Kitsur ha-Melakhat ha-Mispar

Compendium of the Art of Number

(c. 1500)

by R. Eliyahu Mizrahi (ca. 1455–1526, Constantinople)

Translation by Scott Weisman

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Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

Kitsur ha-Melakhat ha-Mispar ("Compendium of the Art of Number") is a Hebrew arithmetic textbook by Rabbi Eliyahu Mizrahi (ca. 1455–1526), the chief rabbi of the Ottoman Empire and one of the foremost Jewish mathematicians of his age. Drawing on the Greek and Arabic traditions, it lays out the practical art of calculation step by step — operations on whole numbers and fractions, ratios and proportions, progressions and roots, and the rules for solving the everyday problems of trade and measurement. Lucid and systematic, it became a standard introduction to arithmetic for Hebrew readers. This English translation is made from the Hebrew text as printed in the first edition (Basel, 1546), where the work was bound together with bar Hiyya's astronomy.

A note on the translation process

This translation was produced with the translation-pipeline. See the linked repository for details on the process.

A note on the references. A scriptural or bibliographic reference given in brackets or parentheses is, in most cases, an **editorial identification of an allusion rather than a citation printed in the source**. Books of this kind quote and weave scripture constantly without naming it, and the identification is supplied here as a convenience to the reader. Where the source does print a reference of its own, it is given as printed. **An identified allusion is not evidence that the author cited that passage** — look at the original before resting an argument on one.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

How this edition was produced

This edition was derived from the 1546 Basel first edition (printer Heinrich Petri), digitized by the New York Public Library. The path from source scan to finished text was as follows:

1. **Source isolation.** The 1546 volume is bilingual: each work is printed once in Hebrew and then again in Latin (the apparatus of Sebastian Münster and Erasmus Oswald Schreckenfuchs), the Latin bound and scanned in

reverse page order after the Hebrew. Only the **Hebrew** portion — physical pages 1–370 of the 598-page scan — was translated; the Latin counterpart of the same works was left in the original.

2. **Batching.** The Hebrew-only source was split into uniform two-page batches in physical scan order.
3. **Translation.** Each batch was rendered into English sequentially in a publication register, using Münster's facing Latin annotations as a parallel witness to disambiguate the difficult sixteenth-century Hebrew type (his apparatus runs through the astronomy and ends at its colophon).
4. **Reconciliation.** Flagged anomalies — uncertain numerals, scrambled type, structural boundaries — were re-examined against the source and corrected. Genuine compositor errors of the 1546 printing were kept verbatim with an editorial `[recte: …]` note rather than silently emended.
5. **Collation.** The reconciled batches were assembled and split into the two distinct works bound in the volume — the astronomy of Avraham bar Hiyya and the arithmetic of R. Eliyahu Mizrahi — each issued as its own edition.

This volume is *Kitsur ha-Melakhat ha-Mispar ("Compendium of the Art of Number")*; its companion, *Sefer Tsurat ha-Arets ("The Form of the Earth")*, is issued separately.

Acknowledgements

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Kitsur ha-Melakhat ha-Mispar (Compendium of Arithmetic)

Title Page

[Manuscript marginal note, modern hand, ink: "Kizur ha Melacha ha Mispar"]

קיצור הַמְּלָאכֶת הַמִּסְפָּר

Kitsur ha-Melakhat ha-Mispar — "Compendium of the Art of Number"

[Latin: **COMPENDIUM ARITHMETICES**, excerpted from the book of *Institutions of Arithmetic* by master Elias Orientalis. (= *Compendium Arithmetices, decerptum ex libro Arithmeticarum institutionum magistri Eliae Orientalis* — i.e. a compendium of arithmetic drawn from the arithmetical textbook of Rabbi Eliyahu Mizrahi, "Elias the Easterner".)]

Sebastian Münster, To the Studious and Fair-Minded Reader

[This page is entirely in Latin — Münster's preface to the arithmetic. Rendered below.]

[Latin: **SEBASTIANUS MUNSTERUS** to the studious and fair-minded reader.]

[Latin: You will readily forgive me, excellent reader, this fault — that I have placed arithmetic after astronomy in a reversed order, when in fact it is arithmetic that holds the chief place, and without which no one could grasp the musical proportions, the geometric measurements, or the astronomical motions, the very subject of this present undertaking. When I was lecturing on the *Sphere of the World* of Rabbi Abraham the Spaniard to my students at Basel, they were obliged to prepare copies for themselves by writing it out, and since this came hard to them, I resolved to have the little book printed in type, so that both I and the professors of the other academies might make use of it; for I have no wish to keep to myself alone what may also profit others. And so it came about that, the *Sphere* being finished, it at length occurred to me that I would be doing a worthwhile thing if I were also to add an arithmetic — of which Schreckenfuchs, a man much devoted to the study of the Hebrew tongue, had earlier prepared an exemplar. But it matters little whether the one or the other comes first in the book, provided you know that arithmetic is to be observed and learned first in mathematical study.]

[Latin: The annotations we have added may serve you in place of a translation — indeed, of a fuller explanation, both of the language and of the art. For, aiming at brevity, we have omitted Schreckenfuchs's exact interpretation; yet it was of no small help to us in our own annotations. Farewell.]

The Compendium of the Art of Number

[Latin outer-column commentary, left of the Hebrew text:] [Latin: Arithmetic is the art by which are known the methods of reasoning and the rules that, easily and without labor, lead one toward the exact investigation of numbers. And indeed it is necessary that we turn the sharpness of our wit and our whole study toward this art, since the other liberal arts cannot be understood without it. We add this too — though it is self-evident — that this science is, as it were, a step by which our mind ascends from corporeal and sensible things, in which from childhood we have grown up, to things foreign to us.]

קיצור הַמְּלָאכֶת [The Compendium of the Art of Number]

A book that gathers up the *Book of Numerical Roots* [*Sefer Charashot Misperiyot* — the foundations of computation] which Rabbi Eliyahu the Easterner [Eliyahu Mizrahi; printed מורחי, recte מזרחי], may his memory be a blessing, composed and abridged.

הַמִּסְפָּר [Number]

It is a wisdom from which are derived the modes of computation and the methods that direct a person toward the knowledge of the numerical investigations [?] easily, without toil or labor.

Since it is therefore incumbent upon us to multiply our study and our diligence continually in this wisdom — for it is shared in common by every branch of wisdom —

it follows of itself that this wisdom is like the rung of the bridge by which our thought crosses from these corporeal, sensible things to the things that are intelligible and abstract; and it crosses [thus from] those corporeal things in which we grew up from our youth, and to which we were habituated, toward other things that are foreign to us, things we were not habituated to —

— our senses, which in their perceiving point [the way] for our souls. And there is no path to the knowledge of the kinds of those things which truly exist, except through these four arts: namely the art of number [arithmetic], geometry

[וְהַהַנְדָּסָה, al-handasa], astronomy [?], and the art of composing melodies [music]. These are the kinds of this wisdom.

We say [further]: just as every one of [these] crafts requires, for its execution, expertise in its craft — and a representation [of the result] is derived from it through the precision of its inquiry [?] — so it is with these arts within the craft of philosophy. And inasmuch as this is so, the two parts of this wisdom — namely the art of number and geometry — are in fact more honored and nearer to first philosophy than the two latter parts,

according to what shall be the subject of the science of number. Now the substance [literally: life] of number is what is shared in common by every kind of matter, whatever matter it may be; and this is not so for the subject of the science of composing melody, for that — although its substance be shared in common with every kind of the [musical] scales [?] — is nevertheless [a kind of] number...

[Latin outer-column commentary, right column:] [Latin:...of which we have no experience, but by reason of their subtlety they are like to our own minds; nay, there is no way to any private discipline of which it may be said that it rests upon truth, except through these four disciplines: Arithmetic, Astronomy, Geometry, and Music — which are, as it were, the parts of this science. And just as in every art there is need of a certain experience and of a settled habit, by which the craftsman is directed toward the accomplishment and end of his investigation, so this work is possessed of these arts in Philosophy. Since this is so, these two parts — Arithmetic and Geometry — are more excellent than the rest, and draw nearer to first philosophy, since their scope and subject...]

Kitsur ha-Melakhat ha-Mispar (Compendium of the Art of Number) — Eliyahu Mizrahi

[it] approaches first philosophy more than [does] the [study of] measure of any of the other disciplines among its companions. For the subject of geometry, indeed, lies outside [number] — being common to all magnitudes, whatever the magnitude may be and of whatever matter it may consist; whereas this is not so for the wisdom of the intellect, whose subject is solely the matter of the celestial spheres, in [their] determinate form alone, and nothing besides this.

[Latin note (Schreckenfuchs/Münster): "…of Arithmetic, since number itself partakes of any other matter, but is not in this way the subject of music, because it partakes of all the species of vocal sounds, although the vocal sound itself is also number. Geometry has for its aim measure, which it shares with any measurable matter. Astronomy, however, considers orbs and spheres. Hence it is clear how necessary these four parts are, and that from among them we especially choose Arithmetic and Geometry, whose scope and subject [coincide with] the quadrant, [together] with the subject of the divine science. And although Arithmetic communicates with and is assimilated to Geometry for the said reasons, it nonetheless precedes it by nature. For…"]

Therefore it necessarily follows from this that, of these four parts, we should choose the two parts that bear solely on incorporeal things and on their imitation [as] the subject of the divine science. Yet, since the wisdom of number — although it is common to, and bound up with, the wisdom of geometry for the reasons we have stated — is nonetheless prior by nature to the wisdom of geometry:

[for] this is so [in] that, when we posit number, we [thereby] posit geometry, but [geometry] is not raised up by the raising-up of geometry [itself]; and [in] that, where geometry is found, number is found, but [number] is not found by the finding of number [alone]. And this is so [in] that, when the geometer [?] exists, there is no doubt that reckoning exists along with him — for so it is with magnitudes.

— figures, whether triangular or quadrilateral, possessing eight sides and their angles [?]. Here, then, is a thing already in use in the art of reckoning, yet the art of measure is not found except by way of reckoning.

מין הספירה — On the Kind of Numeration

Now, the one who begins with the art [of number] must first grasp four things. First, the symbols of the figures — that is, the principal signs, which are the signs the teachers use, [each] according to its place, in counting one after another by means of the art, and according to the figures; for these signs are simple [i.e. uniform] in every land, in the West and in most of the lands of the East. Therefore we have chosen, in our composition, these signs, which are the figures of all [peoples]:

1	2	3	4	5	6	7	8	9	0

And if you wish to use the [letters] instead, together with the [Hebrew] characters — see, here they are for you: א ב ג ד ה ו ז ח ט [= 1 2 3 4 5 6 7 8 9], and the zero (0). Now, as for the numbers: behold, they are of two kinds — a kind of quantity, and a kind of quality. As for the kind of quantity, this is indicated in [the figures] themselves, in their very essence; and as for —

[Latin note (Schreckenfuchs/Münster): "In numeration it is necessary to have certain characters by which the numbers may be expressed according to their diversity, whether they be Indian or Hebrew characters. The Indian [characters] are now divulged throughout the whole West and through a good part of the East, which the Jew also uses in this book, even if he does not refer [to them], whether he uses these or the Hebrew figures. We shall point out, moreover, in the notes, the quantity and quality in numeration. By quantity understand the absolute signification..."]

