacred Tribunal against the Aristarchuses of our own age, returning to life from the underworld, that they have striven to move from its place, as it were, not only the Earth but Sacred Scripture itself. For the literal sense is, so to speak, a kind of place and foundation, in which its other senses rest; and in the following chapter we shall teach that, so long as nothing stands in the way, it is to be retained immovably.

CAPVT XXXVIII. Responsiones Copernicanorum ad Auctoritates Sacras circa Solis Motum ac Terrae Quietem afferuntur ac refutantur, tanquam non solum parum solidae, sed etiam periculosae.

Chapter 38. The replies of the Copernicans to the sacred authorities concerning the motion of the Sun and the rest of the Earth are brought forward and refuted, as being not only of little solidity but also dangerous.

I. The form and beauty of the Copernican hypothesis so bewitched the innovators and its followers, by some allurement or other, that they did not hesitate both to twist the words of Sacred Scripture to an alien sense and to despise the judgement of the holy Fathers and of churchmen. Copernicus himself, of course, raised their spirits and led the way by his example, in the preface of his work to Paul III, as we shall soon see. [Margin: Authors of the replies to the sacred authorities.] In devising replies to the sacred authorities he was followed by Johannes Kepler, in the Introduction to the *Commentaries on Mars*, in the notes to chapter 1 of his *Mysterium Cosmographicum*, and in the *Epitome of Copernican Astronomy*, book 1, from page 138; by Christoph Rothmann, in Tycho's letters, page 130; and by Paulus Antonius Foscarinus, in

the letter to Sebastianus Fantonus, General of the Carmelite Order, published in Italian at Naples in the year 1615, and afterwards rendered into Latin by David Lotaeus and published at Lyon in the year 1641 together with Galileo's dialogues, at the expense of Io. Antonius Huguetan [Jean-Antoine Huguetan] — in which letter the Author professes a conciliation of the authorities of Sacred Scripture and of theological propositions with Copernicus's opinion on the mobility of the Earth and the stability of the Sun. From these, some things were taken for the same conventicle by Redemptus Baranzanus [Redento Baranzano], part 1 of the *Uranoscopia*, doubt 10, member 3; by Petrus Herigonius [Pierre Hérigone], volume 5 of the *Cursus Mathematicus*, in the *Theorics of the Planets*, at the end of book 2, or from page 626; and by Jacob van Lansbergen, physician, in the *Apologia* for the commentaries of Philips van Lansbergen on the diurnal and annual motion of the Earth, against Libertus Fromondus, theologian of Louvain, and Io. Baptista Morinus [Jean-Baptiste Morin], mathematician of Paris, published at Middelburg in the year 1633. [Margin: The 5 chief heads of the replies.] Their replies, moreover, are reduced to 5 heads. The *first* is the lack of skill in astronomy, or in the whole of mathematics, in some ecclesiastical persons, so that they call the judgement and censure of those persons on these matters into contempt or suspicion. The *second* is that a knowledge of physical and astronomical matters was neither handed down by the Holy Spirit in the divine writings, nor is to be cared for by churchmen. The *third* (which is next to the second) is that if any things are found said in passing in the Scriptures about the motion of the Sun or the stability of the Earth, they do not belong to the dogmas of the Faith, especially if they are said by someone who does not bear the person of God. The *fourth* is that the passages customarily gathered from Sacred Scripture against the motion of the Earth and the stability of the Sun ought to be taken not in the proper literal sense, nor in the rigour of truth, but in a popular sense, or as to appearance, or figuratively. The *fifth* is the confirmation of the fourth from many other passages of Scripture in which physical or astronomical matters are touched on, which, it is agreed, are nevertheless to be taken not according to the rigour of the letter but with a more benign interpretation. Let us begin with the first head, setting up questions for each one, to be resolved in this chapter and the following.

Quaestio 1. An Physicae & Astronomiae peritia necessaria sit Personis Ecclesiasticis, ad ferendam validam & obligatoriam censuram de rebus physicis & astronomicis; ex regulis Fidei ac sacrorum Canonum.

Question 1. Whether skill in physics and astronomy is necessary for ecclesiastical persons in order to pass a valid and binding censure on physical and astronomical matters, by the rules of the Faith and of the sacred Canons.

[Margin: Authors for the affirmative side.] II. There is no doubt that Copernicus would answer us in the affirmative. For since he recognised to how great a danger he was exposing the novelty of his hypothesis, he said in the preface to Paul III, Supreme Pontiff: *"Although I know that the thoughts of a philosopher are*

far removed from the judgement of the common crowd, because his pursuit is to search out the truth in all things, so far as that is permitted to human reason by God, yet I consider that opinions wholly foreign to rightness are to be shunned. And so, when I thought to myself how absurd an ἀκρόαμα [Greek: akroama, "a thing heard", a doctrine] *those would think it who know that this opinion — that the earth is placed immovable in the middle of the heaven, as its centre — has been confirmed by the judgements of many centuries, if I on the contrary were to assert that the earth moves, I hesitated long with myself whether I should bring to light my commentaries written for the demonstration of its motion,"* etc. Though he acknowledged these things, he nevertheless adds that he had dedicated his work *On the Revolutions* to that Supreme Pontiff for this reason: as to one most highly cultivated in these arts, and so that he should not seem to fear the judgement of anyone fit to judge of these matters. [Margin: Why Copernicus dedicated his work to Paul III.] His words are: *"But so that the learned and the unlearned alike might see that I shun the judgement of no one at all, I preferred to dedicate these night-labours of mine to Your Holiness rather than to anyone else; because even in this most remote corner of the earth in which I live, you are held most eminent both for the dignity of your rank and for your love of all letters, and of mathematics too, so that by your authority and judgement you can easily check the bites of slanderers, although it is a proverb that there is no remedy against the bite of a sycophant."* But what will you do, Copernicus, if some who are steeped in mathematics not at all, or only very slightly, have spoken against you because of the authority of Sacred Scripture, which resists your hypothesis? [Margin: The arrogant saying of Copernicus.] He answers without hesitation: *"If perhaps there will be* ματαιόλογοι [Greek: mataiologoi, "idle talkers"] *who, though ignorant of all mathematics, nevertheless take on themselves to judge of these matters, and on account of some passage of Scripture badly twisted to their purpose dare to censure and attack this undertaking of mine, I do not regard them at all, so much so that I even despise their judgement as rash."* Lest this arrogant saying should seem harsher than is just, he tries to soften it with an example, saying: *"For it is not unknown that Lactantius, otherwise a celebrated writer but little of a mathematician, speaks quite childishly about the shape of the earth when he mocks those who declared that the earth has the shape of a globe. So it ought not to seem surprising if such people laugh at us too. Mathematics is written for mathematicians."*

III. With this standard raised, as it were, and the cap of liberty once thrown up, nothing stood in the way of minds like his flocking to the same place. Accordingly Kepler, at the very beginning of chapter 1 of his *Mysterium Cosmographicum*, in an opening certainly praiseworthy, professes as follows: *"Although it is pious, right at the beginning of this disputation about Nature, to see whether anything is said contrary to the sacred writings, I nevertheless judge it untimely to raise that controversy here before I am challenged [prius atq. folliciter]. This I promise in general: that I will say nothing that is injurious to the Sacred Writings, and that if Copernicus is convicted of anything together with me, I will hold him for nothing. And this has always been my mind, ever since I began to know Copernicus's books of*

the Revolutions." [Margin: Kepler requires skill in astronomy in religious men who are to judge of this matter.] But in the notes on this chapter, expounding the little word *untimely*, he says: *"Copernicus meets this scruple in the preface to Paul the Third, Supreme Pontiff, but a little too rigidly; and for that address he paid the penalty in the end, more than 70 years having elapsed since the publication of the book and since his death. For he has been SUSPENDED, says the censure, UNTIL HE BE CORRECTED; but I think that this too is to be understood: UNTIL HE BE EXPLAINED. For how he is not contrary to Scripture — his purpose, of course, being very far different — I have tried to show by reasons and examples in the Introduction to the Commentaries on Mars. I have also explained the words of Copernicus himself more clearly at the end of book 1 of the Epitome of Astronomy; and I hope that in these places the religious will be satisfied, provided they bring to this judgement such a mind and such a knowledge of astronomy that the glory of the visible works of God can safely be entrusted to their patronage."* But how much he grieved over the censures passed on Copernicus he indicates not much later, saying of every most religious person: *"Let him consider with himself whether the causes of the conciliation sought between the tongue and the finger of God were just enough, weighty enough; and whether it is expedient, that conciliation being repudiated, to suppress by censures this fame of the divine works of immense beauty."* Afterwards he concludes that the censures are vain, and that their invalidity, once discovered, brings it about that they have no force even in matter in which they ought to have force; for that is what that allegorical clause of Kepler's means: *"The edge of the axe, however, once struck against iron, is afterwards of no use even against wood. Let whoever it concerns take this to heart."* In the Introduction to the *Commentaries on Mars*, however, he offers the unlearned this advice in a freer style: *"But whoever is too dull to be able to grasp the science of astronomy, or too weak to believe Copernicus without offence to his piety, I advise him to dismiss the school of astronomy, to condemn, if he pleases, any of the tenets of the philosophers whatever, to mind his own affairs and, giving up this wandering through the world, to go home to cultivate his own little field; and, raising to this visible heaven the eyes with which alone he sees, to pour himself out with his whole heart in thanksgiving and praise of God the Creator, certain that he renders God no less worship than the astronomer, to whom God has granted that he sees more keenly with the eye of the mind,"* etc. [Margin: Kepler taunts the Office of the Sacred Inquisition.] Not content with this, he shows a little later, when he adds the following, with how much obstinacy he prefers his own judgement to the judgement of the Fathers and of the Church: *"To the tenets of the Saints about these natural matters, however, I answer in one word: in theology the weight of authorities is to be weighed, but in philosophy the weight of reasons. Holy, then, is Lactantius, who denied that the Earth is round; holy is Augustine, who, granting its roundness, nevertheless denied the Antipodes; holy is the Office of the people of today, who, granting the smallness of the Earth, nevertheless deny its motion. But holier to me is the truth, I who demonstrate from philosophy — with due respect to the doctors — that the Earth is round, and inhabited round about by Antipodes, and of most contemptible smallness, and finally that it is carried*

through the stars." As if Lactantius were held in the Catholic Church to be a saint, or a Doctor of infallible authority — a man whom all we Catholics admit to have erred in many things! Or as if the smallness of the Earth necessarily drew its motion along with it — when Kepler himself, with Copernicus, moves through the annual orb not the Earth alone, but the elementary sphere together with the lunar heaven, a machine much larger than the Sun! And how is due respect to the Doctors of the Church kept, if, on the question of how the divine writings are to be understood, you obstinately set your own judgement before theirs? Or with what face do you dare to say that the motion of the Earth has been demonstrated by you, when you have brought forward only probable arguments, or arguments that are indeed necessary, but only on the hypothesis of the Earth's motion, showing that if it moved the celestial phenomena could conveniently be saved?

Not content with this either, Kepler, at the end of book 1 of the *Epitome of Astronomy*, page 140, does not hesitate to brand with the mark of superstition those who have not subscribed to Copernicus. For he says: *"In the present age all the most eminent of the philosophers and astronomers concur with Copernicus. This ice has been broken; we are winning by the votes of the better sort; for the rest, almost nothing stands in the way but superstition, or fear of the Cleantheses."* By Cleantheses he means either the accusers or the judges of the Sacred Inquisition; for on the same page he had said that Aristarchus of Samos was accused of sacrilege before the Areopagites by Cleanthes, because, in asserting that the Earth moves, he had moved the sacred things of Vesta. In the *Hyperaspistes*, page 195, the same author calls it a wicked servitude to serve ecclesiastical decrees — if, understand, they have been made badly; and he seems tacitly to censure the sacred censors as punishing themselves while they punish others, or as not daring to defend the marks set by the sacred censures. For what else does that quip mean: *"But let Perilus teach his own bull to bellow. And unless all conjecture deceives me, he dreads the brow of Silenus, which Aegle herself, foolish, painted with vermilion"*? Finally, in the preface to book 4 of the same *Epitome*, he recites this fragment from his letter of apology: *"How severely were all the astronomers rebuked by the first Christians! Did not Eusebius write of one that he preferred to desert Christianity (because excommunicated, I suppose) rather than his profession? Did not those who taught that there are Antipodes seem to Tertullian and Augustine to be too wise? And Virgil, Bishop of Salzburg, was indeed removed from office because he had dared to assert it."* Here, however, he either lies or is mistaken in his history; for Virgil was neither removed from office nor accused for the bare assertion of the Antipodes, as we shall see below. [Margin: The danger of erring in censures through lack of skill in astronomy.] Lastly, to strengthen the opinion of these men, one could adduce the danger of passing censures, through lack of skill, against some truth demonstrable by natural reason — in the way that the Athenian judges of old, through ignorance of the causes from which eclipses of the Moon arise, would not tolerate Anaxagoras, nor those who taught that the Moon is deprived of its light by the shadow of the earth, as we related from

Plutarch's *Nicias* in book 5, chapter 1, number 1. Accordingly, how anxious Saint Augustine was lest his brethren, through lack of skill, should seek from the divine writings a defence for manifest errors in philosophy is already clear from what was said in chapter 36, number 2; and it is a wonder that his words should have been unknown to the Copernicans, since at first sight they support them wonderfully.

[Margin: The negative answer, and the reasons for it.] IV. Notwithstanding these things, however, to the question proposed at the beginning of number 3 I answer in the negative. *First*: when some physical or mathematical question has devolved, directly or indirectly, into a question about the Catholic Faith — as has this one, in which it is debated whether the motion of the Earth and the stability of the Sun are against Sacred Scripture — then, for those who preside over the Holy Office of the Inquisition to be able to pass a sufficient judgement, that is, one not only licit and prudent but also valid and binding, it suffices that they should have come to know, either by themselves or through other learned men, that a proposition on the one hand is not evidently true by physical or mathematical evidence, and on the other hand is against the plain and literal sense of Sacred Scripture in that part in which it is Sacred Scripture, or against some proposition of a canonical writer. [Margin: 1. Reason for the Sacred Inquisitors.] For this by itself is enough for them reasonably to be able to pass censure against the said proposition, because of that universal necessity by which all we Catholics are bound: that so long as no evident and manifest inconvenience follows from Sacred Scripture understood to the letter, we should understand it to the letter, and not follow another sense repugnant to the literal one. How much more, then, does it suffice if some proposition not only is not evidently true, but is against physical evidence, and has been hissed off as absurd, commonly in the Academies and also by some professors of astronomy; and if, on the other hand, it is established that the holy Fathers, the interpreters of the divine writings and the Doctors of the Church taught the contrary from Sacred Scripture? And so indeed the matter stood, and stands, in our case. For the Sacred Congregation deputed by the Supreme Pontiff Paul V, and afterwards by Urban VIII, a man most knowledgeable in astronomical matters, included, besides many Cardinals themselves very skilled in philosophy, theology or canon law and imbued to no slight degree with knowledge of the sacred writings, many others also appointed for this inquiry; and a most diligent inquiry had already been made beforehand by the Qualifiers. It was already certain, too, that the Copernican hypothesis not only had been demonstrated by no one (since the Copernicans themselves admit that all the celestial appearances can be explained, saving the rigour of astronomical truth, by either hypothesis — that is, of the Earth either moved or at rest), but also was against the evidence of common sense, and against that plain, obvious and literal sense of Sacred Scripture which the unanimous consensus of the Fathers and of the sacred interpreters had followed for one thousand six hundred years. Nothing, therefore, could keep them from being able to pass censure against

the hypothesis of Copernicus. Nor was it required that the persons already mentioned should be professors and masters of astronomy, since it was already commonly known in the Schools that the Copernicans had not demonstrated that their hypothesis is so in fact, but only that it is possible, and that by it the celestial phenomena are explained quite neatly and readily; and on account of this neatness and usefulness in composing and using astronomical tables, that hypothesis was permitted as a mere hypothesis, that is, provided that no one asserts that it is so in fact.

[Margin: 2. Reason.] V. *Second*: just as Kepler, for judging of astronomical matters, requires no common skill in astronomy and appeals to the court of Urania, so in matters of Faith and Religion Catholics must require no common skill in the divine writings, and appeal to the ecclesiastical court. Now this controversy, although materially, as they say in the Schools, it is astronomical and physical, is nevertheless formally a controversy about a matter pertaining to Faith and Religion, as I shall teach below; and this is clear even from the fact that what is disputed now is not whether the hypothesis of Copernicus is possible, or fit for saving the celestial phenomena, but whether it is against Sacred Scripture according to the received unanimous interpretation of the Fathers. And this skill is had far more abundantly and clearly from the sacred books themselves and the holy Fathers, and from the rules of theology and of the sacred Canons, than from mathematicians and astronomers as such. [Margin: 3. Reason.] *Third*: even if the Sacred Inquisitors or any judge in a cause of the Faith holding authority from the Supreme Pontiff, were to pronounce some censure against some proposition in a doubtful matter — one so far demonstrated in neither direction, or demonstrated indeed but not held to be so — and yet were to do so on suspicion of novelty and of some danger in matters of the Faith, the presumption would have to be in their favour, and that censure would have to be observed at least in the external forum, at any rate if it could be observed without grave inconvenience and without a fear of the kind that befalls a steadfast man. It would nevertheless be permitted to work for the said judges, once they recognized the demonstration of the truth for themselves or through others, to remove the censure. I assume this doctrine here as most certain, partly from the common doctrine on laws and on presumptions, partly from the particular doctrine of canon law on ecclesiastical censures.

VI. [Margin: 4th reason.] *Fourth*: not only the Sacred Inquisitors, whom Kepler seems to brand with inexperience in astronomical matters, but many men highly expert in astronomy have judged the Copernican hypothesis both absurd in itself and contrary to the authority of the Divine Scriptures. [Margin: Tycho's judgement on the sense of Sacred Scripture in this matter.] Thus Tycho, in a letter to Rothmann of the year 1589 February 21, admonishes him as follows: "There is no reason for you to persuade yourself that the physical absurdities which accompany the Copernican hypothesis have been sufficiently refuted by him. Much less will the pleas you put forward in excusing those things to which Sacred Scripture asserts the contrary be able to earn a place.

For the authority and reverence of the Divine Scriptures both is and ought to be greater than that it should be fitting to draw them on in this way, like a buskin." [Margin: And of Clavius.] Our own Clavius too, certainly no ordinary astronomer, in chapter 1 of the *Sphere*, where he taught by reasons that the Earth is immobile, added on page 196: "This opinion is also favoured by the sacred Scriptures, which in very many places affirm that the Earth is immobile and testify that the Sun and the other stars move; for we read in Psalm 103, etc." [Margin: And of Nicolaus Mulerus.] Nicolaus Mulerus [Nicolaus Mulerius] conducted himself with equal reverence in the *Isagoge* to the *Frisian Tables*, page 318. There, having mentioned the opinion of the Pythagoreans on the Sun's place at the centre of the world and on the Earth's motion, he added: "But since this question does not belong to this place, we do not wish to examine it here, testifying only this: that the authority of Sacred Scripture is so great with us, and our mind is moved by so great a reverence for it, that we do not dare to go over to the opinion of the Pythagoreans against its plain sense."

[Margin: The prudence and piety of Pierre Gassendi.] We also have a remarkable example of prudence and moderation in Pierre Gassendi. In his second letter *De motu impresso a motore translato* he had set out many arguments for the motion of the Earth and had solved not a few of the arguments usually raised against it; yet he first kept himself within the limits of a mere hypothesis, and then, in a doubtful matter, subscribed to the decrees of the Sacred Congregation. For he concludes that letter, written in the year 1640 on the Ides of December to Petrus Puteanus [Pierre Dupuy], thus: "You do not require me to repeat that I did this not to assert motion for the Earth, but, out of love of the truth, to hint that its rest must be established by a firmer reason." And a little after: "Accordingly I am of a mind to revere that decree by which some cardinals are said to have approved the rest of the Earth. For although the Copernicans might defend the passages of sacred Scripture which attribute standing or rest to the Earth and motion to the Sun, or hold that they are to be explained of the appearance itself, as they put it, etc., nevertheless, because those passages are explained otherwise by men whose authority in the Church is, as is well known, so great, for that reason I myself stand with them, and on this occasion I do not blush to make my understanding captive. Not that I therefore think it to be an article of faith; for (so far as I know, at least) this has not been asserted by them, nor promulgated and received throughout the universal Church; but because their judgement must be held a prejudgement, which could not be of the greatest weight among the faithful [?]." [Margin: The author's vote.] Nor can our own testimony, such as it is, be rejected on the ground that we have by no means weighed or understood the arguments of the Copernicans, if what we have taught in this whole Section is read through. Therefore let no one count us among those whom Copernicus and Kepler despise as unfit to judge of these matters.

VII. [Margin: 5th reason.] *Fifth*: as regards the Antipodes denied by Lactantius, he is not to be counted among the holy Fathers, and he denied the

Antipodes for no other reason than that he thought the heaven to be semispherical, and the Earth not round but itself hemispherical too, as is clear to anyone who reads him in book 3 of the *Divine Institutes*, chapter 24. [Margin: Why Lactantius denied the Antipodes.] Nor must it be denied that some other Fathers thought the heaven to be like a vault, and hemispherical. [Margin: Fathers for whom the heaven was hemispherical.] Among these, Ascanius Martinengus, in the *Glossa magna* on chapter 1 of Genesis, numbers Saint Chrysostom, *On the Incomprehensible Nature of God*, chapter 2; Procopius, Acacius and Severianus on book 1 of Genesis; Theophylact on the Epistle to the Hebrews; Theodoret, *On Matter and the World*; and Euthymius on Psalm 103. But this is no obstacle to us at all, because the Fathers who acknowledged the roundness of the heaven and of the Earth are far more numerous; and so there was no unanimous consensus of the Fathers about this figure, whereas about the stability of the Earth there was.

[Margin: Why Saint Augustine denied the Antipodes.] As for Saint Augustine, he did indeed deny the Antipodes in book 16 of the *City of God*, chapter 9, although he granted the roundness of the Earth. Because in the preceding chapters he had taught that all the human beings who inhabit the Earth are descended from Adam, he thought that the lower hemisphere of the Earth, which lies opposite the Holy Land, was either covered with water, or that men could not have migrated to those lands across so vast an expanse of seas as he supposed to lie between, since no history existed of such a migration. In this he followed not so much Lactantius, as Ludovicus Vives [Juan Luis Vives] says in that place, as Cicero in book 6 of the *Republic*, who held that an immense Ocean is poured between us and those antipodal lands — lest he should admit that there are some men not descended from Adam, and therefore neither redeemed by Christ nor belonging to the City of God. I do not transcribe all the holy Doctor's words, since they are readily available. But neither does this support our adversaries, because this was not the unanimous opinion of the Fathers, nor was it deduced from any positive passage of Scripture taken literally. What moved Saint Augustine was only a negative authority: Scripture, while it describes the propagation of the sons of Noah over the earth by families and heads, makes no mention of a land of the Antipodes to which they migrated. For he says: "Therefore, among those peoples of men of that time, who are gathered to have been divided into seventy nations and as many languages, let us seek, if we can, to find that City of God on pilgrimage on earth, etc." So, as Vives says, Saint Augustine had no slight reason for denying the Antipodes.

VIII. [Margin: Whether, and why, Bishop Virgil was deposed for asserting the Antipodes.] Finally, as regards Virgil, Bishop of Salzburg: as Baronius relates in volume 9, at the year of Christ 744, he was a very religious man from Ireland who, out of zeal for spreading the Faith, came with Sidonius into Baioaria, or Bavaria, and was known to Boniface, who presided over that Church as its prelate. A certain minister of his had baptized someone in debased Latin with this formula: *Baptizo te in nomine Patria et Filia et Spiritua Sancta*.

When Boniface, Bishop of Mainz, judged that the person should be baptized again, Virgil and Sidonius, out of modesty, thought that he should not be reproved before others, but that recourse should be had by letter to the Roman Pontiff Zachary; and Zachary wrote back to Boniface that the man was not to be rebaptized, and Boniface acquiesced in the judgement of the Supreme Pontiff, as Baronius reports in the same place. But at the year 748 Baronius relates a threefold calumny brought against Virgil before Boniface: namely, that he was sowing hatred between Boniface and Odilo, Duke of Bavaria; that he boasted that he had obtained the vacant bishopric from the Apostolic See; and finally that he asserted "that a new world is found, which is lit by another Sun and another Moon." Baronius shows that these were calumnies: when Boniface had written to the Pontiff about them, and the Pontiff had given him power to convene a provincial council and, if Virgil were proved guilty, to expel him from the Church and deprive him of the priesthood, not only was Virgil not deprived, but he was even raised to the episcopate, persevered in apostolic work with Boniface, spread the Gospel far and wide among the neighbouring peoples, and was afterwards enrolled among the saints by Gregory IX with solemn rite.

[Margin: Pope Zachary's rescript to Boniface concerning Virgil.] And lest anyone should think that Pope Zachary erred on a question of law — indeed, so that it may be clear that he did not err even on a question of fact, since he ordered Virgil to be punished not absolutely but only on a supposition — here is part of Zachary's letter to Boniface, which Baronius quotes in full at the year 748 [ab Annum] [recte: ad Annum]: "It has also been intimated by Your Holiness that that Virgil (we do not know whether he is called a priest), malignantly opposed to you because he was being charged by you with erring from the Catholic doctrine, was sowing hatred between you and Otilo, Duke of the Bavarians, saying that he had received licence from us that he should obtain the diocese of one of those four bishops, now deceased, whom your fraternity ordained there. This is by no means true, for iniquity has lied to itself. As for the perverse doctrine which he has spoken against the Lord and against his own soul — namely, that there is another world and other men beneath the earth, and another Sun and Moon — if he is convicted of so confessing, convene a council, expel him from the Church and deprive him of the honour of the priesthood, etc." Yet he was not expelled from the Church by Boniface; and so far was he from being deprived of the priesthood that he was rather made Bishop of Salzburg. He had therefore been accused of the said crimes by calumny; for because he was acceptable to God, it was necessary that temptation should prove him. Ignorantly, then, did Kepler above, and Jacob van Lansbergen in the *Apologia*, tract 3, assert that he was cast out of his bishopric; and far more imprudently do they say that this was done because he asserted the Antipodes. For, as Baronius adds in the margin to Zachary's letter: "To have doubted about the Antipodes is not heresy; but to have held that there are several worlds is repugnant to divine Scripture, and is therefore

shown to be heresy." On this story see also M. Velserus [Markus Welser], book 5 of the *Res Boicae*, Aventinus in the *Annales Boiorum*, and Mazzonus [Jacopo Mazzoni] in the *Defence of Dante*, book 3, chapter 33.

Quaestio 2. Vtrum Scriptura Doceat res Physicas aut Astronomicas.

Question 2. Whether Scripture teaches physical or astronomical matters.

IX. [Margin: What Sacred Scripture teaches.] It is not the purpose of sacred Scripture, the Copernicans say, nor of the Holy Spirit, to instruct men in physical or mathematical matters, but in those which pertain to morals and to eternal salvation. In their favour seems to be the Apostle's saying in Romans 15: "For whatever things were written were written for our instruction, that through patience and the comfort of the Scriptures we might have hope"; and 2 Timothy 2: "All scripture divinely inspired is profitable for teaching, for reproving, for instructing in history, that the man of God may be perfect, furnished for every good work." Finally there is the pronouncement of Christ the Lord himself on the love of God and of neighbour: "On these two commandments hangs the whole law and the Prophets"; and the saying of Isaiah 48: "I am the Lord God, teaching useful things," where the gloss adds: "not subtle ones." Accordingly Saint Augustine, in book 2 of *De Genesi ad litteram*, chapter 8, said: "It is also commonly asked what form and figure of the heaven is to be believed according to our Scriptures. For many dispute much about those things which our forebears, with greater prudence, passed over, since they would be of no profit to learners for the blessed life and — what is worse — take up much precious time that ought to be spent on wholesome things. For what does it matter to me whether the heaven, like a sphere, encloses the earth on every side, balanced in the middle of the mass of the world, or covers it from above on one side like a disc? But since the faith of the Scriptures is at stake, etc." And not much later: "Briefly, it must be said that our authors knew about the figure of the heaven what the truth holds, but that the Spirit of God, who spoke through them, did not wish to teach men these things, which would profit no one's salvation."

X. [Margin: Reply to Question 2.] To the question proposed, however, one must answer with a distinction. It must be granted that the per se and primary end of Sacred Scripture is not to instruct men in matters pertaining to mere physics or mathematics, or to natural or civil history and the other natural arts and faculties, stopping there. [Margin: For what end Sacred Scripture teaches natural things.] Yet it must be denied that it does not, from time to time, teach in passing some things pertaining to these sciences and natural faculties, so as to use them as means either to ground some doctrine on morals or on things conducive to eternal salvation, or to show the divine omnipotence, wisdom, providence, etc. "For the invisible things of him, from the creation of the world, are clearly seen, being understood by the things that are made; his eternal power also, and divinity," as the Apostle declares, Romans, chapter 1. Therefore, if it is established from the Sacred Scriptures who created heaven

and Earth with the remaining elements and mixed bodies, and in what order, and how great a wisdom and power shines forth in the founding, preservation and governance of these things, in their motions and other effects, and how far all creatures depend on his nod, the exposition of these things is referred to that end at which all divinely inspired Scripture aims. That is why the Apostle wisely added: "for instructing in history, that the man of God may be perfect, furnished for every good work." In these words he embraces not only civil history mixed with ecclesiastical, such as is found in the books of Judges, Kings, Paralipomenon, Tobit, Esther, etc., but also natural history, which is handed down at the beginning of sacred Genesis, in some chapters of Job and Ecclesiasticus, and also in some Psalms — toward the end I have mentioned. And so, to our purpose: when the speed and most constant course of the Sun, or the stability of the Earth hanging in the middle of the air, is asserted in the Scriptures, the Fathers and every pious reader of the divine codices [?] rise from this to the admiration and praise of God no less than when he learns from the same Sacred Letters that the Earth was uncovered from the waters and in places made higher than they, and prepared with so many plants and animals for the service of men; that the surges of the seething and raging sea are confined within fixed bounds of shore; that the rivers not only enter the sea in such a way that it does not overflow, but return to the place from which they come forth, to flow again; that the winds and snows are brought out of the treasuries of God; that the waters are bound up in the clouds; and many other things like these, which belong to natural history and physical science. And if these things are subtle, they are not so subtle that they are not also useful.

[Margin: Sacred Scripture does not teach falsehood.] *Second*: even if Sacred Scripture did not ordain the knowledge of such things to the end I have mentioned, far be it, nevertheless, that we should grant that in propositions of this kind it is deceived or deceives, or says what is false. And it would say what is false if, while the Sun stands still and the Earth goes round, it nevertheless said absolutely, as it does say, that the Sun moves and that the Earth is stable and in no way inclines. [Margin: Tycho's view on the truth of Sacred Scripture in physical matters.] Hence Tycho aptly added, in the letter already named above, page 147: "For although they — that is, the divine Letters — in physical matters and in certain others for the most part accommodate themselves to the capacity of the common people, far be it nevertheless that on that account we should hold that they speak in so vulgar a way that we do not believe they also set forth true things. Thus Moses, although in the first chapter of Genesis, treating of the creation of the world, he does not relate the inner secrets of astronomy, since he was writing for an unlearned people, nevertheless puts forward nothing that could not be granted by the astronomers themselves as well." How much knowledge of natural things sacred Scripture has handed down to us, and from how many errors and false opinions of the philosophers it has freed us, is clear from Saints Basil and Ambrose in their books *Hexaemeron*; from Junilius, Bede, Molina, Suárez and others who wrote on the work of the

six days; from Philo's *De opificio mundi*; and also from the books of Franciscus Valesius [Francisco Vallés] *De sacra philosophia*.

Quaestio 3. An Assertiones de Solis Motu & Stabilitate Terrae, quae habentur in Sacris Scripturis, ad Fidei & Religionis quaestiones pertineant.

Question 3. Whether the assertions about the motion of the Sun and the stability of the Earth that are found in the Sacred Scriptures pertain to questions of Faith and Religion.

XI. The Copernicans indeed deny it; but they ought to have used the distinction of Saint Thomas, *Prima Secundae*, question 1, article 6, and of the other theologians. Some things are of the Faith per se and directly, and of this kind are those commonly called Articles of Faith: for example, that God is one in nature and three in Persons; that CHRIST is God and man; the future resurrection of the dead, etc. [Margin: What things are of the Faith per se and directly, and what by accident and indirectly.] Others are of the Faith indirectly and by accident, insofar as they are ordained, as means or dispositions, to the things that are of the Faith per se. Of these Saint Thomas gives the example of Abraham, of whom we are bound to believe that he had two sons, one by the bondwoman and the other by the free woman. It is in this second way, then, that the propositions of Sacred Scripture on the Earth's standing still and the Sun's motion pertain to the Faith. [Margin: Reply to Question 3.] Moreover, those assertions pertain to a question of Faith not only insofar as they are ordained to the knowledge and praise of God, but also because, if they were not true, the truth of the whole of Sacred Scripture could be called into doubt and suspicion of falsehood; and so not even the remaining propositions, which per se concern the Faith and the morals necessary to eternal salvation, could be solidly proved and confirmed by the testimony of the divine Letters. Hence it is that Saint Augustine, in that very book 2 of *De Genesi ad litteram*, chapter 8 — in which he had raised the question "What form and figure of the heaven is to be believed according to our Scriptures", and had said that knowledge of this matter does not of itself pertain to salvation and the blessed life — nevertheless added: "But since the faith of the Scriptures is at stake, because of th- at reason which I have mentioned more than once — lest anyone who does not understand the divine utterances, when he has either found something of this kind about these matters in our books or heard it from them, which seems to oppose reasons he himself has grasped, should in no way believe them when they advise, narrate or pronounce other useful things — it must be said briefly about the figure of the heaven, etc." It therefore pertains to a question of Faith that we know in what way what Sacred Scripture says about these matters is true — whether in the plain and obvious, or literal, sense, or in some figurative sense — and that we defend its truth and authority. That authority is so great that it is even of faith that Tobias's dog, wagging its tail for joy at the return of the younger Tobias, ran ahead like a messenger,

because Sacred Scripture narrates this in Tobit 11; and yet the motion of that tail concerns the Faith far less than the motion of the Sun and the standing still of the Earth.

XII. [Margin: The decree of Trent on the true interpretation of Sacred Scripture.] From what has been said it is now clear how the decree of the holy Council of Trent, session 4, on the edition and use of the sacred books, is to be understood, where it is said: "Furthermore, in order to restrain petulant spirits, it decrees that no one, relying on his own prudence, in matters of faith and of morals pertaining to the edification of Christian doctrine, shall, wresting the sacred Scripture to his own senses, dare to interpret the sacred Scripture itself contrary to that sense which holy Mother Church has held and holds — whose it is to judge of the true sense and interpretation of the holy Scriptures — or even contrary to the unanimous consensus of the Fathers, even though such interpretations were never at any time to be published." For the holy Council must be judged to have included under these words everything that is referred to the Faith, both per se and directly and by accident and indirectly, according to the doctrine of Saint Augustine and Saint Thomas set out above. This is all the more so since a similar decree was confirmed, in a more universal formula and without restriction, by Pius IV in the Bull found at the end of the Council of Trent, where the formula of the profession of Faith is given, by which we are bound not to interpret Sacred Scripture except according to the unanimous consensus of the Fathers, as I have already said in chapter 36, number 1.

Saint Augustine is to be understood in the same way when, in book 2 of *De Genesi ad litteram*, chapter 10, he denies that churchmen ought to be occupied with these questions. For he is speaking of those on whom other ministries, more necessary to the salvation of souls, have been enjoined; not of the Doctors of the Church in the case where the authority of the divine Letters must be defended even down to one iota or one tittle, nor of those who have been appointed by their prelates to unravel these questions for the common benefit of the Church, so as to help others by the shortcut of their own labours. The holy Doctor's words, moreover, are these: "Some brethren also raise the question about the motion of the heaven, whether it stands still or moves. For if it moves, they say, how is it a firmament? But if it stands still, how do the stars, which are believed to be fixed in it, go round from the East to the West, the northern stars completing shorter circuits near the poles? etc. To them I answer that these things are investigated by very subtle and laborious reasonings, in order to perceive truly whether it is so or not so; and for taking up and treating these I have no time now, nor ought those to have it whom we desire to see instructed for their own salvation and for the necessary benefit of holy Church." Yet to defend the truth of sacred Scripture he adds that the name "firmament" does not necessarily signify immobility, since it can signify firmness and the impassable boundary of the upper and lower waters; and he adds: "Nor, if the truth should persuade us that the heaven stands still, are we prevented by the circuit of the stars from being able to understand this; for it has been found by those very

men who have investigated this most curiously and with the greatest leisure that, even with the heaven not moving, if the stars alone turned, everything that has been observed and comprehended in the revolutions of the stars could have come about."

As for Saint Thomas's saying in Opusculum 10, that "it seems safer that the things which the philosophers have commonly held, and which are not repugnant to our faith, should neither be asserted as dogmas of faith nor denied as contrary to it, lest occasion be given to the wise of this world to despise the doctrine of the faith": this does not favour the Copernicans. They do not teach what the philosophers have commonly held, but things so contrary to the common philosophy that they seem to be repugnant to the faith as well, by the very fact that they are repugnant to the divine Letters. I therefore wonder that Jacob van Lansbergen, in his *Apologia*, tract 3, used — or rather abused — this authority of Saint Thomas on behalf of Copernicus.

CAPVT XXXIX.

Chapter 39.

Quaestio 4. An Assertiones Sacrae Scripturae de Solis Motu & Stabilitate Terrae accipiendae sint secundum sensum literalem, An Figurate vel ad sensum vulgi, seu quoad apparentiam. Vbi de sensus literalis vsu Regulae Ecclesiasticae traduntur.

Question 4. Whether the assertions of Sacred Scripture about the motion of the Sun and the stability of the Earth are to be taken according to the literal sense, or figuratively, or according to the sense of the common people, that is, as regards appearance. Here the ecclesiastical rules on the use of the literal sense are set out.

I. This is the chief controversy between us and the Copernicans. For they contend that the Sacred Letters, in these physical and mathematical matters, accommodate themselves to the common sense of the people; and since to the people the Sun seems to move and the Earth to stand still — because the Sun seems much smaller than the Earth — Sacred Scripture had to adapt itself to them, leaving it meanwhile to more learned men to reconcile the tongue of God with the finger of God from the harmony and beauty of the divine works, judged by reason rather than by sense: that is, to reconcile the sacred expressions with effects worthy of the omnipotence of God. So Kepler, in the notes to chapter 1 of the *Mysterium Cosmographicum*: "There is indeed a tongue of God; but there is also a finger of God. And who would deny that the tongue of God is adapted both to its purpose and, on that account, to the popular tongue of men? In the most evident matters, therefore, to twist the tongue of God so that it refutes the finger of God in nature — this every most religious man will most carefully avoid. Let him to whom the praises of our Creator and Lord are a concern read — let him read, I say, my fifth book of the *Harmonics*; and, having grasped the most exquisitely harmonic polity of the motions, let him consider with himself whether the causes of the reconciliation sought between

the tongue and the finger of God were just enough and weighty enough, or whether it is expedient to reject the reconciliation and suppress with censures this report of the immense beauty of the divine works." The same author too, in the *Epitome of Copernican Astronomy*, at the end of book 1, having observed that astronomers also speak according to the sense of the eyes when they say that the planets stand still and retrogress, that the Sun keeps its solstices, etc., goes on to infer: "How much less will it be required of the divinely inspired Scriptures that, setting aside the common usage of speech, they should weigh their words by the rule of natural science, and by abstruse and ill-timed expressions about matters beyond the grasp of those to be instructed disturb the simple people of God, and thereby block for themselves the way to their genuine and far more sublime goal." And so, everywhere, do the other supporters of the Copernican sect whenever something from the sacrosanct books is objected to them.

II. [Margin: Reply to Question 4.] But I say that in these and all other propositions and expressions of Sacred Scripture the proper literal sense is to be followed, and their plain and obvious meaning, whenever this is not manifestly repugnant to some truth known from a more certain revelation or divine tradition, or from a definition of the Supreme Pontiff, or by the natural light. For if it is so repugnant, they are to be taken figuratively and interpreted in another sense. [Margin: Sacred Scripture is to be taken in its proper literal sense when that sense is not manifestly false.] When I say that the sense of the letter is to be followed, however, I do not exclude the other, mystical senses — allegorical, tropological and anagogical; but they may be used only in such a way that the literal sense is nevertheless not denied, but is either expressly set first as a foundation or implicitly and tacitly presupposed. When I have proved this proposition, it will then remain to show that, if the assertions of Sacred Scripture on the motion of the Sun and the immobility of the Earth are taken as uttered only with respect to appearance and the sense of the common people, and not with respect to truth, they are not only not taken literally but are taken in a plainly false sense — and that without any necessity, since their literal sense is repugnant to no truth evidently or certainly known from elsewhere. [Margin: Of how many kinds is the sense of Scripture?] Here, however, I presuppose the very celebrated division of the sense of Sacred Scripture into the literal or historical, of which one is called simple or proper and the other figurative; and into the spiritual or mystical, which is subdivided in three ways: namely into the allegorical, concerning something future in the Church of God militant; the anagogical, concerning what is to come in the other life; and the tropological or moral sense — although some confuse this last with the literal but figurative sense, and St Jerome, in question 12 to Hedibia, and St Augustine, in the book *On the Usefulness of Believing*, chapter 3, distribute these senses somewhat differently. On the common division of these senses, however, see above all St Thomas, Part 1, question 1; Sixtus Senensis, book 3 of the *Bibliotheca Sancta*; Possevinus, book 2 of the *Bibliotheca selecta*, chapter

18; Bellarmine, Controversy 1, book 3, chapter 3; and Salmeron, volume 1, Prolegomena 7 and 8.

[Margin: The reply is proved.] With these things presupposed, our reply is proved, first, because the literal sense is principally intended by God, who is the author of the divine letters, according to that saying of St Thomas, Part 1, question 1, article 10: "Since the literal sense is the one the author intends, and the author of Sacred Scripture is God, who comprehends all things at once in his intellect, it is not unfitting, as Augustine says in *Confessions* 12, if even according to the literal sense there are several senses in one letter of Scripture." [Margin: The authority of St Thomas.] The literal sense, simply speaking, is the one taken according to the proper and obvious meaning; it stands to the improper or figurative sense as any literal sense stands to the mystical, because the figurative is more hidden than the proper. The reason is that, as Sixtus Senensis says in book 3 of the *Bibliotheca Sancta*, under the title "On the use of historical exposition": "The historical kind of exposition is above all supremely necessary for instruction in those things which we are to believe, etc. For no exposition brought forward to prove dogmas has force and strength except one that explains the meaning of the letter truly, purely and genuinely." [Margin: Sixtus Senensis.] Our own Salmeron also inculcates the same most earnestly in volume 1, Prolegomenon 9, canon 33. [Margin: Salmeron.] There, having said that "one must by no means depart from the words of Sacred Scripture according to the rule of grammar, nor from the proper meaning of the words, unless some valid reason occurs that compels a figurative expression", he adds: "Otherwise the whole of Sacred Scripture would perish, and its literal sense with it, and it would hold only empty allegories and ill-timed interpretations, which each person would attach to it at his own will and pleasure; and so Scripture would always say what we wish, not what the Holy Spirit wished us to understand." He confirms this from the jurisconsults, under the title *De verborum significatione*, and from Marcellus the jurisconsult on the law *Non aliter*, L. 3 *de legatis*: these lay down that one must not depart from the proper sense of words unless it is gathered from the most manifest indications, and from the absurdity of the things themselves, that the speaker's mind was otherwise.

So too Bellarmine, Controversy 1, book 3, chapter 3: [Margin: Bellarmine.] "It is agreed between us and our adversaries that effective arguments must be sought from the literal sense alone; for it is certain that the sense which is gathered immediately from the words is the sense of the Holy Spirit." Antonio Possevino too, in book 2 of the *Bibliotheca selecta*, chapter 18, praises and lists those "who, either in ancient times or in this age, have interpreted the Divine Scripture according to the literal sense, which is the solid foundation of all the senses." [Margin: Possevinus.] But lest we seem to beg testimony only from our own household or from more recent writers, let us see the mind of the more ancient Fathers on this literal sense.

III. [Margin: St Basil.] St Basil, in Homily 9 on the *Hexaemeron*, calls those who drag Scripture into mere allegories, or in any way take away the truth of

the letter, "interpreters of their own dreams", and adds: "But I, when I hear 'grass', understand grass; and plant, and fish, and wild beast, and cattle — all these I take just as they have been said." Before him, however, Tertullian, in the book *On the Flesh of Christ*: [Margin: Tertullian.] "Certainly it is most perverse that, when we name the flesh, we should understand the soul, and, when they signify the soul, we should interpret them as the flesh. All things will be in danger of being taken otherwise than they are, and of losing what they are while they are taken otherwise, if they are named otherwise than they are." And further down he says: "The good faith of names is the safeguard of their properties." Above all, St Augustine teaches that the foundation of the literal notion must be laid beneath the other senses, in the sermon on Abraham and the sacrifice of Isaac, saying: [Margin: St Augustine.] "We admonish as much as we can, and we enjoin, that when you hear the mystery of Scripture expounded as it records the things that were done, you first believe that the deed was done just as it is read, lest, with the foundation of the deed withdrawn, you seek as it were to build in the air." For with this laid down, as the same author teaches in the book of *Eighty-three Questions*, "when deeds are allegorized, the things done do not lose their credibility." And in book 1 of *De Genesi ad litteram*, chapter 1, speaking of the manner of interpreting sacred Genesis, he says: "In the narration of things done, then, it is asked whether all of them are to be taken according to the figurative understanding only, or are also to be asserted according to the faith of things done"; and he solves the question, saying: "If, then, that Scripture is to be examined in both ways, let us ask how what is said is meant, apart from the allegorical signification, etc." And in chapter 21: "When we read the divine books, amid so great a multitude of true understandings which are drawn out of a few words and fortified by the soundness of the Catholic faith, let us choose above all that which it is clear the author we are reading meant, etc." The same holy Doctor, in book 3 of *On Christian Doctrine*, chapter 9: "As to follow the letter, and to take signs for the things signified by them, is a mark of servile weakness, so to interpret signs uselessly is a mark of error wandering astray." And in chapter 10: "To this observance, by which we beware of following a figurative — that is, a transferred — expression as if it were proper, must be added this one too: that we do not wish to take a proper expression as if it were figurative." And in chapter 39: "When the sense, if taken according to the proper meaning of the words, is absurd, one must certainly inquire whether perhaps what we do not understand has been said by this or that trope." Therefore, if it is not absurd, it is to be taken according to the proper meaning of the words.

[Margin: Argument from the waters above the heavens.] Moreover, Sacred Scripture tells us that there are waters above the heavens no more often, and in no clearer words, than it tells us that the Sun moves and the Earth is stable; and for that reason some of the Fathers — but rather few — understood those passages about the waters and the heavens not according to the proper sense of the letter but figuratively. St Augustine, however, took those waters according

to the proper sense of the letter to such a degree that in book 2 of *De Genesi ad litteram*, chapter 5, he pronounced that golden saying: "In whatever manner and of whatever kind the waters there may be, let us by no means doubt that they are there. For the authority of this Scripture is greater than all the capacity of human ingenuity." Further, the Apostle, in Galatians 2, relates that he withstood Cephas to his face because he was to be blamed: fearing to eat with the Gentiles in the presence of the Jews, he was in a way compelling the Gentiles by his example to judaize, that is, to abstain from the foods which the Mosaic law forbade. St Jerome, interpreting this passage, had said that Saints Peter and Paul did this by prior agreement; that St Paul did not truly think St Peter had done wrong, but that Peter wished to be rebuked in this way by St Paul so as to lift the ill will of the Jews from himself, etc.; and thus that St Paul had rebuked him as to appearance and outward sound. St Augustine withstood St Jerome, and contended in several letters to him that this way of interpreting sacred Scripture must not be admitted. These letters are found also among the Letters of St Jerome, volume 2, from letter 86 to 96; and letter 96 reads as follows: "For once an officious lie is admitted into the summit of so great an authority, no particle of those books will remain which, as it seems to anyone either difficult for morals or incredible for faith, will not be referred, by that same most pernicious rule, to the purpose and design of a lying author." He presses this reason in the remaining letters. And what he says of the officious lie he would certainly say of our case: for just as Peter not only seemed blameworthy but was truly blameworthy, and was not only apparently rebuked by St Paul but truly rebuked, as Augustine proved in letter 86 among Jerome's, so the Sun truly moves, and does not only seem to move, because Scripture says both; and there is nothing absurd in interpreting Scripture according to the proper sense of the letter.

[Margin: St Gregory the Great.] St Gregory the Great was plainly of the same mind in interpreting the divine letters. In book 1 of the *Moralia*, chapter 21, he likens the literal sense to drink, and the figurative or mystical to food, saying: "For Sacred Scripture is sometimes food for us, sometimes drink. It is food in the more obscure passages, because it is as it were broken by being expounded and swallowed by being chewed; it is drink in the more open passages, because it is drunk down just as it is found." And in book 31, chapter 23, explaining that saying of Job ("if my land cries out against me, and with it its furrows weep"), he says: "In this matter, because the order of the history fails, the mystical understanding shows itself to us as though the doors were now open, as if it plainly cried out: since you recognize that the sense of the letter has failed, it certainly remains for you to return to me without hesitation" — that is, to the mystical sense. But if the literal and historical sense can be retained, it must certainly be retained. Day would fail me if I wished to heap up here the testimonies of the other Fathers for our cause. It is enough for me to have pointed out that Ori- gen, because he shrank from the literal sense in explaining the divine letters, fell into intolerable errors, and clung so stubbornly

to his allegories that St Jerome, in the preface to the tenth vision of Isaiah, addressed to Bishop Amabilis, wrote of him: "Origen roams at large through the free spaces of allegory, and makes his own ingenuity the sacraments of the Church." The same must likewise befall those who, devoted to Platonic Ideas or to Pythagorean mysteries and philosophemes, have contrived the workmanship of the world not as Sacred Scripture sets it before us, but as they themselves fashioned it, contemplating archetypal reasons that were most beautiful to their own minds, however ill-fitting and absurd to others. Now we must clear away certain things that seem to stand in the way of our reply.

IV. [Margin: The 1st objection is cleared away.] *First*: against the literal sense and its followers stand those two passages of the Apostle, 1 Corinthians 7, "Now all these things happened to them in figure", and 2 Corinthians, chapter 10, "For the letter kills, but the spirit gives life." But the first passage does not mean that the precepts and ceremonies of the law are to be taken in a figurative sense only, with the proper meaning of the literal sense excluded; it means that they can be taken in an allegorical sense concerning Christ and the Church, while the literal notion is nevertheless retained, or kept safe. The second passage means that the old law, understood and observed according to the bare letter, with the spiritual sense of grace foreshadowed through Christ neglected, kills, that is, does not avail for salvation; but that the Spirit of grace, which is had from the New Testament, gives life — as Salmeron, among others, rightly expounds in volume 1, Prolegomenon 22. [Margin: The 2nd objection is cleared away.] *Second*: the literal sense is manifold, as St Augustine teaches in *Confessions* 12, chapters 26, 30 and 31; in book 1 of *De Genesi ad litteram*, chapters 19 and 21; in book 11 of the *City of God*, chapter 19; and in book 3 of *On Christian Doctrine*, chapter 27; so too St Thomas, Part 1, question 1, article 10, and many others cited by Salmeron, volume 1, Prolegomenon 8. But this does not prevent the proper literal sense from being taken in preference, if nothing absurd follows from it, even if nothing unfitting follows from the figurative sense. This rule, however, must always be kept: if no absurdity is demonstrated in the proper meaning of the literal sense, that sense holds first place; but if its absurdity has been proved from elsewhere, let us look for another sense, by which the truth of the sacred utterance may be kept safe. The same St Augustine taught this clearly in book 1 of *De Genesi ad litteram*, chapter 21, expounding the verse of Psalm 103, "stretching out the heaven like a skin": [Margin: A notable judgement of St Augustine for the use of the literal sense.] "But someone says: how is what is written in our letters, 'Who stretches out the heaven like a skin?', not contrary to those who attribute the figure of a sphere to the heaven? Let it indeed be contrary, if what they say is false; for that is true which the divine authority says, rather than what human weakness conjectures. But if perhaps they are able to prove it by such evidence that one ought not to doubt it thereafter, it must be demonstrated that what is said with us about the skin is not contrary to those true reasons." This is the rule we have so often inculcated above.

V. [Margin: The 3rd objection is cleared away.] *Third*: the sacred interpreters not seldom grant that Moses, in speaking of God's workmanship, accommodated himself to an uneducated people; and among others St Thomas Aquinas, in the First Part of the *Summa*, question 70, article 1, reply to 3, said: "It must be said that according to Ptolemy the luminaries are not fixed in the spheres, but have a motion separate from the motion of the spheres, etc. But according to the opinion of Aristotle the fixed stars are in their orbs and are not moved except by the motion of the orbs, according to the truth of the matter; yet the motion of the luminaries is perceived by sense, not the motion of the spheres. Moses, however, condescending to an uneducated people, followed what appears to the senses." Therefore no derogation is done to Sacred Scripture if it is said to have asserted the motion of the Sun and the rest of the Earth because they appear so to the common people, so as to accommodate itself to the common capacity of men. But it is one thing, of two things both of which are true, to pass over one in silence as hard to understand or not obvious to the senses — such as St Thomas thinks the motion of the celestial spheres to be — and to express the other, which is plain to the senses, such as the motion of the Sun; it is quite another thing to affirm what is false because what is true is not obvious to sense, and therefore not easy for the common people to understand. But if the Sun were immobile and the Earth moved, and Scripture nevertheless affirmed that the Sun moves and that the Earth does not move, it would not only be silent about the truth but would in addition assert a falsehood, which is unfitting. Here the saying of St Augustine in the book *On Lying* applies: "There can be no just cause for a lie to be told; but there can be a just cause for the truth sometimes to be kept silent." Let St Thomas be consulted, Part 1, question 67, article 4; question 68, article 3; question 69, article 2, reply to 3; and question 70, article 1, reply to 4, on how he explains the expressions of Moses without falsity, keeping some literal sense intact.

VI. [Margin: The 4th objection is cleared away.] *Fourth*, you will press: if the Sun stood still and the Earth really moved, Scripture would nevertheless have spoken in the same way as now, or at least could have spoken so, in order to adapt itself to the capacity of men. For who would have believed David, or Job, or Solomon, or the Son of Sirach, saying, "The Earth moves and returns in its circles", and conversely, "You who founded the Sun upon its stability; it shall not be inclined for ever and ever", or such παράδοξα [Greek: παράδοξα (paradoxa, "paradoxes" — things contrary to received opinion)], wholly foreign to the senses of men? But I deny that in that case Scripture would have spoken so, or could have spoken as it does now, with no indication anywhere from God, in another Scripture or in some natural effects manifest to sense mediately or immediately, of the rest of the Sun and the motion of the Earth — indications from which we would be compelled to take the other expressions about the motion of the Sun and the rest of the Earth figuratively. Besides, is it not most difficult to grasp, and not obvious to the senses, that there are waters above the heavens, and that the light of the Sun existed before the Sun? And yet

Scripture did not keep silent about these things; and much less did it keep silent about the mystery of the Most Holy Trinity, of the Incarnation, of the Eucharist, of the resurrection of the dead. On what ground, then, is it said so confidently that Scripture would have kept silent about that motion of the Earth, precisely because it is above or against the sense of men? Finally, you deny that anyone would believe Scripture asserting the motion of the Earth, because of the absurdity and παραδοξίαν [Greek: παραδοξίαν (paradoxian, "paradoxicality" — strangeness against received opinion)] of this motion; and yet you want Aristarchus or Copernicus to be believed when they assert so great an absurdity? Is not this itself a horrible monster of intolerable absurdity — not to grant to God speaking what you would grant to a little man speaking? Either, then, be silent about what you claim God ought to be silent about, or rather speak what God affirms by speaking.

VII. [Margin: The 5th objection is cleared away.] *Fifth*: the Copernicans bring forward many propositions of Sacred Scripture pertaining to cosmographical, physical or mathematical matters, in which it is nevertheless established that Scripture accommodates itself to the capacity and sense of the common people, and that the literal sense, according to the proper meaning of the language, cannot be taken. To all of these we reply in general that nothing is ever affirmed in them that is not so, or denied that is so. Next, if it is established that they are to be taken in a sense other than the literal and proper one, this must certainly be established either from other, clearer passages of Scripture, or from a definition of the Church, or from some evident proposition known by the natural light. But in none of these ways is it established that the passages adduced in chapter 36 for the motion of the Sun and the rest of the Earth are to be taken otherwise. We shall nevertheless run through these other passages, so that the difference in interpreting may appear more clearly.

Loca Sacrae Scripturae ad Physicam aut Mathesim pertinentia, quae Copernicani Figurate aut ad sensum Popularem accipienda esse contendunt, Examinantur.

The passages of Sacred Scripture pertaining to physics or mathematics which the Copernicans contend are to be taken figuratively, or according to the popular sense, are examined.

VIII. [Margin: Firmament.] Genesis 1 says: "And God called the firmament heaven." But if the heaven filled with fixed stars moves, how is it called a firmament? I answer: from its firmness, which does not necessarily signify local immobility. But see what I said in Section 1, chapter 2, question 1. [Margin: The great luminaries.] In the same chapter 1 of Genesis it is said: "And God made two great luminaries: the greater luminary to rule the day, and the lesser luminary to rule the night, and the stars." Here Moses seems to have spoken according to the sense of the people; otherwise the Moon is not a great luminary, since Saturn and Jupiter, etc., are planets much larger than the Moon; nor should the Sun or the Moon have been distinguished from the stars, as

though they themselves were not stars. But I have already expounded this passage at length, with the Fathers, in Section 1, chapter 4, question 5, which is found on page 231. There we taught that the Moon, considered as a luminary, is greater than all the other stars except the Sun, because it is like the Sun in illuminating, at least when it is full, and lights up the whole hemisphere at once far more than all the rest taken together. Therefore, even according to the proper sense of the letter the Moon is a great luminary, though smaller than the Sun — granted that in respect of its body, or as a planet, it is not great, or greater than all the others; and Tycho says almost the same in the *Epistolae Astronomicae*, page 148. Next, as I said in the same place, it is astronomically evident, from the Moon's apparent diameter and distance, that it is smaller than some of the planets; and therefore, if Moses had called the Moon a great planet (which, however, he did not do), we would be compelled to have recourse to its apparent magnitude. But no evidence compels us to say that the motion of the Sun and the stability of the Earth are asserted in the Scriptures with respect to appearance alone. Nor, when "the stars" is added, is it denied that the Moon and the Sun are stars with this name taken as generic, but only with it taken as specific; or else "and the rest" is to be understood.

[Margin: The geometrical measurement of the Brazen Sea.] 3 Kings, chapter 7, says: "He made also a molten sea of ten cubits from brim to brim, round in compass; its height was five cubits, and a line of thirty cubits encircled it round about." Jacob van Lansbergen, in tract 3 of the *Apologia*, tries to drag this passage to the sense of the common people, on the ground that if the diameter of this round vessel was 10 cubits, its circumference could not have been only 30 cubits but must have been greater, since — from what we said in book 1, chapter 4 — the diameter of a circle is to its circumference as 100 to 314. To this objection I reply in three ways. *First*: it is not established from Scripture whether that line — in Italian *il cordoncello* — lay in the same plane as the brim and the ten-cubit diameter; for it could have been somewhat lower than the brim, and so have been exactly 30 cubits, and somewhat less than the circumference. *Second*: even if the line is taken for the circumference of the brim, a round number is nevertheless set down truly and according to the proper sense of the letter, as often happens elsewhere; for this does not deny the more exact number, which had to be 31 2/5 if the diameter was 10; or, if the circumference was 30, the diameter had to be 9 174/314. *Third*: one would have to have recourse to this sense of a round signification, accommodated to the people, for this reason: that the ratio of the diameter to the circumference has been demonstrated by Archimedes and others; whereas none of the Copernicans has so far demonstrated that the Sun does not really move and that the Earth does move.

[Margin: The Sun coming forth from its chamber.] In Psalm 18 the Sun is said to come forth like a bridegroom from his chamber — not because it has slept the whole night, since it is said that there is no one who hides himself from its heat, and so it illuminates the other hemisphere by night as well, but in view

of the analogy between the splendour of its rising and the new adornments of a bridegroom. And in Psalm 23 the Earth is founded upon the seas and prepared upon the rivers [Margin: The Earth upon the waters.] — not because it can float upon the waters, but because, as St Augustine says in book 2 of *De Genesi ad litteram*, chapter 1, most riverbanks, and places receding from the shore, occupy a higher place than the water because of swellings of the ground and hills.

[Margin: The heaven stretched out like a skin.] In Psalm 103, moreover, God is said to have stretched out the heaven like a skin: that is, with the ease with which tents made of skins are stretched out; or because he clothed the world with the heaven as with a skin; or because a skin, too, in bellows and inflated wineskins, is so stretched that it tends to roundness. Therefore this is not repugnant to the roundness of the heaven. This is the exposition of St Augustine in book 2 of *De Genesi ad litteram*, chapter 8.

[Margin: The Earth stands for ever.] Ecclesiastes 1 says: "A generation passes away, and a generation comes; but the Earth stands for ever." This passage, as I said in chapter 36, has a twofold literal sense. One concerns the standing of the Earth as opposed to generation and corruption, signifying that the Earth is subject to generation and corruption neither as a whole nor in the greatest part of its depth; this is the only sense the Copernicans embrace. The other sense concerns standing as opposed to local motion. For the Earth is like a stage, on whose surface men succeed one another; but at length, having presented their brief spectacle, they pass away by local motion too, while the body passes into the bowels of the earth, and the soul formerly passed into the bowels of the same earth, but now passes either into its bowels or into heaven. But the Earth, exempt from all motion, stands for ever; and this sense is the principal one and conforms to the Hebrew root, as our own Pineda shows at length and learnedly on chapter 1 of Ecclesiastes.

[Margin: Put out into the deep.] Lastly, Christ the Lord says to Peter, in Luke 5, "Put out into the deep" — literally "into the high", because the sea seems higher than the shore through a deception whose cause the opticians explain, although because of the swelling of the waves it can sometimes really be higher than the shore. But if in this case we are compelled to take refuge in an optical expression and the popular sense, this happens because it is presupposed, as evidently known from elsewhere, that the sea is not really higher than the shores. Now, it is not evidently established that the Earth moves and the Sun stands still, so that we should be bound on that account to draw the propositions about the motion of the Sun and the standing of the Earth to a figurative sense or to appearance. *Altum* could also be taken for "deep". Let there now be:

VNICA CONCLVSIO.

The Single Conclusion.

PROPOSITIONES SACRAE SCRIPTURAE, IN QUIBUS SOLIS MOTUS, & TERRAE IMMOBILITAS ASSERITUR, ACCIPIENDAE SUNT AD LITERAM, SECUNDUM PROPRIUM SENSUM.

The propositions of Sacred Scripture in which the motion of the Sun and the immobility of the Earth are asserted are to be taken according to the letter, in their proper sense.

IX. The conclusion is proved by this single syllogism. *Every proposition that is found uttered by a canonical writer in Sacred Scripture is to be taken in the literal and proper sense whenever, in that sense, there is no repugnance with other propositions of the same Sacred Scripture that are equally or more certain, or with a definition of the Supreme Pontiff of the Catholic Roman Church, or with some proposition certain and evident by the natural light. But the propositions of Sacred Scripture in which the motion of the Sun and the stability of the Earth are asserted were uttered by a canonical writer, and are repugnant to none of the things enumerated in the major. Therefore they are to be taken in the literal and proper sense.*

[Margin: Proof of the major.] The MAJOR has been sufficiently proved from number 2 to 7. And as regards the definition of the Church made through the Roman Pontiff, it has already been most learnedly proved by Bellarmine, Controversy 1, book 3, from chapter 3 to 10. There, from the whole antiquity of the Fathers and the custom of the Church, and from the authority conferred by Christ on St Peter and his successors, he establishes that in a doubt whether a Sacred Scripture is to be received among the canonical books or not, or concerning the sense and interpretation of some proposition found in it, recourse must be had to a single, certain, visible rule, which is the Supreme Pontiff of the Catholic Roman Church; and this is confirmed from the decree of the Council of Trent, session 4, on the edition and use of the sacred books.

[Margin: Proof of the minor as to its three parts.] The MINOR is proved as regards the canonical writer. For the propositions asserting the motion of the Sun and the stability of the Earth were uttered by the author of the Book of Judges and of Paralipomenon, by David, by Ecclesiastes, by Ecclesiasticus, by Isaiah, and by Job himself, or by God speaking to Job — not, however, by the friends of Job — as is clear from what was said in chapter 36. All these authors and books are listed among the canonical books by the Council of Trent, and are defined as to be held for such, session 4. That they are not repugnant to another Scripture or to a definition of the Church is proved, both because no such proposition can be brought forward to which they are repugnant, and because, although it has not yet been defined by the Supreme Pontiff, it has nevertheless been defined by those deputed by him that it is rather the assertions of the motion of the Earth and the stability of the Sun that are repugnant to Sacred Scripture, as will be clear from what is to be said in the following chapter. Finally, that those propositions are not repugnant to any proposition certain and evident by the natural light is proved, *first*, because it is rather the propositions opposed to them that are repugnant to physico-mathematical demonstrations, as was said in chapter 35, conclusions 2 and 3; *second*, because if the celestial phenomena alone are considered, all of them can be saved in astronomical

rigour, whether the Earth is supposed to rest and the Sun to move, or the Earth to move and the Sun to rest, as was said in chapter 35, conclusion 1. For no one has so far demonstrated the motion of the Earth or the rest of the Sun; at most it has been demonstrated that, if the Earth moves through the annual orb with the diurnal motion and the libration of the axis, and the Sun at the centre of the world moves with no motion other than that of rotation about its own centre, all the celestial phenomena can occur, as to appearance, in the same way as if the Sun moved about the Earth resting at the centre of the world; and that for some of them a more probable account of such phenomena is given by the hypothesis of Copernicus, granted that in other respects it contains a greater absurdity, inwardly and outwardly. Finally, the Copernicans themselves admit that in astronomical rigour the same things would appear in the heaven whether the Earth stands and the Sun moves, or the Earth is carried round while the Sun stands still: Copernicus, book 1, chapter 5, says: "For between things moved equally in the same direction no motion is perceived — between the thing seen, I mean, and the one seeing. Now it is the Earth from which that celestial circuit is viewed and reproduced to our sight. If, then, some motion is assigned to the Earth, that same motion will appear in all things that are outside it, but in the opposite direction, as if they were passing by; and such, in the first place, is the daily revolution." And in chapter 8: "Why then should we not admit that the appearance of this daily revolution belongs to the heaven, and the reality to the Earth?" etc. Therefore, if the Earth is supposed to be at rest, the same motion will nevertheless appear in the celestial bodies. [Margin: Kepler's testimonies for the equipollence of the hypotheses of the Earth at rest and in motion.] And Kepler, in the introduction to the *Commentaries on Mars*: "The opinion of Brahe on the form of the world ought surely to be commended not a little to the learned. For, taking as it were a middle way, on the one hand it frees the astronomers, as far as can be done, from the useless apparatus of so many epicycles, and embraces with Copernicus the causes of the motions that were unknown to Ptolemy, giving some place to physical speculations by receiving the Sun into the centre of the planetary system; on the other hand it serves the common run of the learned, and eliminates the motion of the Earth, so hard to believe — although through it the theories of the planets are entangled in many difficulties in astronomical speculations and demonstrations, and celestial physics is not a little disturbed." That physics, namely, which Kepler feigns for himself in his mysteries — not the physics which God and the Intelligences understand, for whom those are no tangles which seem tangles to the weakness of our imagination. Meanwhile, however, Kepler admits that in the Tychonic hypothesis, which frees the Earth from all motion, the phenomena are saved.

But, you will say, if you listen to Kepler in his notes on chapter 1 of the *Mysterium Cosmographicum*, the harmonic laws in the motions of the planets do not seem able to hold together with the rest of the Earth and the motion of the Sun. I reply that this is utterly false, as Kepler himself at length admits and attests in book 5 of the *Harmonice*, chapter 3, where he speaks thus: "Although I cannot

do otherwise than put in its place the opinion of Copernicus on the world as the only one, and, if it were possible, persuade everyone of it; yet because the matter is still new among the common run of students, and the doctrine that the Earth is one of the planets and is carried among the stars around an immobile Sun is most absurd to the hearing of most people: let those, then, who are offended by this opinion know that these harmonic speculations hold their place in the hypotheses of Tycho Brahe as well, because that author holds in common with Copernicus all the rest that pertains to the disposition of the bodies and the tempering of the motions," etc. — see this in his own text if you want more. Meanwhile it also appears that that beauty of the Copernican hypothesis contends with the foulness of its absurdity, so that it is most difficult to believe. Again, in the *Hyperaspistes*, book 3, chapter 22, where he speaks of Chiaramonti and fears that Chiaramonti may turn the decrees of the Holy Inquisition against him, he says: "If he should seize the weapon of Jove and hurl his thunderbolt at the hypotheses which I follow; if he should bring an action against me, as Cleanthes once did against Aristarchus, for violated religion, because the sacred things of Vesta were moved from their place: the equipollence demonstrated in the *Commentaries on Mars* and inculcated in the *Harmonice* will amply satisfy the astronomers." Therefore, Kepler, you admit that the hypothesis of Brahe, demonstrated by you, which halts the Earth and moves the Sun, is equipollent to the Copernican hypothesis, which moves the Earth.

[Margin: Galileo's testimonies for the same equipollence.] Further, Galileo, in Dialogue 2, *On the Cosmic System*, Italian page 107, Latin page 81, speaking of the diurnal motion, says: "And this motion, as far as this first appearance goes, in no way prevents it from being able to belong to the Earth alone just as well as to the whole rest of the world, the Earth excepted. For the same appearances would be seen in the one position as in the other." And at Italian page 267, Latin 201, when Simplicio had said: "Whatever has been brought forward in confirmation of the Earth's motion concludes, in sum, nothing else than that the reasons asserting the stability of the Earth are not necessary; but meanwhile, from the contrary side, no demonstration has been produced that necessarily proves and concludes the mobility of the Earth," Salviati, who sustains the person of Galileo in those dialogues, replied: "I have never taken it upon myself to lead you away from your opinion, Simplicio; much less would I dare to pronounce a definitive sentence on so great a dispute." So much for the celestial phenomena. As for his saying, at the end of dialogue 4, that the flux and reflux of the sea give a most evident testimony for the motion of the Earth, with what vanity and falsity this was said is clear from what was said in chapter 14. Finally, that the motion of the Earth is not proved by Galileo from the circular motion of heavy bodies, etc., we have sufficiently demonstrated in chapter 17.

CAPVT XL.

Chapter 40.

Quam Censuram mereantur & retulerint a Doctis viris, sed praecipue a Sacra Congregatione Cardinalium Officio Inquisitionis a Summo Pontifice deputatorum, Assertores Motus Telluris & Immobilitatis Solis.

What censure the assertors of the motion of the Earth and of the immobility of the Sun deserve, and have received, from learned men, but above all from the Sacred Congregation of Cardinals deputed by the Supreme Pontiff to the Office of the Inquisition.

I. I shall first review some astronomers, philosophers and theologians who have condemned the hypothesis of Copernicus in more explicit formulae. [Margin: Tycho.] Tycho, in the *Epistolae Astronomicae*, page 147, judges it "absurd and contrary to Sacred Scripture". [Margin: Tassonus.] Alexander Tassonus, in book 4 of the *Diverse Thoughts* [diversarum Contemplationum], question 25, asserts that it is "against nature, against sense and physical reasons, against astronomy and mathematics, and against religion". [Margin: Mersennius.] Marinus Mersennius, however, in his *Questions on Genesis*, chapter 1, verse 10, article 7, rejects it in such a way as nevertheless to deny that it has so far been condemned as a heresy by the Catholic Church. [Margin: Gassendus.] Petrus Gassendus, at the end of the second letter *De motu impresso a motore translato*, does indeed condemn and repudiate that opinion, but he adds: "Not that I therefore consider it to be an article of faith," etc. [Margin: Scheinerus.] Our own Scheinerus, in the *Disquisitiones Mathematicae*, pages 28 and 35: the Copernican motion is "beyond all belief, and cannot be defended without innumerable absurdities; and it follows from it that many passages of Sacred Scripture, and the common forms of speech of the astronomers, must be twisted into a perverse sense without any necessity, when everything can be defended most conveniently in the plain sense". [Margin: Delphinus.] From the Franciscan family, Io. Antonius Delphinus, in his book *On the Celestial Globes and Motions*, chapter 18, calls the opinion of the Earth's motion "foolish, insipid, and long since hissed out of the schools". But others brand it with graver censures. [Margin: Kircher.] For Athanasius Kircher, in his *Magnes*, page 539, affirms that it is "dangerous in faith". [Margin: Lipsius.] Indeed Iustus Lipsius [Justus Lipsius], book 2 of the *Physiologia*, disputation 19 — which Georgius Polaccus too reports in his *Anticopernicus Catholicus*, assertions 1 and 45 — and, with Lipsius, [Margin: Tellez.] our own Balthassar Tellez, in his *Summa of Philosophy*, part 2, disputation 46, section 2, call the opinion of the Earth's motion a heresy. [Margin: Serarius.] Nicolaus Serarius of our Society does the same on chapter 10 of Joshua, question 14, where, speaking of Copernicus, he says: "Although he dedicated these revolutions of his to the Supreme Pontiff Paul III, so that they might escape all reprehension, nevertheless, if these hypotheses were seriously asserted as true, I do not see how they could be immune from

heresy. For Scripture always attributes rest to the Earth and motion to the Sun and Moon; or, if ever those stars stand still, it signifies that this happens by a great miracle." [Margin: Tannerus.] Moreover, Adamus Tannerus, a most learned theologian of our Society, in his dissertation *On the Heaven*, question 9, says that it is a certain and true assertion that the stars, not the Earth, are moved from east to west; and that the opposite, condemned as heresy or error by Philastrius and by Diodorus of Tarsus, can no longer be safely defended after the decree of the Cardinals. [Margin: Fromondus.] Likewise Fromondus in his *Vesta*, [Margin: Acarisius.] and Acarisius in his, subscribe to the same censures, which we shall shortly bring out from the decree of the Cardinals. [Margin: Inchofer.] Melchior Inchofer of our Society, in his *Tractatus Syllepticus*, from chapter 8 to 12, holds: 1. that the consensus of the Fathers is unanimous that the Sun moves; 2. that it is of faith that the Sun moves circularly; 3. that it is more probable that it is of faith that the Earth is at the centre of the world and does not move — of faith, I say, at least indirectly, inasmuch as from its opposite there follows something contrary to faith, namely that the passages of Scripture asserting the motion of the Sun and the stability of the Earth are false. [Margin: Polaccus.] Georgius Polaccus agrees with him in every respect in his *Anticopernicus Catholicus*, assertions 126, 127, 128, 129 and 130. [Margin: Pineda.] Our own Pineda, on chapter 1 of Ecclesiastes, page 114, calls the opinion of the Earth's motion "an absurd falsity"; [Margin: Lorinus.] and Lorinus of our Society asserts that this same falsity has been demonstrated by him from the divine letters, on the same chapter 1 of Ecclesiastes. [Margin: The author's opinion.] For my part, I do not dare to add anything to, or take anything from, the censure passed against this opinion by the Sacred Congregation of Cardinals, which I shall report a little below; rather, I subscribe to it entirely, as having been pronounced most prudently and for the most just reasons.

II. There now follows a copy of the decree issued under Paul V against the books of Copernicus, of Didacus Astunica [Diego de Zúñiga] and of Paulus Antonius Foscarinus, extracted from the Chancery of the Holy Office of the Inquisition at Rome; Tannerus too reports a great part of it, in his dissertation *On the Heaven*, question 9. To it we shall subjoin the emendation of the books of Copernicus promulgated at Rome in the year 1620, and the abjuration which Galileo was compelled to make in the year 1633 under Urban VIII. A copy of the abjuration was shown to us by the Most Reverend Father Pallavicinus, Inquisitor of Mantua, and Georgius Polaccus also recites it, in his *Anticopernicus Catholicus*, after assertion 123, from a letter of the Most Eminent Cardinal of St Onuphrius sent in the same year to the Most Reverend Inquisitor of Venice.

Extractus Decreti Sacrae Congregationis Eminentissimorum S.R.E. Cardinalium sub Paulo V. Editi V. Martij Anni M DC XVI.

Extract of the decree of the Sacred Congregation of the Most Eminent Cardinals of the Holy Roman Church, issued under Paul V on 1616 March 5.

And since it has also come to the notice of the Congregation named above that the false Pythagorean doctrine, altogether opposed to Divine Scripture, of the mobility of the Earth and the immobility of the Sun — which Nicolaus Copernicus in *De revolutionibus orbium caelestium*, and Didacus Astunica on Job, also teach — is now being spread abroad and accepted by many, as may be seen from a certain printed letter of a certain Carmelite Father, entitled *Lettera del R. P. Maestro Paolo Antonio Foscarini Carmelitano, sopra l'opinione de i Pittagorici, & del Copernico della mobilità della Terra, e stabilità del Sole, & il nuovo Pittagorico Sistema del Mondo* (Letter of the Reverend Father Master Paolo Antonio Foscarini, Carmelite, on the opinion of the Pythagoreans and of Copernicus concerning the mobility of the Earth and the stability of the Sun, and the new Pythagorean System of the World), Naples, Lazzaro Scorriggio, 1615, in which the said Father tries to show that the doctrine mentioned above, of the immobility of the Sun at the centre of the world and the mobility of the Earth, is consonant with the truth and not opposed to Sacred Scripture: therefore, lest an opinion of this kind creep any further to the ruin of Catholic truth, the Congregation has decided that the said Nicolaus Copernicus, *De revolutionibus orbium*, and Didacus Astunica on Job are to be suspended until they are corrected; that the book of Father Paulus Antonius Foscarinus, Carmelite, is to be altogether prohibited and condemned; and that all other books teaching the same thing are likewise to be prohibited — as by the present decree it prohibits, condemns and suspends them all, each in its respective way. In witness of which the present decree was signed and sealed by the hand and seal of the Most Illustrious and Most Reverend Lord Cardinal of St Cecilia, Bishop of Albano, on 1616 March 5. Rome, from the press of the Apostolic Camera, in the year 1616.

P., Bishop of Albano, Cardinal of St Cecilia.

Place of the seal.

Register, folio 90.

Fr. Franciscus Magdalenus Capiferreus [Francesco Maddaleni Capiferro], Secretary, of the Order of Preachers.

Monitum Sacrae Congregationis Ad Nicolai Copernici Lectorem eiusque Emendatio, Permissio, & Correctio.

Admonition of the Sacred Congregation to the reader of Nicolaus Copernicus, with its emendation, permission and correction.

The Fathers of the Sacred Congregation of the Index judged that the writings of Nicolaus Copernicus, the noble astrologer, *On the Revolutions of the World*, were to be altogether prohibited, for this reason: that he does not hesitate to treat principles about the position and motion of the terrestrial globe that are repugnant to Sacred Scripture and to its true and Catholic interpretation (something by no means to be tolerated in a Christian man), not by way of hypothesis, but to assert them as most true. Nevertheless, because there are in those writings many things most useful to the commonwealth, they came

with unanimous consent to the opinion that the works of Copernicus printed up to this day were to be permitted, as they have permitted them — but with those passages corrected, according to the emendation subjoined, in which he argues about the position and motion of the Earth not by way of hypothesis but by asserting it. Those copies, however, that are to be printed hereafter are permitted only with the passages already mentioned emended as follows, and with a correction of this kind prefixed to Copernicus's preface.

LOCORUM, QUAE IN COPERNICI LIBRIS VISA SUNT CORRECTIONE DIGNA, EMENDATIO.

Emendation of the passages in the books of Copernicus that have been judged worthy of correction.

In Praefatione circa finem.

In the Preface, near the end.

There, at *si fortasse* ("if perhaps"), delete everything up to the words *hi nostri labores* ("these labours of ours"), and adjust it thus: *Caeterum hi nostri labores* ("But these labours of ours").

In Cap. 6. lib. 1. pag. 6.

In chapter 6 of book 1, page 6.

There, at *Si tamen attentius*, correct: *Si tamen attentius rem consideremus; nihil refert, an Terram in medio Mundi, an extra medium existere, quoad solvendas caelestium motuum apparentias existimemus: omnis enim* etc. ("If, however, we consider the matter more attentively, it makes no difference whether we suppose the Earth to exist in the middle of the world or outside the middle, as regards resolving the appearances of the celestial motions; for all," etc.)

In Cap. 8. eiusdem Libri.

In chapter 8 of the same book.

This whole chapter could be struck out, because it treats professedly of the truth of the Earth's motion, while it refutes the reasons of the ancients proving the Earth's rest. Since, however, it seems to speak problematically, let it be emended as below, so that students may be satisfied and the sequence and order of the book may remain intact.

First, on page 6, delete the short passage from *Cur ergo* up to the word *provehimur*, and let the passage be corrected thus: *Cur ergo non possum mobilitatem illi formae suae concedere, magisque quod totus labatur mundus, cuius finis ignoratur scirique nequit, et quae apparent in caelo, perinde se habere, ac si diceret Virgilianus Aeneas,* ("Why then can I not grant to the Earth a mobility belonging to its form, and the more so since the whole world is slipping by, whose limit is unknown and cannot be known; and that the things which appear in the heaven behave just as if Virgil's Aeneas were saying,")

Second, on page 7, at the passage *Addo*, let it be corrected in this way: *Addo etiam difficilius non esse contento et locato, quod est Terra, motum adscribere, quam continenti* ("I add, too, that it is no more difficult to ascribe motion to what is contained and located, which is the Earth, than to what contains it").

Third, on the same page, at the end of the chapter, the passage *Vides* is to be deleted up to the end of the chapter.

In Cap. 9. pag. 7.

In chapter 9, page 7.

The beginning of this chapter, up to the passage *Quod enim*: correct it thus: *Cum igitur Terram moveri assumpserim: videndum nunc arbitror, an etiam illi plures possint convenire motus, quod enim* etc. ("Since, then, I have assumed that the Earth moves, I think we must now see whether several motions too can belong to it; for," etc.)

In Cap. 10. pag. 9.

In chapter 10, page 9.

The passage *Proinde*: correct it thus: *Proinde non pudet nos assumere* ("Hence we are not ashamed to assume"). And a little below, there, *Hoc potius in mobilitate Terrae verificari* ("that this is rather verified in the mobility of the Earth"): correct it to *Hoc consequenter in mobilitate Terrae verificari* ("that this is consequently verified in the mobility of the Earth"). On page 10, at the end of the chapter, delete those last words: *Tanta nimirum est divina haec D. O. M. fabrica* ("So great, indeed, is this divine workmanship of God, Best and Greatest").

In Cap. 11.

In chapter 11.

Let the title of the chapter be adapted in this way: *De Hypothesi triplicis motus Terrae, eiusque demonstratione* ("On the hypothesis of the threefold motion of the Earth, and its demonstration").

In lib. 4. cap. 10. pag. 122.

In book 4, chapter 10, page 122.

In the title of the chapter delete the words *horum trium siderum* ("of these three stars"), because the Earth is not a star, as Copernicus makes it.

Fr. Franciscus Magdalenus Capiferreus, of the Order of Preachers, Secretary of the Sacred Congregation.

Rome, from the press of the Reverend Apostolic Camera, 1620.

Literae Eminentiss. Cardinalis S. Onufrij Ad Reu. P. Inquisitorem Venetum.

Letter of the Most Eminent Cardinal of St Onuphrius to the Reverend Father Inquisitor of Venice.

VERY REVEREND FATHER.

CAPVT XL.

The treatise of Nicolaus Copernicus *De revolutionibus orbium caelestium* has been suspended by the Congregation of the Index, because it maintains that the Earth moves and not the Sun, but that the Sun stands at the centre of the world (an opinion which is contrary to Sacred Scripture); and Galileo Galilei, a Florentine, was prohibited many years ago by this Sacred Congregation of the Holy Office from holding, defending or teaching the said opinion in any manner, by word or in writing. Nevertheless the same Galileo dared to compose a book entitled *Galilaeus Galilaei Lynceus*; and, without disclosing the said prohibition, he extorted a licence to print it (as in fact he did print it). And while supposing, at its beginning, middle and end, that he wished to treat the already-mentioned opinion of Copernicus hypothetically, he nevertheless (although he could not treat of it in any manner) treated of it in such a way that he rendered himself vehemently suspect of adhering to such an opinion. For this reason, having been examined and shut up in the prison of the Holy Office, he was condemned by the sentence of these Most Eminent Lords of mine to abjure the said opinion, to remain in formal prison at the discretion of their Eminences, and to perform other salutary penances — as Your Reverence will see in the copy of the sentence and of the abjuration written below. It is sent to you so that you may notify it to your vicars, and so that knowledge of it may reach them and all professors of philosophy and mathematics; in order that, knowing how the said Galileo was dealt with, they may grasp the gravity of the error he committed, so as to avoid it, and also the penalties which they would suffer by falling into it. In closing, may the Lord God preserve Your Reverence. Rome, 1633 July 2.

Your Reverence's,
as a brother,
the Cardinal of St Onuphrius.

Sententia in Galilaeum & Abiuratio eiusdem.

The sentence against Galileo, and his abjuration.

[Margin: Names of the cardinals of the Sacred Congregation.] We, Gaspar, of the title of the Holy Cross of Jerusalem, Borgia [Gaspar de Borja];

Brother Felix Centinus [Felice Centini], of the title of St Anastasia, called of Ascoli;

Guidus, of the title of St Mary of the People, Bentivolus [Guido Bentivoglio];

Brother Desiderius Scaglia [Desiderio Scaglia], of the title of St Charles, called of Cremona;

Brother Antonius Barberinus [Antonio Barberini], called of St Onuphrius;

Laudivius Zacchia [Laudivio Zacchia], of the title of St Peter in Chains, called of St Sixtus;

Berlingerius, of the title of St Augustine, Gypsius [Berlinghiero Gessi];

Fabricius, of St Lawrence in Panisperna, Verospius [Fabrizio Verospi], called the Priest;

Franciscus, of St Lawrence in Damaso, Barberinus [Francesco Barberini], and

Martius, of St Mary the New, Ginettus [Marzio Ginetti], Deacons; by the mercy of God Cardinals of the Holy Roman Church, Inquisitors General throughout the whole Christian Commonwealth against heretical depravity, specially deputed by the Holy Apostolic See.

[Margin: The form of the sentence against Galileo.] Whereas you, Galileo, son of the late Vincenzio Galilei of Florence, aged seventy years, were denounced to this Holy Office in the year 1615 for holding as true the false doctrine taught by many, namely that the Sun is at the centre of the world and immobile, and that the Earth moves, even with a diurnal motion; also, that you had certain disciples whom you were teaching the same doctrine; also, that you kept up a correspondence about it with certain mathematicians of Germany; also, that you had published certain letters entitled *On the Sunspots*, in which you set out the same doctrine as true; and that you answered the objections drawn from Sacred Scripture, which were raised against you from time to time, by glossing the said Scripture according to your own sense; and whereas there was afterwards produced a copy of a writing in the form of a letter, said to have been written by you to a certain former disciple of yours, in which, following the hypotheses of Copernicus, you include several propositions contrary to the true sense and authority of Sacred Scripture:

This Holy Tribunal, therefore, wishing to provide against the disadvantages and damages that arose from this and were spreading to the ruin of the holy Faith, by order of Our Lord and of the Most Eminent Lords Cardinals of this supreme and universal Inquisition, had the two propositions on the stability of the Sun and the motion of the Earth qualified by the theological Qualifiers, as follows:

That the Sun is at the centre of the world and immobile with respect to local motion is a proposition absurd and false in philosophy, and formally heretical, because it is expressly contrary to Sacred Scripture.

That the Earth is not the centre of the world, nor immobile, but moves even with a diurnal motion, is likewise a proposition absurd and false in philosophy, and, considered theologically, at the least erroneous in faith.

But since it then pleased us, for the time being, to proceed gently with you, it was decreed in the Holy Congregation held before Our Lord on 1616 February 25 that the Most Eminent Lord Cardinal Bellarmine should enjoin you to abandon altogether the said false doctrine; and that, if you refused, you should be commanded by the Commissary of the Holy Office to give up the said doctrine, and not to teach it to others, nor defend it, nor treat of it; and that, if you did not acquiesce in this command, you should be thrown into prison. And in execution of the same decree, on the following day, in the Palace, before the above-mentioned Most Eminent Lord Cardinal Bellarmine, after you had been gently admonished by the same Lord Cardinal, you were commanded by the Lord Commissary of the Holy Office then in office, in

the presence of a notary and witnesses, to desist altogether from the said false opinion, and that in future it should not be lawful for you to defend or teach it in any way, neither by word nor in writing; and when you had promised obedience, you were dismissed.

And so that so pernicious a doctrine might be removed entirely and creep no further to the grave detriment of Catholic truth, a decree issued from the Sacred Congregation of the Index by which the books treating of a doctrine of this kind were prohibited; and that doctrine was declared false and altogether contrary to Sacred and Divine Scripture. And since this book, published at Florence in the year just past, has lately appeared, whose inscription showed that you were its author — its title being *Dialogo di Galileo Galilei delle due massime Sisteme del Mondo, Tolomaico, e Copernicano* (Dialogue of Galileo Galilei on the two chief systems of the world, Ptolemaic and Copernican) — and since the Sacred Congregation learned at the same time that, through the printing of the said book, the false opinion of the motion of the Earth and the stability of the Sun was gaining strength more and more every day, the said book was diligently examined, and in it was found an open transgression of the said command that had been intimated to you, in that in the same book you had defended the said opinion, already condemned and declared to be such in your presence. For in the said book, by various circumlocutions, you take pains to persuade that it is left by you as undecided and expressly probable; and this too is a most grave error, since an opinion that has already been declared and defined to be contrary to divine Scripture can in no way be probable.

Therefore, by our order, you were summoned to this Holy Office, where, examined under oath, you acknowledged the said book as written by you and committed by you to print. Likewise you confessed that about ten or twelve years ago, after the command had been given you as above, you began to write the said book. Likewise, that you sought licence to publish it, without however indicating to those who gave you that permission that you had been commanded not to hold, defend or teach such a doctrine in any way.

You confessed likewise that the writing of the said book is composed in many places in such a way that a reader might think that the arguments brought for the false side were so phrased as to be more able to constrain the intellect by their efficacy than to be easily refuted. You excused yourself on the ground that you had fallen into an error so foreign (as you said) to your intention because you had written in the form of a dialogue, and through the natural complacency that everyone takes in his own subtleties, and in showing himself cleverer than men commonly are in devising ingenious and apparently probable arguments even in favour of false propositions.

And when a suitable term had been assigned to you for making your defence, you produced a certificate in the hand of the Most Eminent Lord Cardinal Bellarmine, procured by you, as you said, to defend yourself from the calumnies of your enemies, who kept saying that you had abjured and had been punished by the Holy Office. In this certificate it is said that you had not abjured, nor been

punished, but only that the declaration made by Our Lord and promulgated by the Sacred Congregation of the Index had been notified to you, in which it is contained that the doctrine of the motion of the Earth and the stability of the Sun is contrary to the Sacred Scriptures, and that it can therefore be neither defended nor held. Therefore, since no mention is made there of two clauses of the command, namely *to teach* and *in any way*, it is to be believed that in the course of fourteen or sixteen years they had slipped from your memory, and that for this very reason you were silent about the command when you sought permission to have the book printed; and that you say this not to excuse your error, but so that it may be ascribed to vain ambition rather than to malice. But this very certificate, produced for your defence, has rather aggravated your case, since it says in it that the said opinion is contrary to Sacred Scripture, and yet you dared to treat of it, to defend it, and to urge it as probable. Nor does the permission artfully and cunningly extorted by you help you, since you did not disclose the command laid upon you.

But since it seemed to us that you had not stated the whole truth about your intention, we judged it necessary to proceed to a rigorous examination of you, in which (without any prejudice to the things you had confessed, and to those that had been brought against you above concerning your said intention) you answered as a Catholic. Therefore, having seen and maturely considered the merits of this case of yours, together with your above-mentioned confessions and excuses, and everything else that was by law to be seen and considered, we have come to the definitive sentence written below against you.

Invoking, therefore, the most holy name of Our Lord Jesus Christ and of his most glorious Mother, Mary ever Virgin, by this our definitive sentence — which, sitting as a tribunal, on the counsel and judgement of the Reverend Masters of Sacred Theology and Doctors of both Laws, our Consultors, we deliver in these writings in the case and cases brought before us between the Magnificent Carolus Sincerus [Carlo Sinceri], Doctor of both Laws, Fiscal Procurator of this Holy Office, of the one part, and you, Galileo Galilei, the defendant here, inquired into, examined and confessed as above by the present written process, of the other — we say, pronounce, judge and declare that you, the above-mentioned Galileo, on account of the things brought out in the written process and confessed by you as above, have rendered yourself to this Holy Office vehemently suspect of heresy: that is, of having believed and held a doctrine false and contrary to the Sacred and Divine Scriptures, namely that the Sun is the centre of the world [orbis terræ] and does not move from east to west, and that the Earth moves and is not the centre of the world, and that an opinion may be held and defended as probable after it has been declared and defined to be contrary to Sacred Scripture; and that you have consequently incurred all the censures and penalties enacted and promulgated against delinquents of this kind by the Sacred Canons and by other constitutions, general and particular. From these it is our pleasure that you be absolved, provided that first, with a sincere heart and unfeigned faith, you abjure, curse and detest before us the

above-mentioned errors and heresies, and any other error and heresy contrary to the Catholic and Apostolic Roman Church, in the form that will be presented to you by us.

And so that this grave and pernicious error and transgression of yours may not remain altogether unpunished, and so that you may be more cautious in future and be an example to others, that they may abstain from offences of this kind, we decree that the book of the *Dialogues* of Galileo Galilei be prohibited by public edict; and we condemn you to the formal prison of this Holy Office for a period to be fixed at our discretion; and by way of salutary penance we enjoin that for the next three years you recite the seven penitential psalms once a week, reserving to ourselves the power of moderating, changing or taking away, wholly or in part, the above-mentioned penalties and penances.

And so we say, pronounce, and by sentence declare, ordain, condemn and reserve, in this and in every other better manner and form in which by law we can and ought.

So we, the undersigned Cardinals, pronounce.

F., Cardinal of Ascoli.
G., Cardinal Bentivolus.
F., Cardinal of Cremona.
Brother Antonius, Cardinal of St Onuphrius.
B., Cardinal Gypsius.
F., Cardinal Verospius.
M., Cardinal Ginettus.

ABIVRATIO GALILÆI.

The abjuration of Galileo.

I, Galileo, son of the late Vincenzio Galilei of Florence, aged seventy years, arraigned in person before this court, and kneeling before you, Most Eminent and Most Reverend Lord Cardinals, Inquisitors General throughout the whole Christian Commonwealth against heretical depravity, having before my eyes the most holy Gospels, which I touch with my own hands, swear that I have always believed, do now believe, and with God's help shall in future believe all that the Holy Catholic and Apostolic Roman Church holds, preaches and teaches. But because by this Holy Office — on the ground that, after I had been juridically enjoined by the same, with a command, to abandon altogether the false opinion that holds the Sun to be the centre of the world and immobile, and the Earth not to be the centre and to move, and that I might not hold, defend or teach the said false doctrine in any way, or in writing; and after it had been notified to me that the said doctrine was repugnant to Sacred Scripture, I wrote and committed to print a book in which I treat that same doctrine, already condemned, and adduce arguments of great efficacy in fa- vour of it, without offering any solution — I have for that reason been judged vehemently suspect of heresy, namely of having held and believed that the Sun is the centre of the world and immobile, and that the Earth is not the centre and moves.

Therefore, wishing to remove from the minds of Your Eminences, and of every Catholic Christian, this vehement suspicion rightly conceived against me, with a sincere heart and unfeigned faith I abjure, curse and detest the above-mentioned errors and heresies, and in general every other error and sect contrary to the said Holy Church; and I swear that in future I shall never again say or assert, by word or in writing, anything on account of which a similar suspicion could be held of me; but that, if I know of any heretic or anyone suspected of heresy, I shall denounce him to this Holy Office, or to the Inquisitor and Ordinary of the place where I may be. I swear moreover and promise to fulfil and observe entirely all the penances that have been imposed on me, or shall be imposed, by this Holy Office. And if it should happen that I contravene any of my said promises, protestations and oaths (which God forbid), I submit myself to all the penalties and punishments that have been enacted and promulgated against delinquents of this kind by the Sacred Canons and by other constitutions, general and particular. So help me God and his holy Gospels, which I touch with my own hands.

I, the above-mentioned Galileo Galilei, have abjured, sworn, promised and bound myself as above; and in witness of this I have subscribed with my own hand the present document of my abjuration, and have recited it word for word, at Rome, in the convent of the Minerva, on this day, 1633 June 22.

I, Galileo Galilei, have abjured as above, with my own hand.

CONCLVSIO Sectionis Huius.

Conclusion of this Section.

So much for the arguments on both sides, and for the censures against the assertors of the immobility of the Sun and the motion of the Earth. How justly and prudently these censures were passed, and how undeservedly some speak against them, or even do not yet acquiesce in them, is clear from what has been said in the whole of this section from chapter 4. This section, intended above all others as a part of this volume and of my work, I congratulate myself, with the greatest thanksgiving, on having brought with God's help to the desired end. For I wished it to be, as it were, an apology for the Sacred Congregation of Cardinals who pronounced the said censures as a tribunal — not so much because I thought that so lofty a summit of authority and eminence needed this labour of mine, especially among Catholics, as out of love for the truth, which is to be urged upon anyone whatever, even a non-Catholic; out of a certain signal zeal and eagerness to keep Sacred Scripture whole and intact; and lastly out of the reverence and observance that I owe, by my own particular Institute, to the holy Catholic and Apostolic Roman Church.

Let there be, therefore,

CONCLVSIO VNICA & Vnicè in hac controuerſia Complectenda.

The single conclusion, and the one alone to be embraced in this controversy.

It must be asserted absolutely that the Earth stands naturally immobile at the centre of the world, and that the Sun moves around it with both a diurnal and an annual motion.

This is proved, first, because the opposite is manifestly repugnant to Sacred Scripture understood according to the letter (as it must be, so long as there is no absurdity), as we have shown from chapter 36 to 39 inclusive; second, because the opposite was condemned as heretical — or, as regards the motion of the Earth, as at the least erroneous in faith — by the Sacred Congregation of Cardinals deputed by the Supreme Pontiffs Paul V and Urban VIII, as is clear from the preceding chapter 40; and finally, because the opposite is contrary to physical evidence and to certain physico-mathematical demonstrations, according to what has been said more than once, especially in chapter 35.

SECTIO QVINTA DE SYSTEMATE MVNDI HARMONICO.

Section Five: On the Harmonic System of the World.

CAPVT I.

Chapter 1.

De Neceſſitate huius Sectionis ac de Auctoribus, qui de Proportionibus Harmonicis tractauerunt.

On the necessity of this Section, and on the authors who have written on harmonic proportions.

I. No one with even a little learning in astronomy who has contemplated the order of the heavens fails to recognise a certain harmony in the intervals and the motions of the planets; and what kind of harmony it is, and how great, cannot be neglected by anyone who professes an accurate treatment of the system of the world and of the heavens. There are, however, many things necessary or useful to the theory and practice of music that are not at all required for this dissertation on the harmonic system. Yet, so that no one's desire and curiosity may seem too little satisfied, I shall at least point out a good number of authors from whom, as from springs, a reader may quench this thirst, importunate though it may be.

These authors were: Aristotle, section 19 of the *Problems*, and there Blancanus, on the mathematical passages of Aristotle, from marginal number 359; Nicomachus in the *Enchiridion* of harmonics; the musical fragments of Heraclides Ponticus in Athenaeus; Aristoxenus in three books of *Harmonic Elements*, translated by Antonius Gogavinus [Antonio Gogava]; Euclid in the *Musical Elements*; Panaetius in his book on geometrical and musical ratios; Vitruvius, book 5, chapter 4, and on that passage Daniel Barbarus [Daniele Barbaro] and Gulielmus Philander [Guillaume Philandrier]; Pliny the Elder, book 2, chapter 22, and on that passage his commentators; Plutarch in the short work *On Music*, with the exposition of Carolus Valgulius [Carlo Valgulio]; Ptolemy in three books of *Harmonics*, translated by Antonius Gogavinus; Porphyry on Ptolemy's

Harmonics; Cassiodorus, *On the Disciplines*, under the heading "On Music"; St Augustine, *On Music*; Boethius in his book *On Music*, with the exposition of Franchinus Gaffurius [Franchino Gaffurio] and of Faber Stapulensis [Jacques Lefèvre d'Étaples]; Apuleius and Psellus [Michael Psellos] on music; Censorinus, *On the Birthday*, chapter 11; Isidore, book 3 of the *Origins*, in the chapter on music; Martianus Capella, book 9 of the *Philology*; Bede in the short work *On Music*; Macrobius, book 2 of the *Dream of Scipio*, from chapter 1 to 5; Guido Aretinus [Guido of Arezzo] in the *Introductorium* of music; Berno, abbot of Cluny; Io. Freschius; Petrus Aron [Pietro Aron]; Otho Theogerus, bishop [Theoger of Metz]; Othomarus Argentinas [Ottmar Luscinius] — all in books on music; Iodocus à Prato, otherwise called Iusquin [Josquin des Prez]; also Io. Richafort [Jean Richafort]; Nicolaus Bursius of Parma [Nicolò Burzio]; Io. Ghiselinus [Johannes Ghiselin]; Adamus Fuldensis [Adam of Fulda]; Iacobus Obrecht, a Netherlander [Jacob Obrecht]; Sixtus Dietrich; Gherardus à Salice; Andreas Sylvanus; Nicolaus Craën; Vincentius Lusitanus [Vicente Lusitano]; Horatius Tigrinus [Orazio Tigrini] in his musical compendium; Antonius à Vinea; Petrus Pontius [Pietro Ponzio]; Stephanus Vanneus [Stefano Vanneo] and Petrus Vanneus, *On the Power of Music*; Io. and Georgius Valla [Giorgio Valla]; Alypius in the *Isagoge* of music; Artusius [Giovanni Maria Artusi], *On the Imperfection of Modern Music*; Damianus Goës [Damião de Góis]; Gregorius Meyer; Io. Spadarus of Bologna [Giovanni Spataro]; Nicolaus Vicentinus [Nicola Vicentino]; Theodoricus Augustanus; Hieronymus Diruta [Girolamo Diruta]; Henricus Glareanus in his *Dodecachordon*; Ludovicus Folianus of Modena [Lodovico Fogliano] in *Musica theorica*; Franchinus Gaffurius; Iosephus Zarlinus [Gioseffo Zarlino] in the *Harmonic Institutions*; Vincentius Galilaeus [Vincenzio Galilei] in the *Dialogues on Music*; Marsilius Ficinus in the compendium of Plato's *Timaeus*, from chapter 27 to 35; Scipio Cerretus of Naples [Scipione Cerreto]; Io. Petrus Aloysius Palestrina [Giovanni Pierluigi da Palestrina]; Ludovicus Zacconus [Lodovico Zacconi]; Aloysius Denticus [Luigi Dentice]; Felix Anerius [Felice Anerio]; Franciscus Salinas [Francisco de Salinas], doctor of the school of Salamanca; Sebastianus Ravallius [Sebastián Raval]; Illuminatus Ayguinus [Illuminato Aiguino]; Augustinus Pisa [Agostino Pisa], on the beat in music; Clavius, in book 5 of the *Elements*, where he treats harmonic proportions; Robertus Flud [Robert Fludd], or de Fluctibus, in the *Microcosm* and the *Macrocosm*, each of which he contends was founded on harmonic laws; Johannes Kepler in five books of *Harmonics*, in the *Mysterium Cosmographicum*, chapter 12, and in the *Epitome of Copernican Astronomy*, book 6, from page 840 and 900; Pierre Hérigone in tome 5 of the *Cursus mathematicus*, from page 573, and more fully in Euclid's *Music*, in the same tome 5, from page 802; Marinus Mersennius on chapter 4 of Genesis, from verse 21, and in the paralipomena of the same work, from page 1854; Marius Bettinus [Mario Bettini], *Apiarium* 10, Progymnasma 1; and, most recently and most learnedly, Athanasius Kircher in the ten books of his *Musurgia*.

II. [Margin: The principal authors on the praises of music.] On the praises of music, and on its admirable power for many things, but above all for stirring and calming the emotions of the mind, and so on: Plutarch, the short work *On Music*; Petrus Vanneus, *On the Power of Music*; Martianus Capella, book 9 of *The Marriage of Philology and Mercury*, chapter 1; Zarlinus, part 1 of the *Harmonic Institutions*, from chapter 2 to 7, and part 2, from chapter 4 to 9; Mersennius on chapter 4 of Genesis, verse 21; Athanasius Kircher passim, but above all books 9 and 10 of the *Musurgia*.

CAPVT II.

Chapter 2.

De Concentu Cælorum In Communi.

On the concert of the heavens in general.

I. We have the most ancient testimony to the concert [concentus] of the heavens from God himself, the founder and harmonizer [Harmoſte] of the heavens, speaking thus with Job: [Margin: Job, chapter 38, verse 37.] *Who shall tell out the reckoning of the heavens, and who shall make the concert of heaven to sleep?* Various interpreters interpret this passage variously, but many refer it to the harmony that shines forth in the motions, the intervals and the arrangement of the stars. [Margin: The concert of heaven from Sacred Scripture.] So the Chaldean paraphrast: *Who has set the stars of heaven in wisdom?*, or: *Who has set the weeks of heaven in wisdom, and who has dwelt in the revolution of heaven?* The Zurich version, on the other hand: *Who by his wisdom will dictate to the aether what it is to do? And who will stay the bottles of heaven?* And Pagninus: *Who has numbered the heavens in wisdom, and who has made the things that come down from heaven to rest?* Again, Caietanus and Vatablus interpret it thus: *Who made the heavens so wisely, in the number in which they are?* And Philippus, cited by our own Pineda on that passage, interprets the concert of heaven as the arrangement and beauty of the heavens, for which men or angels praise God; so the concert is taken there objectively, for the object that rouses the rational creature to song and to the praises of God — as is said in Psalm 18: *The heavens tell the glory of God.*

Thomas Aquinas and Lyranus add, moreover, that the saying can be referred to the preceding verse, in which God had said: *Who gave the cock understanding?* For he goes on: *Who shall tell out the reckoning of the heavens?* — as if he were saying: who will tell the cock the proportion of the heavenly motions, so that from it he may discern the hours fixed for his singing? [Margin: The praises of the cock.] For with his song he seems to mark off, at every three unequal hours, the four watches of the night, but above all the fourth. On this see the praises of the cock in Pliny, book 10, chapter 21. The word *Concentus*, moreover, is in Hebrew *Nebel*, from the root *Nabal*, which means "to flow down", and, by synecdoche, a vessel in which flowing water is caught and carried, such as wineskins are; but since from little vessels of this kind there were made certain

musical instruments, the word is therefore taken for the psaltery, or for the *Nablum*, from the same root *Nabal*, and hence for concert, as the Vulgate has it; or, as Symmachus has it: *Who will adorn the organs of heaven?*; or, with St Augustine: *Who has bent down the organs of heaven to the earth?* Most of the rabbis cited by Pineda, however, interpret it as clouds full of water, or as the wineskins and bottles of heaven, and the Royal Bible as *the down-flowing of the heavens* — above all when stormy rains fall with a great noise, which God suddenly makes to sleep when he abruptly brings back fair weather. Furthermore, that passage of Joshua, chapter 10, *Sun, do not move*, and *the Sun stood still*, is in Hebrew *dom* and *vaijdom*, that is, as the Royal Bible has it, *be silent* and *it was silent*: there the motion of heaven is signified metaphorically by sound or voice, and its rest by silence; for the fabrications that the rabbis invent on that passage have been sufficiently refuted there by our own Serarius.

[Margin: The symphonism of the elements.] Also relevant to our subject are those words of Wisdom 19, verse 17: *For while the elements are changed into one another, as in an organ the sound of the quality is changed, and all keep their own sound; from which it can be judged with certainty by the very sight.* Among our own writers, Io. Lorinus and Io. Pineda interpret this passage admirably; but it is illustrated most of all by what Father Athanasius Kircher teaches in book 10 of the *Musurgia* on the symphonism of the elements, from page 367 to 373. Among many other things he adduces the tetrachord of Orpheus, in which, on the testimony of Brienius [Manuel Bryennios], Orpheus referred the Hypate to Earth, the Parypate to Water, the Paranete to Air and the Nete to Fire.

II. [Margin: The harmony of the world and of the heavens, from the Fathers.] Nor indeed are the Fathers out of tune with the Scriptures. On the contrary, [Margin: St Augustine.] St Augustine, on the harmony of the whole world, said in book 11 of the *City of God*, chapter 18: *God adorned the order of the ages like a most beautiful poem, with certain antitheses, as it were; and just as contraries set against contraries give beauty to speech, so the beauty of the age is composed by a certain eloquence, not of words but of things, through the opposition of contraries.* On that verse of Psalm 148, *His confession is above heaven and earth*, he also says: *What is the confession of God? That he himself confesses? No — but that all things confess him, all things cry out; the beauty of all things is, in a manner, the voice of those who confess God. Heaven cries out to God: you made me, not I; the earth cries out: you founded me, not I*, and so on. [Margin: St Ambrose.] St Ambrose chimes in with him in book 4 of the *Hexaemeron*, chapter 1, and in the preface on the Psalms, saying: *The angels praise the Lord, the powers of the heavens sing psalms to him; and a more explicit account holds that the very axis of heaven turns with a certain sweetness of perpetual concert, so that its sound is heard at the farthest parts of the earth, where certain secrets of nature lie; nor does this seem foreign to the usage of nature.*

[Margin: Philo.] Before him, Philo, in his book *On Dreams*, wrote most eloquently: *Heaven, by the perpetual concert of its motions, gives forth a most sweet harmony. If it could reach our ears, it would rouse in us uncontrollable loves and a*

mad desire; goaded by it, we would forget the things necessary for life, giving up food and drink, but would be refreshed, like candidates for immortality, by the divine songs of consummate music. Moses, having heard such songs when he was rapt out of the body, is said to have tasted neither bread nor water for forty days and as many nights. And a little later: *Thus heaven, the archetypal instrument of music, seems to me to have been fashioned for no other purpose than that hymns might be sung skilfully and musically to the parent of all things.* [Margin: Boethius.] After Philo let Severinus Boethius follow, most learned in musical proportions, in book 1 *On Music*, chapter 2. There he divides music three ways, namely into mundane, human and instrumental, comprehending under the mundane the admirable order and consonance of divine providence, especially in the arrangement of the heavens; and among other things he says: *How can it be that so swift a machine of heaven should move in a silent and noiseless course, even if that sound does not reach our ears?* [Margin: St Isidore.] What of Isidore, bishop of Seville, in book 3 of the *Origins*, chapter 16? Does he not say: *Without music no discipline can be perfect, for nothing exists without it. For the world itself is said to be composed by a certain harmony of sounds, and heaven itself revolves under the modulation of harmony*? [Margin: Bede.] But Bede ascribes much more to this music, in tome 1, in the *Musica theorica*, page 406, where he says: *While the celestial music is made up of the subtler things, it is rendered most sonorous without any incongruity; for it is poured secretly from the higher things down to the lower, as far as our hearing, although through habit we do not perceive it — as is the case with those who live near the* καταβαθμὸν *[Greek: katabathmon, "the descent"], that is, the descent of the Nile. But if someone were born in another world (if that were possible), as St Augustine asserts, and afterwards came into this world, he would hear it without any impediment, and it would please him beyond measure.*

[Margin: St Anselm.] St Anselm says similar things in book 1 of *De Imagine Mundi*: *The seven orbs of the heavens revolve with the sweetest harmony, and the sweetest concerts are produced by their circling. This sound does not reach our ears because it is made beyond the air, and its magnitude exceeds our narrow hearing.* [Margin: Licentius the poet.] To these must be added Licentius, a Christian poet and disciple of St Augustine, who, as is found in St Augustine, letter 39, and in Sixtus Senensis, book 5 of the *Bibliotheca*, annotation 105, sang thus of God:

He fitted numbers to the heavens, and bade them ply their sounding modes and lead out matching dances.

III. Let us now hear those outside the Church [exteros]. The most ancient of them, Pythagoras, is said to have won such fame precisely because he taught that the whole world is harmonically arranged, [Margin: Athenaeus.] as Athenaeus asserts, book 14, chapter 13; but so did Plato and Archytas, [Margin: Plutarch.] of whom Plutarch said at the end of his short work *On Music*: *But now, friends, you have passed over what is the greatest of all, and what shows most clearly that music is to be cultivated. For Pythagoras, Archytas, Plato and the rest of the ancient philosophers declared that the motions of all things and the revolutions of the stars*

could neither come to be nor subsist without music; for they contend that God the craftsman fashioned all things with harmony. [Margin: Plato.] Certainly Plato, in book 10 of the *Republic*, said that a single Siren sits on each of the celestial orbs; [Margin: Cicero.] and from Platonic springs flowed that saying of Cicero in book 1 of the *Tusculans*: *When Archimedes bound the motions of the Moon, the Sun and the five wandering stars into a sphere, he did the same as the God of Plato's Timaeus who built the world, so that a single revolution governed motions most unlike in slowness and swiftness.* But he says it more fully in the *Dream of Scipio*, where Scipio himself says: *What is this, I said, what is this sound, so great and so sweet, that fills my ears?* His father Paulus answers him at once: *This is the sound that, joined by unequal intervals which are yet marked off in fixed proportion, is produced by the impulse and motion of the orbs themselves; tempering high notes with low, it evenly produces concerts. For such great motions cannot be set going in silence, and nature brings it about that the extremes sound low at one end and high at the other.* [Margin: The praise of the number seven.] *For this reason that highest course of the starry heaven, whose revolution is swifter, moves with a high and rapid sound, but this lunar and lowest course with the lowest sound. For the ninth, the Earth — of the spheres, that is, or globes — remaining immobile, always clings to the lowest seat, embracing the middle place of the world. But those eight courses, in which the force of the modes is the same, produce seven sounds distinguished by intervals — a number that is the knot of almost all things. Learned men, imitating this with strings and songs, have opened for themselves a way back to this place, as have others who, with outstanding gifts, have cultivated divine studies in human life. Filled with this sound, the ears of men have grown deaf (nor is there any duller sense in you), as where the Nile rushes down from the highest mountains at the place called the Catadupa: the people who live near that place lack the sense of hearing because of the greatness of the noise. But this sound, from the most rapid revolution of the whole world, is so great that men's ears cannot take it in, just as you cannot look straight at the Sun, and your sight and sense are overcome by its rays.*

[Margin: Macrobius.] Subscribing to this, Macrobius, book 2 on the *Dream of Scipio*, chapter 1, speaks of that sound as a real one, in this world: *For sound must arise from the very circling of the orbs, because air that is struck, set in motion by the very intervention, gives off from itself the force of a crash, nature herself compelling the violent collision of two bodies to end in sound.* And after a few words: *From these things it is gathered by unassailable reasoning that musical sounds proceed from the revolution of the spheres: for sound must arise from motion, and the reason that is in divine things becomes in the sound the cause of melody. Pythagoras, first of all men of the Greek nation, conceived this in his mind, and understood that something composed sounds from the spheres.* He goes on to relate by what means Pythagoras, hearing the blows of hammers in a blacksmith's workshop, discovered the harmonic proportions. And in chapter 3: *Hence Plato in his Republic, when he treated of the rotation of the celestial spheres, says that Sirens sit on each of the orbs, signifying that by the motion of the spheres song is offered to the divinities; for Siren, in its Greek sense, means "singing to God". The theologians, too,* [Margin: The nine Muses.]

the nine Muses — the musical songs of the eight spheres, and the one greatest concord, which consists of them all — held to be. He adds, moreover, from Hesiod, that the eighth of the Muses, who presides over the starry sphere, is Urania, and that the ninth is Calliope, the concert arising from the sound of all the spheres, called Calliope from the beauty of her voice; and he concludes: *That the Muses are the song of the world even the Etruscans know, who called them Camenae, as if Canenae, from singing* [canendo]. *Therefore the theologians too, approving that the heaven sings, applied musical sounds to sacrifices*, and so on; and soon after: [Margin: Strophe and antistrophe, an imitation of the twofold motion of heaven.] *In the very hymns of the gods, too, metres were applied in tuneful verses by strophe and antistrophe, so that by the strophe the direct motion of the starry orb, and by the antistrophe the contrary regression of the wandering stars, might be proclaimed; from these two motions the first hymn in nature, to be dedicated to God, took its beginning.* The half-line of Angelus Politianus [Angelo Poliziano] alludes to this passage of Macrobius and of Plato:

To each its harmless Siren.

[Margin: Pliny.] Further, Pliny, book 2, chapter 3, speaking of the motion of the stars, says: *Whether the sound of so great a mass, whirling in unceasing rotation, is immense and therefore easily exceeds the capacity of our ears, I for my part could not easily say — any more, by Hercules, than whether the tinkling of the stars carried round together, as they revolve their orbs, is a concert of sweet and incredible charm. To us who live within it, the world glides by silently, day and night.* In the same book, chapter 22, he relates the Pythagorean distribution of the celestial intervals according to the musical canons, of which more below. [Margin: Psellus.] Michael Psellus, at the beginning of his *On Music*, says: *The ancients say that music contains all things.* [Margin: Censorinus.] Censorinus, who is older than he, in *On the Birthday*, chapter 11, towards the end, said of Pythagoras: *Moreover, he related many things that musicians treat of: he showed that the stars and this whole world are* ἐναρμόνιον *[Greek: enarmonion, "harmonious"].* [Margin: Dorylaus.] *That is why Dorylaus wrote that the world is the organ of God.* The rest concerning Pythagoras we shall report from Censorinus in chapter 7, number 2.

[Margin: Martianus Capella.] Fittingly, then, Martianus Capella, in *The Marriage of Philology and Mercury*, after book 8, which is entirely on astronomy, immediately adds book 9, which is on music and is so entitled; there he introduces Harmony speaking thus of herself: *Long since, indeed, hating the earth-born and disdainful of mortals, I strike the starry orbs of heaven, in which it is forbidden to expound the precepts of art, since the very rapidity of the revolving machine sings and recognises a melody consonant with the all-sounding beats of the measures.* [Margin: Ptolemy.] But we had almost forgotten our Ptolemy, who in book 1 of the *Harmonics*, chapter 2, lays down: *It is the aim of the harmonist everywhere to preserve the rational positions of the canon, in no way conflicting with the senses, according to the opinion of the majority; just as it is the astronomer's aim to preserve positions of the celestial motions consonant with the observed revolutions.* In book 3, chapter

8, he measures the configurations of the celestial aspects against musical laws, and in chapters 9, 10, 11 and 12 he compares the celestial motions to harmonic ones. [Margin: Caelius Rhodiginus.] Caelius Rhodiginus [Lodovico Ricchieri], book 5 of the *Antiquae lectiones*, chapter 25, asks, or wonders: *How does it come about that the whirling rotation of such great orbs is carried round in a silent and noiseless circuit — even if, with us not hearing it, a wonderful delight of sounds passes by?* [Margin: Petrus Gregorius Tholosas.] Similarly Petrus Gregorius Tholosas [Pierre Grégoire], book 12 of the *Syntaxes*, chapter 7, defining celestial music, says: *Celestial music is that which arises from the disposed order of the orbs and planets, by a concordant and meeting motion, in a defined space.* [Margin: Zarlinus.] Iosephus Zarlinus, too, in part 1 of the *Harmonic Institutions*, chapter 6, brings forward with his learning many things on the sound of heaven — unheard by us though it is, through habituation — and on the harmony in the motions and distances of the heavens. It now remains to settle, from what has been said, the question whether the sound and concert of the heavens is to be taken properly or metaphorically.

An Cæleſtia corpora Realem Sonitum ſuo motu producant?

Whether the celestial bodies produce a real sound by their motion.

IV. The affirmative opinion is attributed to Pythagoras by Macrobius, as we have already seen, book 2 on the *Dream of Scipio*, chapter 3; but long before that by Aristotle, who treats this very controversy in book 2 of *De caelo*, from text 52. For in text 54 he says: *That about which doubt has been raised, and which led the Pythagoreans to say that a consonance arises from these bodies being carried*, and so on — that is, from the stars, although at the beginning he did not name them, namely in text 52, where he says: *It is clear from this that the saying that a harmony arises from these bodies being carried, as though the sounds became consonant, was said wittily and finely by those who asserted it; yet the truth is not so.* Censorinus, too, very eloquently attributes the same to Pythagoras, in the book *On the Birthday*, chapter 11; we shall bring forward his words below, chapter 7, number 2.

[Margin: Authors for a real sound of the heavens.] This persuasion of Pythagoras about celestial music — to speak with the Conimbricenses, book 2 of *De caelo*, chapter 9, question 1 — took hold of the minds of very many by its sweetness, so much so that most, especially among the pagans, asserted that a sound is roused by the mutual collision of the celestial bodies, which we do not perceive, whether because of the distance, or because of habituation, or because our hearing has been dulled from infancy. Among them were — openly, indeed — Philo, in the book *On Dreams*; Cicero, in the *Dream of Scipio*; Pliny, book 2, chapter 3, although he seems to have doubts; Macrobius, book 2 on the *Dream of Scipio*, chapters 1 and 3; Martianus Capella, book 9 of the *Marriage of Philology*; Bede, in the *Musica theorica*; St Anselm, book 1 of *De Mundi imagine*; Caelius Rhodiginus, book 5 of the *Antiquae lectiones*, chapter 25; and Iosephus Zarlinus, part 1 of the *Harmonic Institutions*, chapter 6 — as is clear from their words reported in numbers 2 and 3. Not a little in favour of the same

opinion, too, are St Ambrose, in the preface on the Psalms; St Isidore, book 3 of the *Origins*, chapter 16; Plutarch, at the end of his short work *On Music*; and finally reason itself, by an induction made from what we experience in other bodies. [Margin: An argument for a real sound of the heavens.] Aristotle did not conceal this argument, *De caelo* 2, text 52: *It seems to some*, he says, *necessary that a sound should arise from the carrying of such great bodies, since it arises also from the carrying of those that are among us, which nevertheless have neither so great a mass nor such a speed as the Sun and the Moon have. How much more, then, when so many stars, so great in multitude and magnitude, are carried with such speed, is it impossible that some immense sound should not arise?* Nor is it an obstacle that this sound is not perceived by us. For the cause is that the sound falls upon us as soon as we are born and occupies our ears, and by its perpetual habituation does not allow itself to be distinguished from its contrary, namely the privation of sound, which never occurs. *Since, however*, Aristotle continues in text 52, *it seemed unreasonable that we do not hear this voice, they say the cause of this is that this sound is present to us as soon as we are born, so that it cannot be manifest by comparison with the contrary silence; for voice and silence are discerned each by the other.* He adds, moreover, the comparison with the ears of smiths, accustomed to the noise; by this he seems to allude to that workshop in which Pythagoras learned the harmonic proportions from the sound of hammers, on the testimony of Macrobius, book 2 on the *Dream of Scipio*, chapter 1. Aristotle's words at the end of text 52 are: *Therefore, just as to blacksmiths, because of habituation, it seems to make no difference, the same also happens to all men. These things, however, as was said at the beginning, are said elegantly and musically; but it is impossible that this should in reality be so.* The Ciceronian comparison, drawn from those who went deaf at the Catadupa because of the roar of the Nile, and to which Bede and Zarlinus subscribe, was already adduced in number 3.

V. [Margin: Authors against a real sound of the heavens.] The contrary opinion, however, which excludes a real sound from heaven, was followed, together with the Philosopher, by the Peripatetics, *De caelo* 2, from text 53 to 56; by the Conimbricenses, *De caelo* 2, chapter 9, question 1; Amicus, treatise 5 on *De caelo*, question 6, doubt 10; Abulensis [Alonso Tostado] on chapter 10 of Joshua, question 14; Vallesius, *De sacra philosophia*, chapter 36; Kepler, *Harmonics*, book 5, chapters 4 and 8; Serarius on chapter 10 of Joshua, question 13; Lorinus on Psalm 18, at verse 5, *Their sound has gone out into all the earth*; likewise Sixtus Senensis, book 5 of the *Bibliotheca sancta*, annotation 105; and Sanctius [Gaspar Sánchez] on chapter 38 of Job, verse 37; and not a few of the Fathers, especially St Basil, in the scholia on Psalm 18, where, explaining *There are no speeches nor words*, he adds: *Neither do they utter speech nor bring forth any word, but by the mere display of their order they call the whole earth and the sea to the praise of God*; and in homily 1 on the *Hexaemeron* he calls the opinion of Pythagoras a crafty imposture, flabby with ruinous decay. But St Ambrose treats it more fully in book 2 of the *Hexaemeron*, chapter 2. Having reported the opinion of the philosophers, who asserted that a most sweet sound is given forth

by the impulse and motion of the celestial globes but is not perceived by us, just as the roar of the Nile is not perceived by the inhabitants of the Catadupa, he adds: *But the truth itself answers them easily. For we who hear thunder, produced by the collision of clouds — would we not hear the revolutions of such great orbs, which, just as they are thought to be carried with a greater motion, so would rouse more vehement sounds?* Although this holy Doctor brings forward a different reason, for this sound's not being imperceptible, and does not resolve it, while he adds: *They add, moreover, that this sound does not reach the earth so that men, captivated by its sweetness and charm — which that most swift motion of the heavens produces from the eastern parts all the way to the west — should not abandon their own business and works, and everything here remain idle, through a certain rapture of the human mind towards the heavenly sounds.* We have already brought forward this reason, drawn from the final cause, from Philo in number 2. The same St Ambrose, in the book *On Isaac and the Soul*, chapter 7, on Aquila's version "Sounding like the Sun", adds: *By this the turning of the heavenly axis, and the courses of the Sun, Moon and stars — the concert of the globes — seem to be expressed. It is held even by some of our own; and since it has not found belief, it does not seem out of place, at least for the charm of its sweetness.* And in the preface to the Psalms of David he says: *The talk of certain people has it that the very axis of heaven turns with a certain sweetness of perpetual concert, so that, if habituation allowed, its sound would be heard in the farthest parts of the earth*, and so on. From this it is clear that he speaks from another's opinion. Besides these, St Irenaeus in his book *Against Heresies*, and St Epiphanius in book 1 of the *Panarion*, also list this among the heresies of the Marcosians: that they ascribed, that is, a real sound to the heavens.

VI. [Margin: Arguments against a real sound of the heavens.] Aristotle's arguments, for their part, are these. In *De caelo* 2, texts 53 and 54, he says: *If the heavens gave forth as great a sound as would befit them, then, even though we did not perceive it by the sense of hearing, we would still perceive it by touch, so to speak, because excessive sounds destroy even inanimate bodies — as the sound of thunder shatters certain stones, or makes them tremble and shakes them. Since, therefore, we perceive such a sound neither by hearing nor by any other sense, we more reasonably say that there is no such sound.* Again, in texts 54, 55 and 56, he teaches that the stars themselves do not produce a sound, because they are not moved of themselves but with the motion of their orbs, as a sailor moves with the motion of his ship; that the orbs themselves are not in air or in fluid fire, or in any medium suited to sound; and, finally, that *in that which is carried continuously and makes no blow, it is impossible for a sound to arise*. For not just any mutual rubbing of two bodies suffices for sound, especially if their surfaces are very highly polished; what is required is a cleaving of air or water, or of some intervening fluid body, struck by an interrupted motion. The Conimbricenses add that, in that case, those who had lacked the use of hearing for some years, through an injury to the organ corrupted by some disease, would perceive the sound of the heavens when they recovered their hearing. I add, for my part, that just as the perpetuity of the stars' motion does not prevent the eye from discerning it, so neither

would the perpetuity of a sound prevent us from hearing it; nor is silence or rest required for this. It seems to me, however, that if any sound were after all produced in the fluid aether, which extends as far as the sphere of the fixed stars — whether by the Firmament itself or by the bodies of the planets themselves — it could not reach us, both because of the thinness of the aether itself, which yields to their motion with almost no resistance, and because of their huge distance from us. Kepler, however, who holds the sphere of the fixed stars to be immobile, settles the matter of the others thus in book 5 of the *Harmonics*, chapter 4: *Now there are no sounds in heaven, nor is the motion so turbulent that a whirring is drawn out of the friction of the heavenly air.*

CAPVT III.

Chapter 3.

De Proportionibus, & præcipuè de Harmonicis.

On proportions, and especially on harmonic proportions.

I. [Margin: Aliquot and aliquant parts.] A PART, according to Euclid, book 5, is a magnitude that measures a greater magnitude as its whole. If it measures it perfectly, so that when repeated some number of times it exactly equals its whole, it is called an *aliquot part* [pars aliquota]: as 2 is with respect to 8. But if, when repeated, it exceeds its whole or falls short of it, as 2 does with respect to 9, it is called an *aliquant part* [pars aliquanta]; for the number two taken four times falls short of nine, but taken five times exceeds it.

[Margin: What ratio and proportion are.] A *ratio* is the mutual relation [habitudo] of two magnitudes of the same kind with respect to quantity. A *proportion*, on the other hand — which the Greeks call ἀναλογίαν [Greek: analogian, "analogy"] and some Latin writers *proportionalitas* — is a likeness of ratios. Thus the relation of 2 to 4 taken by itself, or of 3 to 6, is called a ratio; but the likeness of relation between the ratio of 2 to 4 and the ratio of 3 to 6 — that is, the fact that 2 is to 4 as 3 is to 6 — is called a proportion. It has, however, now become customary for a ratio itself to be called a proportion as well.

[Margin: Rational and irrational proportion.] Proportion is divided, first, into *rational* and *irrational*. A rational proportion is one that can be expressed in numbers, such as the proportion of a line three palms long to a line of six palms, since it can be expressed by the numbers 3 and 6. Hence such a proportion holds between two commensurable quantities, that is, quantities having some common aliquot part which can measure each of them exactly — as unity is such a part with respect to any whole number. And therefore between numbers, or discrete quantities, there cannot fail to be commensurability and rational proportion. An irrational proportion is one that cannot be expressed in numbers; it is found only between continuous quantities. Euclid demonstrates, at the end of book 10, that such a proportion, or incommensurability, holds between the diagonal and the side of any square: for no aliquot part can be

assigned common to those lines, nor can their proportion be expressed in any numbers.

[Margin: Proportion of equality and of inequality, greater or lesser.] Second, proportion is divided into the proportion of *equality*, such as holds between 20 and 20 or between 100 and 100, and of *inequality*, which holds between two unequal quantities, as between 20 and 10. The latter is subdivided into the proportion of *greater* and of *lesser inequality*, according as the greater quantity is compared with the lesser, as antecedent with consequent — as 4 with 2 — or the lesser with the greater, as 2 with 4, putting 2 in the antecedent of the proportion and 4 in the consequent.

[Margin: The 5 kinds of rational proportions of inequality.] Third, rational proportion, of greater as well as of lesser inequality, is subdivided into five kinds, of which the first three comprise the simple and the other two the compound. The simple kinds of greater inequality are the *multiple*, the *superparticular* and the *superpartient*; the compound are the *multiple superparticular* and the *multiple superpartient*. If you add to these the prefix *sub*, you will have as many kinds of rational proportion of lesser inequality, and they will be, and will be called, the *submultiple*, *subsuperparticular*, *subsuperpartient*, *submultiple superparticular* and *submultiple superpartient*.

II. [Margin: Multiple proportion.] A *multiple* proportion is the relation of a greater quantity to a lesser which measures that greater exactly, such as the proportion between 4 and 2, between 20 and 4, between 100 and 5. Its species are determinate and run on to infinity — namely double, triple, quadruple and so on — according as the greater quantity contains the lesser twice, three times, four times, and so on.

[Margin: Superparticular proportion.] A *superparticular* proportion is when the greater quantity contains the lesser once only, and in addition one aliquot part of that lesser — whether this part is a half, a third or a fourth, and so on to infinity. The species of these proportions are distinguished by the particle *sesqui*, as their characteristic mark, with the number of the aliquot part added. Thus a proportion is called *sesquialtera* when the greater contains the lesser once and in addition half of the lesser, as is the proportion of 3 to 2, or 9 to 6, or 30 to 20. In Greek it is called ἡμιόλιος [Greek: hēmiolios, "half-and-whole"], because ἥμη [recte: ἡμι-] [Greek: hēmi-, "half"] means half and ὅλος [Greek: holos, "whole"] means whole, as if you were to render it *totimedia* or *totidimidia*, "whole-and-a-half". But if the greater contains the lesser once and besides a third, or a fourth, a fifth, a hundredth or a thousandth part of the lesser, the proportion is called *sesquitertia*, *sesquiquarta*, *sesquiquinta*, *sesquicentesima*, *sesquimillesima*, and so on for the rest. It is worth setting out here some examples of them, comparing the upper number with the lower.

Superparticularium Proportionum Exempla.

Examples of superparticular proportions.

Proportion					
Sesquialtera, ἡμιόλιος [hēmiolios]	3 / 2	9 / 6	15 / 10	36 / 24	45 / 30
Sesquitertia, ἐπιτρίτη [epitritē]	4 / 3	12 / 9	20 / 15	24 / 18	100 / 75
Sesquiquarta	5 / 4	15 / 12	30 / 24	60 / 48	100 / 80

Reſiduum Tabulæ Præcedentis.
The rest of the preceding table.

Proportion					
Sesquiquinta	6 / 5	18 / 15	30 / 25	48 / 40	90 / 75
Sesquisexta	7 / 6	21 / 18	35 / 30	70 / 60	98 / 84
Sesquiseptima	8 / 7	24 / 21	40 / 35	72 / 63	96 / 84
Sesquioctava	9 / 8	27 / 24	45 / 40	72 / 64	99 / 88
Sesquinona	10 / 9	30 / 27	40 / 36	80 / 72	100 / 90
Sesquidecima	11 / 10	44 / 40	77 / 70	121 / 110	132 / 120
Sesquicentesima	101 / 100	202 / 200	303 / 300	404 / 400	505 / 500

III. [Margin: Superpartient.] A *superpartient* proportion is when the greater quantity contains the lesser quantity once only and, in addition, several aliquot parts of the lesser which, taken together, do not make up a single aliquot part, unity excepted; otherwise it would turn out to be superparticular. Such is the proportion between 8 and 5. The species of this proportion are usually distinguished by two characteristic marks. One of them, expressed by the particles *bi-*, *tri-*, *quadri-*, *quintu-*, *sextu-*, *septu-*, *octu-* and so on, indicates the number of aliquot parts that the greater quantity contains; the other names the proportion of those aliquot parts to the lesser quantity, which they measure. For example, the proportion of the number 8 to 5 is called *supertripartiens quintas* [supertripartient of fifths], because 8 contains 5 once and, besides, three fifth parts of five, which are three units; and so for the rest. Look at some examples of them in the following table, comparing the upper number with the lower.

Exempla Proportionis Superpartientis.
Examples of superpartient proportion.

Proportion				
Superbipartiens tertias	5 / 3	20 / 12	50 / 30	100 / 60
Superbipartiens quintas	7 / 5	35 / 25	63 / 45	98 / 70
Supertripartiens quartas	7 / 4	21 / 12	70 / 40	98 / 56
Supertripartiens quintas	8 / 5	40 / 25	80 / 50	96 / 60
Superquadrupartiens quintas	9 / 5	27 / 15	49 / 35	99 / 55
Superquintupartiens sextas	11 / 6	44 / 24	77 / 42	99 / 54
Supersextupartiens septimas	13 / 7	65 / 35	91 / 49	130 / 70
Superseptupartiens octavas	15 / 8	60 / 32	90 / 48	120 / 64
Superoctupartiens nonas	17 / 9	51 / 27	85 / 45	170 / 90
Supernoncupartiens decimas	19 / 10	57 / 30	95 / 50	190 / 100

Proportiones Compoſitæ.

Compound proportions.

IV. [Margin: Multiple superparticular.] A *multiple superparticular* proportion is when the greater quantity contains the lesser some number of times — say twice, three times or four times, and so on — and in addition one aliquot part of the lesser. Such is the proportion of 9 to 4: for nine contains four twice and, besides, a fourth part of four. It is therefore called *dupla sesquiquarta* [double sesquiquartal]: double because it contains the lesser twice, *sesquiquarta* because it contains in addition a fourth part of the lesser. The species of this proportion are therefore determinate and run on to infinity, compounded from the species of the multiple — double, triple, quadruple and so on — and from the species of the superparticular, namely *sesquialtera*, *sesquitertia*, *sesquiquarta* and so on. When these are combined, the result is the proportion *dupla sesquialtera*, *dupla sesquitertia*, *dupla sesquiquarta* and so on, or *tripla sesquialtera*, *tripla sesquitertia*, *tripla sesquiquarta* and so on, and likewise for the rest. Look at some examples of them in the following small table, comparing the upper number with the lower.

Exempla Proportionis Multiplicis Superparticularis.

Examples of multiple superparticular proportion.

Proportion				
Dupla sesquialtera	5 / 2	25 / 10	60 / 24	100 / 40
Dupla sesquitertia	7 / 3	35 / 15	70 / 30	98 / 42
Tripla sesquialtera	7 / 2	35 / 10	70 / 20	98 / 28
Tripla sesquitertia	10 / 3	40 / 12	60 / 18	100 / 30
Tripla sesquiseptima	22 / 7	66 / 21	110 / 35	176 / 56
Decupla sesquitertia	31 / 3	93 / 9	248 / 24	341 / 33

V. [Margin: Multiple superpartient.] A *multiple superpartient* proportion is when the greater quantity contains the lesser some number of times and, in addition, aliquot parts of the lesser that do not make up a single aliquot part. Such is the proportion of 11 to 3: for eleven contains three three times and, besides, two third parts of three, which do not make one aliquot part of three; otherwise the proportion would be multiple superparticular. The species of this kind are determinate and run on to infinity, and they take their names partly from the species of the multiple proportion — namely from double, triple, quadruple and so on — and partly from the species of the superpartient proportion. Combining the names in this way gives *dupla superbipartiens tertias*, *dupla supertripartiens quartas*, *dupla superquadripartiens quintas* and so on; likewise *tripla superbipartiens tertias*, *tripla supertripartiens quartas* and so on. See a specimen of them in the following table, comparing the upper number with the lower.

Exempla Proportionis Multiplicis Superpartientis.

Examples of multiple superpartient proportion.

Proportion				
Dupla superbipartiens tertias	8 / 3	32 / 12	80 / 30	96 / 36
Dupla supertripartiens quartas	11 / 4	44 / 16	110 / 40	220 / 80
Dupla superquadrupartiens quintas	14 / 5	42 / 15	140 / 50	280 / 100
Tripla superbipartiens tertias	11 / 3	33 / 9	110 / 30	220 / 60
Tripla supertripartiens quartas	15 / 4	60 / 16	90 / 24	120 / 32
Dupla superquintupartiens sextas	17 / 6	51 / 18	85 / 30	170 / 60
Quintupla supertripartiens quintas	28 / 5	56 / 10	84 / 15	280 / 50

VI. That there cannot be more rational proportions than the five already mentioned, of greater as well as of lesser inequality, is established as follows. Any commensurable quantities whatever (between which there is a rational proportion) have to one another the proportion that a number has to a number, as Euclid demonstrated in book 10 of the *Elements*, proposition 5; and a greater number cannot be compared with a lesser in any ways other than the five already mentioned. For either the greater contains the lesser some number of times exactly, that is, with no remainder; or it contains it once and, besides, a single aliquot part of the lesser; or it contains it once and several aliquot parts of the lesser which do not make one aliquot part; or it contains the lesser several times and a single aliquot part of the lesser; or, finally, it contains the lesser several times and, besides, several aliquot parts of the lesser which do not make one aliquot part. As for the denominators of proportion, see our Clavius on book 5 of Euclid, from page 540 in my copy, of the Roman edition of the year 1589.

De Proportionalitate Arithmetica, Geometrica, & Harmonica.

On arithmetic, geometric and harmonic proportionality.

VII. Boethius, Iordanus Nemorarius [Jordanus de Nemore] and many other arithmeticians call the proportionalities already mentioned *medietates* [means].

[Margin: Arithmetic proportionality.] ARITHMETIC proportionality, or mean, is when three or more numbers proceed by the same difference, as do these numbers: 4, 7, 10, 13, 16; for each of them exceeds its antecedent by three. If this happens continuously, it is called continuous proportionality; but if the numbers are set with a jump and an interruption — for example 4, 7 and 8, 11 and 30, 33 — it is called discrete.

[Margin: Geometric proportionality.] GEOMETRIC proportionality, or mean, is when three or more numbers keep the same proportion among themselves; and this is properly an *analogia* [analogy], or proportionality. Such is the subtriple in these numbers: 2, 6, 18, 54, 162. If it proceeds continuously in this way it is called continuous; but if it is interrupted it is called discrete, as in these: 2, 6 — 54, 162.

[Margin: Musical proportionality.] HARMONIC, or musical, proportionality or mean is when three numbers are so arranged that the proportion of the greatest to the least is the same as that of the difference between the two greater

to the difference between the two lesser. Such is the case in these numbers: 3, 4, 6. For the proportion of 6 to 3 is double, while the difference of the greatest is 2 and of the least 1, between which the proportion is likewise double. But the numbers already mentioned proceed neither by the same difference, as happens in arithmetic proportionality, nor by the same proportion, as in geometric. Another example of harmonic proportionality is in these numbers: 42, 12, 7. It is called musical, or harmonic, because it mostly contains those proportions in which the musical consonances consist. For in the first example, between 6 and 4 there is the *sesquialtera* proportion, which makes the consonance called *diapente*, or the fifth; between 4 and 3 there is the *sesquitertia* proportion, which makes the consonance called *diatessaron*, or the fourth; and finally, between the extreme numbers 6 and 3 there is the double proportion, which makes the consonance called *diapason*, or the octave; and so with most of them. Kepler, however, in book 3 of the *Harmonics*, chapter 3, contends that the definition of harmonic proportion already mentioned is empty, because he takes the cause of harmony from the division of the circle into aliquot parts, as will be said in chapter 4.

VIII. [Margin: Properties of the three proportionalities already mentioned.] Further, the three proportionalities already mentioned are distinguished from one another by several notable properties, which Clavius lists on book 5 of Euclid, from page 555. [Margin: 1st property.] The *first* is that in the geometric the proportion of the greatest of the three numbers [?] to the middle is the same as that of the middle to the least; in the arithmetic, by contrast, the proportion of the greatest to the middle is less than that of the middle to the least; while in the musical the proportion of the greatest to the middle is greater than that of the middle to the least. [Margin: 2nd property.] The *second* is that the arithmetic has equal differences between its terms but unequal proportions; the geometric, on the contrary, unequal differences between its terms but equal proportions; and the harmonic, lastly, neither equal differences nor equal proportions. [Margin: 3rd property.] The *third* is that in the arithmetic the sum of the extremes is double the middle; in the geometric and harmonic, however, the sum of the extremes exceeds double the middle by the amount by which the difference of the greater exceeds the difference of the lesser. [Margin: 4th property.] The *fourth* is that in the geometric the number produced by multiplying the first into the third is equal to the square of the middle; in the arithmetic the extremes multiplied together produce a number that falls short of the square of the middle by the number made by multiplying the difference of the lesser into the difference of the greater. [Margin: 5th property.] In the harmonic, finally, the number produced by multiplying the extremes together exceeds the square of the middle by the number made by multiplying the difference of the lesser by the difference of the greater. For the rest, see Clavius: he on page 597, and likewise Kircher in book 3 of the *Musurgia*, chapter 2, teach how to find three numbers in harmonic proportionality, as follows.

CAPVT III.

Tres Numeros in Proportionalitate Harmonica reperire.

To find three numbers in harmonic proportionality.

From three numbers in arithmetic proportionality, multiply the middle by the extremes, and you will have the extremes of a harmonic proportionality; the extremes of the arithmetic, multiplied by each other, produce the middle of the harmonic, as you see in the four following examples.

Arithmet.	1 2 3	3 7 11	4 6 8	10 60 110
Harmon.	2 3 6	21 33 77	24 32 48	600 1100 6600

How such proportions are continued, and how other problems bearing on this are solved, are set out in the same places by Clavius and Athanasius Kircher; by Zarlinus, part 1 of the *Harmonic Institutions*, from chapter 31 to 44; and by Ludovicus Folianus [Lodovico Fogliano], *Musica theorica*, section 1, especially from chapter 9. For we are not doing that now; we are only pointing out the sources from which one may judge whether harmonic proportion or proportionality — the musical mean — is found, or is to be sought, in the motions and intervals of the stars. For, as Petrus de Apono [Pietro d'Abano] said on the *Problems* of Aristotle, *the mean is what produces harmony*: of three strings, the middle one, stretched according to the ratios of the extreme ones, produces a concert most sweet to the ears. I am pleased, however, to add at the end of the chapter, in each of the five kinds of proportion, three examples of harmonic proportionality set up among three terms, as can be seen in the following table.

		Radices	Proportionalitates Harmonicæ
In the multiple genus, from	*Dupla*	1 2	3 4 6
	Tripla	1 3	2 3 6
	Quadrupla	1 4	5 8 20
In the superparticular genus, from	*Sesquialtera*	2 3	10 12 15
	Sesquitertia	3 4	21 24 28
	Sesquiquarta	4 5	36 40 45
In the superpartient genus, from	*Superbipart. tertias*	3 5	12 15 20
	Supertripart. quartas	4 7	44 56 77
	Superquadrup. quintas	5 9	35 45 63

		Radices	Proportionalitates Harmonicæ
In the multiple superparticular genus, from	*Dupla sesquialtera*	2 5	14 20 35
	Tripla sesquitertia	3 10	39 60 130
	Dupla sesquiquarta	4 9	52 72 117
In the multiple superpartient genus, from	*Dupla superbipart. tertias*	3 8	33 48 88

		Radices	Proportionalitates Harmonicæ
	Dupla supertripart. quartas	4 11	60 88 165
	Dupla superquadrup. quintas	5 14	98 14 166 [recte: 95 140 266]

CAPVT IV.

Chapter 4.

De Conſonantiarum ac Diſſonorum Interuallorum Inuentoribus, Inuentione, Numero, & Nomenclatura : deq. Monochordi ſimplici diuiſione.

On the inventors, the discovery, the number and the naming of the consonances and of the dissonant intervals; and on the simple division of the monochord.

I. How close a kinship there is between the pastoral art, the hammering or smith's art, and the art of music can be traced back to their very first origin, since it is agreed that these three arts were born in one and the same house, that of Lamech, who was of the line of Cain, in births close together, if not in a single birth. For the sacred writer Moses relates of Lamech in Genesis 4: [Margin: The origin of music from Jubal.] *Who took two wives, the name of the one Adah, and the name of the other Zillah. And Adah bore Jabal [?], who was the father of those who dwell in tents and of herdsmen; and his brother's name was Jubal, who was the father of those who sing to the harp and the organ. Zillah also bore Tubal-cain, who was a hammerer and smith in every work of brass and iron.* It is as though even then music were exerting its powers between the soft kind of songs and sounds that suits herdsmen and the hard kind that suits those who hammer metals; and we were given to understand that neither gentle nor fierce natures can be rightly ruled or tamed without some harmony. Although Berosus the Chaldean and Josephus the Hebrew affirm that the musical consonances were discovered by Jubal while he listened with an attentive ear to the sound of Tubal-cain's hammers, it is said that afterwards — whether that art was shipwrecked in the Flood or lay hidden among a few Hebrews — it arose again under Mercury, the inventor of the tortoise-shell lyre, as Diodorus Siculus and Lucian hold; or under Apollo, the inventor of the lyre, as Lactantius prefers; or from Amphion, as Pliny the Elder has it. At length, however, it was reborn at its original birthplace, in a smithy, and, as if half-buried, it was roused again by the sound of hammers and revived.

[Margin: Pythagoras, the second inventor of the consonances.] For, as *Macrobius* reports (book 2 on the *Dream of Scipio*, chapter 1), and Boethius after him, Pythagoras was passing by a workshop in which smiths were softening red-hot iron with blows, and heard something harmonious strike his ears from the sounds of the hammers answering one another in a fixed order and from the alternation of a high and a low sound. To find out where that consonance came from — whether from the strength of the hammerers' arms and their force, or from the weight of the hammers themselves — he ordered the smiths

to exchange hammers among themselves. When he heard the same harmony, he did not doubt that it was to be ascribed to the difference of the weights, especially since, when other weights were added, he had obtained different sounds that were not so consonant. Having therefore examined the weights of the hammers, he found that sound stands to sound as weight stands to weight of the same material. Then, turning from the hammers to strings made of sheep's gut and ox sinews, he hung weights on them in the proportion he had detected in the hammers; and when these too gave back a similar consonance, he laid down that universal axiom: [Margin: The musical axiom.] *As magnitude is to magnitude in matter of the same species, so is sound to sound.*

II. Further, among the many consonances that Pythagoras found by the device we have described, he accepted only those that are simplest; for, as Heraclides says in his *Introduction to Music*, he wished to remove rashness and inconstancy from things and, like a bee, to sip the purest and choicest dew of concords. He therefore held that the consonances of voices were to be sought not from the untrustworthy and inconstant judgement of the ears, but from fixed causes and from the laws of numbers: for he thought simplicity the mother of constancy, and composition and mixture the seedbed of inconstancy and uncertainty. Hence it came about that he received into the canon only those consonances that arise from multiple or superparticular proportion, up to the quadruple inclusive and no further: namely those between 2 and 1, between 3 and 2, between 4 and 3, between 3 and 1, and between 4 and 1. Boethius too lists almost only these consonances (book 2, from chapter 16), with the tone added — not as a consonance, as he himself affirms in book 1, chapter 16 — namely in the ratio of 9 to 8, or 18 to 16, and so on. Further, from that quaternary of numbers within which Pythagoras confined himself arose the *Tetractys*, or Pythagorean Quaternary, by which, as by a perennial fountain of the intellectual beauty of the soul, they used to swear — or rather, by him who through the Tetractys displayed the perpetual perfection of the soul. On this Zarlinus [Gioseffo Zarlino] writes much and learnedly in part 2 of the *Harmonic Institutions*, chapter 2, and Kepler in book 3 of the *Harmonics*, from page 4.

[Margin: Musicians called Canonists and Harmonists.] From the same source arose the sect of those musicians who were called *Canonici* [Canonists], because in determining the harmonic intervals and consonances they gave more weight to the ratios of numbers than to the senses; they were also called Pythagoreans. Opposed to this sect was the sect of the Harmonists, or Aristoxenians, who attributed more to the sense of hearing than to the ratios of numbers abstracted from sensible matter. Between them *Ptolemy*, taking a middle way, gave as much to the senses as Aristoxenus did and as much to reason as Pythagoras did, and therefore approved more consonances than Pythagoras, but not all those that Aristoxenus approved. So Ptolemy, in book 1 of the *Harmonics*, chapter 1, having said that the harmonic faculty of the soul consists in discerning the difference of lowness and highness of sounds, and that sound is an affection of struck air, adds: [Margin: Ptolemy, the reconciler of sense with reason.]

Hearing and reason, then, are the judges of harmony, but not on the same terms: hearing is related to the affection as a whole and as it is in itself, reason to its form and its cause. For in general it is proper to the senses to find the approximate and to receive the exact, whereas reason, on the contrary, receives the approximate and finds the exact. Then, from chapter 6, he examines and corrects the Pythagoreans; from chapter 9, the Aristoxenians; and from chapter 13, Archytas the Pythagorean; and from chapter 15, and through the whole of book 2, he labours to reconcile reason with hearing. The same author, in book 1, chapter 2, having compared the harmonist with the astronomer in this respect — that each must investigate a priori the reasons of the apparent motions in such a way that what appears to the senses is preserved — adds: *But first of all, both the Pythagoreans and the Aristoxenians were deceived. For the Pythagoreans, not following the comparison made by the ears in all the cases where they ought to have, fitted to the differences of sounds ratios that agree very poorly with some experiments made by the senses; and so they brought calumny upon this judge, that is, upon reason, among the men of the other sect. The Aristoxenians, on the other hand, giving very great weight to what they had received through the senses, as it were misused reason in passing, and so on.*

III. But not even Ptolemy himself emended Pythagoras and Aristoxenus in such a way as to satisfy later musicians. For since the Pythagoreans received only five consonances, called *diapason*, *dia-* tessaron*, *diapente*, *diapason diapente* and *disdiapason* — or, as the moderns say, the octave, fifth, fourth, twelfth and fifteenth — and the Aristoxenians had added the *ditonus*, the *semiditonus*, and the major and minor *hexachord* — that is, the major third, the minor third, the major sixth and the minor sixth — Ptolemy rejected these last as discordant, although all musicians with good ears accept them. [Margin: Ptolemy's errors, and the cause of the errors.] Conversely, he admitted among the consonances the diapason with the semiditonus, the diapason with the ditonus, and the diapason diatessaron — that is, to speak with the practitioners, the minor tenth, the major tenth and the eleventh, which are concordant. But besides these he added the interval between 6 and 7, and the one between 7 and 8, and others like them, which the ears and the practice of singing abhor; and he omitted the minor and major thirteenth, which all musicians accept today. Kepler (book 3 of the *Harmonics*, page 8) holds that the cause of Ptolemy's error was that he sought the principles of the consonances from abstract numbers, insofar as they are numbers; and yet no precise reason can be given by numbers why these numbers 1, 2, 3, 4, 5, 6, and so on, concur in the harmonic intervals, while 7, 11, 13 and the like do not. For what delights the hearing in music comes from sound, and sound is produced from the continuous, not the discrete, quantity of bodies. Therefore, as Zarlinus notes (part 1, chapter 41), and Valgulius [Carlo Valgulio] on Plutarch's *Music*, and Kepler (book 3 of the *Harmonics*, page 9), the causes of the consonances must be sought from continuous quantity, since number is not their proximate cause, but the proportion between high and low sounds — granted that this proportion, being rational, is expressed by numbers,

and that the differences of sounds are found more exactly by numbers than by dividing a continuous quantity with compasses.

IV. [Margin: From what source Kepler determined the consonances.] For the reasons already mentioned, therefore, Kepler, searching for the cause of the consonances in continuous quantity, sought it from the divisions of the circle into aliquot parts that are made geometrically and scientifically — that is, from regular plane figures that can be demonstrated — in such a way that those sides of the figures which are knowable and have their own demonstration are consonant when compared with the whole circumscribed circle; as we shall set out in the Scholia of this chapter. Meanwhile let us set out the more common doctrine.

Definitiones & Axiomata Quædam ad Harmonicen ſpectantia.

Some definitions and axioms bearing on harmonics.

V. [Margin: What consonance is.] *Consonance*, in Greek συμφωνία [symphōnia, "sounding together"], is a ratio of number between a high and a low sound, pleasant to hear, as is gathered from section 19 of the *Problems* of Aristotle; or, as *Severinus Boethius* defines it (book 1, chapter 8): *it is a mixture of a high and a low sound, falling sweetly and uniformly on the ears.* To this definition subscribe *Glareanus* (book 3 of the *Dodecachordon*, chapter 9), *Zarlinus* (part 2 of the *Harmonic Institutions*, chapter 12) and *Kircher* (book 3 of the *Musurgia*, in the preface). *Boethius*, however, said otherwise: *Consonance is the concord, brought into one, of voices unlike one another.* He did this, as some think, so that by the first definition he might adhere to Plato, who referred consonance to likeness, and by the second to Nicomachus, who referred it to unlikeness. But *Daniel Barbarus* [Daniele Barbaro], on book 5 of Vitruvius, chapter 4, said: *Consonance, or concent, is a tempered mixture of low and high sounds which reaches the ears with pleasure, arising from a comparison either multiple or superparticular.* [Margin: Dissonance.] According to Boethius and Kircher, *dissonance*, in Greek ἀσυμφονία [asymphonia, "not sounding together"], *is the harsh and unpleasant striking upon the ear of two sounds mixed with each other.*

[Margin: What a phthongus is.] *Phtongus*, in Greek φθόγγος [phthongos, "note"], is, according to Boethius, Glareanus and Kircher in the places cited above, *the fall of the voice*; or a certain definite sound of a single string, which, if it is tuneful and suited to melody, is called ἐμμελὲς [emmeles, "melodic"], and if otherwise, ἐκμελὲς [ekmeles, "unmelodic"]. For Euclid it is *a concordant fall of the voice upon one pitch*; but for Proclus it is *a sound lacking interval*.

[Margin: Unison.] The *unison*, in Greek ἰσόφωνος [isophōnos, "equal-sounding"], is a sound entirely equal to itself or to another; or the repetition of the same sound or voice; or the twin sound of equal strings or voices. As such, it is incapable of slackening and tightening, because it is in music what unity is in arithmetic and the point in geometry.

[Margin: Harmonic space and interval [?].] *Harmonic space* is a body of sounds compared with another similar body that is tighter or slacker. A harmonic *interval*, in Greek διάστημα [diastēma, "interval"], is a certain mutual relation of the spaces of the low and the high; or the distance between a low and a high sound.

[Margin: What a high and a low sound are.] A *high sound* is one produced by the frequent and quick vibration of air compressed through the collision of bodies; a *low* one, by the slow and sluggish vibration of slack air through the collision of bodies. This causal definition can be taken only comparatively, since not even the high and the low are said absolutely, but comparatively, just like large and small, fast and slow, and so on. On this occasion those skilled in music agree on the four following axioms. [Margin: Four musical axioms on the high and the low.] 1. *A part, compared with the whole, gives a higher sound.* 2. *A harmonic whole, compared with its part, gives a lower sound*: namely because the air is struck more tightly by the part than by the whole, and vibrates more frequently. 3. *A high sound consists of more motions than a low one.* 4. *The slack, compared with the taut, gives a lower sound.*

[Margin: Tension of the voice, intension, and so on.] *Tension*, in Greek τάσις [tasis, "tension"], is the state of a voice or string set at a pitch fit for singing. *Intension* of a voice or string, in Greek ἐπίτασις [epitasis, "tightening"], is the motion from a low to a high sound; *remission*, in Greek ἄνεσις [anesis, "slackening"], is the motion from high to low. *Elevation*, in Greek ἄρσις [arsis, "raising"], is the pronunciation of a syllable or word with the acute accent; *deposition*, or θέσις [thesis, "setting down"], is pronunciation with the grave accent.

[Margin: What a simple consonance is.] A *simple consonance* is one that does not resolve into another consonance; a *composite* one is one that resolves into two consonances. On this account the *diapason*, *diapente* and *diatessaron*, and also the *ditonus*, the *semiditonus* and the major and minor *hexachord*, are called simple; whereas the *diapason diapente*, the *disdiapason*, the *ditonus* with the *diapente*, and others of this kind are called composite.

[Margin: Perfect consonance.] A *perfect consonance* is one that so delights the hearing that the appetite comes to rest in it and desires nothing further; an imperfect one, when it does not delight so much as to keep the hearing from desiring something further — as *Folianus* teaches (section 2 of the *Musica theorica*, chapter 5). Further, among the perfect consonances themselves there are certain degrees. Those that arise from multiple or superparticular proportion, do not exceed the number three, and are simple, are the most perfect of all and in the first degree: namely the *diapason*, whose proportion is 2 to 1, and the *diapente*, whose proportion is 2 to 3 — although of these two the diapason is the more perfect. Those that arise from the multiple or superparticular genus, but extend to the number four, or stay within three but are composite, are in the second degree: namely the *diatessaron*, whose proportion is 4 to 3, the *diapason diapente*, that is 3 to 1, and the *disdiapason*, that is 4 to 1. Lastly, in the third

degree are those that are simple and arise from the superparticular genus of proportion, but extend beyond four up to six inclusive: such are the *ditonus*, whose proportion is 5 to 4, and the *semiditonus*, whose proportion is 6 to 5. All the rest are imperfect, either because they do not arise from the multiple or superparticular genus, or because they lie outside the senary. For although not all the proportions of numbers contained within the senary produce perfect consonances, all perfect consonances, considered in their least and radical terms, are nevertheless contained within the senary; [Margin: Praises of the senary.] which is why *Zarlinus* (part 1 of the *Harmonic Institutions*, chapters 15 and 16) and *Athanasius Kircher* (book 4 of the *Musurgia*, chapter 4, from page 186) deservedly extol them. Among the perfect consonances, those that are simple arise from the proportions between the numbers of the senary taken in order, as you see in the following small table — granted that Zarlinus later excludes the ditonus and semiditonus from the perfect, that is, from the more perfect, consonances.

Consonance	Numbers
Diapason	(1 2
Diapente	(2 3
Diatessaron	(3 4
Ditonus	(4 5
Semiditonus	(5 6

But Kepler (book 3 of the *Harmonics*, chapter 2) sets the perfect consonances at only 6, or 7 with the unison, and (chapter 12) the imperfect at 6.

[Margin: What a dissonant interval is.] Dissonant intervals that are nonetheless musical and concordant are the differences of the consonances, or parts of them, which, although considered in themselves they are not consonances, are nevertheless steps by which one ascends or descends from one consonance to another; and they serve aptly for distinguishing and composing the consonances, as cartilages serve the bones. [Margin: How the difference of consonances is investigated.] Further, to find the difference between two consonances and assign to it its species of proportion, the greater number of one consonance must be multiplied by the lesser number of the other, and again the greater number of the other consonance by the lesser of the other; for the two numbers so produced will be the terms of the proportion sought, that is, the difference of those consonances. For example, let the difference be sought between the diapente, that is 3, 2, and the diatessaron, that is 4, 3. Multiplying 3 by 3 gives 9; multiplying 2 by 4 gives 8. This proportion of 9 to 8, then — the sesquioctave — is the difference between the diapente and the diatessaron, and it is the one we shall show below to be called the major tone; and so with the others.

VI. It remains for us to gather into a table the perfect and the imperfect consonances, and then the dissonant but concordant intervals, omitting the unison, which is the principle of the consonances, not a consonance. [Margin: Explanation of the following tables.] In this table we shall set the Graeco-Latin,

Greek and Italo-Latin names; then the numbers of the proportions in their least and radical terms. If each of these two numbers is multiplied by the same number, other consonances of the same species can be had, finite but without end: thus if you multiply the terms of the diapente, 3, 2, by 10, they will make 30, 20, between which there is likewise the same species of proportion and the same consonance. Finally we shall add both the denomination of the species of each proportion and the genus of the proportion itself, indicated by these five marks: M, S, s, MS, Ms. Of these, M signifies *multiple*; capital S, *superparticular*; small s, *superpartient*; MS, *multiple superparticular*; and Ms, *multiple superpartient*.

[Margin: Points to note in the nomenclature of the consonances.] Care must be taken, however, not to treat the words *diapason*, *diapente* and *diatessaron* as nominative cases of the singular number and inflect or decline them, as some unskilled people do, thus: diapason, diapasondos; diatessaron, diatessarondos; diapente, diapentes. For in Greek they are genitive cases of the plural number with the preposition διὰ [dia, "through"]; and so in Greek they are written διὰ πασῶν [dia pasōn]; διὰ πέντε [dia pente]; διὰ τεσσάρων [dia tessarōn]. The first of these means *through all*, or *around all*; the second, *through five*; and the third, *through four*. For the diapason goes through all the concords and contains radically within itself all the more perfect consonances; and because after seven strings we return to it, the practitioners call it the *octave*. The diapente is called the *fifth* because it lies between notes that stand five degrees of the strings apart, as will be clear from the diatonic system, which we shall set out in chapter 6, numbers 3 and 5; just as the diatessaron is called the *fourth*, the disdiapason the *fifteenth*, and so on. Finally, in the judgement of Macrobius (book 2 on the *Dream of Scipio*, chapter 1), the diapason consists of six tones; the diapente of three tones and a hemitone; the diatessaron of two tones and a semitone; the diapason diapente of nine tones and a hemitone; and the disdiapason of twelve tones. *Censorinus* agrees with him (*De die natali*, chapter 11); and that this is true will be clear from what is to be said in chapter 6, number 3, at the end of the diatonic system. With these things explained, here now is the table.

I. TABVLA CONSONANTIARVM.

Table 1: The consonances.

	Ordo	Græcolatina	Græca	Italo-latina	Termini in minimis num.	Species	Genus
Perfect	1	*Diapason*, queen of the consonances	Διὰ πασῶν	Octave	2 1	Dupla	M
	2	*Diapente*	Διὰ πέντε	Fifth	3 2	Sesquialtera	S
	3	*Diatessaron*, tetrachord	Διὰ τεσσάρον	Fourth	4 3	Sesquitertia	S
	4	*Diapason diapente*	Διὰ πασῶν ἐπὶ διὰ πέντε	Twelfth	3 1	Tripla	M
	5	*Disdiapason*, or bisdiapason	Δὶς διὰ πασῶν	Fifteenth	4 1	Quadrupla	M

	Ordo	Græcolatina	Græca	Italo-latina	Termini in minimis num.	Species	Genus
	6	*Ditonus*, enharmonic third	Δίτονος	Major or hard third	5 4	Sesquiquarta	S
	7	*Semiditonus*, chromatic third, sesquitone	τριημιτόνιον	Minor or soft third	6 5	Sesquiquinta	S
Imperfect	8	*Hexachordum* major, or tone with the diapente	ἑξάχορδον μέγα	Major or hard sixth	5 3	Superbipartiens tertias	s
	9	*Hexachordum* minor, semitone with the diapente	ἑξάχορδον μικρὸν	Minor or soft sixth	8 5	Supertripartiens quintas	s
	10	*Diapason with the ditonus*	δίτονος διὰ πασῶν	Major tenth	5 2	Dupla sesquialtera	MS
	11	*Diapason with the semiditonus*	ἡμιδίτονος καὶ διὰ πασῶν	Minor tenth	12 5	Dupla superbipartiens quintas	Ms
	12	*Diapason diatessaron*	διὰ πασῶν, καὶ διὰ τεσσάρων	Eleventh	8 3	Dupla superbipartiens tertias	Ms
	13	*Diapason with the major hexachord*	διὰ πασῶν καὶ ἑξάχορδον μέγα	Major thir-teenth	10 3	Tripla sesquitertia	MS
	14	*Diapason with the minor hexachord*	διὰ πασῶν καὶ ἑξάχορδον μικρόν	Minor thir-teenth	16 5	Tripla sesquiquinta	MS
Disputed, and admitted by few	15	*Semidiapason*	ἡμὶ διὰ πασᾶν	False octave	4096 2187	✠ ✠	s
	16	*Semidiapente*, or sesquiquinta	ἡμὴ διὰ πέντε	False fifth	64 45	Superdecunonu-partiens quadragesimas quintas	s
	17	*Tritonus*	τρίτονος	Hard fourth	45 32	✠ ✠	s
	18	*Ditonus with the diapente*	δίτονος καὶ διὰ πέντε	Major seventh	15 8	Superseptupar-tiens octavas	s
	19	*Semiditonus with the diapente*	ἡμιδίτονος καὶ διὰ πέντε	Minor seventh	9 5	Superquadrupar-tiens quintas	s
	20	*Sesquisexta*; Ptolemaic			7 6	Sesquisexta	S
	21	*Sesquiseptima*; Ptolemaic			8 7	Sesquiseptima	S
	22	*Disdiapason with the ditonus*; in Zarlino and Kepler	δὶς διὰ πασῶν ἐπὶ δίτονος		5 1	Quintupla	M

Ordo	Græcolatina	Græca	Italo-latina	Termini in minimis num.	Species	Genus
23	*Disdiapason with the diapente*; in Zarlino	δὶς διὰ πασῶν ἐπὶ διὰ πέντε		6 1	Sextupla	M

II. TABVLA. INTERVALLA DISSONA, Sed Concinnitati feruientia continens.

Table 2, containing the dissonant intervals that nevertheless serve concinnity [concinnitas].

Nomenclatura varia	Termini minimi	Species	Genus
Tonus maior [major tone], ἐπόγδοος [epogdoos, "one-eighth over"]; to the practitioners the major or perfect second. It is the difference between the diapente and the diatessaron	9 8	Sesquioctava	S
Tonus minor [minor tone]; the imperfect second. It is the difference between the diatessaron and the semiditonus, or between the diapente and the major hexachord	10 9; 20 18	Sesquinona	S
Semitonium maius [major semitone]: the difference of the minor tone and the semiditonus. By some it is called ἀποτομὴ [apotomē, "a cutting-off"]	54 50	Superbipartiens vigesimasquintas	s
Semitonium minus [minor semitone]: the difference between the ditonus and the diatessaron, or between the diapente and the minor hexachord	16 15	Sesquidecimaquinta	S
Semitonium minimum [least semitone]: the difference between the ditonus and the semiditonus, or between the major and the minor hexachord. It is also called δίησις [diēsis], or the Pythagorean diesis, or the Pythagorean λεῖμμα [leimma, "remnant"], as in Boethius and Glarean. To Vitruvius and Capella it is the fourth part of a tone; to Aristotle, the smallest element of music	25 24	Sesquivigesimaquarta	S
Semitonium medium [middle semitone], in Herigone, following Euclid, and in Mersenne: namely, what remains if you subtract the major semitone from the major tone	135 128	Superseptupartiens centesimas vigesimasoctavas	s
Semitonium maximum [greatest semitone], in the same Herigone, in Euclid's *Music*	27 25	Superbipartiens vigesimasquintas	s

Nomenclatura varia	Termini minimi	Species	Genus
Diesis enharmonica [enharmonic diesis], so called by the same Herigone; but to Mersenne it is the major apotome. In Aristotle and Suidas it is written δίεσις [diesis]; but Boethius and Glarean write δίησις	128 125	Supertripartiens centesimas vigesimasquintas	s
Limma Platonicum [Platonic limma], or the Pythagorean semitone and Pythagorean diesis, to Macrobius; but by Daniele Barbaro and Herigone it is called the Pythagorean semitone, while to Kircher it is the Pythagorean limma	256 243		s
Apotome Platonica [Platonic apotome]: what is left when the limma is taken from the major tone; Mersenne calls it the minor apotome	2187 2048		s
Comma maius [major comma], Κομμα [komma]: the difference of the major and the minor tone	81 80	Sesquioctogesima	S
Comma minus [minor comma], in Herigone	10240 10125		s
Schisma, σχίσμα [schisma]: half of a comma	4352 4330		s
Diaschisma, διὰ σχίσμα [dia schisma]: half of the diesis, or a diesis improperly so called	162 160		s

From what has been said it is clear that *diesis* is a highly equivocal name, since some have used it for one species of semitone and others for another. Macrobius, moreover (book 2 on the *Dream of Scipio*), says that in the usage of more recent writers the diesis is a sound smaller than a semitone. Practitioners nowadays, wherever they want half a note to be used, set the sign of the diesis.

Quæſtio 1. Quas nam Conſonantias acceptarint Authores infraſcripti ?

Question 1. Which consonances did the authors listed below accept?

VII. To answer the question more briefly, in place of names we shall set numbers as indices of the consonances, to be looked up in column 1 of Table 1. Besides the consonances, almost all admitted the tone, the semitones, and the diesis or limma, and not a few the apotome.

- The Pythagoreans, Boethius, Martianus Capella, Macrobius: 1.2.3.4.5.
- Aristoxenus and Vitruvius, according to Daniele Barbaro: 1.2.3.4.5.8.9.19.
- Vitruvius himself, book 5, chapter 4: 1.2.3.4.5.12.
- Daniele Barbaro himself: 1.2.3.4.5.6.7.8.9.10.11.12.18.
- Ptolemy: 1.2.3.4.5.12.20.21.
- Euclid, and Herigone following him: 1.2.3.6.7.8.9.
- Iosephus Zarlinus [Gioseffo Zarlino]: 1.2.3.4.5.6.7.8.9.10.22.23.
- Ludovicus Folianus [Lodovico Fogliano]: 1.2.3.4.5.6.7.8.9.10.11.12.13.14.
- Marinus Mersennius [Marin Mersenne]: 1.2.3.5.6.7.8.9.18.19.
- Athanasius Kircher: 1.2.3.4.5.6.7.8.9.15.16.17.18.19.

- Johannes Kepler, in his musical scale: 1.2.3.4.5.6.7.8.9.10.11.12.13.14, as will be clear from what is to be said in chapter 6, number 5 — granted that earlier he had counted among the perfect consonances the unison and six other consonances, namely 1.2.3.4.5.8.22, as is clear from his book 3, chapter 2.

Quæſtio 2. Cur Conſonantiæ oriantur magis à Proportionibus Maioris, quàm Minoris inæqualitatis ?

Question 2. Why do consonances arise from proportions of greater rather than of lesser inequality?

VIII. For it is commonly said by musicians that the diapason arises from the double proportion rather than from the subdouble, and the diapente from the sesquialter rather than from the subsesquialter, although between the same terms 2 and 1 there is both the proportion of 2 to 1, the double, and that of 1 to 2, the subdouble, and so on. Ludovicus Folianus (section 2 of the *Musica Theorica*, chapter 3) and Zarlino (part 2 of the *Harmonic Institutions*, chapter 50) reply that the cause is this: in a proportion of greater inequality the greater term is compared with the lesser in a more perfect manner, namely as the container, which has the character of form, with the contained, which has the character of matter; and also that the relation of excess, or majority, is positive and real, whereas the relation of defect, or minority, is nothing but a privation, and is not a real relation but a relation of reason.

Quæſtio 3. An Tonus diuidi poſſet in duo Semitonia æqualia.

Question 3. Whether the tone could be divided into two equal semitones.

IX. Most deny it: first of all Macrobius (book 2 on the *Dream of Scipio*, chapter 5), Boethius (at the beginning of book 3), Glarean (book 1 of the *Dodecachordon*, chapter 10), Daniele Barbaro (on book 5 of Vitruvius, chapter 4), and Kircher (book 3 of the *Musurgia*, axiom 10). For just as a semivowel is not so called because it divides a vowel into two equal parts, so the semitone is so called because it lies between the extremes, not, however, because it is the exact half of the tone. The reason is that the tone belongs to the superparticular genus, since its proportion is 9 to 8, and no superparticular proportion can be divided into two equal parts. Barbaro therefore reproaches Aristoxenus because, following the judgement of the ears rather than the reasons of numbers, he divided the tone into two equal parts; and Barbaro says that it is cut into two unequal parts, of which one is called the major hemitone, or apotome, and the other the minor hemitone, or diesis. But Carolus Valgulius [Carlo Valgulio], on Plutarch's *Music*, defends Aristoxenus, and teaches that although the unit lying between 9 and 8 cannot be cut into two whole parts, yet the string itself, or chord, on which, as on a rule, concords are formed by divisions duly made, can be cut anywhere, because magnitude is continuous, and so into two equal parts. Nor, he teaches, was Aristoxenus so unskilled in arithmetic, or ignorant of the

doctrines of the Pythagoreans, since he wrote whole volumes on arithmetic and had as his teacher Xenophanes, a noble Pythagorean. But Father Athanasius Kircher (book 3 of the *Musurgia*, chapter 12, propositions 1 and 2) teaches that the tone can be divided into two equal parts in irrational numbers, but not in rational ones. He adds, moreover, that whoever, with Philolaus and Boethius, posit a half of the comma, namely the schisma, cannot deny a half of the tone. Consult him if you want more on this matter.

Quæſtio 4. Quare Diapaſon totam diuerſitatem Conſonantiarum in ſe contineat, aut terminet, & ſit Regina Conſonantiarum.

Question 4. Why the diapason contains within itself, or bounds, the whole diversity of the consonances, and is the queen of the consonances.