— as for [the kind of] quality [as opposed to] quantity, it is indicated by their place. And we mean by "quantity" that in which we do not look at the [figures] except [as] the number of the units included in them — only that, [namely] whether the units are other [than units]: tens, or hundreds, or [some other rank]. And this is like the sign for "three," by way of example: for [in the case of quantity] we do not look at them except [as] the three[ness] alone — that is, [in] that they are compounded of three units; and we do not look at the units [to ask] whether they are other [than units] — and then [it would be] "thirty"; or whether they are tens, and then [it is] "thirty"; or whether they are hundreds, and then [it is] "three hundred" — and the like of these.

And we mean by "quality" that in which we look at the units included in them [to ask] of what kind they are: whether they are units, or [are] of the tens, or of the hundreds, or [proceed] beyond this among the ranks. For the first figure, [namely] the one indicating "one," [signifies] according to its quantity; [but] if it stands in the first place, then according to the disposition of the signs it indicates the rank of the units, and so it shall be a single [unit]. And if it stands in the second place, it indicates the ranks [of the tens] —

— the ranks of the tens, and so it shall be [a ten]. And if it stood in the third place, it indicates the rank of the hundreds, and so it shall then be [a hundred]. And in this way, perpetually, according to the multitude of places the ranks indicating the incorporeal things are multiplied. Yet [the figure's] quantity is always one [and the same]: that [same] "one" which, at one time, shall be [a count] of tens, and at one time of hundreds.

And likewise the second figure, indicating "two," [as to] its quantity: if it stood in the first place, it indicates the ranks of the units, and it shall be two units; and if [it stands] in the second place, it indicates the tens, and it shall then be two tens

— that is, twenty; and if it stood in the third place, it indicates the rank of the hundreds, and it shall then be two hundreds — that is, two hundred. And so for all of them: the ranks are multiplied by the multiplication [of place], and the [qualities] are exchanged, [while] the quantity is multiplied and the [single] rank remains one in itself. That is: if you set, in the second place, the sign "two" [for example], it indicates two tens; and if you set in it the sign "three," it indicates three tens —

[Latin note (Schreckenfuchs/Münster): "...[and so] consequently, ascending from one place to another, the quality of the numbers is continually changed. But their quantity always remains the same, except that the unit, according to [its] quality as we said, is changed from one [to] ten and [to] a hundred. Further, the second notula [little mark], namely 2, signifies a binary if it be in the first place; and in the second place it signifies two tens, that is, twenty; and in the third place, two hundreds, that is, 200. And in this manner all the notulae are multiplied and varied according to one and another situation of place. For example: 2 written in the second place signifies twice ten; and 3, thrice ten; and 9, nine times ten — where to the quantity of the numbers is added the quality of the tens. Further, the notula which [signifies] hundreds..."]

מין הספירה — On the Kind of Numeration *(continued)*

(Foliation runs 219, 220, 221, 221 — the last two leaves both bear "221"; the sequence is continuous, not a gap.)

[fol. 219]

the tens; and if you place the sign *tet* [9 = ט], it points to nine tens [= 90].

Now I have already explained the matter to you clearly, in [terms of] how these figures behave according to their positions. The numbers — in their quantity and in their quality — all take their sign from the ten figures, for the tenth of them is the *galgal* ["wheel," the round mark): that is, the figure that has no quantity at all. It is called in the foreign tongue *gulla* [the round one / nought], and is derived from the [word for] "nothing" in the [various] languages. In the tongue of Ishmael [Arabic] its figure is called *ṣifra* [cipher], which signifies a thing that has no quantity, and likewise in their languages it stems from the [word for] privation. In the tongue of Greece [*yavan*] it is the figure called *ouden* [οὐδέν, "nothing"]. Indeed, even though it carries no quantity in itself, it is nonetheless a cause of quality in the other figures that follow it — that is, the figures that come after it acquire, beyond their [own] quantity, a further degree of quality according to the place and the position they occupy.

For instance, if you wish to write ten, you put first the figure of "nothing" [the cipher] and then the figure of unity, in this fashion: **10**. Here, although the cipher possesses no quantity, it nonetheless confers a degree upon the unity that follows it.

If you wish to write a hundred and one [101], you place the cipher in the middle. The first figure will signify the units of that same quantity, while the cipher [itself] yields no quantity, [yet] it confers a degree…

[Latin note (Schreckenfuchs/Münster), fol. 219 outer column: "…also signifies [the cipher]. It is a certain small circle, which the Latins call *nulla* — for *nulla* is derived from that which is nothing [*nihil*]. The Saracens, however, call it *ciphra*, a word that signifies privation; and the Greeks [call it] *ouden*, which likewise sounds 'nothing.' And although it signifies nothing in itself and is in no way a quantity, it is nevertheless a cause of quality in the other figures following it — that is, the marks that follow it have, beyond their quantity, also a degree of quality according to their differing position and place. Thus, if you wish to denote ten, you put first the nought-mark and then the mark of unity, in this manner — **10** — where, although the cipher (o) has no quantity, it nevertheless assigns a degree to the unity that follows. If you wish to write a hundred…"]

[fol. 220]

which the figure signifies; and if it does not point to a quantity, it is nonetheless an indicator of a degree. And so the first figure is drawn forward by the others that follow it, [each] figure pointing to a degree drawn after the [preceding] degree. This is what was lacking to it [the bare figure until placed].

And so, if you wish to write a hundred and one [101], you place a cipher in the middle, since the first figure has no [intrinsic] value — whether it points to one figure or to others after it. For in our case the first figure points to one [unit]; and in its position the other figures and the cipher are arranged in sequence, pointing to the degree of the tens, yet the [cipher] does not point to any count [of its own], since it has no quantity. It points only to a degree, in the manner we have set forth. The first figure points to a count of one, by reason of its being the first figure; and in its [third] position the [following] figure points to a hundred, the other [figure] that comes after [the first] being the cipher, [in] position three. For behold, the use of the cipher is to increase the places of the degrees — nothing else apart from this.

And so, by this method, you are already able to write any number you wish with these signs that are written here, [drawn] from them.

[Latin note (Schreckenfuchs/Münster), fol. 220 outer column: "…a hundred and one: you put a cipher in the middle, and the first numeral will signify the units of that same quantity, whereas the cipher makes no degree of the ten [*denarius*] but is without signification; and the mark following it signifies that same unit as a hundred [*centenarius*], when it is placed in the third position. The cipher therefore helps to augment the place and the degree, which would not happen without it. In this manner you will be able to write whatever number you wish with these marks, of which nine signify quantity and the tenth signifies nothing at all. Therefore, the marks of the numbers having been set down and explained according to their varying position and place — whatever numbers there finally be — it remains for us to set forth this kind by a clearer way: namely, how [one comes] to…"]

[fol. 221]

their knowledge by their quantity, and the tenth of them [the cipher] without quantity at all, [yet] pointing to a degree without any quantity whatsoever.

And after I have explained well to you the matter of these signs in their places and their positions, and the knowledge of the rest of [the art of] numeration with these signs — there is in them [enough] numeration — there already remains [nothing] but to make known to you the figure that points to [enables] reckoning, which you may reach by [this] matter, for the knowledge of this kind [of numeration].

Before that, I shall rouse [your attention], by way of pointing out and making known to you the use [reason] of why there are nine signs, no more and no less, in counting. For they suffice for every number that there may be — whether [the numbers] be *peratim* ["particulars" = digits] alone, or *kelalim* ["universals" = articles] alone, or compounded of *peratim* and *kelalim* together.

The *kelalim* [articles] taken alone are the numbers [reckoned] by themselves, such as the tens, from one up to nine [10–90]. The *kelalim* compounded with the *peratim* together are the figures compounded from the others — such as the tens [*ʿaśarot*], the hundreds [*meʾot*], the thousands [*alafim*], and the ten-thousands [*rivavot*], and their [further] elevations.

And the *kelalim* together with the *peratim* are the numbers compounded from the others, such as [25] **כ"ה**: behold, [it is made] of "twenty" [*kaf* = 20] of them — which are like two [*bet* = 2] in reckoning — and the [digit-]five [*he* = 5], [these being] of the others by themselves.

[The worked composites here: **25** = 5 + 20 = **כ"ה**, and **203** = 3 + 200 = **ר"ג**, matched against the Latin examples opposite.]

[Latin note (Schreckenfuchs/Münster), fol. 221 outer column: "...to the knowledge of it. Yet first I shall show why there are only nine numbering-marks. Indeed it is settled that every number is either *particular* — that is, a digit — or *universal* — that is, an article — or composed of digits and articles. **Digits** are those which ascend from unity up to nine [1–9]. **Articles**, by contrast, are like the unities [but of higher order] — such as the tens [*denarii*], hundreds [*centenarii*], thousands [*millenarii*]. **Articles combined with digits** yield a composite number, which is to say [one] constituted from unities and tens, as is **25**; or from unities and hundreds, as is **203**. Articles are contained under the unities to which they are similar, since the ten [*denarius*] stands in its place just as the unit stands in the number-series. So of a hundred, a thousand, and ten thousand you should judge..."]

[fol. 221 *(second leaf so numbered)*]

and their elevations; and those that are *kelalim* [articles] are [contained] under the others to which they are similar, since they correspond to them. For the ten [behaves] like one [*kemo aleph*] in reckoning; and so the hundred [*me'ah*], the *rivavah* [ten-thousand], and the *harkavah* [composition] are like two [*kemo bet* = 2] in reckoning; and so [are] the [two] hundreds [*resh* = 200] in reckoning, and the thousands, and their elevations.

And the **thirties** [*sheloshim*] are like three [*gimel* = 3] in reckoning, and likewise the **three thousands** [*sheloshet alafim*] and the **three ten-thousands** [*ḥag rivavot* / three myriads]. The **nineties** [*tishʿim*] are like nine [*tet* = 9] in reckoning, and likewise the **nine hundreds** [*teshaʿ me'ot*], the **nine thousands** [*teshaʿ alafim*], and the **nine ten-thousands** [*teshaʿ rivavot*]; and so on of this kind without end.

And the *kelalim* [articles] together with the *peratim* [digits] taken together are likewise compounded with the others; for behold, [a number] such as **כ"ח** [*kaf-ḥet*] points to [a composite degree], by way of example: for among them there are [components] like two [*bet* = 2] of them, as in our example, while others among them stand by themselves.

And so [a number] such as [203] **ר"ג** — behold: among them there are [components] like two [*bet*, the "two hundred"], and likewise the [single] three [*gimel*], which is of the others by themselves [the bare digit 3] together. They are required by this [composition], so that in this fashion there shall be the numeration of every number. [And all this is] included within the nine other [figures] alone — the signs of the numeration, *tet* [9 = ט] alone — and…

מין," opening the next leaf]

[Latin note (Schreckenfuchs/Münster), fol. 221 *(second)* outer column: "…[so judge of the higher orders]. **Item:** 20, in computation, are as 2. Judge the same of 200, of 2000, and 20000. So **30** are as if 3 in computation — say, [so] of 300, of 3000, of 30000. And **90** are as if 9, wherever they fall in number, as 900 and 9000 and 90000, and so on in succession. **Item:** articles are comprehended together with digits under the unities, as in **25** there are **20** as if two, and **5** as if unities by themselves. So in **203**, the *ducenta* [two hundred] are as two, and the *tria* [three] as unities standing by themselves. **Hence it follows** that the quantity of all numbers is comprehended under the nine unities; and for that reason there are only nine marks."]

Kitsur ha-Melakhat ha-Mispar — Addition (הקיבוץ)

מין הקיבוץ — The Operation of Gathering (Addition)

הקיבוץ — Addition (הקיבוץ)

He said: This is the reckoning [הוא החשבון] of its numbers.

[Latin note (Schreckenfuchs/Münster): "The Hebrews call addition *congregatio* (a gathering). It is the reduction of many given numbers — equal or unequal, like or unlike — into one number."]

What is addition [הקיבוץ]? It is the joining of several numbers — whether rows [or a sequence] or distinct numbers — into a single number.

And from where do we begin? We begin at the fourth pathway, the operation of addition [מלאכת הקיבוץ]. And I say that whenever you wish to combine [להוסיף] one of several numbers with another, set them down one beneath the other [קצתם על קצת], so as to bring them all back to a single number. See, then, that you arrange all the orders of the numbers [סדרי המספרים]: place each one under the one of the same kind, so that you open with every rank [מדרגה] — placing every rank under the like rank: the units' rank [מדרגת ת...] under the units [מדרגה], the tens' [מדרגת ר"ל] under the like, [tens] under the tens, the hundreds' [מאות] under the hundreds [תחת ה...], and the tens [עשרות] under the tens [תחת ה...].

[Latin note (Schreckenfuchs/Münster): "And here it is necessary, when you have several numbers which you wish to reduce into one sum, that you write them in order one beneath another, observing the rank, situation, and place — units namely under units, the denarius [ten] under the denarius, and so forth."]

So too, the hundreds [המאות] under the tens [העשרות], and the tens [העשרות] under the hundreds [the ranks]. After this you add [תחבר] all the signs [הסימנים] that come together — whether they be rows [or a sequence] [המתחלפים] — that is, the ranks that are upon them [בעלי איבגד"א]...

[Latin note (Schreckenfuchs/Münster): "And the number arising from this addition will be one of three [kinds] already mentioned: namely a *digit* (digitus) or an *article* (articulus), or composed of both. If a digit results, write it below, under the drawn line — the first number under the first, or units under units."]

After [אחר] the [tens'] rank [ר"ל], in another [rank], then [add] the number [המספר]. The question [השאלה]: from what does the result come out — whether it issues from another [rank] — according to what they are, [according to] whether they will be digits to the [units] [פרטיים], and if [they make] whole [tens] [כללים]; and if [the issue] is a digit [הפרט] under the [units], then there is another reckoning [...]; and if a [whole] [כלל] is gathered together [שניתם], then [carry] it; and if it makes a digit [פרטים], write it under the [units] you have written down [תכתבנו].

below [למטה], with its mark [באותה], the [units'] rank [המדרגה] itself [עצמה]; and afterward [you carry] what remains, the line [קו], under each rank of all the addends [הנקבצים] [ר"ל]. If there be among the addends [הנקבצים] other ranks [במדרגת האחרים], then write [תכתוב] also the sum [הסך] under [it] [חסר], in the rank of the others [במדרגת האחרים], under the [units'] reckoning [חקר] the *mubril* [המובריל — i.e. the carried-over remainder].

And if [there are] among the addends [הנקבצים] tens [העשרות] in the rank of the tens [במדרגת העשרות], write them [כתוב בם] the sum [הסך] under [it] in the rank of the tens [במדרגת העשרות], under the [units'] reckoning [חקר] the *mubril* [המובריל], and so for every rank [ובכן בכל מדרגה ומדרגה].

And if there be a [whole] sum [הסך] entirely [בכל] without [לבר] — see [ראו] that we have explained [שנבים [recte: שנבאר]] that it falls [מ...] in the [units] — from where comes this rank [מורגה [recte: מדרגה]] itself [הוא], and within it [ובפני] its rank [מדרגתה] is reached [יונח] each one [כל אחד]. And [I] say [ואמר] [ר"ל]: if there be among the addends [הנקבצים] other [ranks] [אחרים], and the sum [וחסך] rises [עלת] upon two [על ב'] — for example [משל] that there be [שהם] in the whole [בכלל], and write [ויכתוב] two signs [שני סימנים] in the second rank [במדרגה השנית], or two enclosures [שתי נסגרות [recte: נסגרות]], and under it [ותחת ד...] the rank of the others [מדרגת האחרים] [you] write [ויכתוב] the cipher [סיפרא — i.e. zero].

And likewise [וכן], if there be among the addends [הנקבצים] tens [עשרות], and the sum [וחסך] rises [עלת] upon two [על ב'] — for example [משל] that they be [שהם] in the whole [בכלל] — write [ויכתוב] two signs [שני סמנים] or enclosures [נסגרות] in the second rank [במדרגה השנית], it [לח], and under it [ותחת] the rank of the tens [מדרגת העשרות], and write [ויכתוב] the cipher [סיפרא].

And if there be a sum [הסך] rising [עלת] upon two [ב'] — like [כמו] [102 = ק"ב] [for example] upon [על] [ר"ל] — for example [משל] that, behold [הנה], it is fitting [ראוי]...

[Latin note (Schreckenfuchs/Münster): "But if the product be an article [articulus] — say a simple or repeated denarius [ten] — write it below the line, under the rank of the tens [denarii]. And in this manner you proceed through all the orders of the ranks. But if the product be a complex [number], it is necessary that you carefully attend to which order and rank it points to — so that, if the sum of the units should ascend to 20 (which are articles), write 2 in the second place, and a cipher [0] in the place of the units. Likewise, if the collected numbers ascend to a sum of 20 tens [denarii], where again you have an article: write 2 in the second place, in respect of this second place, and a cipher [0]; mark this in the second place. Likewise, if the aggregate makes up two species of articles — say, if there be 120, you write..."]

for [we] see [נראי] that you should write [שתכתוב] two signs [שני סימנים] [ב"ז = ?] [or] enclosures [נסגרות] in the rank [במדרגה], the second [השנית], it [לח], in [the] *be'er* [בעאר [recte: בעמוד?]] in the whole [בכלל]. For [I] say [שב...]: that we see [שבזרינו] [recte: שבחרנו] or write [או ירשום] [12 = ר"ל] [or write] [the marks] [סימנים], pointing [מורים] upon [על] [12 = י"ב] upon [על] the rank [המדרגה], the second [השנית], it [לח], and under [it] [ותחת] the rank of the addition [המדרגה החיבור [recte]], the addend [הנקבצת], and write [ויכתוב] the cipher [סיפרא].

[Latin note (Schreckenfuchs/Münster): "...you write the binary [2] in the second place, since it signifies 20, and a unit in the third place, since it is an article of hundreds [centenarius]. Or let 12 be signified in the second order of numbers, and under the augmented rank write a cipher [0]. But if the product be both an article and a digit, the article is written in its own place and the digit in the place where the addition was made. And in this manner you will be able to reduce all numbers into one number, nor will you mind however much they be multiplied."]

And if there be a sum [הסך] entirely [בכל], and a digit [ופרט] together [יחד], behold [הנה] write [ויכתוב] the [whole] [הכלל] in its place [במקומו] according to [לפי] its rank [מדרגתה] just as [כאשר] [you do] in the four [בארבעה], and the digit [והפרט] under [תחת] the rank [המדרגה] of the addend [הנקבצת].

And by this way [ובזה הדרך] you will be able [תוכל] to combine [לחבר] any [אי] numbers [זרים [recte]] that there be [שיהיה] into [אל] a single number [מספר אחר]; and however much [וכמו מה] you multiply them [שירכבו] the ranks [המדרגות], the *me'urgarot* [המורגרות], there is [אין] nothing [חשש] in this [בזה].

[Latin note (Schreckenfuchs/Münster): "Example of addition:" — followed by the worked column:]

Addend
1101
3931
9755
57052
28067

[Latin note (Schreckenfuchs/Münster): "Here we begin to add from the units, so that if the product grows to an article, the place for marking may be supplied in the following..."]

המשל בזה — The Worked Example for This

We wished [ורצינו] to gather [לקבץ] these: a thousand [אלת] and a hundred [ומאה] and one [ואחד] [= 1101], and next [ואחר], and three thousand [וג' אלפים] nine hundred [תשע מאות] thirty-one [31 = ל"א] [= 3931]; and nine thousand [וט' אלפים] seven hundred fifty-eight [758 = תשנ"ח]; and fifty-six [ונ"ו] thousand [אלפים] and fifty-two [ונ"ב] [= 57052]; and twenty-eight [וכ"ח] thousand [אלפים] and sixty-six [66 = וס"ו] [= 28067].

Behold [הנה] we have arranged [נסדר] our numbers [מספרינו]. And we begin [ונתחיל] from the last [מהתאחרי [recte: מהמתאחרים — the rightmost]], because [מפני]...

because [מפני] that if there be [שאם יהיה] the sum [הסך] [of] the units [העולה [recte] ת], entirely [בכל], behold [הנה] the place [מקום] of its addition [הוספתו], behold it [הוא] [במ...] in the rank [במדרגה] the fifth [הנמשבת [recte: החמישית]] to it [לח], and let us add [ולבן [recte: ולכן]] the addend [נקבצ...]. The others [האחרים] in it [בלם [recte: בכם?]], and behold them [והנה הם] [י"ר [recte] = ?], and the cipher [וחצף [recte: וחסר?]] the sum [הסך] of the addition [העולה] from the others [מהאחרים], and we have added it [וגמשוה [recte: וגמרנוה?]]... [ק] under another [סר [recte] אחר], under [תחת] [the rank] [the addends] [הנקבצים], and write [ונכתוב] under it [תחתיו] the sum [הסך = חסך] [ר"ל], one [א'] [in] the rank [מדרגת] of the others [האחרים], and that [וחי [recte: וזה]] which [אשר] from it [במ...] [is] it [לח], to be [להיות] that they are [שהם] tens [עשרות]. We see [ראוי [recte: ראינו]] [ש] that we return it [שיכתבזוב [recte]] in the rank [במדרגה] the addend [הנקבצת], it [לה], that [the issue] [זה], a digit [נקבץ [recte: פרט?]] in the rank [במדרגה], the *me'urgarot* [the tens'] [העשרות], and they [והם] are tens [עשרים], and so that they be [ולהיות שהם] a whole [כל] in every rank [בכל מדרגה], and there is no [ואין] there [שם] another [אחרים] in the whole [בכלל], let us write [לבן [recte: לכן] נכתבנו] the cipher [סיפרא = סימרא] under [תחת] the rank [המדרגה] of the addends [הנקבצת] [ו"ב = ?], in the rank [כמדרגת] the fifth [הגמשברת [recte: החמישית]], it [לח], according to [לפי] what they are [מה שכרם].