X. The diapason, in Greek διὰ πασῶν [dia pasōn], that is "through all", or "to all", or "around all", is so called because all the consonances are either contained within it or bounded by it. For although various harmonic proportions are found beyond it, yet according to sense, and judged by the ears, they seem to have almost no difference from those contained within the diapason — indeed, not even from the diapason itself; nor is any wholly new consonance found in which something of the diapason is not felt. For the ditonus seems to affect the sense in the same way as the diapason ditonus, and the semiditonus as the diapason semiditonus; so too the diatessaron and the diapason diatessaron, and so with the other consonances. Hence it comes about that, once the seven simple consonances, set apart from one another by a perceptible difference, are complete — namely the diapason, diapente, diatessaron, ditonus, semiditonus, and the major and minor hexachord — there is a repetito [?] of the consonances, and their whole diversity is bounded by the diapason. And because everyone easily discerns it and is refreshed by it, Aristotle (section 19, problem 35) calls it καλλίστη συμφωνία [kallistē symphōnia], that is, the most beautiful consonance. [Margin: The diapason, the most beautiful of the consonances.] Moreover, this too belongs to it uniquely: added to a like consonance, it generates other consonances without end, for example the didiapason [recte: disdiapason], the trisdiapason, the quaterdiapason, and so on. But if you add a diapente to a diapente, no consonance is produced, just as none is produced from others of the same species added together, as Aristotle teaches (section 19, problem 42). For the rest in praise of the diapason, read Aristotle, section 19, problems 17 and 32.

Quæſtio 5. Quo Artificio Diſcerni poſſint ipſo auditu Conſonantiæ omnes : & quomodo Monochordi Diuiſio ſit facienda.

Question 5. By what art all the consonances can be discerned by hearing itself, and how the division of the monochord is to be made.

XI. We learned above from Macrobius that Pythagoras hunted out the consonances from the sound of different hammers, and afterwards from chords — that is, strings or gut strings — stretched and hung with different weights, in the proportion he had detected in the hammers, so that the tensions of strings of the same species and thickness came out different according to the difference of the weight hung on them. But the monochord is in every way superior, because it is difficult to find several strings exactly alike in species, thickness and uniformity, or to stretch them with precisely as great a difference as the exact distinction of the consonances requires. [Margin: Notion and definition of the monochord.] The monochord is so called from μόνος [monos], which means single or solitary, and from χορδὴ [chordē], which means string, because it is an instrument containing a single stretched string; Ptolemy and Boethius call it the harmonic rule [Regula Harmonica]. Suidas calls this instrument *Magadea*, from the nominative Μαγὰς [Magas], and says that it is a squared piece of wood, hollow inside, containing strings fitted for grasping the variety of notes; for it can consist of two strings, one of which always gives the whole, and the other the part due to the consonance. But because the method with a single string is simpler, Guido Aretinus [Guido of Arezzo] defined it thus: *The monochord is a long squared piece of wood, hollow within, with a string drawn over it, by whose sound we grasp the varieties of notes.* [Margin: What the magas is.] However, the *magas* — or, as others have it, in Glarean above and in Kircher (book 4, chapter 1), the *magadis* — is mostly taken both for the twin fixed support, or little hemispherical bridge, that bounds and holds up the ends of the whole string, and for a third little bridge, or movable stool, that is set under the string at the point where the division of the string has been made by marking, according to the proportion due to the consonance sought. More recent writers call it the *cursor*, because it is moved forwards and backwards while we hunt for the consonances. The *pectis*, or plectrum, is what we strike the string with, so that by its vibration it gives out its sound. From *magas*, moreover, is formed the verb μαγαδίζειν [magadizein], which means to play on a string, forming various consonances on it.

[Margin: The first way of dividing the monochord.] The first use of the monochord, then, is for us to explore by it the individual consonances separately, or even the dissonant intervals, by a simple division of the string into only two parts — a division, I say, not real, but equivalent to a real one: namely by marking on a straight line set beneath the stretched string itself and of the same length, so that the place of the movable *magas* may be known. The string is to be divided into as many equal parts as the sum of the two numbers that determine the proportion due to the consonance. Thus, if you wish to explore the *diapason*, whose proportion is between 2 and 1, then since 2 and 1 make 3 you must divide the string into three equal parts; for if you strike the part of the string containing two parts, and immediately the other, containing a single such part, you will hear this consonance. So, if you wish to discern the *diapente*, whose proportion is as 3 to 2, then since 3 and 2 make 5 you will divide the

whole string into 5 equal parts; and with the *magas* placed where two parts end on one side and three on the other, by striking each you will learn this consonance. And so with the rest, as the following small table will show. In it the first column contains the number of equal parts into which the whole string is to be divided by marking, with the help of compasses or of folded paper; the second, the number of parts of the longer partial string; and the third, of the shorter. Struck together, or immediately one after the other, they will present to purified ears the consonance [?] sought, or else a dissonant interval that nevertheless serves concinnity [?].

Monochordi Diuiſio Pro Conſonantijs
Division of the monochord for the consonances.

Consonantiae	Equal parts of the whole string	Parts of the divided string: longer	Parts of the divided string: shorter
Diapason	3	2	1
Diapente	5	3	2
Diatessaron	7	4	3
Diapason diapente	4	3	1
Disdiapason	5	4	1
Ditonus	9	5	4
Semiditonus	11	6	5

Pro Diſſonis Interuallis
For the dissonant intervals.

Dissona interualla	Equal parts of the whole string	Parts of the divided string: longer	Parts of the divided string: shorter
Tonus maior [major tone]	17	9	8
Tonus minor [minor tone]	19	10	9
Semitonium minimum [least semitone]	49	25	24
Semitonium minus [minor semitone]	31	16	15
Semitonium medium [middle semitone]	263	135	128
Semitonium maius [major semitone]	104	54	50
Semitonium maximum [greatest semitone]	52	27	25

RESIDVVM TABVLÆ PRÆCEDENTIS.
The rest of the preceding table.
Monochordi Diuiſio Pro Conſonantijs
Division of the monochord for the consonances (continued).

Consonantiae	Equal parts of the whole string	Parts of the divided string: longer	Parts of the divided string: shorter
Hexachordum maius [major hexachord]	8	5	3
Hexachordum minus [minor hexachord]	13	8	5
Diapason with the ditonus	7	5	2
Diapason with the semiditonus	17	12	5
Diapason diatessaron	11	8	3
Diapason with the major hexachord	13	10	3
Diapason with the minor hexachord	21	16	5
Semidiapason	6283	4096	2187
Semidiapente	109	64	45
Tritonus	77	45	32
Ditonus with the diapente	23	15	8
Semiditonus with the diapente	14	9	5
Sesquisexta	13	7	6
Sesquiseptima	15	8	7
Disdiapason with the ditonus	6	5	1
Disdiapason with the diapente	7	6	1

Pro Diſſonis Interuallis
For the dissonant intervals (continued).

Dissona interualla	Equal parts of the whole string	Parts of the divided string: longer	Parts of the divided string: shorter
Diesis enharmonica [enharmonic diesis]	253	128	125
Limma Platonicum [Platonic limma]	499	256	243
Apotome Platonica [Platonic apotome]	4235	2187	2048
Comma maius [major comma]	161	81	80
Comma minus [minor comma]	20365	10240	10125
Schisma	8682	4352	4330
Diaschisma	322	162	160

[Margin: The second way.] The second way is when a single string is divided into equal parts in such a manner that the same division serves, if not all, at least several consonances — say the seven simple ones. In that case you will divide the string into 120 equal parts. For if you strike the whole string, and then its half, namely the partial string consisting of 60 parts, you will hear

the *diapason*; and you will hear the same thing another way if you strike on one side the string having 80 parts and on the other the one having 40. If, however, you strike the whole string and then the partial one having 80 parts, you will perceive the *diapente*. But if you strike the whole string, and immediately the partial one having 90 parts, you will hear the *diatessaron*. If, however, you strike the whole string and then the partial one having 96 parts, you will hear the *ditonus*. But if you strike the whole string, and immediately the partial one having 100 parts, you will perceive the *semiditonus*. Again, if you strike the whole string and then the partial one that has 72 parts, you will hear the *major hexachord*. Finally, if you strike the whole string, and immediately that one of the partial strings that has 75 parts, you will perceive the *minor hexachord*. And so your ear will become master of the seven simple consonances, which Virgil calls the "seven distinctions of notes" [Septem discrimina vocum]. For these, see the synopsis in the following small table.

Diuiſio Monochordi pro 7. Conſonantijs Simplicibus.

Division of the monochord for the 7 simple consonances.
The whole string is to be divided into 120 equal parts.

	Parts	For the consonance
The whole string is to be struck, and immediately with it or after it is to be struck the string of	100	*Semiditonus*
	96	*Ditonus*
	90	*Diatessaron*
	80	*Diapente*
	75	*Major hexachord*
	72	*Minor hexachord*
	60	*Diapason*

We shall speak below about monochords in the diatonic, chromatic and enharmonic genera, after the difference of these genera has been explained, namely in chapter 6, numbers 3 and 4. There you will learn that a diatonic monochord suited to the consonances and to the dissonant but concinnous intervals is to be divided into 9216 equal parts according to Boethius, Glarean and Zarlino; or into 3600 parts according to Folianus; or into 2160 or 720 parts according to Kepler.

[Margin: The third way.] The third way is by the figure that Ptolemy (book 1 of the *Harmonics*, chapter 11) calls the *Helicon*, that is, by the square ABCD. Divide its side BD in half at F, and CD in half at E. Then divide the side BD into four equal parts, BG, GF, FH, HD. Again, divide the same side BD into three equal parts, BK, KI, ID. From the points of division of the side BD draw, to the opposite side AC, perpendiculars parallel to the sides: GL, KM, FN, IO, HP. And finally, from B draw the straight line BE to E, the midpoint of the side CD. For if the parallels just mentioned, together with the

sides to which they are parallel, are strings of the same species and tension, and the chordotomum, or magas, makes BE a continuous line, you will find the consonances or concinnous intervals written below.

Diagram. The square of the Helicon, set into the right column. A square with A at the top left, B at the top right, C at the bottom left and D at the bottom right. Five horizontal lines cross it, parallel to AB and CD; they meet the left side AC at L, M, N, O, P and the right side BD at G, K, F, I, H, from top to bottom. A straight line runs from B down to E on the bottom side CD, crossing the five horizontals at Q, R, S, T, V, from top to bottom.

- A (top left corner): A
- B (top right corner): B
- L, M, N, O, P (left side, top to bottom): L, M, N, O, P
- C (bottom left corner): C
- G, K, F, I, H (right side, top to bottom): G, K, F, I, H
- D (bottom right corner): D
- Q, R, S, T, V (on the line from B to E, top to bottom): Q, R, S, T, V
- E (on the bottom side): E

TABELLA PRO HELICONE.

Table for the Helicon.

- *Unison*: CE with ED.
- *Diapason*: AB with CE; or OT with TI.
- *Diapente*: CE with TI; or NS with CE; or AB with OT.
- *Diatessaron*: TI with SF; or CE with VH; or AB with NS.
- *Ditonus*: PV with CE; or MR with OT.
- *Semiditonus*: NS with PV; or AB with MR.
- *Major hexachord*: PV with VH; or MR with CE.
- *Minor hexachord*: AB with PV.
- *Diapason diapente*: AB with VH; or ED with QG; or NS with SF.
- *Disdiapason*: AB with SF; or OT with QG.
- *Diapason with the ditonus*: MR with TI; or PV with SF.
- *Diapason with the diatessaron*: AB with TI; or OT with SF.
- *Diapason with the major hexachord*: MR with SF.
- *Major tone*: VH with TI; or MR with OT.
- *Minor tone*: MR with NS.
- *Major semitone*: OT with PV.

I pass over the remaining methods as suited to fewer consonances: namely the mesolabe [Mesolabrum], about which Zarlino (part 2, chapter 25); and the square divided into eight parallelograms with an oblique line such as BE, about which

Athanasius Kircher (*Musurgia*, book 4, chapter 5, Lemma 2); and others pointed out by Glarean (*Dodecachordon*, book 1, chapters 17 and 18) or by Ludovicus Folianus (section 2, chapters 14 and 15).

I add only this. When one string is struck, another string — not struck, but in unison with it or akin to it in consonance — vibrates, provided it lies within the sphere of the first string's vibrations. You can test this by laying a bent slip of straw [paleola] on the second string: you will see it jump. This effect was known to Macrobius (book 2 on the *Dream of Scipio*), and Kepler tries to give its cause (*Harmonics*, book 3, page 14); so too Fracastorius [Girolamo Fracastoro] (*De Sympathia*, chapter 11), and the natural philosophers [Physiologi] in the question of action at a distance.

SCHOLIA.

Scholia.

[Margin: Kepler's doctrine of the origin of the consonances.]

I. Kepler, then, setting out to investigate the origin of the consonances from the divisions of the circle, lays down at the start of book 1 of the *Harmonics* that the demonstration of a figure in a circle is the derivation of a quantity to be known — that is, measured or described — from the diameter, through the possible intermediate steps.

[Margin: Proper and improper demonstration of figures in a circle.]

The *proper* demonstration of such a figure is when the number of its angles, or of the angles of a kindred figure with double or half [?] the number of sides, becomes the middle term for determining the ratio of the side to the diameter. The *improper* demonstration is when the ratio of the side to the diameter cannot be determined geometrically from the number of angles applied directly, unless one brings in the side of another figure that has neither double nor half the number of sides.

[Margin: Several degrees of knowability, both of the expressibles and of the inexpressibles.]

He further establishes several degrees of knowledge [gradus scientiae]. The first degree of knowability [scibilitas] is when some line can be shown to be equal to the diameter, or a plane equal to the square of the diameter. The second degree is when, the diameter (or likewise its square) having been divided into some definite number of equal parts, the proposed line or plane is shown to be equal to such a part or parts. That plane is then called *expressible* [Effabile], and the line *expressible in length* [Effabilis longitudine]: for number is, so to speak, the language of geometers. The third degree is when a line is inexpressible in length but its square is expressible; the line is then also called *expressible in square* [effabilis potentia]. The remaining degrees he calls simply *inexpressible*, rather than irrational or surd.

Among these the fourth degree — though the first of the inexpressibles — is when neither the line nor its square is expressible, but the square can still be transformed into a rectangle whose sides are at least expressible in square. Such a line is called a *mese* [Mese, "medial"], because it is a mean proportional between two expressible lines commensurable in square only; its square is

called a *meson* [Meson, "medial area"], whether it has the form of a square or is transformed into a rectangle. From this kind of plane, and from the expressible plane, other species and degrees arise. The fifth degree of knowledge, then, is when two lines, neither of them expressible nor medial [mesae], and wholly incommensurable with each other, nevertheless make the sum of their squares, and their common rectangle, expressible. The sixth degree is when two lines that are neither expressible, nor medial, nor commensurable with each other nevertheless make one of the two expressible and the other a *meson* — that is, either the sum of the squares or the common rectangle. The seventh degree is when, of two inexpressible and incommensurable lines, neither is expressible — that is, neither the sum of the squares nor the common rectangle — yet each is a *meson*. For the other degrees, which arise from combinations of these, consult Kepler himself up to proposition 34, where he begins to determine the knowable sides of figures according to the degrees already mentioned.

[Margin: Tetragon.] [Margin: Octagon.] [Margin: Sixteen-sided figure.] [Margin: Triangle.] [Margin: Hexagon.] [Margin: Dodecagon.] [Margin: Figure of 24 sides.] [Margin: Decagon.]

He shows that the diameter of the circle is knowable in the 1st degree; the side of the tetragon inscribed in the circle in the 3rd degree, but its square [Quadratum] in the 2nd. The side of the octagon, and of the octagonal star, is knowable only in the 8th degree, but the sides of the two joined together in the 6th degree. The side of the sixteen-sided figure is knowable in degrees far below the 8th degree. The side of the triangle inscribed in the circle is knowable in the 3rd degree, and that of the hexagon in the 2nd; but their planes, or areas, are *mesa* and stand to each other in double ratio. The sides of the dodecagon and of the dodecagonal star, if joined together, are knowable in the 5th degree; if separated, in the 8th. The plane of the dodecagon, however, is expressible. Likewise the regular figure of 24 sides, and all the figures that arise from it by continually doubling the number of sides, together with their stars, have knowable sides, but in a degree below the 8th. Further, the sides of the decagon and of the decagonal star joined together are knowable in the 5th degree, but separated in the 8th; joined with the diameter of the circle, in the 4th degree.

[Margin: Pentagon.] [Margin: Fifteen-sided figure.] [Margin: Heptagon.]

The sides of the pentagon and of the pentagonal star are knowable separately in the 8th degree, but joined, in the 4th as well as in the 6th degree. The planes of the decagon, the pentagon, the icosagon [Icosigonum] and the rest of this class fall either [?] into the remoter degrees of knowledge. The sides of the fifteen-sided figure, and of the star that arises from it, have no proper demonstration, just as those of the thirty-sided figure have none; and moreover the heptagon is not knowable, nor does it have a proper demonstration. Hence the division of arcs of the circle into 3, 5, 7, etc. equal parts, and in any ratio whatever that is not a continued doubling of those demonstrated before, cannot yield knowable sides of figures.

Having demonstrated this, he concludes that there are four classes of knowable figures: three having a proper demonstration, and a fourth having an improper one. In the division of the circle the diameter holds first place, as being expressible in length. In 2nd place is the hexagon, because its side, being equal to the semidiameter, is expressible in length. In 3rd place are the tetragon and the triangle, whose sides are expressible in square only. In 4th place are the sides of the dodecagon and the decagon, and of the stars of the same name, because they are among the lines inexpressible in square and the composites of the first species. In 5th place are the sides of the pentagon and the octagon, and also of the pentagonal and octagonal stars, etc.

[Margin: Congruent figures.]

In book 2 of the *Harmonics* he treats of the geometrical congruence or incongruence — or unsociability [insociabilitas] — of figures, and he concludes that the figures congruent in plane angles or in solid angles, or in plane and solid angles together (that is, those whose angles meet at one point in such a way as to leave no gap between the sides), are only the twelve listed below.

Left	Right
1 Triangle	7 Dodecagon
2 Tetragon	8 Icosagon
3 Pentagon	9 Pentagonal star
4 Hexagon	10 Octagonal star
5 Octagon	11 Decagonal star
6 Decagon	12 Dodecagonal star

[Margin: Kepler's doctrine of the origin and number of the consonances.]

II. In book 3 of the *Harmonics* he inquires into the origin of the harmonic ratios, and the nature and differences of the things that pertain to song, and he first lays down these axioms.

1. The diameter of the circle, and the sides of the radical figures explained by him in book 1 that have a proper demonstration, determine a part of the circle consonant with the whole circle, whether the circumference of the circle is stretched round in a circle or straight like a single string. And therefore the consonances are infinite, because the demonstrable figures are infinite — granted that some of them have been selected.
2. By whatever degree the demonstration of a side stands removed from the first, by that same degree the consonance with the whole circle of the part of the circle cut off by the side recedes from the most perfect consonance, that of the unison.
3. The indemonstrable sides of figures and of stars determine a part of the circle dissonant from the whole circle; and so do sides that are indeed demonstrable, but not by a proper demonstration.
4. Figures that have kindred demonstrations of their sides beget kindred harmonies.

5. Strings or arcs of the circle of equal tension, having to one another, by reason of their length, the same ratio as holds between a part or the remainder of the circle and the whole circle, also have the same consonance or dissonance, even if it is contained between other terms or sounds.
6. When two strings have given out identical sounds, a third voice consonant with one of them is consonant with the other too; or, if dissonant with one, it is dissonant with the other too.
7. When two strings or voices have given out identical sounds, a third voice that is identisonant [identisona] with one of them is also identically consonant with the other.

On these premises he affirms: 1. that after the unison, which he calls the most perfect consonance, the half with the whole circle begets the most perfect and simplest consonance, in the first degree — and an identical one on the opposite side — because it is made by the diameter of the circle. 2. If the greater part of the circle does not have a continued double ratio to the whole circle, and is consonant with the whole circle, the lesser part will be dissonant from the whole. 3. Strings in continued double ratio are all identically consonant with one another. 4. A string that is consonant with either of the multiples of a continued double ratio is consonant with the other as well; and if it is dissonant with one, it is dissonant with the other too.

In chapter 2, further, he deals with the harmonic section of a string — when, that is, the whole string is cut into parts such that each is consonant both with the whole and with the others. He teaches that there are only seven such sections, namely those whose ratio is indicated by the numbers below, and which he had treated in the *Mysterium Cosmographicum*, chapter 12.

Harmonic sections: if one part of the string is to the other part as

Vt	Ad	Vt	Ad
1	1	5	1
2	1	3	2
3	1	5	3
4	1		

After this, in chapter 3, he treats of the trinity of concordant sounds, or of harmonic means; in chapter 4, of the origin of the melodic [concinna] intervals; in chapter 5, of the natural division of the consonant intervals into melodic ones; in chapter 6, of hard and soft song; in chapter 7, of the division of a single octave in each kind of song; in chapter 8, of the number and order of the smallest intervals; in chapters 9 and 10, of the notes and characters of voices and strings, and of the syllables Ut, re, mi, fa, sol, la; in chapter 11, of the composition of systems; in chapter 13, of naturally melodious song; in chapters 14 and 15, of the Modes, or Tones, of melodies; and in chapter 16, of figured song.

CAPVT V.

Chapter 5.

De Muſicæ ac Melodiarum Cantuumque Diuiſionibus varijs, ac Generibus.

On the various divisions and kinds of music, of melodies and of songs.

[Margin: Divisions of music, 1 and 2.]

I. Music is twofold: one theoretical, the other practical. Theoretical music, on the authority of Boethius (book 1, chapter 2), is threefold. *Cosmic* [Mundana] music considers the harmony of the whole world and of its parts — the symphony [Symphonismus] and symmetry of the elements and of the heavens. *Human* music deals with the proportions of the body and the soul, with each other and with their own parts — that is, with the harmony of the microcosm. *Instrumental* [Organica] music contemplates the harmony that results from sounds and voices, whether in a natural or an artificial way. Zarlino, however (part 1, chapter 5), first divides music in two, into *animastic* [Animastica] and *instrumental*, and then subdivides the animastic into cosmic and human, and the instrumental into natural and artificial.

[Margin: Division 3.]

[Margin: What natural and artificial music are.]

The third division, then, is of instrumental music into the *natural*, which considers the sounds produced naturally by the force of the throat or larynx, the lungs, the palate, the tongue, the teeth and the lips — for these are the natural instruments of the sound and voice of animals — and into the *artificial*, which considers sound made, or able to be made, by various manufactured instruments.

[Margin: Division of musical instruments.]

Of these instruments the first kind comprises the pneumatic ones, or ἐμφυσώμενα [Greek: emphysōmena, "blown into"], that is, instruments that sound by the force of breath and are animated by air, whether it be the breath of living creatures or wind and air. Such are reed-pipes [calami], hemlock-stalk pipes [cicutae], syrinxes, shepherds' or soldiers' pipes, in Italian *subioli* or *cifoli*; pipes with three or six holes [tristomae, hexastomae], in Italian *flauti*; and *litui*, or horns; bagpipes [Cornamusae], in Italian *corni*, *cornetti*, *cornamuse*; and serpent horns, in Italian *biscioni*; and shawms [Tibiae], in Italian *piue* or *pifferi*; and bladder-pipes [Fistulae vtriculares], in Italian *piue sordine*; and trumpets, buccinas and war-trumpets [Classica], in Italian *trombe*; and slide trumpets [Tubae ductiles], in Italian *tromboni*; and *zooglossa* pipes, with which we imitate the various voices of animals; and *anthropoglossa* pipes, with which we imitate human speech and laughter. But first among them are the instruments that are called, by antonomasia [ἀντονομαστικῶς, antonomastikōs], *organs*, or by Vitruvius *musical canons* [Canones Musici]: built of many pipes or tubes of lead or of cypress wood, and blown by bellows.

The second kind comprises the instruments called ἔγχορδα [Greek: enchorda, "stringed"] or ἔντατα [Greek: entata, "strung taut"] — that is, chordal or sinew-strung instruments, consisting of one or more strings, which sound when struck with the fingers, the nails, plectra, etc. Such are psalteries, harps, lutes [Testudines], lyres, barbitons, *cheles*, sambucas, pandoras, mandoras, nablas, citterns [Cytharae], pectides, harpsichords [Clauicymbala], and the Turkish three-string, commonly *il colachon* or *colasson*. For lutes, pandoras and mandoras are called *leuti* in Italian; citterns, *cetre*; Spanish citterns, *chitariglie*; theorbos, *chitaroni*. The small four-string *chelys* is *il violino*; the six-string *chelys* is *la viola*; the larger four-string *chelys*, *il violone*;

the smallest *chelys*, or *lynterculus*, *il lirino*; the twelve-string *chelys*, or lyre, *lira* or *lirone*. The clavichord, or clavicymbal, or manichord, is called *clavicembalo*, *manacordo*, *spinetta*. Finally, the third kind comprises the percussion instruments, in Greek κρούστικα [Greek: kroustika, "struck"] or κρούστα [Greek: krousta, "struck"]: cymbals, sistrums, drums [Tympana], bells, clappers [Crotala], and xylorgans [Zylorgana], that is, instruments which have wooden cylinders in place of pipes. On musical instruments there have written excellently Mersennius (on Genesis, chapter 4, verse 21, question 56, or from page 1515), Othomarus Luscinius [Ottmar Luscinius] (*Musurgia*, book 1), and our own Kircher (book 2, chapters 3 and 6, and book 6 of the *Musurgia*). There Kircher also sets out, from his own experiments and Mersennius's, the proportions of strings, pipes, etc., and what voices strings of different metals, of gut and of thread give out; or what sounds different woods give when struck, and in what proportion.

[Margin: Division 4, from Isidore.]

Fourth, music is divided by Isidore (*Etymologies* [Origines], book 3) into the *harmonic*, which consists of the songs of voices; the *organic*, which arises from breath; and the *rhythmic*, which takes its numbers from the impulse of the fingers. Some, however, include under the rhythmic both the metrical art of syllables, for building verse, and the art of dancing; others separate the metrical from the rhythmic, as can be seen in Zarlino (part 1, chapter 5).

[Margin: Division 5, from Martianus Capella.]

Fifth, music is divided according to Martianus Capella (*On the Marriage of Philology*, book 9) into three kinds. The first is called εἰδικὸν [Greek: eidikon, "specific", "of form"], which is consonant from sound, numbers and words that are alike and persevering; the part of it that pertains to melody is called harmonic, the part that pertains to numbers rhythmic, and the part that pertains to words metrical. The second kind is ἐργαστικὸν [Greek: ergastikon], that is, operative; and the third ὑπερεργαστικὸν [Greek: hyperergastikon], that is, super-operative, or ἑρμηνευτικὸν [recte: ἑρμηνευτικόν] [Greek: hermēneutikon, "interpretive"], that is, enunciative. On these consult that author, for this division is very obscure and accepted by almost no one.

[Margin: Division 6, into the Dorian, Phrygian, Lydian modes, etc.]

II. Sixth, music is divided into various modes, but chiefly into the Dorian, Phrygian, Lydian, Aeolian, Ionian and Iastian, and into modes mixed from

these, according as these modes were invented by the Dorians, the Phrygians, etc., or were more in use in those regions — although Alypius in his *Isagoge* says the Dorian was invented by Thamyras, the Phrygian by Marsyas, the Lydian by Amphion, the Hypodorian by Philoxenus, the Hypolydian by Polymnastus, and the Mixolydian by Sappho. In assigning the difference between them, however, there is an astonishing and almost inextricable confusion. See, if you like, the interpreters of Aristoxenus and Ptolemy; Glarean (book 1 of the *decachordon*, chapters 2 and 7); Mersennius (on Genesis, chapter 4, verse 24, from page 1664); and Kircher (*Musurgia*, book 3, chapter 15, and book 7, page 554), who collect the opinions of others. Here it is enough to know that the Dorian was a grave and severe mode, in which Re–Sol reigned; the Phrygian religious and hard, in which Mi–La; the Lydian wanton, and the Ionian soft, in which Ut–fa. Hence Pliny the Elder (book 2, chapter 22) said that Saturn moves in the Dorian, Jupiter in the Phrygian, Mercury in the Lydian. Further, all the modes, simple and mixed, are brought down to 12 according to Kircher, or to 15 according to Euclid, Cassiodorus and Capella.

[Margin: The musical modes, 12.]

The *Dorian*: in Lucian δεμνὸς [recte: σεμνός] [Greek: semnos, "solemn"], sacred and grave; in Apuleius warlike, and most apt for setting heroic verse, on account of its gravity joined with briskness.

The *Hypodorian*, or plagal [subiugalis] of the Dorian: gloomy and too grave.

The *Phrygian*: in Lucian ἔνθεος [Greek: entheos, "inspired"], in Apuleius religious; full of severe indignation, and so also called σκόλιος [Greek: skolios, "crooked"]; fit for tragic iambics and for carrying minds out of themselves, as Plato has it (book 3 *On Justice*) and Aristotle (*Politics* 8, chapter 5).

The *Hypophrygian*: humble, and composed for weeping, on account of its sad lament.

The *Lydian*: cheerful, drunken, threatening, and suited to revellers; and so in Lucian βακχικὸς [Greek: bakchikos], that is, bacchic and mad. For that reason Plato rejected it (dialogue 5, *Republic*).

The *Ionian*, or Iastian: soft, dissolute, wanton.

The *Hypolydian*: tearful, devout, or, as others say, wailing.

The *Mixolydian*: soul-bending, and leading to various affections.

The *Hypomyolydian* [?]: full of natural delight.

The *Aeolian*: gentle and of wonderful sweetness, and fit for lyric poets; the more recent writers call it the "foreign" mode.

The *Hypoaeolian*: it too has a remarkable sweetness.

The *Ionian*: in Lucian γλαφυρὸς [Greek: glaphyros, "polished", "elegant"], in Apuleius wanton; apt for iambics and trochaics. Plato condemned it for its excessive softness (dialogue 5, *Republic*); our musicians call it the Fifth.

The *Hypoionian* corrects the softness of the Ionian and corresponds to the Sixth of the more recent writers.

But according to Euclid, Cassiodorus and Capella there are 15, and they are numbered thus: *Dorian, Iastian, Phrygian, Aeolian, Lydian, Hypodorian, Hypoiastian, Hypophrygian, Hypoaeolian, Hypolydian, Hyperdorian, Hyperiastian, Hyperphrygian, Hyperaeolian, Hyperlydian*. But on these Kepler says more (*Harmonics*, book 3, chapters 14 and 15).

[Margin: Division 7, into the diatonic, chromatic and enharmonic genus.]

III. Seventh, music is divided into three most celebrated genera, namely the diatonic, the chromatic and the enharmonic — in Greek διατονικὸν [Greek: diatonikon], χρωματικὸν [Greek: chrōmatikon], ἐναρμονικὸν [Greek: enarmonikon]. So Vitruvius has it (book 5, chapter 4), naming the three genera of modulation διάτονον [Greek: diatonon], χρῶμα [Greek: chrōma, "colour"] and ἁρμόνιαν [Greek: harmonian] — on which see Daniele Barbaro; so too Ptolemy (*Harmonics*, book 1, chapter 13), who attributes this division to Archytas the Pythagorean; Martianus Capella (*On the Marriage*, book 9, the chapter on the genera of tetrachords), where he calls these three genera of tetrachords or of modulations ἐναρμόνον [recte: ἐναρμόνιον] [Greek: enarmonon], χρῶμα and διάτονον; Boethius (book 5); Macrobius (book 2 on the *Dream of Scipio*, chapter 4); Zarlino (part 2, chapters 9 and 16); and Athanasius Kircher (*Musurgia*, book 3, page 119, and more fully in the whole of chapter 13). This division is taken from the different ways of ordering the tetrachords in the ascent from a low sound to a high one.

[Margin: The diatonic genus.]

The *diatonic*, on Plutarch's testimony, is the most ancient and the most natural; Zarlino attributes its invention either to Terpander of Lesbos [Therpandro Lesbio] or to Pythagoras. This genus ascends through two tones and a minor semitone; and because it proceeds by spaces a tone apart and abounds in tones, it is called *diatonum*, or diatonic. This genus is severe, grave and constant, displaying manly character and bearing; it is very much in use today, and was in use also in the times of Martianus Capella and Macrobius.

[Margin: The chromatic.]

The *chromatic* genus was invented by Timotheus of Miletus, the lyric poet, as Suidas and Boethius report; and for this reason Aristotle says in the *Metaphysics* that had there been no Timotheus, we should have been forced to do without many melodies. *Chroma*, further, means "colour" in Greek; the genus is so named because it brings various colours into the diatonic, and it stands between the diatonic and the enharmonic as variety of colour stands between white and black. It proceeds by two semitones, a major and a minor, and a semiditone, that is, three semitones. It is far more artful than the diatonic and suited to stirring various affections; hence Vitruvius says of it: *"Through subtle skill and the frequency of its measures it has a sweeter delight."* Barbaro, however, calls it soft and plaintive.

[Margin: The enharmonic.]

The *enharmonic* was invented by Olympus, on the testimony of Aristoxenus and of Plutarch (*On Music*). As it is the most excellent of all and the fullest of

authority, so it is of the highest art and skill, and on account of its difficulty it was used by very few; and for that reason it was called by antonomasia [ἀντονομαστικῶς] the *harmonic*, or enharmonic. It proceeds by a diesis and a ditone. Kepler, treating these matters lightly at the end of book 3, chapter 6 of the *Harmonics*, thinks that the diatonic corresponds to hard song and the chromatic to soft song; or that there were two species of the diatonic, the hard and the soft, which were mixed together in the chromatic; but that nothing in the music in use corresponds to the enharmonic, except the vibrations of the human voice, the tremulant of organs, the mordent on the strings of the pandora, and the like. Finally, Macrobius (book 2 on the *Dream of Scipio*, chapter 4) says: *"Since there are three genera of musical melody, the enharmonic, the diatonic and the chromatic, the first has fallen out of use because of its excessive difficulty, while the third is disreputable for its softness; hence the middle one, that is, the diatonic, is assigned to the music of the world in Plato's teaching."*

Aristoxenus, further, subdivided the diatonic into the soft and the intense [Incitatum], and the chromatic into the soft, the sesquialter and the tonic. These subdivisions, like those of Didymus, Archytas and Eratosthenes, are rejected by Ptolemy (*Harmonics*, book 1, chapters 12 and 13, and book 2, chapters 13 and 14), Boethius (book 5, chapters 15, 16 and 17) and Zarlino (part 2, chapter 16). Kircher, however (*Musurgia*, book 3, chapter 13), says that the more expert musicians accept five subordinate genera, or species, of the diatonic — namely the Pythagorean, the soft, the syntonic, the toniac and the equal; three species of the chromatic, the ancient, the soft and the syntonic; and finally two of the enharmonic, the ancient and the Ptolemaic. He sets out patterns of these in tetrachords, with their ratios, in larger and smaller numbers.

Here, however, on account of what is to be said in the next chapter, it should be noted beforehand that the lowest of the strings was called by the Greeks *hypate hypaton*; the second *parhypate hypaton* [hypathon]; the third *lichanos hypaton* [Lychanon]; and the highest *hypate meson*. We shall distinguish them in order by the initial numbers I, II, III, IV.

Tetrachordon Diatonicum Diatonum seu Pythagoricum (the diatonic, or Pythagorean, diatonic tetrachord)

Strings	Number
I.	6144
Tone: sesquioctava ratio, as 9 to 8	
II.	6912
Tone: sesquioctava ratio, as 9 to 8	
III.	7777 [?]
Minor semitone: 13-superpartient of 243 [tredecup. 243], as 256 to 243	
IV.	8192

Tetrachordum Diatonicum Molle (the soft diatonic tetrachord)

Strings	Number
I.	63
Sesquiseptima, 8 to 7	
II.	72
Sesquinona, 10 to 9	
III.	80—45
Sesquivigesima, 21 to 20	
IV.	84—48

Tetrachordum Diatonicum Incitatum seu Syntonum (the intense, or syntonic, diatonic tetrachord)

Strings	Number
I.	36
Minor tone: sesquinona, 10 to 9	
II.	40
Major tone: sesquioctava, 9 to 8	
III.	45
Minor semitone: sesquidecimaquinta, 16 to 15	
IV.	48

Tetrachordum Diatonicum Toniacum (the toniac diatonic tetrachord)

Strings	Number
I.	168
Tone: sesquioctava, 9 to 8	
II.	189
Sesquiseptima, 8 to 7	
III.	216
Sesquivigesimaseptima, 28 to 27	
IV.	224

Tetrachordum Diatonicum Aequale (the equal diatonic tetrachord)

Strings	Number
I.	9
Sesquinona, 10 to 9	
II.	10
Sesquidecima, 11 to 10	
III.	11
Sesquiundecima, 12 to 11	
IV.	12

Tetrachordum Chromaticum Antiquum (the ancient chromatic tetrachord)

Strings	Number
I.	6144
Trisemitone	
II.	7296
Semitone	
III.	7776
Minor semitone	
IV.	8192

Tetrachordum Chromaticum Molle (the soft chromatic tetrachord)

Strings	Number
I.	105
Sesquiquinta, 6 to 5	
II.	126
Sesquiquartadecima, 15 to 14	
III.	135
Sesquivigesimaseptima, 28 to 27	
IV.	140

Tetrachordum Chromaticum Syntonum, seu Incitatum (the syntonic, or intense, chromatic tetrachord)

Strings	Number
I.	66
Sesquisexta, 7 to 6	
II.	77
Sesquiundecima, 12 to 11	
III.	84
Sesquivigesimaprima, 22 to 21	
IV.	88

Tetrachordum Enharmonicum Antiquum (the ancient enharmonic tetrachord)

Strings	Number
I.	6144
Ditone	
II.	7776
Diesis	
III.	7984
Diesis	
IV.	8192

Tetrachordum Enharmonicum Ptolemaicum (the Ptolemaic enharmonic tetrachord)

Strings	Number
I.	276
Sesquiquarta, 5 to 4	
II.	345
Sesquivigesimatertia, 24 to 23	
III.	360
Sesquiquadragesimaquinta, 46 to 45	
IV.	368

These were the foundations of the other tetrachords, out of which the whole system, or musical scale, was composed: it consisted of five tetrachords, the fourth string of the first being the first of the second tetrachord, the fourth of the second the first of the third, and so on, as we shall set out in the examples of the next chapter, where we shall explain the names and order of the strings.

[Margin: 8. The division into hard and soft.]

IV. *Eighth*, the genus of melodies is divided into hard song and soft song [Cantum Durum ac Molle], a division well known among recent writers. Hard song is that in whose system the intervals are ordered from the lowest voice by the major third and major sixth, that is, by the consonances 5 to 4 and 5 to 3, which are also called hard and harsh. Soft song is that in which the intervals are ordered from the lowest voice by the minor third and minor sixth, that is, by the consonances 6 to 5 and 8 to 5, which are called the soft third and the soft sixth. Examples of these are in Kepler, book 3 of the *Harmonics*, chapter 6; anyone who wants them may consult him.

[Margin: 9. The division of song into plain and figured.]

V. *Ninth*, music is divided into plain song, or firm song, and figured, or harmonic, song. Plain song is that in which the passage from sound to sound is made by the simple raising and lowering of the voice, without any search for consonance as such and without variation of time; or, that in which the difference of high and low is attended to, but without a symmetry of several voices consonant in harmony through various durations of time. Figured song is that whose melody is made by several voices consonant with one another in harmony, and by durations of time fittingly ordered in voice or sound; and because this is done by means of certain notes and figures, this song is called *figured*. This kind is thought to have been invented in the last centuries. Plain music, moreover, is subdivided into the Boethian, the Gregorian and the Aretine; all of them, for their gravity and clarity, are very well suited to divine worship and to stirring devotion. Boethius, imitating the Greeks, set up 15 divisions on the monochord, distributing 15 strings into four tetrachords and admitting a semitone between the first and second point; among the Latins, Saints Ambrose and Augustine followed him. Later St Gregory the Great, about the year 594, devised the seven letters of the alphabet A B C D E F G, repeating them up to the number 15. But Guido Aretinus [Guido d'Arezzo], about the year of the Lord 1024, established the hand, or musical scale, with

20 letters and the six syllables *ut, re, mi, fa, sol, la*; of these, a little in the next chapter.

CAPVT VI.

Chapter VI.

De Chordis Vocibus ac Notis Musicis, earumq. nomenclatura, & distributione in Systemate Harmonico seu in Scala Musica.

On the strings, voices and musical notes, their nomenclature, and their distribution in the harmonic system, or musical scale.

I. In the modulation of voices or sounds three principal accidents are usually considered. *First*, the natural quality of the voice or sound, by which there are four notable species of voices: namely, *Discantus*, or *Cantus*; *Altus*, or *Contratenor*; *Tenor*; and *Basis* — in Italian *Soprano, Contralto, Tenore* and *Basso*; for Julius Pollux calls the last *Basis*. *Second*, the continuous quantity in the duration, or stay, of time that we dwell on in pronouncing some word or sustaining a sound. For this the notes serve that Ioannes de Muris [Johannes de Muris], or Muria, of Paris is said to have invented about the year of the Lord 1320. Their order is such that each preceding note requires a stay twice as long as the one following. For the value, name and properties of these notes, see the following small table, with the time of the primum mobile, which assumes the ordinary pace of the hand raised and lowered, from our own measurements made with a pendulum.

Musical notes	Times	Names	Properties	Time of the primum mobile
[note-shapes]	8	Maxima	It sleeps	18″ 40‴
[note-shapes]	4	Longa	It lies down	9″ 20‴
[note-shapes]	2	Breve	It sits	4″ 40‴
[note-shapes]	1	Semibreve	It walks	2″ 20‴
[note-shapes]	½	Minim	It hurries	1″ 10‴
[note-shapes]	¼	Semiminim	It runs	0″ 36‴
[note-shapes]	⅛	Chroma	It flies	0″ 18‴
[note-shapes]	1/16	Semichroma	It flies away	0″ 9‴
[note-shapes]	1/32	Biscroma	It vanishes	0″ 4½‴

Before notes of this kind, however, neither Guido Aretinus nor others had notes distinguishing durations; they used mere thickish points, or little circles. From this the art was also called *counterpoint* [Contrapuncti Ars]: as a note is now set against a note, so then a point was marked against a point.

II. *Third*, in sounds and voices one considers the raising and lowering from low to high or from high to low — that is, the change of the voices through certain steps, by which the voice as it were ascends or descends. To distinguish these steps the ancients used either different parts of the same string, some

longer and some shorter, or different strings of different tension, to which they also gave fixed names. They arranged them so that, out of 15, 16 or 18 strings, they nonetheless set up five tetrachords, because the last of the first tetrachord was the first of the second, and so on — except the first of the fourth tetrachord, which is not shared with the last of the third; and for that reason the fourth tetrachord is called that of the disjunct strings. And so they constituted the greatest diatonic system. Vitruvius, book 5, chapter 4, supplies us with the names of the five tetrachords, saying: *"There are five tetrachords: the first, the lowest, which in Greek is called* ὕπατον *[hypaton, "highest"]; the second, the middle, which is called* μέσον *[meson, "middle"]; the third, the conjunct, which is called* συνέμμενον *[synemmenon, "joined"]; the fourth, the disjunct, which is named* διεζέυγμενον *[diezeugmenon, "disjoined"]; the fifth, which is the highest, is called in Greek* ὑπερβόλεον *[hyperbolaeon, "exceeding"]."* Martianus Capella subscribes to this (book 9, in the chapter on the tones), as does Boethius (book 1, chapter 20). You have the names of the strings, the explanations of the names, and their number according to various authors in the following table, in which we have written ὑπατῶν [hypatōn] with Glarean, and not ὑπάτων [hypatōn], as some write it corruptly.

CHORDARVM, Nomina secundùm tropos Musicos (the names of the strings according to the musical modes)

Order	Greek names	Graeco-Latin	Meaning
18	νήτη ὑπερβολαίων	Nete hyperbolaeon	The last of the high strings, or of the exceeding
17	παρανήτη ὑπερβολαίων	Paranete hyperbolaeon	The second-to-last of the high, or of the exceeding
16	τρίτη ὑπερβολαίων	Trite hyperbolaeon	The third of the high, or of the exceeding
15	νήτη διεζευγμένων	Nete diezeugmenon	The last of the disjunct
14	παρανήτη διεζευγμένων	Paranete diezeugmenon	The second-to-last of the disjunct
13	τρίτη διεζευγμένον	Trite diezeugmenon	The third of the disjunct
12	παραμέση	Paramese	Next to the middle string, corresponding to *b durum*
11	νέτη συνεμμένων	Nete synemmenon	The last of the conjunct
10	παρανήτη συνεμμένων	Paranete synemmenon	The second-to-last of the conjunct
9	τρίτη συνεμμένων	Trite synemmenon	The third of the conjunct
8	μέση	Mese	The middle
7	λιχανὸς μεσῶν	Lichanos meson	The index finger, or the extended, of the middle strings
6	παρυπάτη μεσᾶν [recte: μεσῶν]	Parhypate meson	The one below the principal, or the second, of the middle strings
5	ὑπάτη μεσὼν	Hypate meson	The principal, or the low, of the middle strings

Order	Greek names	Graeco-Latin	Meaning
4	λιχανὸς ὑπατῶν	Lichanos hypaton	The index finger of the principal, or low, strings
3	παρυπάτη ὑπατῶν	Parhypate hypaton	The second, or the one below the principal, of the low strings
2	ὑπάτη ὑπατῶν	Hypate hypathon	The principal of the principal, or the low of the low
1	προσλαμβανόμενος	Proslambanomenos	The added, or acquired, note

So, in counting the strings of the Greek system, some list only 15: for example, Ptolemy (*Harmonics*, book 2, chapters 5 and 11), Glarean (*Dodecachordon*, book 1, chapter 19), Ludovicus Folianus (*Musica Theorica*, section 3, last chapter), Zarlino (*Harmonic Institutions*, part 2, chapter 28), Mersennius [Marin Mersenne] (on chapter 4 of *Genesis*, verse 24, page 1670), and Kepler (*Harmonics*, book 3, chapter 11). All of these leave out strings 9, 10 and 11 — that is, the third, second-to-last and last of the conjunct strings — and put the paramese immediately after the mese, numbering the rest in the same order as above. But Vitruvius (book 5, chapter 4), and on him Daniel Barbarus [Daniele Barbaro] and Philander [Guillaume Philandrier]; Euclid, and with him Hérigone (volume 5 of his *Cursus Mathematicus*, in the *Music* of Euclid); Martianus Capella (book 9, *On the Marriage of Philology and Mercury*, in the chapter on the modes); and Athanasius Kircher (*Musurgia*, book 3, chapter 13, page 144) list 18 strings, in the same order as I do — although Capella differs from us here and there in naming them, as is clear from the following small table; in the others he agrees with us.

Nomenclatura peculiaris Martiani Capellae (Martianus Capella's own nomenclature)

Order	Name
4	ὑπάτων διάτονος
7	μέσων διάτονος
9	τρίτη συνζευγμένων
10	συνζευγμένων διάτονος
11	νήτη συνζευγμένων
14	διεζευγμένων διάτονος
17	ὑπερβολαίων διάτονος

Biancani (*Loca mathematica Aristotelis*, on section 19 of the *Problems*) and Bettini (*Apiaria*, apiary 10, progymnasma 1, proposition 1), however, do not list all the strings but only the 8 principal ones in the common octachord: *hypate, parhypate, lichanos* [Lychanon]*, mese, paramese, trite, paranete, nete*. But before we add the Aretine notes to these strings and arrange the greatest system according to the three genera — namely the diatonic, the chromatic and the enharmonic — a few things must first be touched on about the notes of the strings that Guido Aretinus devised and increased up to the number of 22 strings.

[Margin: Guido Aretinus, the restorer of music.]

III. Before Guido Aretinus, most Europeans used to employ in church chant eight straight lines, as eight strings, whose beginnings were marked with Greek letters; and on those lines points were marked, as notes are now, but not in the spaces between them. So Vincenzo Galilei teaches in his dialogue on music, and our Kircher shows it from manuscripts of the Vatican Library and of the library of Messina (*Musurgia*, book 5, chapter 1). Later Guido Aretinus, a Benedictine monk from Arezzo in Etruria and master of the monastic choir, while he was at Pomposa, a town of the Duchy of Ferrara, in the year of the Lord 1024, devised and found a new manner of singing, easy and pleasant, which Europe still uses now, together with the harpsichords [clauicymbalis] that were also invented. He gained acceptance for it and taught it through the whole of Italy up to the year 1028, with the approval of the Supreme Pontiffs John XX and Benedict VIII, by whom he was summoned to Rome and received with honour.

First, since he saw that between the eight lines used by his predecessors the spaces were idle and empty of notes — every step went from line to line — he reduced the lines to 5, but set notes in the spaces, so as to take in more intervals with fewer lines. *Second*, to distinguish the three principal tetrachords he substituted for the Greek strings these six syllables, *ut, re, mi, fa, sol, la*, through which the ascent is made from the lowest, *ut*, to the highest, *la*. He chose them from that strophe of the hymn of St John the Baptist:

UT queant laxis RE-sonare fibris, MI-ra gestorum FA-muli tuorum; SOL-ve polluti LA-bii reatum, Sancte Ioannes.

("That your servants may with free voices sound out the wonders of your deeds, loose the guilt of their polluted lips, O holy John.")

Of these, the first tetrachord is expressed by *ut, re, mi, fa*; the second by *re, mi, fa, sol*; the third by *mi, fa, sol, la*. *Mi, fa*, or *fa, mi*, is a semitone; the others, next to one another, are major and minor tones. *Third*, he distributed these syllables over five tetrachords, corresponding to the five fingers of the hand and to as many keys [clauibus], in such a way that he constituted 20 strings and, with an admirable economy, represented every difference of tones and semitones. He nevertheless kept the seven letters devised earlier by St Gregory the Great, namely A B C D E F G; when these are completed the cycle returns to A. But before the first A he set Γ, that is, a capital gamma, to signify that the Greeks were the inventors of music, and to add a tone to complete the diapason, or octave, which two conjoined tetrachords do not fill.

On this hand, or musical scale, Guido himself wrote a book called *Micrologus*, or *Introductorium*, and dedicated it to Theobaldus, bishop of Arezzo. In the dedicatory letter he promises that from it as much skill in singing can be drawn within a month as anyone could scarcely acquire in many years by the old method; and at the end he adds this closing formula: *"End of the Micrologus of Guido, at the age of 34 years, under Pope John XX,"* etc.

Although the greater part of Europe has accepted those six syllables, some have nonetheless been content with four only, *ut, re, mi, fa*, as Mersenne reports

(on chapter 4 of *Genesis*, page 1679). Others have proposed seven, namely *ut, re, mi, fa, sol, la, bi*, as Ericius Puteanus [Hendrik van der Putte] in his *Musathena*, so as to distinguish the seven *phthongi* [phthógos], the notes that properly make up the octave, just as the Greeks are said once to have distinguished them by their seven vowels αιυεηοω. Some in the Low Countries, too, on Kepler's testimony (*Harmonics*, book 3, chapter 9), use these seven: *bo, ce, di, ga, lo, ma, ni*. Yet in the year 1547, on the testimony of Maillardus (chapter 10, *On the Tones*), these eight were in use in the Low Countries: *ut, re, mi, fa, sol, la, sy, o*.

With these things set out first, here now is the musical scale, or greatest musical system, corresponding to the Greek strings and tetrachords, with the ratios of the intervals; this system indeed we have compiled from what is handed down by Guido himself in the *Introductorium*; by Barbaro and Philander on book 5 of Vitruvius, chapter 4; by Folianus in the last chapter of section 3; by Zarlino, part 2 of the *Harmonic Institutions*, chapters 30, 33 and 36; by Glarean, book 1 of the *Dodecachordon*, chapters 5 and 19; by Hérigone in volume 5 of his *Cursus Mathematicus*, in the *Music* of Euclid; by Mersenne on chapter 4 of *Genesis*, from page 1668; and by Kircher, book 3 of the *Musurgia*, chapters 8, 9 and 13, and book 4, chapters 2 and 3. We have, moreover, supplied from others what some have omitted, and corrected what more than one writer confuses. Although Guido's scale could be fitted to the chromatic and enharmonic genera, it is usually adapted only to the diatonic. And although among the ancients the greatest system was contained within the disdiapason, or double octave, such as it is in the following scale from the string *proslambanomenos* to the string *nete hyperbolaeon* — since the number 9216 to 2304 is quadruple, which constitutes the double octave, just as 9216 to 4608, or 4608 to 2304, is double and constitutes the octave — Guido nevertheless added other strings and increased this system up to 22 strings.

SYSTEMA Maximum Diatonicum, cum Divisione Monochordi Diatonici, & cum Typo Scalae Musicae Guidonis Aretini (the greatest diatonic system, with the division of the diatonic monochord and with the pattern of the musical scale of Guido Aretinus)

Graeco-Latin names of the strings	Number of the string: Greek and Boethius	Number of the string: Guido	Division of the monochord: ratios	Musical keys	Notes of the mutation of the voices	Single intervals
		22	1536	E e	la	
						Tone
		21	1728	D d	la, sol	
						Tone
		20	1944	C c	sol, fa	
						Minor semitone
		19	2048	b b	mi	

Graeco-Latin names of the strings	Number of the string: Greek and Boethius	Number of the string: Guido	Division of the monochord: ratios	Musical keys	Notes of the mutation of the voices	Single intervals
						Major semitone, apotome
		18	2187	B b	fa	
						Minor semitone
Nete hyperbolaeon	15	17	2304	A a	la, mi, re	
						Tone
Paranete hyperbolaeon	14	16	2592	g	sol, re, ut	
						Tone
Trite hyperbolaeon	13	15	2916	f	fa, ut	
						Minor semitone
Nete diezeugmenon	12	14	3072	e	la, mi	
						Tone
Paranete diezeugmenon	11	13	3456	d	la, sol, re	
						Tone
Trite diezeugmenon	10	12	3888	c	sol, fa, ut	
						Minor semitone
Paramese	9	11	4096	b	mi	
						Major semitone
Here the disjunction of the strings takes place		10	4374	b	fa	
						Minor semitone
Mese	8	9	4608	a	la, mi, re	
						Tone
Lichanos meson	7	8	5184	G	sol, re, ut	
						Tone
Parhypate meson	6	7	5832	F	fa, ut	
						Minor semitone
Hypate meson	5	6	6144	E	la, mi	
						Tone
Lichanos hypaton	4	5	6912	D	sol, re	
						Tone
Parhypate hypaton	3	4	7776	C	fa, ut	
						Minor semitone
Hypate hypaton	2	3	8192	B	mi	
						Tone

Graeco-Latin names of the strings	Number of the string: Greek and Boethius	Number of the string: Guido	Division of the monochord: ratios	Musical keys	Notes of the mutation of the voices	Single intervals
Proslambanomenos	1	2	9216	A	re	
						Tone
		1	10368	Γ	ut	

Two ruled columns of the leaf carry labels that span several rows and are given here rather than in the body of the table:

- At the left, vertical labels for four tetrachords: *Tetrachord of the hyperbolaeon* (from Nete hyperbolaeon to Nete diezeugmenon); *Tetrachord of the diezeugmenon* (from Nete diezeugmenon to Paramese); *Tetrachord of the meson* (from Mese to Hypate meson); *Tetrachord of the hypaton* (from Hypate meson to Hypate hypaton).
- At the right, an unheaded column with the strings of the *Tetrachord of the synemmenon* (so labelled vertically): Nete synemmenon, Paranete synemmenon, Trite synemmenon, Mese — set against the rows from Paranete diezeugmenon down to Mese.

IV. Let us now examine this system: we shall find in it, besides the tones and semitones, the first five species of the more perfect consonances, as will at once be clear from the following table.

Consonance					Whole string of Boethius				Whole string of Guido			
Diapason	ut	2	to	1	thus 9216	to	4608	and	10368	to	5184	
Diapente	ut	3	to	1	thus 9216	to	6144	and	10368	to	6912	
Diatessaron	ut	4	to	3	thus 9216	to	6912	and	10368	to	7776	
Diapason diapente	ut	3	to	1	thus 9216	to	3072	and	10368	to	3456	
Disdiapason	ut	4	to	1	thus 9216	to	2304	and	10368	to	2592	

[Margin: 1. Corollary.]

From this it is clear, *first*, that in both scales the diapason ends at the eighth string, the diapente at the fifth and the diatessaron at the fourth string; but the diapason-diapente at the twelfth of Boethius and at the thirteenth of Guido, and the disdiapason at the fifteenth of Boethius but at the sixteenth of Guido. From this it is clear why these consonances have been called the octave, the fifth, the fourth, the twelfth and the fifteenth: namely because the strings are consonant with the whole string, which is set in first place, at the eighth position, the fifth and so on, if they are counted inclusively from the first.

[Margin: 2. Corollary.]

It is clear, *second*, that between strings 1 and 8, which make the diapason, there are five whole tones and two semitones, that is, 6 tones; between strings 1 and 5, which make the diapente, three tones with a semitone; between strings 1 and

4, which make the diatessaron, two tones and a semitone; and between strings 1 and 12 of Boethius, or 1 and 13 of Guido, which make the diapason-diapente, 9 tones and a semitone; and finally, between strings 1 and 15 of Boethius, or 1 and 16 of Guido, which make the disdiapason, 12 tones. This whole doctrine of the tones and semitones contained in the consonances already mentioned is set out expressly by Macrobius (book 2 on the *Dream of Scipio*, chapter 1), and in large part by Censorinus (*On the Birthday*, chapter 11) and by Pliny the Elder (book 2, chapter 22).

[Margin: 3. Corollary.]

It is clear, *third*, that the remaining eleven consonances, out of the 14 we enumerated in chapter 4, Table 1, are not found exactly in this scale. The reason is that this scale takes account of tones and semitones, and it is not so much a division of one string as a comparison of many different strings. If the remaining eleven consonances were to be found in Guido's scale, then the numbers you see in the small table below would have to be found in it.

	Interval								
For	the ditone	as	5	to	4	thus	10368	to	8294
For	the semiditone	as	6	to	5	thus	10368	to	8640
For	the major hexachord	as	5	to	3	thus	10368	to	6220
For	the minor hexachord	as	8	to	5	thus	10368	to	6485
For	the major tenth	as	5	to	2	thus	10368	to	6147
For	the minor tenth	as	12	to	5	thus	18368	to	4320
For	the eleventh	as	8	to	3	thus	10368	to	3891
For	the major thirteenth	as	10	to	3	thus	10368	to	3112
For	the minor thirteenth	as	16	to	5	thus	10368	to	3240

Chromatic system, or division of the chromatic monochord [SYSTEMA CHROMATICVM ſeu Diuiſio Monochordi Chromatici]

Tetrachord	String	Proportion of the intervals	Synemmenon
Tetrachord of the hyperbolaeon	Nete hyperbolaeon	2304	
		Trihemitone	
	Paranete hyperbolaeon	2736	
		Semitone	
	Trite hyperbolaeon	2916	
		Semitone	
	Nete diezeugmenon	3072	
Tetrachord of the diezeugmenon		Trihemitone	3456 Nete synemmenon
			Trihemitone
	Paranete diezeugmenon	3648	
		Semitone	
	Trite diezeugmenon	3888	

Tetrachord	String	Proportion of the intervals	Synemmenon
		Semitone	
	Paramese	4096	
			4104 Paranete synemmenon
			Semitone
	Disjunction of the strings	Tone	4374 Trite synemmenon
			Semitone
Tetrachord of the meson	Mese	4608	4608 Mese
		Trihemitone	
	Lichanos meson	5472	
		Semitone	
	Parhypate meson	5832	
		Semitone	
	Hypate meson	6144	
Tetrachord of the hypaton		Trihemitone	
	Lichanos hypaton	7296	
		Semitone	
	Parhypate hypaton	7776	
		Semitone	
	Hypate hypaton	8192	
		Tone	
	Proslambanomenos	9216	

Enharmonic system, or division of the enharmonic monochord [SYSTEMA ENHARMONICVM ſeu Diuiſio Monochordi Enharmonici]

Tetrachord	String	Interval	Synemmenon
Tetrachord of the hyperbolaeon	Nete hyperbolaeon	2304	
		Ditone	
	Paranete hyperbolaeon	2916	
		Diesis	
	Trite hyperbolaeon	2994	
		Diesis	
	Nete diezeugmenon	3072	3456 Nete synemmenon
Tetrachord of the diezeugmenon		Ditone	
	Paranete diezeugmenon	3888	
		Diesis	Ditone
	Trite diezeugmenon	3992	
		Diesis	
	Paramese	4096	4374 Paranete synemmenon
		Tone	4491 Trite synemmenon
Tetrachord of the meson	Mese	4608	4608 Mese
		Ditone	
	Lichanos meson	5832	
		Diesis	
	Parhypate meson	5988	

Tetrachord	String	Interval	Synemmenon
		Diesis	
	Hypate meson	6144	
Tetrachord of the hypaton		Ditone	
	Lichanos hypaton	7776	
		Diesis	
	Parhypate hypaton	7984	
		Diesis	
	Hypate hypaton	8192	
		Tone	
	Proslambanomenos	9216	

In the systems already mentioned, but especially in the diatonic, Martianus Capella (book 9, the chapter *What a system is*) considers eight perfect species of systems: 1. from Proslambanomenos to Mese; 2. from Hypate hypaton to Paramese; 3. from Parhypate hypaton to Trite diezeugmenon; 4. from Lichanos hypaton to Paranete diezeugmenon; 5. from Hypate meson to Nete diezeugmenon; 6. from Parhipate meson to Trite hyperbolaeon; 7. from Lichanos meson to Paranete hyperbolaeon; 8. from Mese to Nete hyperbolaeon. Each species thus comprises an octochord.

V. Kepler, however, constitutes his consonances differently. I would therefore like to add here, from the third book of his *Harmonices*, chapter 11, the Greatest System. It contains the perfect and imperfect consonances, with their intervals, through two diapasons or through the disdiapason, which Ptolemy also called the perfect system; and it contains both perfect and imperfect consonances. To tell these apart we have added another small table of our own.

The Keplerian system [SYSTEMA KEPLERIANVM.]

Key	Interval	Number	Guido's key and syllables	String
B				
gg		540		
	Semitone			
ffg		576		
	Limma			
ff		607		
	Semitone			
ee		648		
	Diesis			
ddg		675	ee la	
	Semitone			
dd		720	dd la sol	
	Semitone			
ccg		768	cc sol fa	
	Limma			
cc		810	bb fa b mi	
	Semitone			

Key	Interval	Number	Guido's key and syllables	String
hh		864	aa la mi re	
	Diesis			
bb		900	g sol re ut	Nete hyperbolaeon
	Semitone			
a		960	f fa ut	Paranete hyperbolaeon
	Semitone			
gg		1024	e la mi	Trite hyperbolaeon
	Limma			
g		1080	d la sol re	Nete diezeugmenon
	Semitone			
fg		1152	c sol fa ut	Paranete diezeugmenon
	Limma			
f		1215	b fa b mi	Trite diezeugmenon
	Semitone			
e		1296	a la mi re	Paramete
	Diesis			
dg		1350	G sol re ut	Mese
	Semitone			
d		1440	F fa ut	Lechanos meson
	Semitone			
cg		1536	E la mi	Parhypate meson
	Limma			
c		1620	D sol re	Hypate meson
	Semitone			
h		1728	C fa ut	Lechanos hypaton
	Diesis			
b		1800	B mi	Parhypate hypaton
	Semitone			
A		1920	A re	Hypate hypaton
	Semitone			
Gg		2048	Γ ut	Proslambanomenos
	Limma			
G		2160		

Under the left panel: Here each line signifies one principal string.

Under the right panel: Here not only the lines but also the spaces between the lines each signify one principal string, or principal sound, in the manner of modern diagrams.

The whole string BG is 2160. Struck [?] with the remaining strings listed below, it gives the consonances listed below, on account of the proportions that we shall set beside them.

BG with	Makes the consonance	For as	to	Thus 2160 to	Order of the strings, numbered from the first, G
B g	Diapason	2	1	1080	g Octave
B d	Diapente	3	2	1440	d Fifth
B c	Diatessaron	4	3	1620	c Fourth

BG with	Makes the consonance	For as	to	Thus 2160 to	Order of the strings, numbered from the first, G
B dd	Diapason-diapente	3	1	720	dd Twelfth
B gg	Disdiapason	4	1	540	gg Fifteenth
B h	Ditone	5	4	1728	h Third, hard or major
B b	Semiditone	6	5	1800	b Third, soft or minor
B e	Major hexachord	5	3	1296	e Sixth, major, hard
B dg	Minor hexachord	8	5	1350	dg Sixth, minor, soft
B hh	Diapason with the ditone	5	2	864	hh Major tenth
B bb	Diapason with the semiditone	12	5	900	bb Minor tenth
B cc	Diapason-diatessaron	8	3	810	cc Eleventh
B ee	Diapason with the major hexachord	10	3	648	ee Major thirteenth
B ddg	Diapason with the minor hexachord	16	5	675	ddg Minor thirteenth
B g	✠ ✠ ✠	3 432/576	1	576	g Most imperfect fourteenth

The same Kepler, moreover, in book 3 of the *Harmonices*, chapter 7, divided the whole string into 720 equal parts and determined the length of the remaining strings up to the octave, for the hard and the soft song, as you see in the first of the small tables that follow. And in chapter 8, dividing the whole string into 2160 parts, he determined the least intervals within one diapason, as you can see in the second small table below.

Table I. Of the length of the strings [Tabula I. Longitudinis Chordarum.]

Order of the strings	For the soft song	For the hard song
8	360	360
7	405	405
6	450	432
5	480	480
4	540	540
3	600	576
2	640	640
1	720	720

Table II. For the least intervals [Tabula II. Pro minimis Interuallis.]

Length of the strings	Intervals, concinnous or nearly so
1080	
	Semitone
1152	
	Limma
1215	
	Semitone
1296	

Length of the strings	Intervals, concinnous or nearly so
	Diesis
1350	
	Semitone
1440	
	Semitone
1536	
	Limma
1620	
	Semitone
1728	
	Diesis
1800	
	Semitone
1920	
	Semitone
2048	
	Limma
2160	

CAPVT VII.

Chapter VII.

An & Quo ordine Musarum voces, & Chordarum sonos Cælestibus sphæris accommodare oporteat.

Whether, and in what order, the voices of the Muses and the sounds of the strings should be fitted to the celestial spheres.

[Margin: Sirens and Muses, presiding over the celestial orbs.]

I. I shall begin with Macrobius, who in book 2 on the *Dream of Scipio*, chapter 3, says: "Plato, in his *Republic*, namely in book 10, when he treated of the revolution of the celestial spheres, says that a Siren sits on each of the orbs, signifying that by the motion of the spheres song is offered to the divinities; for *Siren*, understood in Greek, means 'singing to God'. The theologians, too, the nine Muses, etc." And a little below: "That the Muses are the song of the world even country people know" — or, as others read, "the Etruscans know" — "who called them Camenae, as if Canenae, from *canendo*, 'singing'. Therefore the theologians too, approving that heaven is tuneful, applied musical sounds to sacrifices, which among some peoples used to be performed with the lyre and the cithara, among others with pipes and other musical instruments. In the very hymns to the gods, too, metres were applied in tuneful verses through strophe and antistrophe, so that by the strophe the direct motion of the starry orb, and by the antistrophe the contrary regression of the wandering stars, should be proclaimed; and from these two motions the first, in nature, to be dedicated to God took its beginning."

[Margin: Strophe and antistrophe, the symbol of the two motions of the heaven.]

That strophe and antistrophe, together with motion in a circle, were used to represent the two motions of the heaven is attested also by Didymus and Victorinus in the scholia on Pindar, by the ordinary gloss on Pindar, and also by Triclinius on Sophocles' *Electra*. Their words are quoted by Mazzonus [Jacopo Mazzoni] in the *Defence of Dante*, book 2, chapter 34. He reconciles Didymus and Victorinus, who say that the strophe ran from right to left, so as to show the motion from east to west, with the Gloss and Triclinius, who say that the strophe ran from left to right: the former spoke of the right and left of the world, the latter of the right and left of a man.

[Margin: The number and names of the Muses.]

But to return to the Muses: writers agree neither on their number, nor on their names, nor on their order and offices. Phurnutus [Cornutus] reported that there were once two, on account of theory and practice. Three, however, are given by Ephorus (as reported by Arnobius), by M. Varro, by St Augustine, and by Zezes [Tzetzes] the grammarian on Hesiod, namely Cephison, Apollonida and Boristhenida, daughters of Apollo; Glareanus (book 2 of the *Dodecachordon*, chapter 14) says this was invented on account of the threefold sound, which is made by the voice, by breath, or by striking. Aratus, however (book 5 of the ἀστρικῶν [astrikōn, "astronomical matters"]), posits four Muses, daughters of Jupiter and the nymph Plusia, namely Archen, Meleten, Thelxinoën and Acœdem. Cicero likewise records four, in book 3 of *On the Nature of the Gods*, but calls them Thelxiopem, Mnemen, Aoeden and Meleten, born of the second Jupiter; and as many born of Jupiter Pierius and Antiopa. Epicharmus, however, in the *Marriage of Hebe*, lists seven Muses, daughters of Pierus and the nymph Pimpleis, namely Nilum, Tritonem, Asopum, Heptapolem, Acheloida, Tipoplum and Rhodian. The more common opinion, however, with Homer, Hesiod and Orpheus, holds that there were nine Muses, daughters of Jupiter and Mnemosyne. Phurnutus does not dissent, saying that the ternary multiplied by itself produces the novenary number of the Muses; nor does Zezes, although he says that their names were once Callichorem, Helicen, Eunicen, Telxinoen, Terpsichoren, Euterpen, Enceladen, Dian and Europen. But Hesiod, in the *Theogony*, lists their names in this order:

> Clio and Euterpe and Thalia and Melpomene, Terpsichore and Erato and Polyhymnia and Urania, and Calliope; and she it is who brings them all forth in order.

[Margin: The discoveries and offices of the Muses.]

Nor do all writers record their discoveries and functions in the same way. Our fellow-citizen Lilius Gregorius Gyraldus [Lilio Gregorio Giraldi], in syntagma 7 of *On the Gods of the Gentiles*, under the word *Musa*, reports the following. Clio discovered history; Thalia φυτουργίαν [phytourgian, "planting"], that is, the art of plants; Euterpe the pipes; Melpomene the ode, that is, song; Terpsichore the dance, that is, the *tripudium*; Erato wedding rites and

dancing; Polymnia agriculture; Urania astrology; Calliope poetry. Further on, however, he interprets them allegorically: Urania as the sublimity of the intelligence; Polymnia as the capacity of memory; Euterpe as the delight of the will; Erato as the love of like things; Melpomene as the depth of thought; Terpsichore as the exercise of the arts; Calliope as the beauty of eloquence; Clio as good fame and glory, which arises from the things already mentioned; and Thalia as the budding of virtues. The same author, however, gives this Greek epigram, rendered by him into Latin:

> Calliope showed the hero the art of song; Clio brought forth the measures of the sweet-sounding cithara; Euterpe the resounding song of the tragic chorus; Melpomene stirred the lyre with sweet harmony; and pleasing Terpsichore made ready to blow the reeds; but Erato found the delightful hymns of the gods; Polymnia joined harmony and dances to numbers; Urania the chorus of the stars and the turnings of heaven; comic life and manners, Thalia, were found by you.

[Margin: Apollo Musagetes.]

Apollo himself, moreover, is called Μουσήγετος [Mousēgetos, "leader of the Muses"], that is, the Leader and Choragus of the Muses, as Phurnutus, Macrobius (book 2 on the *Dream of Scipio*, chapter 3) and Proclus on Plato report. The words of this Proclus are: "Apollo is worshipped as Musagetes, and is the unity directed to harmony in the universe; here, in turn, is the chorus of the Muses of the whole novenary number; and by these two the whole world is bound together with indissoluble bonds."

II. As regards the order of the Muses in relation to the spheres

[Margin: The order of the Muses in the heavens.]

of heaven with which they are compared, different writers hold different views. Plato, for his part, determined nothing on this point. Nor did Cicero, in his book on the *Dream of Scipio*, although he indicated his mind obscurely in these words: "All things are joined together for you in nine orbs, or rather globes. One of them, the celestial and outermost, which embraces all the rest, is itself the supreme God, holding and containing the others; in it are fixed those eternal courses of the stars that revolve. Beneath it are set seven, which turn backwards with a motion contrary to that of the heaven, etc." And a little below: "And in the lowest orb the Moon, kindled by the rays of the Sun, revolves" — the lowest, that is, of the moving orbs, for shortly afterwards he adds: "For the one that is in the middle and ninth, the Earth, does not move and is the lowest, and towards it all weights are borne by their own inclination."

[Margin: Cicero's opinion.]

Therefore, according to Cicero's mind, the lowest of the Muses is to be assigned to the Earth, and the highest to the sphere of the fixed stars or to the whole heaven, as containing in itself all the motions of the others.

[Margin: And Macrobius's.]

This passage of Cicero is explained by Macrobius (book 2 on the *Dream of Scipio*, chapter 3). He teaches that the Muses are to be ascribed to the celestial spheres in exactly the order in which Hesiod listed them, so that Clio is assigned to the lowest sphere and Urania to the eighth, while Calliope is the most beautiful voice, as arising from the concord of all the spheres. Macrobius's words are these: "The theologians, too, held that the nine Muses are the musical songs of the eight spheres and the one greatest containment, which consists of them all. Hence Hesiod, in the *Theogony*, calls his eighth Muse Urania, because after the seven wandering spheres that lie beneath, the eighth, the starry sphere set above them, is called by its proper name heaven. And to show that the ninth and greatest is the one produced by the concordant totality of sounds, he added: Καλλιόπη θ' ἥ δε προφερεστάτη ἐστὶν ἁπασάων [Kalliopē th' hē de propherestatē estin hapasaōn, "and Calliope, who is the foremost of them all"]. By the name itself he shows that the very sweetness of the voice is called the ninth Muse, for Καλλιόπη [Kalliopē] is, in its Greek interpretation, 'of the best voice'. And to indicate more pointedly that she is the one that consists of all, he gave her a word of totality, namely προφερεστάτη ἁπασάων [propherestatē hapasaōn, "foremost of all"]."

[Margin: Glareanus's opinion.]

Glareanus held the same view of Calliope, in book 2 of the *Dodecachordon*, chapter 12, in these words: "But the ninth orb must be understood as the symphony blended from these eight phthongi, and for this reason called Calliope, as Plato philosophizes about the Muses in the *Republic* — not rashly, in the judgment of all the learned; Cicero's treatment in the *Dream of Scipio* is the same." And in chapter 14 he affirms that, according to Plato in book 10 of the *Republic*, the Sirens assigned to the individual spheres were called the eight Muses on account of the eight sounds, while the symphony blended from them was called by Plato the ninth Muse, and by Hesiod Calliope, as the most excellent of all. He also praises Christophorus Landinus [Cristoforo Landino], because he interpreted better than Servius, with reference to this totality and excellence of the voices comprised in Calliope, that verse of Virgil in *Aeneid* 9:

> You, O Calliope, I pray, breathe upon me as I sing.

Here the best of poets used the plural number in addressing Calliope, as being the one who comprehended all the others in herself or under herself — which in *Aeneid* 7 he did not dare to do when he invoked Erato. Then, in the system of the world, together with Hesiod, Glareanus assigns Clio herself to the Moon, Euterpe to Mercury, Thalia to Venus, Melpomene to the Sun, Terpsichore to Mars, Erato to Jupiter, Polhymnia to Saturn, Urania to the fixed stars, and Calliope to the last orb of all.

[Margin: The opinion of Marsilio Ficino.]

But Marsilius Ficinus [Marsilio Ficino], on Plato's dialogue on poetic frenzy — and with him, subscribing to his view, Iosephus Zarlinus, in part 2 of the *Harmonic Institutions*, chapter 29 — applies Thalia to the Moon; Euterpe to Mercury, Erato to Venus, Melpomene to the Sun, Clio to Mars, Terpsichore to

Jupiter, Polhymnia to Saturn, Urania to the sphere of the fixed stars; and he sets Calliope over the individual spheres and over all of them together, to indicate the harmony that arises from them all.

But Gyraldus, in syntagma 7 of *On the Gods of the Gentiles*, under the word *Musa*, attributes Thalia to the Earth, as silent and having no harmony; to the Moon, Clio and the Hypodorian mode; to Mercury, Calliope and the Hypophrygian; to Venus, Terpsichore and the Hypolydian; to the Sun, Melpomene and the Dorian; to Mars, Erato and the Phrygian; to Jupiter, Euterpe and the Lydian; to Saturn, Polhymnia and the Mixolydian; to the fixed stars, Urania and the Hypermixolydian. This opinion is subscribed to, in Marinus Mersennius on chapter 4 of Genesis, verse 21, page 1704, by an unnamed author in the following distichs:

> First of all Thalia, who lies deaf in the lap of the Earth, makes the silences of night bud forth with song. Persephone (a) and Clio breathe; the Hypodorian therefore is born; hence it begets the Prosmeledes (b), sprung from it. And the following string gives the Hypo-Phrygian, which Calliope herself brings forth; the interpreter (c) of the gods himself brings it forth. The third shows the beginnings of the Hypolydian on the strings: Terpsichore comes to meet it; kindly Paphis (d) sets it in order. Melpomene and Titan (e), believe me, establish the mode which is said to be the Dorian, in fourth place. Erato wishes to assign the fifth string to the Phrygian, and so does Mars, who loves not peace, but always battles. You will have the Lydian measure of Euterpe and of Jupiter, which the sixth string, holding it sweetly, bade to be. Saturn and Polyhymnia drive the seventh string, from which the Mixolydian takes its beginning. While Urania searches out the friendly eighth, the Hypermixolydian turns the pole with art.

[Margin: a. It is Lu[?].]
[Margin: b. Proslambanomenos.]
[Margin: c. Mercury.]
[Margin: d. Venus.]
[Margin: e. The Sun.]

It is pleasing, therefore, to gather these opinions into one synopsis in the following small table.

Spheres (Glareanus, after Hesiod, Plato, Cicero and Macrobius)	Muses (Glareanus, after Hesiod, Plato, Cicero and Macrobius)	Ficinus and Zarlinus (after Plato)	Gyraldus and others in Mersennius
The ninth, or the eight together	Calliope	Calliope	✠
✳ Of the fixed stars	Urania	Urania	Urania
♄ Of Saturn	Polyhymnia	Polyhymnia	Polyhymnia
♃ Of Jupiter	Erato	Terpsichore	Euterpe
♂ Of Mars	Terpsichore	Clio	Erato
☉ Of the Sun	Melpomene	Melpomene	Melpomene

Spheres (Glareanus, after Hesiod, Plato, Cicero and Macrobius)	Muses (Glareanus, after Hesiod, Plato, Cicero and Macrobius)	Ficinus and Zarlinus (after Plato)	Gyraldus and others in Mersennius
♀ Of Venus	Thalia	Erato	Terpsichore
☿ Of Mercury	Euterpe	Euterpe	Calliope
☾ Of the Moon	Clio	Thalia	Clio
The Earth	✠	✠	Thalia

Thus all agree in the assignment of three Muses — Urania, Polyhymnia and Melpomene; in the rest not all agree.

[Margin: Our opinion, and a critique from Aristotle and Pliny.]

III. You will now ask us for our own view. Our answer is that these analogies and harmonies were determined by the authors already mentioned "more prettily than truly", to use the words of Aristotle (book 2 of *De caelo*, text 52); or that, with Pliny (book 2, chapter 22), it is said "that Saturn moves in the Dorian, Mercury in the phthongus, Jupiter in the Phrygian, and similarly in the rest, with a subtlety more pleasing than necessary". And Macrobius himself, in book 2 on the *Dream of Scipio*, chapter 4, says that to pursue these matters more subtly "is the mark of one showing off, not of one teaching". Of the same opinion are Glareanus (book 2 of the *Dodecachordon*, chapter 13, at the end), Zarlinus (part 2 of the *Harmonic Institutions*, chapter 29, at the end), Mersennius (on chapter 4 of Genesis, page 1704) and Kircherus [Athanasius Kircher] (book 10 of the *Musurgia*, page 381).

[Margin: Our first reason.]

The reason that moves me, however, is not a single one. First, we made plain enough under number one how inconstant the tradition was about the number, the names and the properties of the Muses.

[Margin: 2nd reason.]

Second, the order of the celestial spheres among the older astronomers was itself also varied, as is clear from the variety of systems we have already set out in book 3, chapter 6, and book 9, section 3. So you would not satisfy even Macrobius by assigning Thalia and Euterpe to Venus and Mercury, since he admits that those planets are also carried above the Sun.

[Margin: 3rd reason.]

Third, if the Sun is the leader of the Muses, and is therefore called Musagetes, how does one of the Muses preside over his sphere — as his deputy, or helper? Again, if no ninth sphere distinct from the others is granted for the primum mobile, but each planet is moved by a single spiral motion, there will be no sphere to which Calliope or another ninth Muse may be assigned; and much less is a Muse of its own to be given to the Earth, since it does not move.

[Margin: 4th reason.]

Fourth, how do the Muses and the Sirens fit together in Plato's opinion, if elsewhere Greece told the fable that the Sirens were defeated by the Muses in a singing contest? It is better, therefore, to dismiss these fables and to recognize,

in place of the Muses, the Peripatetic Intelligences — which the theologians also recognize — as the governors of the heavens.

De Chordarum & Cælestium sphærarum Analogia.

On the analogy of the strings and the celestial spheres.

[Margin: The inventor of the lyre: Mercury, or Apollo?]

IV. Before we deal with the strings of the celestial lyre, we must briefly inquire who the inventor of the lyre was, and how many strings it had. For if we believe Diodorus Siculus, Mercury devised a three-stringed lyre, but Orpheus made it a tetrachord. Terpander, on the testimony of Plutarch in his short work *On Music*, added to it laws and names for the strings — granted that Suidas, Boethius (book 1, chapter 20) and Aristotle (section 19, problem 32) attribute a seven-stringed lyre to him. But Macrobius (book 1 of the *Saturnalia*, chapter 19) relates that the tetrachord was attributed to Mercury, to signify the same number of regions of the world, or seasons of the year; and he adds: "Just as the lyre of Apollo, of seven strings, gives us to understand the motions of as many celestial spheres, among which nature has set the Sun as governor." But Hyginus, in his *Poetic Astronomy*, affirms that Mercury was the inventor of the seven-stringed lyre, on account of the number of the seven Pleiades, or Atlantids; that when he had driven off Apollo's cattle, he complied with Apollo in order to satisfy him as he pursued him, and agreed that Apollo should name himself the inventor of the seven-stringed lyre; and that he was rewarded besides by Apollo with that rod with which Mercury, a little later, reconciling two serpents, used to carry the caduceus in return for the lyre. But even before Hyginus, Homer, in the hymn he dedicated to Mercury, recognizes him as the inventor of the seven symphonies through as many strings, made from the guts of birds, in this verse:

> Επτὰ δὲ συνφώνος ὀίων ἐτανύσατο χορδὰς [Hepta de synphōnos oiōn etanysato chordas, "and he stretched seven concordant strings of sheep"] — that is: "And he stretched seven consonant strings made from the guts of sheep."

Horace agrees with this (book 3 of the *Odes*, Ode 11), addressing Mercury and his tortoise-shell:

> And you, tortoise-shell, skilled to resound with seven strings.

In imitation of this, or to imitate the 7 symphonies, whether celestial or human, Pan is believed to have been the first to join seven reeds into one wind instrument. Ovid mentions a pipe of this kind in *Metamorphoses* 2, in this half-line:

> … a pipe unequal with its seven reeds:

And Corydon in Virgil, *Eclogue* 2:

> I have a pipe made of seven unequal hemlock stalks.

Granted that the well-known Menalcas in Theocritus boasts of a pipe of nine voices, or reeds, made by himself, in this verse:

> Συρίγγαν ἐπόησαν καλὰν ἐγω ἐννεάφωντον [recte: ἐννεάφωνον]
> [syringan epoēsan kalan egō enneaphōnton, "a fine pipe of nine voices"]

that is:

> I myself made a beautiful syrinx of nine voices.

But to return to the seven-stringed lyre: Glareanus (book 2 of the *Dodecachordon*, chapter 12) holds that its seven strings signify the seven species of consonances into which the diapason is divided, and which reign in the two octochord instruments; and to this he draws that line of Virgil in *Aeneid* 6:

> And the Thracian priest, too, in his long robe, answers the measures
> with the seven distinct notes.

[Margin: 7 species of the diapason.]

These species are: the diapason; the diapason-diapente; the disdiapason; the diapason with the ditone; the diapason with the semiditone; the diapason with the major hexachord; and the diapason with the minor hexachord — unless by these you understand the 7 simple consonances that we listed in chapter 4, table 1.

V. But whatever may be the case about the inventor and purpose of the seven-stringed lyre, or of the nine-stringed lyre or pipe, it is certain that some authors, not to be despised, recognized in the celestial spheres intervals and differences of deep and high notes, and therefore fitted to them the strings of the lyre or of the harmonic system. Cicero led the way for them from the Platonic source, in the *Dream of Scipio*. When Scipio asks, "What is this, I said, what is this sound, so great and so sweet, that fills my ears?", Cicero brings in his father Paulus answering in these words:

[Margin: Cicero's opinion on the high sound of the highest sphere.]

"It is the sound, he said, which, joined by unequal intervals, yet distinguished in a fixed proportion, is produced by the impulse and motion of the orbs themselves; tempering the high with the low, it produces harmonies evenly. For such great motions cannot be roused in silence, and nature brings it about that the extremes sound low at one end and high at the other. For this reason that highest course of the starry heaven, whose revolution is swifter, moves with a high, roused sound, while this lunar and lowest course moves with the deepest. For the Earth, the ninth, remaining immobile, always stays fixed in the lowest seat, embracing the middle place of the world. But those eight courses, in which two have the same power, produce seven sounds distinguished by intervals — a number that is the knot of almost all things. Learned men, imitating this with strings and songs, have opened for themselves a way back to this place."

[Margin: Macrobius's opinion.]

Explaining these words, Macrobius (book 2 on that *Dream*, chapter 4) teaches that a sharper sound comes from a great and rapid collision of the air, and a deeper one from a slow and more sluggish one. He confirms this both by the example of a rod swung through the air and by that of strings, which sound high if they are stretched with a tighter pull, and lower if with a looser one;

and of pipes and flutes, in which, he says, holes that are narrower, or farther from the mouth of the one blowing, give a deeper sound than those that are more open, or nearer the mouth. From this he concludes that the highest sound is owed to the supreme, starry orb, both on account of its size, by which it extends without measure, and on account of the very great speed of its daily motion towards the west; whereas the deepest sound is owed to the Moon, on account of the narrowness of its orb and the slowness of its motion. He also concludes that, of the 9 spheres described by Tullius [Cicero], the Earth is immobile and the other eight are mobile, but that there are only seven sounds, because Mercury and Venus are moved with the same motion as the Sun; and that for this reason Cicero said that two have the same power.

[Margin: Glareanus's exposition of Cicero's text.]

Glareanus, however (book 2 of the *Dodecachordon*, chapter 13), thinks that the passage of Cicero is corrupt and was not well enough understood by Macrobius; and that the word *Septem*, "seven", is to be referred not to the sounds but to the intervals, so that the eight courses produce as many sounds, but sounds separated from one another by seven intervals. Or rather, it should be read thus: "But those eight courses, in which the power of the extremes is the same, produce seven intervals with distinct sounds." For by the laws of music eight phthongi, or principal notes, produce seven intervals, or seven species of the diapason.

[Margin: The cause of the diversity of opinions on this.]

VI. Now, from the sources of Plato and Cicero, three different opinions have flowed. The first assigns to the supreme sphere the highest of the strings — that is, either the *mese*, which is the highest of the first nine strings in the harmonic system, or the *nete hyperbolaeon*, which is the highest in the ancient system of 15 strings. The second assigns to the supreme sphere the deepest of the strings, namely the *proslambanomenos*, or at least the *hypate hypaton*. The third, finally, either adheres to both under different respects, or strives to reconcile both — granted that it leans towards one rather than the other. The reason for these different opinions is both the twofold motion that appears in the heaven, and the differing distance and size of the orbs. For if one considers the common motion, called that of the primum mobile, towards the west, the fixed stars move faster than Saturn, and Saturn faster than the remaining lower planets; and on that account the highest sound is owed to the sphere of the fixed stars. But if one considers the apparent proper motion towards the east, the sphere of the fixed stars is the slowest, then Saturn, and so on; so, on account of this cause of motion, the deepest sound seems to be allotted to them. Conversely, since the length of a string and the size of bodies give a deeper sound than shortness and smallness, as Aristotle teaches (section 19, problem 8), a deeper sound seems to be attributed to the spheres and stars more remote from the Earth. It is disputed besides whether the string giving the deepest sound ought to be placed in the lowest place of the musical system and of the tetrachords, and the highest string in the highest place; or rather the deepest in the highest

place and the highest in the lowest. Glareanus discusses this question learnedly in book 1 of the *Dodecachordon*, chapter 5, and book 2, chapter 8.

[Margin: 1st opinion: of Cicero, Macrobius and Guido Aretinus.]

VII. The first opinion is that of Guido Aretinus, whom many have followed. As is clear from the systems set out in chapter 5, after number 3, he put in the lowest place the deepest string, Gamma-ut, which the proslambanomenos follows, and in the highest degrees of the scale the higher sounds. This agrees with Cicero (book 6 of the *Republic*, or the *Dream of Scipio*) and with Macrobius: namely, that the upper bodies, as being swifter in the daily motion, give a higher but smaller sound, while the lower bodies, being slower, give a greater but deeper so- und. This opinion was followed by Gulielmus Philander [Guillaume Philandrier] on Vitruvius, book 5, chapter 4, who attributes it to the ancients; by Gyraldus, in syntagma 7 of *On the Gods of the Gentiles*, under the word *Musa*; by Valla on Pliny, book 2, chapter 22; by Father Athanasius Kircher, book 10 of the *Musurgia*, page 393; and by the unknown author in Mersennius on chapter 4 of Genesis, verse 21, page 1704. For they give the deepest string to the Moon or to the Earth, and the highest to Saturn or to the sphere of the fixed stars.

[Margin: Of Philander, Gyraldus, Valla, Kircher.]

These writers are certainly supported, on the one hand, by the order of the strings on the cithara, which is the reverse of the order of the strings on the lyre. For on the lyre the deepest and thickest string is in the highest place, whether the lyre is a trichord, a tetrachord or a hexachord; but on the cithara the deepest and thickest of the strings is in the lowest seat, and the highest in the highest — an order that our organs still keep, in which the thicker pipes are placed in the lower row and the more slender in the upper. And, on the other hand, the human voice too begins more often not from the highest but from the lowest, and from a deeper sound, and rises little by little, by a natural transition, to a higher one. For the deeper voice is, as it were, the foundation, without which the upper and higher voices seem to degenerate into chatter, unless they are sustained by the strength and firmness of a deeper voice. Although the highest voice may seem to soothe the hearing more sweetly, yet, as Glareanus noted above, no song is more pleasing than when the lowest voice sounds firmly; and the upper voices achieve almost nothing without the Bass of the lower voices — for so Julius Pollux names the deeper voices. Nor today is the consonance of the diatessaron admitted unless it is propped up from below by the support of the diapente, that is, the fifth, or else of the third. Mersennius, too, although on chapter 4 of Genesis, page 1704, he rejects fictions of this kind, nevertheless on page 1705 describes, from the secrets of the Cabbalists, the ten-stringed cithara of David with the following analogy, in which he assigns the higher sound to the highest spheres and spirits. Here it is in a table.

[Margin: The strings of the Davidic cithara according to the Cabbalists.]

Strings	Names of the strings	Notes	Bodies	Spirits
1	Parhypate meson	F fa ut	The elements	The blessed soul
2	Lichanos meson	G sol re ut	☾	Angels
3	Mese	A la mi re	☿	Archangels
4	Paramese	B fa b mi	♀	Principalities
5	Trite diezeugmenon	C sol fa ut	☉	Virtues
6	Paranete diezeugmenon	D la sol re	♂	Powers
7	Nete diezeugmenon	E la mi	♃	Dominations
8	Trite hyperbolaeon	F fa ut	♄	Thrones
9	Paranete hyperbolaeon	G sol re ut	✳	Cherubim
10	Nete hyperbolaeon	A la mi re	The first moved ("1. Mobile")	Seraphim

[Margin: 2nd opinion.]

VIII. The second opinion assigns the deeper sound to the upper spheres: either because it takes them to be slower, when their proper motion is considered, or because, even if the greater speed of the common motion confers sharpness on them, the length of the interval and the size of the bodies nevertheless produce a deeper and greater sound — just as with the strings of many-stringed instruments, in which the thicker and larger string produces a deeper but more sonorous voice.

[Margin: Of Nicomachus.]

This was the opinion of Nicomachus in the *Enchiridion of Harmonics* — as Boethius and Philander report — who accordingly assigned the hypate to Saturn and the nete to the Moon. Servius Honoratus, too, explains Virgil's phrase "through the friendly silences of the silent Moon" in this way: "'Of the silent Moon': either he means night, in the poetic manner, or he spoke on a physical account. For there are seven circles: of Saturn, Jupiter, Mars, the Sun, Venus, Mercury and the Moon. And the first, that is, Saturn, sounds vehemently, the rest less according to their order, as we have heard in the cithara, whose last string sounds less."

[Margin: And of Servius the grammarian.]

There, by the last string of the cithara, Servius means the one that is nearer to our ears, and in this respect he compares it with the Moon — not according to the order of upper and lower position, for the Moon is below and Saturn above, whereas on the cithara the highest and thinnest string, of least sound, is above, and the deepest below. Therefore Angelus Politianus [Angelo Poliziano], in the last chapter of the *Miscellanea*, neither understood Servius nor justly censured him, when he says: "This, then, will at last hold good, if he takes the last not for the lowest but for the highest, which in Greek is called the hypate, from which a deeper sound is roused; just as from the other, outermost and thinnest string — which I for my part would gladly call the last — a high voice is brought forth, because it is stretched forward."

But Politianus is more justly censured by Glareanus (book 2 of the *Dodecachordon*, chapter 8), where he teaches that Servius spoke truly and according to the nature and series of the celestial orbs (if there were any sound there), and

indeed according to the strings of the cithara as well; for on it the hypate, the deepest of the strings, holds the lowest place, and the highest string the last, which, even if it is stretched to the utmost, is not for that reason heard the most. "For granted," says Glareanus, "that in one and the same string its sharpness strikes the hearing more when it is stretched hard than its depth does when it is stretched more loosely; yet among different strings, the one that is larger and thicker is heard more, and makes a greater sound, than one that is more slender, even if the latter is stretched more and sounds higher — just as the sound of a man with a good voice is heard more than the wailing or crying of a little boy. Hence too it is that the higher voices achieve almost nothing without the base and firmness of the deeper ones."

[Margin: And of Angelo Poliziano.]

It appears, however, that Politianus judged rightly as far as this goes, in granting the deeper sound to the hypate; and that, by putting it in the highest place, he subscribed to Servius, who attributes it to Saturn.

[Margin: And of Boethius.]

Of the same opinion, finally, was Severinus Boethius. For he not only recognized the hypate as the deepest of the strings, but also assigned it to the upper spheres. In book 1 of his *Music*, chapter 20, he says of the strings: "And among these, the one that was the deepest was called hypate, as being greater and more venerable; hence they also call Jupiter Hypatos. They call a consul, too, by the same name, on account of the excellence of his dignity; and it was attributed to Saturn on account of the slowness of his motion and the depth of his sound." So too, in book 4, chapter 10, he puts the proslambanomenos in the highest place and the nete in the lowest.

[Margin: Glareanus's opinion.]

And Glareanus, a follower of Boethius, absolutely denies that there is sound in the heavens, or any true account of symphony, as is clear from book 2 of the *Dodecachordon*, chapter 14, from the paragraph *Ceterum* to the end. Yet on the supposition either that there were some sound in the heaven, or that one should philosophize about the celestial bodies as about terrestrial sounding ones, he inclines to Boethius's view. For in book 1 of the *Dodecachordon*, chapter 5, he says: "I am not unaware that to many this order of the strings laid down by Guido seems inverted, and entirely contrary to the natural course of the heaven from which this formula was taken, since the upper bodies of the celestial orbs give a deeper sound, as being greater. And the divine Severinus Boethius, too, seems to have observed this everywhere in his demonstrations." And a little later: "Whether, then, we also invert them, so that the upper are the deeper — namely so that we put the tetrachord hypaton in the highest place, with the proslambanomenos, which Severinus sometimes does — either on account of the name [?] hypatos, which means 'principal', or because it so seemed to the first teachers of this art. For many old instruments, too, such as trichords, tetrachords and hexachords, still have it so, etc." And in book 2, chapter 8, he says that Servius spoke "both truly and according to the nature and series of the

celestial orbs (if there is any sound there)"; and it is clear from what has been said that Servius gave Saturn the deepest and most vehement sound. And in chapter 13, after recounting Cicero's opinion, he departs from it and adds: "Others, on the contrary, ascribed the higher sounds to the lower bodies, and the deeper, or greater, to the upper, because larger bodies also give a greater sound and smaller ones a smaller — an opinion that seems to me far more probable, since bodies in the heaven are as they are in this sensible world." And he adds a twofold type: one Ciceronian, in which he assigns the shorter and higher strings to the upper spheres; the other Boethian, in which he assigns the longer and deeper strings to the upper spheres. We shall display these types within a table, together with others. In chapter 14, however, he chose the Boethian type alone — as does Mario Bettini in the *Apiaria*, apiary 10, progymnasma 1, proposition 1, inasmuch as he applies the hypate to the fixed stars, the parhypate to Saturn and the nete to the Moon.

[Margin: And of Bettini.]

This opinion is favoured, moreover, by the order of the lyre, in which the thicker string, of deeper sound, holds the highest place, and the string of higher sound holds the lowest — as is also kept, for the most part, in other instruments.

[Margin: 3rd opinion.]

IX. The third opinion shows itself indifferent. Zarlino holds it, in part 2 of the *Harmonic Institutions*, chapter 29, where he sets out two systems: one according to Cicero, but assigning the mese to the fixed stars and the proslambanomenos to the Moon; the other according to Boethius. Within the three opinions already mentioned, however, there is still some variety as regards the whole order of the strings, as will appear more clearly from the following tables.

CHORDARVM CÆLESTIVM ORDO

The order of the celestial strings

	According to Cicero and Macrobius: Glareanus, Zarlino and Gyraldus	According to Cicero and Macrobius: Gulielmus Philander	According to Cicero and Macrobius: Father Athanasius Kircher	According to Nicomachus and Boethius: Glareanus and Zarlino	According to Nicomachus and Boethius: Mario Bettini
✳	Mese				Hypate ut
♄	Lichanos meson	Nete	Nete	Lichanos hypaton A re	Parhypate re
♃	Parhypate meson	Paranete	Paranete	Hypate meson B mi	Lichanos mi
♂	Hypate meson	Paramese	Paramese	Parhypate meson C fa ut	Mese fa
☉	Lichanos hypaton	Mese	Mese	Lichanos meson D sol re	Paramese sol
♀	Parhypate hypaton	Licanos hypaton	Lichanos hypaton	Mese E la mi	Trite re re

	According to Cicero and Macrobius: Glareanus, Zarlino and Gyraldus	According to Cicero and Macrobius: Gulielmus Philander	According to Cicero and Macrobius: Father Athanasius Kircher	According to Nicomachus and Boethius: Glareanus and Zarlino	According to Nicomachus and Boethius: Mario Bettini
☿	Hypate hypaton	Parhypate hypaton	Parhypate hypaton	Trite synemmenon F fa ut	Paranete mi mi
☾	Proslam-banomenos	Hypate hypaton	Hypate hypaton	Paranete synemmenon G sol re ut	Nete fa fa
The Earth			Proslam-banomenos	Nete synemmenon a la mi re	

Foot of the table: Add to these what I reported from the Cabbalists at the end of number 7.

[Margin: The author's opinion.]

X. What our own opinion on this matter is can already be clear from what was said at the beginning of number 3: namely, that these things are investigated more charmingly than truly, or with a subtlety more pleasing than necessary, as we said there with Aristotle and Pliny. And Macrobius, very prudently, in book 2 on the *Dream of Scipio*, chapter 4, when he had this controversy in hand, concluded with these words:

[Margin: An excellent sentence of Macrobius.]

"To illuminate, as I think, the obscurity of Cicero's words, the treatise on music that we have given succinctly, with such brevity as was permitted, will suffice. For to search out the netes and hypates and the names of the other strings, and to stitch together the minute subtleties of tones or limmas, and what in sounds is taken for a letter, what for a syllable, what for a whole word, is the part of one who makes a display, not of one who teaches. Nor, because Cicero made mention of music in this place, must we on this occasion go through all the treatises that there can be about music — treatises which, as far as my opinion goes, I do not think have any end."

[Margin: Glareanus's absolute sentence.]

Imitating him, I believe, Glareanus concluded near the end of chapter 13 of book 2 of the *Dodecachordon*: "Not without reason did this fiction seem to Aristotle more pleasing to tell than likely to be true"; and a little later: "But this indulgence is granted to antiquity, which thought that human minds were to be raised in any way it could to the contemplation of celestial things." Zarlino himself too, at the end of chapter 29 of part 2, concludes with Pliny that these things are investigated with a pleasure more subtle than necessary; and Marinus Mersennius, on chapter 4 of Genesis, verse 21, page 1704, rejects all these fictions

as put forward without sufficient proof. And long before them, St Ambrose, in the book *On Isaac and the Soul*, chapter 7, denied that fictions of this kind about the harmony of the heavens find credence, although on account of the pleasantness of the discourse they are heard with favour and delight — as we showed in chapter 2 from his own words.

[Margin: Our 1st reason.]

Nor am I moved only by the reason that there is no real sound in the heavens — what I thought about this I said in the same chapter 2. First, rather: even if there were some sound, the truer order of the planets is not the one that the authors already mentioned suppose. And it may indeed seem astonishing that Fathers Mersennius and Kircher, more recent astronomers, place Mercury nearer the Moon than Venus, unless you say they spoke according to the opinion of the ancients. But since these two planets are carried now below and now above the Sun, do they not measure out by their motions the whole portion of that system which runs from their apogee to their perigee? Similarly Mars too, which at times becomes nearer the Earth than the Sun: does he not occupy in great part, by his own ascent and descent, that interval through which the Sun, Venus and Mercury ascend and descend, and mingle himself with them, so that you cannot ascribe to him a distinct string or interval?

[Margin: [?] Reason.]

Second: since it is more probable that the planetary heavens are fluid, and that the planets move in them of themselves and not by the motion of orbs, as was said in section 1, chapter 2, question 2, and chapter 7, a greater sound could indeed be attributed to the sphere of the fixed stars, as being solid, as to a greater body; but a greater sound could not be attributed to Saturn, Jupiter and Mars than to the Sun, since the Sun is greater than those three planets.

[Margin: 3rd reason.]

Third: since there are eight different species of harmonic systems, as I showed from Martianus Capella in chapter 5, at the end of the chromatic and enharmonic system, it is very uncertain which of them should be assigned to the celestial system.

[Margin: 4th reason.]

Fourth: if one were free and had the leisure to play at these things, account would have to be taken both of the satellites of Saturn and Jupiter, and of the differing position that the planets hold at the apogee and perigee of the eccentric and of the epicycle, and of the differing altitude that the fixed stars hold.

[Margin: 5th reason.]

Fifth: as regards speed and slowness of motion — since in reality they are not moved by any other real motion than toward the west, although because of their slowness the lower ones seem to recede toward the east — a higher sound would certainly have to be assigned to the upper ones, as being absolutely swifter in this motion, except when they become retrograde. On the other hand, the three upper planets, because of the greater mass of their bodies, would

require a deeper sound than the three lower ones. Thus one reason destroys the other; and therefore this analogy, sung up with a charm rather oratorical and academic, loses all its harmony through its very discord — to say nothing of the disagreement among the authors themselves in distributing the strings over the heaven.

CAPVT VIII.

Chapter 8.

An & Qua ratione Distantijs Siderum Interualla Systematis Harmonici debeant accommodari.

Whether, and by what method, the intervals of the harmonic system should be fitted to the distances of the stars.

Achilles Tatius, in the *Isagoge* to Aratus's *Phaenomena*, in the chapter on the order of the spheres, 7, says: "Of their harmonic motion, as has been said, Aratus treats in the *Canon*, and Eratosthenes in the *Mercurius*, and also Hypsicles, Thrasyllus and Adrastus of Aphrodisias; the authors of this doctrine were the Pythagoreans, who think that all things are moved in harmony and order." Boulliau, too, at the end of the Prolegomena to the *Astronomia Philolaica*, names the *Harmonicon caeleste* of Franciscus Vieta [François Viète], which Petrus Puteanus [Pierre Dupuy] had lent for use to Father Marinus Mersennius, from whom, how- ever, it was stolen. But since we do not know their opinion, we shall come down to the views of others.

[Margin: 1st opinion, of the Pythagoreans.]

I. The first opinion was that of the Pythagoreans, which we shall try to fish out from Pliny, Plutarch and Censorinus. Pliny, then, in book 2, chapter 21, says: "Many have tried to track down the intervals of the stars from the Earth, and they have reported that the Sun is distant from the Moon nineteen parts, each as great as the distance of the Moon itself from the Earth. But Pythagoras, a man of shrewd mind, inferred that from the Earth to the Moon is one hundred and twenty-six thousand stadia; from the Moon to the Sun double that; and from there to the twelve signs triple; and our own Gallus Sulpicius was of this opinion too."

[Margin: The distance of the Moon from the Earth and from the Sun according to the ancient mathematicians.]

Partly twin to this passage is the one in Plutarch, book 2 of the *De placitis* (*On the Opinions of the Philosophers*), chapter 31, where he says: "Empedocles: that the Moon is twice as far from the Sun as from the Earth. The mathematicians: that the Moon is distant from the Sun nineteen parts, each as great as its own distance from the Earth. Eratosthenes: that the Moon is seven hundred and eighty thousand stadia distant from the Earth." I should think, however, that these mathematicians, who determined the distance of the Moon from the Sun as nineteen times the distance of the Moon from the Earth, were Aristarchus

and Hipparchus and their followers. For Aristarchus, in the book *On the Sizes and Distances of the Three Bodies*, proposition 7, had already demonstrated "that the distance by which the Sun is distant from the Earth is greater than eighteen times the lunar distance from the Earth, but less than twenty times": that is, about nineteen times. And Hipparchus put the distance of the Moon at about 64 terrestrial semidiameters, and that of the Sun at 1200, which is to 64 about as 19 to 1.

It is surprising, then, that Eratosthenes gave the lunar distance only 780,000 stadia, that is, 97,500 Italian miles. For, from what was said in book 2, chapter 7, Eratosthenes put 80,181 stadia in the diameter of the Earth and 40,090½ in the semidiameter; and when 780,000 stadia are divided by 40,090½, there come out, for the lunar distance, 19 whole terrestrial semidiameters.

[Margin: Pythagoras's earlier opinion, most absurd.]

But Pythagoras must have blundered far more shamefully, since he granted this distance no more than 126,000 stadia, which, divided by 8, make 15,750 Italian miles — that is, scarcely 4 semidiameters of the Earth. The lunar parallax protests against this, for we have already proved, in book 4, chapter 14, that it requires more than 51 semidiameters of distance from the Earth. If this distance were doubled, the distance of the Sun would come out at 8 terrestrial semidiameters, and if tripled, the distance of the fixed stars would be 12 semidiameters of the Earth. So if we followed this ancient opinion of Pythagoras, the Sun and the fixed stars would be subject to a parallax of several degrees, against the evidence of observations. With these rudiments set out first, however, let us pass on to his other opinion.

[Margin: Pythagoras's later opinion.]

II. Having finished chapter 21, then, Pliny, in book 2, chapter 22, goes on to say: "But Pythagoras at times, on a musical principle, calls the distance of the Moon from the Earth a tone; from the Moon to Mercury, half of that space; and from Mercury to Venus about as much; from Venus to the Sun one and a half; from the Sun to Mars a tone, that is, as much as to the Moon from the Earth; from Mars to Jupiter a half, and from Jupiter to Saturn a half, and from there one and a half to the zodiac [Signiferum]: so that seven tones are made, which they call the diapason harmony, that is, the universality of concord. In it Saturn is moved in the Dorian, Mercury in the phthongus, Jupiter in the Phrygian, and similarly in the rest" — a subtlety more pleasing than necessary.

Censorinus, in the book *De die natali*, chapter 11, sets out this view more explicitly indeed, but in the distance of Saturn from the fixed stars he differs from Pliny unless the text is corrected. For he says: "Pythagoras taught that this whole world was made on a musical principle, and that the seven stars wandering between heaven and Earth, which govern the births of mortals, have a well-proportioned motion [Greek: ἔυρυθμον (eurythmon, "well-rhythmed")] and intervals suited to the musical diastemata, and give out various sounds, each according to its altitude, so concordant that they make indeed a most sweet melody, but one inaudible to us because of the greatness of the sound, which

the narrowness of our ears could not take in." And a little after: "Pythagoras indicated how many stadia there were between the Earth and each of the stars. By the stadium in this measurement of the world is to be understood chiefly the one they call Italian, of 625 feet" — that is, of 125 paces, which make an eighth part of an Italian mile. He then goes on to say: "Thus he thought that from the Earth to the Moon is about 126 thousand stadia, and that this is the interval of a tone; from the Moon to the star of Mercury, which is called Stilbon [Greek: στίλβων (stilbōn, "the Gleamer")], half of that, as it were a hēmitonion [Greek: ἡμιτόνιον (hēmitonion, "half-tone")]; from there to Phosphoros [Greek: φόσφορον (phosphoron, "the Light-bearer")], which is the star of Venus, about as much, that is, another hēmitonion; from there further to the Sun, three times as much, as it were a tone and a half; and so the star of the Sun is three tones and a half distant from the Earth, which is called the diapente, and from the Moon two and a half, which is called the diatessaron; and from the Sun to the star of Mars, whose name is Pyroeis [Greek: πυρόεις (pyroeis, "the Fiery")], there is as much interval as from the Earth to the Moon, and this makes a tone [Greek: τόνον (tonon, "a tone")]; from there to the star of Jupiter, which is called Phaethon [Greek: φαέθων (phaethōn, "the Radiant")], half of that, which makes a hēmitonion; as much from Jupiter to the star of Saturn, whose name is Phainon [Greek: φαίνων (phainōn, "the Shiner")], that is, another hēmitonion; from there to the highest heaven, where the signs are, likewise a hēmitonion [?]. And so from the highest heaven to the Sun the diastema is a diatessaron, that is, of two tones and a half; and to the summit of the Earth from the same heaven there are six tones, in which is the symphony of the diapason [Greek: διὰ πασῶν (dia pasōn, "through all")]."

Zarlino, too, described the numbers and proportions of Censorinus in part 1 of the *Harmonic Institutions*, chapter 6, putting six tones from the Earth to the highest heaven — not on his own opinion, but on that of Pythagoras. For he himself denies that there is a tone from the Earth to the Moon, since the Earth, being immobile, is unfit to render any sound, and so to sound in concert with the Moon.

[Margin: Pliny to be corrected.]

Georgius Valla, in book 1 of his *Music*, chapter 2, refers that "sesquiplum" in Pliny not to a tone but to a semitone, so that from Saturn to the fixed stars there are three quarters of a tone. Glareanus, however, in book 2 of the *Dodecachordon*, chapter 13, affirms that all the old codices of Pliny have not *seven* but *six* tones, and that accordingly "semitonium" should be read for "sesquiplum"; he denies, moreover, that the diapason can arise from seven tones on any opinion of the musicians, and much less on that of Aristoxenus. Certainly, if we consult Guido's scale, taken from Boethius but enlarged and corrected, which we set out in chapter 6, number 3, we shall see that from the first string to the eighth, which make the consonance of the diapason, there are indeed seven intervals, but only 6 tones, namely 5 whole ones and two semitones [hemitonia]. Finally, Censorinus, a most diligent author, cannot be reconciled with Pliny in any

other way. Since he affirms that the Pythagorean stadia are equal to the Italian, and so Pythagoras puts no more than 126,000 stadia from the Moon to the Earth, that is, 15,750 Italian miles, which are about 4 semidiameters of the Earth, we shall easily construct the following small table, with parallaxes agreeing with the Pythagorean intervals.

PYTHAGORICA INTERVALLA

The Pythagorean intervals, from Pliny corrected, and from Censorinus

Body	Distances of the planets from one another: interval	Distances of the planets from one another: semid. of the Earth	And from the Earth: semid. of the Earth	Horizontal parallax
The Earth				
	Tone	4		
☾	—	—	4	14° 28′
	Semitone	2		
☿	—	—	6	
	Semitone	2		
♀	—	—	8	
	Tone and a half	6		
☉	—	—	14	4° 6′
	Tone	4		
♂	—	—	18	
	Semitone	2		
♃	—	—	20	
	Semitone	2		
♄	—	—	22	
	Semitone	2		
✳	—	—	24	2° 23′

[Margin: 2nd opinion, of the Platonists.]

III. The second opinion was that of the Platonists rather than of Plato himself. They rejected the distances of the planets determined by Archimedes for no other reason than that these did not preserve the proportions due to music, as Macrobius relates in book 2 on the *Dream of Scipio*, chapter 3, where he has this: "Rightly, then, everything that lives is captivated by music, since the celestial soul by which the universe is animated took its origin from music. While it drives the body of the world to its spherical [?] motion, it produces a sound which is separated by unequal intervals, yet intervals distinguished by reason in due proportion, just as it was woven together from the beginning. But these intervals, which in the soul, being incorporeal, are judged by reason alone, not by sense — it must be asked whether this balanced measurement has also been preserved in the very body of the world. And Archimedes indeed believed that he had found out the number of stadia by which the Moon was distant from the surface of the Earth; Mercury from the Moon; Venus from Mercury; the Sun from Venus; Mars from the Sun; Jupiter from Mars; Saturn from Jupiter. And he thought he had also measured by reason the whole space from the

orb of Saturn up to the starry heaven itself. This measurement of Archimedes, however, was rejected by the Platonists, as not preserving the double and triple intervals. And they laid down that this is to be believed: that as great as the distance is from the Earth to the Moon, twice that is from the Earth to the Sun; and as great as it is from the Earth to the Sun, three times that is from the Earth to Venus; and as great as from the Earth to Venus, four times that is from the Earth to the star of Mercury; and as great as it is to Mercury from the Earth, nine times that is from the Earth to Mars; and as great as from the Earth to Mars, eight times that is from the Earth to Jupiter; and as great as it is from the Earth to Jupiter, twenty-seven times that is from the Earth to the orb of Saturn. Porphyry inserted this Platonic conviction into the books in which he shed some light on the obscurities of the *Timaeus*; and he says that they believe these intervals to be in the body of the world after the image of the weaving of the soul — intervals which are filled out with epitrites, hemiolii and epogdoi, with semitones and the limma — and that so the harmony comes about."

Marsilius Ficinus [Marsilio Ficino] also reports these proportions, in the *Compendium* on Plato's *Timaeus*, chapter 34, and asserts that they seem to him the more probable, and in keeping with Plato's mind in the *Timaeus* and in books 8 and 10 of the *Republic*. To weigh these intervals, then, we shall take the lunar distance from the Earth as very nearly 60 terrestrial semidiameters, according to what was said in book 4, chapter 4. With this posited, the Platonic intervals will be as can be seen in the following small table.

INTERVALLA PLATONICA.

The Platonic intervals

Body	Ratio to the distance from the Earth immediately preceding	Distance from the Earth, in units of which the lunar distance is one	Distances from the Earth, in units of which the lunar distance is 60
The Earth			
☾	1	1	60
☉	Double the lunar	2	120
♀	Triple the solar	6	360
☿	Quadruple the Venerean	24	1440
♂	Ninefold the Mercurial	216	12960
♃	Eightfold the Martial	1728	103680
♄	Twenty-sevenfold the Jovial	46656	2799,360

[Margin: The absurdity of the Platonic intervals.]

The distance of the Sun from the Earth, then, would be only 120 terrestrial semidiameters, which conflicts with what we demonstrated in book 3, chapter 7; and the distance of Saturn from the Earth would be about 2,799,360 semidiameters of the Earth, that is, in the ratio of 23,328 to 1 to the solar distance, whereas it ought to be only as 10 to 1, or about tenfold, from what was said in book 7, section 6, chapters 1 and 2. So farewell to this Pythagorean and

Platonic harmony, which does not reconcile reason with the experience of the senses and with observations, although it ought to have been reconciled, as Ptolemy rightly decides in book 1 of the *Harmonics*, chapters 1 and 2. For it is not enough that these proportions agree with reason even if they cannot be approved by sense, as Zarlino seems to have admitted in part 1, chapter 6.

[Margin: 3rd opinion, Bettini's.]

IV. The third opinion is that of our Father Mario Bettini, to be read in the *Apiaria*, apiary 10, progymnasma 1, propositions 1 and 3, with their scholia. He supposes, first, that the Sun is distant from the centre of the Earth by 1145 terrestrial semidiameters and is in the middle of the planetary system; and that its distance, together with the remaining distance to the Empyrean, makes a diapente, so that the Sun's distance from the Earth is to be tripled to obtain the semidiameter of the whole world, as an entire monochord: namely 3435 terrestrial semidiameters. From this it follows that, ascending from the Earth to the Sun, the Sun occupies the place of the fifth string, and the note befits it that is called *Sol* in Guido's musical scale, matching the name of the Sun.

Next he determines the distances of the planets, and indeed of the heavens above the planets, according to the consonances of the more recent musicians admitted in the common use of the octochord — but of an octochord purely Dorian and having a diatonic depth. Further, to serve the ease of the arithmetical operation and the order of the planets found by the more recent astronomers, and at the same time so that the harmonic consonances should depart as little as possible from the observations, he takes some planets at their mean distance, some at apogee, some at perigee. With these things supposed, he determines the intervals of the planets and of the heavens from the Earth in semidiameters of the Earth, as you see in the following table.

In it, as I said, the distance of the Empyrean from the centre of the Earth, or the whole string, is 3435 terrestrial semidiameters. The distance of the Moon at apogee from the Earth, so that it may be the major tone according to the ancients, which is the ninth part of the whole string, is 381 6/9 terrestrial semidiameters. The distance of Venus at perigee from the Earth makes the consonance of the ditone, namely the fifth part of the whole string, which is 687 terrestrial semidiameters; so it comes about that between the Moon at apogee and Venus at perigee he puts the minor tone. The distance of Mercury at perigee from the Earth makes the consonance of the diatessaron, which is the fourth part of the whole string, or 858 3/4 terrestrial semidiameters; so it comes about that between Mercury and Venus, both at perigee, he puts the major semitone, or apotome. The mean distance of the Sun from the Earth, moreover, is, as has been said, 1145 semidiameters of the Earth, and with the remainder of the whole string, which is 2290, it makes a diapente, and with the whole string a diapason-diapente; hence it comes about that between Mercury and the Sun there is a major tone, as he himself thinks.

The mean distance of Mars from the Earth occupies the end of the major hexachord, that is, two fifths of the whole string, and so 1374 terrestrial

semidiameters. To hunt down the distance of Jupiter, he subtracts the major hexachord, which is 1374, from the whole string of 3435, and 2061 remain; of this he takes the ninth part as a major tone, namely 229, and adds it to the distance of Mars, and the distance of Jupiter from the Earth becomes 1603 terrestrial semidiameters. The distance of Saturn he makes half of the whole semidiameter of the world, or of the whole string, that is, 1717½ terrestrial semidiameters, so that it acquires the consonance of the diapason with the whole string. The distance of the fixed stars he makes two thirds of the whole string, so that they too sound a diapente. The distance of the crystalline heaven he determines by the consonance of the diatessaron, but in such a way that from the Earth to the crystalline there are three quarters of the whole string, that is, 2574 3/4 semidiameters of the Earth. Hence it comes about that the distance between the Empyrean and the crystalline is to the whole string as 1 to 4, which is the disdiapason. From these figures we have put together the following table, taken from his express words, but with many consonances added that arise from his intervals, which he appends.

HARMONICA INTERVALLA EX P. MARIO BETTINO

Harmonic intervals according to Father Mario Bettini, shown on the line AB as the entire string of the monochord of the world

	Bodies	Distance from the Earth, in semidiameters of the Earth	Proportions with consonances or concinnous intervals	Proportions with consonances or concinnous intervals (continued)
A	The Earth			
D	☾ at apogee	381 6/9	AB to BD, as 9 to 8: a tone	
E	♀ at perigee	687	AB to BE, as 5 to 4: a ditone	AB to AE, as 5 to 1: a disdiapason with a ditone

RESIDVVM TABVLÆ PRÆCEDENTIS.

The remainder of the preceding table

	Bodies	Distance from the Earth, in semidiameters of the Earth	Proportions with consonances or concinnous intervals	Proportions with consonances or concinnous intervals (continued)
F	☿ at perigee	858 3/4	AB to BF, as 4 to 3: a diatessaron	AB to AF, as 4 to 1: a disdiapason
C	☉ at mean distance	1145	AB to BC, as 3 to 2: a diapente	AB to AC, as 3 to 1: a diapason-diapente
H	♂ at mean distance	1374	AB to BH, as 5 to 3: the major hexachord	AB to AH, as 5 to 2: a diapason with a ditone
I	♃ at perigee	1603	AB to BI, as 3435 to 1832	
G	♄ at perigee	1717 1/1 [recte: 1717 ½]	AB to BG, as 2 to 1: a diapason	AB to AG, as 2 to 1: a diapason

	Bodies	Distance from the Earth, in semidiameters of the Earth	Proportions with consonances or concinnous intervals	Proportions with consonances or concinnous intervals (continued)
K	✳ The fixed stars	2290	AB to BK, as 3 to 1: a diapason with a diapente	AB to AK, as 3 to 2: a diapente
L	The crys-talline	2574 3/4	AB to BL, as 4 to 1: a disdiapason	AB to AL, as 4 to 3: a diatessaron
			AF to AG, as 1 to 2: a diapason	
			AK to AC, as 2 to 1: a diapason	
			AH to AF, as 2 to 1: a diapason	
B	The Empyrean	3435		

[Margin: Father Bettini's 1st error.]

V. Although the symmetry just described is far more concinnous than either the Pythagorean or the Platonic, it nevertheless contains many inconsistencies and fallacies, and introduces a very great and altogether dissonant discord between astronomy and harmonics. *First*, he assigns the Moon so great a distance as no astronomer has ever assigned or can assign while the parallaxes are preserved. For no one has ever found, or will detect, in the Moon a horizontal parallax greater than 1° 43′ or less than 51 1/5′; the former of these implies a distance from the Earth of 33 1/2 terrestrial semidiameters, the latter about 67, as is certain from geometry and from what was said in book 4, chapter 14. And Father Bettini himself approves this way of determining the distances of the stars by parallaxes, in the *Apiaria*, apiary 8, progymnasma 3, proposition 9, and progymnasma 4, and in volume 2 of the *Aerarium Philosoph.*, page 73.

[Margin: 2nd error.]

Second, from the distance of the Moon of 381 6/9 terrestrial semidiameters that he lays down, it would follow that the Moon could never be eclipsed by the shadow of the Earth. This is contrary to the experience of every age, and contrary to all who are even lightly versed in astronomy; indeed it is contrary to what the same Father teaches, very learnedly, on lunar eclipses caused by the Earth's shadow, in the *Apiaria*, apiary 8, progymnasma 2 and progymnasma 3, proposition 11. For no one has ever carried the height of the Earth's shadow beyond 282 semidiameters of the Earth, as is clear from what was said in book 3, chapter 11, problem 8. Nor may you say that this removes only the eclipse of the Moon at apogee, and not below the apogee. For when the 282 semidiameters, which at the most belong to the shadow, are taken from the lunar distance of 381, there remain 99 terrestrial semidiameters — and no one has laid down so great a difference between the Moon's perigee and apogee. And yet we know that the Moon, even at apogee, has at times run into a total eclipse, and so has been far below the apex of the Earth's shadow.

[Margin: 3rd error.]

Third, not even a total eclipse of the Sun could occur if the Moon were 381 and the Sun 1145 semidiameters of the Earth distant from the Earth, as he lays down; for while the apparent diameters of the luminaries are preserved, such an eclipse would be impossible, as is clear from book 5, chapter 9. And yet Father Bettini himself admits a total eclipse of the Sun, in the *Apiaria*, apiary 8, progymnasma 2, proposition 10, and he is bound to admit it from the history of eclipses, which we treat in book 5, chapter 20.

[Margin: 4th error.]

Fourth, the Moon's distance of 381 semidiameters of the Earth cannot hold together with the distance of the Sun of 1145 that he posits, as is proved by Aristarchus's problem founded on the dichotomy phase of the Moon, which Father Bettini himself praises in volume 1 of the *Aerarium*, page 629. For if the Moon's distance is set at 381 6/9, or nearly 382, it follows that the Sun is distant from the Earth by at least 7299 terrestrial semidiameters, and in reality by much more, if the angle observed by us at the time of dichotomy is taken, as is clear from what was demonstrated in book 3, chapter 7, problem 3.

[Margin: 5th error.]

Fifth, he puts Jupiter and Saturn much nearer to the Sun and the Earth than the prosthaphaereses and the commensurations of the orbs demonstrated by Copernicus and others require. For these require that the distance of Jupiter be very nearly fivefold the distance of the Sun from the Earth, and that of Saturn about tenfold the Sun's distance from the Earth, as we showed in book 7, section 1, chapters 1 and 2. Yet according to Father Bettini the distance of Jupiter is to the distance of the Sun less than as 14 to 11, and that of Saturn less than as 16 to 11.

[Margin: 6th error.]

Sixth, he assigns to the fixed stars a distance from the Earth of 3435, so small that no astronomer could ever ascribe it, since from it there would follow a horizontal parallax of one whole minute and a half, which all astronomers know to be false. All these errors arose from his wanting the Sun's distance both to stand in the fifth place and to give out the note *Sol*, and to make a diapente from the highest heaven with the whole semidiameter of the world; and wanting the Moon to make a tone, that is, to contain in its distance one ninth of that whole interval, and so on.

[Margin: Other incongruities.]

I pass over the fact that he assigns no worthy consonance to Jupiter, certainly not the least of the planets; and that no definite consonance answers to 4 planets outside their perigee position, to the Moon outside apogee, or to the Sun and Mars outside their mean distance — as though in fact for the most part they did not keep the harmony. This is hardly consistent in one who has taken upon himself to demonstrate harmonic proportions kept by God in the intervals of the planets. Finally, so that the Sun may occupy the place of the fifth string, he

has taken the Earth as the first string; yet the Earth, like the Empyrean, is unfit to represent a sound because of its immobility.

[Margin: 4th opinion, Kepler's.]

VI. The *fourth* opinion is that of *Johannes Kepler*, in book 5 of the *Harmonics* [Kepler's *Harmonice Mundi*], chapter 4. There he sets out the extreme intervals from Tycho's observations, that is, the distances of the planets from the Sun when they are at aphelion, that is, most distant from the Sun, and when they are at perihelion. And if the two extreme intervals of the same planet are considered, he admits that harmony is found in none of the planets except Mars and Mercury, as is clear from the table he gives, as here below. To understand it one must suppose, from chapter 3 of the same book, that the *extreme convergences* of two planets are when their apsides are nearest, namely at the perihelion of the upper and the aphelion of the lower planet; and that the *extreme divergences* are the opposite apsides of two planets, namely the aphelion of the upper and the perihelion of the lower. Now here is the table.

INTERVALLA COMPARATA CVM HARMONICIS.

The intervals compared with the harmonic ones

Planet	Apsis	Distance, in units of which the radius of the annual orb is 1000	Letter	Proportions of each
Saturn	Aphelion	10052	a	More than a minor tone: 10000/9000
	Perihelion	8968	b	Less than a major tone: 10000/8935
Jupiter	Aphelion	5451	c	No concinnous proportion, but
	Perihelion	4949	d	about as 11 to 10, or half of 6/5
Mars	Aphelion	1665	e	If it were 1665/1388, it would be the harmonic 6/5
	Perihelion	1382	f	If it were 1665/1332, it would be the harmonic 5/4

RESIDVVM TABVLÆ PRÆCEDENTIS.

The remainder of the preceding table

Planet	Apsis	Distance, in units of which the radius of the annual orb is 1000	Letter	Proportions of each
The Earth	Aphelion	1018	g	If it were 1020/980, it would be the diesis 25/24;
	Perihelion	982	h	so it does not possess the diesis
Venus	Aphelion	729	i	Less than a sesquicomma
	Perihelion	719	k	More than a third part of the diesis
Mercury	Aphelion	470	l	More than an overflowing diapente,

Planet	Apsis	Distance, in units of which the radius of the annual orb is 1000	Letter	Proportions of each
	Perihelion	307	m	243/160; less than the harmonic 8/5

[Margin: Use of the table.]

So the extreme intervals of no single planet allude to harmonies, except those of Mars and Mercury. For in Mars, if its perihelion of 1382 were 1388, it would be to its aphelion of 1665 as 5 to 6, and so it would be the consonance of the semiditone, or minor third; yet it is not far from it. In Mercury, however, although to Kepler they seem to allude to the diapente, or to the harmony 8/5, which is the minor hexachord, to me its extreme intervals seem rather to allude to the diatessaron, which is between 4 and 3; or else this interval is to be disregarded.

"But if you compare the extreme intervals of different planets with one another," says Kepler, "some light of harmony shines out," as is clear to anyone who studies the first column of the preceding table and the letters marking its extremes. a to d, that is, the aphelion of Saturn to the perihelion of Jupiter, has the proportion of 10,052 to 4949, that is, nearly 2 to 1; so their extreme divergences make a little more than a diapason. But their convergences, that is, b to c, or the perihelion of Saturn to the aphelion of Jupiter, have the proportion of 8968 to 5451, which is not indeed as 5 to 3, but midway between this and 8 to 5, that is, between the major sixth and the minor sixth. Thus the extreme divergences of Jupiter and Mars, c f, embrace nearly a disdiapason, or the proportion 4 to 1; and their convergences, d, e, nearly a diapente with a diapason, or 3 to 1. Again, the extreme divergences of the Earth and Mars, e and h, have somewhat more than 5 to 3, which is the major sixth; and their convergences, f and g, have an overflowing diatessaron, since the diatessaron is as 4 to 3. Again, the extreme convergences of the Earth and Venus, g and k, have an overflowing diatessaron. Between Venus and Mercury, however, the divergence, i and m, is a little less than a diapason with a semiditone, that is, than 12 to 5; and the convergence, k and l, is a little more than a diapente, or than 3 to 2.

[Margin: Kepler's absolute judgement on the harmony of the intervals.]

Kepler, however, adds that these intervals, insofar as they are lengths without motion, are not fitly examined for harmonies, whose subject is rather motion itself, as to its speed and slowness. Therefore, if we seek harmonies, they are not to be sought in the intervals insofar as these are the semidiameters of the orbs, but insofar as they are the measure of the motions, that is, rather in the motions themselves — especially since for the semidiameters of the orbs only the mean intervals, or the mean distances from the Sun, can be taken, and among these harmony shines out less than in the extremes of the aphelia and perihelia. For this reason Kepler absolutely denies that harmonic proportions are to be sought among the intervals of the planets as such, and apart from motion.

Quinta & Nostra Opinio, & Authoritates pro illa.

The fifth opinion, which is ours, and the authorities for it.

[Margin: 5th and true opinion.]

VII. The fifth opinion, not unlike the preceding and truer, is that the distances of the planets and of the fixed stars, whether from the Earth, from the Sun, or from one another, are not to be determined from harmonic proportions or musical intervals. Beyond doubt this was the opinion of Aristotle [Margin: Aristotle's.], in book 2 of *De caelo*, text 52, where, speaking of the Pythagoreans who required harmony in the motion and intervals of the heavens, he said: "This is said prettily indeed and elegantly, yet the truth is not so." It was the opinion, too, of Pliny the Elder [Margin: Pliny's.], book 2, chapter 22, where, after reporting Pythagoras's opinion on the intervals of the stars according to tones, semitones and tones-and-a-half, he concludes that these and similar things said by him are "a subtlety more pleasant than necessary."

[Margin: Macrobius's.]

Macrobius too was of the same opinion, in book 2 on the *Dream of Scipio*, chapter 4, judging this inquiry to belong "to one who makes a show, not to one who teaches." We drive home their judgements as often as the opportunity comes back to us. [Margin: Zarlino's. Glareanus's.] In this, moreover, Zarlino subscribed to Aristotle and Pliny, in part 2 of the *Harmonic Institutions*, chapter 29, and so did Glareanus, in book 1 of the *Dodecachordon*, chapter 5, and book 2, chapter 13, where he says: "But that the intervals of the celestial orbs in the heaven itself stand in the same ratio as the notes in the diapason, in whatever kind of modulation we may finally set them, does not seem likely to me." And after some intervening matter he adds: "Boethius, a true judge of this business, when he saw these things so wonderfully varied among the ancients, and Pliny not afraid to call them a subtlety more pleasant than necessary, so tempered these matters as nevertheless to set both opinions before the reader's eyes." Finally, declaring his own opinion, he says: "But to confess at last what we hold for certain, we say in general: not without reason did this fiction seem to Aristotle more pleasant to tell than likely to be true. For if anyone should wish to apply these things, so to speak, he will not, by Hercules, find the intervals of the planets agreeing with the musical spaces, nor will he detect that any definite ratio of sound is present, either in the subject or in the efficient cause, as the physicists hold. But this indulgence is granted to antiquity, which thought that human minds were to be raised by any means whatever to the contemplation of heavenly things."

[Margin: Mersennius.]

Nor is there any doubt that Mersennius, on chapter 4 of Genesis, verse 21, page 1558, was of the same opinion, since he says there: "I know there is no need for the distance of the planets, both from one another and from the Earth, to be as exact as it is in their motions and sizes, in order that music may represent their economy; and yet I wish it could be done exactly, so that through voices

and instruments we might have some commerce with the heavenly bodies." And on page 1559 he sets little value on those very allusions to consonances that we reported from Kepler in the preceding number, because the opinions of astronomers on the distances of the stars, among which a musician might choose, are diverse. Again, on page 1703, he reported the opinion of certain astrologers — whose, I do not know — who say that the diapason and the disdiapason are ruled by the Sun, the diapente by Venus, the diatessaron by Mercury, and the diapason-diapente by Jupiter; and on the following page he says that he rejects all of this, and adds: "In vain, therefore, will the harmonists labour if they seek a tone from the Earth to ☾; a semitone from the Moon to ☿, and as much from it to ♀; a diapente from the Earth to the Sun; a diatessaron from the Moon to the Sun; a tone from the Sun to ♂, from there a semitone to Jupiter, as much from ♃ to ♄, and from ♄ to the firmament; and consequently a diatessaron from the Sun to the firmament, and a diapason from the Earth to the firmament. For the experience of geometrical astronomy teaches well enough that those spaces are wrongly set up, as is plain from article 3 and from what is said elsewhere on the distance of the heavens from one another. And the same is to be concluded about the proportion of the motions."

[Margin: Father Athanasius's praise of music, and his opinion.]

Most recently our Father Athanasius Kircher — who has deserved best of all in exalting music to the heavens, and as reigning in the whole of nature, especially in book 10 of the *Musurgia*, which he entitles *Decachordon Naturae*, register 2, paragraph 2, or page 381 — inquiring what the music of the celestial bodies is and in what it consists, concludes that their harmonic concent consists "not in the periodic numbers of the motions, not in that perceptible collision of the celestial bodies, but in nothing else than their admirable disposition and a certain ineffable proportion conspiring into union, by which these bodies of the world so correspond to one another that, if one were removed or changed, the harmony of the whole would rightly perish, etc. This harmony, as we have said, consists both in a certain admirable disposition and a most well-proportioned spacing of one bo- dy of the world from another; and in the most exact analogy of the quantity or size of each, fitted to the attaining of its end." The end intended by God, however, was not the delight, perceptible to the hearing, of a sound excited by the heavens, but the production, preservation and advancement to its final end of sublunary nature, especially of living things, each in its own way — which Kircher goes on to expound there excellently and at length — and therefore so great a distance was assigned to the planets that they might produce effects suited to sublunary natures. For if the Moon or the Sun were much nearer than they are, the former would moisten and chill too much, the latter would dry and heat too much. And had Jupiter, wholesome and most temperate, with its four companions, not been set between Saturn and Mars, dissonant, pestilent and malign planets, or the beneficent Sun and Venus on this side of them, great and intolerable harms would have followed.

Pliny the Elder had noted this before Kircher, in book 2, chapter 8. Having said that the star of Saturn is "of a cold and stiffening nature", and a little after that "the third star, that of Mars, which some call the star of Hercules, fiery and burning from the nearness of the Sun, revolves in about two years", he at once adds: "And so Jupiter, set between the two, is tempered by each of them, by the excessive heat of the one and the cold of Saturn, and is made wholesome." And Cicero's famous Paulus hints at this very thing in the *Dream of Scipio*, when he says: "of which one globe is held by the star that on earth they call Saturn's. Next comes that brightness, favourable and wholesome to the human race, which is called Jupiter's; then the ruddy one, dreadful to the lands, which you call Mars's." What we stammer about these things, as a sample to point out divine Providence, is to be understood also — if indeed it can be understood by us — of innumerable other ratios and proportions chosen by God as ordered to an end.

Rationes pro Quinta & Nostra Opinione.

Reasons for the fifth opinion, which is ours.

[Margin: 1st reason.]

VIII. *First*: the intervals and motions of the celestial bodies either produce no sound at all — especially in the planetary system, where the planets travel in a fluid and most tenuous aether — or at any rate they produce no sound perceptible to us, as experiment shows. And so Pliny the Elder said truly, in book 2, chapter 3: "To us who live within it, the world glides by in silence, alike by day and by night." Therefore they are not ordered by God to that end, and consequently those harmonic proportions that sound in concord to our ears in human voices, so as to produce a concinnity pleasant to the ears, are not to be sought in them. For when the end is removed, the order and proportion owed to it is removed; and much more so in the intervals themselves taken apart from motion, since, as Kepler rightly said, the proper subject of harmony is not immobile quantity but mobile quantity, that is, the very motion of the bodies.

[Margin: 2nd reason.]

Second: even if the celestial bodies gave out by their motion a sound perceptible to us, the consonances of those sounds would still not stand within the bounds and limits within which the consonances of human voices stand — which, because of either our weakness or the necessity of nature, we confine within the bounds of the Boethian system or of Guido's scale. For since the lungs, throat, palate, tongue, teeth and lips are not ordered to this end alone, that we may modulate sweetly in forming the voice, but to many other ends as well, these organs had to be so formed that they could descend in depth, or rise in height, only to a definite limit. But among other animals there are some that can form a deeper or a higher voice than we can; and in strings and in organ pipes many more and subtler differences can be found. How much

more in the motions of the stars, intermingled with so great a variety! In vain, therefore, would we hold the laws of that concent to the far too curtailed and slender standard of our harmony; it would be just as if we wished to confine to our measures the songs of angels in the innumerable bodies which, God permitting, they could assume.

[Margin: 3rd reason.]

Third: if harmonic proportion is to be sought in the celestial intervals or motions, it is chiefly because some symmetry can be found in them, or even a proportion double, triple, quadruple, quintuple, etc., if not most exactly, then at least with fractions that spoil that proportion little. But granting (though not yet conceding) the proportions just mentioned, harmony still does not follow from them as such, but only as they are in the qualities of voices and sounds. For Panaetius rightly said, in his book on the ratios of geometry and music: "The faculty of consonances is observed not in the magnitudes of voices but in their qualities." Otherwise, wherever quantities are found, discrete or continuous but reduced to discrete by division or marking off, in which there are, besides unity, the numbers 2, 3, 4, 5, 6, 8, 12 — by which all 14 consonances listed in the first table of chapter 4 are contained — harmony would reign there, taken properly or quasi-properly; and so very many arts would be confounded with one another.

Nor indeed must every symmetry be weighed against the canon of the musical monochord, or against the standard of harmonic ratios. For who would demand this of an architect in buildings, of an apothecary in compounding drugs, of a commander-in-chief in drawing up a battle line, of a shipbuilder in putting ships together, or of God himself and of Nature in the structure of the human body, or of animals or plants? It would surely be ridiculous to require that the nerves, the intestines and the tendons should have in the body of animals the measure that harmonic proportions require; or that the five principal vessels or instruments of the soul — namely the brain, the heart, the liver, the kidneys and the spleen — should be so commensurate with one another as to quantity or motion that, for example, the brain, in itself or with another member, should make a diapason, the heart a diapente, the liver a diatessaron, the kidneys a diapason-diapente, the spleen a disdiapason. For a proportion of this kind would be unsuited to the end to which they are ordered, which is far different from the delight of the ears. The same judgement, then, applies to those who pursue in the celestial intervals and motions the symmetry proper to music, that is, symphonism. Nor is there any doubt that many proportions unsuited to producing consonances, such as those that arise from the numbers 7, 9, 14 compared with others, are nevertheless most suited to other effects intended by God and by Nature in the heaven and in other bodies.

[Margin: 4th reason.]

Fourth: either harmonic proportions are to be sought in any intervals of the planets whatever — and not this, for most of them do not have them, and so for the most part the work of the divine Harmost would lack the harmony that the

followers of the Pythagoreans commend so highly — or in the more notable intervals, for instance in the greatest, mean and least distance of the stars from one another, or from the Sun, or from the Earth, or from all of these. Neither can this be maintained. First, the diversity of opinions about these intervals of the stars among the more recent astronomers stands in the way. Second, not even in the opinion of a single astronomer are all the greatest, mean or least intervals of the planets so tempered together that the commonly accepted musical consonances are found in them. Finally, the apogees of the planets have never all concurred with apogees, nor perigees with perigees, in one place under the fixed stars; or if this has ever happened or will happen, it does not suffice for a harmony worthy of God, for it would have to happen always or for the most part. In the same way it does not suffice for the excellence of some arch-musician if the voices and tones are so ordered by him that they are consonant twice or three times and dissonant a thousand times. And I say nothing for the present of the variety of the genera and modes of music indicated in chapter 5, which is so great that it is not certain which genus suits the heavens.

[Margin: 5th reason.]

Fifth, finally: no one is so senseless, or of so stubborn and brazen a brow, as not to grant to Ptolemy, in book 1 of the *Harmonics*, chapters 1 and 2, that sense and reason must be the judges of harmony — but in such a way that sense, a posteriori, first finds what is near the truth, while reason, considering the causes, finds and determines a priori what is exact; or, if reason has first found what is exact, that must still be received by sense and approved as concinnous. So we shall not, with the Pythagoreans, attribute too much to reason, nor, with the Aristoxenians, too much to the senses. And that judgement of his from book 1, chapter 2, must be driven home once more, where he says: "It is the harmonist's purpose everywhere to preserve the rational positions of the canon, in no way ever conflicting with the senses, according to the opinion of the majority; just as it is the astronomer's to preserve the positions of the celestial motions, consonant with the observed revolutions, and these too taken, indeed, from things evidently and universally more apparent." But if we should wish the intervals of the planets, from harmony's rules to determine, they will often conflict with observations made evidently by the senses; nor will the commensurations of the orbs, or the prosthaphaereses resting on them, represent such places of the planets as are detected through astronomical instruments and accurate observations. And they will conflict so far as to be a long way from the closeness that this business would require, for reason to have a foundation for the view that God willed to attain other ends through the celestial intervals and motions in such a way that He nevertheless also set before Himself this end too — namely, the beauty of harmonic ratios, to be wrought by Himself and to be contemplated by the intellectual creature. [Margin: Objections are dissolved.] Since, then, the observations repeatedly protest against the harmonic laws and are dissonant with them, it must be said either that those ends could

not be attained together through these means, or vertè [?] that God did not will it. Accordingly, the assertions of Scripture, of the Fathers and of the wise concerning the harmony and concent of the heavens are not to be taken in a proper or quasi-proper sense — as though nothing else of harmonic ratio were lacking to the heavens except a sound perceptible to us — but in a metaphorical sense, according to analogy and a certain accommodation. In this way too, in political and household government, that saying of Ecclesiasticus 32 is to be understood: "Have they made you ruler? Do not be lifted up, etc., and do not hinder music" — that is, the order and subordination of offices, or the peace and concord of citizens or of the members of a household.

CAPVT IX.

Chapter IX.

An Magnitudo & Densitas Corporum Cælestium ex Harmonicis Proportionibus Determinata fuerit.

Whether the size and density of the celestial bodies were determined from harmonic proportions.

I. This too we deny, for the reasons adduced in chapter 8 from number 8, if what we said of the intervals is applied to the sizes and densities of the planets — with the exception, however, of the argument a posteriori, taken from sensible experiment, concerning their density; for we have no such experiment by which we could convict of falsity those who would fashion the density and rarity of the planets from harmonic proportions. Nor indeed did Kepler, in book 4 of the *Epitome of Astronomy*, define those from harmonic ratios, but only from geometrical ones, as is clear from his pages 484 to 489. He said only that Saturn is twice as high as Jupiter, one and a half times heavier, and one and a half times rarer, and thus twice as high as it is heavier, and twice as ample as it is rarer, and so proportionally for the others; and that the Sun is the densest of all. The proportions of density that he devises we have already reported in book 7, section 1, chapter 1, number 7; and his various propositions bearing on this we reviewed in book 7, section 6, chapter 8, number 16, to which we refer the reader — on condition that he consults scholium 2 of the same chapter. The same author, moreover, in book 5 of the *Harmonice*, chapter 4, expressly denies that the proportions of the bodies of the planets are harmonic.

Father Antonius Maria de Reita [Anton Maria Schyrleus de Rheita], in his *Oculus Enoch et Eliae*, book 4, chapter 2, member 4, asserts that the Earth is to the Sun as a square root is to its square, that is, as 10 to 100, so that the Sun is a hundred times larger than the Earth, and things like this, on which see our book 7, section 6, chapter 8, scholium 3; and from this Father Athanasius Kircher, in book 10 of the *Musurgia*, page 379, says that, if these things were true, certain harmonic proportions could be gathered. That, I say, by no means agrees with the more exact observations of the diameters and distances obtained

by us and by others, as is clear from what was said in book 7, section 6, from chapter 10. And Kircher himself, on the same page and on page 381, holds suspect the observations of Rheita on which he built those proportions, and says: "But I very much doubt whether Rheita did not choose, for the quantity of the said apparent diameters assumed at pleasure, so beautiful and handsome a proportion, rather than actually observe it."

CAPVT X.

Chapter X.

An & Qua Ratione Motus Planetarum ex Proportionibus Harmonicis Determinati fuerint à DEO.

Whether, and in what way, the motions of the planets were determined by God from harmonic proportions.

I. Here, to be sure, Kepler triumphs: in book 6 of the *Epitome of Copernican Astronomy*, pages 477, 501 and 900; in the *Mysterium Cosmographicum*, chapters 14, 20, 21; but above all in book 5 of the *Harmonice*, in whose proem he says that he had begun to speculate about this business 22 years before, and had spent the best part of his life in astronomy to this end, namely to show that "the whole nature of harmony, however great it is, with all its parts explained in book 3, is found among the celestial motions". From Kepler, moreover, Petrus Herigonius [Pierre Hérigone] selected a very few things in volume 5 of his *Cursus Mathematicus*, from page 573; Mersennius, on chapter 4 of Genesis, pages 1558 and 1704; and Athanasius Kircher more, in book 10 of the *Musurgia*, from page 376. But before Kepler, Ptolemy, in book 3 of the *Harmonics*, from chapter 9, took up this very argument to demonstrate, and it is therefore fair to hear him first.

[Margin: The teaching of Ptolemy on the harmony of the celestial motions.]

II. Thus Ptolemy, in book 3 of the *Harmonics*, chapter 9. As regards the motion in longitude, he likens the risings and settings of the stars to the deepest voices, and their meridian ascents to the highest. This is, first, because just as those who practise the voice begin from the deepest notes and, when they have ascended to the highest, descend until they end in the deepest, and so move from silence through the highest voice to silence, so the stars, from rising through the meridian to setting, pass from privation of appearance to privation of appearance; and second, because the lower places, to which the limits of rising and setting are likened, give out the deepest sounds, while the upper ones, resembling the meridians, give out the highest. In chapter 10, as regards the motion in altitude, he says that in the motion toward the greatest distance from the Earth the diatonic genus reigns, because in it two intervals are not smaller than the remaining one; in the motion of least distance, the enharmonic, because in it both of the two intervals become smaller than the remaining one; and finally, in the motion of mean distance, the chromatic, because in

it the lichanos, or index string, cuts the tetrachord in the middle and stands midway between the diatonic and the enharmonic. In chapter 11 he compares the motion in latitude to the changes that take place in the tones without passing from one genus to another; and he compares the Mixed Lydian and the Hypodorian, as the extreme modes of the tones, with the greatest latitude to the north and the south, and the Dorian, as the middle one, with the planets when they lack either latitude, being in the ecliptic, or declination, being in the equator. In chapter 12 he likens the remaining order of the tetrachords to the remaining aspects to the Sun: namely, the tetrachords of the disjunct strings, that is, the diezeugmenon, to the intervals running from occultation to the appearances of the stars, and also to the oppositions with the Sun, such as acronychal risings and full moons; and the tetrachords synemmenon to the quadratures; and so on for the rest. In chapter 13, the whole circle having been divided into 360 parts, he assigns to the trine aspect 120 parts, to the quadrate 90, to the sextile 60, and to the opposition 180; and so, taking from this 180 parts for the proslambanomenos, he gives 20 to the highest of the mediae, 90 to the last of the diezeugmenon, and 60 to the last of the hyperbolaeon. In chapter 14 he investigates the consonances diapente, diatessaron and some others through the division of the circle into 360 parts, according as certain aspects have, to the whole circle or to a portion of the circle owed to another aspect, a sesquitertian, sesquialter, etc., proportion. Finally, in chapter 15, he says that the sound of Saturn and Mars, the maleficent planets, makes the consonance diatessaron with each of the beneficent ones: that is, the last of the excellentes of Saturn is consonant with the last of the disjunct of Jupiter, and the last of the conjunct of Mars with the mese, or middle, of Venus; and that the sound of Saturn belongs rather to the solar sect, that of Mar- s, to the lunar sect; and that therefore all the configurations of Saturn with Jupiter are beneficent, while of the configurations, or aspects, of Saturn with the Sun only the trines are beneficent, as being more consonant than the rest. So too, of the configurations of Mars with Venus and the Moon, only the triangular are beneficent; further, the configurations of Saturn with the Moon and Venus are bad, and all those of Mars with the Sun dangerous. Let it suffice for us to have pointed these things out, so that it may appear that Ptolemy in these matters partly plays, rather, in a poetic vein — as it seemed to Kepler too, in the appendix to the *Harmonice* — and partly hunts out a harmony in the heaven from the astrological faculty, or vanity.

III. Kepler, however, in book 4 of the *Epitome of Copernican Astronomy*, page 477, and in book 3 of the *Harmonice*, chapter 6, teaches that the least number suitable for determining all the parts of the monochord, so as to constitute the double diapason system — that is, of the soft and the hard song — is one of 720 parts. And since, from the ancient observations of Aristarchus and from more recent ones, the apparent diameter of the Sun at apogee subtends 30′ ("which", he says, "is the 720th part of the whole circle"), this diameter was determined at so great a quantity in order that the first body, that is, the Sun, the chorus-leader of celestial music, might divide the circle for the earth-dwellers

and the contemplating creature according to the harmonic laws, that is, into 720 parts. This number can be divided into very many aliquot parts, namely by two, three, four, five, six, eight, nine, ten, twelve, fifteen, sixteen, eighteen, twenty, twenty-four and forty-eight; that is, in order, into parts of 360, 240, 180, 124, 90, 80, 72, 60, 48, 45, 40, 36, 30, 15. The consonances he determines from the sides of the figures that can be inscribed in a circle and demonstrated, we have already taught in chapter 4, scholia 1 and 2. In book 4 of the *Harmonice*, chapters 5 and 6, and in book 6 of the *Epitome of Astronomy*, page 901, he teaches that sublunary nature is sensibly aroused and stimulated to greater activity as often as the planets are configured with one another harmonically, that is, as often as their aspects and radiations occur at such a distance from one another under the zodiac as the harmonic proportions require. Of these aspects we shall speak in the following chapter; for now we are dealing with the motions themselves. On these Kepler writes in book 6 of the *Epitome*, from page 90, and in the whole of book 5 of the *Harmonice*, especially from chapter 3, in which he affirms as a most certain matter that the proportion between the periodic times of any two planets is precisely the sesquialter of the proportion of their mean distances from the Sun, provided that the arithmetical mean between the two diameters of the elliptical orbit is a little less than the longer diameter.

[Margin: The proportion [?] between the periods [?] and the distances [?] of the planets [?].]

Thus if, for example, from the period of the Earth, which is one year, and from the period of Saturn, which is 30 years, you take the third part of the proportion, that is, the cube roots, and make the double of this proportion by squaring the roots, the most exact proportion of the intervals of Saturn and the Earth from the Sun will come out. For the cube root of 1 year is 1, and its square is 1; but the cube root of 30 years is a little more than 3, and the square number a little more than 9; therefore the mean distance of Saturn from the Sun is a little more than nine times the mean distance of the Earth from the Sun. This Keplerian proposition is accepted by Hérigone as well, in volume 5 of the *Cursus Mathematicus*, page 573; and Wendelen noted the same proportion between the periods and the intervals of the satellites of Jupiter from Jupiter, in a most learned letter that he wrote out to me. Kepler goes on, and in chapter 4 admits that in the periodic times of the planets compared with one another there are no harmonic proportions; and to this Mersennius, on chapter 4 of Genesis, page 1558, and Athanasius Kircher, in book 10 of the *Musurgia*, page 377, readily subscribe. The periodic motions of the planets about the Sun, gathered from all the times spent in all the degrees of the whole circuit, long, middling and short, are, according to Kepler in the same place, as below.

Period	Days	Sixtieths of a day	Hence mean daily motion ′	″	‴
Saturn	18769	12	2	0	27
Jupiter	4332	37	4	59	8
Mars	686	59	31	26	33

Period	Days	Sixtieths of a day	Hence mean daily motion ′	″	‴
Earth with ☾	365	15	59	8	11
Venus	224	42	96	7	39
Mercury	87	58	245	32	25

Again, in the same chapter 4, he admits that the daily journeys of any two planets, whether the slowest or the fastest are taken, have no harmonic proportion if their true journeys in the aethereal air are considered; but if their apparent motions are taken, as seen from the Sun itself, and how great a daily arc in the ecliptic the angle made at the Sun subtends, he says that the following consonances, or concinnous intervals, are found. He takes the daily motions from the Tychonic astronomy.

Harmonies of pairs: Diverg.		Con-verg.		Apparent daily motions		Mo-tion	Let-ter	Proper harmonies of each: between		Ra-tio	Har-mony
				♄	Aphe-lion	1′ 46″	a	between	1′ 48″	is 4/5	major
a		b			Per-ihe-lion	2′ 15″	b	and	2′ 15″		third
d	1/3	c	1/2	♃	Aphe-lion	4′ 30″	c	between	4′ 35″	is 5/6	minor
c		d			Per-ihe-lion	5′ 30″	d	and	5′ 30″		third
f	1/8	e	1/5	♂	Aphe-lion	26′ 14″	e	between	25′ 21″	is 2/3	diapente
e		f			Per-ihe-lion	38′ 1″	f	and	38′ 1″		
h	5/12	g	2/3	Earth	Aphe-lion	57′ 3″	g	between	57′ 28″	is 15/16	semi-
g		h			Per-ihe-lion	61′ 18″	h	and	61′ 18″		tone
k	3/5	n	5/8	♀	Aphe-lion	94′ 50″	n	between	94′ 50″	is 24/25	diesis
n		k			Per-ihe-lion	97′ 37″	k	and	98′ 47″		
m	1/4	l	3/5	☿	Aphe-lion	164′ 0″	l	between	164′ 0″	is 5/12	diapason
					Per-ihe-lion	384′ 0″	m	and	394′ 0″		with the minor third

[Margin: Explanation of the preceding table.]

In the preceding little table, then, the first column indicates the harmonic proportions between the daily motions of two planets about the Sun, either

diverging — that is, comparing the aphelion of the higher planet with the perihelion of the lower — or converging — that is, comparing the perihelion of the higher with the aphelion of the lower. To signify this, letters of the alphabet have been added which, when looked up in the second column, signify the motion of a planet at aphelion or at perihelion. For example, in the first column, under the title "diverging", you see a and d, and set against them this fraction, 1/3. Now a in the second column signifies Saturn's aphelion motion, and d Jupiter's perihelion motion; therefore between Saturn's daily aphelion motion, which is 1′ 46″, and Jupiter's perihelion motion, which is 5′ 30″, the proportion is like that between 1 and 3. For when the motions are resolved into seconds, the daily motion of Saturn is 106″, and of Jupiter 330″, between which the proportion is as 1 to 3. But between b and c, that is, between the converging daily motions of Saturn's perihelion and Jupiter's aphelion, the proportion is the one indicated by this fraction, 1/2, namely double, or as 2 to 1; for the motion of Saturn's perihelion is 135″ in seconds, and that of Jupiter's aphelion 270″, between which there is a most perfect diapason. In the others, however, except Jupiter with Mars, the proportions of the motions are so close to harmonic ones that, if strings were stretched so, the ears could not easily discern the imperfection of the consonance. He therefore concludes that there are perfect harmonies: between ♄ perihelion and ♃ aphelion, the diapason; between ♃ perihelion and ♂ aphelion, the diapason with the soft third, nearly; between ♂ perihelion and the Earth's aphelion, the diapente; between the perihelia of these same, the soft sixth; between the aphelia of the Earth and ♀, the hard sixth; between the peri- helia, the soft sixth; and between ♀ aphelion and ☿ perihelion, or also between the perihelia, the disdiapason. From these and other considerations — though not without many cautions — he tries in chapter 5 to draw the proportions of the planetary motions to the places of the system, that is, to the keys of the musical scale, in the genus of the hard and of the soft song. In chapter 6 he contends that musical tones, or modes, are in a certain way expressed by God in the extremes of these same motions; and in chapter 7, that universal harmonies of all six planets are given, fourfold, like common counterpoints. And in chapter 8, although he admits that there are no sounds in the heaven, that the motions in which he considers the harmonies are not true but only apparent from the Sun, and that there is no solid natural cause for comparing the apparent motions of the planets with human voices, he nevertheless, by I know not what lure of congruence and analogy, assigns the bass to Saturn and Jupiter, the tenor to Mars, the alto to the Earth and Venus, and the descant to Mercury.

Stepping from here to the eccentricities, in chapter 9 — a very long one — with many axioms devised and ingeniously coordinated to his purpose, he tries to show that the eccentricities of the orbs must have been determined from harmonic ratios, so that the extremes of the aphelion and perihelion motions could represent the harmonic proportions discussed above; that therefore the inscriptions and circumscriptions of the orbs in the five bodies, or about the five

Regular bodies, had to give way to these harmonic ratios; and that the intervals of the planets could not have been built so exactly from the Regular bodies, lest the harmonies of the extreme motions about the Sun should perish. Finally, in chapter 10, since he seems to himself to have detected a harmony between the extreme motions of the planets — not the true ones, but those apparent to the Sun, that is, seen from the Sun — he concludes that the Sun is the palace of the whole of nature, and that there is in it some mind hidden from us which can perceive those harmonies, since they arise only from motions subtending angles at the Sun. From this he tries to confirm the rest of the Sun and the motion of the Earth, without which a great part of those harmonies perishes.

The same author, however, taught how to investigate harmonic proportions in the motions as seen both from the Sun and from the Earth, in book 6 of the *Epitome of Astronomy*, page 901. We have given that teaching of his on another occasion, in book 7, section 5, chapter 8, number 7, so that we have no need to repeat it here.

[Margin: The author's opinion.]

IV. *Our* opinion, however — from which Mersennius, on chapter 4 of Genesis, from page 1558 and 1704, and Kircher, in book 10 of the *Musurgia*, are not far removed — is that Kepler's efforts already mentioned contain more ingenuity than solid learning or true doctrine. *First*, a great part of them rests on the immobility of the Sun at the centre of the world, on a certain mental power of the Sun that apprehends the harmonies, and on the annual motion of the Earth about the Sun — a hypothesis we have already rejected in the last chapters of the preceding section. *Second*, the first three reasons adduced in chapter 8, number 8, tell against it here. *Third*, harmony properly so called, or harmonic proportions, are not found as exact as was fitting and possible for God, had He set harmonic ratios before Himself in the archetype in determining the motions: not in the periods of the planets compared with one another, nor in most of the daily motions as seen from the Earth — where the creature that contemplates the divine works, accustomed to sensible harmonies, dwells — nor in the motions as regards latitude, nor between the extreme motions themselves as apparent from the Sun; and finally, those extreme motions of two planets, for instance Saturn's perihelion and Jupiter's aphelion, very rarely concur, and for the most part those motions proceed without these harmonies. Since this is so, it seems rather that we must assert that the harmony which Scripture, the Fathers and very many of the wise recognize in the heaven is to be understood only analogically and metaphorically, through a certain accommodation and likeness. Just as, namely, in sensible harmony a concent pleasing to the ears nevertheless arises from unequal and diverse sounds and voices, so from the motions of the celestial bodies, though diverse and unequal, there nevertheless follows an admirable order toward the end sought by God, and a conspiring of the means toward that same end — most pleasing to the angelic and human intellect that contemplates these things more deeply.

The rest, however, which has been sought out with such ingenuity by Kepler and others to establish in the heavens some harmony of such a kind that nothing else is lacking to it but a sensible sound, seems to be mere symbolism, poetic or oratorical rather than philosophical. So Kepler himself, in the appendix to the *Harmonice*, page 253, judged of the symbolisms of Ptolemy and of Robert Flud [Robert Fludd], or de Fluctibus; and Mersennius, on Genesis chapter 4, page 1558, judged the very same of Kepler's analogies, calling them symbolisms that are at most oratorical, that is, such as orators may use to amplify, by metaphors of this kind, the divine providence in celestial things. And he adds that the eternal geometry of God is no more concerned with harmonic proportions than with the temperings of figures, colours, flavours and odours. Just as we cannot, from the image of the divine geometry impressed on our mind, give a reason why this figure or mixture of colours rather than another delights the eyes, and why these flavours or odours rather than others are pleasant to the palate or the nostrils — nor in these matters must we have recourse to the image of geometry — so neither in giving a reason why these sounds rather than others delight the hearing; and much less is the reason for the celestial motum [?], and for their intelligible beauty, to be given from these things which are pleasant to the hearing alone.

CAPVT XI.

Chapter XI.

An Siderum Aspectus vim habeant ac determinationem ex Configurationibus Harmonicis.

Whether the aspects of the stars have their force and determination from harmonic configurations.

I. I taught much on this argument in book 7, section 5, chapter 8, since aspects are affections of the planets in longitude, which that section dealt with; and meanwhile I seemed to report Kepler's teaching on this matter in such a way that, by not disapproving it, I approved it, because the divisions of the circle by aspects seem to have a greater kinship with harmonic consonances than the motions of the planets have. Moreover, Kepler, in book 4 of the *Harmonice*, chapter 5, propositions 1, 2, 3 and 4, says that the kinship of the radiations, or aspects, with the circle and its arcs, and with the regular figures, is greater than that of the consonances; that the congruence of the figures inscribable in a circle can do more toward constituting efficacious configurations than toward the consonances; and that toward the same, congruence can do more than scibility. What congruence is, and what the scibility of figures is, has been sufficiently indicated here in the scholia of chapter 4.

[Margin: What is an efficacious configuration?]

Further, in the same chapter 5, an efficacious configuration is defined by Kepler: "When the rays of two planets make such an angle as is apt to stimulate

sublunary nature and the inferior faculties of living beings, so that each becomes busier about its own work at the time of the configuration." He then assumes a twofold axiom: namely, that the arc of the circle of the zodiac cut off by the side of a congruent and scible figure or star measures the angle of the efficacious configuration; and that the angle of a scible and congruent figure or star is the module, or measure, of the efficacious configuration. With these laid down — since he had demonstrated in book 1 that the diameter of the circle, the tetragon and the trigon, the hexagon, the octagon and the octagonal star, the dodecagon and the dodecagonal star, and the pentagon and the decagon with the pentagonal and decagonal star, are scible, and in book 2 that they are congruent — he therefore constitutes 13 efficacious configurations. Among them the most efficacious is the conjunction, and to it corresponds the whole circle of 360°. Next comes the opposition, because the two rays concur in the same line, which is the most perfect congruence, and to it corresponds the semicircle of 180°. Next, the quadrate aspect, to which corresponds the quadrant of the circle, of 90°; after it the trine, or trigonal, aspect, to which corresponds a third of the circle, that is, 120°; next the sextile, or hexagonal, aspect, to which corresponds a sixth of the circle, that is, 60°; and then the remaining Keplerian aspects, in the order of efficacy that we represent at once in the little table below.

Efficacious configurations, from Kepler, book 4 of the Harmonice, chapter 5.

Arc of the zodiac intercepted (degrees)	Names of the aspects	Sign	Scible and congruent figures, from whose sides the aspects arise	Harmonies once believed by Kepler
0 or 360	Conjunction	☌		
180	Opposition	☍	Diameter of the circle	diapason
90	Quadrate	□	Tetragon	diatessaron
120	Trine	△	Trigon	diapente
60	Sextile	✶	Hexagon	semiditone
45	Octile, or Sequadrus		Octagon	minor hexachord
135	Trioctile, or Sesquadrus		Octagonal star	
30	Semisextile		Dodecagon	
150	Quincunx		Dodecagonal star	
72	Quintile		Pentagon	ditone
108	Sesquintile, or Tridecile		Decagonal star	
144	Biquintile		Pentagonal star	major hexachord
36	Semiquintile, or Decile		Decagon	

[Margin: Kepler's old axiom condemned by Kepler.]

II. In book 4 of the *Harmonice*, chapter 6, Kepler likewise recounts the occasion for introducing new aspects into astrology. He had treated it in his book *De stella nova*, chapters 8, 9 and 10, and in his replies to the physicians

Helisaeus Roslinus [Helisäus Röslin] and Philippus Feselius [Philipp Feselius], who attacked these new aspects; and finally in what he had said about these aspects in the *Ephemerides*, from page 33 to 36. For in the year 1606 he had taken as an axiom *that God the Creator either took the laws for ordering the aspects from the harmonies of song below the octave, or tempered the ears of man, the judges of those concords, to the celestial aspects.* This axiom, which was coined by no one else but himself, as is clear from page 34 of the *Ephemerides*, he now refutes, on the ground that there would then have to be as many aspects as there are simple consonances up to the diapason. For the quadrate aspect would have to correspond to the diatessaron itself; the trine to the diapente; the opposite to the diapason; the quintile to the major third, or ditone; the sextile to the minor third, or semiditone; the biquintile to the major sixth, or major hexachord; and the Sequadrus to the minor sixth, or minor hexachord. For if you take away from the whole string as great a portion as any of the aspects already mentioned takes away from the circle, the remainder of the string makes with the whole string the consonance already mentioned that is assigned to each aspect. But, Kepler says, it has been found by meteorological observations that sublunary nature is stimulated also by the semisextile aspect, which intercepts a twelfth part of the circle, although, when a twelfth portion is taken from the string, the remainder of eleven parts does not sound in consonance with the whole string; and, on the contrary, that it is not stimulated sensibly or manifestly by the sesquadrus, which intercepts three eighths of the circle, or 135°, although, when three eighths are taken from the string, the remaining five parts are consonant with the whole. Roused by this, he corrects his axiom in this place and teaches that the harmonic proportions are not the causes of the aspects, and that their efficacy does not come from the harmonic section of the circle as such. For granted that the harmonies have a great kinship with the aspects, and that the origin of both is from the figures inscribable in a circle, yet the harmonies arise from them in one way and the aspects in another. For in however many ways the circle is divided demonstratively, all those ways can be transferred to a straight line, that is, to the string, and to any part of it, and so the consonances are distinguished. But not every demonstrative division of the circle, made, say, into three or four arcs, can be transferred to any arc of it whatever. For a straight line, whether cut short or prolonged, remains straight; but a circle cut short does not remain straight. From this it follows that there is a proportional section of any two straight lines whatever, and yet there is none of any two arcs whatever of one circle.

[Margin: The falsity of the axiom already mentioned.]

III. Kepler, having become more cautious than his former self, teaches these and other things in that chapter 6. Among them the following are notable, which I am pleased to set out in his own words, with a little explanation of our own added. *For a true mathematical and causal comparison of the concords with the aspects to exist, the axiom must be overturned completely, since it is not only insufficient but also diametrically contrary to the truth. For no aspect properly*

corresponds to any lesser concord, except that the opposite corresponds to the concord diapason; but each corresponds to the greater concords, the companions of the lesser, out of the triad of each section. That is, the aspects are defined by the same segments of the circle as the greater concords also are, while the lesser concords are defined by the residues of the circle. For example, the trigonal aspect does not correspond to the concord diapente but to the diapason epidiapente. That is, the trine aspect is between two planets a third part of the zodiac, or 120°, apart, and not by the residue, or 240°; and between 3 and 1 is the consonance diapason epidiapente. He goes on: *The quadrate does not correspond to the diatessaron but to the disdiapason; the quintile not to the major third, but to the one compounded of it and the diapason.* For the quadrate aspect is between things 90°, or one quarter, apart, and not by the residue, or three quarters of the zodiac; and between 4 and 1 is the consonance disdiapason. Likewise the quintile aspect is between planets 72°, or one fifth, apart, and not by two fifth parts of the zodiac; and between 5 and 1 is the consonance diapason with ditone, which not everyone accepts among the true consonances. He likewise continues, saying: *The sextile corresponds not to the minor third but to the disdiapason epidiapente; the biquintile not to the major sixth, as we supposed above, but to the one compounded of the major third and the diapason; the sesquadrus not to the minor sixth but to the one compounded of the diatessaron and the diapason, as is clear from the same proportion of part to whole in each case.* That is, the sextile aspect is between bodies a sixth part of the zodiac apart; the biquintile between bodies two fifth parts of the zodiac, that is 144°, apart; and the sesquadrus between bodies three eighth parts of the zodiac, that is 135°, apart. Now between 6 and 1 is the consonance disdiapason with diapente; between 5 and 2 lies the diapason with ditone; and between 8 and 3 the diapason diatessaron. Let the preceding table, therefore, be corrected as in the following table, to which we have added three consonances omitted by Kepler.

Aspect	Degrees of the zodiac intercepted	Consonances with the whole circle
Opposition	180	diapason
Quadrate	90	disdiapason
Trine	120	diapason epidiapente
Sextile	60	disdiapason epidiapente
Octile	45	✠
Sesquadrus	135	diapason diatessaron
Semisextile	30	✠
Quincunx	150	diapason with semiditone
Quintile	72	disdiapason with ditone
Sesquintile	108	diapason with major hexachord
Biquintile	144	diapason with ditone
Semiquintile	36	✠

Consult now the first table of consonances in our chapter 4, and you will see that there are not a few consonances to which no aspect is assigned here,

and conversely that there are some aspects here against which the crosses set beside them, ✠, indicate that no consonance corresponds.

IV. These and similar considerations compelled Kepler to admit, in the same book 4, chapter 6, that harmonic consonances and aspects do have something in common as regards their origin from the divisions of the circle, but that music and meteorology are nevertheless born from these divisions in different ways, for different causes concur in constituting the aspects; and that nature makes a choice of those aspects that are furnished with more prerogatives. But let his words, I ask, be noted: *What, then, is it that sets a limit to the number of aspects? And why is no semiquadrate, or octile, and no decile or tridecile introduced, except only after the principal ones? Why is the sesquadrus, ennobled by a musical kinship, either omitted or held cheap. The semisextile, a stranger in music, is not merely inserted but even displayed among the first?* He answers himself at once: *Because it is not music that forms the aspects; geometry forms both kinds, though the one by some laws and the other by others. For whatever comes from a noble figure, one that has some singular privileges in geometry, is both harmonic in music and efficacious in meteorological things. But meteorology and music are like different peoples, sprung from the same fatherland, geometry.* And he adds that the harmonic proportions, though they arose from the circle, went out from the circle and founded colonies, while the aspects remained within their fatherland, the circle, and use no laws other than the circle's, taken from the regular and congruent plane figures inscribed in a circle. The rest of what he says in chapter 7 about the faculty of sublunary nature, about a certain soul of the whole universe, and about a soul of the Earth that perceives the force of the aspects and is stimulated to act, neither belongs to this place nor is to be admitted by Catholics without correction. Here, however, we must select from book 6 of the *Epitome of Copernican Astronomy* the aspects that Kepler finally accepted, and in the degree of dignity that he later recognized in them by the very observation of meteorological effects.

[Margin: The dignity of the aspects.]

V. In book 6 of the *Epitome of Astronomy*, then, page 843, he teaches that in the first degree of the aspects are placed: the *conjunction*, as the beginning of all; the *opposition*, as occurring in all three divisions of the circle; the *quadrate*, as occurring in two, the area of its figure being expressible; the *sextile*, because its side is expressible; the *semisextile*, because its side is among the inexpressibles of the more perfect order, and because, repeated twelve times, it encloses a stable plane; and the *trine*, since its side is expressible in square [potentia]. On these, consult what was said in scholium 1 of chapter 4.

In the second order, he says, are the *quintile* and the *biquintile*, because, although their sides are inexpressibles of a lower order, they nevertheless share among themselves in the divine proportion, which he explained in book 1 of the *Harmonice*, and the figures themselves excel in congruence into solid figures; and the *quincunx*, because its figure is fruitful in the congruence of planes. He says, however, that the *decile* and *tridecile* already fall short of congruence, and that the *octile* and *sesquadrus* are the most worthless, because they are formed

from sides that are neither expressible nor use the divine proportion, nor in turn are they raised up by any notable congruence of the figure. He therefore places them in the third degree, in book 4 of the *Harmonice*, chapter 6. To the other demonstrable figures he assigns no aspects, because their efficacy vanishes and has no support from observation, since not even all the aspects of the third degree are wholly beyond doubt. Let this, however, be the final table of the aspects according to Kepler's mind.

The order of the aspects in force and dignity

Orders of the aspects	Names of the aspects	Sign	Degrees of the zodiac intercepted	Consonances
I.	Conjunction	☌	0 or 360	unison
	Opposition	☍	180	diapason
	Quadrate	□	90	disdiapason
	Sextile	✶	60	disdiapason epidiapente
	Semisextile		30	none
	Trine	△	120	diapason epidiapente
II.	Quintile		72	disdiapason with ditone
	Biquintile		144	diapason with ditone
	Quincunx		150	diapason with semiditone
III.	Decile		36	none
	Tridecile		108	diapason with major hexachord
	Octile		45	none
	Sesquadrus		135	diapason diatessaron

It deserves consideration, however, that the other consonances to which some aspects correspond are either the beginning or species of the consonance diapason, which Kepler did not notice; and that, of the consonances that belong to the diapason and are enumerated in the first table of chapter 4, there is none that does not have its aspect in the zodiac, except the one compounded of the diapason and the minor hexachord, such as is between 16 and 5. This is so, first, because the number 360 is not divided into 16 whole parts, since a sixteenth part of that number is 22½; and second, because these parts, taken five times, make 112½, a number too near to the number 108, which belongs to the diapason with the major hexachord, so that it is no wonder if the efficacies of the two cannot be distinguished. Yet I do not think that this affinity with the diapason was set before God as a motive for determining those aspects, since even without it, indeed without any consonance at all, there are three aspects, namely the *semisextile*, the *decile* and the *octile*. For the rest: in the mixtures of colours, flavours and odours, and of the temperaments owed to innumerable plants and animals, in the climacteric years, in the rhythms of the pulse, in the species of fevers and in very many other things, there are definite degrees, mostly expressible by numbers, if their nature were perfectly known. And yet, because those degrees can have proportions owed to harmonic numbers, we must not on that account be anxious to search out music in them beyond the

bounds of metaphor and of a somewhat looser analogy; and so too neither in the aspects of the planets, nor in their motions, nor in their sizes, nor in their intervals. As though nothing had been made wisely by God that He had not subjected to the laws of harmony.

LIBRI NONI FINIS.

End of the Ninth Book.

LIBER DECIMVS PROBLEMATVM ASTRONOMICORVM COMMVNIVM.

The Tenth Book: Of Common Astronomical Problems.

SECTIO PRIMA DE TRIANGVLORVM ANALYSI.

The First Section: On the Analysis of Triangles.

PROHOEMIVM.

Preface.

Although it is our purpose, God willing, to hand down in Volume 2 of this *Almagest*, and in its first book, the whole theory and practice of measuring triangles according to the various forms of trigonometrical analysis, together with the tables needed for it, it has nevertheless seemed good, in this vestibule, as it were, of the common problems, to set out first an epitome of this practice by means of logarithms and mesologarithms, not, however, neglecting sines, tangents and secants, with examples for each, by which each precept is illustrated. The reader can then have recourse to them, and we shall not need to explain by examples the problems to be handed down in the remaining sections; instead a single example of the same rule can serve for very many problems to be solved by it. We suppose here, moreover, that the reader has procured for himself the logarithmic tables of arcs or angles of Adrianus Vlacq [Adriaan Vlacq], or of Father Bonaventura Cavalieri, principal professor of mathematics in this University of Bologna, which he published again in a new form at my urging, at Bologna in the year 1643. For the logarithms of absolute numbers, however, one must have the tables of John Napier, reformed by Henry Briggs and enlarged by Adriaan Vlacq of Gouda, which give logarithms for all numbers increasing in natural series from unity up to 100000; or at least the tables of Henriconis, which go up to 20000. If anyone lacks these tables, however, let him by all means use the old form, and the tables of sines, tangents and secants, which are indeed found everywhere. But the most correct are those published by our Clavius, or by Franciscus Scothen, or by Philippus Lansbergius [Philips van Lansbergen]; and the most minute, running to every ten seconds of each minute of the first and last degrees of the quadrant, are those of Georgius Ioachimus Rheticus [Georg Joachim Rheticus] in his great *Canon* and of L. Valentinus Otho [Valentin Otto] in the *Opus Palatinum*; and finer still are those

that Bartholomaeus Pitiscus [Bartholomäus Pitiscus] has in his great *Canon*, since they contain the sines, tangents and secants for every single second of the first minute, and so on. Finally, we suppose the reader already practised in the doctrine of triangles, and we offer here nothing other than an epitome of the rules, especially of those in which logarithms are substituted in place of sines, tangents and secants.

CAPVT I. In Quo Definitiones, & Præcepta quædam generalia pro vſu Logarithmorum.

Chapter 1. In which are given definitions and certain general precepts for the use of logarithms.

[Margin: What logarithms are, and how many kinds there are.]

LOGARITHMS are artificial numbers substituted for natural ones, suited to performing multiplication by addition alone and division by subtraction alone; indeed, if the residue of a logarithm is employed, they are suited to performing multiplication and division, and hence the rule of three, or of proportions, by addition alone, which is a huge saving of labour and gain of time. Further, some logarithms are for natural numbers taken absolutely; others are logarithms of arcs of the circle, or of the angles that the arcs of the circle measure. For example, in the tables of Father Bonaventura Cavalieri the logarithm of the number twelve is 10 791 812. But the logarithm of twelve minutes is 75 429 065, and that of twelve degrees is 93 178 789. Now those that are substituted in place of sines are called logarithms; those in place of tangents[50], mesologarithms; and finally those in place of secants, tomologarithms. Thus the second logarithm is substituted for the second sine, and the second mesologarithm for the second tangent. Granted, the whole trigonometry of arcs or angles can be completed by logarithms and mesologarithms alone.

[Margin: The logarithm of the radius.]

The *logarithm of the radius* is the artificial number substituted for the natural number of the radius, or whole sine, or semidiameter of the circle; or it is unity followed by as many ciphers as the assumed radius has ciphers and unity together. Therefore, if the radius assumed in the tables of sines is 10,000,000, that is, unity with seven ciphers, as most tables have it, the logarithm of the radius will be unity with eight ciphers, thus: 100,000,000. And so the logarithm of the radius always exceeds the radius by a single cipher. But when logarithms of absolute numbers are employed in rectilinear triangles, the logarithm of the radius is supposed to have as many ciphers as in the logarithms of arcs or angles.

[Margin: What the residue of a logarithm is.]

The *residue of a logarithm* (understand: of the radius), or arithmetical complement, is the residue that remains when the logarithm is subtracted from the logarithm of the radius. For example, since the logarithm of the number 12 taken absolutely is, as we said, 10 791 812, if you subtract this number from the logarithm of the radius, which is 100,000,000, there re-

[50] printed *Tagentium*, for *Tangentium*

mains this residue: 89 708 188. Likewise, since the logarithm of 12°, or of an angle subtending an arc of 12°, is, as we said, 93 178 789, when this is likewise subtracted from the logarithm of the radius, 100 000 000, there remains this residue: 06 821 211. For a cipher must be written at the beginning of any logarithmic series if no other numerical figure has to be put in its place. You see, then, that the residue of a logarithm is obtained very easily: the last figure of the given logarithm, on the right, is subtracted from ten, but all the others always from nine, since by the rules of subtraction all the ciphers of the logarithm of the radius have to be changed into nines. Further, for brevity, we shall indicate the residue of a logarithm by these marks, *Res.L.*, just as we indicate a logarithm by the letter L alone and a mesologarithm by the letter M; and the second logarithm, or logarithm of the complement, by this mark, L2, and the second mesologarithm, or mesologarithm of the complement, by this mark, M 2.

[Margin: Prior knowledge of the species.]

The *species* of an angle is said to be known when it is known whether it is right, acute or obtuse; the species of an arc, when it is known whether it is a quadrant of the circle, or greater or less than a quadrant.

PRÆCEPTA.

Precepts.

[Margin: 1st precept.]

I. In the addition as much as in the subtraction of logarithms, if at the beginning of the sum or of the remainder, on the left, one or more ciphers come out in place of units or of numbers, they must be written nonetheless, even if no figure of a unit or of a number stands before them, as in the two following examples.

ADDITION			
Numbers	2	Log.	03010300
	4	Log.	06020600
Sum		Log.	09030900

SUBTRACTION		
Logarithm of the radius		100000000
Of degrees 12	Log.	93178789
Residue	R.	06821211

One series must always be written above the other in such a way that the last figure of the one stands above the last figure of the other, on the right, in the way usually done in the rule of addition, by which a sum is collected from several numbers set one above another.

[Margin: 2nd precept.]

II. If, after the addition of any logarithms, or mesologarithms, or tomologarithms, or of these mixed together, the sum is to be called a logarithm, it must not contain more terms or figures than there are ciphers in the logarithm of the radius. So if any figure is surplus, it must be thrown away from the beginning of the sum, whether that figure is a one, or a two, or any number at all. Therefore, since in our tables and those of Father Cavalieri the logarithm of the radius contains eight ciphers after the unit, a logarithmic sum must not contain more than 8 figures, as you see in the examples written below; in the first two of them a two is thrown away from the left-hand place, and in the third a one.

	L2 89471612	M 105713821	M2 117581222
	L 91184134	L 91147332	L 72637773
	L 99938612	L 89196231	
Uncorrected sum	L 280594358	L 286057384	L 190218995
Corrected sum	L 80594358	L 86057384	L 90218995

[Margin: 3rd precept.]

III. If the said sum after addition is to be called a double logarithm, or a doubled logarithm, then a single one must be kept in the first place of the sum. Thus if, in the first of the three examples given above, the sum 280594358 were to be called, in the rules to be given below, not the logarithm of some arc or angle but the double logarithm, indicated by these marks, LL, the sum would have to be 180 594 358. But in the third example no change would need to be made; instead the sum 190 218 995 would have to be kept for the double logarithm. It is called the double logarithm, however, not because it is double the sum that would remain once the one or the two had been thrown away from the first place, but because it is double the logarithm that is to be used to find some arc or angle, which must be doubled to obtain the arc or angle sought, as will be shown in the examples of the rules.

[Margin: 4th precept.]

IV. If the said sum after addition is to be called a mesologarithm, or a second mesologarithm, and it contains more figures than there are ciphers in the logarithm of the radius, then the second figure either exceeds 3 or does not exceed 3. If it exceeds it, whatever figure is surplus beyond the number of the said ciphers must be thrown away from the first place; if it does not exceed it, a single one must be kept in the first place, which holds the surplus figure, as in the examples written below.

M 10892371	L 9995841	T 102030402
M 104719922	M 82158137	L 92562314
L 92606330	T 117728665	

Uncorrected sum	M 306249963	M 2 299845213	M 194592716
Corrected sum	M 106249963	M 2 99845213	M 94592716

You see that in the first sum a 1 has been kept in place of the 3, because the second figure, 0, does not exceed 3; but in the others the surplus two or one has been thrown away, because the second figure is 9, which exceeds three.

[Margin: 5th precept.]

V. Finally, if after addition the said sum is to be called a tomologarithm, a single one must be kept before figures equal in number to the ciphers that the logarithm of the radius has, as you see in the following examples.

	T 135362739	L 99931420
	L2 99623978	L 93157900
		M 102163024
Uncorr. sum	T 234986717	T 295252344
Corrected sum	T 134986717	T 195252344

[Margin: 6th precept.]

VI. When a logarithm is smaller than would allow another logarithm to be subtracted from it, a single one must be added before the first figure, on the left, of the smaller logarithm. This is the same as if you had set the logarithm of the radius above the smaller one. For example, since the logarithm of forty, which is 16 020 600, cannot be subtracted from the logarithm of twelve, which is 10 791 812, then in place of 10 791 812 one must use the logarithm increased on the left by a one, thus: 110 791 812; and then, after 16 020 600 has been subtracted, there will be left the residue 94 771 212.

[Margin: 7th precept.]

VII. If the given arc or angle is greater than 90°, use in its place its supplement to 180°, and for this supplement look up its logarithm, or sine, and so on.

[Margin: 8th precept.]

VIII. If the sum of the logarithms of absolute numbers ought not to be the logarithm of a whole unit, but it is certain

from elsewhere that, after the solution of the triangle, some quantity less than a unit must come out, say less than one foot, or a fraction, then one will have to use a privative logarithm according to the cossic rules, and subtraction must be employed instead of addition, so that the numerator and denominator of the fraction to be found may be had. Choose, then, 10, or 100, or 1000 as your denominator, and look up its logarithm; from this subtract the logarithmic sum found before. For the residue, then, look up the corresponding number in the table of logarithms of absolute numbers: it will be the numerator of the fraction, to be set above the denominator. But the matter will become clearer by an example. Let the logarithmic sum be 12 041 200, which in the

tables corresponds to the number 16, and yet let it be certain from elsewhere that the side of the triangle sought ought not to be, say, 16 feet, but not even one whole foot. Choose, then, the denominator 1000, whose logarithm is 30 000 000. If you subtract from this 12 041 200 (which was the sum collected), there remains 17 958 800, which is the logarithm of the number 62. So when the numerator 62 has been written above the chosen denominator 1000, this fraction results: 62/1000. It signifies that the side sought is not, say, of one foot, but contains sixty-two thousandth parts of one foot; and the same holds for the other measures.

[Margin: What the base is.]

Further, in the following rules, by the name *base*, or hypotenuse, is to be understood the line subtending, or opposite, the right angle, when the triangles are right-angled; in the others, however, the third side after the two others that have been named.

CAPVT II. In Quo Regulæ 13. Triangulorum Planorum ſeu Rectilineorum, RECTANGVLORVM, vt eſt

Chapter 2. In which are 13 rules for plane, or rectilinear, triangles that are RIGHT-ANGLED, as is

Diagram. A right triangle set in the left column under the chapter title, completing its closing words "as is". The base runs horizontally from A on the left to C on the right; a vertical side rises from C to B; the hypotenuse joins A and B. The right angle is at C.

- A (lower left): end of the base and vertex of the hypotenuse
- C (lower right): vertex of the right angle
- B (top right): top of the vertical side

I. Regula. Datis Baſi, & Angulo Adiacente, Quærere Latus oppoſitum angulo dato. *Rule 1. Given the base and an adjacent angle, to find the side opposite the given angle.*

[Margin: By logarithms.]

To the logarithm of the base add the logarithm of the given angle, and the sum will be the logarithm of the side sought.

EXEMPLVM. *Example.*

Let base AB be 600 feet; its logarithm is	L	27781513
Let angle A be 30°; the logarithm of these is	L	96989700
The sum of these, with the unit taken away according to what was said in chapter 1, precept 2, is	L	24771213

Look up, then, in the tables of Vlacq or of Cavalieri, or in ours, the logarithm 24 771 213, and opposite it there will correspond 300. Therefore side BC,

opposite angle A, is 300 feet, of such feet as base AB is 600 feet. Likewise, in all the rules for plane triangles, the sides or bases sought will be found with the help of the logarithmic table, by looking up, opposite the logarithm that has been found, the number that corresponds to it.

If angle B were given, A would nevertheless also be given, because A is the complement of B to the remaining right angle, by proposition 32 of the first book of Euclid's *Elements*.

[Margin: By sines.]

OR: Let it be done that as the radius, or whole sine, is to the base, so is the right sine of the given angle to the side opposite the given angle.

EXEMPLVM. *Example.*

Let angle A be 30°; its right sine is	5 000 000
Let base AB, by which the sine is to be multiplied, be of feet	600
The sum from the sine multiplied by the base is	3 000 000 000
Which, divided by the radius	10 000 000
Gives the quotient, that is, side BC, of feet	300

II. Regula. Datis Basi & vno Laterum, Quærere Angulum oppositum Lateri dato. *Rule 2. Given the base and one of the sides, to find the angle opposite the given side.*

[Margin: By logarithms.]

To the residue of the logarithm of the base add the logarithm of the given side, and the logarithm of the angle sought will result. How the residue of a logarithm is obtained was explained in chapter 1.

EXEMPLVM. *Example.*

Let base AB be 600 feet; its logarithm is	L	27781513
When this is subtracted from the logarithm of the radius, that is, from 100 000 000	—	—
There remains the residue of the logarithm of the base	RL	72218487
Let side BC be 300 feet; the logarithm of these is	L	24771213
The sum of these two is the logarithm	L	96989700

Look up, then, among the logarithms of arcs, arealiter [?], the logarithm 96 989 700: for you will see that opposite it, in the first column of the tables, there corresponds 30°. You will conclude, then, that angle A is of so many degrees.

[Margin: By sines.]

OR: Let it be done that as the base is to the radius, or whole sine, so is the given side to the right sine of the angle sought.

EXEMPLVM. *Example.*

Let radius AB be	10 000 000
Let side CB be of feet	300
When the radius is multiplied by the side, let the sum be	3 000 000 000
Which is to be divided by base AB, taken to be of feet	600
The quotient will be the right sine BC	5 000 000
To this sine corresponds angle A, of degrees	30

III. Regula. Datis Latere & Angulo Adiacente, Quærere Basim. *Rule 3. Given a side and an adjacent angle, to find the base.*

[Margin: By logarithms.]

To the logarithm of the given side add the residue of the second logarithm belonging to the given angle, and the logarithm of the base will result.

EXEMPLVM. *Example.*

Let angle A be 30°; its second logarithm is	L 2.	99375306
When this is taken away from the logarithm of the radius, there remains the residue	RL 2.	00624694
Let side AC be 520 feet; the logarithm of these is	L	27160033
The sum of these two is the logarithm	L	27784727
To this logarithm corresponds base AB, of 600 44/100 feet		

[Margin: By sines.]

OR: Let it be done that as the radius is to the given side, so is the secant of the given angle to the base sought.

EXEMPLVM. *Example.*

Let angle A be 30°; its secant will be	11 547 005
Let side AC be of feet	520
When the secant is multiplied by the side, let the sum be	6 004 442 600
When this is divided by the radius	10 000 000
The base AB, or quotient, is of feet 600 44426/100000	

IV. Regula. Datis Latere & Angulo adiacente, Quærere Latus alterum. *Rule 4. Given a side and an adjacent angle, to find the other side.*

[Margin: By logarithms.]

To the logarithm of the given side add the mesologarithm of the given angle, and the logarithm of the side sought will result.

EXEMPLVM. *Example.*

Let side AC be 520 feet; its logarithm is	L	27160033
Let angle A be 30°; the mesologarithm of these is	M	97614394
The sum, with the unit taken away from the first place, is the logarithm to which there corresponds in the tables side BC, of 30 [recte: 300] feet	L	24774427

[Margin: By sines.]

OR: Let it be done that as the radius is to the given side, so is the tangent of the given angle to the other side.

EXEMPLVM. *Example.*

The tangent of angle A, 30°, is	5 773 503
Side AC is of feet	520
When the tangent is multiplied by the side, let the sum be	3 002 221 560
Which, divided by the radius	10 000 000
Gives the quotient, or side BC, of feet	300 1/5

V. Regula. Datis Laterib. ambobus, Quærere Angulum vtrumq Acutorum.
Rule 5. Given both sides, to find each of the acute angles.

[Margin: By logarithms.]

To the logarithm of the side opposite the angle sought add the residue of the logarithm of the other side, and the sum will be the mesologarithm of the said opposite angle. When this angle is subtracted from 90°, the remainder will be the other acute angle.

EXEMPLVM pro Angulo A. *Example for angle A.*

Let side BC be 300 feet; its logarithm is	L	24771213
Let side AC be 520 feet; the residue of its logarithm is	RL	72839967
The sum is the mesologarithm	M	97611180

To which there corresponds in the tables of arcs the angle 29° 59′.

[Margin: By sines.]

OR: Let it be done that as the side adjacent to the angle sought is to the radius, so is the side opposite the same angle to the tangent of the opposite angle.

EXEMPLVM. *Example.*

The radius is for us of parts	10 000 000
Side BC is of feet	300
When the radius is multiplied by the side, let the sum be	3 000 000 000
When this is divided by side AC, of feet	520

The quotient comes out as the tangent 5 769 230 4/5, namely that of angle A, 29° 59″.

VI. Regula. Datis Latere & Angulo Oppoſito Quærere Baſim. *Rule 6. Given a side and the opposite angle, to find the base.*

[Margin: By logarithms.]

To the logarithm of the given side add the residue of the logarithm of the given opposite angle, and the logarithm of the base will result.

EXEMPLVM. *Example.*

Let side BC be 300 feet; its logarithm is	L	24771213
Let angle A be 30°; the residue of the logarithm of these is	RL	03010300
Therefore base AB is 600 feet, because the sum is the logarithm	L	27781513

[Margin: By sines.]

OR: Let it be done that as the radius is to the given side, so is the second secant of the given angle to the base sought.

EXEMPLVM. *Example.*

The second secant of angle A, 30°, is	20 000 000
Side BC is of feet	300
When the second secant is multiplied by the side, let the sum be	6 000 000 000
Which, divided by the radius	10 000 000
There remains the quotient, or base AB, of 600 feet	

VII. Regula. Datis Latere & Angulo Oppoſito Quærere Latus alterum. *Rule 7. Given a side and the opposite angle, to find the other side.*

[Margin: By logarithms.]

To the logarithm of the given side add the second mesologarithm of the given angle, and the logarithm of the other side will result.

EXEMPLVM. *Example.*

Let side BC be 300 feet; its logarithm is	L	24771213
Angle A, 30°; the second mesologarithm of these is	M2	102385606
The sum (with the unit taken away from the beginning) is the logarithm	L	27156819

To which there corresponds side AC, of 520 feet.

[Margin: By sines.]

OR: Let it be done that as the radius is to the second tangent of the given angle, so is the given side to the other side sought.

EXEMPLVM. *Example.*

The second tangent of angle A, 30°, is	17 320 508
Side BC is of feet	300
When the second tangent is multiplied by the side, the sum is	5 196 152 400
Which, divided by the radius	10 000 000
The quotient is, approximately, 520 feet, or 519 6/10	

VIII. Regula. Datis Lateribus, Quærere Baſim. *Rule 8. Given the sides, to find the base.*

Find the acute angles by the fifth rule, and then find the base by the sixth. OR: the sum of the squares of the sides will be a square whose square root will be the base. For example, if side AC is 520 feet, its square, when that number is multiplied by itself, is 270 400; and if side BC is 300 feet, its square is 90 000. The sum of these squares is 360 400, and its nearest square root is 600. Therefore base AB is approximately 600 feet. The foundation of the rule is proposition 47 of the first book of Euclid.

IX. Regula. Datis, Baſi & Latere, Quærere Latus alterum. *Rule 9. Given the base and a side, to find the other side.*

Find by rule 2 the angle opposite the given side, and from there, by rule 7, the other side. Or subtract the square of the side from the square of the base: there will be left a square number whose square root will be the side sought.

X. Regula. Datis Baſi & Angulo adiacente, Quærere Latus adiacens Angulo dato. *Rule 10. Given the base and an adjacent angle, to find the side adjacent to the given angle.*

Find by rule 1 the side opposite the given angle, and from this, by rule 7, the side adjacent to it.

XI. Regula. Datis Baſi & vno Laterum, Quærere Angulum adiacentem Lateri dato. *Rule 11. Given the base and one of the sides, to find the angle adjacent to the given side.*

Find by rule 2 the angle opposite the given side, and subtract the angle found from 90°: for the angle sought will be left.

XII. Regula. Dato vno Angulorum Acutotorum, eoque ſubtracto à Gradibus 90. notus ſit alter Acutorum. *Rule 12. When one of the acute angles is given and is subtracted from 90°, the other of the acute angles becomes known.*

XIII. Regula. Datis tantummodo duobus acutis angulis in Triangulo rectilineo Rectangulo, non poteſt concludi certa quantitas Baſis aut Laterum; quia ſaluis prædictis angulis poteſt Triangulum Rectangulum eſſe maius ac maius in infinitum. *Rule 13. When only the two acute angles of a rectilinear right-angled triangle are given, no definite size of the base or of the sides can be concluded, because, with the said angles unchanged, the right-angled triangle can be larger and larger without limit.*

CAPVT III. In quo Regulæ 9. Triangulorum Planorum OBLIQVANGVLORVM, Quorum Schema ſit

Chapter 3. In which are 9 rules for plane OBLIQUE-ANGLED triangles, whose diagram is

Diagram. A plain triangle set in the left column under the chapter title, completing its closing words "whose diagram is". The base runs from A on the left to B on the right; the third vertex, C, stands at the top, nearly above B.

- A (lower left): end of the base
- B (lower right): end of the base
- C (top, above B): the vertex opposite the base

I. Regula. Datis Lateribus duobus & Angulo vni eorum Oppoſito, Quęrere Angulum Oppoſitum reliquo dato Lateri, priùs tamen ſpecie præcognitum. *Rule 1. Given two sides and an angle opposite one of them, to find the angle opposite the other given side, its species, however, being known beforehand.*

[Margin: By logarithms.]

To the logarithm of the given angle add the residue of the logarithm of the side opposite it, and the logarithm of the remaining side; the result will be the logarithm of the angle sought, if the angle is to be acute, or of the angle to be subtracted from 180° so that an obtuse angle results, if the angle sought is to be obtuse.

EXEMPLVM. *Example.*

Let the given angle A be 60° 36′; their logarithm is	L	99401248
Let the opposite side BC be 2240 feet; the residue of their logarithm is	RL	66497520
Let side AB be 1800 feet; their logarithm is	L	32552725
The sum (with the unit taken away from the beginning) is the logarithm	L	98451493

To this corresponds the acute angle C, 44° 26′.

[Margin: By sines.]

OR: Let it be done that as the side opposite the given angle is to the sine of the given angle, so is the other given side to the sine of the angle sought.

EXEMPLVM. *Example.*

The sine of angle A, of 60° 36′, is	8 712 138
Side AB is of feet	1800
The sine multiplied by side AB gives the sum	15 681 848 400
Which, divided by side CB, which is of feet	2240
Gives the quotient 7000 825 40/224, which is the sine of angle C, 44° 26′.	

II. Regula. Datis Latere & duobus Angulis, quorum vnus ſit Oppoſitus lateri dato, Quęrere Latus Oppoſitum alteri angulo dato. *Rule 2. Given a side and two angles, one of which is opposite the given side, to find the side opposite the other given angle.*

If the angles to be used are acute, proceed as below.

[Margin: By logarithms.]

To the logarithm of the given side add the residue of the logarithm of the angle opposite it, and the logarithm of the other given angle; the result will be the logarithm of the side sought.

EXEMPLVM. *Example.*

Let side AB be 1800 feet; their logarithm is	L	32552725
Let angle C be 44° 26′; the residue of its logarithm is	RL	01548530
Let angle A be 60° 36′; their logarithm is	L	99401248
The sum (with the unit taken away) is the logarithm	L	33502503

To this corresponds side CB, of 2240 feet.

[Margin: By sines.]

OR: Let it be done that as the sine of the given angle opposite the given side is to that same side, so is the sine of the other given angle to the side sought.

EXEMPLVM. *Example.*

The sine of the remaining angle A, of 60° 36′, is	8 712 138
Side AB is of feet	1800
The sine multiplied by side AB gives the sum	16 681 848 400
Which, divided by the sine of angle C, of 44° 26′	7 000 789
The quotient gives side CB, of 2240 feet.	

But if the angle opposite the side to be found is obtuse, proceed as below.

Diagram. A triangle set in the right column beside the paragraph on triangle GAC. Its base runs from G on the left to C on the right, with the vertex A above, nearer G. Two dotted lines descend from A to the base, ending at P and at E; P and E are lettered below the base, P nearer G.

- A (top): vertex of the angle GAC
- G (lower left): end of the base
- P (below the base, left of centre): foot of the first dotted line from A
- E (below the base, right of P): foot of the second dotted line from A
- C (lower right): end of the base

Consider triangle GAC, in which, from the given angle GAC, which is obtuse, the greatest side, GC, is to be found. For since angle G or C is given, the sum of angle GAC and G or C, subtracted from 180°, will leave the third angle, G or C. Now suppose side AC is given; drop the perpendicular AP, and in triangle APC, right-angled at P, with the base AC and the angle C, find side AP by rule 1 for right-angled triangles. Then, C being subtracted from 90° and angle CAP thus obtained, find side PC with the base AC by the same rule 1. Next, in triangle APG, with side AP, now found, and with angle G, which is known, find side GP by rule 7 for right-angled triangles; joined to side CP, it will make up the side sought, GC.

[Margin: In this figure angle A should have been obtuse.]

[Margin: Otherwise.]

But if the sides comprehending angle GAC are given: G being subtracted from 90°, GAP is known. With it, and with the base GA, find side GP by rule 1 for right-angled triangles; and with the base AC and the angle CAP, known as above (that is, by subtracting angle C from 90°), find side PC by the same rule 1 for right-angled triangles. For the sum of the sides GP and PC will be the side sought, GC, opposite the obtuse angle.

III. Regula. Datis Lateribus duobus & Angulo Verticali, hoc eſt à datis Lateribus comprehenſo, Quærere Angulum quemuis Baſi adiacentem. *Rule 3. Given two sides and the vertical angle, that is, the angle comprehended by the given sides, to find either angle adjacent to the base.*

[Margin: Preparation for the calculation.]

[Margin: Use the figure placed at the beginning of the chapter.]

Three things must first be prepared: namely, the *sum of the given sides*; then their *difference*, by subtracting the smaller from the greater; and lastly the *half-sum of the angles at the base*, which is obtained by subtracting the vertical angle from 180° and dividing the remainder by 2. Thus if side AC is 1800 feet and CB 2240 feet, the *sum of the sides* is 4040 feet, and their difference is 440. And if the vertical angle C is 74° 58′, this, subtracted from 180°, leaves the sum of angles A and B taken together, 105° 2′; half of this, or the half-sum, is 52° 31′. With these prepared, proceed by logarithms or by sines, as below.

[Margin: By logarithms.]

To the residue of the logarithm of the sum of the sides add the logarithm of the difference of the sides and the mesologarithm of the half-sum of the angles at the base; the result will be the mesologarithm of the angle that is to be added to the already mentioned half-sum of the angles, so that the greater angle results, or subtracted from it, so that the smaller angle at the base remains.

EXEMPLVM. *Example.*

Let the sum of the sides be 4040 feet; the residue of their logarithm is	RL	63936186
The difference of the sides, 440 feet; their logarithm is	L	26434527
The half-sum of the angles at the base, 52° 31′; their mesologarithm	M	10115281 1
The sum (with the unit taken away, according to chapter 1, precept 4) is the mesologarithm	M	91523524

To this corresponds the angle 8° 4′ 45″, which, added to the half-sum, 52° 31′, makes angle A, 60° 35′ 45″; but subtracted from it, it makes angle B, 44° 26′ 15″.

[Margin: By sines.]

OR: Let it be done that as the sum of the given sides is to their difference, so is the tangent of the half-sum of the angles adjacent to the base to the tangent of the angle that is to be added to that same half-sum, so that the greater angle results, or subtracted from it, so that the smaller remains.

EXEMPLVM. *Example.*

The tangent of the half-sum of the angles, 52° 31′, is	13 040 106
The difference of the sides is	440
The tangent multiplied by the difference of the sides gives the sum	5 737 646 640
Which, divided by the sum of the sides	4040
Gives the quotient, that is, the tangent of the angle of 8° 4′ 45″	1 420 011 ½

This, added to the half-sum of the angles or subtracted from it, gives the same as above.

IV. Regula. Datis Lateribus tribus, Quærere Angulum quemlibet. *Rule 4. Given the three sides, to find any angle.*

[Margin: Use the figure set out just before.]

First, onto the greatest side, GC, drop from the opposite angle A the perpendicular AP; then, taking PE equal to GP, draw the straight line AE from A to E. Two triangles thus arise, APG and APC, right-angled at P. In each of them, besides the right angle, the base is known, AG and AC respectively, since these are given sides of triangle GAC. Therefore, if side GP, and side PC in the other triangle, were known, the angles G and C could be found by the rules for plane right-angled triangles; and their sum, subtracted from 180°, will leave angle GAC. But for sides GP and PC to become known, one must find the segment EC — namely the segment in whose remainder, GE, the perpendicular AP falls at the half, P. For once EC has been found and subtracted from the whole greatest side GC, the remainder GE will be left known; its half, GP, will be one side of triangle APG, while the other half, PE, added to the line EC, will make up side PC. In this way you will have data sufficient to solve each of the two right-angled triangles. The whole effort of this rule, then, is directed to finding the segment EC; for which proceed as follows.

[Margin: By logarithms.]

To the residue of the logarithm of the greatest side, GC, add the logarithm of the sum of the smaller sides, AC and AG, and the logarithm of the difference of those same smaller sides; the result will be the logarithm of the segment EC.

EXEMPLVM. *Example.*

Let side GC be 2483 feet; the residue of its logarithm is	RL	66050233
Side AC, 2240 feet, and AG, 1800 feet; their sum, 4040; its logarithm	L	36063814
The difference of the sides is 440 feet; its logarithm	L	26434527
The sum (with the unit taken away from the beginning) is the logarithm	L	28548574

To this corresponds the segment EC, of 716 feet. When these are subtracted from side GC, of 2483 feet, GE remains, of 1767 feet, whose half, GP, is 883½, and PE is just as much. Therefore, when PE, 883½, and EC, 716, are added together, side PC comes to 1599½. So, to solve the right-angled triangle APG and to find angle G by rule 11 for right-angled triangles, there are given the base AG, of 1800 feet, and the side GP, of 883½ feet. And to solve the right-angled triangle APC and to find angle C by the same rule, there are given the base AC, of 2240 feet, and the side PC, of 1599½ feet.

[Margin: By sines.]

The same segment EC will be obtained if it is done that as the greatest side is to the sum of the smaller sides, so is the difference of the smaller sides to the segment EC — that is, to the segment of the greatest side at the half of whose remainder the perpendicular falls.

V. Regula. Datis Lateribus duobus, & Angulo Verticali, Quærere Latus tertium ſeu Baſim. *Rule 5. Given two sides and the vertical angle, to find the third side, or base.*

First find the angles at the base by rule 3, then the third side by rule 2.

VI. Regula. Datis Lateribus duob. & Angulo vni eorum Oppoſito, Quærere Latus tertium, dummodo nota ſit ſpecies reliquorum angulorum. *Rule 6. Given two sides and an angle opposite one of them, to find the third side, provided the species of the remaining angles is known.*

First find, by rule 1 of this chapter, the angle opposite the other of the given sides, and then the third side by rule 2.

VII. Regula. Datis Angulis duobus, & Latere vni eorum Oppoſito, Quærere Latus Oppoſitum tertio angulo. *Rule 7. Given two angles and a side opposite one of them, to find the side opposite the third angle.*

The third angle is known, because it is the supplement of the other two, taken together, to two right angles. Therefore, if the sum of the two given angles is subtracted from 180°, the third side can then be found by rule 2 of this chapter.

VIII. Regula. Datis Angulis duobus, & Latere vni eorum Adiacente, Quærere Latera reliqua. *Rule 8. Given two angles and a side adjacent to one of them, to find the remaining sides.*

Since the third angle becomes known when the sum of the two given angles is subtracted from 180°, the remaining sides can certainly be found by rule 2 of this chapter.

IX. Regula. Omnia Quæſita in Regulis præcedentibus, inueniri poſſunt per regulas Triangulorum Rectangulorum, reducto Triangulo Obliquangulo ad duo Triangula Rectangula, vt fuſiùs explicabo in Libro Trigonometrico. *Rule 9. Everything sought in the preceding rules can be found by the rules for right-angled triangles, by reducing the oblique-angled triangle to two right-angled triangles, as I shall explain more fully in the Trigonometric Book.*

CAPVT IV. In Quo Regulæ 16. Triangulorum Sphæricorum RECTANGVLORVM, Cuiuſmodi eſt Triãgulum infraſcriptum ABC, Rectangulum ad A, in quo arcus BC, ſubtenſus angulo recto A; vocetur Baſis.

Chapter 4. In which are 16 rules for spherical RIGHT-ANGLED triangles, such as the triangle ABC drawn below, right-angled at A, in which the arc BC, subtending the right angle A, is to be called the base.

Diagram. A spherical triangle in the right column under the chapter title. B stands at the top, A at the lower left, C at the lower right; the side BC is a long curved arc, AC a shallower curve, and AB a short, nearly straight side. Two small dots and a comma-like mark stand inside the angle at A.

- B (top): vertex
- A (lower left): vertex of the right angle
- C (lower right): vertex

I. Regula. Datis Baſi & Angulo Adiacente, Quærere Latus Oppoſitum Angulo dato. *Rule 1. Given the base and an adjacent angle, to find the side opposite the given angle.*

[Margin: By logarithms.]

To the logarithm of the base add the logarithm of the given angle, and the result will be the logarithm of the side sought — with the unit, however, taken away in the leftmost place, according to what was said in chapter 1, precept 2.

EXEMPLVM. *Example.*

Let the base BC be 60° 30′; its logarithm is	L	99396968
Let angle B be 23° 30′; its logarithm is	L	96006997
Therefore AC is an arc of 20° 18′ 27″, since the sum of the logarithms is	L	95403965

[Margin: By sines.]

OR: Let it be done that as the radius is to the sine of the base, so is the sine of the given angle to the sine of the side sought.

EXEMPLVM. *Example.*

The sine of the base BC, of 60° 30′, is	8 703 557
The sine of angle B, of 23° 30′, is	3 987 491
One sine multiplied by the other gives the sum	34705 345 205 487
Which, divided by the radius, or whole sine	10 000 000
Gives the quotient 3 470 534 521/1000, namely the sine of the arc AC, 20° 18′ 27″.	

II. Regula. Datis Baſi & Angulo Adiacente, Quærere Latus Adiacens angulo dato. *Rule 2. Given the base and an adjacent angle, to find the side adjacent to the given angle.*

[Margin: By logarithms.]

To the second mesologarithm of the base add the residue of the second logarithm of the given angle, and the result will be the second mesologarithm of the side sought.

EXEMPLVM. *Example.*

Let the base BC be 66°; its second mesologarithm is	M 2.	96485831
Let angle C be 35°; the residue of its second logarithm is	RL 2.	00866355
Therefore AC is 61° 28′ 30″, since the sum is the second mesologarithm	M 2.	97352186

[Margin: By sines.]

OR: Let it be done that as the radius is to the secant of the given angle, so is the second tangent of the base to the second tangent of the arc, or side, sought.

EXEMPLVM. *Example.*

The secant of angle C, of 35°, is	12 207 746
The second tangent of the base BC, of 66°, is	4 452 287
The secant multiplied by the tangent gives the sum	54352 388 813 102
Which, divided by the radius	10 000 000
Gives the quotient 5 435 238 88/100, that is, the second tangent of side AC, 61° 28′ 30″.	

III. Regula. Datis Baſi & Angulo Adiacente Quadrante minoribus; Quærere Angulum alterum Baſi Adiacentem. *Rule 3. Given the base and an adjacent angle, each less than a quadrant, to find the other angle adjacent to the base.*

[Margin: By logarithms.]

To the second logarithm of the base add the mesologarithm of the given angle, and the result will be the second mesologarithm of the angle sought. But if the data are greater than a quadrant, consult the scholia of this chapter.

EXEMPLVM. *Example.*

Let the base BC be 60° 30′; its second logarithm is	L 2.	96923388
Let angle B be 23° 30′; its mesologarithm is	M	96383019
Therefore angle C is 77° 54′ 53″, since the sum, with the unit taken away according to chapter 1, precept 4, is the second mesologarithm	M 2.	93306407

[Margin: By sines.]

OR: Let it be done that as the radius is to the second sine of the base, so is the tangent of the given angle to the second tangent of the angle sought.

EXEMPLVM. *Example.*

The second sine of the base BC, of 60° 30′, is	4 924 236
The tangent of angle B, of 23° 30′, is	4 348 124
The second sine multiplied by the tangent gives the sum	21 411 188 693 264
Which, divided by the radius	10 000 000
Gives the quotient 2141118 87/100, that is, the second tangent of angle C, 77° 54′ 53″.	

IV. Regula. Datis Baſi & Latere, Quærere Angulum Oppoſitum Lateri dato. *Rule 4. Given the base and a side, to find the angle opposite the given side.*

[Margin: By logarithms.]

To the logarithm of the given side add the residue of the logarithm of the base, and the result will be the logarithm of the angle sought — with the unit taken away, if it is surplus, according to chapter 1, precept 2.

EXEMPLVM. *Example.*

Let the base BC be 60°; the residue of its logarithm is	RL	00624694
Let side AB be 20°; its logarithm is	L	95340517
Therefore angle C is 23° 15′ 42″, since the sum is	L	95965211

[Margin: By sines.]

OR: Let it be done that as the radius is to the second secant of the base, so is the sine of the given side to the sine of the angle sought.

EXEMPLVM. *Example.*

The second secant of the base BC, of 60°, is	11 547 005
The sine of side AB, of 20°, is	3 420 202
The second secant multiplied by the sine gives the sum	39 493 089 595 010
Which, divided by the radius	10 000 000
Gives the quotient 3 949 308 9595/10000, that is, the sine of angle C, 23° 15′ 42″.	

V. Regula. Datis Baſi & Latere Quadrante minoribus: Quærere Angulum Adiacentem Lateri dato. *Rule 5. Given the base and a side, each less than a quadrant, to find the angle adjacent to the given side.*

[Margin: By logarithms.]

To the second mesologarithm of the base add the mesologarithm of the given side, and the result will be the second logarithm of the angle sought. What is to be done when the base or the side is greater than a quadrant will be stated in the scholia of this chapter.

EXEMPLVM. *Example.*

Let the base BC be 60° 30′; its second mesologarithm is	M 2.	97526420
Let side AB be 30°; its mesologarithm is	M	97614394
Therefore, of angle B, 70° 56′ 5″, on account of the second logarithm	L 2	95140814

[Margin: By sines.]

OR: Let it be done that as the radius is to the tangent of the given side, so is the second tangent of the base to the second sine of the angle sought.

EXEMPLVM. *Example.*

The second tangent of the base BC, of 60° 30′, is	5657728
The tangent of side AB, of 30°, is	5773 503
One tangent multiplied by the other gives the sum	32 664 909 581 184
Which, divided by the radius	10 000 000
Gives the quotient 3 266 490 96/100, that is, the second sine of angle B, 70° 56′ 5″.	

VI. Regula. Datis Baſi & Latere Quadrante minoribus, Quærere Latus alterum. *Rule 6. Given the base and a side, each less than a quadrant, to find the other side.*

[Margin: By logarithms.]

To the second logarithm of the base add the residue of the second logarithm of the given side, and the result will be the second logarithm of the side sought. But if the data are greater than a quadrant, consult the scholia of this chapter.

EXEMPLVM. *Example.*

Let the base BC be 60°; its second logarithm is	L 2.	96989700
Let side AB be 20°; the residue of its second logarithm is	RL 2.	00270142
Therefore side AC is 57° 51′ 15″, since the sum is the second logarithm	L 2.	97259842

[Margin: By sines.]

OR: Let it be done that as the radius is to the second sine of the side, so is the secant of the base to the secant of the side sought.

EXEMPLVM. *Example.*

The secant of the base BC, of 60°, is	20 000 000
The second sine of side AB, of 20°, is	9 396 926
The secant multiplied by the second sine gives the sum	187 938 520 000 000

which, divided by the radius	10 000 000
gives the quotient 18 793 852, that is, the secant of side AC, 57° 51′ 15″.	

VII. Regula. Datis Latere & Angulo Adiacente, Quadrante minoribus, Quærere Latus Alterum. *Rule 7. Given a side and an adjacent angle, each less than a quadrant, to find the other side.*

[Margin: By logarithms.]

To the logarithm of the given side add the mesologarithm of the given angle, and the result will be the mesologarithm of the side sought. But if the data are greater than a quadrant, consult the scholia of this chapter.

EXEMPLVM. *Example.*

Let side AB be 20°; its logarithm is	L	95340517
Let angle B be 50° 10′; its mesologarithm is	M	100787534
Therefore side AC is 22° 17′ 18″, since the sum, with the unit taken away according to chapter 1, precept 4, is	M	96128051

[Margin: By sines.]

OR: Let it be done that as the radius is to the sine of the given side, so is the tangent of the given angle to the tangent of the remaining side.

EXEMPLVM. *Example.*

The sine of side AB, of 20°, is	3 420 202
The tangent of angle B, of 50° 10′, is	11 988 184
The sine multiplied by the tangent gives the sum	41 002 010 893 168
Which, divided by the radius	10 000 000
Gives the quotient 4 100 201 [?]89/1000, that is, the tangent of side AC, 22° 17′ 19″.	

VIII. Regula. Datis Latere & Angulo Adiacente, Quærere Angulum Oppositum Lateri dato. *Rule 8. Given a side and an adjacent angle, to find the angle opposite the given side.*

[Margin: By logarithms.]

To the second logarithm of the given side add the logarithm of the adjacent angle, and the result will be the second logarithm of the angle sought.

EXEMPLVM. *Example.*

Let side AB be 20°; its second logarithm is	L 2	99729858
Let angle B be 50° 10′; its logarithm is	L	98853109
Therefore angle C is 43° 48′ 46″, for its second logarithm is	L 2	98582967

[Margin: By sines.]

OR: Let it be done that as the radius is to the sine of the given angle, so is the second sine of the given side to the second sine of the angle sought.

EXEMPLVM. *Example.*

The second sine of side AB, of 20°, is	9 396 926
The sine of angle B, of 50° 10′, is	7 679 110
One sine multiplied by the other gives the sum	72 160 024 415 860
Which, divided by the radius	10 000 000
Gives the quotient 7 216 002 44/106[51], that is, the second sine of angle C, 43° 48′ 46″.	

IX. Regula. Datis Latere & Angulo Adiacente, Quadrante minoribus, Quærere Baſim. *Rule 9. Given a side and an adjacent angle, each less than a quadrant, to find the base.*

[Margin: By logarithms.]

To the second logarithm of the given angle add the second mesologarithm of the given side, and the result will be the second mesologarithm of the base. But if the data exceed a quadrant, consult the scholia of this chapter.

EXEMPLVM. *Example.*

Let angle B be 50°; its second logarithm is	L 2	98080675
Let side AB be 36°; its second mesologarithm is	M 2	101387390

[51] so printed; a second reading of the page confirms the denominator's third figure as 6. The product 72 160 024 415 860 divided by the radius gives 7 216 002.44…, so the denominator should be 100; the printed 6 may be a foul sort.

Therefore the base BC is 48° 30′, since the sum, with the unit taken away according to chapter 1, precept 4, is the second mesologarithm	M 2	99468065

[Margin: By sines.]

OR: Let it be done that as the radius is to the second sine of the given angle, so is the second tangent of the given side to the second tangent of the base.

EXEMPLVM. *Example.*

The second sine of angle B, of 50°, is	6 427 876
The second tangent of side AB, of 36°, is	13 763 819
The sine multiplied by the tangent, and the sum divided by the radius	10 000 000
Gives the quotient 8 847 253, that is, the second tangent of the base BC, 48° 30′.	

X. Regula. Datis Latere & Angulo Oppoſito, ſingillatim Quadrante minoribus, Quærere Latus alterum ſpecie tamen præcognitum. *Rule 10. Given a side and the opposite angle, each singly less than a quadrant, to find the other side, its kind, however, being known beforehand.*

[Margin: By logarithms.]

To the mesologarithm of the given side add the second mesologarithm of the opposite angle, and the result will be the logarithm of the other side, less than a quadrant. But if you are certain from elsewhere that this side must be greater than a quadrant, then subtract the side found from 180°, and the remainder will be the side sought. When, however, one of the data is greater than a quadrant, consult the scholia of this chapter.

EXEMPLVM. *Example.*

Let the given side AC be 40° 10′; its mesologarithm is	M	99263778
Let angle B be 40° 30′; its second mesologarithm is	M 2	100685011
Therefore side AB is 81° 13′ 6″, since the sum is the logarithm	L	99948789

[Margin: By sines.]

OR: Let it be done that as the radius is to the second tangent of the given angle, so is the tangent of the given side to the tangent of the side sought.

From here on, for brevity's sake, we shall refrain from examples by sines: for it is clear enough from what has been said so far how one is to work with them.

XI. Regula. Datis Latere & Angulo ipſi Oppoſito, Quærere Baſim ſpecie tamen præcognitam. *Rule 11. Given a side and the angle opposite it, to find the base, its kind, however, being known beforehand.*

[Margin: By logarithms.]

To the logarithm of the given side add the residue of the logarithm of the opposite angle, and the result will be the logarithm of the base, less than a quadrant. Or, if the base must be greater than a quadrant, you will obtain it by subtracting the arc found from 180°.

EXEMPLVM. *Example.*

Let the given side AB be 20°; its logarithm is	L	95340517
Let angle C be 23° 20′; the residue of its logarithm is	RL	04022173
Therefore BC is 59° 42′ 30″, on account of the logarithm	L	99362690

[Margin: By sines.]

OR: Let it be done that as the radius is to the second secant of the given angle, so is the sine of the given side to the sine of the base sought.

XII. Regula. Datis Latere & Angulo Oppoſito, Quærere Angulum reliquum ſpecie notum: dummodo ſingula Datorum ſint Quadrante minora. *Rule 12. Given a side and the opposite angle, to find the remaining angle, known in kind, provided that each of the data is less than a quadrant.*

[Margin: By logarithms.]

To the residue of the second logarithm of the given side add the second logarithm of the opposite angle, and the result will be the logarithm of the remaining angle, if acute. But if it must be obtuse, you will obtain it by subtracting the angle found from 180°. And if any of the data is greater than a quadrant, consult the scholia of this chapter.

EXEMPLVM. *Example.*

Let side AB be 20° 30′; the residue of its second logarithm is	RL2	00284124
Let angle C be 24° 10′; its second logarithm is	L 2	99601655
Therefore angle B is 76° 55′, on account of the logarithm	L	99885779

[Margin: By sines.]

OR: Let it be done that as the radius is to the secant of the given side, so is the second sine of the given angle to the sine of the angle sought.

XIII. Regula. Datis Lateribus, Quærere Baſim. *Rule 13. Given the sides, to find the base.*

[Margin: By logarithms.]

To the second logarithm of one given side add the second logarithm of the other side, and the result will be the second logarithm of the base.

EXEMPLVM. *Example.*

Let side AB be 20°; its second logarithm is	L 2	99729858
Let side AC be 70° 10′; its second logarithm is	L 2	95340517
Therefore the base BC is 71° 15′ 10″, on account of the second logarithm	L 2	95070375

[Margin: By sines.]

OR: Let it be done that as the radius is to the second sine of the given side, so is the second sine of the other side to the second sine of the base.

XIV. Regula. Datis Lateribus Quadrante minoribus, Quærere Angulum Oppoſitum cuiuis Laterum. *Rule 14. Given the sides, each less than a quadrant, to find the angle opposite either of the sides.*

[Margin: By logarithms.]

To the mesologarithm of the side opposite the angle sought add the residue of the logarithm of the remaining side, and the result will be the mesologarithm [?] of the angle sought. And if any of the data is greater than a quadrant, consult the scholia of this chapter.

EXEMPLVM. *Example.*

Let side AB be 20°; its mesologarithm is	M	95610659
Let side AC be 70°; the residue of its logarithm is	RL	00270142
Therefore angle C is 21° 10′ 38″, on account of the mesologarithm	M	95880801

[Margin: By sines.]

OR: Let it be done that as the radius is to the tangent of the side opposite the angle sought, so is the second secant of the remaining given side to the tangent of the angle sought.

XV. Regula. Datis Angulis duobus obliquis Quadrante minoribus, Quærere Baſim. *Rule 15. Given the two oblique angles, each less than a quadrant, to find the base.*

[Margin: By logarithms.]

Add together the second mesologarithms of the oblique angles, and the result will be the second logarithm of the base. And if the data are singly greater than a quadrant, consult the scholia of this chapter.

EXEMPLVM. *Example.*

Let angle C be 36°; its second mesologarithm is	M 2	101387390
Let angle B be 55° 20′; its second mesologarithm is	M 2	98398377
Therefore the base BC is 72° 9′ 8″, on account of the second logarithm	L 2	99785767

[Margin: By sines.]

OR: Let it be done that as the radius is to the second tangent of either given angle, so is the second tangent of the other given angle to the second sine of the base.

XVI. Regula. Datis Angulis obliquis, Quærere Latus cuiuis angulorum Oppoſitum. *Rule 16. Given the oblique angles, to find the side opposite either of the angles.*

[Margin: By logarithms.]

To the second logarithm of the angle opposite the side sought add the residue of the logarithm of the remaining oblique angle, and the result will be the second logarithm of the side sought.

EXEMPLVM. *Example.*

Let angle C be 23° 10′; its second logarithm is	L 2	99634877
Let angle B be 78° 0′; the residue of its logarithm is	RL	00095956
Therefore side AB is 19° 58′, on account of the second logarithm	L 2	99730833

[Margin: By sines.]

OR: Let it be done that as the radius is to the second sine of the angle opposite the side sought, so is the second secant of the remaining given angle to the second sine of the side sought.

SCHOLIA. *Scholia.*

Diagram. A small woodcut at the head of the scholia, right column. Two arcs meet at A on the left and C on the right, forming a lens. R stands at the upper arc, towards the right; B at the lower arc, left of centre. A third arc runs from R down to B, dividing the lens into a left part lettered P and a right part lettered V.

- A (left point of the lens): vertex

- P (inside the left part): the triangle ARB
- R (upper arc, right of centre): vertex
- V (inside the right part): the triangle RCB
- C (right point of the lens): vertex
- B (lower arc, left of centre): vertex

[Margin: The use of the vicarious triangle.]

I. We have said more than once in the preceding rules that the data are each required to be less than a quadrant. Otherwise, if one or both of them is greater than a quadrant, that is, of more degrees than 90, those rules do not hold for the given triangle unless they are transferred to another triangle, its vicarious triangle [Triangulum Vicarium]; once that triangle is solved, the given triangle will also remain solved. First, therefore, one must continue two sides, or a side and the base, until they complete a semicircle, and so the vicarious triangle arises. For example, in the figure set out above, suppose the proposed triangle is the spherical triangle ARB, right-angled at R, to be solved by rule 2 of the preceding chapter, which requires the base AB to be less than a quadrant and the adjacent angle ABR likewise acute, that is, less than a quadrant; and yet AB is, say, 100°, and angle ABR 120°. Let the base AB and the side AR be continued as far as C, so that ABC and ARC are both semicircles. Then triangle V will be the vicarious triangle of triangle P. And by the very fact that, besides the right angle R, two other data are given in P, two other data will also be given in V. For if angle A is given, angle C is also given, since the two opposite angles made by semicircles of equal circles are equal. If side AR is given, then, when it is subtracted from 180°, side RC will be given. Similarly, if the base AB is given, then, when it is subtracted from 180°, the base BC will be given. Finally, if angle ABR is given, then, when it is subtracted from 180°, angle RBC will be given. And so in the vicarious triangle V sufficient data are had to solve it by the same rule by which the proposed triangle P was ordered to be solved. Thus in our case the base BC would be 80°, and the adjacent angle RBC 60°. Therefore, by rule 2 for spherical right-angled triangles, the side BR, adjacent to angle B, can be sought.

[Margin: The solution of the proposed triangle from the solution of the vicarious triangle.]

II. Once the vicarious triangle V is solved, the other triangle P, the one first proposed, remains solved. For if the common side BR was sought, then once it is found in V it is also found for triangle P, because the side is the same. But if side AR was sought, then once side RC is found and subtracted from 180°, side AR becomes known. If, on the other hand, AB was sought, then once BC is found and subtracted from 180°, the base AB will become known. But if angle ABR was sought, then once RBC is found and subtracted from 180°, angle ABR will be known. Finally, if angle A was sought, it is enough to find angle C, for C is equal to angle A. The whole skill lies in continuing the sides, or the base and one of the sides, so that in the vicarious triangle you have sufficient data to solve it, and from its solution the proposed triangle remains solved. For

this continuation we shall indeed give some rules in our *Trigonometry*, God willing; but practice itself and the drawing of the figure will easily instruct a skilful geometer in this matter. And from this you will learn how you should use the vicarious triangle in spherical oblique-angled triangles too, where there is need.

CAPVT V. In quo Regulæ 14. Triangulorum Sphæricorum OBLIQVANGVLORVM, Cuiuſmodi eſt ABR; vel eius Vicarium V, ſeu BRC, in ſequenti Schemate.

Chapter 5. In which are 14 rules for spherical OBLIQUE-ANGLED triangles, such as ABR, or its vicarious triangle V, that is, BRC, in the following figure.

Diagram. A small woodcut in the left column under the chapter title, the same design as the figure at the head of the scholia on the previous page. Two arcs meet at A on the left and C on the right, forming a lens; a third arc runs from R on the upper arc down to B on the lower arc, dividing the lens into a left part lettered P and a right part lettered V.

- A (left point of the lens): vertex
- P (inside the left part): the triangle ABR
- R (upper arc, right of centre): vertex
- V (inside the right part): the vicarious triangle BRC
- C (right point of the lens): vertex
- B (lower arc, left of centre): vertex

I. Regula. Datis Lateribus duobus, & Angulo vni eorum Oppoſito, Quærere Angulum alteri Oppoſitum, ſed ſpecie præcognitum. *Rule 1. Given two sides and the angle opposite one of them, to find the angle opposite the other, its kind, however, being known beforehand.*

[Margin: By logarithms.]

To the residue of the logarithm of the side opposite the given angle add the logarithm of the given angle and the logarithm of the remaining side, and the result will be the logarithm of the angle sought.

EXEMPLVM. *Example.*

Let side BR be 40° 20′; the residue of its logarithm is	RL	01889391
Let angle C be 36° 15′; its logarithm is	L	97718150
Let side RC be 50°; its logarithm is	L	98842540
Therefore angle RBC is 44° 24′ 55″	L	98450081

[Margin: By sines.]

OR: Let it be done that as the sine of the side opposite the given angle is to the sine of the given angle, so is the remaining given side to the sine of the angle sought, if acute. And if you were certain from elsewhere that the angle

sought must be obtuse, you will obtain it by subtracting the angle found from 180°.

II. Regula. Datis Lateribus duobus ſingillatim Quadrante minoribus, & Angulo vni eorum Oppoſito, ac Specie anguli oppoſiti reliquo dato Lateri, Quærere Baſim. *Rule 2. Given two sides, each singly less than a quadrant, the angle opposite one of them, and the kind of the angle opposite the remaining given side, to find the base.*

[Margin: By logarithms.]

First, to the residue of the second logarithm of the given angle add the second mesologarithm of the side adjacent to it, and the result will be the second mesologarithm of a certain arc, which is to be called the First.

Then, to the residue of the second logarithm of the side adjacent to the given angle, add the second logarithm of the First Arc and the second logarithm of the remaining given side, and the result will be the second logarithm of the Second Arc.

Third, if the angles opposite the given sides are of the same kind, that is, both acute or both obtuse, the sum of the First and Second Arcs will make up the base sought. But if they are of different kinds, subtract the smaller of the arcs found from the larger, and the remainder will be the base sought. If the data are greater than a quadrant, use the vicarious triangle, according to what was said in the scholia of chapter 4.

EXEMPLVM. *Example.*

[Margin: Step 1.]

Let angle C be 54° 34′; the residue of its second logarithm is	RL2	02367553
Let side RC be 40°; its second mesologarithm is	M 2	100761875
Therefore the First Arc is 25° 56′ 30″	M 2	103129418
The residue of the second logarithm of side RC, of 40°, is	RL2	01157460
The First Arc, of 25° 56′ 30″; its second logarithm is	L 2	99538755
The second logarithm of side RB, of 60°, is	L 2	96989700
Therefore the Second Arc is 35° 56′ 24″	L 2	97685915

[Margin: Step 2.]

[Margin: Step 3.]

Now the angles C and RBC are both acute; therefore the sum of the arcs, the First of 25° 56′ 30″ and the Second of 35° 56′ 24″, which is 61° 52′ 54″, makes the base BC.

[Margin: By sines.]

OR: Let it be done that as the radius is to the secant of the given angle, so is the second tangent of the side adjacent to the given angle to the second tangent of the First Arc. Then let it be done that as the second sine of the side adjacent

to the given angle is to the second sine of the remaining given side, so is the second sine of the First Arc to the Second Arc, which is to be added to the first arc if the angles opposite the given sides are of the same kind, so that the base is obtained; otherwise the difference of the arcs found will give the base.

III. Regula. Datis Lateribus duob. ſingillatim Quadrante minoribus, & Angulo vni eorum Oppoſito, & Specie anguli oppoſiti reliquo Lateri, Quærere Angulum Verticalem, ſeu comprehenſum à Lateribus datis. *Rule 3. Given two sides, each singly less than a quadrant, the angle opposite one of them, and the kind of the angle opposite the remaining side, to find the vertical angle, that is, the angle contained by the given sides.*

[Margin: By logarithms.]

First, to the mesologarithm of the given angle if it is acute, or of its supplement to 180° if it is obtuse, add the second logarithm of the side adjacent to this angle, and the result will be the second mesologarithm of a certain angle, which is to be called the First Angle.

Then, to the mesologarithm of the side adjacent to the given angle, add the second mesologarithm of the other side and the second logarithm of the First Angle found, and the result will be the second logarithm of an angle, which is to be called the Second Angle.

Third, if the angles opposite the given sides are of the same kind, the sum of the First and Second Angles will make up the vertical angle sought; but if they are of different kinds, the difference of the angles found will be the size of the vertical angle. But if the given sides are greater than a quadrant, use the vicarious triangle, which is treated in the scholia of chapter 4.

EXEMPLVM. *Example.*

Let angle C be 54° 34′; its mesologarithm is	M	101478013
Let side RC be 80°; its second logarithm is	L 2.	92396702
Therefore the First Angle is 76° 17′	M 2.	93874715
The mesologarithm of side RC, of 80°, is	M	1075[?]6812
The second mesologarithm of side RB, of 60°, is	M 2.	97614394
The second logarithm of the First Angle, of 76° 17′, is	L 2.	93749696
Therefore the Second Angle is 39° 4′	L 2.	98900902

Now the angles C and RBC, opposite the given sides, are both acute; therefore the sum of the First and Second Angles found, that is, of 76° 17′ and 39° 4′, which is 115° 21′, makes the vertical angle BRC.

[Margin: By sines.]

OR: Let it be done that as the radius is to the tangent of the given angle, so is the second sine of the side adjacent to the given angle to the second tangent of the First Angle. Then let it be done that as the tangent of the side adjacent

to the given angle is to the second tangent of the remaining given side, so is the second sine of the First Angle found to the second sine of the Second Angle. Now if the angles opposite the given sides are of the same kind, the sum of the First and Second Angles found will be the vertical angle; otherwise the difference of the angles found will be the vertical angle.

IV. Regula. Datis Lateribus duobus, quorum ſaltem vnum ſit Quadrante minus, & dato Angulo Verticali acuto, Quærere Baſin. *Rule 4. Given two sides, at least one of which is less than a quadrant, and the vertical angle, acute, to find the base.*

[Margin: By logarithms.]

First, to the second mesologarithm of the smaller of the given sides add the residue of the second logarithm of the acute vertical angle, and the result will be the second mesologarithm of an arc, which is to be called the First Arc.

Then, to the residue of the second logarithm belonging to the First Arc found, add the second logarithm of the smaller of the given sides and the second logarithm of the difference between the other side and the First Arc found; and the result will be the second logarithm of the base. The base must agree in kind with the difference already mentioned; otherwise the supplement of the supposed base to 180° must summi [?], so that the true base is obtained. But if the vertical angle is obtuse, or both of the given sides are greater than a quadrant, use the vicarious triangle, which is treated in the scholia of chapter 4.

EXEMPLVM. *Example.*

Let angle BRC be 37° 13′; the residue of its second logarithm is	RL2	00988938
Let side RB be 60°; its second mesologarithm is	M 2	97614394
Therefore the First Arc is 54° 3′ 30″, for the sum is	M 2	98603332
The residue of the second logarithm of the First Arc, of 54° 3′ 30″, is	RL2	02312166
Side RC, of 80°, and its difference from the First Arc is 25° 56′ 30″; the second logarithm of this is	L 2	99538755
The second logarithm of side BR, of 60°, is	L 2	96989700
Therefore the base BC is 40° 1′ 49″	L 2	98840621

That is, less than a quadrant, just as the difference of the First Arc and side RC was less than a quadrant.

[Margin: By sines.]

OR: Let it be done that as the radius is to the sine of the smaller of the given sides, so is the sine of the remaining side to an arc, which is to be called the First. Then let it be done that as the radius is to the First Arc, so is the versed sine of the vertical angle to the Second Arc; add this to the versed sine of the difference of the sides, and the result will be the versed sine of the base sought.

V. Regula. Datis Lateribus duobus, quæ ſimul ſumpta ſint minora Semicirculo; & dato Angulo Verticali, Quærere Angulum vtrumq. ad Baſim. *Rule 5. Given two sides which taken together are less than a semicircle, and the vertical angle, to find each angle at the base.*

[Margin: By logarithms.]

To the residue of the second logarithm of the half-sum of the sides add the second logarithm of their half-difference and the second mesologarithm of half the vertical angle, and the result will be the mesologarithm of the half-sum of the angles at the base.

Then, to the residue of the logarithm of the half-sum of the sides add the logarithm of their half-difference and the second mesologarithm of half the vertical angle, and the result will be the mesologarithm of the half-difference of the angles sought. This, therefore, added to the half-sum of the angles at the base, will give the larger angle, and subtracted from the same half-sum it will leave the smaller angle. But if the given sides together are greater than a semicircle, employ the vicarious triangle, which is treated in the scholia of chapter 4.

EXEMPLVM. *Example.*

Let side RC be	40°	0′	
And side RB	60°	0′	
The sum of the sides will be	100°	0′	half-sum 50°
And the difference of the sides	20°	0′	half-difference 10°
The vertical angle BRC	112°	0′	half-angle 56°

The residue of the second logarithm of the half-sum, 50°, is	RL2	01919325
The second logarithm of the half-difference, 10°, is	L 2	99933515
The second mesologarithm of half the vertical angle, 56°, is	M 2	98289874
Therefore the half-sum of the angles at the base is 45° 56′ 30″, a single unit being kept in the mesologarithm, according to precept 4 of chapter 1	M	100142714
Then		
The residue of the logarithm of the half-sum of the sides, 50°, is	RL	01157460
The logarithm of the half-difference of the sides, 10°, is	L	92396702
The second mesologarithm of half the vertical angle, 56°	M 2	98289874
Therefore the half-difference of the angles is 8° 41′ 35″	M	91844036

Add this to the half-sum, 45° 56′ 30″, and the larger angle C becomes 54° 38′ 5″. Subtract it, and RBC, the smaller, becomes 37° 14′ 55″.

[Margin: By sines.]

OR: Let it be done that as the second sine of the half-sum of the sides is to the second sine of their half-difference, so is the second tangent of half the

vertical angle to the tangent of the half-sum of the angles sought. Then let it be done that as the sine of the half-sum of the sides is to the sine of their half-difference, so is the second tangent of half the vertical angle to the tangent of the half-difference, which is to be added to the half-sum of the angles itself to make the larger angle, and subtracted from it to leave the smaller of the angles sought.

VI. Regula. Datis Lateribus duobus, Quorum alterum ſaltem ſit Quadrante minus, & Angulo Verticali acuto; Quærere Angulum quemuis ad Baſin. *Rule 6. Given two sides, at least one of which is less than a quadrant, and the vertical angle, acute, to find either angle at the base.*

[Margin: By logarithms.]

First, to the residue of the second logarithm of the vertical angle add the second mesologarithm of the side opposite the angle sought, and the result will be the second mesologarithm of the First Case [Primus Casus]. Then, to the residue of the logarithm of the First Case add the second mesologarithm of the vertical angle and the logarithm of the difference between the First Case and the other given side, and the result will be the second mesologarithm of the angle sought.

EXEMPLVM. *Example.*

Let the vertical angle BRC be 50°; the residue of its second logarithm	RL2	01919325
Let side BR be 36°; its second mesologarithm is	M 2	101387390
Therefore the First Case is 25° 2′, for the sum is	M 2	103306715
Side RC, 70°		
The First Case, 25° 2′; the residue of its logarithm	RL	03735102
The difference of the First Case and side RC, 44° 58′, whose logarithm is	L	98402322
The second mesologarithm of angle BRC, 50°	M 2	99238135
Therefore angle C is 36° 3′ 26″, for	M 2	101375559

[Margin: By sines.]

OR: Let it be done that as the radius is to the secant of the vertical angle, so is the second tangent of the side opposite the angle sought to the second tangent of the First Case. Then let it be done that as the second tangent of the vertical angle is to the second secant of the First Case, so is the sine of the difference between the First Case and the other side to the second tangent of the angle sought.

VII. Regula. Datis Angulis duobus & Latere vni eorum Oppoſito, Quærere Latus alteri angulo dato Oppoſitum, ſpecie tamen præcognitum. *Rule 7. Given two angles and the side opposite one of them, to find the side opposite the other given angle, its kind, however, being known beforehand.*

[Margin: By logarithms.]

To the residue of the logarithm of the angle opposite the given side add the logarithm of the given side and the logarithm of the

remaining given angle, and the result will be the logarithm of the side sought, less than a quadrant. But if it must be greater than a quadrant, subtract the side found from 180° and you will have the side sought.

EXEMPLVM. *Example.*

Let angle C be 38° 10′; the residue of its logarithm is	RL	02090459
Let side RB be 42°; its logarithm is	L	98255109
Let angle RBC be 54° 15′; its logarithm is	L	99093281
Therefore side RC is 61° 29′ 17″, for the sum is	L	99438849

[Margin: By sines.]

OR: Let it be done that as the sine of the angle opposite the given side is to the sine of the given side, so is the sine of the remaining given angle to the sine of the side sought, less than a quadrant; or (if it must be greater than a quadrant) of the side to be subtracted from 180°, so that the side sought is obtained.

VIII. Regula. Datis Angulis duobus acutis, & Latere vni eorum Oppoſito, nec non Specie Lateris oppoſiti alteri angulo, Quærere Baſim adiacentem datis angulis. *Rule 8. Given two acute angles and the side opposite one of them, and also the kind of the side opposite the other angle, to find the base adjacent to the given angles.*

[Margin: By logarithms.]

To the residue of the second logarithm of the angle adjacent to the given side add the second mesologarithm of the given side, and the result will be the second mesologarithm of the First Arc.

Then, to the logarithm of the First Arc add the mesologarithm of the angle adjacent to the given side and the second mesologarithm of the remaining given angle, and the result will be the logarithm of the Second Arc.

Third, if the given side is less than a quadrant, the sum of the First and Second Arcs found will make up the base sought. But if the given side is greater than a quadrant, the sum made of the Second Arc and of the supplement of the First Arc to a semicircle will make up the base sought.

EXEMPLVM. *Example.*

Let side RB be 40°; its second mesologarithm is	M 2	100761865
Angle RBC, 35° 25′; the residue of its second logarithm	RL2	00888641

Therefore the First Arc is 34° 22′, for	M 2	101650506
The logarithm of the First Found, 34° 22′, is	L	97516538
The mesologarithm of angle RBC, 35° 25′, is	M	98519312
The second mesologarithm of angle C, 52°, is	M 2	98928098
Therefore the Second Arc is 18° 16′ 30″, for	L	94963948

Added to the First Arc, this makes the base BC 52° 38′ 30″.

[Margin: By sines.]

OR: Let it be done that as the radius is to the secant of the angle adjacent to the given side, so is the second tangent of the given side to the second tangent of the First Arc. Then let it be done that as the tangent of the angle adjacent to the given side is to the second tangent of the remaining given angle, so is the sine of the First Arc to the sine of the Second Arc, which agrees in kind with the side not given. Let the rest, as regards the sum of the arcs, be done as above.

IX. Regula. Datis duobus Angulis acutis, & Latere Oppoſito vni eorum, ac Specie Lateris oppoſiti alteri angulo dato, Quærere Angulum Tertium. *Rule 9. Given two acute angles and the side opposite one of them, and the kind of the side opposite the other given angle, to find the third angle.*

[Margin: By logarithms.]

To the second logarithm of the given side add the mesologarithm of the angle adjacent to it, and the result will be the second mesologarithm of an angle, which is to be called the First Angle.

Then, to the logarithm of the First Angle add the second logarithm of the remaining given angle and the residue of the second logarithm of the angle adjacent to the given side, and the result will be the logarithm of an angle, which is to be called the Second Angle; and it must agree in kind with the remaining side.

Third, if the given side is less than a quadrant, the sum of the first and second angles found will make up the third angle sought. But if the given side is greater than a quadrant, then to have the angle sought use the sum made of the Second Angle and of the supplement of the First Angle to 180°.

EXEMPLVM. *Example.*

Let side RB be 40°; its second logarithm is	L 2	98842540
Let angle RBC be 54° 34′; its mesologarithm	M	101478013
Therefore angle 1 is 42° 53′, a unit being removed, etc.	M 2	100320553
The logarithm of angle 1, 42° 53′, is	L	98328331
The second logarithm of angle C, 37° 13′, is	L 2	99011062
The residue of the second logarithm of angle RBC, 54° 34′	RL2	02367553
Therefore angle 2 is 69° 12′, for the sum is	L	99706946

[Margin: Remember precept 4 of chapter 1.]
Therefore, when angles 1 and 2 are joined, angle BRC becomes 112° 5′.
[Margin: By sines.]
OR: Let it be done that as the radius is to the second sine of the given side, so is the tangent of the angle adjacent to the same side to the second tangent of the First Angle. Then let it be done that as the second sine of the angle adjacent to the given side is to the second sine of the remaining given angle, so is the sine of the First Angle to the sine of the Second Angle, which agrees in kind with the side not given. Therefore, when the angles found are joined as above, the result will be the third angle sought.

X. Regula. Datis Angulis duobus, quorum vnus ſaltem ſit acutus, & Baſi ijs adiacente, quæ ſit Minor quadrante, Quærere Angulum Baſi Oppoſitum.
Rule 10. Given two angles, at least one of which is acute, and the base adjacent to them, which is less than a quadrant, to find the angle opposite the base.
[Margin: By logarithms.]
To the mesologarithm of the acute angle, or (if both are acute) of the smaller of the acute angles, add the second logarithm of the base, and the result will be the second mesologarithm of the First Angle.
Then, to the residue of the logarithm of the First Angle add the second logarithm of the smaller of the given angles and the logarithm of the difference between the larger of the given angles and the First Angle found, and the result will be the second logarithm of the angle sought. But if the base is greater than a quadrant, use the vicarious triangle, as in the scholia of chapter 4.

EXEMPLVM. *Example.*

Let angle A be 37° 13′; its mesologarithm is	M	98805277
Let the base AB be 60°; its second logarithm is	L 2	96989700
Therefore the First Angle is 69° 12′, for	M 2	95794977
The residue of the logarithm of angle 1, 69° 12′	RL	00292694
Angle A, 37° 13′; its second logarithm	L 2	99011062
Angle ABR, 112° 6′, and its difference from angle 1 is 42° 54′, whose logarithm	L	98329691
Therefore the vertical angle ARB is 54° 24′ 30″, for	L 2	97633447

But if the First Angle found were greater than the remaining angle, the larger of the acute ones, then the vertical angle, that is, the angle sought, must be obtuse; therefore, to obtain it, subtract the angle found second from 180°.
[Margin: By sines.]
OR: Let it be done that as the radius is to the sine of the smaller of the given angles, so is the sine of the remaining given angle to the First Found [Inventum primum]. Then let it be done that as the radius is to the First Found, so is the versed sine of the base to the Second Found. Third, add the Second Found to

the versed sine of the difference between either of the given angles and the supplement of the other to 180°, and the result will be the versed sine of the vertical angle sought.

XI. Regula. Datis Angulis duobus ſimul duos rectos non excedentibus, & Baſi adiacente ipſis, Quærere Latus vtrumq. vnico actu. *Rule 11. Given two angles which together do not exceed two right angles, and the base adjacent to them, to find each side in a single operation.*

[Margin: By logarithms.]

First, collect the sum of the angles at the given base and their difference, and finally half the given base. Then, to the residue of the second logarithm of the half-sum of the angles add the second logarithm of their half-difference and the mesologarithm of the half-base, and the result will be the mesologarithm of the half-sum of the sides sought.

Third, to the residue of the logarithm of the half-sum of the given angles add the logarithm of their half-difference and the mesologarithm of half the given base, and the result will be the mesologarithm of the half-difference; this, added to the half-sum of the sides, will make the larger side, and subtracted, the smaller.

EXEMPLVM. *Example.*

Let angle ARB be	54°	34′	
Let angle C be	37°	13′	
Their sum	91°	47′	half-sum 45° 53′ 30″
Their difference	17°	21′	half-difference 8° 40′ 30″
The base AR, 80°			half-base 40°

The residue of the second logarithm of the half-sum of the angles, 45° 53′ 30″, is	RL2	01573149
The second logarithm of the half-difference of the angles, 8° 40′ 30″, is	L2	99950126
The mesologarithm of the half-base, 40°, is	M	99238135
Therefore the half-sum of the sides is 50° 38′ 44″, on account of the mesologarithm	M	100861410
Then		
The residue of the logarithm of the half-sum of the angles, 45° 33′ 30″, is	RL	01438600
The logarithm of the half-difference, 8° 40′ 30″, is	L	91784810
The mesologarithm of the half-base, 40°, is	M	99238135
Therefore the half-difference of the sides is 9° 59′ 47″	M	92461545

This half-difference, added to the half-sum of the sides, makes side AB 60° 38′ 31″; subtracted, it leaves the smaller side BR, 40° 38′ 57″.

[Margin: By sines.]

OR: Let it be done that as the second sine of the half-sum of the given angles is to the second sine of their half-difference, so is the tangent of the half-base to the tangent of the half-sum of the sides. Then let it be done that as the sine of the half-sum of the given angles is to the sine of their half-difference, so is the tangent of the half-base to the tangent of the half-difference of the sides, which is to be added to the half-sum of the sides itself to make the larger side, and subtracted to make the smaller.

XII. Regula. Datis Angulis duobus, quorum ſaltem vnus ſit Acutus, & Baſi Adiacente, quæ ſit minor Quadrante, Quærere Latus vtrumuis. *Rule 12. Given two angles, at least one of which is acute, and the base adjacent to them, which is less than a quadrant, to find either side.*

[Margin: By logarithms.]

To the second logarithm of the base add the mesologarithm of the angle that is given and is opposite the side sought, and the result will be the second mesologarithm of the First Arc, for so the first quantity found is to be called. Then, to the residue of the second logarithm of the First Arc found add the second mesologarithm of the base, and the second logarithm of the difference between the second given angle and the First Arc found, when the side sought is opposite an acute angle; or the second logarithm of the sum made of the second of the given angles and the First Arc found, when the side sought is opposite an obtuse angle; and the result will be the second mesologarithm of the side sought. But if the difference or sum already mentioned is greater than a quadrant, subtract the side found from 180° to have the side sought. But if both of the given angles are greater [?] than an acute angle, or the base is greater than a quadrant, have recourse to the vicarious triangle, which is treated in the scholia of chapter 4.

EXEMPLVM. *Example.*

Let angle ARB be 112° 6′, and let side AB be sought.		
Its complement to 180° is 67° 54′, whose mesologarithm	M	103914120
Let the base AR be 40°; its second logarithm is	L 2	98842540
Therefore the First Found is 27° 55′ 38″, for	M 2	102756660
Then		
Let angle A be 54° 3[?]′; the sum of it and the First Found is 82° 29′ 38″, whose second logarithm is	L 2	91161800
The residue of the second logarithm of the First Found, 27° 55′ 38″, is	RL2	00537750
The second mesologarithm of the base, 40°, is	M 2	100761865
Therefore side AB is 80° 15′, on account of the second mesologarithm	M 2	92461415

[Margin: By sines.]

OR: Let it be done that as the radius is to the tangent of the angle opposite the side sought, so is the second sine of the base to the second tangent of the

First Found. Then let it be done that as the second tangent of the base is to the secant of the First Found, so is the second sine of the difference between the First Found and the second of the given angles, if the side sought is opposite an acute angle — or the second sine of the sum made of the First Found and the other of the given angles, if the side sought is opposite an obtuse angle — to the second tangent of the side sought (if the said sum or difference does not exceed 90°), or of its supplement to 180° (if it does exceed it).

XIII. Regula. Datis Angulis tribus, Quærere Latus quoduis tanquam baſis.
Rule 13. Given the three angles, to find any side as a base.

First, you must determine which side it is that you want to find, and call it the base, and the angle opposite it the vertical angle. Once these are determined, subtract the vertical angle from 180°, and let the remainder be called the Complement of the Vertical. Further, subtract the smaller of the remaining angles from the larger, and let the remainder be called the Difference of the angles. Add this to the Complement of the Vertical, and the result will be the Mystic Sum [Summa mystica], for so I call it for the sake of distinction. Subtract the same difference from the same Complement of the Vertical, and the Mystic Difference will result. Take half of each, and you will have the Mystic Half-sum and also the Mystic Half-difference, which are to be used as we shall say shortly.

[Margin: By logarithms.]

Second, add together these four: the logarithm of the Mystic Half-sum, the logarithm of the Mystic Half-difference, and the residue of the logarithm of each given angle singly; and the result will be a doubled second logarithm, that is, a certain sum, in which the redundant unit will have to be kept, according to what was said in chapter 1, precept 2.

Third, take half of the doubled logarithm, and you will have the second logarithm of half the side sought, as of a half-base; therefore double this will be the whole side sought, that is, the whole base.

EXEMPLVM. *Example.*

In the triangle P, set out at the beginning of the chapter, let the base AR be sought.

Let the vertical angle ABR be	120°
Its complement to two right angles	60°
Let angle BRA be	45° 20′
And angle A	37° 10′
The difference of the angles is	8° 10′
Joined to the complement of the vertical, 60°, this gives the Mystic Sum	68° 10′
Subtracted from the same, it gives the Mystic Difference	51° 50′
Therefore the Mystic Half-sum is	34° 5′

And the Mystic Half-difference	25° 55′

With these prepared,

The logarithm of the Mystic Half-sum, 34° 5′, is	L	97484967
The logarithm of the Mystic Half-difference, 25° 55′, is	L	96405445
The residue of the logarithm of angle BRA, 45° 20′, is	RL	01480030
The residue of the logarithm of angle A, 37° 10′, is	RL	02188656
Therefore the double of the second logarithm is	L L2	197559098
Its half is the second logarithm	L 2	98779549

To this corresponds in the tables the arc 40° 58′ 20″; doubled, therefore, it gives the side AR, that is, the base, 81° 56′ 40″.

[Margin: By sines.]

OR: Let it be done that as the radius is to the second secant of either of the angles adjacent to the base sought, so is the second secant of the other of the said angles to an arc, which is to be called the Found [Inventum]. Then let it be done that as the radius is to the arc found, so is the difference of two versed sines, of which more shortly, to the versed sine of the base sought. One of the said versed sines is to be the versed sine of the vertical angle; the other, the versed sine of the difference between either of the two angles adjacent to the base and the supplement to 180° of the other angle, likewise adjacent to the base.

XIV. Regula. Datis Lateribus Tribus, Quærere Angulum quemlibet tanquam Verticalem. *Rule 14. Given the three sides, to find any angle as the vertical angle.*

First, determine which of the three angles you want to know, and call it the vertical angle; call the side opposite it the base; the others keep the common name of sides. Then add the base to the aggregate of the sides, and let the sum of these be called the Mystic Sum; from the same aggregate of the sides take away the base, and let the remainder be called the Mystic Difference. Take half of each of them, and you will have the Mystic Half-sum and also the Mystic Half-difference, whose use I shall teach shortly.

[Margin: By logarithms.]

Second, then, collect these four into one sum: namely the logarithm of the Mystic Half-sum, the logarithm of the Mystic Half-difference, and the residue of the logarithm of each side taken singly; and the result will be the double of the second logarithm, of which more shortly.

Third, take half of the double of the second logarithm, and you will have the second logarithm of the half vertical angle; doubling it, you will have the vertical angle sought.

EXEMPLVM. *Example.*
In triangle V, in which angle C is sought.

Let side BC be	60°	20′	
Let side RC be	40°	20′	
Their aggregate is	100°	40′	
Let the base BR be	50°	20′	
The Mystic Sum	151°	0′	and the Mystic Half-sum 75° 30′
The Mystic Difference	50°	20′	and the Mystic Half-difference 25° 10′

With these prepared,

The logarithm of the Mystic Half-sum, 75° 30′	L	99859416
The logarithm of the Mystic Half-difference, 25° 10′	L	96286472
The residue of the logarithm of side BC, 60° 20′	RL	00610204
The residue of the logarithm of side RC, 40° 20′	RL	01889391
The sum, the double of the second logarithm	LL 2	198645483
Its half is the second logarithm	L 2	99322721

To this corresponds the half-angle 31° 10′ 34″. Therefore the whole angle C is 62° 21′ 8″.

[Margin: By sines.]

OR: Let it be done that as the radius is to the second secant of one side containing the angle sought, so is the second secant of the other side containing the same angle to the First Found. Then let it be done that as the radius is to the First Found, so is the difference of the versed sines (namely of the base and of the difference of the sides) to the versed sine of the angle sought.

INDEX REGVLARVM PRÆDICTARVM TRIGONOM. *Index of the trigonometric rules already given.*

Given	together with	Sought	Rule
Plane right-angled triangles			**Chapter 2**
Base and	an adjacent angle	the side opposite the given angle	1
		the side adjacent to the given angle	10
	one side	the angle opposite the given side	2
		the angle adjacent to the given side	11
		the other side	9
One side and	the angle adjacent to it	the base	3

Given	together with	Sought	Rule
		the other side	4
	the opposite angle	the base	6
		the other side	7
Two sides		the base	8
		either of the acute angles	5
Plane oblique-angled triangles			**Chapter 3**
Three sides		any angle	4
Two sides and	the vertical angle, that is, the angle they contain	the base	5
		either angle at the base	3
	the angle opposite one of the sides	the base	6
		the angle opposite the remaining given side	1
A single side and	two angles, one of which is opposite the given side	the side opposite the other angle	2
		the side opposite the third angle	7
	two angles, one of which is adjacent to the given side	the remaining sides	8
Spherical right-angled triangles			**Chapter 4**
Base and	the angle adjacent to the base	the side opposite the given angle	1
		the side adjacent to the given angle	2
		the other angle adjacent to the base	3
	one side	the angle opposite the given side	4
		the angle adjacent to the given side	5
		the other side	6

RESIDVVM INDICIS REGVLARVM. *The rest of the index of rules.*

Given	together with	Sought	Rule
Spherical right-angled triangles			**Chapter 4**
One side and	the angle adjacent to it	the other side	7
		the angle opposite the given side	8
		the base	9
	the angle opposite it	the other side	10
		the base	11
		the other angle	12
Two sides		the base	13

Given	together with	Sought	Rule
		the angle opposite either of the sides	14
Two angles		the base	15
		the side opposite either angle	16
Spherical oblique-angled triangles			**Chapter 5**
Two sides and	the angle opposite one of them	the angle opposite the other side	1
		the base	2
		the vertical angle	3
	the vertical angle	the base	4
		each angle at the base	5
	an acute vertical angle	the angles at the base	6
Two angles and	the side opposite one of them	the side opposite the other of the angles	7
		the base	8
		the third angle	9
	the base adjacent to them	the angle opposite the base	10
		each side	11
	a base less than a quadrant	each side	12
All three angles		any side	13
All three sides		any angle	14

SECTIO SECVNDA PROBLEMATVM PRIMI MOBILIS.

Section Two: Problems of the Primum Mobile.

PROHOEMIVM. *Preface.*

We call Problems of the Primum Mobile those that are general and common to all stars, whether fixed or wandering, and also to the points or parts themselves of the ecliptic, the equator, the meridian, the horizon, the vertical, and the other great circles that are usually marked out in the highest heaven, and to the angles arising from the intersection of the circles already mentioned with one another — whether that highest heaven is in reality the primum mobile or not. Further, for the notions of the terms, and for the definitions of the arcs or angles to be used below, the definitions we gave in Book 1 of this work, chapter 22, should be read again.

[Margin: Advice for the practice of the following problems.]

As for the order of the problems, although it can vary, we shall follow the one that can, as it were, lead the reader by the hand from simpler and better-known phenomena to others less known and more involved. And although, on the occasion of figures explained once for one or a few problems, we think it right to give other related problems as well, so that we are not forced to insert and explain the same figure too often, everyone will nevertheless be satisfied if they consult the alphabetical index of the problems already explained at the end of this book, so that they may be readily at hand without confusion of order. In

place of the examples that someone might want, the general examples given in the first section for solving any triangles will suffice — especially in chapter 4, where we taught how to solve spherical right-angled triangles, and in chapter 5, where we taught how to solve spherical oblique-angled ones; nor will there be any need except to send the reader to the rules given there, or else to the examples given in other books of this work of ours — apart from a very few cases, very frequent in astronomy, which we shall illustrate with examples. Finally, we shall teach the construction of the Tables of the Primum Mobile from the problems proper to them, in such a way, however, that we add the principal authors who have already constructed the tables already mentioned.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

Auctores, vnde aliqua ſequentium Problematum. *Authors from whom some of the following problems come.*

Although, God granting, we are going to present many new problems devised by us, it is nevertheless right to praise those authors who have handed down to us not a few of the following problems, granted that they did so in another and not infrequently more perplexing way. Such are Ptolemy, books 1 and 2 of the *Almagest*; Geber [Jābir ibn Aflaḥ], books 1 and 2 of his *Astronomy*; Copernicus, books 1 and 2 of the *Revolutions*; Regiomontanus, in 63 problems of the Primum Mobile and 31 precepts of Directions; Erasmus Reinholdus [Erasmus Reinhold], in 60 precepts of Directions; Petrus Apianus, in 100 propositions of the Primum Mobile; Erasmus Osualdus, in 70 propositions of the primum mobile; Daniel Santbech, in 159 propositions of Astronomical and Geometrical Problems; Io. Antonius Maginus, from book 5 to book 12 of the *Primum Mobile*, where he has 199 problems, besides 55 canons in his work on Directions; Longomontanus, book 2 of the *Spherics*, giving 59 problems; Clavius, here and there in the *Astrolabe*; Bonaventura Cavalieri, in the *Centuria* of problems; Hérigone, in the common problems; Petavius [Denis Pétau], in the *Uranologion*; Bartholomaeus Pitiscus [Bartholomäus Pitiscus], book 1 of the *Astronomical Problems*, where see problem 16, and the single book of *Geographical Problems*; Villebrordus Snellius [Willebrord Snellius], at the end of his doctrine of triangles, where he solves 8 spherical problems — besides others who have inserted some problems of this kind here and there in their works, such as Tycho in volumes 1 and 2 of the *Progymnasmata*, and those who have treated comets trigonometrically.

1. Problema. Meridianam Lineam, & Poli atque Aequatoris Altitudinem Inuenire. *Problem 1. To find the meridian line, and the altitude of the pole and of the equator.*

[Margin: Problem 1.]

I taught the finding of the meridian line sufficiently in Book 1 of this *Almagest*, chapter 10, and the finding of the altitude of the pole in the same book, chapter

12; so go back there. It was right, however, to mention them here first, since these two are the most universal foundations of astronomical observations and of most problems. The other methods, less necessary but still useful, or very ingenious, we shall give in the Section of Geographical Problems. Now if you subtract the altitude of the pole from 90°, you will have the distance of the pole from the vertex, which is always as great as the elevation of the equator; just as the distance of the equator from the vertex, that is, from the zenith, is as great as the altitude of the pole.

2. Probl. Declinationem Solis Maximam vel Minimam Obſeruare, & ex vtraq. vel vtralibet Obliquitatem Eclipticæ deducere, Quæ ſemper tanta eſt, quanta Diſtantia Tropicorum ab Aequatore; quantaq Diſtantia Polorum Eclipticæ & Aequatoris. *Problem 2. To observe the greatest or the least declination of the Sun, and from both, or from either, to deduce the obliquity of the ecliptic, which is always as great as the distance of the tropics from the equator, and as great as the distance between the poles of the ecliptic and of the equator.*

[Margin: Problem 2.]

In the adjoining figure, let the horizon be HR, and the equator the circle AQ, whose axis is BP; and let the meridian be HZRN, in which the zenith, that is, the vertex of the observer, is Z, and the nadir N; and the poles of the equator B and P.

Diagram. A small woodcut in the left column beside the text of problem 2: a circle (the meridian HZRN) with the horizontal diameter HR, the equator AQ drawn through the centre, the axis BP drawn through the centre, and a further chord from C towards the lower right. Small stars stand on the circle at K, G, C and M, marking the stars and solar positions discussed in problems 2–4. Labels, clockwise from the top:

- K (top, left of Z): the Sun's place at the summer solstice, a star at K
- Z (top): zenith
- G (top, right of Z): a star at G
- B (upper right): pole of the equator
- M (right, above R): a star at M
- R (right): horizon
- Q (lower right): equator
- N (bottom): nadir
- P (lower left): pole of the equator
- H (left): horizon
- C (left, above H): the Sun's place at the winter solstice, a star at C
- A (upper left): equator

First, then, suppose you have observed the Sun's greatest meridian altitude HK — namely the one it has at noon on the day of the summer solstice — and have corrected it by subtracting the refraction and adding the solar parallax. If from the Sun's altitude so corrected, that is, its true altitude, you subtract the altitude of the equator HA, the arc AK will be left, that is, the greatest

declination of the Sun. I gave examples of this practice in Book 3, chapter 27, number 3.

Second, you will obtain the same result even without the altitude of the equator, by two meridian altitudes of the Sun: the greatest, namely at noon of the summer solstice, when the Sun is at K, and the least at noon of the winter solstice, when it is at C, correcting both by adding the parallax and subtracting the solar refraction. For if you subtract the least, HC, from the greatest, HK, the distance of the tropics CK will remain; half of it, AK or AC, will be the greatest declination sought, as I taught with examples in Book 3, chapter 27, number 2. Indeed, if you add this, that is CA, to the least true altitude of the Sun, HC, you will have the altitude of the equator HA.

3. Probl. Declinationem quamlibet Solis & cuiuſuis ſideris ex Altitudine Meridiana; 4. Probl. Vel Altitudinem Meridianam ex Declinatione deducere. *Problem 3. To deduce any declination of the Sun and of any star from the meridian altitude; Problem 4. Or the meridian altitude from the declination.*

[Margin: Problem 3.]

Observe the meridian altitude of a star with an astronomical quadrant, or any other instrument for altitudes, and correct it if need be by adding the parallax and subtracting the refraction, and you will have the true altitude. Then go back to the figure of problem 2. For if the star observed was between your vertex and the part of the horizon farther from the visible pole of the world, the altitude of the equator HA, subtracted from the greater altitude HK of the star standing at K, will leave the northern declination of the star, AK, if the observer is northern, or the southern, if southern. Conversely, if the star is at C, and its true altitude HC is less than the altitude of the equator HA, the former taken from the latter will leave the declination of the star AC — southern if the observer is northern, northern if southern. But if the star was between the vertex and the part of the horizon nearer the visible pole of the world, as if it is at G or at M, the altitude of the pole RB, taken from the true altitude RG of the higher star, will leave GB, its distance from your pole, which taken from 90° will leave the declination AG of star G, towards the same pole. But if the altitude RM of star M is less than the altitude of the pole RB, then, the former being subtracted from the latter, BM will be left, the distance from the pole, which, again taken from 90°, will leave the declination QM of star M, likewise towards your pole. See the example in Book 6, chapter 11, problem 1. The declination, and indeed also the right ascension, of a star, even one placed outside the meridian, can also be observed with equatorial armillae set in a position analogous to the celestial sphere; but the method given above is much easier and more evident.

[Margin: Problem 4.]

But if the true declination of a star were known, not changed about noon — say the arc CA or AK — then, by subtracting the former and adding the latter to the altitude of the equator HA, the true meridian altitude of the star would

become known; and by taking the parallax from it and adding the refraction, the apparent meridian altitude will be known; and so for the rest.

Probl. Solis Locum in Ecliptica inquirere Data ipſius 5. Declinatione Meridiana, & Obliquitate Eclipticæ. 6. Declinatione Meridiana & Aſc. Recta Solis. 7. Obliquitate Eclipt. & Aſc. Recta Solis. *Problem. To find the place of the Sun in the ecliptic, given its: 5. meridian declination and the obliquity of the ecliptic; 6. meridian declination and the right ascension of the Sun; 7. the obliquity of the ecliptic and the right ascension of the Sun.*

[Margin: Problem 5.]

In the adjoining diagram, let the equator be AIL, and the ecliptic AVLPA; and let the arcs of the meridians by which the declination of the points of the ecliptic, and so of the Sun, is measured be GDP, ♋IV♑ and VHS.

Diagram. A woodcut in the right column: a lens-shaped figure formed by the straight equator AIL and the two arcs of the ecliptic, the upper arc through G, ♋ and V and the lower arc through P, ♑ and S, crossed by three meridian arcs GDP, ♋IV♑ and VHS. Labels:

- A (left point), with ♈ below it: beginning of Aries
- L (right point), with ♎ beside it: beginning of Libra
- G, ♋, V (upper arc, left to right): points of the ecliptic; ♋ the beginning of Cancer
- P, ♑, S (lower arc, left to right): points of the ecliptic; ♑ the beginning of Capricorn
- D, I, H (on the equator, left to right): feet of the three meridian arcs

First, then, if the declination is nil, the Sun is at the beginning of Aries, A, or of Libra, L, which you will tell apart from the difference of season. But if the declination is the greatest and summer one, ♋I, the Sun will be at the beginning of Cancer; if the winter one, ♑I, at the beginning of Capricorn. Outside these cases one must know beforehand in which quadrant of the ecliptic

[Margin: Order of the problems.]

[Margin: Order of the problems.]

it lies — whether in the first, AG♋, namely between the vernal equinox and the summer solstice, as when the Sun is at G. For then, in triangle ADG, right-angled at D, from the given declination GD and the angle of the obliquity of the ecliptic GAD, you will gather the Sun's longitude from the beginning of Aries, AG, that is, the base, by Rule 11 of spherical right-angled triangles. But if it is in the 2nd quadrant, as at V, find by the same rule the base VL in triangle VHL, right-angled at H, since the Sun's declination VH and the obliquity of the ecliptic HLV are given; for when the base VL is taken from 180°, the Sun's elongation from the beginning of Aries, AGV, will be left, and so its place in the ecliptic. Here, however, we suppose the straight lines AIL, GP, VS, etc., to be substituted for arcs of great circles. If the Sun is at S, a point of the third quadrant, look likewise for the base SL in triangle SHL, right-angled at H, in which the declination SH and the obliquity HLS are given; add this base to

180°, and the result will be the Sun's longitude from the beginning of Aries. Finally, if the Sun is at P, a point of the fourth quadrant of the ecliptic, look by the same rule for the base AP in triangle ADP, right-angled at P [?]; for the declination DP and the obliquity DAP are given. Take this base from 360°, and what is left will be the Sun's longitude from the beginning of Aries, reckoned according to the succession of the signs.

[Margin: Problem 6.]

But if the Sun is at G, and the declination GD and the right ascension AD are given, you will gather the base AG, that is, the longitude of the Sun, by Rule 13 of spherical right-angled triangles, since in triangle ADG, right-angled at D, two sides are given. Or, if the Sun is at V, its right ascension AH, taken from 180°, will leave the side HL; with it and with the declination VH you will gather, by the same Rule 13, the base VL, which, taken from 180°, gives its longitude; and so for the other cases.

Diagram. A woodcut in the left column, of the same design as the figure of problem 5 on the preceding page: a lens-shaped figure formed by the straight equator AIL and the two arcs of the ecliptic, the upper arc through G, ♋ and V and the lower arc through P, ♑ and S, crossed by three meridian lines GDP, ♋I♑ and VHS. Labels:

- A (left point), with ♈ beside it: beginning of Aries
- L (right point), with ♎ beside it: beginning of Libra
- G, ♋, V (upper arc, left to right): points of the ecliptic; ♋ the beginning of Cancer
- P, ♑, S (lower arc, left to right): points of the ecliptic; ♑ the beginning of Capricorn
- D, I, H (on the equator, left to right): feet of the three meridian lines

[Margin: Problem 7.]

But if the Sun is at G, and the obliquity GAD and the Sun's right ascension AD are given, you will gather the base AG by Rule 9 of spherical right-angled triangles. The rest you now understand.

Probl. Aſcenſionem Rectam Solis, ſeu Puncti Eclipticæ, in quo reperitur Sol, inueſtigare, Datis 8. Longitudine Solis, & Obliquitate Eclipticæ 9. Declinatione, & Longitudine Solis 10. Obliquitate Eclipticæ, ac Declinat. Solis *Problem. To find the right ascension of the Sun, or of the point of the ecliptic in which the Sun is found, given: 8. the longitude of the Sun and the obliquity of the ecliptic; 9. the declination and the longitude of the Sun; 10. the obliquity of the ecliptic and the declination of the Sun.*

[Margin: Problem 8.]

First, in the figure of problem 5, let the Sun be at G, a point of the first quadrant, and let its longitude be given, that is, the base AG, together with the obliquity of the ecliptic GAD. Then in triangle ADG, right-angled at D, you will gather the side AD, that is, the Sun's right ascension, by Rule 2 of spherical right-angled triangles. But if the Sun is in the 2nd quadrant, as at

V, its longitude AV, taken from 180°, will give the base VL; with it and with the obliquity VLH, by the same Rule 2, you will gather the side HL, which, taken from 180°, leaves the Sun's right ascension AH. If, however, the Sun is in the 3rd quadrant, as at S, take 180° from the longitude and the base SL will remain; with it and with the obliquity HLS, look by the same Rule 2 for the side HL, which, added to 180°, makes the Sun's right ascension. Lastly, if the Sun is in the 4th quadrant, as at P, the Sun's longitude taken from 360° will give the base AP; with it and with the obliquity DAP, look for the side AD, which, taken from 360°, leaves the right ascension of ☉.

[Margin: Problem 9.]

Second, let the declination and the longitude be given. In all four cases already mentioned, the base will be given from the longitude, and from the declination the side adjacent to the base, say the side GD, or VH, etc.; with these, by Rule 6 of spherical right-angled triangles, find the other side, which will be either the right ascension itself or its argument, as was said in the three later cases of problem 8.

[Margin: Problem 10.]

Third, let the obliquity of the ecliptic be given, and the side opposite it, that is, the declination — say the angle GAD and the side GD. Then by Rule 10 of spherical right-angled triangles you will have the other side, which will be the right ascension, as AD, or its argument, as in the other three cases already mentioned.

Probl. Obliquitatem Eclipticæ explorare, Datis 11. Declinatione, ac Longitudine Solis 12. Declinatione, & Aſc. Recta Solis 13. Longitudine & Aſcenſ. Recta Solis *Problem. To find out the obliquity of the ecliptic, given: 11. the declination and the longitude of the Sun; 12. the declination and the right ascension of the Sun; 13. the longitude and the right ascension of the Sun.*

[Margin: Problem 11.]

The first method is if the declination of the Sun is given, say the side GD (namely in the figure of problem 5), and the longitude, that is, the base AG, from astronomical tables or ephemerides, or from elsewhere. For by Rule 4 of spherical right-angled triangles you will obtain the angle GAD, that is, the obliquity of the ecliptic; and you will work in the same way when the Sun's place is in the other quadrants of the ecliptic, for you will always know from the longitude which base is to be used.

[Margin: Problem 12.]

The second method is, given the Sun's declination GD and right ascension AD — that is, two sides: for then by Rule 14 of spherical right-angled triangles you will learn the angle of the obliquity of the ecliptic; and so in the other three cases, which have been dealt with more than once above.

[Margin: Problem 13.]

The third method is if the Sun's longitude AG, that is, the base, and the right ascension AD are given: for by Rule 5 of spherical right-angled triangles you

will obtain the angle of the obliquity of the ecliptic; and you will proceed in the same way in the other three cases.

We pass over the remaining methods by which the distance of the poles could be verified from the declination and latitude of other stars, or from their longitude or right ascension, since that distance is rather presupposed for those properties, as being much more certain.

14. Probl. Angulum Eclipticæ & Meridiani determinare, Datis Eclipticæ Graau quouis, & Obliquitate; ipſiuſq. Tabulam conſtruere. Idem verò Angulus vocatur quoque Eclipticæ & Circuli Horarij, vel Declinationis. *Problem 14. To determine the angle of the ecliptic and the meridian, given any degree*[52] *of the ecliptic and the obliquity; and to construct its table. This same angle is also called the angle of the ecliptic and the hour circle, or the circle of declination.*

[Margin: Problem 14.]

The meridian, cutting the ecliptic at the solstitial points, makes four right angles with it; but at the equinoctial points it makes two acute angles, vertically opposite each other, and two other, obtuse, ones. Any one of the acute angles is found by subtracting the obliquity of the ecliptic from 90°; the obtuse, by adding it to 90°. Outside these positions trigonometry is needed; but it is enough to find this angle for any degree of the first quadrant of the ecliptic — namely, in the figure of problem 5, for any degree of the arc AG♋. Let there be, then, the spherical triangle ADG, right-angled at D, in which the obliquity of the ecliptic, that is, the angle GAD, is given, and the longitude AG of the given point G, namely the base. Then by Rule 3 of spherical right-angled triangles you will find the angle AGD that the meridian, of which GD is a portion, makes with the ecliptic; and the supplement of this angle to 180° will be the angle DGV. The same angle could be found given the obliquity of the ecliptic and the declination of the given point of the ecliptic; or from the distance from the nearer equinox and the declination; or from the distance and the right ascension; but the way I have taught is simpler and comes first. Scheiner, however, in the *Rosa Ursina*, from page 356 to 363, suggests two other methods, mechanical ones: one by the analemma, from the astrolabe of Io. Roias [Juan de Rojas]; the other by an instrument he devised himself.

[Margin: The table's construction, and its authors.]

Once a table of this angle has been constructed, then, for every degree of the quadrant AG♋, it will serve the three remaining quadrants; for that angle is of the same size at any four degrees of the ecliptic equally distant from the nearer equinoctial or solstitial point — say at the degrees G, V, S, P. Moreover, a table of this kind is given by Copernicus, book 2, chapter 3; by Lansbergen, page 99 of the *Tables*; by Io. Antonius Maginus, in the *Directions of the Primum Mobile*, page 199; by Boulliau, in the *Philolaic Tables*, page 140; and by Scheiner, in the *Rosa Ursina*, page 355,

[52] printed *Graau*, for *Gradu*

[Margin: Order of the problems.]
[Margin: Order of the problems.]
supposing an obliquity of 23° 30′; Kepler, in the *Rudolphine Tables*, folio 24; and Longomontanus, in the *Astronomia Danica*, book 1 of the *Spherics*, folio 60, from Tycho's obliquity. Five other methods could have been added, of which we shall speak from problem 76 on; but this one is simpler and more common.

15. Probl. Declinationem Solis, aut cuiuſuis puncti aut gradus Eclipticæ Inquirere, Datis Obliquitate & Gradu quouis Eclipticæ, ac Tabulam Declinationum harum conſtruere. *Problem 15. To find the declination of the Sun, or of any point or degree of the ecliptic, given the obliquity and any degree of the ecliptic; and to construct a table of these declinations.*

[Margin: Problem 15.]

Let the figure of problem 5 be repeated here, in which the spherical triangle ADG is right-angled at D; and in it let the base AG be given, from the given degree of the ecliptic in the first quadrant, and also the angle of the obliquity GAD. Then by the first rule of spherical right-angled triangles you will find the side GD, that is, the declination belonging to the degree G; and so for the other degrees of the first quadrant A♋. For this it will be enough to construct a table, since it will serve the remaining quadrants: the declination of any four points of the ecliptic equally distant from the nearer equinoctial or solstitial point is equal.

[Margin: The table's construction, and its authors.]

Moreover, a table of the declination of the Sun, and so of the points of the ecliptic, has already been constructed and handed down to us by Ptolemy, book 1 of the *Almagest*, chapter 13; Orontius [Oronce Finé], book 2 of the *Cosmographia*, chapter 4; Copernicus, book 2, chapter 2; Reinhold and Regiomontanus, in the *Tables of Directions*, page 1 or 2 of the tables; Clavius, in the *Sphere*, page 264; Maginus, in the *Tables of Directions*, page 34; Tycho, volume 1 of the *Progymnasmata*, page 80; Longomontanus, book 1 of the *Spherics*, folio 55; Boulliau, in the *Philolaic Tables*, page 140; the *Rudolphine Tables*, folio 24; Lansbergen, page 94; and Griembergerus [Christoph Grienberger], in Scheiner's *Rosa Ursina*, page 391. This last table is worked out to every 5′ of the degrees, whereas those of Reinhold, Clavius and Tycho go to every 10′, and the rest to single degrees — each of them, however, according to the obliquity of the zodiac that its author thought had held, or held, in his own time.

16. Probl. Aſcenſionem Rectam cuiuſuis Puncti Eclipticæ Inueſtigare, Datis Eclipticæ Obliquitate, ac puncto, Eiuſque Tabulam componere. *Problem 16. To find the right ascension of any point of the ecliptic, given the obliquity of the ecliptic and the point; and to compose its table.*

[Margin: Problem 16.]

Above, in problem 8, we taught how to derive the Sun's right ascension from the Sun's longitude, that is, from the given point of the ecliptic, and from

the obliquity of the ecliptic. In exactly the same way the right ascension of any degree of the first quadrant of the ecliptic, AG♋, is gathered, and so a table is constructed which serves for the whole ecliptic. For, given a point of the ecliptic in the 2nd quadrant, ♋L — say V — and hence its distance from the solstitial point ♋, the right ascension AD of the point G, equally distant from the same point ♋, is sought; and this (that is, HL, which is equal to it), subtracted from 180°, the degrees of the whole semicircle AIL, leaves AH, namely the right ascension of the point V; and so for the rest, as was said in problem 8.

[Margin: The table's construction.]

Tables of this kind have been supplied to us by Ptolemy, book 2 of the *Almagest*, chapter 8; Orontius, book 2 of the *Cosmographia*, chapter 3; Regiomontanus, in the *Directions*, page 52; Copernicus, book 2, chapter 3; Reinhold, in the *Tables of Directions*, from page 86, and in the *Prutenic Tables*, page 1; Clavius, in the *Sphere*, from page 330; Maginus, in the *Tables of Directions*, page 54; Boulliau, in the *Philolaic Tables*, page 141; Tycho, volume 1 of the *Progymnasmata*, page 86; Lansbergen, page 97 of the *Tables*; Longomontanus, book 2 of the *Spherics*, folio 57; Santbech, page 103; the *Rudolphine Tables*, folio 24; and Grienberger, in Scheiner's *Rosa Ursina*, page 391, a different table for each different obliquity of the ecliptic. Grienberger's goes to every 5′ of the degrees; Reinhold's and Tycho's to every 10′; the others', except Ptolemy's, to single degrees.

Probl. Aſcenſionem Rectam Sideris Obſeruare 17. More antiquo per Clepſydras 18. Per diſtantiam Lunæ à Sole, & Sideris à Luna 19. Per diſtantiam Veneris à Sole, & Sideris à Venere 20. Per Diſtãtiam Sideris à Saturno, aut Ioue 21. Per diſtantiam ipſius Sideris à Sole 22. Per Aſc. Rectam Solis, & tempus inter eam ac tranſitum Sideris per medium cæli 23. Per Aſc. Rectam vicinæ ſtellæ, & tranſitum Sideris per M. C. *Problem. To observe the right ascension of a star: 17. in the ancient manner, by water clocks; 18. by the distance of the Moon from the Sun, and of the star from the Moon; 19. by the distance of Venus from the Sun, and of the star from Venus; 20. by the distance of the star from Saturn or Jupiter; 21. by the distance of the star itself from the Sun; 22. by the right ascension of the Sun, and the time between it and the star's transit through mid-heaven; 23. by the right ascension of a neighbouring star, and the star's transit through the M. C. (mid-heaven).*

[Margin: Problems 17, 18, 19, 20, 21, 22, 23.]

I have already given all these problems, in the same order, in Book 6, chapter 11, from number 4 to 11, with examples added on fixed stars. They can also be applied to the planets, provided that their observed and apparent distances are converted into true ones by applying parallax and refraction, according to what is to be said in the Section on parallaxes and refractions. There is therefore no need to repeat them here.

24. Probl. Aſcenſionis Rectæ, ac Declinationis Siderum præſertim Fixorum Incrementa ac Decrementa, creſcente Longitudine eorum, diſcernere ac Determinare. *Problem 24. To distinguish and determine the increments and decrements of the right ascension and declination of the stars, especially the fixed stars, as their longitude increases.*

[Margin: Problem 24.]

This whole business, likewise, I dealt with in Book 6, chapters 13 and 19, where I showed the new finding we had detected: namely, that there are some fixed stars whose right ascension decreases continuously for many years, although their longitude increases; and which stars of this kind there are, and how long the time of this decrease lasts, etc.

Probl. Declinationem Sideris extra Meridianum poſiti Obſeruare 25. Per Altitudinem, Azimuthumq. Sideris, cum Altitud. Poli 26. Per Altitudinem Azimuthumq. Sider. & Tempus 27. Per Altitudinem Sider. Tempus & Altitud Poli 28. Per Azimuthum Sideris, Tempus, & Altitud. Poli *Problem. To observe the declination of a star placed outside the meridian: 25. by the altitude and azimuth of the star, with the altitude of the pole; 26. by the altitude and azimuth of the star, and the time; 27. by the altitude of the star, the time, and the altitude of the pole; 28. by the azimuth of the star, the time, and the altitude of the pole.*

[Margin: Problem 25.]

We have indeed given these very problems in Book 6, chapter 11, numbers 3 and 4. But because of other, related problems to be drawn shortly from the same diagram, it seemed good to repeat them here somewhat more distinctly. Here, however, let us presuppose the method of finding or observing the azimuth of a star, which is to be set out a little below, that is, from problem 34 on. For these problems, then, in the following figure, let the horizon be HR, and the meridian HZRQ, in which the vertex, or zenith, is Z, and the pole of the equator EOQ is P; and let ZA be the quadrant of a vertical circle, or circle of altitude, on which the star is either at B, on this side of the equator, or at G, a point of the equator, or at I, beyond the equator (which you will soon tell from the size of the declination). And from the pole of the world let arcs of circles of declination be drawn through the star as far as the equator or beyond, namely PBN, PG and PCI. The azimuth of the star will then be the arc HA, which the angle AZH measures; and the altitude of the pole will be PR, whose complement to 90° will be the distance of the pole from the vertex, PZ.

[Margin: In this figure the point G ought to be on the vertical ZA.]

Diagram. A woodcut in the right column beside the setting-out of problems 25–28: a circle (the meridian HZRQ) with the horizontal diameter HR (the horizon); the equator EOQ drawn as an arc from E at the upper left through O below the centre to Q at the lower right; the pole P at the upper right; arcs of circles of declination drawn from P through B, G and C; and the vertical quadrant from Z towards A. Labels:

- Z (top): zenith

- E (upper left): the equator's end on the meridian
- B (below the top): the star on this side of the equator
- P (upper right): pole of the equator
- N (left, below E): where the arc PBN meets the equator
- G (centre left): the star at a point of the equator
- C (centre left, beside G): on the arc PCI
- I (lower left): the star beyond the equator
- H (left): horizon
- A (inside, near H): foot of the vertical quadrant ZA on the horizon
- O (below the centre): on the equator
- R (right): horizon
- Q (lower right): the equator's end on the meridian

First, then, let the star be at B. In the triangle BZP let the distance of the pole from the vertex, PZ, be given from the altitude of the pole; and from the true altitude of the star, AB (the observed altitude, that is, corrected by adding the parallax and subtracting the refraction), let its complement to 90°, BZ, be given; and the angle BZP, comprehended by these sides, which is the supplement to 180° of the azimuthal angle

[Margin: Order of the problems.]

[Margin: Order of the problems.]

that was observed, AZH. Then by Rule 4 of spherical oblique-angled triangles you will find the base BP, which, subtracted from 90°, will leave the true declination of the star, BN — northern if P is the north pole, southern if it is the south. But if the star is at G, the complement of the true altitude will be GZ, and the rest as above; and the base PG will be found to be of 90°, which will be a sign that the star has no declination but lies on the equator itself. If, however, the star is at I, the complement of the true altitude will be IZ, and the rest as above; but the base PI will come out greater than a quadrant, and you must then take 90° from it, so that what remains is the star's declination towards the other pole, IC. For the rest, you will solve the triangles ZGP or ZIP by the same Rule 4.

[Margin: Problem 26.]

Second, let the star be at B, and let its true altitude AB be given, and its complement to 90°, BZ; and also the azimuth AZH, and its supplement to 180°, BZP; and, from the time before or after noon converted into parts of the equator, the angle ZPB. Then by Rule 7 of spherical oblique-angled triangles you will find the side BP, the complement to 90° of the declination BN. But if the star is at G, the complement of the altitude will be GZ, and the rest given as above; but by the same Rule 4 you will find the side GP to be a quadrant, and therefore there will be no declination. If the star is at I, and the complement of the altitude is IZ, the rest being given as above in the triangle ZIP — that is, the azimuthal angle at Z and the angle of the time IPZ — you will find the side PI greater than a quadrant; and when 90° are subtracted from it, the star's declination towards the other pole, IC, will be left.

[Margin: Problem 27.]

Third, let the star be at B; and let its true altitude AB be given, and its complement to 90°, BZ, and PZ, the complement of the altitude of the pole; and the angle BPZ, from the time before or after noon converted into parts of the equator. Then by Rule 2 of spherical oblique-angled triangles you will find the base BP; when this is subtracted from 90°, the declination BN remains. But if the base PG is found to be a quadrant — namely from the angle GPZ, the complement of the altitude GZ and the side ZP — there will be no declination. If, however, the base is found to be greater than a quadrant, as PI — namely in triangle ZIP, from the angle IPZ, the complement of the altitude IZ and the side ZP — its excess over 90° will be the star's declination IC, but towards the other pole.

[Margin: Problem 28.]

Fourth, let the star be at B; and let there be given, as above, the arc ZP, as a base known from the altitude of the pole, and the angles adjacent to it: BZP, known from the azimuth, and BPZ, from the time. Then by Rule 11 or 12 of spherical oblique-angled triangles you will find the side BP, the complement of the declination NB; or by the same rule you will find, from the angle GPZ or IPZ and the rest given as above, the quadrantal arc PG, the sign of no declination, or PI, greater than a quadrant, from which, when 90° are taken away, CI will be known, the declination towards the other pole.

[Margin: Other methods indicated.]

Other problems could indeed arise if the angle B, or G, or I were given; but that angle is not usually known beforehand here — rather, it is derived from the data already mentioned, as I shall teach shortly.

Probl. Angulum in centro Sideris factum à Circulo Verticali & Declinationis Inquirere, Datis 29. Declinatione & Altitudine Sideris ac Poli 30. Declinatione & Altitudine Sideris atque Azimutho 31. Declinatione, & Altitudine Sideris, ac Tempore 32. Altitudine Sideris, ac Poli, & Azimutho 33. Altitudine Sideris, Azimutho, ac Tempore *Problem. To find the angle made at the centre of a star by the vertical circle and the circle of declination, given: 29. the declination and the altitude of the star and of the pole; 30. the declination and the altitude of the star, and the azimuth; 31. the declination and the altitude of the star, and the time; 32. the altitude of the star and of the pole, and the azimuth; 33. the altitude of the star, the azimuth and the time.*

In the preceding figure ZP is the complement of the altitude of the pole, and the angle at Z is known, since it is the supplement to 180° of the azimuth AZH. Now if the star is at B, with the declination BN towards your pole, the complement of the declination is BP, and that of the altitude BZ. But if it is at G, the declination is nil, and its complement is GP, of 90°, while the complement of the altitude is GZ; if, however, it is at I, the complement of the altitude is IZ, and IP is made up of the declination IC and the quadrant CP. So the triangle to be solved will be either BZP, or GZP, or IZP; and the angle at the pole P is

to be known from the time before or after noon, converted into parts of the equator. Therefore, from the solution of one triangle, BZP, you will learn how the other two are to be solved, so that we are not too long.

[Margin: Problem 29.]

First, then, let the complement of the declination, BP, be given, and that of the altitude, BZ, and the base ZP. Then by Rule 14 of spherical oblique-angled triangles you will find the angle ZBP that the vertical ZA makes at the centre of the star B with the circle of declination PN.

[Margin: Problem 30.]

Second, let the complement of the declination, BP, be given, and that of the altitude, BZ, and from the azimuth the angle at Z. Then by Rule 3 of spherical oblique-angled triangles you will learn the angle ZBP.

[Margin: Problem 31.]

Third, let the complement of the declination, BP, be given, and that of the altitude, BZ, and from the time the angle BPZ; for by the same Rule 3 of spherical oblique-angled triangles you find the angle ZBP.

[Margin: Problem 32.]

Fourth, let the complement of the altitude, BZ, be given, and that of the pole, ZP, and from the azimuth the angle BZP; for by Rule 5 or 6 of spherical oblique-angled triangles you will find the angle ZBP.

[Margin: Problem 33.]

Fifth, let the complement of the altitude, BZ, be given, and from the azimuth the angle BZP, and from the time the angle BPZ. Then by Rule 9 of spherical oblique-angled triangles you will learn the angle ZBP. And the same rules will serve you in the other triangles, GZP or IZP, if you use data of the same kind. Five other problems could have been added, from five other combinations of the data already mentioned; but it is enough to have indicated these, for I shall explain the rest from problem 89 on.

Probl. Azimuthum ſideris Inueſtigare, Datis 34. Declinatione, & Altitudine Sideris ac Poli 35. Declinatione, Altitudine Sideris, ac Tẽpore 36. Declinatione Sid. Altitud. Poli, ac Tẽpore 37. Altitud. Poli, & Sideris, ac Tempore *Problem. To find the azimuth of a star, given: 34. the declination, and the altitude of the star and of the pole; 35. the declination, the altitude of the star and the time; 36. the declination of the star, the altitude of the pole and the time; 37. the altitude of the pole and of the star, and the time.*

Let the figure set out at problem 25 be repeated here, in which the azimuth, that is, the angle AZH, is sought. This becomes known if the angle AZP is known, since that is its supplement to 180°. Further, if the declination is less than a quadrant, as BN, its complement will be BP, and that of the altitude BZ; and the angle BPZ will be known from the time after or before noon, converted into parts of the equator; and so the triangle BZP will have to be solved. But if there is no declination, its complement GP will be of 90°, and that of the altitude GZ, and the angle GPZ known from the time; and the

triangle GZP is to be solved. If, however, the declination is towards the other pole, as CI, the side IP will have to be compounded from the declination IC and the quadrant CP, and the complement of the altitude will be IZ, and the angle IPZ known from the time; and the triangle IPZ is to be solved. It is therefore enough to indicate how, by solving the triangle BZP, the angle BZP, the complement of the azimuth AZH, can be found.

[Margin: Problem 34.]

First, then, let the complement of the declination, BP, be given, as the base, and that of the altitude, BZ; and from the altitude of the pole the side ZP. Then by Rule 14 of spherical oblique-angled triangles you will find the angle BZP.

[Margin: Problem 35.]

Second, let the complement of the declination, BP, be given, and that of the altitude, BZ; and from the time the angle BPZ. Then by Rule 1 of spherical oblique-angled triangles you will learn the angle BZP.

[Margin: Problem 36.]

Third, let the complement of the declination, BP, be given, and the complement of the altitude of the pole, ZP; and from the time the angle BPZ. Then by Rule 5 or 6 of spherical oblique-angled triangles you will learn the angle BZP.

[Margin: Problem 37.]

Fourth, let ZP be given from the altitude of the pole; and from the altitude of the star its complement, BZ; and from the time the angle BPZ. Then by Rule 3 of spherical oblique-angled triangles you will obtain the angle BZP, which is to be subtracted from 180° so that the azimuth AZH becomes known.

Probl. Azimuthum Sideris Obſeruatione colligere 38. Immediatè per inſtrumentum 39. Mediante Altitudine, & Declinatione Solis aut Fixi Sideris tranſeuntis per eumdem Verticalem: Eaq. occaſione Declinationem & Aſc. Rectam ſideris Inquirere. *Problem. To gather the azimuth of a star by observation: 38. directly, by an instrument; 39. by means of the altitude and declination of the Sun, or of a fixed star passing through the same vertical; and on that occasion to find the declination and right ascension of the star.*

[Margin: Problem 38.]

The first method of observing the azimuth is by the azimuthal quadrant, above a horizontal plane

[Margin: Order of the problems.]

[Margin: Order of the problems.]

set up perpendicularly and turning about a centre, from which a horizon has been described in the same plane, with the degrees marked out on its circumference. It is better to wait at a predetermined degree for the arrival of the star than to take the azimuth and the altitude of the star together by moving the quadrant. But a still more certain method, surely, is by our thread triangle, already described in Book 5, chapter 15, number 16: placing it at the boundary of the predetermined degree which the star is next going to reach, and, as soon as the star has arrived there, obtaining the star's altitude through

some companion with a separate instrument. But since you may happen to be in some place where no plane marked in degrees together with a horizon is at hand, and yet you need to observe the azimuth of some planet or fixed star, so that its declination and right ascension may be derived from it and its place in the heavens known, take the method used by us and by Father Francesco Maria Grimaldi.

[Margin: Problem 39.]

[Margin: The method of observation.]

The second method, then, is by a thread hung perpendicularly from above and fixed below to the horizontal plane, together with a second thread hung from the same place and stretched downward, not perpendicularly, but so that it makes a triangle with the plane of the floor, and the two threads present two sides of the thread triangle. This second thread must be tied to some heavy body in such a way that it can be moved to the right or to the left and afterwards fixed firmly in place — of which shortly. And the position must be chosen such that, at the hour of the planet's or star's transit suitable for the observation, the two threads lie in the plane of the vertical circle through which the planet, etc., is to pass; or else, by moving the thread stretched obliquely above the floor, you must at once wait for the planet to pass through the plane enclosed between the two threads, the line of sight being directed through both threads together onto the centre of the star. At the same moment the altitude of the planet must be taken by someone else, and by a third companion the altitude of some known fixed star, so that the moment of time can be known. Afterwards, with nothing in the threads changed, one must wait until another fixed star of known declination reaches the same plane enclosed by the two threads already mentioned, and at that moment its altitude must be taken. And so the observation will have been completed, fit for determining the azimuth of the planet, or of the former star, and indeed also its right ascension and declination.

[Margin: Step 1.]

For in the figure just preceding, let the planet have been observed at I, a point of the vertical plane AZ; and from the altitude of the fixed star observed at that time let the time before or after noon have been gathered, by a problem to be given in Section 3; converted into parts of the equator, this will show the angle IPZ. And let the apparent altitude of the planet, AI, be converted into the true one, by adding the parallax to it and subtracting the refraction, if it has any. Afterwards let the fixed star B, whose declination BN is known from the astronomical tables, have reached the plane AZ of the same vertical, and let its altitude AB have been observed at that time. Then in triangle BZP, from the complement of the altitude BZ, the complement of the declination BP, as base, and the complement of the altitude of the pole PZ, you will find by Rule 14 of spherical oblique-angled triangles the angle BZP, the complement of the azimuth AZH.

[Margin: Step 2.]

Next, in triangle IZP, with the vertical angle BZP just found, that is IZP, with the complement of the altitude of the pole ZP, and with the complement of the planet's true altitude IZ, you will find by Rule 4 of spherical oblique-angled triangles the base IP. This will be either a quadrant, and a sign of no declination; or greater than a quadrant, and then its excess over 90° will give the declination CI, towards the other pole; or less than a quadrant, and then its complement to 90° will give the declination towards your own pole.

[Margin: Step 3.]

Third, from the right ascension of mid-heaven, of which we shall speak from problem 108 on, and from the angle at P, taken as above from the observed time — added to the right ascension of mid-heaven if the time was before noon, or subtracted if it was after noon — you will obtain the right ascension of the planet.

Probl. Altitudinem Sideris cuiuſuis Inquirere, Datis 40. Azimutho, & Declinatione ipſius, ac Poli Altitudine 41. Azimutho, Declinatione, & Tempore 42. Azimutho, Altitud. Poli, & Tempore 43. Altitud. Poli, Declinatione, & Tempore. *Problem. To find the altitude of any star, given: 40. its azimuth and declination, and the altitude of the pole; 41. the azimuth, the declination and the time; 42. the azimuth, the altitude of the pole and the time; 43. the altitude of the pole, the declination and the time.*

These problems too are served by the figure explained at problem 25, in which the altitude of the pole RP, subtracted from 90°, gives the arc ZP; the azimuth AZH, subtracted from 180°, makes the angle AZP known; and from the time before or after noon, converted into parts of the equator, the angle at P becomes known — for instance BPZ, if the star is at B; and then BP, the complement of the declination BN, is useful for solving the triangle BZP. But if the star is at G, the angle known from the time is GPZ, and the complement of the nil declination, for solving the triangle GZP, is a quadrant. If, however, the star is at I, the angle known from the time is IPZ; and the side PI, made up of the quadrant PC and the declination CI, is for the solution of the triangle IPZ. But it will be enough to indicate the solution of the triangle BPZ, for finding BZ, the complement of the altitude; for from data of the same kind, by the same rules, the other two can be solved, and either GZ, the complement of the altitude AG, or IZ, etc., can be found.

[Margin: Problem 40.]

First, then, let the angle Z be given from the azimuth, and the side ZP from the altitude of the pole, and the complement of the declination, BP: for by Rule 2 of spherical oblique-angled triangles you will find the base BZ, the complement of the true altitude AB.

[Margin: Problem 41.]

Second, let the angle Z be given from the azimuth, and the angle BPZ from the time, and the complement of the declination, BP: for by Rule 7 of spherical

oblique-angled triangles the side BZ, the complement of the altitude AB, will become known.

[Margin: Problem 42.]

Third, let the angle Z be given from the azimuth, and the angle BPZ from the time, and from the altitude of the pole the base ZP adjacent to them: for by Rule 11 or 12 of spherical oblique-angled triangles you will learn the side BZ, the complement of the altitude AB.

[Margin: Problem 43.]

Fourth, let the side ZP be given from the altitude of the pole, and the complement of the declination, BP, and from the time the included angle BPZ: for by Rule 4 of spherical oblique-angled triangles you obtain the base BZ, which is the complement of the true altitude AB.

[Margin: A note on the apparent altitude.]

And if you wish to convert the true altitude into the apparent one, take the parallax away from it and add the appropriate refraction.

Probl. Altitudinem Solis, vel cuiuſuis Sideris in Verticali Primario Inquirere, Datis 44. Altitudine Poli, & Declinatione Sideris 45. Altitudine Poli, & Tempore 46. Declinatione Sideris, ac Tempore *Problem. To find the altitude of the Sun, or of any star, on the prime vertical, given: 44. the altitude of the pole and the declination of the star; 45. the altitude of the pole and the time; 46. the declination of the star and the time.*

In the figure added here (though it was devised for other uses as well), let the horizon be HOR and the meridian HVRN, in which the zenith is V and the pole P, from which the equator AOQ has been described, with the parallels CD and KN. And let VM be the quadrant of a vertical that is not the prime vertical, but VTO that of the prime vertical — namely of the one that passes through the common section of the equator and the horizon, that is, through the points of equinoctial rising or setting. For a star cannot be on the prime vertical unless it is on some parallel on this side of the equator — in an oblique sphere, that is; for in a right sphere the prime vertical is the equator itself. Let the star, then, be at T, and consider the triangle TVP, right-angled at V, in which TV, the complement of the altitude OT, is sought.

Diagram. A circle representing the meridian HVRN, set in the right column beside the opening of problems 44–46. The horizontal diameter is the horizon, H at its left end and R at its right, with O on it; V, the zenith, stands at the top and N at the bottom. The equator AOQ runs from A at the upper left down to Q at the lower right; parallels to it run from C (top, left of V) and from K (left) down towards D and N on the lower right. Arcs curve down from V to the left (the verticals VM and VTO); two dotted lines descend across the circle. The pole P is on the rim at the upper right.

- C (rim, just left of V): end of the parallel CD
- V (top): the zenith
- A (rim, upper left): end of the equator AOQ

- B (inside, below C): point, not named on this page
- P (rim, upper right): the pole
- K (rim, left): end of the parallel KN
- S (inside, right of K): point, not named on this page
- I (inside, below S): point, not named on this page
- T (near centre): the star on the prime vertical
- H (left end of the horizon)
- M (inside, just below H): end of the vertical VM
- O (on the horizon, right of centre): common section of equator and horizon
- R (right end of the horizon)
- D (rim, lower right): end of the parallel CD
- Q (rim, below D): end of the equator AOQ
- N (bottom): foot of the meridian, end of the parallel KN

[Margin: Problem 44.]

First, if the complement PV is given from the altitude of the pole, and from the star's declination the complement of the declination, or base, TP, you can find by Rule 6 of spherical right-angled triangles the side TV, that is, the complement of the altitude OT.

[Margin: Problem 45.]

Second, if the side PV is given from the altitude of the pole, and from the time before or after noon, converted into parts of the equa-

[Margin: Order of the problems.]

[Margin: Order of the problems.]

tor, the angle TPV, then by Rule 7 of spherical right-angled triangles you will be able to obtain the side TV, the complement of the altitude OT.

[Margin: Problem 46.]

Third, if the complement of the declination is given, that is, the base TP, and from the time the angle TPV, then by Rule 1 of spherical right-angled triangles you will learn the side TP [?], which is the complement of the altitude OT — the true altitude, that is; and if you add the refraction to it and take away the parallax, you will have the visible, or apparent, altitude of the star on the prime vertical.

47. Probl. Diſtantiam Solis, vel Sideris cuiuſuis à Verticali Primario Inuenire. *47. Problem. To find the distance of the Sun, or of any star, from the prime vertical.*

[Margin: Problem 47.]

Find the azimuth of the Sun or of any star by problems 34 to 39, and subtract it from 90°: the distance of the star from the prime vertical will remain. As in the figure just preceding: if the star was on the vertical VM, say at B, or S, or I, and the azimuthal angle MVH has been found, this angle, subtracted from 90°, will leave the angle MVO, which measures the arc of the horizon MO, that is, the distance of the star from the prime vertical VTO.

Probl. Longitudinem Sideris Inuenire, Datis 48. Diſtantia Polorum, Declinatione, & Aſcenſione Recta Sideris 49. Diſtantia Polorum, Declinatione, & Latitudine Sideris 50. Diſtantia Polorum, Latitudine, & Aſcenſione Recta Sideris 51. Declinatione, Latitudine, & Aſcenſione Recta Sideris. *Problem. To find the longitude of a star, given: 48. the distance of the poles, and the declination and right ascension of the star; 49. the distance of the poles, and the declination and latitude of the star; 50. the distance of the poles, and the latitude and right ascension of the star; 51. the declination, latitude and right ascension of the star.*

In the following diagram let ABCD be the solstitial colure, drawn through the beginning of Cancer, E, and of Capricorn, F. In it the north pole of the equator AVC is B, but that of the ecliptic ETF is G; and the south pole of the former is D, of the latter H. Through the poles of the equator let the circle of declination BλDθ be drawn. Now a star can have a northern declination as well as a northern latitude, as when it is at P or at Z: for when the quadrant of latitude GPQ or GZβ is drawn through it, the latitude of the star P will be PQ and its declination Pλ, while the latitude of the star Z will be Zβ and its declination Zθ, both northern. Conversely, it can have both southern, and then the circles or quadrants of latitude must be drawn through the star from the southern pole of the ecliptic, H: as, if the star is at Y, when the quadrant HYX is drawn, its latitude will be YX and its declination Yλ, both southern. Likewise the declination can be something and the latitude nil, as when the star is at R, to which the quadrant of latitude GR belongs, and its declination is Rλ. Conversely, the latitude can be something and the declination nil, as when the star is at λ and its latitude is λT. Finally, the latitude can be of one kind and the declination of the other: as when the star is at S, for its declination Sλ is northern but its latitude ST southern; and vice versa if the star is at ε, its declination εθ will be southern and

Diagram. A circle, the solstitial colure ABCD, filling the lower half of the left column. The equator runs from A on the left to C on the right; the ecliptic runs from E, beside the sign of Cancer, on the upper left, down through the centre, where the sign of Aries stands, to F, beside the sign of Capricorn, on the lower right. B and G stand at the top, H and D at the bottom. Stars, each marked with a small asterisk, are set at K, P, M, Z, I, ε, N, L, Y and O. R, S, T and X are clustered left of centre, with λ below them and V on the equator below T; θ, ε and β stand right of centre. A dotted line runs from G down towards R and T. Which drawn line carries which letter is not certain from the woodcut as seen.

- B (top): north pole of the equator
- G (top, right of B): north pole of the ecliptic
- K (rim, upper left, star)
- P (inside, below K, star)
- E (rim, left): beginning of Cancer, with the sign ♋ beside it
- M (rim, right, star)

- Z (inside, right of centre, star)
- I (rim, left, below E, star), with a small sign beside it [?]
- R, S, T, X (clustered left of centre)
- A (left end of the equator)
- λ (below S)
- V (on the equator, below T)
- ♈ (centre, on the equator): Aries
- ♎ (below ♈): Libra
- a sign right of centre [?], with θ beside it
- C (right end of the equator)
- N (rim, right, below C, star)
- ε (below θ, star)
- β (below ε, to the left), with a further letter beside it [?]
- L (rim, lower left, star)
- Y (inside, lower centre, star)
- F (rim, lower right): beginning of Capricorn, with the sign ♑ beside it
- O (rim, below F, star)
- H (bottom): south pole of the ecliptic
- D (bottom): south pole of the equator

its latitude εμ northern. We have thought these things worth pointing out so that beginners may know how they ought to draw, through a star placed in various positions on the sphere, the arcs of declination and of latitude from the poles already mentioned, and how to tell them apart, and how to apply to other cases what we shall teach in one case or another. Now when the right ascension of a star is nil, or 360°, it will be at the beginning of Aries; when 180°, at the beginning of Libra; when 90°, at the beginning of Cancer; and when 270°, at the beginning of Capricorn.

[Margin: Problem 48.]

[Margin: Case 1.]

First, let the star be at P; and let there be given the distance of the poles, BG (which is always as great as the obliquity of the ecliptic), and PB, the complement of the declination Pλ, and the right ascension, not greater than 180°. If it is less than 90°, add it to 90°; but if greater, subtract the excess from 180°; and either way you will have the angle PBG, enclosed by the given sides BG and BP. Therefore by Rule 5 or 6 of spherical oblique-angled triangles you will find the angle PGB, namely the argument of the longitude of the star, which always lies in the quadrant of the ecliptic of the same kind as the one to which its right ascension belongs on the equator. So if the right ascension was less than 90°, the angle PGB itself will be the longitude of the star, counted from the beginning of Aries in the order of the signs; but if the right ascension was greater than 90°, the angle PGB, added to 90°, will make up the longitude of the star that is sought.

[Margin: Case 2.]

But if a northern star is at Z — that is, when the right ascension is greater than 180° — then, if it is less than 270°, its complement to 270°, but if greater, its excess over 270°, will be the angle ZBG in the triangle BZG, in which there are given BG, the distance of the poles, and BZ, the complement of the declination Zθ. Therefore by the same Rule 5 or 6 of spherical oblique-angled triangles you will find the angle ZGB, which is to be added to 90°, if the right ascension is less than 270°, to give the longitude of the star; but if the right ascension was greater than 270°, the supplement of the angle ZGB to 180°, added to 270°, will produce the longitude of the star that is sought. And one must operate in the same way in the other cases, if the data are the same, whether the star is northern or southern.

[Margin: Problem 49.]

Second, let the star be at P, and let there be given the distance of the poles, BG, and PB, the complement of the declination Pλ, and PG, the complement of the latitude PQ. For with the three sides of the triangle BPG given, the angle PGB will become known by Rule 14 of spherical oblique-angled triangles. It is to be subtracted from 90°, so that the longitude sought remains, if it is known from elsewhere that the star belongs to the first quadrant of the ecliptic; or added to 90°, to give the longitude of a star belonging to the second quadrant. By the same Rule 14 you will solve the triangle BZG if the star is at Z, since BG is given, as above, together with the complements of the declination, BZ, and of the latitude, GZ: for the angle BGZ, once found and added to 90°, will make up the longitude of a star belonging to the third quadrant of the ecliptic; but if the star belongs to the fourth quadrant, the supplement of the angle BGZ to 180°, added to 270°, will produce its longitude.

[Margin: Problem 50.]

Third, let the star be at P, and let there be given the distance of the poles, BG, and the complement of the latitude, PG, and, from the right ascension, the angle PBG, as I said in problem 48. Then by Rule 3 of spherical oblique-angled triangles you will find the angle PGB, which, subtracted from or added to 90°, makes the longitude of the star, according as the right ascension was either less or greater than 90°, but less than 180°. But if the right ascension is greater than 180° and the star is at Z, you will solve by the same Rule 3 the triangle BGZ, in which there are given BG, as above, and the complement of the latitude, GZ, and, from the right ascension, the angle ZBG. Therefore the angle BGZ, once found and added to 90° if the right ascension is less than 270° — or, if it is greater, the supplement of the angle found to 180°, added to 270° — will produce the longitude of the star.

[Margin: Problem 51.]

Fourth, let the star be at P, but let there be given, from the right ascension, the angle PBG, as above, and the complement of the declination, BP, and the complement of the latitude, GP; for then by the first rule of spherical oblique-angled triangles you obtain the angle PGB, which is the argument of the longitude, as above. In the same way, if the star is at Z, and from the right

ascension the angle ZBG is given, with the complement of the declination, BZ, and the complement of the latitude, GZ, you will find by the same first rule of spherical oblique-angled triangles the angle BGZ, the argument of the longitude, according to the condition of the right ascension, as above.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

Probl. Latitudinem Sideris inquirere, Datis 52. Diſtantia Polorum, Declinatione & Aſc. Recta ſideris 53. Diſtantia Polorum, Declinatione, & Longitudine ſideris 54. Diſtantia Polorum, Aſc. Recta, & Longitudine Sideris 55. Declinatione, Aſc. Recta, & Longitud. Sid. *Problem. To find the latitude of a star, given: 52. the distance of the poles, and the declination and right ascension of the star; 53. the distance of the poles, and the declination and longitude of the star; 54. the distance of the poles, and the right ascension and longitude of the star; 55. the declination, right ascension and longitude of the star.*

[Margin: Problem 52.]

Look at the figure set out at problem 48. First, let the star be at P, and let there be given the distance of the poles, BG, and the complement of the declination, BP, and, from the right ascension, the angle PBG, according to what was said at problem 48. Then in triangle PBG, by Rule 4 of spherical oblique-angled triangles, you will find the base PG, namely the complement of the northern latitude PQ. If this base were a quadrant, like GR, there would be no latitude; but if it were greater than a quadrant, like GS, the excess over 90° would be the southern latitude ST. So in triangle BZG, if the star is at Z, from BG, and the complement of the declination, BZ, and the angle ZBG, known from the right ascension, the base GZ — namely the complement of the northern latitude Zβ — can be known by Rule 4 of the same triangles.

[Margin: Problem 53.]

Second, in triangle PBG, let there be given PB, the complement of the declination of the star P, and BG, the distance of the poles, and the angle PGB, known from the longitude: for if the star belongs to the first quadrant of the ecliptic, that angle is the complement of the longitude to 90°; if to the second quadrant, it is the excess of the longitude over 90°. Therefore by Rule 2 of spherical oblique-angled triangles you will find the base PG, which is the complement of the latitude PQ. Likewise, if the star is at Z, and BG is given, as above, and BZ, the complement of the declination, and, from the longitude, the angle ZGB — which becomes known by taking 90° from a longitude belonging to the third quadrant, or, if it belongs to the fourth quadrant, by adding to 90° the complement of the longitude to 360° — then by the same Rule 2 you will learn the base GZ, which, subtracted from 90°, will give the northern latitude Zβ.

[Margin: Problem 54.]

Third, let the star be at P, and let there be given the distance of the poles, BG, and the adjacent angles, namely PBG, from the right ascension, in the way

stated at problem 48, and PGB, from the longitude, according to what was said at problem 53: for by Rule 11 or 12 of spherical oblique-angled triangles the side PG, the complement of the latitude PQ, will become known, etc.

[Margin: Problem 55.]

Fourth, let the star be at P, but let there be given the angles PBG, from the right ascension, and PGB, from the longitude, and the complement of the declination, BP: for by Rule 7 of spherical oblique-angled triangles the side PG can be found, which is the complement of the latitude PQ. And so for the rest.

Probl. Aſcenſ. Rectam Sideris Inquirere, Datis 56. Diſtantia Polorum, Declinatione, & Longitudine Sideris 57. Diſtantia Polorum, Declinatione, & Latitudine Sideris 58. Diſtantia Polorum, Longitudine ac Latitudine Sideris 59. Declinatione, Longitud. & Latitud. Sideris *Problem. To find the right ascension of a star, given: 56. the distance of the poles, and the declination and longitude of the star; 57. the distance of the poles, and the declination and latitude of the star; 58. the distance of the poles, and the longitude and latitude of the star; 59. the declination, longitude and latitude of the star.*

[Margin: Problem 56.]

Here too use the figure already set out at problem 48, in which let the star be imagined at P; and let there be given the distance of the poles, BG, and the complement of the declination, BP, and, from the longitude, the angle PGB, according to what was said at problem 53. Then by Rule 3 of spherical oblique-angled triangles you will learn the angle PBG. If you take 90° from it (when the longitude belongs to the first quadrant of the ecliptic), you will have the right ascension of the star; but if the longitude belongs to the second quadrant, add to 90° the supplement of that angle to 180°, and the right ascension will result. Now if the star is at Z, and the distance BG is given, and the complement of the declination, BZ, and, from the longitude, the angle ZGB, as above, find by Rule 3 of spherical oblique-angled triangles the angle ZBG; add to 90° its supplement to 180°, if that longitude belongs to the third quadrant of the ecliptic; or, if to the fourth, add the angle ZBG to 270°; and the right ascension of the star will result.

[Margin: Problem 57.]

Second, let the star be at P; and let there be given the distance of the poles, BG, the complement of the declination, BP, and the complement of the latitude, PG: for by Rule 14 of spherical oblique-angled triangles the angle PBG will become known, which is the argument of the right ascension; but the quadrant of the equator to which it belongs must be known beforehand, etc.

[Margin: Problem 58.]

Third, let the star be at P; and let there be given, as above, BG, and the complement of the latitude, GP, and, from the longitude, the angle PGB, according to what was said at problem 56: for then by Rule 5 or 6 of spherical oblique-angled triangles the angle PBG will become evident, which is the

argument of the right ascension, likewise according to what was said at problem 56.

[Margin: Problem 59.]

Fourth, let the star be at P; and let there be given the complements of the declination, BP, and of the latitude, GP, and, from the longitude, the angle PGB, as above: for by Rule 1 of spherical oblique-angled triangles you will obtain the angle PBG, namely the argument of the right ascension.

60. Probl. Aſcenſionum Rectarum Tabulam pro Planetis aut Stellis alijs Latitudinem habentibus Conſtruere. *60. Problem. To construct a table of right ascensions for the planets, or for other stars that have latitude.*

[Margin: Problem 60.]

Use problem 58, and construct the tables in this way. Further, Regiomontanus in his *Directions* gives two tables of the mediations of the heavens: one from page 16, for stars whose latitude does not exceed 8°; the other, at page 40, a general one for any latitude. He assumes an obliquity of the ecliptic of 23° 30′, and teaches their use in problems 3 and 4. Likewise Maginus gives a general table, according to a threefold obliquity of the ecliptic, at page 55 of his *Directions of the Primum Mobile*, and at page 56 one for a latitude not exceeding 7° only; he teaches their use in canon 8. Finally Argolus [Andrea Argoli], in volume 1 of his *Ephemerides*, from page 259, gives the same for latitudes up to 9°, assuming an obliquity of the ecliptic of 23° 31′ 30″. Now they call mediations of the heavens what others call right ascensions.

Probl. Declinationem Sideris inquirere, Datis 61. Diſtantia Polorum, Aſc. Recta, & Longitud. Sideris 62. Diſtantia Polorum, Aſcenſione Recta & Latitud. Sideris 63. Diſtantia Polorum, Longitud. & Latit. Sid. 64. Aſc. Recta, Longitud. & Latitud. Sideris *Problem. To find the declination of a star, given: 61. the distance of the poles, and the right ascension and longitude of the star; 62. the distance of the poles, and the right ascension and latitude of the star; 63. the distance of the poles, and the longitude and latitude of the star; 64. the right ascension, longitude and latitude of the star.*

[Margin: Problem 61.]

Let the figure set out at problem 48 be repeated here, in which, first, let the star be at P, and let there be given the distance of the poles, BG, and, from the right ascension, the angle PBG, according to what was said at problem 48, and, from the longitude, the angle PGB, according to what was said at problem 53. Then in triangle PBG, by Rule 11 or 12 of spherical oblique-angled triangles, you will learn the side BP; when this is subtracted from 90°, the declination Pλ will be left. So in triangle BZG, if the star is at Z, with BG given, and the angles ZBG, from the right ascension, and ZGB, from the longitude, you will obtain the side BZ; when this is taken from 90°, the northern declination Zθ will remain. But if the side obtained were a quadrant, there would be no declination; if it exceeded a quadrant, the excess would be a southern declination.

[Margin: Problem 62.]

Second, let the star be at P; and let there be given the distance of the poles, BG, and, from the right ascension, the angle PBG, and PG, the complement of the latitude PQ: for by Rule 2 of spherical oblique-angled triangles you will find the base BP, the complement of the declination Pλ.

[Margin: Problem 63.]

Third, let the star be at P; and let there be given the distance of the poles, BG, and, from the longitude, the angle PGB, according to what was said at problem 53, and the complement of the latitude, PG: for by Rule 4 of spherical oblique-angled triangles you obtain the base BP, which is the complement of the northern declination Pλ. Similarly in triangle ZBG, if the star is at Z — that is, when the latitude is northern and the longitude belongs to the third or fourth quadrant of the ecliptic — with PG [?] given, as above, and, from the longitude, the angle ZGB, and the complement of the latitude, GZ, you will learn the base BZ, the complement of the northern declination Zθ; and so for the rest.

[Margin: Problem 64.]

Fourth, let the star be at P; and let there be given, from the right ascension, the angle PBG, and, from the longitude, the angle PGB, and the complement of the latitude, PG: for by Rule 7 of spherical oblique-angled triangles you will obtain the side BP, the complement of the declination Pλ.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

65. Probl. Declinationum Tabulam pro Planetis, alijſq. Sideribus Latitudinem habentibus, Conſtruere. *65. Problem. To construct a table of declinations for the planets and other stars that have latitude.*

[Margin: Problem 65.]

Consult problem 63 and, following it, construct a table for each degree of longitude and of latitude for one quadrant of the ecliptic; for the same declination will hold for any degree of longitude equally distant from the nearer equinox, the same latitude being given.

[Margin: The authors of this table.]

Such a table was indeed published by Io. Antonius Maginus in the Tables of the *Directions of the primum mobile*, from page 36; it extends up to 83 degrees of latitude. But it is founded on the distance of the poles, that is, the Tychonic obliquity of the ecliptic, 23° 31′ 30″, as Maginus himself admits in the same place, in Canon 4. So if anyone uses another obliquity, that table will need some correction. The same author, in his new Tables of Directions, page 3 of the tables, gave a table of the equation of declinations for the planets, but extended only up to 8 degrees of latitude, and likewise founded on the Tychonic obliquity. Argoli, for his part, gives the same table in volume 1, before the Ephemerides, at page 246, up to a latitude of 9 degrees. Regiomontanus, for his part, in the Tables of Directions, pages 2 and 14 of the tables, gives one table

for latitude up to 8°, and another, general one; he teaches the use of both there in problems 1 and 2, assuming an obliquity of the ecliptic of 23° 30′.

Probl. Angulum Circuli Declin. & Latitud. in centro Sideris factum indagare, Datis 66. Polorum diſt. Declin ac Latitud. Sideris 67. Polorum diſt. Declin. & Longitud. Sideris 68. Polorum diſt. Decl. & Aſc. Recta Sideris 69. Polorum diſt. Latitud. & Longitud. Sid. 70. Polorum dſt. Lat. & Aſcenſ. Recta Sider. 71. Polor. diſt Longitud. & Aſc. Recta Sideris 72. Longitud. Latitud. & Declinat. Sideris 73. Longit. Latitud. & Aſc. Recta Sideris 74. Longitud. Decl. & Aſc. Recta Sideris 75. Latitud. Declin. & Aſc. Recta Sideris *Problem. To find the angle made at the centre of a star by the circle of declination and the circle of latitude, given: 66. the distance of the poles, and the declination and latitude of the star; 67. the distance of the poles, and the declination and longitude of the star; 68. the distance of the poles, and the declination and right ascension of the star; 69. the distance of the poles, and the latitude and longitude of the star; 70. the distance of the poles, and the latitude and right ascension of the star; 71. the distance of the poles, and the longitude and right ascension of the star; 72. the longitude, latitude and declination of the star; 73. the longitude, latitude and right ascension of the star; 74. the longitude, declination and right ascension of the star; 75. the latitude, declination and right ascension of the star.*

You will solve all these problems by taking as your example the single triangle BPG of the figure set out at problem 48. But since these problems are very rarely used, it will be enough to give the principal and easier one, which is 66.

[Margin: Problem 66.]

Let the star, then, be at P, and let the three sides be given: namely the base BG, the distance of the poles; BP, the complement of the declination; and GP, the complement of the latitude. For by Rule 14 of spherical oblique-angled triangles you will find the angle BPG, which the arcs Bλ, of declination, and GQ, of latitude, make at the centre of the star P. For the other data it is enough to have indicated the angles and sides corresponding to the problems, numbered in order, and the rules of spherical oblique-angled triangles given in Section 1, chapter 5, by which the angle P can be found — as you see done in the following small table.

[Margin: Problems 67, 68, 69, 70, 71, 72, 73, 74, 75.]

Order of the problems	Data for the angle BPG	Rules
66	Sides BG, BP, GP	14
67	Sides BG, BP; angle PGB	1
68	Sides BG, BP; angle PBG	5 or 6
69	Sides BG, PG; angle PGB	5 or 6
70	Sides BG, PG; angle PBG	1
71	Side BG; angles PGB, PBG	9
72	Sides PG, PB; angle PGB	3

Order of the problems	Data for the angle BPG	Rules
73	Side PG; angles PGB, PBG	9
74	Side PB; angles PGB, PBG	9
75	Sides PG, PB; angle PBG	3

Set vertically beside the rules column: Oblique-angled triangles, Section 1, chapter 5.

Probl. Angulum circuli Declinationis cum Ecliptica inueſtigare, Datis 76. Declinatione ac Longitudine Sideris 77. Declinatione, & Aſc. Recta Sid. 78. Declinatione Sid. & Obliquitate Ecliptica 79. Longitud. & Aſc. Recta Sid. 80. Longitud. Sid. & Obliquitate Ecliptica 81. Aſc. Recta Sid & Obliquit. Eclip. 82. Latitud. complem. & Angulo circuli Declin. ac Latit. in centro Sideris 83. Ijſdem Datis, ſed alio modo. *Problem. To find the angle of the circle of declination with the ecliptic, given: 76. the declination and longitude of the star; 77. the declination and right ascension of the star; 78. the declination of the star and the obliquity of the ecliptic; 79. the longitude and right ascension of the star; 80. the longitude of the star and the obliquity of the ecliptic; 81. the right ascension of the star and the obliquity of the ecliptic; 82. the complement of the latitude, and the angle of the circle of declination and of latitude at the centre of the star; 83. the same data, but in another way.*

The problem can indeed be solved from the data assigned to problem 80, in the way in which we taught in problem 14 how to find the angle of the ecliptic and the meridian, since the meridian is one of the circles of declination, and every circle of declination is, with respect to some horizon, a meridian. So this business could be settled by the table of the angle of the ecliptic and the meridian. Nevertheless, taking the occasion of the figure set out at problem 48, which is to be recalled here, we have decided to complete this practice by a more fruitful inquiry. For it we shall consider the triangle R♈λ, or θ♈μ, right-angled at λ, or at θ.

Diagram. A circle set in the right column, the figure recalled from problem 48: a near-horizontal diameter A–C, a line from E through the centre point ♈ to F, curved arcs descending from B and G to H and D, and a dotted line from G down towards X. Small stars mark K, P, M, I, Z, L, Y, N, O, T and ε. A Capricorn sign ♑ stands beside F; three other small sorts could not be identified on the scan.

- B (top of the rim)
- G (rim, upper right of B)
- K (rim, upper left)
- P (inside, below K)
- M (rim, right)
- E (rim, left), with the sign ♋ just inside it
- [?] (a small sign inside, right of I)
- I (rim, left, below E)

- Z (inside, right, below M)
- R, X, S, T (clustered left of centre)
- [?] (a small sign left of θ)
- θ (right of centre)
- A (left end of the diameter), C (right end)
- ♈ (centre)
- λ (below S), V (right of λ)
- ♎ (below ♈)
- N (rim, right, below C)
- ε (inside, below θ)
- β (below ε, to the left)
- [?] (a sort right of β, with a long descending stroke)
- L (rim, lower left)
- [?] (a small sign touching F)
- F (rim, lower right)
- Y (inside, lower left)
- O (rim, lower right, below F)
- H (bottom left), D (bottom)

In the first triangle the declination of the point of the ecliptic is the arc Rλ, which becomes known from the longitude of the star through the table of declinations treated in problem 15; the obliquity of the ecliptic is the angle at ♈; and the base R♈ is the longitude of the star if it is less than 90°, or the supplement of the longitude to 180° if it is greater than 90° but less than 180°. The side λ♈, in turn, is the right ascension if that is less than 90°, or its supplement to 180° if it is greater than 90° and less than 180°. But if either the longitude or the right ascension exceeds 180°, the triangle θ♈μ must be used. In it the declination is θμ; the obliquity of the ecliptic is the angle θ♈μ; and the base ♈μ is the excess of the longitude over 180° if the longitude is less than 270°, or the complement of the longitude to 360° if it is greater than 270°. Likewise the side ♈θ is the excess of the right ascension over 180°, or its complement to 360°, according as the right ascension is less or greater than 270°. We shall, however, give examples of the problems only in the triangle R♈λ, indicating the data and the rules of spherical right-angled triangles given in Section 1, chapter 4, by which the angle λR♈ can be found. I add, however, that if in the triangle PQR, right-angled at Q, the complement of the latitude PQ is given, together with the angle P from problems 66 and following, the angle R can be obtained by Rule 8 of spherical right-angled triangles; or the base PR by Rule 9, and with it the angle R by Rule 3.

[Margin: Problems 76, 77, 78, 79, 80, 81, 82, 83.]

Order of the problems	Data for the angle λR♈	Rules
76	Side Rλ; base R♈	5
77	Sides Rλ and λ♈	14
78	Side Rλ; angle ♈	12

Order of the problems	Data for the angle λRϒ	Rules
79	Side λϒ; base Rϒ	4
80	Base Rϒ; angle ϒ	3
81	Side λϒ; angle ϒ	8
82	Side PQ; angle P	8
83	Side PQ; angle P	9 and 3

Set vertically beside the rules column: Right-angled triangles, Section 1, chapter 3.
[Margin: Order of the problems.]
[Margin: Order of the problems.]

Probl. Angulum Circuli Latitudinis & Aequatoris inquirere, Datis 84. Declinat Sid. & Angulo Declinat. ac Latitud. in centro Sideris 85. Longitud. & Asc. Recta Sideris 86. Longitud Sid. & Obliquit. Eclipt. 87 Asc. Recta Sid. & Obliquit. Eclipt. *Problem. To find the angle of the circle of latitude and the equator, given: 84. the declination of the star, and the angle of the circles of declination and of latitude at the centre of the star; 85. the longitude and right ascension of the star; 86. the longitude of the star and the obliquity of the ecliptic; 87. the right ascension of the star and the obliquity of the ecliptic.*

Look at the figure immediately preceding, and in it at the triangle VXϒ, right-angled at X, in which the angle V is sought; or at the triangle βδ♋, right-angled at β, in which the angle δ is sought. The angle of the obliquity of the ecliptic is ♋; the base is V♋ in the first triangle and δ♋ in the second, known from the right ascension; and the side X♋, or β♋, is known from the longitude of the star, according as either of them is greater or less than 90° or 270°, following the precepts just given for problem 76 and the rest. We shall give the worked example in the triangle X♋V, for finding the angle XV♋, according to the data corresponding to the problems set out in order, indicating the rules of spherical right-angled triangles given in chapter 4 of Section 1, by which the acute angle at V, which the circle of latitude makes with the equator, can be found. The same angle, however, can also be found in the triangle λYV, if the angle Y is given (on which see from problem 66 on) together with the complement of the declination, λY.

[Margin: Table of this angle.]

Maginus gives a table of this angle in the *Directions of the primum mobile*, page 36, and teaches its use in Canon 3.

[Margin: Problems 84, 85, 86, 87.]

Order of the problems	Data for the angle XV♋	Rules
84	Side X♋; base V♋	4
85	Side X♋; angle ♋	8
86	Base V♋; angle ♋	3
87	Side λY; angle Y	8

Set vertically beside the rules column: Section 1, chapter 4.

88. Probl. Angulum Eclipticæ ac Verticalis circuli factum in centro fideris Latitudinem habentis inquirere. *88. Problem. To find the angle of the ecliptic and the vertical circle made at the centre of a star that has latitude.*

[Margin: Problem 88.]

This problem serves hardly any other use than that of eliciting the parallaxes of the luminaries or of comets, and so we solved it in various ways in Book 5, chapter 10, from problem 3 to 5 — besides the mechanical methods, by means of the analemma or the astrolabe, which our Scheiner teaches in the *Rosa Ursina*, from page 144 to 149.

Probl. Angulum Eclipticæ & Verticalis. in centro Solis aut Sid. Latitud. carentis factum, Per angulum Verticalis ac circuli Declinationis ibidem factum, explorare Datis præter Locum eius in Ecliptica 89. Altitudine Poli, Tempore ac Declinat. Sid. 90. Altitud. poli, Tempore, & Altitud. Sid. 91. Altitud. Poli, Tempore, & Azimutho Sid. 92. Altit. Poli Declinat & Altitud Sideris 93. Altitud. Poli, Declinat. & Azimutho Sid. 94. Alt. Poli, Altitud. & Azimutho Sideris 95. Tempore, Declinat & Altitudine Sideris 96. Tempore, Declinat. & Azimutho Sideris 97. Tempore, Altitud. & Azimutho Sideris 98. Declinatione, Altitud. & Azimutho Sid. *Problem. To explore the angle of the ecliptic and the vertical made at the centre of the Sun, or of a star lacking latitude, by way of the angle of the vertical and the circle of declination made at the same place, given, besides its place in the ecliptic: 89. the altitude of the pole, the time and the declination of the star; 90. the altitude of the pole, the time and the altitude of the star; 91. the altitude of the pole, the time and the azimuth of the star; 92. the altitude of the pole, the declination and the altitude of the star; 93. the altitude of the pole, the declination and the azimuth of the star; 94. the altitude of the pole, and the altitude and azimuth of the star; 95. the time, and the declination and altitude of the star; 96. the time, and the declination and azimuth of the star; 97. the time, and the altitude and azimuth of the star; 98. the declination, altitude and azimuth of the star.*

[Margin: Explanation of the diagram.]

For all these, describe the horizon HCR and the meridian HVRA, in which let V be the zenith and P the north pole of the world; and the ecliptic BSA, at a given point of which let the star S stand. Through S let a quadrant of a vertical circle, VC, be drawn, so that on it the true altitude of the star is CS, and its complement SV; and let the altitude of the pole be PH, whose complement to 90° is PV. Now when an arc of a great circle is drawn from the pole P to the centre of the star, that is PS, it will be either the complement of a northern declination; or, if the star is on the equator, a quadrant; or, if it declines to the south, the sum of a quadrant and the southern declination.

Diagram. A circle set at the right of the short-measure lines in the right column: a curve from H across to R (the horizon), a curve from A rising to B

(the ecliptic), a curve from P across to S, and a line from V down through S to C.

- V (top of the rim)
- P (rim, upper left)
- B (rim, right, above R)
- H (rim, left)
- S (inside, right; small italic)
- R (outside the rim, right, at the end of a line)
- A (rim, lower left)
- C (inside, right, below S)
- M (below the bottom of the rim)

Moreover the angle CVR is the measure of the given azimuth CR, and its supplement to 180° is the angle SVP. And when the time before or after noon is given and converted into parts of the equator, the angle SPV is given. Therefore in the triangle VSP, from any three of the five quantities already mentioned, the angle VSP, which the vertical makes with the circle of declination, can be gathered. Further, when the point S of the ecliptic is given, the angle PSA is also given, which the meridian, or circle of declination, makes with the ecliptic, by problem 14 or by problems 76 to 81. So by adding the angle VSP to it, the angle VSA will result; and when this is subtracted from 180°, there will remain the angle VSB, which the vertical makes with the ecliptic. The whole business, therefore, now consists in finding the angle VSP by solving the spherical oblique-angled triangle VSP. This will be done by the following rules, already set out in chapter 5 of Section 1, applied in order to the data of the problems proposed, as the following small table indicates.

[Margin: Problems 89, 90, 91, 92, 93, 94, 95, 96, 97, 98.]

Order of the problems	Data for the angle VSP	Rules
89	Sides VP and PS, and angle SPV	5 or 6
90	Sides VP and VS, and angle SPV	1
91	Side VP, and angles SPV and SVP	10
92	Sides VP, PS and SV	14
93	Sides VP and PS, and angle SVP	1
94	Sides VP and SV, and angle SVP	5 or 6
95	Sides PS and SV, and angle SPV	3
96	Side PS, and angles SPV and SVP	9
97	Side SV, and angles SPV and SVP	9
98	Sides SV and SP, and angle SVP	3

Set vertically in a ruled strip beside the rules column: Spherical oblique-angled triangles, Section 1, chapter 5.

99. Probl. Differentiam Aſcenſionis Rectæ ſeu Diſtantiam Æquatoriam inter duo Sidera Inuenire, Datis Diſtantia eorum inter ſe, & vtriuſq. Declinatione. *99. Problem. To find the difference of right ascension, or the equatorial*

distance, between two stars, given their distance from each other and the declination of each.

[Margin: Problem 99.]

In the following figure, through the south pole of the equator, A, and the north pole, B, let two semicircles of declination, AEB and AQB, be drawn, intercepting the portion EQ of the equator.

[Margin: Case 1.]

First, let both stars be northern, one at C, the other at D, their distance being CD. Then in the triangle CBD the remaining two sides will be given — CB, as being the complement of the declination EC, and BD, the complement of the declination QD — and the base CD will be given. Therefore by Rule 14 of spherical oblique-angled triangles the angle B will be known, which measures the difference of right ascension, namely the arc EQ.

[Margin: Case 2.]

Second, let both stars be southern, one at F, the other at G, with the distance given, that is the base FG, and the sides AF, the complement of the declination FQ, and AG, the complement of the declination GE; for by the same Rule 14 you will find the angle A,

[Margin: Order of the problems.]

[Margin: Order of the problems.]

which measures the arc EQ, that is, the difference of right ascension.

[Margin: Case 3.]

Third [?], let one star be with declination, as at F, the other without declination, as at E, and let their distance, that is the given base, be EF. For in the triangle EAF, from that base, the quadrant AE, and the side AF, the complement of the declination FQ, you will obtain by the same Rule 14 the angle A, which measures the arc EQ.

[Margin: Case 4.]

Fourth, let one star be northern, as C, the other southern, as F. For in the triangle CBF, given the base, that is their distance CF; the side CB, which is the complement of the northern declination EC; and the side BQF, which is the sum of the quadrant BQ and the southern declination FQ, you will find by the same Rule 14 the angle B, which measures the equatorial distance, or difference of right ascension, that is the arc EF. We are speaking here of the true distance: how an apparent, or observed, distance should be converted into the true one, when parallax and refraction intervene, or at least refraction, has been explained in Book 6, chapter 11, problem 8.

Diagram. A lens-shaped figure set in the left column beside the short-measure lines: two semicircles of declination meeting at B above and A below, with the straight line E–Q across the middle and arcs drawn from C to D, from C to F, from E to F and from G to F, and a further arc from the upper left down to F.

- B (top point)
- D (right arc, below B)

- C (left arc, below the level of D)
- E (middle of the left arc)
- Q (middle of the right arc)
- F (right arc, below Q)
- G (left arc, below E)
- A (bottom point)

100. Probl. Aſcenſionem Rectam Sideris notæ Declinationis inueſtigare per Diſtantiam ab alio ſidere notæ Declinationis & Aſc. Rectæ. *100. Problem. To find the right ascension of a star of known declination through its distance from another star of known declination and right ascension.*

[Margin: Problem 100.]

Find by problem 99 the difference of right ascension, and add it to the known right ascension if the star in question is more easterly, as when it is at D; subtract it if it is more westerly, as when it is at C; and the right ascension of the star sought will result, or remain. But if the sum from the addition exceeds 360°, discard them; and if the subtraction has to be of a larger number from a smaller, because the difference of right ascension is greater than the right ascension of the known star, add 360 to the smaller number and subtract the difference from the sum.

101. Probl. Declinationem Sideris inueſtigare, Datis Declinatione alterius ſideris, & Diſtantia eorum inter ſe, cum Differentia Aſc. Rectæ. *101. Problem. To find the declination of a star, given the declination of another star and their distance from each other, with the difference of right ascension.*

[Margin: Problem 101.]

The four cases of problem 99 can arise here, but all of them by Rule 2 of spherical oblique-angled triangles. First, let the star be C, whose declination EC is known, and so also its complement CB; let the other be D; and let their distance CD be given, with the angle B, which measures the difference of right ascension between them. For by the said Rule 2 the base BD will be found, which is the complement of the declination DQ. Conversely, if the declination DQ is known, and its complement BD, you will find the base BC, the complement of the declination EC. Likewise in the triangle CBF: if CB is known, and the distance CF and the angle B are given, and the base found turns out greater than a quadrant, as BQF, then once the quadrant BQ is subtracted the declination FQ, towards the other pole, will be known; but if the base found were the quadrant AE, in the triangle EAF, there would be no declination. If you do not understand the rest, you are too dull.

102. Probl. Differentiam Longitudinis, ſeu Diſtantiam Eclipticam inter duo ſidera inuenire, Data diſtantia eorum inter ſe, & Latitudine vtriuſq. *102. Problem. To find the difference of longitude, or the ecliptic distance, between two stars, given their distance from each other and the latitude of each.*

[Margin: Problem 102.]

Consult problem 99, changing the names, and taking A and B to be the poles of the ecliptic, EQ an arc of the ecliptic, and AEB and AQB the semicircles of latitude. Carry out the rest as there.

103. Probl. Longitudinem ſideris notæ Latitudinis inueſtigare, Per Diſtantiam ab alio ſidere notæ Latitudinis ac Longitudinis. *103. Problem. To find the longitude of a star of known latitude through its distance from another star of known latitude and longitude.*

[Margin: Problem 103.]

Find the difference of longitude by problem 102, and add it to the known longitude if the star in question is more easterly; subtract it if more westerly. If in the first case the sum exceeds 360°, discard them, keeping the remainder; but if in the second case the subtraction cannot be made, add 360° to the given longitude, and then subtract the difference of longitude from the sum. In this way you will have the longitude of the other star.

104. Probl. Latitudinem Sideris inueſtigare, Datis Latitudine alterius ſideris eorumq. tum Diſtantia, tum Differentia Longitudinis. *104. Problem. To find the latitude of a star, given the latitude of another star, together with both their distance and their difference of longitude.*

[Margin: Problem 104.]

Do everything as in problem 101, changing the names of declination into latitude and of right ascension into longitude.

105. Probl. Diſtantiam duorum Siderum inter ſe inquirere, Datis vtriuſq. Latitudine, ac Differentia Longitudinis, vel vtriuſq. Declinatione ac Differentia Aſcenſionis rectæ. *105. Problem. To find the distance of two stars from each other, given the latitude of each and the difference of longitude, or the declination of each and the difference of right ascension.*

[Margin: Problem 105.]

Use the figure of problem 99, in which, by Rule 4 of spherical oblique-angled triangles, you will find the base — that is, the distance of the stars sought — from the given sides, namely the complements of the two latitudes, and the vertical angle, that is, the difference of longitude; or from the complements of the two declinations and the vertical angle, namely the difference of right ascension. For example, in the triangle CBD, given CB, the complement of the latitude EC, and BD, the complement of the latitude DQ, and the angle B, which measures the difference of longitude, find the distance CD. It can happen, however, that one of the sides already mentioned is a quadrant, say EF, in the triangle EAF, because one star, E, has no latitude or declination; or that it is greater than a quadrant, as BQF in the triangle CBF, because the latitude or declination of one star is northern and that of the other southern. Nevertheless, by the same Rule 4 you will find the base, or distance, EF or CF, and so on.

How an apparent distance is converted into the true one has been explained in Book 6, chapter 11, problem 8, and it belongs to the section on parallaxes and refractions.

106. Probl. Longitudinem ac Latitudinem Stellæ inquirere, Data eius Diſtantia à duabus ſtellis notæ Longitudinis ac Latitudinis. *106. Problem. To find the longitude and latitude of a star, given its distance from two stars of known longitude and latitude.*

107. Probl. Aſcenſionem Rectam, & Declinationem Stellæ inquirere, Data eius Diſtantia à duabus ſtellis notæ Aſcenſionis Rectæ, & Declinationis. *107. Problem. To find the right ascension and declination of a star, given its distance from two stars of known right ascension and declination.*

[Margin: Problem 106.]

[Margin: Problem 107.]

I have already taught these problems sufficiently in Book 6, chapter 12, problem 2. But for problem 107 one must change the names and functions of longitude into right ascension, of latitude into declination, of the ecliptic into the equator, and of the poles A, B of the ecliptic into the poles of the equator; for everything else is to be done exactly as there. And from the operations to be carried out there it will be clear how great a saving of labour results if the distance between those stars which are used to learn the conditions of the third has been known beforehand. To this end we have presented, in the same Book 6, chapter 10, the distances of so many fixed stars, observed with care partly by Tycho and partly by ourselves.

[Margin: Notice about a double problem to be inserted here.]

Insert here problems 131 and 132, which for good reasons I have put off to that place.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

Probl. Aſcenſionem Rectam Medij Cæli, ſeu Punctum Aequatoris Culminans Indagare, Datis præter Aſc. Rect. Sideris aut Puncti Eclipt. 108. Tempore ante vel poſt meridiem 109. Tempore ante vel poſt tranſitum Stellæ per Meridianum 110. Altitudine Poli, Altitudine, & Declinatione Sideris 111. Altitud. Poli Altitud. & Azimutho Sid. 112. Altit. Poli. Declinat. & Azimutho Sid. 113. Azimutho, Altitudine, & Declinat. Sid. *Problem. To find the right ascension of mid-heaven, that is, the culminating point of the equator, given, besides the right ascension of the star or of the point of the ecliptic: 108. the time before or after noon; 109. the time before or after the transit of a star across the meridian; 110. the altitude of the pole, the altitude, and the declination of the star; 111. the altitude of the pole, the altitude, and the azimuth of the star; 112. the altitude of the pole, the declination, and the azimuth of the star; 113. the azimuth, the altitude, and the declination of the star.*

Let the figure set out at problem 25 be taken up again here. Its meridian is HZR, its horizon HR, its equator EQ, and AZ is a quadrant of a vertical circle. In it the star is either at B, with BP the complement of its declination; or at G, with no declination, so that GP is a quadrant; or at I, so that IP is the sum of the declination IC, toward the other pole, and the quadrant CP. The visible pole is P.

Diagram. A circle set in the left column beside the text of problem 108, recalling the figure of problem 25: the horizon H–R runs across it, with arcs drawn from the pole P on the right-hand rim. Labels read:

- Z (above the rim at the top, right of centre): one end of the vertical quadrant AZ
- E (rim, upper left): the culminating point of the equator
- B (inside, near the top): a star with declination, BP its complement
- P (rim, upper right): the visible pole
- N (inside, below E)
- G (inside, left of centre): a star with no declination
- C (immediately right of G)
- I (below G): a star whose declination IC lies toward the other pole
- H (outside the rim, left) and R (outside the rim, right): the horizon HR; the meridian HZR
- A (inside, lower left): the foot of the vertical quadrant AZ
- O (inside, below centre)
- Q (rim, lower right): the equator EQ

[Margin: Problem 108.]

First, then, let the star be at B, and from the time before or after noon, converted into parts of the equator, let the angle ZPB be given, which measures the arc of the equator corresponding to it. For this angle, added to the given right ascension of a western star, or subtracted from it if the star is eastern, will make up or leave the point E culminating, that is, passing through the meridian, or the right ascension of mid-heaven. Thus, if the star were at G, the angle known from the time would be GPE, measuring the arc EG of the equator; but if the star were at I, that angle would be IPE, measuring the arc EC of the equator, to be added to the right ascension of a western star or subtracted from that of an eastern one.

[Margin: Problem 109.]

Second, let a star of known right ascension cross the meridian, and let the time from its transit, or before its transit, be counted with the help of the pendulum, on which see Book 2, chapter 20. That time, converted into parts of the equator and added to the star's right ascension if the time is after the transit, or subtracted from it if before, will give the right ascension of mid-heaven for the moment at which you began counting the time before the transit, or finished counting it after the star's transit across mid-heaven. The same could, however, also be obtained with the equatorial armillae, with which the angle

already mentioned, or the culminating point of the equator itself, would be observed.

[Margin: Problem 110.]

Third, let the star be at B, and in the triangle BZP let there be given PZ, the complement of the altitude of the pole; BZ, the complement of the true altitude of the star; and BP, the complement of the declination — or, if there is no declination, a quadrant, or else the sum of the declination toward the other pole and the quadrant. For by Rule 14 of spherical oblique-angled triangles you will find the angle at P, which is to be added to the given right ascension of a western star, or subtracted from it if the star is eastern, so as to obtain the right ascension of mid-heaven.

[Margin: Problem 111.]

Fourth, in the same triangle let PZ and BZ be given as above, and the angle BZP, the supplement of the azimuth AZH to 180°. For by Rule 5 or 6 of spherical oblique-angled triangles you will find the angle at P; use it as above.

[Margin: Problem 112.]

Fifth, in the same triangle let PZ and BP be given, and the angle BZP, as above. For by Rule 3 of spherical oblique-angled triangles you will find the angle at P; use it as above.