After this [אחר זה] the addend [נקבוץ] rank [מדרגת] of the hundreds [המאורת [recte: המאות]], and they [וחם [recte: והם]] are nine [ט'], and to be [ולהיות] that they are [שהם] entirely [בכל] joined [מחובר] with the others [עם האחרים], write [נכתב] the others [האחרים], that they [שהם] are nine [ט'], under [תחת] another [the next] [ה], the *me'urot* [המאורת — hundreds] that are [שהיא] in the rank [מדרגתה] and the *re'ah* [וראה [recte]]... that it [שהיא] in the whole [בכלל] we have written it [נכתבחו [recte: נכתבנוה]] in the rank [במדרגת] the [...]; it [לה].

After this [אחר זה] the addend [נקבוץ] [of] the rank [מדרגת] of the thousands [האלפים] [ורחם [recte: והם]]...

[Latin note (Schreckenfuchs/Münster): "...the order of the numbers. The units taken together make 16, which are written under the line of the numbers to be added — 6 in the first place and ten in the second place, whose quantity is indeed a number, but in order that it may become ten it is written in the second place. Then let all the denarii [tens] be added together and they are collected as 20: a cipher [0] is therefore written under this grade of addition, since no units extend there, and a 2 in the following place, as already shown. If again we add together the hundreds [centenarii] we obtain 19, which number is composed of an article and units: write 9 under the hundreds and the unit under the following order. Then collect the millenarii [thousands], which are 29; and write the digit under their order and the article under the following order. After this collect..."]

Kitsur ha-Melakhat ha-Mispar — The Operation of Gathering (Addition), continued

and [collect] them as one, and write it down. This [carried figure] is one of the thousands, so write it likewise. Once you have written everything down as we have explained, you will have the sum.

[Latin note (Schreckenfuchs/Münster): …collect into one [ten of] thousands and you will find 9, which you should write under the same place since they are a digit, and you will have the sum of all the said numbers.]

[Latin worked example, left column — column of addends with their total:]

Addends
1101
3931
9755
57052
28067
99906

After this, once our gathering of these figures came out to its result, and there remained 9, with the others standing beside it according to their places — we have now explained it. Now [we set down] the sum of all these figures in their proper arrangement, just as the reckoners reckoned them: ninety-thousand and קֵ"א [= 91 [?]], and רח"ג [?] — nine thousand and מ[א]ות ל"א [hundred (and) 31 [?]]; and רחט [?] thousand תשנ"ח [= 758]; and רח"ג [?] thousand, ג"ב [?]; and רחנ"ח [?] thousand ס"ז [= 67]; they are ninety and nine thousand תת"ק[ו] [= 906 [?]].

The Scales [of Verification] — This Is the Checking

המאזני זה התמין [recte: המאזנים זה המין — "The Scales: This Is the Operation (of Checking)"]

The "scales" [מאזנים, the balances — i.e., the methods of proof by casting out] by which this checking is done — indeed the earlier authorities have already written about them. There are two kinds: the first is upon ג›ר"ר› [?] … by means of the nine [casting out nines], the other upon ג›ר"ר› [?] … by means of the seven [casting out sevens].

These checks — the first of them, by which you cast out all the figures טֵ 'טֵ' [nine by nine], ר"ל של"א [these are running-residue markers] …

[Latin note (Schreckenfuchs/Münster): The weighing-out, or proof, of this operation [species]. The ancients wrote of two modes of proof: one is done by the nine [novenarius], the other by the seven [septenarius]. The first mode is done thus: Cast away the quantity of all the numbers added together at once, by nine, without regard to quality…]

המין החכאה — among them [the residues of] the addition and the remainder that remains. If it does not arrive at [a multiple of] the whole, keep it.

Again you cast out by nine the sum, ט 'ט' [nine by nine], and do not let [anything] fall among them — neither among the figures of the addition nor among the remainder — until [you reach a multiple] that does not arrive at the whole, and keep it. And if the two remainders are equal — only [if they are] equal — then it is correct.

But [even] if they are equal, it is possible that the result is not in fact correct. For one is obliged to reckon whether an error [שגיאה, mistake] arose — whether by an excess or by a deficiency [בתוספת … בחסרון, in adding too much or too little] — and for this reason these are called the "scales of deceit" [מאזני מרמה, deceptive balances]: another [residue], which is not [the true] obligatory one, can remain there beside it.

But the checking — the second of them — is upon the seventh [casting out by sevens]. And [the procedure] begins from the figure of the highest [places]. The remainders — reckon them for the tens, and join them with the figures nearest to them, ל [the next place over]; and when you reckon them for the others, cast out the rising [carried] amount in their measure. And what remains — until you have cast out all the figures, [taking] each one for the tens.

[Latin note (Schreckenfuchs/Münster): …of quality, and keep the remainder [residue]. Then do the same with the sum produced by the addition. And if those two remainders are unequal, know that you have erred. But if they are equal, it can be that you have acted correctly — though it can also be that you have erred. For it can happen that the error falls out [evenly] by nine, through addition or subtraction; for that reason this proof is uncertain or deceptive, since the true and the false are not distinguished except in one way [respect]. Another mode of proof is done by the seven: cast out [the remainder] sought, both from the numbers being added together and from the sum produced. Moreover, when you cast out at the last or next-to-last place by seven, the residue that remains is taken for as many denarii [tens] as the number has units…]

the remainder that does not arrive at [a multiple of] the whole, [taking it] by sevens — keep them.

Again I tell you: count, and begin from the last figure, which is the figure of the highest [largest] place. Add to it the figures of the later [places], and reckon the remainders for the tens, and join the others; and cast it out in their measure ז' [by seven], and the remainder that does not arrive at [a multiple of] the whole by sevens — keep them. Reckon for the tens, and join with the figures, the remainders; and reckon them for the others, and cast out in their measure ז' [by seven], and keep their remainder — and likewise [for] the figures of the first [places].

But if the remainder of the second check [the seven] should come out for you, [check whether] from the proof it corresponds to the check of the first [the nine]. It is possible that it is correct — and [if so] it stands alone [confirmed].

And if it is some other remainder than [that] of the scales [the first check], one is not obliged [to conclude an error]; rather, cast it out by itself.

But [as for] these scales of the seven — that result which is reached by this checking, when [you check] by [casting out] the seven: join it together with the figure of the highest [largest] place, according to what is said — that you cast out by sevens all the figures of the last [highest places], and cast it out, the rising [carried] amount אֹתָן [= 69 [?]] ט"ס [those] ס› 'ט› [?] just as [explained]…

[Latin note (Schreckenfuchs/Münster): …the number, however, is at once carried to the right; the next [place] is held [reckoned] in place of those units. But if, in that order [place] of the numbers, after the sevens have been cast out some residue should remain, it will stand in the place of the tens [denarii] with respect to the number on the right of the following [place]. Therefore, casting out in this manner the sevens from the numbers added together at once, and from the number of the whole sum: if the residue be equal, the addition itself will be correct. Yet in this mode too one can easily err. There is, therefore, a third and more certain mode — namely, to derive [the residue] from the numbers added together and from the sum of the numbers, [casting out] by nines as many times as you can, and keep the residue of the nines [novenarius] together with [it].]

by fours [the residue], in their explanation. There are [some] in your figure and [some] in others that do not arrive [at a multiple of the whole] — keep them in your figure.

And take also the sum, the total, for the remainder from the figures, and reckon them as they are written down in the figure of the last [places]. As for the figure of the whole — when you make your account: if there was among them a [completed] total at all. Cast out one after another for the ninth — that is, by the others, from the last; and join them with the others, so that the seven [residue] should not arise [out of place]. And reckon them by the nines: if they [agree] with the sevens and the others — if they are equal to the nines and the others in your figure — then behold, it is correct [צדקת]; but if not, it is false [לאו כזבת, "indeed it is a lie"].

The Operation of Striking [Multiplication] — This Is the Combining of Numbers

המין החכאה [recte: המין ההכאה — "The Operation of Striking" = Multiplication]

ההכאה — *[Gloss, small type:* this is the combining of numbers.*]*

[These are numbers] which are of two kinds, where they are [either] not laid out [composite] in the whole — that is, the one [unit] alone — or [a number] that has [further] orders [places]. The kinds of striking [multiplication] are divided into four parts: singulars with singulars — and these are the explained [stated] [products]; singulars with wholes; and the explained [product] of a singular with a singular, and of a whole with a whole — and this comes…

[Latin note (Schreckenfuchs/Münster): **Multiplication.** The author uses, for the term "multiplication," the word "percussion" [striking]. For the number that is multiplied is broken up [discutitur] into as many equal parts as the multiplying number has units. In which manner of speaking the Germans too [follow], when they say of men in a matter of dispute "zerschlagen" [struck/beaten down] — this is [the sense].]

Kitsur ha-Melakhat ha-Mispar — Multiplication (ההכאה) …the multiplication of wholes by wholes and of wholes by parts; the multiplication of parts by parts; the multiplication of wholes by parts; and the multiplication of parts by parts and of mixed numbers by mixed numbers — for already, when whole numbers are multiplied by fractions, as when one-third is multiplied by one-third, the result is one-ninth: a part by a part [yielding] a part, and the multiplied quantities differ. As each is multiplied by its fellow, the product of them increases, while their species varies, so that nine of them rises to [a value], and from them falls [recte: results] one of the five compound kinds [of multiplication].

So consider the kinds of multiplication, and you will find that the path of multiplication runs through all the kinds. There is, however, [one] that is the multiplication of wholes — the first kind — and there is the multiplication of parts by parts. And this is what is set down at the foot of the column [recte: the board]; for it is reckoned just like the others in the same matter. The multiplication is such that it is set out in the way explained, [when] the result is greater [recte:...]. From the necessity [recte: from the one obligated], for every kind of multiplication of parts: at the outset the multiplication of parts is [carried out] together with [its] fellow-parts, [each] one [factor] standing with [the other] parts; and the result of the multiplication, set down, [is reckoned] by kinds. And the multiplication-by-parts that remains is such that we set [the value], reaching it by the foot [of the board], to [its] proper place.

[Latin note (Schreckenfuchs/Münster), left column, p. 231:] *Est ergo multiplicatio collectio numerorum, qui aequaliter ab uno numero proueniunt, q[ui] in ea proporti[on]e se habent ad multiplicante[m] qua multiplicas, se habet ad unitate[m]; ut quater quinq[ue] sunt 20, ubi quatuor partes uicenarij sunt aequales.* ("Multiplication, then, is a gathering-together of numbers which proceed equally from a single number, and which stand to the multiplier in the same proportion as the multiplier stands to unity; thus four times five is 20, where the four fifth-parts of the twenty are equal.")

Multiplicat autem digitus digitum, digitus articulu[m], item digitus digitum & articulum. Omnium horum multiplicatio pendet à cognitione primae speciei, qua scil. digitus multiplicat digitum, hoc est omnes gradus denarior[um], centenarior[um] &c. considerantur haud secus quàm si essent absolutae unitates. Unde necessaria est cognitio multiplicationis digitorum, deinde accedatur ad alias species. ("A digit multiplies a digit; a digit, an article [a round ten]; likewise a digit multiplies both a digit and an article. The multiplication of all these depends on knowledge of the first kind, namely a digit multiplying a digit — that is, all the ranks of tens, hundreds, and so on are treated exactly as if they were absolute units. Hence knowledge of the multiplication of the digits is necessary, after which one proceeds to the other kinds.")