[Margin: Problem 113.]

Sixth, in the same triangle let there be given, as above [?], the angle BZP and the sides BZ and BP. For then by Rule 1 of spherical oblique-angled triangles the angle at P will be given; use it as above in problem 110.

Probl. Aſcenſionalem Differentiam; ipſiuſq. 114. Tabulam, & Amplitudinem Ortiuam & 115. Occiduam Puncti Eclipticæ aut Sideris Determinare, Data eius Declinatione, & Altitudine Poli. *Problems 114 and 115. To determine the ascensional difference and its table, and the rising and setting amplitude of a point of the ecliptic or of a star, given its declination and the altitude of the pole.*

[Margin: Explanation of the figure.]

Consider the figure below, in which the meridian is HCQ, the horizon HMR and the equator AMQ, while its southern parallel is KBN and its northern parallel CDT. To the points where these parallels cut the horizon, B and D, let the arcs of declination PIB and PDL be drawn from P, the northern pole of the equator, as far as the equator or through it. Then DL will be the northern declination of the point D, or of a star rising or setting there; ML will be the ascensional difference; and MD the rising or setting amplitude of the Sun, or of another planet, or even of a fixed star. BI, on the other hand, will be the southern declination of a point of the ecliptic, or of a star, rising or setting at B; its ascensional difference will be the arc IM of the equator, and its rising or setting amplitude the arc BM of the horizon. Finally, when the altitude of the pole PR is subtracted from 90°, the depression of the equator RQ is given,

to which the elevation HA is equal; and either of the two measures the acute angle at M.

[Margin: Problems 114, 115.]

First, then [?], let the northern declination DL be given, and the acute angle M, already known from the altitude of the pole. For in the triangle MDL, right-angled at L, you will find by Rule 10 of spherical right-angled triangles the side LM, which is the ascensional difference, and by Rule 11 the base MD, that is, the rising or setting amplitude.

Diagram. A circle set in the right column beside the text of problems 114–115: the horizon H–R runs across it, crossed by the equator and two parallels, with arcs of declination drawn from the pole P on the upper right rim. Labels read:

- C (rim, top) and Q (rim, lower right): the meridian HCQ
- A (rim, upper left) and M (centre, on the horizon): the equator AMQ
- K (rim, left, below A), B (on the horizon, left of centre) and N (rim, bottom): the southern parallel KBN
- D (on the horizon, right of M) and T (outside the rim, lower right): the northern parallel CDT
- P (rim, upper right): the northern pole of the equator
- I (inside, left of centre): the foot on the equator of the arc PIB
- L (inside, below D): the foot on the equator of the arc PDL
- H (outside the rim, left) and R (outside the rim, right): the horizon HMR

Second, if the southern declination BI of a point or star B is given, and the acute angle M, known from the altitude of the pole, then in the triangle BIM, right-angled at I, look for the side IM, that is, the ascensional difference, by Rule 10 of spherical right-angled triangles, and by Rule 11 of the same, the base BM, that is, the rising or setting amplitude. Further, if the declination of the Sun or of a star were the greatest possible for it, it would produce the rising or setting amplitude so called without qualification.

[Margin: Authors of the table.]

Tables of the ascensional difference are given by Regiomontanus in the *Directions*, page 44; by Reinhold in the same work, page 102; by Copernicus, Book 2, chapter 7; by Clavius in the *Sphere*, page 340; by Maginus in the *Small Directions*, page 670, and in the *Tables of Directions*, table 4 of the Analogy, whose use he teaches in canon 10; by the *Rudolphine Tables*, folio 24; and by Argoli in the *Ephemerides*, volume 1, page 223. A particular way of constructing this table is taught in the same place by Reinhold, in precept 23. Maginus, for his part, adds in canon 10 that the difference of the semidiurnal arc from 90° is itself the ascensional difference, and that it can thus be had from the tables of the semidiurnal arc.

Probl. Angulum Horizontis & Circuli Declinationis in centro Sid. aut puncto Eclipticæ Oriente vel Occidente factum inuenire, Datis 116. Altitud. Poli & Declinatione Sideris 117. Altitud. Poli, & Aſcenſionali differentia 118. Altitud. Poli, & Amplitudine Ortiua 119. Declinatione;

& Aſcenſ. Differentia 120. Declinatione, & Amplitudine Ortiua 121. Aſcenſionali differ. & Amplitud. Ortiua *Problem. To find the angle made by the horizon and the circle of declination at the centre of a star, or of a point of the ecliptic, as it rises or sets, given: 116. the altitude of the pole and the declination of the star; 117. the altitude of the pole and the ascensional difference; 118. the altitude of the pole and the rising amplitude; 119. the declination and the ascensional difference; 120. the declination and the rising amplitude; 121. the ascensional difference and the rising amplitude.*

On this angle, however, when the star or point is off the horizon, I shall speak from problem 188 onward.

In the figure immediately preceding, consider the triangle MLD, right-angled at L, in which what is sought is the angle

[Margin: Order of the problems.]

[Margin: Order of the problems.]

MDL, which the horizon HR makes with the circle of declination PL at the centre of the star D as it rises or sets, or at some given point of the ecliptic. The acute angle at M, for its part, is known from the altitude of the equator, which is the complement of the altitude of the pole. Similarly, in the triangle BIM, right-angled at I, the angle IBM is sought. Either angle can be found by the following rules of spherical right-angled triangles set out in Section 1, chapter 4, as the small table below shows, arranged in order, according to the data, for the problems proposed.

[Margin: Problems 116, 117, 118, 119, 120, 121.]

Order of the problems	Data for the angle MDL	For the angle IBM	Rules
116	Angle M and side DL	Angle M; side BI	12
117	Angle M; side ML	Angle M; side MI	8
118	Angle M; base MD	Angle M; base BM	3
119	Side DL; side LM	Side BI; side IM	14
120	Side DL; base MD	Side BI; base BM	5
121	Side LM; base MD	Side IM; base BM	4

I pass over two less convenient methods: by the distance of the rising or setting point from the meridian, or by the angle of the ecliptic and the meridian together with the meridian altitude.

122. Probl. Arcum Semidiurnum ac Seminocturnum Puncti Eclipticæ vel Sideris cuiuſuis, inquirere, Datis Aſcenſionali Differentia, vel Altitudine Poli ac Declinatione Puncti aut Sideris: & Tabulam conſtruere pro huiuſmodi arcu. *122. Problem. To find the semidiurnal and seminocturnal arc of a point of the ecliptic or of any star, given the ascensional difference, or the altitude of the pole and the declination of the point or star; and to construct a table for an arc of this kind.*

[Margin: Problem 122.]

If the ascensional difference is not given, find it by problem 114 or 115 from the given altitude of the pole and declination of the point of the ecliptic or of the star. Then look at the figure set out at problem 114. If the declination is toward the visible pole P, namely LD, add the ascensional difference ML to the arc AM of the equator, which is 90°, and the result will be the semidiurnal arc AML, through which the point or star D must revolve until, from its true rising, appearing at D, it reaches the meridian. The same difference ML, subtracted from 90°, will give the seminocturnal arc. But if the point or star is at B, declining toward the other pole, subtract the ascensional difference IM from 90° for the semidiurnal arc, and add it for the seminocturnal.

I said "from the true rising" because refraction visibly raises the star, so that it rises somewhat earlier at B or D, and sets later. Therefore, if the true rising and setting have to be converted into the apparent, or the apparent into the true — and the semidiurnal arc with them — the refraction of the star, or even its parallax, must be known beforehand, and this problem must be corrected, as we shall say in Section 3.

[Margin: Authors of the table.]

Further, a table of the semidiurnal arc for every degree of the ecliptic, from 34° to 55° of the altitude of the pole, is given by Maginus in the *Directions*, folio 134, and in the *New Tables*, page 315. But it can also be had from the tables of semidiurnal time, to be reviewed in the next section, if the time is converted into parts of the equator.

123. Probl. Aſcenſionem ac Deſcenſionem Obliquam Puncti Eclipticæ, vel Sideris cuiuſuis determinare, Datis aut Aſcenſionali Differentia, aut Alt. Poli ac Declinat. & Tabulam Aſcenſionum Obliquarum conſtruere.
123. Problem. To determine the oblique ascension and descension of a point of the ecliptic or of any star, given either the ascensional difference or the altitude of the pole and the declination; and to construct a table of oblique ascensions.

[Margin: Problem 123.]

If the ascensional difference is not given, find it by problem 114 or 115, or by the tables listed there. Then find the right ascension of the point of the ecliptic by problems 8 to 16, or by the tables listed at problem 16; or the right ascension of the star by problems 25 to 28, or 56 to 60, or by the tables listed at problem 60. With these obtained, look at the figure set out at problem 114. In it, if the declination of the point or star is toward the pole visible to you — as when the star or point is at D — the ascensional difference, that is, the arc LM, added to the right ascension will make the oblique descension, and subtracted from it will make the oblique ascension at the given altitude of the pole. But if the point or star declines toward the opposite pole — as when it is at B — the ascensional difference MI, added to the right ascension, will make up the oblique ascension, and subtracted from it will leave the oblique descension.

It is by this device, then, that the tables of oblique ascensions for the degrees of the ecliptic have been made, for many if not all degrees of altitude of the

northern pole. These can nevertheless also serve for the degrees of altitude of the southern pole, if the descension is exchanged for the ascension, or the ascension for the descension, according to the rules of Book 1, chapter 23, rules 7 to 12. There I also taught how the oblique ascensions of the remaining quadrants are very easily gathered from the oblique ascensions of one quadrant of the ecliptic.

[Margin: Authors of the tables of oblique ascensions.]

Tables of oblique ascensions, further, are given by Ptolemy, Book 2, chapter 8 — but for every ten degrees of the ecliptic, and from the first clime up to an altitude of the pole of 54½°, where the longest day is of 17 hours; by Regiomontanus in the *Directions*, from page 54, from 1° to 60° of polar altitude; by Copernicus, Book 2, chapter 10 — but only for every six degrees of the ecliptic, and from 39° to 57° of altitude of the pole; by Erasmus Reinhold in the *Directions*, page 133, who extends Regiomontanus's table from 61° of polar altitude up to 89°, and at page 174, where he gives tables of oblique ascensions from 16° to 66° of altitude of the pole — but assuming the greatest obliquity of the ecliptic, the Ptolemaic one of 23° 52′, whereas in the rest, just like Regiomontanus, he had assumed an obliquity of 23° 30′, and Copernicus, for his part, 23° 28′; by Clavius in the *Sphere* [?], page 341, from 36° to 60° of altitude of the pole, assuming an obliquity of the ecliptic of 23° 30′; and by Maginus in the *Directions of the primum mobile*, from page 59, from 1° to 66° of altitude of the pole, assuming an obliquity of the ecliptic now the greatest, 23° 52′, now the least, 23° 28′. From page 93, however, Maginus constructs another table for an obliquity of the ecliptic of 23° 31′ 30″, from 1° to 79° of altitude of the pole, and he teaches the use of these tables in canons 11 and 12. This later table he carried over into the *New Tables of Directions*, from page 7, and teaches its use in canon 7 — but only up to 55° of altitude of the pole. Longomontanus gives one in Book 2 of the *Spherics*, page 70, but only for an elevation of the pole of 56°; Argoli, in volume 1 of the *Ephemerides*, page 282, but only for altitudes of the pole of 41° 50′ and 45°; Boulliau in the *Philolaic Tables*, from page 144, beginning from 1° of altitude of the pole and running up to 67°, and for the degrees of four signs, since for the remaining altitudes he gives the ascensional difference. It is worth while, however, to consult Reinhold, precept 25 of the *Directions*, or Clavius above, on the construction of these tables.

124. Probl. Data Poli Altitudine & Aſcenſione aut Deſcenſione Obliqua; Inuenire Punctum Eclipticæ ipſi congruens. *124. Problem. Given the altitude of the pole and the oblique ascension or descension, to find the point of the ecliptic corresponding to it.*

[Margin: Problem 124.]

Look up, in the body of the tables named in the preceding problem, the given oblique ascension or descension under the given altitude of the pole. At the head of the table you will have the sign of the ecliptic, and in the left margin the corresponding degree of the sign.

[Margin: Order of the problems.]
[Margin: Order of the problems.]

125. Probl. Eclipticæ punctum, cum quo Stella Oritur, aut Occidit, inuenire, Datis eius Aſcenſ. ac Deſcenſ. Obliqua, & Altitud. Aequatoris. *125. Problem. To find the point of the ecliptic with which a star rises or sets, given its oblique ascension and descension and the altitude of the equator.*

[Margin: Problem 125.]

Look up, by problem 124, the point of the ecliptic corresponding to the oblique ascension or descension. That is precisely the point with which the star rises, if you have used the ascension, or sets, if you have used the descension. Or else have recourse to problem 5, which I gave in Book 6, chapter 20, in which, however, the obliquity of the ecliptic is also assumed to be known. For stars whose latitude is not greater than 7°, canon 10 of Maginus's *New Directions* can also serve this purpose, together with the table of the difference of rising that he gives in the same place, from page 124 to 312, for altitudes of the pole from 1° to 50°.

126. Probl. Eclipticæ punctum, cum quo Stella oritur aut occidit inquirere, Data Latitudine Longitudine & Declinatione ipſius, atq. Altitudine Poli cum Obliq. Eclipt. *126. Problem. To find the point of the ecliptic with which a star rises or sets, given its latitude, longitude and declination, and the altitude of the pole with the obliquity of the ecliptic.*

[Margin: Problem 126.]

Let the meridian be ALG, the horizon GHL, and the equator FHI, whose northern pole is B and southern pole Z; and let the ecliptic be EKO, whose northern pole is C and southern V. Let the star rise or set at D or at Q, and through it let there be drawn from their poles the quadrants of declination BN and ZR, and of latitude CM and VT. Finally, let BC and ZV be the distance of the poles, that is, as great as the obliquity of the ecliptic. What is now sought is the point K of the ecliptic with which the star D or Q rises or sets.

Diagram. A circle set in the left column beside the text of problem 126, with two small circles on its rim, at upper right and lower left, drawn about the poles of the equator; a horizontal line G–L crosses it, with oblique lines for the equator and ecliptic and quadrants drawn from the poles. Labels read:

- A (above the rim, top), G (outside the rim, left) and L (outside the rim, right): the meridian ALG; G and L also end the horizon GHL
- E (rim, upper left), K (right of centre, on the horizontal line) and O (outside the rim, lower right): the ecliptic EKO
- F (rim, left, below E), H (left of centre, on the horizontal line) and I (rim, lower right): the equator FHI
- B (inside the upper-right small circle): the northern pole of the equator
- Z (inside the lower-left small circle): the southern pole of the equator
- C (right of the upper small circle): the northern pole of the ecliptic

- V (rim, lower left): the southern pole of the ecliptic
- D (on the horizontal line, right of K) and Q (left of centre, on the horizontal line): the star rising or setting
- N (lower right, right of P) and R (inside, left, below T): ends of the quadrants of declination BN and ZR
- M (lower right, below P) and T (inside, upper centre): ends of the quadrants of latitude CM and VT
- S (below H) and P (inside, lower right)

With these laid down: first, in the triangle BDL, right-angled at L, there is given, as a side, the altitude of the pole BL, and, as the base, the complement of the declination BD. Therefore by Rule 4 of spherical right-angled triangles you will find the angle BDL. Second, by problem 66, find the angle BDC — since in this triangle the three sides are given: BC, the distance of the poles; BD, the complement of the declination; and CD, the complement of the latitude — and subtract it from the angle BDL; for there will remain the angle CDL, to which the angle MDK is equal. Third, in the triangle MDK, right-angled at M, the angle already mentioned is given, and the side MD, namely the latitude of the star. Therefore by Rule 7 of spherical right-angled triangles the side MK will become known. Added to the longitude of the star, determined at M by the arc CM, it will give the point K with which the northern star D sets; subtracted, the point with which it rises. But if the star is southern, at Q, the arc TK must be sought in the same way; this, however, subtracted from the longitude T, will give the point of co-setting, and added to it, the point of co-rising. The rest concerning the risings and settings of the stars we shall give in Section 3.

Probl. Punctum Eclipticæ, cum quo Sidus tranſit per Medium cæli, ſeu Mediationem cæli inueſtigare, Data Recta Aſcenſione Sideris: ſeu Arcum Eclipticæ coaſcendentem in Sph. Recta Dato Arcui Aequatoris inuenire, & Tabulas Mediationum condere. 127. 128. 129. 130. *Problems 127, 128, 129, 130. To find the point of the ecliptic with which a star passes through mid-heaven, that is, the mediation of the heavens, given the right ascension of the star; or to find the arc of the ecliptic that rises together with a given arc of the equator in the right sphere; and to construct tables of mediations.*

[Margin: Problem 127.]

This can be obtained in four principal ways. The first and quicker way is by the tables of right ascensions, which some call the mediations of the heavens, of which I spoke at problem 60. For if you look up the given right ascension in the body of those tables, you will find at the head of the table the sign, and at the left side the degree of the sign, with which the star culminates, that is, passes through mid-heaven, even in the oblique sphere; or else the arc of the ecliptic that rises together with the given arc of the equator in the right sphere.

[Margin: Problem 128.]

The second way is by trigonometry. For it, bring back here the figure set out at problems 48 and 76, in which, for the sake of one example, let the star

be at P, and let its right ascension be given, and from it the arc λ♎, which is either the right ascension itself, if this does not exceed 90°, or its argument, if it does, according to the rules given at those same problems. Therefore in the triangle R λ ♎, right-angled at λ, the side λ♎ is given, and moreover the angle ♎, which is the obliquity of the ecliptic. Hence by Rule 9 of spherical right-angled triangles the base R♎ will not remain hidden; it will be either the arc of the ecliptic sought, if the right ascension did not exceed 90°, or, if it did, its argument, belonging to a similar quadrant.

[Margin: Problem 129.]

The third way is likewise by trigonometry, if the angle of the circle of declination with the ecliptic is given, by problems 76 and following, together with the obliquity of the ecliptic. For in the triangle R♎λ of the same figure the angles at R and at ♎ will be given; therefore by Rule 15 of spherical right-angled triangles the base R♎ will become known.

[Margin: Problem 130.]

The fourth way is also trigonometric, if the right ascension of the star is given, and the angle of the ecliptic and the circle of declination, treated from problem 76 onward. For in the triangle already mentioned the side λ♎ will be given, and the opposite angle R; and so by Rule 11 of spherical right-angled triangles the base R♎ will become known. From these a skilled geometer will understand how to proceed in the remaining cases.

[Margin: Authors of the tables.]

For transit of this kind through mid-heaven, that is, for mediation, tables had been given by Maginus, in the *Introduction to the Ephemerides*, page 145 of the Latin and 166 of the Italian, but for the year 1581, and relying on the Copernican places of the stars; by Longomontanus, Book 2 of the *Spherics*, chapter 4, at the end of problem 2, but for the year 1630, relying on the Tychonic places of the stars; by Argoli as well, volume 1, chapter 29, for the year 1640, likewise relying on the Tychonic places; and finally by Franciscus Montebrunus, at the end of volume 2 of the *Ephemerides*, for the year 1650, but using Lansbergen's places of the stars.

131. Problema. Longitudinem ac Latitudinem Stellæ inuenire, aut Aſcenſionem Rectam & Declinationem, per Duas Stellas notæ Longitud. ac Latitud. vel Aſc. Rectæ ac Declinationis, in eadem recta linea cum ſtella ignoti loci exiſtentes, & Diſtantiam ignotæ ab vna notarum. *131. Problem. To find the longitude and latitude of a star, or its right ascension and declination, by means of two stars of known longitude and latitude, or of known right ascension and declination, which stand in the same straight line with the star of unknown place, and by means of the distance of the unknown star from one of the known ones.*

[Margin: Problem 131.]

Let the poles of the ecliptic be V and R, and let GH be a portion of the ecliptic. Let the observed star be I, in a straight line with two known fixed stars,

A and B, or C and O, through which let the semicircles of latitude VBR, VOR, VIR, VCR and VAR be drawn.

Diagram. A lens-shaped figure of semicircles of latitude set in the right column beside the text of problem 131, all meeting at V at the top and R at the bottom, crossed by a horizontal line G–H; stars are drawn at A, O, I, C and B. Labels read:

- V (top point) and R (bottom point): the poles of the ecliptic
- G (outside, left end of the horizontal line) and H (outside, right end): the portion GH of the ecliptic
- I (left of centre, with a star): the observed star
- A (left, with a star) and B (lower right of centre, with a star): one pair of known fixed stars
- C (lower left, with a star) and O (upper centre, below a star): the other pair of known fixed stars
- P (right of centre, above the line): with VP a quadrant and PB the latitude of B
- M (left, just above the line), N and S (small, on the line right of M), L (below the line, right of C)

[Margin: Step 1.]

Now, if the star I is in a straight line with the stars A and B, consider the triangle VAB, in which the sides are given: AV, the complement of the latitude AG of the star A; and VB, which in our case is the sum of the quadrant VP and the latitude PB. The angle AVB is known from the difference of the longitudes of the stars A and B. Therefore, by Rule 4 of

[Margin: Order of the problems.]

spherical oblique-angled triangles, find the base AB, that is, the distance of the stars, unless it is already known from the observations, ours or others', adduced in Book 6, chapter 10. Then, by Rule 5 or 6 of the same triangles, find the angles VAB and VBA.

[Margin: Step 2.]

Second, subtract the distance of the unknown star I from either A or B, and AI or IB will remain. Therefore, in the triangle VAI, from the givens VA, the complement of the latitude of the star A, the distance AI, and the included angle VAI, you will find by Rule 4 of oblique spherical triangles the base VI, which is the complement of the latitude IS of the star I; and by Rule 5 or 6 the angle AVI, which measures the difference of longitude, to be added to the longitude of the star A, the more westerly, so that the longitude of the star I may become known. Similarly, in the triangle VIB, from the given distance IB, the side VB, and the angle VBI already found, you will find by Rule 4 the base VI, that is, the complement of the latitude of the star I, and the angle BVI, which measures the difference of longitude, to be subtracted from the longitude of the star B, the more easterly, so that the longitude of the star I may become known. From these you will gather how to proceed in other similar cases.

[Margin: On right ascension and declination.]

Moreover, whatever has been said so far of longitude and latitude holds entirely of right ascension and declination, if the names and functions of the poles, and of the arcs or angles, of longitude are exchanged for those of right ascension, and those of latitude for those of declination.

132, Longitudinem ac Latitudinem Stellæ inuenire, aut Aſc. Rectam & Declinat. ope 4. Fixarum notæ Longitud ac Latitud. vel Aſc. Rectæ & Declinat. Quarum binæ ſint in eadem recta Linea cum Stella ignoti loci, & alteræ binæ in altera recta linea cum eadem Stella. *132. To find the longitude and latitude of a star, or its right ascension and declination, by means of four fixed stars of known longitude and latitude, or of known right ascension and declination, two of which are in the same straight line with the star of unknown place, and the other two in another straight line with the same star.*

[Margin: Problem 132.]

In the preceding problem the distance of the star from one of the fixed stars was required; here it is not required, and the business can be completed by means of two threads stretched crosswise before the eye, and a straight line in which, together with the star of unknown place, there are two stars on one side and two on the other. This is easier still if the distances of the fixed stars from one another are already known from Book 6, chapter 10. Otherwise an analysis of 7 triangles must be swallowed; but the tedium of the calculation is relieved by the ease of the observation, which needs no instruments. So, in the figure of the preceding problem, let everything be set out as it is there, and let the star be at I, standing in the straight line AIB with the stars A and B, and in the straight line CIO with the stars C and O.

[Margin: Step 1.]

First, then, find by the preceding problem the distance AB, unless you already know it, and the angle VBA.

[Margin: Step 2.]

Second, in the triangle CRO, find by the same problem 131 the distance CO, unless it is known, and the angle RCO, whose supplement to a semicircle is the angle VCO.

[Margin: Step 3.]

Third, in the triangle BLP, right-angled at P, in which there are given the angle B already found and the side BP, namely the latitude of the star B, find the angle PLB by Rule 8 of spherical right-angled triangles, and the side PL by Rule 7.

[Margin: Step 4.]

Fourth, in the triangle CMN, right-angled at M, in which there are given the angle found at C and the side CM, known from the latitude of the star C, find by the same Rule 8 the angle MNC, and by Rule 7 the side MN.

[Margin: Step 5.]

Fifth, when the sides PL and MN have been joined, let their sum be subtracted from the whole MP, that is, from the difference of longitude of the stars C and B, and the portion NL will be known.

[Margin: Step 6.]

Sixth, in the triangle NIL there are given the portion NL, from step 5, and the angles at N and L, from steps 3 and 4; therefore find the side NI by Rule 10 of oblique-angled spherical triangles.

[Margin: Step 7.]

Seventh, in the triangle NIS, right-angled at S, there are given the angle N, by step 4, and the base NI, by step 6. Therefore find by Rule 1 of spherical right-angled triangles the side IS, which is the latitude of the star I, and by Rule 2 of the same the side NS. If you add to it the side MN, known from step 4, the arc MS will result, to be added to the longitude of the star C, the more westerly, so that the longitude of the star I may become known. From this you will learn the method to be kept in other cases,

[Margin: On right ascension and declination.]

and how the right ascension and declination are to be gathered, if the names and functions of longitude are exchanged for those of right ascension, and those of latitude for those of declination. And let V and R be the poles of the equator, and GH a portion of it, cut at G, M, S, L and P by the circles of declination drawn through all the stars, and so on. Since these things are easy, and the reader is supposed to be already well practised in what precedes, there is no reason to set them out more fully here.

Probl. Punctum Eclipticæ Culminans, eiuſque Altitudinem meridianam inuenire, Datis 133. Aſcenſione Recta Medij Cæli 134. Solis loco; Azimutho, & Altitudine, cum Angulo Eclipticæ & Verticalis per Solem ducti. 135. Solis loco, Declinatione, ac Tempore 136. Puncto Eclipticæ Oriente aut Occidente, Anguloque ipſius cum Horizonte, & Amplitudine Ortiua, aut Occidua. *Problems. To find the culminating point of the ecliptic and its meridian altitude, given: 133. the right ascension of mid-heaven; 134. the place of the Sun, its azimuth and altitude, with the angle of the ecliptic and of the vertical drawn through the Sun; 135. the place of the Sun, its declination, and the time; 136. a point of the ecliptic rising or setting, with its angle with the horizon, and its rising or setting amplitude.*

[Margin: Problem 133.]

First, if the right ascension of mid-heaven is given, by one of the methods of problems 108 to 113, or is known in some other way, look it up in the body of the tables of right ascensions: for at the head you will have the sign, and in the left margin the degree of the sign, that is, the culminating point, the one passing through the meridian.

[Margin: Problem 134.]

Second, for problem 134. In the following figure [?] let the meridian be HVRC, the horizon HBR, and the ecliptic EBC, on which let the Sun be at S.

Through the Sun's centre let there pass the quadrant VSA of the vertical circle, and from the northern pole of the world P the portion PS of a great circle, which will be either the complement of the northern declination, if the Sun is in the northern signs; or the sum of a quadrant and the southern declination, if it is in the southern signs; or a quadrant, if it is on the equator. Therefore, in the triangle ESV, the portion ES of the ecliptic must be obtained; added to the place of the Sun S, if the Sun is in the western quadrant, or subtracted, if in the eastern, it will give the culminating point [?] of the ecliptic.

Diagram. A circle set in the right column beside the text of problem 134, crossed by a near-horizontal line from H to R, with arcs for the ecliptic, the vertical through the Sun, and a great circle from the pole P. Labels read:

- H (outside the rim, left), V (rim, top), R (outside the rim, right) and C (outside the rim, lower right): the meridian HVRC
- H, B (below the line, right of centre) and R: the horizon HBR
- E (outside the rim, upper left), B and C: the ecliptic EBC; E is also the culminating point, with HE its meridian altitude
- S (inside, upper left): the Sun
- V, S and A (inside, left, on the line): the quadrant VSA of the vertical circle
- P (rim, upper right): the northern pole of the world, with PS a portion of a great circle
- M (above the rim, left of V), O (outside the rim, left, below E), K (inside, right, just above the line) and I (below B, near the rim)

You will obtain ES because there are given SV, the complement of the Sun's altitude; SVE, the Sun's azimuth; and ESV, the angle of the ecliptic and the vertical (or else found from problems 89 to 98). Therefore by Rule 11 or 12 of oblique spherical triangles the side ES will become known, and EV, the complement of the meridian altitude HE of the point E.

[Margin: Problem 135.]

Third, for problem 135, consider the triangle SPE, in which the angle PSE is given: for from the place of the Sun on the ecliptic the angles of the ecliptic and the meridian, that is, the circle of declination, are given by problem 14, or 76 to 83. And from the time before or after noon, converted into degrees of the equator, the angle SPE is given; finally, from the declination of the Sun, SP is given. Therefore by Rule 11 or 12 of oblique-angled spherical triangles the side ES will become known; added to the place of the Sun when it is western, or subtracted from the place of the Sun when it is eastern, it will give the culminating point of the ecliptic. And when VP, the complement of the altitude of the pole, is taken from the arc EP, there will remain EV, the complement of the meridian altitude HE of the culminating point E.

[Margin: Problem 136.]

Fourth, for problem 136, look at the triangle HBE, right-angled at H, in which, from the rising or setting point B, the angle HBE, which the ecliptic makes with the horizon, is given by the tables or by the problems that follow.

Moreover, from the rising or setting amplitude the arc HB is given. This arc either is, according to some, the rising or setting amplitude itself; or it is made up of that amplitude and a quadrant, if the point B is on this side of the equator; or it is the remainder of a quadrant from which the rising or setting amplitude of the point B, situated beyond the equator, has been taken. With these given, find by Rule 9 of spherical right-angled triangles the base BE, and by Rule 7 the side HE, which is the meridian altitude of the culminating point E. This point is known if you add the base BE to the place of the point B when it is setting, or subtract it from the place of the point B when it is rising. It will help, however, to use a globe, so that you may see more quickly how to proceed.

[Margin: Order of the problems.]

137. Probl. Punctum Eclipticæ imum Cæli obtinens, eiuſque Depreſſionem inuenire. *137. Problem. To find the point of the ecliptic that occupies the lowest point of the heavens [imum caeli], and its depression.*

[Margin: Problem 137.]

Find by problems 133 to 136 the culminating point and its meridian altitude. For the point of the ecliptic diametrically opposite to it, that is, 180° distant from the culminating point, will be the one found in the hidden part of the meridian; and its depression will be as great as the meridian altitude of the culminating point.

138. Probl. Puncti Eclipticæ Culminantis Altitudinem Meridianam inuenire, Data eius Declinatione, & Altitudine Poli. *138. Problem. To find the meridian altitude of the culminating point of the ecliptic, given its declination and the altitude of the pole.*

[Margin: Problem 138.]

In the figure of problem 134, let the given declination of the culminating point E of the ecliptic be either northern, EO, or southern, EM; and let the point of the equator passing through the meridian be either M, whose altitude is HM (which, of course, is known if you subtract the altitude of the pole from 90°), or O, whose altitude is HO, known in the same way. Therefore subtract the southern declination EM from the altitude HM of the equator, or add the northern declination EO to the altitude HO of the equator, and you will have the meridian altitude HE of the point E.

139. Probl. Punctum Eclipticæ Oriens vel Occidens inuenire, Datis Solis aut puncti noti Eclipticæ Culminantis Altitud. meridiana, cum angulo Eclipticæ & Meridiani. *139. Problem. To find the rising or setting point of the ecliptic, given the meridian altitude of the Sun, or of a known culminating point of the ecliptic, with the angle of the ecliptic and the meridian.*

[Margin: Problem 139.]

In the figure immediately preceding, the triangle HBE is right-angled at H, and in it there are given the altitude HE of the Sun or of the point E, and

also the angle HEB, of the ecliptic and the meridian. Therefore by Rule 9 of spherical right-angled triangles obtain the base BE; add it to the place of the Sun or of the point E, and you will have the rising point B of the ecliptic; subtract it, and you will have the setting point. If the rising amplitude of the point B were given, and from it the arc HB, this could be done another way; but that amplitude is not usually given unless the point B is known first.

140. Probl. Punctum Eclipt. Oriens vel Occidens inuenire, Datis Tempore à Meridie, & loco Solis, cum Altitudine Poli. *140. Problem. To find the rising or setting point of the ecliptic, given the time from noon and the place of the Sun, with the altitude of the pole.*

[Margin: Problem 140.]

Under the given altitude of the pole, look up the semidiurnal arc in its own tables, and also the oblique ascension corresponding to the place of the Sun in the tables of oblique ascensions, which are treated at problem 123. Finally, convert the given time from noon into degrees of the equator, and combine these three into one sum, casting off 360° if the sum exceeds them. Then enter the body of the tables of oblique ascensions with the sum already mentioned, or with its remainder, under the given altitude of the pole: for at the head you will have the sign, and in the left margin the degree, that is, the rising point of the ecliptic, from which the setting point is 180° distant.

141. Probl. Punctum Eclipticæ Oriens & Occidens inuenire, Datis Altitudine Aequatoris, Obliquitate Eclipticæ, & Tempore ante vel poſt meridiem ex Sidere notæ Aſcenſ. Rectæ. *141. Problem. To find the rising and setting point of the ecliptic, given the altitude of the equator, the obliquity of the ecliptic, and the time before or after noon taken from a star of known right ascension.*

[Margin: Problem 141.]

If the star from which the time is known is eastern, you will look for the rising point of the ecliptic; if western, the setting point; for the one is 180° from the other along the ecliptic. Now in the accompanying diagram let the meridian be HARK, the horizon GFR, and the equator AGQ; and first let the intersection with the equator of the portion ZO or TF of the ecliptic standing above the horizon be at C, whether vernal or autumnal. Through the star from which the time was taken, whether it is on the ecliptic or off it, as at L, let there be drawn the circle of declination PLD, through the poles of the equator P and D, which will determine the right ascension of that same star at G.

[Margin: Step 1.]

First, then, the arc GC of the equator will be known, namely the distance of the point G from the nearer equinox; and from the time before or after noon, converted into degrees of the equator, the arc GA is known, whose complement to 90° is the arc GB. Add to it the arc GC, and the arc CB becomes known.

[Margin: Step 2.]

Second, if the star is eastern and the intersection C vernal, or western and the intersection C autumnal, use the triangle CBO, in which, besides the side or base CB, there are given the angle BCO, the angle of the obliquity of the ecliptic, and the angle CBO, the supplement to two right angles of the angle ABH, which measures the altitude of the equator. Therefore by Rule 11 or 12 of oblique-angled spherical triangles the side CO will become known, that is, the distance of the point O, rising or setting, from the nearer equinox.

Diagram. A circle set in the right column beside the text of problem 141, crossed by a near-horizontal line from H to R, with arcs for the equator, the ecliptic and the circle of declination, and a small star drawn below L. Labels read:

- H (outside the rim, left end of the line), A (top), R (outside the rim, right end of the line) and K (below the rim, bottom): the meridian HARK
- G (just above the line, left of centre), F (just below the line, left of centre) and R: the horizon GFR, as printed
- A, G and Q (bottom, right of K): the equator AGQ; G is also where the circle of declination meets the equator
- Z (above the rim, top left), O (below the line, right of B), T (top, right of A) and F: the portions ZO and TF of the ecliptic
- C (inside, upper half, left of centre): the intersection of the ecliptic and the equator above the horizon
- L (inside, right of C, with a star): the star off the ecliptic
- P (rim, upper right) and D (rim, lower left): the poles of the equator, on the circle of declination PLD
- B (below the line, right of F): the point of the arcs GB and CB and of the angles CBO, ABH and FBC
- M (inside, lower half, below O): the intersection below the horizon
- S (bottom, right of Q)

But if the star is eastern but the intersection C autumnal, or western and the intersection vernal, use the triangle CBF, in which, besides CB, there are given the obliquity of the ecliptic BCF and the angle FBC, from the altitude of the equator. Therefore by Rule 11 or 12 of the same triangles find FC, the distance of the point F, rising or setting, from the nearer equinox.

But if the intersection of the ecliptic and the equator is below the horizon, as at M, the right ascension will give the arc GM, namely the distance of the point G from the nearer equinox, and the time, as above, will give the arc AG, and from it GB. When GB is subtracted from the arc GM, the arc BM will be known. With it, in the triangle BMO or BMF, and with the angles set up — that of the obliquity at M, and that of the altitude of the equator, or its complement, at B — you will find by the same Rule 11 or 12 the arc MO or MF, that is, the distance along the ecliptic of the rising or setting point from the nearer equinox M.

Probl. Punctum Eclipticæ Oriens aut Occidens inuenire duobus modis, Datis Loco Solis, & Tempore apparenti poſt meridiem, cum Altitudine Poli. 142. 143. *Problems 142 and 143. To find the rising or setting point of the ecliptic in two ways, given the place of the Sun and the apparent time after noon, with the altitude of the pole.*

[Margin: Problems 142 and 143.]

I taught the first of these ways in Book 5, chapter 10, number 5, and the second in the same place, number 6, by the tables of houses. And opposite the rising point lies the setting point, 180° distant from it.

Probl. Punctum Aequatoris Oriens & Occidens inuenire, Datis 144. Aſc. Recta Medij Cæli 145. Aſc. Recta ſideris, eiuſq. Diſtãtia Aequatoria à Meridiano. *Problems. To find the rising and setting point of the equator, given: 144. the right ascension of mid-heaven; 145. the right ascension of a star and its equatorial distance from the meridian.*

[Margin: Problem 144.]

Add 90° to the right ascension of mid-heaven, and you will have the rising point of the equator; subtract 90° from the same right ascension of mid-heaven, and you will have the setting point of the equator.

[Margin: Problem 145.]

If the equatorial distance of a star from the meridian, known either from the time before or after noon (as happens with the Sun) or from some other source, is a quadrant, that is, 90°, the right ascension of the star itself will give the rising point of the equator if the star is eastern, and the setting point if it is western. But if the distance is less than a quadrant, its complement to 90°, added to the right ascension of an eastern star, will give the rising point of the equator; subtracted from the right ascension of a western star, it will give the setting point. But if the distance of the star from the meridian exceeds a quadrant of the circle, its excess over 90°, subtracted from the right ascension of an eastern star, will give the rising point of the equator; added to the right ascension of a western star, it will give the setting point.

[Margin: Order of the problems.]

146.Probl. Aſcenſionem Obliquam Horoſcopi, ſeu Puncti Eclipticæ Aſcendentis inuenire, Dato Loco Solis, & Tempore à meridie. *146. Problem. To find the oblique ascension of the horoscope, that is, of the ascending point of the ecliptic, given the place of the Sun and the time from noon.*

[Margin: Problem 146.]

Find the right ascension of the given place of the Sun by the tables reviewed at problem 16, and add to it the hours from noon converted into degrees of the equator; the result will be the right ascension of mid-heaven, casting off, however, 360° if it exceeds them. Add to this ascension a quadrant, that is, 90°, and the result will be the oblique ascension of the horoscope.

Probl. Angulum Eclipticæ & Horizontis inueſtigare, Datis 147. Puncti Eclipticæ Culminantis Altitudine meridiana, & Angulo Eclipt. ac Meridiani 148. Puncti Eclipt. Culminantis angulo Eclipt. ac Meridiani & Puncti Eclipt. in Horizõte poſiti Amplitudine ortiua vel occidua 149. Puncti Eclipt. Culminantis angulo Eclip. ac Merid. & Puncto Eclipticæ Oriente aut Occidente. 150. Puncti Eclipticæ Culminantis Altitudine meridiana, & puncti Eclipt. Orientis, vel Occidentis Amplitud. ortiua, vel occid. 151. Puncti Eclipticæ Culminãtis Altitud. meridiana, & Pũcto Eclipticæ Oriente vel Occidente 152. Puncti Eclipticæ Orientis vel Occidentis Amplitud. ortiua vel occidua, & Puncto Eclipt. Culminante. *Problems. To investigate the angle of the ecliptic and the horizon, given: 147. the meridian altitude of the culminating point of the ecliptic, and the angle of the ecliptic and the meridian; 148. the angle of the ecliptic and the meridian at the culminating point of the ecliptic, and the rising or setting amplitude of the point of the ecliptic standing on the horizon; 149. the angle of the ecliptic and the meridian at the culminating point of the ecliptic, and the rising or setting point of the ecliptic; 150. the meridian altitude of the culminating point of the ecliptic, and the rising or setting amplitude of the rising or setting point of the ecliptic; 151. the meridian altitude of the culminating point of the ecliptic, and the rising or setting point of the ecliptic; 152. the rising or setting amplitude of the rising or setting point of the ecliptic, and the culminating point of the ecliptic.*

Let the figure set out at problem 134 be repeated here, in which the culminating point of the ecliptic is E, its meridian altitude HE, and the angle of the ecliptic and the meridian HEB, while the rising or setting point of the ecliptic is B. When B is given together with the point E, the difference along the ecliptic shows the base EB in the triangle EHB, right-angled at H; in this triangle, from the given point B, the rising amplitude is known, and from it, in turn, the side HB. Now the angle HBE is sought, which can be had by the rules of spherical right-angled triangles written below, according to the order of the givens.

Diagram. The woodcut of problem 134 set again in the left column beside the text of problems 147–152: a circle crossed by a near-horizontal line from H to R, with arcs for the ecliptic, the vertical, and a great circle from the pole P. Labels read:

- E (outside the rim, upper left): the culminating point, with HE its meridian altitude and HEB the angle of the ecliptic and the meridian
- B (below the line, right of centre): the rising or setting point, with EHB the triangle right-angled at H and HB its side on the horizon
- H (outside the rim, left end of the line) and R (outside the rim, right end): with PR the altitude of the pole and BPR the triangle right-angled at R (problem 153)
- P (rim, upper right): with PB the complement of the declination of B (problem 153)
- C (outside the rim, lower right): in the angles PBC and RBC (problem 153)

- S (inside, upper left): the nonagesimal degree, with BS a quadrant (problems 154–155)
- A (inside, left, on the line) and V (rim, top): with ASB the triangle right-angled at A, AS the altitude of the nonagesimal, SV its distance from the vertex, and VA the circle of altitude (problems 154–155)
- M (above the rim, left of V), O (outside the rim, left, below E), K (inside, right, just above the line) and I (below B, near the rim)

Order of the problems	Data for the angle EBH	Rules
147	Angle HEB, side HE	8
148	Angle HEB, side HB	12
149	Angle HEB, base EB	3
150	Side HE, side HB	14
151	Side HE, base EB	4
152	Side HB, base EB	5

[Margin: Problems 147, 148, 149, 150, 151, 152.]

153.Probl. Angulum Eclipticæ & Horizontis inuenire, Datis Puncto Eclipticæ Oriente vel Occidente, ipſiuſq. Declinatione & Alt. Poli. Et Tabulas huius Anguli cõſtruere. *153. Problem. To find the angle of the ecliptic and the horizon, given the rising or setting point of the ecliptic, its declination, and the altitude of the pole; and to construct tables of this angle.*

[Margin: Problem 153.]

In the preceding figure let B be the rising or setting point of the ecliptic. The complement of its given declination is the base PB in the triangle BPR, right-angled at R, in which the side PR is given, which is the altitude of the pole. Therefore by Rule 4 of spherical right-angled triangles find the angle PBR, and add it to the angle PBE, known from the point B (for it is as great as the angle of the ecliptic and the meridian, treated at problem 14, or the angle of the ecliptic and the circle of declination, treated from problem 76 onward); and the angle EBR will result, whose supplement to two right angles is the angle sought, EBH. Or, if from the angle PBC, known from the angle of the ecliptic and the meridian, that is, the circle of declination, you subtract the angle PBR, there remains RBC, equal to the angle sought, EBH.

It is chiefly by this device that the table of the angle of the orient is constructed for each altitude of the pole — that is, of the angle of the ecliptic and the horizon made in the east. The angle made in the west by the ecliptic and the horizon is always equal to it, speaking of angles of the same kind and comparing acute with acute, as Regiomontanus shows in the *Epitome of the Almagest*, Book 2, proposition 35. And in the same place, proposition 36, he demonstrates that the external eastern angle taken together with the internal western, or the internal eastern together with the external western, are equal to two right angles.

[Margin: Authors of the table [?] of this angle.]

Furthermore, a table of such angles is given by Copernicus, Book 2, chapter 10, from a polar altitude of 39° to 57°, for every sixth degree of the signs; by Maginus, in the *Tables of Directions of the primum mobile*, from folio 200 on, for each degree of the rising signs, and from an altitude of the pole of 30° to 51°; and by Kepler, in the *Rudolphine Tables*, folio 26, for each altitude of the pole and for every third degree of the rising signs. This last table was reproduced by Reinerus [Vincenzo Renieri] in the new *Medicean Tables*, page 324, and by Boulliau in the *Philolaic Tables*, page 158. On their use see what I said in Book 5, chapter 10, number 8.

154. Probl. Angulum Eclipticæ & Horizontis inuenire, Data Nonageſimi Eclipticæ gradus ab Horizonte computati Altitud. aut Diſtantia à vertice. *154. Problem. To find the angle of the ecliptic and the horizon, given the altitude, or the distance from the vertex, of the nonagesimal degree of the ecliptic reckoned from the horizon.*

[Margin: Problem 154.]

In the figure immediately preceding, let S be the nonagesimal degree reckoned from the horizon point B, so that the base BS is 90°. Therefore in the triangle ASB, right-angled at A, if the altitude AS of the nonagesimal is given, known either by itself or through its complement SV, which is the distance of the nonagesimal from the vertex, the angle sought, ABS, will also be given by Rule 4 of spherical right-angled triangles.

155. Probl. Diſtantiam Nonageſimi Eclipticæ gradus à Vertice vel eius Altitudinem inuenire Modis 7. & de Tabula Diſtantiæ huius vel Altitud. *155. Problem. To find the distance of the nonagesimal degree of the ecliptic from the vertex, or its altitude, in 7 ways; and on the table of this distance or altitude.*

[Margin: Problem 155.]

The altitude of the nonagesimal degree of the ecliptic, reckoned from the horizon, subtracted from 90°, gives its distance from the vertex. The altitude of the nonagesimal itself can be found in as many ways as the angle of the ecliptic and the horizon can be found — not, however, by the ways that presuppose the altitude of the nonagesimal — namely by problems 147, 148, 149, 150, 151, 152 and 153. The reason is that in the preceding figure the distance of the nonagesimal S along the ecliptic from the horizon, namely the arc BS, is a quadrant, since it amounts to 90°; while the circle of altitude VA makes a right angle with the other arc of the horizon, AB, just as it does with BS. Therefore the angles at A and S are right, and so, by the properties of spherical triangles, AB is a quadrant, and the angle ABS is the measure of the altitude AS. Hence the tables of the angle of the ecliptic and the horizon, treated at the end of problem 153, serve as they stand for the altitude of the nonagesimal. Besides these, Nicolaus Mulerius, in the *Frisian Tables*, page 448, gives a table of the distance of the nonagesimal both from the vertex and from the meridian; so, by subtracting the distance from the vertex from 90°, the altitude of the nonagesimal remains. But he

gives the table from an altitude of the pole of 42° to 56°, whereas our own Petavius [Denis Pétau], in Book 8 of *De doctrina temporum*, page 735, extends it from an altitude of 31° to 51°.

[Margin: Order of the problems.]

Probl. Angulum Meridiani & Verticalis per Nonageſimum Eclipt. gradum ducti inueſtigare, ideſt Azimuthum Nonageſimi; Datis 156. Altitudine puncti Eclipticæ Culminantis, & Arcu Eclipticæ inter Meridianum ac Nonageſimum 157. Diſtantia Nonageſimi à Vertice, & Arcu Eclipt. inter Meridianũ ac Nonageſimum 158. Diſtantia Nonageſimi à Vertice, & Altitudine puncti Eclipt. Culminantis 159. Angulo Eclipticæ & Meridiani noto ex puncto Eclip.. culminante, & Diſtantia Nonageſimi à Vertice 160. Angulo eodem, & Arcu Eclipticæ inter Meridianum ac Nonageſimum 161. Angulo eodem, & Altitudine puncti Eclipticæ Culminantis. *Problems. To investigate the angle of the meridian and the vertical drawn through the nonagesimal degree of the ecliptic, that is, the azimuth of the nonagesimal, given: 156. the altitude of the culminating point of the ecliptic, and the arc of the ecliptic between the meridian and the nonagesimal; 157. the distance of the nonagesimal from the vertex, and the arc of the ecliptic between the meridian and the nonagesimal; 158. the distance of the nonagesimal from the vertex, and the altitude of the culminating point of the ecliptic; 159. the angle of the ecliptic and the meridian, known from the culminating point of the ecliptic, and the distance of the nonagesimal from the vertex; 160. the same angle, and the arc of the ecliptic between the meridian and the nonagesimal; 161. the same angle, and the altitude of the culminating point of the ecliptic.*

Taking up again the figure we set out at problem 153, consider the triangle EVS, right-angled at S. It is right-angled because the vertical circle AV, drawn through the nonagesimal S, makes a right angle at S with the ecliptic EC, since it is the same as the circle of latitude, as is clear from what was said in Book 5, chapter 10, Theorem 1. In this triangle the angle EVS is sought, from any two of the four remaining parts of the triangle. These are: the base EV, which is the complement to 90° of the altitude HE of the culminating point E; the side ES, that is, the arc of the ecliptic lying between the meridian and the nonagesimal S; the side SV, namely the distance of the nonagesimal from the vertex, which is the complement of the altitude AS, known from problem 155; and finally the angle SEV, which the ecliptic makes with the meridian, and which is known by problem 14 if the point E of the ecliptic is known. Which of the angles at E serves, an inspection of the astronomical globe will show. With this laid down, we shall indicate in our usual way the rules of spherical right-angled triangles by which the angle EVS can be found, according to the givens that serve the problems set out in order.

Order of the problems	Data for the angle EVS	Rules
156	Base EV and side ES	4
157	Side VS, side ES	14

Order of the problems	Data for the angle EVS	Rules
158	Base EV, side VS	5
159	Side VS, angle SEV	12
160	Side ES, angle SEV	8
161	Base EV, angle [?] SEV	3

[Margin: Problems 156, 157, 158, 159, 160, 161.]

Probl. Arcum Eclipticæ inter Meridianum & Nonageſimum, necnon inter Meridianum & Horizontem inuenire, Datis 162. Angulo Eclipticæ & Meridiani, ex puncto Culminante, & Altitudine puncti Eclipticæ Culminantis 163. Angulo eodem, & Altitudine Nonageſimi 164. Angulo eodem, & Angulo Meridiani ac Verticalis per Nonageſimum ducti 165. Altitudine puncti Eclipticæ Culminantis, & Nonageſimi. 166. Altitud puncti Eclipt. Culmin. & Angulo Meridiani ac Verticalis per Nonageſimum ducti 167. Altitudine Nonageſimi, & Angulo Meridiani ac Verticalis per Nonageſimũ ducti 168. Angulo Eclipticæ ac Meridiani, & Angulo Eclipticæ ac Horizontis 169. Angulo Eclipt. & Merid. & Altitudine puncti Eclipticæ Culminantis 170. Angulo Eclipticæ & Merid. & Puncti Aſcendentis vel Deſc. Amplitud. Ortiua vel Occidua 171. Angulo Eclipticæ, & Horizontis; & Altitud. puncti Eclipt Culmin. 172. Angulo Eclipticæ & Horizontis; & puncti Aſcend vel Deſcendentis Amplitudine Ortiua, vel Occidua 173. Altitudine puncti Eclipticæ Culminantis; & puncti Aſcendentis, aut Deſcendentis Amplitudine Ortiua vel Occidua. *Problems. To find the arc of the ecliptic between the meridian and the nonagesimal, and also between the meridian and the horizon, given: 162. the angle of the ecliptic and the meridian, from the culminating point, and the altitude of the culminating point of the ecliptic; 163. the same angle, and the altitude of the nonagesimal; 164. the same angle, and the angle of the meridian and the vertical drawn through the nonagesimal; 165. the altitude of the culminating point of the ecliptic, and that of the nonagesimal; 166. the altitude of the culminating point of the ecliptic, and the angle of the meridian and the vertical drawn through the nonagesimal; 167. the altitude of the nonagesimal, and the angle of the meridian and the vertical drawn through the nonagesimal; 168. the angle of the ecliptic and the meridian, and the angle of the ecliptic and the horizon; 169. the angle of the ecliptic and the meridian, and the altitude of the culminating point of the ecliptic; 170. the angle of the ecliptic and the meridian, and the rising or setting amplitude of the ascending or descending point; 171. the angle of the ecliptic and the horizon, and the altitude of the culminating point of the ecliptic; 172. the angle of the ecliptic and the horizon, and the rising or setting amplitude of the ascending or descending point; 173. the altitude of the culminating point of the ecliptic, and the rising or setting amplitude of the ascending or descending point.*

Let the figure set out at problem 153 be called back here. In it, if the arc ES is obtained, which lies between the meridian HVR and the nonagesimal S, that arc, added to the quadrant SB, will make up the arc EB of the ecliptic

between the meridian and the horizon. Conversely, if the arc EB of the ecliptic intercepted between the meridian and the horizon is obtained, then, when the quadrant SB, that is, 90°, is taken away, the arc ES between the meridian and the nonagesimal will remain.

[Margin: For the arc ES.]

Furthermore, the arc ES can be obtained in the triangle [?] ESV, right-angled at S, from pairs of the four givens — EV, the complement of the altitude HE of the culminating point; VS, the complement of the altitude of the nonagesimal; the angle SEV of the ecliptic and the meridian, known from the culminating point E; and the angle EVS of the meridian and the vertical VA drawn through the nonagesimal S — if the rules of spherical right-angled triangles written below are applied.

Diagram. The woodcut of problems 134 and 147–155 set again in the right column beside the text for problems 162–173: a circle crossed by a near-horizontal line from H to R, with arcs for the ecliptic, the vertical, and great circles from the pole P. Labels read:

- E (outside the rim, upper left): the culminating point, with HE its altitude, SEV the angle of the ecliptic and the meridian, and HEB the same angle in the triangle EBH
- V (rim, top): the vertex, with EV the complement of the altitude of E, VS the complement of the altitude of the nonagesimal, and VA the vertical through S
- S (inside, upper left): the nonagesimal, with ES the arc of the ecliptic from the meridian to the nonagesimal, SB a quadrant, and EVS the triangle right-angled at S
- H (outside the rim, left end of the line) and R (outside the rim, right end): with HVR the meridian, EBH the triangle right-angled at H, and HB the rising or setting amplitude of B
- B (below the line, right of centre): the rising or setting point, with EB the arc of the ecliptic between the meridian and the horizon, and HBE the angle of the ecliptic and the horizon
- A (inside, left, below the line): with AS the altitude of the nonagesimal
- C (outside the rim, lower right): with EC the ecliptic
- M (above the rim, left of V), P (rim, upper right), O (outside the rim, left, below E), K (inside, right, just above the line) and I (below B, near the rim)

The arc EB, on the other hand, is obtained in the triangle EBH, right-angled at H, from pairs of the four givens: HE, the altitude of the culminating point E; HEB, the angle of the ecliptic and the meridian, known from the culminating point E; the angle HBE, of the ecliptic and the horizon; and HB, the rising amplitude of the point B when it is rising, or the setting amplitude when it is setting; or let the rules of spherical right-angled triangles written below be followed. Remember, however, that if the rising amplitude is less than a

quadrant, the nonagesimal is in the western quadrant; if greater, in the eastern. The contrary of these happens with the setting amplitude.

[Margin: For the arc EB.]

Order of the problems	Data for the arc ES	Rules
162	Angle SEV, base EV	2
163	Angle SEV, side VS	10
164	Angle SEV, angle SVE	16
165	Base [?] EV, side VS	6
166	Base EV, angle SVE	1
167	Side SV, angle SVE	7
	Data for the base EB	
168	Angle HEB, angle HBE	15
169	Angle HEB, side HE	9
170	Angle HEB, side HB	11
171	Angle HBE, side HE	11
172	Angle HBE, side HB	9
173	Side HE, side HB	13

[Margin: 162, 163, 164, 165, 166, 167.]

[Margin: 168, 169, 170, 171, 172, 173.]

A table for this arc ES, however, constructed from a polar altitude of 51° to 55°, is given by Mulerius in the *Frisian Tables*, page 448.

[Margin: Table of this arc.]

[Margin: Order of the problems.]

Probl. Amplitudinem Ortiuam Horoſcopi, vel Occiduam puncti Eclipticæ occidentis Inuenire, Datis 174. Azimutho Nonageſimi, de quo Prob. 156. 175. Angulo Eclipt. & Horizontis, & Altitud. puncti Eclipticæ Culminantis 176. Angulo Ecl. & Horiz. & Arcu Eclipticæ inter Meridianum & Horizontem 177. Angulo Ecl. & Horiz. & Angulo Eclip. ac Meridiani 178. Angulo Eclipt. & Meridiani, & Altitud. puncti Eclipticæ Culminantis 179. Angulo Eclipticæ & Meridiani, & Arcu Eclipt. inter Merid. & Horizontem. 180. Altitudine puncti Eclipticæ Culminantis, & Arcu Eclipt. inter Merid. & Horiz. *Problems. To find the rising amplitude of the horoscope, or the setting amplitude of the setting point of the ecliptic, given: 174. the azimuth of the nonagesimal, treated at problem 156; 175. the angle of the ecliptic and the horizon, and the altitude of the culminating point of the ecliptic; 176. the angle of the ecliptic and the horizon, and the arc of the ecliptic between the meridian and the horizon; 177. the angle of the ecliptic and the horizon, and the angle of the ecliptic and the meridian; 178. the angle of the ecliptic and the meridian, and the altitude of the culminating point of the ecliptic; 179. the angle of the ecliptic and the meridian, and the arc of the ecliptic between the meridian and the horizon; 180. the altitude of the culminating point of the ecliptic, and the arc of the ecliptic between the meridian and the horizon.*

[Margin: Problem 174.]

Look at the figure immediately preceding, and at the explanation given for the arc EB. If, then, the azimuth AVH is given, by problems 156 to 161, since it measures the arc HA, add that arc to 90°, that is, to the quadrant AB; and the result will be the rising amplitude of the point B when it is rising, if the nonagesimal is in the eastern quadrant, or when it is setting, if it is in the western. Conversely, subtract the arc HA from 90°, and you will have the rising amplitude of the point B when it is rising, if the nonagesimal is in the western quadrant, or when it is setting, if it is in the eastern. For the other problems use the triangle EHB, right-angled at H, and the givens and rules of spherical right-angled triangles that follow.

Order of the problems	Data for the arc BH	Rules
175	Angle EBH and side HE	10
176	Angle EBH, base BE	2
177	Angle EBH, angle HEB	16
178	Angle HEB, side HE	7
179	Angle HEB, base BE	1
180	Side HE, base BE	6

[Margin: Problems 175, 176, 177, 178, 179, 180.]

181. Probl. Nonageſimum Eclipticæ gradum aut Punctum ab Horizonte numeratum Inuenire. *181. Problem. To find the nonagesimal degree or point of the ecliptic, reckoned from the horizon.*

[Margin: Problem 181.]

Find, by one of problems 139 to 143, the rising or setting point of the ecliptic; then subtract 90° from the longitude of the rising point, or add 90° to that of the setting point. For what remains, or results, will be the nonagesimal degree of the ecliptic reckoned from the horizon. As in the preceding figure: if from the ecliptic place, that is, the longitude, of the rising point B you subtract the quadrant BS, the longitude of the nonagesimal point S will be known; but if B is the setting point, add the quadrant BS to it.

Probl. Nonageſimi Situm Inuenire, hoc eſt, ſitne in Meridiano, an in Quadrante Orientali, vel Occid. Eclipticæ, Datis 182. Nonageſimo ipſo puncto Eclipticæ; 183. Puncto Eclipticæ Oriente 184. Puncto Eclipt. Oriente, & Culminante 185. Puncto Eclipticæ Culminante 186. Puncti Eclipticæ Orientis Amplitudine Ortiua. *Problems. To find the situation of the nonagesimal, that is, whether it is on the meridian, or in the eastern or the western quadrant of the ecliptic, given: 182. the nonagesimal itself, as a point of the ecliptic; 183. the rising point of the ecliptic; 184. the rising and the culminating point of the ecliptic; 185. the culminating point of the ecliptic; 186. the rising amplitude of the rising point of the ecliptic.*

[Margin: Problem 182.]

First: if the given nonagesimal is the beginning of ♋ or ♑, the nonagesimal will be on the meridian, as it will be whenever it passes through the vertex. But if the nonagesimal is neither of the beginnings mentioned, and the inhabitant is outside the Torrid Zone: from the beginning of ♋ to the beginning of ♑, that is, in the descending signs, it will be in the western quadrant for northerners, and in the eastern for southerners. But if the nonagesimal is in the ascending signs, namely from the beginning of ♑ to the beginning of ♋, it will be in the eastern quadrant for northerners, and in the western for southerners.

Within the Torrid Zone, however: if the nonagesimal is from the beginning of ♋ to the beginning of ♑, it will be in the eastern quadrant with respect to northerners and in the western with respect to southerners, if its declination is greater than the geographical latitude of the inhabitant and of the same kind; but if it is less, or of a different kind, it will be in the western with respect to northerners and in the eastern with respect to southerners. If, on the other hand, the nonagesimal is from the beginning of ♑ to the beginning of ♋, it will be in the western quadrant with respect to northerners and in the eastern with respect to southerners, if its declination is greater than the geographical latitude and of the same kind; but if it is less, or of a different kind, it will be in the eastern with respect to northerners and in the western with respect to southerners.

Finally, with respect to those who live under the equator, the nonagesimal of the first or third quadrant of the ecliptic is in the western quadrant, while that of the second or fourth is in the eastern.

[Margin: Problem 183.]

Second: if the rising point of the ecliptic is the beginning of ♈ or ♎, the nonagesimal will be on the meridian, and it will be the beginning of ♋ or ♑. But if the rising point is neither of the beginnings mentioned, and the inhabitant is outside the Torrid Zone: if the rising point of the ecliptic is within the northern semicircle, the nonagesimal will be in the eastern quadrant with respect to northerners, and in the western with respect to southerners. On the contrary, if the rising point is within the southern semicircle [?], the nonagesimal will be in the western quadrant with respect to northerners [?], and in the eastern with respect to southerners.

Within the Torrid Zone, however, subtract 90° from the rising point, and you will have the nonagesimal. With this, then, proceed as in the part of the preceding problem that concerns the Torrid Zone.

[Margin: Problem 184.]

Third: if the rising point and the culminating point of the ecliptic are exactly 90° apart, the nonagesimal will be on the meridian, and will culminate. But if they are more than 90° apart, the nonagesimal will be in the eastern quadrant; if less than 90°, in the western.

[Margin: Problem 185.]

Fourth: if the culminating point of the ecliptic is the beginning of Cancer or Capricorn, the nonagesimal will be on the meridian. But if it is neither of these, and the inhabitant is outside the Torrid Zone: from the beginning of ♋ to the beginning of ♎, that is, in the descending signs, the nonagesimal will be in the western quadrant with respect to northerners, and in the eastern with respect to southerners; but from the beginning of ♎ to the beginning of ♋, the nonagesimal will be in the eastern with respect to northerners, and in the western with respect to southerners.

Within the Torrid Zone, however: if the culminating point is in the descending signs, that is, from the beginning of ♋ to the beginning of ♎, the nonagesimal will be in the eastern quadrant with respect to northerners and in the western with respect to southerners, if the declination of the culminating point is greater than the geographical latitude of the inhabitant and of the same character; but if it is less, or of a different character, it will be in the western with respect to northerners and in the eastern with respect to southerners. But if the culminating point is within the ascending signs, the nonagesimal will be in the western quadrant with respect to northerners and in the eastern with respect to southerners, if the declination of the culminating point is greater than the geographical latitude and of the same kind; but if it is less, or of a different kind, it will be in the eastern with respect to northerners and in the western with respect to southerners.

Finally, with respect to those who are in the right sphere: if the culminating point is in the 1st or 3rd quadrant of the ecliptic, the nonagesimal is in the western quadrant; but if in the 2nd or 4th quadrant, the nonagesimal is in the eastern quadrant.

[Margin: Problem 186.]

Fifth: if the rising amplitude of the rising point of the ecliptic is a quadrant, the nonagesimal will be on the meridian; and it will be the beginning of Cancer when the beginning of Libra is rising, or of Capricorn when the beginning of Aries is rising — and this, indeed, for anyone whatever. For an inhabitant outside the Torrid Zone, however: if the amplitude just mentioned is greater than a quadrant, the nonagesimal will be in the eastern quadrant with respect to northerners, and in the western with respect to southerners; on the contrary, if it is less than a quadrant, the nonagesimal will be in the western quadrant with respect to northerners, and in the eastern with respect to southerners. Within the Torrid Zone, however, since the rising point of the ecliptic is given, use the second part of problem 183.

What has been said about the problems concerning the nonagesimal, or related to it, from problem 133 to this one, 186, [Margin: Order of the problems.]

taken together with what was said in Book 5, chapter 10, contains a complete and very clear doctrine for eclipses of the Sun and other astronomical matters. Besides these, some matters still remain for Section 3, concerning the time at which a star is in the nonagesimal. But first we must complete Section 2.

[Margin: Note on [?] the use of the nonagesimal [?] for [?] eclipses of the Sun, etc.]

187. Probl. Arcum Meridiani inter Sidus culminans & Eclipticæ punctum culminans inuenire, Data vtriuſq. Declinatione. *187. Problem. To find the arc of the meridian between a culminating star and the culminating point of the ecliptic, given the declination of each.*

[Margin: Problem 187.]

If the declination of the star and that of the culminating point of the ecliptic are of the same kind, that is, both northern or both southern, the difference of the declinations will be the arc sought; but if they are of different kinds, the sum of the declinations will be. Another method, by the analysis of triangles, will be given below at problem 216. For the present problem, however, the figure set out at problem 162 serves after a fashion, in which the culminating point of the ecliptic is E. For let us imagine that the equator passes through H and that the star is in O: then the arc sought, OE, will be the difference between the northern declination HE of the point E and the northern declination HO of the star O. Likewise, if the equator passes through V and the star is in M, the arc sought, EM, will be the amount by which the southern declinations EV and MV differ from each other. Conversely, if, with the equator drawn through O, the star is in H, the arc sought will be HE, made up of the southern declination HO of the star and the northern declination OE of the culminating point, and so on.

Probl. Angulum Horizontis, & Circuli Declinationis, per Sidus aut punctum Eclipt. extra Horizontem poſitum ducti inuenire, Datis 188. Altitudine Poli, & Diſtantia Aequatoria Sideris aut puncti à Meridiano. 189. Altitudine Sideris aut Puncti, eiuſque Azimutho & Declinatione 190. Altit. & Azimutho Sideris aut puncti, & Altit. Poli 191. Altitudine, Azimutho, & diſtantia Sid. aut puncti à Meridiano 192. Altitudine, ac Declinat. Sid. vel Puncti, cum Alt. Poli 193. Altitud. ac Declinat Sid. vel Puncti, eiuſque diſtantia à Meridiano. *Problems. To find the angle of the horizon and the circle of declination drawn through a star or a point of the ecliptic placed outside the horizon, given: 188. the altitude of the pole, and the equatorial distance of the star or point from the meridian; 189. the altitude of the star or point, and its azimuth and declination; 190. the altitude and azimuth of the star or point, and the altitude of the pole; 191. the altitude, the azimuth, and the distance of the star or point from the meridian; 192. the altitude and declination of the star or point, with the altitude of the pole; 193. the altitude and declination of the star or point, and its distance from the meridian.*

Above, at problems 116 to 121, we sought this angle, but with the star, or any point, standing on the horizon; now we seek the same angle with the star or point of the ecliptic placed outside the horizon. For this, in the following figure, let the meridian be HVRQ, in which the vertex is V, and let the visible

pole of the world, say [?] the northern, be P, from which the equator EBQ is described. Let the star be in M, or in S, or in C, above the horizon, or below it, in I or in T; and from the pole P let arcs of declination be drawn through the centre of the star as far as the equator and the horizon, such as PMCKI TSP; and through the centre of the star let quadrants of the circle of altitude be drawn from V, namely ACV and FMV.

[Margin: What I say of a star holds for a point of the ecliptic.]

Diagram. A circle set in the left column beside the text of problems 188–193: the meridian HVRQ crossed by a horizontal line from H to R (the horizon), with the equator EBQ, arcs of declination from the pole P, and quadrants of altitude from the vertex V. Labels read:

- H (outside the rim, left end of the line), V (rim, top), R (outside the rim, right end of the line) and Q (outside the rim, lower right): the meridian HVRQ
- V (rim, top): the vertex, from which the quadrants of altitude ACV and FMV are drawn
- P (rim, upper right): the visible pole, with PR the altitude of the pole and VP its complement
- E (outside the rim, top, left of V), B (centre, below the line) and Q: the equator EBQ
- M (inside, upper right) and L (inside, upper centre): the star M with its declination LM and its altitude FM
- S (inside, right, above the line) and G (inside, lower centre): the star S with its declination GS and its altitude FS
- C (inside, left, above the line) and A (below the line, right of D): the star C with its southern declination CL and its altitude AC
- I (inside, lower left of centre) and T (below the line, right of centre): the star below the horizon, with IG, AI, TZ and VZ
- Z (inside, right, below S; a z-shaped sort): with the depression TZ and the quadrant VZ
- D (inside, just right of H, below the line), O (inside, right, on the line) and F (right of O, below the line): the angles at D and O and the right angles at F
- K (outside the rim, lower left) and N (rim, bottom): the other pole K and the nadir N, with the triangle KIN

With this laid down, the altitude of the pole will be PR, and its complement to 90° will be VP. The declination of the star M will be LM, whose complement is MP; that of the star S will be GS, and its complement SP; while the southern declination CL of the star C, added to a quadrant, will make up the arc PC. The equatorial distance from the meridian (which, for the Sun, is known from the time before or after noon, converted into degrees of the equator) will be the angle DPE for a star placed in M or C, and its supplement to 180° will be the angle DPR; but for the star S the distance will be GPE, and its supplement to 180° will be the angle OPR. Finally, the altitude of the star M will be FM,

and its complement MV; while the altitude of the star S is FS, its complement SV; and the azimuth of both is the angle HVF, whose supplement to 180° gives the angle FVR. But for a star placed in C the altitude is AC, its complement CV, and its azimuth HVA, whose supplement to 180° is AVR.

[Margin: Problem 188.]

Now, if the star is in C or in M, the acute angle at D is sought, which can be found from the givens of problem 188. For in the triangle DPR, right-angled at R, the side PR, the altitude of the pole, is given, and the angle DPR, the complement of the star's distance from the meridian; therefore by Rule 8 of spherical right-angled triangles you will find the angle D. So too, if the star is in S, or in T, or in I, the acute angle O is sought in the triangle OPR, in which PR, the altitude of the pole, is given, and the angle OPR, the complement of the star's distance from the meridian; therefore by Rule 8 of the same, find the angle O.

[Margin: Problem 189.]

[Margin: 190, 191, 192, 193.]

For the other problems, however, the angle made by the circle of declination and the vertical at the centre of the star C, M or S must be found by problems 29 to 33. Then, if the star is in C, use the triangle DAC, right-angled at A; and with the angle C found and the altitude AC, find the angle D by the same Rule 8. But if the star is in M, use the triangle DMF, right-angled at F, and by the angle M, found as before, and the altitude FM, find the angle D by the same Rule 8. Finally, if the star is in S, use the triangle OSF, right-angled at F, in which the angle S, found as above, is given, and the altitude FS; therefore by the same Rule 8 you will obtain the acute angle O.

Probl. Diſtantiam Stellæ aut Puncti Ecliptici à Meridiano inquirere, Datis 194. Aſc. Recta Stellæ & Aſc. Recta Medij cæli 195. Altitud. Poli; & Stellæ Altitudine vel Profunditate, ac Declinatione 196. Altit. Poli, Azimuth & Altitudine Stellæ, aut Profunditate 197. Alt. Poli Azimuth ac Declinatione ſtellæ 198. Azimutho, Altitudine vel Profundit. ac Declinatione Stellæ.
Problems. To find the distance of a star or of a point of the ecliptic from the meridian, given: 194. the right ascension of the star and the right ascension of mid-heaven; 195. the altitude of the pole, and the altitude or depression [profunditas] of the star, and its declination; 196. the altitude of the pole, the azimuth, and the altitude or depression of the star; 197. the altitude of the pole, the azimuth, and the declination of the star; 198. the azimuth, the altitude or depression, and the declination of the star.

[Margin: Problem 194.]

For problem 194: if the star is in the eastern hemisphere, subtract the right ascension of mid-heaven from its right ascension; but if it is in the western hemisphere, subtract the right ascension of the star from the right ascension of mid-heaven, taking 360° in addition where the subtraction cannot otherwise be made; and what remains will be the distance of the star from the meridian. Understand the same of a point of the ecliptic.

[Margin: Problem 195.]

For problem 195, look at the figure immediately preceding, and in it the star in four places, two above and two below the horizon. For if the star is in M, use the triangle VMP, and seek the distance from the meridian, that is, the angle MPV, from the givens VP, the complement of the altitude of the pole; VM, the complement of the altitude of the star, as the base; and MP, the complement of the declination; and do this by Rule 14 of spherical oblique-angled triangles. But if the star is in C, use the triangle VCP, and by the same Rule 14 seek the angle CPV from the givens VP, as above; VC, the complement of the altitude of the star; and CP, made up of the quadrant LP and the declination LC towards the other pole. But if the star is below, as in T, use the triangle VTP, seeking the angle of distance TPV by the same Rule 14 from the givens VP, as above; TP, the complement of the declination; and VT, made up of the depression TZ of the star and the quadrant VZ. But if it is in I, use the triangle KIN, in which KN is as great as VP, the complement of the altitude of the pole, since K is the other pole and N the nadir; and KI is the complement of the declination IG, while IN is the complement of the depression AI. Therefore by the same Rule 14 you will find the angle IKN, the supplement to 180° of the angle IKE, which measures the distance of the star from the meridian.

[Margin: Problem 196.]

For problem 196: the supplement of the azimuth to 180° is the vertical angle contained by the complement of the al- [Margin: Order of the problems.]

titude of the pole and by the complement of the altitude or depression of the star, or by the aggregate of the quadrant and the depression. Therefore by Rule 5 or 6 of spherical oblique-angled triangles you will find the angle opposite the altitude or depression, which measures the distance sought if the star is in M, C or T; or it is the supplement to 180° of the distance sought if the star is in I.

[Margin: Problem 197.]

For problem 197, the vertical angle is sought that measures the distance sought if the star is in M, C or T, or its supplement to 180° if the star is in I. This angle is contained by the complement of the altitude of the pole and by the complement of the given declination; and the angle opposite the complement of the declination is given, namely the complement of the azimuth. Therefore, by Rule 3 of spherical oblique-angled triangles, seek the vertical angle already mentioned.

[Margin: Problem 198.]

For problem 198, the angle opposite the complement of the declination is given from the complement of the azimuth; the complement of the altitude or depression is given, or the aggregate of the depression and the quadrant; and the angle opposite this complement or aggregate is sought by Rule 1 of spherical oblique-angled triangles. This angle measures the distance of the star from the meridian if the star is in C, M or T; or it is the supplement of the distance to 180° if the star is in I.

199. Probl. Angulum in Centro Sideris Orientis vel Occidentis, factum à Circulo Latitudinis & Horizontis inuenire, Datis Latitudine, ac Declinatione Sideris cum Altitudine Poli, & Diſtantia Polorum Eclipticæ ac Aequatoris. *199. Problem. To find the angle at the centre of a rising or setting star made by the circle of latitude and the horizon, given the latitude and declination of the star, with the altitude of the pole and the distance between the poles of the ecliptic and of the equator.*

[Margin: Problem 199.]

First, by problem 66, seek the angle at the centre of the star made by the circles of latitude and of declination. Second, by problem 116, seek the angle at the centre of the star made by the circle of declination and the horizon, and let it be called the second angle. Third, inspect the astronomical globe: if the arc of declination falls within the angle sought, you will add the first angle to the second; but if it falls outside, you will subtract it, and the angle sought will result, or remain.

200. Probl. Angulum ad Polum Aequatoris factum à Coluro Solſtitiorum, & à Circulo Declinationis Stellæ aut puncti cæleſtis inuenire, Data Stellæ aut puncti prædicti Aſcenſione Recta. *200. Problem. To find the angle at the pole of the equator made by the solstitial colure and by the circle of declination of a star or celestial point, given the right ascension of the star or point already mentioned.*

[Margin: Problem 200.]

If the right ascension is less than 90°, subtract it from 90°. But if it is greater than 90° but less than 270°, subtract 90° from the ascension. Finally, if it is greater than 270°, subtract 270° from the ascension; and the angle sought will remain. This angle is nil if the right ascension is exactly 90° or 270°, and right if the right ascension is nil or 180°. When the angle found is obtuse, if you are seeking an acute one, subtract it from 180°.

201. Probl. Angulum ad Polum Eclipticæ factum à Coluro Solſtitiorum, & circulo Latitudinis Stellæ aut puncti Cæleſtis inueſtigare, Data Lõgitudine Stellę aut pũcti. *201. Problem. To find the angle at the pole of the ecliptic made by the solstitial colure and by the circle of latitude of a star or celestial point, given the longitude of the star or point.*

[Margin: Problem 201.]

Do everything as in problem 200, exchanging the names and roles of the right ascension for the names and roles of the longitude; or see the explicit rules given in Book 6, chapter 13, from number 4 to the end of the chapter.

202. Probl. Angulum ad Polum Mundi factum à Coluro Aequinoctiorum, & circulo Declinationis ducto per Stellam aut punctum cæleſte inuenire, Data Aſc. Recta ſtellę aut puncti. *202. Problem. To find the angle at the pole of the world made by the equinoctial colure and by the circle of declination drawn through a star or celestial point, given the right ascension of the star or point.*

[Margin: Problem 202.]

This angle is as great as the right ascension, if the ascension is less than 180°; but if it is greater, subtract 180° from it, and what remains will be the angle sought. The angle will, however, be nil if the right ascension is nil or 180°.

203. Probl. Angulum ad Polum Eclipticæ factum à Coluro Aequinoctiorum, & circulo Latitudinis ducto per Stellam vel punctum cæleſte, inuenire, Data Longitudine puncti aut Stellæ. *203. Problem. To find the angle at the pole of the ecliptic made by the equinoctial colure and by the circle of latitude drawn through a star or celestial point, given the longitude of the point or star.*

[Margin: Problem 203.]

This angle is nil if the longitude is nil or 180°; but it is as great as the longitude when the longitude is less than 180°. For if it is greater, subtract 180° from the longitude, and the remainder will be the angle sought.

204. Angulum qui fit ad Polum Mundi, à Meridiano & à Coluro Solſtitiorum, quando hi circuli non coincidunt; Dato Temporis momento, aut reliquis ad Aſc. Rectam Medij cæli requiſitis; inueſtigare. *204. To find the angle formed at the pole of the world by the meridian and by the solstitial colure, when these circles do not coincide, given the moment of time, or the other things required for the right ascension of mid-heaven.*

[Margin: Problem 204.]

Let the right ascension of mid-heaven be given, or found by problems 108 to 113. If it is 90° or 270°, there will be no such angle, but the meridian will coincide with the solstitial colure; if it is nil, or 180°, the angle already mentioned will be 90°, that is, right. But if the right ascension of mid-heaven is less than 90°, subtract it from 90°; if it is greater than 90° but less than 180°, subtract 90° from it; if greater than 180° but less than 270°, subtract it from 270°; finally, if it is greater than 270°, subtract 270° from it, and you will obtain the acute angle sought.

205. Probl. Angulum qui fit ad Polum Mundi, à Meridiano & Coluro Aequinoctiorum, inuenire, Datis quæ in Probl. 204. *205. Problem. To find the angle formed at the pole of the world by the meridian and the equinoctial colure, given what is given in problem 204.*

[Margin: Problem 205.]

Seek the right ascension of mid-heaven by the problems set out from 108 to 113. If it is nil, or 180°, there will be no such angle, but the meridian will coincide with the equinoctial colure; if it is 90° or 270°, that angle will be 90°, that is, right. But if it belongs to the 1st quadrant of the equator, the angle will be as great as the right ascension of mid-heaven; if to the 2nd, subtract it from 180°; if to the 3rd, subtract 180° from it; if to the 4th, subtract it from 360°; and so the acute angle sought will be known.

Probl. Diſtantiam Poli Eclipticæ à Vertice inuenire, Datis. 206. Altitudine Nonageſimi 207. Altitud. Poli; Diſtantia Polorum, & Angulo ad Polum Mundi facto à Coluro Solſtitiorum, & Meridiano. *Problems. To find the distance of the pole of the ecliptic from the vertex, given: 206. the altitude of the nonagesimal; 207. the altitude of the pole, the distance between the poles, and the angle made at the pole of the world by the solstitial colure and the meridian.*

[Margin: Problem 206.]

In the adjoining diagram the meridian is HVRM, the horizon HBR, and the pole of the world P, from which the small polar circle CZ is described; and on its circumference lies the pole of the ecliptic, now at C, now at I, now at Z, now at L. A portion of the ecliptic is EBM; and the portion ZVNA of the great circle drawn from the pole Z through the vertex V makes a right angle with the ecliptic at N. Since this circle passes from the vertex to the horizon, it becomes at once the circle of latitude and the circle of altitude, and it marks at N the nonagesimal degree of the ecliptic, counted from the point B of the horizon. Therefore, since the arc of latitude ZN from the pole of the ecliptic Z to the ecliptic

Diagram. A circle set in the right column beside the text of problems 206–207: the meridian HVRM, crossed by the horizon HBR and by the ecliptic EBM, with a small circle about the pole of the world P carrying the positions C, I, Z and L of the pole of the ecliptic, and the arc ZVNA from Z through the vertex. Labels read:

- E (outside the rim, upper left, at the end of a line): the ecliptic EBM
- V (rim, top): the vertex
- O (inside, upper left): not named in the text of this page
- N (inside, left): the nonagesimal, where ZVNA meets the ecliptic at right angles
- D (inside, left of centre): not named in the text of this page
- C (inside, on the small circle), I (inside the small circle, top), Z (outside the rim, upper right; a z-shaped sort) and L (rim, right, below P): positions of the pole of the ecliptic on the small polar circle
- P (on the small circle, right): the pole of the world
- H (outside the rim, left) and R (rim, right): the horizon HBR and the meridian HVRM
- A (inside, lower left, below the horizon): the end of the arc ZVNA
- B (inside, lower centre): the point of the horizon from which the nonagesimal is counted
- M (outside the rim, lower right): the meridian HVRM and the ecliptic EBM

[Margin: Order of the problems.]

is a quadrant, and VA is likewise a quadrant, then if the complement NV of the altitude AN of the nonagesimal, which is the arc common to both quadrants, is taken away, there will remain the distance of the pole Z from the vertex, that is, the arc ZV, as great as the altitude AN of the nonagesimal.

[Margin: Problem 207.]

Second, if in the triangle VPZ the complement PV of the altitude PR of the pole is given, and the distance between the poles PZ, and, by problem 204, the angle VPZ, which is formed by the meridian and the solstitial colure (of which colure PZ is a portion), then by Rule 4 of spherical oblique-angled triangles the base ZV will certainly become known, namely the distance of the pole of the ecliptic from the vertex. In the same way, if the pole of the ecliptic were at C, from the givens VP and PC and the angle VPC, as above, you will gather the distance of C. But if the pole of the ecliptic is on the meridian itself, say at I, that is, when the beginning of Capricorn culminates, then the distance between the poles IP, less the complement VP of the altitude of the pole, will leave the distance VI; but if it is at L, that is, when the beginning of Cancer culminates, the distance between the poles PL, added to the complement PV, will make up the distance VL.

Probl. Angulum in centro Sideris culminantis, factum à Circulo Latitudinis & Meridiano, inuenire, Datis 208. Diſtantia Polorum; Declinatione, & Latitudine Sideris 209. Diſtantia Polor. Declinat. & Angulo ad Mundi Polum Meridiani & Coluri Solſtit. 210. Diſtantia Polor. Declinat. & Angulo ad Eclipt. Polum facto à Circulo Latitud. & Coluro Solſtitiorum 211. Diſtantia Polorum, Latitudine, & Angulo ad Mundi Polum ſupradicto. 212. Diſtantia Polorum, Latitudine, & Angulo ad Eclipt. Polum ſupradicto 213. Diſtantia Polorum, & Angulis ad Polos Mundi & Eclipticæ ſupradictis 214. Declinat. & Angulis ad Polos prædictis. 215. Latitudine, & Angulis ad Polos prædictis. *Problems. To find the angle at the centre of a culminating star made by the circle of latitude and the meridian, given: 208. the distance between the poles, and the declination and latitude of the star; 209. the distance between the poles, the declination, and the angle at the pole of the world of the meridian and the solstitial colure; 210. the distance between the poles, the declination, and the angle at the pole of the ecliptic made by the circle of latitude and the solstitial colure; 211. the distance between the poles, the latitude, and the angle at the pole of the world mentioned above; 212. the distance between the poles, the latitude, and the angle at the pole of the ecliptic mentioned above; 213. the distance between the poles, and the angles at the poles of the world and of the ecliptic mentioned above; 214. the declination, and the angles at the poles already mentioned; 215. the latitude, and the angles at the poles already mentioned.*

Look at the figure immediately preceding, in which the culminating star is at V, whether at the vertex or away from the vertex. Let the portion PV of the meridian be the complement of the given declination, VZ the complement of the given latitude, and PZ the given distance between the poles. Let the angle at the pole of the world made by the meridian and the solstitial colure, known from problem 204, be VPZ, since PZ is a portion of the solstitial colure; and finally let the angle at the pole of the ecliptic made by the circle of latitude and the solstitial colure, known by problem 201, be VPZ. Now in the triangle PVZ

the angle at V is sought; and it will be found by the following rules of spherical oblique-angled triangles, which correspond in order to the problems proposed.

[Margin: Problem: 208, 209, 210, 211, 212, 213, 214, 215.]

Order of problems	Givens for the angle PVZ	Rules
208	sides PZ, PV, ZV	14
209	sides PZ, PV, angle VPZ	5 or 6
210	sides PZ, PV, angle VZP	1
211	sides PZ, VZ, angle VPZ	1
212	sides PZ, VZ, angle VZP	5 or 6
213	side PZ, angles VPZ, VZP	10
214	side PV, angles VPZ, VZP	9
215	side VZ, angles VPZ, VZP	9

Section 1, chapter 5.

216. Probl. Pũctum Eclipticæ Culminans cum Sidere notæ Longitudinis ac Latitudinis; Arcumq. Meridiani inter Sidus & punctũ Eclipticæ Culminans, & inde Declinationem Sideris inuenire, Dato Angulo in centro Sideris Culminantis facto à Meridiano, & circulo Latitudinis. *216. Problem. To find the culminating point of the ecliptic together with a star of known longitude and latitude, and the arc of the meridian between the star and the culminating point of the ecliptic, and hence the declination of the star, given the angle at the centre of the culminating star made by the meridian and the circle of latitude.*

[Margin: Problem 216.]

We devised this problem, and the others from problem 201 that serve it, both for other purposes and especially for the sake of the lunar parallaxes, which are the foundations of the distance both of the Moon and of the other planets from the Earth. Let there be, then, in the preceding diagram the Moon, or any star whatever, culminating at V; let its latitude VN be given, and from its longitude let the point N of the ecliptic be known. In the triangle ONV, right-angled at N, besides the side VN, let the angle NVO be given, from the angle PVZ, which is equal to it and has been found by problems 209 to 215 (for in problem 208 the declination is presupposed known, which is what we are seeking here). With these given, then, gather by Rule 7 of spherical right-angled triangles the side NO, and by Rule 9 the base OV. Then, if the point N, which is known, is eastern, as it lies in the figure, subtract the side NO from it; but if it is western, add it; and the culminating point O of the ecliptic will be known, and its declination from the table of problem 15. Comparing this declination with the base OV that has been found, you will easily gather the true declination of the star V. For if the point O is the beginning of Aries or of Libra, the arc OV itself will be the declination of the star V; but if the point O is in the northern semicircle of the ecliptic, and the star is more northern than it, the aggregate of the declination of the point O and the base OV will make up the declination of

the star V; if, however, the point O is in the southern semicircle, and the star is more northern than it, the difference between the declination of the point O and the base OV will give the declination of the star V. From these you will understand the remaining cases.

[Margin: Note for the observation of the parallaxes of the Moon.]

Although this declination can be obtained by means of the tables indicated at problem 65, yet if anyone lacks them, or if they are not constructed for so great a latitude as would be needed, this problem will be very useful. Once the true declination is known, and the altitude of the equator, the true meridian altitude of the star is known; compared with the meridian altitude observed exactly with some instrument, it yields the parallax of the star belonging to that altitude, if the star has any, provided that the star is beyond the hazard of sensible refractions, as I showed by many examples in Book 4, chapter 14.

217. Probl. Eclipticæ Arcum ſemper Apparentem determinare, Datis Obliquitate Eclipticæ, & Altitudine Poli, excedente tamen complementum Obliquitatis Eclipticę. *217. Problem. To determine the arc of the ecliptic that is always visible, given the obliquity of the ecliptic and the altitude of the pole, the altitude, however, exceeding the complement of the obliquity of the ecliptic.*

[Margin: Problem 217.]

First consider the figures of the following diagram, in which the meridian is HVPD and the horizon HBR, and the equator AQ is described from the northern pole of the world P, whose altitude is PR and the complement of whose altitude is PV, since the zenith or vertex is V. Let the ecliptic be CBD; the remaining semicircles signify parallels of the equator, among which are the tropics: that of Cancer, CN, and that of Capricorn, KD. The greatest declination of the Sun, or the measure of the obliquity of the ecliptic, is the arc AC, or QN; and its complement to 90° is the arc CVP.

[Margin: Case 1.]

First, then, if, as in the 1st of the figures, the altitude of the pole PR is less than the complement CVP of the obliquity of the ecliptic, every parallel that the Sun describes is cut by the horizon, and therefore all the points of the ecliptic rise and set every day in such a horizon.

[Margin: Case 2.]

Second, if, as in the 2nd of the figures, the altitude of the pole PR is equal to the complement CVP of the obliquity of the ecliptic, the tropics graze the horizon and are not cut by it. Thus the tropic of Cancer CN, standing wholly above the horizon, touches the horizon at R with a single point of itself, N; the tropic of Capricorn KD, wholly hidden, touches the horizon at H; and the remaining solar parallels are cut by the horizon. The result is that, when the Sun is at the beginning of Cancer and is describing its tropic, the day among the northern peoples is exactly 24 hours long and there is no night, because the Sun does not wholly set, but only half of it, the centre of the solar disc being at R (I say this, abstracting for now from refraction and parallax). Conversely,

there is a night of 24 hours among the southern peoples who have an altitude of the south pole as great as the complement of the obliquity of the ecliptic, for the whole parallel of Cancer is hidden from them. On the same day, moreover, the circles, the Ar- [Margin: Order of the problems.]

ctic and the Antarctic, pass, the former through the vertex, the latter through the nadir.

Diagram. Three circles set side by side across the left column: "the figures of the following diagram" for the three cases of problem 217, numbered 1, 2 and 3 in the text from left to right by their order on the page. Each circle is crossed by a horizontal line (the horizon HBR or its like) and by oblique parallels of the equator. Labels read:

- First figure (case 1): K (rim, upper left); A, C and V (along the rim at the top); P (outside the rim, right, above the line); H (left end of the line); B (centre, on the line); R (right end of the line); D (below the rim, bottom); Q (rim, lower right, beside D); N (outside the rim, lower right)
- Second figure (case 2): C (outside the rim, upper left); V (top); P (outside the rim, upper right); A (outside the rim, left, above the line); H (left end of the line); K (inside, left); B (centre, below the line); N (outside the rim, right, above the line); R (right end of the line), where the tropic of Cancer CN touches the horizon; Q (outside the rim, right, below the line); D (rim, lower right)
- Third figure (case 3): C, V and P (top); O (outside the rim, upper left); A (outside the rim, left, above the line); H (left end of the line); S (inside, upper centre); G (inside, left, on the line; a small sort); B (centre), the beginning of Aries; I (inside, right of B; a slanting sort); T (inside, below B); N (outside the rim, right, upper); R (right end of the line); O (outside the rim, right, below R); M (outside the rim, lower right); D (rim, lower right)

[Margin: Case 3.]

The third case, the one our problem requires, is represented in the 3rd of the figures; for in it the altitude of the pole PR is greater than the arc CVP, that is, than the complement of the obliquity of the ecliptic. Therefore the horizon cuts only those parallels that lie within the parallel OR and HM; and the tropic of Cancer CN stands so far above the horizon that it does not touch it. Now look at the ecliptic CBD and its two arcs: one before the beginning of Cancer, let it be SC; the other following the beginning of Cancer and equal to the former, let it be CS. For when the Sun has reached the point S, for the whole time it spends in the arc SC and in the arc CS it will never set for such a horizon, because the parallels within CN and OR are not cut by the horizon. We must therefore find the arc of the ecliptic that is always visible, which consists of the arcs SC and CS. It will be found if the points S of the ecliptic are known, one preceding and the other following the beginning of Cancer, through which passes the parallel OSR, touching the horizon. These points will become known from the declination of the parallel OSR; and this declination

is AO, or QR, as great, that is, as the complement PQ of the altitude PR of the pole to the quadrant. Now of the points S one lies in the first quadrant of the ecliptic and the other in the second, and they are equally distant from the beginning of Cancer; therefore, once the former is known, the other is known at once.

Go back, then, to problem 15; and with the given declination of the parallel OSR, that is, with the complement of the altitude of the pole, namely SG, and with the angle GBS of the obliquity of the ecliptic opposite it, seek in the spherical triangle GBS, right-angled at G, the base BS, by Rule 11 of spherical right-angled triangles. For that will be the elongation of the point S from the beginning of Aries, B, and it will accordingly give the point S that precedes the beginning of Cancer. But if you subtract the base BS from 180°, you will have the elongation of the later point S from the same beginning of Aries. You have an example below, in Section 3, problem 14, in the first step; although this can be completed without trigonometry, by seeking in the body of the table of the Sun's declinations the declination already mentioned, that is, the complement of the altitude of the pole. For at the head and foot of the table you will have the sign, and in the margin the degrees sought, which enclose on either side the arc of the ecliptic that is always visible. How much time the Sun spends in traversing this arc you will learn from astronomical tables or from ephemerides; but on this time, which must in every case be corrected on account of refraction, since refraction increases it greatly, I shall speak in the following section, problems 13 and 14. Further, the arc always hidden, namely TD taken twice, is as great as the arc always visible, SC taken twice.

In another way, not very different, though without demonstration and figure, Reinhold accomplishes this in his *Tabulae directionum*, precept 41. "Multiply," he says, "the sine of the complement of the altitude of the pole by the radius, and divide the sum by the sine of the obliquity of the ecliptic; for the quotient will be the sine of the base BS, whose complement to the quadrant is the arc SC."

218. Probl. Stellæ aut Puncti Eclipticæ ſitum reſpectù hemiſphærij Orientalis aut Occidentalis diſcernere, ſeu vtrùm ſit in parte Orientali, an Occidentali, Datis Aſc. Recta ipſius, & Aſcenſ. Recta Medij Cæli ad Tempus datum. *218. Problem. To determine the position of a star or of a point of the ecliptic with respect to the eastern or the western hemisphere, that is, whether it is in the eastern or the western part, given its right ascension and the right ascension of mid-heaven at a given time.*

[Margin: Problem 218.]

Subtract the right ascension of the given star or point from the right ascension of mid-heaven (taking 360° in addition if the subtraction requires it), and the elongation of the star or point from the meridian will remain. If this is less than 180°, the star or point will be in the western hemisphere; if greater, in the

eastern; if nil, it will be in mid-heaven; if 180°, it will be in the angle of the earth, that is, at the lowest mid-heaven.

219. Probl. Vtrùm Stella vel Punctum Eclipticæ, ſit ſupra an infra Horizontem, diſcernere, Datis Aſc. Recta ipſius & Arcu Semidiurno, & Aſc. Recta Medij Cæli. *219. Problem. To determine whether a star or a point of the ecliptic is above or below the horizon, given its right ascension and semidiurnal arc, and the right ascension of mid-heaven.*

[Margin: Problem 219.]

With the right ascension of the star and the right ascension of mid-heaven, gather by problem 218 the elongation of the star or point from the meridian, and its position. Then, if the star is in the western hemisphere and its elongation from the meridian is less than the semidiurnal arc (known by problem 122), the star will be above the horizon; if greater, it will be below; if equal, it will be on the western horizon. But if the star or point is in the eastern hemisphere, then when 180° is subtracted from its elongation from the meridian (taking in the circle of 360° if need be), the elongation from the angle of the earth will remain; and if this is less than the seminocturnal arc, it will be below the horizon; if equal, it will be on the eastern horizon.

You will determine this in another and quicker way if you know the rising point of the ecliptic, by problems 140 to 143, and the point of the ecliptic with which the star rises and with which it sets, by problem 125; for in this way you will know which half of the ecliptic is below the horizon and which above. And if the point with which the star rises is in the eastern part but has not yet risen, the star will be below; but if the point with which it sets is in the western part and has not yet set, the star will be above the horizon, and so on.

220. Probl. Altitudinem vel Profunditatem Solis, aut ſtellæ cuiuſuis inueſtigare, Datis Altitud. Poli; loco Solis aut Stellæ, cum eius Declinatione & arcu Semidiurno, ſiue Parallelus Sideris ſecet, ſiue non Horizontem. *220. Problem. To find the altitude or depression of the Sun or of any star, given the altitude of the pole, and the place of the Sun or star, with its declination and semidiurnal arc, whether the parallel of the star cuts the horizon or not.*

[Margin: Problem 220.]

I have chosen to transcribe this problem here, without a figure, from Cavalieri's *Centuria*, problems 3 and 17.

[Margin: Step 1.]

First, then, with the altitude of the pole and the declination of the star, seek by the table, or by problem 122, the semidiurnal arc of the star, and from the moment of time the distance of the star from the meridian, by problems 194 to 198.

[Margin: Step 2.]

Second, if the parallel of the star cuts the horizon (which you will learn from its declination and the altitude of the pole), join together these five: namely,

the logarithm of the binary, that is, of the number 2; the second logarithm of the altitude of the pole; the second logarithm of the declination of the star; the logarithm of the half-sum made from the semidiurnal arc and the distance of the star from the meridian; and finally the logarithm of the half-difference between the semidiurnal arc and the distance of the star from the meridian. For the sum of these 5, the superfluous units being discarded (according to what was said in Section 1, chapter 1, precept 2), will be the logarithm of the altitude or depression sought. If the arc found by this logarithm exceeds 90°, its excess over 90° will be the depression; but if it does not exceed it, it will be either the altitude or the position on the horizon itself.

[Margin: Step 3.]

Third, if the parallel of the star does not cut the horizon (which happens when the declination of the star exceeds the complement of the altitude of the pole), or even if it does cut it (for this rule is universal), the second mesologarithm of the declination of the star, joined with the second logarithm of the distance from the meridian, will make the mesologarithm of an arc, which is to be called the first arc. Next, the logarithm of the declination; the residue of the second logarithm of the first arc found; and the logarithm either of the sum made from the first arc and the altitude of the pole (if the distance of the star from the meridian is less than a quadrant in a northern parallel, or greater in a southern one), or of the difference between the first arc and the altitude of the pole (if the distance of the star from the meridian is greater than a quadrant in a northern parallel, or less in a southern one): these three, I say, joined together, make the logarithm of the second arc. If this arc is greater than 90°, its excess is the depression of the star; if it is less, the arc is that of the altitude.

[Margin: Order of the problems.]

221. Probl. Altitudinem aut Profunditatem Solis inquirere, Datis Loco Solis Ecliptico, puncto Eclipticæ Oriente, cum Angulo Eclipticæ & Horizontis. *221. Problem. To find the altitude or depression of the Sun, given the Sun's place in the ecliptic and the rising point of the ecliptic, with the angle of the ecliptic and the horizon.*

[Margin: Problem 221.]

Let the figure set out earlier for problem 162 be repeated here. First, let the Sun's place in the ecliptic be at S, and the rising point of the ecliptic at B. Their difference gives the base BS in the triangle ASB, right-angled at A, in which the angle ABS of the ecliptic and the horizon is given. Therefore by the first rule of spherical right-angled triangles you will find the Sun's altitude AS. Second, let the Sun be at I, below the horizon. The difference, namely, between its place I and the rising point B will give the base BI in the triangle BKI, right-angled at K, in which the angle KBI of the ecliptic and the horizon is given. Therefore by the same first rule you will find the Sun's depression KI; for VKI is a portion of a vertical circle.

Diagram. A circle set at the right of the left column, beside the text of problem 221: the figure "set out earlier for problem 162", here repeated and used for problems 221 to 223. It is crossed by a near-horizontal line from H to R (the horizon) and by arcs and straight lines drawn from P and from V. Labels read:

- M (outside the rim, top, left of V)
- V (top of the rim)
- E (outside the rim, upper left)
- O (outside the rim, left, below E)
- S (inside, upper left)
- P (rim, upper right)
- H (left end of the line)
- K (inside, right, above the line)
- R (right end of the line)
- A (inside, left, below the line)
- B (inside, below the line, centre)
- I (inside, near the rim, lower right)
- G (outside the rim, lower right)

222. Probl. Altitud. aut Profunditatem Solis inquirere, Datis Loco Solis in Ecliptica, pũcto Eclipticæ Culminante, ipſiuſq. Diſtantia à Vertice, & Angulo Eclipticæ ac Meridiani. *222. Problem. To find the altitude or depression of the Sun, given the Sun's place in the ecliptic, the culminating point of the ecliptic and its distance from the vertex, and the angle of the ecliptic and the meridian.*

[Margin: Problem 222.]

In the preceding figure, if the Sun is at S, use the triangle SEV. In it the side ES is given from the Sun's place S and the culminating point E; the distance EV of the point E from the vertex is given; and, from the angle of the ecliptic and the meridian that belongs to the point E, the angle contained by the given sides, that is, SEV, is given. Therefore by the 4th rule of spherical oblique-angled triangles seek the base SV; when it is subtracted from 90°, the Sun's altitude AS becomes known. But if the Sun is below, as at I, use the triangle IEV. In it the difference between the Sun's place I and the point E gives the side EI; the side EV is given, as above; and so is the contained angle IEV. Therefore by the same 4th rule you will find the base VI, which will be greater than the quadrant VK; when the quadrant is taken away, then, the Sun's depression IK will remain.

223. Probl. Solis Altit. aut Profunditatem inueſtigare, Datis Altitudine Poli Declinatione ac Diſtantia Solis à Meridiano. *223. Problem. To find the altitude or depression of the Sun, given the altitude of the pole, the declination, and the distance of the Sun from the meridian.*

[Margin: Problem 223.]

If the Sun is at S, above the horizon, use the triangle SVP. In it the side PS is given from the declination: it is either a quadrant, if the Sun is on the equator; or the complement of the declination, if the Sun is northern; or the aggregate of the declination and a quadrant, if the Sun is southern. VP is also given, the complement of the altitude of the pole; and so is the contained angle SPV, known from the Sun's equatorial distance from noon, that is, from the time before or after noon converted into parts of the equator. Therefore by the 4th rule of spherical oblique triangles the base SV will be found [?], which is the complement of the altitude sought, AS.

But if the Sun is below, at I, use the triangle IPV, in which the side IP is given from the declination, as above; VP, the complement of the altitude of the pole; and the angle IPV, from the distance from the meridian. Therefore by the same 4th rule seek the base VI, which is of course greater than the quadrant VK; when the quadrant is taken away, the remainder will be the Sun's depression KI.

224. Probl. Angulum Horarij circuli & Eclipticæ, inueſtigare, Datis puncto Eclipticæ per quod tranſit Horarius, eiuſq. puncti Declinatione. *224. Problem. To find the angle of an hour circle and the ecliptic, given the point of the ecliptic through which the hour circle passes, and the declination of that point.*

[Margin: Problem 224.]

Since an hour circle is one of the circles of declination, and a circle of declination is the meridian of some people, seek the angle of the ecliptic and the meridian by problem 14, or by problems 76 to 83. See also, if you like, the practical instrument for this in Scheiner, *Rosa Ursina*, page 363.

225. Probl. Arcum Horizontis reſectum à circulo Horario inuenire, Datis Alt. Poli, & ipſo Horario. *225. Problem. To find the arc of the horizon cut off by an hour circle, given the altitude of the pole and the hour circle itself.*

[Margin: What the hour arc is.]

[Margin: Problem 225.]

In the following scheme let the meridian be HVRN, the horizon HBR, and the equator ABQ, described from the poles of the world P and O. Let the quadrant of the equator AB, and BQ, be divided into 6 parts, each containing 15 degrees, that is, one hour, so that a circle drawn through the poles of the world and such divisions is a circle of declination in such a way that it is also an hour circle. Thus the circle OBP is the circle of the sixth hour after or before noon; and the circle OLP, for example, if AL is equal to LB, is the circle of the 3rd hour after or before noon; and the circle OIP is that of the 9th hour before or after noon, in accordance with what was said more fully in Book 1, chapter 21, number 7.

Diagram. A circle set at the right of the right column, beside the text of problem 225, the "following scheme" used for problems 225 and 226. The rim is the meridian HVRN. It is crossed by a vertical diameter from V to N (the

vertical VBN of problem 226), a horizontal diameter from H to R (the horizon HBR), and the diameter from A to Q (the equator ABQ), with arcs and lines drawn from the poles P and O. Labels read:

- V (top)
- A (outside the rim, upper left)
- P (rim, upper right)
- L (inside, upper left)
- G (inside, right of L, by the vertical)
- H (left end of the horizontal)
- R (right end of the horizontal)
- K (inside, left, below the horizontal)
- B (inside, centre, below the horizontal)
- M (inside, right, below the horizontal)
- I (inside, lower right)
- O (rim, lower left)
- S (inside, below B, by the vertical)
- Q (rim, lower right)
- N (bottom)

Now the arc HK, or MR, cut off by such a circle is sought. Consider, then, the triangle HOK, right-angled at H. In it HO is given, the depression of the south pole, as great as the given altitude PR of the north pole. There is given besides the angle HOK, as great as OPA, determined by the hours before or after noon converted into parts of the equator, that is, by the arc AL; and these hours the hour circle itself, which is named from them, displays. Therefore by the 7th rule of spherical right-angled triangles seek the arc HK of the horizon, cut off by such an hour circle.

[Margin: Another way to conclude the same.]

But if you seek MR, use the triangle MPR, right-angled at R, in which the altitude of the pole RP is given, and the angle MPR, the supplement to two right angles of the angle IPA, which is known from the kind of hour circle, that is, from the hours before or after noon converted into parts of the equator. Accordingly, by the same 7th rule seek the side MR. The arc BK or BM, however, can also be found by problem 227, and when it is subtracted from 90°, the arc HK or MR is obtained.

226. Probl. Arcum Verticalis circuli cuiuſuis, reſectum ab Horario circulo inuenire, Datis præter Horarij ſpeciem, Azimutho, ſeu Declinatione Verticalis à Meridiano, & Altit. Poli. *226. Problem. To find the arc of any vertical circle cut off by an hour circle, given, besides the kind of hour circle, the azimuth, that is, the declination of the vertical from the meridian, and the altitude of the pole.*

[Margin: Problem 226.]

In the preceding figure let the vertical be VBN. What is sought, then, is the arc GV, cut by the hour circle OGP, or the arc NS, cut by the hour circle OIP. Use, therefore, the triangle VGP or ONS, in which the angle at V or at N is

right if the vertical is the prime vertical; otherwise it is known, because it is the complement of the given azimuth. There is given besides the complement of the altitude of the pole, which is PV, or of the depression of the other pole, which is ON; and from the kind of hour circle, that is, from the hours before or after noon converted into parts of the equator, the angle GPV is known, or SON, the supplement of the angle SOA to two right angles. Therefore, either by the 7th rule of spherical right-angled triangles, or by the 11th or 12th of the oblique-angled, you will learn the arc sought, GV or SN.

Probl. 227. 228. Arcum Horarium, ſeu Horizontis interceptum à Circulo Horario & puncto veri ortus occaſuſue Solis inueſtigare, Data Hora Aſtronomica, Italica, aut Babylonica & Altitudine Poli. *Problems 227 and 228. To find the hour arc, that is, the arc of the horizon intercepted by an hour circle and the point of the true rising or setting of the Sun, given the astronomical, Italian or Babylonian hour, and the altitude of the pole.*

[Margin: Problem 227.]

First, if the astronomical hour is given, the hour circle is also given, and from it the portion of the equator between the hour cir- [Margin: Order of the problems.]

cle and the meridian or the horizon.

Therefore, in the figure of problem 225, let the Sun be at L, and let the hour be the 3rd before or after noon. For the circle OLP will be given, answering to this hour and intercepting the arc AL of the equator, of 3 hours, that is, of 45 degrees. The complement of this arc to 90 degrees is the arc LB, in the triangle KBL, right-angled at L, in which the angle KBL is given, which the altitude of the equator measures, that is, the complement of the altitude of the pole. Therefore by the 9th rule of spherical triangles you will seek the base BK, which is called the hour arc; for the point of the true, that is, the equinoctial, rising or setting is B.

[Margin: Problem 228.]

Or seek the arc HK by problem 225; when it is subtracted from the quadrant HB, the arc KB will remain. Likewise in the triangle BIM, right-angled at I: when the hour before or after noon is given and converted into parts of the equator, and 90 degrees, that is, the quadrant AB, are subtracted, the side BI will be given, and the angle IBM, namely the altitude of the equator. Therefore by the same 9th rule you will obtain the desired arc BM; or certainly by problem 225, if you take the arc MR from the quadrant BR.

Second, if the hour is Italian or Babylonian, convert it into an astronomical hour by means of the semidiurnal arc, and do the rest as above. Or certainly, let the Italian or Babylonian hour be halved, that is, let half of it be taken, and let it be turned into parts of the equator. For in this way you will have the distance from the meridian of the hour arc answering to such an hour, namely the arc AL or AI. From these the arc BL or BI will also be known; and with these, as above, in the triangle KBL or BIM, and with the angle at B, you will find by

the 9th rule already mentioned the desired arcs BK or BM. The reason why the Italian or Babylonian hour is to be divided in two is given by our Clavius [Christoph Clavius], proposition 17 of his *Gnomonica*.

229. Probl. Poſitionis Arcum inuenire, ideſt Altitudinem Poli cõſpicui ſupra Circulum poſitionis ſtellæ aut puncti, Datis Altitudine poli, Declinatione ſtellæ ſeu puncti, & Diſtantia ipſius Aequatoria à Meridiano. & de Tabulis Poſitionum. *229. Problem. To find the arc of position, that is, the altitude of the visible pole above the circle of position of a star or point, given the altitude of the pole, the declination of the star or point, and its equatorial distance from the meridian. And on the tables of positions.*

[Margin: Problem 229.]

The circle of position, according to Regiomontanus's *Directions*, problem 20, and the common notion of the astrologers, as I also taught in Book 1, chapter 21, number 10, is a great circle drawn through the two common sections of the meridian and the horizon, and through a star or point of the celestial, or even of the terrestrial, spherical surface.

[Margin: The circle of position and the arc of position are not the same.]

But the arc of position is an arc of a great circle drawn from the visible pole of the world to the circle of position orthogonally, that is, at right angles. This arc is called the elevation of the visible pole of the world above the circle of position, and among the astrologers it is the foundation of directions. For it most of them usually hand down various rules and tables; but you would scarcely find anyone who knows how, or is willing, to dispatch this problem by a geometrical figure without tables, or to explain the foundation of the tables thoroughly. Indeed, Magini and some others confuse their readers when they confuse circles of position with arcs of position.

Let there be, then, in the adjoining figure the meridian HAPR, the horizon HBR, and the equator ABQ. And first let the star be at S, through which the semicircle of position HSR is to be drawn; and from the visible pole of the world P let the quadrant PSD of a circle of declination be drawn through the star; and from the same pole P the arc PC, falling perpendicularly on the semicircle of position HSR. For this arc will be the measure of the altitude of the pole P above the circle of position.

Diagram. A circle set at the right of the left column, beside the text of problem 229: the "adjoining figure", with the meridian HAPR as its rim, the horizon HBR as the horizontal line, the equator ABQ as the oblique line from A to Q, and arcs through H and R (semicircles of position) and lines from the pole P. A small star marks the point S. Labels read:

- A (outside the rim, upper left)
- S (inside, upper left, above the star)
- P (rim, upper right)
- C (inside, right of centre, above the line)
- D (inside, left, above the line)

- H (left end of the line)
- B (inside, centre, below the line)
- I (small, beside B at the right)
- R (right end of the line)
- O (inside, lower right)
- M (inside, lower left of centre)
- Q (rim, lower right)

[Margin: Step 1.]

To find this arc, then: first, in the triangle SPR seek the angle PRS by the 5th or 6th rule of spherical oblique-angled triangles. For in it the sides are given: PR, the altitude of the pole; and PS, which is either the complement of the declination DS, as in the figure, or a quadrant, or the aggregate of the quadrant and the declination of the star or point toward the other pole. There is given besides the vertical angle, that is, the angle contained by the given sides, namely SPR, since it is the supplement to 180 degrees of the angle SPA, which is known from the equatorial distance DA of the star S from the meridian.

[Margin: Step 2.]

Second, in the triangle PCR, right-angled at C, seek by the first rule of spherical right-angled triangles the arc of position CP, since in it the base PR is given, which is the altitude of the pole, and the angle CRP adjacent to it, found by the 1st step. But if the star or point is at M, first, in the triangle PMR, seek the angle MRP by the 5th or 6th rule of the oblique-angled, since PM is given, the aggregate of the quadrant PI and the southern declination MI; and PR, the altitude of the pole; and the angle MPR, the supplement to two right angles of the angle MPA, which is known from the distance from the meridian. Then, in the triangle POR, with the base PR and the angle MRP found, that is, ORP, seek the arc of position PO; and so for the rest.

[Margin: The authors of these tables.]

But if anyone unskilled in trigonometry, or weary of it, wants to find the arc of positions by means of tables, let him consult the tables of positions that Regiomontanus constructed: either the particular ones for altitudes of the pole of 42°, 45°, 48° and 51°, from page 176, up to the 32nd degree of declination, which is about the greatest declination of the planets; or the general one from page 224. Their use and construction he teaches, though very obscurely, in problems 20 and 29. Or consult Giovanni Antonio Magini, in his older *Tables of Directions*, from folio 137, where the general table of positions begins; or from folio 148, where the particular tables of positions begin, from a degree of the altitude of the pole up to 46°, for declination up to 31°, which hardly any planet exceeds. He teaches the use of these tables in the same place, canons 30 and 33. In his more recent *Tables of Directions*, published on Tychonic principles, he gives two general tables of positions from page 341, and two particular ones, which he calls "directorial", for altitudes of the pole of 42° and 45°, from page 379; their construction and use he teaches in the same place, canons 16, 17 and 18. From these Argoli and some others put together their own.

230. Probl. Horarium Stellæ aut Puncti circulum, ſeu ſitum inquirere, hoc eſt quot Horis Temporalibus diſtet à Meridiano, Datis Diſtantia eius à Meridiano & Arcu Semidiurno, aut Seminocturno. *230. Problem. To find the hour circle, that is, the position, of a star or point, that is, by how many temporal hours it is distant from the meridian, given its distance from the meridian and its semidiurnal or seminocturnal arc.*

[Margin: Problem 230.]

Gather the distance of the star or point from the meridian, if it is above the earth, or from the lowest heaven, if it is below the earth, and do so by problem 194; and gather the semidiurnal arc, if it is above, or the seminocturnal, if it is below the earth, by problem 122. Then multiply the distance of the star or point from the meridian by 6, and divide the sum by the semidiurnal or the seminocturnal arc; for the quotient will give the temporal hours by which the star is distant from the meridian or from the lowest heaven. Unless you prefer to use the general Ptolemaic table that Magini sets out in his *Tables of Directions*, folio 162, teaching its use in canon 35.

231. Probl. Aſcenſionem, aut Deſcenſionem Horariam Stellæ, aut Puncti Eclipticæ, ſeu Circulum Poſitionis Horarium inuenire. *231. Problem. To find the horary ascension or descension of a star or of a point of the ecliptic, that is, the horary circle of position.*

[Margin: Problem 231.]

Every point of the ecliptic and every star passes through the angle of the east with the oblique ascensions, through the angle of the west with the oblique descensions, and through the meridian with the right ascensions. Since this is so, that point or star, by the motion of the primum mobile, while it is moved from one angle to another, that is, from the beginning of one house to the beginning of another, produces a variation in ascension or descension as great as the ascensional difference at such an altitude of the pole. Again, since six temporal hours are assigned to a quadrant of the primum mobile, the ascensional difference turns out different for the individual hours, that is, less or greater according to the greater or lesser distance of the star from the meridian. Therefore, if the ascensional difference is distributed into six equal parts, and if from the right ascension of the star, in the first half of the ecliptic, the sixth part of the ascensional difference is taken away six times in succession, while in the remaining half it is added, the horary ascensions will result, for six whole

[Margin: Order of the problems.]

hours before noon. For the descensions and the hours after noon the contrary must be done; that is, subtraction in place of addition, and addition in place of subtraction. Or certainly use the table of horary ascension that Magini gives in his *Directions*, page 190, teaching its use in canons [?] 36 and 37.

The remaining problems, which serve judicial astrology, I have decided, for just reasons, to suppress.

232. Probl. Angulum Verticalis & Circuli Latitudinis in centro Sideris factum, inuenire. *232. Problem. To find the angle made at the centre of a star by the vertical and the circle of latitude.*

See the Appendix to Book 3, chapter 7, rule 3, problem 1.

SECTIO TERTIA IN QVA PROBLEMATA TEMPORIS.

Section Three, in which are the problems of time.

1. Problema. Diem Primi Mobilis à Die Sidereo & Solari diſcernere, ipſiuſq. Horas ac Minutias in partes Aequatoris, & has viciſſim in Tempora conuertere, ac Tabulas pro vtriſque exactas conſtruere. *Problem 1. To distinguish the day of the primum mobile from the sidereal and the solar day; to convert its hours and minutes into parts of the equator, and these in turn into times; and to construct exact tables for both.*

[Margin: Problem 1.]

What the primum mobile is, and how it is given, we discussed sufficiently in Book 9, Section 2, chapter 3. How great the difference is between one day of the primum mobile, one sidereal day, or day of the fixed stars, and one solar day, we examined most minutely in Book 3, chapter 31; and there we brought forward the cause of the difference, and the method of constructing 4 tables, of which two suit the day of the primum mobile and two the solar day. For one sidereal day is not longer than a day of the primum mobile by a hundred-and-twentieth part of a second of time, that is, of a beat of the human artery; and therefore this small addition escapes every subtlety of the senses, so that in practice, for a single day at least, it is to be disregarded.

[Margin: The root of the 1st table.]

The first table, then, is for converting degrees and minutes of the equator into hours and minutes of hours of the primum mobile. In it 4 minutes of time are given for each degree, 4 seconds for each minute of the equator, 4 thirds of time for each second, and so on.

[Margin: The root of the 2nd table.]

The second table is for converting hours and minutes of the primum mobile into degrees, or parts, and minutes of the equator. In it 15 degrees are given for each hour; 15 minutes of the equator for each minute of time; 15 seconds of the equator for each second of time; 15 thirds of the equator for each third of time; and so on.

[Margin: The root of the 3rd table.]

The third table is for converting degrees and minutes of the equator into hours and minutes of time of the mean solar day. In it there must be attributed to each degree $3^{m}\ 59^{s}\ 20½'''$ of time; to each minute of the equator, $3^{s}\ 59'''\ 20½''''$ of time; to each second of the equator, $3'''\ 59''''\ 20½'''''$ of time; and to each third of the equator, $3''''\ 59'''''\ 20½''''''$ of time.

[Margin: The root of the 4th table.]

The fourth table, finally, is for converting hours and minutes of the solar day into degrees and minutes of the equator. In it one must assign to each hour 15° 2′ 27″ 51‴, or 15° 2′ 28″ (granted that Reinhold, choosing a round number, gives 15° 2′ 30″); to each minute of time, 15′ 2″ 28‴ of the equator; to each second of time, 15″ 2‴ 28⁗; and finally to each third of time, 15‴ 2⁗ 28⁗′. See the form of these tables in the small tables that follow.

CONVERSIO. *Conversion.*

I. Parts of the equator into hours and minutes of the day of the primum mobile.

Gr.	Ho.	′
Mi.	′	″
Sec.	″	‴
Ter	‴	⁗
1	0	4
2	0	8
3	0	12
4	0	16
5	0	20
10	0	40
15	1	0
30	2	0
60	4	0
90	6	0
180	12	0
360	24	0

II. Hours and minutes of the day of the primum mobile into degrees and minutes of the equator.

Ho.	Gr.	M	Gr.	′
		S	′	″
		T	″	‴
		Q	‴	⁗
1	15	1	0	15
2	30	2	0	30
3	45	3	0	45
4	60	4	1	0
5	75	5	1	15
6	90	6	1	30
9	135	10	2	30
12	180	20	5	0
15	225	30	7	30
18	270	40	10	0
21	315	50	12	30
24	360	60	15	0

III. Parts of the equator into hours and minutes of the solar day.

Gr.	Ho.	′	″	‴
M	′	″	‴	⁗
S	″	‴	⁗	⁗′
T	‴	⁗	⁗′	⁗″
1	0	3	59	20
2	0	7	58	41
3	0	11	58	1
4	0	15	57	22
5	0	19	56	42
10	0	39	53	24
15	0	59	50	6
30	1	59	40	12
60	3	59	20	24
90	5	59	0	36
180	11	58	1	12
360	23	56	2	24

IV. Hours and minutes of the mean solar day into degrees and minutes of the equator.

Ho.	Gr.	′	″	M	Gr.	′	″	‴
Mi.	′	″	‴	S	′	″	‴	⁗
Sec	″	‴	⁗	T	″	‴	⁗	⁗′
Ter	‴	⁗	⁗′	Q	‴	⁗	⁗′	⁗″
1	15	2	28	1	0	15	2	28
2	30	4	56	2	0	30	4	56
3	45	7	24	3	0	45	7	24
4	60	9	52	4	1	0	9	52
5	75	12	19	5	1	15	12	20
6	90	14	47	6	1	30	14	48
9	135	22	11	10	2	30	24	40
12	180	29	34	20	5	0	49	20
15	225	36	58	30	7	31	14	0
18	270	44	22	40	10	1	38	40
21	315	51	44	50	12	32	3	20
24	360	59	8	60	15	2	28	0

[Margin: Order of the problems.]

Auctores Tabularum præmiſſarum. *The authors of the tables set out above.*

The first table, complete, is found everywhere in the books on the sphere: the Alphonsine Tables, page 32; the Prutenic Tables, page 3; Orontius [Oronce Finé], Book 4 of the *Sphere*, chapter 5; Clavius in the *Sphere*, page 231; Blancanus [Giuseppe Biancani] in the *Sphere*, Book 1, part 1, chapter 5; Magini in the *Directions*, canon 16, and in the *Second Movables*, canon 9.

The second table, complete, is given by the Alphonsine Tables, page 32; Clavius in the *Sphere*, page 231; Lansbergen in the *Tables*, page 4; and Orontius, cited above.

The third table is given by Magini in the *Directions*, canon 16, and in the *Second Movables*, canon 9.

The fourth table is given by Reinhold in the *Directions*, precept 51.

2. Probl. Diſcernere Vſum temporis Primi Mobilis & Diei Solaris. *Problem 2. To distinguish the uses of the time of the primum mobile and of the solar day.*

[Margin: Problem 2.]

It matters greatly to know this difference, so that we do not confuse one time with another. The general rule is this: whenever we consider a time measured by the revolution of the equator viewed in itself — whether it is the measure of one revolution of the equator or of its parts — we should use the first or the second of the tables already mentioned. The same must be done when we consider arcs of the equator intercepted between two fixed points of the equator. But if the time is referred to the solar day and its parts, the third and fourth tables are to be used; and if the uses of both times are mixed, tables 1 and 2 must be used in part, and 3 and 4 in part.

[Margin: Case 1. Example.]

For example, suppose that from the difference of right ascension you are seeking the difference in time between the arrival of one fixed star and of another at the meridian, just as if the Sun did not exist in nature, or moved with no other motion than the fixed stars. Then the first table is to be used, because sidereal time differs insensibly from the time of the primum mobile. Conversely, suppose that from the time of the primum mobile between the transit of one fixed star and of another through the same meridian (known beforehand from the whole revolution of one fixed star to the same meridian) you wish to know the difference of right ascension: then Table 2 will have to be used.

[Margin: Case 2. Example.]

But if, from the Sun's distance from the meridian, you wish to know the solar time, or the hours of the Sun before or after noon, or the semidiurnal time, the 3rd table must be employed. Conversely, if from solar time, or the hours of the solar day, you want to know the arc of the equator, or the semidiurnal arc, use Table 4.

[Margin: Case 3. Example.]

When, finally, both kinds of time are mixed in practice, or an arc of the equator is to be gathered from both times, tables 1 and 2 must be employed in part, and 3 and 4 in part. Suppose, for instance, that from the distance of a fixed star from the meridian, and from its right ascension, the right ascension of the midheaven — that is, the point of the equator passing through the middle of the heavens — is to be gathered; and that then, from the right ascension

that the Sun has at that time of day (known in whatever way), compared with the right ascension of the midheaven, you wish to know the time in common use, namely the hour of the solar day reckoned from noon. You must then convert by the 3rd table into time the parts of the equator that correspond to the difference found between the right ascension of the midheaven and the right ascension of the Sun — granted that, to find the right ascension of the midheaven, you used an arc of the equator that abstracts from the Sun's motion. And conversely, suppose the time is counted up to the transit of a star through the meridian, and the time before or after the transit of the Sun through the meridian, so that from the difference or the sum of the two, together with the right ascension of the Sun, you may gather the right ascension of the fixed star. The time of the star's transit is then to be converted into parts of the equator by the 2nd table, but the time of the Sun's transit into parts of the equator by the 4th table.

I do not yet see these rules noticed by any of the astronomers. Magini and Reinhold, in the places cited under problem 1, were content only to acknowledge that the operation is more precise if we use table 3 in place of 1, and table 4 in place of 2. And yet, if this is neglected, there is a danger of erring at times by a whole half degree in the arc of the equator, and by two minutes in the time, when a semicircle is under consideration, as mostly happens; for in a whole circle the error committed could be one degree in the one case and 4 minutes in the other.

3. Probl. Minutias Diei in Horas ac Minutias Horarias, Parteſq. Aequatoris; & has viciſſim in Diei Minutias conuertere; ac Tabulas pro his condere. *Problem 3. To convert minutes of the day into hours and minutes of time, and into parts of the equator; and these in turn into minutes of the day; and to construct tables for them.*

[Margin: Problem 3.]

Since the Arabs, imitating Ptolemy, divide the day itself into 60 minutes, we shall make the first table if we multiply the given minutes of the day by 24 hours and divide the total by 60. For if no fraction remains, we shall have hours; otherwise we should divide the quotient again by 60, so that the second quotient gives the minutes of time.

[Margin: Table 1.]

From these, by the 2nd or the 4th table of problem 1, we shall then easily find the corresponding parts of the equator, and so make the second table. Alternatively, we shall obtain the same thing by multiplying the minutes of the day by 360 and dividing the total by 60.

[Margin: Table 2.]

Conversely, we shall multiply the given hours and minutes by 60 and divide the total by 24, for the quotient will show the minutes of the day that correspond to the hours and minutes of time; and in this way the third table can be made.

[Margin: Table 3.]

[Margin: Table 4.]

Lastly, the fourth table will be made if the parts of the equator are multiplied by 60 and the total is divided by 360 degrees.

[Margin: The authors of these tables.]

Tables 1 and 3, complete, are given by the Prutenic Tables, page 9½, and by Lansbergen, page 7 of the *Tables*. Table 2 is given by the Alphonsine Tables, page 32. Table 4, moreover, is given by the Prutenic Tables, page 3½; the Alphonsine Tables, page 32½; and Magini in the *Second Movables*, canon 9.

4. Probl. Tempus Aequale in Apparens, aut Apparens in Aequale conuertere, iuxta inæqualitatem Dierum Naturalium. *Problem 4. To convert equal time into apparent, or apparent into equal, according to the inequality of the natural days.*

[Margin: Problem 4.]

This was treated at length in Book 3, chapters 31 and 32, to which we refer the reader.

5. Probl. Horas Aſtronomicas, Babylonicas, Italicas & Europæas inuicem commutare. Nec non Aequales Horas in Temporales ſeu Inæquales. *Problem 5. To exchange astronomical, Babylonian, Italian and European hours for one another; and also equal hours for temporal, or unequal, hours.*

[Margin: Problem 5.]

The first conversion is treated sufficiently in Book 1, chapter 28; the second exchange in Book 1, chapter 29.

6. Probl. Horas quibus Luna noctu lucet, determinare. *Problem 6. To determine the hours during which the Moon shines by night.*

[Margin: Problem 6.]

This too we have treated thoroughly in Book 4, chapter 4, problem 2.

7. Probl. Semidiurnum ac Seminocturnum Tēpus Verum ſiue Solis ſiue cuiuſuis ſideris, inuenire Modis 4. Datis Declinatione ſideris, & Altitudine Poli vbi Sideris parallelus Horizontem ſecat; & Tabulas Condere. 8. 9. 10. Vel datis Aſcenſione Recta & Aſcenſ Differentia, aut Aſc. Obliqua ſideris ac Tabulas condere vel examinare. *Problem 7 [with 8, 9, 10]. To find the true semidiurnal and seminocturnal time, whether of the Sun or of any star, in 4 ways. Given the declination of the star and the altitude of the pole where the star's parallel cuts the horizon; and to construct tables. Or, given the right ascension and the ascensional difference, or the oblique ascension of the star, both to construct tables and to examine them.*

[Margin: Problem 7.]

The first way is mechanical, and uses an astronomical or a geographical globe, as I taught in Book 1, chapter 27, number 3; but it is not so exact. The second is carried out most exactly by problem 122, given in section 2.

[Margin: Problem 8.]

For the semidiurnal arc, once found, is to be converted into hours and minutes of time of the solar day by table 4, indicated in problem 1 of this section, rather than by table 2; but if the star is a fixed one, table 2 is to be employed. This time, subtracted from 12 hours, will leave the seminocturnal time. Furthermore, as great as the semidiurnal time is among the northern peoples, so great on the same day is the seminocturnal time among the southern peoples who have an equal alti- [Margin: Order of the problems.]

tude of their own pole; and as great as the seminocturnal time is for the northern peoples, so great is the semidiurnal for the southern peoples just mentioned.

[Margin: Problem 9.]

The third way is completed without trigonometry, by tables of the right ascensions of the Sun or of any star, together with tables of right ascensions. For when the ascensional difference corresponding to the place of the Sun, or of another star, in the ecliptic has been found for the given altitude of the pole, it is added to the right ascension if the star is on this side of the equator, and subtracted if it is beyond; and the result, or what remains, is the semidiurnal arc among the northern peoples, which is converted into hours, and so on, as was stated above for the 2nd way.

[Margin: Problem 10.]

The fourth way is carried out with the help of tables of oblique ascensions. For, for the given altitude of the pole, one looks up the oblique ascension of the star corresponding to its place in the ecliptic, and also the oblique ascension corresponding to the diametrically opposite place, that is, the one 180 degrees away from it in the ecliptic; and the former is subtracted from the latter (adding 360 degrees if need be). What remains is the diurnal arc, and this, cut in two, gives the semidiurnal arc, to be converted into hours as above. See the example in Book 1, chapter 27, number 4.

[Margin: The authors of this table.]

Tables of the semidiurnal time, moreover, are given by the Alphonsines, page 30, from a polar altitude of 36° to 45°, and for every third degree of the signs. Clavius in the *Sphere*, page 409, from an altitude of 36° to a polar altitude of 56°, and for every third degree of the signs. Magini in the Latin *Introduction to the Ephemerides*, page 25, and in the Italian, 32, for polar altitudes from 35° to 58°, but for each single degree of the signs; he adds as well a table of the rising and setting of the Sun, and of noon, in Italian hours. Franciscus Iunctinus [Francesco Giuntini], at the end of the *Speculum Astrologicum* and in its resolved tables, page 4, for polar altitudes of 36° to 55°, and for every third degree of the signs. Iosephus Moletius [Giuseppe Moletti] in Book 1 of Ptolemy's *Geography*, page 78, exactly as Iunctinus. Origanus in volume 1 of the *Ephemerides*, page 91, where he gives two tables for the single polar altitude of Frankfurt, 52° 20′, one according to the Prutenic obliquity of the ecliptic, the other according to the Tychonic; then, on page 91, he gives a table for polar altitudes from 6° to 89°,

and for every third degree of the signs, constructed according to an obliquity of the ecliptic of 23° 28′. And finally Argoli, in volume 1 of the *Ephemerides*, page 97, gives one like Magini's.

11. Probl. Semidiurnum Tempus Verum Solis determinare, vbi Parallelus Solis Horizontem non Secat, Seu Tempus continuatorum dierum abſq nocte, Datis Declinatione Sideris, & Altitudine Poli, & Obliquit. Eclipticæ. *Problem 11. To determine the true semidiurnal time of the Sun where the Sun's parallel does not cut the horizon, or the time of continuous days without night; given the declination of the star, the altitude of the pole, and the obliquity of the ecliptic.*

[Margin: Problem 11.]

Seek, by problem 217 of section 2, the arc of the ecliptic that is always visible. For as many degrees as such an arc contains, so many natural days, very nearly, will the Sun spend above the given horizon, making a continuous day. But if you wish to work more accurately, consult astronomical tables or ephemerides, and see how many days are needed for the Sun to pass from the degree in which the always-visible arc of the ecliptic begins to the degree in which it ends, taking one of those limits inclusively and the other exclusively. For instance, if that arc is of 20 degrees, of which the first is the 22nd degree of Gemini and the last the 11th degree of Cancer, seek the time in which the Sun runs from the 22nd degree of Gemini inclusive to the 11th degree of Cancer exclusive. This time, divided in two, can be called semidiurnal in the sense already given, and so on.

12. Probl. Semidiurnum Tempus Solis Apparens, aut cuiuſuis Sideris, Computatis Refractionibus & Parallaxibus inuenire Datis horizontali Refractione, Parallaxi & Declinatione Solis, vnà cum Alt. Poli: vbi Parallelus Sideris Horizōtem ſecat. *Problem 12. To find the apparent semidiurnal time of the Sun, or of any star, taking account of refractions and parallaxes; given the horizontal refraction, the parallax and the declination of the Sun, together with the altitude of the pole, where the parallel of the star cuts the horizon.*

[Margin: Problem 12.]

How great the force of refraction is in raising a star in appearance has been stated sufficiently in Book 1, chapter 36, and more fully in Book 3, chapter 9, in the scholia. It follows, then, that the Sun seems to rise somewhat sooner than it really rises, and to set somewhat later; and so, since the semidiurnal arc is increased at both ends, the apparent day is lengthened and the night shortened. The semidiurnal arc must therefore be corrected. To do this exactly, consider the following figure, in which, as usual, HVR is the meridian, HBR the horizon, ABQ the equator, CE its northern parallel and KD its southern.

Diagram. A circle set at the right of the right column, beside the opening of the construction of problem 12: the "following figure". V stands at the top of the rim, with C and A at the upper left, K at the left above H, H at the left, P at the upper right, R at the right, E and Q at the lower right and D at the

bottom. A near-horizontal line runs from H to R (the horizon HBR), with O, B, a small unread mark, and G standing along it from left to right. T stands below and left of O; L and F below G; N and M below the line; Z lower down, near the rim between M and Q. Straight lines run from V and from P to points on and below the line HR. Labels read:

- V (top of rim): the zenith end of the vertical; on the meridian HVR
- C (upper left of rim): end of the northern parallel CE
- A (upper left of rim): end of the equator ABQ
- K (left of rim, above H): end of the southern parallel KD
- P (upper right of rim): the pole of the world
- H (left): meridian and horizon
- R (right): meridian and horizon
- O (on the horizon, left of B): apparent rising of the star N
- B (on the horizon): point of the horizon and the equator
- [?] (on the horizon, immediately right of B): a very small mark that does not resolve on the scan; the text names a point I on the horizon, where the star M rises in appearance
- G (on the horizon, right of B): apparent rising of the star F
- T (below the horizon, left): where the star N would rise
- L (below G): where the star F would rise
- F (below G, right of L): the star on the northern parallel
- N (below the horizon, left): the star on the southern parallel
- M (below the horizon, centre): the star on the equator
- Z (near the rim, lower right): end of the quadrant of declination PFZ
- E (lower right of rim): end of the northern parallel CE
- Q (lower right of rim): end of the equator ABQ
- D (bottom of rim): end of the southern parallel KD

Now let the star be at the point F of the northern parallel, so that it would rise at L if refraction and parallax did not prevent it; but by the force of the corrected refraction let it rise apparently at G, when it is really at F, because refraction raises it visibly in the vertical VF by as much as the arc FG. And from the pole of the world P, through the star F, let the quadrant of declination PFZ be drawn. For in the triangle VFP three sides will be given: namely VP, the complement of the altitude of the pole; FP, the complement of the northern declination FZ; and VGF, made up of the quadrant VG and GF, the horizontal refraction of the star, first corrected by subtracting the star's parallax. Therefore, by [proposition] 14 of the oblique-angled spherical triangles, seek the angle VPF, which measures the arc ABZ of the equator, and so the semidiurnal arc as made apparent by the force of refraction. Further, the difference between the true semidiurnal arc AM and the apparent AZ will be the arc MZ, which the angle MPZ measures.

Similarly, if the star is at the point M of the equator, which would otherwise rise when it reached B, but by the force of the corrected refraction MI rises apparently at I, use the triangle VMP. In it VP is given, as above; PM is

a quadrant; and VM is made up of the quadrant VI and MI, the horizontal refraction of the star, corrected by subtracting the horizontal parallax. Therefore by the same [proposition] 14 you will seek the angle VPM, which is the measure of the semidiurnal arc AM; on account of refraction this is greater than the true semidiurnal arc AB by as much as the arc BM of the equator. Finally, if the star is at the point N of the southern parallel, which would rise on its arrival at T but rises apparently at O by the force of the horizontal refraction NO, employ the triangle VPN. In it the side VP is known, as above; PN consists of the quadrant PB and the southern declination BN; and VN consists of the quadrant VO of the vertical and the corrected horizontal refraction NO. Then by [proposition] 14 of the same triangles investigate the angle VPN, which measures the arc AB of the equator, that is, the apparent semidiurnal arc.

If the Sun is the star you are using, you will convert the semidiurnal arc into solar hours; if a fixed star, into hours of the day of the primum mobile. Further, from the time counted from the altitude of the fixed stars to the physical rising of the Sun, or from this to noon, compared with the semidiurnal quantity, we find that around the equinoxes the Sun usually rises sooner by two and sometimes by three minutes; but the variety of the refractions produces a remarkable diversity in this.

13. Probl. Semidiurnum Tempus Solis Apparens, computatis Refractione & Parallaxi Inuenire, vbi Parallelus Solis Horizontem non ſecat, ſeu diem continuam plurium dierum naturalium apparentem; Datis horizontali Refractione, & Parallaxi; ac Declinatione Solis; vnà cum Alt. Poli. *Problem 13. To find the apparent semidiurnal time of the Sun, taking account of refraction and parallax, where the Sun's parallel does not cut the horizon, or the apparent continuous day of several natural days; given the horizontal refraction and the parallax, and the declination of the Sun, together with the altitude of the pole.*

[Margin: Problem 13.]

Let the figure set out in section 2, problem 217, case 3, be taken up again. In it the parallel grazing the horizon is OR, and its true declination is AO, or GS, or QR — as great, that is, as the complement of the altitude of the pole to a quadrant. But if the Sun's horizontal refraction and parallax are known, subtract the parallax from the refraction, and there will remain the corrected refraction, which is to be subtracted from the declina- [Margin: Order of the problems.]

tion QR, since refraction, by raising the star, increases its declination — or, to speak more precisely, it makes the Sun, although it is on a parallel nearer the equator and having a smaller declination (say, below the parallel OSR), nevertheless seem to be on the parallel OSR. It follows that the arc BS of the ecliptic is smaller, and SC larger; and SC doubled makes the arc of the ecliptic that is always visible. Therefore, by problem 217 of section 3 [?], but using the declination, or the complement of the altitude of the pole, corrected by subtracting the horizontal parallax, seek the arc BS. You will then have the

apparent distance of the Sun S from the beginning of Aries, and so the point of the ecliptic that begins the always-visible arc; and the point S that ends the same arc is the same distance from the beginning of Libra. So the arc of the ecliptic enclosed within these points will not be hidden — namely, the arc of the ecliptic that is always visible by the force of the corrected refraction. From the ephemeris, moreover, the time will be known during which the Sun stays in that arc, which comes to nearly as many days as there are degrees in that arc.

Diagram. Three small circles set side by side across the left column, within the text of problem 13; each is crossed by a horizontal line from H to R (the horizon) and by oblique lines. The letters are very small. Labels read:

- First circle: K, A, C, V (along the upper left of the rim); P (right, above the line); H (left); B (centre); R (right); N (lower right); D, Q (together at the bottom)
- Second circle: C (upper left); V (top); P (upper right); A, K (left, above the line); H (left); B (below the centre); N, R (right); Q (right, below R); D (bottom)
- Third circle: V, P (together at the top); C (upper left); O (inside, below C); A (left); S (inside); H (left); G (a small letter just right of H, on the line); B, I, T (inside, below the centre); N, R (right); O (right, below R); M (right); D (bottom)

The contrary happens with the arc of the ecliptic that is always hidden, namely the arc TD taken twice: for it is made shorter than the true arc by as much as the true arc SC, taken twice, is made longer by the force of refraction.

14. Probl. Refractionem Horizontalem Solis, conuenientem ei in Altitudine Poli tanta, vt parallelus eius Horizontem non ſecet, inuenire, Datis Altitudine Poli, Declinatione Solis vera, & Arcu Eclipticæ ſemper apparente, aut ſemper latente ex obſeruatione cognito vel Die primæ aut vltimæ apparitionis Solis &c. *Problem 14. To find the horizontal refraction of the Sun that corresponds to it at an altitude of the pole so great that its parallel does not cut the horizon; given the altitude of the pole, the true declination of the Sun, and the arc of the ecliptic always visible or always hidden, known from observation, or the day of the Sun's first or last appearance, etc.*

[Margin: Problem 14.]

I have decided to insert this problem here, although it otherwise belongs to refractions, because of its affinity with the preceding one, and to illustrate it with a famous example reported by the observers themselves, the Dutch sailors, in the voyage to Nova Zembla [Novaya Zemlya] undertaken in 1596.

[Margin: The Dutch observation of the Sun always hidden.]

In qno [?] year, when they were at a northern polar altitude of 76 degrees, they saw the Sun on 2 November, new style, almost whole for the whole day, but not quite whole; on the 3rd they barely saw the upper edge of the Sun; and on the 4th they saw nothing of it at all, until 24 January 1597, when they began to see the upper limb of the Sun; and on the 25th they saw the whole

Sun standing clear — although, by problem 12, they did not expect the Sun until 11 February. They add, too, that from 30 April 1597 the Sun was always visible. Not knowing, then, the cause of so great a portent, they recorded it in writing as a wonderful thing in part 3 of the *Dutch Navigation*. From there Magini took it, in Book 6 of the *Primum Mobile*, problem 16, and so did Kepler in the *Astronomiae Pars Optica*, page 138, both rightly ascribing it to a very great refraction caused by the thickness of the vapours on that horizon.

For neither — as Kepler rightly argues — could those Palinuruses, who repeated the altitude of the pole so often, have committed so great an error as would be needed to defend this difference, which, as we shall soon see, exceeds 2 degrees; nor, as the disciples of Bodinus [Jean Bodin] imagined, was the place or shore where they were held fast in the bonds of the ice floating in the ocean, so that during that night of nearly three months it migrated sixty German miles towards the south. For they testify that they were then likewise at the same polar altitude of 76°, on 4 November of the year 1576[53] and on 24 January of the year 1579[54].

[Margin: Step 1: for the arc always visible, and the continuous day.]

In order, then, to investigate the enormous quantity of this refraction: first, by problem 217 of section 2, we shall investigate the true arc of the ecliptic that is always hidden, and always visible, at the given polar altitude of 76°, whose complement to 90° is 14°; and the declination GS, or TI, is as great. With this, and with the obliquity of the ecliptic — that is, the acute angle B — of 23° 30′, I find each of the arcs BS and BT to be 37° 21′. The first point of the ecliptic, S, distant from the beginning of Aries, B, and beginning the always-visible arc of the ecliptic, is therefore Taurus 7° 21′; and the point ending that arc is Leo 22° 39′. Now in the year 1597 the Sun was at Taurus 7° 21′ on 27 April, and at Leo 22° 39′ on 15 August; therefore from 27 April to 15 August the Sun ought always to have been visible above the horizon in question.

[Margin: For the arc always hidden, and the continuous night.]

Conversely, since from the arc BT of 37° 21′, reckoned from the beginning of Libra, one finds Scorpio 7° 21′, where the Sun was in the year 1596 on 30 October; and since, taking 37° 21′ before the beginning of Aries, one finds Aquarius 22° 39′, where the Sun was on 11 February 1597: it follows that the Sun ought to have been hidden on that horizon, with perpetual night, from 30 October inclusive to 11 February of the following year exclusive. And yet it was hidden only from 4 November to 25 January.

[Margin: Step 2: for the refraction.]

Second, converting the problem: the Dutch observation gives the time of the first occultation, which fell between 3 and 4 November, with the Sun placed at Scorpio 11° 44′ for the western shore of Nova Zembla; to this place of the Sun a declination of 15° 24′ corresponds. If you subtract from this the declination TI, corresponding to the parallel that grazes the horizon — that

[53] so printed; the passage's own dates are 1596–97, see 24 January 1597 above
[54] so printed; the passage's own date for this day is 24 January 1597

is, the complement of the altitude of the pole, which is 14° — there remains a refraction of 1° 24′. But between 24 and 25 January following, the Sun was at Aquarius 5° 5′, to which a declination of 19° 2′ corresponds; if you subtract 14° from this, there remains a refraction of 5° 2′. Magini, granted, in the place cited, gathers this second refraction as 4° 58′, because he uses the place of the Sun for day 25, without reducing it to the meridian of the western shore of Nova Zembla; while Kepler, using another and more obscure method, gathers the refraction now as 4° 3′, now as 4° 14′. Finally, on 30 April 1597 the Sun was at Taurus 9° 51′ for the western shore of Nova Zembla, and its declination is 14° 49′; subtract, then, 14 degrees, and a refraction of 49′ remains. Magini, granted, gathers 40′, because he uses a place of the Sun at Taurus 9° 25′. But since the Sun ought always to have been visible before 27 April, as is clear from the 1st step, I suspect that there is an error in the number of days, or that clouds stood in the way for several preceding days, or that they had sailed towards the south.

[Margin: Summary of the rule.]

Accordingly this problem 14 is solved by finding the declination of the Sun at the time of its first or last appearance at the given polar altitude, and comparing that declination with the complement of the altitude of the pole; for the difference between the complement and that declination is the horizontal refraction, which must, however, be corrected by subtracting the Sun's horizontal parallax.

Probl. Temporis momentum ex Obſeruatione Solis inquirere, Datis 15. Altitudine poli 16. Alt. Poli, Soliſq. Altitud. ac Loco in Eclipt. 17. Alt. Poli, Declinatione, & Altitudine Solis 18. Alt. Poli Azimutho, & Declinatione Solis 19. Alt. Poli, Altitud. ac Azimutho Solis 20. Azimutho, Altitud. ac Declinatione Solis *Problem. To find the moment of time from an observation of the Sun, given: 15. the altitude of the pole; 16. the altitude of the pole, and the Sun's altitude and place in the ecliptic; 17. the altitude of the pole, and the declination and altitude of the Sun; 18. the altitude of the pole, and the azimuth and declination of the Sun; 19. the altitude of the pole, and the altitude and azimuth of the Sun; 20. the azimuth, altitude and declination of the Sun.*

In the following scheme the meridian is HZRQ, the horizon HR, the equator EQ, and the north pole P, whose altitude PR, subtracted from 90 degrees, gives PZ, the distance of the pole from the zenith. The quadrant of the vertical circle, or circle of altitude, is AZ. The Sun's place is either at B, a point of the northern parallel — and then its northern declination is BN, and the complement of this to 90 degrees is BP — or at G, a point of the equator, and then no declina-

Diagram. A circle set at the right of the right column, beside the opening of the joint problems 15–20: the "following scheme". The horizon runs across it from H to R, and lines run from P and from Z across the circle. Labels read:

- Z (above the rim at the top): the zenith
- E (upper left of rim): end of the equator EQ
- N (just below E): end of the declination BN

- B (on an arc near the top): the Sun's place on the northern parallel
- P (right of rim): the north pole
- G (inside, left of centre): the Sun's place on the equator
- C (inside, right of G): not named in the text on this page
- I (below G): not named in the text on this page
- H (left): meridian and horizon
- A (on the horizon, below H): end of the vertical quadrant AZ
- R (right): meridian and horizon
- O (below the centre): not named in the text on this page
- Q (lower right of rim): meridian and equator

[Margin: Order of the problems.]

tion exists, but GP is a quadrant of the circle of declination; or at I, a point of the southern parallel, and then its southern declination is CI, which together with the quadrant CP makes up the arc ICP. Finally, the azimuth of the Sun — that is, the distance from the meridian of the vertical circle drawn through the Sun's centre — is the arc HA of the horizon, which the angle HZA measures; when that angle is subtracted from 180°, the angle AZP becomes known.

[Margin: Problem 15.]

First, then, if only the altitude of the pole is given, and you have equatorial armillae, set them so that the meridian of the armillae, which is HZRQ, coincides with the meridian line, and the equator of the armillae, ECQ, stands as far from the vertex or zenith as the altitude of the pole, PR; for in this way they will be set up analogously to the heavens. Then turn the circle of declination until one pinnule shades the other, or until the circle of declination itself, with its convex side, exactly shades the concave side opposite it. Note the arc of the equator between the meridian and the circle of declination, and convert the intercepted degrees into solar hours: in this way you will have the hours and minutes of hours before noon or after noon, according as the Sun was observed in the eastern or the western hemisphere.

[Margin: Problem 16.]

Second: if you lack armillae, use an astronomical globe. Raise the pole to the altitude due to it, and raise the point of the ecliptic in which the Sun then stands above the horizon by as much as the altitude of the Sun given by the observation requires. Through that point of the ecliptic draw, from the pole of the world, the semicircle of declination, fixed to the poles of the world so that it can be turned round; for the arc of the equator intercepted between this semicircle and the meridian, converted into solar hours, will give the time before or after noon. But if you wish to work more accurately, have recourse to the analysis of triangles by the following methods.

[Margin: Problem 17. The principal one.]

Third: let the Sun be at B, on this side of the equator. For in the triangle BZP the distance PZ of the pole from the vertex is given; and BP, the complement of the Sun's declination, which is known from the table of declinations if the Sun's place in the ecliptic is known from the ephemeris for the approximately

known hour; and finally the base BZ is given, since it is the complement to 90° of the observed altitude, corrected by adding the parallax and subtracting the solar refraction. Therefore, by [proposition] 14 of the oblique-angled spherical triangles, the angle BPZ will become known. It measures the arc NE of the equator, which is to be converted into solar hours so that you have the time before or after noon, according as the Sun was observed in the eastern or the western quadrant.

But if the Sun is in the equinoctial points, G, use the triangle GZP and seek the angle GPZ by the same 14; for the sides PZ, as above, and GP, which is a quadrant, are given, and GZ, the complement of the corrected solar altitude. But if the Sun is at I, beyond the equator, employ the triangle IZP, seeking the angle IPZ, which measures the arc CE; for the side PZ is given, and IP, made up of the declination and a quadrant, and the base IZ, the complement of the corrected altitude of the Sun. Then the arc of the equator, known from the angle P and converted into solar hours, gives the time before or after noon.

This problem is the most important of all those of this order, because it is enough to observe the altitude of the Sun and to correct it as has been said: the altitude of the pole is already given, and from the Sun's place, known in the ephemeris, the Sun's declination is given by the table, on which see section 2, problem 15. One must, however, know the hour beforehand roughly, so that from the two places of the Sun at noon — of the present day and of the preceding day if the Sun is in the eastern quarter, or of the following day if it is in the western — you may have a more exact place of the Sun for the said hour, and from it the corrected declination.

[Margin: Problem 18.]

Fourth: if, besides the side PZ, there are given the angle at Z, the supplement of the azimuth to 180°, and the side opposite it — namely BP, the complement of the declination, or GP, a quadrant, or IP, made up of the declination and a quadrant, according as the Sun was at B, at G or at I — seek, by [proposition] 3 of the oblique spherical triangles, the angle at P; convert it into solar hours before or after noon, according as the Sun was observed in the eastern or in the western hemisphere.

[Margin: Problem 19.]

Fifth: if, besides PZ, known as above, there is given the corrected altitude of the Sun, and from it its complement BZ, or GZ, or IZ, according as the Sun was at B, or G, or I; and the angle at Z, contained by the given sides and known from the azimuth, of which it is the supplement to 180°: seek by [proposition] 5 or 6 of the oblique spherical triangles the angle at P, and

[Margin: Order of the problems.]

convert it into hours before or after noon, and so on.

[Margin: Problem 20.]

Sixth: if there are given, from the azimuth, the angle at Z, and the complement of the corrected altitude, BZ, or GZ, or IZ; and either BP, the complement of the declination, or GP, a quadrant, or IP, the aggregate of the quadrant and

the declination: seek by [proposition] 1 of the oblique spherical triangles the angle at P, to be converted into hours before or after noon. Therefore in the four problems set out above, the triangle BZP, or GZP, or IZP must be solved, according as the Sun was at B, or at G, or at I.

[Margin: Other methods indicated.]

I pass over the remaining methods, which depend on a given angle at B, or G, or I, because this angle is not usually known except from the data already mentioned; and once these are in hand, the moment of time is found more readily through them. Since, however, these problems are in frequent use in astronomy, I will give an example in the one that is the easiest and most certain of all, namely in 17.

EXEMPLVM Probl. 17. *Example of problem 17.*

Let the altitude of the Sun be observed on 1645 April 10, at a polar altitude of 45°, in the morning, with an astronomical quadrant or otherwise, and let it be 10° 20′, at about the 6th hour before noon. First, then, I find from the ephemeris that the Sun's place at noon of April 10 is Aries 20° 54′, and on the preceding day 19° 56′; so the Sun's daily motion was 59′ [?]. Of these, about 15′ belong to 6 hours, to be subtracted from the place of the 10th day, so that the Sun's place at the 6th hour before noon is Aries 20° 39′. With this, from the table of the Sun's declinations, I find a northern declination of 8° 5′.

Next, since the Sun is at about its mean distance from the Earth, I find in my own tables that the Sun's parallax for the observed altitude of 10° 20′ is, by my reckoning, 24⅓″ (for Tycho's is 2′ 56″), and the refraction 9′ 40″. So, adding my parallax and subtracting the refraction from the apparent altitude, the corrected altitude of the Sun comes out at 10° 10′ 44″, that is, very nearly 10° 10′.

Therefore in the triangle BZP the side PZ is given, 45°, and the side BP, 81° 55′, as the complement of the declination BH, 8° 5′; finally the base BZ is given (much larger than the scheme shows), 79° 50′, as the complement of the corrected altitude AB, 10° 10′. With these, by [proposition] 14 of the oblique-angled spherical triangles, I find the angle BPZ, 82° 22′; to which, by Table 3 given in problem 1 of this section, correspond $5^{h}\ 28^{m}\ 34^{s}$ of solar time before noon. Working by logarithms, the example stands as follows.

Quantity	Arc	Logarithm	Value
PZ	45° 0′	Residual log.	015051
PB	81° 55′	Residual log.	000434
Aggregate	126° 55′		
BZ	79° 50′		
Sum	206° 45′		
Difference	47° 5′		
Half-sum	103° 22′	Log.	999925
Half-difference	23° 32½′	Log.	960143

Quantity	Arc	Logarithm	Value
Sum		Doubled log.	1975553
Half of the logarithm		Second log.	987776

To this second logarithm corresponds the angle 41° 11″ [?]. Therefore the vertical angle BPE is, as I said, 82° 22′.

SCHOLIA. *Scholia.*

[Margin: For observing the altitude.]

I. Although the more usual and more certain way of observing the altitude of the Sun uses a quadrant, or a square, or an astronomical ring, it can nevertheless also be observed by means of a gnomon casting a right or a versed shadow, or both. For these, look at the figure in

[Margin: Order of the problems.]

which there is a ruler AB parallel to the horizon, from which a style or ruler AC is raised perpendicularly, a little less high than a Roman foot, so that it casts a rather long shadow, but still a distinctly bounded one. And with the Sun at S, let the outermost ray DE fall from the upper limb D through the tip C of the ruler; then AE will be the right shadow.

[Margin: First method: by the right shadow.]

The ruler AB, however, must be divided into equal parts as small as possible, just like the ruler AC, so that you know exactly the parts of the right shadow AE in the parts in which AC is known. For in this way, in the triangle CAE, right-angled at A, the two sides about the right angle will be given; from them, by [proposition] 5 of the right-angled plane triangles, seek the angle AEC, which measures the altitude of the Sun's upper limb. If you subtract from it the Sun's apparent semidiameter SD, which is very nearly 16′, you will have the apparent altitude of the Sun's centre S.

Diagram. A figure set at the right of the left column beside the opening lines of the page, serving the three ways of observing the Sun's altitude by shadows. A small circle (the Sun) stands at the upper left, with lines running down from two points of its rim; below, a horizontal base line, an upright rising from its left end, a short horizontal at the top of the upright joined to a second upright, a long oblique line to the right end of the base, and an arc rising from the base. Labels read:

- S (centre of the small circle): the Sun's centre
- D (rim of the circle, right): the Sun's upper limb
- M (rim of the circle, left): the Sun's lower limb
- H (top of the left upright): end of the ruler FH; edge of the window CH
- F (top of the right upright): top of the wall FG
- C (on the left upright, below H): tip of the ruler AC; edge of the window CH
- V (on the oblique line, right of the upright through F): end of the versed shadow FV

- A [?] (lower left, the foot of the upright; printed without a visible crossbar): foot of the ruler AC
- E (on the base, right of A): end of the right shadow AE
- G (on the base, beside E): foot of the wall FG
- B (right end of the base): end of the line MB

[Margin: Second method: by the versed shadow.]

II. Now, for the versed shadow, let there be a wall or small beam FG, erected perpendicular to the horizontal plane AB and divided into very small equal parts; to its top F let another, shorter ruler FH be fixed, about a foot long or at most two feet, so that it makes a perfectly right angle F with FG, and divided into parts of the same kind as those in FG. With the Sun placed at S, the outermost ray MV will fall from the lower edge M through the end H, bounding the versed shadow FV. So, by counting the parts it takes up, the sides FV and FH about the right angle will be known; therefore by the same 5 you will find the angle H, which measures the altitude of the Sun's lower limb M. If you add to it the Sun's apparent semidiameter MS, you will have the apparent altitude of the Sun's centre.

[Margin: Third method: by both shadows.]

III. Finally, let there be a wall ACH erected perpendicular to the horizontal plane AB, in which there is a small window CH, with a plate on either side ending in sharp edges C and H; and let the length of the window, or the distance between the plates, CH, be known in the parts in which the height AC is known. For when the Sun shines directly on the window and sends its rays into the room (which ought to be thoroughly dark), the ray DE, grazing the lower edge of the window, will make or bound the right shadow AE; with it and with the side AC you will find, as above, the angle AEC, that is, the altitude of the Sun's upper limb. The ray MB, grazing the upper edge of the window, will bound the versed shadow, but a hidden one; in its place use the side AB and the whole height ACH, and seek by the same rule 5 of triangles the angle ABH, that is, the altitude of the Sun's lower limb. The difference between the altitudes of the upper and the lower limb will be the measure of the Sun's apparent diameter; and half that difference, added to the altitude of the lower limb, will give the apparent altitude of the Sun's centre.

Probl. Temporis momentum ex Obſeruatione Stellarum Fixarum, abſq. Trigonometria 21. deducere, Datis 22. Aſc. Recta Stellæ ac Solis 23. Aſc. Recta Stellæ ac Solis & Altitudine Stellæ ac Poli 24. Aſc. Recta Stellæ ac Solis, & Altitud. Poli. *Problem. To deduce the moment of time from an observation of the fixed stars, without trigonometry, given: 21. — 22. the right ascension of the star and of the Sun; 23. the right ascension of the star and of the Sun, and the altitude of the star and of the pole; 24. the right ascension of the star and of the Sun, and the altitude of the pole.*

[Margin: Preparation for the problems.]

The right ascension of the Sun, which is required here, becomes known from astronomical tables, or from an ephemeris constructed for your meridian or reduced to it: take the Sun's place in the ecliptic, and with it consult the tables of right ascensions given in problem 16 of section 2. Moreover, if you want to know the moment of time as it is a part of the day of the primum mobile, it is enough to gather the right ascension from the place of the Sun that it held at the preceding noon. But if, as is commonly done, you want the time and the hour of the solar day, you must know approximately, or roughly, the hour after noon of the preceding day; this is to be multiplied by 60 and the result divided by 24, for the quotient will give the minutes to be added to the place that the Sun had in the ecliptic at the preceding noon. With this place of the Sun, the Sun's right ascension is to be sought.

[Margin: Order of the problems.]

[Margin: Problem 21.]

First: if a fixed star crosses the meridian — shown by threads hanging perpendicular over the meridian line, or otherwise — the right ascension of the star will likewise be the right ascension of the midheaven. From it subtract the right ascension of the Sun (adding 360° if need be), and convert the remainder into solar hours and minutes, so that you have the time counted from the preceding noon. Therefore, if some astronomical observation can conveniently be made, by waiting for the moment of transit of a known fixed star through the midheaven, the moment of time can be gathered at once.

[Margin: Problem 22.]

Second: if a fixed star of known right ascension is distant from the meridian, let the time that elapses between the observation made and the transit of the fixed star through the meridian be counted by means of the pendulum (on which see Book 2, chapter 20, at the end of proposition 11), and let that time be converted into parts of the equator by Table 2, given in problem 1 of this section. These parts subtract from the right ascension of the fixed star itself if the star was eastern, or add if it was western, and you will have the right ascension of the midheaven; from this subtract the right ascension of the Sun, and convert the remainder into solar hours and minutes of hours, so that you have the time from noon.

[Margin: Problem 23.]

Third: use a very large astronomical globe, raising the pole to the given altitude of the pole and turning the globe until the fixed star whose altitude is given reaches that given altitude above the horizon, measuring it with a special pair of compasses or with a quadrant arc fitted to the globe. For if you draw the semicircle of declination from the pole of the world through the star, it will cut off the arc of the equator lying between this semicircle and the meridian. Subtract this arc from the right ascension of the star if it is eastern, or add it if it is western, and the right ascension of the midheaven will result; subtracting from it the right ascension of the Sun, and turning the remainder into solar hours and minutes, you will have the time from noon.

[Margin: Problem 24.]

Fourth: employ equatorial armillae set up astronomically, as the altitude of the pole requires. For the circle of declination, turned so that the star's rays pass through the dioptric pinnules, will cut off the arc of the equator between itself and the meridian; use this arc as in problem 23.

Probl. Temporis momentum ex Stellæ Fixæ obſeruatione Deducere Trigonometricè, Datis præter Aſcenſionem Rectam Solis & Stellæ Fixæ 25. Alt. Poli, Declinatione ac Altitudine Stellæ 26. Alt. Poli, Azimutho, & Declinat. Stellæ. 27. Alt. Poli, Azimutho, & Alt. ſtellæ. 28. Azimutho, Altitudine ac Declinat. Stellæ. *Problem. To deduce the moment of time trigonometrically from an observation of a fixed star, given, besides the right ascension of the Sun and of the fixed star: 25. the altitude of the pole, and the declination and altitude of the star; 26. the altitude of the pole, and the azimuth and declination of the star; 27. the altitude of the pole, and the azimuth and altitude of the star; 28. the azimuth, altitude and declination of the star.*

How the right ascension of the Sun is to be gathered, I taught in the preparation for problem 21. Once it is known, use the figure set out in problem 15, and carry out for the fixed star everything I said there about the Sun, so as to have the angle at P, the pole of the world; for this angle measures the arc of the equator intercepted between the meridian and the circle of declination drawn through the star. This arc, then, you will subtract from the right ascension of the fixed star itself if the star is in the eastern hemisphere, and add it if in the western, and you will obtain the right ascension of the midheaven. From this subtract the right ascension of the Sun (adding 360°, if the subtraction requires it), and convert the remainder into solar hours and minutes of hours; in this way you will have the exact time from the preceding noon.

[Margin: Problems 25, 26, 27, 28.]

Therefore you will solve problem 25 as problem 17, problem 26 as problem 18, problem 27 as problem 19, and problem 28 as problem 20. Yet of them all the easiest and most certain is 25, for it is enough to observe the altitude of the star without the azimuth; the rest are given easily, namely the distance of the pole from the vertex, since it is the complement

[Margin: Order of the problems.]

of the altitude of the pole; and the other side is given from the declination of the fixed star, since it is either the complement of the declination, if the star is on this side of the equator; or a quadrant, if the star is on the equator; or the aggregate of the quadrant and the declination of a fixed star placed beyond the equator. These two sides contain the required angle at the pole of the world. The base, moreover, is the complement of the observed altitude, corrected by subtracting the refraction; although it is advisable to use stars raised beyond the hazard of sensible refractions, namely those having an altitude of not less than 20°. With these given, then, the angle at the pole is obtained by [proposition] 14 of the oblique spherical triangles. Since, however, this problem 25 is the

most used in most of the astronomical observations made at night, I am glad to add an example of it.

EXEMPLVM Problematis 25. *Example of problem 25.*

Let the altitude of Arcturus in the east be observed on 1645 April 10, at a polar altitude of 45°, as 44° 20′, at about the 9th hour after noon. First, the ephemeris gives the Sun's place at noon as Aries 20° 54′. The 9 hours multiplied by 60 make 540; divided by 24 hours, these yield 22½′[55], the minutes to be added to the Sun's place; and so at the 9th hour the Sun's place in the ecliptic is Aries 21° 16′. To this place, from the table of right ascension constructed for an obliquity of the ecliptic of 23° 30′, there corresponds a right ascension of the Sun of 19° 39′. From the corrected tables of the fixed stars, moreover, the right ascension of Arcturus for this time is found to be 209° 57′ 11″, and its northern declination 21° 5′ 27″, whose complement is 68° 54′ 33″. With these prepared, the triangle BPZ will be solved as follows, in the figure of problem 17, which is set on page 578.

Quantity	Arc	Logarithm	Value
PZ	45° 0′ 0″	Residual log.	015051
BP	68° 54′ 33″	Residual log.	003011
Aggregate	113° 54′ 33″		
Base BZ	45° 40′ 0″		
Sum	159° 34′ 33″		
Difference	68° 14′ 33″		
Half-sum	79° 47′ 16″	Log.	999306
Half-difference	34° 7′ 16″	Log.	974892
		Doubled second log.	1992260
Half vertical angle	23° 50′ 0″	Second log.	996130
Angle BPZ	47° 40′ 0″		

Since, then, half of the doubled second logarithm is the second logarithm of the half-angle BPZ, 23° 50′, the whole vertical angle BPZ, or NPE, is in any case 47° 40′. When these are subtracted from the right ascension of Arcturus in the east, 209° 57′, there remains the right ascension of the midheaven, 162° 17′. Subtracting from this the right ascension of the Sun, which is 19° 39′, there is left an arc of 142° 38′, which, converted into solar hours, gives 9^h 29^m after noon on April 10.

29. Probl. Temporis momentum deducere, Datis Alt. Poli Aſc. Recta Solis, & duabus Stellis in eodem ſimul Verticali obſeruatis, quarum nota ſit Aſc. Rec. & Declinatio, eaq. occaſione, ſi lubeat, Altitudinem Fixarum inuenire. *Problem 29. To deduce the moment of time, given the altitude of the pole, the right ascension of the Sun, and two stars observed at the same time on the*

[55] so printed; 540 divided by 24 gives 22½

same vertical circle, whose right ascension and declination are known; and on that occasion, if one wishes, to find the altitude of the fixed stars.

In the following figure the meridian is HVRQ, the horizon HOR, the equator AOQ, and the altitude of the pole PR, whose complement to 90° is PV. The quadrant of the vertical circle is VT, on which stand the two stars F and G together; through them let the quadrants of declination PFK and PGM be drawn.

[Margin: Problem 29.]

First, in the triangle FPG, seek by [proposition] 4 of the oblique spherical triangles the base FG, and by 5 or 6 the angle PGF; for in it are given the side FP, the complement of the declination of the star F, the side GP, the complement of the declination of the star G, and the contained angle GPF, since it is the difference of the right ascensions of the two stars.

[Margin: Order of the problems.]

Second, in the triangle VGP, seek by 3 of the same triangles the angle GPV, that is, MPA, and by 2 the base VG; for there are given the sides VP, the distance of the pole from the vertex, and GP, the complement of the declination of the star G, and the angle VGP, already found.

Third: subtract the angle MPA, that is, the arc MA, from the right ascension of the star G if it is eastern, or add it if western, and the right ascension of the midheaven will become known; subtract from this the right ascension of the Sun, and the remainder, converted into solar hours, will give the desired time from the preceding noon. Fourth: subtract the base VG from 90°, and there will remain the true altitude TG of the star G; if you add to it the base FG, found in the first step, the altitude TF of the star F will result.

Diagram. A circle set at the right of the right column, beside the solution of problem 29: the "following figure" of that problem. The horizon runs across it from H to R, the equator from A through O to Q, an arc of the vertical circle from V down through F and G, and lines run from P to K and from P through G to M. Labels read:

- V (top of the rim): the vertex
- A (upper left of the rim, a short dash before it): end of the equator AOQ
- F (inside, below V): the star F
- P (upper right of the rim): the pole
- K (inside, left): end of the declination quadrant PFK
- G (inside, below F): the star G
- H (left of the rim): meridian and horizon
- M (inside, left of centre): end of the declination quadrant PGM
- R (right of the rim): meridian and horizon
- O (below the centre): intersection of horizon and equator
- T (inside, right of O): foot of the vertical quadrant VT
- Q (lower right of the rim): meridian and equator

30. Probl. Temporis momentum Data Aſc. Recta Solis ac Medij Cæli aut Obliqua Horoſcopi, ſeu puncti Eclipt. Aſcendentis. *Problem 30. The moment of time, given the right ascension of the Sun and of the midheaven, or the oblique ascension of the ascendant, that is, of the rising point of the ecliptic.*

[Margin: Problem 30.]

If the right ascension of the midheaven is not given, subtract 90° from the oblique ascension of the ascendant; there will remain the right ascension of the midheaven. From this subtract the right ascension of the Sun, and convert the remainder into solar hours and minutes, so that you have the time from noon.

31. Probl. Temporis momentum definire, Dato Temporis momento in alio loco Terræ, cuius nota ſit differentia longit. à tuo loco. *Problem 31. To determine the moment of time, given the moment of time at another place on the Earth whose difference of longitude from your place is known.*

[Margin: Problem 31.]

Suppose the beginning of an eclipse was observed at Paris at $10^h\ 30^m$ after noon. Then, since Bologna in Italy differs in longitude by 11°, to which correspond 44 minutes of time, and Bologna is more easterly, you will conclude that the beginning took place at Bologna at $11^h\ 14^m$; but if Bologna were more westerly, those 44^m would have to be subtracted.

32. Probl. Tempus Ortus, Occaſuſue Solis veri aut Apparentis inueſtigare. *Problem 32. To investigate the time of the rising or setting of the Sun, true or apparent.*

[Margin: Problem 32.]

Seek the true semidiurnal time by problems 7 or 8, etc., or the apparent one by problem 12; and that very time will be the time before noon at which the Sun's centre will rise, and after noon at which it will set, truly or apparently.

33. Probl. Tempus Apparens Ortus & Occaſus Limborum Solis definire. *Problem 33. To determine the apparent time of the rising and setting of the Sun's limbs.*

[Margin: Problem 33.]

Consult problem 12: subtract the Sun's apparent semidiameter from the Sun's horizontal refraction, and carry out the rest as there; for you will have the semidiurnal time at which the upper limb of the Sun will apparently both rise before noon and set after noon. Again, add the Sun's apparent semidiameter to the Sun's horizontal refraction and do the rest as there; for you will obtain the semidiurnal time at which the lower limb of the Sun will apparently rise before noon, and at which it will set after noon.

34. Probl. Tempus requiſitum vt totus Solis diſcus oriatur ſupra Horizontem definire. *Problem 34. To determine the time required for the whole disc of the Sun to rise above the horizon.*

[Margin: Problem 34.]

Find the time of the apparent rising of each edge of the Sun by problem 33; for the difference between the risings of the two limbs will be the required time. Or at least observe it, [Margin: Order of the problems.]

by counting the time with the help of the pendulum explained in Book 2, chapter 20. We have done this very often, as we shall relate, God willing, in volume 3 among the solar observations, and we found it to be 186^s or 184^s of time on the equinoctial days, but 320^s at the time of the summer solstice and 380^s at the time of the winter solstice. See also, if you please, the small table set out for this in the Appendix to Book 3, chapter 10, towards the end.

[Margin: Our observation concerning the rising of the Sun's disc.]

35. Probl. Tempus, quo Sol aut Luna oritur priùs habitanti in Montis aut Turris cacumine, quàm habitanti ad pedem Montis aut Turris, Datis Semidiametro Terræ, Altitudine Montis aut Turris, Altitudine Poli, ac Declinatione Solis, Methodo Petri Nonij ſed fallaci. *Problem 35. The time by which the Sun or the Moon rises earlier for someone living on the summit of a mountain or tower than for someone living at the foot of the mountain or tower, given the Earth's semidiameter, the height of the mountain or tower, the altitude of the pole and the declination of the Sun: by the method of Petrus Nonius, but a fallacious one.*

[Margin: Problem 35.]

Let ANCQ be a vertical circle passing through the centre [?] of the rising Sun, and under this circle let TKR be a great circle of the Earth, with a tower or mountain TD standing on it, whose summit is D and foot T. Let the rational horizon of the inhabitant at T be ABC, and his sensible horizon ETG; for the inhabitant at K, let the rational horizon be PBQ and the sensible horizon HKM, touching the Earth at K. Let the lines NB and BL be perpendicular, NB to the horizon AC and BL to the horizon PQ. Now it is evident that at the moment when the Sun M rises physically for the inhabitant at K, it also rises for the inhabitant at D; and when it rises at G, it rises sensibly for the inhabitant at T.

With these premises, the first thing sought is the arc MG, that is, the difference between the sensible horizons, or the depth of the Sun M below the horizon ETG when the Sun rises at M. You will find this arc as follows. In the triangle BKD, right-angled at the point of contact K by [proposition] 18 of the third book of Euclid, there are given BK, the Earth's semidiameter, known in paces, and BTD, consisting of the Earth's semidiameter BT and the height TD. Therefore by [proposition] 11 of the plane right-angled triangles find the angle DBK, or NBL. This angle is equal to the angle CBQ: for each of them together with the angle LBC makes up a right angle, the first the angle NBC, the second the angle LBQ; hence, when the common angle LBC is taken away, there remains NBL equal to CBQ, which measures the arc QC, that is, the distance of the rational horizon PQ from the horizon AC. But the distance of the sensible horizon HM from the sensible horizon EG is just as great, because the arc MQ, by which PQ is distant from HM, is equal to the arc CG, by which

AC is distant from EG, since the parallels of each pair are distant from each other by the same terrestrial semidiameter, BK and BT. Therefore MG is given, the depth of the Sun M below the horizon, which we were seeking.