For there is no [recte:...] multiplication, and the path to its knowledge belongs to the one who is obligated by what compels us to set [it] in order — to incorporate into the board every kind of the multiplication of parts together with parts — so that you may know which parts will [yield a value], as a person grows accustomed by himself.

Behold, here is the board, which comprehends [all of] this:

Multiplication board of the digits. A triangular ("comprehensive") multiplication table of the single digits 1–9. The first factor labels the rows down the right-hand margin (1=א through 9=ט); each row lists, from right to left, the products of that factor with the successively larger digits, up to 9×9. Products are written in the printer's round-marker notation: within a cell the units-letter stands on the right and the tens-letter (carrying the round mark ◦, here transcribed ס) on the left — so ◦30 = ג◦ ,20 = ◦כ ,10 = א, etc. (This is the convention the Latin note explains: כ◦ [kaf with round mark] is the same as 20.) The table reads right-to-left in the source; it is rendered below as a multiplication grid (row factor × column factor). Lower-triangular cells (already given by symmetry) are left blank as in the source.

×	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1 (א)	2 (ב)	3 (ג)	4 (ד)	5 (ה)	6 (ו)	7 (ז)	8 (ח)	9 (ט)
2 (ב)		6 (ו)	8 (ח)	10 (א◦)	12 (א◦ב)	14 (א◦ד)	16 (א◦ו)	18 (א◦ח)
3 (ג)			12 (א◦ב)	15 (א◦ה)	18 (א◦ח)	21 (ב◦א)	24 (ב◦ד)	27 (ב◦ז)
4 (ד)				20 (ב◦)	24 (ב◦ד)	28 (ב◦ח)	32 (ג◦ב)	36 (ג◦ו)
5 (ה)					30 (ג◦)	35 (ג◦ה)	40 (ד◦)	45 (ד◦ה)
6 (ו)						42 (ד◦ב)	48 (ד◦ח)	54 (ה◦ד)
7 (ז)							56 (ה◦ו)	63 (ו◦ג)
8 (ח)								72 (ז◦ב)
9 (ט)								[numeral: ?] (תא)

[Latin note (Schreckenfuchs/Münster), right column, p. 232, beside the table:] *Tabula multiplicationis digitorum inter se, in qua situs literarum haud aliter quàm in cifris est aduertendus, ut כ◦ idem est quod 20. & 36 ג◦ו.* ("Table of the multiplication of the digits among themselves, in which the position of the letters is to be observed exactly as with the [Arabic] ciphers: thus כ◦ [kaf with round mark] is the same as 20, and ג◦ו [gimel with round mark + vav] [is] 36.")

And after you already know this, by the foot [of the board], here is [the matter]: already you will be able to enter by the way of true knowledge, and to say, when [it is] told to you in the case [recte:...], that it [the product] is a gathering of many numbers that are not [explicitly written out] but only set down [once] as a total. For if not so, no kind of number would be subtracted [recte: read off] at its end [recte: corner]; and therefore it is fitting for us that it be written in a column [or: in a row].

[Latin note (Schreckenfuchs/Münster), right column, p. 232:] *Multiplicatio, inquit, est additio multorum numerorum, qui tamen expresse non scribuntur per summas* *suas, sed semel duntaxat significantur, & alius multiplicans ducitur in multiplicandum.* ("Multiplication, he says, is the addition of many numbers, which, however, are not written out expressly by their several sums, but are indicated only once; and the one number, the multiplier, is carried into the other, the multiplicand.")

By the foot [of the board], another method [for] the multiplied number.

A worked example: you will set down two columns, [showing] by how many times [a number is taken], when the number is multiplied — that being [the case] when it is gathered together with the rows — as, for instance, if you wish to gather three numbers, each one of which is sixteen. Behold, [this is] from the addition: to write sixteen in one column, [and] another beside it, to make known that the gathering is together with the rows, that it is sixteen. And there is no need to write "sixteen" a second time in [the second] column. [Likewise for the] other rows: just as it is [done] when one is obligated to do so, by the gathering of the numbers — this with this.

After this, you will write another number beneath it, [namely the number] three, to make known to us by how many times — [the number] that is to be multiplied and gathered, as [explained]. And here is the thing [stated] in these [terms]: [it is] enlarged again by means of two other rows from the number sixteen, for there is no [need to] reckon by the foot of the column that it [is repeated]. So, in [the case of] three rows; or that we be drawn [along] six [times] ten [= sixteen], one [taking] after [the other] three [times]. After this, you will continue to subdivide [recte: read off] beneath the two first rows; and you write the number — its result — from what is explained.

[Latin note (Schreckenfuchs/Münster), left column, p. 233:] *Exemplum habes inter sedecim, quos per 16 16 16 simul additos in summa[m] istam 48 rediges, aut per ductione 3 in 16 quod fit per multiplicationem. Ergo in multiplicatione scribimus numerum multiplicandu[m] duntaxat semel & non toties quoties illum uolumus augeri, et numeru[m] multiplicate[m] scribimus sub eo in hunc modum:* ("You have an example with sixteen, which by 16 16 16 added together you will reduce into that sum, 48; or by drawing 3 into 16, which is done by multiplication. Therefore in multiplication we write the multiplicand only once, and not as many times as we wish it to be increased, and we write the multiplier beneath it in this manner:")

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16
3
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[Latin note continues:] *In additione uero opus est tribus numeris sic dispositis:* ("But in addition there is need of three numbers arranged thus:")