Secondly, then, with this depth, the declination of the Sun and the altitude of the pole, find the semidiurnal arc and time by problem 12, using the arc MG just found in place of the horizontal refraction, adding it to the vertical quadrant, and carrying out the rest as there. Compare this time with the semidiurnal time for the given day (for if the declination is given, one or other of the days on which that declination belongs to the Sun, before or after the nearer solstice, is also given) and for the given altitude of the pole: the difference of this time from the former will be the time by which the Sun rises earlier for the inhabitant at D than for the inhabitant at T. But that holds if the Sun's rays are supposed unrefracted, as Nonius supposes them in his book *De crepusculis*. Since in reality they are refracted in the denser air around the Earth, by their force the Sun will rise much sooner for the summit of the mountain; and so this problem is liable to an optical fallacy, which we shall correct in the following Section, problem 36.

[Margin: The error of Petrus Nonius.]

Diagram. A large circle set at the right of the left column, with a small circle (the Earth) drawn about T inside it. A vertical runs from N at the top through D, T and B; two horizontals run from E to G and from A to C; lines run from P through T and K to M, from H down towards Q, and from K up to L.

- P, H (upper left of the rim): ends of the lines through the Earth
- N (top): the zenith end of the vertical NB
- L (upper right): end of the perpendicular BL
- G, C (right of the rim, G above C): ends of the horizons ETG and ABC
- E, A (left of the rim, E above A): the other ends of those horizons
- D (inside, above the small circle): the summit of the tower or mountain
- T (centre of the small circle): its foot
- K (right edge of the small circle): the second inhabitant's station
- B (below T): the Earth's centre in the construction
- R (inside, below the small circle): on the Earth's great circle TKR
- M (lower right of the rim): the Sun rising for the inhabitant at K
- Q (below M): end of the rational horizon PBQ

[Margin: Order of the problems.]

36. Probl. Crepuſculi apparentis Quantitatem inquirere, Data Profunditate Solis ſub Horizonte ad initium matutini, aut finem veſpertini Crepuſculi cum Altit. Poli & Declinatione Solis. *Problem 36. To find the length of the apparent twilight, given the Sun's depth below the horizon at the beginning of morning twilight or the end of evening twilight, together with the altitude of the pole and the declination of the Sun.*

[Margin: Problem 36.]

How the Sun's depth at the beginning or end of an observed twilight is gathered, and how great it is, was stated in Book 1, chapter 31, and in its Scholium 2. With that depth given, therefore, and added to the quadrant of the vertical circle, use problem 12 and its figure, in which let FG be the depth, added to the quadrant VG, so that the side VF is known; let the side FP be the complement of the Sun's declination, and VP the complement of the altitude of the pole. Therefore, in the triangle VFP, find by [proposition] 14 of the oblique spherical triangles the angle FPV, which measures the arc ABZ of the equator. From this subtract the semidiurnal arc; the remainder, converted into solar hours or minutes of time, will give the length of the twilight. I know that others investigate this with the help of the analemma, as Biancani does in the *Sphaera*, Book 10, or "On the Sun", chapter 16, or else by plane triangles. But the method I have proposed here is the most exact of all.

37. & 38. Probl. Tempus inuenire, quo Sol aut Sidus quoduis venit ad Verticalem Primarium, aut ad quemuis Verticalem, Datis Altitud. Poli, Declinatione Sideris, & Azimutho ipſius Verticalis. *Problems 37 and 38. To find the time at which the Sun or any star comes to the prime vertical, or to any vertical, given the altitude of the pole, the declination of the star, and the azimuth of the vertical itself.*

[Margin: Problem 37.]

In the next figure the meridian is HVRN, the horizon HR, the equator AOQ, and the prime vertical quadrant VO, in which the Sun is at T. Now in the triangle TVP, right-angled at V, there are given PV, the complement of the altitude of the pole PR, and TP, the complement of the declination. Therefore by [proposition] 5 of the right-angled spherical triangles seek the angle TPV; converted into hours and minutes, either solar or of the primum mobile according as the star is the Sun or a fixed star, it will give the time both before and after noon at which the star comes to the prime vertical.

[Margin: Problem 38.]

Now let the star be at the point B of another vertical. In the triangle BPV find the angle BPV, to be converted into time, so that you have the time before and after noon at which the star will arrive at the vertical VM. Seek it, I say, by [proposition] 3 of the oblique-angled spherical triangles: for in that triangle there are given VP, the complement of the altitude of the pole; BP, the complement of the star's declination; and the angle BVP, because it is the supplement of the given azimuth to two right angles. But if the star is beyond the equator, at I, use the triangle IPV, seeking the angle IPV, since there are given VP, the angle at V as above, and IP, consisting of a quadrant and the star's declination.

Diagram. A circle (the sphere) set at the right of the right column. HR runs across it; dotted lines descend from V through T and O and from M towards Q; straight lines run from P to K, from P to A, and from P through T towards I and M; an arc runs from C down through B and S to I.

- V (top of the rim): the zenith, on the meridian HVRN
- C (upper left, outside the rim)
- A (upper left of the rim): end of the equator AOQ
- B (inside, below C): the star on another vertical
- P (right of the rim, high): the pole
- K (left of the rim, below A)
- S (inside, right of K; italic)
- T (inside, above O): the Sun on the prime vertical
- I (inside, at the left; small): the star beyond the equator
- H, R (left and right of the rim): ends of the horizon HR
- M (inside, below H): on the vertical VM
- O (centre)
- D (lower right of the rim)
- Q (lower right, below D): end of the equator AOQ
- N (foot): on the meridian

39. Probl. Tempus inuenire, quo Sidus veniet ad datam altitudinem in Verticali, Datis Alt. Poli, & Declinat. Sideris. *Problem 39. To find the time at which a star will come to a given altitude in a vertical, given the altitude of the pole and the declination of the star.*

[Margin: Problem 39.]

In the figure immediately preceding, let the star be at B, and consider the triangle BPV, in which there are given PV, the complement of the altitude of the pole; BV, the complement of the star's given altitude; and BP, the complement of the declination. Therefore by [proposition] 14 of the oblique-angled spherical triangles you will find the angle BPV, which, converted into time, will give the desired time before or after noon at which the star will arrive at the given altitude.

[Margin: Order of the problems.]

40. & 41. Probl. Tempus inuenire, quo Sol, aut quoduis Sidus perueniet ad Nonageſimum Eclipticæ gra[?]um, dato die & in data Altitudine Poli. *Problems 40 and 41. To find the time at which the Sun, or any star, will arrive at the nonagesimal degree [?] of the ecliptic, on a given day and at a given altitude of the pole.*

[Margin: Problem 40.]

If the star is not the Sun, consult Book 4, chapter 4, problem 8, and take what was said there of the Moon as said of any other planet or fixed star. If it is the Sun, seek the semidiurnal time by problem 12. Then seek the oblique ascension of the point of the ecliptic in which the Sun is, and of the point of the ecliptic 90 degrees distant from it in the order of the signs; and take the arc of the equator intercepted between the two oblique ascensions, converted into solar time, away from the semidiurnal time, if the semidiurnal time is the greater: the remainder will be the time before noon at which the Sun will be in

the nonagesimal degree of the ecliptic, counted from the rising horizon. But if the semidiurnal time is the smaller, subtract it from the arc already mentioned, converted into time; the remainder will be the time after noon at which the Sun will come to the nonagesimal. Finally, if the arc already mentioned were equal to the semidiurnal arc, the Sun will be in the nonagesimal at the very moment of noon.

[Margin: Problem 41.]

You will obtain the same result another way, by the doctrine of triangles, from the figure already set out in Section 2, problem 206, in which N is the nonagesimal degree of the ecliptic and the Sun is in it. *First*, then, in the triangle NPZ, find by [proposition] 14 of the oblique spherical triangles the angle PNZ. For there are readily given the base PZ, which is the distance between the poles of the ecliptic and of the equator, as great as the obliquity of the ecliptic; and the sides N VZ [?], which is a quadrant of the circle of latitude, and NP, which is the complement of the Sun's declination if the Sun is on this side of the equator, or a quadrant if it is on the equator, or the sum of a quadrant and the declination if it is beyond the equator.

[Margin: Step 1.]

Secondly, in the triangle NVP, seek by [proposition] 3 of the oblique spherical triangles the angle NPV: for there are given the angle VNP, already found, and the side PN, from the Sun's declination; and VP, which is the complement of the altitude of the pole, is supposed known. Lastly, convert the angle NPV into solar hours and minutes of hours, for you will have the time before or after noon at which the Sun is in the nonagesimal: before noon if the nonagesimal (namely the given place of the Sun) is in the eastern quadrant, but after noon if it is in the western quadrant. For the situation of the nonagesimal in this respect consult problem 182 of Section 2; or, if you please, gather it easily by looking at a globe and turning it.

[Margin: Step 2.]

Diagram. A circle set at the right of the left column, with a small circle drawn about P at the right of the rim; a curve runs from E at the upper left over V to the small circle. HR runs across; a line runs from N through D and C to P; another from H through A and B towards M; lines descend from V.

- E (outside, upper left): end of the curve
- V (top of the rim): the zenith
- Z (outside, upper right; small): the pole of the ecliptic, per the text
- O (upper left of the rim)
- N (inside, below O): the nonagesimal degree, with the Sun in it
- D (inside, right of N)
- C (inside, right of centre)
- I (inside, upper right)
- P (inside the small circle at the right): the pole of the world
- L (right of the rim, below P)
- H, R (left and right of the rim)

- A (inside, lower left)
- B (on the rim, below)
- M (outside, lower right)

42. Probl. Tempus inuenire, quo arcus Eclipticæ datus, tranſit per Verticalem Primarium aut quemuis alium, Datis Declinatione, & Aſcenſ. recta extremorum Eclipticæ arcum concludentium & Alt. Poli, & quandoq. Azimutho Verticalis. *Problems 42 and 43. To find the time in which a given arc of the ecliptic passes through the prime vertical or through any other vertical, given the declination and right ascension of the endpoints of the ecliptic that enclose the arc, the altitude of the pole, and sometimes the azimuth of the vertical.*

[Margin: Problem 42.]

If the given arc of the ecliptic has one or both of its endpoints in the southern semicircle of the ecliptic, it never passes whole through the prime vertical for an inhabitant of the north. If the arc is the whole northern semicircle, it passes through the prime vertical in 12 equal hours. If it is less than the northern semicircle, but each of its endpoints is equally distant from the beginning of Cancer — as if the arc lay between 20° of Taurus and 10° of Leo — the arc of the equator enclosed between the right ascensions of the two endpoints, converted into hours of the solar day, will give the desired time. But if it is less than the northern semicircle and its endpoints are not equally distant from the beginning of Cancer, look at the following figure, in which the meridian is HVRN; the horizon HMR; the equator AOQ; the ecliptic GOE; the prime vertical VMN, making right angles with the meridian at V and N; and let P be the north pole of the world.

[Margin: Order of the problems.]

Now let the arc FK of the ecliptic be given, whose beginning F has a greater declination than its end K: it will pass through a higher point of the vertical, say through B, than the end, which passes through the point K. The contrary would happen if the beginning of the arc had a smaller declination than the end, for it would pass through a less high point. Now let quadrants of declination be drawn through the points B and K, and let them be PBI and PKL. With this set up, *first*, in the triangle BVP, right-angled at V, investigate by [proposition] 5 of the right-angled spherical triangles the angle BPV; for there are given the base BP, which is the complement of the greater declination, and the side VP, the complement of the altitude of the pole.

[Margin: Step 1.]

Secondly, in the triangle KVP, right-angled at V, seek the angle KPV by the same [proposition] 5; for there are given the base KP, the complement of the smaller declination, and VP, as above. Then subtract the smaller of the angles found from the greater; there will remain the angle BPK, measuring the arc IL of the equator, which is called the equation or prosthaphaeresis.

[Margin: Step 2.]

Thirdly, subtract the smaller of the right ascensions of the given endpoints of the ecliptic from the greater, and the remainder will be an arc of the equator, to be held in mind here for the time being. From this arc, then, subtract the prosthaphaeresis already mentioned if the beginning of the arc is more northerly, that is, has the greater declination; but add it if the beginning has a smaller declination than the end. What remains or results, converted into hours and minutes of the solar day, will give the time required for the arc already mentioned to pass through the prime vertical.

[Margin: Step 3.]

But if the vertical is not the prime vertical, there must be given, besides what has been said, the azimuth HVM, and from it the angle MVP, which is the supplement of the azimuth to two right angles. With this angle, with VP, the complement of the altitude of the pole, and with BP, the complement of the greater declination, find in the triangle BVP the angle BPV; and in the triangle KVP seek, with KP, the complement of the smaller declination, the angle KPV, by [proposition] 3 of the oblique-angled spherical triangles. Having subtracted the smaller of the angles found from the greater, complete the rest as in the third step of problem 42.

[Margin: Problem 43.]

Diagram. A circle set at the right of the right column, at its head. HR runs across and a vertical runs from V to N; lines run from A through O to Q, from G through F, L and M towards E, from P through B to I, and from P through K towards L.

- V (top of the rim), N (foot): on the meridian HVRN and the prime vertical VMN
- G (upper left of the rim), E (lower right of the rim): the ecliptic GOE
- A (left of the rim, high), Q (outside, lower right): the equator AOQ
- H, R (left and right of the rim): the horizon
- P (right of the rim, high): the north pole of the world
- F (inside, below G): the beginning of the given arc FK
- K (inside, right of L): its end
- B (inside, right of F): where F crosses the vertical
- I, L (inside, left): ends of the quadrants of declination PBI and PKL
- M (inside, below L): on the prime vertical VMN
- O (inside, right of M): the intersection in GOE and AOQ

44. Probl. Tempus inter tranſitus duorum Siderum aut Punctorum Eclipticæ per Medium cæli determinare, Datis eorum Aſc. Rectis. *Problem 44. To determine the time between the transits of two stars or of two points of the ecliptic through the midheaven, given their right ascensions.*

[Margin: Problem 44.]

The difference of the right ascensions, converted into time by the tables indicated in problem 1, will give the required time; although this very interval can be discerned in the stars by observation, if the time elapsing between their

transits is counted with the help of the pendulum described in Book 2, chapter 20.

45. Probl. Temporis momentum inuenire, quo Eclipticæ punctum datum, aut Sidus quoduis à Sole diſtinctum, tranſit per Medium cæli, dato die, Datis Altitudine poli, Aſcenſione recta Puncti aut Sideris, & Solis, & Semidiurno tempore Solis. *Problem 45. To find the moment of time at which a given point of the ecliptic, or any star other than the Sun, passes through the midheaven on a given day, given the altitude of the pole, the right ascension of the point or star and of the Sun, and the semidiurnal time of the Sun.*

[Margin: Problem 45.]

Subtract the right ascension of the Sun from the right ascension of the point or star (taking 360 degrees for the subtraction, if need be), and the remainder, converted into time, will give the time of the transit already mentioned, counted from the preceding noon. Now if this time is less

[Margin: Order of the problems.]

than the semidiurnal time, or greater than its complement to 24 hours, the point or star will arrive at the midheaven by day; but if the time already mentioned is greater than the semidiurnal time, or less than its complement to 24 hours, it will arrive by night.

46. Probl. Diem, quo Stella cum Sol[?] Cælum Mediat, ſeu tranſit cum illo Meridianum, inuenire. *Problem 46. To find the day on which a star mediates the heavens with the Sun [?], that is, passes through the meridian with it.*

[Margin: Problem 46.]

Seek, by problems 126 to 130 of Section 2, or by the tables indicated there, the mediation of the heavens, that is, the point of the ecliptic with which the star culminates. Then from astronomical tables or an ephemeris find the day on which the Sun will be in that degree of the ecliptic; for on that day the star will pass through the meridian with the Sun.

47. Probl. Temporis momentum datæ diei inuenire, quo Stella aliqua vel punctum Eclipticæ abſolutè oritur aut occidit; Datis Aſcenſione Recta Solis & Aſcenſione Obliqua puncti aut ſtellæ, & Semidiurno tempore Solis. *Problem 47. To find the moment of time on a given day at which a star or a point of the ecliptic rises or sets absolutely, given the right ascension of the Sun, the oblique ascension of the point or star, and the semidiurnal time of the Sun.*

[Margin: Problem 47.]

We call absolute rising the ascension above the horizon, and absolute setting the descension, without any regard to the moment of the Sun rising or setting together with the star, according to what was said in Book 6, chapter 21. From the oblique ascension of the point or star, then — given, or found by problem 123 of Section 2 — subtract the sum of 90 degrees and the right ascension of the Sun; and the remainder, converted into solar time, will give the time,

reckoned from the preceding noon, at which the point or star will rise within the given day. For the setting, seek the oblique ascension of the point of the ecliptic diametrically opposite the given point in which the star is, or in which its longitude is determined, and from it subtract the sum of 90 degrees and the right ascension of the Sun: the remainder, converted into hours, will give the moment of the rising of the opposite point, that is, of the setting of the proposed point or star. Further, if the remainder already mentioned is less than the Sun's semidiurnal time, or greater than the complement of the semidiurnal time to 24 hours, the rising or setting will be by day; but if it is greater than the Sun's semidiurnal time, but not greater than the complement of the semidiurnal time to 24 hours, the rising or setting will be by night — according as you have solved the problem for the rising or for the setting.

48. & 49. Probl. Idem inueſtigare duobus alijs modis. *Problems 48 and 49. To investigate the same in two other ways.*

[Margin: Problems 48 and 49.]

I set out these two methods earlier, in Book 6, chapter 20, problem 7, and we illustrated it with an example; so turn there.

50. Probl. Tempus inuenire, quod intercedit inter duorum Siderum aut punctorum Ortus Occaſuſue abſolutos. *Problem 50. To find the time that elapses between the absolute risings or settings of two stars or points.*

[Margin: Problem 50.]

For the time between two risings of points or stars within the same natural day, gather their oblique ascensions by problem 123 of Section 2; for [?] the time between settings, their oblique descensions. Now subtract the smaller from the greater, or the earlier from the later, of the oblique ascensions; the remainder, converted into time, will be the time from the rising of the point or star whose oblique ascension is smaller to the rising of the point or star whose oblique ascension is greater. But if you subtract that same time from 24 hours, there will remain the time from the rising of the point or star whose oblique ascension is greater to the rising of the one whose oblique ascension is smaller. What I have said of risings holds for settings, if you use oblique descensions. And if you use an oblique ascension on the one side and an oblique descension on the other, you will have the time between the rising of one and the setting of the other; for which see also the rules given in Book 6, chapter 21, page 467.

[Margin: Order of the problems.]

51. Probl. Diem Ortus & Occaſus reſpectiui, ſed Veri ſiue Matutini ſiue Veſpertini, cuiuſuis ſtellæ inuenire. *Problem 51. To find the day of the respective rising and setting of any star — but the true one, whether morning or evening.*

[Margin: Problem 51.]

See Book 6, chapter 20, problem 6, where there are also examples; for the definitions of these risings and settings, see chapter 21 in the same place.

52. Probl. Tempus inuenire, quo Stella aliqua olim verè Oriebatur vel Occidebat. *Problem 52. To find the time at which some star formerly truly rose or set.*

[Margin: Problem 52.]

Seek the sense of the problem, and its solution, in Book 6, chapter 20, problem 8.

53. Probl. Arcum Viſionis ſeu Fulſionis & interuallum Eclipticæ pro Ortu & Occaſu Heliaco ſiderum determinare. *Problem 53. To determine the arc of vision, or of shining, and the interval of the ecliptic, for the heliacal rising and setting of the stars.*

[Margin: Problem 53.]

See Book 6, chapter 20, problem 9, and the first part of that problem.

54. Probl. Diem Ortus & Occaſus Heliaci ſeu apparentis, cuiuſuis Sideris determinare. *Problem 54. To determine the day of the heliacal, or apparent, rising and setting of any star.*

[Margin: Problem 54.]

See Book 6, chapter 20, the whole of problem 9, where I gave an example in Sirius and in Procyon; for an understanding of the definitions, consult chapter 21 in the same place. And you have examples of the risings and settings in the ancient poets or historians explained in the same place, in the whole of chapter 22.

55. Probl. Tempus inuenire quo ſtella perueniet ad Aequatorem, aut Tropicum, aut etiam illum tranſgre[?]ietur, ſi ſit capax talis aduentus. *Problem 55. To find the time at which a star will reach the equator or a tropic, or even pass beyond [?] it, if it is capable of such an arrival.*

[Margin: Problem 55.]

See Book 6, chapter 19, problem 1, where it is also taught which star is capable of the arrival already mentioned.

56. Probl. Tempus inuenire, quo ſtella capax tranſitus per Verticem dati loci, tranſitura ſit per illum. *Problem 56. To find the time at which a star capable of a transit through the zenith of a given place will pass through it.*

[Margin: Problem 56.]

In Book 6, chapter 19, problems 2 and 3, it is taught which stars are capable of a transit through the zenith of a given place, and in problem 4 the time at which it has passed or will pass through the zenith.

57. Probl. Tempus inuenire, quo ſtella oriens & occidens, euaſura ſit inter Semper Apparentes, aut Latentes, & inter eas duratura. *Problem 57. To find*

the time at which a star that rises and sets will pass among the stars that are always visible, or always hidden, and will remain among them.

[Margin: Problem 57.]

See Book 1, chapter 17, and then Book 6, chapter 19, problems 6 and 7; for from these you will gather a sufficient way of solving this problem.

58. Probl. Tempus inuenire, quo ſtella capax perueniendi ad Mundi Polum ſit illuc peruentura, aut iam peruenerit. *Problem 58. To find the time at which a star capable of reaching the pole of the world will reach it, or has already reached it.*

[Margin: Problem 58.]

Consult Book 6, chapter 19, problem 5, where we also teach which star is capable of such an arrival.

59. Probl. Tempus indagare, quo ſtella idonea peruentura ſit ad circulum Polarem, ſiue intra illum ſit, ſiue extra. *Problem 59. To find out the time at which a suitable star will reach the polar circle, whether it is inside it or outside.*

[Margin: Order of the problems.]

[Margin: Problem 59.]

In Book 6, chapter 19, problem 11, we solved this problem on the occasion of the following one.

60. Probl. Tempus inuenire, quo Stellæ ad id idoneæ Aſcenſio Recta creſcet, & quo decreſcet; quamuis ſemper Longitudo creſcat: aut creuit decreuitq. olim. *Problem 60. To find the time at which the right ascension of a star suited to this will increase, and at which it will decrease, although its longitude always increases; or at which it increased and decreased in the past.*

[Margin: Order of the problems.]

[Margin: Problem 60.]

We have already taught this new problem in Book 6, chapter 19, problem 11, with examples given.

SECTIO QVARTA IN QVA PROBLEMATA GEOGRAPHICA.

Section Four, in which are the geographical problems.

1. Probl. Proportionem Paralleli ad Maximum ſphæræ circulum inuenire, & Tabulam ipſius condere, Data Paralleli Declinatione. *Problem 1. To find the proportion of a parallel to a great circle of the sphere, and to construct a table of it, given the declination of the parallel.*

[Margin: Problem 1.]

Look at the following figure. In it let ABC be the terrestrial meridian, described about the Earth's centre D; let AEC be the semicircle of the equator, whose pole is B; and let AD be the sine of the left-hand quadrant AE of the equator, falling perpendicularly on the axis BD of the equator. Further, let TGH

be a parallel to the equator, whose quadrant is TG and whose semidiameter is TI. Now since the axis DB passes through the centre D of the great circle AEC and through the centre I of the parallel TGH, and stands on the plane of the parallel at right angles, making the angles TIB and TID right angles, TI will be the right sine of the arc TB; and TB is the complement of the arc AT, which is known from the declination of the parallel. Moreover, the circumferences of circles are to one another as their semidiameters are to one another, by theorem 5 of book 11 of the *Mathematical Collections* of Pappus of Alexandria. Therefore the semicircumference AEC of the equator will be to the semicircumference TGH of the parallel as the semidiameter AD is to the semidiameter TI. But AD is the radius, or whole sine, of the quadrant AE; and TI is the right sine of the arc TB, and also the whole sine of the quadrant TG. Let it be done, therefore: as the whole sine AD, of 100,000 parts, is to the sine of the arc TB, so is the quadrant AE of the great circle, of 90 degrees, to the arc TG, in parts of which AE is 90.

Diagram. A half-ellipse set in the left column beside the text: the meridian runs from A at the left over T, B and H to C at the right, and the lower curve runs back under the base through E. A vertical runs down from B through I and G to D; the horizontal AD forms the base line; TI runs across from T to the vertical; a shaded arc runs from I to H.

- B (top): pole of the equator
- T (upper left): end of the parallel's semidiameter TI
- I (on the vertical, under B): centre of the parallel
- H (upper right): point of the parallel
- G (on the vertical, below I)
- A (left end of the base)
- D (right end of the base, under G): the Earth's centre
- C (right)
- E (bottom, below the base)

EXEMPLVM. *Example.*

Let the parallel TGH decline from the equator AEC by the arc AT, of 63 degrees; its complement TB will then be 27 degrees, whose sine TI is 45,399 parts. Let it be done, then: as AD, of 100,000 parts, is to TI, of 45,399 parts, so is the quadrant AE, of 90 degrees, to another; and there will come out the arc TG, that is, the quadrant TH of the parallel, of 40° 51′ 33″. Therefore, although TG is of 90 degrees with respect to the parallel, yet with respect to the equator it contains only 40° 51′ 33″ of degrees as large as the degrees of the equator are. In other words, if TG were transferred to AE, it would not equal AE, but of the 90 equal parts of AE it would equal only 40° 51′ 33″ parts. Granted that Moletius and Vernerus reckon 40° 53′ 3″.

By this device a table is constructed for the single degrees of declination by which the parallels stand off from the equator, showing the proportion of the quadrant of a parallel to the quadrant of a great circle — such a table as Iosephus

Moletius [Giuseppe Moletti] and Io. Vernerus [Johannes Werner] have left us on chapter 23 of Ptolemy's *Geography*, page 64, and Orontius Finaeus in Book 5 of his *Cosmography*, chapter 1.

[Margin: The authors of these tables.]

To this table it is usual to append a second, containing the proportion of one degree and minute included in any parallel to the degree and minute of a great circle. It is made by multiplying the degrees of the great circle found in the quadrant of the parallel by 60 and dividing the sum by 90. As in the preceding example: if one makes, as 90 is to 40° 51′ 33″, so 60 to another, one will find that one degree of the parallel already mentioned contains 27′ 15″ 24‴ of a degree of the great circle. Such a table is given by Vernerus, Moletius and Orontius in the places cited a little earlier, and also by Egnatius Dantes [Egnazio Danti] in the *Astrolabe*, part 2, chapter 37, and by Io. Schonerus [Johannes Schöner] in his booklet *On the Distances of Places*.

A third table, finally, ought to be attached to these, containing the miles included in one degree or one minute of a parallel, of the kind of which one degree of a great circle, or one minute of it, consists, according to the opinion or view chosen on the size of the terrestrial globe that we indicated in Book 2, chapter 7.

[Margin: 3. Table.]

Præambulum ad Problemata de Magnitudine Orbis Terræ. *Preamble to the problems on the size of the globe of the Earth.*

The size of the globe of the Earth — that is, of the globe compacted of water and earth, which it has been thought fit to call the Terraqueous globe — is investigated in two general ways. The first is through the investigation of the diameter. For once the quantity of the terrestrial diameter has been found in stades, leagues or known miles, if one makes, as 7 to 22, or (which is better) as 100 to 314, so the diameter found to another, there will come out the number of stades, leagues or miles comprised in the circumference of a great circle of the Earth, such as the meridian, the equator, and any vertical circle, or any circle drawn through the poles of the world and described on the surface of the Earth. The second way proceeds from the investigation of the stades, leagues or miles comprised in one or a few degrees of a great circle of the Earth to the quantity of the whole circumference. Once that is known, if one makes, as 22 to 7, or rather as 314 to 100, so the stades, leagues or miles comprised in the circumference to another, the stades, leagues or miles included in the diameter of the terrestrial globe will be known. And once the diameter and circumference are known, how the area of a great circle, the whole surface and the solidity of the terrestrial globe are found is clear from what was said in Book 1, chapters 4 and 6, and from the example given in Book 2, chapter 7, number 8.

2. Probl. Diametri Terreſtris Quantitatem Inueſtigare Methodo Maurolyci, non tamen omnino exacta. *Problem 2. To investigate the quantity of the terrestrial diameter by the method of Maurolycus — not, however, an entirely exact one.*

[Margin: Problem 2.]

Franciscus Maurolycus, Abbot of Messina, in dialogue 3 of his *Cosmography*, proposes roughly this method, [Margin: Order of the problems.]

[Margin: Order of the problems.]

and from him Silvius Bellus, in *De Distantiis*, part 3, proposition 12, though somewhat differently. In the accompanying diagram, let BCD be a great circle of the terrestrial globe — say, a vertical — and on it a very high mountain AB, such as he takes Etna to be, from which there is a view out over the sea along the visual ray AC, touching the terraqueous globe at C. Then let the perpendicular height AB of the mountain above the level of the sea be found by hypsometry [Greek: ὑψομετρίαν (hypsometrian, "measurement of heights")], that is, the measurement of heights; and by embadometry [Greek: ἐμβαδομετρίαν (embadometrian, "measurement of ground")], or the measurement of the ground, the length BC as far as the limit of vision. This length cannot be so great that it could not be taken, without danger of notable error, for the straight line CE, perpendicular to AG. Given, then, in the triangle AEC, right-angled at E, the sides CE and AB (that is, approximately AE), if each is multiplied by itself, the squares of the sides CE and AE will result; their sum, by proposition 47 of the first book of Euclid, is equal to the square of the subtending side AC, and its square root will give the quantity of AC itself. But by the penultimate proposition of the third book of Euclid's *Elements*, the square of the line AC is equal to the rectangle made by the whole DA and the external straight line AB; hence this rectangle will be known. And AB, the height of the mountain, is known; therefore the whole AD will be known, and if you subtract AB from it, the diameter BD of the Earth becomes known.

Diagram. A circle set at the right of the left column. A vertical runs from A above the circle through B, E and G to D; a line runs from A down to C, where it touches the circle; EC runs across from the vertical to C.

- A (top, above the circle): summit of the mountain
- B (top of the circle): foot of the mountain
- E (on the vertical, just below B)
- C (right): the point of tangency
- G (centre)
- D (foot of the vertical)

Willebrord Snel disapproves of this method in book 2 of the *Eratosthenes Batavus*, chapter 17, because refraction visibly raises the limit C for the observer at A. Nor can this evil be met if sailors, on coming to C, begin to see the summit A of the mountain and note the limit and the distance CB, as Biancani advises in book 4 of the *Sphaera*, treatise 3, chapter 5. I, rather, do not altogether approve the method: both because it is very difficult to know the height of the

mountain exactly without prior knowledge of the Earth's semidiameter; and because, with BC, a curve, assumed for CE, and AB for AE, the problem limps on both feet in geometry and is liable to an error not to be despised, as I have shown by an example in my Geographical book — especially in investigating the height of mountains, and in the slope of the mountain and of the plain lying below it down to the sea. Nor is it approved by Clavius himself, in book 8 of the *Geometria Practica*, proposition 34. Silvius Bellus, moreover, is mistaken in thinking that Maurolycus orders AC to be measured; for Maurolycus plainly wants the curve BC to be measured, which he then orders to be taken for a straight line, just as if it were EC.

[Margin: The error of Silvius Bellus.]

3. Piobl. Diametrum Terræ inueſtigare methodo P. Chiſtophori Griembergeri Societatis IESV. *Problem 3. To investigate the diameter of the Earth by the method of Father Chistophorus [?] Griembergerus, of the Society of Jesus.*

[Margin: Problem 3.]

Father Griembergerus, with German candour, communicated this invention of his to Father Mario Bettini, who published it in *Apiarium* 2, *Progymnasma* 3, proposition 7, scholium 2. Reduced to a few words, it runs as follows.

[Margin: Step 1. A suitable construction of a wall on the shore.]

First, on a shore of the sea running straight for up to 40 or 50 miles, let a wall ABMN be built, proceeding in the plane of the same vertical circle; this can be done by skilful architects and builders.

Diagram. A long masonry wall standing on the waves, with a water-line ruled beneath. At each end stands a gabled level, each lettered E, with a plumb-line hanging inside. Hatched rules lie along the top of the wall: the lower course lettered A, C, D, I, K and M from left to right, and a raised course above it lettered G at the left, F and H in the middle, and L at the right. B stands at the lower left corner of the wall and N at the lower right.

- E (left gable): the level
- G (left, raised course): end of the rule GF
- F (middle, raised course): end of the rule GF
- H (middle, raised course): end of the rule HL
- L (right, raised course): end of the rule HL
- E (right gable): the level
- A (upper left corner of the wall)
- C (top course, left)
- D (top course)
- I (top course)
- K (top course, right)
- M (upper right corner of the wall)
- B (lower left corner, at the water)
- N (lower right corner, at the water)

Let this wall stand above the shore, when the sea is at its calmest, so that AB is one or two palms. On the wall let a rule AC be laid which, by means of the level E, is exactly balanced to the horizon, and along its upper side let a line AD be marked to serve as the fiducial line. Then lay on it a rule GF which over a large part of itself fits against and touches AC itself, and let the wall go on being built in such a way that between the rule GF and the wall there can be inserted, without any gap, a rule DI of the same thickness as AC. On this in turn let a rule HL be laid, fitting against and touching DI itself over a large part of itself, and let the wall proceed so that between DI and the rule HL there can be inserted, without a gap, a rule KM equal in thickness to DI. And so, by means of three or four rules with a perfectly straight edge, the same straight line will be continued through the whole length AM of the wall. Let this length be measured exactly with ten-foot rods, and the measure reduced to thousandths, or at least hundredths, of one palm. Further, if this wall runs on for about 50 miles over the shore, equally high with respect to the surface of the calm sea, it will be found higher at its end MN, etc., than at its beginning AB; and the level E, placed on the last rule KM, will incline its plumb-line somewhat to the left, so that it tends towards the centre of the Earth.

[Margin: Step 2.]

With the wall built, now use, *secondly*, the following figure. In it let BOPG be the vertical circle on the surface of the Earth in whose plane the wall lies, representing a great circle of the Earth, and let the centre of the Earth be C. Through C let the diameter BP be drawn, continued with the initial and perpendicular height AB of the wall. Let the wall be ABMN, whose length AM is known; and in it let BN be understood to be drawn from B, parallel and equal to AM, touching the globe of the Earth at B and making there, with the surface of the sea, the right angle ABN. Now from M let the straight line MN be drawn, equal to AB and making, by means of an exact square, the right angle AMN; and let the point N be marked on the wall. Through N and through the centre C let the diameter GCL be drawn, continued through N to Q, the point that terminates AM continued as far as Q. And from N, letting down a plumb-line, let the straight line LN be measured.

Diagram. A circle set at the right of the right column. Above it A, M and Q stand on a horizontal; BN is a horizontal touching the circle at B; a vertical runs from A through B and C to P at the foot of the circle; a diameter runs from G at the lower left through the centre C and L, past O on the circle, through N to Q; MN joins M to N.

- A (upper left): top of the wall's height AB
- M (top): end of the wall AM
- Q (upper right): where GN continued meets AM continued
- B (top of the circle): point of tangency of BN
- N (right of B): end of the tangent BN
- L (near the circle, upper right): point of the diameter GCL
- O (on the circle, upper right)

- C (centre)
- G (lower left, on the circle)
- P (foot of the circle)

[Margin: Step 3.]

Thirdly, multiply the parts of the tangent BN, which is known from AM, by themselves, and you will have its square. By proposition 36 of the third book of Euclid, the rectangle under the whole GN and the external segment LN, terminated by the tangent, is equal to that square. Therefore, the square of BN being known, the rectangle already mentioned will be known; and since its side LN is known in parts of the same kind, and so is the square of LN, if you divide the square of the tangent BN by the square of LN, the quotient will show how many times LN is contained in the whole GN. Or, if you divide the square of BN by the line LN, the whole GN will become known; and with LN taken from it, the diameter GL of the Earth will be known.

So, most ingeniously, Griembergerus. But it is extremely difficult for the thickness of the rules to be so equal that one of them does not fall short by as much as a hundredth of an inch. So if any rule is 10 feet long, and at each application falls short of straightness by one hundredth of an inch, then after 1200 applications, that is, after 2400 paces, it will fall short by one foot; and after 50,000 paces, by 20 feet and more. But grant that the workmen have the eyes of a lynx in making and applying the rules: this wall will be so costly that it is not worth undertaking. For LN will be only about 250 ancient Roman paces, or 1250 feet, if we suppose BN to be 50 miles, or 50,000 such paces — paces of which, from what was said in Book 2, chapter 7, there are 5,174,000 in the semidiameter CB and 10,348,000 in the diameter GL. But let there be 5,000,000 in CB, in round figures. Then from BN and CB, in the triangle CBN, right-angled at B, the angle BCN is found, by [proposition] 5 of plane right-angled triangles, to be 34′ 20″, whose secant is 100,005. Therefore by [proposition] 3 of the same triangles you will find the base CN to be 5,000,250 paces. With CL, of 5,000,000 paces, taken away, there remains, as I said, LN, of 250 paces, that is, 1250 ancient Roman feet — a height that no tower in Europe is found to have. For the measures of the taller towers, as I have collected them with great diligence, are those set down below.

[Margin: The difficulty of this practice, and the insane height of the wall at its end.]

[Margin: Order of the problems.]

[Margin: Order of the problems.]

[Margin: The height of some notable towers.]

Height of the towers	Feet	Ancient Roman
Modena	Feet of Modena 175	262
Bologna, the Asinelli	Feet of Bologna 234	312
Cremona	Feet of Cremona 332	362
Genoa, the lantern with its rock	Palms of Genoa 440	480

Height of the towers	Feet	Ancient Roman
Strasbourg	Feet of Strasbourg 765	705

Who, though, could safely raise so great a height of wall over the shore of the sea without its tottering? Or at what expense, I ask, would so insane a substructure stand? It would be better to build a single tower at least 500 feet high, which could serve both this very purpose and other uses.

4. Probl. Diametrum Terræ indagare Methodo P. Marij Bettini S. I. sed fallaci. *Problem 4. To find the diameter of the Earth by the method of Father Mario Bettini, S.J. — but a fallacious one.*

[Margin: Problem 4.]

Father Bettini, in *Apiarium* 2, *Progymnasma* 3, proposition 7, proposes his own method, which he prefers to the way of Maurolycus and Eratosthenes. "In the following diagram," he says, "let there be the globe of the Earth and its great circle BMEK, and its diameter BE, on which let a tower, or a rock, or a mountain AB, of known height, stand perpendicularly. From its foot B let the visual line BC be stretched out by a dioptra to the edge C of the sensible horizon, so that by means of a square it makes with AB the angle ABC a right angle. Now the moderate reach of vision over the plane of the horizon, namely BC, is 35 Italian miles" (he says). Now about the centre Q, through the end-point C, let the semicircle ACF be described, and let AE be continued as far as F, so that AF is the diameter of this semicircle; and let C be joined to its extremities by the straight lines AC and FC. For by proposition 31 of the third book of Euclid the angle ACF will be a right angle, and the perpendicular BC let fall from C will, by proposition 13 and by the corollary of proposition 8 of the sixth book of the *Elements*, be a mean proportional between AB, which is known, and BF. Let BC, then, which is 35,000 paces, be multiplied by itself, and the sum will be 1,225,000,000. When this is divided by the paces contained in AB, the quantity of the straight line BF will come out. From this, finally, let EF, which is equal to AB, be subtracted, and the diameter BE of the Earth will remain.

Diagram. Two concentric circles set at the right of the left column, the inner one drawn as a globe with outlines of land. A vertical runs from A at the top of the outer circle through B at the top of the inner circle, the centre Q, and E at the foot of the inner circle, to F at the foot of the outer circle. C stands on the outer circle at the right; lines run from A to C, from B to C and from F up to C. P lies on a line inside, running towards C; K stands at the left of the inner circle and M at its right.

- A (top of the outer circle)
- B (top of the inner circle): foot of the tower AB
- C (right, on the outer circle): edge of the sensible horizon
- P (inside, towards C)

- K (left of the inner circle)
- Q (centre)
- M (right of the inner circle)
- E (foot of the inner circle)
- F (foot of the outer circle)

[Margin: The fallacy of this method.]

But it is false that the common and middling distance of vision BC is 35 Italian miles. For if the height of the eye B is nothing above the convexity of the Earth, which is concentric with the sea, there is hardly any such distance; and if there is some height, the distance of vision varies with the varying height of the eye, as we have already shown in Book 2, chapter 10. There you will also see Father Bettini excused, who erred in this along with some great men — especially since he himself, in scholium 1, suggests a way, difficult indeed but not impossible, of measuring the line BC. This would be if C were the top of a rock PC, or of a ship's mast, from which the visual line BC were directed to B; and from there the angle BCQ were taken with a quadrant, whose complement to a right angle is the angle BQC, the secant of which is QC. For, the height PC being known, one would make: as the excess PC of the secant QC is to the paces included in PC, so is the tangent BC of the same angle BQC to the paces included in BC. But the tower would have to be on the shore of the sea, like the Lantern of Genoa, and one would have to sail towards it until, from C, the top of the ship's mast, its foot B came into view, and then take the angle PCB with a most exact instrument.

5. 6. & 7. Probl. Diametrum Terræ inueſtigare triplici Methodo P. Chriſtophori Clauij Soc. IESV. *Problems 5, 6 and 7. To investigate the diameter of the Earth by a threefold method of Father Christoph Clavius, of the Society of Jesus.*

Father Clavius, in Book 8 of the *Geometria Practica*, proposition 34, intending to correct the method of Maurolycus, teaches three ways of measuring the Earth's diameter; Kepler touches on two of them, from Clavius, in Book 1 of the *Epitome of Copernican Astronomy*, page 29.

[Margin: Problem 5.]

The *first* is as follows. In the following figure let ADG be a great circle of the Earth, whose diameter is AG and centre C; and let AB be a tower or mountain of known height, from whose top B the surface of the calm sea is viewed. Let the angle B be taken with an instrument by means of the visual line BD, touching the surface of the sea at D; Clavius thinks this is done most surely at the moment when the centre of the Sun, or a star, rises apparently from the horizon, or sets. For, the angle B being known, and AF drawn perpendicular to AB, there will arise the triangle ABF, right-angled at A, in which the angle BFA will be known, as the complement of the angle B to a right angle. Hence, AB being given, you will find by [proposition] 6 of plane right-angled triangles the base BF, and by [proposition] 7 the side AF, which is equal to the side FD

by the corollaries of proposition 36 of the third book of Euclid. Hence, FD being added to the base BF, the whole tangent BD will be known; and its square is equal to the rectangle comprised under the whole GB and the segment AB, by proposition 36 of the third book of Euclid. Therefore, when that square is divided by the height AB, the quotient will give the length of the whole BG; subtract AB from it, and there will remain the diameter AG of the Earth, known in parts of which AB was known.

Diagram. A circle set at the right of the right column. A vertical runs from B above the circle through A at its top and the centre C to G at its foot. A horizontal runs from A through F to E at the right; a line runs from B through F to D, where it touches the circle; a dotted line runs from C up towards F, and a line joins C to D.

- B (above the circle): top of the tower or mountain AB
- A (top of the circle): its foot
- F (on the horizontal AE, where BD crosses it)
- E (right end of the horizontal)
- D (on the circle): point of contact of BD
- C (centre)
- G (foot of the circle)

[Margin: Problem 6.]

Secondly, another way. With the height AB known, and the angle B known by the dioptra, and a straight line CD drawn from the Earth's centre through the point of contact D, there will arise the triangle BCD, right-angled at the contact D, by proposition 18 of the third book of Euclid; in it the angle BCD will be known, as the complement of the angle B to the other right angle, by proposition 32 of the first book of the *Elements*. Now let there be another line AE, touching the Earth at A, equal to BD and cutting BD at F; and from C let a straight line be understood to be drawn to F. Then AF will be equal to FD by the corollaries of proposition 36 of the third book of Euclid. Hence in the triangles ACF and DCF, right-angled at the contacts A and D, the sides containing the right angles — namely AC and CD, and AF and DF — will be equal; and therefore, by proposition 4 of the first book of the *Elements*, the angle ACF will be equal to the angle DCF, and known, as being half of the angle BCD already found above. With these known beforehand: *first*, in the triangle ABF, right-angled at A, the height AB is given, and the acute angle F, namely the complement of the angle B; therefore by [proposition] 7 of plane right-angled triangles, seek [?] the side AF. *Secondly*, in the triangle ACF, right-angled at A, AF is given and the opposite angle ACF; therefore by the same [proposition] 7 seek the side AC, that is, the semidiameter of the Earth, whose double gives the diameter AG.

[Margin: Step 1.]

[Margin: Step 2.]

[Margin: Problem 7.]

Thirdly, another way. With the height AB of the mountain or tower known, and the angle B as above, the angle BCD will be known, as being the complement of the angle B to a right angle. Then for the angle BCD find the secant CB (since its tangent is BD); from this secant cut off the whole sine AC, and there will remain the excess AB of the secant over the whole sine. Let it be done, therefore: as this excess is to the paces or feet of the known height AB, so is the whole sine, or radius, AC to another; and there will come out the semidiameter AC of the Earth, known in paces or feet.

8. Probl. Diametrum Terræ inuenire Methodo P. Pauli Cafati Soc. IESV. *Problem 8. To find the diameter of the Earth by the method of Father Paulus Casatus [Paolo Casati], of the Society of Jesus.*

[Margin: Problem 8.]

Father Paulus Casatus, while reading through our studies on the size of the Earth, at once found this method, with the skill in geometrical matters that he possesses. I have already reported it in Book 2, chapter 7, number 4, and, drawn together into fewer words, it is as follows. In the figure set immediately before, and already explained, the height AB of a tower situated at the sea is supposed known; and it is supposed that, a ship at sea or some other visible sign having been viewed from B at its base D, the angle B has been taken, from which its complement to a right angle, that is, the angle BFA, is known. In the triangle right-angled at A, therefore, let AF and BF be found, in parts of which AB is known; and since AF is equal to FD, as the same man demonstrated in the same place, if AF, that is FD, is added to BF, there will be known

[Margin: Order of the problems.]

[Margin: Order of the problems.]

the tangent BD, in the triangle CDB, right-angled at the contact D; and since the angle B is given in this triangle, the side CD — that is, the semidiameter of the Earth — can at once be found by [proposition] 4 of plane right-angled triangles. One could also seek the tangent of the acute angle C, which is the complement of the angle B to a right angle, and make: as BD, the tangent, is to the same BD known in paces or feet, so is the whole sine AC to the semidiameter of the Earth in paces or feet.

9. Probl. Diametrum Terræ inquirere Methodo Kepleriana. *Problem 9. To investigate the diameter of the Earth by Kepler's method.*

[Margin: Problem 9.]

Johannes Kepler, in Book 1 of the *Epitome of Copernican Astronomy*, page 28: "Since the Earth," he says, "is round, plumb-lines hung in distant places must nod towards each other, and towers built to the perpendicular must nod away from each other at their tops and stand farther apart from each other than at the bottom." In the following figure, then, above the surface of the Earth DFBE, let there be two mountains AB and MF, or towers, or citadels, very distant from each other, yet such that the summit of one can be seen from

the other. From A, along the visual line AM, let M be sighted and the angle MAB observed; again, from M let A be sighted and the angle FMA taken. For when AB and MF are produced to C, the centre of the Earth, the angle C will be known, as the supplement of the angles A and M, taken together, to two right angles. Now let the distance between the places, namely FB, be measured in paces or miles, if it is not known from elsewhere. For in this way, in the isosceles triangle FCB, the base FB will be given, and the angle C, and either of the angles at the base, as half of the supplement of the angle C to two right angles. Therefore, given the two angles C and CFB, with the opposite side FB, you will find by [proposition] 2 of plane oblique-angled triangles the side CB, opposite the angle F, that is, the semidiameter of the Earth; and when it is doubled, the whole diameter of the Earth will be known.

Diagram. A small figure set at the right of the left column. M at the upper left and A at the upper right are joined by a straight line, and a second line runs from A down towards G. An arc, the surface of the Earth, runs from D at the left through F and B to E at the right. Lines from M through G and F, and from A through B, converge below on C.

- M (upper left): summit of the mountain MF
- A (upper right): summit of the mountain AB
- G (below M, on the line MF)
- D (left end of the arc)
- F (on the arc): foot of MF
- B (on the arc): foot of AB
- E (right end of the arc)
- C (foot of the figure): centre of the Earth

10. Probl. Diametrum Terræ inquirere Methodo P. Nicolai Cabæi Soc. IESV, & D. Alphonſi Iſæi. *Problem 10. To investigate the diameter of the Earth by the method of Father Nicolaus Cabaeus [Niccolò Cabeo], of the Society of Jesus, and of D. Alphonsus Isaeus.*

[Margin: Problem 10.]

Each of these two (which is nothing new), unaware of the other's discovery, once pointed out to me the method I shall set out below; and Father Cabeo later presented it in Book 1 on the *Meteorologica*, text 7, question 1. Let there be a hemisphere of the Earth, with its great semicircle ABH and its diameter AH; and let there be a tower on the seashore, or a ship's mast, MP, whose perpendicular line is produced to the Earth as far as C, so that the whole is PMC. From another place on the sea, or on a seashore raised hardly at all above the sea — say from the place F — let the point G of the tower or mast be marked along a line parallel to the horizon, or lying most exactly at the level of the horizon, namely FG. This will be done more surely by means of a lamp placed at G and moved up or down by another person until it can be observed from F along the horizontal line FG. Then at F let a pole or small beam, or another ship's mast, BK, be erected perpendicularly from B; and, with the eye moved

to G, let another point or sign K be marked along another horizontal GK, which makes a right angle at G with CP. Let its distance FK be measured most exactly in equal small parts, going down to inches, or even to small parts of an inch (although neither Cabeo nor Isaeus states this); and let FB likewise be measured, if F is raised above the shore or the surface of the sea; and then the straight line FG, either with stretched cords or by trigonometry, taking the angle K and using the side KF in the triangle KFG, right-angled at F. Finally, when KB is produced to C, there will arise the triangle KGC, right-angled at G; from which FG, the perpendicular let fall and now known, will be a mean proportional between KF and FC. Let it be done, therefore: as KF is to FG, so FG is to FC; and let that small height FB, which will be at least as great as the observer's stature, be subtracted from FC; and BC will remain, that is, the semidiameter of the Earth, known in the parts already mentioned.

Diagram. A figure set at the right of the left column. A semicircle stands on the base AH, with C, its centre, on the base. A vertical runs from C up through B, the top of the semicircle, and F to K. A horizontal runs from F to G, and a line runs from K down to G. A line runs from C through M on the semicircle and through G to P at the upper right.

- K (top of the vertical): the sign on the pole BK
- P (upper right): top of the mast MP
- F (on the vertical above B): the observer's place
- G (on the line CP): the point marked on the tower or mast
- B (top of the semicircle)
- M (on the semicircle): foot of the mast MP
- A (left end of the base)
- C (centre, on the base): centre of the Earth
- H (right end of the base)

[Margin: The fallaciousness or slipperiness of this method and of the preceding ones.]

But if, because of the distance FG, the sign K is not a certain point, and an error of one inch of a foot is committed which cannot be discerned, an error of more than 300 Italian miles could be committed in the semidiameter of the Earth, as I demonstrated in my book on geography. And I say the same of all the preceding problems, from 2 to this 10, if the angles taken deviate from the truth by even half a minute. It therefore seems better to take refuge in those problems by which the arc of the Earth is investigated, and from it the circumference and the diameter.

11. Probl. Terræ Ambitum, Eratoſthenis modo Inueſtigare, & ipſius Errores corrigere. *Problem 11. To investigate the circumference of the Earth by the method of Eratosthenes, and to correct his errors.*

[Margin: Problem 11.]

Eratosthenes of Cyrene — the man famous for the duplication of the cube, the measurer of the pyramids of Egypt, prefect of the royal library of Alexandria,

and, to use the words of Pliny the Elder, Book 2, chapter 108, "subtle in every branch of letters, and in this one surely skilled beyond the rest," namely in the measurement of the Earth — seemed to have accomplished it in such a way that Pliny, in the same place, calls it a presumptuous venture, but one grasped by reasoning so subtle "that one would be ashamed not to believe it." For he noticed, or learned by report, that at Syene at noon on the summer solstice the Sun lit up the bottoms of wells, and that no upright shadows were then cast by gnomons set up above the horizon; from this he concluded that the Sun was then vertical to that place. He believed, moreover, the surveyors' report that 5000 stadia lay between Syene and Alexandria in Egypt (which he thought were situated under the same meridian). He therefore inquired how many degrees and minutes the arc of the meridian between Alexandria and Syene contained, by means of a gnomon set in a scaphe at Alexandria and the observation of its solstitial shadow at noon; from this he concluded that the arc was the fiftieth part of the whole circle, namely 7° 12′. Multiplying 5000 by 50, therefore, he concluded that there were 250 000 stadia in the whole circumference of the Earth, and 694⅗ in one degree. But if 700 are given to one degree, the whole circle comes to 252 000 stadia. From this two opinions have flowed concerning Eratosthenes' numbers, which I have already reported in Book 2, chapter 7, scholium 2, indicating errors committed by Eratosthenes, some of which I set out in Book 3, chapter 27, number 7, where I also examined Eratosthenes' method.

[Margin: The fallacy of this method is fourfold.]

It is not necessary, therefore, to repeat these things here again; it is enough to indicate that of the four things assumed by Eratosthenes, two are doubtful and two false. For it is false that Alexandria and Syene are under the same meridian, since according to Ptolemy's *Geography* Syene is farther east by 1½ degrees. From this difference, and the difference of the polar altitudes, a greater arc between those places is gathered than Eratosthenes gathers, so that he should have assigned to one degree of a great circle only 649 21/46 stadia, as I showed in the book on geography. It is false, moreover, that two rays of the Sun, one issuing from its centre and the other from its edge, reach the Earth sensibly parallel, as I have already shown in Book 3, chapter 27, number 7; and so he should have added to the arc of 7° 12′ the apparent semidiameter of the Sun, namely 15′ or 16′. It is doubtful, however, whether Syene lies exactly under the Tropic of Cancer, and whether the Sun at the solstitial noon stands over its zenith, since the space of the Earth within which there are then no shadows and the bottoms of wells are lit up occupies 300 stadia, on the testimony of Cleomedes, Book 1, and Strabo, Book 17. It is doubtful, moreover, whether there are 5000 stadia between Syene and Alexandria, as Cleomedes, Book 1, Strabo, Book 2, and Pliny, Book 2, chapter 73, report, adding as many stadia found between Meroë and Syene. For the same Pliny, Book 6, chapter 29, says that explorers sent by Nero found 874 miles between Syene and Meroë, that is, 6992 Roman stadia; and Pliny and Censorinus use these stadia in Eratosthenes'

measurement — although it may be more probable that Eratosthenes, or the royal surveyors, used the Alexandrian stadium. This ambiguity is confirmed by Hipparchus, who, as Pliny says, thought that 25,000 stadia should be added to Eratosthenes' circumference of the Earth, and 69 stadia to one degree.

[Margin: Eratosthenes' measurement emended by Hipparchus.]

[Margin: Order of the problems.]

[Margin: Order of the problems.]

12. Probl. Terræ Ambitum inquire modo Poſſidonij, eiuſq. Errores corrigere. *Problem 12. Investigate the circumference of the Earth by the method of Posidonius, and correct his errors.*

[Margin: Problem 12.]

Posidonius of Apamea, who flourished about the year 40 before Christ, and who is praised for his manifold learning by Cicero in the book *De natura deorum*, by Strabo, Book 16, and by Pliny, Book 7, chapter 30, believed that Rhodes and Alexandria are under the same meridian and are 5000 stadia apart; and that Canopus, the star set on the rudder of Argo, was raised in his time above the horizon of Alexandria at its meridian altitude by 7° 30′. Climbing, therefore, a certain lookout on the island of Rhodes in order to measure the arc between Rhodes and Alexandria from the meridian altitude of Canopus, he saw Canopus graze the horizon and rise not at all. He therefore concluded that that arc was 7° 30′, and so that the whole great circle of the Earth contains 240,000 stadia, and one degree 666⅔ stadia, as Cleomedes reports in Book 1 of *De mundo*, chapter 10, and Proclus at the end of the *Sphere*. In this reasoning some things are doubtful and some false. It is false that at that time the true meridian altitude of Canopus at Alexandria was 7° 30′, although Geminus also reports this in the *Elements of Astronomy*, and Pliny, Book 2, chapter 70. This is so, first, because the horizontal refraction, which the ancients left out, had to be subtracted from this altitude — a refraction which Kepler, in the *Astronomia optica*, page 149, suspects was about 3 degrees, because of the vapours of the Nile and the rains towards the Mountains of the Moon. It is so, secondly, because the difference in the altitude of the pole between Rhodes and Alexandria is only 5° 2′, if one compares what Ptolemy states in Book 2 of the *Almagest*, chapter 6, and Book 5, chapter 12, and in Book 1 of the *Geography*, chapter 11, and Book 4, chapter 5; or it is a little greater, if the altitude of the pole at Rhodes is corrected from the observation of the equinoctial shadow made there and reported by Vitruvius, Book 9, chapter 8. It is false, secondly, that Alexandria and Rhodes are under the same meridian, since Rhodes is farther west by 2°, or by 1° 45′, as is clear from ancient and recent geographical maps — although Ptolemy, in Book 5 of the *Almagest*, chapter 3, says that they lie almost under the same meridian. It is doubtful, however, whether there are 5000 stadia between these two places; for I suspect this equality and roundness of the numbers between Rhodes and Alexandria, between Alexandria and Syene, and between Syene and Meroë. Besides, if we listen to the differing opinions on the interval between Rhodes

and Alexandria reported by Strabo, Book 2, and by Pliny, Book 5, chapter 31, it is very uncertain. Finally, it is doubtful whether Posidonius observed Canopus in the city of Rhodes, or rather at a place 400 stadia away from it; or at Cnidus, where it had been observed by Eudoxus, as Strabo says in the same place; or near Lindus, with mountains lying between, as Snel suspects in Book 1 of the *Eratosthenes Batavus*, chapter 14; and whether, as often happens, that star, with the vapours of the horizon standing in the way, could not be seen before it was one or two degrees from the horizon.

13. Probl. Terræ Ambitum metiri artificio Marini Tyrij & Ptolemæi. *Problem 13. To measure the circumference of the Earth by the device of Marinus of Tyre and of Ptolemy.*

[Margin: Problem 13.]

This is done by means of the meteoroscope — that is, the instrument by which high things are observed from a low place, or lower things from a high one — indicated by Ptolemy in Book 1 of the *Geography*, chapter 3: by taking the angle of position between two places of known polar altitude, and measuring the distance between them in the same vertical. Let one place be D and the other E, and through them from the pole of the Earth, C, let the quadrants of declination, or of geographical latitude, CDA and CEB be drawn as far as the arc AB of the equator; and let the parallel of the place D be DF, and that of the place E, GE. Let the observer be at D, and let him, sighting E, take the angle of position CDE. It is now proposed to find the arc DE in the triangle CDE, in which there are given the observed angle CDE; the side CD, which is the complement of the altitude of the pole at the place D; and the side CH, the complement of the altitude of the pole at the place E. Therefore by [proposition] 2 of spherical oblique-angled triangles the arc DE can be found in degrees and minutes. Hence, if the stadia or miles contained in DE are known, and these are multiplied by 360 and the sum is divided by the degrees and minutes of the arc DE, the size of the Earth's circumference will be known. Similarly, if the place D is sighted from E, then from the angle CED so taken, and from the complements CE and CD already mentioned, DE will be found. Although Ptolemy adduces no observation of his own or of Marinus, he nevertheless hints at this method in Book 1 of the *Geography*, chapters 7 and 11, and Book 7, chapter 5, where 500 stadia were found by Marinus and by himself in one degree of a great circle of the Earth, and 180,000 in the whole circumference — figures to which not a few have subscribed, as I reported in Book 2, chapter 7, scholium 3.

Diagram. A figure set at the right of the left column: a spherical triangle. Two arcs descend from C at the apex, the left one through G and D to A, the right one through E and F to B, and an arc AB joins their feet. Parallels run across from G to E and from D to F, and a straight line joins D to E.

- C (apex): the pole of the Earth
- G (on the left arc): end of the parallel GE of the place E

- E (on the right arc): the second place
- D (on the left arc): the first place, where the observer stands
- F (on the right arc): end of the parallel DF of the place D
- A (foot of the left arc): on the equator
- B (foot of the right arc): on the equator

14. Probl. Terræ Ambitum venari, modo Almæonis, & multorum Arabum atque Coſmographorum. *Problem 14. To hunt down the circumference of the Earth by the method of Almaeon and of many Arabs and cosmographers.*

[Margin: Problem 14.]

One must explore the altitude of the pole at two places situated under the same meridian, or take at both places the meridian altitude of a fixed star on the same nights, or at any rate within a space of three months, provided that the star is high enough not to be subject to refractions. In this way the arc of the meridian lying between the two places will be established from the difference of the altitudes of the pole, or of the star. Then one must survey the distance between the places in stadiasms or miles, proceeding by a straight route, multiply the distance found by 360, and divide the sum by the degrees found in the arc already mentioned, so that the size of the Earth's circumference becomes known. This method was used by Ismael Abilfedea and other most skilled geometers at the command of Almaeon — or rather Alammon [al-Maʾmūn], king of the Arabs — proceeding along the same meridian in the plains of Zinar, or Fingar, along the straight route of the Red Sea; and they found that one degree of the terrestrial meridian contained 56⅔ miles — but miles each of which consists of 4000 cubits, as Alfraganus [al-Farghānī] relates in the *Rudiments of Astronomy*, difference 8. Why these Arabian miles make 68 ancient Italian miles, as Fernel and Maurolycus also teach, and Kepler, and who subscribed to this opinion, I have said in Book 2, chapter 7, scholium 4 — although our Clavius, in chapter 1 on the *Sphere*, page 211, and Barocius [Francesco Barozzi], in Book 1 of the *Cosmographia*, page 38, took those 56⅔ as Latin miles.

[Margin: The opinion of Adriaan Metius and of al-Battānī.]

Using this method, Adrianus Metius [Adriaan Metius], in his book on the use of the globe, says that Franeker in Friesland and Deventer are situated under the same meridian and differ in the altitude of the pole by exactly one degree, and that between them there are 15 common German miles. Another similar example, but of doubtful reliability because of the huge interval, you have in Kepler, precept 57 of the *Rudolphine Tables*; and Albategnius [al-Battānī], in chapter 6, says that 85 miles were found in one degree of a great circle of the Earth, or the journey of two days for a man travelling light.

15. Probl. Terræ Ambitum conquirere artificio Io. Fernelij. *Problem 15. To seek out the circumference of the Earth by the device of Io. Fernelius [Jean Fernel].*

[Margin: Problem 15.]

Jean Fernel, no less skilled in measuring than in healing, relates in Book 1 of the *Cosmotheoria*, chapter 1, in the scholia, that on August 25 he observed the meridian altitude of the Sun at Paris, and from it found the altitude of the pole to be 48° 38′. Then from the table of declinations of the ecliptic he gathered how great the meridian altitude of the Sun would be at a place one degree distant from Paris towards the north, and found that it would be as follows:

August, day	Altitude of the ☉
27	47° 26′
28	47° 5′
29	46° 41′

Then, having prepared a carriage with a certain wheel whose single revolutions, counted by single strokes of a drum, would show the route straight north from Paris, he arrived on the 29th at a place where the meridian altitude of the Sun was observed by him to be 46° 41′. He therefore deduced that this place was one degree distant from Paris; and, having counted the revolutions of the wheel already mentioned, he found that 68 96/1000 miles were contained in that degree. For the circumference of the wheel was 4 paces, or 20 feet, and its diameter 6 feet and 6 common digits across, and the revolutions came to 17024.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

16. Probl. Terræ Ambitum ingenio Nonij & Iacobi Mazzonij concludere. *Problem 16. To determine the circumference of the Earth by the ingenuity of Nonius and of Jacopo Mazzoni.*

[Margin: Problem 16.]

Petrus Nonius, in Book 2 of *De observatione*, chapter 18, pointed out this method, hidden from many, after rejecting Eratosthenes' method because of the paralogism indicated above; but Jacopo Mazzoni of Cesena, a man most learned in every field, set it in a better light in Book 1 of his Italian *Defence of Dante*, chapter 77, on the occasion of the passage which the poet Dante had imagined from the surface of the Earth to its centre — an example of which fiction and Greek vanity I pointed out in Dionysiodorus, in Book 2, chapter 7, scholium 9. But Mazzoni illustrated this device with no figure or demonstration. Let ABOD, then, be the terrestrial meridian, whose centre is C; and let ERF be the visible hemisphere of the Sun, with its diameter ESF and its centre S, from which at noon on the day of the solstice the ray SL descends straight onto the style AL of a gnomon; and let it be produced to C. At either end of the terrestrial space IK, within which there is then no shadow of gnomons, let there be the gnomons GI and HK, along whose length the rays from the extreme edges of the Sun pass to the centre of the Earth, namely the ray TGIC and the ray VHKC. Through the tips of the same gnomons let two other rays be drawn from the opposite edge of the Sun, striking the Earth at M and N, namely the

ray XHN and the ray ZGM. Again, let two further rays be drawn touching the Sun and the Earth, namely EB and FD; and to these let there be drawn from C through M the straight line CMP, and through N the straight line CNQ. For the arc IAK of the Earth will be the one that both lacks all shadow and is measured by the angle ICK, which at the same time measures the apparent diameter EF of the Sun, since that diameter is comprehended by the same angle TCV: for because of the huge distance of the Sun from the Earth, the points E and T, no less than F and V, can be taken as the same. Now the arc AK will be irradiated by all the points of the solar portion RV, and the arc AI by all the points of the portion RT, by the axioms of optics. So although the arc AK is not wholly lit by the points of the portion RT, the gnomon AL preventing it, nor the arc AI wholly illuminated by the points of the portion RV, and AL therefore casts a certain penumbra, it will nevertheless hardly be sensible, since the brightness of the points of the other portion overcomes it. Outside that arc, however, there will be some pure shadow cast by the gnomons GI and HK, since they are supposed to stand at the extremes and to bound the space that lacks shadow simply so called.

Diagram. A tall figure set at the right of the left column. At the top is a horizontal frame line with S at its centre, and beneath it a semicircle, the Sun's hemisphere, hangs from the upper left to the upper right, with R at its lowest point. At the upper left three letters stand one below another — E, T, and a third that is not certainly legible — and at the upper right likewise F, V, and a third not certainly legible. A vertical runs from S through R and L down to the circle. Long lines run down from the upper corners and cross before reaching two small dark gnomons, G at the left and H at the right, beside L. At the foot is a circle, the Earth, with B at the left, D at the right, C at its centre and O at its foot; on its upper arc stand I, A and K, with M at the left and N at the right; lines run from C through M to P at the left and through N to Q at the right.

- S (top centre): centre of the Sun's hemisphere
- E (upper left): end of the Sun's diameter
- T (upper left, below E): edge of the Sun
- X (upper left, below T)
- F (upper right): end of the Sun's diameter
- V (upper right, below F): edge of the Sun
- Z (upper right, below V)
- R (lowest point of the semicircle)
- G (left gnomon)
- L (between the gnomons): top of the style AL
- H (right gnomon)
- P (left, on the line CMP)
- Q (right, on the line CNQ)
- I (on the Earth's arc, left of A)
- A (top of the Earth's circle): foot of the style AL
- K (on the Earth's arc, right of A)

- M (left, on the circle)
- N (right, on the circle)
- B (left of the circle)
- C (centre of the circle): centre of the Earth
- D (right of the circle)
- O (foot of the circle)

Now let that space IAK be measured, and let the angle TCV be known from the apparent diameter of the Sun at the summer solstice; for in this way it will be known how many stadia or miles are in the minutes enclosed in the arc IAK, and accordingly how many in 60 minutes, or in one degree, and finally how many in 360°. Mazzoni, moreover, supposes that 300 stadia were found in the arc IAK around Syene and Cyrene — as Cleomedes, Book 1 of *De mundo*, chapter 10, Pausanias, and Pliny, Book 2, chapter 73, and Book 6, chapter 29, report from Eratosthenes; and from Posidonius he takes the diameter of the Sun at apogee to be the seven-hundred-and-fiftieth part of the whole circumference of the solar heaven, that is, 28′ 48″. And so he concludes that there are 625 stadia in one degree, and 225,000 in the whole circumference of the Earth. But the diameter of the Sun at apogee is somewhat larger.

17. Probl. Terræ Ambitum inueſtigare Geometriſmo Villebrordi Snellij. *Problem 17. To investigate the circumference of the Earth by the geometrism of Willebrord Snel.*

[Margin: Problem 17.]

Willebrord Snel, many years ago, published two books on the measurement of the Earth, to which he gave the title *Eratosthenes Batavus*. In them he first investigated with much diligence the measure of the ancient Roman foot, which he contends is equal to the present-day Leiden or Rhineland foot, contained twelve times in the Leiden perch. Then, as in the following figure, he chose two towers, that of Leiden, L, and that of Zoeterwoude, S, because between them there is a field spread out into a large and convenient plain. In it he drew, by staking out, the straight line LS, and another, AB, cutting it at right angles; and, having marked the stations A and B, he measured the interval AB and found it to be 87 Leiden perches, 0 [?] feet and 5 digits.

[Margin: Step 1.]

Diagram. A figure set at the right of the right column: a rhombus with B at the top, L at the left, S at the right and A at the foot; its diagonals LS and BA cross at D.

- B (top): station B
- L (left): the tower of Leiden
- D (centre): crossing of LS and AB
- S (right): the tower of Zoeterwoude
- A (foot): station A

Then, using a quadrant that showed every two minutes of each degree, from A he sighted the station B along the side of the quadrant, and through

the movable dioptra the tower now L, now S, and observed the angles BAL and BAS, which make up the angle LAS. Again from B, sighting the station A, and from there the towers L and S, he obtained the angles LBA and SBA, which make up the angle LBS. Since, then, in the triangle ABL the angles LAB and LBA were known, their sum subtracted from 180 degrees showed the angle ALB, by [proposition] 32 of the first book of the *Elements*; likewise in the triangle ABS the sum of the angles BAS and ABS, taken from 180 degrees, showed the angle ASB. Therefore in the triangle ABL, with all the angles known, and the side AB, he found the sides AL and BL by [proposition] 2 of plane oblique-angled triangles; likewise in the triangle ABS, with all the angles also known, and the side AB, he found the sides AS and BS. Afterwards in the triangle LAS, with the sides AL and AS known, and the included angle, he investigated the distance LS by [proposition] 5 of plane oblique triangles; and he confirmed it by exploring the same distance in the triangle LBS, in which the sides LB and SB were known, and the included angle LBS. Although, since LS was drawn perpendicular to AB, the same interval could have been rendered more surely by [proposition] 1 of right-angled triangles, in ADL and ADS — by finding, from the bases AS and AL already found and the angles DAS and DAL, the opposite sides DS and DL, and compounding the interval LS from them. This he finally found to be 1029 Leiden perches, 3 feet and 3 digits, and such, he found, was the distance between the tower of Leiden and the town of Zoeterwoude. This distance, acquired as a great side, he used in investigating the distances of other neighbouring towns, as we shall shortly say in the second step.

[Margin: Step 2.]

Secondly, then, having similarly taken the angles of position of Leiden and Zoeterwoude with The Hague and the neighbouring villages, and so, with three angles known as above, and the side LS, he obtained the distance between Leiden and The Hague; and by this method, with manifold geometry, he obtained the intervals between the places of Holland written below, which I have used to correct the differences of longitude in my book on geography.

Ends of the distances	Leiden perches	Leiden feet
Leiden and Zoeterwoude	1029	3 3/12
Leiden, The Hague	4103	3
The Hague, Gouda	7594	3
Leiden, Gouda	5897	8
Gouda, Dordrecht	5897	8
The Hague, Dordrecht	10112	7
Leiden, Dordrecht	10633	1
or	10634	7

[Margin: Order of the problems.]

[Margin: Order of the problems.]

Ends of the distances	Leiden perches	Leiden feet
Remainder of the preceding table		
Gouda, Utrecht	7847	5
Leiden, Utrecht	11628	8
or	11631	8
The Hague, Rotterdam	5616	8
Leiden, Rotterdam	6972	3
Leiden, Oudewater	7975	1
or	7981	8
Oudewater, Montfoort	1566	1
The Hague, Haarlem	10725	1
Leiden, Haarlem	7040	4
Amsterdam, Utrecht	9201	0
Leiden, Amsterdam	9725	8
Amsterdam, Alkmaar	8193	0
Leiden, Alkmaar	14750	0

Thirdly, and finally, having observed with a large quadrant the altitudes of the pole of many of the places already mentioned, and also the angles of position, he investigated by the analysis of spherical triangles the arc of the vertical circle lying between pairs of places, and especially between Leiden and Alkmaar, whose very great distance he had obtained. Let the figure set out in Problem 13 be repeated here, and let Leiden be D, whose latitude, known from the altitude of the pole, is AD, 52° 10′ 30″, and its complement DC 37° 49′ 30″. Let Alkmaar be E, whose altitude of the pole is 52° 40′ 30″, as Snel gives it, and its complement 37° 19′ 30″; and let the angle of position CDE be 14° 58′. The arc DE of the vertical circle will then come out at 0° 31′ 4″. Since, then, by the second step, this arc contains 14,750 Leiden perches, that is, 177,000 ancient Roman feet, or 35,400 paces, it follows that one degree of a great circle of the Earth contains 68 400/1000 ancient Roman miles, and the whole circle 24,624, as I reported in Book 2, chapter 7. I have shown, however, in my book on geography that the Roman foot which Snel compared with the Leiden foot was more recent, and larger, than Vespasian's. But because of the over-complicated operation into which Snel was forced, in order at last to find a great distance corresponding to about half a degree, it is no wonder if he fell short of that truer measure which we have reached by a simpler and surer way, as we shall explain in the following problem.

18. Probl. Ambitum Terræ Methodo Noſtra & P. Franciſci Mariæ Grimaldi. *Problem 18. The circumference of the Earth by our own method and that of Father Francesco Maria Grimaldi.*

[Margin: Problem 18.]

Although I have already given this problem in Book 2, chapter 7, from number 6 on, with a notable example added, I think it should be repeated here, condensed into a few words, for the completeness of the doctrine and its unity

with what precedes. We carried it out, moreover, in four steps. *First*, with great diligence I sought out the measure of the Roman foot in use under Vespasian, or even under Augustus, and compared it with the Bologna foot and with the feet or palms of many notable cities, as appears in the table of measures placed at the beginning of our book on geography — which I shall perhaps transcribe at the end of this Section. I found that 1495 ancient Roman feet make 1200 present-day Bologna feet, and so that the Bologna foot is to the Roman as 5 to 4, with an insensible difference, and likewise the pace to the pace and the mile to the mile. We set down a measure of this kind in Book 2, chapter 7, number 2.

[Margin: Step 1.]

Secondly, along the bank of the Reno, where there is an open and great plain, *Father Grimaldi*, with several companions and helpers, measured the interval from the small tower of the gate of San Felice as far as a certain quadrilateral column standing near the gate, on the bank of the river; it came to 1088 2/5 Bologna paces, a cord being stretched in a straight line and held up at intervals, and ten-foot rods being used. Having acquired this great side, and having observed the angles of four other stations — namely, of the Asinelli Tower; of the church of the Most Holy Virgin painted by St Luke, standing on the Mount of the Guard, commonly *della Guardia*, a definite part of it being chosen; of our country house, situated on Monte Paderno; and finally of the tower of Modena, which is plainly seen from that house, as it is also from part of the church already mentioned and from the summit of the Asinelli tower — by an accurate analysis of only five rectilinear triangles we determined that between the tower of Modena and our country house there are 21,176 Bologna paces, 26,470 Roman; and between the tower of Modena and the Asinelli of Bologna 20,404 Bologna paces, that is, 25,505 ancient Roman. This agrees very closely with the 25 miles which the *Antonine Itinerary* puts between Modena and Bologna — whether the Roman surveyors neglected the 505 paces, content with a round number, or rather used other termini, since the towers already mentioned did not then exist. You have the angles already mentioned, and the scheme of the stations used, in Book 2, chapter 7, number 7.

[Margin: Step 2.]

Thirdly, in a window of the country house of our College of Bologna, which stands in the Serra, or Monte Paderno, a thread triangle was erected perpendicularly, such as we described in Book 5, chapter 15, number 16, and turned towards the tower of Modena, which could conveniently be seen from there by day; and it was kept in the same plane of the vertical circle passing through that window and the tower already mentioned. When the Fidicula of the heavenly Lyre reached that plane, its altitude was taken, and it was 20° 17′. The northern declination of the Fidicula was then 38° 30′; and on the same mountain the altitude of the pole had already been examined earlier and found to be 44° 27′.

[Margin: Step 3.]

Fourthly, Father Grimaldi went to Modena and observed most diligently the altitude of the pole in our College at Modena, which is very near the tower of that city, and found it to be 44° 37′. Therefore, from the complements of the polar altitudes and of the declination of the Fidicula, according to the scheme shown in Book 2, chapter 7, number 6, the arc of the vertical circle drawn through the tower of Modena and the window already mentioned was found to be 0° 17′ 35″. Since, then, by the second step, there are between these two places 21 176/1000 Bologna miles, and 26 470/1000 ancient Roman miles, there are accordingly in one degree 72 256/1000, or 72 1/4, Bologna miles, but 90 8/25 Roman; and in the whole circumference of the Earth 26,010 Bologna miles, 32,512 Roman, as we said in the same place.

[Margin: Step 4.]

19. Probl. Terræ Ambitum Noſtra alia methodo tentare. *Problem 19. To attempt the circumference of the Earth by another method of our own.*

[Margin: Problem 19.]

Let two places on the Earth be chosen, very distant indeed, but such that the interval between them in miles or stadia can be known, either by proceeding along a straight route with ten-foot rods, or by geometry, in the way that Snel did in Problem 17 and we in Problem 18. Let the altitude of the pole be observed exactly in each place, and the beginning of some total eclipse of the Moon — or rather of the total obscuration — but with the diligence which we required in Book 5, chapter 15; and let each observer obtain the moment of the beginning from the altitude of a fixed star, using Problem 25 of Section 3. For when the smaller time is subtracted from the greater, and the difference converted into parts of the equator, the difference of geographical longitude will be known. With these obtained, let the figure of Problem 13 be taken up again, and in it let one place be D and the other E. For in the triangle DCE the sides CD and CE will be known, since they are the complements of the polar altitudes, and the angle C, since it measures the arc AB of the equator, that is, the difference of the longitudes. Therefore, by [proposition] 4 of spherical oblique-angled triangles, the vertical arc DE drawn through both places will be found — that is, how many degrees and minutes it contains. And since the miles or stadia belonging to this arc will be known, those which are in one degree, and in 360 degrees, or the whole circumference of the Earth, will also become known.

20. Probl. Terræ Ambitum ὁλοσχερῶς, ſeu Eminus diſcernere. *Problem 20. To discern the circumference of the Earth in the gross [Greek: ὁλοσχερῶς], or from afar.*

[Margin: Problem 20.]

This can be done by means of a geographical globe, and of a sea voyage from one place to another very distant one, made as straight as possible, and as diligently as possible from

[Margin: Order of the problems.]

[Margin: Order of the problems.]

the measures customary among shipmasters, ascertained. For when the legs of a pair of compasses are spread from the one place to the other, and so transferred onto the equator of the globe, the degrees, and very nearly the minutes, contained in the arc of the vertical circle passing through those two places will be known; and so, from the miles or leagues found on that voyage, how many miles are owed to one degree and to the whole circumference.

[Margin: Example of the problem.]

For example, Ferdinandus Magellanus [Fernão de Magalhães], in the year 1519, on September 20, set sail with favourable winds from the port of Sanlúcar, which lies beyond Seville, and on the 26th reached the island of Tenerife by an almost straight course, having covered 960 present-day Italian miles, as Antonius Pigafetta [Antonio Pigafetta] and Odoardus Barbosa [Duarte Barbosa] attest from the daily observation of the shipmasters; and the globes of the more recent geographers show an arc of 12 degrees between that port and Tenerife. It follows that about 80 miles are contained in one degree of a great circle of the Earth. But if that course is corrected and reduced to a straight path, we are certain that somewhat fewer are contained — especially since he reached another port, 24 degrees distant from Tenerife, after covering 1680 miles counted from Tenerife, from which 70 miles to one degree would follow. But it is better to use Problem 18, if anyone has been granted the opportunity of such places.

Problemata. 21. 22. 23. 24. 25. 26. Altitudinem Turris Sex modis præcipuis Geometricè metiri per Altimetriam. *Problems 21, 22, 23, 24, 25, 26. To measure the height of a tower geometrically, by altimetry, in six principal ways.*

[Margin: Problem 21.]

I said "geometrically", for there is no one who does not know how to measure it mechanically, with a stretched rope or with rods. For these, then, let there be in the following figure a tower AB, lit by the Sun and casting a shadow as long as AD, bounded by the ray BD; and let the straight line AD be measured with ten-foot rods. Let a pole XZ be erected perpendicularly at the place X — that is, at such a position that its tip is bounded by the ray BD, so that a tiny part of it is lit at Z, or rather a small ball fixed on its tip. Let the part DX of the shadow be measured with ten-foot rods, and let the length of the pole be known in the same kind of units in which DX is known. For if AD makes a right angle with AB, or is brought to one by means of a horizontal level, two equiangular triangles will arise, namely BDA and ZDX: for the right angle DXZ will equal the right angle DAB, and the angle ADB will be common to both triangles. Therefore, by [proposition] 32 of the first book of the *Elements*, the complements to the other right angle, that is, the angles DZX and DBA, will be equal to each other. Therefore, by [proposition] 4 of the sixth book of Euclid's *Elements*, let it be done: as DX, known in feet, is to XZ, known in feet, so DA, known in feet, is to AB; and thus the height of the tower AB will be known in feet. Indeed, in place of the pole XZ a man of known stature

could be used, who would walk to and fro until the ray of the Sun grazed the top of his head.

Diagram. A figure set within the width of the left column. At the right stands a tall tower, drawn with two barred windows and a round-headed door at its foot, with A at its foot and B at its top beside a small cross-like finial. A base line runs from D at the left to A, with T, O, V and X lettered below it. At the left a vertical rises from D to Y, and from Y a quadrant hangs to the right, with I on its arc and H below it. Long lines run from B down to D and from Y down towards A, crossing near the middle of the figure. A vertical stands at X, with Z at its top on the line BD. Above T stands E, with a quadrant whose vertical side ends at G, with K on its arc and L at the end of its horizontal side; below E hangs a small quadrant with R at the left and S at the right. A horizontal line runs from E to C on the tower. Dotted lines run from L down to V and from V up towards F. A second quadrant stands at F, with M at the top of its vertical side, P on its arc and Q at the right; lines run from E and from F up to B.

- Y (top left): the station Y, top of the cord DY
- I (on the arc at Y)
- H (below I, left)
- B (top of the tower)
- Z (top of the pole XZ)
- G (top of the vertical side of the quadrant at E)
- K (on the arc of the quadrant at E)
- M (top of the vertical side of the quadrant at F)
- P (on the arc of the quadrant at F)
- E (the observer's eye, the station E)
- L (end of the horizontal side of the quadrant at E)
- F (the station F)
- Q (right of F, below the line)
- C (on the tower, at the end of the line EC)
- R (left end of the hanging quadrant below E)
- S (right end of the hanging quadrant below E)
- D (lower left corner)
- T (on the base line, below E)
- O (on the base line, below L)
- V (on the base line, where the dotted lines meet)
- X (on the base line, foot of the pole XZ)
- A (foot of the tower)

[Margin: Problem 22.]

Secondly, measure the shadow AD as above, provided it is at right angles to AB; and, taking your station at T, turn the dioptra RE of a hanging quadrant onto the extreme ray BD of the Sun — the one, that is, that bounds the shadow — so that the ray passes through the holes of the pinnules, or rather grazes their upper side; and observe on the quadrant the arc TER. For it will be equal

to the angle DBA, because of the parallelism of the perpendicular lines BA and ET; and the angle TES will be equal to the angle ADB, by [propositions] 29 and 32 of the first book of the *Elements*. Therefore, by [proposition] 4 of plane right-angled triangles, given AD and the angle ADB, the height AB will become known.

[Margin: Problem 23.]

Thirdly, let there be in the plane AD a plane mirror TX, parallel to the horizon, on which there falls from the top of the tower, B, a ray, or image, of B itself; let it be BV. You, then, move back from the mirror towards E until, with your eye placed at L, you see the image of the point B at V by the reflected ray VL; and, dropping a plumb line from L, measure the height of the eye OL in parts as small as possible, but equal, and also the distances OV and AV. For here too you will obtain two equiangular triangles, namely VBA and OLV, since the right angles VAB and LOV are equal, and the angle of incidence BVA, by the axioms of catoptrics or anacamptics, is equal to the angle of reflection OVL; and so the remaining angle VBA is equal to the remaining angle OLV. Let it be done, therefore, by [proposition] 4 of the sixth book: as OV is to OL, so VA is to AB; and the height AB will be known.

[Margin: Problem 24.]

Fourthly, stretch yourself out on your back, parallel to the horizon, and let your eye be at D and your feet at T; and let a companion be present who, with the help of a plumb line, fixes a pole of known length upright to the horizon, moving it to and fro in the plane DA, which is at right angles to the tower AB, until Z, the tip of the pole XZ, is in the same straight line BD — the line of sight that runs from the top of the tower, B, to your eye; and let both the distance DX and the distance DA be known. For, from what was said in Problem 21, as DX is to XZ, so DA is to the height of the tower AB.

[Margin: Problem 25.]

Fifthly, since the plane AD is sometimes not parallel to the horizon, place at the height of your eye (while you stand upright) a quadrant EGKL, or a geometrical square, so that the side GE stands perpendicular to the horizon and the other side, EL, is parallel to it; and, turning the dioptra towards the top of the tower, observe the angle KEL, that is, BEC, by the arc KL. You could also obtain it by means of a hanging quadrant ERS, with its centre placed at E and the eye at R; for the angle TER, by [proposition] 15 of the first book of the *Elements*, equals the angle GEK, and so their complements to a right angle, namely TES and KEL, are equal. With these angles noted, measure the height ET and the interval TA, or rather EC. For in the triangle BEC, right-angled at C, given the side EC and the adjacent angle BEC, you will be able to find, by [proposition] 4 of plane right-angled triangles, the height CB; and if to it you add the height ET — or rather AC, if from E you have marked the point C along the line of sight and have measured AC — the whole height AB will be known.

[Margin: Problem 26.]

Sixthly, if you cannot get to the foot of the tower, because water, or an enemy army, or some other obstacle lies between, nor measure the whole distance AD or EC, choose two of the stations available: one at E, where, as above, take the angle BEL by means of a quadrant, or a square, or a ring; the other at F, where take the angle PFQ. Then measure the interval EF between the stations. For in the triangle BEF, besides the side EF, all the angles will be known: BEF is known from the earlier observation; the angle BFE, by [proposition] 13 of the first book of the *Elements*, is the supplement to two right angles of the angle PFQ, known from the later observation; and the third, EBF, is the supplement of these two to two right angles, by [proposition] 32 of the first book of the *Elements*. Therefore, by [proposition] 2 of plane oblique-angled triangles, you will find the side BE; and with it and the angle BEC, in the triangle BEC, right-angled at C, find by [proposition] 1 of right-angled triangles the height BC. This will differ from the height AB by not much more than the height of your eye, ET, if TA is, at least to the senses, a plain of a kind.

But if you can neither get to A nor have an interval EF sufficient for a proper operation, draw from D a cord DY at right angles to the line of sight DA, with the help of a set-square or gnomon, and measure the distance DY with ten-foot rods. Then at the station Y, by means of a quadrant parallel to the horizon, take the angle DYA by the arc HI, sighting, that is, the foot of the tower, A. For in the triangle DYA, right-angled at D, given the side DY and the angle DYA, you will obtain by [proposition] 4 of right-angled triangles the interval AD, with which you may then operate as in the earlier Problems 21, 22 or 24.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

27.Probl. Diſtantiam Inſulæ, aut alterius loci ex littore maris, vel ripa fluminis conſpicui, metiri. *Problem 27. To measure the distance of an island, or of some other place, visible from the shore of the sea or from the bank of a river.*

[Margin: Problem 27.]

You will solve this problem by the latter part of Problem 26. For in the figure set out above, let A be an island or some other place, and suppose you wish to measure the interval AD between it and the shore D. For if on the shore you set up the angle YDA by means of a set-square, and, having measured the distance DY, take at the station Y, on the shore or the bank, the angle HYI, that is, DYA, you will find the interval DA, as was said there.

28.Probl. Montium Altitudinem per partes ope normæ metiri. *Problem 28. To measure the height of mountains by parts, with the help of a set-square.*

[Margin: Problem 28.]

Let there be a mountain APB, whose perpendicular height AP is to be found; but suppose that above the plane AB you have a gnomon, or set-square, MNO, containing a most exact right angle N, whose height NB is as great as your

stature from foot to eye. Starting from the foot of the mountain, B, and setting the side NB of the square upright to the horizon with the help of a plumb line,

Diagram. A figure set within the width of the left column. A frame has A at its lower left and B at its lower right, with P at the top of its left side and G and C below P. The hatched slope of a mountain falls from P at the upper left to B at the lower right, and three set-squares stand on it as steps: at the top one with E and D, whose vertical DF runs down to F, with S beside it; in the middle one with I and H, whose vertical HK runs down to K; and at the right one with M and N, whose vertical runs down through O to B. Horizontal lines run from G through S, and from C through K, to the right side. Each square carries a small quadrant arc in its corner. A small letter stands on the slope under K, partly lost in the hatching and not certainly legible.

- E (top square, left end of its upper side)
- D (top square, corner)
- P (top of the left side): the summit of the mountain
- F (foot of the vertical DF)
- G (left side, below P)
- S (beside F, on the line from G)
- I (middle square, left end of its upper side)
- H (middle square, corner)
- K (foot of the vertical HK)
- [?] (on the slope under K)
- M (lowest square, left end of its upper side)
- N (lowest square, corner)
- C (left side, below G)
- O (on the right side, below N)
- B (lower right corner): the foot of the mountain
- A (lower left corner)

look through the pinnules of the side MN at a sign L on the mountain, placed there by a companion — some ball, or a paper disc coloured red. Then climb to the sign L and set up the square in the same way, so that it has the position IHK, and look through the side HI at a sign placed at S. Afterwards climb to S and set up the square so that it has the position EDF, with the side F upright to the horizon; and through the side ED sight the sign P; and go on in this way, step after step, if P were not the summit of the mountain. Then let the height of the side BN be known: for as many such stations as there have been, so many times will the side BN be contained in the height AP. For BN is equal to AC, by [proposition] 34 of the first book of the *Elements*, since ACNB is a parallelogram, all its angles being right — because AC and NB are perpendicular to AB, and the angle N is right by the construction of the square; and so the straight line NML, continued in imagination to C, is parallel to AB, by [proposition] 28 of the first book of the *Elements*. Similarly HK will equal CG, and FD will equal GP; and so the three NB, HK and DF together will equal the three AC, CG and GP taken together, that is, the whole

perpendicular height of the mountain, AP, above the horizontal AB — not, however, the whole height absolutely above the surface of the Earth concentric with the surface of the sea, unless AP stood on the shore of the sea, or were the height of a cliff, or of a promontory, and so on.

[Margin: Note, for avoiding error in the heights of mountains.]

The problem can also be solved by climbing to L, placing the square there upright to the horizon, and looking back along the other side at a small ball N fixed on a pole BN — provided the pole is upright to the horizon by means of a plumb line — and so on successively: for as many stations of observation as there were up to the summit P, so many times would the length of the pole be contained in the height AP.

[Margin: Another way.]

29. Probl. Montis aut Turris Altitudinem abſolutam metiri, Data ſemidiametro Terræ, & Angulo in vertice ipſius obſeruato, ſi inde liber ſit proſpectus in Mare, aut ſuperficiem Mari concentricam. *Problem 29. To measure the absolute height of a mountain or a tower, given the semidiameter of the Earth and the angle observed at its top, if from there there is a free view onto the sea, or onto a surface concentric with the sea.*

[Margin: Problem 29.]

I call absolute height the perpendicular height, not above a straight and horizontal line, but above that surface of the Earth which is concentric with the surface of the sea and coincides with it — or which is as far from the centre of the Earth as that surface is. In the following scheme, then, let HAR be the hemisphere of the terraqueous globe, HCR its rational horizon, and AC the semidiameter perpendicular to it, in which let AB be the height of the mountain. From its top, by means of a very large quadrant that shows not only the minutes of each degree but at least the tens of seconds, with the pinnules turned down onto the horizon below, which no obstacle obstructs, let the angle B be observed along the line of sight BG, touching the globe of land and sea at G; and, drawing the straight line CG in imagination, let the triangle BCG be considered, right-angled at G, by [proposition] 18 of the third book of Euclid's *Elements*.

[Margin: What the absolute height of mountains is.]

Diagram. A small figure set at the right of the right column. A semicircle stands on the diameter HR, with C at its centre. A vertical rises from C through A on the arc to B above it. A horizontal chord runs from F at the left to G at the right; a line from B touches the arc at G, and a line runs from C to G.

- B (top): the summit of the mountain
- A (top of the arc): the foot of the mountain's perpendicular
- F (left end of the chord)
- G (right end of the chord): the point of contact
- H (left end of the diameter)
- C (centre): the centre of the Earth

• R (right end of the diameter)

For in it the side CG is given, which is the semidiameter of the Earth, known not only in miles but also in paces or feet; and so is the opposite angle, whose complement to a right angle is the other angle, BCG. Therefore, by [proposition] 3 of plane right-angled triangles, find the base CB; and if you subtract from it the Earth's semidiameter AC, the height of the mountain, AB, will remain. From this, moreover, you will gather how exact the knowledge of the Earth's semidiameter must be: for if paces or feet in it are neglected, the measure of the height AB will also be crude, and it will come out differently according to the different opinions about the semidiameter of the Earth. By this device we measured the height of Monte Paderno near Bologna.

[Margin: Corollary.]

30. Probl. Montis Altitudinem abſolutam indagare, Datis Semidiametro Terræ, & Circuli magni Arcu inter Montis perpendiculum, atque oculum primò detegentem Montis apicem. *Problem 30. To find the absolute height of a mountain, given the semidiameter of the Earth and the arc of a great circle between the perpendicular of the mountain and the eye that first discovers the peak of the mountain.*

[Margin: Problem 30.]

In the figure immediately preceding, let AB be the perpendicular height of the mountain, and let someone sailing, or swimming, towards it arrive at G, where he first discovers the summit B, which a little before was hidden, so that the line of sight BG touches the eye G, placed almost on the surface of the sea. For in the triangle CBG, right-angled at the point of contact G by [proposition] 18 of the third book of Euclid, the angle BCG is given, which the given arc AG measures, and the side CG, which is the semidiameter of the Earth. Therefore, by [proposition] 3 of plane right-angled triangles, the base BC will be found; and by subtracting from it the Earth's semidiameter AC, the height sought, AB, will become known. The arc AG, moreover, is observed very easily if one sails towards A in the plane of the meridian, towards the poles of the world, and, when the ship has come within sight of the top B, the altitude of the pole is taken, the altitude of the pole on the mountain AB being known; or else the meridian altitude of the same fixed star is taken in both places in the same year, for the difference of the altitudes will give the arc AG. But if that arc is given in miles, let them be converted into degrees and minutes, and let the rest be completed as above.

EXEMPLVM *Example.*

In Montibus Ætna, Pico, & Caucaſo. *On the mountains Etna, the Peak and the Caucasus.*

Maurolycus, in the third dialogue of the *Cosmographia*, page 75, and certain Knights of Malta have reported to us that the summit of Etna is discerned right

from the island of Malta, and even beyond, from a distance of 216 Italian miles and more; and to these, by what was said in Book 2, chapter 7, correspond 3 degrees of a great circle — where we also showed that CG, the semidiameter of the Earth, is 4139 Bologna miles. Let it be done, then: as the radius CG, of 100,000 parts, is to 4139 miles, so the secant BC of the arc AG, 3°, of 100,138 parts, is to 4144 71/100 miles. For when AC, 4139 miles, is subtracted, there will remain the height of Etna, AB, of 5 7/10 Bologna miles — granted that Snel, in Book 2 of the *Eratosthenes Batavus*, puts only 4, because of refractions, which, as Father Kircher also thinks in a letter sent to me on 1647 February 17, intervene and make either the peak of the mountain visible from the island, or the island from the peak of the mountain, when otherwise they could not be seen. But the height of Mount Athos, of which I shall speak in Problem 35, and which cannot be diminished by refractions, to me very probable makes this height. How this height, and that of the mountains that follow, is to be corrected once the intervention of refraction is granted, I shall say in Section 6, Problem 50.

[Margin: The height of Etna.]

[Margin: Order of the problems.]

[Margin: Order of the problems.]

Aloysius Cadamustus [Alvise Cadamosto], in his voyages, in Ramusius [Giovanni Battista Ramusio], volume 1; Io. Hugonis Lintschotanus[56] [Jan Huyghen van Linschoten], in his; Snel, in Book 2 of the *Eratosthenes Batavus*, chapter 17, and in Book 2 of the *Tiphys Batavus*, near the end; and our Fournier, in Book 19 of the *Hydrographia*, chapter 8 — all report that the Peak, the mountain of the island of Tenerife, one of the Fortunate Islands, is seen by those sailing towards the south pole, and by those sailing back towards the north, from a distance of 4 degrees of a great circle; indeed, that its summit is seen like a temple from the island of Madeira [Materia], that is, from a degree of 4° 10′. Let it be done, then: as AC, 100,000, is to 4139 miles, so the secant BC of 4 degrees, 100 244, is to 4149 1/10 miles. For there remains the height of the Peak, AB, of 10 1/10 Bologna miles — as much as Snel also gathers, disregarding the fractions. But if refraction should diminish it, see Section 6, Problem 50.

[Margin: The height of the Peak.]

Aristotle, in Book 1 of the *Meteorologica*, text 63, or summa 4, chapter 1, among the signs of the height of the Caucasus, sets down this as well: "For it is seen from the so-called Deeps, and by those sailing into the Lake." The Deeps are the parts of the Euxine Sea near the Cimmerian Bosporus, so called because they have no bottom, sailors having sounded for it in vain; the Lake is the Maeotic marsh. He means, then, that those sailing from the Euxine into the Maeotis through the Bosporus see the Caucasus. In Ptolemy the Cimmerian Bosporus lies almost on the same parallel as the Caucasus, and is distant from it in longitude by at least 12 degrees. The altitude of the pole of the Caucasus is 47 degrees; that of the Bosporus, at its mouth, 47 1/2 degrees. From these,

[56] printed "Lintſchoſanus", long s for t

by Problem 102, to be set out below, the distance of the Bosporus from the Caucasus on a vertical circle is deduced as 9° 32′, whose secant is 101,400. Now let it be done: as the radius CA, of 100,000 parts, is to 4139 miles, so the secant BC, of 101 400 parts, is to 4196 97/100 miles. Therefore AB, the height of the Caucasus, would be at least 57 97/100 Bologna miles, or 58 — unless it is to be corrected for refractions, according to what will be said in Section 6, Problem 50. The more recent geographers, however, set the Caucasus less far from the Bosporus, and so on their reckoning the height of the Caucasus would decrease.

[Margin: A passage of Aristotle on the Caucasus explained.]

[Margin: The height of the Caucasus.]

31. Probl. Montium Altitudinem metiri more antiquiorum Geometrarum, duplicem tamen fallaciam continente. *Problem 31. To measure the height of mountains in the manner of the older geometers — a manner, however, that contains a double fallacy.*

[Margin: Problem 31.]

In the following figure, already set out in Book 2, chapter 4, number 8, let CB be a quadrant of the Earth, described from the centre A, and on it the height of the mountain MP. Let the perpendicular PM be continued as far as A, and let the surveyor's eye be at C, the point of contact made by the tangent CK. On this tangent let the side CD of the geometrical quadrant CDF stand perpendicularly; and, with the eye set at C, let the alidade [dioptra] CE be raised and the summit M sighted through it, and let the arc EF, divided in the usual way into degrees and minutes, show the angle ECF. For in the triangle GCK, right-angled at C, if the distance CK is known, you can find the height GK by [proposition] 4 of plane right-angled triangles. But if CK is not known, find it by Problem 26. In this way Dicaearchus and Zenagoras, in Pliny the Elder, Book 2, chapter 65, and in Plutarch's *Aemilius*, seem to have measured the height of Cyllene, Pelion, Olympus, Satabyrium and other mountains; and indeed they find it far too small — namely, for Olympus and Pelion, 1250 paces; for Cyllene, 14 stadia, or 1750 paces; and for Satabyrium, 15 stadia, or 1875 paces. Certainly our Biancani, in the *Sphaera*, part 3, treatise 3, chapter 1, appendix 1, affirms that by this method he found the height of Monte Baldo, which rises out of the territory of Verona, to be 804 paces, supposing the distance CK from the territory of Parma, where he was staying, to be 70 Italian miles, and so the angle GCK to be 39 1/2 minutes.

Diagram. A figure set at the right of the left column. From the centre A at the lower left a straight line rises to M at the upper right; an arc of the Earth's surface runs from C at the upper left down to B at the lower right, crossing the line AM at P, with K on AM above P. At C stands a small quadrant, its side running up to D and its arc through E; lines run from C to G and M at the top and along the tangent to K, with F on that line below E. A vertical line falls to B.

- D (upper left, above C): the upper end of the quadrant's side CD

- E (top, right of D): on the quadrant's arc
- G (top): the top of the height GK
- M (upper right): the summit of the mountain
- C (left): the surveyor's eye, the point of contact
- F (below E): on the quadrant's arc, on the line CK
- K (right, on AM): the foot of the height GK on the tangent
- P (on AM, at the arc): the foot of the mountain on the Earth's surface
- A (lower left): the centre of the Earth
- B (lower right): the end of the quadrant CB

But in this practice a double fallacy lies hidden, and I am surprised that it was not mentioned by Orontius [Oronce Finé] in his *Geometria practica*, Book 2; nor by Clavius, Book 3; nor by Maurolycus in dialogue 3 of the *Cosmographia*; nor by Daniel Santbech in his problems, from proposition 77 to 82; nor by Gemma Frisius in the *Radius Astronomicus*, chapter 10; nor by Io. Taisnerus [Johannes Taisnier] in *De usu annuli*, Book 1; nor by Magini, book 2, *De altitudinibus*; nor by Sylvius Belus [Silvio Belli]; nor by many other geometers; nor, finally, by Biancani himself. First: if the distance of the station C from the mountain PM is considerable, and of many miles, a notable slope of the ground usually intervenes, increasing or diminishing the apparent height of the mountain. Second: the height taken in the manner already mentioned is not the absolute height, but the height above the horizontal CK, which is imagined; and usually — unless the foot of the mountain is on the seashore — there remains a great part belonging to the height of the mountain and lying hidden below the horizontal CK, down to that surface of the Earth which is concentric with the convex surface of the sea. Indeed, the height GK is not perpendicular to such a surface, nor does its line tend to the centre of the Earth. And yet, when the question in cosmography is the perpendicular height of mountains, we commonly seek the height that stands perpendicularly on the convex surface of the terrestrial globe and rises from that same surface, concentric with the surface of the sea — especially when we wish to know their absolute height, and how much it hinders the perfect roundness of the Earth, and how great a space is visible all round from the summit of the mountains, and how great the slope is from there down to the level of the sea. Some years after I had written this, however, I came upon Father Cabeo [P. Cabæi], Book 1 of the *Meteorologica*, text 63, question 1, where he mentions this fallacy; but not even he supplied a sufficient remedy for it, as I shall presently show.

[Margin: Error, or oversight, of some geometers.]

[Margin: First fallacy.]

[Margin: Second fallacy.]

32. Probl. Altitudinem Montium, more antiquo menſuratam corrigere, & ad abſolutam reducere, data Terræ ſemidiametro. *Problem 32. To correct the height of mountains measured in the old way, and to reduce it to the absolute height, given the semidiameter of the Earth.*

[Margin: Problem 32.]

Look at the figure set out and explained just above, in which let ABC be a quadrant of the Earth's globe, with its centre A. Let the observer at C, by means of the quadrant CDF standing perpendicularly on the horizontal plane CK, sight the summit M of the mountain PM through the alidade and along the line of sight CM, and take the angle ECF subtending the arc EF; and with this angle and the distance CK, in the triangle CKG, right-angled by supposition at K, let him gather the height GK, according to what was said in Problem 31. The principal error of the geometers in this measurement, then, is to take the height GK for the absolute height MP, and to suppose that GK falls perpendicularly on the surface CPB. This error must now be corrected, and we shall accomplish it in two steps. First, in the triangle ACK, right-angled at C, CK is supposed known, and the semidiameter of the Earth AC; therefore, by [proposition] 8 of plane right-angled triangles, we shall find the base AK, and by [proposition] 5 of the same, the angle AKC. This, joined to the right angle GKC, will make the angle PKG, and its supplement to two right angles, by [proposition] 13 of the first book of the *Elements*, will show the angle GKM. Second, then, in the triangle GKM we shall find the side KM by [proposition] 2 of plane oblique-angled triangles. For in it are given the side GK, found by Problem 31; the angle GKM, just found; and the angle KGM, because by [proposition] 32 of the first book of Euclid it equals the right angle CKG and the acute angle GCK, already observed, taken together. And so, by the same proposition, the angle M opposite the side GK becomes known, because it is the supplement to two right angles of the sum made of the angles GKM and KGM. Next, we shall add the side KM to the straight line AK, found by the first step, and the whole AM will result; and if we subtract from it the Earth's semidiameter AP, there will remain the absolute height of the mountain, PM, measured along the perpendicular tending to the centre of the Earth. Here, however, the eye C is supposed not to be raised at all above the Earth; for if it were raised, there would have to be added to GK as great a portion as the height of the observing eye is thought to be, so that we may correct the first fallacy.

[Margin: Step 1.]

[Margin: Step 2.]

EXEMPLVM. *Example.*

Thus I showed in the Geographical Book that the hill of Parma, near Forum Naevii, from which Father Biancani was sighting the summit of Monte Baldo, is higher than the ground of Bologna by about 40 paces; and let the ground of Bologna now be set higher than the level of the Adriatic Sea by at least 16 Roman paces. And so that place of Parma will be higher than the already mentioned sea by about 56

[Margin: Example of the correction.]

paces; and these, added to the height GK of 804 paces reckoned by Biancani, make it 860 paces. But since in Magini's *Italia* Monte Baldo is 75 modern miles

distant from the centre of Parma, which make 95 ancient Roman miles, then, if the angle KCG observed by Biancani was 41′, it follows by Problem 31 that the height KG above CK is 1134 Roman paces; and the height of the hill of Parma, added to these, makes KG 1190 paces.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

With these supposed, there is given, first, the side CK, 95 000 Roman paces, and AC, by what was said in Book 2, chapter 7, 5 174 000 Roman paces. Therefore, by [proposition] 5 of right-angled triangles, the angle AKC will come out 88° 57′, and by [proposition] 8 of the same, AK will come out 5 174,879 1/2 paces. The angle GKM comes out 1° 3′; and the sum of the right angle GKC and the observed angle GCK, 41′ — that is, the quantity of the angle KGM — is 90° 41′; and so the angle M is 88° 16′. With these, and with the side GK of 1190 paces, KM is found to be 1190 1/2 paces; and these, added to the paces of AK already found, make 5 176,070 paces. Finally, when AP, 5 174 000 paces, is subtracted from them, there remains the absolute height of Monte Baldo, PM, of 2070 ancient Roman paces, or 1654 of the modern Italian paces of this region of Italy of ours — that is, twice as great as Biancani gathered. So great an error can be committed because of the fallacies already mentioned. But the height comes out greater still if you take the distance between Forum Naevii, where Father Biancani was, and Monte Baldo from Magini's *Italia*, as 85 miles.

[Margin: Biancani's error in measuring Monte Baldo.]

Now Father Cabeo, in Book 1 on Aristotle's *Meteorologica*, text 64, question 1, says that he was present at Biancani's observation, but that PK must be added to the height KG, the secant AK being found first from the given tangent CK and the radius AC. Strictly, however, it is KM that must be added to PK; and at a much greater distance of the eye C from the mountain, KM can come out many paces greater than GK.

[Margin: A small error of Cabeo in the same matter.]

33. Probl. Montis altitudinem abſolutam, ex & Mari aut littore ſpectabilis, inueſtigare, 34. Data Terræ ſemidiametro ſola, aut cum Diſtantia in Milliarib. inter perpendiculum montis & menſorem. *Problems 33 and 34. To find the absolute height of a mountain visible from the sea or the shore, given the semidiameter of the Earth alone, or together with the distance in miles between the perpendicular of the mountain and the surveyor.*

[Margin: Problem 33.]

Let AGB be a terrestrial hemisphere, whose diameter is ACB and whose centre is C; from C let the straight line CGM be raised as far as the summit M of the mountain GM. Let the surveyor be at K, whether he is at sea or on the shore, and let him raise a very long pole, or the mast of a ship, KN, perpendicular to the convexity AGKB, its straightness continued with CK. At its foot let the angle MKN, which the fiducial line KM makes with the pole KN, be observed

with a quadrant; then, from the top N of the pole or of the ship's mast, let the angle MNK be observed with a quadrant. For from the given height of the pole or mast KN, and from the adjacent angles, whose sum taken from 180 degrees shows the angle KMN, the side KM can be known by [proposition] 2 of plane oblique-angled triangles. With it, in the triangle CKM, and with the semidiameter of the Earth CK and the included angle CKM — which is the supplement to two right angles of the observed angle MKN — the base CM is to be found by [proposition] 5 of plane oblique-angled triangles; and if the semidiameter of the Earth CG is subtracted from it, the height of the mountain, GM, will be known.

Diagram. A figure set at the right of the left column. A semicircle stands on the diameter AB, with C at the centre of the base. A hatched mountain rises from G on the arc to its summit M at the upper left; a dotted line runs from M across the top through L to N at the upper right. A line from C rises through G to M, and another from C runs up through K on the arc to N. Lines run from M down to K and across to N; R and P lie on the lines below a small arc near N, which carries O.

- M (upper left): the summit of the mountain
- L (top, between M and N)
- N (upper right): the top of the pole or mast
- O (right, on the small arc near N)
- R (middle, on the line from M)
- P (right of R): with R, the ends of the arc PR
- G (left, on the arc): the foot of the mountain
- K (on the arc, right of centre): the surveyor's station
- A (lower left): end of the diameter
- C (bottom centre): the centre of the Earth
- B (lower right): end of the diameter

But if the distance GK is given in miles, it is to be converted into the degrees and minutes corresponding to it, according to the proportion of the circumference that corresponds to the given semidiameter; and the arc GK will be known, and the angle GCK subtending it. It will suffice, then, to observe the angle MKN by means of the arc PR, so that you have, by [proposition] 13 of the first book of the *Elements*, its supplement to two right angles, that is, the angle CKM; and the sum of this and of the angle C, taken from 180 degrees, will give the angle CMK. Therefore, in the triangle CMK, the angles CMK, opposite the given side CK, and CKM, opposite the side sought, CM, will be given; and so, by [proposition] 2 of plane oblique-angled triangles, CM will be found, and by subtracting from it the Earth's semidiameter CG, the remainder will be the height of the mountain, GM. Or, indeed, in the triangle MGK seek the height GM, since the angle GMK is given, with the side opposite it, GK, and the angle MKG. For when the angle C is taken from two right angles, and the angle MKC is taken from half the remainder, that is, from CKG, MKG remains.

[Margin: Problem 34.]

35. Probl. Montis altitudinem abſolutam metiri Datis Semidiametro Terræ, Montiſque Vmbra in littus aut Mare proiecta, & Momento temporis, quo Mons proijcit tantam Vmbram. Vbi notatu digna de Monte Atho. *Problem 35. To measure the absolute height of a mountain, given the semidiameter of the Earth, the mountain's shadow cast onto the shore or the sea, and the moment of time at which the mountain casts so long a shadow. With points worth noting about Mount Athos.*

[Margin: Problem 35.]

Very famous is the shadow of Mount Athos, which on the day of the solstice it casts as far as the island of Lemnos, commonly called Stalimene, which lies eighty-six thousand paces from it, or 87 000 — that is, onto a bronze heifer set up for the purpose in the forum of the town of Myrene [Myrina], which is on that island, as Solinus reports in chapter 21, Pliny the Elder in Book 4, chapter 12, Plutarch in the book *On the spots in the orb of the Moon*, Statius in Book 5 of the *Thebaid*, from the verse beginning "Lemnos, where fire-bearing…", and Valerius Flaccus in Book 1 of the *Argonautica*, from the verse beginning "At the rising of the Sun…". From this shadow Jacopo Mazzoni, in the *Comparison of Plato and Aristotle*, section 3, chapter 5, and in the *Defence of Dante*, Book 3, chapter 12, elicits a varying and rather inconsistent height for this mountain, and falls into a kind of ageōmetria [Greek: ἀγεομετρίαν [recte: ἀγεωμετρίαν] (ageōmetrian, "want of geometry")], about which I have said more in the Geographical Book. Certainly, from the length of such a shadow alone the height of this or of any mountain cannot be known. But if the moment at which it casts the shadow is known, it certainly can. Look, then, at the figure of Problem 33, in which let GM be the height of the mountain, and let the length of the shadow, GK, be given; according to one's opinion about the circumference of the Earth, let it be converted into the arc GK, which shows the angle GCK. Next, from the given moment of time, and so from the time from noon, and from the declination belonging to the Sun on that day, and from the altitude of the pole at the place where the shadow ends, let the true altitude of the Sun be gathered, by Problem 43 of Section 2. For if the parallax of the Sun is taken from it, and the refraction and the apparent semidiameter of the Sun are added, the visible altitude of the upper limb of the Sun will be given, and its complement to 90 degrees will be the angle MKN — since the line KN tends to the zenith of the place K, where the shadow ends, while the line MK, bounding the shadow, is the ray falling on the upper limb of the Sun, and the arc RP measures the visible distance of that solar limb from the zenith. When, therefore, the angle MKN is known, its supplement to two right angles, CKM, will also be known, in the same triangle in which, as I said, the angle C is known by means of the shadow, and so is the supplement of both to two right angles, namely the angle CMK. The semidiameter of the Earth, CK, is also given; therefore, by [proposition] 2 of plane oblique-angled

triangles, the side CM can be found, and when the semidiameter of the Earth CG is subtracted from it, the height of the mountain, GM, remains, known.

[Margin: The shadow of Mount Athos.]

Would that the ancients had handed down to us the hour at which the shadow of Mount Athos reached that heifer of the town of Myrene! It is certain, however, from the situation of the two places, that this could not have happened at noon, as Mazzoni once suspected; nor near noon, nor at any time before noon, since Myrene lies further east than that mountain, and on very nearly the same parallel; nor yet at the moment of setting itself, almost, when the ends of the shadows fade away and cannot be made out. With these things set down, from the altitude of the pole of Myrene, which is 41° 10′, and from the geographical globe, with the Sun's place at the beginning of Cancer, you will gather that the shadow of Mount Athos could not have fallen on Lemnos except when the Sun was sinking towards its setting, far from noon. We must therefore find this moment of time, since no historians have recorded it for us. This we shall do once we are given the azimuth of the vertical circle passing through the summit of Mount Athos and the town of Myrene, or Myrina; for in the plane of this vertical lies the ray MK, which bounds the shadow cast from M to K. The azimuth we shall find by means of the following Lemma, and afterwards the time, and finally the altitude of the Sun.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

Lemma. Data duorum locorum Altitudine Poli, & Differentia Longitudinis, inuenire Azimuth Verticalis per vtriuſq. verticem ducti, & Poſitionis Angulos. *Lemma. Given the altitude of the pole of two places and the difference of longitude, to find the azimuth of the vertical circle drawn through the zenith of each, and the angles of position.*

I. EXEMPLVM *First Example.*

Pro Angulis poſitionis Montis Atho, & oppidi Myrinæ. *For the angles of position of Mount Athos and of the town of Myrina.*

In the following figure let HMBR be the meridian passing through Myrina, M, and HOR its horizon; let NAP be the meridian passing through Mount Athos, which lies further west, and NOP its horizon; let MAV be the western vertical semicircle, drawn through A and M; let EQ be the equator, KG the tropic of Capricorn, and CF that of Cancer, on which the Sun is at S, near setting, with BS the complement of its declination, 66° 30′; and let B be the north pole of the world. Now, by Ptolemy, Book 3 of the *Geography*, chapter 12, the polar altitude of Mount Athos, PB, is given as 41° 10′, and its complement BA as 48° 50′; the polar altitude of Myrina, RB, as 40° 56′, and its complement BM as 49° 4′; and the difference of longitude, that is, the angle ABM, as 1° 20′. Therefore, in the triangle ABM, given the sides AB and MB and the included

angle, the angle of position BAM is found, by [proposition] 5 of spherical oblique-angled triangles, to be 102° 38′, and BMA 76° 30′.

Diagram. A figure set at the right of the left column: a circle of the sphere crossed by several great-circle arcs — the meridians, their horizons, the vertical, the equator and the two tropics.

- M (top): Myrina
- C (upper left): on the tropic of Cancer
- E (upper left): on the equator
- K (upper left): on the tropic of Capricorn
- A (inside the circle, below M): Mount Athos
- B (upper right): the north pole of the world
- R (right): on the horizon HOR
- P (right): on the meridian NAP
- H (left): on the horizon HOR
- N (left): on the meridian NAP
- S (near the centre): the Sun, near setting
- O (centre): where the horizons meet
- F (lower right): on the tropic of Cancer
- Q (lower right): on the equator
- G (lower right): on the tropic of Capricorn
- V (bottom): the end of the vertical MAV

II. EXEMPLVM. *Second Example.*

Pro Tempore, quo Sol die Solſtitiali tranſit per Verticalem communem Atho & Myrinæ, & pro Altitudine Solis. *For the time at which the Sun, on the day of the solstice, passes through the vertical common to Athos and Myrina, and for the altitude of the Sun.*

In the same figure consider the triangle MBS, with the Sun at S. In this triangle there is given MB, the complement of the altitude of the pole of Myrina, which is 49° 4′; BS, the complement of the Sun's declination on the day of the summer solstice, which is 66° 30′; and the angle of position BMA, of 76° 30′. Therefore, by [proposition] 3 of spherical oblique-angled triangles, the angle MBS will be found to be 82° 2′, which measures the arc of the equator counted from noon; that is, a solar time of 5^{h} 27^{m} 6^{s} from noon, at which the Sun reached the already mentioned vertical common to both places — or 2^{h} 3^{m} before the true setting of the Sun. Then, by [proposition] 2 of the same, the base MS will be found to be 69° 4′; therefore the true altitude, OS, of the centre of the Sun at that time — that is, the complement of the arc MS — is 20° 56′.

III. EXEMPLVM. *Third Example.*

Pro abſoluta Altitudine Montis Atho. *For the absolute height of Mount Athos.*

Let us now return to Problem 35, to solve it, and take up again its figure from Problem 33. First, the angle MKN is 68° 45′. For the true altitude of the centre of the Sun was found at the end of the second example to be 20° 56′; when our parallax of 20″ is subtracted from it, and the refraction of 4′ is added, together with the apparent semidiameter of the Sun, 15′ 25″, the visible altitude of the upper limb of the Sun, which sends out the ray MK, comes out 21° 15″ 5″[57]. Therefore the visible distance of the Sun from the zenith of Myrina is 68° 44′ 55″, or in round figures 68° 45′. The angle CKM, therefore, is 111° 15′. The length of the shadow GK, from the foot of Mount Athos to the shore of the island of Lemnos, is given as 86 000 ancient Roman paces, from Solinus, chapter 21; and to these, on our opinion of the circumference of the Earth, correspond 57′ — that is, the angle MCK — even though from the altitudes of the pole and the difference of longitude set by Ptolemy an arc of distance of 1° 2′ 39″ follows. But since, because the places are so close together, it is easy to slip into error over the difference of longitude, we shall rather follow the shadow.

Hence the sum of the angles C and MKC comes out 112° 12′, and the remainder to two right angles is the angle CMK, 67° 48′. With these, and with the side CK of 5174 Roman miles, according to what was said in Book 2, chapter 7, the side CM is found in the triangle MCK, by [proposition] 2 of plane oblique-angled triangles, to be 5209 miles. When CG, 5174 miles, is subtracted, then, there remains the height of Mount Athos, GM, of 35 Roman miles, or 28 Bologna miles.

[Margin: The huge height of Mount Athos.]

Rightly, then, did Bellonius [Pierre Belon] say in his description of Greece: "Athos, of all things the most memorable, huge, famous, very hard of access, and of such height that the winds do not climb to its summit," and so on. On the winds Pomponius Mela agrees with him; and on its circuit of 150 miles, Pliny the Elder, Book 4, chapter 10, who also places on its summit the town of Acrothon, later Uranopolis, or the city of heaven, as being nearer to heaven than to earth, and Apollonia, whose inhabitants were called Macrobii, that is, long-lived. Finally, "not without reason have they noted among the wonders so long a shadow from so great a height," says Solinus, chapter 21.

36. Probl. Montis Altitudinem veſtigare, Datis ſemidiametro Terræ & Atmoſphæræ refractiuæ; Refractione horizõtali Solis, & Temporis differentia, qua Solis centrum priùs oritur vertici, quàm imo montis, in data Poli altitudine datoq. die. Qua occaſione de admirabili altitudine Montis Caucaſi & Caſij. *Problem 36. To track down the height of a mountain, given the semidiameter of the Earth and of the refractive atmosphere, the horizontal refraction of*

[57]so printed, the 15 with the seconds mark; the sum above gives 21° 15′ 5″

the Sun, and the difference of time by which the centre of the Sun rises for the summit earlier than for the foot of the mountain, at a given altitude of the pole and on a given day. On which occasion, on the wonderful height of Mount Caucasus and of Casius.

[Margin: Problem 36.]

From the centre of the Earth, A, describe in the heaven of the Sun, S, a semicircle BCD of a vertical circle; beneath it let there be the semicircle HKL, and another, EFG, described from the same centre; and let BAD be the vertical line, perpendicular to the horizon AC. On this line, if, as Nonius conceives it, HKL is the semicircle of the Earth, the height of the mountain will be HM, whose summit will be lit by the ray SKM, touching the Earth at K, before the foot of the mountain, H, is lit, afterwards, by the horizontal ray. Now if the ray drawn from the centre of the Sun, which lights the summit of the mountain at the very first moment, were unrefracted, as Nonius supposes, as cited in Section 3, Problem 35, the height of the mountain could be found by the solution of a single triangle, as follows. From A let the straight line AS be drawn to the centre of the Sun, S, and to the point of contact K the straight line AK, making the angle AKS a right angle, by [proposition] 18 of the third book of Euclid. Then, from the given altitude of the pole, the declination of the Sun, and the time before noon at which the Sun shines on the summit of the mountain, let the depression of the Sun below the true horizon, which is the arc CS, be found by Problem 223 of Section 2; and thus the angle CAS will be known. But since in the triangle AKS, besides the right angle AKS, the angle S is given, which is very nearly the horizontal parallax of the Sun — for TK is supposed very small with respect to AT, and, because AS is so great, the rays SK and ST are sensibly parallel — if you subtract the angle S from a right angle, the angle KAS will remain known. Subtracting from it the angle CAS already found, the angle KAC will remain; and this, taken in turn from the right angle MAC, makes the angle MAK known. With it, and with the semidiameter of the Earth, AK, in the right-angled triangle AKM, you will find the base AM by [proposition] 3 of plane right-angled triangles; and when the semidiameter AH is subtracted from it, the height of the mountain, HM, will be known. But

[Margin: First method, for unrefracted rays.]

Diagram. A figure set at the right of the right column. A large semicircle stands on the vertical BD at the left, its horizontal radius running from A to C on the right. Down the vertical, from the top, stand B, M, R, O, H, E, A, G and L, with D at the foot. Small arcs about A pass through H and L and through E and G. Lines run from A out to P and to S at the lower right; a line from S rises through K towards M; a dotted horizontal runs from E through K to V on the right.

- B (top of the vertical)
- M (on the vertical, below B): the summit of the mountain
- R (on the vertical, below M)
- O (on the vertical, below R)
- H (on the vertical, below O): the foot of the mountain

- E (on the vertical, below H): an end of the semicircle EFG
- A (on the vertical): the centre of the Earth
- G (on the vertical, below A): an end of the semicircle EFG
- L (on the vertical, below G): an end of the semicircle HKL
- D (foot of the vertical)
- P (upper middle)
- I (just left of K)
- K (middle): the point of contact of the ray SKM
- T (below K)
- F (below T, near A): on the semicircle EFG
- V (right, end of the dotted horizontal)
- C (right, end of the horizontal AC)
- S (lower right): the centre of the Sun since the ray by which the summit of the mountain first sees the Sun is refracted in the denser air around us, we need another method.

[Margin: Order of the problems.]
[Margin: Order of the problems.]
[Margin: Second method, for refracted rays.]

Let HKL, then, be the surface of the denser air, and EFG that of the Earth, and let the angle MAK be found as in the first method. Next: the ray placed immediately below SK does not touch at K, but cuts and enters the surface of the air HKL at a point as close as possible to K; nor does it go straight on to M, but is refracted towards the perpendicular AK, so that the refracted ray is KR and the angle MKR measures the given refraction. Therefore, if the horizontal refraction of the Sun is subtracted from the right angle AKM, the angle AKR will remain. With this angle, with the angle HAK found above, and with the semidiameter of the atmosphere AK, find the side AR by [proposition] 8 of plane oblique-angled triangles; subtract from it the semidiameter of the Earth, AE, and the remainder will be the height of the mountain, ER. If this is equal to or less than the height of the refractive air — that is, than the excess of the semidiameter of the atmosphere over the semidiameter of the Earth — it will be the correct height of the mountain. But if it is greater, that is a sign that the ray KR, having entered near K, afterwards passed out of the denser air into the rarer and underwent a double refraction: one towards the perpendicular at K, the other away from the perpendicular at I. Therefore, when the perpendicular AIP is drawn, the ray KIR, refracted once, will be refracted again from I to O; the ray refracted the second time will be IO; and because the inclination is sensibly the same, the second refraction, OIR, will be as great as the first, RKM, as can be gathered from what was said in Book 5, chapter 5, number 5.

Now the triangle AIK is isosceles. Therefore, by [proposition] 5 of the first [?] book of Euclid, the angle AIK is as great as AKI, which was known above; and its supplement to two right angles, by [proposition] 13 of the first book of Euclid, is the angle AIR. The supplement to two right angles of this angle and of the angle ARI, taken together, is by [proposition] 32 of Euclid the angle RAI.

When the second refraction, RIO, is subtracted from the angle AIR itself, the angle AIO becomes known, in the triangle AIO, in which there is also known the angle OAI, the same as RAI, and the base AI, that is, the semidiameter of the atmosphere. Therefore, by [proposition] 8 of plane oblique-angled triangles, you will find the side AO; subtract from it the terrestrial semidiameter AE, and the corrected height of the mountain, EO, will be known.

EXEMPLVM. *Example.*

In Monte Caucaſo. *On Mount Caucasus.*

Aristotle, *Meteorologica* Book 1, summa 4, chapter 1, text 63, says this: "The Caucasus is the greatest of the mountains towards the summer rising, both in its extent and in its height. Signs of its height are that it is seen both from the so-called Deeps and by those sailing into the lake. Moreover, its summits are lit by the Sun up to a third part of the night, both from dawn and again from evening."

On these words the height of the Caucasus has been discussed learnedly, above all by Jacopo Mazzoni in the *Comparison of Plato and Aristotle*, section 3, chapter 5, and section 4; by Biancani in his *Loca mathematica Aristotelis* on this text; and by Cabeo on Book 1 of the *Meteorologica*, text 63, question 2. Among other heights of the Caucasus, Mazzoni gathered a height of at least 76 or 78 Italian miles, though Biancani contends that he was mistaken, while Cabeo holds it is not greater than 15.

I, however, suppose, first, that Aristotle is not speaking of a third part of the mountain, as it seemed to the interpreter Alexander. This is both because, since the whole height of the mountain was uncertain, not even a third part of it could have been marked out with certainty; and because he would then have said nothing unusual or wonderful, since there is no mountain whose fourth part, then its third part, and afterwards its half is not lit by the Sun before its lowest part. Second, I suppose that Aristotle is speaking of a third part of the night, considered as much at dawn as in the evening twilight. For in the Greek it stands in the genitive case, which the interpreter wrongly turned into the ablative, reasoning that, because the Greeks lack a sixth case and use the genitive in its place, here too τῆς νυκτὸς [Greek: tēs nyktos, "of the night"] was to be turned into the sixth case. Let it stand, then, that the Caucasus is regularly lit through a third part of the night, both before the rising and after the setting of the Sun proper to the horizon on which the Caucasus stands perpendicularly. These things being laid down:

[Margin: Step 1.]

First: the polar altitude of the Caucasus, from the third map of Asia and from Book 5, chapter 9, of Ptolemy's *Geography*, is between 47° and 48°. The shortest night of the whole year (to take it as small as possible), that is, the

solstitial and summer night, is there of 8^h 13^m. Its third part is 2^h $44½^m$[58]; added to the 7^h and 53½ minutes of the semidiurnal arc, this makes 10^h 38^m before noon, to which corresponds an arc of the equator, or angle at the pole of the world, of 159° 32′. Hence, from the complement of the altitude of the pole, which is 42° 30′, and the complement of the Sun's summer declination, which is 66° 30′, there follows, by Problem 223 of Section 2, the depression of the Sun below the true horizon, that is, the angle CAS: 13° 10′ 30″.

[Margin: Step 2.]

Second: in the triangle AKS, right-angled at K, the angle S is given from Book 3, chapter 8 — that is, the horizontal parallax of the Sun at apogee, 27″. If you subtract this from a right angle, SAK remains, 89° 59′ 33″. Subtracting from it the angle CAS, 13° 10′ 30″, leaves KAC, 76° 49′ 3″; and this, taken from the right angle HAC, leaves MAK, 13° 10′ 57″. Then subtract the horizontal refraction of the Sun, MKR — which Tycho, because of the excessive parallax of the Sun at apogee, 2′ 54″, made 34′, as Kepler notes in the *Rudolphine Tables*, precept 197; but had he used our parallax of 27″, he would have made it 31′ 33″, or at most 32′, as we shall teach in Problem 25 of the last section. Subtracting, I say, the refraction of 31′ 33″ from the right angle MKA leaves the angle RKA, 89° 28′ 27″. In the triangle RKA, in which RAK was found to be 13° 10′ 57″, ARK is therefore 77° 20′ 36″.

[Margin: Step 3.]

Third, then: from the angles already mentioned, and taking the smallest height of the refractive air, TK, which Kepler in the *Astronomiae Pars Optica* sets at 2 Italian miles, and AT, from Book 2, chapter 7, at 4139 miles — so that the whole AK is 4141 miles — I find AR to be 4244 miles.

[Margin: Step 4.]

Fourth: with this, with AI, 4141 miles, and with the angle ARK, 77° 20′ 36″, I find the angle AIR to be 89° 28′ 30″; and so the supplement of these to two right angles, that is, the angle RAI, is 13° 10′ 54″.

[Margin: Step 5.]

Fifth, and finally, I subtract the second refraction of the emerging ray, which is 31′ 33″ — that is, the angle OIR — from the angle AIR, found to be 89° 28′ 30″, and AIO remains, 88° 56′ 56″. In the triangle AIO the angle OAI was also 13° 10′ 54″; therefore AOI comes out 75° 46′ 2″. With these angles, and with AI, 4141 miles, AO is found to be 4174 miles; if you subtract from these AE, 4139 miles, there remains the height of the Caucasus, EO: 35 Italian miles. This is not much greater than the height of Mount Athos. It is great indeed, though a greater height is to be asserted on other grounds, by Problem 30, and finally from what will be said in Problems 50 and 51 of Section 6; and it is not unworthy of Aristotle's assertion, whatever has been said against it on this matter so far by those who have not considered the force of refractions. Now if you increase the height of the refractive air, the height of the Caucasus will

[58] so printed; a third of 8^h 13^m is strictly 2^h $44⅓^m$

come out greater by nearly the same increment: if you make it 10 miles, the Caucasus would be 43 miles high; if 20, the Caucasus would be 53 miles high, and so on.

[Margin: The height of the Caucasus.]

From this the height of Mount Casius in Syria comes out much more tolerable. From its summit the rising Sun is seen in the fourth watch of the night, or at the second cockcrow, as Mela, Book 1, chapter 49, Pliny the Elder, Book 5, chapter 22, Solinus, chapter 37, and Ammianus Marcellinus, Book 22, have consistently affirmed. But we have treated these matters at length in the Geographical book, where I also gave several other methods of investigating, by probable conjecture, the height of mountains from the ascent, or from the fall of the rivers running down from them. I only give this warning: some mountains at sea are not seen, although they are higher than others which are seen, because small mountains stand in the way. For let ABC be the hemisphere of the Earth, G its centre, and the eye at B: seeing the summit K of a mountain, it will nevertheless not see the summit Q of the higher FQ, because the lower mountain MN stands in the way, and so on.

[Margin: The height of Mount Casius.]

[Margin: Higher mountains are nevertheless hidden by lower ones.]

Diagram. A small figure at the right of the right column, beside the passage on hidden mountains. A rectangle stands on the base AC, with a semicircle drawn on AC inside it; lines run from G, at the middle of the base, to the upper corners and to points along the top; a mountain profile runs from M at the top to Q at the upper right and down the right side towards C.

- K (upper left corner)
- B (top, middle): the eye
- M (top, right of B): summit of the lower mountain
- Q (upper right): summit of the higher mountain
- N (inside, below M)
- F (inside, lower right)
- A (lower left corner)
- G (base, middle): the centre of the Earth
- C (lower right corner)

37. Probl. Data altitudine abſoluta Montis aut Turris, & ſemidiametro Terræ, inueſtigare Altitudinem abſolutam alterius Mõtis aut Turris, inde ſpectabilis, & notâ diſtantia ab altero. *Problem 37. Given the absolute height of a mountain or tower and the semidiameter of the Earth, to find the absolute height of another mountain or tower visible from it, its distance from the first being known.*

[Margin: Problem 37.]

In the figure of Problem 33, let the height KN be known and GM unknown; and let the distance GK be known in degrees — or indeed in miles, but converted into degrees — so that from the arc GK the angle MCN is known. Then let

there be observed with a quadrant, sighting the summit M from N, the angle MN

[Margin: Order of the problems.]

C. For thus, in the triangle MCN, two angles will be given, C and N — indeed M as well, as their supplement to two right angles — and also the side CN, made up of the height KN and the semidiameter of the Earth CK. Therefore, by [proposition] 2 of plane oblique-angled triangles, the side CM will become known; subtract from it the semidiameter of the Earth, CG, and the height GM will be known.

38. Probl. Data ſemidiametro Terræ, & Diſtantia inter perpendicula duorum Montium, aut Turrium, quorum vertex ex alterius vertice videri poſſit; inuenire Altitudinem aut vtriuſque, aut minimam illius, qui altero maior ſupponatur. *Problem 38. Given the semidiameter of the Earth and the distance between the perpendiculars of two mountains or towers, the summit of one of which can be seen from the summit of the other, to find the height either of both, or the least height of the one that is supposed greater than the other.*

[Margin: Problem 38.]

Look at the attached figure, in which FMG is the hemisphere of the Earth and C its centre; A is the summit of one mountain — say, the greatest of the Graian Alps — and AL its perpendicular height, which is to be continued to C. Let the height of the other — say, of Haemus — be EH, equal to AL, with the line continued as far as C; and through the summits of both let the straight line AMH be drawn, touching at M the orb of the Earth, or the surface of the sea. Let this line be bisected by the perpendicular CM, which makes right angles at M, by [proposition] 10 of the first book of Euclid. For both the arc LME — known from the given distance, converted into degrees and minutes — and the angle ACH will be bisected (although in this figure the opposite is represented, just as the height of the mountain AL is not equal to EH either). Therefore its half, ACM, will be known, in the triangle ACM, right-angled at M, in which the semidiameter CM is given. Hence, by [proposition] 3 of plane right-angled triangles, the base CA will be known; and when CL is subtracted, AL will be known, which is supposed equal to EH. And this is the true height of the mountains AL and HE, which are supposed equal. It may nevertheless be found in the same way even if AMH does not touch the Earth at M, provided that it is parallel to AMH. Taken either way, however, this height is the least of those that ought to be granted to the greater height EO, of the mountain which is supposed greater than the mountain AL.

Diagram. A figure in the left column, at the right of the opening lines of Problem 38. A semicircle stands on the base FG with its centre C at the middle of the base. Straight lines run from C up to A at the upper left, to M at the top of the arc, and to H at the right; the straight line from A through M to H touches the arc at M. A further line runs from A to O at the upper right, and from O down to H. L and E stand where the lines CA and CH cross the arc.

- O (upper right, above H)
- A (upper left): the summit of one mountain
- L (on the arc, below A): the foot of the mountain AL
- M (top of the arc): the point of contact of AMH
- H (right): the summit of the second mountain
- E (on the arc, below H): the foot of the mountain EH
- F (left end of the base)
- C (middle of the base): the centre of the Earth
- G (right end of the base)

E X E M P L V M. *Example.*

De Alpibus & Hæmo. *On the Alps and Haemus.*

[Margin: The height of Haemus and of the Alps.]

Livy, decade 4, book 10, relates that Philip, King of Macedon, having heard that the Alps of Italy could be seen from Haemus, a mountain of Thrace — as Strabo, Mela and Pliny the Elder have it — climbed it; but, his progress hindered by clouds, he came down again disappointed of his wish. Mount Haemus, moreover, is called by Pinto the Chain of the World, or the Silvered Mountain. Now the Alps that divide Italy from Greece, or from Illyricum, are distant from Mount Haemus by at least 10 degrees of a vertical circle; and so the angle ACM is 5 degrees, and CM is 4139 modern Italian miles, from what was said in Book 2, chapter 7. Therefore LA is 15 such miles. But perhaps they are not the Italian Alps, but the mountains of Illyricum or of Dalmatia, which are seen from Haemus.

39. Probl. Data ſemidiametro Terræ, & Altitudine oculi ſupra conuexum Terræ Mariq. commune eleuati : Inuenire Arcum Terreſtrem ab oculo viſibilem verſùs vnam partem, ſeu Diſtantiam viſùs reſpectù obiecti nihil à conuexo prædicto eleuati. *Problem 39. Given the semidiameter of the Earth and the height of the eye raised above the convex surface common to land and sea, to find the terrestrial arc visible from the eye in one direction, or the distance of sight with respect to an object raised not at all above that convex surface.*

[Margin: Problem 39.]

I treated this problem in Book 2, chapter 10, and in its scholium; indeed, I added a table in which, supposing the terrestrial degrees visible from the eye, I found the height of the eye in ancient Roman miles. Now I wish to construct another small table, in which, assuming the height of the eye in both ancient Roman miles and Bologna Italian miles, I have found the arc already mentioned and placed it in the second column. When this arc is doubled, one has the portion of the terrestrial globe visible all round from such an eye — provided, that is, that the view runs free both in one direction and in the other as far as the horizon, and that the horizon is raised neither by mountains, nor by woods, nor by any other swelling of the land, but ends either in the very surface of the

sea, or in a land surface, but one no farther from the centre of the Earth than the surface of the sea.

[Margin: Order of the problems.]

Height of the eye in miles: ancient Roman	Height of the eye in miles: Bologna Italian	Arc of the Earth visible in one direction	Portion of the terrestrial globe visible
1¼	1	1° 12′	2° 24′
2½	2	1° 47′	3° 34′
3¾	3	2° 10′	4° 20′
5	4	2° 30′	5° 0′
6¼	5	2° 48′	5° 36′
7½	6	3° 4′	6° 8′
8¾	7	3° 19′	6° 38′
10	8	3° 34′	7° 8′
11¼	9	3° 46′	7° 32′
12½	10	3° 58′	7° 56′
13¾	11	4° 10′	8° 20′
15	12	4° 21′	8° 42′
16¼	13	4° 32′	9° 4′
17½	14	4° 42′	9° 24′
18¾	15	4° 52′	9° 44′
20	16	5° 1′	10° 2′
21¼	17	5° 11′	10° 22′
22½	18	5° 20′	10° 40′
23¾	19	5° 28′	10° 56′
25	20	5° 37′	11° 14′
26¼	21	5° 45′	11° 30′
27½	22	5° 52′	11° 44′
28¾	23	6° 1′	12° 2′
30	24	6° 9′	12° 18′
31¼	25	6° 17′	12° 34′
32½	26	6° 24′	12° 48′
33¾	27	6° 31′	13° 2′
35	28	6° 39′	13° 18′
36¼	29	6° 46′	13° 32′
37½	30	6° 52′	13° 44′
38¾	31	7° 0′	14° 0′
40	32	7° 7′	14° 14′
41¼	33	7° 13′	14° 26′
42½	34	7° 20′	14° 40′
43	35	7° 26′	14° 52′
44¼	36	7° 32′	15° 4′
45½	37	7° 38′	15° 16′
47	38	7° 44′	15° 28′
48¼	39	7° 50′	15° 40′
50	40	7° 56′	15° 52′
51¼	41	8° 2′	16° 4′
52½	42	8° 8′	16° 16′
54	43	8° 14′	16° 28′

Height of the eye in miles: ancient Roman	Height of the eye in miles: Bologna Italian	Arc of the Earth visible in one direction	Portion of the terrestrial globe visible
55¼	44	8° 19′	16° 38′
56½	45	8° 25′	16° 50′
58	46	8° 30′	17° 0′
59¼	47	8° 36′	17° 12′
60½	48	8° 41′	17° 22′
62	49	8° 47′	17° 34′
63¼	50	8° 52′	17° 44′

[Margin: Order of the problems.]
[Margin: Order of the problems.]

But from Kepler's *Epitome of Astronomy*, page 23.

A visible signal, if it is so many paces high	**is seen in a calm sea from so many Italian miles**	**A signal so many paces high**	**is seen from so many Italian miles**
⅕	1	21	12
⅗	2	37	16
1	3	59	20
2	4	84	24
4	5	114	28
5	6	131	30
7	7	149	32
9	8	189	36
12	9	233	40
15	10	364	50

40. Probl. Data Altitudine obiecti, puta Montis, Turris, aut mali nautici; ac Semidiametro Terræ. Ex quanta diſtantia videri poſſit eius apex ab oculo nihil à terra mariue eleuato : aut Quanta debeat eſſe altitudo obiecti, à tali oculo viſibilis. *Problem 40. Given the height of an object — say, of a mountain, a tower, or a ship's mast — and the semidiameter of the Earth: from how great a distance its top can be seen by an eye raised not at all above land or sea; or how great the height of an object visible from such an eye must be.*

[Margin: Problem 40.]

On this too enough was said in Book 2, chapter 10, and in its scholium 2, to which the preceding tables also contribute.

41. Probl. Data altitudine tam Obiecti quàm oculi ſupra Terram, inuenire maximam Diſtantiam, ex qua videri poterit obiectum : Si detur ſemidiameter Terræ. *Problem 41. Given the height both of the object and of the eye above the Earth, to find the greatest distance from which the object can be seen, if the semidiameter of the Earth is given.*

[Margin: Problem 41.]

For this problem likewise see Book 2, chapter 10, scholium 3.

42. Probl. Data diſtantia Fontis à mari, inuenire Altitudinem Libramenti, ſeu fontis, neceſſarium, vt flumen per eumdem tramitem rectum decurrens, poſſit ad Mare deſcendere. *Problem 42. Given the distance of a spring from the sea, to find the height of fall, or of the spring, needed for a river running down by the same straight course to be able to descend to the sea.*

[Margin: Problem 42.]

See Book 2, chapter 10, scholium 4, with the warning given there, and chapter 4 of the same book, number 9.

43. & 44. Probl. Data diſtantia duorum Locorum, & Altitudinibus Poli, inuenire Differentiam Longitudinis Geographicæ,Angulos Poſitionis, & Azimutha communis Verticalis. *Problems 43 and 44. Given the distance of two places and their altitudes of the pole, to find the difference of geographical longitude, the angles of position, and the azimuths of the common vertical.*

[Margin: Problem 43.]

In the figure immediately following, the poles of the world are A, the southern, and B, the northern. Through them are drawn the semicircles of the meridians, or of declinations, BEA, the western, and BQA, the eastern, intercepting the arc of the equator EQ — that is, the difference of longitude, or distance of the meridians, which either angle, A or B, measures.

[Margin: First case.]

Let there be on these, then, two northern places, C and D, through which let the arc CD of a vertical circle be drawn. For in the triangle CBD the angle B, that is, the difference of longitude, can be found by [proposition] 14 of spherical oblique-angled triangles: for there are given the distances of the zeniths from the pole B — that is, the complements of the polar altitudes, BC and BD — and the base CD, from the distance of the places, which is to be converted from miles into degrees according to the opinion chosen in Book 2, chapter 7. Likewise, if two southern places are given, such as F and G, the complements of the polar altitudes, AF and AG, will be given, and the base from the distance FG; with these, find the angle A.

[Margin: Second case.]

Next let one place be given on the equator, as at E, and the other on this side of it or beyond, as at F; then use the triangle EAF, in which there are given AF, the complement of the altitude of the pole, AE, a quadrant, and the base EF, from the known distance; and so, by the same [proposition] 14, finding [?] the angle A, which measures the difference of longitudes.

[Margin: Third case.]

Finally, let two places be given, one on this side of the equator, at C, the other beyond it, at F; then use the triangle CAF, and find by the same [proposition] 14 the angle A, since there are given the base CF, from the distance converted

into degrees, AF, the complement of the altitude of the pole, and AC, made up of the quadrant AE and of the latitude, or polar altitude, EC.

The angles of position, however — say at C and at D, and so on — you will find by the same [proposition] 14, if you make the base the arc opposite the angle sought; and their supplement to 180° will be the azimuth.

[Margin: Problem 44.]

Diagram. A figure at the right of the right column, beside the cases of Problems 43 and 44. Two semicircles of meridians form a tall lens, meeting at B at the top and at A at the foot. On the left meridian stand, from the top, C, E and G; on the right meridian D, Q and F. Arcs join C to D, E to Q (the equator), and G to F; further arcs run from C down to F and from E to F.

- B (top): the northern pole
- D (right meridian, upper): a northern place
- C (left meridian, upper): a northern place
- E (left meridian, middle): on the equator
- Q (right meridian, middle): on the equator
- F (right meridian, lower): a southern place
- G (left meridian, lower): a southern place
- A (foot): the southern pole

45. & 46. Probl. Data duorum locorum Altitudine Polari, & Differentia Longitudinis, Inuenire Diſtantiam eorumdem & Azimutha communis Verticalis. *Problems 45 and 46. Given the polar altitude of two places and the difference of longitude, to find their distance and the azimuths of the common vertical.*

[Margin: Problem 45.]

In the preceding figure, if the places are C and D, the complements of the polar altitudes, BC and BD, are given, and from the difference of longitude the angle B; therefore, by [proposition] 4 of spherical oblique-angled triangles, the base CD becomes known, to be converted afterwards into miles according to the opinion on the size of the Earth's circumference chosen in Book 2, chapter 7. But if one place is on this side of the equator and the other beyond it, as C and F, you can use the triangle CBF, in which there are given the angle B, that is, the difference of longitude, BC, the complement of the altitude of the pole, and BF, made up of the quadrant BQ and of the latitude, or altitude of the pole, FQ. You can also use the triangle CAF, in which there are given, from the difference of longitude, the angle A, AF, the complement of the altitude of the pole, and AC, the sum of the quadrant AE and of the altitude of the pole, or EC; and in either triangle you can find the distance CF by the same [proposition] 4.

The angles of position, however — that is, those situated at the base CD, or CF — you will find by [proposition] 5 or 6 of spherical oblique-angled triangles; and their supplements to 180° will be the azimuths.

[Margin: Problem 46.]

47. & 48. Probl. Data vnius loci Altitudine Poli & Diſtantia ab altero loco, cum Differentia Longitudinis, Inuenire alterius loci Altitudinem Poli, Poſitionis angulos, & Azimutha verticalis per vtrumque locum ducti. *Problems 47 and 48. Given the altitude of the pole of one place and its distance from another place, together with the difference of longitude, to find the altitude of the pole of the other place, the angles of position, and the azimuths of the vertical drawn through both places.*

[Margin: Problem 47.]

In the preceding diagram, let there be two northern places, C and D; and in the triangle CBD let there be given the altitude of the pole of the place C, and hence its complement BC; the distance CD, converted into degrees; and the angle opposite it, B, that is, the difference of longitude. For by [proposition] 2 of spherical oblique-angled triangles you will find the base BD, which is the complement of the altitude of the pole for the place D; by [proposition] 1 of the same, the angle of position CDB; and by 3, the angle of position DCB. And you will proceed by the same rules if the two places are C and F, or E and F. Further, the supplement of the angle of position to 180° is the azimuth, and sometimes the angle of position itself coincides with the azimuth.

[Margin: Problem 48.]

49. Probl. Meridianam lineam inuenire ope duarum Vmbrarum Rectarum. *Problem 49. To find the meridian line by means of two right shadows.*

[Margin: Problem 49.]

I have already given this problem, with a suitable diagram, in Book 1 of this work, chapter 10, adding two scholia for finding it more exactly. Go back there, then, and remember that the Sun must be so high as to be clear of the sensible refractions, that is, higher than 25°; and take care not to neglect even the least thing in this.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

50. 51. 52. Probl. Meridianam Lineam per vmbras rectas aut aliter inuentam examinare. *Problems 50, 51, 52. To examine a meridian line found by right shadows or otherwise.*

[Margin: Problem 50.]

In Book 1, chapter 10, scholium 3, we indicated three ways of making this correction. The *first* is by equal altitudes of the Sun, one before noon and one after noon, especially on the day of the summer solstice, and by equal azimuths on either side, counting them from the meridian line, which is as yet uncertain. For if, at an equal distance from the meridian — that is, in an equal azimuth on either side — the Sun's afternoon altitude is found exactly equal to its morning altitude, that is evidence that the meridian line is correct. Otherwise it must be corrected: the part of it facing south must be made more easterly if the Sun's

morning altitude is greater, or more westerly if it is less, than the afternoon altitude. How much it must be changed I shall teach in Problem 53.

[Margin: Problem 51.]

Second, you will achieve the same thing more surely on any clear night if, in an equal azimuth on either side, counted from the meridian once it has been found, you observe with an exact quadrant two altitudes of the same fixed star: one before the star's transit through the meridian, the other after the transit. But you must choose a star that is higher than 20 or 25 degrees above the horizon, so that it is not subject to sensible refraction, since the evening refraction is usually less than the morning one and the two do not exactly compensate each other. Nor, however, should the star be near the meridian; otherwise the increase and decrease of its altitude could become imperceptible. Now if the altitudes already mentioned are equal, the meridian line will be exact; but if they are unequal, its part towards the south must be drawn back towards the quarter in which the altitude was greater. How much it must be drawn back I shall say in Problem 53.

[Margin: Problem 52.]

Third, with the help of a pendulum that shows the times of the primum mobile precisely, such as we described in Book 2, chapter 20, let the time be counted from an eastern altitude of a fixed star, taken by another companion (the star appearing far from the horizon and from the meridian, but farther from the horizon), until its transit through the plane of the supposed meridian, indicated by two perpendicular threads or by that thread triangle which we described in Book 5, chapter 15, problem 8. Next let the time be counted from the transit through the supposed meridian until the western altitude of the same star equal to the first, observed by the companion as above. Or at any rate, when the later time has become equal to the earlier, let the star's altitude be taken at once, with the instrument already prepared, so that no delay slips in. For if equal altitudes and at the same time equal times are found, they will be evidence of a perfectly correct meridian.

[Margin: Correction of the meridian line.]

Otherwise the southern part of the meridian line must be bent towards the side in which the greater time was counted — bent, I say, by as many degrees or minutes as correspond on the equator to the half-difference of the times counted. But if you have used equal times, and the altitudes taken at their ends are unequal, you will bend the meridian towards the quarter in which the altitude was greater. How much it must be bent I shall teach in the problem immediately following.

53. Probl. Meridianæ lineæ Fallaciam per duas altitudines Solis aut Stellæ Fixæ in Azimuthis, aut temporibus æqualibus captas, vt ſuprà inuentam Corrigere. Data præter Altitudines & Azimuthum, Declinatione Sideris.

Problem 53. To correct the error of a meridian line found, as above, by two altitudes of

the Sun or of a fixed star taken in equal azimuths or at equal times; given, besides the altitudes and the azimuth, the declination of the heavenly body.

[Margin: Problem 53.]

In the following figure let the supposed meridian be AVQ, the horizon HOR, and the equator AOQ, whose pole is P; let the vertex be V, and VT the quadrant of a vertical circle, in which the centre of the Sun or star stands once at G and once at F, now on this side of the meridian, now beyond it. And through the centres of the body let the quadrants of declination PGM and PFK be drawn from P. Now suppose it is established by Problem 50 or 51 that the meridian line must be corrected, because in equal azimuths the body's true altitude TG was found greater than its altitude TF.

[Margin: Step 1.]

First, in the triangle VGP, find the angle VPG by the first [proposition] of spherical oblique-angled triangles. For in it are given the side GP, which is the complement of the body's declination; the opposite angle GVP, which is the supplement to two right angles of the given azimuth HVT; and the side opposite the angle sought, namely VG, which is the complement of the true altitude. For I suppose that the apparent altitude, if it is the Sun's, has been corrected by adding the parallax, and that the body is clear of the sensible refractions.

Diagram. A circle representing the supposed meridian, with V at the top. A and P stand on the circumference at the upper left and upper right; H and R at the left and right ends of the horizon line, which passes through O; Q on the circumference at the lower right. A line runs from V down through F and G to T near the horizon; lines run from P through F towards K and through G towards M; a line runs from A through O towards Q.

- V (top): the vertex
- A (upper left, on the circle)
- P (upper right, on the circle): the pole
- F, G (inside, on the line VT): the two positions of the body
- K, M (near the horizon line, left of centre): the feet of the quadrants of declination PFK and PGM
- H (left) and R (right): the ends of the horizon HOR
- O (on the horizon line)
- T (on the horizon line): the foot of the vertical quadrant VT
- Q (lower right, on the circle)

[Margin: Step 2.]

Second, in the triangle VFP, find the angle VPF by the same first [proposition], since in it are given FP, the complement of the body's declination; the opposite angle FVP, from the equal and known azimuth, as above; and VF, the complement of the true altitude. Then subtract the smaller angle VPF from the greater VPG, and keep half the difference, or remainder.

[Margin: Step 3.]

Third, taking a centre on the meridian line and describing a circle of large radius in the plane of the horizon, divide the circle so that near the meridian to be corrected you have some degrees on either side, and single minutes of the degrees, by transversal lines, as is done in Tycho's quadrants, or in ours, to be described in volume 2, book 2. And in the arc through which the meridian line passes, take as many degrees or minutes as there are in the half-difference of the angles found in steps 1 and 2; and, at that distance taken from the meridian, mark two points in the arc, one on either side. Now if the altitude of the body was greater towards the east than towards the west, draw a straight line through the eastern point marked and through the centre of the circle; but if the altitude was greater towards the west, draw it through the western point. That line will be the corrected meridian line.

But if you have used Problem 52 with unequal times, convert the half-difference of the time into degrees of the equator, and you will have the half-difference of the angles at the pole P, to be applied as above. For although the angles made at the meridian — one by the circle of declination, the other by the vertical circle drawn through the centre of the same body at the same time — are not equal, yet when one of them is corrected the other is corrected too.

Finally, if you have used Problem 52 but with equal [?] times, and have found unequal altitudes, choose either of the altitudes, say TG, and in the triangle VGP find, by the first [proposition] of spherical oblique-angled triangles, the azimuthal angle GVP; for there are given GP, the complement of the declination; GV, the complement of the altitude; and the angle GPV, from the time counted and converted into degrees of the equator. Then in the triangle VFP, using the other altitude, find the angle FPV as above, and, subtracting the smaller from the greater, apply the half-difference to correcting the meridian line as above.

54. Probl. Meridianam Lineam inuenire, Datis Declinationibus & Aſcenſionibus Rectis duarum Stellarum in eodem Verticali apparentium, vnà cum vnius Altitudine vera. *Problem 54. To find the meridian line, given the declinations and right ascensions of two stars appearing in the same vertical, together with the true altitude of one of them.*

[Margin: Problem 54.]

Although anyone about to observe the declinations and right ascensions of the stars must presuppose a meridian line, yet if someone trusts declinations observed by another astronomer, or indeed his own, but has lost the meridian he once had, he will be able to test the usefulness of this problem. Let there be, then, in the preceding figure two stars, F and G, appearing in exactly the same vertical TV (which can best be observed with the help of our thread triangle, described in Book 5, chapter 15, problem 8); and then let the altitude of one of them, say of the star F, be taken by you or by your companion; and meanwhile let the thread triangle be kept in the same position, or that plane by whose help was determi- [Margin: Order of the problems.]

ned the vertical in which the two stars appeared.

[Margin: Step 1.]

First, then, in the triangle FPG find the angle GFP by [proposition] 5 or 6 of spherical oblique triangles. For there are given the complements of the declinations, FP and GP, and the included angle FPG, since it is the difference of the given right ascensions.

[Margin: Step 2.]

Second, in the triangle VFP there are likewise given the sides FP, the complement of the declination of the star F, and FV, the complement of the true altitude — that is, of the altitude corrected by subtracting the refraction, if the star F was subject to it. The included angle VFP is also given, since it is the supplement to two right angles of the angle GFP already found. Therefore, by [proposition] 5 or 6 of spherical oblique triangles, find the azimuthal angle FVP, whose supplement to 180 degrees will give the angle HVT, that is, the azimuth itself.

[Margin: Step 3.]

Third, draw a straight line through the plane in which the stars already mentioned appeared, and, taking a centre on that line, describe a circle with a radius of two or three feet, such that at least those of its degrees which you judge to be nearer the meridian are subdivided into minutes. For if in this circle you count off from the plane of the vertical the degrees and minutes found in the angle HVT, and draw a straight line through the end of that arc and the centre of the circle, that will be the meridian line. The arc already mentioned, however, must be counted towards the west if the stars were eastern, but towards the east if they were western.

55. Probl. Meridianam Lineam inuenire per Maximam vtrimque Digreſſionem ſtellæ Fixæ ſemper apparentis. *Problem 55. To find the meridian line by the greatest digression, on either side, of a fixed star that is always visible.*

[Margin: Problem 55.]

Tycho strongly recommends this method, as one he used himself. Let some fixed star near the pole be chosen, then, both of whose digressions, to the east and to the west, can be observed on the same night; and let it be observed on either side with the help of an azimuthal quadrant or of our thread triangle (on which see Book 5, chapter 15, problem 8). On either side let straight lines be marked in the planes of the two verticals in which the star reached its greatest digression; for these lines, when produced, will intersect at a point. Taking that point as centre, let an arc be described that is contained between the two lines, and let the arc be divided in half. For if you draw a straight line through the point of this division and through the centre of the arc, that will be the meridian line. Related to this method is another, but far too laborious, which Father Emmanuel Maignan gives on page 137 of his *Perspectiva horaria*.

56. Probl. Meridianam Lineam inuenire duabus eiuſdem ſideris Altitudinibus æqualibus eadem die vel nocte captis. *Problem 56. To find the meridian line by two equal altitudes of the same heavenly body taken on the same day or night.*

[Margin: Problem 56.]

With an azimuthal quadrant set exactly in place, take a rising altitude of some star by night, or of the Sun by day, and draw a straight line through the plane of the vertical in which the body stood. Then, when you are certain from the magnetic needle or some other indication that the star has passed the meridian and is inclining towards the west, follow the same star, taking its altitudes again and again until you reach the one that is equal to the first, rising one; and through the plane of this vertical draw a second straight line until it intersects the first, and do the rest as was said at the end of the preceding problem. You will do better, however, if you use a fixed star, or the Sun indeed but on a solstitial day, clear of the danger of refractions, and determine the planes of the verticals with the help of our thread triangle described in Book 5, chapter 15, problem 8; or if, with the help of a double plumb line at rest, you mark two solar shadows intersecting each other when the Sun has reached an equal altitude on either side.

57. Probl. Meridianam Lineam inuenire, Datis Altitud. Poli ac Stellæ, huiuſq. Declinatione,& ſitu circuli altitudinis ſtellæ. *Problem 57. To find the meridian line, given the altitudes of the pole and of a star, the star's declination, and the position of the star's circle of altitude.*

[Margin: Problem 57.]

In the figure of Problem 53 let the star be F; and by the thread triangle (on which see Book 5, chapter 15, problem 8), or by an azimuthal quadrant, let a plane be determined. When the star has come to that plane, let its altitude be taken, and let a straight line be drawn marking the plane. Then, having taken the altitude TF of the star F, find in the triangle FVP the angle FVP by [proposition] 14 of plane oblique-angled triangles. For FV is given, the complement of the star's altitude; FP, the complement of the declination; and VP, the complement of the altitude of the pole. The supplement to 180 degrees of the angle FVP will be the azimuthal angle HVT; do the rest as in Problem 54, step 3.

58. Probl. Meridianam Lineam deſignare, Data duorum locorum, quorum alter ex altero ſit ſpectabilis, Altitudine Poli, & aut Diſtantia in Milliaribus, aut Differentia Longitudinis. *Problem 58. To lay out the meridian line, given the altitude of the pole of two places, one of which can be seen from the other, and either their distance in miles or their difference of longitude.*

[Margin: Order of the problems.]

[Margin: Problem 58.]

Consider the figure placed below, in which there are two places on the Earth, B and C. Through them let quadrants of declination PBA and PCQ be drawn from P, the pole of the equator AQ, and let those places be joined by the arc BC of a vertical circle. The angle PBC must now be found, if the place where the observer stands is B, or PCB if it is C; and this by [proposition] 5 or 6, or by 14, of spherical oblique triangles. For in the triangle BPC are given the sides PB and PC, since they are the complements of the altitudes of the pole, and either the angle P, which measures the difference of longitude, or the arc BC, which is the distance converted into degrees and minutes.

Diagram. A spherical triangle with the pole P at its apex and the curved base AQ (the equator) below. B stands on the side PA, C on the side PQ. A straight line runs from B through K up to F; the chord BOC and the arc BC join the two places; a small arc runs from G through K to O about the centre B.

- P (apex): the pole of the equator
- A (lower left) and Q (lower right): the ends of the equator AQ
- B (on PA): the observer's place
- C (on PQ): the other place
- F (above K): the end of the meridian line BF
- K (on the arc GKO and the line BF)
- O (on the chord BC and the arc GKO)
- G (left end of the arc GKO)

Now suppose the observer's place is B, and at it let some plane standing perpendicular to the horizon be directed towards C — or a thread triangle (on which see Book 5, chapter 15, problem 8), or the rule of a dioptra — so that the straight line BOC can be drawn, representing the chord of the arc BC. Then, taking B as centre and describing from it the arc GKO, let the arc KO be taken in it, of as many degrees and minutes as were found in the angle PBC. For if you draw the straight line BF through the points B and K, that will be the meridian line.

59. Probl. Meridianam Lineam inuenire per vnicum Vmbræ punctum, Data Poli Altitudine, & Declinatione Solis, ac Altitud. *Problem 59. To find the meridian line by a single point of shadow, given the altitude of the pole and the declination and altitude of the Sun.*

[Margin: Problem 59.]

From the centre A, on a precisely horizontal plane, describe the circle BCI with as large [?] a radius as you have found, from a test made beforehand on the preceding day, to be required for the shadow of the style AB to reach its circumference; and from A erect the style AB perpendicularly.

Diagram. A large circle with V at the top, P at the upper left and Q at the upper right on the circumference; D at the left and R at the right joined by a horizontal diameter through A; K on the circumference at the lower right and E at the lower left. A small inner circle about A carries B above A, C at its upper left, L to the left, I below and M to the right, with a dotted line from C

through A towards I. Lines run from V through S to K, from P to S, from S through G, and from Q through G towards H; curved arcs run from D through H towards E and from E through M towards R; B is joined to S and to C.

- V (top): the vertex
- P (upper left) and Q (upper right), on the circle
- S (upper right, inside): the Sun
- B (above A): the tip of the style AB
- C (upper left of the inner circle): the point of the shadow on the circumference
- G (right, inside)
- D (left) and R (right): the ends of the horizon DHR
- L, A, M, I: on and about the inner circle BCI, A being its centre
- K (lower right, on the circle): the foot of the vertical circle VK
- H (lower, inside): on the horizon DHR
- E (lower left)

Then, when about two or three hours before or after the moment of noon — known in any way whatever — the Sun S, raised above the horizon DHR by the arc KS of the vertical circle VK, has cast the ray SBC through the tip B of the style to C, and the shadow AC has reached the circumference of the circle already mentioned, mark the point C on it; and at the same moment let a companion be present to take the exact altitude of the Sun. But if it does not reach the circumference, but falls inside or

[Margin: Order of the problems.]

[Margin: Order of the problems.]

outside it — the tip of the shadow, that is — you can nevertheless, once the end of the shadow has been marked, mark on the circumference itself, by means of a thread stretched from the centre A of the style through that end, the point C, which lies in the same straight line with the tip of the shadow and with the centre of the style. Indeed, with the style removed, you can mark with the help of the thread the point I, diametrically opposite the point C, although this is not at all necessary.

With these points marked, let the meridian DVRE be understood as described from A, as from the centre of the Earth, with V as its vertex, and P the pole of the equator EHQ; and from P let the quadrant of declination PSG be drawn through the centre of the Sun S. For in the triangle PSV the three sides will be given: PS, the complement of the Sun's declination; VS, the complement of the altitude taken, but corrected by adding the parallax and subtracting the solar refraction; and PV, the complement of the altitude of the pole. Therefore, by [proposition] 14 of spherical oblique-angled triangles, the azimuthal angle SVP will become known to you. It measures on the horizon the arc DHK, whose supplement to a semicircle is the arc KR, similar to the arc CL or IM, taken on the circumference of the circle BCI beginning from the point marked by the shadow. For if you draw the straight line LM through the end L or M of the arc already mentioned and through the centre of the style, that will be the

meridian line, coinciding with the horizontal meridian DAR. But it must be possible to take the arc already mentioned accurately from the circle already mentioned, or from another one subdivided into single minutes; and for the day on which the operation is to be done you must have the Sun's declination known, not only for the noon of that day but also for the hour before or after noon, known approximately — especially if the day is very distant from both solstices.

Another method, by means of the almucantar circle [circuli Almuncatarath] — certainly ingenious, but mechanical, and not attaining trigonometric precision, though very sure for gnomonic operations — is taught by Father Emmanuel Maignan in his *Perspectiva horaria*, page 128. But for astronomical operations, and indeed for nautical ones, the more suitable method is the one I gave above, taken from Athanasius Kircher's *Magnes*, page 500, but developed and perfected. For a plane, moreover, that is either perpendicular to the horizon or inclined, the same Father Maignan suggests certain mechanical methods on pages 131 and 133.

Problema. 60. 61. 62. 63. 64. Meridianam Lineam inuenire alijs 5. Modis, licèt non omnibus ita exactis. *Problems 60, 61, 62, 63, 64. To find the meridian line by 5 other methods, though not all of them so exact.*

[Margin: Problem 60.]

First, it can be done by the shadow of a gnomon together with an astrolabe or analemma, as our Clavius teaches in Book 1 of the *Gnomonice*, proposition 23, in the scholia; since this is readily available, there is no reason for us to copy it out here. Or it can be done by the method that Father Emmanuel Maignan gives on page 134 of the *Perspectiva horaria*.

[Margin: Problem 61.]

Second, by the shadow of a gnomon or of a style erected perpendicularly on the plane of the horizon, observed on the days of the equinox. For if at different hours two or more points marked by the tip of the shadow are noted, and a straight line is drawn through them, it will mark out the equator or a parallel of the equator. Therefore, if another line is drawn to it at right angles, that will be the meridian line, as Father Emmanuel Maignan demonstrated in his *Perspectiva horaria*, page 136. Nor do refractions stand in the way, if the observed points are noted when the Sun is very high.

[Margin: Problem 62.]

Third, the decrease of the right or versed shadow can be observed in the morning until the shadow becomes least. For if you draw [?] a straight line through the end of the least shadow and its beginning, that is, the centre of the style, that will be the meridian — found, however, rather crudely, because for some time near noon the decrease or increase of the shadow is not discerned. Or [it can be done] if the points of the rising and setting of the Sun on the same day are observed by two planes, or by straight lines intersecting each other on

the horizontal plane, and the arc described from the point of intersection and intercepted between them is divided in half.

[Margin: Problem 63.]

Fourth, it can be done with the help of the magnetic needle, but only loosely [eminus], because in many places these needles decline from the true position of the meridian, and much more so if iron objects are not far off, or stones approaching the nature of the magnet, which draw the needle towards themselves. If, however, you had once obtained a meridian line, and with its help the magnetic declination, you could afterwards recover the meridian if it were lost, taking account of the declination at least in the nearest years; for there are some who think that the declination in the same place varies after many years.

[Margin: Problem 64.]

Finally, when trees are cut on a plane parallel to the horizon, veins of the tree are seen on the surface of the trunk, arranged as if in a circle, in such a way that towards one pole of the world the intervals between their circumferences are widest, and towards the other narrowest. Therefore a straight line drawn through the narrowest and the widest position of them will indicate the meridian, after a fashion.

65. & 66. Probl. Meridiana Linea inuenta, quotuis alias in plano Horizontis aut Verticalis parietis deſignare. *Problems 65 and 66. Once a meridian line has been found, to lay out as many others as you like on the plane of the horizon or of a vertical wall.*

[Margin: Problem 65.]

First, if you want to lay out another meridian line on the plane of the horizon, but near the first, draw a straight line parallel to the first by [proposition] 31 of Book 1 of Euclid. But if you want one far off, erect on the horizontal plane a gnomon as exactly perpendicular to the horizon as possible; and when a companion gives the signal that the shadow is now falling on the first line, you will mark the end of the shadow of your gnomon, and through that point and the centre on which the side of the gnomon stands perpendicularly you will draw a straight line; for that will be the meridian line. And the same is done if the point of noon is indicated by some sundial.

[Margin: Problem 66.]

Second, if you want a meridian line on the plane of a wall perpendicular to the horizon, fix a style in it; and when the moment of noon arrives, as indicated by another meridian line already shaded by its own style, or by a sundial, mark on the wall the point that ends the shadow of the style. For if you draw through that point a straight line perpendicular to the horizon, that will be the meridian with respect to the style already mentioned.

Probl. 67. 68. 69. 70. Altitudinem Poli per Altitudines Meridianas Solis aut Fixæ alicuius inquirere modis 4.abſq. Trigonometria. *Problems 67,*

68, 69, 70. To find the altitude of the pole by the meridian altitudes of the Sun or of some fixed star, in 4 ways, without trigonometry.

It must be noted that every altitude of the Sun observed with a quadrant, an astronomical square, a sextant, a ring or an astrolabe must be corrected by adding the parallax due to that altitude and subtracting the refraction, if there is any, so that the apparent altitude may be converted into the true one. This being laid down:

[Margin: Problem 67.]

First, let the meridian altitude of the Sun's centre be observed on the two solstitial days of the same year, or on two that follow each other most closely; and, both being corrected, let the smaller be subtracted from the greater, and half the difference added to the smaller of the meridian altitudes. This gives the altitude of the equator; if you subtract it from 90 degrees, the altitude of the pole will remain — the more correct, the nearer to noon the moment of the solstice fell, which you will learn from the ephemerides.

[Margin: Problem 68.]

Second, if the obliquity of the ecliptic, that is, the Sun's greatest declination, is known and certain from elsewhere, it is enough to observe the meridian altitude of the Sun's centre at either solstice, but chiefly at the summer one, and to correct it as above. For the obliquity of the ecliptic, added to the Sun's true winter altitude or subtracted from its summer one, will give the altitude of the equator; and this, subtracted from 90 degrees, will give the altitude of the pole.

[Margin: Problem 69.]

Third, if it is the day of an equinox, the altitude of the Sun's centre, observed and corrected as above, will give the altitude of the equator, provided the equinox falls at noon (on which consult the ephemerides or astronomical tables). Otherwise, for as many hours before or after noon as it falls, you must add as many minutes to the Sun's altitude already corrected, or subtract them: add them, that is, if the vernal equinox falls after noon or the autumnal before noon; but subtract them if the vernal falls before, or the autumnal after, noon. Then subtract this altitude, thus corrected again, from 90 degrees, and the altitude of the pole will remain.

[Margin: Problem 70.]

Fourth, on any day, having observed and corrected the meridian altitude of the centre of the Sun or of a fixed star, find from an ephemeris or by tables the Sun's place at that noon, and with it, in the tables indicated in section 2, problem 15, find the Sun's declination; and this (as also the declination [?] of the fixed star, already known), if it is southern, add; if northern, subtract from the true

[Margin: Order of the problems.]

[Margin: Order of the problems.]

altitude of the Sun or of the fixed star, and the result will be the altitude of the equator, which is to be subtracted from 90 degrees so that the altitude of the pole remains.

Probl. 71. 72. 73. 74. Altitudinem Poli ex Altitudine Meridiana Stellæ Circumpolaris deducere modis 4. abſq. Trigonometria. *Problems 71, 72, 73, 74. To derive the altitude of the pole from the meridian altitude of a circumpolar star, in 4 ways, without trigonometry.*

[Margin: Problem 71.]

The *first* and most excellent way is to observe the greatest and the least meridian altitude of the same fixed star — one that turns about the pole in such a way that it never sets, and that at its least altitude is so far removed from the horizon as to be beyond the hazard of sensible refractions; or at any rate the refraction must be known, so that it may be subtracted from that altitude. Then subtract the least of the altitudes from the greatest, divide the resulting difference in half, and add half of it to the least of the altitudes: this gives the altitude of the pole. It is not necessary that both altitudes be taken on the same night, but it is expedient that they be taken within the same year. You have a diagram of this practice and some examples in Book 1, chapter 12, and in Book 6, chapter 4, in the Appendix.

[Margin: Problem 72.]

Secondly, if the declination of some fixed star is certain, it is enough to observe its greatest meridian altitude above the pole, or its least below it. For if the star is between the vertex and the equator, its declination — added to the greatest altitude if it lies towards the hidden pole, or subtracted if towards the visible pole — will give the altitude of the equator; and this, subtracted from 90 degrees, will leave the altitude of the pole. But if the star is between the visible pole and the vertex, the complement of the declination to a quadrant, subtracted from the greatest altitude, will leave the altitude of the pole. And if it is the least of the meridian altitudes, the complement of the declination to a quadrant, added to the altitude, will produce the altitude of the pole.

[Margin: Problem 73.]

Thirdly, if one knows the approach by which the Pole Star each year approaches the pole of the world, or recedes from it, and its distance from the pole of the world at some past moment of years gone by, it is enough to observe its meridian altitude, either the greatest or the least. For if you subtract its distance from the pole from the greatest altitude, or add it to the least — that distance being first corrected as the annual approach or recession requires — you will have the altitude of the pole. For an example of this, see Book 6, chapter 4, in the Appendix.

[Margin: Problem 74.]

The *fourth* way, though not so exact, and serving only this present century, is to take the altitude of the Pole Star at the moment when the Fidicula of Lyra, or the bright star in the mouth of Canis Major, crosses the meridian; for at that time [?] the altitude of the pole is as great as that of the Pole Star.