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16
16
16
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[Latin note continues:] *Itaq[ue] in multiplicatione subijcimus duobus numeris lineã, sub qua scribimus productum ipsum, idq[ue] sic:* ("And so in multiplication we place a line under the two numbers, beneath which we write the product itself, thus:")

```
16
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And this is the comprehensive multiplication that includes every kind of the products — except that, if it be wholes by wholes, or parts and wholes brought together with parts — [then the value is reached] by the foot [of the board], [so that] the number's residue [recte:...] comes out. As is explained, [you write] one row beside [it], and if [the factors] are wholes brought together with wholes, or with parts and parts together — behold, when the rows are many, the [values] resting beneath [them are] read off [recte: subtracted] in the gathering; and they must also be set down distinctly, [the various] differences, to the explanation — and I [will] say that the multiplication of wholes by wholes, or of wholes and parts together with parts: in the way [that] each one [stands] with [the other] parts, the number's [product] is always read off by the ranks, in the first column. And the part is what is enlarged by the number of times that the number is multiplied — the one set beneath the other. And here is the part, with every rank, from the ranks of the upper column. And you write, in every rank of all the ranks, from the [value of that] rank itself, and [from the next] rank; and if there were parts [taken] by the foot, on the rank [recte:...]. The result rises from the others on the last rank, and rises from the tens on the rank of the tens, and so [too] on every rank — on every rank, by the foot [of the board].

[Latin note (Schreckenfuchs/Münster), left column, p. 234:]

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16
3
——
48
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Hic iam caute agendu[m] est in positione numerorum prodeuntiu[m] ex multiplicatione, & uidendum an is sit digitus, uel articulus uel numerus ex utroq[ue] co[m]positus. Item quot sunt numeri multiplicantes, tot ueniunt signandi ordines numerorum sub linea. Sed tunc positio numerorum est quoq[ue] mutãda iuxta *situm numeroru[m] multiplicantium. Positis itaq[ue] numeris, multiplicante scilicet & multiplicando, in ordines suos, duc primam figuram multiplicante[m] in omnes figuras multiplicãdi & productu[m] pone sub linea in tertiam seriem numeroru[m]. Digitum & cifram an notabis, articulum uê...* ("Here one must now proceed carefully in the placing of the numbers arising from multiplication, and observe whether the result is a digit, or an article [a round ten], or a number composed of both. Likewise, as many as the multiplying numbers are, so many ranks of numbers must be marked under the line. But then the placing of the numbers must also be changed according to the position of the multiplying numbers. So, the numbers — the multiplier and the multiplicand — having been set in their ranks, carry the first figure of the multiplier into all the figures of the multiplicand, and place the product under the line in the third row of numbers. You will mark a digit and a cipher [zero]; an article, however…" [the sentence breaks off, continuing onto the following page].)

Multiplication (continued) — the methods of multiplying multi-digit numbers

[Latin note (Schreckenfuchs/Münster): *…ro reservabis et addes summae quae ex sequentis numeri multiplicatione consurget. Cumque negocium absolveris, invenies in tertia serie numerorum summam quae emergit ex ductione secundi numeri in primum.* — "…you will hold [it] in reserve and add it to the sum that arises from the multiplication of the next figure. And when you have completed the operation, you will find in the third row of figures the total that emerges from the multiplication of the second number into the first."]

in the third row, which the second [figure] multiplies. So [carry] from the carry between the second row and the third row. And if [the multiplicand] was a whole number standing alone, then write its product, and reckon it according to the rank [of the figures]. If you have completed the operation, you will find in the third row the total that the multiplication produced. And if it was [composed of] wholes and parts together, reckon [each] according to its rank, and write down the total. ✻ According to the rank [b].

When you multiply parts by their positions, write down the total as we explained, according to the rank of the operation; and afterward [proceed] to the corners — that is, you already know that the carry takes [its value] from the rank of the figure, so that it goes out from the figure of the number, and emerges with the part that is in the second row, which is the number that is in the third row.

If you multiply wholes [standing] alone, or wholes and parts together — [whether] with wholes alone, or with parts together — the procedure is one and the same for them: that which is multiplied afterward, the figure of the second [number] with all the ranks of the upper figure, you write down [b]. The procedure for the third [step] is according to what we said.

After this, when the tens of the second figure reach all the ranks of the upper figure, then write down also that which goes out into the fourth figure, beneath the third figure, at the mouth of the carry, so that it begins at the head of the figure.

the fourth, from the positions of the tens, because the tens were exchanged from the tens.

After this, when the figure of the hundreds of the second figure reaches all the ranks of the upper figure, you write down [the result], reckoning it in the fifth figure according to the order that we explained — only that it begins at the fifth figure, from the positions of the hundreds; and likewise proceed in this way.

After this, gather all the remainders that are beneath the line of the multiplier, for it is the sum that the number in the upper figure produced [together] with the number in the second figure.

[Latin note (Schreckenfuchs/Münster): *Multiplicatis omnibus figuris superioris ordinis, et rite sub linea subiecta signatis, reducuntur producta in unam summam per additionem.* — "When all the figures of the upper rank have been multiplied, and have been duly marked beneath the subjoined line, the products are reduced to a single sum by addition."]

[Latin note (Schreckenfuchs/Münster): *Exemplum istud primum est valde facile, ubi duntaxat unus multiplicans ducitur in tres figuras lineae superioris, unde sub protracta linea una tantum numerorum series procreatur, quae totius multiplicationis est summa.* — "This first example is very easy, where merely one multiplier is drawn into the three figures of the upper line; from which, beneath the drawn line, only a single row of numbers is produced, which is the sum of the whole multiplication."]

Worked example — the first method:

A worked example of the first procedure. And it is like this: understand this — that this is [done] with this. The figure of the upper [row], and you multiply it; and likewise the figure beneath, [setting] the figure beneath the figure in its rank, as in the operation [described].

The numbers set out (320 × 5 = 1600; reads right-to-left in source):

Operand / line	Value
Upper number (multiplicand)	320
Multiplier	5
Product	1600

The figure that is in the upper row — and you raise it; place the figure beneath the figure in its rank, as in the operation. ✻ Understand this: 5 [ה'] with the figure 2 [ב'] that is in the second row, and it comes to 10 [י']; and as for the hundreds — for it is a whole [number] — they are joined to their companions [i.e. added together] with the result that the multiplication produced; the figure 5 [ה'] with the figure 3 [ג'], and you write down beneath the line the number in the rank of the operation, as we explained.

After this, understand: 5 [ה'] with the 3 [ג'] that is in the upper figure, and it comes to 15 [ט"ו]; we joined to them the 1 [א'] that we held [in reserve], and it comes to 14 [י"ד]; and we wrote it down [b] beneath the line in the rank that follows; and 7 [ז'], which is 1 [א'] in the rank that follows it, and we set down [b] the total that came from raising the 5 [ה'] together with the 3 [ג']. The hundreds and the tens — these are three hundred and twenty; these are six hundred [recte: and they are six hundred].

The second method:

A worked example of the second procedure: and it is [from] the corner of the parts [i.e. of the partial/place-shifted figures], whether [the multiplication is] of parts; or of parts together with parts; or of parts with wholes; and of parts in particular — and behold, here it is [as an example] within it:

[Latin note (Schreckenfuchs/Münster): *Ostendit hoc exemplum qualiter sit agendum quum vel soli articuli, vel articuli cum digitis multiplicant solum articulos, vel articulos cum digitis. Tunc, inquit, omnes figurae multiplicatae, quae scribuntur in inferiori numerorum serie, sunt ducendae in figuras superioris lineae, ut in proposito exemplo 6 primum sunt ducendae in 5, deinde in 2, et ultimo in unum, et productum ordinate scribendum sub linea protracta.* — "This example shows how one must proceed when either the articuli [tens] alone, or the articuli together with the digiti [units], multiply articuli alone, or articuli together with digiti. Then, he says, all the multiplied figures, which are written in the lower row of numbers, are to be drawn into the figures of the upper line; as in the proposed example 6 is first to be drawn into 5, then into 2, and last into 1, and the product is to be written in order beneath the drawn line."]

Understand this: the figure that is in the second [row]…

The numbers set out for this worked example (236 × 125 = 29500; reads right-to-left in source):

Operand / line	Value
Second / lower figure (multiplier)	125
Upper figure (multiplicand)	236
Partial product (6 × 125)	750
Partial product (3 × 125, shifted one place)	375
Partial product (2 × 125, shifted two places)	250
Total	29500

Understand this: the figure that is in the second [row] is raised against the third [rank], and you write down [the result]. The figure that is in the third [row] [b], in the rank of the others — and the thirties [ושלשים] that we kept, as we explained. ✻ Again, understand this — that the figure in the upper [row] is raised, [comes] to 12 [י'ב], joined with the 3 [ג'] that is in the second [row], and it comes to 15 [ט"ו]; and we set down the parts in the figure of the third [row], at the rank of the operation as in the operation; and 16 [recte: the hundreds and tens].

we kept. Again, understand it: that the figure in the upper [row] is raised, and we wrote it down; with that — the figure in the second [row] — it is raised, [comes] to 6 [ששה], and we wrote it down [b] in the rank of the third [row], as in the operation. The figure in the fourth [row], as in the rank of the tens — and likewise [continue].

After this, when we wrote down [the result] for the raising of the 3 [ג'] that is in the figure, the lower [one], with all the ranks of the upper figure; and behold, the 3 [ג'] that is in the figure, the lower [one], was raised with the [figure] that is in the upper [row], and it comes to 9 [ט'], and you write down [b] in the figure of the fourth [row], as in the rank of the tens, and likewise [continue]. We kept it; again, understand it: the 3 [ג'] with the 5 [ה'] that is in the figure, the upper [one], and it comes to six [ששה], the multiplications. With the 2 [ב'] that is in the second [row], and you write down [b] in the figure of the fourth [row], as in the rank of the multiplication.

[Latin note (Schreckenfuchs/Münster): *...cta. Deinde 3, quae scribuntur secundo loco, ducendae sunt in omnes superiores figuras, et quod emergit scribendum est sub priori producto, sic tamen quod prima figura signetur in loco secundo qui est ordo seu gradus denariorum, id quod te docet exemplum adiectum. Tertio, ducenda est figura tertia inferioris lineae in omnes notulas superioris lineae, et producti prima notula scribenda sub ordine centenariorum. Quarto et ultimo, hae tres lineae ex multiplicatione productae in unam summam per additionem congregandae, et tunc apparebit totius multiplicationis expressus numerus.* — "...[completed]. Then the 3, which is written in the second place, must be drawn into all the upper figures; and what emerges is to be written beneath the prior product, yet in such a way that the first figure is set in the second place, which is the order or rank of the tens — that is what the appended example teaches you. Thirdly, the third figure of the lower line is to be drawn into all the marks of the upper line, and the first mark of the product is to be written under the order of the hundreds. Fourth and last, these three lines produced from the multiplication are to be gathered into a single sum by addition, and then the expressed number of the whole multiplication will appear."]

After this, when we wrote down [the result], for the raising of the 2 [ב'] that is in the figure, the lower [one], with all the ranks of the upper figure, in their due order. Understand it: that the figure in the second [row], with the [figure] that is in the upper [row], it comes to 10 [recte: 200 = ר']; and as for the hundreds — for it is a whole [number] standing alone — we kept it, and you write down [b] in the figure of the parts, in the rank of the hundreds.

Again, take note: pair the [residue] 2 in the lower rank with the [residue] 2 in the upper rank, and it comes to 4. Multiply it by the entry beside it [the other residue], and it comes to 200. When you cast them out in the rank [of the fourth column], the result is verified. — Again, take note: [pair] the [residue] in the upper rank, and it comes to 2; cast them out in the fifth rank, and it is verified.

After this, we gathered the three rows that lie beneath the line, and the result comes to 9[?] thousand and 100[?] — and this is the total[?] תתר"ל [?] [the closing total, an abbreviated single-letter numeral, remains abraded]. With the help of HaKadosh Baruch Hu [הב"ה], it is verified.

[Latin note (Schreckenfuchs/Münster): The proof of multiplication is done in this manner. Cast out 9 from the number to be multiplied as many times as you can, and keep the remainder. Do the same with the number you are multiplying by. Then multiply the one remainder by the other, and from the product cast out 9 as many times as you can, and whatever remains, keep this. Thirdly, cast out 9 from the sum of the multiplication as many times as you can; and if the remainder kept is equal to the reserved remainder, you have not erred in your operation. You will find an example of both in the next example. — Example: in the next example you will find both [residues] 7.]

הָאוֹנְזִים שֶׁל זֶה הַמִּין [?] — The Verification of This Kind [of Operation]

The verification by which you know whether this kind of operation is true is this: cast out [by nines] beginning from the highest rank, multiply [the residue] by way of its place-rank, and what remains keep; and likewise cast out the residue again by way of the place of [its] rank, and what remains keep. After this, take note: with what you have kept, ascend [reckon] by casting out the nines, and what remains keep. Again, ascend by casting out [from the residue] toward the ninth-place [unit], and there remains, if it is without remainder; and the kept remainder is verified [against] the reserved one. — Again cast out by way of its rank.

[Latin note (Schreckenfuchs/Münster):...] [The casting-out residues here are heavily abbreviated single-letter numerals; the legible ones are recovered above, with the closing total abraded.]

[Smaller-type rubric at head of column: הַחִסּוּר — "Subtraction"]

הַחִסּוּר — Subtraction

Subtraction is the diminishing of one number out of another, larger one. That is to say: the taking of a smaller number out of a larger number — I mean, the subtracting of one number out of another that is larger than it. My intent in this is that, over every rank, you must look carefully beforehand: that the lower rank, [set] beneath the upper one which is greater than it — by the tens, by the hundreds, by the thousands [each in its degree] — be the lesser. After the ranks of the thousands, the upper is greater than the one after it by the rank of the thousands; and likewise the lower [rank] — and so on without end — must be, in itself and in its sum, the lesser.

[For] the upper number to exceed the lower, it does not suffice that the [whole] rank merely exceed the [whole] rank; for it may already fall out in your path that you find this rank lesser, when the two numbers are reckoned [compared] by their two respective ranks — the larger number by its upper rank, and the smaller number by its lower rank. Should a single rank fall short [of the one above it] when you set all [the digits] within the ranks, beneath the [given] rank — for example, [comparing] the upper totals against the lower rank, in degrees 4[?], 200[?], 30[?] — [then the rank-by-rank rule governs].

[Latin note (Schreckenfuchs/Münster): Subtraction is the subducting of one number from another, larger number — which you must understand of all the orders taken together at once; otherwise it would be necessary that all the figures of the lower number be smaller than the figures of the upper number through their successive degrees, by units, tens, hundreds, etc. But when only the thousand-figure of the upper order is smaller than the thousand of the lower, while all the remaining figures in the lower order are larger than the figures of the upper order, it will suffice that the lower number be subtracted from the upper. The same is to be understood when in the upper order the last figure is a hundred-figure, and the hundred of the lower order is larger, while all the remaining figures are of the upper number.]

the other ranks against the other ranks, and the tens against the tens [each rank against its like].

After this, take note: subtract every lower rank from what lies above it, rank by rank, from the upper [number], and what is taken, write it in the third rank. After this, take note again: the second [figure] against the rank that corresponds to it — that is, by way of comparison, so that it be set [aligned], such that between this rank and the second [the difference] remain in the third rank. If a [given] rank falls short of those above it — and if the rank should itself fall short — write for it, in the [given] rank, a [cipher]; and if it falls short [so that] the lower rank is greater than it, you take a [borrowed] figure for it by the third rank, against the rank that is itself in its degree, and you carry it to the upper rank with the diminished [borrowed] tens from the [next] column. What is reckoned for it, subtract from the upper [rank]: one [unit] from the next rank — borrow it toward the rank that is reckoned, the lower remainder — subtract it from the lower [rank], and afterward proceed by the rule.

[Latin note (Schreckenfuchs/Münster):... of the lower number. Moreover, the form of positioning the numbers of this species is this. Let the larger number, from which the subtraction is made, be placed in the upper place; and the number to be subtracted, write thus beneath it, so that digit be under digit, denarius [ten] under denarius, etc.; and then let the subtraction be made, and whatever residue remains, let it be written in the third order — with a line, however, interposed between them. And if the upper and lower [figures] are equal, there will be no need of a third order of numbers. Likewise, when the lower figure is larger than the upper, subtract the lower from a denarius [ten], and write the remainder together with the upper figure beneath the line, in the third order. And as often as you do this, add a unit to the next lower figure of the number, or subtract it from the next upper figure of the number.]

הַמָּשָׁל — The Example [rubric, foot of p. 241]

הַמָּשָׁל בָּזֶה — The Example Concerning This

If we wish to subtract a number — and it is the number [1339] אלף ש"לט — from a larger number greater than it, namely the number ב' אלפי' שמ"ח[?] [2348[?]; the Latin Exemplum gives 2345], then, after the lower rank falls short of what lies above it: the upper figure being [a 5] and the lower being [a 9], so that the 9 is greater than the 5, write [borrow]. Behold: join the 5 that is in the upper rank with the 1 that is left over after subtracting the 9 [from ten], and it comes to 6; and write it. Afterward proceed against the other ranks in themselves.

After this, carry the 3[?] that is in the [units] rank toward the last rank, lower, and subtract; or subtract one from the next [rank], drawn toward the [proper] rank, so that the upper rank [governs] — for in every case the lower is subtracted from the upper. And behold, they [the two threes in the tens] are equal; so write a cipher [0] beneath it in the third rank, at the rank of the tens.

After this, subtract the 3 that is in the rank of the hundreds from the 3 that is in the rank of the hundreds of the upper [number], and they are equal [0]; so write a cipher [0] beneath it in the third rank, at the rank of the hundreds.

[The remaining thousand-figure stands as 1; the difference is therefore 1006.]

[Latin note (Schreckenfuchs/Münster): Example.]

Worked subtraction example. A vertical subtraction arranged with the larger number above, the subtrahend beneath it, a horizontal rule, and the remainder below — exactly the layout the Latin note describes (digit under digit, with the line interposed). The values follow the Hebrew worked example on the same page (subtrahend 1339 = אלף ש"לט; the printed minuend reads ב' אלפי' שמ"ח = 2348[?], while the Latin Exemplum here gives 2345 — see the parsing note).

	Value
Minuend	2345
Subtrahend	1339
Remainder	**1006**

Kitsur ha-Melakhat ha-Mispar — Subtraction (conclusion) and Division (opening)

After this, subtract [נגרע] what is ranked [שכמדורגת] in the thousands [האלפים] of the upper [number], and the remainder is one [or: "and the rest"] [והנשאר א']. We write it in the third column according to the rank [כמדרגת] of the thousands [האלפים]; and you ascend [ותעלה] to the third column. This is what remains of the lower-row subtraction [המחסר] from the upper column [מהטור העליון].

[Latin note (Schreckenfuchs/Münster): *In probatione subtractionis sic procedes. Abijce quoties potes a superiori et inferiori numerorum ordine novenarium, et utriusque relictum serva. Deinde subtrahe inferioris residuum a residuo superioris; et si illud superius minus fuerit adde ei 9, deinde fac subtractionem et residuum serva. Tertio abijce a tertio numero 9 quoties poteris, et si residuum aequale fuerit priori residuo non errasti.* — "In the verification of subtraction you proceed thus. Cast out nines from the upper and from the lower order of numbers as many times as you can, and keep the remainder of each. Then subtract the remainder of the lower from the remainder of the upper; and if that upper [remainder] is the smaller, add 9 to it, then perform the subtraction and keep the remainder. Third, cast out nines from the third number as many times as you can; and if the remainder equals the prior remainder, you have not erred."]

The verification of the species [מין] of subtraction [חסיר]:

Its verification, by which you will know whether this subtraction is correct, is this: cast out nines [שתשליך מהמותר] from the upper remainder, and keep the rest of the nines [ט']; and what remains of the nines, keep it [שמרה]. Again, cast out [תשליך] the nines from the remainder [הטור].

So too cast out [the nines] from the ninths [תשיעיות], and keep its remainder. After this, cast out nines again [אחר זה]; keep [שמור] [the result]. Subtract the lower keeper [המשמור השפל] from the upper keeper [מהשמור העליון]; and if the keeper [שמור] of the upper is larger, then [subtract directly]. But if the upper is smaller, either you have erred, or [you must] cast out — for if the keeper of the upper is smaller, add to it [9] [תשמור העליון אם קטן ממנו]. After this, take the lesser keeper from the upper, and what remains is the keeper of the rest [שמור הנשאר].

After this, you go [תשלך] to the figures [אותיות] of the third column. The third [column] — to the ninths [לתשיעיות], and what remains — provided there is no difference [שנוי] in the keeper [לשמור], [it shows] in your verification [שבענינך] [whether you have erred] [שטעית?].

פתח ז' החלוק — Petaḥ (Gate) 7: Division [chapter heading; printed garbled "פ"ט ז חחלוק", recte פתח ז' החלוק]

[Body heading, large square type:] **החלק [recte: החלוק] — Division**

[Latin note (Schreckenfuchs/Münster): *De divisione. Divisio ostendit dati numeri partes aequales respectu alterius numeri minoris eo. Velut nostri definiunt, est propositis duobus numeris, maioris in tot partes distributio, quod sunt unitates in minori. Per ipsam enim cognoscitur proportio minoris numeri ad maiorem. Sunt autem hic duo numeri necessarij, dividendus qui superiori loco, et divisor qui infra dividendum ponitur, ultima scilicet figura sub ultima.* — "On division. Division reveals the equal parts of a given number with respect to another number smaller than it. As our [authorities] define it: given two numbers, it is the distribution of the larger into as many parts as there are units in the smaller. For by it is known the proportion of the smaller number to the larger. Here two numbers are required: the dividend, which is placed in the upper position, and the divisor, which is placed beneath the dividend — namely, its last figure beneath the last figure."]

Division [החלוק] makes known [מודיע] the parts [חלקי] of a number relative to [למספר] another number: from a number, as many equal parts are set out [מונח השרים] as there are units in a smaller number [למספר מונח קטן] within it, taking the larger down [יגרע] to another smaller number [מספר אחר קטן] within it [ממנו].

I state the general rule [הכולל] for knowing this: arrange [שתסדר] the dividend number [המספר המחולק] in the first column [בטור ראשון], and set [והחתיר] the divisor number [המספר המחלק] in the second column [בטור שני], such that its last figure [מדרגתו האחרונה] falls beneath the last figure [תחת המדרגה האחרונה] of the upper column [שבטור העליון].

After this, you draw [המשיך] [a line], and beneath it you write [ותכתוב תחתיו] the quotient of the division [היוצא מהחלוקה], which is called by the name "part" [ויקרא בשם חלק]; and its last figure [מדרגתו האחרונה] falls beneath the first figure [תחת המדרגה הראשונה] of the second column [שבטור השני], which is the divisor [שהוא

המחלק] [recte; the dividend/divisor terms are abraded but follow the Latin: dividend above, divisor below]. [Then proceed to work] the dividend [המחולק] and the divisor [והמחלק], and the part [והחלק].

Now, by way of example [דרך משמוש]: the rule is that you reckon [שתחשוב] across every figure [בכל מדרגות] of the dividend [המחולק] just as for the next, and that you begin [ושתתחיל] from the last figure [מהמדרגה האחרונה] of the dividend [של המחולק].

And if the divisor [המחלק] leaves a remainder [פרט], then separately [לבד] you subtract [תחסר] the number of the times [מספר הפעמים] that the divisor measures it [אשר ימנה אותו המחלק].

[Latin note (Schreckenfuchs/Münster): *digitis, incipiasque ab ultima figura. Et quando divisor est digitus tantum, quaeres numerum in superiori figura quem aliquoties numeret divisor, et numerum illum quadratum scribe sub linea atque sub gradu numeri dividendi. Haud secus ages cum reliquorum graduum figuris. Quod si figura dividens maior fuerit figura dividenda, non scribatur directe sub ea, sed figura dividenda habeatur pro denario et sequens pro digito, et in eis simul iunctis quaeratur numerus quotus, inventusque scribatur sub priori figura, nempe quae ad dextram signatur. Item si divisor constat ex digito et articulo tunc quaeratur numerus aliquotus qui ductus in divisorem ultimum reddat numerum, qui...* — "...digits, and you begin from the last figure. And when the divisor is only a digit, you seek a number in the upper figure that the divisor measures some number of times, and write that resulting number beneath the line and beneath the rank of the dividend's figure. You proceed no differently with the figures of the remaining ranks. But if the dividing figure is larger than the figure being divided, let it not be written directly beneath it; rather let the figure being divided be taken for a ten and the following [figure] for a digit, and in these joined together let the quotient be sought; and once found, let it be written beneath the prior figure — namely the one marked to the right. Likewise, if the divisor consists of a digit and an article [a ten], then let a quotient number be sought which, multiplied into the last [figure] of the divisor, yields a number that..."]

The dividend [המחולק] and the quotient number [והמספר החוזר] you write [כתבתה] beneath the line [תחת הסר], opposite [בנגד] that same figure [אותה המדרגה]; and so for every rank — a different path [דרך אחר] for each [לכל]. That is to say [ר"ל]: the number of the times [שמספר הפעמים] that the divisor [המחלק] is found [אשר ימצא] in each and every figure [כל מדרגה ומדרגה] of the dividend's figures [ממדרגות המחולק] is written [יכתב] beneath the line [תחת הסר], opposite the completed figure [בנגד המדרגה הגמורה] [reading המורת as הגמורה/הנגמרת] of the dividend [(מהמחל(ק].