Probl. 75. 76. 77. 78. Data Proportione Vmbræ ſeu Rectæ ſeu Verſæ ad Gnomonem, inquirere Altitudinem Veram Solis: & viciſſim Data

vera altit Solis, Vmbræ rationem ad Gnomonem: & Triplicem Tabulam Gnomonicam conſtruere, Erroreſq. multorum in hoc Geometrarum Corrigere. *Problems 75, 76, 77, 78. Given the proportion of the shadow, whether right or versed, to the gnomon, to find the true altitude of the Sun; and conversely, given the true altitude of the Sun, [to find] the ratio of the shadow to the gnomon; and to construct a threefold gnomonic table, and to correct the errors of many geometers in this matter.*

The first table is for the right shadow [Vmbra Recta]. For it, look at the following figure. In it let the horizontal line be AB, and erected perpendicularly on it the style, or gnomon, AC, through whose tip C there passes, from the upper limb D of the Sun S, the ray DCE, which ends the shadow AE. This is called the right shadow because the gnomon is upright [rectus] to the horizon. The gnomon AC must be of known size, say one or two feet; or, if it is too tall for the end of its shadow to be distinguished, a globe must be fixed on top of the tip C of the gnomon. For it was for this purpose that Manilius the mathematician placed a gilded ball on the obelisk that stood in the Campus Martius at Rome, to gather the shadow, as Pliny the Elder relates (Book 36, chapter 10). Furthermore, the gnomon AC, or a ruler or straight line equal to the gnomon, must be divided into equal parts as small as possible, say 12, or 60, or 100, or 1000, etc. — and indeed the shorter the gnomon, the smaller the parts — so that with the help of very sharp compasses the length of the shadow AE can be determined in parts of the kind into which the gnomon is divided, that is, the proportion of the shadow to the gnomon. Similarly, if a gnomon HF is fixed at right angles into a wall FG upright to the horizon, and the ray MHV from the Sun's lower limb M passes through its tip H, FV will be the versed shadow [Vmbra verſa]. So that it can be known, the gnomon HF must be divided into equal parts, 12, or 60, or 100, or 1000, etc.

Diagram. Set at the right of the narrow lines in the left column. At the upper left a small circle with S at its centre, D on its rim at the upper right and M on its rim at the left. Two straight lines run down to the right from the circle to a horizontal base line: the base line ends at A [?] at the left and B at the right. A vertical line rises from A [?] to C. A short horizontal segment runs from H to F, with a vertical line from F down towards the base. A quarter-arc curves down through the lower part of the figure to the base line. V stands on one of the descending lines at the right of the vertical through F; E and G stand together on the base line, to the right of A [?].

- S (centre of the circle): the Sun
- D (upper right on the circle): the Sun's upper limb
- M (left on the circle): the Sun's lower limb
- C (on the vertical from the base): the tip of the gnomon AC
- H (upper left of the short horizontal segment): the tip of the gnomon HF
- F (right end of the short horizontal segment)
- V (lower right, on a descending line)
- A [?] (left end of the base line): the foot of the gnomon

- E, G (on the base line, right of A [?])
- B (right end of the base line)

[Margin: Problem 75.]

On these assumptions: if the right shadow AE is given in terms of parts of which the gnomon AC is known, then in the triangle CAE, right-angled at A, two sides about the right angle are given. Therefore, by [proposition] 5 of right-angled plane triangles, one can find both the angle AEC, which measures the altitude of the Sun's upper limb, and the angle ACE, which measures the distance of the same limb from the vertex. These are the true ones if the foot A of the gnomon, or its tip C, were at the centre of the Earth and AB were the rational horizon; but the seen or apparent ones if, as commonly happens, A or C is on the surface of the Earth and AB is the sensible horizon. In that case the parallax must be added to the altitude already mentioned and the Sun's refraction subtracted; conversely, the refraction must be added to the distance from the vertex and the parallax subtracted, so that the one becomes the corrected or true altitude, the other the true distance of the limb D from the vertex. Finally, from the altitude of the limb thus corrected the Sun's apparent semidiameter must be subtracted, and to the distance from the vertex it must be added, so that the true altitude of the Sun's centre, or its true distance from the vertex, may be had.

[Margin: Problem 76.]

But if the versed shadow FV is given, known in parts of which the gnomon HF is known, the sides about the right angle F are likewise given. Therefore, by the same [proposition] 5 of right-angled plane triangles, one can find both the angle FHV, which measures the altitude of the Sun's lower limb M, and the angle FVH, which measures the distance of the same limb from the vertex: the true ones if F is at the centre of the Earth and HF is the rational horizon, but the seen ones if F is on the surface of the Earth. In that case each must be corrected in the way stated in Problem 75. But to the corrected altitude the Sun's apparent semidiameter must be added, and from the distance from the vertex it must be subtracted, so that the true altitude of the Sun's centre, or its distance from the vertex, may be had. And so the 2nd table is constructed.

[Margin: Construction of table 2.]

[Margin: Problem 77.]

Conversely, if the true altitude of the Sun is given, and A is supposed to be at the centre of the Earth, the Sun's apparent semidiameter will have to be added to it, and the angle AEC will become known. With it and with the gnomon, that is, the opposite side AC, you will find the right shadow AE by [proposition] 7 of right-angled plane triangles. But if the Sun's true distance from the vertex is given, and you subtract from it the Sun's apparent semidiameter, you will have the angle ACE; with it and with the adjacent side AC you will find the right shadow AE by [proposition] 4 of the same triangles. But if A is on the surface of the Earth, everything must be done as above, except that the parallax must first be added to the altitude and the refraction subtracted, while from

the distance from the vertex the parallax must be subtracted and the refraction added. In this way, by Problem 75 or 77, a gnomonic table can be constructed in which the Sun's altitude can be found at once from the right shadow, and the proportion of the right shadow to the gnomon from the Sun's altitude.

[Margin: Construction of table 1.]

[Margin: Problem 78.]

For the versed shadow, on the other hand, subtract the Sun's apparent semidiameter from the given altitude of the Sun, and you will have the angle FHV; with it and with the gnomon FH, find the versed shadow FV by [proposition] 4 of right-angled plane triangles. But if the Sun's distance from the vertex is given, add to it the Sun's apparent semidiameter, and you will have the angle FVH; with it and with the opposite gnomon HF, find the versed shadow FV by [proposition] 7 of the same triangles. But if A or F is on the surface of the Earth, the altitude must first be corrected by adding the parallax to it and subtracting the refraction; and the distance from the vertex by subtracting the parallax from it and adding the solar refraction. And so, by Problem 76 or 78, a gnomonic table can be constructed that shows the versed shadow from a given altitude of the Sun, or the altitude of the Sun from a given versed shadow.

[Margin: Construction of table 2.]

[Margin: Order of the problems.]

[Margin: Order of the problems.]

[Margin: Table 3.]

Third, and last, a table is constructed for the three more notable shadows, namely the winter [Brumali], the equinoctial and the solstitial or summer shadow, and these at noon: either by deriving from such a shadow the Sun's meridian altitude at the equinoxes and at each solstice, or by deriving the shadow from this altitude. For these, let the following diagram serve as well. In it let the rational or astronomical horizon be HQR, and in it let the centre of the Earth be Q; but let the sensible or physical horizon be FGN, on which the gnomon GD is erected perpendicularly; and from the centre of the Earth Q let the semicircle of the meridian HZR be described.

Diagram. Set across the left column. A semicircle of the meridian stands on a lower straight line HQR, with a second straight line FGN drawn parallel to it a little above. From the point D, at the tip of a short dark upright gnomon standing on FGN at G, lines fan out up to the right to the arc, where small circles mark the Sun at three places; further lines run from those places through D down to the left, to points on FGN. A vertical line runs from Z at the top of the arc down through D to the gnomon; dotted lines run from Q under the gnomon up to the right towards the arc.

- Z (top of the arc)
- C, ♋ (top of the arc, right of Z): the Sun's upper edge at the beginning of Cancer, with the sign of Cancer
- I (in a small circle on the arc): the Sun's centre at the beginning of Cancer

- L (on the arc, further right), followed by two small sorts [?] [?]: the limb L at the equinox
- M (on the arc, right): the Sun's limb M
- ♑ (right of the arc, below M): the sign of Capricorn
- O (in a small circle on the arc, lower right): the Sun's centre at the beginning of Capricorn
- P (on the arc, left)
- D (centre): the tip of the gnomon
- F (left end of the line FGN), N (right end)
- B, b, A, a, S, f, G (along FGN, left to right): the ends of the shadows and the foot G of the gnomon
- H (left end of the lower line), Q (beneath the gnomon): the centre of the Earth, R (right end)

[Margin: Description of the solstitial and equinoctial shadow.]

Now when the Sun's centre is at O, the beginning of Capricorn, the ray of the limb M passing through the tip D will be MDb, ending the right winter shadow Gb. The angle DbG will measure the visible altitude NM of the limb M, which is certainly greater than that given by the angle DBG, measuring NO, the visible altitude of the centre O — an altitude less than the one measured by the angle OQR, measuring RO, the true altitude of the centre O. But when the Sun's centre is at ♈ or ♎, either of the equinoctial points — that is, at the beginning of Aries or of Libra — the ray LDa will end the equinoctial shadow Ga, and the angle DaG will measure the visible altitude NL of the limb L, greater than the visible altitude N♈ of the Sun's centre, measured by the angle DAG; and this is less than the true altitude R♈, measured by the angle ♈QR. Finally, when the Sun's centre is raised to I, the beginning of Cancer, the upper edge C will send its ray through D to f, ending the solstitial shadow Gf; and the angle DfG will be the measure of the visible altitude NC, which will be, or will appear, greater than the visible altitude NI, measured by the angle DSG; but this is less than the true altitude RI, measured by the angle IQR.

De Auctoribus Tabularum Gnomonicarum. *On the authors of gnomonic tables.*

The construction of the gnomonic table, the 3rd in particular, was taught by Ptolemy in Book 2 of the *Almagest*, chapters 5 and 6, and by Georgius Purbachius [Georg von Peuerbach], and after him by Daniel Santbech. In that table the gnomon is supposed to be of 1200 parts; at its head are the hundreds of parts, in the left margin the parts of the right shadow from 1 up to 100, and in the following columns the degrees, minutes and seconds of the Sun's distance from the vertex. Orontius Fineus, on the other hand, in Book 4 of his *Cosmographia*, chapter 4, presents a gnomonic table in which he takes the gnomon as 12 parts; in its left margin [?] stand the single degrees of the whole quadrant, and in the other columns the parts of the right shadow, of which the gnomon is 12.

Erasmus Reinhold, however, in his *Tables of Directions*, having taught in precepts 46 and 47 how to find the shadow from the Sun's altitude and the Sun's altitude from the shadow, afterwards, from page 17 of the tables, gives the fecund table [Canonem fœcundum], in which the radius, or whole sine, is supposed to be of at least 10 000 000 parts. At its head and foot are placed the degrees up to 45; in the right and left margins the minutes — or rather, towards the beginning and the end of the quadrant, single decades of minutes — of the Sun's distance from the vertex; and in the body of the columns are placed the tangents of any arc or angle, which serve for the quantity of the right shadow and for its proportion to a gnomon of 10 000 000 parts. For in the earlier figure, set out before Problem 75, if, with the centre placed at C and the radius AC taken as the semidiameter or whole sine, you describe the quadrant AV, the shadow AE will turn out to be the first tangent of the angle ACE, but the second tangent of the angle AEC, which is its complement to a quadrant. Then at the end of that book, on page 235, he gives a table of the threefold meridian shadow — namely the solstitial, the equinoctial and the winter shadow — for single degrees of the latitude, or altitude of the pole; and indeed a double one: the earlier, in which the gnomon is taken as 10 000 000 parts, the later, in which the gnomon is of 60 parts.

Afterwards Giuseppe Moletti, in Book 1 of Ptolemy's *Geography*, set out a double small table: one that gives, for the single degrees of the Sun's altitude, the quantity of the right shadow in parts of which the gnomon is 60; the other, in which the gnomon is taken as 100 000 parts. This is a fecund table showing the right shadow in the body of the columns, but only for whole degrees of altitude. Again, Maginus, in Book 5 of *De dimetiendi ratione*, page 118, gives a gnomonic table in which the gnomon is taken as 1000 parts; at its head are the hundreds of parts of the shadow, in the left margin the single parts from 1 to 100, and in the body of the following columns the degrees, minutes and seconds of the Sun's distance from the vertex. Finally our Clavius, in Book 5 of the *Gnomonice*, proposition 2, and on page 475, sets out a table he constructed himself, in which the gnomon is taken as 12 parts. At its head are placed the whole degrees of the Sun's altitude, in the left margin the single minutes of the degrees, and in the remaining columns the right shadow in whole parts and in sexagesimal minutes. At the foot of the table, however, are the degrees of the Sun's altitude, and in the right margin the minutes of the degrees, but the shadow shown in the body is the versed shadow. For as great as the right shadow of a given altitude is, so great is the versed shadow of the complement of the given altitude to a quadrant; or, as great as the right shadow of one altitude of the Sun is, so great is the versed shadow of that altitude which is the complement of the former altitude to a quadrant — as Clavius demonstrates there, and Orontius above.

De Errore aut Incuria prædictorum Auctorum. *On the error or carelessness of the authors already mentioned.*

Moreover, all the authors already mentioned supposed the tip of the gnomon to be at the centre of the Earth, neglecting parallax and refraction; and they used the ray of the Sun issuing from the Sun's centre, as if the right or versed shadow were bounded by this ray, neglecting the difference that can arise from the Sun's apparent semidiameter when the Sun's altitude is to be derived from the shadow, or the shadow from the Sun's altitude. Granted that this threefold error — of parallax, of refraction and of the solar semidiameter — may be disregarded in the construction of dials and in certain other geographical operations, and is no proof of error or ignorance in such great men, I still cannot approve their carelessness. For they gave occasion of erring not only to others but even to themselves — as I shall show below of Ptolemy, Maginus and others — in astronomical operations, and especially in investigating by tables of this kind the altitude of the pole, the obliquity of the ecliptic, or the days or hours of the equinoxes and solstices. This error can sometimes grow, in the altitude of the pole, beyond 45 minutes; at the time of the equinoxes, to almost two whole days; and at the moments of the solstices, to an error of still more days.

79. 80. 81. Probl. Data Vmbra Meridiana Solari ſiue Solſtitiali, ſiue Aequinoctiali, ſiue quacumque, ideſt vmbræ ratione ad Gnomonem, inuenire Altitudinem Poli. Vbi de Errore in hoc non ſolùm Priſcorum ſed etiam recentium Coſmographorum. *Problems 79, 80, 81. Given the Sun's meridian shadow — solstitial, or equinoctial, or any other whatever, that is, the ratio of the shadow to the gnomon — to find the altitude of the pole. Where [we treat] of the error in this of not only the ancient but also the recent cosmographers.*

[Margin: Problem 79.]

First, let the ratio of the solstitial shadow, either summer or winter, to the gnomon be given; and with it, by Problem 75 or 76, find the true altitude of the Sun's centre. From it subtract the Sun's greatest declination if the shadow is a summer one, or add it if a winter one; and subtract the sum or remainder from 90 degrees, for the altitude of the pole will remain.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

[Margin: Problem 80.]

Secondly, let the equinoctial shadow be given; and with it, by Problem 75 or 76, find the true altitude of the Sun and subtract it from 90 degrees, for what remains is the altitude of the pole.

[Margin: Problem 81.]

Thirdly, let any meridian shadow whatever be given; and with it, by Problem 75 or 76, find the true altitude of the Sun. Then, with the Sun's place in the ecliptic, find in the table indicated in section 2, problem 15, the Sun's declination; if it is southern, let it be added to the Sun's altitude, if northern, subtracted from it. For what comes out of this, subtracted from 90 degrees, gives the altitude of the pole. But the precept must be illustrated by examples, and the errors of some cosmographers called out of the shadow into the light.

I. EXEMPLVM. Pro Altitudine Poli Alexandriæ ex Vmbra Æquinoctiali.
Example 1. For the altitude of the pole at Alexandria from the equinoctial shadow.

[Margin: Look at the figure before Problem 75.]

Vitruvius affirms (Book 9, chapter 8) that at Alexandria in Egypt the equinoctial shadow was observed to stand to the gnomon as 3 to 5. Therefore if, by [proposition] 5 of right-angled plane triangles, one makes: as 3 is to the whole sine of 100 000 parts, so is 5 to another, the result will be 166,666⅔, namely the tangent of the angle AED, that is, of 59° 2′ 10″, which measures the visible altitude of the Sun's upper limb. But we need the true altitude, and that of the Sun's centre. We shall therefore add our parallax of 6″ — for no sensible refraction intervenes — and subtract the Sun's apparent semidiameter of 16′, and there will remain the true altitude of the Sun's centre, 58° 46′ 16″: that is, the altitude of the equator, if at the moment when such a shadow was observed there was a true equinox. And when these are taken from 90 degrees, there remains the altitude of the pole at Alexandria, 31° 13′ 44″.

[Margin: Error of Gemma Frisius and of Ptolemy.]

Gemma Frisius, however, in chapter 21 of the *Radius Astronomicus*, and Snel, in Book 1 of the *Eratosthenes Batavus*, chapter 8, from the analysis of the triangle already mentioned and the ratio of the shadow to the gnomon as 3 to 5, rightly enough gather the angle AED as 59° 2′; but they wrongly conclude from it that this is the true altitude of the equator, and that the altitude of the pole at Alexandria follows from it as 30° 58′ — so as not, that is, to depart from Ptolemy, who in Book 5 of the *Almagest*, chapter 13, and in Book 4 of the *Geography*, chapter 13, puts the same altitude of the pole, 30° 58′. From this we gather that he fell into the same error, especially since in Book 2 of the *Almagest*, chapters 3 and 5, he teaches no other way of gathering the altitude of the pole than either by the longest day or by the equinoctial or solstitial meridian shadow, with no mention made of the Sun's apparent semidiameter, none of parallax or of refraction.

II. EXEMPLVM. Pro Altitudine Poli Romæ ex Vmbra Æquinoctiali.
Example 2. For the altitude of the pole at Rome from the equinoctial shadow.

[Margin: See the figure before Problem 75.]

The same Vitruvius (Book 9, chapter 8) and Pliny the Elder (Book 2, chapter 72) report that the equinoctial shadow at Rome at noon was observed to stand to the gnomon as 8 to 9. Therefore if, by [proposition] 5 of right-angled plane triangles, one makes: as the shadow AE, which is 8, is to the whole sine of 100,000 parts, so is the gnomon AC, which is 9, to another, the result will be 112 500, namely the tangent of the angle AEC, of 48° 22′, which measures the visible altitude of the Sun's upper limb. But if you add our parallax of 10″ and subtract the refraction of 1′ 57″ and the Sun's apparent semidiameter of 16′, the true altitude of the Sun's centre will be 48° 4′ 13″; and when these are taken from a quadrant, there remains the altitude of the pole at Rome, 41° 55′ 47″, or

41° 56′, which is the amount Clavius puts in his table. But it is not certain that such a shadow was observed at the very moment of the equinox.

[Margin: Maginus's slip.]

Maginus, for his part, in Book 11 of the *Problems of the Primum Mobile*, problem 10, gathers from the ratio of the shadow to the gnomon as 8 to 9 a visible altitude of the Sun's limb of 48° 7′. Supposing a refraction of 2′ 15″ and a parallax of 2′ 45″, he turns this into a true altitude of 48° 7′ 30″; but, neglecting the 30″, [he keeps] 48° 7′, and concludes its complement, namely 41° 53′. Yet he ought to have put the altitude of the limb at 48° 22′; or else he ought to have called the altitude of 48° 7′ the altitude not of the limb but of the centre.

Probl. Altitudinem Poli adipiſci, Datis 82. Die maxima aut minima, & Obliquitate Eclipticæ 83. Semidiurno arcu, vel Differentia Aſcenſionali, & Declinatione Solis aut Stellæ 84. Semidiurno arcu vel Differ. Aſcenſionali & Amplitudine ortiua Solis aut Stellæ 85. Amplitudine ortiua, & Declinatione Solis aut Sideris. *Problem. To obtain the altitude of the pole, given: 82. the longest or the shortest day, and the obliquity of the ecliptic; 83. the semidiurnal arc, or the ascensional difference, and the declination of the Sun or of a star; 84. the semidiurnal arc or the ascensional difference, and the rising amplitude of the Sun or of a star; 85. the rising amplitude, and the declination of the Sun or of a heavenly body.*

In the adjoining figure let the meridian be HCPN and the horizon HMR; and let P be the visible pole of the equator AMQ. Let the Sun be at D, a point common to the parallel CT and to the horizon, on this side of the equator, through which [passes] the quadrant of declination PDL; but if the Sun were at B, a point of the parallel KN beyond the equator, draw the arc of declination PB.

Diagram. Set at the right of the short lines in the right column. A circle (the meridian) with a horizontal diameter from H at the left to R at the right, on which stand B, M and D. Oblique straight lines cross the circle: one from A at the upper left down to Q at the lower right, others joining K, P, C, T and N on the rim; I and L stand at intersections inside the circle, I above the diameter and L below it.

- C (top of the circle)
- A (upper left, on the circle)
- K (left, on the circle)
- P (upper right, on the circle): the visible pole
- H (left end of the diameter), R (right end): the horizon HMR
- B, M, D (on the diameter, left to right)
- I (inside, above the diameter)
- L (inside, below the diameter)
- T (lower right, on the circle)
- Q (bottom, right of centre, on the circle)
- N (bottom, on the circle)

[Margin: Problem 82.]

First, then, if the longest artificial day is given, the Sun will rise at D, a point of the tropic lying on this side of the equator, and PD will be the complement of the Sun's greatest declination, or of the obliquity of the ecliptic. And when half the artificial day has been converted into parts of the equator corresponding to the solar time, the angle APL will be known, which measures the greatest semidiurnal arc AL; and its supplement to 180 degrees will be the angle DPR. With this and with the base PD, in the triangle DPR, right-angled at R, find the altitude of the pole PR by [proposition] 2 of right-angled spherical triangles.

[Margin: By the longest day.]

[Margin: Otherwise.]

Or otherwise: in the triangle MLD, right-angled at L, find the angle LMD by [proposition] 14 of the same triangles. For the side ML is given, if you subtract 6 hours from the greatest half-day and convert the remainder into parts of the equator; and so is the side LD, which is the Sun's greatest declination, or the obliquity of the ecliptic. Now the angle LMD is equal to the angle HMA, which measures the altitude HA of the equator; therefore, when it is subtracted from 90 degrees, the remainder will be the altitude of the pole, since this is as great as the distance of the equator from the vertex.

[Margin: By the shortest day.]

But if the shortest day is given, the Sun will rise at B; and then, when the complement of half that day to 6 hours has been converted into parts of the equator, the arc IM will be known, and BI will be the Sun's greatest declination, as great as the obliquity of the ecliptic. Therefore in the triangle BIM, right-angled at I, with the sides given, you will find by the same [proposition] 14 the angle BMI; subtract it from 90 degrees, and the altitude of the pole will be known.

[Margin: Problem 83.]

Secondly, let the semidiurnal arc of the Sun or of a star be given, say the arc AML (for I now call the semidiurnal arc of a star the arc that it uses up from its rising to the meridian, whether it is really day or night). For when the quadrant AM of the equator is subtracted, ML, the ascensional difference, will remain — unless it had been given directly. With it and with the declination LD of the heavenly body, find by [proposition] 14 of right-angled spherical triangles the angle LMD, and subtract it from 90 degrees, so that the altitude of the pole becomes known. But if the semidiurnal arc were less than a quadrant, the Sun or the star will rise, for example, at B, and the triangle BMI would have to be used, as I said of the shortest day. The altitude of the pole itself could, however, also be found by tables of semidiurnal arcs, the Sun's place being given.

[Margin: Problem 84.]

Thirdly, let there be given, from the semidiurnal arc, the ascensional difference ML or MI, and the rising amplitude MD or MB. For in the triangle LMD or IBM, right-angled at L or at I, one side and the base MD or MB will be given; therefore by [proposition] 5 of right-angled spherical triangles you will

find the acute angle M, and when it is subtracted from 90 degrees you will have the altitude of the pole.

[Margin: Problem 85.]

Fourthly, let the rising amplitude MD be given as the base, and the northern declination LD as a side; or the rising amplitude MB, and the declination BI as a side. For in the triangle MLD or MIB, right-angled at L or at I, by [proposition] 4 of right-angled spherical triangles you will obtain the acute angle at M, whose complement to 90 degrees will be the altitude of the pole.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

[Margin: A caution concerning the problems already mentioned.]

It must be noted, however, that because of refraction — and also, if it is the Sun, of parallax — the rising amplitude and the semidiurnal arc vary somewhat; and therefore, if one of these is given by observation, the altitude of the pole deduced from it is not quite so exact.

86. Probl. Altitudinem Poli definire, Dato Arcu Eclipticæ ſemper Apparente, & Obliquitate Eclipticæ. *Problem 86. To define the altitude of the pole, given the arc of the ecliptic that is always visible, and the obliquity of the ecliptic.*

[Margin: Problem 86.]

How the arc of the ecliptic that is always visible is investigated was stated in section 2, problem 217, whose figure can serve for the present business. I add, however, that the arc of the ecliptic that is always visible is not to be reckoned from the time when a day without any night begins, or ends, but from the time when the centre of the Sun is on the horizon and moves in the circle of perpetual apparition. This arc being given, then, take its half, and find the second sine of that half; multiply it by the sine of the obliquity of the ecliptic, and divide the sum by the whole sine, or radius. For the quotient will be the sine of the angle that measures the altitude of the equator; subtract this from 90 degrees and you will have the altitude of the pole.

87. Probl. Altitudinem Poli determinare, Datis Climate & Obliquit. Eclipticæ. *Problem 87. To determine the altitude of the pole, given the clime and the obliquity of the ecliptic.*

[Margin: Problem 87.]

A clime is understood to be given when the longest day is given on the parallel passing through the middle or through the end of the clime. The extremes of the climes are usually distinguished by a difference of half an hour, although the parallel through the middle of a clime does not differ from the parallel through the beginning or end of the clime in the longest day of the year except by a quarter of an equal hour. And so it goes, beginning from the equator, which is the first of the parallels and their beginning, as far as the frigid zones, in which the longest day is greater than 24 hours. As many half-hours, therefore, as the longest day exceeds 12 hours — that is, the equinoctial day

— so many is the number of the completed clime. Hence, if the first clime is given, the longest day on the parallel through its middle is $12^h\ 15^m$, but on the parallel that bounds it, $12^h\ 30^m$. If, on the other hand, the 7th clime is given, then, since seven half-hours make $3^h\ 30^m$, these added to 12 hours make the longest day on the bounding parallel, $15^h\ 30^m$. The number of the climes being given, therefore, the longest day is given; and its half, converted into parts of the equator, will give the greatest semidiurnal arc. With this and with the obliquity of the ecliptic — that is, the Sun's greatest declination — or its complement, find the altitude of the pole by Problem 82, set out a little earlier. But if the longest day exceeds 24 hours, turn to Problem 86. And if it is exactly 24 hours, the altitude of the pole will be 66° 30′.

88. Probl. Data Poli Altitudine, determinare Clima, immò & Parallelum Climatis. *Problem 88. Given the altitude of the pole, to determine the clime, and indeed the parallel of the clime.*

[Margin: Problem 88.]

Find the greatest semidiurnal arc — the Sun, that is, holding its greatest declination — either by Problem 122 of Section 2 or by the tables indicated there; double it, and from the sum throw away 180 degrees. Convert the remainder into time of the equator, that is, into hours and minutes. For as many half-hours as there are in that remainder, so many is the number of the climes within which the place of the given altitude of the pole does not lie. If nothing is left over beyond the half-hours, the place will be on the parallel bounding the clime indicated by the number of half-hours. But if some minutes of an hour remain, either they will be 15^m, and then the place will be on the middle parallel of the following clime; or they will be fewer than 15^m, and it will lie between the parallel already mentioned and the parallel bounding the next clime towards the equator; or, finally, they will be more than 15^m, and it will lie between the middle parallel already mentioned and the parallel bounding the clime that follows towards the visible pole. Thus, since here at Bologna the altitude of the pole is 44° 29′ 30″, and so the greatest semidiurnal time is $7^h\ 35^m$ and the longest day $15^h\ 10^m$ — granted that, owing to the refractions, it is apparently lengthened at both ends, so that it seems to be 15^h and nearly 15^m — therefore, when the 12 hours are thrown away, there remain $3^h\ 10^m$, that is, 6 half-hours and 10^m. We are therefore outside the 6th clime and are in the 7th, but within the parallel drawn through the middle of the seventh and the one drawn through the end of the sixth.

Probl. Altitudinem Poli determinare ex Altitudine Solis aut Fixarum extra Meridianum capta, Datis 89. Altitudine, ac Declinatione Sideris, & Azimutho 90. Altitudine, ac Declinatione Sideris, & Diſtantia Aequatoria à Meridiano 91. Altitudine, Declinatione, & Aſcenſione Recta Sideris, vnà cum Aſcenſ. Recta Medij Cæli 92. Altitudine, Azimutho, & Diſtantia

Aequatoria Sideris à Meridiano 93. Altitudine, Azimutho, & Aſc. Recta Sid. cum Aſc. Recta Medij Cæli 94. Declinatione, Azimutho, & Diſtantia Sideris Aequatoria à Meridiano 95. Declinatione, Azimutho, & Aſc. Recta Sid. cum Aſcenſ. Recta Medij Cæli. *Problem. To determine the altitude of the pole from an altitude of the Sun or of the fixed stars taken outside the meridian, given: 89. the altitude and the declination of the star, and the azimuth; 90. the altitude and the declination of the star, and its equatorial distance from the meridian; 91. the altitude, the declination and the right ascension of the star, together with the right ascension of the midheaven; 92. the altitude, the azimuth, and the equatorial distance of the star from the meridian; 93. the altitude, the azimuth, and the right ascension of the star with the right ascension of the midheaven; 94. the declination, the azimuth, and the equatorial distance of the star from the meridian; 95. the declination, the azimuth, and the right ascension of the star with the right ascension of the midheaven.*

For all these, let the following figure serve, in which the meridian is HZQ, the horizon HR, and the equator EOQ, whose pole is P. Let the Sun, or any other star whatever, be either at B, on this side of the equator, so that the arc BP is the complement of the declination to 90 degrees; or at G, a point of the equator, and then GP is a whole quadrant; or at I, beyond the equator, and then the arc IP is made up of the declination IC and the quadrant CP. Hence what will be said of the triangle ZBP, using the side BP, is to be understood of the triangles ZGP and ZIP, when the side GP or IP, known from the declination, is to be used. Further, the angles at the pole of the world P, made by the meridian and another circle of declination, measure the equatorial distance of the star from the meridian. This can be known either by counting the time that passes from the taking of the star's altitude or azimuth until its transit through the midheaven, and converting it into parts of the equator; or from the difference between the right ascension of the star and the right ascension of the midheaven. The ascension of the midheaven can be observed by waiting until one fixed star of known right ascension passes through the meridian, and at the same moment having another companion take the altitude or azimuth of another star whose right ascension is likewise known. Finally, the transit through the meridian and the position of the azimuth are best marked by our thread triangle, on which see Book 5, chapter 15, Problem 8.

Diagram. Set at the right of the short lines in the right column. A circle (the meridian HZQ) with Z at the top, H at the left and R at the right of a horizontal line (the horizon) on which A stands; P, the pole, at the upper right on the rim, with arcs drawn towards it from E, N and B at the upper left; G and C close together inside the circle, I below them, O below the centre, Q at the lower right on the rim.

- Z (top of the circle)
- E (upper left, on the circle)
- B (upper middle, inside)
- P (upper right, on the circle): the pole
- N (left, below E)

- G, C (inside, left of centre)
- I (inside, below G)
- H (left end of the horizon), R (right end)
- A (on the horizon, left of centre)
- O (below the centre)
- Q (lower right, on the circle)

Now, the azimuth HZA being given, its supplement to 180 degrees is given, namely the angle AZP, which from here on we shall call, for brevity, *the angle Z*; just as the angle known from the distance from the meridian we shall call *the angle P*. And the complement of the altitude AB to 90 degrees is the side BZ. We shall therefore use the triangle BZP. But what we say of it holds for the two others, with the side BP and the angle P changed; and in it we shall always investigate the arc ZP, which is the complement of the polar altitude PR.

[Margin: Problem 89.]

First: since the side BZ is given from the altitude, and BP from the declination, and the angle Z from the azimuth, you will obtain the ba- se PZ, that is, the complement of the altitude of the pole, by [proposition] 2 of oblique-angled spherical triangles.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

[Margin: Problem 90.]

Secondly: since the side BZ is given from the altitude, and BP from the declination, and the angle P from the distance from the meridian, you will obtain the base PZ by the same [proposition] 2.

[Margin: Problem 91.]

Thirdly: since BZ is given from the altitude, and BP from the declination, and the angle P from the difference between the right ascensions of the star B and of the midheaven, you will have the base PZ by the same [proposition] 2.

[Margin: Problem 92.]

Fourthly: since BZ is given from the altitude, and the angle Z from the azimuth, and P from the distance from the meridian, you will obtain the base PZ by [proposition] 8 of oblique-angled spherical triangles.

[Margin: Problem 93.]

Fifthly: since BZ is given from the altitude, and the angle Z from the azimuth, and the angle P from the difference of the right ascensions of the star B and of the midheaven, you will acquire the base PZ by the same [proposition] 8.

[Margin: Problem 94.]

Sixthly: since BP is given from the declination, and the angle Z from the azimuth, and the angle P from the equatorial distance from the meridian, the base PZ will become known by [proposition] 8 of the same triangles.

[Margin: Problem 95.]

Seventhly: since BP is given from the declination, and the angle Z from the azimuth, and the angle P from the difference of the right ascensions of the star and of the midheaven, you will therefore have the base PZ by the same

[proposition] 8; and when it is subtracted from 90 degrees, the altitude of the pole RP will be known.

Probl. Altitudinem Poli venari per duas ſtellas, in eodem Verticali exiſtentes, quarum nota ſit Declinatio & Aſcenſio Recta, & præterea Datis 96. Altitudine vnius Stellarum prædictarum 97. Azimutho vnius Stellarum prædict. 98. Aſc. Recta Medij cæli. *Problem. To hunt out the altitude of the pole by means of two stars standing on the same vertical, whose declination and right ascension are known, and given in addition: 96. the altitude of one of the stars already mentioned; 97. the azimuth of one of the stars already mentioned; 98. the right ascension of the midheaven.*

In the following figure the meridian is HVQ, the horizon HR, the equator AOQ, and the quadrant of the vertical circle VT, on which are the two stars F and G. Through their centres, from the pole of the world P, let the quadrants of the circle of declination PFK and PGM be drawn.

For the three problems that follow, then, the first thing to be found is the angle VFP or VGP, and it is found easily. For in the triangle FPG the sides FP and GP are given, being the complements of the given declinations; and so is the angle FPG, since it is the difference of the given right ascensions. Therefore, by [proposition] 5 or 6 of oblique-angled spherical triangles, you will find the angle FGP, that is VGP, and the angle GFP; and when this is subtracted from 180 degrees, the angle VFP becomes known. These having been found:

Diagram. Set at the right of the short lines in the left column. A circle (the meridian HVQ) with V at the top, H at the left and R at the right of the horizon line; A at the upper left on the rim and Q at the lower right, joined by the equator through M and O; P, the pole, at the upper right on the rim, with lines drawn to it from K and from G; F inside below V; T on the lower part of the vertical through O.

- V (top of the circle)
- A (upper left, on the circle)
- F (inside, below V)
- P (upper right, on the circle): the pole
- K (left, inside)
- G (inside, right of centre)
- H (left end of the horizon), R (right end)
- M (on the equator, left of centre)
- O (below the centre)
- T (below and right of O)
- Q (lower right, on the circle)

[Margin: Problem 96.]

First: if the altitude of the star F is given, find VP, the complement of the altitude of the pole, by [proposition] 4 of oblique-angled spherical triangles. For in the triangle VFP there are given VF, the complement of the altitude TF; FP, the complement of the declination of the star F; and the included angle VFP.

But if the altitude of the star G was taken, use the triangle VGP, in which VG is the complement of the altitude and GP the complement of the declination of the star G, together with the included angle VGP found above; and by the same [proposition] 4 find VP.

[Margin: Problem 97.]

Secondly: if the azimuth is given, its supplement to 180 degrees is the angle FVP. With it, and with the angle VFP already found, and with FP, the complement of the declination of the star F, the higher one, find by [proposition] 7 of oblique-angled spherical triangles the side VP, which is the complement of the altitude of the pole.

[Margin: Problem 98.]

Thirdly: if the right ascension of the midheaven is given, the difference between it and the right ascension of the higher star F will give the angle VPF. With it, and with the angle VFP found above, and with the adjacent arc FP, which is the complement of the declination of the star F, find VP by [proposition] 11 or 12 of oblique-angled spherical triangles; and when it is taken from 90 degrees, the altitude of the pole RP will be known.

99. Probl. Poli Altitudinem Inquirere, Data Aſcenſione Recta duorum Siderum, quorum vnum ſit in Merid. quando alterum eſt in Horizonte, & huius Declinatione. *Problem 99. To find the altitude of the pole, given the right ascension of two stars, one of which is on the meridian when the other is on the horizon, and the declination of the latter.*

[Margin: Problem 99.]

In the following scheme the meridian is HKQ, the horizon HR, and the equator AMQ, whose pole is P; and the star rising or setting is at the point B of the horizon, through which let the quadrant PBC be drawn, so that its given declination is BC and the complement of the declination BP. But if the star were on the equator, that is, at the point M, a whole quadrant would have to be used in place of BP; and if beyond the equator, the sum of the declination and the quadrant would have to be used. It is enough, however, to indicate the problems in the triangle BPR, right-angled at R. In it, besides the base BP, the angle BPR is given, which is as great as the difference of the right ascensions of the star B and of the star L, which appears between the pole, roughly known, and the horizon; or it is the supplement of that difference to 180 degrees when the star K appears between the visible pole and the equator. Therefore, by [proposition] 2 of right-angled spherical triangles, obtain the altitude of the pole RP. But note that the star at B appears sooner because of horizontal refraction, and that it is therefore better to observe it when it appears raised about half a degree above the horizon; for then it will really be on the horizon. And if at that moment another known star is on the meridian, you will be able to dispatch this problem.

Diagram. Set at the right of the short lines in the right column, beside Problem 99. A circle (the meridian HKQ) with H at the left and R at the right

of the horizon line; K at the top, P at the upper right and L at the right on the rim, K and L each marked with a small star sign; A at the upper left on the rim, joined to Q at the lower right through M (the equator AMQ); B on the horizon right of centre, marked with a small star sign, with the line PBC through it down to C.

- K (top of the circle, star sign)
- A (upper left, on the circle)
- P (upper right): the pole[59]
- L (right, on the circle, star sign)
- H (left end of the horizon), R (right end)
- B (on the horizon, right of centre, star sign): the rising or setting star
- M (on the equator, below the horizon)
- C (bottom, on the circle)
- Q (lower right, on the circle)

100. Probl. Poli Altitudinem inquirere, Datis duabus Stellis notæ Declinationis & Aſc. Rectæ ſimul orientibus, aut occidentibus. *Problem 100. To find the altitude of the pole, given two stars of known declination and right ascension rising, or setting, at the same time.*

[Margin: Problem 100.]

In this second diagram the meridian is HPQ, the horizon HR, and the equator AMQ, whose pole is P; and the stars rising or setting at the same time are O and B. Through them let the quadrants of declination PBD and POC be drawn; for the declination of the star O will be CO and its complement OP, while that of the star B will be DB and its complement BP. These being given in the triangle BPO, with the vertical angle BPO, which measures the difference of the right ascensions, you will find:

[Margin: Step 1.]

first, the angle POB, by [proposition] 5 or 6 of oblique-angled spherical triangles.

[Margin: Step 2.]

Secondly, in the triangle OPR, right-angled at R, in which the base OP is given from the declination of the star O, and the angle POR has been found, find the altitude of the pole PR by the first [proposition] of right-angled spherical triangles. It may happen, however, that the arcs PO and PB, both or one of them, are a quadrant, or are made up of a quadrant and the declination, if both stars or one of them is either on the equator or beyond it. It is advisable, moreover, to wait for two stars that appear raised together about 30′ above the horizon; for then they will really be on the horizon.

Diagram. Set at the right of the short lines in the right column, beside Problem 100. A circle (the meridian HPQ) with H at the left and R at the right of the horizon line; A at the upper left and P at the upper right on the rim; O

[59] no star sign at P, where the meridian line meets the rim.

and B on the horizon right of centre; M below the horizon left of centre; C and D below; Q at the lower right on the rim.

- A (upper left, on the circle)
- P (upper right, on the circle): the pole
- H (left end of the horizon), R (right end)
- O, B (on the horizon, right of centre): the two stars
- M (below the horizon, left of centre)
- C (below M)
- D (lower right, inside)
- Q (lower right, on the circle)

101. Probl. Data Vnius loci Altitudine Poli, & Differentia longitudinis inter ipſum & alterum locum, cum alterutro Poſitionis angulo; Inuenire alterius Altitudinem Poli, ac Diſtantiam horum Locorum. *Problem 101. Given the altitude of the pole of one place, and the difference of longitude between it and another place, with either of the angles of position: to find the altitude of the pole of the other, and the distance of these places.*

[Margin: Problem 101.]

In the next figure let there be two northern places, C and D, through which, and through the northern pole of the world B and the southern A, let two semicircles of meridians BCA and BDA be drawn, intercepting the arc EQ of the equator, that is, the difference of longitude, which either angle B or A measures. And let the arc CD of a vertical circle be drawn, which contains the distance between the two places; for the angles of position will be BCD and BDC. Therefore in the triangle CBD, if the altitude of the pole of your place C is given, its complement will be CB. With it, and with the angle B, known from the difference of longitude, and with the angle of position, observed or given, find BD, the complement of the altitude of the pole of the place D, and the distance CD, by [proposition] 11 or 12 of oblique-angled spherical triangles. But if the altitude of the pole of the place D is given, its complement BD will be given; with it, and with the angles B and CDB of position, find by the same [proposition] 11 or 12 the arc CB, which is the complement of the altitude of the pole at the place C, and the distance CD. From these you understand what is to be done in the triangle CFB if one place is northern and the other southern; for then BF will be made up of the quadrant BQ and the latitude QF.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

Diagram. Set at the right of the short lines in the left column. Two curved meridians join B at the top and A at the bottom in a lens shape; D stands on the right meridian and C on the left, joined by a straight line (the vertical arc CD); E at the left and Q at the right of a horizontal line across the middle (the arc EQ of the equator); F on the right meridian below the equator and G on the left, with straight lines drawn from C, E and G to F.

- B (top): the northern pole of the world

- D (right meridian, upper)
- C (left meridian, upper)
- E (left, on the equator)
- Q (right, on the equator)
- F (right meridian, below the equator)
- G (left meridian, lower)
- A (bottom): the southern pole

102. Probl. Data Altitudine Poli duorum locorum & alterutro Poſitionis angulo; inuenire Differentiam Longitudinis, ac Locorum Diſtantiam. *Problem 102. Given the altitude of the pole of two places and either angle of position: to find the difference of longitude and the distance of the places.*

[Margin: Problem 102.]

In the triangle CBD, given CB and DB, the complements of the altitudes of the pole, and the angle BCD or BDC, you will find the angle B, that is, the difference of longitude, by [proposition] 3, and the base CD, or the distance, by [proposition] 2 of right-angled spherical triangles. The angles of position, moreover, are known to ship-masters by loxodromy and by the line that the rhumbs of the winds make with the meridian line; and as long as they use the same wind, they are believed to sail along nearly the same circle — but more on these matters in the book on geography. I add that if the difference of longitude is given, that is, the angle B, and, from the altitudes of the pole, the complements BC and BD, then the distance CD is to be found by the fourth [proposition] of oblique-angled spherical triangles.

103. Probl. Data Diſtantia duorum locorum inter ſe, ac Differentia longitudinis, cum alterutro Poſitionis angulo, inuenire vtriuſque Altitudinem Poli. *Problem 103. Given the distance of two places from each other, and the difference of longitude, with either angle of position: to find the altitude of the pole of each.*

[Margin: Problem 103.]

In the figure of Problem 101 let CD be given, in degrees and minutes, or in miles but converted into degrees and minutes; and the difference of longitude B, and the angle of position C or D. For in the triangle CBD you will be able to find the side either opposite or adjacent to the angle of position, by [proposition] 7 or 8 of oblique-angled spherical triangles; and so the complements of the polar altitudes, BC and BD, can be known. When these are subtracted from 90 degrees, the altitude of the pole will remain.

104. Probl. Data duorum locorum Diſtantia & vnius Altitudine Poli, cum angulo Poſitionis, inuenire alterius Altitudinem Poli, & Differentiam Longitudinis. *Problem 104. Given the distance of two places and the altitude of the pole of one, with the angle of position: to find the altitude of the pole of the other, and the difference of longitude.*

[Margin: Problem 104.]

In the same preceding figure and its triangle CBD, if the polar altitude of the place C is given, its complement CB is given; if that of the place D, the complement BD is given. Therefore, with the distance CD, and with the angle of position C or D, either included by the sides or opposite to one of the given sides, you will find the angle B, or the difference of longitude, by [proposition] 5 or 3 of oblique-angled spherical triangles; and you will find the complement of the altitude of the pole, that is, either CB or DB, by [proposition] 4 or by [proposition] 2 of the same. But if the distance of the places is taken from the travelled interval and is too great — say, more than 300 or 400 miles — Ptolemy declares (*Geography* Book 1) that long journeys are not to be trusted, because of their various windings, which are not reduced with certainty to a straight route by subtracting a third part, as Marinus of Tyre thought, or a sixth, as Kepler does (chapter 16 of the *Rudolphine Tables*).

105. Probl. Data duorum locorum Diſtantia, & vtriuſque angulo Poſitionis, inuenire vtriuſq. Altitudinem Poli, & Differentiam Longitudinis. *Problem 105. Given the distance of two places and the angle of position of each: to find the altitude of the pole of each, and the difference of longitude.*

[Margin: Order of the problems.]

[Margin: Order of the problems.]

[Margin: Problem 105.]

Keeping to the same figure of Problem 101 and to the triangle CBD (for what is said of it is easily understood of the other cases): since the base CD is given, from the distance converted, if need be, into degrees and minutes, and both angles at the base, namely the angles of position C and D, the angle B, which measures the difference of longitude, can indeed be found by [proposition] 10 of oblique-angled spherical triangles, and by [proposition] 11 or 12 each side, CB and DB, which is the complement of the altitude of the pole. But beware of long journeys; for then it is better to use eclipses and Problem 106 or 107, which are to be given below. Conversely, if the places are very near, it is better to use this problem or Problem 104.

[Margin: A caution on the use of these problems.]

Monitum Pro reliquis modis inueſtigandæ Altitudinis Poli. *A caution concerning the remaining ways of investigating the altitude of the pole.*

For the remaining ways and problems given by means of the figure of Problem 101, see Problems 43 to 49. The others we have neglected for this reason: they presuppose the altitude of the pole as known beforehand, and they have neither such ease nor such certainty that it would be worth testing by them an altitude of the pole once known. Nor is it worth the trouble to turn to them, leaving aside these 32, which we have given from Problem 67 to 104, with a few inserted as occasion offered. We shall therefore move on to other

problems, for the difference of longitude, already found above in several ways, and now to be found in other ways.

[Margin: For the altitude of the pole the 32 problems given are amply sufficient.]

Problem. Longitudinis Differentiam, ſeu Diſtantiam Meridianorum inueſtigare 106. 107. 108. per Eclipſes Luminarium, modis 3. Et vtræ Eclipſes ſint ad id meliores. *Problem 106, 107, 108. To investigate the difference of longitude, or the distance of the meridians, by eclipses of the luminaries, in 3 ways; and which eclipses are better for it.*

[Margin: Problem 106.]

The *first way* is by eclipses of the Moon observed accurately at two places on the Earth, at both of which the beginning, or the end, or the middle (in partial eclipses of six digits), or the beginning of total obscuration, or the beginning of emersion from the shadow has been observed with the care we called for in Book 5, chapter 15. For the difference of time, converted into parts of the equator, will give the difference of longitude. You have some examples of this practice, with their scheme, in Book 1, chapter 13. The same can, however, be obtained by lunar eclipses of which one phase was observed at one place and another at the other, provided that at one of the two, or at some other place, the duration of the whole eclipse, or of the stay in the shadow, or the half-duration or half-stay, was also observed. For by adding or subtracting the duration or the stay, or the half-duration or the half-stay, the observation of the other place will easily be reduced to the same kind of phase; and so the difference of longitude will be obtained by the first way. For example, suppose that at Rome the beginning of the eclipse was observed, and at Bologna the middle and the end. The difference between the middle and the end will be the half-duration of the eclipse; doubled, it will give very nearly the whole duration; and this, subtracted from the time of the end observed at Bologna, will give the beginning of the eclipse at Bologna; or, added to the time of the beginning observed at Rome, it will give the end of the eclipse with respect to Rome. By comparing, then, the time of the end in both places, or the time of the beginning, this problem can be completed by the preceding one.

[Margin: Problem 107.]

The *second way* is by the observation of an eclipse at a place of unknown longitude, and by the time due to the same eclipse on the meridian for which very exact tables have been constructed. For let the beginning of a lunar eclipse have been observed at Goa at 10^h after noon, and from Lans- [Margin: Order of the problems.]

[Margin: Order of the problems.]

bergen's tables let the beginning of the same eclipse have been due at Goes at $4^h\ 50^m$ after noon. From this it follows that Goa is more easterly than Goes by $5^h\ 50^m$, that is, by $77°\ 30'$. Yet since astronomers usually go astray or

disagree in observing the beginning and end of eclipses, our Georges Fournier (*Hydrographia* Book 12, chapters 27 and 28) puts little trust in them.

[Margin: Problem 108.]

The *third way* is by eclipses of the Sun, provided that the difference of longitude, which otherwise appears from them, is corrected by computing the parallax. For this purpose, however, eclipses of the Moon — especially the immersions or emersions of total ones — are preferred by Hipparchus (in Strabo, Book 1); by Ptolemy (*Geography* Book 1, chapter 4), and there, in their commentaries, by Vernerius [Johannes Werner], Moletti, Maginus and Bercius; by al-Battānī (chapter 6); by Clavius (on the *Sphere*, chapter 2, page 284); by Tycho (in his *Letters*, page 73); by Peter Apian (*Cosmographia* Part 1, chapter 10); by Orontius (*Cosmographia* Book 5, chapter 3); by Iunctinus [Francesco Giuntini] (chapter 2 on the *Sphere*, page 469); by Maurolycus (*Cosmographia*, dialogue 3, page 67); by Daniel Santbech (proposition 14, on observation); by Cavalieri (in the *Centuria*, problem 47); by Biancani (in the *Introduction to Geography*); and by Hérigone (volume 4 of the *Cursus Mathematicus*, page 485). Kepler, by contrast (*Rudolphine Tables*, chapter 16, precept 61), affirms that solar eclipses are better suited to this purpose than lunar ones, because the beginnings and ends of solar eclipses are discerned much more certainly; and he says that the method of removing the parallaxes in his tables is most certain and most refined. He repeats the same in chapter 31, precept 176, where he also teaches this method:

[Margin: Which eclipses are better for it?]

"From the observation of each place, for the apparent time at it, let the true places of the Moon from the Sun be sought. When these are compared with each other — if they are of different sides, their sum, or if of the same side, their difference — divided by the true hourly motion of the Moon from the Sun, will yield the corresponding hours and minutes. Let these be added to the apparent time of the place for which the place of the Moon came out the earlier; let the time so altered be compared with the observed time of that place, and the difference of the meridians will be evident." He goes on, and illustrates the precept with an example, as follows.

[Margin: Kepler's precept for the difference of meridians from a solar eclipse.]

EXEMPLVM. *Example.*

At Graz let the end of a solar eclipse have been observed at exactly $12^h\ 47^m$, in the year 1598 February 25, when the Moon truly followed the Sun by 52′ 29″ ♓. But at Uraniborg the beginning was observed at $10^h\ 10^m$, when the Moon truly preceded the Sun by 26′ 8″. The sum is 1° 18′ 37″; divided by the hourly motion of the Moon from the Sun, which was 33′ 54″, it gives a quotient of $2^h\ 19^m$. And since at Uraniborg the Moon's place precedes, add the time found to its time: it becomes $12^h\ 29^m$. But at Graz the Moon reached that later place at $12^h\ 47^m$. The difference of these meridians would therefore be 18^m of time.

He warns, however, in chapter 16, that if the visible latitude becomes large, as it did at the end of the eclipse already mentioned, the longitude obtained from it is less reliable. And in the Catalogue of the *Rudolphine Tables*, page 33, he puts the difference between Graz and Uraniborg at only 14 minutes of hours, following, of course, partly the itinerary distance and partly observations of lunar eclipses — as is clear from his reasoning in chapter 16, from page 36 to 40, and from page 11 of his *Ephemerides*, which I have examined more fully in my book on geography. All things considered, therefore, this method by solar eclipses is not only more laborious but also more uncertain, because of many perplexities, and more hazardous — granted that their beginnings, or at least their ends, especially those of large eclipses, are in fact discerned more clearly than those of lunar eclipses, which suffer from the smoke and the shadowy fringe [vmbrago]; this, however, can be told apart from the true eclipse by telescopes and by certain other signs from small spots, as I taught in Book 5, chapters 5 and 15. Moreover, neither I hitherto, nor Father Bonaventura Cavalieri, chief Professor of Mathematics at Bologna, in a letter on this controversy sent to me while I was staying at Parma, in the year 1645 October 8, could understand how the true place of the Moon from the Sun at Graz could be found for a given time by tables that suppose, in their epochs, the meridian of Uraniborg or another one different from that of Graz — unless the distance of the meridians is known beforehand, which here, however, is being sought as unknown. Father Cavalieri suspected, however, that Kepler thought this problem was to be solved not by tables but by the analysis of triangles, from the lunar latitude and the sum of the visual semidiameters of the Sun and the Moon.

[Margin: Kepler's caution.]

[Margin: Kepler's inconsistency.]

[Margin: In the end, lunar eclipses are preferred.]

For in the following figure let ABC be the ecliptic, in which B is the centre of the Sun, and OML, etc., its disc; and DC the apparent path of the Moon, in which let P be the centre of the Moon at the beginning of the observed eclipse, when the Moon's disc began to graze the Sun's disc, or to pass apparently under it, at L.

Diagram. Set in the right column. A straight line (the ecliptic) runs from A at the left to a point at the right where it meets an oblique straight line rising from D at the lower left (the Moon's apparent path). A large circle stands about the ecliptic line, with G, B and E on the line near its lower edge; three equal circles stand along the oblique line, at F (lower left), at M (middle, its lower half shaded) and at P (right); straight lines run from B down to the circles.

- A (left end of the ecliptic)
- G (on the ecliptic, left of B): foot of the perpendicular FG
- B (on the ecliptic): the centre of the Sun
- E (on the ecliptic, right of B)
- I (above P): foot of the perpendicular PI

- L (between the Sun's disc and the circle P): where the Moon's disc first grazes the Sun's
- C (right end, where the two lines meet)[60]
- O (between the Sun's disc and the circle F)
- M (on the shaded circle in the middle)
- F (lower-left circle): the Moon's centre at the end of the eclipse
- P (right circle): the Moon's centre at the beginning of the eclipse
- D (lower-left end of the Moon's path)

For when the perpendicular PI is drawn to the ecliptic — this is the apparent latitude, and is known by observation — and BP is the known aggregate of the Sun's visual semidiameter BL and the Moon's IP, then in the triangle BPI, right-angled at I, given the side IP and the base BP, the side BI can certainly be known by [proposition] 9 of right-angled plane triangles; it is the Moon's lead [anteceſſio] in the ecliptic with respect to the Sun. So too at the end of the eclipse: with the Moon's centre placed at F, and the perpendicular FG drawn and given — namely the visible latitude — and given the sum of the semidiameters BO and OF, the side BG, or the Moon's lag [ſucceſſio], can be known. But this lead and lag is apparent, not true; and yet Kepler requires the true one, which — whether it is gathered from the true latitude of the Moon or from the true longitude of the luminaries — cannot be known unless tables adapted to the meridian of each place are to hand, or unless recourse is had to other particular observations, and to methods not included in that precept of the *Rudolphine Tables*,

109. Probl. Diſtantiam Meridianorum inueſtigare per Horologia in principio ac fine itineris inſpecta. *Problem 109. To investigate the distance of the meridians by clocks inspected at the beginning and at the end of a journey.*

[Margin: Problem 109.]

At the beginning of the journey by which you depart from one place, have a portable clock which shows the hours of that place exactly, by striking or by the fall of sand, and whose motion is never interrupted or varied on the journey by jolting or by a change of air. Then, at the end of the journey, inspect that same clock. If it shows the same hours as are observed at the second place, both places lie under the same meridian. But if the portable clock shows more hours or minutes than are reckoned at the second place, the second place will be more westerly than the one you left; if fewer, more easterly. Cavalieri admits, however (in the *Centuria*, problem 47), that it is difficult on a journey to keep a clock exactly in its state — that is, so that it imitates exactly the daily revolutions of the primum mobile or of the Sun, and their parts. Yet this method is not spurned by Maurolycus (*Cosmographia*, dialogue 3, page 67½), Gemma Frisius (*De usu globi*, chapter 19), Biancani (in the *Introduction to Geography*, before the

[60] the printed sort may be G: one reading of the page finds a round letter with a vertical spur at its lower right, another C with no spur; C is kept because the text names the ecliptic ABC and the Moon's path DC.

Table of Longitudes) and Hérigone (volume 4 of the *Cursus Mathematicus*, page 482). Hérigone adds that the imperfection which can arise from the clock's construction can be corrected by the method of Bulengerus, by means of a large equinoctial sundial, with which tables of equations may be constructed and the irregular motion of the clock reduced to regularity; and the clock is to be kept immune from changes of the air. But the reliability of the clock can be tested far more exactly by means of the pendulum described by us in Book 2, chapter 20, if the traveller halts his journey from time to time.

110. Probl. Diſtantiam Meridianorum Inueſtigare per Eclipſes Satellitum Iouis. *Problem 110. To investigate the distance of the meridians by eclipses of the satellites of Jupiter.*

[Margin: Problem 110.]

Since those four little wandering stars which were detected around Jupiter by Galileo with the telescope, and called the Medicean stars, or the Jovial Companions, perform their epicycles and circuits in such a way that, on the upper side, they fall some- [Margin: Order of the problems.]

[Margin: Order of the problems.]

times into the shadow of Jupiter's globe and suffer eclipse — as is clear from the scheme set out in Book 7, Section 1, chapter 3 — and since they complete very many revolutions within a year: if the moments of these eclipses are observed with a perfect telescope at two places on the Earth far apart, and the time is then deduced from an altitude of the fixed stars taken by Problem 25 of Section 3, the distance of the meridians can be known from the difference of time converted into parts of the equator, and it can be confirmed frequently, since eclipses of these are much more frequent than those of the luminaries. So that observers might be ready, Galileo once set about ephemerides of the motions and eclipses of these satellites, and afterwards so did his pupil Dom Vincentius Renerius [Vincenzo Renieri], Olivetan[61], chief mathematician of the University of Pisa; he sent me a specimen and sample of them more than once. But alas, just as he was about to put them to the press, he was carried off by death outside his monastery, and his writings on this subject could never be found.

That Galileo was the inventor of this device for observing longitudes, and that he proposed it to the Catholic King in the year 1631, is attested by Michaël Florentius Langrenus [Michael Florent van Langren] at the beginning of his *Selenographia* — although the same Galileo had also proposed it to the States of Holland, through Signor Deodatus of Lucca, as Father Bonaventura Cavalieri once told me. Yet I would not wish anything taken from Petrus Herigonius [Hérigone], who asserts in volume 5 of the *Cursus Mathematicus*, page 872, that this same idea came to him two years before Galileo published it. On a moving ship, however, it is hard to use a telescope with the steadiness and firmness that

[61]printed "Obuetanus", the b with a broken bowl; read as Oliuetanus

would be needed to observe the true moment of these eclipses. For the rest of what bears on this, go back to Book 7, Section 1, chapter 4, and its scholia. Besides, unless the ship-masters are very skilled, there is a danger that they will take for these satellites little fixed stars near Jupiter.

111. Probl. Diſtantiam Meridianorum Inuenire per Eclipſes, aut Illuminationes Lunarium Macularum. *Problems 111 and 112. To find the distance of the meridians by the eclipses or the illuminations of the lunar spots.*

This, if I am not mistaken, is the invention of Michael Florent van Langren, which he indicated at the beginning of his *Selenographia*. When he was Cosmographer to the Catholic King, he proposed it to the Councillors for the affairs of the Indies in the year 1632 January 7 and in the year 1633 May 10. In it he also indicates and promises a method of observing the place of the Moon exactly, without the entanglements of parallaxes and refractions; and he states that he had observed the times and the differences of right ascension of certain fixed stars by counting the time by the oscillations of a pendulum, as early as the year 1627, before Galileo had published anything on this matter — which, as I have related in Book 2, chapter 20, happened to me also.

[Margin: Problem 111.]

The *first way*, then, is if one observes in eclipses of the Moon not so much their beginnings and ends as the moments at which certain named particles on the Moon, seen through the telescope and thought to be spots, or islands, or peaks, begin to be darkened or to be lit, and this at different places on the globe of the Earth at the same moment. For example, if the moment of time is observed at which the spot of Grimaldi (called that of Posidonius by Langren) begins to be darkened: when the times of the two observations are compared, their difference will give the distance of the meridians. And since there are very many islands or peaks, as it were of rocks, on the Moon's disc, one phase after another can be observed during the whole time of the eclipse in which some part of the Moon is visible, as was done by Langren and afterwards by Hevelius. Hevelius too approves this method, on page 486 of his *Selenographia*, adding on page 490 another method by the inclinations of the Moon's horns observed at different places.

[Margin: Problem 112.]

The *second way* is if, on each of the nights on which the Moon shines for us, the lunar particles are observed with the telescope from two places on the Earth, together with the moments at which certain peaks begin to be lit that had not before been lit by the Sun — provided that both observers use the same particle, distinguished from the others by its name or some other sign. For the difference of the times will make the distance of the meridians plain. But this very thing is difficult on a moving ship, since the eye wavers along with the telescopes; and the moments of these illuminations are hard to discern, since the Moon's disc is lit slowly, and not wholly except in about 15 days, that is, in 396 hours; and so one minute out of about thirty of the Moon's disc in 13 hours.

113. Probl. Diſtantiam Meridianorum Inueſtigare methodo Orontij per Lunæ locum, ſed fallaci. *Problem 113. To investigate the distance of the meridians by the method of Orontius, through the place of the Moon — but a deceptive method.*

[Margin: Problem 113.]

Orontius Finaeus, in the little work he wrote and entitled *On finding the longitude of places otherwise than by eclipses*, teaches a certain method in which he supposes that the Moon, when it reaches the meridian, lacks parallax, and that its true longitude and latitude therefore do not differ from the apparent — which is false. For as regards longitude this is not true, except when the nonagesimal point of the ecliptic, counted from the horizon, is on the meridian, namely when the Moon's centre is at once on the meridian and at the beginning of Cancer or of Capricorn — unless perhaps the Moon is at the zenith. This error in Orontius I afterwards found noted in Petrus Nonius, in his book *On the Errors of Orontius*, chapter 15; and so there is no reason for us to delay the reader with these entanglements by recounting it at length.

[Margin: Orontius's error.]

114. Probl. Diſtantiam Meridianorum venari methodo Verneri, Gemmæ Friſij, Apiani, & Santbechij, ſed fallaci. *Problem 114. To hunt out the distance of the meridians by the method of Werner, Gemma Frisius, Apian and Santbech — but a deceptive method.*

[Margin: Problem 114.]

Johannes Werner (on Book 1 of Ptolemy's *Geography*), Peter Apian (Part 1 of the *Cosmographia*, at the end of chapter 10), Gemma Frisius (*De usu globi*, chapter 18) and Daniel Santbechius [Daniel Santbech] (in the book *On the Observation of Phenomena*, proposition 14) direct that at the place whose longitude is unknown one take the distance of the Moon's centre from some fixed star not more than 5° distant from the ecliptic, and observe the moment of time then; and divide that distance by the hourly motion of the Moon, so as to obtain from it the moment of the conjunction of the Moon with that star, nearly true, neglecting the parallax, which they think does not appreciably hinder this business. Next they want the true place of the Moon and of the star to be found by tables fixed to some place, or meridian, so that the time may be known at which the Moon ought to be in conjunction with that star. For by comparing this time with the time of the conjunction due at the place of unknown longitude, the difference of the times indicates the difference of longitude. But Petrus Nonius (*On the Errors of Orontius*, chapter 15) rightly warns that an error of about one degree can be committed, because of the Moon's parallax in longitude; and so the visible conjunction can precede or follow the true one by two hours, to which 30° on the equator correspond, and so great an error could be committed in the difference of longitude. At the place where the Moon is observed, therefore, one must find the parallax of the Moon's longitude from the altitude of the pole and the visible altitude of the Moon, and know from the time of the observation the nonagesimal degree of the ecliptic; if the visible place of the Moon lies

between it and the ascending degree, the parallax is to be subtracted from it; if outside it, added, so as to obtain the true place and the true distance of the Moon from the fixed star. But Nonius adds that if, all things computed, an error of 15 minutes is committed in the Moon's motion, one can go wrong by 7½° in the distance of the meridians.

115. Probl. Diſtantiam Meridianorum tentare Methodo Hieronymi Ruſcelli. *Problem 115. To try for the distance of the meridians by the method of Hieronymus Ruscellus.*

[Margin: Problem 115.]

Hieronymus Ruscellus [Girolamo Ruscelli], on Book 1 of Ptolemy's *Geography*, chapter 4, attempts to investigate the longitude of places by means of an instrument that shows the positions of the Moon with respect to any fixed star, toward one of the 32 quarters of the winds marked on the horizon; and, the moment of time being taken then, he orders it to be computed at what time the Moon ought to have had such a position with respect to a meridian of known longitude, so that from the difference of the times there may be derived

[Margin: Order of the problems.]

[Margin: Order of the problems.]

the difference of longitude of the places. But besides omitting the parallax, he also errs in teaching that the instrument should be set on the meridian line as indicated by the magnetic needle alone, since the needle usually declines from the position of the meridian. And Maginus, on that chapter 4, concludes that all these ways are slippery, and that a slight error is easily committed which nevertheless produces a great deception in the longitude.

116. Probl. Diſtantiam Meridianorum inueſtigare methodo Longomontani & Kepleri, ſed corrigenda. *Problem 116. To investigate the distance of the meridians by the method of Longomontanus and Kepler — but a method that needs correcting.*

[Margin: Problem 116.]

Longomontanus in the *Astronomia Danica* (Book 1 of the *Theorics*, at the end of chapter 9) teaches this method more fully, and Kepler in the *Rudolphine Tables* (precept 61) more briefly. *First*: by the problem given in our Book 4, chapter 4, problem 8, find the moment of time at which, on the day of the observation to be made, the Moon will be in the nonagesimal. But this method supposes the longitude of the place known; so proceed otherwise. If the Moon is nearly on the ecliptic, wait for the moment at which both horns of the Moon appear on the same vertical circle; or, if the Moon is full, await the moment at which the lunar spots have exactly the same situation with respect to the vertical circle as you know, from a particular observation and from rules about this situation, they must have in the nonagesimal; or, if the Moon is at the beginning of Cancer or of Capricorn, wait until its centre is on the meridian — for by all these signs the Moon will be in the nonagesimal. At that moment

take the altitude of a fixed star, so as to know the moment of time from it by Problem 25 of Section 3; and at the same time observe the distance of the Moon's centre from some fixed star placed near the ecliptic.

[Margin: First step.]

Second: for that moment of time, gather from the most correct astronomical tables possible the true longitude of the Moon with respect to the meridian to which the roots of the tables are fixed, and the longitude of the fixed star already mentioned. When the smaller is subtracted from the greater, the difference of longitude will remain, and so very nearly the distance between the Moon and the fixed star (since the star chosen is one nearest the ecliptic, and the situation in the nonagesimal excludes all parallax of longitude).

[Margin: Second step.]

Third: divide this difference by the true hourly motion of the Moon, which you will easily gather from the daily motion in the ephemerides, or by the tables of the secondary movables; and the hours and minutes corresponding proportionally to that difference will come out. Convert them into parts of the equator, and you will have the distance of the meridians. The meridian of the observation will be more easterly than the meridian of the tables if the distance of the Moon from the fixed star is found smaller by the tables than by observation, and more westerly if it is greater. Longomontanus then adds 15 of the more notable stars nearer the ecliptic, with their latitude and longitude for the completed year 1620; the construction of a radius or astronomical staff suited to taking small distances between the Moon and the stars; and finally an example, which is as follows.

[Margin: Third step.]

EXEMPLVM. *Example.*

In the year 1617 October 6, Old Style, let the distance of the Moon's centre from Aldebaran have been observed as 10°, when the Moon was in the nonagesimal; and from the altitude of the stars and the altitude of the pole let that moment have been 11^h *after noon at the place of unknown longitude. For that moment the Danish tables give the place of Aldebaran in* ♊ *4° 26′, and the place of the Moon, with respect to the meridian of Copenhagen, in* ♉ *27° 20′. The difference of these places on the ecliptic is 7° 6′, and very nearly so great a distance of the Moon from Aldebaran ought to be observed if the place of observation were under the meridian of Copenhagen; but it was observed as 10°. The distances therefore differ by 2° 54′. Now on that day the daily motion of the Moon is 14° 41′ and the hourly 31′ 44″. Say therefore: if 31′ 44″ require 1 hour, how many hours do 2° 54′, or 174′, require? Surely* $4^h\ 45^m$*, that is, 71° 15′ of the equator. By just as many degrees, 71° 15′, then, the place of observation is more easterly than Copenhagen, because the observed distance is greater than the one deduced from the Copenhagen tables. Hence, if the longitude of Copenhagen from the Fortunate Isles is, as Longomontanus supposes, 36° 40′, it follows that the longitude of the place of observation is 107° 55′.*

117. Probl. Longomontani Methodum præcedentem Corrigere. *Problem 117. To correct the preceding method of Longomontanus.*

[Margin: Problem 117.]

The method already described is faulty, because — owing both to the true latitude of the fixed star and to the visible latitude of the Moon, altered by the parallax of altitude — the observed ecliptic distance can be notably smaller than the true one; especially since, among the 15 stars that Longomontanus proposes for use, he puts the bright star of Aries, whose northern latitude is, for him, 9° 57′; Aldebaran, whose southern latitude is 5° 31′; and the lower head of the Twins, whose northern latitude is 6° 38′. To correct this method, then, at the moment when one observer takes the visible distance of the Moon from the star, and a second the altitude of the same star, or of another suitable for drawing out the moment of time, a third must take the visible altitude of the Moon's centre when it stands in the nonagesimal. With these acquired: *first*, let everything be carried out as in the preceding problem; and, the time being equated according to the supposed difference of the meridians, let the true latitude of the Moon be sought by astronomical tables or ephemerides, together with the parallax of altitude corresponding to the observed altitude. Let this parallax be added to a true southern latitude, or subtracted from a northern one that is nevertheless greater than the parallax, and the visible latitude of the Moon will be had, of the same kind as the true latitude. But if the parallax of altitude is greater than the northern latitude, their difference will be the Moon's visible southern latitude. I speak, however, of an observer who is on this side of the ecliptic, toward the north; for if beyond it, the names of northern latitude must be changed into southern, and so on.

[Margin: First step.]

Second: from the given latitude of the fixed star and its visible distance, and from the visible latitude of the Moon, gather by Problem 102 of Section 2 the angle at the pole of the ecliptic, that is, the difference of longitude, or the ecliptic distance of the Moon from the fixed star; and use this difference in the same way as you used it in Problem 116. In this way you will obtain a more accurate distance of the meridians.

[Margin: Second step.]

EXEMPLVM Præcedens correctum. *Example: the preceding one corrected.*

In the case of the preceding problem, let the altitude of the Moon, standing in the nonagesimal, have been observed as 40°. Since in that year 1617, on October 6, or on the 16th by the New Style, at 11^h after noon at Copenhagen the true latitude of the Moon was northern, 4° 45′, and was decreasing, and since the place of observation was found more easterly by 4^h 45^m, the true latitude at the place of observation was certainly only 4° 42′. The Moon was then about 128° distant from the apogee; therefore the parallax of altitude (which, when the Moon is in the nonagesimal, is the same as the parallax of latitude) corresponding to the observed altitude of 40° is 47′ 58″, or 48′. When these are taken from the Moon's true latitude of 4° 42′, the visible northern

latitude of the Moon remains 3° 54′, and its complement to the quadrant is 86° 6′. At the same time the latitude of Aldebaran was southern, 5° 31′, and its visible distance from the Moon's centre was 10°. Therefore, in the following figure, let semicircles of latitude be drawn from the poles of the ecliptic P and Q, one, PDQ, through the centre of the Moon D, and the other, PRQ, through the centre of Aldebaran R.

Diagram. Set in the right column, beside the short lines of the example. Two curved arcs (the semicircles of latitude) join P at the top and Q at the bottom in a lens shape. On the left arc stand D and, below it, A; on the right arc M above, B in the middle and R below. A straight line runs from A across to B (the ecliptic arc AB); straight lines run from D to M and from D down toward R.

- P (top): the pole of the ecliptic
- M (right arc, upper)
- D (left arc): the centre of the Moon
- A (left arc, below D): on the ecliptic
- B (right arc): on the ecliptic
- R (right arc, lower): the centre of Aldebaran
- Q (bottom): the other pole of the ecliptic

Since in the triangle DPR there are given DR, the distance, 10°; DP, the complement of the Moon's visible latitude, 86° 6′; and PR, 95° 31′ (the quadrant PB being added, of course, to the star's latitude BR), the angle P is found by [proposition] 14 of oblique-angled spherical triangles to be 3° 23′. So great, then, is AB, namely the ecliptic distance of the Moon from Aldebaran — and not 2° 54′, as Longomontanus thought. Therefore, since the hourly motion of the Moon was 31′ 44″, surely $6^h\ 20^m$ are due to 3° 23′, to which 95° on the equator correspond, and not 71° 15′. Longomontanus therefore erred by

[Margin: A notable error of Longomontanus.]

[Margin: Order of the problems.]

23° 45′ in the distance of these meridians, through neglecting to take the latitudes into account.

118. & 119. Probl. Meridianorum Diſtãtiam Methodo Io. Bapt. Morini inquirere, ſed fallaciter. *Problems 118 and 119. To seek the distance of the meridians by the method of Jean-Baptiste Morin — but deceptively.*

Jean-Baptiste Morin, Royal Mathematician at Paris, in 1634 put his method before judges delegated for the purpose, and set it down in a book entitled *On the Science of Longitudes*, resting for the most part on the methods of Orontius and Werner. Among those judges was Pierre Hérigone, who declared, with the others, that these ways were useless for navigation. He reports and refutes them in volume 4 of the *Cursus Mathematicus*, at the end of the *Histiodromia*, that is, from page 489 on, and in volume 5, at the end, from page 862 on. The chief of them are two.

The first is as follows. In the semicircle of the meridian AEB, under which the place of unknown longitude lies, let a fixed star, say C, be observed crossing the

meridian together with the Moon's centre D. For once the right ascension of the star is given or found, the same will be the right ascension of the midheaven, and of the centre of the Moon as well. At the same moment let the meridian altitude AD of the Moon be observed; and from this and the altitude of the pole BF let the declination of the Moon be gathered by Problem 3 of Section 2. With this and with the right ascension of the Moon, let the place of the Moon in the zodiac be sought by Problem 48 of Section 2. From this, look up in the ephemerides the hour with respect to the place to whose meridian the ephemerides are computed, and the degree of the zodiac in which the Sun stands at the time of the observation, and so, through this degree, the right ascension of the Sun. Compared with the right ascension of the midheaven mentioned above, it will give the hour at the place of observation; and from the difference of the two times, the difference of geographical longitude between the place of observation and the place of the tables or ephemerides will become known.

[Margin: Problem 118.]

Diagram. Set in the left column beside the first way. A semicircle (the meridian) on a straight base. E stands at the top, D at the upper left, C at the left beside a small dot, F at the right on the arc, G inside above the base, A at the lower left end of the base and B at the lower right. A straight line runs from G up to the arc near E, and another from G toward F.

- E (top of the arc)
- D (upper left): the Moon's centre
- C (left, with a dot): the fixed star
- F (right, on the arc)
- G (inside, above the base)
- A (lower left end of the base)
- B (lower right end of the base)

The second way is as follows. Let the altitude of the pole FB be given; and while the Moon D crosses the meridian ADB, let the altitude of a star G be observed, beyond the hazard of sensible refractions. For given the complement of the altitude GE, and the complement of the pole's altitude FE, and the arc FG known from the declination of the star, the angle GFE in the triangle EFG will become known by [proposition] 14 of oblique-angled spherical triangles. It is to be subtracted from the right ascension of the star G if the star is eastern, or added if it is western, so that the right ascension of the midheaven is had. The rest, then, is carried out as in the first way.

[Margin: Problem 119.]

But in these and other ways Morin errs: he uses the declination of the Moon deduced from the observed altitude as if it were the true one, when it is the visible one and subject to a notable parallax, which pours a conspicuous error into the distance of the meridians.

120. Probl. Diſtantiam Meridianorum inueſtigare Methodo Herigonij, & ea occaſione Declinationem Lunæ inuenire. *Problem 120. To investigate the distance of the meridians by the method of Hérigone, and on that occasion to find the declination of the Moon.*

Hérigone, in volume 4 of the *Cursus Mathematicus*, in the *Histiodromia*, proposition 23, page 486, sets out this lemma first. Let AI be a portion of the equator, AF of the ecliptic, and DEC of the Moon's path; let the Moon be at C, and let its circle of declination be IC.

[Margin: Problem 120.]

Diagram. Set in the left column beside the lemma of Problem 120. A stands at the upper left with the sign of Aries beside it, and C at the upper right. Two curves sag from the region of A to C and cross near the bottom middle, where the sign of the ascending node stands above the crossing and E below it. D at the left and F at the right lie on a nearly horizontal line through E. I is the bottom point, where straight lines from A and from C meet.

- A ♈ (upper left): the beginning of Aries
- C (upper right): the Moon
- ☊ (above the crossing): the ascending node
- D (left)
- E (below the crossing)
- F (right)
- I (bottom point)

First, then, in the triangle AFI, right-angled at I, from the obliquity of the ecliptic, that is, the angle IAF, and from the right ascension of the Moon AI, let the angle IFA, the side IF and the base AF be investigated by [propositions] 8, 7 and 9 of right-angled spherical triangles.

[Margin: First step.]

Second: from the base AF let there be subtracted the longitude of the Dragon's Head ☊ from the beginning of Aries, which is AE, and EF will be left known. For in the triangle CEF — in which the angle CEF, that is, the greatest latitude of the Moon, is in addition supposed given, and the angle EFC, the supplement of the IFA found to two right angles — the base will be given together with the adjacent angles. Therefore by [proposition] 11 or 12 of oblique spherical triangles seek the side CF; joined to the side IF, it will give the true declination of the Moon. Let the rest be done as in Problems 118 and 119. But the place at which the Moon is observed in the meridian must be known, so that there is no error in its longitude greater than 15 or 20 degrees, in order that the calculation of the longitude of the node ☊ and of the right ascension of the Moon can be transferred to it from the meridian to which the tables are fixed. With this assumed, no error is to be feared in the place of the node greater than 8″ or 10″, nor in the declination of the Moon greater than 43″ or 45″. Hérigone admits, however, at page 495, that this method of his has hardly any use in navigation, and that it is better to have recourse to loxodromic tables; and at page 496 he affirms that, owing to the diversity of the hypotheses of the best astronomers,

the place of the Moon is not so certain that an error of 18′ is plainly avoided — and from that an error of more than 8 degrees can overflow into the distance of the meridians.

[Margin: Second step.]

Monitum de Reliquis Modis inquirendi Diſtantiam Meridianorum. *A caution on the remaining ways of seeking the distance of the meridians.*

The remaining ways I have not thought fit to copy here from my Geographical Book, as being less certain — especially those which rest on the declination of the magnetic needle, and on the ship's course while the ship follows the guidance of the magnetic needle, granted that this latter is more certain than the former. For here we pursue only those problems that can serve astronomy; but the inquiry into longitude that pilots hunt by histiodromy and by loxodromic lines and angles can hardly serve astronomy, because they are often at sea away from islands and mainlands, or in places unsuited to astronomical observations. If anyone nevertheless wants this method, let him have recourse to Problem 102. For pilots, by observing the lines of the rhumbs and the angles made by them with the meridian line, and correcting these according to the declination of the magnetic needle, acquire the angles of position from one place to the other; and with the altitudes of the pole taken at both places, they find the difference of longitude. I hear also that Doctor Io. Marci [Jan Marek Marci] has published at Prague a little work on finding the difference of longitude by the true motion of the Moon, a truly learned one; but since I do not have that little work at hand, I am forced to go without so great a fruit, and cannot communicate it to my reader — just as I cannot communicate the ways that Robertus Dudlæus [Robert Dudley] likewise hands down by means of the Moon in *De arcanis maris*, Book 1, chapters 5 and 6, a book I was allowed to see only in haste.

Probl. 121. 122. 123. 124. 125. Dato temporis Momento, definire, Vbi tunc meridies, vbi media nox; Vbi dies, vbi nox ſit, intra Zonas ſaltem Temperatas & Torridam; & Quibus oriatur tunc Sol, quibus occidat; & Quota ſit hora cuiuſuis ſpeciei in quouis loco dato, ac viciſſim In quo loco terrarum ſint tales ac tot horæ & minutiæ horarum. *Problems 121, 122, 123, 124, 125. Given a moment of time, to determine where it is then noon and where midnight; where it is day and where night, at least within the temperate zones and the torrid; for whom the Sun is then rising and for whom it is setting; and what hour of any kind it is in any given place; and conversely, in what place on Earth there are such and so many hours and minutes of hours.*

[Margin: Problems 121, 122, 123, 124, 125.]

All these and other problems are solved by our Geographical Cross, which we published at Bologna in 1643, and which we shall publish again, God willing, in our Geographical Book, which will be the fourth of volume 2.

[Margin: Order of the problems.]

SECTIO QVINTA DE PARALLAXIBVS.

Section Five. On Parallaxes.

Although in the earlier books the occasion — indeed the necessity — of treating parallaxes presented itself more than once, it was nevertheless not proper to work through there everything that bears on the completeness of this doctrine. I therefore decided to set up this special Section on parallaxes, in which, discussing all the kinds of parallax in order, I would point out what we have already explained, and pursue the rest, which has been more advisedly put off to this place. This the more so because I have seen not a few things omitted by other authors: for most of them treated parallaxes only on the occasion of eclipses of the Sun, or of the Moon's distance from the Earth. I have listed their names in Book 5, chapter 10, number 1.

[Margin: Authors who have written on parallaxes.]

CAPVT I.

Chapter I.

De Parallaxi Altitudinis vel Diſtantiæ à Vertice in Communi. *On the parallax of altitude, or of distance from the vertex, in general.*

What the parallax of altitude, or of distance from the vertex, is, I taught in Book 1, chapter 35, adding five rules, or axioms, pertaining to this kind; and in its Scholium 3 I demonstrated which angle is the parallactic angle of altitude, that is, the angle that measures the quantity of this parallax at least indirectly, or by equivalence. That was as much as was then sufficient for the apprenticeship of the reader, who was to be grounded in that book in certain common principles and properties of the sphere. Afterwards, in Book 5, chapter 10, on the occasion of solar eclipses, I handed down to the reader, by then more advanced in astronomy, not only the definitions of this parallax, and what the true and the visible place of a star is, and what the true and the visible altitude and distance from the vertex are, but also other definitions, and selected theorems and problems bearing on the parallaxes that serve eclipses of the Sun. Lastly, in Book 8, Section 1, chapter 17, since the parallaxes of comets had to be dealt with, and since in a treatment in itself quite intricate and varied we did not want readers to have to beg this doctrine from elsewhere, we set out first the definitions of all the kinds of parallax and the more notable axioms of parallaxes; and in the same chapter, number 2, we explained what the parallax of altitude is formally, and what it is taken equivalently, that is, which angles are the measure of parallaxes, or equivalent to that measure. From the places already mentioned, then, taken together, and from what is to be said here, especially in chapter 12, let the reader seek this doctrine whole, just as if it had all been copied into this chapter.

I add, however, that Regiomontanus, in Problem 1 of *De cometis*, and Chiaramonti, in Book 1, chapter 1 of the *Antitycho*, and some others, are accustomed to draw from the centre of the Earth a line parallel to the line of the visible

place, and to use the arc of the Firmament intercepted between this parallel and the line of the true place as the parallax of altitude — understand it equivalently, as I set out in the same Book 8, Section 1, chapter 17, number 2, with a diagram suited to the purpose. Moreover, when the visible place or the visible altitude is distinguished from the true place or the true altitude, refraction is not considered in this Section — refraction, which can use up the whole parallax or diminish it — but parallax alone, by itself. For what happens to these through the mixture or concurrence of parallax with refraction must be told in the following Section.

CAPVT II.

Chapter II.

In quo Problemata ad Parallaxim Altitudinis ſpectantia. *In which are the problems bearing on the parallax of altitude.*

Since the altitude of a star is the complement of its distance from the vertex, and the distance from the vertex is the complement of the altitude to the quadrant, that is, to 90°, it is enough to give the problems bearing on altitude; for from these, what pertains to the distance from the vertex at once becomes known.

1. Probl. Diſtantiam Sideris à centro Terræ inuenire, Data Parallaxi Horizontali. *Problem 1. To find the distance of a star from the centre of the Earth, given the horizontal parallax.*

[Margin: Problem 1.]

In the following diagram, let BH be a quadrant of a great circle of the Earth, described from the Earth's centre A, under the quadrant KG of the vertical circle, which is to be conceived in the heaven of the star. In it let the star appear at L to the eye at B, placed on the surface of the Earth's convexity, along the straight line BL of the physical horizon, parallel to the astronomical horizon AG; on these let the vertical line ABK fall perpendicularly, tending toward the zenith of the observer B; and let the straight line AL be drawn from the Earth's centre to the centre of the star. Then in the triangle ABL, right-angled at B by construction and by the property of a perpendicular line, the angle L, the horizontal parallax of altitude, is given. With it, and with the opposite side, that is, the Earth's semidiameter AB, taken as unity, seek the base AL by [proposition] 6 of right-angled plane triangles — that is, the distance of the star from the Earth's centre — and it will be known in terrestrial semidiameters, that is, in parts of which the Earth's semidiameter is one. In this way, from the Moon's horizontal parallaxes, we drew out its varying distance from the Earth's centre in Book 4, chapter 14, with another diagram set out first in chapter 13.

Diagram. Set in the right column beside the construction of Problem 1. A quadrant arc runs from G at the lower left up through L and C to D near the top; a vertical straight line runs from A at the bottom right up through B to K

at the top; a horizontal base runs from G through H to A. A smaller arc about A passes through B toward H. Straight lines run down toward A from C, L and B, and lines from B fan out to E, I and F, at the right of B and below it.

- D (top of the arc)
- K (top of the vertical line): toward the zenith
- C (on the arc, upper left)
- B (on the vertical line): the observer's eye
- L (on the arc, left): the star's apparent place
- E (right of B)
- I (below E)
- F (below I)
- G (lower left end of the arc and base)
- H (on the base)
- A (bottom right): the Earth's centre

2. Probl. Parallaxin Sideris Horizontalem inueſtigare, Data eius diſtantia à centro Terræ in partibus, qualium Semidiameter terræ eſt vna. *Problem 2. To investigate the horizontal parallax of a star, given its distance from the centre of the Earth in parts of which the Earth's semidiameter is one.*

[Margin: Problem 2.]

In the preceding triangle ABL, right-angled at B, given the base AL, that is, the distance of the star in parts of which AB is one, you will certainly be able to investigate the opposite angle ALB, namely the horizontal parallax of the star, by [proposition] 2 of right-angled plane triangles. By this reasoning, in Book 3, chapter 8, we obtained the horizontal parallaxes of the Sun, taking its greatest, least and mean distance from the Earth's centre from various astronomers; and we did the same for the five lesser planets in Book 7, Section 6, chapter 7. Let the reader go there, then, if he wants an example; for there is no leisure to linger here — as there is none in the following problems either, which we did not wish to omit here, so that the whole doctrine of parallaxes might be had collected together, but which we ought not to treat anew, only point to as worked through elsewhere.

[Margin: Order of the problems.]

3. & 4. Probl. Parallaxim Datæ altitudini Veræ aut Viſæ conuenientem inuenire, Data Sideris diſtantia à Centro Terræ in ſemidiam. terræ. Vbi de Tabularum geminarum Auctoribus ſtructura & Vſu. *Problems 3 and 4. To find the parallax corresponding to a given true or visible altitude, given the star's distance from the centre of the Earth in semidiameters of the Earth. Where the twin tables are treated: their authors, construction and use.*

[Margin: Problem 3.]

In the preceding figure let the star be at C, AC its distance from the centre of the Earth, and CB its distance from the eye of the observer B. For in the triangle ABC there are given AB, namely one semidiameter of the Earth, and AC, the

distance of the star in terrestrial semidiameters. *First*, then, if the true altitude GC of the star is given, which the angle GAC measures, then by subtracting it from 90° the angle CAB will be known, contained by the known sides AB and AC; therefore by [proposition] 3 of oblique-angled plane triangles the angle ACB will become known, that is, the parallax corresponding to the given true altitude. By this device the table of parallaxes of the true altitude is constructed. These parallaxes are to be used when you wish to convert the true altitude of a star, known from astronomical calculation, into the visible altitude, that is, the altitude apparent to the eye at B, by taking the parallax away from the true altitude. Such is the table of parallaxes of the Sun and Moon given by Ptolemy in Book 5, chapter 18; by Copernicus in Book 4, chapter 24; by Reinhold in the *Prutenic Tables*, page 120; by Maginus in the *Secunda Mobilia*, page 678, and in the *Supplement to the Ephemerides*, pages 59 and 293; by Tycho in volume 1 of the *Progymnasmata*, pages 80 and 120; by Longomontanus in the *Astronomia Danica*, Book 1 of the *Theorics*, chapter 8; by Giuseppe Moletti in his Tables, page 82; by Mulerius in the *Frisian Tables*, page 421; by Kepler in the *Rudolphine Tables*, page 98; by Reinerus [Vincenzo Renieri] in the *New Medicean Tables*, pages 311 and 364; by Lansbergen in his tables, from page 46; by Pétau in Book 8 of *De doctrina temporum*, page 737; by Boulliau in the *Philolaic Tables*, page 138; and by Wendelen in the *Idea Tabularum Atlanticarum*, page 34.

[Margin: First case: for the true altitude.]

[Margin: Authors of these tables.]

Second, let the visible altitude LBC be given. Added to 90°, it makes the angle ABC; with this, and with the sides AB and AC given as above, you will find by the first [proposition] of oblique-angled plane triangles the parallactic angle ACB, that is, the parallax corresponding to the visible altitude. The same can, however, be found by right-angled triangles, producing the side CB to E, until the perpendicular AE from A meets it. For in the triangle ABE, right-angled at E, given the semidiameter of the Earth as one, and the angle ABE, that is, CBK, which is the complement of the visible altitude, you will find the side AE by the first [proposition] of right-angled plane triangles. Then in the triangle ACE, right-angled at E, from the side AE now found and the given distance AC, you will seek the parallax ACE corresponding to the visible altitude. In this way the second table of parallaxes of altitude is constructed, which is to be used when the visible altitude is to be converted into the true one, by adding the parallax to the visible altitude. A table constructed for true altitudes can, however, serve for visible ones, if you keep our precept handed down in Book 3, chapter 8, number 7.

[Margin: Second case: for the visible altitude.]

[Margin: Problem 4.]

5. & 6. Probl. Diſtantiam Sideris inquirere, Data Altitudine ipſius vera aut viſa, cum Parallaxi altitudinis. *Problems 5 and 6. To seek the distance of a star, given its true or visible altitude together with the parallax of altitude.*

[Margin: Problem 5.]

In the triangle ABC of the preceding figure, if the true altitude is given, its complement is the angle CAB. With this, and with the angle C of the parallax, and so with the remainder to two right angles, ABC, and with the semidiameter of the Earth AB as one, seek the distance of the star AC by [proposition] 1 of oblique plane triangles. But if the visible altitude CBL is given, add it to 90°, and the angle ABC will result; with this, and with the parallactic angle C, and with the semidiameter of the Earth AB as one, seek the distance AC.

[Margin: Problem 6.]

7. & 8 Probl. Data Parallaxi Horizontali, inuenire Parallaxim cuiuis altitudini viſæ conuenientem, & viciſſim Data Parallaxi conueniente altitudini viſæ; inuenire Parallaxim Horizontalem. *Problems 7 and 8. Given the horizontal parallax, to find the parallax corresponding to any visible altitude; and conversely, given the parallax corresponding to a visible altitude, to find the horizontal parallax.*

[Margin: Problem 7.]

First: if the horizontal parallax is given, even if the distance of the star from the Earth is not known, let the proportion be made: as the radius, or whole sine, is to the sine of the horizontal parallax, so is the second sine of the given visible altitude to the sine of the parallax corresponding to that altitude.

Second: if the parallax corresponding to a given visible altitude is given, let the proportion be made: as the second sine of the given altitude is to the given parallax, so is the radius to the horizontal parallax. These problems are very useful for the parallaxes of the five lesser planets, for which tables are not usually constructed, except perhaps of the horizontal parallaxes, as we did in Book 7, Section 6, chapter 7.

[Margin: Problem 8.]

[Margin: Note for the parallaxes of the five planets.]

The reason for these problems is this. In the preceding figure LB falls perpendicularly on AB, and CE on AE, by construction; therefore AB comes out as the right sine of the horizontal parallax L, and AE as the right sine of the parallax ACE, while the bases subtending the right angles, namely AL and AC, come out as the radius, or whole sine — as is plain from the definitions of sines. Thus in the triangle ABE, if AB is the radius, AE will be the right sine of the angle ABE, that is, of its vertically opposite angle CBK, which is the complement of the visible altitude. With these laid down, there arises a lemma that is the foundation of Problems 7 and 8. For I say, with our Cysatus [Johann Baptist Cysat], *De cometis*, chapter 3, lemma 4: *As AB, the whole sine of the right angle AEB, is to the same AB, the right sine of the horizontal parallax ALB, so is AE, the sine of the angle ABE, the complement of the visible altitude, to the same AE, the sine of the parallax ACB; and conversely.* This lemma is founded on [proposition] 4 of Book 6 of Euclid's *Elements*, since the triangle ABE is considered in two ways: first, as the side AB in it is the whole sine and the side AE the right sine

of the angle ABE; second, as AE is the right sine of the parallax C, and AB in turn the sine of the horizontal parallax L. In fact, therefore, there are two equiangular triangles, having as homologous those sides that are subtended by equal angles.

[Margin: Lemma on which Problems 7 and 8 are founded.]

Diagram. Set in the right column beside the proof of the lemma; the same construction as the figure on p. 641. A quadrant arc runs from G at the lower left up through L and C to D near the top; a vertical line runs from A at the bottom right through B to K; the base runs from G through H to A; a smaller arc stands about A. Straight lines run down toward A from C, L and B, and lines from B fan out to E, I and F.

- D (top of the arc)
- K (top of the vertical line)
- C (on the arc, upper left): the star
- B (on the vertical line): the observer's eye
- L (on the arc, left)
- E (right of B): the foot of the perpendicular AE on CB produced
- I (below E)
- F (below I)
- G (lower left end of the arc and base)
- H (on the base)
- A (bottom right): the Earth's centre

9. Probl. Data Parallaxi conueniente vni Altitudini Viſæ minori, inuenire Parallaxin conuenientẽ alteri viſæ Altit.datamaiori. *Problem 9. Given the parallax corresponding to one smaller visible altitude, to find the parallax corresponding to another, greater, given visible altitude.*

[Margin: Problem 9.]

Let the proportion be made: as the second sine of the smaller altitude is to the second sine of the greater altitude, so is the right sine of the parallax corresponding to the smaller altitude to the right sine of the parallax corresponding to the greater visible altitude. For as Snel demonstrated in *De cometa anni 1618*, chapter 4, page 26; Chiaramonti in Book 3 of *De novis stellis*, chapter 19, and in the supplement to the *Antitycho*, part 2, chapter 35; and our Cysat in *De cometis*, chapter 3, lemma 5: *As the second sine of the smaller visible altitude is to the second sine of the greater, so is the sine of the parallax at the smaller visible altitude to the sine of the parallax at the greater.* For by proposition 1 of Clavius's rectilinear triangles, in these triangles the sines of the angles opposite the sides are to one another as side is to side, and so on.

[Margin: Lemma for Problem 9.]

10. Probl. Datis duabus Parallaxibus duarum Altitudinum Viſarum, inquirere Parallaxin Horizontalem. *Problem 10. Given two parallaxes of two visible altitudes, to seek the horizontal parallax.*

[Margin: Problem 10.]

First, subtract each of the visible altitudes from 90°; for the visible distances from the vertex will be left, and for these seek their right sines separately. Likewise, for the given parallaxes seek their right sines one by one. *Second*: let the proportion be made: as the difference of the sines corresponding to the visible distances from the vertex is to the difference of the sines corresponding to the parallaxes, so is the whole sine to the sine of the horizontal parallax. This practice rests on the second part of lemma 5, demonstrated by Cysat in *De cometis*, chapter 3; that part runs as follows: *As the difference of the sines of the complements of the star's visible altitudes is to the difference of the sines of the parallaxes, so is the whole sine to the sine of the greatest, or horizontal, parallax.*

[Margin: Order of the problems.]

11. Probl. Data Differentia duarum Parallaxium, cum proportione quam habent earum Sinus; vtramq. Parallaxim ſingillatim notam facere. *Problem 11. Given the difference of two parallaxes, together with the ratio that their sines have, to make each parallax known individually.*

[Margin: Problem 11.]

Let the proportion be made: as half the difference of the terms of the given ratio is to the tangent of half the difference of the parallaxes, so is the sum made up of half the difference of the terms of the ratio and the consequent term of the same ratio to the tangent of the angle that is to be added to half the difference of the parallaxes, so that the greater parallax becomes known, or to be subtracted, so that the smaller becomes known. See Clavius, proposition 7 on rectilinear triangles, and Cysat, *De cometis*, chapter 3, lemma 6. How the difference of parallax is to be understood will be stated below, on another occasion, in chapter 9.

12. Probl. Data Differentia duarum ignotarum Parallaxium altitudinis, & duabus Altitudinibus, ex quibus illa eſt obſeruata, reperire ſingillatim vtramq. Parallaxim, ſingulæ altitudini viſæ debitam. *Problem 12. Given the difference of two unknown parallaxes of altitude, and the two altitudes from which it was observed, to find each parallax individually, the one owed to each visible altitude.*

[Margin: Problem 12.]

First, seek the sines of the complements of the given altitudes to the quadrant, that is, of the visible distances from the vertex. Add these into a sum and take half of the sum; from this half subtract either of the sines, and the difference between that sine and the half-sum of the sines will remain. *Second*, let the proportion be made: as the difference between either sine and the half-sum of the sines is to the half-sum of the sines, so is the tangent of half the difference of the parallaxes to the tangent of half the sum of the parallaxes. If to this half-sum you add half the difference of the parallaxes, the greater parallax will result, owed to the smaller altitude; if you subtract it, the smaller parallax will result, owed to the greater visible altitude. For the theory of this problem see

Snel, *De cometa anni 1618*, chapter 4, and Chiaramonti, *De stellis novis*, Book 3, proposition 2.

13. Probl. Data Summa duarum Parallaxium ſingillatim ignotarum, & duabus Altitudinibus viſis, ex quibus ſumma illa fuit obſeruata, reperire vtramq. Parallaxim altitudinis ſingillatim. *Problem 13. Given the sum of two parallaxes that are individually unknown, and the two visible altitudes from which that sum was observed, to find each parallax of altitude individually.*

[Margin: Problem 13.]

Let the proportion be made: as the half-sum of the second sines corresponding to the altitudes is to the difference between either of these sines and their half-sum, so is the tangent of half the sum of the parallaxes to the tangent of half the difference of the parallaxes — which is to be added to their half-sum if you want the greater parallax, or subtracted if you want the smaller. This practice rests on the proposition demonstrated by the authors cited above, if its terms are converted.

14. Probl. Parallaxim Altitudinis Lunæ à Sole inquirere. *Problem 14. To seek the parallax of altitude of the Moon from the Sun.*

[Margin: Problem 14.]

Subtract the parallax of altitude of the Moon from the parallax of altitude of the Sun when the Sun stands at the same altitude as the Moon, and the parallax of the Moon from the Sun will be left.

15. Probl. Parallaxim Altitudinis Phænomeni inueſtigare, Datis Latitudine ac Longitudine ipſius obſeruata eodem momento in duobus terræ locis; quorum nota ſit altitudo Poli & diſtantia Meridianorum; Dato inſuper Momento temporis, & Eclipticæ puncto Culminante in vtroq. Meridiano, & Obliquitate Eclipticæ. *Problem 15. To investigate the parallax of altitude of a phenomenon, given its latitude and longitude observed at the same moment in two places on the Earth whose pole altitude and distance of meridians are known; given, moreover, the moment of time, the point of the ecliptic culminating in each meridian, and the obliquity of the ecliptic.*

[Margin: Problem 15.]

I have already given this problem, with its example, in Book 8, Section 1, chapter 19, numbers 11, 12 and 13. We owe this problem, however, to Scipione Chiaramonti.

16. Probl. Altitudinis Parallaxim inueſtigare, Datis Phænomeni ab eadem Fixa ſtella Diſtantijs viſis eodem momento temporis, ex duobus locis terræ, quorum nota ſit altitudo poli, ac diſtantia Meridianorum, & Momento temporis, cum Declinatione atq. Aſcenſione Recta ſtellæ Fixæ. *Problem 16. To investigate the parallax of altitude, given the visible distances of a phenomenon*

from the same fixed star at the same moment of time, from two places on the Earth whose pole altitude and distance of meridians are known, and given the moment of time, together with the declination and right ascension of the fixed star.

[Margin: Problem 16.]

This problem likewise was proposed by Father Francesco Maria Grimaldi while we were discussing the comet of the year 1618, and was solved by him at once; I have already set it out in Book 8, Section 1, chapter 22, number 25.

Monitum de Reliquis ad hoc caput ſpectantibus. *A notice on the remaining problems that belong to this chapter.*

The remaining problems that belong to this chapter contain ways of observing phenomena, especially new ones, and of gathering from them whether they have a parallax, or how great a one; for just reasons it seemed best to defer these to chapters 9, 10 and 11. Some also belong to chapters 6, 7, 8 and 9, insofar as the parallax of altitude can be confirmed from a given parallax of declination, of right ascension, of latitude or of longitude; but the parallax of altitude is in fact usually put first, as the foundation for investigating the remaining parallaxes.

CAPVT III.

Chapter III.

De Parallaxi Profunditatis. *On the parallax of depth [parallaxis profunditatis].*

Although this kind can be brought back to the parallax of distance from the vertex, it would not rightly be called a parallax of altitude above the horizon, since through it the star appears below the horizon, unless refraction prevents this; and distinguishing it is useful for phenomena, especially those nearer the Earth, if they are viewed from very high mountains. I should like, further, what I said about the sensible horizon, with respect to an eye raised above the Earth, in Book 1, chapter 8, scholium 2, to be read again.

17. Probl. Parallaxin Maximam Profunditatis Sideris in Horizonte Aſtronomico ſeu vero conſtituti explicare ac inuenire, Data Diſtantia Sideris à Centro Terræ. *Problem 17. To explain and find the greatest parallax of depth of a star set in the astronomical, or true, horizon, given the star's distance from the centre of the Earth.*

[Margin: Problem 17.]

In the present figure, about the Earth's centre H let the circle of the Earth be described, with its great circle AI, etc., and the vertical semicircle VRM, under which let AI, etc., lie. In this semicircle let the star be at R, a point of the astronomical, or true, horizon HR, which is parallel to the physical horizon AB; through these horizons let the vertical line VAHM be drawn at right angles. From R, the centre of the star, let the straight line RIC be drawn, touching the Earth at I and therefore making, by [proposition] 18 of Euclid's third [book], a

right angle with the semidiameter HI at the point of contact I; and parallel to RIC let HN be drawn. For CIR will be the sensible horizon with respect to the eye at C, just as AB is with respect to the eye at A, which is not raised. Lastly, just as the horizontal parallax with respect to the eye at A is the angle ABH, so with respect to the eye at C, and at I, the horizontal parallax will be the angle HRI; but with respect to the raised eye at C it is to be called the parallax of depth, not of altitude, because

Diagram. Set in the right column beside the construction of Problem 17. A semicircle runs from V at the top round the right side through B and R to M at the bottom; a smaller circle, the Earth, stands about H. A vertical line runs down the left through V, C, A, H and M. The horizontal HR runs from the Earth's centre to the star; a straight line runs from C through I to R; a line runs from A to B; a dotted line runs from A down to the right; a line runs from H down to N on the lower right.

- V (top of the vertical line and the semicircle)
- C (on the vertical line, below V): the raised eye
- A (on the vertical line, at the Earth's surface): the eye not raised
- I (right of A, on the Earth's circle): the point of contact
- B (on the semicircle, upper right): end of the physical horizon AB
- H (on the vertical line): the Earth's centre
- R (on the semicircle, right): the star
- N (on the semicircle, lower right): end of HN, drawn parallel to RIC
- M (bottom of the vertical line and the semicircle)

[Margin: Order of the problems.]

the star R will appear to the eye at C to be distant from the vertex V not only by a quadrant, as if the eye were at C and the star at B, but by more than a quadrant, by as much as the angle HRI; namely by the whole angle VCR, which by [proposition] 32 of Euclid's first [book] is equal to the two interior and opposite angles, namely the right angle CHR and the acute angle HRC. Therefore the angle R will be the amount by which the visible distance exceeds the quadrant. Further, the angle HRC is equal to the angle RHN, which measures the arc RN, because the lines CR and HN are parallel, and by [proposition] 29 of the first [book] of Euclid's *Elements*. Therefore the star will be thought to be below the horizon HR by the whole arc RN, although in reality it is supposed to be in that horizon.

Further, once the horizontal parallax has been found by Problem 2 with respect to the eye at I, which is not raised, this same will be altogether the horizontal parallax — but of depth — with respect to the raised eye at C, because the angle is identical.

18. Probl. Parallaxim Maximam Profunditatis Sideris infra Horizontem verum poſiti diſcernere atq. inuenire, Data Diſtantia Sideris à centro Terræ. *Problem 18. To distinguish and find the greatest parallax of depth of a star placed below the true horizon, given the star's distance from the centre of the Earth.*

[Margin: Problem 18.]

In the other figure, which follows, let three semicircles of vertical circles be described about the centre of the Earth: the first, BTM, on the Earth's surface; the second, AKL, in the heaven of the star; the third, FGI, in the highest heaven. Let one observer be at B, not raised at all; let his vertical line, drawn from the zenith to the nadir, be FCI, his sensible horizon BH and his rational horizon CG, each of them orthogonal to FCI. Now let the star be at S, and from its centre let the straight line STA be drawn, touching the Earth at T and making with the semidiameter CT a right angle at the point of contact T, by [proposition] 18 of Euclid's third [book]. For this same line will be the sensible horizon with respect to the raised eye at A, and along it the eye at A will first see the star S below the horizon (provided, that is, that refraction does not intervene, which we do not consider here in this section). The true depth of the star below the true horizon, however, will be the arc KS, which the angle KCS measures; but the greatest, or horizontal, parallax will be the angle CST, and it will better be called the parallax of depth. And this same will be the horizontal parallax, but of altitude, with respect to the eye at T, which is not raised, because the same line ATS is the sensible horizon for both eyes, A and T, and the triangle CTS is right-angled at T. Therefore, if the horizontal parallax is known by the problem 2 handed down in chapter 2, the greatest parallax of depth will also be known, because the parallactic angle is identical; and this, added to the true depth of the star known from elsewhere, will give the visible depth. But if one chooses to consider the places of the star in the highest heaven, its true place will be under Q, but its visible place under R; just as, if the star is at O, its true place is under P, but its place as seen from the eye at B is under H.

Diagram. Set in the left column beside the construction of Problem 18. Three concentric semicircles about C open to the right: the inner BTM (the Earth's surface), the middle AKL (the heaven of the star) and the outer FGI (the highest heaven). The vertical line F–A–B–C–M–L–I runs down the left side. A horizontal runs from C to G; a line runs from B out to H; a line runs from A through T to S; further lines run from C out to the outer arcs at the upper and lower right.

- F (top of the vertical line): on the outer semicircle FGI
- A (on the vertical line, below F): the raised eye, on the middle semicircle AKL
- B (on the vertical line, below A): the observer not raised, on the inner semicircle BTM
- C (on the vertical line): the centre of the Earth
- M (on the vertical line, below C): end of the inner semicircle BTM
- L (on the vertical line, below M): end of the middle semicircle AKL
- I (bottom of the vertical line): end of the outer semicircle FGI
- O (upper right): a position of the star
- P (upper right, beside O): the true place of a star at O in the highest heaven

- H (upper right, on the outer arc): end of the sensible horizon BH; the visible place of a star at O
- T (centre of the figure): the point of contact of STA
- K (right of T, on CG): on the middle semicircle
- G (right end of the horizontal from C): end of the rational horizon CG
- S (lower right): the star
- Q (lower right, beyond S): the true place of the star S in the highest heaven
- R (lower right, below Q): the visible place of the star S in the highest heaven

19. Probl. Data Sideris Diſtantia à centro Terræ ac Semidiametro Terræ; inuenire Altitudinem oculi neceſſariam, vt poſſit videre Sidus tunc primò, cùm erit verè in Horizonte Rationali, per radios irrefractos. *Problem 19. Given the star's distance from the centre of the Earth and the semidiameter of the Earth, to find the altitude of the eye needed for it to see the star for the first time just when the star is truly in the rational horizon, by unrefracted rays.*

[Margin: Problem 19.]

Let the figure set out in Problem 17 be repeated here. In it, seek the horizontal parallax HRI by Problem 2 of chapter 2; for this will be equal to the angle CHI. For CHR and RIH are both right angles, and equal; therefore, taking away the common angle IHR, which on the one side with IHC, and on the other with HRI, completes one of the right angles, by [proposition] 32 of Euclid's first [book], the angle CHI is left equal to the angle HRI. Then in the triangle CHI, right-angled at the point of contact I, there are given the angle CHI and the semidiameter of the Earth HI, in paces, or miles, etc., as I suppose; therefore by [proposition] 3 of right-angled plane triangles you will find the base HC. From this subtract the semidiameter of the Earth HA, and the altitude of the eye sought will be left.

20. Probl. Data Sideris Diſtantia à cẽtro Terræ, ac vera Profunditate ſub Horizonte vero, ac Semidiametro terræ, inquirere oculi Altitudinem primò viſuri ſidus per radios irrefractos. *Problem 20. Given the star's distance from the centre of the Earth, its true depth below the true horizon and the semidiameter of the Earth, to seek the altitude of an eye that is to see the star for the first time by unrefracted rays.*

[Margin: Problem 20.]

Let the figure explained in Problem 18 be repeated here. Having sought the horizontal parallax CST by Problem 2 of chapter 2, join it to the star's true depth, that is, to the angle SCK, and subtract the sum of these from 90°: the angle TCK will be left; and this in turn, subtracted from 90°, that is, from the right angle ACK, will leave the angle ACT. In brief, therefore, the angle ACT is as great as the aggregate of the star's true depth and its horizontal parallax. Therefore in the triangle ACT, right-angled at T, with the angle ACT and

the terrestrial semidiameter CT, investigate the base CA by [proposition] 3 of right-angled plane triangles; from this subtract the semidiameter CB, and the altitude of the eye sought, AB, will remain.

21. Probl. Data Oculi Altitudine ſupra terram, Semidiametro terræ ac Diſtantia Sideris à centro Terræ, Inuenire Profunditatem tum veram, tum apparentem Sideris primò ab oculo per radios irrefractos viſibilis. *Problem 21. Given the altitude of the eye above the Earth, the semidiameter of the Earth and the star's distance from the centre of the Earth, to find the depth, both true and apparent, of a star first visible to the eye by unrefracted rays.*

[Margin: Problem 21.]

In the same figure of Problem 18, and in its triangle CAT, right-angled at T, there are given CT, the semidiameter of the Earth, and CA, made up of the terrestrial semidiameter CB and the given altitude of the eye BA; with these, seek the angle CAT by [proposition] 2 of right-angled plane triangles. For its complement to 90° will be the angle ACT, which together with OCK completes the right angle BCK; therefore the angle CAT will be equal to the angle OCK. Then in the triangle CTS, right-angled at T, given CT as one semidiameter of the Earth, and the distance CS, seek the parallactic angle CST by the same [proposition] 2; and when this is taken from 90°, the remaining angle SCT, to the right angle, will be known. From it take away the angle OCK already found, and KCS will be left, measuring the true depth KS of the star below the true horizon; if you add to it the parallax CST, the visible depth below the true horizon will become known. Therefore, from first to last, the apparent depth below the true horizon is as great as the angle ACT, and so it is enough to have found that angle, in the Sun as well as in the Moon and the other stars. So that you may see, then, how great this depth could be if the unrefracted rays of a star reached our eyes, consult the small table placed in Section 4, Problem 39, where, for a given altitude of the eye up to 50 Italian miles, the visible arc of the Earth is set down, which here would be the arc BT, that is, the angle BCT; and that some mountains are 30 miles high perpendicularly is gathered from Section 4, Problem 35, Example 3. But because the star is raised in appearance by the force of refraction, it happens that the star can appear at so great a depth to an eye still less high, or that, to an eye so high, it can appear at a greater depth.

CAPVT IV.

Chapter IV.

De Parallaxi Orbis Annui. *On the parallax of the annual orb.*

Enough has been done by us on this in Book 6, chapter 7, from number 7, and again expressly in Book 9, Section 4, chapter 28, where we also pointed

out the errors of many who take only half the parallax. We therefore refer the reader there, so as not to repeat the same things here in vain.

[Margin: Order of the problems.]

CAPVT V.

Chapter V.

De Parallaxi Diſtantiæ inter duo Phænomena. *On the parallax of distance between two phenomena.*

The parallax of distance is *the difference between the apparent, or visible, distance of two phenomena and their true distance*. It arises from the parallax of altitude or of depth, either of both visible bodies or at least of one. When both phenomena are planets, or a comet appearing in the planetary heaven or below it, this parallax arises from the parallax of altitude of both. But if one of them is a fixed star, then, since the parallax of the fixed stars is commonly held by astronomers to be imperceptible, not even from it does a parallax of distance arise. For in the following scheme let the observer's vertex, or zenith, be A, and the nadir C; through these points let two semicircles of vertical circles, ABC and ADC, be drawn, enclosing the portion BD of the true horizon. And let there be two stars having equal altitude, one at H, the other at G, and let their true distance be the arc HG. For if each of them undergoes a sensible parallax of altitude, and H appears at E and G at F, their visible distance will be the arc EF, certainly greater than the true one; but if only one of them, say the star H, undergoes the parallax of altitude HE, the visible distance will be EG, likewise greater than the true one. Again, if the stars are in the true horizon, one at B, the other at D, and each undergoes a sensible parallax of depth, and the former is seen to be at K and the latter at M, the visible distance will be KM, but the true one BD; whereas if only one of them, say B, undergoes such a parallax, the visible distance will be the arc KD. From these it will not be difficult to understand the remaining cases.

Diagram. Set in the left column beside the opening of Chapter V. Two arcs, the vertical semicircles ABC and ADC, meet at the zenith A at the top and the nadir C at the bottom, forming a lens. On the left arc, from the top, stand H, E, B and K; on the right arc, from the top, G, F, D and M. Straight lines join H to G, E to G, E to F, B to D, K to D and K to M.

- A (top): the observer's zenith
- H (left arc, upper): the true place of the first star
- G (right arc, upper): the true place of the second star
- E (left arc, below H): the apparent place of H
- F (right arc, below G): the apparent place of G
- B (left arc, middle): a star in the true horizon
- D (right arc, middle): a star in the true horizon
- K (left arc, below B): the apparent place of B

- M (right arc, below D): the apparent place of D
- C (bottom): the nadir

22. 23. 24. Probl. Data Parallaxi veræ Altitudinis, aut Profunditatis, vtriuſque, aut vnius duorum Phænomenorum, & Diſtantia ipſorum vera, inuenire Diſtantiam eorumdem viſam ſeu apparentem. *Problems 22, 23 and 24. Given the parallax of the true altitude, or of the depth, of both or of one of two phenomena, and their true distance, to find their visible, or apparent, distance.*

[Margin: Problem 22.]

First: in the figure set out above, let each star, H and G, undergo a parallax of altitude. Therefore in the triangle HAG seek the angle A by [proposition] 14 of oblique-angled spherical triangles. For in it are given the base HG, which is the true distance; the side HA, which is the complement of the true altitude of the phenomenon H; and the side AG, the complement of the true altitude of the star G — for by the very fact that the parallax of the true altitude is given, the true altitude cannot be unknown. Then in the triangle EAF seek the visible distance EF, as the base, by [proposition] 4 of oblique spherical triangles. For in it the vertical angle A has already been found, and the sides AE and AF are given, if to the complements of the true altitudes there is added to each its own parallax of altitude, HE on the one side and GF on the other.

[Margin: Problem 23.]

Second: let only one of the stars be subject to a sensible parallax, namely the star H, while the star G is aparallactic; and then in the triangle HAG, as above in the first part of Problem 22, seek the angle A. Then in the triangle AEG, by [proposition] 4 of oblique spherical triangles, investigate the base EG, namely the apparent distance, since in it are given the vertical angle A; the side AG, the complement of the true altitude of the star G; and the side AE, made up of the complement of the true altitude of the star H and its parallax EH.

[Margin: Problem 24.]

Third: let the parallax of depth of the star B be BK, and that of the star D none, and let the true distance be BD. With this and with the quadrants CB and CD, seek the angle C by [proposition] 14 of oblique spherical triangles. Afterwards, in the triangle CKD, seek by [proposition] 4 the base KD, that is, the apparent distance, since the angle C is now given, and the side CD is a quadrant, while the side CK is the remainder of the quadrant CB once the parallax of depth BK has first been taken away. From these you will easily solve the remaining cases.

25. 26. 27. Probl. Data Parallaxi viſæ altitudinis aut Profunditatis datæ, & Diſtantia viſa duorum Phænomenorum; inuenire Diſtantiam eorumdem veram. *Problems 25, 26 and 27. Given the parallax of a given visible altitude or depth, and the visible distance of two phenomena, to find their true distance.*

[Margin: Problem 25.]

First: in the figure set out a little earlier, let there be two stars, H and G, each subject to parallax. Then in the triangle EAF seek the angle A by [proposition] 14 of oblique spherical triangles, since in it are given the base EF, which is the visible distance, and the sides AE and AF, because they are the complements of the visible altitude. Then in the triangle HAG seek by [proposition] 4 of the same the base HG, which is the true distance, since there are given the vertical angle A, just found, and the side AH, which [?] is the complement of the visible altitude BE if from the complement EA you take away the parallax EH; in the same way the side AG is given, by taking the parallax GF away from the complement AF of the visible altitude.

[Margin: Problem 26.]

Second: let only one of the stars, say H, undergo the parallax HE; and then in the triangle EAG seek the angle A by [proposition] 14 of oblique spherical triangles. For in it are given the base EG, which is the visible distance; AG, the complement of the true altitude; and AE, the complement of the visible altitude. Then in the triangle HAG seek, by [proposition] 4 of the same, the base HG, namely the true distance; for you have already obtained the vertical angle A, and there are given the side AG, the complement of the altitude, true and visible at once, and AH, if from the complement EA of the visible altitude BE you take away the parallax EH.

[Margin: Problem 27.]

Third: in the triangle KCD let there be given the visible distance KD, the quadrant CD, and KC, the complement of the visible depth; and therefore by [proposition] 14 of oblique spherical triangles seek the angle C. With it, then, in the triangle BCD, the angle C being now given together with the quadrants BC and BD, seek by [proposition] 4 of the same triangles the true distance BD; and so for the rest.

CAPVT VI.

Chapter VI.

De Parallaxi Declinationis & Aſcenſionis Rectæ. *On the parallax of declination and of right ascension.*

The parallax of declination, and of right ascension, is *the difference between the true and the apparent, or visible, declination or right ascension*, arising from the parallax of altitude or of depth. Now if the phenomenon is in the eastern hemisphere, the parallax increases the right ascension; if in the western, it decreases it — unless it is there at the end of Pisces or the beginning of Aries, and here at the end of ♍ or the beginning of ♎; for then it can decrease it in one respect and increase it in another. As for the declination, five cases can be given in which it is diminished or increased because of the parallax; I shall explain them more clearly by the following diagram. Let the horizon be HQR and the semicircle of the meridian HVR, in which the vertex is V and the visible

pole of the world B. Let the altitude of the pole be BR; given this, VB, the distance of the pole from the vertex, is given, if BR is subtracted from 90°. Let the quadrant of the equator be AQ, and that of altitude VC. The *first* case, then, is when both the true place of the star, E, and the visible place, D, are on this side of the equator, and the visible declination GD is less than the true FE. The *second*, when the true place is on this side, at D, but the visible place I is on the equator itself; and the true declination is GD, but the visible is none. The *third*, when the true place is on this side, at D, but the visible beyond, at M; and the true declination GD is of one kind, the visible MO of the other. The *fourth*, when the true place is at the point I of the equator, but the visible beyond, at M; and the true declination is none, but the visible is MO. The *fifth*, when both the true place M and the visible place N are beyond the equator, and the true declination MO is less than the visible NK.

[Margin: Case 1.]
[Margin: Case 2.]
[Margin: Case 3.]
[Margin: Case 4.]
[Margin: Case 5.]
[Margin: Order of the problems.]

Diagram. The diagram promised for the five cases of Chapter VI, set across the head of the left column. A semicircle stands on the straight diameter H–R, the horizon HQR, with V at the top of the arc. A lies on the arc at the upper left and B on the arc at the right. Below A, near the left part of the arc, stand F and G, and lower down M and N; E and D stand inside the figure below V, I and a small round letter o near the centre, and K below them. C and Q stand on the diameter, C left of centre and Q right of it. A straight line runs from V down to C, a straight line from A down to Q, and several curved lines run from B across to the left part of the arc.

- V (top of the arc): the vertex
- A (arc, upper left): end of the quadrant of the equator AQ
- B (arc, right): the visible pole of the world
- E (below V): the true place of the star in the first case
- D (below E): the visible place in the first case, the true place in the second and third
- F (left, near the arc): foot of the true declination FE
- G (below F): foot of the visible declination GD
- I (centre): a point on the equator
- o (beside I): the point O of the text
- M (lower left): a place beyond the equator
- N (below M): the visible place in the fifth case
- K (below I and o): foot of the visible declination NK
- H (left end of the diameter): the horizon HQR
- C (on the diameter, left of centre): end of the quadrant of altitude VC

- Q (on the diameter, right of centre): end of the quadrant of the equator AQ
- R (right end of the diameter): the horizon HQR

Probl. Parallaxim Declinationis Veræ & Aſc. Rectæ Veræ inuenire, vt vtraq. in Viſam commutetur, Datis 28. Parallaxi Altitudinis veræ datæ; Altitudine Poli, & Declinatione vera Sideris. 29. Parallaxi Altitudinis veræ, Altitud. Poli, & Azimutho 30. Parallaxi Altitudinis veræ, Declinatione vera, & Azimutho Sideris. *Problem. To find the parallax of the true declination and of the true right ascension, so that each may be converted into the visible, given — 28. the parallax of the true altitude, the altitude of the pole, and the true declination of the star; 29. the parallax of the true altitude, the altitude of the pole, and the azimuth; 30. the parallax of the true altitude, the true declination, and the azimuth of the star.*

[Margin: Problem 28.]

First: let the star be truly at E but apparently at D. By [proposition] 14 of oblique-angled spherical triangles, seek the angle EVB in the triangle EVB, in which are given EV, the complement of the true altitude; VB, the complement of the altitude of the pole; and EB, the complement of the true declination FE.

[Margin: Case 1.]

Then in the triangle DVB, by [proposition] 4 of the same, seek the base DB, which is the complement of the visible declination; when it is taken away from 90°, the visible declination GD will be known. For in that triangle are given the vertical angle at V, already found; VB, as above; and VD, if to the complement of the true altitude you add the parallax of altitude DE. Lastly, in the triangle EBD, by [proposition] 14 of the same, seek the angle EBD, which measures the parallax of right ascension FG, since in that triangle are given EB and DB, the complements of the true and of the visible declination, and the parallax of altitude DE.

[Margin: Case 2.]

But if the star is at D and appears at I, the quadrant IB will come out for you, and so the visible declination will be none. The parallax of right ascension, on the other hand, will be GI, whose measure is the angle DBI, to be sought with DB, IB and the parallax ID. The rest as above, using first the triangle DVB, then IVB, and lastly IDB.

[Margin: Case 3.]

If, however, the star is at D but appears at M, MB will come out for you greater than a quadrant, and the parallax of right ascension will be GO, etc., as above; but use the triangles DVB, MVB and MDB.

[Margin: Case 4.]

If, again, the star is at I but appears at M, the true declination will be none, IB a quadrant, and MB greater than a quadrant; and the parallax of right ascension will be IO. Carry out the rest as above, using the triangles IVB, MVB and MIB.

[Margin: Case 5.]

Finally, if the star is at M but appears at N, both MB and NB will be greater than a quadrant, the true declination will be MO and the visible NK, and the parallax of right ascension will be OK. Complete the rest by the same rules of triangles as above, using the triangles MVB, NVB and NMB.

[Margin: Problem 29.]

Second: let the star be at E and appear at D. First, then, in the triangle EVB, by [proposition] 4 of oblique spherical triangles, seek EB, the complement of the true declination, since in it are given VB, the complement of the altitude of the pole; EV, the complement of the true altitude; and EVB, the supplement of the azimuth to 180°.

[Margin: Case 1.]

Afterwards, by the same [proposition] 4, seek DB, the complement of the visible declination, in the triangle DVB, since VB and DVB are given, as above, and DV, if to the complement EV you add the parallax of the true altitude. Finally, in the triangle EBD, its three sides being now known, seek by [proposition] 14 of the same the angle EBD, that is, the measure of the parallax of right ascension FG.

[Margin: Case 2.]

But if the star is at D and appears at I, use the triangles DVB, IVB and IDB, performing the rest as above; and IB will come out for you a quadrant, etc.

[Margin: Case 3.]

If, however, the star is at D but appears at M, use the triangles DVB, MVB and MDB; the rest as above.

[Margin: Case 4.]

Now if the star is at I but appears at M, use the triangles IVB, MVB and MIB.

[Margin: Case 5.]

If, finally, it is at M but appears at N, use the triangles MVB, NVB and MNB. The rest as above, by the same rules.

[Margin: Problem 30.]

Third: let the star be at E but appear at D. First, then, in the triangle DVB, by [proposition] 2 of oblique spherical triangles, seek VB, since there are given in it EVB, the supplement of the azimuth to two right angles; EV, the complement of the true altitude; and EB, the complement of the true declination. Then, by [proposition] 4 of the same, seek in the triangle DVB the base DB, the complement of the visible declination, since there are given VB, now found; the azimuthal angle DVB; and DV, made up of EV, the complement of the altitude, and the parallax DE.

You now understand the remaining cases well enough — although it is more difficult for the azimuth to be given without the altitude of the pole than for the altitude of the pole to be given without the azimuth.

[Margin: Corollary.]

From what has been said it is clear how the true declination and right ascension are converted into the visible.

Probl. Parallaxim Declinationis Viſæ & Aſc. Rectæ Viſæ Inuenire, vt commutentur vtraque in Veram, Datis 31. Parallaxi Altitudinis viſæ datæ, Altitudine Poli, & Declinatione viſa Sideris. 32. Parallaxi Altitudinis viſæ datæ, Altitudine Poli, & Azimutho 33. Parallaxi Altitudinis viſæ datæ, Azimutho, & Declinatione viſa Sideris. *Problem. To find the parallax of the visible declination and of the visible right ascension, so that both may be converted into the true, given — 31. the parallax of the visible altitude, the altitude of the pole, and the visible declination of the star; 32. the parallax of the visible altitude, the altitude of the pole, and the azimuth; 33. the parallax of the visible altitude, the azimuth, and the visible declination of the star.*

[Margin: Problem 31.]

First: in the preceding figure let the star be at E but observed at D; and [let there be given] DV, the complement of the visible altitude; BD, the complement of the visible declination; and BV, the complement of the altitude of the pole. For in the triangle DVB, by [proposition] 14 of oblique spherical triangles, you will seek the angle DVB. With it, then, in the triangle EVB, by [proposition] 4 of the same, seek EB, the complement of the true declination, since VB is given, as above, and EV, if from the complement DV of the visible altitude you take away the parallax DE. Finally, in the triangle DEB, by [proposition] 14 of the same, seek the angle EBD, measuring [?] the parallax FG of right ascension, since in that triangle the sides DB and EB are now known from the complements of the visible and the true declination, together with the parallax DE.

From these, moreover, and from what was said in Problems 28 and 29, it will not be difficult to know how one should proceed in the other four cases enumerated at the beginning of this chapter.

[Margin: Problem 32.]

Second: if in the already mentioned triangle DVB there are given DV, the complement of the visible altitude; VB, the complement of the altitude of the pole; and, from the azimuth HVC, its supplement to 180°, EVB; then by [proposition] 4 of oblique spherical triangles you will seek DB, the complement of the visible declination. Then, in the triangle EVB, with EVB and VB known as above, and EV, the complement of the true altitude, known by subtracting the parallax DE from the arc DV, you will seek by the same [proposition] 4 EB, the complement of the true declination. Finally, in the triangle EBD, by [proposition] 14 of the same, you will investigate, as above, the angle EBD, which measures the parallax FG of right ascension; and you will proceed likewise in the other four cases.

[Margin: Problem 33.]

Third: if in the same triangle DVB there are given DV, the complement of the visible altitude; DB, the complement of the visible declination; and, from the azimuth, the angle DVB; then by [proposition] 2 of oblique spherical triangles seek the arc VB. With it, then, in the triangle EVB, by [proposition] 4 of the same, seek EB, the complement of the true declination; for there are

given the angle at V, as above; VB; and EV, once the parallax DE has been taken away from the complement DV. Finally, by [proposition] 14 of the same, seek as above the angle EBD, which measures the parallax of right ascension, etc.

34. & 35. Probl. Parallaxim Declinationis & Aſcenſionis Rectæ tam viſæ quàm veræ, aliter inueſtigare; per triangula ſphærica Rectangula, Datis præter Azimuthum, & Altitudinem Poli, Declinatione vera aut viſa, & Parallaxi Altitudinis veræ aut viſæ, Si Verticalis circulus per Aequatorem tranſeat. *Problems 34 and 35. To investigate in another way the parallax of declination and of right ascension, both visible and true, by right-angled spherical triangles; given, besides the azimuth and the altitude of the pole, the true or the visible declination and the true or the visible parallax of altitude, if the vertical circle passes through the equator.*

It must be recalled that, if the true declination and right ascension are to be converted into the visible, there must be given the true

[Margin: Order of the problems.]

declination and the true right ascension, and the parallax belonging to the true altitude; but if the visible is to be converted into the true, there must be given the visible declination and right ascension, and the parallax of the visible altitude. For either case, however, one must first seek the angle that the equator makes with the vertical circle drawn through the centre of the star. This is easily done in the triangle AIV of the preceding figure, by [proposition] 8 of right-angled spherical triangles; for in it are given the azimuth AVI and the adjacent side AV, which is as great as the altitude of the pole, and so the angle AIV, to which CIQ is equal, will not remain hidden. Once these angles have been obtained, the preceding problems of this chapter will easily be completed.

[Margin: Problem 34.]

First, then, let the star be truly at E but appear at D, and let its true declination FE be given. For in the triangle FIE, right-angled at F, from the given side FE and the angle FIE, as above, the base IE is sought by [proposition] 11 and the side FI by [proposition] 10 of right-angled spherical triangles. Then, when the given parallax DE has been taken away from the base IE, there remains the known base ID, in the triangle GDI, right-angled at G. With it and with the angle I there are sought, by [propositions] 4 and 5 of right-angled [triangles], the side GD, which is the visible declination, and the side IG; when IG is subtracted from the side IF, the parallax of right ascension FG remains.

[Margin: Problem 35.]

Second: let the star again be at E and appear at D, but let the visible declination DG be given. Then in the triangle GDI, right-angled at G, with the given side GD and the opposite angle GID, found at the beginning, the side GI is sought by [proposition] 10 of right-angled spherical triangles, and by [proposition] 11 the base DI, which is joined to the parallax DE to make IE. And then in the triangle FIE, with the base IE and the angle I, the true declination FE is sought

by [proposition] 1 of right-angled spherical [triangles], and by [proposition] 2 the side FI; if from it you take away the side GI already found, there remains the parallax of right ascension, namely the arc FG.

From what has been said, an industrious geometer can easily understand how he ought to proceed in the other 4 cases recounted at the beginning of the chapter. Yet, so as not to fail beginners, I shall point it out.

[Margin: Case 2.]

If the star is at D and appears at I, there is now no need to seek the visible declination, since it is none, but only the arc GI, which is the parallax of right ascension. This is done by [proposition] 6 of right-angled spherical triangles, since there are given the base ID, which is the parallax of altitude, and GD, which is the true declination. But if the true declination is to be sought, then GD, the true declination, is sought by [proposition] 1 of the same, since in the same triangle the base ID is given, as above, and the angle I is supposed already found at the beginning.

[Margin: Case 3.]

If, however, the star is at D but appears at M, and the true declination GD is given, then with it, in the triangle GID, right-angled at G, and with the acute angle I, GI is sought by [proposition] 10 and the base ID by [proposition] 11 of right-angled spherical triangles. Then in the triangle MIO, right-angled at M, with the angle I and the side IM (which is known if from the parallax MD you take away the arc ID that has been found), the visible declination MO, towards the other side, is sought by [proposition] 7 of the same, and by [proposition] 9 the base IO, which, joined to the arc GI, makes the parallax of right ascension GO. But if the visible declination MO is given, the triangle MIO must first be solved in the way we taught for GID, and then GID in the way we taught for solving MIO.

[Margin: Case 4.]

If, again, the star is at I and appears at M, there is nothing to seek but the apparent declination MO, by [proposition] 7, and the base IO, that is, the parallax of right ascension, by [proposition] 9 of right-angled spherical triangles; because in the triangle MIO, right-angled at O, the angle I and the parallax of altitude MI are given.

[Margin: Case 5.]

If, finally, the star, situated at M, appears at N, and the true declination MO is given, then with it and with the angle I, in the triangle MIO, right-angled at O, seek by [proposition] 10 of right-angled [triangles] the side OI, and by [proposition] 11 the base MI. Then join to the side MI the parallax of altitude MN, to make the base IN; with it and with the angle I, in the triangle NIK, right-angled at K, seek the visible declination NK by [proposition] 1 and the side IK by [proposition] 2 of the same; for if you take IO away from the whole IK, the remainder OK will be the parallax of right ascension. Conversely, if the visible declination NK is given, with it and with the angle I seek by [proposition] 10 the side KI and by [proposition] 11 the base NI; and when the

parallax of the visible altitude MN has been taken away from the side NI, the base MI will remain. With it, in the triangle MIO, and with the angle I, seek the true declination MO by [proposition] 1 and the side IO by [proposition] 2 of the same triangles; for when IO is subtracted from the whole KI, the parallax of right ascension OK will remain.

[Margin: The imperfection of the older trigonometry.]

This method is far more accurate, and no more difficult, than the one used by the ancients and proposed by Magini (Book 12 of the *Primum mobile*, Problem 14) and by Tycho (*Progymnasmata*, volume 1, pages 158, 162, 164). For, because of its nearness and smallness, they used an arc of a circle that is not a great circle, parallel to the equator, in place of an arc of a great circle, and employed it for the parallax of right ascension.

CAPVT VII.

Chapter VII.

De Parallaxi Latitudinis ac Longitudinis. *On the parallax of latitude and of longitude.*

The parallax of latitude and of longitude is *the difference between the true and the visible latitude and longitude*, arising from the parallax of altitude or of depth. Since it was dealt with at length in Book 5, chapter 10, on the occasion of eclipses of the Sun, the reader will not mind recalling that doctrine from there, just as if it had been copied out here.

CAPVT VIII.

Chapter VIII.

De Parallaxi Motûs. *On the parallax of motion.*

The parallax of motion is *the difference between the true and the visible motion*, and it is of as many kinds as motion is. Thus, for example, the Moon in the western quadrant of the ecliptic seems slower, and in the eastern faster, than it really is, because of its apparent motion in longitude, which there seems slower and here faster by force of the parallax. Moreover, it can happen that a phenomenon is so near the Earth that the apparent arc of its diurnal circle under the equator is of only 90°, and that to an inhabitant under the equator it seems to move for 6^h from its rising to its apparent setting, while a fixed star, completing its semicircle, spends 12^h from its rising to its setting; and so that phenomenon would seem twice as swift as the fixed stars, faster than it really is. This can be explained by the following diagram — drawn, however, for other purposes in Book 8, Section 1, chapter 15 — in which the celestial equator is HEFG; under it the terrestrial [circle] CBDO; and between the two the circle of the phenomenon LMP; and let the sensible horizon with respect to the eye B be IBK, cutting off from the circle of the phenomenon the quadrant LNM.

Diagram. Set in the right column within the first paragraph of Chapter VIII. A large circle carries E at the top, F at the right and G at the bottom, with H standing just outside it at the left; T and Q lie on it at the upper left and upper right, I and K at the left and right, R and S at the lower left and lower right. A straight line joins I to K, passing through L, B and M. About the centre A stand two small concentric circles: B above A, V below B inside the inner circle, c at the left and D at the right on the line running from H to F through the centre, o below A and P lower on the vertical, with N above B. Straight lines run from E through N down to the centre and on through P to G, from the centre out to T, Q, R and S, and from E to L and to M.

- E (top of the large circle): on the celestial equator HEFG
- F (right): on the celestial equator
- G (bottom): on the celestial equator
- H (left, outside the circle): on the celestial equator
- T (large circle, upper left): not named in the text on this page
- Q (large circle, upper right): not named in the text on this page
- R (large circle, lower left): not named in the text on this page
- S (large circle, lower right): not named in the text on this page
- I (large circle, left): end of the sensible horizon IBK
- K (large circle, right): end of the sensible horizon IBK
- L (on the line IK, left of B): where the phenomenon is seen to rise
- N (above B): on the quadrant LNM of the phenomenon's circle
- M (on the line IK, right of B): where the phenomenon is seen to set
- B (above the centre): the eye, on the terrestrial circle CBDO
- V (below B, inside the inner circle): not named in the text on this page
- c (left of the centre): the point C of the terrestrial circle CBDO
- D (right of the centre): on the terrestrial circle CBDO
- A (centre): not named in the text on this page
- o (below A): the point O of the terrestrial circle CBDO
- P (below o, on the vertical): on the circle of the phenomenon LMP

For it will be seen to rise at L and to set at M, having completed — by the motion of the primum mobile, which it is sup- [Margin: Order of the problems.]

posed to imitate — the arc LNM, of 6^h, while meanwhile a fixed star rising at I and setting at K completes almost the semicircle IEK in very nearly 12^h. And so the diurnal motion of that phenomenon will seem twice as swift as the motion of the fixed stars; but its nocturnal motion, on the contrary, one and a half times as slow, because it must complete the arc MPL, of 270°, or 18^h, while the fixed star completes the arc KGI, of very nearly 180°, or 12^h. On the same occasion I gave there, among other things, the following problems.

36. Probl. Data Semidiametro Terræ, & dato Tempore, quo Phænomenon apparet ſupra Horizontem Phyſicum, & interim mouetur ſub Aequatore

ad motum Primi Mobilis, inuenire quanta debeat eſſe ipſius Altitudo ſupra terram. *Problem 36. Given the Earth's semidiameter, and given the time for which a phenomenon appears above the physical horizon while meanwhile it moves under the equator with the motion of the primum mobile, to find how great its altitude above the Earth must be.*

[Margin: Problem 36.]

See Book 8, Section 1, chapter 15, Problem 1; it would be pointless to repeat it here.

37. Probl. Data Terræ Semidiametro, & Altitudine Phænomeni ſupra terram ſub Aequatore moti ad Primi Mobilis motum, Inueſtigare Tempus, quo apparere ſeu morari poteſt ſupra horizõtem Phyſicum. *Problem 37. Given the Earth's semidiameter and the altitude above the Earth of a phenomenon moved under the equator with the motion of the primum mobile, to investigate the time for which it can appear, or stay, above the physical horizon.*

[Margin: Problem 37.]

See Book 8, Section 1, chapter 15, Problem 2; here it had only to be proposed.

38. Probl. Data Diſtantia Phænomeni à centro terræ, veroq. ipſius Parallelo, in quo mouetur diurno motu; dataq. loci terræ Phænomenon obſeruantis Latitudine, ac Semidiametro terræ: Inueſtigare Differentiam inter moram Phænomeni ſupra Horizontem Phyſicum & Aſtronomicum in motu diurno. *Problem 38. Given the distance of a phenomenon from the centre of the Earth and its true parallel, in which it moves with the diurnal motion; and given the latitude of the place on Earth observing the phenomenon, and the Earth's semidiameter: to investigate the difference between the phenomenon's stay above the physical and above the astronomical horizon in its diurnal motion.*

[Margin: Problem 38.]

See Book 8, Section 1, chapter 15, Problem 7; let it suffice to have proposed it here.

39. Probl. Data Mora Phænomeni ſupra Horizontem tum Phyſicum, tum Aſtronomicum loci, cuius data ſit Latitudo, & data vera Declinatione paralleli, in quo verſatur Phænomenon, vnà cum Semidiametro terræ, Inueſtigare Diſtantiam Phænomeni à centro terræ. *Problem 39. Given the stay of a phenomenon above both the physical and the astronomical horizon of a place whose latitude is given, and given the true declination of the parallel in which the phenomenon moves, together with the Earth's semidiameter, to investigate the distance of the phenomenon from the centre of the Earth.*

[Margin: Problem 39.]

See Book 8, Section 1, chapter 15, Problem 8, where what we only propose here is explained.

CAPVT IX.

Chapter IX.

De Differentia Parallaxium Altitudinis, Locorumq. Viſorum. *On the difference of the parallaxes of altitude, and of the visible places.*

[Margin: What the difference of the parallaxes is.]

Although the difference between the true and the visible place of a phenomenon is the parallax, the difference between two visible places of the same phenomenon, viewed from the same or from different places on Earth at the same moment of time, is nevertheless not always the difference of the parallaxes: sometimes it is their sum, though more often it is their difference. Of this difference Longomontanus barely indicated a little (Book 2 of the *Spherics*, chapter 10, Problem 1); Cysat more (*De Cometis*, chapter 3, Lemma 8), and Chiaramonti in the *Antitycho* (Book 1, chapter 3) — in whom, however, an ambiguity must be guarded against, since more than once he calls the difference of two visible places the difference of the parallaxes. But in place of these, let there be some problems.

40. Probl. In Verticali circulo communi duobus terræ locis, Determinare Triplicem Situm Phænomeni, in quorum vno differentia duorum locorum viſorum ſit differentia parallaxium inæqualis, in altero æqualis, in tertio autem ſit Aggregatum parallaxium Altitudinis. *Problem 40. In a vertical circle common to two places on Earth, to determine the threefold position of a phenomenon: in one of these the difference of the two visible places is the difference of the parallaxes, being unequal; in another, being equal; and in the third it is the sum of the parallaxes of altitude.*

[Margin: Problem 40.]

In the present figure, let there be described from the Earth's centre A the terrestrial vertical semicircle CBE and the celestial vertical quadrant DS, and another quadrant below it, in the heavens or in the air, in which the phenomenon moves. Let one place on Earth, and the observer, be at B, through which let the vertical line AD be drawn from A, and let the horizon ES be orthogonal to it. Then let the other place on Earth be at G, through which let another vertical line, AGL, be drawn from A, and let the two places be connected by the chord GB. With these laid down:

Diagram. Set in the right column within the opening of Problem 40. A straight vertical line rises from A on the base to D at the top right; the base runs from S at the left through T, C and A to E at the right, and a short vertical stroke rises at its right end with a small m beside its top. A large arc runs from D at the top down to S at the lower left, carrying from the top Z, then H, F and I close together, then M and L, X, N, O, P and Y down its left side; Z, X and Y each stand beside a small star mark. A second arc inside it passes through K, Q and R. V stands beside the vertical line below D; B stands on the vertical

line above A, with G lower left of B, and an arc runs through G and B. Straight lines run from A, from B and from G up to the points on the arcs.

- D (top of the vertical line): the vertex of the place B; end of the celestial quadrant DS
- S (left end of the base): end of the celestial quadrant DS and of the horizon ES
- E (right end of the base): end of the terrestrial semicircle CBE and of the horizon ES
- A (on the base): the Earth's centre
- C (on the base, left of A): end of the terrestrial semicircle CBE
- B (on the vertical line above A): the first place on Earth and its observer
- G (lower left of B): the second place on Earth
- L (on the large arc): the vertex of the place G; the true place in the second case
- K (inner arc): the phenomenon in the first case
- F (on the large arc): the true place in the first case
- H (beside F): the place seen from B in the first case
- I (beside F): the place seen from G in the first case
- Q (inner arc): the phenomenon in the second case
- M (on the large arc): the place seen from B in the second case
- R (inner arc): the phenomenon in the third case
- N (on the large arc): the true place in the third case
- P (on the large arc): the place seen from B in the third case
- O (on the large arc): the place seen from G in the third case
- Z (top of the large arc, beside a star mark): not named in the text on this page
- X (left of the large arc, beside a star mark): not named in the text on this page
- Y (lower left, beside a star mark): not named in the text on this page
- V (beside the vertical line below D): not named in the text on this page
- T (on the base, beside S): not named in the text on this page
- m (top of the short stroke at the right): not named in the text on this page

[Margin: Case 1, of the sum of the parallaxes.]

First: let the phenomenon be at K, between the vertices D and L of the two places; and through its centre let the straight line AKF be drawn from the centre of the Earth, and through the same point the straight lines BKH and GKI from the two places. Then the true place of the phenomenon under the highest heaven will be the point F; but the place seen by the eye at B will be H, and by the eye at G, I; and the difference of the visible places will be the arc HI, which will be equal to the sum of the parallaxes of altitude. For the parallax with respect to the eye B is the angle AKB, while with respect to the eye G it is the angle AKG — if not formally, at least dimensively and equivalently, as was shown in Book 1, chapter 35, Scholium 2, and in Book 8, Section 1, chapter 17,

number 2. It is clear, moreover, that if the phenomenon were equally distant from each vertex, the parallaxes would be equal.

[Margin: Case 2, of a single parallax.]

Second: let the phenomenon be under the vertex of one place, say at Q, under the vertex L of the place G, through which let the vertical line AGL be drawn, and from the place B the line of the visible place, BQM. For with respect to the place G there will be no parallax, nor any difference between the true and the visible place, since to that place the phenomenon appears in the vertex; but with respect to the place and eye B, the difference of the true place L from the visible place M will be the parallax itself, not merely equal to it, and the angle AQB measures it.

[Margin: Case 3.]

Third: let the phenomenon be outside the vertex of either place, towards the same side, that is, outside the arc intercepted between the vertices of the two places — say at R, through which let three straight lines be drawn: one, ARN, marking the true place N under the highest heaven; another from the place B, BRP, marking the visible place P with respect to the eye B; and the third, GRO, marking the visible place O with respect to the eye G. For the difference of the visible places will be the arc PO, to which the difference of the parallaxes, GRB, will be equal. For the parallax with respect to B is the angle ARB, but with respect to G it is the angle ARG; when this is subtracted from the angle ARB, there remains the difference of the parallaxes, namely GRB. And this third case is more frequent than the others.

[Margin: Order of the problems.]

Probl. 41. 42. 43. Data, vel per Problema 44. aut 45. inuenta Differentia locorum viſorum Phænomeni, ſub eodem verticali ex duobus terræ locis ſimul viſi, & Diſtantia Phænomeni apparente à vertice alterutrius loci, cum arcu diſtantiæ locorum; Inueſtigare ſeorſim Parallaxes Altitudinis cuiuis loco conuenientes, in triplici caſu Problematis 40. Secluſa Refractione.

Problems 41, 42, 43. Given — or found by Problem 44 or 45 — the difference of the visible places of a phenomenon seen at the same time from two places on Earth under the same vertical, and given the apparent distance of the phenomenon from the vertex of either place, together with the arc of the distance between the places: to investigate separately the parallaxes of altitude belonging to each place, in the threefold case of Problem 40, refraction being set aside.

[Margin: Problem 41.]

[Margin: Case 1.]

First: if the phenomenon appears between the vertices of the two places — that is, if to one place it appears towards one region of the world and to the other towards the opposite region — look at the figure set out before. In it, for the given arc BG, seek the chord in the tables of sines, that is, the sine of half the said arc, doubled. With this chord, in the triangle BKG, seek the sides BK and BG by no. 2 of the plane oblique-angled triangles, in the parts of

which the radius AG is known. For in that triangle the angle GKB is given, because, from what was said in Problem 40, it is equal to the difference of the visible places; and the angle KBG or KGB is given. For if the visible distance of the phenomenon from the vertex D is given, which is the angle HBD, its supplement to two right angles is the angle KBA; but if the visible distance from the vertex L is given, which is the angle LGI, its supplement to two right angles is the angle KGA, by Euclid I.13. Moreover, once the arc GB is known, the angle A is also known, and its supplement to two right angles is, by I.32 of the same, the sum of the angles ABG and AGB. These angles stand at the base BG of the isosceles triangle ABG; therefore either of them, by I.5 of the same, is half the said sum. So, when ABG is subtracted from the angle KBA, and AGB from the angle KGA, the angles KBG and KGB will be known. After this, in the triangle KBA, with the radius AB, the side BK and the angle ABK, seek the parallax AKB with respect to the place B, by no. 3 of the plane oblique-angled triangles; and by the same, in the triangle AGK, with the radius AG, the side GK and the angle AGK, seek the parallax AKG with respect to the place G.

[Margin: Problem 42.]

[Margin: Case 2.]

Second: if the phenomenon is under the vertex L of one place, say G — as when it is at Q — there is no need of trigonometry, because ML, the difference of the visible places, is equal to the parallax BQG. Therefore, since the arc GB is known, and the arc DL is similar to it, if the arc DL is taken from the visible distance of the phenomenon from the vertex D, that is, from the arc MD, ML will remain, and so the parallax BQG with respect to the place B; for at G there will be no parallax.

[Margin: Problem 43.]

[Margin: Case 3.]

Third: if the phenomenon appears to both places towards the same region, and is therefore outside the arc of the vertical circle intercepted between the vertices of the places — as when it is at R — then, if its visible distance from the vertex D is given, that is, the angle DBR, its supplement to two right angles is given, which is ABR. From this subtract the angle ABG, known as I taught above in Problem 41, and GBR remains. But if the visible distance from the vertex L is given, the angle LGR, its supplement to two right angles, AGR, together with AGB, known as noted above, will make up the sum to be taken from four right angles, or 360°, so that the angle RGB becomes known, by Euclid I.13. Therefore, with either of these, and with the difference of the visible places, that is, the angle BRG, and with the base BG, that is, the chord of the arc BG, known in the parts in which the radius is AG — it is taken from the tables of sines — you will find the side GR by no. 2 or no. 7 of the plane oblique-angled triangles. For the third angle will be the supplement of the other given angles to two right angles, by Euclid I.32. Then, in the triangle AGR, with the radius AG, the side GR and the angle QGR, the supplement of AGR to two right angles, find the parallax ARG for the place G by no. 3 of the

same triangles; add it to the angle BRG, that is, to the difference of the visible places, and the whole parallax ARB for the place B will result. See an example in Book 8, Section 1, chapter 21, number 7, and chapter 19, number 2.

44. Probl. Data Diſtantia duorum terræ locorum ſub eodem verticali, ac Phænomeni Diſtantijs à vertice, eodem momento temporis vtrobique viſis in eodem verticali, Inueſtigare Differentiam locorum vtrobique viſorum; vt per eam, & per problema 41. 42. aut 43. inquiri poſſint Parallaxes Altitudinis. *Problem 44. Given the distance between two places on Earth under the same vertical, and the distances of a phenomenon from the vertex, seen at the same moment of time at both places in the same vertical: to investigate the difference of the places seen at the two, so that by means of it, and of Problem 41, 42 or 43, the parallaxes of altitude can be found.*

[Margin: Problem 44.]

Look at the figure immediately preceding. In it, if the phenomenon is at Q, under the vertex of one place, the arc of distance BG, that is DL, taken from the arc DM, the visible distance from the vertex D, leaves the difference of the visible places — indeed the very parallax of altitude with respect to the place B. But if the phenomenon is within both vertices, as at K — that is, it appears to one observer towards one region and to the other towards the opposite — then subtract the angle BAG, known from the arc of distance BG, from 180°; to this remainder, that is, to the angles ABG and AGB taken together, add the angles of the visible distances, DBK and LGK, and subtract the whole sum from four right angles, or 360°. For the sum of the angles KBG and KGB will then be known; finally subtract it from 180°, and the angle BKG will remain, that is, the difference of the visible places.

But if the phenomenon is outside the vertex of either place, and appears to both towards the same region, then subtract from 180° the angle BAG, known from the given arc BG, and add the remainder, bisected — that is, the angle ABG — to the angle DBR of the visible distance from the vertex; and the sum is to be taken from 180°, so that the angle GBR becomes known: keep it. Similarly, subtract the angle RGL of the other visible distance from the vertex from 180°; the angle AGR will remain. Join it to the angle AGB, and take the sum from four right angles; the angle BGR will remain. Join this to the angle GBR kept back, and take the total from 180°: there will remain the angle sought, BRG, that is, the difference both of the visible places and of the parallaxes.

45. 46. 47. Probl. Datis Phænomeni Diſtantijs ab eadem Fixa ſtella viſis ex duobus terræ locis, in eodem verticali circulo omnibus his exiſtentibus, Inuenire Differentiam locorum viſorum in triplici caſu problematis 40. ſecluſa Refractione. *Problems 45, 46, 47. Given the distances of a phenomenon from the same fixed star, seen from two places on Earth, all these lying in the same*

vertical circle: to find the difference of the visible places in the threefold case of Problem 40, refraction being set aside.

[Margin: Problem 45.]

[Margin: Case 1.]

First: in the figure set out before, let the phenomenon be at K, within the two vertices and below the star F, appearing at I to the eye G and at H to the eye B. Then the sum of the visible distances from the fixed star, that is, the arc HI, will be the difference of the visible places, and equal to the sum of the parallaxes. But if, with respect to one place, say G, it is above the star Z, and with respect to the other, B, below it, subtract the lesser of the visible distances from the fixed star, IZ, from the greater, HZ; and HI, the difference of the visible places, will remain.

[Margin: Problem 46.]

[Margin: Case 2.]

Second: let the phenomenon be at Q, under the vertex L of the one place G. Whether the star, like X, is below the phenomenon for both eyes, or, like Z, below it with respect to one and above it with respect to the other, yet within the vertices — in either case the lesser of the visible distances from the star, subtracted from the greater, will give the difference of the visible places, or the arc ML. But if the star is below with respect to one and above with respect to the other, but outside the vertices, the visible distances from the star are to be added together to give the difference of the visible places. Further, if the star lies towards the same region for both places, it will be outside the vertices; if towards opposite regions, it will be within the vertices.

[Margin: Problem 47.]

[Margin: Case 3.]

Third: let the phenomenon be at R, outside the arc LD intercepted between the vertices of the places, and let the fixed star be, with respect to both observers, either lower than the phenomenon, as Y is, or higher, as X is. Then in either case the lesser visible distance from the fixed star, taken from the greater, will give the difference of the visible places, OP. But if the star is below the phenomenon with respect to one observer and above it with respect to the other, then the sum of the visible distances from the star will be the difference of the visible places, or the arc OP.

[Margin: Order of the problems.]

48.Probl. Dato Phænomeni vero Motu, ſiue communi tantùm, ſiue proprio, & communi, cum diſtantijs eius ab eadem Fixa ſub eodem circulo maximo, viſis ex eodem terræ loco, in eodem quadrante hemiſphærij, dummodo ſtella Fixa ſit extra arcum, quem ſuo motu deſcribit Phænomenon ab vna ad alteram obſeruationem: Inueſtigare Differentiam parallaxium liberarum, aut liberatarum à Refractione. Immò hoc ipſum præſtare, etiam quando ſtella Fixa eſt intra prædictum arcum, Phenomeni motu deſcriptum. *Problem 48. Given the true motion of*

a phenomenon — either the common motion only, or the proper and the common together — with its distances from the same fixed star under the same great circle, seen from the same place on Earth in the same quadrant of the hemisphere, provided that the fixed star is outside the arc which the phenomenon describes by its motion from the one observation to the other: to investigate the difference of the parallaxes, either free of refraction or freed from it. And indeed to accomplish this same thing even when the fixed star is within the already mentioned arc described by the phenomenon's motion.

[Margin: Problem 48.]

For in this way Lemma 8 [?] of our Father Cysat, chapter 8 of *De Cometis*, must be limited; take this rule from it. Subtract the lesser of the visible distances from the greater; the remainder will be the visible motion. From this take the true motion, and the remainder will be the difference of the parallaxes. He shows this by an induction made from the following cases.

[Margin: Summary of the problem.]

[Margin: Case 1, of a phenomenon rising and approaching the star.]

The first is when the phenomenon, rising from the horizon, approaches by its true proper motion against the order of the signs a star more westerly than itself. In this case the visible distance decreases more and more, both because of the approaching proper motion and because of the diminution of the depressing parallax. For imagine that a comet near the horizon is visibly distant from Spica, which is more westerly, in the same vertical circle — and, for ease, in the equator — by 15° 20′, and at that time has a parallax of 16′; then, after 3 hours, having been raised, it has a parallax of 9′, having meanwhile advanced towards the west by its true motion by 3′. It will now seem distant from Spica by only 15° 10′, because of the 20′ the proper motion has taken away three, and the decrease of the parallax 7′. The visible motion, therefore, will appear to be 10′; if you subtract from it the true motion of 3′, 7′ will remain, which is the very difference of the parallaxes. And the same will happen, the proportion being kept, with all other numbers you may imagine in this first case.

[Margin: Case 2, of a phenomenon rising and receding from the star.]

The second case is that of a phenomenon rising, but receding from a more easterly star because of its motion against the order of the signs. For then the visible distance of the phenomenon from the star is smaller near the horizon than far from it, both because of the recession from the star by force of the motion and because of the lessened parallax, which earlier, by depressing the phenomenon towards the star, was diminishing the distance. For example, if earlier it seemed distant 15° 10′, having a parallax of 16′, and after three hours has a parallax of 9′, and by its true motion has advanced 3′ against the order of the signs, it will now seem distant 15° 20′, because the true motion has taken 3′ from the approach to the star, and the difference of the parallax 7′. Therefore the difference of the visible distances, that is, the visible motion, will be 10′; and when the true motion of 3′ is taken from it, 7′ will remain, that is, the difference of the parallaxes.

[Margin: Case 3, of a phenomenon descending and approaching the star.]

The third case is that of a phenomenon descending from the meridian towards the horizon and approaching a star more westerly than itself. For then the visible distance will be smaller near the horizon than far from it, both because of the approaching true motion and because of the increase of the parallax towards the star; and in this case the rule already given will hold, as will be clear to anyone who works it out.

[Margin: Case 4, of a phenomenon descending and receding from the star.]

The fourth case is that of a phenomenon descending, but receding from a more easterly star. In this case the visible distance will be greater near the horizon than far from it, partly because of the phenomenon's motion against the order of the signs, partly because of the greater parallax towards the same region as the true motion; and in this case too a careful computer will recognise that the said rule holds.

[Margin: The remaining 4 cases, of a phenomenon going in the order of the signs.]

The remaining four cases are those of a phenomenon moving by its proper motion in the order of the signs, and so, while rising, approaching a star more easterly than itself or receding from a more westerly star; or, while descending, receding from the star or approaching it, according as the star is more easterly or more westerly. It is enough to have indicated these, so that it may be clear to a careful examiner that the rule given holds in all of them: that the difference of the visible distances is always equal to the visible motion, and always greater than the true motion, and that the true motion, subtracted from the visible, leaves the difference of the parallaxes — provided that the star is not within the arc of the phenomenon's true motion.

[Margin: What is to be done if the star is within the arc of the phenomenon's motion.]

For if at the beginning the star were within the arc which the phenomenon traversed by its true motion from the first observation to the second — which would happen if the phenomenon moved very swiftly against the order of the signs — then the sum of the two visible distances from the fixed star and of the motion of the fixed star will be the visible motion of the phenomenon; and when the phenomenon's true motion is subtracted from this, the difference of the parallaxes will remain. For if the fixed star stood still, and the phenomenon, from the first observation to the second, reached it exactly by its visible motion, the visible motion would certainly be as great as the visible distance had been at the first observation; but if it did not merely reach the resting fixed star but passed beyond it, it would certainly cover by its visible motion a space as great as the earlier and the later visible distances taken together. But meanwhile the fixed star moves with the diurnal motion towards the same part towards which the phenomenon moves by its own proper motion, to say nothing of the motion it shares with the star; therefore the visible motion comprises, besides the two visible distances, the whole arc which the fixed star has completed as well. Finally, if the phenomenon's motion is in the order of the signs, contrary

to the motion of the primum mobile, and so overcomes it that it leaves the star, previously more easterly, behind it towards the west, the sum of the visible distances from the fixed star, subtracted from the motion of the fixed star, leaves the visible motion of the phenomenon. Care must be taken, however, that a difference of the parallaxes gathered in a circle other than the vertical is not carried over to the difference of the parallaxes of altitude — a matter in which, as I showed in Book 8, Section 1, chapter 20, numbers 16 and 17, Father Cysat went wrong.

49.Probl. Datis diſtantijs duabus viſis à polo Mundi, in Meridiano eodem obſeruatis, Phænomeni alium quàm primi Mobilis motum non habentis, Inuenire aut Aggregatum, aut Differẽtiam Parallaxiũ vtrique altitudini Meridianæ conuenientem: Diſſimulata interim Refractione. *Problem 49. Given two visible distances from the pole of the world, observed in the same meridian, of a phenomenon that has no motion other than that of the primum mobile: to find either the sum or the difference of the parallaxes belonging to the two meridian altitudes, refraction meanwhile being disregarded.*

[Margin: Problem 49.]

If the phenomenon at its greatest altitude still remains between the pole and the observer's vertex, the difference between its visible distances from the pole of the world will be the sum of the parallaxes. But if it reaches the vertex exactly, its distance from the pole will then be true and visible at once; the other visible distance, below the pole, will therefore be greater than this by as much as the parallax, and so the difference of the visible distances from the pole will be equal to the parallax belonging to the lower altitude. Finally, if at its highest meridian altitude it passes beyond the vertex towards the equator, the difference of the visible distances from the pole will be the difference of the parallaxes. Longomontanus demonstrated this with a brief and concise diagram in Book 2 of the *Spherics*, chapter 10, Problem 1; and we, with Tycho, have confirmed and illustrated it with examples in Book 8, chapter 11, from Problem 64 onwards. Let the reader therefore turn back there.

Monitum duplex & Theorema *A twofold caution, and a theorem*

1.Theorema. Differentia Diſtantiarum viſarum à vertice, per ſe quidem nec eſt differentia parallaxium, nec earum ſumma, nec ipſi parallaxi æqualis: Sed poteſt eſſe nullum, & quodlibet horum. *Theorem 1. The difference of the visible distances from the vertex is not of itself either the difference of the parallaxes, or their sum, or equal to the parallax itself; but it can be none of these, and any one of them.*

I add this against certain people who confuse the difference of the distances from the vertex with the difference of the parallaxes. That it can be any one of the three already mentioned is clear from the threefold case of Problem 40; that it can likewise be none of them is clear if the difference of the visible distances

from the vertex shows, to sense, a whole quadrant: for none of the parallaxes can be of 90°, and the sum of two parallaxes does not exceed 3° even in the Moon, etc.

[Margin: Order of the problems.]

2.Theorema. Differentia differentiarum, quæ ſunt inter duas altitudines veras & inter duas viſas eiuſdem Phænomeni ijſdem duobus momentis, æqualis eſt Differentiæ Parallaxium. *Theorem 2. The difference of the differences that lie between two true altitudes and between two visible altitudes of the same phenomenon, at the same two moments, is equal to the difference of the parallaxes.*

This proposition will be explained and made plain by an example. Let the true altitude of the Moon be 10°, but the visible 8° 54′ 41″, and so the parallax 65′ 19″. Next let the Moon's true altitude be 40°, but the visible 39° 8′ 43″, and so the parallax 51′ 17″: the difference of the parallaxes will then be 14′ 2″. Now the difference of the true altitudes is 30°, and of the visible ones 30° 14′ 2″; the difference of these, therefore, is 14′ 2″ — just as great, of course, as that of the parallaxes. And you will find the same on examining any two altitudes, visible and true, at the same moment, compared with two other altitudes, true and visible, at another single moment — provided that the phenomenon keeps the same distance from the centre of the Earth in both.

For the rest that seems to belong to this chapter, see above, chapter 2, Problems 11, 12, 13.

CAPVT X.

Chapter X.

De Modis Obſeruandi, An aliquod Phænomenon, præſertim nouum, habeat Parallaxin aliquam Altitudinis, & quantam, per Obſeruationes in eodem loco terræ factas. *On the methods of observing whether some phenomenon, especially a new one, has any parallax of altitude, and how much, by observations made at the same place on Earth.*

Although observations of this kind do not suffice for absolute evidence about the place and distance of a phenomenon, they are nevertheless easier, because they do not depend on the uncertain distance between meridians — as do those observations which are made of the same phenomenon, say a comet or a new star, at the same time by different observers in different meridians. Of these observations we shall speak in the following chapter.

50. Probl. Phænomeno & Obſeruatore in Aequatore poſitis, explorare Altitudinis Parallaxin, ſi non alio motu quàm primi Mobilis intra vnum diem moueatur, ſecluſa Refractione. *Problem 50. With the phenomenon and the observer placed at the equator, to explore the parallax of altitude, if the phenomenon*

moves within one day with no motion other than that of the primum mobile, refraction being set aside.

[Margin: Problem 50.]

In this figure let three quadrants be described from the Earth's centre C: one, AB, terrestrial; another, VR, of the celestial equator in the highest heaven; and a third, HN, in which the phenomenon is to appear.

Diagram. Set in the left column within the opening of Problem 50. A base line runs from F at the left through C and A to N and R at the right, and a vertical line rises from C towards V at the top. A large arc runs from V at the top down to R at the lower right, carrying in order E, D, I, P, Q, G, K and T; a middle arc runs from H at the left through M and L down to N; a small inner arc runs from O through B down to A. S stands small beside V at the top. Small star marks stand beside S, I, Q, M, L, K and T. Straight lines run from C and from B out to points on the arcs.

- C (on the base, left): the Earth's centre
- A (on the base): end of the terrestrial quadrant AB
- B (on the inner arc): the observer; end of the terrestrial quadrant AB
- V (top of the large arc): the vertical point, the zenith; end of the quadrant VR
- R (right end of the base): end of the quadrant VR and of the horizontal line CR
- H (left end of the middle arc): end of the quadrant HN, under V
- N (on the base, right): end of the quadrant HN, where the phenomenon rises
- L (on the middle arc): the phenomenon (Problem 50); its least altitude (Problem 51)
- M (on the middle arc): the phenomenon at its greatest altitude (Problem 51)
- K (on the large arc): end of the visible distance VK
- G (on the large arc): the true place on the line CLG
- E (on the large arc): the true place on the line CME (Problem 51)
- D (on the large arc): end of the visible distance VD (Problem 51)
- P (on the large arc): the pole of the world (Problem 51)
- S (beside V): not named in the text on this page
- I (on the large arc): not named in the text on this page
- Q (on the large arc): not named in the text on this page
- T (on the large arc): not named in the text on this page
- O (start of the inner arc): not named in the text on this page
- F (left end of the base): not named in the text on this page

Let the observer at B wait for the phenomenon until, rising from N to L, it is beyond all danger of sensible refraction, that is, until it is higher than about 40°; and with a most exquisite instrument, one showing even the seconds themselves if possible, let him take its visible distance from the vertex, VK, that is, the angle VBK. For when the straight line CLG is drawn from the centre of the Earth

through the centre of the star, the star's true place will be under G. At the same moment let a companion begin to count the times of the primum mobile with an exact pendulum — on which see our Book 2, chapter 20 — and continue until the star comes to H, under the vertical point V. For if that time, converted into degrees and minutes of the equator, is equal to the phenomenon's visible distance VK, the phenomenon will have no sensible parallax; but if it is less than the said distance, it will have a parallax of altitude as great as the small portion of the equator corresponding to the difference between the parts of the equator answering to the time and the visible distance from the vertex; and this parallax will be the angle CLB. The observer, however, must be certain that he is under the equator, must have the position of the equator known, and must not be deceived about the moment at which the phenomenon reaches the zenith V; and these three things together carry great difficulty.

51. Probl. Data Phænomeni à Refractione liberi, nunquamq. occidentis, & ſemper circa polum Mundi conſpicui, nec intra diem vnum variantis declinationem, Altitudine meridiana maxima & minima; Diſcernere Aggregatum aut Differentiam Parallaxium altitudinis, aut etiam parallaxin ipſam, ſi quam habuerit. *Problem 51. Given the greatest and the least meridian altitude of a phenomenon that is free of refraction, never sets, is always visible about the pole of the world and does not vary its declination within one day: to discern the sum or the difference of the parallaxes of altitude, or even the parallax itself, if it has any.*

[Margin: Problem 51.]

In the preceding figure let there be quadrants of three meridians: the terrestrial AB, that of the highest heaven VR, and below it HN, in which the phenomenon is. Let the horizontal line be CR and the vertical VC. Let the observer be at B, and let him observe the least visible altitude of the phenomenon at L and the greatest at M, by means of the visible distances from the vertex, the greatest VBK and the least VBD. Now, when the straight lines CLG and CME are drawn from the Earth's centre through the centre of the star, its true places will be G and E. Let the pole of the world be P, whose altitude RP we suppose to be the true one, differing insensibly from the visible or observed altitude, since this pole is supposed to be in the highest heaven, above the fixed stars themselves. With these laid down, subtract the least distance of the phenomenon, VD, from the distance of the pole from the vertex, that is, from the arc VP; there will remain its upper visible distance from the pole, which is DP. Conversely, from the greatest visible distance of the phenomenon, VK, take the distance of the pole, VP; there will remain the lower visible distance of the same phenomenon from the pole of the world, which will be PK. Now if the distances DP and PK are equal, the phenomenon will have no parallax sensible to your instrument. But if DP is less than PK, let it be subtracted from PK, and their difference will be the difference of the visible distances from the pole of the world, and will be equal to the sum of the parallaxes CLB and CMB

taken together, as Longomontanus, who will be reviewed shortly, teaches. But if the phenomenon at its greatest altitude were at the vertex V itself, then the difference of the visible distances from the pole P will be equal to the parallax BLC itself; and if at its greatest altitude it likewise passed beyond the vertex V, the said difference would be equal to the difference of the parallaxes. See the examples of Longomontanus in Book 2 of the *Spherics*, chapter 10, Problem 1, and in the Appendix to the *Astronomia Danica*, chapter 10; and of Tycho, *Progymnasmata* volume 1, page 417 and from page 443 onwards. On page 516 Tycho rightly warns that there must be a very notable difference, in degrees themselves, between the least and the greatest altitude of the phenomenon; otherwise, from a small difference an insensible difference or sum of the parallaxes could follow. Moreover, as the same author teaches on page 445, it can be gathered whether the two visible distances of the phenomenon from the pole of the world are equal: they are, if half the difference of the meridian altitudes, added on the one side to the lesser altitude and taken on the other from the greater, yields the same altitude of the pole that has been exactly explored by other means. How the parallax of altitude is drawn out of the sum or the difference of the parallaxes has been stated in chapter 1, Problems 11, 12, 13.

[Margin: Order of the problems.]

52. Probl. Datis quæ in Problemate 51. Diſcernere an Phænomenon habeat Parallaxin minorem, an maiorem Parallaxi nota, & debita alicui Planetæ in tali altitudine viſa. Secluſa interim Refractione. *Problem 52. Given what is given in Problem 51: to discern whether the phenomenon has a parallax smaller or greater than the known parallax that is due to some planet at such a visible altitude. Refraction set aside for the present.*

[Margin: Problem 52.]

First, by Problem 51, gather the phenomenon's sum of parallaxes, or its parallax, or its difference of parallaxes, according to the threefold case of that same problem. Next, from tables or from elsewhere, gather the planet's parallax of visible altitude when it is, or is supposed to be, at the two visible altitudes at which the phenomenon was observed.

[Margin: Case 1.]

Combine the planet's two parallaxes into one sum when the phenomenon at its greatest altitude was between the vertex V and the pole P. In that case, if the sum of the planet's parallaxes is equal to the phenomenon's sum of parallaxes, the parallaxes of both — that is, of the planet and of the phenomenon — will be equal, and so will their distance from the Earth. But if the sum of the planet's parallaxes is less than the sum already mentioned, the phenomenon will have greater parallaxes; if greater, smaller parallaxes than the planet.

[Margin: Case 2.]

If, however, the phenomenon at its greatest meridian altitude reached the vertex V, then a comparison of its parallaxes with the planet's will show at once which is the smaller.

[Margin: Case 3.]

Finally, if the phenomenon at its greatest meridian altitude passes beyond the vertex — that is, lies in the arc intercepted between the upper semicircle of the equator and the vertex — then subtract the smaller of the planet's parallaxes from the greater, and compare the remainder with the difference of the phenomenon's parallaxes. If they are equal, the parallaxes of both, and their distances from the Earth, will also be equal; but if the remainder is smaller, the phenomenon's parallaxes will be greater, and if it is larger, they will be smaller, than those of the planet observable at the same altitudes.

You have three examples in Tycho, *Progymnasmata* volume 1, in the new star of Cassiopeia: compared with the Moon on pages 331 and 446, with the Sun on page 423, and with Saturn on page 425. But for what was certain about that star's parallax, see what has been said in Book 8, Section 2, chapters 6 and 7.

53. Probl. Obſeruata Phænomeni circa Polum Mundi motu primi Mobilis euntis, nunquamque occidentis, nec declinationem mutantis, Diſtantia ab ijſdem ſtellis Fixis, cum quibus tranſit Meridianum in infima & ſumma altitudine; Diſcernere an Phænomenon habeat Parallaxin altitudinis ſenſibilem, & maiorem an minorem parallaxi Planetæ debita, Data Poli altitudine & Declinatione Fixæ coobſeruatæ. Secluſa Refractione. *Problem 53. Having observed the distance of a phenomenon — one that goes about the pole of the world by the motion of the primum mobile, never sets, and does not change its declination — from the same fixed stars with which it crosses the meridian at its lowest and its highest altitude: to discern whether the phenomenon has a sensible parallax of altitude, and whether it is greater or smaller than the parallax due to a planet, given the altitude of the pole and the declination of the fixed star observed with it. Refraction set aside.*

[Margin: Problem 53.]

An observation of this kind is more useful when the phenomenon at its greatest altitude comes too close to the vertex. Further, from the declination of the fixed star crossing the meridian together with the phenomenon, and from the altitude of the pole, the distance of the fixed star from the pole of the world is gathered. From this, together with each of the two distances of the phenomenon from the fixed star, the twin distance of the phenomenon from the pole of the world is gathered. From this, finally, by Problem 51, the sum or the difference of the parallaxes, or the phenomenon's parallax itself, is gathered; and by Problem 52, whether it has a greater or a smaller parallax than a planet seen at an equal altitude. For example, in the figure immediately preceding, let the distance of the phenomenon L from the pole star Q, observed with a sextant or otherwise, be the arc KQ, and let the distance PQ of the pole star from the pole of the world P be known: then the visible distance of the phenomenon from the pole of the world, the arc KP, will be known. Again, if the observed distance of the phenomenon M from the same pole star I is the

arc DI, its distance from the pole P will be the arc DP, made up of the distance from the star I and the distance of the star I from the pole P. See the examples in Tycho, *Progymnasmata* volume 1, from page 416 on.

54. Probl. Dato Aggregato, aut Differentia Parallaxium Ignotarum, duabus Phænomeni altitudinibus viſis conuenientium; Datiſq. Parallaxibus Planetæ in ijſdem altitudinibus ipſi conuenientibus; Parallaxes Altitudinis vtriuſque Phænomeni notas facere. Secluſa Refractione. *Problem 54. Given the sum or the difference of the unknown parallaxes belonging to two visible altitudes of a phenomenon, and given the parallaxes belonging to a planet at the same altitudes: to make known the phenomenon's parallaxes of altitude at both. Refraction set aside.*

[Margin: Problem 54.]

Once the sum or the difference of the parallaxes has been acquired, by the method we taught in Problem 51 or 53, or from elsewhere, and once the planet's parallax at each of the two visible altitudes is known from tables or otherwise, combine the planet's parallaxes into one sum and set up the following proportion.

[Margin: Case 1.]

As the sum of the planet's parallaxes is to the planet's smaller parallax, so let the sum of the phenomenon's unknown parallaxes be to the phenomenon's smaller parallax. When that is subtracted from the sum of the unknown parallaxes, the phenomenon's greater parallax will also become known.

[Margin: Otherwise.]

Otherwise: as the sum of the planet's parallaxes is to the planet's greater parallax, so let the sum of the phenomenon's parallaxes be to the phenomenon's greater parallax. When that is subtracted from the sum of the unknown parallaxes, the smaller remains known.

[Margin: Case 2.]

But if you use the difference of the parallaxes: as the difference of the planet's parallaxes is to the planet's greater or smaller parallax, so let the difference of the phenomenon's parallaxes be to the phenomenon's greater or smaller parallax.

Even without the planet's parallaxes, however, both of the phenomenon's parallaxes can be investigated by Problem 11, 12 or 13.

55. Probl. Experiri An Phænomenon habeat aliquam ſenſibilem Parallaxin, aut ſaltem Lunari maiorem, per obſeruationem ipſius in eodem verticali cum duabus Fixis; & alteram deinde obſeruationem eiuſdem Phænomeni ac Fixarum earumdem; dummodo Phænomenon non alio quàm primi Mobilis motu moueatur: nec Refractio interueniat; aut præcognita ſit, & detracta. *Problem 55. To find out whether a phenomenon has some sensible parallax, or at least one greater than the Moon's, by an observation of it in the same vertical as two fixed stars, and then a second observation of the same phenomenon and the same fixed stars — provided that the phenomenon is moved by*

no motion other than that of the primum mobile, and that refraction does not intervene, or else is known beforehand and taken away.

[Margin: Problem 55.]

[Margin: Digges's problem, corrected by Tycho and by us.]

This problem belongs partly to the little book of the Englishman Thomas Diggeseus [Thomas Digges], which he entitled *Alae seu Scalae Mathematicae* [*Wings or Ladders of Mathematics*], chapter 10; partly to Tycho, who corrects him in *Progymnasmata* volume 1, page 676; and partly to us. It runs as follows. Wait until the phenomenon falls into the plane of a vertical circle in which two fixed stars are found at the same time (which, given so great a multitude and density of fixed stars and so great a variety of vertical circles, will not be difficult to bring about). This, on Digges's authority, you will achieve with a ruler of wood or metal, at least 6 inches wide and 5 feet long, whose sides and straight edges are exactly parallel to one another. Set it upright, perpendicular to the horizon. Then, when you are about to look at the phenomenon, step back and place your eye in such a position that one side of the ruler appears with its edge to cut the phenomenon in two; and note carefully whether, in the same way and at the same moment, the edge of the ruler intersects two fixed stars through their centres. For then you will be certain that the phenomenon and the two fixed stars are in the plane of the same vertical. After some hours of the same night, or of the following one, when those fixed stars have declined markedly from the earlier plane, incline the ruler so that its edge passes through those two fixed stars. If it then passes through the centre of the phenomenon as well, the phenomenon will have no sensible parallax; but if the phenomenon lies outside the edge of the rule, and nearer to the horizon than the fixed stars, it will have a sensible parallax. Tycho rightly warns, however, that this is to be understood of a phenomenon that is moved by no motion other than that of the primum mobile; that it is extremely difficult to hold the eye in the same plane so that nothing wavers to one side or the other of the ruler's edge; and that this observation is better carried out with a hanging thread to which a lead weight is attached, which first

[Margin: Order of the problems.]

holds it in a perpendicular position and can then be swung round to the inclined plane in which those two fixed stars lie — although Tycho fears that here too the eye may be somewhat deceived. We, however, say that this is done more safely with our thread triangle, sufficiently described in Book 5, chapter 15, number 16: first in a position perpendicular to the horizon, and afterwards inclined. For two threads set apart from each other, one of which is covered by the other, show that the eye is in the same plane with them.

Further, Digges affirms that by this method, or rather by that method of his own, he observed on several nights the new star seen in Cassiopeia's Chair in the year 1572; that it always appeared to him to be in one and the same straight line with two little stars, one of which is in the knee of Cassiopeia and the other in the left shoulder of Cepheus; and that in the month of February of the year

1573 it did not, through any whole night, digress from the line of those stars by more than the semidiameter of its own apparent disc. From this he gathers that its parallax could not have reached two minutes.

[Margin: Digges's problem 2, without instruments.]

He adds, however, a second method, without any instrument, for gathering whether a phenomenon has a parallax greater than the Moon's, and applies it to the same new star. That star, he says, was never seen to recede from the little star that is in the foot of Cassiopeia's Chair by three times the diameter of the Sun. But if it had had a parallax greater than the Moon's, and of at least one degree, then when it moved about near the horizon it ought to have stood three times the Sun's diameter away from that star, since, when it was in the same vertical but above that little star, it seemed to be distant from it by about a single solar diameter, or 30′. This problem, however, being rough in itself, is to be refined by the problems that follow.

56. Probl. Explorare, An Phænomenon habeat parallaxim ſenſibilem, aut etiam Lunari vel Solari maiorem, per diſtantias ipſius à Polo Mundi, vel ab vna aliqua ſtella Fixa captas, eàdem vel diuerſis noctibus: Dummodo non habeat motum proprium, à motu primi Mobilis diuerſum, aut habeat, ſed diſcretè à motu viſo præcognoſcibilem aut præconceſſum; nec refractio interſit, aut cognita ſit & detracta. *Problem 56. To explore whether a phenomenon has a sensible parallax, or even one greater than the Moon's or the Sun's, by its distances from the pole of the world, or from some one fixed star, taken on the same night or on different nights — provided that it has no proper motion different from the motion of the primum mobile, or, if it has one, that this can be known beforehand distinctly from the visible motion, or is granted beforehand; and that refraction does not enter in, or else is known and taken away.*

[Margin: Problem 56.]

[Margin: Authors or practitioners of this problem.]

This problem is customary in observations of comets and new stars, and it was used above all by Thaddaeus Haggecius [Tadeáš Hájek] in his *Dialexis*, in Tycho, *Progymnasmata* volume 1, page 513; by Maestlinus [Michael Mästlin], in the same author, page 545; by Kepler in his book on the new star of Serpentarius [*De stella nova*], chapter 15; by our Cysat, *De Cometa anni 1618*, chapter 3, proposition 4; by Santutius, *De Cometa anni 1577*, chapter 8; by Snellius [Snel], on the comet of the year 1618, chapter 4; and by Longomontanus in the Appendix to the *Astronomia*, chapter 10. But they do not add the limitations that we have set down — and yet without them, as we have shown in Book 8, chapter 11, nothing certain can be gathered in this matter, and this will become plain from the present problem itself.

[Margin: Method 1, Kepler's.]

The first method is Kepler's, at the place just cited. For it, in the next diagram, let A be the centre of the Earth and B the eye on the surface of the terrestrial hemisphere MBN; let KEFP be the ecliptic of the fixed stars, and

LCDO the heaven, or as it were the heaven, of the phenomenon. Now let the phenomenon be observed by taking its distances from some known fixed stars, from which, by the laws of triangles, the visible place of the phenomenon in longitude and latitude can be deduced. Let this place on the ecliptic be at G, in the east, and to it let the line of the true place, ACG, be drawn, passing through the centre C. Then let the same place be awaited after some days or weeks, until it visibly becomes western, at H; to it let the straight line ADH be drawn from the centre of the Earth through the phenomenon D. At both times let the altitude of the phenomenon be taken, or its distance from the point G and from H, directly or by means of some star. For if it has a parallax equal to the Moon's, or a sensible one, it will not appear at both times under the same place among the fixed stars: at its rising it will be seen lower down, at E, along the line BCE, and at its setting at F, along the line BDF — given, however, the condition that Kepler adds there on page 78, namely *if it stays at rest under the same place of the fixed stars*; that is, if it is moved by no motion other than that of the primum mobile.

Diagram. A small figure set into the left column beside the account of Kepler's method. Three concentric semicircles stand on one base line; straight lines run up from the centre A and from B.

- E (top, left): on the line BCE, where the phenomenon is seen at its rising
- G (top, left, above E): the place on the ecliptic at the end of the line ACG
- C (on the middle semicircle, left): the phenomenon at the first observation
- B (on the inner semicircle, at the middle): the eye on the Earth's surface
- D (on the middle semicircle, right): the phenomenon at the second observation
- H (top, right): the place at the end of the line ADH
- F (top, right): on the line BDF, where the phenomenon is seen at its setting
- K, L, M, A, N, O, P (along the base, left to right): A the centre of the Earth; MBN the terrestrial hemisphere; KEFP the ecliptic of the fixed stars; LCDO the heaven of the phenomenon

[Margin: Method 2, the more common.]

The second method I have already taught in chapter 9, Problems 45, 46 or 47, or in chapter 10, Problems 51 and 53.

57. Probl. Datis duabus Phænomeni Altitudinibus viſis & exquiſitè obſeruatis, vtràq. ante vel poſt meridiem, cum ſuis Azimuthis, & cum Tempore inter vtramq. obſeruationem interiecto, Parallaxin eius in circulo altitudinis Inueſtigare: dummodo non habeat alium motum, quàm primi Mobilis, & altitudines viſæ per detractionem Refractionis præcognitæ, corrigantur. *Problem 57. Given two visible altitudes of a phenomenon, exquisitely observed, both before noon or both after, together with their azimuths and with the time elapsed between the two observations: to investigate its parallax in the circle of altitude — provided that it has no motion other than that of the primum*

mobile, and that the visible altitudes are corrected by taking away the refraction known beforehand.

[Margin: Problem 57.]

[Margin: The problem that is Regiomontanus's second.]

Among the 16 problems on comets that Regiomontanus left us for the study of comets, this is the second; Tycho also proposes it from that source in volume 2, from page 147 and at page 242, and so does Chiaramonti in the *Antitycho*, Book 2, chapter 16. What I said about the motion of the primum mobile alone is a limitation expressed by Regiomontanus, who says: *But let us imagine that the comet is moved, in so small a space of time, only according to the motion of the primum mobile.* Now, then, for the analysis of this problem, in the following figure let BZS be a portion of the meridian and BPC half of the western horizon. In the first observation let the true place of the phenomenon be at L but the visible place at M, and through these two places let the quadrant ZLMK of a vertical circle be drawn. Next, in the second observation, let the phenomenon be truly at G but apparently at O, and through these places let another quadrant of a vertical circle, ZGO etc., be drawn. The two parallaxes of altitude will then be the arcs LM and GO. Afterwards, through the two true places L and G, let the semidiurnal arc ALGD be drawn, parallel to the equator, in which the phenomenon moves with the motion of the primum mobile; and from the difference of the observed time between the two observations let the arc LG be known. Through the second, visible, place O let another semidiurnal arc QNOS be drawn, parallel to the first. Now let G and O be joined to the pole of the world H by the arcs of declination HG and HO, and to the earlier true place L let the arc of declination HL be drawn, which will be equal to the arc HG by 10 of Book 2 of Theodosius's *Spherics*. From H let the arc of declination HN be drawn to the parallel QOS — for this too will be equal to the arc HO by the same 10 — but let it be drawn so that the angle LHN is equal to the angle GHO. Let the true place L be joined to N by the arc LN, and the visible place M to N by the arc MN; finally, from M let the perpendicular arc MF be drawn to LN; and through N, from the vertex Z, let the quadrant ZNP be drawn. But if both observations were made before noon, BEC will be the eastern horizon, and the first true place of the phenomenon will be at G and the visible at O, while the second true place is at L and the visible at M; the rest as above.

Diagram. A figure across the right column in the course of Problem 57. A large semicircular outline stands on a base line; Z stands at the top and H at the upper right; several arcs cross from the left side to the right-hand end, and others run down from Z and from H.

- Z (top): the vertex; on the meridian BZS
- H (upper right): the pole of the world
- A (left side, upper): start of the semidiurnal arc ALGD
- Q (left side): start of the semidiurnal arc QNOS
- L (left side): the true place at the first observation
- M (left side, below L): the visible place at the first observation

- F (beside M): foot of the perpendicular arc MF on LN
- N (left side, lower): on the parallel QNOS
- G (inside, lower centre): the true place at the second observation
- B (left end of the base): end of the meridian portion BZS and of the horizon BPC
- K (below B): end of the quadrant ZLMK
- P (bottom): on the horizon BPC; end of the quadrant ZNP
- O (bottom): the visible place at the second observation
- [?] (bottom, between O and S): a small letter, E or F
- [?] (right-hand end of the base, upper): a small letter, C or G
- D (right-hand end, below it): end of the semidiurnal arc ALGD
- S (lower right): end of the meridian portion BZS and of the arc QNOS

[Margin: Step 1.]

First, therefore: because in the intervening time the point L is carried to G by the motion of the primum mobile, the point N must also be carried over to O, since the angles GHL and NHO are equal to each other: for to one and the same common angle N- [Margin: Order of the problems.]

HG the equal angles GHO and LHN have been added by construction; so the whole GHL is equal to the whole NHO, and the arc NO is similar to the arc LG of the true motion. Now from the second observation the complement of the visible altitude, ZO, is given; also given is the first azimuth, BZO, which, subtracted from 180°, leaves known the adjacent angle OZH. With this, with ZO, and with ZH, the complement of the altitude of the pole, one finds in the triangle OZH the side HO, the angle HOZ and the angle ZHO, which measures the distance of the point O from the meridian.

[Margin: Step 2.]

Second: the angle GHL is known from the time between the two observations, converted into parts of the equator. But the arc NO of the parallel is similar to this arc LG; therefore the angle NHO is equal to the angle LHG, as we were also saying above, and so it too is known. When it is taken from the angle ZHO already found, the angle ZHN is left known in the triangle ZNH, in which are given, as above, the arc ZH, the complement of the altitude of the pole, and the arc HN, since by construction, and by 10 of the second book of Theodosius's *Spherics*, it is equal to the arc HO found. So neither the arc ZN, nor the angle NZH, nor ZNH will be hidden. When NZH is subtracted from 180°, then, the angle NZB, adjacent to it, will be known; and when the azimuthal angle BZK is subtracted from this, MZN will be left known.

[Margin: Step 3.]

Third, then: in the triangle MZN, in which the arc ZN and the angle MZN have already been found, and the complement MZ of the visible altitude in the other observation is given, find the side MN and the angles ZNM and ZMN. Then combine the angle ZNM with ZNH found earlier, and the whole angle HNM will be known.

[Margin: Step 4.]

Fourth: the angles ZHO and ZHL have long been known by the first step, from the distance from the meridian known through the time. If you subtract the latter from the former, LHO remains known; and when LHG, known from the time elapsed between the observations, is subtracted — subtracted, I say, from the angle LHO — the angle GHO is left known. By construction, however, the angle LHN is equal to it. For the two triangles GHO and LHN have been set up with equal sides, and with equal angles comprised by equal sides — the side HL equal to the side HG, the side HN to the side HO, and the angle LHN to the angle GHO. So the base LN is also equal to the base GO, and the angle LNH to the angle GOH.

[Margin: Step 5.]

Fifth, then: in the triangle LHN find the base LN. For in it the angle LHN has been found, since it is equal to the angle GHO; the side HN, which is equal to the side HO found by the first step; and the angle LNH, since it is equal to the angle GOH found by the same first step. Since, therefore, the LN found is equal to the base GO, and this is a parallax of altitude, this parallax will be known.

[Margin: Step 6.]

Sixth: subtract the known angle LNH from the whole, or composite, angle HNM found above, and the angle MNL will be known in the same triangle MNL, in which the side MN has already been found by the third step and LN by the fifth. Hence the side, or base, ML will not be hidden — namely the other parallax of altitude. *For the sake of these we have wearied ourselves thus far*, says Regiomontanus; for who would not be wearied by so manifold an analysis of triangles? Then he adds: *All these things suppose that the comet, in the time between the two considerations, is not moved except according to the motion of the primum mobile; and even if it is moved meanwhile, in so short a time its proper motion will not be very sensible. But the arc HG — that is, the distance of the comet from the pole of the world, or the complement of the declination — will vary far less sensibly.* Regiomontanus perhaps supposed this because, trusting the Peripatetic doctrine about the sublunary seat of comets, and so about their great parallax, he did not think that an error of a few minutes, arising from the proper motion, in the parallax being searched out needed attending to. But how did he know that the comet's proper motion is small? For if it is small, someone will say, it will not produce even a sensible variation; or if it does produce one, it is large, and that variation arose from the comet's irregular proper motion, not from a true parallax. Therefore this method, sought out with such subtlety of mind, is useless for an absolute conclusion about the comet's parallax and distance unless it is propped up by some other aid; it holds only for a hypothetical conclusion, namely if the comet is granted no proper motion, or one imperceptible over a few hours. Besides which, the visible altitudes ought first to have been corrected, once the refraction was known.

58. Probl. Regiomontani Problema prædictum Tychonico adminiculo certius reddere, Datis Altitudine, & Declinatione viſa Phænomeni & Altitudine Poli. Sed ſupponendo Altitudinem viſam non indigere correctione Refractionis, aut iam correctam eſſe. *Problem 58. To make the problem of Regiomontanus already mentioned more certain with a Tychonic aid, given the visible altitude and declination of the phenomenon and the altitude of the pole — but supposing that the visible altitude does not need correction for refraction, or has already been corrected.*

[Margin: Problem 58.]

Tycho saw, in volume 2, page 147, that the problem of Regiomontanus already mentioned is not exact because of the proper motion — which almost all comets have, sensible even over an interval of a few hours — unless the difference that arises from the proper motion is removed. He therefore devised a method of reducing the azimuths so that they show the place of the comet as if it were unmoved, or carried by no motion other than that of the primum mobile, by the following problem. He also uses it in volume 1, page 755, and so does Chiaramonti in *De sede sublunari cometarum*, part 2, chapter 21.

[Margin: Here two observations as close together as possible are supposed, so that the altitude may not vary sensibly; and yet the phenomenon's proper motion in so tiny a time is taken as known beforehand, so that the declinations may be corrected by it, etc.]

In this other figure, then, let HAC be the semicircle of the meridian and ALC the horizon, and let the comet be seen at D in the first observation and at F in the second. Through these two places describe from the vertex A two quadrants of vertical circles, ADI and AFK; and from the pole of the world B two quadrants of circles of declination, BDE and BFG, down to the equator MEGL. With these described:

[Margin: Step 1.]

First: in the triangle ADB all the sides are given, since AB is the complement of the altitude of the pole, AD the complement of the visible altitude of the phenomenon, and BD the complement of the visible declination. Find, therefore, the angles BAD and ADB, and if need be the angle ABD — although in Regiomontanus's problem this angle was already presupposed known, from the distance in time from noon converted into parts of the equator, just as the angle ABF was, from the distance from the meridian when the phenomenon was observed the second time. So when ABD is taken from ABF, the angle DBF is left known; this is the difference of right ascension, as Tycho says, changed in the interval between the one observation and the other — just as the angle ABF becomes known from the angle EBG of the intervening time, if this had been observed, when ABD is added to it.

Diagram. A figure set into the right column in the course of Problem 58. A semicircle stands on a base line; A stands at the top and B on the arc at the upper right; arcs run down from A to the base and across from B towards the left.

- A (top): the vertex
- B (on the arc, upper right): the pole of the world
- M (on the arc, left): start of the equator MEGL
- D (inside, below A): the comet at the first observation
- F (inside, below D): the comet at the second observation
- E (inside, left): on the equator, end of the quadrant BDE
- G (inside, left, small): on the equator, end of the quadrant BFG
- H (left end of the base): end of the meridian semicircle HAC
- I (on the base): end of the quadrant ADI
- K (on the base): end of the quadrant AFK
- L (on the base): on the equator MEGL and the horizon ALC
- [?] (right-hand end of the base): a letter, C or G

[Margin: Step 2.]

Second: in the triangle AFB, too, the sides are given — AB, as above, and BF, the complement of the second declination, likewise the visible one — and the angle ABF is already known; therefore the arc AF will not be hidden. Compared with the complement of the altitude observed the second time, it will show either no difference or some difference; Tycho, however, found an insensible difference in the example of his comet, volume 2, page 149. Next, the angle BAF will not be hidden either. Subtracted from the angle BAD found by the first step, it leaves known the angle IAK, which measures the arc IK — that is, the difference of the azimuths between the place of the comet moved and at rest. So when the arc IK is added to the azimuth of the later observation, the azimuth will be reduced to what it would be if the phenomenon had not moved at all, or had moved with no motion other than that of the primum mobile. With the azimuths so corrected, the rest is to be carried out as Regiomontanus prescribed. If anyone wants examples of it, let him consult Tycho's volume 2, from page 149 to 158, or us in Book 8, Section 1, chapter 18, numbers 9 and 24. But let him remember that for the azimuths Tycho sometimes takes the distance of the vertical circle in which the phenomenon lies from either primary vertical — that is, not only from the meridian but also from the one that passes through the intersections of the equator and the horizon. So in the scheme already mentioned the azimuth of the phenomenon D is the arc IL, and that of the phenomenon F the arc KL.

[Margin: Tycho's calculations in volume 2 are less exact than in volume 1, because refraction is omitted.]

Next, let him notice that Tycho, on pages 146 and 150, uses visible altitudes of the comet — one of 28° 56′, the other of 12° 12′ — without applying the correction for refraction; and so Chiaramonti rightly censures him in Book 2 of the *An-* [Margin: Order of the problems.]

titycho, chapter 17. For from the visible altitude of 12° 12′ he ought to have subtracted 9′, according to the little table of Tychonic refractions. Moreover, throughout volume 2 Tycho used 34° 7′ for the complement of the altitude of the pole, or the arc AB, which he nevertheless later found to be 34° 5′ 30″

because of the refractions of the Sun at the winter solstice — refractions he had not yet scented out when he wrote volume 2. For this volume was written first, although it was published later by his heirs.

59. Probl. Datis altitudine Phænomeni viſa ante vel poſt meridiem, cum Azimutho, & Momento temporis tum huius Azimuthi ac altitudinis, tum eo quo Phænomenon tranſit per Meridianum; & Altitudine Poli, Inueſtigare Parallaxim altitudinis; dummodo interuallo horum temporum Phænomenon non moueatur niſi motu primi Mobilis, aut motus proprius ſit notus; & Refractio nulla ſit, aut iam detracta altitudini viſæ. *Problem 59. Given the visible altitude of a phenomenon before or after noon, with its azimuth and the moment of time both of this azimuth and altitude and of the phenomenon's transit through the meridian, and given the altitude of the pole: to investigate the parallax of altitude — provided that in the interval between these times the phenomenon is not moved except by the motion of the primum mobile, or its proper motion is known; and that there is no refraction, or it has already been subtracted from the visible altitude.*

[Margin: Problem 59, and the 3rd of Regiomontanus.]

This is the third problem of Regiomontanus in his little work *De Cometis*, from which Tycho drew it, volume 1, page 756. In the accompanying scheme let HVQ be the meridian, HOR the horizon and AOQ the equator; and let the true place of the phenomenon be at F, but its visible place at G. Through these two places draw both the quadrant VFGT of a vertical circle and the quadrants PFK and PGM of circles of declination — the former from the vertex V, the latter from the pole of the world P. With these set down, in the triangle VPF the arc VP, the complement of the altitude of the pole, is known; so is the angle FVP, if the azimuthal angle AVT is subtracted from 180°; and so is the angle VPF, from the true time between the observation and the phenomenon's transit through the meridian, converted into parts of the equator. So the side VF, which is the complement of the true altitude of the phenomenon, will not be hidden from the trigonometer. When the arc VF, then, is subtracted from the complement VG of the visible altitude, the parallax of altitude FG will remain known.

Diagram. A figure set into the left column in the course of Problem 59. A circle, with V at the top and a line across its lower half from H to R; lines run down from V and across from P.

- V (top): the vertex
- F (inside, below V): the true place of the phenomenon
- G (inside, below F): its visible place
- P (on the circle, upper right): the pole of the world
- [?] (on the circle, upper left; a sort shaped like a 4 after a short dash): A or 4
- K (inside, left): end of the quadrant PFK

- H (left end of the line across): on the meridian HVQ and the horizon HOR
- M (on the line HR): end of the quadrant PGM
- O (below the line HR): on the horizon HOR and the equator AOQ
- T (below the line HR, right): end of the quadrant VFGT
- R (right end of the line across): on the horizon HOR
- Q (on the circle, lower right): on the meridian HVQ and the equator AOQ

[Margin: What is to be done if the phenomenon moves with a proper motion.]

But if, from the beginning of its apparition up to the end, or up to the time of the observation already mentioned, the phenomenon has had a proper motion that has been ascertained, distribute that motion over the intervening times. For example, if it completed 90° in 60 days, assign one and a half degrees to each day and 3′ 45′ [?] to each hour. Or if it advances with a uniformly difform motion, correct the distribution already mentioned as the motion proper to the day of the observation requires; and to the hours between the transit through the meridian and the moment of the observation add or subtract as much as that proper motion requires, compared with the motion of the primum mobile before or after noon and with the phenomenon's tendency in the succession of the signs or against the succession of the signs, so that you have the angle VPF corrected. But either Regiomontanus did not notice that the true proper motion of the phenomenon cannot be discerned from the visible motion without prior knowledge of the parallax, or he disregarded this as something slight. Besides, if the proper motion is transverse and mixed from motion in longitude and in latitude, it can hardly be fitted to correcting the motion of the primum mobile.

60. Probl. Datis Phænomeni apparentis tam ante ortum, quàm poſt occaſum Solis, Altitudinibus viſis antemeridiana, & pomeridiana æqualibus, cum vtroque Azimutho, & Tẽpore obſeruatarum altitudinum, & Altitudine poli; Parallaxin altitudinis Inquirere, dummodo aut motum alium non habeat, quàm motum primi Mobilis, aut motus eius proprius ſit præcognitus; & Refractio nulla ſit, aut dempta iam altitudini viſæ. *Problem 60. Given the visible altitudes, before noon and after noon, and equal, of a phenomenon appearing both before the rising and after the setting of the Sun, with both azimuths and the time of the observed altitudes, and given the altitude of the pole: to inquire the parallax of altitude — provided either that it has no motion other than the motion of the primum mobile, or that its proper motion is known beforehand; and that there is no refraction, or it has already been taken from the visible altitude.*

[Margin: Order of the problems.]

[Margin: Problem 60, and the 4th of Regiomontanus.]

Let us go on to the 4th problem of Regiomontanus, in his little work *De Cometis*. For it, in the following diagram, let ABCD be the horizon and DVB the semicircle of the meridian, in which P is the visible pole of the world and V

the vertex or zenith; and let AKC be the arc that the phenomenon describes by the motion of the primum mobile. Let its first true place be at G, a point before noon, and its second at M, a point after noon; but let it be seen in the first case at O, in the second at N. Through these draw two quadrants of vertical circles, VGOH and VMNR, and to the true places two arcs of declination, PG and PM. Now since the visible altitudes HO and RN are supposed equal, the true altitudes HG and RM must also be equal, since an equal parallax belongs of itself to an equal altitude of the same phenomenon. So their complements too, the arcs VG and VM, will be equal to each other. Indeed, Regiomontanus says, it follows from this that GK and MK are equal to each other, and PG and PM to each other. From the time of the two observations the arc GKM is known, and the angle GPM, whose halves are GPB and MPB. From the azimuths, moreover, are known the angle GVK, whose supplement to two right angles is PVG, and the angle MVK, whose supplement to two right angles is the angle PVM. So in the triangle PVG, from the known angles PVG and VPG and the arc PV, which is the complement of the altitude of the pole, you will be able to find the side VG; when it is subtracted from the complement VO of the visible altitude, the parallax of altitude GO will become known. Likewise in the triangle PVM, from the known angles PVM and VPM and the arc PV, you will gather the side VM; when it is taken from the complement VN of the visible altitude, the parallax of altitude MN will be left. And if you like, you will be able, by solving the same triangles, to investigate the complement of the true declination, both PG and PM.

Diagram. A figure set into the right column in the course of Problem 60. A circle with a vertical line from D at the top to B at the bottom, carrying P and V; two arcs run down from V to the lower left and lower right, and a line crosses the lower part of the circle from A.

- D (top): end of the meridian semicircle DVB
- P (on the line DB): the visible pole of the world
- V (on the line DB, below P): the vertex or zenith
- A (left end of the line across): on the horizon ABCD and the arc AKC
- M (where the left arc meets the line across): the second true place
- G (where the right arc meets the line across): the first true place
- G (right end of the line across)
- N (below M, with a small star): the second visible place
- K (on the line DB, below the line across): on the arc AKC
- o (below G, set small, with a small star): the first visible place
- R (lower left, outside the circle): end of the quadrant VMNR
- B (bottom): end of the meridian semicircle DVB
- H (lower right, outside the circle): end of the quadrant VGOH

Finally Regiomontanus advises that if the phenomenon has a proper motion, the intermediate time — that is, the arc GKM — should be corrected, and from it the angles at P likewise, as was said in the preceding problem. This problem, moreover, is useful for phenomena that cannot be observed when they cross the

meridian because the Sun is above the horizon, provided they can be observed both before the rising and after the setting of the Sun, by waiting for them until they have reached the azimuth in which the visible altitude is equal to the altitude seen earlier; or the altitudes may be taken at equal azimuths. But then one must take, at the same moment, the altitude of a known fixed star, so that from it, by the usual rules of the primum mobile given in Section 3, each moment of time may become known.

61. Probl. Si Phænomenon fuerit cum duabus Fixis ſub eodem verticali citra Meridianum, & capiatur eodem momento diſtantia ipſius à vertice & ab vtraq. Fixarum. Deinde, cùm motu Firmamenti delatum fuerit ad alium verticalem vltra Meridianum tantum quantum prior verticalis citra Meridianum fuerat, capiatur diſtantia à vertice & ab ijſdem Fixis, & altitudo alterutrius Fixarum: Parallaxes vtriuſq temporis ſeorſim eruere, ſecluſis interim Refractionibus, aut correctis per eas diſtantijs & altitudinibus. *Problem 61. If the phenomenon is under the same vertical as two fixed stars on this side of the meridian, and its distance from the vertex and from each of the fixed stars is taken at the same moment; and then, when it has been carried by the motion of the firmament to another vertical as far beyond the meridian as the former vertical was on this side of it, its distance from the vertex and from the same fixed stars is taken, together with the altitude of one or other of the fixed stars: to elicit the parallaxes of each time separately — refractions set aside for the present, or the distances and altitudes corrected by them.*

[Margin: Problem 61.]

This problem is Thomas Digges's, in his *Dialexis*, where it is his 17th — very evident indeed, [Margin: Order of the problems.]

but, because of the slenderness of the angles, slippery in the practice of calculation, as Tycho warns, volume 1, page 666. In the next scheme let B be the vertex, from which the quadrant BIA of the meridian descends into the portion DAE of the horizon; and from the same vertex let the vertical quadrant BD be drawn for the earlier observation, in which let the true place of the phenomenon be G, but the apparent place K, and the visible distances from the fixed stars KH and KI. Next, for the second observation, let BLE be the quadrant of another vertical, whose distance AE from the meridian is as great as DA, that of the earlier one (granted that in this figure it appears greater); and in it let the apparent place of the phenomenon be L, as far from the true place G as it was in the first observation — the place being marked with the same letter; just as the stars I and H, and the trace of the earlier visible place, ought to be marked on the same transverse arc IK of a great circle. Next let the visible place L of the phenomenon be connected with the fixed stars by the arcs LI and LH of a great circle; and from B let the arc BH be drawn, the vertical to H. From these, the task is to investigate the parallaxes GK and GL.

Diagram. A figure set into the left column in the course of Problem 61. B stands at the top; an arc runs down the left side from B to D, and a curved base

runs from D through A to E; on the right an arc runs from B down to H, with L below it and I inside to the left.

- B (top): the vertex
- I (on the left arc, with a star): a fixed star in the first observation
- G (on the left arc, with a dot): the true place of the phenomenon
- K (on the left arc, with a star): the apparent place in the first observation
- H (on the left arc, with a star): a fixed star in the first observation
- D (left end of the base): on the horizon DAE, end of the vertical quadrant BD
- A (bottom of the base): end of the meridian quadrant BIA
- I (inside, left): the star I in the second observation
- G (small, near H): the true place in the second observation
- K (smaller, beside G near H, set tilted along the arc)
- H (right): the star H in the second observation
- L (below H): the apparent place in the second observation
- E (right end of the base): end of the vertical quadrant BLE

[Margin: Step 1.]

First, then, in the triangle ILH the three sides are given: namely the visible distance IL of the phenomenon L from the star I; its distance HL from the star H; and the distance IH of the fixed stars between [?], which is known either from elsewhere or from the very visible distances from the phenomenon in the earlier observation. So the angle IHL will not be hidden.

[Margin: Step 2.]

Second: in the triangle BLH, if the altitude of the star H has been taken, the three sides are known — namely LH, the visible distance of the phenomenon from the star H; BL, the complement of the visible altitude of the phenomenon; and BH, the complement of the visible altitude of the star H. So the angles BHL and HBL will not be unknown. But if the altitude of the star I has been taken, in the triangle IBL there will be given BI, the distance IL, and BL, as above; and neither the angle IBL nor BIL will be hidden.

[Margin: Step 3.]

Third: if you subtract the angle IHL from the angle BHL, the angle BHG is left. With it, and with the angle HBL — that is, HBG — and with the side BH, find the sides BG and HG. For when BG is subtracted from the arc BL, GL, the later parallax, will be known; and when KH — that is, the earlier visible distance of the phenomenon K from the star H — is subtracted, below I, GK, the earlier parallax, will be known. But if the altitude of the star I was taken, use the triangle BIG. For in it are given BI, the complement of the altitude of the star I; the angle IBG, that is, IBL; and the angle BIG, namely when HIL is subtracted from the angle BIL. Then find the arcs BG and IG. For BG, subtracted from BL, leaves the parallax GL; and the arc IG, subtracted from the earlier distance IK, leaves the earlier parallax GK.

62. Probl. Conſiſtente Phænomeno ſub eodem Verticali cum duabus Fixis, & captis eius diſtantijs ab vtraq. Fixarum; rurſuſq. in ſecunda obſeruatione, non habita ratione Verticalium, captis Phænomeni diſtantijs ab ijſdem Fixis, vnà cum altitudine phænomeni, & alterutra Fixarum: Parallaxes vtriq. tempori congruas Inueſtigare, ſecluſa interim Refractione, aut per eam correcto loco viſo. *Problem 62. The phenomenon standing under the same vertical as two fixed stars, and its distances from each of the fixed stars having been taken; and again, in the second observation, without regard to the verticals, the phenomenon's distances from the same fixed stars having been taken, together with the altitude of the phenomenon and of one or other of the fixed stars: to investigate the parallaxes belonging to each time — refraction set aside for the present, or the visible place corrected by it.*

[Margin: Problem 62.]

This problem differs from the preceding one only in that in the second observation the fixed stars and the visible place of the phenomenon have another situation. It is Digges's 19th, and Tycho reports it in volume 1 of the *Progymnasmata*, page 668, as does Longomontanus in Book 2 of the *Spherics*, chapter 10, problem 2. We, however, have drawn the scheme together into the following form. In it let ABC be the meridian and ALC the horizon; B the vertex and D the pole of the world; let BN be the vertical quadrant of the first observation, in which let one of the fixed stars be at H and the other at O; and in the same quadrant let the true place of the phenomenon be at P, but the apparent place at Q. Now let those stars, together with the phenomenon, have been carried by the force of the primum mobile into another situation, which on the same great circle shows the star H at H and the star O at G, and the true place of the phenomenon at F, with the earlier parallax EF, but the place seen later at K, with the parallax FK. Through FK, then, draw the vertical BM, and through G the vertical BL; and connect K with E, G and H by the arcs KE, KG and KH.

Diagram. A figure set across the right column in the course of Problem 62. A semicircle with B at the top and a curved base from A to C; a line runs down from near B to N, and lines run from B down to M and to L.

- B (top): the vertex
- D (on the arc, upper right): the pole of the world
- E (inside, left, with a star): the earlier apparent place carried into the second situation
- F (inside, with a dot): the true place in the second situation
- G (under B): the star O in the second situation
- H (under B, beside G): the star H in the second situation
- I (on the line to N, with a dot): not named in the prose of this page
- O (on the line to N, with a star): the other fixed star in the first observation
- P (on the line to N, with a dot): the true place in the first observation
- Q (on the line to N, with a star): the apparent place in the first observation
- K (lower left, inside): the place seen later

- A (left end of the base): on the meridian ABC and the horizon ALC
- M (below the base): end of the vertical BM
- L (below the base): end of the vertical BL, on the horizon ALC
- N (below the base, right): end of the vertical quadrant BN
- C (right end of the base): on the meridian ABC and the horizon ALC

[Margin: Order of the problems.]

[Margin: Step 1.]

With these described: First, since from the first observation the visible distance QO is known, which in the other situation is EG, and the visible distance OH, which is GH; and since from the second observation the visible distances KG and KH are known, the triangle KGH will be of known sides, and so the angles KHG and GKH can be known.

[Margin: Step 2.]

Second: in the triangle EKH the distance EH seen earlier is known, and the distance HK, with the angle KHE; so the arc KE will become known, with the angles EKH and HEK.

[Margin: Step 3.]

Third: in the triangle KBG, KB is known, the visible distance of the phenomenon K from the vertex; and BG, likewise the distance of the star G from the vertex; and KG, the distance of the phenomenon from the star G. Therefore the angle BKG will become known. Added to the angle GKH, it will make up the angle FKH; and this, taken from the angle EKH, will leave the angle EKF.

[Margin: Step 4.]

Fourth, then: in the triangle EKF, in which besides the angle EKF the angle KEF and the arc KE are known, both the earlier parallax EF and the later parallax KF will become known.

CAPVT XI.

Chapter XI.

De Modis colligendi an, aut etiam quantam Parallaxin Altitudinis habeat Phænomenon, præſertim nouum, ex Obſeruationibus in diuerſis terræ locis factis. *On the methods of gathering whether a phenomenon, especially a new one, has a parallax of altitude, or even how much, from observations made at different places on Earth.*

In almost all the following problems the distance of the two places on a great circle of the Earth is supposed known, or the distance of the meridians lying between the two places of the Earth's surface at which, or from which, the same phenomenon is observed; or at any rate the altitude of the pole at each place is known, or both are known — even if this is not always expressed in the title of the problems. And whenever in these or similar schemes a single true place of the phenomenon is marked, it is a sign that the observation is supposed to have been made at the same moment of time in both places.

63. Probl. Data Semidiametro terræ, Differentia altitudinis Poli duorum locorum ſub eodem Meridiano, vel Longitudinis ſub Aequatore, vel Diſtantiæ ſub quouis verticali, & obſeruato eodem die naturali Phænomeno in Horizonte vtriuſq. loci, Altitudinem Phænomeni & Parallaxin Altitudinis decernere, ſecluſa interim Refractione horizontali, aut per eam correcto loco viſo. *Problem 63. Given the Earth's semidiameter, and the difference of the altitude of the pole of two places under the same meridian, or of longitude under the equator, or of distance under any vertical, and the phenomenon having been observed on the same natural day on the horizon of each place: to determine the altitude of the phenomenon and the parallax of altitude — horizontal refraction set aside for the present, or the visible place corrected by it.*

[Margin: Problem 63.]

From the Earth's centre T let there be described the semicircle NKQ of the hemisphere, of a great circle under the portion of the meridian

[Margin: Order of the problems.]

ABC, or, for the second case, under the equator ABC. Then, with the straight line TKB drawn, bisecting the terrestrial semicircle at K, where one observer is to be, and standing perpendicularly on the horizontal diameter NTQ, let two places on Earth, G and M, be assumed, equally distant on either side from the point K; or, if they are already given, let their distance, that is the arc GM, be bisected at K by the straight line TKB. For GK, or KM, will be the half-difference of the altitude of the pole, if the places are under the same meridian; or of the longitude, if they are under the equator.

Diagram. A figure set into the left column in the course of Problem 63. A vertical line runs from B at the top down to T at the base; a small semicircle stands on the base from G to M, and lines from G and M cross the vertical at L and, higher, at I, reaching the arcs above.

- O (top, left of B): end of the tangent MIO
- B (top): end of the straight line TKB
- R (top, right of B): end of the tangent GIR
- I (on the vertical, below B): the phenomenon's apparent place when G and M are further from K
- A (upper left): end of the tangent MLA
- C (upper right): end of the tangent GLC
- E (left end of the middle arc): end of the arc EIH
- H (right end of the middle arc): end of the arc EIH
- L (on the vertical, lower): the phenomenon's apparent place
- D (left end of the lower arc): on the arc through L
- F (right end of the lower arc): on the arc through L
- K (on the vertical, below L): the point of one observer, on the terrestrial semicircle
- G (left, on the small semicircle): a place on Earth
- M (right, on the small semicircle): a place on Earth
- N (left end of the base): end of the horizontal diameter NTQ

- T (bottom): the Earth's centre
- Q (right end of the base): end of the horizontal diameter NTQ

[Margin: By 18 of the third book of Euclid.]

Now from G and M let the straight lines GLC and MLA be drawn out, touching the Earth at G and M and making right angles there with the Earth's semidiameters TG and TM. Where they intersect each other on the straight line TKB — namely at L — there will be the place of the phenomenon as it appears on the physical horizon of each observer, which horizon the tangents already mentioned represent. And TL will be the distance of the phenomenon from the Earth's centre. With this interval an arc can be described, either of the heavens or of the elementary region, in which the phenomenon is supposed to lie under the meridian or the equator. Its horizontal parallax, moreover, will be the arc AB, which the angle TLM measures, though not immediately; or the arc BC, which the angle TLG measures, from what was said in chapter 1. Similarly, if the places G and M are further from K, and the tangents drawn out from them are GIR and MIO, the place of the phenomenon appearing on both physical horizons will be I, its distance IT, the arc in which it will lie EIH, and the horizontal parallax OB on one side and BR on the other, which the angles MIT and GIT measure. With this set down, let the phenomenon be at L. For in the triangle GLT, right-angled at G, there is given the side GT, which is one semidiameter of the Earth, or assumed as one, or in the miles contained in it; there is given also the angle GTK, which the arc GK measures — that is, the half-difference of the altitude of the pole if the places G and M are under the same meridian, or of the longitude if they are under the equator. Therefore, by the rules of right-angled triangles, neither the distance of the phenomenon, that is the base TL, will be hidden, nor the angle GLT, which measures the horizontal parallax. And in the same way, from the triangle GIT you will elicit the altitude TI, or the distance of the phenomenon from the Earth's centre, and the parallactic angle GIT — if, that is, a straight line is drawn from I to the point of contact G, which, however, you must imagine nearer to the point N than it is now, when the phenomenon was being considered at L; just as the twin straight line from T to G must also be supplied.

EXEMPLVM *Example.*

Let G, the island of São Tomé, be under the equator, and M another place 165° distant from it. Half of these degrees, that is the arc GK, or the angle GTL, will be 82° 30′. Let the Earth's semidiameter TK, or TG, be put at 4139 of our Italian miles (as we have found). For by the analysis of the right-angled triangle TGL the interval TL is found to be 27,571 miles, that is, 6 terrestrial semidiameters and 2737 miles besides; and the parallax GLT, at its greatest, 8° 38′. But so that it may appear how great a distance between two places on Earth is required for the phenomenon to be above the Moon — whose least distance truly lies at about 52 semidiameters of the Earth — let the following little table stand. Its first column contains the distance of two places set on the same great

circle of the Earth; the second, the altitude of the phenomenon needed for it to be seen at the same time from each place, on the physical horizon of each. For if it is seen on that horizon, there is no doubt that, raised above the horizon, it can be seen from the same places, and much more by those living within G and M, towards K, and in other places less distant from each other than the arc GKM.

Arc of distance on a great circle	Semidiameters of the Earth	Italian miles
7° 30′	0	10
15° 0′	0	36
30° 0′	0	146
45° 0′	0	341
60° 0′	0	641
75° 0′	0	1078
90° 0′	0	1715
105° 0′	0	2660
120° 0′	1	0
135° 0′	1	2538
150° 0′	2	3575
165° 0′	6	2737
172° 30′	14	1200
176° 15′	29	2329
177° 30′	47	1067
178° 45′	90	1960

64. Probl. Datis Diſtantijs Phænomeni à Vertice, & ab eadem, vel ijſdem ſtellis Fixis, ex duobus Terræ Locis ſub eodem verticali, eodem momento obſeruatis; vnà cum Diſtantia locorum terræ; Inueſtigare Parallaxim Phænomeni abſolutam, vel comparatam cum Parallaxi Lunari; Methodo Tychonica, ſed corrigenda. *Problem 64. Given the distances of the phenomenon from the vertex and from the same fixed star or stars, observed at the same moment from two places on Earth under the same vertical, together with the distance between the places on Earth: to investigate the parallax of the phenomenon, absolute or compared with the lunar parallax — by the Tychonic method, but one needing correction.*

[Margin: Order of the problems.]

[Margin: Problem 64.]

Tycho, intending to demonstrate in volume 2, from page 123 to 127, that the comet of the year 1577 was not just below the Moon — that is, not distant from the Earth by only 50 terrestrial semidiameters, but by many more — describes in the following figure, from the Earth's centre T, the terrestrial circle HANu, in which A is Uraniborg and B Prague in Bohemia, from which the comet C was seen. By Tycho it was seen at Uraniborg in the year 1577 on November 16, around the sixth hour of the evening, 17° 50½′ distant from the bright star of the Vulture, D; but by Tadeáš Hájek at Prague on the same evening, at hour 6, 17° 50′ distant from the bright star of the Vulture; and he says this was done by

each observer with the aid of a radius. He assumes the altitude of the pole at Prague as 50° 7′ and at Uraniborg as 55° 53′, and the difference as 5° 46′, which the arc AB represents. For a slight difference in the longitude of the places does little harm to the demonstration, especially since in both places the comet was very nearly in the same vertical as the bright star of the Vulture.

Diagram. A figure set across the right column in the course of Problem 64. An outer semicircle stands on a base from S to the right-hand end; inside it a smaller concentric semicircle; at the base a full small circle about the centre T; straight lines run out from the small circle to the arcs.

- R (top of the outer arc): on the outer arc
- D (small, beside R): not named in the prose of this page
- D (on the outer arc, right of R): the bright star of the Vulture
- Q (outer arc, upper left): on the outer arc
- M (outer arc, left): on the outer arc
- O (outer arc, left, below M): on the outer arc
- P (outer arc, left, lowest): on the outer arc
- L (inner arc, left): on the inner arc
- A (top of the small circle): Uraniborg
- G (right of A): on the small circle
- C (inner arc, right): the comet
- E (outer arc, right): on the outer arc
- F (outer arc, right, below E): on the outer arc
- m (italic, outside the arc to the right of F): not named in the prose of this page
- I (inside the small circle, left): not named in the prose of this page
- R (inside the small circle, under G)
- H (on the small circle, left): on the terrestrial circle HANu
- N (on the small circle, right): on the terrestrial circle HANu
- S (left end of the base): on the outer arc
- V (on the base, left): on the base
- T (centre of the small circle): the Earth's centre
- X (on the base, right): on the base
- Z (right end of the base, with a hooked foot): on the outer arc
- u (bottom of the small circle): on the terrestrial circle HANu

[Margin: Tycho's method.]

Having supposed, then, a distance BC of the comet of 50 semidiameters of the Earth: since the chord AB, corresponding to 5° 46′, is 10,060 parts of which TA is 100,000, BC will be 5,000,000. But the distance of the comet from the Vulture's bright star, as seen from A, is, he says, the angle DAF, and as seen from B, the angle DBE. Next he supposes that the same star is seen at D from the places A and B along the lines AD and BD, which are sensibly parallel because of the immense distance of the fixed stars. He further supposes that the angles at A and B which those lines make with the chord AB are right angles, *because those lines tend to the centre of the Earth as if they were one line, and, as lines from*

the centre to the circumference, fall normally on AB, according to the laws of straight lines subtended in a circle. So he says; and he goes on. From the comet's daily motion towards the Vulture, 2½°, and from the difference of meridians, 15^m of time, he gathers that if he himself had observed at the same moment as Tadeáš at Prague — that is, at 5^h 45^m at Uraniborg, when it was the 6th hour at Prague — the comet would have been seen at both places with no difference whatever, 17° 52′ distant from the Vulture. That this would have been impossible if the comet had been below the Moon, distant from the Earth by only 50 terrestrial semidiameters, he tries to show by the following reasoning.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

EXEMPLVM *Example.*

[Margin: Step 1.]

First, in the triangle AGB, with the perpendicular BG drawn to ACF, the visible distance DAC is given, and its complement GAB to the right angle DAB; this complement is 72° 9½′. Therefore the angle GBA is 17° 50½′, since the angle at G is a right angle. And AB is 10,060 parts; therefore by the rules of triangles the side BG is very nearly 9,576 parts — for this is how, by the doctrine of sines, the corrupt passage of Tycho is to be emended.

[Margin: By Euclid I.32.]

[Margin: Step 2.]

Second, in the triangle BGC, from the side BG, 9,576, and the base BC, 5,000,000, the angle GCB comes out 6′ 35″ — the angle by which the angle of the distance seen at A would be greater than that seen at B; for GBC would come out 89° 56′ 25″. If this is added to the angle ABG, 17° 50½′, the whole ABC will be 107° 43′ 55″; and if you take the right angle ABD from it, there remains the angle DBC, 17° 43′ 55″, which represents the distance of the comet from the Vulture as seen at Prague, when at Uraniborg it was seen as 17° 52′. Yet when the observation taken at the same moment is considered, there was no difference between the visible distances. Therefore, for the same visible distance to hold at both places, AB must bear no sensible proportion to the comet's distance BC, and the lines AC and BC must have been sensibly parallel; and so the comet was far above 50 semidiameters of the Earth, and much higher than the Moon. So far Tycho.

Deteguntur fallaciæ in præcedenti Pſeudoapodixi. *The fallacies in the preceding pseudo-proof are exposed.*

[Margin: Tycho's error.]

But here, surely, our Homer was nodding — which is no wonder, since it sometimes happens to great men. And so first Chiaramonti, in the *Antitycho*, Book 2, chapters 12 and 13, and then Galileo in the *Trutinator*, page 22, attacked Tycho's fallacy. For since the northern declination of the bright star of the Vulture is, for Tycho, 7° 52′, and the distance of the equator from Uraniborg's vertex R is 55° 54′, it follows that the Vulture's distance from the vertex of

Uraniborg is 48° 2′. But if DAB had to be a right angle, then, since AB is less than 6°, the distance RAD of the star from the vertex would have to be less than 3°, says Galileo; and he rightly adds that only the vertical line RA falls perpendicularly on the Earth at the point A. Chiaramonti, for his part, says that if the lines DA and DB fell perpendicularly on the Earth, they would have to meet at the Earth's centre, and so could not make right angles with AB — which is entirely true. Chiaramonti further shows, from the data Tycho gives, that the comet's distance from the vertex of Uraniborg was then 73° 30′ 30″, and from the vertex of Prague 67° 44′. Therefore, since at the Moon's distance of 43 semidiameters a parallax of 1° 15′ 30″ belongs to the former distance from the vertex, and one of 1° 13′ 26″ to the latter, the difference of the parallaxes could have been no more than 2′ 4″, and not 6′ 35″ — if Ptolemy's parallax table, in Book 5 of the *Almagest*, chapter 18, is correct. And what of Kepler here, in the *Hyperaspistes* of Tycho, pages 50 and 95? After many attempts at excuses, to be sure, he nevertheless admits that Tycho erred and assumed a false scheme in place of the true one, both because of the confused and varied intentions of his mind, and because he was distracted among a number of affairs.

65. Probl. Corrigere Tychonis Schema, Refractione interim diffimulata.
Problem 65. To correct Tycho's scheme, refraction being disregarded for the present.

[Margin: Problem 65.]

Chiaramonti goes on, in the *Antitycho*, Book 2, chapter 13, page 190. In the preceding Tychonic scheme, drawing from Prague, B, the straight line BD, and Am tangent to the Earth at A, he shows that the angle DAB was not a right angle but only 37° 13′. But earlier, on page 187, he had demonstrated from Tycho's data that on November 16, at 5^{h} 45^{m} at Uraniborg, the comet stood 73° 30′ 30″ from the vertex of Uraniborg. Therefore, if the comet stood 17° 50′ 30″ from the Vulture, as Tycho will have it, it follows that the bright star of the Vulture then stood 55° 40′ from the vertex of Uraniborg; for it was not on the meridian, as Galileo supposed above to make his argument more than sufficient. With this lemma truly demonstrated, then — as will be plain to anyone who examines Chiaramonti's calculation with me — let the arc RD, or the angle RAD, be 55° 40′. Therefore the remaining angle DAm, up to the right angle RAm, is 34° 20′. But the angle mAB is equal to the angle AuB in the alternate segment, by Euclid III.34; and this angle AuB stands on the arc AB, 5° 46′ (the amount assumed for the distance of Prague from Uraniborg on the great circle of the Earth). It is therefore half the angle ATB, which is at the centre, and so it is 2° 53′. The whole DAB, therefore, is 37° 13′, which was to be shown, and not a right angle, as Tycho supposed.

[Margin: By Euclid I.32.]

[Margin: By Euclid III.20.]

Hence GB comes out much shorter than Tycho computed; for it comes out 3,337 parts of which the Earth's semidiameter is 100,000, and not 9,576, as Chiaramonti shows on page 191. Hence, with the base BC put, as Tycho puts it,

at 50 terrestrial semidiameters, it follows from the side BG, 3,337 parts of which BC is 5,000,000, that in the right-angled triangle BGC, right-angled at G, the parallactic angle BCG is at most 2′ 18″, and not 6′ 35″, as Tycho wrongly gathers. The comet could therefore be distant from the Earth by 50 semidiameters of the Earth, and be below the Moon, and yet between its visible distances from the Vulture at Prague and at Uraniborg there could be no sensible difference, or scarcely 2′ 18″.

[Margin: Tycho's various small errors, or doubtful errors.]

Let us now consider that Tycho and Hájek used the astronomical radius, a very deceptive instrument, as he himself later admitted in volume 1 and in the *Mechanica*; and the difference of meridians between Uraniborg and Prague, which by my reckoning from eclipses is only 11′; and some deviation of the comet from the same vertical under which the Vulture is placed together with Uraniborg and Prague; and the uncorrected altitude of the pole, since later, having detected the refractions at the winter solstice, he found from a circumpolar star the altitude of the pole at Uraniborg to be 55° 54′ 30″, not 55° 53′ as he uses it in volume 2; and later the altitude of the pole at Prague was found by him to be 50° 4′, as appears from the last Tychonic observations published by Willebrord Snel, and not 50° 7′, as Tycho supposed in volume 2, page 253. Besides this, the refraction of the comet C was left out, which Tycho had not yet scented; for the comet, being nearly 74° from the vertex, was owed a refraction of at least 7′, from Tycho's little table of lunar refractions. From all these differences, then, whether certain or probable, a much greater difference could arise between the distances of the comet from the Vulture seen at Prague and at Uraniborg, and so the comet could be far lower than the Moon.

[Margin: Tycho's untrustworthy narrative.]

And what of the fact that Tycho, in questionable good faith, pretends that he observed the comet on the same day, November 16, so that he might compare his observation with Hájek's — when Tycho himself, in volume 2, chapter 1, in the history of the observations of this comet of the year 1577, had asserted that from the 15th day four days were empty of observations because of mists and the darkness of the air? We too have noted this, with Chiaramonti in the *Antitycho*, chapter 13, in Book 8, Section 1, chapter 18, number 22, where we showed that Kepler laboured in vain in defending either Tycho's lie or his boast. But before we leave Prague, let us weigh another example, and the method of Longomontanus, which he uses in solving the problem just proposed.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

66. Probl. Idem Problema ſoluere Methodo Longomontani, ſed multò clariùs propoſita. *Problem 66. To solve the same problem by the method of Longomontanus, but set out much more clearly.*

[Margin: Problem 66.]

Let the figure of Problem 64 be repeated here; and in it, to avoid confusion, let Prague be on the other side, at A, and Copenhagen at I. Let the vertical lines be TR for Prague and TQ for Copenhagen; and let the comet of the year 1607 be seen at L, through which are drawn the line of the true place, TLM, and the lines of the visible places, ALP and ILO. Let the visible distance of the comet from the vertex of Prague be the angle PAR, and from the vertex of Copenhagen the angle OIQ (for the comet is supposed to have been sensibly under the same vertical as Prague and Copenhagen). Next let LT be given, the height of the comet from the centre of the Earth, which must be examined to see whether it answers to the observed distances from the vertex and at the same time to the observed distance from the same fixed star; let it be given, I say, in parts of which the Earth's semidiameter AT is known. With these laid down:

[Margin: Step 1.]

First, in the triangle TLA, the sides LT and AT are given, and the angle TAL, the complement of the visible distance from the vertex of Prague; seek, then, the parallactic angle TLA.

[Margin: Step 2.]

Second, in the triangle TIL, two sides, TL and IT, are given, and the angle TIL, which is the complement of the visible distance from the vertex of Copenhagen; so seek the parallactic angle TLI, subtract it from the angle TLA, and there will remain the difference of the parallaxes, ILA. If this is equal to the difference between the comet's distances from the same fixed star seen from A and from I, then the supposed height TL of the comet, or phenomenon, will also be right. But if the difference of the parallaxes is greater than the difference of the visible distances, the height TL was supposed too small; if less, too large. See the example in Book 8, Section 1, chapter 21, number 7.

67. Probl. Datis eiuſdem Phænomeni ſub eodem verticali Altitudinibus obſeruatis ex duobus terræ locis notæ diſtantiæ, & Differentia Parallaxium ex diſtantijs tunc ab eadem ſtella Fixa vel à vertice viſis vtrobique; Parallaxes altitudinis, cuique diſtantiæ viſæ congruentes, ac ſimul diſtantiam Phęnomeni à centro terræ determinare; Secluſa interim Refractione, vel per eam correctis altitudinibus ac diſtantijs. *Problem 67. Given the altitudes of the same phenomenon under the same vertical, observed from two places on Earth of known distance, and the difference of the parallaxes from the distances then seen at both places from the same fixed star or from the vertex: to determine the parallaxes of altitude belonging to each visible distance, and at the same time the distance of the phenomenon from the centre of the Earth — refraction set aside for the present, or the altitudes and distances corrected by it.*

[Margin: Problem 67.]

We have already taught this whole problem in chapter 9; but it is worth completing here the part of it that rests on the difference of the parallaxes, since that is more often knowable, and setting it against the Tychonic fallacy in the

same figure, so that opposites placed side by side may stand out the more clearly. Let the figure of Problem 64, then, be put back in this place. In it, one place, A, and the other, I, are on the surface of the Earth, and there is the phenomenon L, whose distance from the vertex R, seen from A and corrected by adding the refraction, is the angle LAR, and its complement LAT; while its distance from the vertex Q, seen from I and corrected, is the angle LIQ, and its complement LIT. The parallax of the phenomenon with respect to the eye at A is the angle ALT, and with respect to the eye at I the angle ILT; and so the difference of the parallaxes is the angle ILA. With the chord AI drawn, let it be known, from the quantity of the arc AI — that is, of the distance of the places on the same great circle of the Earth — in parts of which the Earth's semidiameter AT, or the radius, is 100,000.

[Margin: Step 1.]

[Margin: By Euclid I.32 and by I.5.]

First, then, since the angle ATI measures the given arc AI, in the triangle ATI, which is isosceles, with equal legs TA and TI, the sum of the angles TIA and TAI at the base will also be known, since it is the supplement of the angle ATI to two right angles; and since those angles are equal to each other, each of them will be known as well. Now, taking TIA from 180°, the adjacent angle AIQ will be known; and if you join to it the given angle QIL, which was the distance of the phenomenon L, seen from I, from the vertex Q, the whole angle AIL becomes known. Since, therefore, in the triangle AIL, besides the side AI, two angles are given, AIL and ALI, the difference of the parallax, whose supplement to two right angles is the angle LAI, seek the side

Diagram. The figure of Problem 64 set again across the right column, in the course of Problem 66 and Problem 67. An outer semicircle stands on a base from S to the right-hand end; inside it a smaller concentric semicircle; at the base a full small circle about the centre T; straight lines run out from the small circle and its centre to the arcs.

- R (top of the outer arc): the vertex of the place A, on the vertical TR
- D (just right of R): not named on this page
- D (on the outer arc, further right): not named on this page
- Q (upper left of the outer arc): the vertex of the place I, on the vertical TQ
- M (left side of the outer arc): end of the line of the true place TLM
- E (right side of the outer arc): not named on this page
- O (left side of the outer arc): end of the line of the visible place ILO
- P (left side of the outer arc, lowest): end of the line of the visible place ALP
- L (left, on the inner arc): the comet, or phenomenon
- A (above the centre): a place on Earth — Prague in Problem 66, Kassel in the Example
- C (right, on the inner arc): not named on this page
- G (right of A): not named on this page

- F (right side of the outer arc): not named on this page
- m (italic, right end of the line through F): not named on this page
- I (inside the small circle, left): a place on Earth — Copenhagen in Problem 66, Uraniborg in the Example
- R (inside the small circle, under G): not named on this page
- H (left on the small circle): not named on this page
- N (right on the small circle): not named on this page
- S (left end of the base): not named on this page
- V (on the base, left): not named on this page
- T (centre of the small circle, on the base): the centre of the Earth
- X (on the base, right): not named on this page
- Z (right end of the base, with a hooked foot): not named on this page
- u (bottom of the small circle): not named on this page

LI, that is, the distance of the phenomenon from the surface of the Earth; and with it, and with IT, and with the angle LIT, the supplement of the angle QIL to two right angles, find the distance TL from the centre of the Earth, in parts of which the chord AI and the radius AT were known. This was one of the things proposed.

[Margin: Step 2.]

Second, in the triangle ATL, in which AT, the Earth's semidiameter, is given, and TL, the distance of the phenomenon, in parts of the same kind, and the angle TAL, the complement of the visible distance from the vertex, find the parallax TLA belonging to the altitude seen from A.

[Margin: Step 3.]

Third, in the triangle TIL, in which TI, the Earth's semidiameter, is given, and TL, the distance of the phenomenon, and the angle TIL, which is the complement of the distance from the vertex Q seen from I, seek out the parallax TLI belonging to the altitude of the phenomenon seen from I; and so the problem will be satisfied.

EXEMPLVM *Example.*

I shall take an example, on purpose, from Chiaramonti's Antitycho, *Book 3, chapter 19, because it is useful to us for judging many matters concerning the height of comets; but I shall fit it to our measures of the Earth's diameter, and I shall use visible altitudes corrected by subtracting the refraction. Let I, then, be Uraniborg, and A Kassel, a city of Hesse, distant on the same vertical by the arc AI, 4° 34′, as Tycho puts it in his letters to Rothmann; and let the comet's visible distance from the vertex of Uraniborg, LIQ, have been 84°, and the difference between the distances from the same fixed star — that is, the difference of the parallaxes, ILA — be one minute.*

[Margin: Step 1.]

First, since AI is 4° 34′, its chord is 7,968 of the parts of which the radius AT is 100,000; therefore, of the miles of which AT, by our reckoning, is 4,139, AI is 330. Next, since ATI is 4° 34′, the sum of the angles TAI and TIA will be 175° 26′, and its half, TIA, 87° 43′, and the angle adjacent to it, AIQ, 92° 17′. If you add QIL, 84°,

to this, the whole LIA becomes 176° 17′. But the angle ILA was supposed to be one minute; with these angles, then, and with AI, 330 miles, LI comes out, the distance of the comet from the surface of the Earth, at 73,173 miles. If you join TI, 4,139, to these, the whole TL would be at most 77,312; for in fact TL is less than the two, LI and IT, taken together. And dividing 77,312 by 4,139, the distance of the comet would come out at most 18 terrestrial semidiameters and 2,810 miles, which is much smaller than the lunar distance. How much smaller, then, if the difference of parallax between Uraniborg and Kassel of 17′, admitted by Tycho, is admitted?

[Margin: By Euclid I.20.]

COROLLARIVM. *Corollary.*

Between two places on Earth not more than 4° or 5° apart there can be a difference of parallax of one minute — that is, such places can see the same phenomenon distant from the same fixed star, disagreeing in the visible distance by a single minute, or even by less than a whole minute — and yet that phenomenon can be much lower than the Moon, and so have a greater parallax than the Moon: namely, if it is much nearer the horizon than the vertex. This is plain from the preceding example, and Chiaramonti notes it in the *Antitycho*, Book 3, chapter 19, theorem 1; he also, in Book 2 of the *Antitycho*, chapter 20, page 223, shows the same thing in another way by two problems worthy of this place. See also the example we have brought forward in Book 8, Section 1, chapter 19, number 3.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

68. Probl. Datis Phænomeni Diſtantijs viſis à duabus quibuſuis ſtellis Fixis, & omnium trium Altitudinibus viſis, vnà cum Diſtantia Fixarum, in duobus Horizontibus; Parallaxes altitudinis cuiq. altitudini cõgruas, notas facere. Secluſis Refractionibus, vel per eas correctis altitudinibus ac diſtantijs. *Problem 68. Given the visible distances of a phenomenon from any two fixed stars, and the visible altitudes of all three, together with the distance between the fixed stars, in two horizons: to make known the parallaxes of altitude belonging to each altitude — refractions set aside, or the altitudes and distances corrected by them.*

[Margin: Problem 68.]

Among other problems that Thomas Digges presents in his *Dialexis*, this is his 20th, which Tycho also reports in volume 1 of the *Progymnasmata*, page 669, asserting that it is the most laborious of all, since it requires the analysis of seven spherical triangles. In the following figure, let the meridian be MAO, and in it the vertex A and the pole of the world Q, and the horizon of the first observer MRO; the first vertical of the phenomenon AHN, in which the true place is G and the apparent place H; the places of the fixed stars K and F, in their verticals AKL and AFR; and the visible distances of the phenomenon from them, HF and HK. Now let the vertex A be carried to the point E — that is, let E be the vertex of the second observer, and BDC his horizon; and let the vertical circles

of the fixed stars be EKS and EFP; and let the vertical of the phenomenon, appearing at I, be EID, the true place of the phenomenon G, and the visible distances from the fixed stars IF and IK; and finally let the distance between the fixed stars be KF. It is now proposed to investigate the parallaxes HG and IG.

Diagram. A figure set across the left column in the course of Problem 68. A large arc rises to A at the top; below, two lower arcs run across the figure from the left to O and C at the right; straight and curved lines run down from A and from E through the fixed stars K and F and the phenomenon at H and I to the lower arcs.

- A (top): the vertex of the first observer, on the meridian MAO
- Q (upper right of A): the pole of the world
- E (right, below Q): the vertex of the second observer
- T (on the arc at the right): not named on this page
- K (middle left): a fixed star
- [?] (small sort between K and F, above them): not read
- F (middle right): a fixed star
- B (far left): on the horizon BDC of the second observer
- M (left, below B): on the meridian MAO and the horizon MRO
- I (below K): the apparent place of the phenomenon for the second observer
- H (below F): the apparent place of the phenomenon for the first observer
- L (lower left): end of the vertical AKL
- S (lower left, on the lower arc): end of the vertical EKS
- D (lower left, on the lower arc): on the horizon BDC; end of the vertical EID
- N (lower middle): end of the vertical AHN
- R (lower right): on the horizon MRO; end of the vertical AFR
- O (right end of a lower arc): on the meridian MAO and the horizon MRO
- C (right end of the lowest arc): on the horizon BDC
- P (bottom): end of the vertical EFP

[Margin: Step 1.]

First, in the triangle AKF, there are given KF, the distance between the fixed stars, and AK and AF, the complements of the visible altitudes of the fixed stars; therefore the angle AKF will be given by trigonometry.

[Margin: Step 2.]

Second, in the triangle KEF, there are given the distance between the fixed stars, KF, and EK and EF, the complements of the altitudes of the fixed stars as seen by the second observer; the angle EKF will therefore be given, and this, subtracted from the angle AKF, will leave the angle AKE.

[Margin: Step 3.]

Third, in the triangle AKE, two sides are given, AK and KE, already known above, with the included angle AKE; so neither the side AE nor the angle AEK will be hidden.

[Margin: Step 4.]

Fourth, in the triangle AFE, there are given the side AE, just found, and AF and EF, the complements of the visible distances of the fixed star F; so the angle FAE will become known.

[Margin: Step 5.]

Fifth, in the triangle HAF, there are given HF, the visible distance of the phenomenon from the fixed star F in the first observation; AF, the complement of the visible altitude FR; and AH, the complement of the visible altitude of the phenomenon. So seek the angle HAF, add it to the angle FAE already found, and the whole angle HAE will be known.

[Margin: Step 6.]

Sixth, in the triangle KEI, there are given the sides KE, the complement of the visible altitude of the star K, and IE, the complement of the visible altitude of the phenomenon for the second observer, and KI, the visible distance of the phenomenon from the fixed star K; so investigate the angle KEI, join it to the angle AEK, and the whole angle AEG will be known.

[Margin: Step 7.]

Seventh, in the triangle AEG there are given the angle AEG, just found, the side AE, found in the 3rd step, and the angle GAE, found in the 5th step, since it is the same as the angle HAE; seek, then, the sides AG and EG. For the former, subtracted from the complement AH of the first visible altitude, will give the parallax GH; and the latter, subtracted from the complement of the altitude of the phenomenon seen by the other observer, will leave the parallax GI known.

But when will it be possible for two observers at different places on Earth to observe so many things exactly at the same moment? For there must be several companions, one of whom takes the altitude of the phenomenon, two others the altitudes of the fixed stars, and two others the distances of the phenomenon from the fixed stars; indeed, if the distances are to be taken more correctly with a sextant, there will have to be seven companions observing at the same moment. But if they observe at a different moment in one horizon, and at a different moment in the other horizon, it must be supposed meanwhile that the phenomenon has no motion other than the motion of the primum mobile.

69. Probl. Data differentia Latitudinis ac Lõgitudinis inter duo loca terræ, ex quibus eodem momento idem Phænomenon obſeruatum eſt; datiſq. tum Azimuthis, tum viſis à vertice Diſtantijs vtrobiq. obſeruatis; Parallaxes altitudinis vtrique diſtantiæ viſæ à vertice congruentes dignoſcere. Secluſis interim, aut computatis Refractionibus. *Problem 69. Given the difference of latitude and of longitude between two places on Earth from which the same phenomenon was observed at the same moment, and given both the azimuths and the visible distances from the vertex observed at each place: to discern the parallaxes of altitude belonging to each visible distance from the vertex — refractions set aside for the present, or computed.*

[Margin: Problem 69.]

Let AB be a portion of the meridian, in which the pole of the world is A.

[Margin: Case 1. If the places differ only in latitude.]

And first let there be two places, C and D, under the same meridian, whose difference of latitude is CD; and let the true place of the phenomenon, observed at the same time, be at E, through which let the arcs of vertical circles be drawn from the vertices C and D to the visible place of the phenomenon — namely the arc CEH to the visible place H, and the arc DEF to the visible place F. For in the triangle CED the side CD is given, which is the difference in latitude, or in the altitude of the pole; and the angle ECD, which is the measure of the azimuth observed by the inhabitant under C; and the angle CDE, which is the supplement to two right angles of the azimuth EDB observed by the inhabitant under D. From these three data, therefore, seek the side CE, which, subtracted from the visible distance CEH of the phenomenon from the vertex, will leave the parallax EH known, belonging to that visible distance. Again seek the side DE, and subtract it from the visible distance DEF of the phenomenon from the vertex, and the remainder will be the parallax EF, belonging to this visible distance from the vertex.

Diagram. A figure set in the right half of the right column in the course of Problem 69. From A at the top an arc of the meridian runs down the right side through C and D to B; arcs from A and from C, L and O run down to the lower left, where several lines meet at E.

- A (top): the pole of the world
- L (upper left, on an arc): a place on the same parallel as C (second case)
- P (left, below L): on the azimuth KLP
- C (right, on the meridian arc): the vertex of a place
- O (inside, between C and E): the vertex of the other place (third case)
- D (right, below C, on the meridian arc): a place under the same meridian as C (first case)
- F (above and left of E)
- E (lower left, where the lines meet): the true place of the phenomenon
- G (below, left of E): the visible place (third case)
- H (below, left of E, beside G): the visible place on the arc CEH
- K (below E): the visible place on the arc LEK (second case)
- R (bottom): end of the meridian AOR (third case)
- B (bottom right): end of the meridian AB; on the azimuths EDB and BCH

[Margin: Case 2. If the places differ only in longitude.]

Second, let there be two places on Earth, C and L, under different meridians but on the same parallel, CL, described from the pole of the world A. From A let the arcs of the meridians, AC and AL, be drawn to the vertices of the places; these will accordingly be equal to each other, since the places are equally distant from the pole of the world. And from them, through the true place E of the phenomenon, let the vertical arcs be drawn — CEH as far as its visible place H,

and LEK as far as the visible place K — and the difference of longitudes will be the angle LAC. With these laid down: in the triangle ACL there are given the sides AL and AC, equal to each other, which are the complement of the altitude of the pole, and the angle LAC included by them; seek, then, both the angles ALC and ACL and the arc LC. Then in the triangle ELC you will have the side LC known, and the angles ELC and ECL; for the angle ELA is the supplement to two right angles of the azimuth KLP observed under the vertex L, and if you take from it the angle CLA found, the angle CLE will remain; likewise the angle ECA is the supplement to two right angles of the azimuth BCH observed under the vertex C, so that when the angle ACL already found is taken away, the angle LCE will be known. In the triangle ELC, therefore, seek the side LE, which, subtracted from the visible distance LEK from the vertex L, will give the parallax EK. Again seek out the side CE; for this, taken from the visible distance CEH from the vertex C, will give the parallax EH.

[Margin: Case 3. If the places differ in longitude and latitude.]

Third, let the two places on Earth differ in longitude and in latitude, and let the vertex of one be C and of the other O, which are joined by the arc CO of a great circle; and let AOR be the portion of the meridian drawn through O; and from the vertices let the vertical arcs be drawn through E, the true place of the phenomenon — CEH as far as its visible place H, and OEG as far as the visible place G. With these marked out, first, in the triangle CAO, there are given the al- [Margin: Order of the problems.]

[Margin: Order of the problems.]

titude-complements of each pole, AC and AO, and the angle CAO contained by them, which the difference of longitude of the places measures. Seek, therefore, the base CO and the remaining angles ACO and AOC. Next, from HCA, the supplement to two right angles of the azimuth ECB, subtract the angle ACO just found, and the angle ECO will remain; conversely, to EOA, the supplement to two right angles of the azimuth EOR, add the angle AOC found, and the result will be the angle EOC in the triangle EOC, in which, besides the angles at O and C, OC is given. Therefore investigate the side CE: subtracted from the arc CEH, that is, from the visible distance from the vertex C, it will leave the parallax EH known. Again seek the side OE and subtract it from the arc OEG, that is, from the visible distance from the vertex O, and the remaining arc EG will be the parallax of altitude belonging to that place. I know that Chiaramonti, in Book 1 of the *Antitycho*, chapter 5, problem 2, gives this same problem, but with another figure; he seems, however, to have used the arc LC of a parallel in place of an arc of a great circle, which can lead some people into error.

70. Probl. Data Diſtantia Phænomeni viſa ab eadem ſtella Fixa, ex duobus terræ locis, ſub eodem Meridiano poſitis, quando Phænomenon tranſit Meridianum, & Momento talis tranſitûs, ac Altitudine Poli vtriuſq. loci, ſcire tum Diſtantiam viſam Phænomeni ab vtroq. vertice, tum Altitudinis

Parallaxes vtriq. loco cõuenientes. Secluſis Refractionibus &c. *Problem 70. Given the visible distance of a phenomenon from the same fixed star, from two places on the earth set under the same meridian, when the phenomenon crosses the meridian, together with the moment of that crossing and the pole altitude of each place: to know both the visible distance of the phenomenon from each vertex and the parallaxes of altitude belonging to each place. Refractions set aside, etc.*

[Margin: Problem 70.]

Consider the right-hand part of the following diagram. In it let the arc of the meridian be ADH, and on it the pole of the world A, and two places on the earth, B and C, the complements of whose latitudes, or of whose pole altitudes, AB and AC, are given; and so, the smaller being taken from the larger, the difference BC is given. Let the true place of the phenomenon crossing the meridian be at D, but its apparent place with respect to the place B be G, and its parallax DG; while its apparent place from C is E, and its parallax DE. And let the fixed star F be joined by four arcs of great circles to the vertices of the places and to the apparent places of the phenomenon, namely by the arcs FB, FC, FE, FG; for EF and GF will be the visible distances from that same star. With these laid down:

[Margin: Step 1.]

First, from the moment of the hour there is given, by the usual tables or the problems of the primum mobile, the culminating point of the ecliptic and its right ascension in mid-heaven; from the tables of the fixed stars, too, the right ascension of the fixed star F is given; and hence the difference between these ascensions is given, and so the arc DAF; and from the same tables of the fixed stars the complement of declination AF is given; and AB is given. Therefore in the triangle BAF the angle A is given with the sides enclosing it; so seek both the side BF and the angle ABF. In this way, in the triangle CBF, the supplement CBF of the angle ABF to two right angles will be known, and the arc BF, while BC was given above.

Diagram. A figure in the right part of the left column, serving two problems: its right-hand part for Problem 70, its left-hand part for Problem 71 ("the left-hand part of the figure immediately preceding"). An arc runs down the middle from A at the top through B, C, D, E and G to H at the bottom; from A two outer arcs run down to L at the left and F at the right, and lines join L and F to B and C.

- A (top): the pole of the world, on the meridian ADH
- B (on the central arc, below A): the vertex of the first place
- C (below B): the vertex of the second place
- L (far left): the fixed star of Problem 71
- M (between L and C): the apparent place of the phenomenon from C (Problem 71)
- K (between M and C): the true place of the phenomenon (Problem 71)
- F (far right): the fixed star of Problem 70

- P (below M): the apparent place of the phenomenon from B (Problem 71)
- D (on the central arc, below the middle): the true place of the phenomenon on the meridian (Problem 70)
- E (below D): the apparent place from C (Problem 70)
- G (below E): the apparent place from B (Problem 70)
- H (bottom): end of the meridian arc ADH

[Margin: Step 2.]

Therefore, second, in this triangle you will seek the arc CF and the angle BCF; subtract the angle from 180 degrees, and you will have the angle ECF.

[Margin: Step 3.]

With these now obtained, third, in the triangle GBF there are given the side BF, found above, and GF, the visible distance from the fixed star from the place B, and the angle GBF, the same as CBF; seek, therefore, BG, the visible distance from the vertex B.

[Margin: Step 4.]

Fourth, in the triangle CFE, from CF found, the visible distance EF and the angle ECF, seek CE, the visible distance from the vertex C; add BC to it, and subtract the whole BE from the arc BG. There will remain EG, the difference of the parallaxes of altitude; from it, since the visible altitudes of the phenomenon have here been found, the quantity of the parallaxes itself is investigated, according to what was said in chapter 2, problems 8 and 9.

71. Probl. Datis quæ dantur in Problemate 70. cum Diſtantijs viſis Phænomeni ab vtroque vertice, etiam ſi fuerit extra Meridianum, Inquirere Parallaxes Altitudinis, Secluſa interim Refractione, aut per eam correctis diſtantijs. *Problem 71. Given what is given in Problem 70, together with the visible distances of the phenomenon from each vertex, even if it is off the meridian: to find the parallaxes of altitude — refraction meanwhile set aside, or the distances corrected by it.*

[Margin: Problem 71.]

Look at the left-hand part of the figure immediately preceding. In it let there first be, under the same meridian ADH, two places B and C, with the complements of the pole altitude, AB and AC, known, and so the difference BC known. Let the true place of the phenomenon be K, but its apparent place from B be P, and from C, M; and let the distances from the fixed star L be, as seen from B, the arc PL, and from C, the arc ML; and let that fixed star be joined with the pole of the world A, and with the vertex of the place B, by the arcs of great circles AL and BL.

[Margin: Step 1.]

First, then, in the triangle ALB there will be given, from the moment of time and the tables of the fixed stars, as in the preceding problem, the angle LAB, which measures the difference between the right ascension of mid-heaven and that of the fixed star, and AL, the complement of declination of the same fixed

star; and AB, the complement of the pole altitude, is also given. Therefore you will seek the arc BL and the angle ABL; when this is subtracted from 180 degrees, the adjacent angle LBC will be known.

[Margin: Step 2.]

Second, in the triangle LBP, from the side BL found, PL, the visible distance of the phenomenon from the fixed star, and BP, the complement of the visible altitude of the phenomenon, you will investigate the angle LBP; subtract it from the angle LBC found above, and the angle KBC will be left known.

[Margin: Step 3.]

Third, in the triangle LBC, from the sides BL and BC found — BC being the difference of latitude of the places — and the contained angle LBC, seek both the side CL and the angle BCL.

[Margin: Step 4.]

Fourth, in the triangle LCM, from the side CL found, the distance LM — the visible distance of the phenomenon from the fixed star from the place C — and CM, the visible distance of the phenomenon from the vertex, investigate the angle LCM; add it to the angle BCL, and the whole angle KCB will result.

[Margin: Step 5.]

Fifth and finally, in the triangle KCB, in which there are now given the angle KCB, the angle KBC found in the 2nd step, and the side BC, as above, seek the arc BK, that is, the true distance of the phenomenon from the vertex B; subtracted from the visible BP, it will leave the parallax KP known. Again, seek the arc CK, which is the true distance of the phenomenon from the vertex C, and subtract it from the visible CM; for the remainder KM will be the parallax of altitude. Nor will it be difficult, by almost the same method, to investigate the parallaxes from the data already mentioned even when the two places differ in longitude, provided the difference of longitude is known.

72. Probl. Data Latitudine ac Longitudine viſa eodem momento Phænomeni ex duobus terræ locis, atq. adeò data Differentia harum parallaxium ad Eclipticam, & eorum Altitudine polari, cum diſtantia Meridianorum, nec non Obliquitate Eclipticæ ſeu diſtantia Polorum Eclipt. & Aequatoris, & inſuper Momento obſeruationis, & inde loco Eclipticæ Culminante vtrobiq. Inuenire Parallaxin Altitudinis, vtrique loco conuenientem. Secluſa tamen Refractione aut per eam correctis locis viſis. *Problem 72. Given the latitude and longitude of a phenomenon seen at the same moment from two places on the earth, and so given the difference of these parallaxes with respect to the ecliptic; and given the pole altitude of the places, with the distance between their meridians, and also the obliquity of the ecliptic, or the distance between the poles of the ecliptic and of the equator, and moreover the moment of observation, and from it the culminating point of the ecliptic at each place: to find the parallax of altitude belonging to each place — refraction, however, set aside, or the apparent places corrected by it.*

[Margin: Problem 72.]

Chiaramonti gives this problem in the *Antitycho*, Book 1, chapter 7, problem 1, with an example of it in Book 3, chapter 4; and in part in the opuscula *De sede sublunari Cometarum*, part 1, chapter 10, problem 2, and part 2, chapters 20 and 21. It requires, however, the solution of a great many spherical triangles, in which the individual errors are small, but, collected together, they can in the end produce a somewhat larger error than the fineness of parallaxes demands, unless it is handled very accurately.

[Margin: Order of the problems.]

[Margin: Order of the problems.]

Moreover, it cannot be adapted to some cases, as the same Chiaramonti admits in Book 1, chapter 7 of the *Antitycho*, in the annotations. So it did not seem right to send the reader into this labyrinth and wear him out. And indeed the labour is worth sparing, since it can scarcely happen that two observers, in distant places on the earth, should by chance observe the same phenomenon — especially a new one — at the same moment of time, or even on the same day at hours of the same denomination or numbering. If, however, there is an interval of a few hours between their observations, and the proper motion of the phenomenon belonging to that interval is known, the observations can be reduced to the same moment. A clear example of this problem may be seen in our Book 8, Section 1, chapter 19, numbers 11 and 12.

73. Probl. Quanta fingi poſſit Maxima Parallaxis, ac Minima Lunæ Diſtantia à centro Terræ, præter eam, quam Aſtronomi recentiores poſuerunt, ſed compenſata ab æquali Refractione. *Problem 73. How great a maximum parallax, and how small a minimum distance of the Moon from the centre of the earth, can be supposed beyond that which the more recent astronomers have laid down, but compensated by an equal refraction.*

From what was said in Book 4, chapter 14, none of the recent astronomers, beginning with Copernicus, has given the Moon, however near its perigee, whether at the syzygies [Copulae] or at the quadratures, a smaller distance from the centre of the earth than 51 ⅕ terrestrial semidiameters, or a greater parallax than 67′ 6″. To this same Moon, however, Tycho, in volume 1 of the *Progymnasmata*, page 124, assigned a horizontal refraction of 33′, while supposing from his observations a maximum parallax at the quadratures of 65′ 36″ and a minimum distance of the Moon of 52 ⅖ terrestrial semidiameters. Someone might, however, suspect that the Moon's parallaxes are much greater, but unobservable for the reason that they are compensated by an equal refraction, and ask us how great a parallax and refraction of this unknown kind may be supposed without violating physical and astronomical experiments. I shall try to satisfy him with distinct replies.

[Margin: Problem 73.]

[Margin: 1st reply, excluding a parallax greater than 23 degrees.]

First, I reply that it is certain that the difference in density between the ethereal air, or purest air, and our own air, which refracts rays sensibly, is more

like the difference between the densities of water and glass than that between air and water, or between air and glass. For, as we shall say in the next Section, problems 17 and 19, the weight, and hence the density, of glass is to the density of water as 60 to 22, or almost triple; while the density and weight of water are to the weight and density of our air as 427,272 to 1 — which we here ask to be granted us. Granted, no one is so senseless as not to concede that there is a greater ratio between ether and air compared together, and between water and glass together, than between ether and air together and between air and glass compared together. Now the maximum refraction of a ray from water into glass, according to Vitellio [Witelo], Book 10, proposition 8, does not exceed 23 degrees. Therefore the refraction of a ray from the purest air into our own air cannot be supposed greater than 23 degrees; and hence neither can the unknown parallax, compensated by an equal refraction, be supposed greater than 23 degrees. And the smaller anyone thinks the excess of the density of air over that of ether to be, compared with the excess of the density of glass over that of water, the smaller than 23 degrees the refraction and parallax of the Moon he will be permitted to suppose.

[Margin: 2nd reply, excluding a parallax greater than 2° 53′.]

Second, I reply that the Moon's parallax cannot be supposed greater than 2° 53′, and hence, by problem 1, the Moon's distance from the centre of the earth cannot be supposed smaller than 20 terrestrial semidiameters. For at a distance from the vertex of only 10 or 5 degrees, no refraction of the Moon was observed by Tycho in clear air; nor is any observable by anyone, since not even at a distance of 45° from the vertex could any refraction of the Moon be detected by Tycho's people or by others. Therefore whatever refraction anyone may suppose beyond Tycho's — but a refraction compensating an equal parallax — will be wholly hidden, and not made up of a refraction partly known, partly unknown. Now at a distance of the luminary from the vertex of 10 degrees, the refraction from water into glass is no greater than 30′, and at a distance of 5 degrees no greater than 15′, by the experiments of Witelo, Book 10, proposition 8. Nor is it permitted to attribute a greater refraction to a ray passing from the ether into our air than from water into glass, by the first reply. Therefore it is not permitted to attribute to the Moon a greater refraction — and hence not a greater parallax — belonging to a distance of 5 degrees from the vertex, than 15′. To this parallax, by problem 5, there belongs a distance from the centre of the earth of at least 20 terrestrial semidiameters; and to this distance, by problem 2, there belongs a horizontal parallax not greater than 2° 53′. Which was to be shown.

[Margin: 3rd reply.]

Third, I reply: suppose someone contends that the Moon's horizontal parallax is 2° 53′, and that its minimum distance can come down to 20 terrestrial semidiameters, and that the residue from Tycho's parallax of 1° 5′ 36″ up to 2° 53′, that is 1° 47′ 24″, lies hidden because, besides the Moon's horizontal refraction of 33′, there is a refraction unknown to us, of 1° 47′ 24″, compensating

just as much parallax, so that the total refraction, composed of the unknown and the known, is 2° 20′ 24″. Such a person cannot be convicted of falsehood from the stay of the Moon visible above the physical horizon, even when the Moon is on the equator; for whatever the parallax took away from that stay by depressing the Moon would be restored by the equal refraction. And when, in Book 8, Section 1, chapter 15, I said that a distance of the Moon smaller than 47 terrestrial semidiameters cannot be supposed, I nevertheless pointed out, in number 14, that this holds on the supposition of no parallax at all, and of a true stay and motion, not an apparent or visible one.

[Margin: What about eclipses?]

Nor could such a person be refuted from the eclipses of the luminaries. For the size of the earth's shadow with the penumbra, and the apparent diameters of the luminaries, would not vary at all through this parallax, and every difference arising from the excess of this parallax over that granted by the astronomers would be compensated by an equal refraction — granted that the real size of the luminaries would turn out much smaller. And although the pure shadow of the earth, not lit through by refracted rays, would turn out much shorter because of the greater refraction, according to what was said in Book 5, chapter 5, problem 1, yet the smaller distance of the Moon from the earth rendered by this hypothesis would also bring it about that the Moon would sometimes, in the middle of total eclipses, disappear entirely and not be lit even by refracted rays — which, as we taught in the same place, is the office of that pure shadow.

[Margin: What about the increments of refractions?]

Moreover, if the lunar refraction were increased beyond the solar, and consequently the solar beyond that of the fixed stars, there would be a greater difference between their horizontal refractions than Tycho now sets, and one more consistent with the difference of inclinations, which, the smaller it is, produces the smaller refraction, other things being equal. For to Tycho the distance of the Sun is nearly twenty times the distance of the Moon, and that of the fixed stars is to the Sun's as 11 ⅔, or at most twelvefold. Therefore, if the Moon's distance were 20 terrestrial semidiameters, the Sun's would be 400 and the fixed stars' 4800; from which, by problem 2, there would follow a horizontal parallax of the Sun of 8′ 36″ and of the fixed stars of 58″. But as it is, Tycho's horizontal parallax of the Sun is 3′ 7″, and of the fixed stars 15″. The remainder, therefore, would be owed to the compensating refraction; and so to the refraction of the Sun, which he sets at 34′, 5′ 29″ would have to be added, and to that of the fixed stars, which he sets at 30′, 43″. The total horizontal refraction of the Moon would thus be 2° 20′ 24″, of the Sun 39′ 29″, and of the fixed stars 30′ 43″. Finally, although the increments of the refractions are greater, and grow steeply near the horizon, while those of the parallaxes are greater near the vertex — so that an addition of refraction as large as the parallax would not sit very neatly at every altitude — yet this incongruity would not be so great that it could evidently be convicted of falsehood. For, with the Moon's distance set at 20 semidiameters and Tycho's lunar refractions added, at the

altitudes from the horizon written below there would follow lunar parallaxes and refractions as in the small table below, with their differences.

Sequitur Tabula Parallaxium ac Refractionum Tychonicarum, deductarum ex Lunæ à terra diſtantia ſemidiametrorum terreſtrium 20. iuxta diuerſas altitudines ab Horizonte. *There follows a table of the Tychonic parallaxes and refractions, derived from a distance of the Moon from the earth of 20 terrestrial semidiameters, at various altitudes from the horizon.*

[Margin: Order of the problems.]

[Margin: Order of the problems.]

The Moon's distance set at 20 semidiameters	Moon's altitude 0°	Moon's altitude 5°	Dif-fer-ence	Moon's altitude 80°	Moon's altitude 85°	Dif-fer-ence
Parallax of the ☾ at this distance	2° 53′ 0″	2° 52′ 10″	0′ 50″	29′ 52″	14′ 56″	14′ 56″
Tychonic parallax	1° 5′ 36″	1° 5′ 26″		11′ 37″	5′ 50″	
Hidden refraction and parallax	1° 47′ 24″	1° 46′ 44″		18′ 15″	9′ 6″	
Tychonic refraction	0° 33′ 0″	0° 14′ 20″		0′ 0″	0′ 0″	
Total refraction	2° 20′ 24″	2° 1′ 4″	19′ 20″	18′ 15″	9′ 6″	9′ 9″

There would therefore be a greater difference of the parallaxes near the vertex, namely 14′ 56″, than near the horizon, namely 50″. Conversely, the difference of the refractions would be greater near the horizon, namely 19′ 20″, than near the vertex, namely 9′ 9″. Therefore the hypothesis already mentioned could not be convicted of falsehood from the increments or decrements required by the refractions and by the parallaxes either. Let the following conclusions stand, then.

I. CONCLVSIO. *Conclusion 1.*

A greater parallax of the Moon than 2° 53′, and a smaller distance from the earth than 20 terrestrial semidiameters, cannot reasonably be supposed. This is clear from what was said in reply 2, which presupposes the first.

II. CONCLVSIO. *Conclusion 2.*

If anyone supposes the Moon's minimum distance from the centre of the earth to be not greater than 20 terrestrial semidiameters, and its horizontal parallax not smaller than 2° 53′, and asserts that whatever exceeds the parallax hitherto granted by the astronomers is due to an equal refraction of the Moon, compensating the parallax, he cannot be convicted of falsehood either from the apparent stay of the Moon above the horizon, or from the eclipses of the luminaries, or from the increments or decrements of the refractions.

This is established from what was said in reply 3, and for anyone who examines the phenomena already mentioned, as we have done. But if anyone

shall evidently convict this hypothesis of falsehood on other grounds, he will have deserved well of astronomy, and I, if I am still alive, shall hear him most gladly.

74. Probl. Quanta ob Refractiones auctas & parallaxium compenſatrices, fingi poſſet maxima Parallaxis & minima Diſtantia à centro Terræ, in Sole, Saturno, ac Fixis. *Problem 74. How great a maximum parallax and how small a minimum distance from the centre of the earth could be supposed in the Sun, Saturn and the fixed stars, on account of refractions increased and compensating the parallaxes.*

[Margin: Problem 74.]

From what was said in problem 73, in reply 2, it is not permitted to suppose a lunar parallax greater than 2° 53′, nor a distance smaller than 20 terrestrial semidiameters. But from what was said in Book 3, chapter 7, on the distances of the Sun; in Book 4, chapter 14, on the distances of the Moon; in Book 6, chapter 7, on the distances of the fixed stars; and in Book 7, Section 6, chapters 2 and 3, on the distances of Saturn from the earth, the Sun's distance is to the Moon's, for Copernicus, Tycho and many of their followers, as 20 to 1; for Kepler as 61 ⅗ to 1; for us as 140 ⅖ to 1. Saturn's distance, moreover, is to the Sun's distance, for all those already mentioned and for us, nearly as 10 to 1, and that of the fixed stars a little greater; so, for a judgement about the fixed stars, it will suffice to decide about Saturn. From these two elements follow the minimum distances given below and, by problem 2, the maximum parallaxes that could be supposed.

In the hypothesis of the authors	Distance of the ☾ from the centre of the earth (terrestrial semidiameters)	Distance of the ☉ (terrestrial semidiameters)	Distance of ♄ (terrestrial semidiameters)	Horizontal parallax of the ☾	Horizontal parallax of the ☉	Horizontal parallax of ♄
Tycho's	20	400	4000	2° 53′ 0″	8′ 34″	0′ 52″
Kepler's	20	1256	12560	2° 53′ 0″	2′ 34″	0′ 17″
Ours	20	2856	28560	2° 53′ 0″	1′ 15″	0′ 9″

75. Probl. Quanta fingi poſſit Refractio horizontalis Lunæ, Solis, Saturni, ac Fixarum ſupra Tychonicam. *Problem 75. How great a horizontal refraction of the Moon, the Sun, Saturn and the fixed stars can be supposed beyond Tycho's.*

[Margin: Problem 75.]

I reply briefly that as great a refraction can be supposed as is the aggregate of Tycho's horizontal refraction and of the horizontal parallax that can be supposed beyond Tycho's horizontal parallax. Therefore, from what was said about parallaxes in Book 3, chapter 8, Book 4, chapter 14, and Book 7, Section 6, chapter 7, and from Tycho's refraction — ☾ 33′, ☉ 34′, Saturn and, nearly, the fixed stars 30′ — we have constructed the following small table.

In the hypothesis of the authors	☾	☉	♄ and ✶
Tycho's	2° 20′ 40″	39′ 27″	30′ 24″
Kepler's	2° 22′ 19″	35′ 32″	30′ 9″
Ours	2° 19′ 4″	34′ 46″	30′ 5″

CAPVT XII.

Chapter 12.

De Diſcrimine Parallaxeos veræ ac per ſe Senſibilis à falſa Parallaxi, vel inſenſibili. Vbi Theoremata 20. recondita ad diſcretionem Parallaxium, profundioremque earum notitiam ſpectantia. *On the distinction between a parallax that is true and sensible in itself, and a false or insensible parallax. With 20 recondite theorems bearing on the discrimination of parallaxes and on a deeper knowledge of them.*

[Margin: What is a true parallax, sensible in itself?]

A true parallax, sensible in itself, is one that arises from a twofold interval, one between the centre of the world and the phenomenon, when each is so great that the difference between the true and the apparent place of the phenomenon can be evidently perceived by some people with some fineness of instruments or observations; and so it is called sensible in itself, granted that, by accident, [Margin: Order of the theorems.]

it could not be perceived by some one observer taken [?] on his own. Such is the parallax of the Moon observed from the surface of the terrestrial globe.

[Margin: What is a parallax insensible in itself?]

A parallax insensible in itself is one that arises from so small an interval between the centre of the world and the observer, or between the centre of the world and the phenomenon, that no observers, even watching from different places, could evidently perceive it with any fineness of instruments or observations. Astronomers judge the parallax of the fixed stars to be of this kind, because of their enormous distance from the centre of the Earth; and the Moon's own parallax would be of this kind if it were observed at about one mile from the centre of the world.

[Margin: How great can the fineness of astronomical observations be?]

How far that evident fineness extends, however, the authors do not agree. For Tycho, in volume 1 of the *Progymnasmata*, p. 516, says that an instrument which reliably yields a difference of 48″ must be very large; and yet on p. 425 he had granted his sextant a precision down to 16″; and in the *Mechanica Astronomica*, pp. 10 and 13, he affirms that his larger quadrants show each ten seconds. Kepler, in the *Epitome of Copernican Astronomy*, Book 4, p. 493, denies that the diligence of any craftsmen is to be trusted to a fifth part of one minute, that is, to 12″. Martinus Hortensius, in his dissertation with Gassendi on Mercury seen under the Sun, denies that a difference of 30″ is evidently observable — a view from which Lansbergen rightly departed in the *Uranometria*, Book 3,

element 7. Galileo, in the third Dialogue on the two systems of the world, p. 287 of the Latin, thinks that instruments could be made whose side would span 30 or even 50 miles, so that each degree would contain a mile, each minute 50 cubits, and each second would subtend a chord of nearly a cubit. But it is one thing that the arcs described from the centre of an instrument can be so divided and subdivided as to show individual seconds with a clear distinction; it is another that the sights can be so made and fitted that, in using the instruments, we can be certain that no possible error exceeds one or two seconds. To us, given the skill and fineness we have so far been able to attain in observing, it seems that this evidence cannot go below 5″; and so we confidently declare a parallax insensible in itself to be one that is less than 5″.

[Margin: What is a false parallax?]

A false parallax is one that either is greater but appears smaller because of an unknown refraction, or is in reality nil, or not sensible in itself to any observers — that is, less than 5″ — and yet appears greater because of a varied motion, by which the phenomenon seems to recede from the same stars, or from the fixed points of the highest heaven. And so, although it arises from motion or from refraction, it is believed to arise from the interval between the centre of the world and the observer, or the phenomenon. Conversely, a parallax can be true and sensible in itself, at least to several people observing the same phenomenon at the same moment in different places on the Earth, and yet not be perceived by some one observer, at whose eye a motion or a refraction keeps the phenomenon all day, or all night, under the same points of the heavens, or at the same distance from the same fixed points.

[Margin: Summary of the foregoing.]

Thus a true parallax could be called, by a causal definition, one that arises from the interval between the centre of the world and either the observer, or the phenomenon, or both; a false parallax, one that arises not from an interval but from motion or refraction, or from illusions of the visible species. Next, a parallax sensible in itself is one that is not less than 5″; an insensible one, one that is less.

[Margin: A collection of axioms and theorems on parallaxes.]

With these laid down — besides the five common axioms on parallaxes given in Book 1, chapter 35, the 15 axioms given to the reader, by then better instructed, in Book 8, Section 1, chapter 17, and the parallactic theorems serving solar eclipses that I gave in Book 5, chapter 10 — there remain some others, bearing on the parallaxes of new phenomena and on a deeper doctrine of them, which we deliberately deferred to the last chapter of this Section; they are to be set out briefly.

1. Theorema. Ignorata Lunæ Parallaxi, ſciri euidenter & exactè nequeunt Diſtantiæ omnium Planetarum ac Fixorum Siderum à Terra, & eorumdem Parallaxes. *Theorem 1. If the Moon's parallax is unknown, the distances of all the*

planets and fixed stars from the Earth, and their parallaxes, cannot be known evidently and exactly.

[Margin: Theorem 1. The Moon's parallax is the foundation of the other parallaxes and of the celestial intervals.]

This is sufficiently proved by induction from all the methods of investigating the parallaxes and distances of the stars that we have so far given, or that the most skilful of the astronomers have devised. For all of them presuppose either the Moon's parallax, from which we derive its distance, and from this the distances of the Sun and the planets, and from the distances their parallaxes; and finally from Saturn's distance we infer the distance of the fixed stars, by the method described in Book 4, chapter 11, together with what we said in Book 3, chapter 7. Or they presuppose the altitude of the pole, which cannot be known exactly except through the true altitudes of the Sun or of the fixed stars; and these cannot be known unless it is established whether they have a parallax, and how much, so that the true altitude may be distinguished from the apparent one. A parallax, in turn, cannot be known except through the distance, which depends either on the Moon's parallax or distance, or on declinations, or right ascensions, or longitudes, or latitudes, or other true places or motions of the stars; and these cannot be distinguished from the apparent ones unless the parallax or distance being sought through them is already presupposed. Moreover, whenever the distance of a phenomenon from fixed stars, and the declination of the fixed stars, is used, astronomers presuppose that the fixed stars have no sensible parallax, and so that their observed and apparent declinations etc. are the same as the true ones; and they gather that the fixed stars lack parallax from their distance as estimated through Saturn's distance. Through the prosthaphaereses, however, and other indications, the proportions of the intervals that lie between the planets can be known.

2. Theorema. Per Obſeruationes Lunæ in vnico Terræ loco factas, cui Luna nunquam tranſeat per verticem, non poteſt ſciri euidenter & exactè Parallaxis Lunæ; etiam ſi Refractio nulla interueniret. *Theorem 2. From observations of the Moon made at a single place on the Earth, through whose zenith the Moon never passes, the Moon's parallax cannot be known evidently and exactly, even if no refraction intervened.*

[Margin: Theorem 2. The lunar parallax is hard to observe.]

For the Moon's parallax cannot be known unless one knows the difference between the Moon's true place — that is, the place reckoned from the centre of the Earth or of the world — and its apparent place, that is, the place observed from the surface of the Earth. This difference, if it can be known evidently in any way, is known chiefly from the observed altitude of the Moon and from a sure prior knowledge of its true altitude. But the true altitude cannot be known unless its true declination and latitude are known beforehand; and to investigate these, the Moon's latitude must be observed, independently of parallax, by someone for whom the Moon appears in the zenith — as we showed,

in Book 4, chapter 12, was nearly done by Ptolemy — or by someone who deduces it from some lunar eclipses, but with the true motion of the Moon, of the Sun and of the lunar nodes presupposed, as we said in the same place. And that motion cannot be evidently separated from the apparent one unless it is established from others' observations that the lunar revolutions behave in the same way in other horizons too (granted that it could be separated from the mean motion), as will be confirmed below from what is to be said about others' observations of phenomena. Granted, if the ecliptic coincided with the horizon, and the apparent motion of the phenomenon were parallel to both, the apparent and the true motion would be the same as regards longitude. Add to this the distance of the equator from the zenith, or the Sun's greatest declination, which is not exactly known unless the parallaxes of the Sun or of the fixed stars are known beforehand; and anyone who denied that these are as small as that, or nil, no one could convict of evident falsehood from observations made at one place on the Earth, since all the phenomena could be saved with respect to one place on the Earth even if a greater parallax were assumed than astronomers have so far assumed. For it cannot be settled with certainty that the fixed stars themselves move only with the motion of the primum mobile, or that they have no proper motion sensible in itself within a few days or years, unless their observations, taken at different places on the Earth, are compared. And so neither Tycho, by his observations alone made at Uraniborg or at Prague, nor any other astronomer outside the torrid zone, or outside the tract over which the Moon sometimes becomes vertical

[Margin: Order of the theorems.]

[Margin: Order of the theorems.]

to the observer, could determine the Moon's parallaxes exactly and evidently. So that you do not think I said this from an itch to carp at Tycho, hear Kepler, in a letter of 1601, written on the Kalends of June (1601 June 1) to Giovanni Antonio Magini, which we have in the supplement to Magini's own *Ephemerides*, p. 253, writing as follows:

[Margin: Kepler's testimony on the uncertainty of the Moon's parallax.]

"As I wish to conclude, there occurs to me what I wanted to write almost in the first place. The theory of the Moon causes Tycho a great deal of difficulty. It seems to me that it must start from the parallaxes that occur because of the sensible distance between the centre and the surface of the terrestrial globe. But the doctrine of parallaxes is confused by the latitudes of the Moon. There would therefore be need of a prior knowledge of the latitudes: each depends on the other" — understand: if the Moon never appears to you in the zenith, or if you do not use the revolutions of the lunar motions, or altitudes of the Moon observed at different places on the Earth. He goes on, however: *"I have therefore considered how the parallax might be investigated by observing, without knowledge of the latitude. Two ways occurred to me. One, if on the same day the Moon is observed once high and once lower, but outside the bound of refractions, when it is near the limits, where the latitude changes little within two hours. It is expedient that this also be done at the beginning of Cancer,*

where the declination too changes little. But when on the same day, after its meridian altitude, the Moon becomes sensibly lower, it acquires a parallax of longitude, which would have to be known beforehand, since it is what is being sought. The other way: that the Moon be observed at distinct times when it is in the nonagesimal degree, at the same limit, at the same distance from the Sun, once higher and once lower; but these three conditions rarely concur. Add that for every single minute in this observation of the altitude, an error of one semidiameter of the Earth is committed, and few of these are contained in the Moon's eccentricity. So, thirdly, I take refuge in this: I ask you to observe the Moon as often as you can in the nonagesimal degree, and to share the observations with us in Germany, together with a most exact altitude of the pole of Bologna. I will see to it that you in turn have our observations. In this way the Moon will sometimes be observed in both places at once, and so the altitudes will become known at various places of the anomaly. For Bohemia and Italy lie, in good part, on the same meridian."

You see him taking refuge in observations made at different places on the Earth, although he indicates that he would not have used them accurately enough, since he thinks that Prague and Bologna are under the same meridian, and that an observation of the Moon's altitude in the nonagesimal apparent to each observer is enough, as though it were then on their meridian; or that the identity of the time at which the observation was made in each place could be known without the places of the Moon, the Sun or the fixed stars, which cannot be known without a prior knowledge of the parallax, if only to exclude it. But since all of Tycho's lunar observations were made before the year 1601, as is plainly established from the appendix to volume 1 of the *Progymnasmata*, it is also clear that neither Tycho nor his followers could then have used the observations that Kepler wished to be made in future — wished, and no more. And in hunting the parallaxes Tycho always used a latitude of the Moon that he could not know through his own observations alone, and it depended on those observed by Ptolemy, as is established from what was said in Book 4, chapters 12 and 14. The same Kepler, however, in the *Astronomiae Pars Optica*, p. 313, had first set out a method of investigating the parallaxes of the Moon and of the stars through different observations taken at different places on the Earth's surface; and yet he added another, through observations made at one place, but supposing that the Moon's true altitude can be known by astronomical calculation, so that it can be compared with the observed one.

3. Theorema. Neq. Parallaxis ſine præcognitione Refractionis aut diſtantiæ à terra, neq. Refractio ſine præcognitione Parallaxeos; aut diſtantiæ à terra, ſiue in Luna, ſiue in alio quouis ſidere, poteſt exactè atq. euidenter ſciri per vllas obſeruationes naturales. *Theorem 3. Neither parallax without prior knowledge of the refraction or of the distance from the Earth, nor refraction without prior knowledge of the parallax or of the distance from the Earth, can be known exactly and evidently by any natural observations, whether in the Moon or in any other star.*

[Margin: Theorem 3. The mutual dependence of parallaxes and refractions.]

For every parallax of any star whatever, if it is to be known independently of the distance from the Earth, presupposes that star's parallax in the circle of altitude to be known beforehand — granted that the parallax of altitude could be confirmed a posteriori through a presupposed parallax of another kind, namely of declination or of right ascension.

[Margin: Proof of the first part, on parallax.]

But the parallax of altitude itself cannot be known except by comparing the star's true altitude with its apparent or observed altitude; and the difference between the true and the observed altitude does not give the correct parallax unless it is certain from elsewhere that no refraction intervenes, or how much refraction must be subtracted from the observed altitude (since refraction visibly raises the star), so that the observed altitude may be corrected and the corrected one compared with the true. Nor is it enough to suppose the observation made outside the hazard of sensible refractions; for that very thing must be known beforehand — namely, how great the apparent altitude of the star is at which it has no sensible refraction. That is why, in all the problems of chapters 10, 11 and 12 that can be solved by observations taken at one or at several places on the Earth, I deliberately warned that the parallax of altitude could be investigated provided there is no refraction, or provided that the apparent altitudes, or the apparent distances from fixed stars or points of the heavens, or the apparent places, are corrected by it. Accordingly Tycho prudently defined the Moon's parallaxes with the refraction subtracted from the apparent altitudes, as I showed by examples in Book 4, chapter 14.

[Margin: Second part, on refraction.]

Refraction itself, moreover, cannot be known unless the refraction in the circle of altitude, or vertical circle, is known; and this cannot be known if you know beforehand neither the parallax of the same star nor its distance from the centre of the Earth, as is proved by sufficient induction. For the refraction that intervenes and apparently raises the star is either equal to the parallax, which apparently depresses it, or greater or less than the parallax. If it is equal, the observed star will appear no higher than its true place; and one cannot conclude that it therefore appears in its true place except because the parallax is known beforehand, by force of which it ought to appear less high, or more depressed, than its true place; and so it is concluded that the whole parallax is corrected by an equal refraction and the star restored to its true place. If the refraction is greater than the parallax, as is supposed in the Sun, the difference between the true altitude computed from tables and the apparent altitude observed with instruments will not give the whole refraction; the parallax will have to be added to it, because the refraction first corrects the parallax and cancels its effect, and then apparently raises the star above its true place, as I shall show with Tycho in the next section, Problem 24. Finally, if the refraction is less than the parallax, it does not emend the whole parallax, nor does it make the star appear absolutely higher than it really is, but only comparatively, relative to

the depression that would otherwise result by force of the parallax; that is, it makes the star appear not as far from the zenith as it would appear if parallax alone intervened. Therefore, for the refraction to become known, one must in such a case add the parallax to the star's true distance from the zenith, so as to have the apparent distance, that is, the distance that would be seen by force of the parallax; for the difference between this and the distance from the zenith observed with instruments will be the refraction, as we shall show in Problem 26 of the next Section. And do not say that the refraction can be known if the height of the refractive air is known, together with the Earth's semidiameter and the angle of inclination; for it will be clear from what is to be said in the next section, from Problem 44 on, that these givens are not enough without the star's distance or parallax. On the contrary, we shall be compelled to hunt out the very height of the refractive air, and the inclination, from a refraction known beforehand.

4. Theorema. Omnibus tamen penſitatis, non licet fingere maiorem parallaxin Lunæ Gradib. 2. 53′. Solis 1′. 15″. & Saturni 9″. Nec Refractionem in Zonis calida & temperatis, ſerenoq. cælo, maiorem Lunæ Gradib. 2. 19′. 4″. Solis, 34′. 46″. & Saturni 30′. 5″. in noſtra quidem hypotheſi, nec multò maior in Tychonica, aut Kepleriana. *Theorem 4. All things weighed, however, one may not suppose a parallax of the Moon greater than 2° 53′, of the Sun greater than 1′ 15″, or of Saturn greater than 9″; nor, in the hot and temperate zones and with a clear sky, a refraction greater than 2° 19′ 4″ for the Moon, 34′ 46″ for the Sun, and 30′ 5″ for Saturn — in our hypothesis, at least; nor much greater in Tycho's or in Kepler's.*

[Margin: Theorem 4.]

All this has been sufficiently proved above, from Problem 73 to 75. We are not speaking, however, of extraordinary refractions at places or times at which thicker vapours are raised.

[Margin: Order of the theorems.]

[Margin: Order of the theorems.]

5. Theorema. Diſtantia viſa Phænomeni ab aliquo ſidere, poteſt eſſe veræ diſtantiæ æqualis, & maior, aut minor quàm vera diſtantia, quamuis magnam Parallaxim altitudinis habeat. *Theorem 5. The apparent distance of a phenomenon from some star can be equal to the true distance, and greater or less than the true distance, however great a parallax of altitude the phenomenon has.*

[Margin: Does parallax increase or diminish the distance from the stars?]

[Margin: Theorem 5.]

This is demonstrated by Scipione Chiaramonti in Book 1 of the *Antitycho*, chapter 8, proposition 1. But without a further diagram it will become evident on an astronomical globe, if you suppose a planet to be at the beginning of Cancer under the meridian, and a fixed star in the equator having no parallax, and the planet to have so much parallax that it appears now in the equator, now

at the beginning of Capricorn, now below the tropic of Capricorn. For in the first case the apparent distance will be less than the true, in the third greater, and in the second equal to the true. These cases, transferred onto a diagram, will at once supply the middle term of the demonstration.

6. Theor. Motus viſus Phænomeni per idem planum euntis, ſed à verticalis plano diuerſum, inferior eſt motu vero. *Theorem 6. The apparent motion of a phenomenon that goes through one and the same plane, but a plane different from that of a vertical circle, is lower than its true motion.*

[Margin: Theorem 6.]

This is against Kepler in the *Hyperaspistes*, p. 13, who concedes it only of the diurnal motion of the primum mobile; but Chiaramonti demonstrates that it is true of the phenomenon's proper motion as well, in Book 1 of the *Antitycho*, chapter 11, and in his *Apologia* for the same work, Book 1, chapter 6. For when the line of motion does not pass through the observer's zenith, the apparent place is always more remote from the zenith than the true place of the phenomenon; and so the apparent path of the phenomenon is always lower than the path of its true motion.

7.Theor. Phænomenon parallaxin habens continuam & ſenſibilem, ſi vero motu deſcribit circulum maximum, à verticali circulo diuerſum; viſo motu non deſcribit circulum maximum. *Theorem 7. A phenomenon having a continuous and sensible parallax, if by its true motion it describes a great circle different from a vertical circle, does not describe a great circle by its apparent motion.*

[Margin: If the true motion is along a great circle, what of the apparent motion?]

[Margin: Theorem 7.]

Chiaramonti demonstrates this in Book 1 of the *Antitycho*, chapter 11, proposition 2; and Kepler admits it in the *Hyperaspistes*, p. 18, and after him Cabæus [Niccolò Cabeo], on *Meteorologica* 1, text 37, question 1. The reason is that the only great circles on a sphere are those that bisect each other at two opposite points, by proposition 11 of the first book of Theodosius's *Spherics*; but the apparent path of the phenomenon in question is always lower than the path of its true motion, by Theorem 2, and the one never cuts the other. Otherwise, if it intersected it, whether at the zenith and nadir or at any other points, the parallax would cease there and would not be continuous; for those points of intersection would be common to the circle of the true and of the apparent motion, and the true and the apparent place would be the same there.

8.Theor. Phænomenon parallaxin habens continuam & ſenſibilem, ſi motu viſo deſcribit circulum maximum à verticali diuerſum; motu vero non deſcribit circulum maximum. *Theorem 8. A phenomenon having a continuous and sensible parallax, if by its apparent motion it describes a great circle different from a vertical circle, does not describe a great circle by its true motion.*

[Margin: If the apparent motion is along a great circle, what of the true motion?]

[Margin: Theorem 8.]

This is the converse of the preceding theorem and is proved in the same way. Otherwise, if both circles — that is, the circle of the apparent and that of the true motion — could be great circles, they would intersect each other at two opposite points, by proposition 11 of the first book of Theodosius's *Spherics*; and if they intersected there, the parallax would not be continuous, etc.

9. Theor. Poteſt Phænomenon magnam & continuam parallaxim habere, & tamen viſo motu deſcribere circulum in ſphæra maximum: & viciſſim. *Theorem 9. A phenomenon can have a great and continuous parallax and yet by its apparent motion describe a great circle on the sphere; and conversely.*

[Margin: An apparent motion along a great circle admits a great parallax.]

[Margin: Theorem 9.]

This is against Kepler in the *Hyperaspistes*, p. 18, but it is Chiaramonti's, in the *Apologia* for the *Antitycho*, p. 53, and Cabeo's, on *Meteorologica* 1, text 37, question 1. Kepler's words are: *"For it is true that, if the comet by its true motion held to the path of a great circle, it did not hold to it by its apparent motion, since it produced a parallax. But with less circumlocution this too is true — and Scipio has not yet refuted it — that if by its apparent motion it marked out an arc of a great circle, it also marked one out by its true motion, because in that way a sensible parallax is taken away from it."* Let those little words *"because in that way"* be noted: for they signify that from an apparent motion along a great circle there necessarily follows the absence of a sensible parallax. But, as Chiaramonti rightly says in the place cited, *"there is no such necessity"*, which he thinks he has shown in Book 1 of the *Antitycho*, chapter 11; I, however, show it as follows. It is not contradictory that some phenomenon should move by its true motion continuously, either along one and the same circle parallel to the equator, or along various circles with a winding course, but in such a way that — by reason of its distance from the Earth in the first case, or of its winding path — it always undergoes just as much parallax as is precisely required for it always to appear in the equator. For just as the parallaxes of altitude continually decrease from the horizon up to the meridian, so too the arcs of the circles of altitude between the equator and that same parallel decrease from the horizon to the meridian. Therefore no contradiction can be shown, from this or from anywhere else, why the distance of the phenomenon from the centre of the Earth could not vary in such a way that it always goes through the plane of the same parallel, and yet the parallactic arc is as great as the arc between the parallel and the equator.

Moreover, because Tycho, in volume 2 of the *Progymnasmata*, from p. 92 on, inferred from the apparent motion of the comet of the year 1577 along a great circle that it was above the Moon, and so had no parallax greater than the Moon's, Kepler judged it to follow from this that the comet had no sensible

parallax; indeed, that this was why Tycho investigated, and why one must investigate, whether a phenomenon goes visibly along a great circle — so as to know from this whether it has a parallax — whereas Chiaramonti had said that the parallax must be examined first, so as to know along what circle the comet went. So Kepler, in the *Hyperaspistes*, p. 12, said:

[Margin: Kepler's perverse order in examining parallax.]

"Hear me, Peripatetic: we attend to whether the comet proceeds in a great circle so that we can decide about its parallax; we seek the parallax so that from it we may learn its place in the diameter of the world as well; the kind of circle comes first in this inquiry, it does not follow."

[Margin: Cabeo judges rightly, but does not defend Tycho adequately.]

Father Cabeo, indeed, judged rightly when he teaches that, with a sensible parallax already presupposed — the Moon's, say — it is rightly inferred that if the body described a great circle by its apparent motion, it does not describe a great circle by its true motion. But he does not rightly defend Tycho against Chiaramonti when he thinks that Tycho, having first presupposed an insensible parallax of the comet, inferred that if it described a great circle by its apparent motion, it also described a great circle by its true motion. For Tycho, in volume 2, first dealt with motion along a great circle from p. 92 to p. 103, deducing from it that the comet had been above the Moon, and then, from p. 110, began to deal with parallax.

10. Theor. Quamdiu non conſtat Vllane, aut Senſibiliſne ſit an inſenſibilis Parallaxis; non poteſt certò conſtare, ex ſolo motu viſo an Phænomenon feratur per circulum maximum motu vero. *Theorem 10. As long as it is not established whether there is any parallax at all, or whether it is sensible or insensible, it cannot be established with certainty from the apparent motion alone whether the phenomenon is carried along a great circle by its true motion.*

This proposition is against Kepler, whose words from the *Hyperaspistes*, p. 12, I reported a little earlier; but it is Chiaramonti's, in Book 1 of the *Antitycho*, chapter 11, proposition 10, and it follows from our Theorems 8 and 9. Chiaramonti explains himself, moreover, in the *Apologia* for the *Antitycho*, p. 33: namely, that if the parallax is unknown, the kind of circle along which the comet goes by its true motion is unknown. Granted, in the *Antitycho*, by more obscure wording, he gave Kepler occasion to suspect that in Chiaramonti's opinion the parallax had to be known beforehand, before the inquiry into the parallax — which would be to argue in a vicious circle, and to overturn the whole of astronomy from its foundations, as Kepler said on p. 12 of the *Hyperaspistes*.

[Margin: Order of the theorems.]

[Margin: Order of the theorems.]

[Margin: Motion along a great circle alone is not a necessary argument for the comet's distance.]

11. Theor. Ex ſolo Phænomeni per circulum maximum motu quamuis regulari ſeu vero, ſeu viſo, non poteſt neceſſariò deduci, Vtrùm illud ſit ſupra [?]n infra Lunam, ſeu vtrùm minorem Lunari, an maiorem Parallaxin habeat. *Theorem 11. From a phenomenon's motion along a great circle alone — however regular that motion may be, and whether it is true or apparent — it cannot necessarily be deduced whether the phenomenon is above or [?] below the Moon, or whether it has a parallax smaller or greater than the Moon's.*

[Margin: Theorem 11.]

For there can be some mobile body in the elementary region that is nevertheless carried with a true motion along a great circle, that is, along the plane of a great circle. Thus balls fired from bombards or fire-spewing tubes travel in a single plane, in such a way that in their descent they are always carried towards the centre of the Earth; and the pendulum [?], which we treated in Book 2, chapter 20, can be set swinging in that same plane and yet, by the inclination of its weight, tend as far as it can towards the centre of the Earth; and finally, on walls erected perpendicular above the horizon and directed towards the centre of the Earth, motions can take place along their planes. Now planes that intersect one another at the centre of the Earth lie in the plane of a great circle of the Earth, by the 6th of the first book of Theodosius' *Spherics*. And it is admitted that this kind of motion, along a great circle, can belong to celestial bodies. Nor, on the other hand, is it inconsistent for an elementary body to be kept to a great circle in its apparent motion, as we said in Theorem 9; and still less is this inconsistent for celestial bodies. Therefore these motions stand indifferently towards bodies below and above the Moon. Tycho's argument, then, by which he inferred from the comet's course along a great circle that it had been above the Moon, is not a necessary one, as we showed at greater length in Book 8, Section 1, chapter 11, number 9.

12. Theor. Ex eo quòd Phænomenon moueattur motu proprio tardiùs quàm Luna, non ſequitur neceſſariò, illud eſſe ſupra Lunam. *Theorem 12. From the fact that a phenomenon moves with its proper motion more slowly than the Moon, it does not necessarily follow that it is above the Moon.*

[Margin: Theorem 12.]

We showed all this in Book 8, Section 1, chapter 11, number 6, where we also showed that Tycho in the end, and Kepler, acknowledged that there is nothing in this argument but mere probability; and against it, as we showed, arguments no less probable, drawn from the motion of comets, are available for a sublunary place of the comet. Granted, on this ground Tycho tried to show that the comet of the year 1577 was higher than the Moon, because, judged by its distances from the stars of Pegasus, it appeared in three hours to have fallen behind the motion of the primum mobile by 3′ 12″, whereas in those hours the Moon, seen at the same longitude and latitude, would have fallen behind it by 15′.

13. Theor. Ex ſola Phænomeni perſeuerantia ſub eâdem ſtella Fixa, vel in eâdem diſtantia ab ijſdem Fixis punctis vel ſtellis obſeruatâ in vnico terræ loco; non poteſt certò colligi Aequalitas in Parallaxi cum Fixis, ſeu Carentia ſenſibilis Parallaxis, aut merus motus primi mobilis; Poteſt autem, ſi obſeruata ſit ex quibuslibet pluribus terræ locis eodem tempore, ſaltem ſecluſa Refractione. *Theorem 13. From a phenomenon's persistence alone under the same fixed star, or at the same distance from the same fixed points or stars, observed at a single place on the Earth, one cannot infer with certainty an equality of parallax with the fixed stars, or the lack of a sensible parallax, or a mere motion of the primum mobile. One can, however, if it is observed from any number of places on the Earth at the same time — at least when refraction is set aside.*

[Margin: Theorem 13.]

[Margin: An argument that is not necessary for the lack of parallax.]

This proposition, as to its negative part, is directed against many in Tycho, *Progymnasmata* volume 1 — namely Hájek, p. 515; Fabricius, 528; Peucer, 539; Mästlin, 545; Digges, 678; Camerarius, 694, these being Tycho's pages — and against Tycho himself, pp. 416 and 515, and in the *Epistolae*, p. 18, and against Longomontanus in the Appendix to the *Astronomia*, p. 32. These concluded that the new star of Cassiopeia of the year 1572 was necessarily very close to the fixed stars, or even had no sensible parallax, because at both meridian altitudes it kept the same visible distance both from the same fixed stars and from the pole of the world — each of them, indeed, relying on the observations of a single place on the Earth. Let Tycho be heard, standing for all the rest, p. 416: *"Both above and below the pole I saw exactly the same arrangement of the stars relative to one another, and the interval precisely the same. This certainly could not have happened if this new star had been subject to any trace of parallax, even the smallest."*

[Margin: Of Tycho, Longomontanus, Hájek,]

[Margin: Fabricius, Peucer, Mästlin, Digges, Camerarius.]

And on p. 515, speaking of the various syllogisms of Thaddaeus Hájek, he says: *"As regards the first syllogism, there is no doubt that phenomena having no parallax necessarily exist above it. So too, what he brings in [?] in the second argument, from the star's distance from the neighbouring fixed stars and from the pole, equal everywhere and always the same, proves sufficiently that it lacked all parallax"*; and shortly after: *"Indeed, if it constantly kept the same interval from the neighbouring fixed stars, however it revolved with the motion of the universe, the consequence will be to admit neither a proper motion of its own nor any motion that arises accidentally by reason of parallax."*

[Margin: The first part of the theorem is proved.]

But there is no such necessity: both from what was demonstrated by Chiaramonti in Book 1 of the *Antitycho*, chapter 10, proposition 14, conceded by Tycho's *Hyperaspistes* on p. 111, and demonstrated by us in Book 8, Section 2, chapter 6; and because a phenomenon of this kind could be very close to the Earth, and so have a large parallax, and have some true proper motion, at least in declination, and yet be held by some internal or external directing force

always [?] in the same line of the apparent place under the fixed stars as long as it is above the visible horizon of one observer — while, if it set below that horizon, it would move with other motions forward and back in longitude and sensibly vary its distance from the fixed stars. For imagine, for the sake of example and clarity, that from a single small loop on the surface of the Earth, where the eye of one observer is, a thread that has first been passed through a luminous globe is drawn out as far as the pole star, and is held stretched without any bending by the force of some Intelligence at the centre of the pole star, and is so carried round with it about the pole: would that globe not always appear under the pole star, though it stood only one semidiameter of the Earth from the eye? Imagine the same with respect to a setting star, say the Dog Star; or — as Aristotle teaches, in Book 1 of the *Meteorologica*, chapter 6, once happened to certain haloes or comets, and as Chiaramonti, not inappropriately, recalls against Tycho in Book 2 of *De tribus novis stellis*, chapter 1, p. 232 — that, attracted by some star, they followed it with a most obedient motion and always appeared under it, with respect to one place on the Earth: suppose this very thing to happen in other phenomena. For no reason can be adduced why it could not happen. Finally, this is confirmed by the stations of the planets, during which they appear motionless for some hours or even days, and in the same place under the fixed stars, notwithstanding parallax: the same thing, therefore, can happen in any phenomenon.

[Margin: The second part of the theorem is proved.]

But if, not with respect to one observer or another, but with respect to several, and indeed any whatever, the same distance from the same fixed stars occurs, observed at the same time, it will be a physico-mathematically evident argument that the phenomenon has no proper motion sensible to anyone, and therefore moves only with the motion of the primum mobile and lacks all parallax sensible in itself. Otherwise, if it were close to the Earth, two visual lines drawn from different places on the Earth's surface through the centre of the phenomenon would necessarily intersect there in such a way as to make a sensible angle at it, and hence a sensible parallax. I said "at the same time"; otherwise, if this happened on different nights or days, and one observer were at Rome on one night and another at Goa or Antwerp, etc., on different nights, that phenomenon could be kept on one night always in the same line of the apparent place as seen at Rome, and on other days in the same line of the apparent place as seen at Goa or Antwerp; and so the individual observers could, by accident, fail to perceive a parallax, although the phenomenon had one that was in itself very sensible — perhaps even perceived by the senses of others, who observed it at different places but at the same time. Therefore, if that identity of distance from the fixed stars, observed in the new star of Cassiopeia in the year 1572 at Uraniborg, Prague, Lucanij and elsewhere, was observed at the same time, then Tycho, in volume 1, p. 123, showed by a stronger argument indeed that it lacked a sensible parallax — leaving aside, however, the consideration of refraction. For if someone were to say that it did have a parallax, sensible in itself

so far as its distance from the Earth goes, but rendered insensible by accident, because the refraction was always as great as the parallax and so prevented the parallax from being perceived, and he is not convicted of falsehood on other grounds, it will not be possible to establish the lack of paral- [Margin: Order of the theorems.]

[Margin: Order of the theorems.]

lax sensible in itself, but only that the parallax was not perceived by the senses.

14. Theor. Ex æquali, ſimilibus plurium dierum momentis, variatione diſtantiæ Phænomeni ab ijſdem Fixis, obſeruata in vnico terræ loco; non poteſt certò colligi merus motus primi Mobilis & cauſam illius varietatis eſſe ipſam Pallaxin,atque adeò nec Quantitas Parallaxis: Eſto id poſſit colligi ſi talis varietas obſeruata ſit in pluribus quibuſuis terræ locis; ſaltem ſecluſa Refractione. *Theorem 14. From an equal variation in a phenomenon's distance from the same fixed stars, at like moments of several days, observed at a single place on the Earth, one cannot infer with certainty a mere motion of the primum mobile, nor that the cause of that variation is the parallax [?] itself, and so neither the quantity of the parallax; granted that this can be inferred if such a variation is observed at any several places on the Earth — at least when refraction is set aside.*

[Margin: Theorem 14.]

[Margin: What if a phenomenon returns daily to an equal variation of distances?]

I explain the theorem first and then prove it. Let there be a phenomenon which at Rome, on September 24, at exactly 7 o'clock after noon, appears to be 1° west of the bright star of the Eagle, near the zenith, and at 12 o'clock, near the horizon, 3°. And let the same phenomenon, on the following night, the 25th, at Rome, at 7 o'clock, again appear to be 1° west of the Eagle, and at 12 o'clock 3°; and so on, every night showing an equal variation at like hours, that is, at every 12 o'clock after noon having an apparent distance of 3° west of the Eagle. Someone could therefore infer from this that such a phenomenon has no motion other than that of the primum mobile, since at like hours, returning after 24 hours, it comes back to the same point under the fixed stars and is not seen to move away from it either forward or backward; and further, since near the zenith it appears less far west of the Eagle than near the horizon, and the Eagle is supposed to move with no motion other than that of the primum mobile, or with none that is sensible within a few days, he could conclude that this variation comes from a parallax sensible in itself, depressing the phenomenon towards the horizon, as parallax does.

[Margin: The first part of the theorem is proved.]

But I say that neither argument is necessary so long as we are not certain that the same alternations are seen in any other horizon. For that return to the same point at like hours, and that variation of the apparent distances, could come from a proper motion of the phenomenon, now slow, now quick, and

now direct, now retrograde, whose alternations returned daily at like hours. For example, the phenomenon could complete 2° in the 5 hours from 7 o'clock to 12 by its proper motion, but afterwards, having sunk below that single observer's horizon, go back towards the Eagle with a slower but regressive motion, in such a way that, having retraced those two degrees in 19 hours, on the following night at 7 o'clock it would again be 1° from the Eagle. And all this could happen even if it were at any distance whatever from the Earth, and so even if it lacked a true parallax sensible in itself. Conversely, however, that alternation could arise from a true parallax of a phenomenon having no motion other than the motion of the primum mobile. Which of these, then, is the cause of that apparent variation cannot be determined with certainty unless we know that exactly the same alternations are observed at any places on the Earth — I do not say at all of them collectively — but at any several of them, and at as many as are required for an induction sufficient to produce physical certainty.

[Margin: The second part of the theorem is proved.]

But if those alternations occurred alike, indeed equally, at like hours anywhere on Earth, this could no longer be attributed to a proper motion. For when the phenomenon, at, say, 18 o'clock after noon at Rome, was going back towards the Eagle, somewhere — indeed in many places — it would be the moments of those hours that lie between 7 and 12, at which it would have to be truly moving away from the Eagle; and it is impossible for the same mobile body, by a continuous true motion, to approach and to move away with respect to the same term towards the same quarter of the world. It would therefore be inferred rightly, and with physical necessity, that that variation and depression, or increase of the apparent distance, comes from parallax; and so some parallax at least could also be known from that difference of distances, if not the whole of it, because of its mixture with refraction.

15. Theor. Ex inæquali Variatione diſtantiæ viſæ Phænomeni ab ijſdem Fixis ſtellis, ſimilibus plurium dierum momentis obſeruata in vnico Terræ loco; non poteſt certò colligi motus proprius Phænomeni potiùs, quàm parallaxis in Longitud. aut Latitud. per ſe ſenſibilis, nec potiùs parallaxis, quàm motus proprius, nedum vtriuſuis determinata quantitas: ſaltem ſi Phænomenon non compleat, nec repetat Reuolutiones ſuas: Poteſt autem ſi prædicta inæqualitas obſeruata fuerit in pluribus quibuslibet terræ locis, ſaltem ſecluſa Refractione, & multò certiùs ſi compleat ac repetat reuolutiones ſuas. *Theorem 15. From an unequal variation in a phenomenon's apparent distance from the same fixed stars, observed at like moments of several days at a single place on the Earth, one cannot infer with certainty a proper motion of the phenomenon rather than a parallax in longitude or latitude sensible in itself, nor a parallax rather than a proper motion, still less a determinate quantity of either — at least if the phenomenon does not complete and repeat its revolutions. One can, however, if the inequality already mentioned has been observed at any several places*

on the Earth — at least when refraction is set aside — and much more certainly if it completes and repeats its revolutions.

[Margin: Theorem 51 [?].]

This seems to be against Kepler, p. 77 of *De Cometis*, against whom I said enough in Book 8, Section 1, chapter 20, number 10, refutation 7. I explain the theorem, and then prove it. Let a phenomenon appear at Rome, on September 24, at 7 o'clock after noon, to be 1° east of the Eagle; on the 25th, at 7 o'clock, 2°; and on the 26th, at 7 o'clock, 3°; and so on, more and more, uniformly or even difformly — say, always at 7 o'clock, on the 24th 1°, on the 25th 1° 20′, on the 26th 1° 40′, or with some other like increment of a uniformly difform motion. From this the observer at Rome could deduce that this comes from a proper motion of the phenomenon in the order of the signs; for it is from like observations, it seems, that the proper motion of the planets in the order of the signs has so far become known — namely from their moving eastward away from some fixed star.

[Margin: The first part of the theorem is proved.]

I say, however, that this argument is not necessary in itself, so long as it is not known whether the same thing happens at like hours in any horizon whatever, with the proportion of motion preserved that is due to the distance of the places and of the meridians. For nothing prevents that variation and inequality from coming wholly from parallax, increased or diminished because of the phenomenon's varying distance from the centre of the Earth, just as much as that apparent variety of motion requires. But if an evident constancy in the phenomenon's apparent size stood in the way of this opinion, or the appearance of a larger diameter at a time when, according to that opinion, it ought to be smaller because of the greater interval, then the argument would no longer be taken from the mere inequality of the distances from the fixed star in longitude, but from elsewhere.

[Margin: The second part of the theorem.]

So if astronomers are certain about the proper motion of the planets, they have acquired that certainty from observations made at several places on the Earth, and after many revolutions of the phenomenon, or by considering the visible increase and decrease of the apparent diameter, and many other conditions; and from these it has been established that the variation is not something accommodated, by means of mere parallax, to the observer of a single horizon. Since, however, for so great a variety of motions to be accounted for by the variation of the parallaxes alone, several conditions would be required whose concurrence is not as probable as a proper motion, a proper motion is therefore inferred from the observations of a single place on the Earth more probably than mere parallax; and if anyone were to infer mere parallax, excluding a proper motion in longitude, he would be doing so neither necessarily nor more probably. Hence, rightly, Tycho, in volume 1 of the *Progymnasmata*, p. 515: *"Although, conversely, it does not follow that a phenomenon which keeps an unequal distance from the fixed stars is for that reason subject to a sensible parallax: for it can*

have a proper motion of its own which alters the distances without any parallaxes, as happens in the star of Saturn, whose parallaxes are scarcely perceptible, and which nevertheless, in one revolution of the heavens, is not equally distant from the fixed stars."

[Margin: Confirmation from the motion of Saturn and Jupiter.]

And Cysat urges the same in his book *De Cometis*, chapter 3, Lemma 8, teaching that a comet can have a proper motion without any parallax; and he gives the example of Jupiter when swift, whose parallaxes, too, he thinks can scarcely be recognised: for if it is 10° 20′ from the fixed star it is approaching, then after 6 hours, he says, its distance from the same fixed star will be observed as 10° 18′, because its motion over 6 hours, when it is swift, is 2 minutes. And

[Margin: Order of the theorems.]

[Margin: Order of the theorems.]

this difference of distances arises only from the true motion, he says.

16. Theor. Ex eo quòd Phænomenon, ex vno terræ loco obſeruatum, in diuerſis altitudinibus viſis, videatur minorem vel maiorem differentiam altitudinum aut parallaxium ſuſcipere, quàm Luna in ijſdem altitudinibus ſuſciperet, non poteſt certò colligi, illud eſſe ſupra vel infra Lunam; ſeu minorem, aut maiorem Lunari Parallaxin habere; & multò minùs ſi Differentia parallaxium non ſit eiuſdem ſpeciei. Poteſt autem, ſi hoc eueniat in quibuſuis terræ locis, ſaltem ſecluſa Refractione. *Theorem 16. From the fact that a phenomenon observed from a single place on the Earth, seen at different altitudes, appears to undergo a smaller or greater difference of altitudes or of parallaxes than the Moon would undergo at the same altitudes, one cannot infer with certainty that it is above or below the Moon, or that it has a parallax smaller or greater than the Moon's; and much less if the difference of the parallaxes is not of the same kind. One can, however, if this happens at any places on the Earth — at least when refraction is set aside.*

[Margin: Theorem 16.]

Let this be said with all due respect to Tycho, volume 2, p. 110; to Kepler, *De stella nova*, chapter 15; to Cysat, *De Cometa anni 1618*, chapter 3, lemma 3, and p. 49; and to Longomontanus, in the Appendix to the *Astronomia Danica*. The first part of the theorem can be gathered from what has been said and what is still to be said; and it is enough to point out that the diversity already mentioned can be attributed no more to parallax than to an anomaly of the proper motion. The second part is clear from the contrary argument: if the same thing is observed at any places on the Earth, that is a physically evident argument of uniformity in place or in motion, and so the diversity must be referred either to parallax or to refraction.

17. Theor. Ad diſcernendas Parallaxes Phænomeni certò & euidenter, cæteris partibus præferendæ ſunt obſeruationes, factæ in diuerſis terræ locis,obſeruationibus in vnico terræ loco factis; & inter illas illæ, quæ factæ

ſunt eodem momento temporis. *Theorem 17. To discern a phenomenon's parallaxes certainly and evidently, observations made at different places on the Earth are to be preferred, other things being equal [?], to observations made at a single place on the Earth; and among the former, those made at the same moment of time.*

[Margin: Theorem 17.]

The first part is against Tycho, *Progymnasmata* volume 1, chapter 7, p. 451, and chapter 8, p. 489, and volume 2, chapter 6, p. 89; and against Kepler in the *Hyperaspistes*, p. 129, and in *De Cometis*, p. 77. For Kepler, relying on Tycho's observations, and Tycho on his own — gathered, as they were, at such great expense and with such diligence, though made at a single place, namely Uraniborg — not only preferred them to the observations of others, but laid down universally that the observations of a single place are more certain than those of different places for drawing out the parallax. Chiaramonti rightly judged the opposite of this in the *Antitycho*, Book 3, chapter 3, and in the *Apologia* for the *Antitycho*, Book 2, chapter 4.

[Margin: Tycho's mistaken opinion.]

Now, Tycho's opinion on that p. 451 is expressed in these words: *"Although — to say how the matter stands — those trials are far stronger and more certain that are carried out by some one observer in the same horizon and with the same instrument carefully made, provided that each thing is in order and not subject to any sensible fault, than if the working-out of the thing sought is looked for from different horizons and sightings."* And in volume 2, p. 489, he thinks he has demonstrated the place of the comet of the year 1577 from observations made at a single place on the Earth so thoroughly *"that they cannot be overthrown with just reasons by anyone who understands these matters"*, and that it therefore needs *"no vote of others, nor observations made elsewhere and decisions deduced from them, by way of confirmation."* But we have already shown sufficiently, in Theorems 2, 14, 15 and 16, that observations made at a single place on the Earth are not sufficient for this — so far are they from being better. And I said "other things being equal", that is, with equal certainty in the altitude of the pole, equal size or quality of the instruments, and equal diligence and circumspection in observing. For a distance between the meridians is not always presupposed; it is enough if, at the same moment, the places of the observations are on the same meridian or vertical. And the fact that human trust mixes itself in here, and that the observations have to be believed, is likewise common to many astronomical conclusions that are derived from more recent observations compared with the observations of the ancients.

The second part, concerning observations made at the same moment, is clear enough, since in them the investigation of the distance from the Earth, or of the parallaxes, depends on no motion of the phenomenon; rather, the parallax and the distance can be gathered from distances purely as seen from the same fixed star, or from the zenith, and from the distance of the places on the Earth on the same great circle, as is clear from what was said in chapter 8 and chapter 11. Akin to these are the observations made at like hours — as

when one observer at Rome observes at the 1st hour of the night and another at Toledo likewise at the first hour of the night — and so for the rest.

18. Theor. Quamuis parua & aliquando vnius tantùm Minuti differentia Locorum viſorum, aut Parallaxium inueniatur ex obſeruationibus eiuſdem Phænomeni, eodem momento factis in diuerſis locis Terræ, ſub eodem verticali poſitis; poteſt tamen eius eſſe magna, & valde maior Lunari, Parallaxis. *Theorem 18. Although a small difference of the apparent places, or of the parallaxes — sometimes of only one minute — is found from observations of the same phenomenon made at the same moment at different places on the Earth situated under the same vertical, its parallax can nevertheless be great, and much greater than the Moon's.*

[Margin: Theorem 18.]

I wanted to point this out here, so that from this too it may be further confirmed how circumspect one must be in pronouncing on the parallax and place of new phenomena. For who, inferring from observations of this kind a parallactic difference of one minute, would not at once cry out that the phenomenon is far beyond the Moon? But it is one thing for the parallax to be smaller than the Moon's, another for the difference of the parallaxes to be so; and we showed above, in chapter 11, Problems 67 and 68, with Chiaramonti, that this difference can be one minute and the phenomenon nevertheless have a parallax of many degrees.

[Margin: The argument of Kepler and Longomontanus is fallacious.]

From this you will gather that the argument is not necessary by which Kepler, in his book on the new star of Serpentarius, chapter 15, p. 81, tries to show that it was above the Moon, on the ground that he at Prague and Fabricius in Frisia observed its place without a difference of six minutes; and I say the same of a similar argument of Longomontanus, in the appendix to the *Astronomia*, chapter 10, for the comet of the year 1607.

19. Theor. Ad Parallaxes Phænomeni eliciendas, meliores ſunt cæteris paribus obſeruationes factæ per Diſtantias à Fixis, quàm per Azimutha & temporum Momenta, quibus capiuntur Altitudines. *Theorem 19. For drawing out a phenomenon's parallaxes, observations made by distances from the fixed stars are better, other things being equal, than those made by the azimuths and moments of time by which altitudes are taken.*

[Margin: Theorem 19.]

Tycho rightly pronounced on this, in volume 1 of the *Progymnasmata*, p. 604, and in the *Epistolae*, p. 295: in the matter of parallaxes, observations founded on the distances of the phenomenon from the fixed stars are more certain than those that rest on altitudes and azimuths together with observed moments of time. For in a short time, and with a slight change in the azimuths, the altitudes undergo a large increase or decrease if they are not near the meridian; and about the meridian the altitudes change little, it is true, but much time slips

by at the same sensible altitude. Distances from the fixed stars, on the other hand, are taken at once, with no dependence on the time, and if they are to be repeated a short time afterwards they do not vary sensibly — at least when they are beyond the hazard of refractions. But since mention of refractions has occurred more than once here and above, there remains a last knot to be untied, arising from the very close entanglement and almost inextricable connection of refractions and parallaxes. For if someone posits much greater refractions than Tycho and others after him did, he must also posit much greater parallaxes, and smaller distances of the stars from the Earth. Next, it is doubtful which of these is to be known beforehand, or can be known beforehand, while the other is unknown. In the meantime, however, what we taught in this Section in Problems 73, 74 and 75, and then in Theorem 3, where we discussed this difficulty sufficiently, can suffice for the solution of this knot.

20. Theor. Etiam ſi Phænomenon habeat lucem totam à Sole, & tota nocte conſpicuum ſit, nullam patiens Eclipſim; non concluditur neceſſariò illud eſſe altius tota vmbra Terræ, imò nec altius Luna. *Theorem 20. Even if a phenomenon has all its light from the Sun and is visible the whole night, suffering no eclipse, it is not necessarily concluded that it is higher than the whole shadow of the Earth — indeed, not even that it is higher than the Moon.*

[Margin: Theorem 20.]

This can be seen against Peter Apian, in Part 2 of the *Astronomicum Caesareum*, in his observation of the third comet, which he saw in the year 1533, in the month of June. There he admits that the comet never had a northern latitude of less than 32 degrees, and yet he says this of it: *"For if it had been near the Earth, and farther from the Sun, it could never have happened that it did not fall into the Earth's shadow, and so it would have remained hidden."* For he supposes that comets shine from no other source than the Sun. But Apian does not say that it was higher than the whole shadow of the Earth, nor even that it was higher than the Moon; he says only that it was not near the Earth with the nearness the ancient Peripatetics assigned, namely in the upper region of the air. It could, however, have been seen the whole night even if it were below the Moon, as is clear from Book 8, Section 1, chapter 15, Problem 7; and from the fact that a great latitude and declination of the phenomenon towards one side, together with a great inclination of the cone of the Earth's shadow towards the opposite quarter, can exempt the phenomenon from every eclipse.

SECTIO SEXTA PROBLEMATA REFRACTIONVM SIDEREARVM IN AERE.

Section Six. Problems of the Refractions of the Stars in the Air.

CAPVT I.

Chapter I.

Quæſtiones præcipuæ de Refractionibus Indicatæ tantùm,& alibi ſoluendæ, aut fuſiùs tractandæ. *The principal questions on refractions — only indicated here, to be solved elsewhere or treated more fully.*

We are not dealing here with the refractions of any luminous body whatever, but only with those of the rays of the stars; and not with these in every denser medium either, but only in our own air, into which the ray of the stars falls as it passes from the purer air, or from the ethereal [?] breath. For the other matters — what happens in refractions in water, glass, crystal and so on, and what serves for determining the burning point of a glass or crystal sphere, for the lenses of telescopes or of spectacles, and for the humours of the eye, all of which we have derived not only from the experiments of others but also from our own — have been worked through diligently by us in our book on optics, which, God willing, will be the third of the Second Volume of this New Almagest. In that book we have also untied many questions about this matter, raised partly by recent writers and partly by ourselves; it has nevertheless seemed good to indicate them here.

The first question is about the cause of refraction: why the ray of a luminous body, in passing from a rarer medium into a denser one, is refracted towards the perpendicular, and in passing from a denser into a rarer one is refracted away from the perpendicular. This cause is treated above all by Alhazen [Ibn al-Haytham], Book 7 of the *Optics*, proposition 8; Vitellio, Book 2, proposition 47; Kepler in the *Astronomiae Pars Optica*, chapter 1, proposition 20; Niccolò Cabeo on Book 3 of the *Meteorologica*, text 8, question 4, p. 95; Pierre Gassendi, *De apparenti magnitudine Solis*, letters 2 and 4, pp. 70 and 134; René Descartes in the *Dioptrics*, chapter 2; Father Emmanuel Maignan, most diligently, in the dioptrics of his *Horaria*, from proposition 10 to 44; Fortunio Liceti, Book 2 of *De natura luminis*, chapter 22; Athanasius Kircher, Book 8 of the *Ars magna lucis et umbrae*, p. 653; and Hérigone, Axiom 5 of the *Dioptrics*.

[Margin: Question 1: on the cause of refraction.]

The second question is whether a greater distance of the luminous body from the centre of the Earth and of the atmosphere causes, other things being equal, a smaller refraction. Kepler seemed to deny this in the *Astronomiae Pars Optica*, pp. 112 and 137, and in the *Rudolphine Tables*, p. 120. But it should certainly have been affirmed. For it is established by experiment that where the inclination of the ray to the surface of the denser medium is smaller, the refraction is smaller, other things being equal; and of two stars placed at equal altitude or depression, all else being equal, the one whose distance from the centre of the Earth is greater has the smaller inclination, as will also be clear from what is to be said below. On the argument against the infinite distance of the fixed stars taken from refraction, see Book 6, chapter 7, in the scholia.

[Margin: Question 2: on the distance of the luminous body.]

The third question is whether a difference in the regions of the Earth makes a difference in the quantity of the refractions. Tycho seems to have doubted this in the *Epistolae*, p. 64; but there, and on pp. 79 and 280, he contends

only that in Hesse they could not have been twice as great as in Denmark. Otherwise, on p. 85, and in the *Progymnasmata*, volume 1, p. 144, and elsewhere, he admits that variation, and greater refractions in the northern tracts — as does Longomontanus too, in Book 2 of the *Spherics*, chapter 11, who admits refractions twice or three times greater in maritime places and those nearer the Arctic pole than elsewhere. Kepler agrees, in the *Astronomiae Pars Optica*, pp. 134 and 136, adding that they are more constant in maritime places than in inland ones, and almost nil on some mountains. And since in Bohemia they were found about two minutes smaller than in Denmark, Scheiner in the *Sol ellipticus*, p. 31, and Martinus Hortensius in his dissertation with Gassendi, think, not without reason, that Tycho's tables of refractions cannot be applied safely to all places without fear of error in a few minutes; and Lansbergen sets down in his table Dutch refractions different from Tycho's. On the monstrous refraction at Novaya Zemlya, see what was said in Section 3, Problems 13 and 14.

[Margin: Question 3: on the difference of places.]

The fourth question is whether a difference of weather — yearly, monthly or daily — changes the refractions. Ptolemy seems to suppose this in Book 8 of the *Almagest*, near the end, where he says that the risings, appearances and occultations of the stars vary with the different changes of the air; and so does al-Battānī, chapter 27, when he says that the autumnal equinox is to be observed rather than the vernal, because of the smaller disturbance of the air. Tycho says the same in the *Progymnasmata*, pp. 79, 124 and 280; and in the *Epistolae* and in the observations of the year '87 he also said that refractions appear greater about the solstices. Kepler too, in the *Astronomiae Pars Optica*, p. 137, thinks that the refractions of the fixed stars were found by Tycho to be smaller than those of the Sun, because he observed the fixed stars on the clearer and longer winter nights, but the solar refractions at the summer solstice, following Tycho's own instruction in the *Progymnasmata*, volume 1, p. 93. We, for our part, have certainly found the morning refractions greater than the evening ones, the winter ones greater than the summer ones, and the spring ones generally greater than the autumn ones, especially while observing the quantity of twilight — which, for that reason too, we think has been found to vary from one author to another. Father Emmanuel Maignan supports this variation in Book 1 of the *Perspectiva Horaria*, from proposition 23 to 25. In proposition 24 he rightly teaches that the air is not always made denser by more intense cold, since ice becomes lighter and rarer by the force of cold, as it floats on top, etc.; and so it can happen that somewhere and at some time the air, congealing under greater cold, is nevertheless rarer than where or when the cold is less, and so the refraction becomes smaller. In proposition 37, moreover, he teaches that no thermoscope can give certain knowledge of the inequality or equality of the astronomical refractions that light undergoes in the air on different days or at different hours — whether you use a glass tube, in which water rises as the cold increases and falls as it decreases (on which see our Bettini,

Apiaria 8, Progymnasma 3), or the little awn of an oat as an indicator of dryness and humidity (on which see Io. Bapt. Porta [Giambattista della Porta], Book 2 of the *Magia naturalis*, chapter 14). For the density of the air varies not only with the coming of cold but also through a mixture of denser or rarer vapours, from which the air shut inside that tube is free. Then again, the changes of the air that occur in the highest part of the refracting atmosphere are one thing, and those in the thermoscopes we keep in our bedrooms or chambers are another — to which the constant breath of our mouths contributes not a little.

[Margin: Question 4: on the difference of times.]

[Margin: Note on the variation of the air, and on the use of the thermoscope.]

The fifth question concerns the shape of the air that refracts the rays of the stars. Most of the recent writers who treat this matter agree that it is not perfectly spherical but as it were elliptical, or fluctuating unevenly like a sea driven by winds, and very rough; Emmanuel Maignan pursues this at length in Book 1 of the *Perspectiva Horaria*, from proposition 26 to 32. From this it follows that the refraction of a solar ray in the air is not a sure index of the vertical circle in which the Sun stands, and that, when that refraction has taken place, the surface of refraction is not always the vertical circle itself.

[Margin: Question 5: on the shape of the air.]

The sixth question concerns the size of the luminous body, or at least a greater intensity of its light: whether, that is, a larger star, or one endowed with a more intense light, undergoes a smaller refraction. For Kepler denies it, in the *Astronomiae Pars Optica*, p. 111, and so does Cabeo on Book 3 of the *Meteorologica*, p. 91; and indeed in the fixed stars, which do not appear brighter than the Sun, Tycho found a smaller refraction. Yet a reason must be given why, if density causes refraction by resisting the direct propagation of light, this resistance is not overcome more by a stronger light. But of this elsewhere.

[Margin: Question 9 [?]: on the size or intensity of the light.]

The seventh question is whether, and why, opacity — which obstructs the propagation of light more — nevertheless does not of itself vary the refractions; and whether hardness, just as it is suited to reflection, also contributes something to varying refraction.

[Margin: Question 7: on opacity and hardness.]

The eighth question is whether the straight line of a ray refracted in the air is a single one, or rather several, one after another. Granted that — as Emmanuel Maignan teaches, not without reason, in Book 1 of the *Perspectiva Horaria* — one refraction after another occurs very frequently, not only from the aether into the air but also from one part of the air into another, denser part, it does not follow from this, as I once suspected together with Father Bonaventura Cavalieri, that the line of refraction, or refracted ray, is curved; rather, it consists of several straight lines, and our eye sees by the last of them. For it is more probable that there is some depth of air of one and the same physical density, and that its density does not increase continuously and uniformly difformly towards us. Nevertheless Albertus Linemannus [Albert Linemann], in his disputation

on uranic refractions, and Io. Graves [John Greaves] of London, as Hevelius reports on p. 197 of the *Selenographia*, affirm that the line is conic — that is, parabolic, elliptical or hyperbolic — because of three regions of air of different density. But not all of these are suited to a sensible refraction.

[Margin: Question 8: on the shape of the refracted ray.]

The ninth question is whether, because the air between the mountains or islands that we view from the sea, or on the sea, is denser than air farther off, the refraction in it can be so great that the summits of mountains, or islands, or the sails of ships, which would otherwise lie hidden below the straight ray touching the surface of the sea, can be seen. Father Athanasius Kircher affirms that this happened to him, in a letter written to me on 1647 February 17. He says that once, when he was on the island of Malta, he saw Etna, which he nevertheless could not make out on the clearest days; and that the same happened to Don Carolus Vintimilius [Carlo Ventimiglia] looking from Monte Pellegrino at some islands or other. Nor does Snel disagree, in the *Eratosthenes Batavus*, Book 2: from this illusion he reduces the height of Etna — which would otherwise, from the distance of the observers, come out at 6 miles — to 4 miles, and he refers to this the sight, from Lilybaeum in Sicily, of ships still lying in the harbour of Carthage. To me it has always seemed hard to believe that the difference of density between that air and the rest, which is nearer to those mountains and islands, is so great; I would rather believe that on the clearest days, because of the glare of the Sun, things could not be made out on the horizon which were made out at other times.

[Margin: Question 9: on things seen on the surface of the sea.]

The tenth question concerns the height of the air that produces sensible refraction. For it is not the same as the height of the exhalations that produce twilight — though this itself is very uncertain among authors, as is clear from what was said in Book 2, chapter 19, and Book 8, Section 1, chapter 14, where I also taught that this very height of the exhalations depends on refraction; and so, since neither can be determined with certainty without the other, both remain uncertain to this day. Maignan contends the same with other arguments in Book 1 of the *Perspectiva Horaria*, proposition 38, but above all because he himself, at Toulouse, on a clear night in the month of August, saw from the 11th hour after noon until midnight, with the Moon below the horizon, a certain bright little cloud spread out near the meridian almost to the zenith; which, all things considered, could not have been illuminated except by the Sun, and so must have been higher than the whole shadow of the Earth. He adds that something similar happened to Michelangelo Ricci when he was staying among the Sabines — a man most learned in mathematics. We think, however, that two extremes are to be avoided: one, that of Tycho and Longomontanus, who confuse the height of the refracting air with the height of the vapours, or rather of the twilight exhalations; the other, that of Kepler, who distinguishes these heights but nevertheless grants the refracting air no greater height than half a

German mile. Whether, and on what hypothesis, the height of this air can be known, if not absolutely, will be told below in the Problems.

[Margin: Question 10: on the height of the refracting air.]

The eleventh question is whether, if the height of the refracting air is unknown, the angle of refraction cannot be known, as Emmanuel Maignan contends in Book 1 of the *Perspectiva Horaria*, propositions 39 to 43; or whether it can at least be known a posteriori, through altitudes of the stars corrected for parallax. But we have already indicated our view on this in Section 5, Theorem 19, and from Problem 73 to the end. Below, however, on the occasion of Tycho's problem, this string will be struck again.

[Margin: Question 11: on the uncertainty of refraction because of the uncertain height of the air.]

The twelfth question concerns the effects of the refraction of the stars' rays in the air. But I have already listed the chief of these in Book 3, chapter 9, in the scholia. Among them is the elliptical shape of the Sun and the full Moon appearing on the horizon, on which I discoursed in Book 3, chapter 2, and Book 4, chapter 15. Likewise the variation of the equinoctial moments, and the deceptiveness of the equinoctial armillae, on which see Book 3, chapter 14, number 8. Likewise the contraction and interruption of the Earth's shadow, and the portion of the Earth illuminated by the Sun being greater than the ancients thought, on which see Book 5, chapter 5. Likewise the contraction — or rather the enlargement — of the height of the vapours, on which see Book 8, Section 1, chapter 14. Likewise the increase of the Sun's diurnal arc, and of the arc of the ecliptic that is always visible, on which see this Book 10, Section 3, from Problem 12 to 15. It remains to propose a question about the apparent size of the stars as changed by the force of refraction.

[Margin: Question 12: on the effects of the refraction of the stars.]

The thirteenth question, then, is whether the stars appear (especially on the horizon) larger or smaller by the force of refraction than they would appear without refraction. That they appear smaller, even at the vertex, was demonstrated by Alhazen, Book 7 of the *Optics*, from proposition 52 to 54; by Vitellio, Book 10, from proposition 51 to 53; by Maurolycus, Book 1 of the *Diaphaneon*, theorem 2; by Kepler too, in the *Rudolphine Tables*, p. 98; and by Wendelen in the *Idea* of the Atlantic tables. As for the cause for which the Sun appears larger on the horizon, Alhazen, Book 7, proposition 55; Vitellio, Book 10, proposition 54; Baccon [Roger Bacon], Part 2 of the *Perspectiva*, distinction 3, chapter 6; the author of the *Perspectiva communis*, Book 1, proposition 82; Kepler in the *Astronomiae Pars Optica*, p. 133, and in the *Epitome of Copernican Astronomy*, p. 81; René Descartes, chapter 6 of the *Dioptrics*; and Hortensius in the dissertation with Gassendi refer it to our estimation. That is, the eye looking upward has nothing from which to estimate the distances of the stars, or of their edges, and so takes them to be nearer; but when it views them on the horizon, the tracts of land or sea lying in between warn the eye of an immense distance, and the estimative faculty judges them larger, because it already knows beforehand that

things seen under the same angle are larger if they are seen at a greater distance. But if you take the diameters of the luminaries with instruments, they appear larger rather on the meridian and nearer the vertex than on the horizon: so says Kepler above all, in the place cited. Yet he, like Wendelen, grants that the Moon's apparent diameter is larger at the vertex than away from the vertex; and Regiomontanus supports this in the *Epitome of the Almagest*, conclusion 1. Pierre Gassendi, on the other hand, in *De apparenti magnitudine Solis humilis et sublimis*, pp. 6, 43, 82 and 85, refers the cause to the dilation of the pupil: shaded, as it were, by the horizontal vapours, and looking with less injury at the rays of the stars, which are tempered and intercepted, it makes the visual angle larger by its opening. Scheiner too favours this dilation, in the *Oculus*. They add, moreover, that if the Sun's apparent diameter is taken while it is on the horizon, either from the shadow of a tablet or from the bright image of the Sun, it is found somewhat smaller than far from the horizon; and so that larger emphasis of the rising or setting Sun is a deception of sight — either because you judge visible things larger from a greater interval, or because of the dilation of the pupil.

[Margin: Question 13: on the apparent size of the stars.]

[Margin: Opinion 1.]

[Margin: Opinion 2.]

Alhazen, however, in Book 7, proposition 51, and Vitellio, in Book 10, propositions 53 and 54, grant that the stars appear larger on the horizon — not always, it is true, but for the most part. This is not because of the first refraction of the rays, from the aether into the air, which takes place towards the perpendicular, but because of the second refraction, which takes place as the rays pass out of the thick and watery vapours into the rest of the air, lying between those vapours and our eye, and purer; for this refraction, they say, takes place away from the perpendicular. And so, just as through a similar refraction the image of a coin placed under water is enlarged at the common surface of water and air, so too is the image of the stars. The same cause is given by Petzanus, the author of the *Perspectiva communis*, Book 3, proposition 12; by Roger Bacon, Part 3 of the *Perspectiva*, distinction 2, chapter 4; and by Bettini, *Apiaria* 8, Progymnasma 4, propositions 1 and 2. There Bettini admirably reconciles with themselves Alhazen, Vitellio and Maurolycus, who teach that the stars appear now larger, now smaller by the force of refraction; and he cites Cleomedes, Book 2 of the *Cyclical Theory*; Giambattista della Porta, Book 1 of *De refractione*, chapter 11; and, in Porta, Seneca, Macrobius and Aphrodisaeus [Alexander of Aphrodisias], who assert that the stars appear larger when the south wind, a vapour-bearing wind, is blowing. To these I add Ptolemy, Book 1 of the *Almagest*, chapter 3; Sacrobosco and Clavius on the *Sphere*, pp. 108 and 109; and Barocius [Francesco Barozzi] in the *Cosmographia*, p. 17, who grant that the stars appear larger absolutely, because of the horizontal vapours. But above all, Posidonius in Strabo, Book 3 of the *Geography*: *"For Posidonius relates that it is commonly said that the Sun sets much larger on the shores bordering the*

Ocean"; and shortly after: *"The cause why the apparent image of its size is increased at rising and likewise at setting on the seas is that more vapours are raised upward from the moist places; the sight, broken through them as through tubes, receives broader images — just as when you look at the setting or rising Sun, or the Moon, through a dry and thin mist, when the star also appears reddish."*

I once replied that the stars appear larger on the horizon absolutely, by the force of the second refraction, made away from the perpendicular because of the watery vapours lying between the ethereal breath outside our atmosphere and the air nearer our eye, which is not as dense as those horizontal vapours are; and that this always happens so, as will be clear by experiment. Nor is this to be attributed to a fallacy of estimation, because of the intervening tract of valleys, plains and seas. For if in a room or a garden you look at the Sun rising or setting, so placed that a hedge, or a wall, or the lower edge of a window blocks all view of the intervening space as far as the horizon, and you see nothing but the Sun, you will nevertheless see it enormously larger than far from the horizon. Nor, again, is it to be attributed to mere dilation of the pupil. For if you try to contract the pupil by voluntary motion, you will still see the image of the rising or setting Sun, or of the full Moon, much larger than at other times; and you will observe the same in Jupiter, Venus and the other stars. Conversely, if about noon you look at the Sun through clouds, or through coloured glass, and dilate the pupil as much as you like, you will never see it as large as on the horizon. Lastly, I, together with Father Grimaldi, often observed the rising Sun with a sextant; with me directing my eye to the right edge of the Sun and him to the left, the Sun's transverse diameter was taken, and it always appeared much larger than at other times. For sometimes it was seen to extend to almost a whole degree, and often to about 45′; in the Moon, too, we noted 38′ and 40′. That it does not appear so great to someone taking its quantity by the shadow or the bright image on a tablet has this cause: the horizontal vapours blunt the rays of the edges and do not allow a clear boundary to be painted by them on the tablet. The second refraction, away from the perpendicular, therefore outweighs the first, which takes place towards the perpendicular — the refraction by whose force the stars would otherwise appear smaller than their just size, but by a difference quite negligible, as Kepler too teaches in the *Astronomiae Pars Optica*, p. 130.

[Margin: Our view.]

For let V be the vertex, and VH and VR two quadrants of a vertical circle, intercepting the arc HR of the horizon, of 90°; and at H and R let there be two stars whose true distance HR is 90°, but whose apparent distance is AB, because of the refraction raising the stars by 34′, as Tycho observed in the Sun. Then in the triangle AVB the sides AV and BV will each be 89° 26′, and with these and the angle V of 90° the arc AB is found to be 89° 59′ 42″. So the difference from the true distance HR is only 18″. But if the arc HR and the angle V were 45°, the difference would be 9″; and if V were of one degree, the difference would not reach even one second. Now let HR be the distance

between the edges of the Sun or of the Moon, since this is not of one degree: by the refraction already mentioned it will certainly not come out smaller by one whole second, which is a quite contemptible difference.

Diagram. A figure beside the demonstration of the triangle AVB, standing at the right of the right column. From V at the top a curved line, the quadrant VR, runs down to R at the lower right, and straight lines run from V down to A and to H at the lower left; a horizontal line joins A and B, and the base runs from H to R, with straight lines crossing the figure between C, F, D, G, B and R.

- V (top): the vertex
- C (upper left, on the line from V): unexplained on this page
- F (left, below C): unexplained on this page
- D (right, middle): unexplained on this page
- G (right, below D): unexplained on this page
- A (left, above H): apparent place of the star at H
- B (right, above R): apparent place of the star at R
- H (lower left): a star on the horizon
- R (lower right): a star on the horizon

The fourteenth question. What if, between our eye and the outermost surface of the air, which causes the first refraction, there lies only air of a single density, and the ray, having entered the denser air, does not leave it before it reaches our eye? Will the distances of the stars, or their diameters, appear larger by the force of refraction, or smaller? To answer this question the following figure must be set out. In it let the hemisphere AOB of the Earth be understood as described from the centre C, its true horizon being ACB but its apparent horizon DE; and in it let there truly be two stars, D and E, and let their true distance, viewed from the centre C, be the arc DE. Through these stars and through the vertex V let two quadrants of vertical circles be drawn, namely HDV and KEV, which are not in the same plane, as you see them here on the paper, but are inclined to each other and intersect in the vertical line CV, their common section. You will conceive this better if you fold this paper along the line CV, bending the plane CVK towards CVH. Now by the force of refraction let star D appear at F and star E at G, and through the apparent places F and G let the arc FG of a great circle be drawn, representing the distance of the apparent places. Let the apparent places be joined by straight lines both with the eye O, by the straight lines OF and OG, and with the centre of the Earth, by the straight lines CG and CF; and let the true places be joined with the centre C by the straight lines CD and CE, and be understood as joined with the eye O by the straight lines OD and OE, which are inclined to each other and lie in the plane of the physical horizon. With this set out, a manifold distinction is needed in answering.

[Margin: Question 14.]

Diagram. A figure beside the setting-out of the fourteenth question, at the right of the right column: a semicircle on the base AB with the centre C at its

midpoint, H and K on the base between A and C and between C and B; above it, lines from H and K rise to the vertex V, crossed by the chord DE and by the chord FG, with straight lines from the small letter o near the centre and from C to D, E, F and G.

- V (top): the vertex
- F (upper left): the apparent place of star D
- I (between F and G): unexplained on this page
- G (upper right): the apparent place of star E
- o (centre): the eye O
- D (left, middle): star D, on the apparent horizon DE
- E (right, middle): star E
- A (lower left): end of the true horizon ACB
- H (base, left of C): foot of the vertical quadrant HDV
- C (base, middle): the centre of the Earth
- K (base, right of C): foot of the vertical quadrant KEV
- B (lower right): end of the true horizon ACB

First, then, I say that if refraction alone, in the vertical circles

[Margin: Reply 1.]

taking place towards the perpendicular, is considered, and the seen places F, G are, as actually happens, seen from the surface of the Earth by the eye O, and the seen distance FG is compared with the true distance DE, as estimated from the centre of the Earth C, then the seen and the true distances are equal if the refraction is precisely as great as the parallax of altitude; and in that case the refracted ray FO is parallel to the unrefracted CD, just as the refracted GO is parallel to the unrefracted CE. But if the parallax is smaller than the refraction, the seen distance is smaller than the true one; and if the parallax is greater than the refraction, the seen distance is greater than the true one. I say the same of the apparent diameters of the stars, seen by the eye O and compared with those that astronomers estimate from the centre C, in so far as diameters are considered as distances between the extreme luminous points.

Second, I say: if the seen places F, G are considered now as seen from O and now as estimated from C, the distance seen from O is always greater than the distance as if seen from C; that is, the angle FOG is greater than the angle FCG. But if the seen places are considered as if from C, and the true places are likewise considered as seen from C, then the seen distance of the true places is greater than the seen distance of the apparent places; that is, the angle DCE is greater than the angle FCG — even if these angles are not in the same plane, as they are represented here, but in different planes — provided that only the refraction towards the perpendicular, in each star's own vertical circle, is considered.

[Margin: Reply 2.]

Third, I say: notwithstanding what has been said, the diameters of the stars can appear larger not only from the double refraction, as was said in the reply to Question 13, but also from a threefold cause. 1. From the trembling of the vapours, on account of which, in the same physical instant, any luminous points

whatever are seen to be now to the right and now to the left, although in reality they occupy a single place at any one instant. 2. From the very swift motion of the primum mobile, that is, the diurnal motion of the stars, which outruns the swiftness of any eye, and of the estimation that relies on the eye, and which causes the extreme and intermediate luminous points of the diameter — of the whole solar disc, or of any star — to be seen in two places at the same time or physical instant, and to occupy both the point from which and the point to which, as it carries the stars on, moment by moment, to different azimuths and to different planes of the almucantar circles [circulorum Almuncatarath]. This swiftness has a greater effect near the horizon, where these changes are greater. In the same way, if because of too great a distance I could not tell the motion of a very swift bird across a space of two palms from rest, the bird, though itself only a palm long, would seem to me to occupy a space of two palms at the same time. And indeed lightning, because of its swiftness, seems to us to occupy a great tract of the sky even when it is short, nor are different exhalations always kindled one after another. 3. If someone were to admit that refraction takes place not only in the vertical circle but in another plane as well, and that the rays of the intermediate points are refracted to the eye in such a way that each of them would be seen farther from the others and larger, some to the right and some to the left, he could give from this an account of this apparent magnitude. But he would defend himself only with difficulty against the common opinion of the opticians, who require that the direct ray, the refracted ray and the perpendicular to which, or from which, the refraction takes place be in the same plane. More on these matters in the *Optics*.

[Margin: Reply 3.]

The fifteenth question is whether the thickness or depth of the denser medium, as such, increases the refraction, as Tycho seems to have held; against him Kepler, in the *Astronomiae Pars Optica*, chapter 4, p. 83. But Kepler attributes this not to the mere thickness but to the greater obliquity and inclination of the rays.

[Margin: Question 15.]

CAPVT II.

Chapter II.

In quo Definitiones & Axiomata Refractionum in communi. *In which the definitions and axioms of refractions in general are given.*

In the following figure let a straight line AB be drawn, representing a plane surface cut by this sheet of paper and cut at right angles, at the point C, by the straight line DCE. From D, with the radius CD, let the arc FCG be described; and from E, with the radius EC, let the arc HCI be described. The first of these represents a concave surface and the second a convex one, with respect to an eye placed at D. For ACB will touch each arc at C and will make with the semidiameters DC and EC a right angle on either side at the point of contact

C, by proposition 18 of Book 3 of Euclid. Indeed, the arcs just mentioned, and the surfaces they represent, will also touch one another at C. Now let there be a luminous body at D, radiating to C by the perpendicular ray DC onto a denser surface — of water, say, or of glass, etc. — whether concave, FCG, or plane, ABC, or convex, HCI. If the ray is reflected, it will be reflected back into itself from C towards D; but if it penetrates these surfaces, it will penetrate them straight through C towards E. Next let there be a luminous body at L, radiating obliquely to C by the ray LC. If this ray is reflected, it will be reflected from C towards K, so that the angle of inclination LCD is equal to the angle KCD, and the angle of incidence LCA is equal to the angle of reflection KCB. But if the ray LC penetrates any one of these surfaces, it will not be propagated straight from C towards M — so that the outer ray LC and the inner ray CM would lie in one and the same straight line LCM. Rather, if it falls on a denser surface, it will be bent (as if it were broken at C) towards the perpendicular DCE and will be propagated along another straight line CR. LC will then be the incident ray and CR the refracted ray; the refracted angle will be ECR, and the angle of refraction RCM. Taken together, these make up the angle ECM, which is equal to the angle of inclination LCD, since it is set vertically opposite it, by proposition 15 of Book 1 of Euclid. Conversely, if the luminous body were in the denser medium, say at R, and passed through C into the rarer medium, it would not go straight from C to P but would be refracted towards L; CL would then be the refracted ray, LCD the refracted angle, and LCP the angle of refraction, which is part of the refracted angle. Finally, if from any point of the reflected ray CK — from K, say — the perpendicular KM is let fall to AB, until it meets the incident ray LC produced as far as the meeting point, KM will be called the Cathetus

Diagram. A figure in the right column for the definitions of reflection and refraction: a horizontal line AB through the point of incidence C, and a vertical line DCE through it; an arc FCG, concave upward, and an arc HCI, concave downward, both touching AB at C; a solid line from L at the upper left to C, continued below C as a solid line to R near E; a dotted line from P through C to M at the lower right; a solid line from C to K at the upper right, with a dotted vertical from K down to M.

- D (top): the luminous point on the perpendicular; centre of the arc FCG
- L (upper left): the oblique luminous point
- P (upper left, beside D): end of the dotted line, where the ray from R would go unrefracted
- K (upper right): on the reflected ray CK
- F (left): end of the concave arc FCG
- G (right): end of the concave arc FCG
- A (left): end of the plane surface AB
- C (centre): the point of incidence
- B (right): end of the plane surface AB
- H (lower left): end of the convex arc HCI

- I (lower right): end of the convex arc HCI
- M (lower right): end of the unrefracted continuation LCM, and foot of the cathetus KM
- E (bottom): on the perpendicular DCE; centre of the arc HCI
- R (bottom, right of E): end of the refracted ray CR

Definitiones. *Definitions.*

1. The perpendicular to a straight line, or to a plane surface, is a straight line making a right angle with the straight line taken by itself, or with a straight line lying in the plane surface. Such is DC with respect to ACB.
2. The perpendicular to a curved line, or to a concave or convex surface, is a straight line that makes a right angle, at the point of contact, with another straight line touching the curve. Such is CD with respect to ACB, which touches the curved surfaces FCG or HCI at the point C. Understand the same of a perpendicular drawn to a parabolic, hyperbolic or elliptical line: for the tangent must first be drawn, or conceived as drawn, and a straight line drawn at right angles at the tangent's point of contact. If the perpendicular is to a circle or a sphere, this line, produced, will fall on the centre of the circle or sphere.
3. The line of incidence, or the incident and direct ray, is a ray from a luminous body terminating at a point of some surface, and that point is called the point of incidence. This ray is either right, that is, coinciding with the perpendicular line, as DC, or oblique and different from the perpendicular, as LC.
4. The reflected ray is a ray distinct from the direct ray, returning from the point of incidence straight or obli- quely towards the origin of the direct ray, that is, making with the direct ray at the point of incidence either no angle or an angle less than two right angles. If the direct ray is perpendicular, as DC, it is reflected back into itself from C towards D, and in the same straight line there are two distinct rays, one DC, the other CD. But if it diverges from the perpendicular, as LC, the reflected ray is CK.

Diagram. The figure of the preceding page printed again, in the left column beside the definitions: a horizontal line AB through the point of incidence C and a vertical line DCE; an arc FCG, concave upward, and an arc HCI, concave downward, both touching AB at C; a solid line from L to C, continued below C to R; a dotted line from P through C to M; a solid line from C to K, with a dotted vertical from K down to M.

- D (top): on the perpendicular
- L (upper left): the oblique luminous point
- P (upper left, beside D): end of the dotted line
- K (upper right): on the reflected ray CK
- F (left): end of the arc FCG
- G (right): end of the arc FCG

- A (left): end of AB
- C (centre): the point of incidence
- B (right): end of AB
- H (lower left): end of the arc HCI
- I (lower right): end of the arc HCI
- M (lower right): end of the dotted line, foot of KM
- E (bottom): on the perpendicular
- R (bottom, right of E): end of the refracted ray CR

5. The refracted ray is a ray proceeding from a ray that falls obliquely on a transparent medium of a density different from the preceding one and penetrates it; it is propagated along a straight line different from the line of the incident ray, that is, one making with the incident ray an obtuse angle at the point of incidence. Such is the ray CR, compared with the incident LC, or the ray CL, compared with the incident RC.
6. The unrefracted ray is a ray proceeding from a ray that falls obliquely on a transparent medium of the same density — or one feigned in the imagination as of the same density — and penetrates it; it is propagated along the same straight line as the incident ray. Such is, or would be, the ray CM with respect to the incident LC, or the ray CP with respect to the incident RC, if the transparent medium of both rays were of the same density.
7. The cathetus of reflection is a straight line drawn from any point of the reflected ray perpendicular to the plane of the mirror, as far as its meeting with the incident ray produced in imagination: such is KM. The cathetus of incidence, by contrast, is a straight line drawn from any point of the incident ray perpendicular to the plane of the mirror, etc.
8. The angle of incidence is the angle made at the point of incidence by the incident line and by a straight line lying in the plane, or touching the surface, on which the ray falls. Such is the angle ACD, if the incident ray is perpendicular, as DC; or the angle ACL, if the ray falls obliquely, as LC.
9. The angle of inclination is the angle made at the point of incidence by the incident line and by the perpendicular. Such is the angle ACD, if the incident ray is AC and the perpendicular DC; or the angle LCD, if the incident ray is LC and the perpendicular DC. Therefore, if the incident ray is DC, the inclination is nil, but the incidence is greatest and strongest; but if the incident ray is AC, at right angles to the perpendicular, the incidence is nil, but the inclination is greatest, and of 90 degrees. Yet Vitellio, Book 10, and Kepler in his *Optics* often use, or name, the angles of inclination and of incidence indiscriminately.
10. The angle of reflection is the angle made at the point of incidence by the reflected ray and by a straight line lying in the plane surface, or touching the curved surface, from which the reflection takes place. Such is the angle KCB, if the reflected ray is CK.

11. The reflected angle is the angle made at the point of incidence by the reflected ray and by the perpendicular drawn through the point of incidence. Such is the angle DCK, if the reflected ray is CK.
12. The angle of refraction is the angle made at the point of incidence by the refracted ray and by the other, unrefracted ray, or the ray assumed as unrefracted. Such is the angle RCM, if the refracted ray is RC and the unrefracted MC; or LCP, if the refracted ray is LC and the unrefracted PC.
13. The refracted angle is the acute angle made at the point of incidence by the refracted ray and by the perpendicular. Such is the angle ECR, if the refracted ray is RC. I said "acute" because the same RC makes with DCE the obtuse angle DCR, but that is not the refracted angle. But if the refracted ray is LC, the refracted angle will be LCD.
14. Refraction towards the perpendicular is when the refracted ray comes closer than the unrefracted ray to the perpendicular produced on the same side, that is, makes a more acute angle with it. Such is the refraction of the ray CR, which comes closer to the perpendicular CE and makes a more acute angle ECR than MC, which makes the angle ECM.
15. Refraction away from the perpendicular is when the refracted ray draws farther than the unrefracted ray away from the perpendicular produced on the same side, that is, makes a less acute angle with it. Such is the refracted ray LC, compared with the unrefracted PC and the perpendicular CD.
16. A dense body is one that has much corporeal substance, and a rare body one that has little substance, in an equal bulk or space. Therefore density is the multitude, and rarity the fewness, of the parts of corporeal substance within the same bulk or within an equal space; and neither is said absolutely, but comparatively.
17. An opaque body is one that is impervious to light, that is, stops the further propagation of light. A transparent or pellucid body, on the other hand, is one that is pervious to light, that is, a body fit to admit the further propagation of light within itself — whether transparency consists in the privation of opacity or in a positive quality contrary to opacity.

Axiomata vel quaſi Axiomata aut Poſtulata. *Axioms, or quasi-axioms, or postulates.*

1. A perpendicular ray, if it is reflected, is reflected back into itself; but if it is not reflected but penetrates a different transparent medium, it passes through unrefracted.
2. The angle of reflection of rays is always equal to the angle of incidence, and the reflected angle to the angle of inclination. So hold Euclid, theorem 1, from hypothesis 3; Alhazen, Book 4, propositions 10, 11, 12 and 18; Vitellio, Book 5, propositions 10 to 20; and René Descartes, *Dioptrics*, chapter 2. This also holds for an elliptical, parabolic and hyperbolic surface, provided that a straight line touching those surfaces is used and

the angles made by the tangent and the incident ray are considered, as geometers commonly consider them — with Apollonius, *Conics*, Book 3, proposition 48; Vitellio, Book 9, proposition 42; Kepler, in the *Astronomiae Pars Optica*, chapter 1, proposition 19; and Cavalieri, *De speculo ustorio*, chapter 7. Granted that Cabeo, on Book 3 of the *Meteorologica*, text 8, question 4, rejects that tangent as something fictitious; yet it is not merely feigned, since it really exists in the intervening medium, say in the air, as the perpendicular does too. And granted that it is not really distinct from the surface and the depth, nature does not on that account cease to operate in relation to such lines. So too we measure the angle of refraction even though one of its lines is the unrefracted ray, which is not such in reality but is feigned as what it would be.

3. A ray falling obliquely from a rarer medium into a denser one is refracted at the point of incidence towards the perpendicular; but a ray falling obliquely from a denser medium into a rarer one is refracted away from the perpendicular. So teach, from a sufficient induction drawn from experiments, Alhazen, Book 7, p. 3 to 14; Vitellio, Book 2, propositions 42 to 45, and Book 10, propositions 3 and 4; Maurolycus at the beginning of the *Diaphaneon*; Porta, *De refractione*, Book 1; Kepler, *Dioptrics*, axioms 2 and 3; and the writers on dioptrics generally.
4. The incident and the reflected ray, compared with each other, just like the incident and the refracted ray compared with each other, are in the same plane, and that plane is called the surface of reflection, or of refraction. So postulate, or even try to prove, Alhazen, Book 4, proposition 13; Vitellio, Book 5, proposition 25, and Book 10, proposition 2; and Kepler in the *Astronomiae Pars Optica*, chapter 3, proposition 16 — who add that the surface of reflection and of refraction is at right angles to the mirror. But whether they demonstrate this without a vicious circle, see Emmanuel Maignan, *Perspectiva horaria*, Book 3, postulate 2, and Book 4, likewise postulate 2.
5. The greater the inclination of the ray, the smaller the angle of reflection, but the greater the angle of refraction. So Vitellio, Book 10, proposition 14; Kepler, *Astronomiae Pars Optica*, chapter 4, proposition 1, and *Dioptrics*, axiom 2. Kepler calls Vitellio's demonstration of this proposition faulty and obscure, and attempts another himself.
6. The refraction of a ray on its way out of a denser transparent medium into a rarer one is as great as on its way into that denser transparent medium from the same rarer one. For the same refracted ray is, under another aspect, the incident ray, and the reverse. So Alhazen, Book 7, proposition 34; Vitellio, Book 10, proposition 9; Kepler, *Dioptrics*, axiom 3, and in the *Astronomiae Pars Optica*, chapter 4, proposition 6; Hérigone, *Dioptrics*, proposition 1; Descartes, *Dioptrics*, chapter 2, number 10; Maignan, *Perspectiva horaria*, Book 4, proposition 2, in the Admonitions. But see what I shall say in chapter 3, proposition 10.

7. The refraction at a circular surface, whether concave or convex, is the same as it would be at a plane surface touching it, if the density of the medium and the inclination of the incident ray are equal. So Descartes, *Dioptrics*, chapter 2, number 12; and it is gathered from the definitions set out above.
8. In the denser medium the angle of inclination is equal to the refracted angle and the angle of refraction taken together; but in the rarer medium the refracted angle is equal to the angles of refraction and of inclination taken together. This is clear from the construction of the figure set out above, from the definitions set out above, and from experiments, for which consult the tables of Vitellio to be given at the end of this Section. Kepler, however, in chapter 4 of the *Astronomiae Pars Optica*, proposition 7, says that in the densest medium the refraction would be equal to the inclination; but he means the greatest, that is, an infinite, density.
9. Other things being equal, the refraction in a different transparent medium is the same whether its depth is great or small. For refraction takes place at once at the surface of the transparent medium of different density; and that density, if it does not vary, does not twist the ray, once refracted, away from the straight line of its first refraction. This is what Kepler meant when he said, in chapter 1 of the *Astronomiae Pars Optica*, p. 10: "Light is not impeded by the solidity of bodies in so far as they are solid"; and he repeats it in chapter 4, proposition 2.
10. The reflected ray and the refracted ray are a straight line, no less than the incident ray. This is to be supposed from experiments, with Euclid, *Catoptrics*, supposition 1; Alhazen, Book 1, propositions 14, 18 and 28, and Book 7, proposition 2; Vitellio, Book 2, theorem 1; Kepler, in the *Astronomiae Pars Optica*, chapter 1, proposition 4; and Emmanuel Maignan, *Perspectiva horaria*, Books 3 and 4, postulate 1.

CAPVT III.

Chapter III.

In Quo Theoremata quædam ſelecta de Quantitate Refractionum, Experimentis aut rationibus ab alijs demonſtrata. *In which certain selected theorems on the quantity of refractions are given, demonstrated by others through experiments or reasoning.*

Although the experiments on refractions depend on the problems to be proposed in chapter 4, I wish nevertheless, before the problems, to add to the axioms and postulates certain theorems — demonstrated as far as could be done in this matter, and accepted by most of those skilled in dioptrics — besides the theorems on the connection of refractions with parallaxes set out in the preceding Section, chapter 12.

Proposition 1. The refractions of crystal and of glass are very nearly the same. So Kepler, axiom 6, and Hérigone, *Dioptrics*, art. 3; and it will be clear from the tables of refractions to be given at the end of this Section.

Proposition 2. The angles of simple refractions are proportional to the angles of inclination, if the density of the medium is considered by itself. For absolutely and in reality the total angles of refraction grow by greater increments of ratio than the angles of inclination. The first part of the theorem is Maurolycus's, *Diaphanorum* Book 1, proposition 10, and Kepler's, in the *Astronomiae Pars Optica*, chapter 4, proposition 8 — but it is to be understood with the limitation, already mentioned, that Kepler himself attached in the corollary to proposition 1. Accordingly he gave the second part of the theorem in proposition 3, giving the reason that a twofold cause of increase is mixed in: one from the density, the other from the obliquity of a ray that has met a greater density at the surface. And in proposition 12 of the *Dioptrics* he said that refractions, weighed exactly, are not proportional to the inclinations. This can be established from the tables to be given at the end. The refractions of crystal, however, are sensibly proportional up to the 15th degree of inclination, or at most the 30th: for 15 degrees Hérigone has it, *Dioptrics*, proposition 2, in the scholium, and for 30 Kepler, *Dioptrics*, axiom 7.

Proposition 3. The refractions of crystal up to the thirtieth degree of inclination are very nearly a third part of the inclination. So Kepler, axiom 8, and Hérigone, *Dioptrics*, axiom 4. Maurolycus, from his own experiments, holds a little differently in Book 1 of the *Diaphanorum*. In proposition 10, in the corollaries, he said that the refraction of a crystalline sphere is to the angle of inclination very nearly as 3 to 8, that is, as 1 to 2⅔; and in proposition 11 he said that within the 70th degree of inclination the refraction is less than half of the inclination. Yet in proposition 10 he had admitted that, if the inclination is greatest, the refraction contains three eighths of one right angle, that is, 33° 45′, which is very nearly a third part of the inclination. For the refraction is much greater near the end of the quadrant of inclination, as we have learned from the more accurate experiments of recent writers and from our own. Cavalieri adds, in *Exercitatio* 6 of the *Geometrica*, proposition 7, that for rays passing out of crystal into the air at an inclination of less than 20 degrees the refraction is very nearly half of the inclination. From the aether into the air, however, it is always less than half of the inclination.

Proposition 4. As the sine of one angle of inclination is to the sine of its refracted angle, so is the sine of another angle of inclination, in the same transparent medium, to the sine of the refracted angle — but considering the simple refraction that arises from the mere difference of specific density. The first part is Hérigone's, *Dioptrics*, proposition 2; Descartes's, *Dioptrics*, chapter 2, number 7; and Emmanuel Maignan's, *Perspectiva horaria*, Book 4, proposition 34. But the second part, or the limitation of the first, is to be added from Kepler, Book 4 of the *Astronomiae Pars Optica*, proposition 1, where, having shown that "light, the more obliquely it falls, is refracted by so much the greater angle",

he adds this corollary: "If the medium itself, on account of its density, were considered by itself, the angles of refraction would become proportional to the angles of incidence." But because another cause is joined to it that increases the refraction, the proposition just stated does not hold of the whole, composite refraction. What this other cause is will be shown shortly by proposition 5.

[Margin: What simple refraction is, and what composite refraction is.]

Proposition 5. When light falls more obliquely, the resistance from the same medium is greater than in a more direct incidence, with respect also to the medium itself. So Kepler, chapter 4 of the *Astronomiae Pars Optica*, proposition 2. For since a ray of light is not a mathematical line but a physical one having breadth, the breadth of a ray falling straight is smaller than that of one falling obliquely; and therefore at the very surface of the denser medium a greater density, that is, a greater portion of the dense body, meets and resists the more oblique ray. From this arises an addition over and above the simple refraction.

Proposition 6. The angles of refraction grow by greater increments of ratio than the obliquity of incidence. So Kepler, chapter 4 of the *Astronomiae Pars Optica*, proposition 3. For by proposition 4 they would grow proportionally; but by proposition 5 a part of the refraction is added from the greater portion of the dense body meeting the oblique ray. This portion grows little at first, but much at a low incidence, in the way the secants do; therefore the whole refraction grows by greater increments than the inclination.

Proposition 7. The sines of the distances from the vertex are not proportional to the sines of the refractions; and near the horizon the refractions grow by precipitate proportions of increments. The first part is Kepler's, in the *Astronomiae Pars Optica*, chapter 4, p. 84; the second is his too, in the same place, proposition 5. The first part is clear from experiments, and from the fact that otherwise the refractions would grow in the same form in all media, which is unacceptable; the second, because they have an addition from the secants, which grow by a precipitous difference near the horizon, that is, near the end of the quadrant.

Proposition 8. The secants of those angles of incidence concur in the measure of the refractions that are formed at the surface of the denser medium. So Kepler, in the *Astronomiae Pars Optica*, proposition 6, and in the *Epitome of Astronomy*, p. 62, where he says that the proportion of two angles of refraction is compounded of two proportions: one between the angles of inclination whose refractions lie within the dense medium — that is, between their sines — and the other between the secants of the same angles. But whether this agrees exactly with observations, see what is to be said against Kepler at the end of Problem 49, reason 4.

Proposition 9. The second secants of the angles of incidence, proportio-

[Margin: Order of the problems.]

nal to the second secants of the refracted angles. So Mersennius, *Optics*, Book 6, theorem 7.

Proposition 10. If in two transparent media there are two lines to the common surface of their contact, together making an angle at the point of

refraction, and light falling first along one is refracted along the other, then, if on returning it falls along the latter, it will in turn be refracted. So Emmanuel Maignan demonstrates in *Perspectiva horaria*, Book 4, proposition 2: for if they do not together make an angle, what was said in axiom 6 is not universally true.

Proposition 11. Whatever combination of different transparent media is made, although in the rare medium there is no angle of inclination at which the incident ray is not refracted in the dense one, yet in the dense medium there is necessarily some angle of inclination at which the incident ray is not refracted in the rare medium, but in the dense medium itself, from which it falls. So Maignan, in the same place, proposition 3. And he adds that in this case the refracted angle comes out greater than a right angle, and that this is how Vitellio, Book 10, proposition 8, is to be understood: in his small table of refractions from glass to air, for an angle of inclination of 80° in the glass, he assigns a refracted angle in the air of 118°; from water to air, a refracted angle of 110°; and from glass to water, one of 98°.

Proposition 12. If a ray of light is transmitted from any rare medium into a similar rare medium through a dense one whose two surfaces, on either side, touching the rare medium on each side, are plane and parallel to each other, then the angle refracted the second time from the perpendicular will be equal to the angle of the first inclination, and the ray refracted the second time will be parallel to the first incident ray produced. So Maignan, above, proposition 4.

Proposition 13. When a ray of light falls from one transparent medium into any other different one, at whatever angle of incidence, then although refraction takes place, some part of it is nevertheless reflected — and indeed with the equality of the angles of incidence and reflection preserved. Likewise Maignan, in the same place, proposition 5, showing it from experiment.

Proposition 14. When a ray falls from a dense medium into a rare one in such a way that it does not go forward into the rare medium, one part of it is refracted within the same dense medium and the other is reflected within the same. Likewise Maignan, in the same Book 4, proposition 6, of the *Perspectiva horaria*.

Proposition 15. If a ray is propagated from a first, rare, transparent medium into a third, dense one, through a second that is denser than the first but less dense than the third, and which moreover has both the surfaces by which it touches each of them plane and parallel to each other, then the refracted angle in the third transparent medium will be equal to the angle that would arise if the second were removed and the first kept, with the same angle of inclination. So Maignan, *Perspectiva horaria*, Book 4, proposition 7. He shows this not only a priori but also a posteriori, from the corrected tables of Vitellio, Book 10, proposition 8, which give the refractions from air into water, from water into glass, and from air to glass. For if the angle of the first inclination is 80°, the refracted angle in water is 50°, which is the size of the second inclination, in

the water; and to this corresponds in glass a refracted angle of 42° 30′. But from the same corrected tables, if the angle of inclination from air into glass is 80°, the refracted angle in the glass will be 42° 30′.

Proposition 16. If a ray is propagated from a first, rare, transparent medium into a third, dense one, through a second denser than both, and which has both the surfaces by which it touches each of them plane and parallel to each other, then the refracted angle in the third transparent medium will not be the same as would arise if the second were removed, other things being equal, but smaller than it; it will, however, be the same as would arise if the second transparent medium were put in place of the third and, in turn, the third in place of the second. So, finally, Emmanuel Maignan, *Perspectiva horaria*, Book 4, proposition 8 — who, by his doctrine of refractions in Books 1 and 4 of that work, has indeed deserved supremely well of this part of dioptrics.

CAPVT IV.

Chapter IV.

In Quo Problemata ad Refractiones ſpectantia. *In which are the problems concerning refractions.*

We shall set out here only a few things about refractions from air into water or glass, or from water into glass, and only as much as suffices for considering the analogy with the refractions from the aether into our air. The rest we refer to the book on optics, where the refractions of the crystalline and glass sphere, of the lenses of spectacles and telescopes, and of the humours of the eye, are treated at length.

1. 2. 3. 4. 5. 6. Probl. Refractiones ex Aere in Aqua, vel in alio liquore contingentes metiri modis 7 *Problems 1, 2, 3, 4, 5, 6. To measure, in 7 ways, the refractions that occur from air into water or into another liquid.*

[Margin: Problem 1, from Ibn al-Haytham, Vitellio and Maignan.]

The first way is by the round instrument of Alhazen [Ibn al-Haytham], Book 7, propositions 2, 10, 11 and 12, and of Vitellio, Book 10, propositions 5, 6, 7, 8. This is certainly laborious, especially since they used the same instrument to measure the refractions produced in glass and crystal as well; and it would be more advisable to measure these separately. Nor is the instrument of Father Emmanuel Maignan, which he describes in *Perspectiva horaria*, Book 4, proposition 46, much easier. We shall therefore proceed to easier ways.

[Margin: Problem 2, from Kepler and Cabeo.]

Kepler gives the second way in his *Dioptrics*, problem 4, and Cabeo in the *Meteorologica*, Book 3, text 8, question 4: namely, by means of a square box. Let it be ABCD. It will be better if it is cubic, made up of four equal and parallel square boards together with a fifth fixed underneath at the bottom of the box, that is, as its base. On one of its sides, say ABHD, describe the quadrant AGD from the centre B, both inside and outside, and divide it into degrees or even

into minutes of degrees. At the centre B erect two styles perpendicular to the plane of the quadrant AGD: one, BM, inside the box, the other, BK, projecting outside the box. Then fill the box with water, or with whatever transparent liquid you please, and set it squarely on the plane of the horizon, so that the sides of the box stand perpendicular to it. With the styles turned towards the Sun R, and the plane of the quadrant placed in the vertical in which the Sun is, observe the degree and minute on which the shadow of the outer style BK falls: it will be, for example, the point G, on which the ray RBG, the boundary of the shadow, falls directly. Meanwhile let the other partner in the observation observe the point on which the shadow of the inner style BM falls. That point will be lower than the outer one — say F — because the ray RB, entering water, which is denser than air, will be refracted in the water towards the perpendicular BD. The difference between the points just mentioned, marked on the arc GF, will be the measure of the angle of refraction FBG; the unrefracted ray will be RBG, and the refracted ray BF. The refracted angle, however, is FBD; and the inclination will be known from the altitude of the Sun's upper limb taken at the same moment, since it is its complement.

Diagram. Woodcut of a cubic box in perspective, set in the right column beside the description of the second way. A ray from R, at the upper right, runs to the top corner B; K stands just left of B on the top edge. An arc on the inner face runs from near G down through F to D.

- R (top right, outside the box): the Sun
- K (top edge, left of B): the outer style
- A (top left corner)
- B (top right corner): the centre of the quadrant
- G (on the inner face, upper)
- F (on the arc, below G)
- D (lower right corner)
- H (lower left of the front face)
- C (bottom left corner)

[Margin: Problem 3, from Bettini.]

The third way is proposed by Father Mario Bettini, *Apiaria* 8, Progymnasma 1, proposition 3, by means of a scaphe, or hemispherical vessel, DCE, at whose bottom stands a style CM, equal to the semidiameter of the hemisphere DCE and perpendicular to the plane of the horizon ACB, so that, with the vessel resting on it, the style makes right angles at C on either side with the horizontal AB. In the other, inner quadrant of the scaphe, say DC, let a quadrantal arc be described from the centre M and divided into degrees and minutes. When the Sun R has risen, let the point G be marked on that arc on which the ray RMG falls, passing through the tip of the style and bounding the shadow. The arc GC will then be the measure of the Sun's apparent distance from the vertex, and at the same time of the inclination of the ray MR by which it departs from the perpendicular MC; while the arc DG will measure the apparent altitude of the Sun's upper limb. Next let the scaphe be filled with water up to the top of

the style, and let the point F be observed on which the refracted ray FM will fall. For if the observation of the points G and F were made at the same moment, GF — that is, the angle FMG — would be the measure of the refraction sought. But it is impossible

[Margin: Order of the problems.]

to do both at the same moment, unless two equal scaphes are available, one full of water and the other of air, and there are two observers. Without these the operation is unreliable, because the Sun moves in the meantime and the inclination of the ray varies. The procedure could nevertheless be carried out with a single vessel and a single observer, if the vessel were used empty of water in the morning and full in the evening, but with the Sun at the same altitude, known through another instrument or indication; or if, on two days with the Sun at the same altitude, the point G were observed once with the vessel empty, and the point F once with the vessel full of water.

Diagram. Woodcut of Bettini's hemispherical vessel (the third way), at the head of the left column. The bowl, its rim stippled, rests on the horizontal line ACB; a style drops from M, at the centre of the rim, to C at the bottom; a line rises from M to R, the Sun, at the upper right; lines run from M to G and F on the inside of the bowl.

- R (upper right): the Sun
- M (centre of the rim): the tip of the style
- D (left end of the rim)
- E (right end of the rim)
- G (left side of the bowl, upper)
- F (left side of the bowl, lower)
- A (left end of the horizontal line)
- C (bottom of the bowl, on the horizontal line)
- B (right end of the horizontal line)

[Margin: Problem 4, from Father Kircher.]

Father Athanasius Kircher suggests the fourth way, near the beginning of Book 8 of the *Ars Magna Lucis et Umbrae*. It uses a scaphe DCE, again full of water and standing upright on the horizon ACB, but with an astronomical quadrant DMH standing on it from outside, so that the side HM and the semidiameter MC lie in the perpendicular HC, which passes through the centre M of the scaphe. Let the limb DRH of the quadrant be divided into degrees and minutes, just like the quadrant EC of the scaphe, which lies in the same plane as DRH. One observer, then, turning the dioptra PR about the centre M towards the Sun, is to note the arc DR that the dioptra cuts off. DR will be the apparent altitude of the Sun, which, subtracted from 90 degrees, will give the arc RH, that is, the angle RMH, which measures the inclination of the ray RP taken from the perpendicular HC. At the same moment the partner in the observation, with his eye applied at K, is to note the point F, which lies on the cathetus of incidence and at which the refracted ray PF ends — the ray propagated from the unrefracted ray PR that passes through the slits or the little

tube of the dioptra — and he is to count the degrees and minutes of the arc FE. For the difference, or excess, of the arc FE over the arc RH, the measure of the true inclination — that is, the arc FG, or the angle FMG, equal to the angle RMS — will be the refraction sought, and the angle CMF will be the refracted angle. I shall give the anaclastic tables that Father Kircher made by this device at the end of the Section, Table V.

Diagram. Woodcut of Kircher's apparatus (the fourth way), in the left column: a hemispherical scaphe with an astronomical quadrant standing on it. The quadrant's arc runs from D at the left up to H at the top; a diameter D–P–M–K–E crosses the rim of the scaphe; a vertical runs from H through M to C; the horizontal line ACB lies beneath. Lines run from R through M, and from M to F and G inside the bowl.

- S (on the quadrant's arc, near the top)
- H (top of the quadrant's arc)
- R (on the quadrant's arc, below S)
- D (left end of the diameter)
- P (on the diameter, left of M)
- M (centre)
- K (on the diameter, right of M)
- E (right end of the diameter)
- G (inside the bowl, right)
- F (inside the bowl, below G)
- A (left end of the horizontal line)
- C (bottom of the bowl, on the horizontal line)
- B (right end of the horizontal line)

[Margin: Problem 5, our own.]

Our fifth way is as follows. Let a box ABP be made of sides of which at least two — namely AP and BK — are plane, opaque, four-sided, equal and parallel, standing perpendicular on the horizontal bottom AC. On the second side, BK, let there be a straight line CK, perpendicular to the horizon and divided into equal parts as small as possible. Let a gold coin be placed on the bottom of the box where the bottom meets the line CK, that is, at the place C, where there is to be a little loop; from it a thread, stretched over the edge D of the side AP, is to be tied to a staff or rule HE perpendicular to the horizon, so that E, D, C lie in one and the same straight line. For an eye placed at E will not see the gold coin, since the opacity of the side AP stands in the way, or at most it will see the extreme edge of the coin. Then fill the box with water up to the top edges DK. For the central ray of the gold coin, which is propagated unrefracted through the water from C to D, will, passing at its exit into the rarer air, be refracted at D, departing from the perpendicular AD etc., so that the refracted ray is DF. With the eye placed at F, therefore, you will see along the ray FDR the point R; note it carefully, taking account of the semidiameter of the coin. With these noted, the height AD and the length AB — known beforehand in the same kind of parts as make up CK — will, in the triangle

ADB, right-angled at A by the construction of the box, give by proposition 5 of plane right-angled triangles the angle ABD, which is equal to the angle next to it at C, and the angle ADB; and the angle ADB will measure the inclination of the ray BD, by which it departs from the perpendicular AD. Again, if from the height of the side CK, which is equal to the height AD, you subtract the parts CR, and use the length AB, that is DK, as above, then in the right-angled triangle DKR you will find, by the same proposition 5, the angle DRK, greater than the angle DCK. The difference of these angles, therefore, will be, by Euclid I.32, the angle CDR, which is equal to the refraction FDE by Euclid I.15. Moreover, the refraction towards the perpendicular is as great as that away from the perpendicular, by axiom 6 of chapter 2. Nor will the enlarged image of the coin disturb the operation, for the intervals between the parts of the line CK are enlarged in the same way.

Diagram. Woodcut of the rectangular box of the fifth and sixth ways, at the head of the right column. To the left stands a vertical staff with E and F at its top and H at its foot; from F a line runs to P and D on the upper left edge of the box and on into it. The box has K at its top right corner, R on its right inner side above C, A at its bottom left and B at its bottom right.

- E (top of the staff)
- F (top of the staff, below E): the eye, or the lamp flame
- H (foot of the staff)
- P (upper left edge of the box)
- D (upper left edge of the box, below P)
- K (top right corner)
- R (right inner side)
- C (foot of the right inner side)
- A (bottom left)
- B (bottom right)

[Margin: Problem 6, our own.]

Our sixth way is the converse of the preceding one. I use the preceding box ABP, but first filled with water. I fix the flame F of a lamp to the staff HE — or rather insert it in a tube, so that it can be moved up and down and stopped at any point I like — and cover it with a plate pierced by a large round hole, so that by this device the image of the pyramidal flame comes out round and imitates a star or a gold coin. Then I move the lamp upwards until the ray FD, propagated from the upper edge of the round flame over the edge D of the box and refracted along DC, ends the straight shadow AC at C, where the bottom AC meets the side CK. Then, with the water let out — that is, with the box full of air alone — I note the point R at which the same ray FD, but now unrefracted along DR, ends the shadow in a straight line. In the right-angled triangle DAB, from the known sides AD and AB, I find as above the angle ABD, that is ACD, and the angle ADC, which, together with the angle CDR that is still to be found, measures the inclination of the ray FD from the perpendicular AD etc., an inclination equal to this angle by Euclid I.15. The complement of the angle

ACD to a right angle will be the angle DCK. Again, having subtracted the parts CR from the side CK, and keeping the length DK, I find in the triangle DKR, right-angled at K, the angle DRK, which I find greater than the interior angle KCD. Their difference, therefore, by Euclid I.32, is the other interior angle CDR, that is, the measure of the refraction towards the perpendicular, whose complement to the inclination is the refracted angle. In this way I measure the refractions of a ray going out of water and of a ray entering water even at night, let alone in cloudy weather; and I need neither a partner nor the altitude of the Sun or of any star.

Probl. 7. 8. 9. 10. 11. Refractionem Radij ex Aere in Vitrum aut Cryſtallum ingredientis factam, metiri modis 5. *Problems 7, 8, 9, 10, 11. To measure, in 5 ways, the refraction undergone by a ray passing from air into glass or crystal.*

[Margin: Problem 7, from Ibn al-Haytham and Vitellio.]

The first way is suggested by Ibn al-Haytham, Vitellio and Father Maignan: by the round instrument mentioned in Problem 1, but not explained by us because it is too laborious.

[Margin: Problem 8, from Kepler and Cabeo.]

The second is the one taught by Kepler and Cabeo. It differs from Problem 2 only in that the box ABCD shown there is filled with a cube of glass or crystal; everything else is to be done in exactly the same way.

[Margin: Problem 9, from Descartes.]

The third way is taught by Descartes, *Dioptrics*, chapter 10. It is very laborious but full of ingenuity, and suited to finding the vertex and the foci of a hyperbolic glass; we therefore reserve it for the book on optics.

[Margin: Problem 10, our own.]

Our fourth way is as follows. Let there be a glass vessel made of square sides, equal and parallel, of the kind that the four-sided flasks roughly are, in which wine is usually carried in carriages or ships, set into the little cells of cases. Let this vessel be represented, in whatever way, by the figure

[Margin: Order of the problems.]

below, ABCD. Let this vessel be filled with water and its bottom BC set on the horizon, so that the sides AB and DC, parallel to each other, stand perpendicular to it. Next, let the side AB, which is to be turned towards the Sun, be covered with a plate of paper or of metal, on which the straight line AB, perpendicular to the horizon, is divided into equal parts as small as possible; and in the same plate let there be a hole G, through which the straight line GM is understood to be drawn parallel to the horizon, its length known in parts of the kind in which the height BG, or CM, equal to it, is known. Now let the Sun's ray S pass through the hole G. If the vessel were not full of water above the point G, the ray would pass straight to K, and the Sun's apparent altitude would be measured by the angle GKI, since IK must be understood as parallel to the horizon. But because it passes through glass, then through water, and

again from the water into glass, the ray will appear to an eye looking from outside to end at the point R; and the refracted ray will be GR, and the angle of refraction KGR. At the same moment as one of the observers marks the point R, let the other take the apparent altitude of the Sun, so that through it the angle GKI becomes known, whose complement to a right angle, namely the angle IGK, equals the angle SGA, that is, the complement of the inclination, to be reckoned from the perpendicular PG. We must now find the angle GRH, made by the refracted ray GR and the straight line HR parallel to the horizon. This angle measures the Sun's apparent altitude under the force of the refraction, and its difference from the angle IKG measures the refraction itself. That angle is easily found by proposition 5 of plane right-angled triangles, in the triangle GRH, right-angled at H: for HR is given, equal to GM, and GH, equal to MR, measured with the help of compasses, the compasses being carried over to the straight line AB, divided into the smallest parts. Nor is it an obstacle that a threefold refraction takes place here — the first from air into glass immediately after the hole G; the second from glass into water; the third from water into glass at the point R. For since the sides of the vessel are supposed to be of equal density, the second refraction, away from the perpendicular, is corrected by the third, towards the perpendicular, and the ray that passes through the water is restored to the position it had when it passed through the first glass side; for the refraction of a ray entering and of a ray leaving is as great, etc.

[Margin: Problem 11, our own.]

We have, however, used a fifth way, so that no one should be in doubt about the fourth. We had a four-sided glass cast, bounded by two square faces, parallel and equal, whose side BG, or CM, as also BC and GM, contains in its length 4½ inches of the ancient Roman foot; while the breadth of the other four surfaces that bound the glass — that is, the lesser thickness of the glass — is very nearly one inch. This glass, fastened at its lower part BC to a brass rule, can be turned with the rule towards the Sun in such a way that, by raising the rule, each of the sides GB and MC can nevertheless be set perpendicular to the horizon with the help of a very fine plumb line. Further, the side, or surface, to be turned towards the Sun is covered with a very thin metal plate, in which the Sun's ray is received through the hole G. The ray does not go straight to K but is refracted upwards to the point R, towards the perpendicular PGM. This point is seen clearly enough on the surface of the side CM, like a little shining star, and is marked there by placing a grain of wax opposite it, or by a thread HR carried through it round the outside of the glass, so that the thread HR is parallel to the horizon. At the same moment the altitude of the Sun is taken by another partner, and so the angle GKI becomes known. Since the sides HR and GH are also known in the smallest parts, as above, the angle GRH is sought, by proposition 5 of plane right-angled triangles, in the triangle GHR, right-angled at H; its difference from the angle GKI is the refraction sought. Further, the angle IGK — that is, the complement of the Sun's altitude GKI to

a right angle — is equal to the angle of incidence SGA; and the complement of this will be the inclination.

Diagram. Woodcut of the square glass vessel in elevation, in the left column. A rectangle with A at top left, D at top right, B at bottom left, and a fourth corner at bottom right; horizontal lines cross it at E–F, G–M (dotted), H–R and I–K. Outside at the left, S, with a small circle below it, sends a ray in at G, which runs down to R; a second line runs from G to K. P stands outside beside G.

- A (top left corner)
- D (top right corner)
- S (outside, upper left): the Sun
- E (left side, upper)
- F (right side, upper)
- P (outside, left of G)
- G (left side): the hole
- M (right side, level with G)
- H (left side, below G)
- R (right side, level with H)
- I (left side, below H)
- K (right side, level with I)
- B (bottom left corner)
- C [?] (bottom right corner; the letter reads as C or G)

You will, however, work more surely and accurately if you apply to the side AB a plate whose upper part, parallel to the horizon, projects outside the glass — to the right, for example — so that its shadow, cast towards K, grazes the top of another plate applied at right angles to the glass at the side DC and projecting outside the glass to the same right side. For it will happen that the shadow cast by the upper part G of the first plate is refracted through the glass and rises to R; you will mark this point more clearly if you apply a sheet of paper to the glass itself at CD. With very sharp compasses, then, mark the interval KR in parts of the kind in which the whole IK, or HR, is known. If at the same moment of time you have observed with an exact instrument the altitude of the Sun's upper limb, that is, the angle IKG, equal to the angle of inclination PGS, you will then be able, by trigonometry, to find the side GI in the right-angled triangle GIK. When HI, which is as great as RK, is subtracted from it, GH will remain; with this and with HR, in the right-angled triangle GHR, find the angle HRG. Subtract it from the angle GKI, and there will remain the refraction sought, KGR, for the observed altitude of the Sun's upper limb, that is, for the given inclination PGS. By these ways, but chiefly by the last, I investigated with Father Francesco Maria Grimaldi the refractions to be given in Table VI at the end of this Section.

12. Probl. Data vna radij Inclinatione, & Angulo refracto, inclinationi vni reſpondente; inuenire Angulum refractum, alteri datæ inclinationi

reſpondentem, in eiuſdem denſitatis diaphano. *Problem 12. Given one inclination of a ray, and the refracted angle corresponding to that one inclination, to find the refracted angle corresponding to another given inclination, in a transparent medium of the same density.*

[Margin: Problem 12.]

Make it so that, as the sine of the given inclination is to the sine of the given refracted angle, so the sine of the other given inclination is to the sine of the other refracted angle. As in the following figure, let MBN be the perpendicular and B the point of incidence; let the rarer medium be within CMD and the denser within CND, so that the ray KB is refracted along BL and the ray AB along BI. Therefore, as the sine KM of the given inclination KBM is to the sine NL of the refracted angle GBL, so the sine AH of the other given inclination ABH is to the sine GI of the other refracted angle GBI. Here, however, we are speaking of simple refraction, arising from the mere difference of specific density, according to what was said in chapter 3, from proposition 4 to 9.

Diagram. Woodcut of a circle in the right column, with horizontal diameter CBD (the surface between the two media) and vertical diameter MHBN (the perpendicular). Incident rays run from K and A on the upper left of the circumference to the centre B, with horizontals K–M and A–H to the perpendicular; refracted rays run from B to I and L on the lower right, with short horizontals to G and N near the foot of the perpendicular.

- K (upper left, on the circumference)
- M (top of the vertical diameter)
- A (left, on the circumference, below K)
- H (on the vertical diameter, below M)
- C (left end of the horizontal diameter)
- B (centre)
- D (right end of the horizontal diameter)
- G (on the vertical diameter, lower)
- N (foot of the vertical diameter)
- I (lower right, on the circumference)
- L (lower right, on the circumference, below I)

EXEMPLVM

Example.

In Vitellio, Book 10, proposition 8, at an inclination of 80 degrees the refracted angle from air to water is 50 degrees. Now let the refracted angle corresponding to an angle of inclination of 45 degrees be sought. Make it, then, that as the sine AH, 9848077, of the inclination ABH of 80 degrees is to the sine GI, 7660444, of the refracted angle GBI of 50 degrees, so the sine, 7072061, of the inclination KBM of 45 degrees is to the sine NL, 5499990 — namely, of the refracted angle NBL, 33° 22′. Therefore the refracted angle corresponding to an inclination of 45 degrees is 33° 22′, considering only the difference of density of the transparent media.

13. Probl. Data Inclinatione, & Refractione illi correſpondente, inuenire Refractionem ſimplicem alteri datæ Inclinationi reſpondentem. *Problem 13. Given an inclination and the refraction corresponding to it, to find the simple refraction corresponding to another given inclination.*

[Margin: Problem 13.]

Subtract the given refraction from the given inclination, and the refracted angle will remain; with it, by Problem [?] 12, find the refracted angle for the other

[Margin: Order of the problems.]

given inclination, and subtract it from this other inclination: the remainder will be the refraction corresponding to the other given inclination — but the simple refraction, not the integral one. Understand all this of a ray entering a denser medium; otherwise, add the refraction to the inclination to obtain the refracted angle.

EXEMPLVM.

Example.

In Vitellio's table, Book 10, proposition 8, a refraction from air into water of 30° corresponds to an inclination of 80°; subtracted from 80°, this leaves a refracted angle of 50°. From these values, for an inclination of 45°, a refracted angle of 33° 22′ is obtained by Problem 12; subtracted from 45°, this leaves a refraction of 11° 38′. Therefore a refraction from air into water of 11° 38′ belongs to an inclination of 45° — if Vitellio's experiments hold good, and if we are speaking of the simple refraction that arises from the mere difference of density.

14. Probl. Datis duobus Angulis refractis in eiuſdem denſitatis diaphano, & Inclinatione vni refractorum reſpondente, inuenire Inclinationem, alteri refracto angulo debitam, ſpectata ſimplici refractione. *Problem 14. Given two refracted angles in a transparent medium of the same density, and the inclination corresponding to one of the refracted angles, to find the inclination belonging to the other refracted angle, considering simple refraction.*

[Margin: Problem 14.]

Make it so that, as the sine of one refracted angle is to the sine of its inclination, so the sine of the other refracted angle is to the sine of the inclination corresponding to its refracted angle. This is said of simple refraction.

15. Probl. Data Refractione integra ſeu compoſita, Inclinationi datæ reſpondente, inuenire Refractionis partes componentes, & ſimplicem refractionem eidem inclinationi conuenientem methodo Kepleri. *Problem 15. Given the integral, or compound, refraction corresponding to a given inclination, to find the component parts of the refraction, and the simple refraction belonging to that same inclination, by Kepler's method.*

[Margin: Problem 15.]

What compound refraction is was stated in chapter 3, from proposition 4 to 9. Now, following Kepler in the *Astronomiae Pars Optica*, chapter 4, proposition 8: subtract the given refraction from its given inclination, and the refracted angle will remain; find its secant. Then make it so that, as the secant of that angle is to the whole sine, so the given compound refraction is to the simple refraction belonging to the same given inclination. He also gives this method in the *Epitome of Copernican Astronomy*, p. 65; but whether it always agrees with observations will become clear from what is to be said in Problem 49 against Kepler, especially in reason 4.

EXEMPLVM.

Example.

For an inclination of 80° of a ray entering from air into water, Vitellio, above, assigns a refraction of 30°. Taking this from 80° leaves a refracted angle of 50°, whose secant is 155,572. Now if one makes it so that, as 155,572 is to the whole sine 100,000, so 30° is to another quantity, 19° 17′ will be found. Therefore the simple refraction belonging to an inclination of 80° is 19° 17′. The remaining addition to the refraction, which is 10° 43′, accrues to it because the medium meets the ray refracted under the water as denser than it meets the direct ray, in the proportion of the secant 155,572 to the radius 100,000.

16. Probl. Data Inclinatione, eiuſque refractione integra ſeu compoſita; reperire Refractiones integras ſeu compoſitas, cæteris inclinationibus in eodem diaphano reſpondentes, methodo Kepleriana. *Problem 16. Given an inclination and its integral, or compound, refraction, to find the integral, or compound, refractions corresponding to the other inclinations in the same transparent medium, by Kepler's method.*

[Margin: Problem 16.]

By Problem 13, find the simple refraction corresponding to any other given inclination. Then subtract the refraction found from its given inclination, just as if it were integral, and there will remain a refracted angle not yet fully known. Find its secant; multiply the excess of the secant over the whole sine by the simple refraction found, and divide the total by the whole sine: the part of the refraction that comes from the secants will result. Next subtract this part again from the refracted angle not yet fully known, and a diminished refracted angle will remain. Find its secant, multiply the excess of the secant over the whole sine by the simple refraction found in the first operation, and divide the total by the whole sine; there will remain the portion from the secants. If it differs insensibly from the portion found before, the problem will be complete; but if they differ by many minutes, let the search for the secant be repeated until no discrepancy remains between the refraction arising from the secants and the one found in the immediately preceding operation. But the obscurity of the rule will be cleared up by an example — a rule set out by Kepler in the

Astronomiae Pars Optica, proposition 8, in a form even more obscure than it is here with us.

EXEMPLVM.

Example.

For an inclination of 50° of a ray entering through the air, let the simple refraction found be 12° 4′. When this is subtracted from 50°, there remains a refracted angle of 37° 56′, not yet fully known, whose secant is 126,787; and its excess over the whole sine, 100,000, is 26,787. This, multiplied by 12° 4′ and divided by 100,000, gives 3° 14′ for the part of the refraction arising from the secants. Again, let 3° 14′ be subtracted from 37° 56′: there remain 34° 42′, whose secant is 121,633, and the excess over 100,000 is 21,633. This, multiplied by 12° 4′ and divided by 100,000, gives 2° 37′, a portion that differs too much from the one found before, 3° 14′. So let 2° 37′ again be subtracted from 37° 56′: there remain 35° 19′, whose secant is 122,554, and the excess over the whole sine is 22,554. This, multiplied by 12° 4′ and divided by 100,000, gives a portion of 2° 43′, still differing from the one just found, 2° 37′. So again subtract 2° 43′ from 37° 56′: there remain 35° 13′, whose secant is 122,402, and the excess over the whole sine is 22,402. This, multiplied by 12° 4′ and divided by 100,000, gives a portion of 2° 42′, differing insensibly from the one just found, 2° 43′. Keep this, then, as the true portion of the refraction arising from the secants, and add it to the simple refraction found in the first place, which was 12′ 4′ [?]; the total refraction will then be 14° 44′, corresponding to an inclination of 50° — granted that Vitellio, from an unreliable experiment, puts it at 15°. Kepler compared the integral refraction deduced a priori in this way, in a small table, with the one Vitellio gave from experiment; we shall give that table at the end of this section as number VIII, together with another small table of refractions in air. But whether this is demonstrative will become clear from what is to be said.

17. Probl. Quãta ſit proportio Vitri ad Aquam in Denſitate, ex ponderis coniectura inueſtigare. *Problem 17. To investigate, by inference from weight, how great the proportion of glass to water is in density.*

[Margin: Problem 17.]

Although it is not entirely evident that density stands to density as weight to weight, other things being equal, this nevertheless seems next to evident. For, as Father Emmanuel Maignan says in Book 1 of the *Perspectiva horaria*, proposition 24, running through all heavy and light things, a large quantity of matter in a small bulk always has heaviness attached to it, while a small quantity of matter in a large bulk always has lightness. From this he rightly concludes that ice is rarer than water: since it floats on water, it is lighter than liquid water of equal bulk, by what Archimedes demonstrated in his book on things carried in water (*On Floating Bodies*). With this reasonably assumed, I weighed the very glass that I have used most often for measuring refractions, and its weight

was exactly 27 ounces. This glass is a parallelepiped as nearly as possible an inch thick, with two square faces, flat and parallel, each side of which is 4½ inches of the ancient Roman foot long. I built a vessel fit to hold exactly this; I weighed it full of water, and when the weight of the vessel was taken away, the water came to 10 ounces exactly, without the addition of any drachma — for the balance that I usually use, the drachmas themselves, and indeed, if one likes, the half-drachmas, it shows distinctly. I therefore concluded that the heaviness of glass is to the heaviness of water in the same bulk as 27 to 10, and consequently that the density of glass is to that of water very nearly as 27 to 10, or nearly as 60 to 22.

[Margin: The proportion of glass to water in weight and density.]

[Margin: Order of the problems.]

[Margin: The proportion of glass to water in weight and density.]

18. Probl. Quanta ſit Proportio Denſitatis Aquæ ad Aeris noſtratis denſitatem, duplici coniectura attentare, vna Kepleri, altera Noſtra. *Problem 18. To attempt, by a twofold conjecture — one Kepler's, the other our own — how great the proportion of the density of water to the density of our air is.*

[Margin: Problem 18.]

Mersennius, in his *Optics*, Book 3, part 4, proposition 32, teaches from Kepler's *Astronomiae Pars Optica*, chapter 4, proposition 10, that water is denser than air 1,533,304,682 times. For Kepler proceeds as follows. The simple refraction of a ray from air into water, at an inclination of 80°, is 19° 17′, by Problem 13. But at the same inclination of 80° the simple refraction of a ray from the aether into our air is 59″. Therefore the simple refraction from the aether into water would be 19° 18′. So the proportion of these refractions is that of unity to 1177⅔. If you form a cube from each term of the proportion, the proportion of the density of air to the density of water will be that of 1 to 1,533,304,682. Yet Kepler adds in the same place, p. 128: "I myself doubt, however, whether it would not suffice to square the terms of the proportion, or rather to multiply them into their own roots, etc.; but neither is this proportion perpetual, nor is it to be ascribed to any part of the air except the one that occupies the highest spaces of the purified sky." From this it is apparent how uncertain and merely arbitrary this method of Kepler's is.

[Margin: Kepler's method for this proportion, but uncertain.]

Here indeed that saying of Horace holds: "It is something to get so far, if one may not go beyond." For in this matter one may proceed by conjectures up to a point, but not beyond. I therefore tried to reach the proportion in density from the proportion in weight, since it is otherwise certain that the same heavy body, when denser, other things being equal, is also heavier than itself when rarer. Accordingly, I hung a bladder of very thin membrane, well inflated, from a balance of such exact sensitivity that it shows the twentieth part of one grain (I speak of the grains with which goldsmiths weigh out small weights of gold), and I found its weight to be 200 grains, that is, 2 drachmas, 2

scruples and 8 grains. But when all the air had been let out of the same bladder and it had been well pressed together, I found it to be 197 grains; so the excess due to the air, arising from the condensation in inflating, was three grains. For although air has some heaviness, as many assert together with Cardano [Girolamo Cardano] and with Kepler in the *Astronomiae Pars Optica*, chapter 4, proposition 3, it still cannot be weighed unless it is condensed, or unless someone moves up into the aether, since the elements do not gravitate in their own place. Further, I found that the same bladder, filled with well water, held 4 pounds and 3 ounces of water. There were therefore 51 ounces of water, that is, 408 drachmas, or 1224 scruples — namely 29,376 grains. Hence the condensed air, or at any rate the excess arising from condensation, was to the water in weight — and, not improbably, in density — as 3 to 29,376, or as 1 to 9792, or nearly 9800. Granted, then, that the air around us, condensed no further than its ordinary thickness, would be lighter and so rarer than when condensed and forced into the bladder; still, it is not so much lighter that it could stand to water in the proportion Kepler wants. Rather, it is about as 1 to 10,000.

[Margin: Our method, of whatever kind, but more probable than Kepler's.]

19. Probl. Proportionem inter Vitri & noſtratis Aeris Denſitatem, ex coniectura ponderis attentare. *Problem 19. To attempt, by inference from weight, the proportion between the density of glass and that of our air.*

[Margin: Problem 19.]

By Problem 17, the weight of water was found to be to the weight of glass in the same bulk as 22 to 60, as nearly as possible; and by Problem 18 the weight of air enclosed in a bladder was found to be to water in the same bulk nearly as 1 to 10,000. Now, as 22 is to 60, so is 10,000 to 27,272. Therefore the weight of the air in question is to the weight of glass — and so, not improbably, the density of the former to the density of the latter — as 1 to 27,272. But now let there be a limit to these subtleties, and let us come to the refractions of the stars from the aether into the air, which are to be investigated by observation; for this is what we chiefly aim at here, intending to treat afterwards, in the book on optics, of refractions from air into glass, with a view to the making of spectacles and telescopes.

Probl. Refractionem in circulo altitudinis, Sideris à Parallaxi ſenſibili immunis, inuenire, Datis præter obſeruatam ſideris Diſtantiam à vertice, vel Altitudinem; 20. Altitudine Poli, Declinatione & Azimutho ſtellæ 21. Altitudine Poli, Declinatione, & Diſtantia Aequatoria Stellæ à Meridiano 22. Altitudine Poli, Azimutho ſtellæ, & Diſtantia Aequatoria à Merid. 23. Stellæ Azimutho, Declinatione, & Diſtantia Aequatoria à Merid. *Problem. To find the refraction, in the circle of altitude, of a star free from sensible parallax, given, besides the observed distance of the star from the vertex, or its altitude: 20. the altitude of the pole, and the declination and azimuth of the star; 21. the altitude of the pole, the declination, and the equatorial distance of the star from the meridian; 22.*

the altitude of the pole, the azimuth of the star, and the equatorial distance from the meridian; 23. the azimuth of the star, its declination, and its equatorial distance from the meridian.

In the accompanying figure let HVRD be the meridian, HAR the true horizon, and VAD the semicircle of a vertical circle, in which a fixed star, by the force of refraction, first appears at the point A of the horizon — for, owing to the lack of sensible parallax, the apparent horizon does not differ at all from the true one in this case. But let its true place be at B; the refraction to be found will then be the arc AB, and it will be called the horizontal refraction [Refractio Horizontalis]. Then from the true place of the star let the arc of declination BP be drawn to the pole of the world P. This arc will be either the complement of the declination, if the star declines toward the visible pole, or the sum of the quadrant and the declination of the star toward the invisible pole; or, if it declines toward neither side, BP will be 90°. But if the star appears above the horizon, as at C, but is truly below it, as at A, draw from A the arc of declination AP.

Diagram. Woodcut of a circle in the right column, representing the meridian. The horizontal diameter is the horizon HAR; the vertical diameter runs from V at the top through C, the centre A and B to D at the foot. P stands on the circumference to the upper right, and straight lines run from P down to the left to the vertical diameter.

- V (top of the vertical diameter)
- P (upper right, on the circumference)
- C (on the vertical diameter, above A)
- A (centre, on the horizontal diameter)
- H (left end of the horizontal diameter)
- R (right end of the horizontal diameter)
- B (on the vertical diameter, below A)
- D (foot of the vertical diameter)

[Margin: Problem 20.]

First, then, from the data of Problem 20: in the triangle VPB the side VP is given, which is the complement of the altitude of the pole, and the side BP, from the declination of the star, known as above; and the angle BVP opposite it, known from the azimuth HVA, of which BVP is the supplement to 180°. Therefore, by [proposition] 2 of oblique-angled spherical triangles, find the third side, or base, VB, which will be the true distance of the star from the vertex; and if the angle at P is obtuse, use the vicarious triangle, according to what was said in Section 1, chapter 4, scholium 1. From this subtract the visible distance VA, which, if the star appears on the horizon at the time the azimuth is observed, as we suppose, will be 90°; and the remainder will be the arc AB, that is, the horizontal refraction. But if the star appears above the horizon, as at C, but is at A, use the triangle VAP, in which VP, AP and AVP are given, as above; and from the base VA, once found, subtract the visible distance from the vertex, VC, and you will have the refraction AC — though no longer the

horizontal one. This first way, however, is useless for distinguishing morning refractions from evening ones, as I shall say at the end of Problem 25.

[Margin: Problem 21.]

Second: in the triangle BVP of the earlier case, if, according to the data of Problem 21, VP and EP are given, as above, together with the angle VPB, that is, the equatorial distance of the star from the meridian — obtained either by reckoning the time from the moment the star appeared until its arrival at the meridian, or in some other way — find the base VB by [proposition] 4 of oblique spherical triangles, take from it the observed distance AV, that is, 90°, and you will have the horizontal refraction AB. But if the star appears above the horizon, use the triangle AVP, in which VP and AP are given, as above, and the included angle APV; with these, by the same [proposition] 4, seek the base VA. Subtract from it the observed distance from the vertex, VC, and the remaining arc AC will be the refraction belonging to such a distance.

[Margin: Problem 22.]

Third: in the earlier triangle BVP, if, according to Problem 22, there are given VP, the complement of the altitude of the pole, and BVP, known from the azimuth as above, and VPE, the equatorial distance from the meridian, seek the base VB by [proposition] 11 or 12 of oblique spherical triangles; subtract from it the quadrant VA, and you will have AB, the horizontal refraction. But if the star appeared at C, use the

[Margin: Order of the problems.]

triangle AVP, and by the same [proposition] 11 seek VA, since VP and the adjacent angles AVP and APV are given, as above; for when the visible distance of the star, VC, is taken from AV, there will remain the refraction AC belonging to such a distance.

[Margin: Problem 23.]

Fourth: in the earlier triangle BVP, if, according to Problem 23, BP is given from the declination, the angle BVP from the azimuth, and the angle VPB of the equatorial distance, find the side VB by [proposition] 1 of oblique spherical triangles; take from it the quadrant AV, and you will have the horizontal refraction AB. But if the star is seen at C, use the triangle VAP: with AP, known from the declination, AVP, the supplement of the azimuth to 180°, and VPA, the equatorial distance from the meridian, find the side VA by the same [proposition] 1; take from it the visible distance VC, and the refraction AC belonging to such a distance of the star from the vertex will be known.

[Margin: Advice on observation.]

How we usually observe the azimuths of the stars, or their equatorial distance from the meridian, we indicated in Book 3, chapter 9.

Probl. 24. & 25. Refractionem Horizontalem Solis aut cuiuſuis Sideris, minorem refractione Parallaxim habentis inquirere; ac Refractionem Solis Tychonicam eiuſque quadruplex vitium emendare. Datis ternis quibuſue, quæ Problemate 20. 21. 22. aut 23. recenſui, & Parallaxi horizontali

ſideris. *Problems 24 and 25. To investigate the horizontal refraction of the Sun, or of any star whose parallax is smaller than its refraction; and to correct Tycho's solar refraction and its fourfold defect — given any of the sets of three quantities that I listed in Problem 20, 21, 22 or 23, together with the horizontal parallax of the star.*

[Margin: Problem 24.]

In the following figure let HVRD be the meridian, HAR the true horizon, and FBM the apparent horizon parallel to it; and from V, the point of the vertex, let VAD, the semicircle of a vertical circle, be drawn, in which the Sun is in reality at E but, by the force of horizontal refraction, appears at B, a point of the physical horizon. For to one observing it from the surface of the terrestrial globe it will seem to be a whole quadrant from the vertex; that is, the arc VB will be taken for a quadrant, although in reality it is less than a quadrant by a deficit as great as the arc AB — which, in any star, is as great as the horizontal parallax, by what was said in Book 2, chapter 2, scholium 2. Then, when the arc EP, which is a portion of a circle of declination, has been drawn from E[62], there will arise the triangle VEP, in which the true distance of the Sun from the vertex, VE, must always be found. If you take the quadrant VA from it, you will have the simple refraction AE, that is, the refraction not corrective of parallax; therefore the horizontal parallax AB must be added to AE itself, so that the whole effect of the horizontal refraction may be had, as Tycho prescribes in volume 1 of the *Progymnasmata*, p. 94. Or rather, take the horizontal parallax AB from the quadrant VA, take the remainder VB from the true distance VE, and you will have the integral horizontal refraction EB.

Diagram. Woodcut of a circle in the left column, representing the meridian. The vertical diameter runs from V at the top to D at the foot. Two horizontal chords cross it: the upper, FBM, the apparent horizon, and the lower, HAR, the true horizon, with B and A where they meet the vertical. K stands on the vertical below V; G and a small o lie between K and B; a small c lies between B and A; E lies on the vertical below A. P stands on the circumference to the upper right, and straight lines run from P down to the left to the vertical.

- V (top of the vertical diameter)
- K (on the vertical, below V)
- P (upper right, on the circumference)
- G (on the vertical, above B)
- o (small, just right of the vertical, beside G)
- F (left end of the upper chord)
- B (on the vertical, at the upper chord)
- M (right end of the upper chord)
- H (left end of the lower chord)
- c (small, on the vertical, between B and A)
- A (on the vertical, at the lower chord)
- R (right end of the lower chord)

[62] printed "eſt E"; construed as ex E

- E (on the vertical, below A)
- D (foot of the vertical diameter)

Further, the true distance of the star, VE, is obtained by one of the four ways given from Problem 20 to 23 inclusive, which there is no need to repeat.

[Margin: Problem 25.]

Now, on the one hand it is certain, other things being equal, that where the inclination of the ray to the surface of the denser medium is smaller, the refraction there is smaller; and that the ray of a star which, at the same distance from the vertex, is farther from the centre of the Earth and of the refracting air is less inclined to the convex surface of the air; and so the refraction of the Sun must be smaller than that of the Moon, the Sun being farther from the Earth. On the other hand, Tycho assigned to the Sun a horizontal refraction of 34′, but to the Moon one of 33′, as is clear from volume 1 of the *Progymnasmata*, pp. 79 and 124. Kepler therefore suspected, in precept 197 of the *Rudolphine Tables*, that Tycho erred in this: that he added to the arc AE a horizontal parallax of the Sun, AB, that was too large, namely 2′ 54″ — as much as he assigns to the Sun at apogee in the same volume 1, p. 80. For that the refractions of the Sun were observed chiefly at the summer solstice, when the Sun is nearest its apogee and through the whole day varies its declination insensibly, is clear from Tycho himself, volume 1 of the *Progymnasmata*, p. 93, and from his observations of the year 1589 sent to me by Father Curtius.

[Margin: First defect of Tycho's refraction.]

But for us the truer parallax of the Sun at that time, from Book 3, chapter 8, is only 27″. Therefore, when 2′ 54″ is taken from 34′, there remains the arc of simple refraction EA, 31′ 6″; if you add 27″ to it, the corrected horizontal refraction of the Sun, drawn from Tycho's observations, comes out at 31′ 33″.

[Margin: Second defect of Tycho's refraction.]

Yet because, owing to a parallax larger than it should be, he also increased the maximum declination of the Sun, as we said in Book 3, chapter 27, number 3, the complement of the maximum declination on the day of the summer solstice, that is, the arc EP, was for Tycho smaller than it should be, and indeed by a minute and a half: for he puts the maximum declination of the Sun at 23° 31′ 30″, whereas it is 23° 30′, as I showed in the same place. Therefore the arc VE too, and so AE, came out a little smaller than it should be — granted, not by as much as the excess of the parallax over the right parallax. So there was indeed a double error, but one was partly corrected by the other, though without Tycho's knowing it.

[Margin: Third defect of Tycho's refraction.]

Finally, Tycho used chiefly Problem 20, which relies on azimuths without the equatorial distance from the meridian; so it is no wonder that he thought the morning refractions equal to the evening ones, although the morning ones are usually greater, since the air is then moister and thicker and not yet cooked through by the heat of the day. Now if, on the day of the solstice, the solstice falls at noon, then the azimuths of the rising and setting Sun will be equal,

and the declinations equal; so on the same horizon entirely equal refractions will be gathered, and yet in reality they can be unequal because of a change in the density of the air. That is why I warned at the end of Problem 20 that this way is useless for distinguishing them. But if the refractions are unequal because of the unequal density of the air, the angle VPE will necessarily be unequal; and so, for distinguishing morning refractions from evening ones, the equatorial distance of the Sun from the meridian, obtained from the time converted into degrees of the equator — which Problems 21, 22 and 23 use — is more useful. Moreover, the nature of the air requires that the summer refractions be somewhat smaller than the equinoctial ones, and these than the winter ones; and we have learned this by experiment, as, God granting, we shall show in volume 3 of the *New Almagest*.

[Margin: Fourth defect of Tycho's refraction.]

But the observer is often, if not on a mountain or a tower, at least in some elevated place, higher than the horizontal line in which the centre of the rising Sun first appears; and so he takes the visible distance of the Sun from the vertex for a quadrant of exactly 90°, and yet it is then greater than 90°. Moreover, it is difficult to make out the centre of the rising Sun or Moon because of its imperfect elliptical figure, more contracted below, as is clear from what was said in Book 3, chapter 2, and Book 4, chapter 15. Therefore, to avoid both inconveniences — which I do not in the least doubt were a hindrance to Tycho — we stretched toward the rising Sun two threads parallel to the horizon and a cubit apart from each other, and so set equidistant from the horizontal plane that the plane they intercepted was parallel to the horizontal plane — and indeed at the height of our eye. With the head resting on a support very well suited to this, the eye unmoved, and the line of sight directed through the two threads so that the farther one was covered by the nearer, and with a green lens tempering the brightness of the Sun, we waited until the centre of the Sun's disc was bisected by both threads at the same moment. At that moment we gave a signal to a companion, who was counting the time with a pendulum, the count having begun from an altitude taken of a fixed star, so that from the time we might obtain the angle at the pole of the world.

EXEMPLVM

Example.

Ex Noſtra Obſeruatione. *From our own observation.*

In this very year in which this volume is sweating under the press, namely 1651, it was decided to repeat the observations of the solar refractions. Among them, on 1651 June 27, in the morning before sunrise, the distance of Aquila from the vertex toward the west was taken, 47° 42′; and at the same moment the counting of the time with the pendulum was begun, up to the rising of the centre of the Sun on the physical horizon of Bologna, marked out by two

threads equally equidistant from the horizon; and immediately, when I had given the

[Margin: Order of the problems.]

signal, Father Francesco Maria Grimaldi ended the count and found that the corrected time of the primum mobile had been $42^{m}\ 33^{s}$, to which correspond 10° 38′ 15″ of the equator. We completed the rest of the working of the calculation as below.

[Margin: First step: for the right ascension and declination of the Sun ☉.]

[Margin: Second step: for the angle at the pole of the world.]

[Margin: Third step: for the refraction.]

Item	Value
The Sun's place at noon on June 26, from our corrected tables, was in ♋ Cancer	4° 31′ 54″
But at noon on the 27th, in Cancer	5° 29′ 2″
Right ascension of the Sun at noon on the 26th	94° 52′ 34″
But at noon on the 27th	95° 58′ 59″
Difference of the right ascensions of the Sun ☉	1° 6′ 25″
Declination of the Sun at noon on June 26	23° 25′ 39″
But at noon on the 27th	23° 23′ 0″
Difference of the declinations of the Sun	0° 2′ 39″
Semidiurnal time of the 27th, in hours	$7^{h}\ 40^{m}\ 0^{s}$
To which there corresponds, from the difference of the declinations	0° 0′ 51″
Therefore the declination of the rising Sun	23° 23′ 51″
And its complement	66° 36′ 9″
To the semidiurnal time add the counted time, $42^{m}\ 33^{s}$; the hours come to	$8^{h}\ 22^{m}\ 33^{s}$
To which there corresponds, from the difference of the right ascensions of the Sun	0° 23′ 1″
Therefore, when Aquila was observed, the right ascension of the Sun ☉ was	95° 35′ 58″
Right ascension of the bright star in Aquila at that moment, from our tables	293° 27′ 54″
Its northern declination	8° 1′ 1″
Its complement	81° 59′ 59″
Complement of the altitude of the pole at Bologna	45° 30′ 30″
Distance of Aquila from the vertex	47° 42′ 0″
Therefore, by Problem 25 of Section 3, the angle at the pole of the world	35° 27′ 0″
When this is added to the right ascension of the western star, the right ascension of mid-heaven becomes	328° 54′ 52″
Its complement to the circle is	31° 5′ 8″
If you add to this the right ascension of the Sun at this moment	95° 35′ 38″
The equatorial distance of the Sun from the meridian becomes	126° 41′ 6″
If you take from this the arc of the equator that rose from the taking of Aquila's altitude to the physical rising of the Sun, and it was	10° 38′ 15″
There remains the true equatorial distance of the Sun at the moment when it physically rose, that is, the angle EPV	116° 2′ 51″
But VP at Bologna is	45° 30′ 30″
And SP, the complement of the true declination	66° 36′ 9″
THEREFORE the true distance EV of the Sun E from the vertex V	90° 31′ 58″
But the visible VB was a quadrant, or	90° 0′ 0″
Therefore the simple refraction of the Sun, EB	0° 31′ 58″
If you add to it our horizontal parallax of the Sun ☉ at that time	0° 0′ 27″
The integral horizontal refraction of the Sun becomes	0° 32′ 25″

Item	Value

If I had added Tycho's parallax of 2′ 54″, but had used Tycho's obliquity of the ecliptic, 23° 31′ 30″, the refraction would have come out at about 34′ 30″. So now, as on other occasions too, we were very glad that our air here at Bologna is of almost the same density at the summer solstice as it is in Denmark, and that the refractions have come out nearly the same for us as for Tycho.

By the same method, from the observations of the preceding years from 1641 up to 1651, we computed the refractions of the Sun, the Moon and the fixed stars about the solstices and equinoxes, as will be seen in Table IX at the end of this Section.

[Margin: Order of the problems.]

26. Probl. Sideris ſupra horizontem apparentis Refractiones in circulo altitudinis inquirere: & Inuenire terminum altitudinis, in quo ceſſant ſenſibiles refractiones, Datis præter Parallaxim ijs, quæ dantur in Probl. 20. 21. 22. aut 23. *Problem 26. To investigate the refractions, in a circle of altitude, of a star that appears above the horizon; and to find the limit of altitude at which sensible refractions cease — given, besides the parallax, the quantities that are given in Problem 20, 21, 22 or 23.*

[Margin: Problem 26.]

In the figure immediately preceding, let the Sun appear at K but be in reality at G. Draw the arc GP; then in the triangle VGP find the arc GV, that is, its true distance from the vertex V, by one of the four ways given from Problem 20 to Problem 23. Next take from the arc VG the visible distance of the Sun from the vertex, namely the arc VK, and there will remain the simple refraction GK; but to this add the parallax belonging to such a distance. For the Sun, being at G, would, if there were no refraction, be seen from the Earth's surface lower down by the force of parallax, say at B. Since, then, it is seen above its true place, at K, the whole refraction is the arc BK, of which the part BG corrects the parallax, while the remainder, GK, raises the star above its true place.

[Margin: First case: for a greater refraction.]

But if the visible distance turns out equal to the true distance VG, that will be a sign that the refraction — say the arc BG — raising the star is equal to, and as great as, the parallax GB that would otherwise depress it; and so G will be the place both seen and true.

[Margin: Second case: for an equal refraction.]

Now if the star is, for example, at G, and yet is seen below, say at B — but not as far below as the parallax, let it be GC, would demand — then the refraction will be smaller than the parallax. In this case the true distance VG, found by one of the four ways given from Problem 20 to Problem 23, must be subtracted from the observed or visible distance VB, and the arc GB will

remain; subtracted from the parallax GC, this will leave the required refraction CB.

[Margin: Third case: for a smaller but sensible refraction.]

Finally, if the star is seen below its true place by as much as the known and sensible parallax demands, and there is no fault in the observation or in the parallax — as, for example, if it is at G and is seen at O, lower by as much as the parallax GO — that is a sign that no sensible refraction intervenes, since nothing diminishes the sensible parallax. And the evidence of this absence of refraction will be if the parallax GO, added to the true distance VG found by one of the four ways already mentioned, makes VO equal to the visible distance. It is reasonable to believe that this is how Tycho found that the ordinary refractions of the Sun and Moon cease to be sensible near 45° of altitude, and those of the fixed stars near 20°, even though neither he nor anyone else has disclosed these problems to us.

[Margin: Fourth case: for a smaller but insensible refraction.]

27. Probl. Refractionem ſimplicem in circulo altitudinis ſecernere à data integra Refractione obſeruata, ſi detur Parallaxis, qua vſus eſt obſeruator in colligenda Refractione integra. *Problem 27. To separate the simple refraction in a circle of altitude from a given observed integral refraction, if the parallax is given that the observer used in deriving the integral refraction.*

[Margin: Problem 27.]

If the integral refraction is greater than the observer's parallax, then when the parallax is subtracted from it the simple refraction will remain. But if they are equal, or the refraction is smaller than the observer's parallax, the given refraction will be simple, and no other separation will be needed — provided that the observer's parallax is correct. But if the refraction is equal to or smaller than the parallax, and the parallax the observer used was larger or smaller than it should be, first find the difference between the correct and the incorrect parallaxes. Add it to the given refraction if the observer's parallax was smaller than the correct parallax; subtract it if larger; and so you will have the correct simple refraction, which will at the same time be integral and will need no separation from another refraction. But if the difference that is to be subtracted from the refraction cannot be subtracted, that is a sign that the observation was disturbed.

EXEMPLVM.

Example.

Tycho — as is found in volume 1 of the *Progymnasmata*, pages 79, 81, 92 and 93 taken together — the Sun's horizontal refractio- [Margin: Order of the problems.]

n he derived as 34′ on the days of the summer solstice, using a parallax of 2′ 54″ for the Sun, which was then almost at its apogee. When I take this from 34′, there remains a simple refraction of 31′ 6″. But since at an altitude of 25°

he puts the parallax at 2′ 38″ and the refraction at 2′ 30″, the simple refraction he deduced was 2′ 30″, for this is how he reasoned: "At the moment when I saw the centre of the Sun through the instruments at an altitude of 24° 59′ 52″, its true altitude by calculation was 25°; but on account of the parallax of 2′ 38″ it ought to have been seen at an altitude of only 24° 57′ 22″. Therefore, by the force of a refraction of 2′ 30″, which corrects the parallax but not the whole of it, it appeared at an altitude of 24° 59′ 52″."

But the parallax Tycho assumed was larger than it should have been: on our hypothesis, on which see Book 3, chapter 8, it was then 18″ at an altitude of 25°. The difference of this from Tycho's 2′ 38″ is 2′ 20″, and this, taken from the refraction of 2′ 30″, leaves a corrected simple refraction of only 10″. By this device we corrected Tycho's simple refractions of the Sun in Table 8 at the end of this Section; but where the integral refraction was greater than the parallax, we added our parallax to Tycho's simple refraction so as to have the integral refraction. Beyond 25° of altitude, however, it was not possible to go.

28. Probl. Refractionem Horizontalem Lunæ, aut ſideris cuiuſuis, minorem parallaxi refractionem habentis, inuenire Datis præter Parallaxim horizontalem ijs, quæ dantur Problemate 20. 21. 22. aut 23. Vel ſi neſciatur ſitne maior an minor, idipſum determinare. *Problem 28. To find the horizontal refraction of the Moon, or of any star whose refraction is smaller than its parallax, given, besides the horizontal parallax, the quantities that are given in Problem 20, 21, 22 or 23. Or, if it is not known whether the refraction is greater or smaller, to determine that very point.*

[Margin: Problem 28.]

The first case is the one we suppose from Tycho's observations, namely when the horizontal parallax is greater than the horizontal refraction. Look, then, at the figure immediately preceding, in which let the Moon be truly at C but appear, by the force of refraction, at B, a point of the physical horizon, at which it would seem to be a whole quadrant from the vertex. Now draw the arc CP, and in the triangle VCP find the true distance VC by one of the ways given from Problem 20 to Problem 23. For if you find it smaller than a quadrant, you will conclude that the refraction BC is smaller than the horizontal parallax AB. Then subtract the true distance VC from the quadrant VA, and subtract the remainder AC from the horizontal parallax AB; for the horizontal refraction BC will remain.

[Margin: First case: for a smaller refraction.]

Second: if the star is at A, a point of the true horizon, but appears by the force of refraction at B, the horizontal refraction AB will be equal to the horizontal parallax BA. The evidence of this will be if, for a star seen at the point B of the physical horizon, the true distance from the vertex, found by one of the four ways already mentioned, is a quadrant.

[Margin: Second case: for the sign of an equal refraction.]

The third case is the one I set out in Problem 24, and its sign is if, for a star appearing in the physical horizon, the true distance from the vertex found by calculation is greater than the quadrant AV — for example, if it is EV.

[Margin: Third case: for the sign of a greater refraction.]

29. & 30. Probl. Refractionem Sideris per obſeruationem altitudinis & declinationis, Data Poli Altitudine, & Parallaxi ſideris, ſi quam habet, veraq. Declinatione, inuenire duobus modis, & Tychonicum Problema ampliare. *Problems 29 and 30. To find the refraction of a star in two ways, by observing its altitude and declination, given the altitude of the pole, the parallax of the star if it has one, and its true declination; and to extend Tycho's problem.*

[Margin: Problem 29.]

Tycho teaches this other way in volume 1 of the *Progymnasmata*, p. 94. He directs that on the day of the summer solstice — on which, that is, the declinations vary insensibly through the whole day — the altitude of the Sun be taken with a large quadrant, so that, when it is subtracted from 90°, the Sun's visible distance from the vertex may be known; and that at the same moment its declination be taken with very large equatorial armillae, properly set up, so that, when this too is taken from 90°, the complement of the declination may be known. And this not once, but from the rising of the Sun up to noon, and from then up to its setting, so that we may be certain of the increase, decrease and cessation of the refractions. When these have been observed, let the figure of Problem 24 be repeated here, in which let the Sun be at C but appear by the force of refraction at G. From these points let there be drawn the arcs CP, which is the complement of the Sun's true declination, and GP, which is the complement of the visible or observed declination.

[Margin: Order of the problems.]

Diagram. Woodcut of a circle in the right column, the figure of Problem 24 repeated, as the text says. The vertical diameter runs from V at the top to D at the foot. Two near-horizontal chords cross it: the upper, F–B–M, and the lower, H–A–R, with B and A where they meet the vertical. K stands on the vertical below V; G stands on the vertical above B, with a small o just to its right; a small c lies between B and A; E lies on the vertical below A. P stands on the circumference to the upper right, and straight lines run from P down to the vertical.

- V (top of the vertical diameter)
- K (on the vertical, below V)
- P (on the circumference, upper right)
- G (on the vertical, above B)
- o (small, just right of G)
- F (left end of the upper chord)
- B (where the upper chord meets the vertical)
- M (right end of the upper chord)
- H (left end of the lower chord)

- c (small, on the vertical between B and A)
- A (where the lower chord meets the vertical)
- R (right end of the lower chord)
- E (on the vertical, below A)
- D (foot of the vertical diameter)

Now in the triangle VGP three sides are given: VP, the complement of the altitude of the pole; VG, the complement of the visible altitude; and GP, the complement of the observed declination. Therefore by [proposition] 14 of oblique spherical triangles you will find the angle VGP; when this is subtracted from 180°, the angle CGP in the triangle CGP will be known. In that triangle GP is given, as above, and CP, the complement of the true declination; therefore by [proposition] 2 of the same triangles the third side, CG, will become known, namely the simple refraction. To this, however, the parallax, say the arc CA, must be added, so that the integral refraction may be had.

[Margin: Problem 30.]

Otherwise: in the triangle GVP, from the three sides given as above, find the azimuthal angle GVP by [proposition] 14 of oblique spherical triangles; then in the triangle CVP find the side VC by [proposition] 2 of the same, for VP, the complement of the altitude of the pole, and CP, the complement of the true declination, are given, and the angle CVP has been found. Afterwards subtract the visible distance VG from the side VC, that is, from the true distance, and the simple refraction GC will remain; if the parallax is added to it, the whole refraction will result. Tycho adds another way, by solving a right-angled triangle after drawing a perpendicular. But it is neither necessary now, for those who are used to logarithms and know how to solve oblique-angled triangles very quickly, nor geometrically exact, since it assumes the external angle VGP equal to the internal opposite angle GCP. So far Tycho. His problem can, however, be adapted to the fixed stars, as being free of parallax, on any night, since their declination does not vary sensibly over one whole night or on neighbouring nights; and it can be adapted to the Sun as well outside the summer solstice, if, according to the hours before or after noon, known in whatever way, the true declination is corrected by the proportional part corresponding to the variation of the whole day. Tycho is, however, deficient in all these problems, because he does not teach what is to be done if the parallax is equal to or greater than the refraction, nor does he teach the sign of these cases, or when sensible refractions cease — which we have supplied, from Problem 26 to Problem 29.

[Margin: Defect of Tycho's problem in the matter of refraction.]

It is worth while, moreover, for testing the reliability of Tycho's table of the Sun's refractions, and for illustrating this problem, to examine one observation out of the 80 observations of Tycho's of the year 1589, from June 11 to 17 — that is, in the new style, from the 21st to the 27th — taken before and after noon with the armillae and the azimuthal quadrants at Uraniborg. These observations, at my request, Father Albertus Curtius [Albert Curtz] of our Society, the man

who by order of the Emperor has been put in charge of bringing out all Tycho's observations, sent me, very generously transcribed from Tycho's journals.

EXEMPLVM

Example.

Ex Tychonicis obſeruationibus. *From Tycho's observations.*

In the year 1589, on June 11, old style, in the evening, with the solstice approaching and due at about midnight, the altitude AG of the Sun's centre was taken with the steel quadrant as 2° 20′, and at that time the declination of the Sun observed with the armillae was 23° 40½′. Therefore the complement of the visible altitude, that is GV, was 87° 40′, and that of the visible declination, or GP, was 66° 19′ 30″; and VP, the complement of the altitude of the pole at Uraniborg, is 34° 5′ 30″. Therefore by [proposition] 14 of oblique spherical triangles the angle GVP was 48° 56′ 40″.

[Margin: First step.]

Next, in the triangle CVP, VP is given, 34° 5′ 30″, and the angle CVP has been found, 48° 57′; and CP, the complement of the true declination, is for Tycho 66° 28′ 30″ but for me 66° 30′. Therefore by [proposition] 2 of oblique-angled spherical triangles there follows the true distance CV of the Sun from the vertex: by Tycho's reckoning 87° 50′ 12″, but by mine

[Margin: Second step.]

[Margin: Order of the problems.]

87° 51′ 47″. When the visible distance VG, 87° 40′, is taken from these, there remains the simple refraction GC: by Tycho's reckoning 10′ 12″, by mine 11′ 47″. If you add to these — to Tycho's, Tycho's parallax at that time, 2′ 54″, and to ours, our parallax, 27″ — the integral refraction comes out: Tycho's 13′ 6″, ours 12′ 4″, much smaller than the refraction that Tycho, in the small table in volume 1 of the *Progymnasmata*, p. 79, gives for an altitude of 2⅓°, for there he gives 19′. Perhaps it is because he used the first way, solving the triangle CPG, where the angle P, being too acute, and GC, being too small an arc compared with the arcs GP and CP, could easily slip an error into the calculation; or because he assumed the angle VGP equal to the angle VCP, as he himself teaches above may be assumed; or because he judged other observations more to be trusted; or because he thought that a small table suited to the morning refractions held good for the evening ones, which, however, are reasonably smaller because of the purer temper of the air, cooked through all day. Wherever this disagreement comes from, I wanted to point it out. For my calculation was clear and accurate to me, and I appeal to the trigonometers.

[Margin: Disagreement of Tycho's small table of refractions with the observations.]

31. Probl. Data Parallaxi Sideris, ſi quam habet, eiuſq. Diſtantia viſa à vertice, ac reliquis ad diſtantiam veram eruendam per Problemata præcedentia requiſitis, Determinare an ſidus ſit infra horizontem verum, an ſupra, an in ipſo horizonte. *Problem 31. Given the parallax of a star, if it has one, its visible distance from the vertex, and the other things required by the preceding problems for deriving the true distance, to determine whether the star is below the true horizon, above it, or on the horizon itself.*

Although this has already been indicated in Problems 26 and 28, a separate rule nonetheless had to be given. First, then, find the true distance of the star from the vertex by Problem 20, 21, 22, 23 or 30; for if it is a quadrant, the star will be on the true horizon; if greater than a quadrant, below the horizon; if smaller than a quadrant, above the true horizon.

[Margin: Problem 31.]

32. Probl. Refractionem Solis Horizontalem ex apparenti ipſius ortu vel occaſu deducere, Data ipſius Parallaxi horizontali, & Altitud. Poli, ac Declinatione Solis. *Problem 32. To deduce the horizontal refraction of the Sun from its apparent rising or setting, given its horizontal parallax, the altitude of the pole and the declination of the Sun.*

First count the time with the help of the pendulum (on which see Book 2, chapter 21) from the apparent rising of the Sun's centre up to noon, or from noon up to its setting, and you will have the apparent semidiurnal time; convert it into degrees of the equator. In this way you will have, in the preceding figure, the angle VPE. With this, with VP, the complement of the altitude of the pole, and with PE, known from the declination — for PE is the complement of the declination if the Sun is on this side of the equator, a quadrant if it is on the equator, or the sum of the declination and a quadrant if the Sun is beyond the equator — with these, I say, find by [proposition] 4 of oblique spherical triangles the true distance of the star from the vertex, that is, the base VE. Subtract the quadrant from it, but add the horizontal parallax to the remainder, and you will have the integral horizontal refraction of the Sun. But if so long a counting of the time is tiresome, take the altitude of a known fixed star about one hour before the rising or after the setting of the Sun, so that by Problem 25 of Section 3 you may have the time before or after noon; but let someone else be on hand at once to begin counting the time up to the apparent rising of the Sun's centre, or from its setting up to the taking of the star's altitude. From this the stretch of time from the apparent rising of the Sun to noon, or from noon to the apparent setting, will be known, and so you will have the apparent semidiurnal time and arc; use it as above.

[Margin: Problem 32.]

33. Probl. Refractionem horizontalem Solis ac Lunæ per Eclipſes Lunæ horizontales attentare methodo P. Bettini, ſed fallaci. *Problem 33. To*

attempt the horizontal refraction of the Sun and Moon by horizontal eclipses of the Moon, by the method of Father Bettini — a fallacious one.

Enough may seem to have been said about this method in Book 5, chapter 5, number 18. Add, however, that if the Moon is truly on the astronomical horizon, diametrically opposite the Sun, the Sun will indeed appear above the physical horizon by the force of refraction, because its horizontal refraction is greater than its horizontal parallax; but the Moon will not appear on the physical horizon, much less above it, because its horizontal refraction, even with the apparent lunar semidiameter added, is smaller than its horizontal parallax.

[Margin: Problem 33.]

[Margin: Order of the problems.]

34. & 35. Probl. Refractiones Solis ac Lunæ extraordinarias ex Eclipſibus horizontalibus aut ortu Solis inexpectato, ſubodorari. *Problems 34 and 35. To scent out extraordinary refractions of the Sun and Moon from horizontal eclipses or from an unexpected rising of the Sun.*

For Problem 34 see Kepler in the *Astronomiae Pars Optica*, p. 137, where from a horizontal eclipse of the Moon seen at Tübingen he infers a refraction of one luminary or the other greater than one degree. For Problem 35, see the same author in the same work, p. 139, and what we said in this Book, Section 3, Problem 14, where we dealt with this fully.

[Margin: Problem 34.]

[Margin: Problem 35.]

36. Probl. Data Refractione Sideris ac Parallaxi, Viſam eius Altitudinem in Veram, ac viciſſim Veram in Viſam ſeu obſeruatori apparentem, conuertere. *Problem 36. Given the refraction and the parallax of a star, to convert its visible altitude into the true one, and conversely the true one into the visible, that is, the altitude apparent to the observer.*

From the visible altitude take the refraction, and to the remainder add the parallax: the true altitude will result. Conversely, from the true altitude take the parallax, and to the remainder add the refraction: the visible altitude of the star will result. Everything is to be done the opposite way if a visible distance from the vertex is to be changed into the true one, or a true one into the visible.

[Margin: Problem 36.]

37. & 38. Probl. Diſtantiam Viſam duorum Siderum in Veram, ac viciſſim Veram in Viſam conuertere, Data Refractione integra & Parallaxi ipſorum (ſi quam habet vtrumq. vel alterum;) & in primo caſu Altitudine viſa ipſorum, in ſecundo autem Altitudine vera. *Problems 37 and 38. To convert the visible distance of two stars into the true one, and conversely the true into the visible, given their integral refraction and parallax (if both or either of them has any), and, in the first case, their visible altitude, but in the second their true altitude.*

Let V be the zenith, from which let the quadrants VA and VB of two vertical circles be drawn, both intercepting the arc AB of the true horizon; let them be produced to H and R and joined by the arc HR. Now let the true place of one star be F, but seen by the force of refraction at C; and the true place of the other G, but seen at D; and let the visible or observed distance be the arc CD, but the true one FG. First subtract the visible altitudes AC and BD separately from 90°, and there will remain the visible distances from the vertex, VC and VD. With these, and with the visible distance between the stars, CD, find the angle V by [proposition] 14 of oblique-angled spherical triangles. Then to the arcs VC and VD add their own refractions, but take away their own parallaxes, so that you have the true distances from the vertex, VF and VG; with these, in the triangle FVG, and with the angle V, find the true distance FG by [proposition] 5 of the same triangles. You have an example in Book 6, chapter 11, at the end of Problem 8, in the distance of the Sun and Sirius — but with Sirius supposed free of parallax.

[Margin: Problem 37.]

Diagram. Woodcut in the right column: a tall spherical triangle with its apex V at the top right. Its left side runs down from V past C and F to A and then H; its right side runs down past D and G to B and then R. An arc joins A to B, and below it an arc joins H to R. Straight lines run from the points on the left side across to points on the right side.

- V (apex, top right)
- C (on the left side, upper)
- F (on the left side, below C)
- D (on the right side, middle)
- G (on the right side, below D)
- A (left end of the arc AB)
- B (right end of the arc AB)
- H (lower left corner)
- R (lower right corner)

But if only one of the stars, say F, has a sensible refraction, and the other, D, does not, the visible distance will be CD, but the true one FD. Then, when the angle V has been found as above from the sides VC, VD and CD, and afterwards, in the triangle VFD, VF — the true distance of the star from the vertex — has been obtained as above, and the parallax of the star D has been taken from VD itself, find the true distance FD by [proposition] 5 of oblique spherical triangles. Or if the star C lacks refraction but the star G has it, the visible distance will be CD but the true one CG; and the triangle CVD must be used first, as above, then the triangle CVG. It can also happen that one star is truly below the horizon at R, and is seen above it by the force of refraction, as at G, while the other is above, at F; and the visible distance is FG but the true one FR. It is enough to have pointed out these cases, for all of them are to be worked out in a similar form.

[Margin: Order of the problems.]

Conversely, let the true distance, say FG, be converted into the visible one, CD, both stars having undergone refraction. Subtract the true altitudes AF and BG, each separately, from 90°, so that you have the true distances from the vertex, VF and VG. With these, and with the true distance FG of the stars from each other, find the angle V in the spherical triangle FVG by [proposition] 14 of oblique-angled triangles. Then from the arcs VF and VG subtract one by one the refractions of their own stars, FC and GD, but add their own parallaxes, if they have any; and with the arcs CV and DV and the angle V already found, find in the triangle CVD, by [proposition] 5 of oblique spherical triangles, the visible distance CD — the distance that is to be seen. From this it is clear how to proceed in the other cases of Problem 37. For in them the angle V must always be sought by means of the true distance of the stars from the vertex and from each other; then, by the angle V and the visible distances from the vertex, the base must be sought, that is, the visible distance of the stars from each other, whether that be CD or FG, etc.

Problems 39 and 40. Given the altitude of the pole and the refraction and parallax of a star in a circle of altitude, to convert its given true declination or right ascension into the visible one, and conversely the given visible one into the true; given, besides, in the first case the true altitude of the star, in the second the visible.

[Margin: Problem 39.]

For these, go back to the fifth section of this book and its chapter 6, and use the figure set out there. For this Problem 39 use Problem 28 of that chapter: to the complement of the given true altitude add the parallax but subtract the refraction, and carry out the rest exactly as there.

[Margin: Problem 40.]

For Problem 40, that is, the conversion of the visible into the true, use Problem 31 of the same chapter: from the complement of the given visible altitude subtract the parallax but add the refraction, and carry out the rest exactly as was said there, in any one of the five cases. For just as parallax, by depressing the star, so refraction, by raising it in appearance, sometimes increases the declination, sometimes diminishes it, sometimes annuls it, and sometimes changes it into the other kind.

Problems 41 and 42. To convert the given true latitude and longitude of a star, with the true altitude, into the visible; or the given visible into the true, with the visible altitude — given, besides, the parallax and refraction of the star in a circle of altitude, the altitude of the pole, and the distance of the poles.

[Margin: Problem 41.]

For both problems go back to Book 5, chapter 10, and use Problem 5 directly or conversely. But for Problem 41, add the parallax to the complement of the true altitude while subtracting the refraction; for Problem 42, subtract the parallax while adding the refraction; and carry out the rest as there.

[Margin: Problem 42.]

Yet the occasion for bringing this problem into practice in astronomy is extremely rare.

Problem 43. To attempt the height of the refractive air, given the semidiameter of the Earth and the horizontal refraction of the Sun, by the method of Father Mario Bettini — but a fallacious one.

[Margin: Problem 43.]

In the present figure let A be the centre of the terrestrial hemisphere BCD, of the refractive air EFG, and of the Sun's heaven LM, in which let the upper limb of the Sun be NHQ; from this let the ray HP, propagated, touch the convex surface of the air at F and pass through it unrefracted. Let another ray be drawn from the higher point S, the Sun being now raised by one minute in its heaven, which enters the air through the point of contact F, or very near it; let it be supposed to go on unrefracted to R, and let SR be that ray. Because of its nearness to F, then, let it make the angle PFR of about one minute. But in reality let it be refracted so that the refracted ray is FC and touches the Earth at C, and let the horizontal refraction be CFR. From the centre A let the straight lines AF and AC be drawn to the points of contact, making the angles AFP and ACF right, by [proposition] 18 of the third [book] of Euclid. Since, then, the horizontal refraction RFC is given, if the assumed minute RFP is added to it, the angle CFP will be known; subtracted from the right angle AFP, it will leave the angle AFC known in the triangle ACF, right-angled at C. Therefore by [proposition] 6 of right-angled plane triangles the base AF will be found, since in that triangle the Earth's semidiameter AC is given, and the opposite angle AFC; and when AV is taken away, the height of the air VF will remain. So Father Bettini, in the *Apiaria*, apiary 8, progymnasma 2, proposition 3: taking from Tycho the horizontal refraction of the Sun, RFC, as 34′, and adding RFP so that PFC is 35′, he subtracts the 35′ from the right angle AFP and obtains the angle AFC, 89° 25′. With this, and with the Earth's semidiameter AC of 3035 5/11 Italian miles, as he himself assumes, he derives VF, the least height of the refractive air, as only 151 paces.

[Margin: Father Bettini's error concerning the height of the refractive air.]

But a great fallacy lies hidden in this method. Let the ray SR be refracted near the contact F: it is nevertheless assumed that, once refracted, it reaches the Earth and touches it at C — although it could pass above the Earth towards I; and between that and the tangent CF there are innumerable other rays refracted from other and yet other points lying at a distance from F towards G. So, for example, the ray refracted at O can be refracted from K, and the ray refracted at C, or rather at Z, can be refracted from the point T, far distant from the point of contact F. Therefore the angle of refraction made at T will not be part of the right angle AFP, nor can it be subtracted from it so that the angle AFC becomes known. What should have been shown is that a ray entering so near the point F that it departs from the right angle by a single minute is refracted there in such a way as to touch the Earth — which, however, cannot be demonstrated while keeping the horizontal refraction of the stars found by Tycho, and the inclination of the ray above the convex surface of the air that is due to it.

Diagram. Woodcut in the right column. A horizontal base line runs from E on the far left through B, A, D and G to N and H at the right, with Q just beyond H. Two concentric semicircles stand on the base about the centre A: the inner (the Earth) through B, C and D, the outer (the air) through E, O, I, F and G. At the right a large arc, part of the Sun's heaven, descends from L at the top through S to H on the base and M below it. Rays run from S and from H up to the left towards F, the point where the outer semicircle is touched, and on beyond it to R and P at the upper left; further lines join F, K, O, C, Z, V and T, and straight lines run from A up to C and to F.

- P (upper left, end of a ray)
- R (upper left, below P)
- I (on the outer semicircle, left of F)
- F (point of contact on the outer semicircle)
- K (right of F, where rays cross)
- O (left, on the outer semicircle below I)
- C (on the inner semicircle, left)
- Z (inside the semicircles, right of C)
- V (top of the inner semicircle, above A)
- T (right of V)
- L (top of the large arc at the right)
- S (on the large arc, above H)
- E (left end of the base)
- B (on the base, foot of the inner semicircle)
- A (centre, on the base)
- D (on the base, right foot of the inner semicircle)
- G (on the base, right foot of the outer semicircle)
- N (on the base, left of H)
- H (on the base, at the foot of the large arc)
- Q (just right of H)
- M (below the base, end of the large arc)

Problem 44. To investigate the height of the refractive air, given the inclination of a ray above the convexity of the air, the inclination of the same ray, once refracted, upon the Earth, and the refraction and the semidiameter of the Earth.

[Margin: Problem 44.]

Kepler, from whom I took this problem and completed it, rightly warns in Book 1 of the *Epitome of Copernican Astronomy*, page 62, and in the *Astronomiae Pars Optica*, chapter 4, proposition 9, that the inclinations of the refracted rays through which we see the stars are the ones we measure on the Earth, not the ones the rays have above the curvature of the refractive air before refraction. Moreover, the inclination of a ray upon the Earth is as great as the distance of the star from the vertex as seen through the refracted ray. On these premises, in the following figure let A be the centre of the terrestrial quadrant BVT and of the quadrant of the air DEM, and let the eye be at B; and let ABD be drawn perpendicular both to the true horizon AM and to the apparent one BG. Let

EB be the refracted ray through which the stars are first seen when they rise physically, touching the Earth at B; for its inclination upon the Earth will be the angle DBE, which in this case is a right angle. Let FE be the ray outside the air, unrefracted, which, produced by supposition straight through the air to O, makes with BE the angle of refraction OEB, equal to its vertically opposite angle GEF. Through the point E let the straight line HI be drawn, touching the convexity of the air at E, and from the centre A let the straight line AEP be drawn perpendicular to it through E; for the inclination of the ray FE above the air will be the angle PEF. Once this is known, if the refraction O

[Margin: Order of the problems.]

EB, that is GEF, which is given, is subtracted from it, the angle PEG will be known; by [proposition] 15 of the first [book] of Euclid it is equal to the angle AEB, which is called the refracted angle. Therefore, Kepler says, let it be made that as the sine of the angle AEB (which he himself, rather equivocally, calls the inclination upon the concave surface of the air) is to AB, the semidiameter of the Earth, so the sine of the angle ABE — the inclination, that is, upon the Earth, which in this case is the whole sine, since B is right — is to AE, the semidiameter made up of AC, the Earth's semidiameter, and CE, the height of the refractive air. But Kepler did not notice that if the eye B is raised above the convex surface of the Earth and the sea, as often happens, the height CE grows by as much again and more.

Diagram. Woodcut in the left column, beside the narrow lines of Kepler's proportion. A vertical line runs from D at the top down through B to A at the foot, and a horizontal line runs from A along the bottom to T and M. Two quadrant arcs are drawn about A: the inner (the Earth) from B down to T, the outer (the air) from D through E down to M. From B a horizontal line runs right through E to G. From the upper left, lines from I and from O run down to E; from E a line runs up to P on the right, and lines run on down to F and H at the right. A long line from A runs up through C, on the inner arc, to E, and another from A runs up through K to H.

- I (top, upper left of E)
- O (upper left, below I)
- P (upper right)
- D (top of the vertical line)
- E (on the outer arc, where the lines cross)
- B (on the vertical line, start of the horizontal BG)
- G (right end of the horizontal line)
- F (right, below G)
- H (right, below F)
- C (on the line from A to E, at the inner arc)
- K (on the line from A to H)
- A (lower left corner, the centre)
- T (on the base, foot of the inner arc)
- M (on the base, foot of the outer arc)

Example.

From Kepler.

Let AB be 860 German miles, and the sine of the angle ABE the whole sine, 100 000; but the angle AEB of 88° 1′, whose sine is 99,940. Multiplying, then, 860 by 100,000 makes 86 000 000; dividing these by 99 940, AE comes out at 860 45,600/99940 German miles. Therefore, when AC, that is 860 miles, is taken away, the height of the air CE remains at 0 46/100 of a mile — that is, a little less than half a German mile.

Problem 45. To find the inclination of a ray above the curvature of the refractive air, given the height of the air, the semidiameter of the Earth and the horizontal refraction of the ray.

[Margin: Problem 45.]

Let everything in the figure of the preceding problem be as set out there. For if the refraction is horizontal, the inclination of the refracted ray EB upon the Earth is the right angle DBE, equal to the angle ABE. Join, then, the Earth's semidiameter AC to the height of the air CE, so that the whole AE is known; then let it be made that as AE is to the sine of the angle ABE, that is, the whole sine, so the semidiameter AB is to the sine of the angle AEB, which is equal to the angle PEG. To this add the horizontal refraction OEB, that is GEF, and you will have the angle FEP, namely the inclination of the ray FE above the curvature of the air DEM.

Problem 46. To find the inclination of a ray above the curvature of the refractive air, given the height of the air, the semidiameter of the Earth, the refraction of the ray, and the distance of the star from the vertex seen through that ray.

[Margin: Problem 46.]

In the same preceding figure consider the triangle ABE, whether it is right-angled or oblique-angled at B. In it are given the side AB, the semidiameter of the Earth, and AE, made up of AC, the semidiameter of the Earth, and CE, the height of the air; and the angle ABE, since it is the supplement to two right angles of the angle DBE, which measures the distance of the star from the vertex seen through the refracted ray BE. Therefore, if B is not a right angle, you will find the angle AEB, that is PEG, by [proposition] 1 of oblique-angled plane triangles; and by adding to it the angle GEF, equal to the given refraction OEB, you will have the inclination sought, PEF.

Problem 47. Given what is given in Problem 45 or 46, to find the length of the refracted ray, or the thickness of the refractive air.

[Margin: Problem 47.]

In the figure immediately preceding, if AB, the semidiameter of the Earth, is given, and AE, compounded of the semidiameter AC and the height CE, and the angle ABE is right, as happens when the refraction is horizontal, then by [proposition] 9 of right-angled plane triangles you will learn the side BE. But if ABE, the complement of the visible distance of the star from the vertex, that is, of the angle DBE, is oblique, then by [proposition] 6 of oblique-angled triangles you will find BE, that is, the length of the refracted ray or the thickness

of the refractive air. Tycho, in volume 1 of the *Progymnasmata*, page 95, and Longomontanus, in Book 1 of the *Spherics*, chapter 11, problem 2, took the height of the twilight vapours or exhalations from Ibn al-Haytham and Witelo as about 12 German miles — though Longomontanus found it to be 13 miles — and took AB, or AC, as 860 miles; with these they derive the thicknesses of the refractive air written below, for the visible distances of the star from the vertex written below.

[Margin: The thickness of the twilight air rather than of the refractive air.]

Visible distance of the star from the vertex (deg.)	Thickness, or length BE, in German miles: Tycho	Thickness, or length BE, in German miles: Longomontanus
90	142	150
75	41½	
60	23	
45	16⅓	18⅓
30	14	

[Margin: The mistake of Tycho and Longomontanus.]

But, as Kepler noted in the *Astronomiae Pars Optica*, pages 79, 117, 129 and 135, the refractive air is a liquid matter, moist and heavier than the twilight exhalations (which he calls the twilight lamps), which are drier and smokier. For that reason the refractive air is not raised as high as the matter of the twilight exhalations; nor is their opacity, sufficient to reflect the ray and make twilight, an argument for a density suited to sensible refractions — for mists and smokes are more opaque, and yet they are not equal to glass or water in refracting.

Problem 48. To find the refraction, given the visible distance of the star from the vertex, the inclination of the ray above the refractive air, together with the height of that same air, and the semidiameter of the Earth.

[Margin: Problem 48.]

Going back to the figure set out in Problem 44, and to its triangle ABE: since in it are given AB, the Earth's semidiameter, and AE, made up of the Earth's semidiameter AC and the height of the air CE, and the angle ABE, which is the supplement to two right angles of the given apparent distance from the vertex, DBE, find the angle AEB — either by [proposition] 2 of right-angled plane triangles, if the visible distance is 90°, in which case the refraction will be horizontal, or by [proposition] 1 of oblique-angled triangles, if DBE is less than 90°. Then subtract the angle AEB from the angle AEO, which is known from the given inclination PEF, being equal to it; for the angle BEO of the refraction sought will remain.

Problem 49. To investigate the height of the refractive air by Kepler's method, through known refractions and inclinations on the Earth, and through the position of a false value differing as little as possible from the true, as he himself thought. But his conclusion is rejected.

[Margin: Problem 49.]

As long as the inclination of the ray above the air is unknown, the height of the air is not known; and while that is unknown, the inclination above the air is unknown — so Kepler thinks in the *Astronomiae Pars Optica*, proposition 9, and in the *Epitome of Copernican Astronomy*, Book 1, page 65. Therefore, in both places,

[Margin: First step.]

he first assumes various heights of the air in parts of which the semidiameter of the Earth is known; and with that, and with the semidiameter of the Earth, and with the inclination of the ray upon the Earth, that is, the visible distance of the star from the vertex, and with the Sun's refraction, the Tycho- [Margin: Order of the problems.]

nic one corresponding to this distance, he investigates the inclination of the ray above the air by Problem 45 or 46. So, in the *Astronomiae Pars Optica*, page 117, in the figure of Problem 44, having assumed CE, the height of the refractive air, as 95 parts, and so the whole AE as 100,095 parts of which AB is 100,000, he derives the inclinations in the air written below; to these I have added the visible altitudes and Tycho's solar refractions.

[Margin: From Tycho, volume 1, *Progymnasmata*, page 79.]

Visible altitude	Inclination on the Earth	Refraction of the Sun ⊙	Inclination in the air
0°	90°	34′ 0″	87° 30′
1°	89°	26′ 0″	87° 19′
2°	88°	20′ 0″	86° 48′
3°	87°	17′ 0″	86° 6′
4°	86°	15′ 30″	85° 17′
5°	85°	14′ 30″	84° 25′
30°	60°	1′ 25″	59° 54½′

[Margin: Second step.]

Secondly, by Problem 15 he seeks the simple refraction corresponding to an inclination of 60°, and finds it to be 43″; and by Problem 13 he finds the refraction corresponding to one degree of inclination, 43‴. Then by Problem 16, by means of the secants and of the inclinations assumed in the air, he investigates the integral refractions until he finds that they agree with Tycho's observations without sensible discrepancy. Now, at a visible altitude of one degree, to which a refraction of 26′ is due according to Tycho, he himself, through the inclination assumed in the air of 87° 19′, resting on a height CE of 95 parts, finds an integral refraction very different from 26′. From this he concludes that the position of 95 parts, or of an inclination of 87° 19′, was false. By the same method, through many windings of calculation from page 117 to page 120, he never satisfies himself, until, as if awakened, he notices that Tycho's table of refractions contains a great dissonance, and decrements of the refractions out of proportion to the decrements of the inclinations — not from any fault in the observations, but because Tycho observed the refractions of the Sun on different days of the year, on which the constitution of the air, its

density and its height varied; and from all of them he chose those he thought more certain and put them into one table, as though he had observed them all on one day.

[Margin: Third step, and a new way of investigating the refractions of altitude.]

Thirdly, therefore, from page 122, so that the variation of the refractions observed within a single day may appear, he sets out an example from Tycho's observations of 1587 January 16, made at Uraniborg from $3^h\ 26^m$ after noon to $3^h\ 58^m$. Within that time of 32^m the declination of the Sun was observed nine times with the armillae, and the visible altitude with the quadrants. From these; from the complement of the altitude of the pole at Uraniborg, which he puts at 34° 5′ 15″; from the true declination of the Sun for that day and hour, south, 18° 45′ 10″ — which, added to a quadrant, completes the other side, that is, the declination arc between the Sun and the north pole of the world, making it 108° 45′ 10″; and from the time after noon, which, converted into parts of the equator, constitutes the angle at the pole of the world made by the meridian circle and the circle of the Sun's declination — from these, I say, and from the declinations corrected for parallax, the Sun's parallax being then assumed as 3′, he derives the refractions as below.

Series of observed altitudes of the Sun	Declination observed	Parallax of declination	Declination without parallax	True declination	Refraction of declination	Addition for the refraction of altitude	Refraction of altitude
3° 50′	18° 35′ 30″	2′ 42″	18° 32′ 48″	18° 45′ 30″	12′ 52″ [?]	1′ 30″	14′ 22″
3° 30′	18° 34′ 0″	2′ 42″	18° 31′ 18″	18° 45′ 28″	14′ 10″	1′ 35″	15′ 45″
3° 10′	18° 32′ 30″	2′ 41″	18° 29′ 49″	18° 45′ 26″	15′ 37″	1′ 37″	17′ 14″
2° 50′	18° 31′ 45″	2′ 40″	18° 29′ 5″	18° 45′ 24″	16′ 19″	2′ 2″	18′ 21″
2° 40′	18° 30′ 30″	2′ 40″	18° 27′ 50″	18° 45′ 22″	17′ 39″ [?]	2′ 12″	19′ 44″
2° 5′	18° 29′ 0″	2′ 39″	18° 26′ 21″	18° 45′ 20″	19′ 0″	2′ 23″	21′ 23″
1° 15′	18° 23′ 30″	2′ 38″	18° 20′ 52″	18° 45′ 16″	24′ 24″	3′ 29″	27′ 53″
1° 0′	18° 22′ 30″	2′ 38″	18° 19′ 52″	18° 45′ 14″	25′ 22″	3′ 37″	29′ 0″
0° 35′	18° 20′ 0″ [2]	38″	18° 17′ 52″	18° 45′ 12″	27′ 20″	3′ 54″	31′ 10″ [?]

[Margin: Fourth step.]

Fourthly, in order to find by Problem 16, from the last refraction, 31′ 10″, the first, 14′ 22″: first, through the inclination ABE of 90° 35′, corresponding to the visible altitude of 35′, and through the radius or whole sine AB, and the assumed angle BEA of 87° 30′, he investigates by Problem 44 the straight line AE, which he finds to be 10,009,000 parts of which AB is 10,000,000. Then, using the first altitude, 3° 50′, that is, the angle ABE of 93° 50′ (at which the refraction was found to be 14′ 22″), he finds by Problems 44 and 16 that the refraction compounded from the proportion of the sines and secants does not come close to the observed 14′ 22″ unless the angle BEA is 88°. For then by Problem 44 the secant AE comes out at 10 005 578 parts of which AB is 10 000 000, and from this, by Problem 16, the integral refraction is derived as 14′ 49″,

which does not differ sensibly from the observed refraction of 14′ 22″. Hence, on pages 124 and 129, he concludes that AE is to AB as 10 005 578 is to 10 000 000. And since there are 860 German miles in AB, multiplying these by 10 000 000 makes 8 600 000 000; dividing these by 10 005 578, AE comes out at 860 48/100 miles. Therefore at Uraniborg the height of the refractive air CE was not greater than half a German mile, or two Italian miles.

Kepler's conclusion is rejected, and some points about the height of the refractive air worth noting are brought forward.

[Margin: The height of the refractive air is established as greater than two miles.]

Although the height of the refractive air in Denmark may seem to have been demonstrated above as not greater than two Italian miles — not geometrically, as Kepler himself admits on page 117 of the *Astronomiae Pars Optica*, but quasi-physically — for many reasons I cannot rest content with it. First, the evident sign of refraction for Kepler himself, in the *Epitome of Copernican Astronomy*, Book 1, page 60, is the elliptical shape of the rising or setting Sun; and I do not doubt that this appears to those observing from mountains much higher than two Italian miles. Nor is it necessary to have recourse to rays refracted by a second refraction on their exit from the denser air, carrying the image of the Sun to mountain ridges higher than this air: even granting that this happened, that very thing would be an argument for a double refraction, by force of which the elliptical figure of the Sun ought to be seen different from how it is seen below the mountain by force of a single simple refraction. Secondly, because in the preceding calculation, by which he deduced the refractions of the Sun, he used a solar parallax of 3′; he afterwards condemned it in the *Rudolphine Tables*, reducing it to a single minute and confessing, on page 120 of those tables, in the last precept, that the refractions of the Sun derived by Tycho were larger than they should be because too great a parallax of the Sun had been assumed. Thirdly, because the same Kepler, in the *Epitome of Copernican Astronomy*, page 65, says openly

[Margin: Order of the problems.]

that for this business an examination of at least three observed refractions is required. He ought therefore, of the nine refractions derived in the third step, to have examined at least three, keeping AE at 10 005 578 parts and the angle AEB at 88°; for perhaps it would not have agreed with some of them. Thus the refraction found by this method differs by 4′ from the Sun's refraction at 15° of altitude, which Tycho determined as 7′ 30″, as Kepler admits in the *Astronomiae Pars Optica*, page 126. Indeed it differs in not a few others as well, as is clear from the table set out there on page 124: for to the inclinations of the solar ray above the air written below he assigns, by this method, compound or integral refractions, which, as you see, differ greatly from the observed ones. From this I conclude that this method of deriving refractions is not so certain and reliable — at least for this purpose, that the height of the air should be derived from it.

Inclination of the ray above the air	Refraction computed by Kepler	Refraction observed by Tycho
45°	0′ 47″	0′ 5″
50°	1′ 0″	0′ 10″
55°	1′ 13″	0′ 35″
60°	1′ 28″	1′ 25″
65°	1′ 53″	2′ 30″
70°	2′ 31″	5′ 0″
75°	3′ 32″	7′ 30″
80°	5′ 36″	10′ 0″
85°	11′ 30″	14′ 40″

From this I conclude that, although the steep increase of the refractions bears a strong analogy to the steep increase of the secants near the horizon, it is nevertheless not certain that nature follows this analogy exactly; and therefore Kepler's method is unreliable, both in determining refractions by means of secants and in constructing this height of the refractive air.

Fourthly, the height of Mount Caucasus, treated below in Problems 50 and 51, suggests that around the Caucasus the refractive air is much higher than two Italian miles, and not less than 20 Italian miles; and so it is more probable that in Denmark too it is much greater than two miles. Kepler himself, moreover, grants in the *Epitome of Astronomy*, Book 1, page 72, that on the highest mountains there is some refraction, especially in summer, though at times none. Nor do the confirmations he adds in the *Epitome of Copernican Astronomy*, page 60, and in the *Astronomiae Pars Optica*, page 135, stand in the way. For even if many mountain ridges covered with almost perpetual snow do not have humid air around them from melted snow, they nevertheless have it from elsewhere — from lakes, from marshes, or from the sea lying not very far off — so that the rays of rising stars, refracted through its vapours, reach the mountain ridges. As for the report that on Olympus, as Aristotle testifies, life could once not be sustained without sponges to moisten the air, and that the same happens now in the mountains of America through which one passes from Nicaragua into Peru: this is not a necessary argument that the air is so rare as to be unfit for refractions, since that effect could come from exhalations that are sulphurous, or that infect the breath with some other malign quality. Next, granted that in that nearer air the density were not so great, yet in the air that lies between the mountains and the extremity of their physical horizon, and through which the rays of rising stars pass, there is, or can be, air much denser from watery vapours raised from humid ground, sea, marshes, pools, rivers and so on. Moreover, against Olympus and the mountains of Nicaragua we set Mount Athos, which in Section 4, Problem 35, we showed to be higher than 28 Italian miles; and yet around its summit stood the towns called Acrothon, Uranopolis and Apollonia, where men lived — men indeed called Macrobii from their longevity, as Pliny testifies, Book 4, chapter 10. Therefore, if Kepler's argument held, the air there is endowed not only with a healthy quality but also

with much humidity, suited to the respiration of animals and so to refractions. Finally, the fact that neither on Olympus nor on Athos, nor on the summit of Cyllene and of Mount Satabyrius, did winds hold sway, and that letters and ashes were found undisturbed when the year came round, is not an argument that the air is free of all vapour or exhalation, but only of the turbulent ones and those not yet evenly attenuated by the Sun. Thus the clouds that flow down in showers, and the exhalations that run out into winds and storms, are in the lowest part of the second region of the air, far below the summits of the higher mountains; the remaining clouds, lighter and drier, with exhalations more thoroughly concocted and brought to an even temper, are in the upper part of the second region of the air, mixed with certain vapours not unsuited to some refraction.

Now, in order to derive a more probable height for this air, we must first set up a new Problem on the height of certain mountains, especially the Caucasus.

Problem 50. To correct, by means of refractions, the given height of the Caucasus and of certain other mountains, given the height of the refractive [?] air, the inclination or refraction of the visible species, the distance of the eye from the mountain's perpendicular in the vertical circle, and the Earth's semidiameter.

[Margin: Problem 50.]

In Section 4 of this Book, Problem 30, I taught how to find the height of mountains, given the arc of a great circle between the mountains seen and an eye on the surface of the sea that first sees the mountain's peak, and given also the Earth's semidiameter. By three examples I derived there the height of Etna as 5 9/10 Bolognese miles, from a given arc of 3°; of the Peak on the island of Tenerife as 10 1/10 miles, from a given arc of 4°; and finally of the Caucasus as 57 97/100 miles, or 58, from an arc of 9° 32′. Snel, on the other hand, in the *Eratosthenes Batavus*, Book 2, suspects that the heights of the Peak and of Etna appear greater because of refractions; and my very dear friend, the most learned Father Athanasius Kircher, in a letter written to me on 1647 February 17, timely warned that heights of mountains found in this way are subject to refractions, and that the species of islands and mountains are refracted in our air and therefore appear higher and nearer than they really are. For often, while he was staying in Malta, Etna was visible to him, which on the following day, however clear, he could not see; and Don Carolus Vintimilius at Palermo had at times seen the island of Luprica standing wholly above the sea from Monte Pellegrino, which at other times he could not see, however clear the sky.

[Margin: Father Athanasius Kircher's salutary warning, and an observation for refractions.]

And although this variation could be referred, wholly or in part, to other causes — for example to the glare of the Sun and the position of the reflected rays, or to exhalations dispersed through the air, or to the disposition of the eye — it can nevertheless, not improbably, be attributed to the variability of the refractions in the thick and vaporous air which, spread over the sea between the eye and the object, is now denser, now rarer. It is therefore our task to

investigate how great a contraction these refractions can produce in a height of mountains supposed from other sources. For this, let us recall here the figure set out in Problem 44, in which let BCT be a vertical quadrant of the Earth, and DEM one of the refractive air; and let AC and AK be semidiameters of the Earth, which the height of the air CE and the given, uncorrected height of the mountain KG meet perpendicularly and in a straight line. Let the mountain's peak G be seen from the eye B at sea along the straight line BG, tangent to the Earth at B, where the angle ABG is right by Euclid III.18; and through the point E let the straight line HI be drawn, tangent to the curvature of the air at E and making the angle PEH right. Now it is certain that the species and rays propagated from the mountain's summit towards the air DEM, so that they can enter it and be refracted to the eye B, must lie above the tangent HEI; and therefore it is certain that the

Diagram. A woodcut in the right column, recalled from Problem 44. A vertical line D–B–A rises from A; a horizontal base line runs from A through T to M. A smaller quadrant arc runs from B through C and K down to T (the Earth's surface), and a larger arc from D through E and F down to M (the upper surface of the refractive air). Straight lines run from A through C to E and on to P, and from A through K and H towards G; BG is a horizontal line from B; a line from I passes through E down towards H (the tangent HEI); a curve runs from O towards E.

- D (top left, on the vertical line): top of the refractive air above B
- I (top, above E): end of the tangent HEI
- O (small letter between D and E): point on a curve
- P (upper right): end of the line from A through E
- E (on the outer arc): point where the ray meets the air
- B (left, on the vertical line): the observer's eye at sea
- G (right): the mountain's peak
- F (on the outer arc, right): point on the arc of the air
- H (beside F): end of the tangent HEI
- C (on the inner arc): foot of the height of the air CE
- K (on the inner arc, lower): foot of the mountain KG
- A (bottom left): centre of the Earth
- T (on the base line): end of the Earth's quadrant
- M (on the base line, far right): end of the quadrant of the air

[Margin: Order of the problems.]

apex of the mountain cannot in fact lie below the point H, but must be somewhat above H. Let it therefore be at F, so that the ray FE, incident at the point E, is inclined to the perpendicular AP at an angle PEF less than 90°; and let the same ray FE, which unrefracted would go on straight to O, be refracted towards B, so that BEO is the refraction, to which the vertical angle GEF is equal by Euclid I.15. With these assumptions, and given what we have stated, you will solve the Problem as follows.

[Margin: First step.]

First, add the Earth's semidiameter AK to the uncorrected height of the mountain KG, and the whole AG will result, known in miles; likewise add AC to the height of the air CE, and the whole AE will result, also known in miles. And since the arc BK between the eye B and the mountain's perpendicular AKG is given, look up its secant AG, and make the proportion: as AG of so many miles is to the secant AG, so is AE of so many miles to AE, the secant of the arc BC. Look up this arc in the tables of secants and subtract it from the arc BK, for the remainder will be the arc CK, which measures the angle CAK.

[Margin: Second step.]

Secondly, consider the triangle AEF, in which are given the side AE, as above; the angle EAF, found by the first step; and the angle AEF, which is the supplement of the inclination PEF to two right angles. Therefore, by Euclid I.32, the third angle AFE will be known, as the supplement of the two already mentioned to two right angles; and so, by [proposition] 2 of oblique-angled plane triangles, the side AF will become known. Subtract from it the Earth's semidiameter AK, and the corrected height of the mountain KF will remain; or subtract AF from AG, given above, and the portion FG will be known, by which the visible height of the mountain exceeds its true height. The image of the mountain, however, will appear nearer, at the surface E, than the mountain itself really is. But if the eye B sees not along the tangent BE but along a higher refracted ray — inasmuch as it sees not the bare apex but some notable portion of the mountain's summit — then we are certain that the true height of the mountain is greater than KF. Conversely, if an eye at F could not by itself see the island B, it would nevertheless see it at F by means of the ray BE, issuing from the air and refracted away from the perpendicular at the point E — and indeed much nearer, namely at E, than it really was.

[Margin: Another method, by means of the refraction.]

But if the refraction is given, look up as above the arc BC, that is, the angle BAE. Its complement to a right angle will be AEB, that is, PEG; add to this the refraction GEF, and PEF will result, whose supplement to two right angles is AEF. With this and the rest, look up AF as above, and so on.

Example: on Mount Caucasus.

In Section 4, Problem 30, we showed that between the Deeps of the Euxine, lying at the mouth of the Cimmerian Bosporus through which one sails into the Maeotian Marsh — that is, between B and the perpendicular AG of the Caucasus — there lies a minimum arc of the vertical circle, BK, of 9° 32′, whose secant is 101 400 of the parts of which the radius AB is 100 000. Therefore, taking AB as 4139 Bolognese miles, as demonstrated in Book 2, chapter 7, the height KG comes out at 57 97/100 miles, since the whole secant comes out at 4196 97/100 miles.

[Margin: First step.]

First, then, let there be given the minimum height of the refractive air, which Kepler set at 2 Italian miles; ACE will then be 4141 miles. Now as 4196 97/100 is to the secant AG of 101 400 parts, so is AE, 4141, to the secant AE, 100,047,

to which corresponds the arc BC of 1° 45′. Subtracting this from the arc BK of 9° 32′ leaves CK, that is EAF, 7° 47′. Now let the inclination PEF be 89°; then AEF will be 91°. Adding to this the angle EAF of 7° 47′ gives a sum of 98° 47′, which, subtracted from 180°, leaves the angle AFE of 81° 13′. With these angles and the side AE of 4141 miles, AF is found to be 4190 miles, by [proposition] 2 of oblique plane triangles — or by dropping a perpendicular from E to AF and resolving the triangle AEF into two right-angled triangles. Subtracting, then, AK, 4139, there remains KF, the corrected height of Mount Caucasus, 51 Bolognese miles; and there remains the excess FG of the visible height over the true height — a height that will be greater still, the smaller the inclination of the ray EF is assumed to be, and if the mountain's summit has been seen by a ray higher than the ray BE, or if the height of the air has been assumed greater.

[Margin: The same, by another method.]

Otherwise, by a given refraction, as follows. Since, from what was said a little earlier, BC, that is BAC, is 1° 45′, AEB, that is PEG, will be 88° 15′. Now suppose the refraction GEF at the surface of the air to be whatever you please. For suppose the horizontal refraction greater than the lunar, but proportional to it, since the more distant objects are from the centre of the Earth, the smaller, other things being equal, is the inclination of the ray. But the ratio of the Sun's distance to the Moon's, from what was said in Section 5, Problem 74, is for us as 140 2/5 to 1; and yet the refraction of the Moon does not exceed the horizontal refraction of the Sun by more than two or three minutes. Therefore, since the apex of the Caucasus is not two semidiameters of the Earth distant from the Earth's centre, and the Moon's distance is to the Caucasus's distance at most as 59 to 1, the ordinary refraction of the Caucasus cannot reasonably be supposed much greater than 35′; and a great density from the vapours of the Euxine and the Maeotis would be needed for the refraction to be 45′. Granting this, PEG will be 91° and AEF 89°, and so KF will come out, as above, at 51 miles — or greater, if a refraction smaller than 45′ is supposed, or a greater height of the air. Thus I hold it certain that the height of the Caucasus is 51 Bolognese miles, so that, as stated above, it is truly called in Aristotle the greatest of the eastern mountains; and perhaps the ark first came to rest on these, or on similar mountains of Armenia, as the first to emerge from the waters of the Flood.

The height of Etna and of the Peak is corrected by refractions.

From what was said in Section 4, Problem 30, there lies between the eye B, beyond Malta, and Etna KG an arc BK of at least 3°, from which the whole AG was found there to be 4144 9/10, and its secant is 100 142. Therefore, if AE is 4141 miles, the secant AE will be 100 045 and the arc BC 1° 43′, and so CK, that is EAF, 1° 17′. From these, and from the inclination PEF of 89°, that is, the angle AEF of 91°, and so the angle AFE of 87° 43′, it follows that AF is 4144 8/10 miles; and so the corrected height of Etna, KF, remains at 5 8/10 miles — that is, scarcely sensibly less than the visible height.

By the same method, from what was said in Section 4, Problem 30, and from what has been supposed here, the corrected height of the Peak comes out at 10

Bolognese miles. For BK is 4°, and its secant AG 100 244, and it was found to be 4149 1/10 miles. Therefore, if AE is 4141, the secant AE will be 100,050 1/2, and its arc BC 1° 49½′, and the arc CK, that is the angle EAF, 2° 10′ 30″. Hence, assuming as above the angle AEF of 91°, AFE will be 86° 49′ 30″, and so the whole AF 4149 miles. Subtracting, then, AK, 4139, there still remains the height of the Peak, KF, of 10 miles — scarcely less than the visible height.

Corollary: against Kepler.

From what has been said here, and in Section 4 from Problem 30 on, it is clear that Kepler strayed far from the truth in the *Epitome of Copernican Astronomy*, Book 17, page 26, where he said that the highest mountains do not exceed half a German mile, or two Italian miles, and that from them the horizon is not seen depressed by more than 2°, whereas from Athos it is seen lower by at least 6°, and from the Caucasus by 8°.

Problem 51. From what has been said about the height of the Caucasus in the preceding Problem, and about the same in Section 4, Problems 30 and 36, to derive a more probable height of the refractive [?] air around the Caucasus, above the sea — the Euxine on one side, the Caspian on the other.

[Margin: Problem 51.]

After so many windings, the outcome of our labours has at last reached the point where, from the height of the Caucasus, we may derive a very probable height of the refractive air around it; this we shall do in distinct propositions.

[Margin: First step.]

First, from Problem 50 of this Section and Problem 30 of Section 4, it is clear that the perpendicular height of the Caucasus, if refractions do not alter its appearance to those sailing through the Cimmerian Bosporus, is not less than 58 Bolognese miles; but if the ordinary refractions in that region are not supposed greater than 45′ — and I have shown that they cannot be so supposed — its height is not less than 51 miles, the height of the air being taken as 2 miles and the inclina- [Margin: Order of the problems.]

tion of the ray at practically its greatest, 89°.

[Margin: Second step.]

Secondly, in Section 4, Problem 36: since Aristotle affirms of the same Caucasus that the Sun is seen there for a third part of the night, both at dawn — that is, before the sunrise due at the foot of the mountain — and after sunset, I demonstrated that, if the height of the refractive air is only 2 Bolognese miles, and Tycho's refraction, corrected by our parallax, is 32′ both for the rising Sun in the air above the Caspian and for the setting Sun above the Euxine and the Maeotis, a double refraction must be used: one of the solar ray entering the curvature of the air, the other of the ray leaving it and reaching the summit of the Caucasus. From both together I showed that the height of the Caucasus cannot be set less, nor much greater, than 35 Bolognese miles; and I added that it increases by about as many miles as are added to the assumed height of the air of 2 miles.

[Margin: Third step.]

Thirdly, therefore: since, by proposition 1 of this problem, the height of the Caucasus is established on other grounds as not less than 51 miles, however much refraction is admitted for this purpose, it follows from this and from proposition 2 that the height of the refractive air must be increased by as many miles as are wanting from the number 35 to 51, namely 16. And therefore it must be set at no less than 18 Bolognese miles, and indeed somewhat larger still, say 20; for as it increases, the true height of the Caucasus also increases, by Problem 50, and from that height in turn the height of the air is increased — at least in that region, where the air is dense and very cold because of the thicker vapours raised from the Caspian, the Euxine and the Maeotian Marsh.

[Margin: How great is the height of the refractive air?]

Elsewhere, however, it is probable that the air — dense indeed, but still fit for a sensible refraction — rises higher, though never above 50 Italian miles, and is nowhere depressed below 2 miles. Within these limits the height of this air varies greatly, according to the variety of the waters and the soil from which the vapours are raised, and of the seasons of the year, and indeed of the same day. Kepler, moreover, notes in the *Epitome of Astronomy*, Book 1, page 72, that in maritime places the height of this air is more constant, while on the highest mountains there is at times almost no refraction, especially in winter, when the air is less vaporous; but when it has boiled up, the height of the air increases — both of the refractive and of the twilight air. And in this sense I grant to Father Emmanuel Maignan, *Perspectiva horaria*, Book 1, proposition 38, that the height of the refractive air is uncertain.

Problem 52. To determine the height of the twilight exhalations by means of rays of the Sun first twice refracted, then reflected, then refracted a third time; and to correct this height, found by the ancients by means of unrefracted rays.

[Margin: Problem 52.]

For Problem 52, which involves an incredible subtlety, see Book 8, Section 1, chapter 14, Problems 4, 5 and 6, where I set out a table of the height of the exhalations according to my own opinion, and showed that their height should not be contracted as much as our friend Wendelen thought.

Problem 53. To find the greatest thickness of the air lying between the twilight exhalations and our eyes.

[Margin: Problem 53.]

Go back to Problem 47, for this is the same as that, except that here, in place of the height of the refractive air, the height of the exhalations must be used, and joined to the Earth's semidiameter, so that the secant is obtained in miles.

Problem 54. To find by how much the arc of the sphere of refractive air illuminated by the Sun is smaller than the arc of the Earth illuminated by the Sun through unrefracted rays, this being given; given the semidiameter of the said sphere and the true semidiameter of the Sun, with its distance from the Earth's centre; and to show that it is not smaller by a single second.

[Margin: Problem 54.]

In the following figure, let ABCD be the orb of the terrestrial sphere together with the refractive air, with centre H; and let EMF be the Sun, at the distance KH. From its extremities let two rays EG and FG come, tangent to the Sun at E and F and, by Euclid III.18, making right angles at E and F with either semidiameter of the Sun, KE and KF. Let the same rays, touching the sphere of the air at A and at C, likewise make right angles at A and C with the semidiameters AH and CH. Let KH be produced to the meeting point G, and from H let HL be drawn parallel to the ray AE; for by Euclid I.29 and I.33 the angle AHL and the two angles at L will be right, and the lines AH and EL equal. Therefore, subtracting the semidiameter AH from the semidiameter EK, the side LK will be known in the triangle HKL, right-angled at L, in which the base HK, the Sun's distance, is given; and so, by [proposition] 2 of right-angled plane triangles, the angle LHK will become known, which measures the arc BI. Added to the arc AI of 90° — as subtending the right angle AHL — it will give the semiarc AB of the air illuminated by the Sun; and this, doubled, will give the illuminated arc ABC, which, compared with the arc of the Earth illuminated by the Sun's unrefracted rays, will show the difference sought.

Example.

In Book 3, chapter 11, we showed that the true semidiameter of the Sun, EK, is about 34 terrestrial semidiameters; and from the Sun's perigee distance HK of 7000 terrestrial semidiameters we showed that the arc AIB is 90° 16′ 8″, as is clear from Problems 1, 4 and 5 of that chapter. Now suppose that AH adds five-sixths of a terrestrial semidiameter to the Earth's semidiameter, so that the height of the refractive air is 3358 miles, of which the terrestrial semidiameter has 4139, from what was said in Book 2, chapter 7. Indeed, for good measure, let us suppose AH to be double the terrestrial semidiameter: for subtracting 2 from EK, which is about 34, LK will remain at 32 of the parts of which HK is 7000. From this will follow the angle LHK, that is the arc BI, of 15′ 42″, and AIB of 90° 15′ 42″, smaller by only 26″ than the terrestrial semiarc illuminated by the Sun's unrefracted rays, which is 90° 16′ 8″. Therefore, if the refractive air is supposed to be only 20 miles high, or at most 50, according to Problem 51, the semiarc of the refractive air illuminated by the Sun will not be smaller by even one second than the semiarc of the Earth illuminated by the Sun's rays taken as unrefracted.

Diagram. A woodcut in the right column. A long, narrow triangle has its apex G at the foot and its upper side running through E, K and F. A semicircle hangs from E to F through M (the Sun, centre K). A vertical line runs from K through M, B, H and D down to G. A small circle about H (the Earth with the refractive air) carries B at its top, A on the left, C on the right and D at the bottom, A and C being where the sides EG and FG touch it. A line runs from L down through I to H, and a line from E through L towards K.

- K (top centre): centre of the Sun
- E (top left): point of tangency of the ray EG on the Sun
- L (on the line from E): foot of the parallel HL

- F (top right): point of tangency of the ray FG on the Sun
- M (on the vertical, below the semicircle's lowest point): point of the Sun's disc
- I (left of B): end of the arc BI
- B (top of the small circle): point on the sphere of the air
- H (centre of the small circle): centre of the Earth
- A (left of the small circle): point of tangency of EG on the air
- C (right of the small circle): point of tangency of FG on the air
- D (bottom of the small circle): point on the sphere of the air
- G (bottom apex): meeting point of the two rays

Problem 55. To find by how much greater the arc of the Earth illuminated by the Sun's refracted rays is than that illuminated by unrefracted rays.

[Margin: Problem 55.]

We have already solved this Problem in connection with the Earth's shadow, lit through or truncated by the Sun's rays, in Book 5, chapter 5, Problem 3; and we showed with Kepler that this arc exceeds the arc illuminated by unrefracted rays by as much as the Sun's horizontal refraction taken on both sides, or doubled, and that this semiarc is greater than the semiarc of the Earth illuminated by unrefracted rays by as much as the Sun's horizontal refraction — that is, in Tycho's reckoning, by mi- [Margin: Order of the problems.]

[Margin: The arc of the Earth illuminated by the Sun's refracted rays.]

nutes 34′; but according to the refraction corrected by our parallax, 31′ 33″. Thus the semiarc illuminated by the Sun is, without refraction, 90° 16′ 8″, from what was said in Book 3, chapter 11, Problem 4; with Tycho's refraction added, 90° 50′ 8″; but with ours, 90° 47′ 41″. And the whole arc is, by Tycho's reckoning, 181° 40′ 16″, but by ours 181° 35′ 22″. Nor does Father Bettini differ much, in *Apiaria* 8, progymnasma 2, proposition 4, where he says that 35′ must be added to the arc of the ancients: because, as we said in Problem 43, he took the angle made by the ray tangent to the air and the ray refracted by the horizontal refraction to be one minute greater than Tycho's horizontal refraction. For the rest, since both Kepler there and Bettini suppose the height of the refractive air to be so small that the arc illuminated by the Sun's rays tangent to the air differs insensibly from the arc illuminated by the Sun's rays tangent to the Earth — and we showed this in Problem 54 — we thought that proposition should be confirmed here. I add, however, that in Book 5, chapter 5, Problem 5, I used, with Kepler in the *Astronomiae Pars Optica*, Tycho's horizontal solar refraction of 34′, as I also did in Book 3, chapter 11, Problem 4. But strictly speaking, because Tycho made this refraction larger than was right — since to the simple refraction of 31′ 6″ he added a parallax of 2′ 54″, when no more than 27″ ought to have been added, as I said in Problem 25 of this section — I wished to point out that this arc needs some correction.

Problem 56. To determine whether, from the horizontal refraction of the fixed stars alone, together with the Earth's semidiameter and the height of the refractive air, the

inclination of the ray of the fixed stars and their distance, or their parallax, can be demonstrated.

[Margin: Problem 56.]

I showed the negative in Book 9, Section 4, chapter 31, from number 6, and I set out the argument taken from this against the Copernican hypothesis. Go back there, then, reader, to obtain a fuller doctrine of refractions; and on that occasion recall what I said in the same place about the parallax derived from the semidiameter of the sphere enclosing the refractive air, and what I shall say in the following problem.

Problem 57. To determine whether the height of the refractive air, or the inclination of a star's ray, can be demonstrated exactly from nothing more than the given semidiameter of the Earth, the horizontal refraction and parallax of some star, and its distance from the centre of the Earth, together with its true distance from the vertex.

[Margin: Problem 57.]

I add this because, having investigated the height of the refractive air with so careful an inquiry in Problems 43, 44, 49 and 51, I might seem less than diligent if I had neglected this method, which found favour with Father Francesco Maria Grimaldi. So, lest anyone think it untried: in the following figure let A be the centre of the Earth, and from it let there be arcs of a vertical circle greater than a quadrant — in the heavens, the star's arc BDC; on the surface of the refractive air, HGK; and on the surface of the Earth, OIL. Let the eye be at O, and let the vertical line AOB fall at right angles on the physical horizon OE and on the rational horizon AD. From A, through the point where the arc HGK and the horizon DE intersect — that is, through the point G — let the straight line AP be drawn, which will be perpendicular to the arc HGK. Now let the Sun or a fixed star truly be at C, and let its true distance from the vertex be the arc BC, which the angle BAC measures. But let it shine along CG and, at G — where it would go on to R if it were not refracted — let it be seen there, by the refracted ray GO, along the straight line OGE, as if it were at E. For the horizontal refraction will be OGR, to which EGC is equal, by Euclid I.15. Finally, let C, the true place of the star, be joined to the eye O by the straight line CO. The task now is to find the semidiameter AG, so that, when the Earth's semidiameter AI is subtracted, the height IG of the air may be known; and some have thought that this can be known from the five data in the title of the Problem.

[Margin: First step.]

For, first, in the triangle OAC there are given AO, the Earth's semidiameter; AC, the star's distance; and the included angle OAC, which is the true distance of the star from the vertex. Therefore, by propositions 3 and 5 of oblique-angled plane triangles, the base OC can be known, and each of the angles ACO and AOC; and when this is subtracted from the right angle AOG, COG becomes known.

Diagram. A woodcut at the head of the right column. A vertical line on the left runs from B at the top down through H and O to A, the centre of the

Earth, at the foot. From B a large arc (the star's vertical circle, BDC) sweeps down the right side through E, M and D to C at the lower right. Two smaller concentric arcs cross the figure about A: an upper one through H, G and K (the surface of the refractive air) and a lower one through O, I and L (the surface of the Earth). A straight line runs from A up through I and G to P at the upper right. From O a horizontal line runs through G to E (the physical horizon), and from A a horizontal line runs to D (the rational horizon). A short line rises from G to R at the upper left. Straight lines join A to C, O to C and G to C. Dotted lines run from V through G to M, from O to M, and from A to M.

- B (top of the vertical line): the vertex on the star's circle
- P (upper right, end of the line from A): end of the perpendicular AP
- H (on the vertical, below B): end of the arc of the refractive air
- R (above and left of G): where the unrefracted ray would continue
- V (on the vertical, below H): end of the dotted line VGM
- G (centre): the point of refraction on the surface of the air
- O (on the vertical, below V): the eye, on the Earth's surface
- E (right, on the large arc): the apparent place of the star
- I (on the line AP, below G): point on the Earth's surface
- M (right, on the large arc, below E): the star above the true horizon (second case)
- A (lower left): centre of the Earth
- D (right, on the large arc, below M): end of the rational horizon AD
- L (foot, on the lower arc): end of the arc of the Earth's surface
- K (foot, on the upper arc): end of the arc of the refractive air
- C (lower right): the true place of the Sun or fixed star

[Margin: Second step.]

Second: with COG, with the base OC, and with the obtuse angle OGC — which is the supplement of the refraction OGR to two right angles, by Euclid I.13 — and so with the angle GCO, which is the supplement of the two already mentioned to two right angles, by Euclid I.32: with these three data, I say, the side OG can be known, by proposition 2 of oblique-angled plane triangles.

[Margin: Third step.]

Third: in the triangle AOG, right-angled at O, from the sides AO, the Earth's semidiameter, and OG, found just now, the base AG can be known by proposition 8 of right-angled plane triangles, and the angle AGO by proposition 5. When AI, the Earth's semidiameter, is subtracted, IG will remain, the height of the refractive air. And since, by Euclid I.15, PGE is equal to the angle AGO just found, and the angle EGC to the horizontal refraction RGO, the whole angle of inclination PGC will certainly be known.

[Margin: Second case of the problem.]

But if the star were above the true horizon, as at M, when it appears at E — which is supposed in the case of the Moon, since it undergoes a horizontal refraction, but one smaller than its parallax — then, drawing the straight lines AM, OM and VGM, one would have to solve first the triangle OAM, according

to what was said in the first step; then the triangle OGM, according to what was said in the second step; and finally the triangle AOG, according to what was said in the third step.

I answer, however, that although one may approach the truth quite closely, it cannot be obtained exactly, with geographical or astronomical rigour, from the five data already mentioned alone. For the refraction of the stars OGR can be known by observation in no other way than by comparing their apparent distance from the vertex with the true one, and from that deriving the simple refraction; and by adding the parallax to it we are taken to have the whole effect of refraction. For example: the true distance of the Sun C from the vertex is by calculation BDC; and yet, appearing to us at E, it seems to be distant by a full quadrant BD, because the angle BOE is a right angle, and so it is believed to be at D. Hence the difference between the true and the supposed distance is the arc DC, namely the simple refraction. Adding to it the arc DE, which the angle EAD measures — that is, AEO (supply here the line AE in the figure) — namely the horizontal parallax, we take the whole effect of refraction, and its measure, to be the arc EDC. But it is not, because otherwise the angle EGC, which is equal to the angle of refraction OGR, would be the exact measure of the arc CDE, although that arc is not described from the centre G. Yet the arc owed to this angle is greater; and the parallax of the star C seen from G along the straight line GC is greater than when it is seen from O along the straight line OC; and anyone at G would see it somewhat lower than someone at O. And therefore the angle of refraction OGR, weighed exactly, equals the simple refraction CD, plus the parallax ED of the star seen from the surface of the Earth, plus the increment of parallax from the surfa- ce of the refractive air, which is nearly equal to the angle OCG. Add to this that, when the refraction is unknown, we do not know the exact parallax, and when the parallax is unknown we do not know the refraction — though within certain limits of ignorance, on which see Section 5, from Problem 73 to 75. Finally, the excessive smallness of the angles ACO and OCG relative to the side AC makes the investigation of the side OG slippery. We have therefore judged that this problem is not much to be trusted; instead, we investigate AG by a safer method, especially in Problem 51. Now follow the tables so often promised.

TABVLÆ ANACLASTICÆ SEV REFRACTIONVM.

Anaclastic Tables, or Tables of Refractions.

Table I. Witelo's, from Book 10 of the Optics, Proposition 8.

Refraction toward the perpendicular.

Angle of inclina- tion	Air to water: angle refracted	Air to water: refraction	Air to glass: angle refracted	Air to glass: refraction	Water to glass: angle refracted	Water to glass: refraction
10°	7° 45′	2° 15′	7° 0′	3° 0′	9° 30′	0° 30′
20°	15° 10′ [?]	4° 30′	13° 30′	6° 30′	18° 30′	1° 30′

Angle of inclination	Air to water: angle refracted	Air to water: refraction	Air to glass: angle refracted	Air to glass: refraction	Water to glass: angle refracted	Water to glass: refraction
30°	22° 30′	7° 30′	19° 30′	10° 30′	27° 0′	3° 0′
40°	29° 0′	11° 0′	25° 0′	15° 0′	35° 0′	5° 0′
50°	35° 0′	15° 0′	30° 0′	20° 0′	42° 30′	7° 30′
60°	40° 30′	19° 30′	34° 30′	25° 30′	49° 30′	10° 30′
70°	45° 30′	24° 30′	38° 30′	31° 30′	56° 0′	14° 0′
80°	50° 0′	30° 0′	42° 0′	38° 0′	62° 0′	18° 0′

Refraction away from the perpendicular.

Angle of inclination	Water to air: angle refracted	Water to air: refraction	Glass to air: angle refracted	Glass to air: refraction	Glass to water: angle refracted	Glass to water: refraction
10°	12° 5′	2° 5′	13° 0′	3° 0′	10° 30′	0° 30′
20°	24° 30′	4° 30′	26° 30′	6° 30′	21° 30′	1° 30′
30°	37° 30′	7° 30′	40° 30′	10° 30′	33° 0′	3° 0′
40°	51° 0′	11° 0′	55° 0′	15° 0′	45° 0′	5° 0′
50°	65° 0′	15° 0′	70° 0′	20° 0′	57° 30′	7° 30′
60°	79° 30′	19° 30′	85° 30′	25° 30′	70° 30′	10° 30′
70°	94° 30′	24° 30′	101° 30′	31° 30′	84° 0′	14° 0′
80°	110° 0′	30° 0′	118° 0′	38° 0′	98° 0′	18° 0′

Table II. The refraction from air to water, reformed by Kepler, chapter 4 of the Astronomiae Pars Optica, proposition 8, according to our Problems 13 and 15.

Distance of the radiant from the vertex, or inclination at the surface of the water	Part of the refraction proportional to the inclinations	Part of the refraction to be added on account of the secants	Composite refraction, demonstrated	Refraction from Witelo's experiment	Difference
10°	2° 25′	0° 1′	2° 26′	2° 15′	0° 11′
20°	4° 49′	0° 10′	4° 59′	4° 30′	0° 29′
30°	7° 14′	0° 35′	7° 49′	7° 30′	0° 19′
40°	3° 39′[63]	1° 23′	11° 2′	11° 0′	0° 2′
50°	12° 4′	2° 42′	14° 46′	15° 0′	0° 14′
60°	14° 28′	4° 40′	19° 8′	19° 30′	0° 22′
70°	16° 52′	7° 19′	24° 11′	24° 30′	0° 19′
80°	19° 17′	10° 43′	30° 0′	30° 0′	0° 0′
90°	21° 43′	14° 47′	36° 30′	0° 0′	

Table III. Father Emmanuel Maignan's, from Book 4 of his Perspectiva horaria, Propositions 45 and 48. The refracted angles from air into water, both from the calculation and proportion of the sines of the inclinations, according to our Problem 12, and from his own ratio, which agrees better with experiments — a refracted angle of 50° at an inclination of 80° being supposed from Witelo.

[63]so printed; 11° 2′ less 1° 23′ gives 9° 39′

Inclination	Refracted, from the calculation of sines	Refracted, by his own ratio	Inclination	Refracted, from the calculation of sines	Refracted, by his own ratio	Inclination	Refracted, from the calculation of sines	Refracted, by his own ratio
1°	0° 46⅔′	0° 41′	31°	23° 38′		61°	42° 51′	
2°	1° 33′		32°	24° 21′		62°	43° 23′	
3°	2° 20′		33°	25° 4′		63°	43° 53′	
4°	3° 7′		34°	25° 47′		64°	44° 21′	
5°	3° 54′	4° 0′	35°	26° 30′	25° 48′	65°	44° 50′	43° 3′
6°	4° 40′		36°	27° 13′		66°	45° 17′	
7°	5° 27′		37°	27° 55′		67°	45° 44′	
8°	6° 13′		38°	28° 37′		68°	46° 10′	
9°	7° 0′		39°	29° 19′		69°	46° 34′	
10°	7° 46′	7° 55′	40°	30° 0′	29° 0′	70°	46° 58′	45° 30′
11°	8° 32′		41°	30° 41′		71°	47° 21′	
12°	9° 18′		42°	31° 22′		72°	47° 43′	
13°	10° 4′		43°	32° 2′		73°	48° 3′	
14°	10° 50′		44°	32° 42′		74°	48° 23′	
15°	11° 36′	11° 47′	45°	33° 22′	32° 3′	75°	48° 43′	47° 48′
16°	12° 22′		46°	34° 2′		76°	49° 1′	
17°	13° 9′		47°	34° 41′		77°	49° 17′	
18°	13° 55′		48°	35° 19′		78°	49° 33′	
19°	14° 40′		49°	35° 57′		79°	49° 47′	
20°	15° 25′	15° 30′	50°	36° 35′	35° 0′	80°	50° 0′	50° 0′
21°	16° 11′		51°	37° 12′		81°	50° 12′	
22°	16° 57′		52°	37° 47′		82°	50° 23′	
23°	17° 42′		53°	38° 24′		83°	50° 32′	
24°	18° 27′		54°	39° 0′		84°	50° 41′	
25°	19° 12′	19° 3′	55°	39° 35′	37° 48′	85°	50° 48′	52° 3′
26°	19° 56′		56°	40° 9′		86°	50° 54′	
27°	20° 40′		57°	40° 43′		87°	50° 58′	
28°	21° 25′		58°	41° 17′		88°	51° 1′	
29°	22° 10′		59°	41° 49′		89°	51° 3′	
30°	22° 50′ [?]	22° 30′	60°	42° 21′	40° 30′	90°	0° 0′ [?]	

Subtract the refracted angle from the inclination, and you will have the refraction, by our Problem 13.

Table IV. Father Marinus Mersennius's, of the refraction from air into crystal, from recent experiments, which he gives in his book of Harmonics; and, from it, Hérigone, volume 5 of the Cursus Mathematicus, proposition 3 of the Dioptrics.

Inclination	Lesser refraction	Greater refraction	Inclination	Lesser refraction	Greater refraction	Inclination	Lesser refraction	Greater refraction
5°	1° 45′	1° 55′	25°	9° 2′	9° 51′	45°	17° 36′	19° 5′
10°	3° 30′	3° 50′	30°	11° 0′	12° 0′	50°	20° 5′	21° 44′
15°	5° 18′	5° 47′	35°	13° 4′	14° 14′	55°	22° 46′	24° 35′
20°	7° 8′	7° 48′	40°	15° 10′	15° 35′	60°	25° 41′	27° 30′
						89°	49° 22′	51° 50′

Table V. Father Athanasius Kircher's, of the Society of Jesus, from Book 8 of his Ars Magna Lucis et Umbrae, *chapter 4, Problem 2.*

Inclination	Air to water	Air to wine	Air to oil	Air to glass
10°	2° 20′	2° 30′	2° 40′	3° 10′
20°	4° 38′	4° 45′	5° 10′	6° 40′
30°	7° 40′	7° 50′	8° 4′	10° 50′
40°	11° 9′	11° 14′	11° 50′	15° 8′
50°	15° 6′	15° 10′	16° 10′	18° 12′
60°	19° 40′	19° 50′	20° 20′	25° 50′
70°	24° 40′	24° 50′	24° 54′	31° 10′
80°	30° 4′	30° 10′	30° 54′	38° 10′

Table VI. From our own experiments, carried out with the greatest care, by Problems 5 and 11.

Inclination	Air to water: angle refracted	Air to water: refraction	Air to glass: angle refracted	Air to glass: refraction
5°	3° 58′	1° 2′	3° 13′	1° 47′
10°	7° 52′	2° 6′	6° 26′	3° 34′
15°	11° 45′	3° 15′	9° 37′	5° 23′
20°	15° 28′	4° 32′	12° 45′	7° 15′
25°	19° 0′	6° 0′	15° 50′	9° 10′
30°	22° 28′	7° 38′	18° 50′	11° 10′
35°	25° 46′	9° 14′	21° 44′	13° 16′
40°	28° 56′	11° 4′	24° 31′	15° 29′
45°	32° 0′	13° 0′	27° 10′	17° 50′
50°	34° 57′	15° 3′	29° 38′	20° 22′
55°	37° 45′	17° 15′	31° 56′	23° 4′
60°	40° 27′	19° 32′	34° 0′	26° 0′
65°	43° 0′	22° 0′	35° 49′	29° 11′
70°	45° 26′	24° 34′	37° 21′	32° 39′
75°	47° 44′	27° 16′	38° 35′	36° 25′
80°	49° 56′	30° 4′	39° 29′	40° 31′
85°	52° 0′	33° 0′	40° 2′	44° 58′
89°	48° 24′	40° 36′	40° 12′	48° 48′

Table VII. Refractions of the fixed stars in air, which Lansbergen, in his Thesaurus Observationum, *assumes rather than confirms by reasoning or observation, in order to reconcile his tables with the observations.*

Places of the observed altitudes	Apparent altitude of the fixed stars above the horizon	Refraction of the fixed stars
Uraniborg	8° 42′	6′ 0″
Goes	13° 8′	6′ 0″
Uraniborg	15° 33′	5′ 50″
Goes	19° 54′	4′ 0″

Places of the observed altitudes	Apparent altitude of the fixed stars above the horizon	Refraction of the fixed stars
Uraniborg	25° 9′	2′ 30″
Frombork	26° 59′	2′ 30″
Middelburg	29° 20′	2′ 0″
Göggingen	32° 46′	2′ 30″

Table VIII. Of the refractions of the stars from Tycho's observation in Denmark — but those of the Sun reformed by Kepler, by Lansbergen, and by us.

Apparent altitude above the horizon	Inclination of the refracted ray upon the Earth	From Tycho, vol. 1 of the *Progymnasmata*, p. 79: Sun	Same, p. 124: Moon	Same, p. 280: fixed stars	Sun, reformed by Kepler in the *Astronomiae Pars Optica*, p. 125, by our Problem 16	Sun and Moon, from Lansbergen's Tables, p. 46	Simple refraction of the Sun, derived from Tycho's observations by our Problem 27	Whole, Tychonic refraction of the Sun, but with our solar parallax added, according to Problem 27
0°	90°	34′ 0″	33′ 0″	30′ 0″	33′ 0″	34′ 0″	31′ 6″	31′ 33″
1°	89°	26′ 0″	25′ 0″	21′ 30″	25′ 34″	26′ 0″	23′ 6″	23′ 33″
2°	88°	20′ 0″	20′ 0″	15′ 30″	18′ 27″	21′ 0″	17′ 6″	17′ 33″
3°	87°	17′ 0″	17′ 0″	12′ 30″	14′ 51″	18′ 0″	14′ 6″	14′ 32″
4°	86°	15′ 30″	15′ 20″	11′ 0″	11′ 30″	15′ 45″	12′ 37″	13′ 3″
5°	85°	14′ 30″	14′ 20″	10′ 0″	9′ 43″	14′ 0″	11′ 37″	12′ 2″
6°	84°	13′ 30″	13′ 50″	9′ 0″	8′ 14″	12′ 40″	10′ 37″	11′ 2″
7°	83°	12′ 45″	12′ 45″	8′ 25″	7′ 6″	11′ 15″	9′ 53″	10′ 18″
8°	82°	11′ 15″	12′ 0″	6′ 45″	6′ 18″	10′ 5″	8′ 23″	8′ 47″
9°	81°	10′ 30″	11′ 20″	6′ 0″	5′ 36″	9′ 5″	7′ 39″	8′ 3″
10°	80°	10′ 0″	10′ 45″	5′ 30″	5′ 2″	8′ 15″	7′ 9″	7′ 32″
11°	79°	9′ 30″	10′ 10″	5′ 0″	4′ 44″	7′ 35″	6′ 40″	7′ 3″
12°	78°	9′ 0″	9′ 35″	4′ 30″	4′ 22″	7′ 5″	6′ 10″	6′ 32″
13°	77°	8′ 30″	9′ 0″	4′ 0″	3′ 50″	6′ 40″	5′ 41″	6′ 3″
14°	76°	8′ 0″	8′ 30″	3′ 30″	3′ 32″	6′ 19″	5′ 12″	5′ 33″
15°	75°	7′ 30″	8′ 0″	3′ 0″	3′ 18″	6′ 0″	4′ 42″	5′ 3″
16°	74°	7′ 0″	7′ 30″	2′ 30″	3′ 4″	5′ 42″	4′ 13″	4′ 34″
17°	73°	6′ 30″	7′ 0″	2′ 0″	2′ 50″	5′ 24″	3′ 44″	4′ 5″
18°	72°	5′ 45″	6′ 30″	1′ 15″	2′ 41″	5′ 7″	2′ 59″	3′ 19″
19°	71°	5′ 0″	6′ 0″	1′ 0″	2′ 31″	4′ 50″	2′ 15″	2′ 35″
20°	70°	4′ 30″	5′ 30″	0′ 0″	2′ 23″	4′ 33″	1′ 46″	2′ 6″
21°	69°	4′ 0″	5′ 0″		2′ 15″	4′ 16″	1′ 17″	1′ 36″
22°	68°	3′ 30″	4′ 35″		2′ 4″	4′ 0″	0′ 49″	1′ 8″
23°	67°	3′ 10″	4′ 10″		1′ 58″	3′ 44″	0′ 29″	0′ 38″
24°	66°	2′ 50″	3′ 45″		1′ 53″	3′ 28″	0′ 10″	0′ 29″
25°	65°	2′ 30″	3′ 20″		1′ 49″	3′ 12″	0′ 10″	0′ 10″
26°	64°	2′ 15″	3′ 0″		1′ 41″	2′ 56″		
27°	63°	2′ 0″	2′ 40″		1′ 35″	2′ 40″		
28°	62°	1′ 45″	2′ 20″		1′ 30″	2′ 24″		
29°	61°	1′ 35″	2′ 0″		1′ 28″	2′ 9″		
30°	60°	1′ 25″	1′ 45″		1′ 25″	1′ 54″		

Apparent altitude above the horizon	Inclination of the refracted ray upon the Earth	From Tycho, vol. 1 of the *Progymnasmata*, p. 79: Sun	Same, p. 124: Moon	Same, p. 280: fixed stars	Sun, reformed by Kepler in the *Astronomiae Pars Optica*, p. 125, by our Problem 16	Sun and Moon, from Lansbergen's Tables, p. 46	Simple refraction of the Sun, derived from Tycho's observations by our Problem 27	Whole, Tychonic refraction of the Sun, but with our solar parallax added, according to Problem 27
31°	59°	1′ 15″	1′ 30″		1′ 22″	1′ 39″		
32°	58°	1′ 5″	1′ 20″		1′ 19″	1′ 24″		
33°	57°	0′ 55″	1′ 10″		1′ 16″	1′ 9″		
34°	56°	0′ 45″	1′ 0″		1′ 13″	0′ 55″		
35°	55°	0′ 35″	0′ 50″		1′ 10″	0′ 41″		
36°	54°	0′ 30″	0′ 45″		1′ 8″	0′ 27″		
37°	53°	0′ 25″	0′ 40″		1′ 5″	0′ 13″		
38°	52°	0′ 20″	0′ 35″		1′ 2″	0′ 0″		
39°	51°	0′ 15″	0′ 30″		1′ 0″	0′ 0″		
40°	50°	0′ 10″	0′ 25″		0′ 57″			
41°	49°	0′ 9″	0′ 20″		0′ 54″			
42°	48°	0′ 8″	0′ 15″		0′ 51″			
43°	47°	0′ 7″	0′ 10″		0′ 49″			
44°	46°	0′ 6″	0′ 5″		0′ 47″			
45°	45°	0′ 5″	0′ 0″		0′ 40″		0′ 5″	

In volume 1 of the *Progymnasmata*, p. 280, Tycho recognized that the refractions of the Moon must be greater than the Sun's, because of the greater inclination of the ray issuing from a nearer star. But he made the Sun's refraction near the horizon greater because of the excessive solar parallax, as Kepler indicates in the last precept of the *Rudolphine Tables* — Kepler who, however, when he sets out in the *Astronomiae Pars Optica*, p. 123, to reform Tycho's refraction of the Sun, uses a parallax of 3′. Lansbergen also erred in making the lunar refractions equal to the solar, when they ought to have been greater.

Table IX. Of the refractions observed by us at Bologna, morning and evening — but mostly in the morning — at the times of the solstices and equinoxes, at a pole height of 44° 29′ 30″.

Apparent altitude	Sun: summer	Sun: equinoctial	Sun: winter	Moon: summer	Moon: equinoctial	Moon: winter	Fixed stars: summer	Fixed stars: equinoctial	Fixed stars: winter
0°	32′ 25″	33′ 0″	33′ 40″	33′ 0″	33′ 40″	34′ 20″	29′ 50″	30′ 10″	30′ 30″
1°	25′ 0″	25′ 30″	26′ 5″	25′ 30″	26′ 6″	26′ 40″	21′ 20″	21′ 40″	22′ 0″
2°	19′ 30″	19′ 55″	20′ 25″	19′ 50″	20′ 26″	20′ 55″	15′ 20″	15′ 40″	16′ 0″
3°	16′ 40″	17′ 0″	17′ 25″	16′ 56″	17′ 26″	17′ 50″	12′ 20″	12′ 40″	13′ 0″
4°	15′ 20″	15′ 38″	15′ 58″	15′ 36″	16′ 0″	16′ 20″	10′ 50″	11′ 10″	11′ 30″
5°	14′ 25″	14′ 51″	15′ 10″	14′ 50″	15′ 12″	15′ 30″	9′ 50″	10′ 4″	10′ 30″
6°	13′ 32″	13′ 46″	14′ 14″	13′ 44″	14′ 6″	14′ 22″	8′ 34″	9′ 0″	9′ 22″
7°	12′ 42″	12′ 54″	13′ 10″	12′ 52″	13′ 12″	13′ 26″	7′ 33″	8′ 0″	8′ 20″
8°	11′ 34″	11′ 44″	11′ 58″	11′ 41″	12′ 0″	12′ 12″	6′ 43″	7′ 5″	7′ 20″

Apparent altitude	Sun: summer	Sun: equinoctial	Sun: winter	Moon: summer	Moon: equinoctial	Moon: winter	Fixed stars: summer	Fixed stars: equinoctial	Fixed stars: winter
9°	10′ 48″	10′ 57″	11′ 10″	10′ 54″	11′ 12″	12′ 24″	6′ 0″	6′ 18″	6′ 25″
10°	10′ 2″	10′ 12″	10′ 24″	10′ 10″	10′ 25″	10′ 36″	5′ 22″	5′ 38″	5′ 42″
11°	9′ 18″	9′ 27″	9′ 38″	9′ 25″	9′ 40″	9′ 50″	4′ 52″	5′ 8″	5′ 15″
12°	8′ 38″	8′ 47″	8′ 58″	8′ 45″	9′ 0″	9′ 10″	4′ 22″	4′ 37″	4′ 44″
13°	8′ 0″	8′ 8″	8′ 18″	8′ 6″	8′ 20″	8′ 30″	3′ 52″	4′ 6″	4′ 12″
14°	7′ 24″	7′ 32″	7′ 42″	7′ 30″	7′ 44″	7′ 53″	3′ 22″	3′ 35″	3′ 40″
15°	6′ 50″	6′ 58″	7′ 8″	6′ 55″	7′ 10″	7′ 19″	2′ 52″	3′ 5″	3′ 10″
16°	6′ 18″	6′ 25″	6′ 34″	6′ 22″	6′ 35″	6′ 44″	2′ 24″	2′ 34″	2′ 38″
17°	5′ 49″	5′ 56″	6′ 5″	5′ 54″	6′ 7″	6′ 15″	1′ 54″	2′ 4″	2′ 8″
18°	5′ 19″	5′ 26″	5′ 35″	5′ 23″	5′ 36″	5′ 33″	1′ 10″	1′ 19″	1′ 22″
19°	4′ 50″	4′ 57″	5′ 6″	4′ 54″	5′ 8″	5′ 15″	0′ 25″	0′ 34″	0′ 36″
20°	4′ 22″	4′ 28″	4′ 36″	4′ 26″	4′ 38″	4′ 45″	0′ 0″	0′ 2″	0′ 4″
21°	3′ 56″	4′ 2″	4′ 10″	4′ 0″	4′ 12″	4′ 18″			
22°	3′ 32″	3′ 38″	3′ 46″	3′ 35″	3′ 48″	3′ 56″			
23°	3′ 10″	3′ 16″	3′ 24″	3′ 13″	3′ 25″	3′ 30″			
24°	2′ 48″	2′ 54″	3′ 2″	2′ 51″	3′ 4″	3′ 10″			
25°	2′ 28″	2′ 33″	2′ 40″	2′ 31″	2′ 41″	2′ 46″			
26°	2′ 10″	2′ 15″	2′ 22″	2′ 12″	2′ 23″	2′ 28″			
27°	1′ 52″	1′ 57″	2′ 4″	1′ 55″	2′ 6″	2′ 10″			
28°	1′ 36″	1′ 41″	1′ 48″	1′ 39″	1′ 50″	1′ 54″			
29°	1′ 22″	1′ 26″	1′ 32″	1′ 24″	1′ 34″	1′ 38″			
30°	1′ 12″	1′ 16″	1′ 22″	1′ 15″	1′ 23″	1′ 36″			
31°	1′ 8″	1′ 12″	1′ 18″	1′ 10″	1′ 19″	1′ 22″			
32°	1′ 2″	1′ 6″	1′ 12″	1′ 4″	1′ 13″	1′ 16″			
33°	0′ 54″	0′ 58″	1′ 3″	0′ 56″	1′ 4″	1′ 7″			
34°	0′ 46″	0′ 49″	0′ 54″	0′ 48″	0′ 56″	0′ 59″			
35°	0′ 40″	0′ 43″	0′ 48″	0′ 42″	0′ 50″	0′ 53″			
36°	0′ 35″	0′ 38″	0′ 42″	0′ 37″	0′ 44″	0′ 46″			
37°	0′ 28″	0′ 31″	0′ 35″	0′ 30″	0′ 36″	0′ 38″			
38°	0′ 20″	0′ 23″	0′ 26″	0′ 22″	0′ 28″	0′ 30″			
39°	0′ 15″	0′ 17″	0′ 20″	0′ 16″	0′ 21″	0′ 23″			
40°	0′ 10″	0′ 12″	0′ 15″	0′ 11″	0′ 16″	0′ 18″			
45°	0′ 5″	0′ 6″	0′ 8″	0′ 6″	0′ 10″	0′ 12″			

The Sun's refractions were worked out on the assumption of our parallax of 27″ at apogee, 28″ at mean distance and 29″ at perigee; but if the parallax is greater, the refraction must be increased too. The Moon's refractions were computed using our parallax appropriate to her at mean distance; when the Moon is at apogee they decrease, and when she is at perigee they increase, roughly in the ratio by which the mean parallax exceeds the least, or is exceeded by the greatest. The refractions of the planets can be taken either from values intermediate between the Moon and the Sun, if the planets are nearer the Earth than the Sun is, or from values intermediate between the Sun and the fixed stars, if they are much higher than the Sun.

APPENDICVLA PRO PARTE HAC II. TOMI I.

A Short Appendix for This Part II of Volume I

Ad Librum Octauum.

On the Eighth Book

[Margin: Trimarchus's opinion on the location of comets.]

Chapter I, No. 3. Among the authors who have treated comets with sufficient care must be counted Hieronymus Trimarchus, in disputation 4 on Book 1 of Aristotle's *Meteorologica*, from section 1 to section 8. In section 3, however, he maintains that comets are below the Moon but above the middle region of the air, because he thinks the heaven is ingenerable and incorruptible; and that those which astronomers saw above the Moon were true stars, beneath which a coma was born, and that the astronomers took them for a single body — provided they were not deceived in the use of their instrument. Caussin too, in Book 1 of *De Domo Dei*, chapter 10, holds that comets are perpetual and are not made anew, but appear.

Chapter XIV. On the altitude of the regions of the air, see what I said in the Appendix to Book 2, chapter 19.

Ad Librum Nonum.

On the Ninth Book

Section I. For this section see Nicolaus Caussinus [Nicolas Caussin], Book 1 of *De Domo Dei*. There, in chapter 1, he deals with the name of heaven; in chapters 2 and 3, with the matter and solidity of heaven, defending its fluidity, and he considers comets to have been shown to lie above the Moon; in chapter 4, that there are no waters above the starry heaven; in chapter 5, that there is no fire in the concave of the Moon; in chapter 6, that the heavens and the stars are of a fiery nature, but of fire of a different species; in chapters 7 and 8, that the heavens are not animate, nor is light the soul of the stars; in chapter 9 he argues for the incorruptibility of the heavens; in chapter 10, that comets are not made anew but appear; in chapter 12, that angels do not move the stars but preside over their motions; and in chapter 14, that there are not several worlds.

[Margin: Fluidity of heaven.]

[Margin: Solidity of the Firmament.]

Mersennius, however, argues for the solidity of the Firmament in his commentary on Genesis, pages 814 and 819. Caussin deals with the Empyrean in Book 4 of the work cited above. Volume 3 of the *Dogmata Theologica* of Dionysius Petavius [Denis Pétau] has also recently appeared, in which he treats learnedly of the work of the six days and maintains the fluidity of heaven.

[Margin: Angels the movers of the heavens.]

Section II, Chapter I. Although Caussin, in Book 1 of *De Domo Dei*, chapter 12, denies that the celestial bodies are moved by angels, Nicolaus Serarius nevertheless teaches that they are moved, and indeed by the Virtues, following St Thomas Aquinas (Opusculum 10, article 17, and *Contra Gentiles* III, chapter

80), in his commentary on Joshua, chapter 10, question 20. And Lorinus [Jean de Lorin], at that place, understands the words of Ecclesiastes 1, *The spirit goes on in its circuit*, of an angel moving the Sun, with [?] St Thomas, Opusculum 10, article 6. Mersennius teaches the same in his commentary on Genesis, page 844.

[Margin: The number of the heavens.]

Section III, Chapter I. On the number of the heavens see Caussin, Book 1 of *De Domo Dei*, chapter 2, and Pétau, Book 1 of *De Opificio sex dierum*, which is in volume 3 of the *Dogmata Theologica*.

Chapter V. What concerns the number of the spheres of Eudoxus and Callippus is learnedly pursued by Boulliau in the Prolegomena to his *Astronomia Philolaica*, page 19.

Section IV, Chapters II and III. See Caussin, Book 1 of *De Domo Dei*, chapter 11, arguing for the immobility of the Earth.

[Margin: The motion of the Earth asserted by Anaximander.]

For the motion of the Earth, on the other hand, Theon of Smyrna in his *Astronomica* cites Anaximander of Miletus, saying: *Eudemus relates in his Astrologica that Oenopides was the first to discover the oblique position of the zodiac and the constitution of the great year; Thales, the eclipse of the Sun;* and Anaximander, that the Earth hangs aloft and moves about the middle of the world.

Chapter X. In Argument 3 I promised to say more about the libration of the Moon in the Appendix to Book 4, because I had not yet decided to reprint the leaves of Book 4 in which, in chapter 9, I have now discussed this libration sufficiently, as far as the observations made so far allowed. For the rest, which depends on future observations, we have reserved either for Volume 3 or for a separate book, which Father Francesco Maria Grimaldi, a most diligent observer of these matters, will perhaps publish, if time and life do not allow it to me.

In the same Argument I named a spot on the Moon the *Lake of Posidonius*, as though it had been so called by van Langren and by us. But in the reprinted leaves it seemed best to name it the *Spot of Grimaldi*, and, out of regard for antiquity, to move Posidonius to the upper part of the Moon.

FINIS TOMI PRIMI.

End of the First Volume.

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[64]printed "Seheineri"

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[65]printed "Parallax.um", a point where a letter should stand

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ERRATORVM CORRECTIO.

Correction of Errors

Page	Column	Line	Errors	Correct to
4	2	39	palatim	paulatim
110	2	26	num.11. & 12.	problem. 64.
233	1	69	ſiue	ſi
237	2	6	interim	interitu
278	1	15	diſtantiam	diſtantiæ
346	1	3	à nobis lacus	à nobis olim lacus
359	1	51	auſtrinus in G.	boreus in G
482	2	48	Degreſſu	Regreſſu
520	2	47	Lechanos	Lichanos
576	1	21	hora	horam
IX	1	12	S. Quintini	Caſtellati Quinti

REGISTRVM.

Register

¶A ¶B

A B C D E F G H I K L M N O P Q R S T V X Y Z.

Aa Bb Cc Dd Ee Ff Gg Hh Ii Kk Ll Mm Nn Oo Pp Qq Rr Sſ Tt Vu Xx Yy Zz.

Aaa Bbb Ccc Ddd Eee Fff Ggg Hhh Iii Kkk Lll Mmm Nnn Ooo Ppp Qqq Rrr Sſſ Ttt Vuu Xxx Yyy Zzz.

Aaaa Bbbb Cccc Dddd Eeee Ffff Gggg Hhhh Iiii Kkkk Llll Mmmm Nnnn Oooo Pppp Qqqq.

All are duernions, except ¶A, which is a ternion, and Qqqq, which is a unernion.

Diagram. The printer's device, engraved, standing between the register and the imprint. A winged cherub's head at the top looks down over a scroll bearing a three-line motto; foliage and scrolled ornaments fill the upper corners. Two putti stand at the sides holding a cartouche that frames a scene of heavy clouds over a rough sea. At the foot, in the scrollwork, is a small shield patterned with lozenges.

- Scroll (top centre), line 1: FLVCTIBVS ET FREMITV
- Scroll, line 2: ASSVRGENS BENACE
- Scroll, line 3: MARINO
- Motto in English: "rising with waves and a sea's roar, O Benacus" — BENACE, a vocative; its last letter stands against the curl of the scroll

Bologna,
from the press of the Heir of Victorius Benatius [Vittorio Benacci], 1651.
With the permission of the superiors.