And if the divisor [המחלק] is larger [גדול] than the last figure of the dividend [מהמדרגה האחרונה שבמחלק], we do not write [לא נכתוב] [the quotient] beneath the line opposite that figure [תחת הסר בנגד המדרגה]; it amounts to nothing [תהיא מאומה]. Rather [אבל] we reckon [נחשוב] that figure as tens [אותה לעשרות], with the next ones ['ומסורבות / ומה שבא אחרי] joined to it ['לה לאחרי]. We combine them together [ונחברם יחד], and seek [ובחפשה] the number of the times [מספר הפעמים] that they measure them [אשר ימנם]. [Then] the dividend [המחולק] and the [quotient] number [והמספר ה...] is written [יכתב] beneath the line [תחת הסר], opposite the figure [בנגד המדרגה] preceding [הקודמת] the last figure [למדרגה האחרונה].

But [ואולם] if the divisor [המחלק] leaves [תניח] a remainder [פרט], then in general [ובכלל] together [יחד] we subtract [נחסור] the number of the times [מספר הפעמים] that the last figure [המדרגה האחרונה] of the dividend [מהמחולק] measures the last figure [את המדרגה האחרונה] of the dividend [מהמחולק], in such a manner [באופן] that it adds [שיוסיף] to the upper [העילה / העליה] — from the remainder [מהנשאר] the number of the times [מספר הפעמים] that they measure [אשר ימנם] the last figure [המדרגה האחרונה] from [מן] the divisor [המחלק], that which is to be subtracted [שיחוסר].

[Latin note (Schreckenfuchs/Münster): *qui auferat sufficienter numerum sibi in dividendo numero respondentem: Deinde ductus in divisorem alium, productum auferatur a numero sibi supraposito, una cum eo quod in posteriori figura residuum mansit, quod respectu sequentis figurae habetur pro denarijs. Numerus autem quotus hinc proveniens scribi debet sub linea atque sub primo gradu divisoris. Idem modus et forma observanda in reliquis figuris, quantumlibet multiplicentur gradus divisoris aut dividendi, atque utiusque. At id per exempla nunc demonstrabimus.* — "...which removes sufficiently the number in the dividend corresponding to it. Then, multiplied into the other [figure of the] divisor, let the product be removed from the number set above it, together with whatever remainder was left in the latter figure — which, with respect to the following figure, is reckoned as tens. The resulting quotient number must be written beneath the line and beneath the first rank of the divisor. The same method and form is to be observed for the remaining figures, however many the ranks of the divisor or the dividend, and of each, may be multiplied. But this we shall now demonstrate by examples."]

that is to be subtracted [שיחוסר] from the last figure [מהמדרגה האחרונה] of the dividend [מהמחולק]. And so [ובכן] for the regrouping [העלה]: the product [המוכאת] of the number of the times [מספר הפעמים] with the preceding figure [עם המדרגה הקודמת] of the divisor [שבמחלק] — that which is to be subtracted [שיחוסר] from the preceding figure [מהמדרגה הקודמת] of the dividend [שבמחולק], with the aid [עם עזר] of the remainder [הנשאר] from the last figure [מהמדרגה האחרונה] of the dividend [שבמחולק] when they are reckoned [בשיחשבו] as tens [לעשרות] in value [בערך].

The preceding figure [המדרגה הקודמת] and the number of the times [ומספר הפעמים] in them [בהם] are written [יכתבו] beneath the line [תחת הסר], opposite [בנגד] the first figure [המדרגה הראשונה] of the divisor [שבמחלק].

And so [ובכן] always [תמיד] a different path [דרך אחר] applies to them [להם], depending on how the figures [שירבו מדרגות] of the divisor [המחלק] or of the dividend [המחולק], or both of them together [שניהם יחד], may be multiplied.

[Latin foot-line: *figuris, quantumlibet multiplicentur gradus divisoris aut dividendi, atque utiusque. At id per exempla nunc demonstrabimus.* — "...figures, however many the ranks of the divisor or the dividend, and of each, may be multiplied. But this we shall now demonstrate by examples."]

Division (חלוק) — worked examples

First example (worked division: 2728 ÷ 8)

An example of the first kind — that is, where the divisor is a single digit.

We seek the number of times that the divisor — the figure 8 — measures off the last place of the upper row [the dividend], which is 2; and we find that it does not go into it at all, since 2 is smaller than it. For that reason we write nothing beneath the rule line opposite the 2, as the rule [requires].

After this we move the divisor to the place that precedes it in the upper row, which is 7, and we combine with it the place reckoned together with [recte: לַח' — i.e. taking the 2 as a 20] — and they make 27. We seek the number of times that the 8, which is the divisor, measures them off, and they are 3; and therefore we write the 3 beneath the rule line opposite the figure of the rank, as the rule [requires]. We then subtract from the upper number, and from the remainder: from the number 24 — which is 8 multiplied [by 3] — opposite the 3, in order to mark what remains over the upper number; and of what we subtracted, namely the 27, there remains nothing of them.

After this we move the divisor to the rank that follows, which is opposite [the next figure], and it is 8...

Diagram. Worked-division layout in the Latin annotation column. The dividend 2728 is written on the top line, the divisor 8 beneath it, and the quotient 341 below the rule line.

- Top line: `2 7 2 8` (dividend)
- Second line: `8` (divisor)
- Rule line beneath
- Quotient line: `3 4 1`

[Latin note (Schreckenfuchs/Münster): **Exemplum primae speciei** — Example of the first kind. — "Since this divisor 8, whose task is to count how many times it goes into the upper figures, cannot be located beneath the last figure 2 of the upper rank — the number to be divided being smaller than the divisor — it is transferred to the prior rank, namely beneath the 7, which with 20 makes 27; in which the eight is found three times. Therefore 3 is written beneath the divisor. Then, the three eights being taken from 27, there remains a residue of 3, which — the seven being cancelled — is marked above the seven."]

[Latin note continues: **Postea** — Afterward...]

and we combine it [the carried 3] so that they make 32. We seek the number of times that the divisor measures them off, and they are 4. And we write the 4 beneath the rule line opposite the figure of the rank, the preceding one. We then subtract from the upper number, so that nothing remains: from the number 32, which is 8 multiplied [by 4], opposite the 4, to mark that nothing remains over them — for nothing of them remains.

After this we move the divisor to the rank that follows, which is opposite [the next figure] of the upper row, and it is 8 — that is, the divisor itself standing opposite the upper 8. And we seek the number of times its figure goes into the upper figure, and they are 1. And we write the 1 beneath the rule line opposite the figure of the rank, so that nothing remains, to mark that nothing remains over them. And to mark that what has been reached is the third rung of the rank, opposite the first figure of the upper number — the third [figure] of the rank — what is reached is the quotient 3 [hundreds]... [Per the procedure the quotient stands as 3-4-1.]

And behold, if the number [the quotient] is the third figure, this is its name [its rank-value], and it is reached from the number 2000 [the leading rank of 2728].

And the times — that is, the quotient figures whose product, [namely] the divisor 8 [taken so many times], yields the number 2728 — are these. When we sum them [3, 4, 1] they make the quotient...

And so the quotient is the figure 341; and it is the last figure of the third [rank].

[Latin note (Schreckenfuchs/Münster): "Afterward let the 8 be transferred to the rank that proceeds next, namely beneath the 2; and it [the eight], in the 2 that with the residue makes 32, is a binary [goes twice over], in which the eight is found four times; and what [is left] when the eight is removed deletes the 2, so that nothing of the residue remains. We therefore write 4 [as] the prior quotient. We proceed not otherwise [than before] with the divisor 8 and the octonary [the eight as] last figure of the upper rank, and for the quotient we write a unit — namely 1 — to the other two quotients; and the absolute [final figure] is therefore the third line, that is, of the thousand[-rank] numbers, which eights are part of the whole sum to be distributed — that is, 341, which multiplied by 8 yield 2728."]

Second example (worked division: 2368 ÷ 148)

An example of the second kind. The second kind is where the divisor divides as both an article and a digit [i.e. is a multi-figure number, here a ten-and-a-unit].

We seek the number of times that the figures of the divisor — its last [highest] one being the one that divides into the upper number — go into it. And [we seek how many times] the number 2 [of the dividend's leading place] contains them, inasmuch as the chosen [first quotient figure], one must subtract its figure-by-figure [reckoning] from the number a number of times, with each rung of the divisor counted against each [figure] of the upper [number] beside it; and there is for it [each comparison]. And we look how many times the two figures [of the divisor] — the digit and the article-of-tens beside it — which are those of the divisor, go into the last [figures] of the upper number. And [for] the second quotient figure...

such that they may be subtracted from the upper number, as we have placed it: beneath the figure of the rank, opposite the first figure of the divisor — [taken] from the upper number at the rank of the divisor — the first [comparison], inasmuch as the [figure] that suffices is the number that is subtracted, the [residue] arising. With each rung of the divisor counted against each [figure] beside it, we seek the number of times; and the quotient figure is 1: [it goes] this many times, and is subtracted from what stands at the upper rank, and there remains a residue. And we reckon, to carry, to mark what stands over...

Diagram. Worked-division layout in the Latin annotation column. The dividend 2368 is on the top line, the divisor 148 beneath it, and the quotient 16 below the rule line.

- Top line: `2 3 6 8` (dividend)
- Second line: `1 4 8` (divisor)
- Rule line beneath
- Quotient line: `1 6`

[Latin note (Schreckenfuchs/Münster): **Exemplum ponitur hic quando diuisor constat duabus figuris, digito scilicet & articulo, ut est in hoc exemplo** — An example is placed here for when the divisor consists of two figures, namely a digit and an article [a ten], as in this example. — "I find here the unit of the divisor [contained] only once in the upper binary [the 2], on account of the following ternary [the 3], in which I do not find the geminated quaternary [twice four], if I take the unit twice in the binary. For the quotient number is multiplied [by] the divisor, and the product is subtracted from the upper number corresponding to it. You may also proceed thus: how many times do I find 148 in 236? — and you will mark the quotient, namely a unit, and there remains a residue of 88. Then change the divisor by one rung, and the figures will stand thus:"]

Over what remains — also as a part [a remainder] in the other [place]: with that which is in the divisor, and over it the 4, and it is subtracted. What is reached is that the figure of the upper number, which is 3, [is set] opposite the first [figure] of the divisor; the upper number, less one, beside the one that remains; for in the divisor, the first of those that divide [stands], and there remains over the third [figure], to mark over the remainder. Also a part [a remainder] arises...

with that which is in the divisor, and over it the 4; and we subtract, so that the figure of the upper number which they are [reaches]: the product is reached from the figure of the rank, the first of the upper number, with the one, and there remains...

to mark over the remainder, and we multiply. The other [figure] remains: also beside that, with that which is in the divisor; and over it the 8, and we come to rest: that the upper figure which they are, the chosen [first], the doubling. Of the upper number which they are: the product is reached from the figure of the rank — for the divisor, the upper number with 9 — and there remains in the divisor, and there remains... and for the 6, reckon over 9, and... over what remains.

After this [it follows] that — since the second [step] is complete [in] the deficiency [remainder] over the upper number, from the summing of the number of the one with every rung of the divisor — it divides [evenly].

[Final section:] When we sought the quotient figure: that which is to its right is in the divisor, the rung remaining of the divisor [taken] from the divisor, the other; for the divisor [goes into] the upper number, and behold them — 8 times... so that they do not exceed the reckonings of the divisor...

Diagram. Continuation of the worked division 2368 ÷ 148 in the Latin annotation column, with the divisor shifted one rung.

- Top line: `8 8 8` (the residue after the first quotient step)
- Second line: `1 4 8` (divisor, shifted)

[Latin note (Schreckenfuchs/Münster): "Where I find the unit eight times in 8, but I do not find 4 just as many times in the 8 placed above it. I will therefore take 6 for the quotient, by which I multiply all the divisors, and I subtract the product from the upper number corresponding to it. Or: notice how many times the divisor 148 is found in 888 — it is found six times, since six times 148 makes 888, which, subtracted from 888, leave no residue. Therefore the number 148 is found in this sum 2368 ten and six times [i.e. sixteen times]. Whoever has learned the kind of division from the Latin tradition will easily understand the procedure which our author, the Jew, observes. By 'rung' [rank], understand the number or numbers placed perpendicularly one above the other. The rest are easy."]

Kitsur ha-Melakhat ha-Mispar — Division (proof) and the opening of Fractions

[p. 251] — The division by "ranks" and "rungs" (continued)

which is in the upper rank [שבטור העליון], to subtract from them eight [8 = ח']; the figure arising from the sum [מחברת] of the eight, together with all the rungs [מדרגות] of the divisor [המחלק] — and not from the figure arising from the sum of the eight together with all the rungs of the divisor. Therefore, in your reckoning [בחשבונך] treat the eight as a rung [כמדרגה] arranged [מסודרת] for it, so that what is below it [שתחתיה] is lacking [חסר], and the eight reaches together with the [other] eight [ח'] that is in the divisor [שבמחלק], and there arises two hundred [ר' 200 =], their cast-out being [גרעונם] from [?] those remaining in the upper rank. There remain two [2 = ב']; and you reckon the two, placing one [אחד] to indicate [להורות] what remains.

After that, take eight [ח'] together with the eight [ח'] that is in the divisor [שבמחלק], and there arises twenty [20 = כ'], their cast-out [גרעונם] [?] from the sum [מחברת] of the eight that is in the upper rank — for that is the sum [החבור] of the eight that is in the upper rank, together with the eight [ח'] that, in the second rung [שבמדרגה השנית] from the upper rank, together with the twenty that remain [המנשארים]; for in the third rung [שבמדרגה השלישית] from the upper rank there remains two hundred [200 = ר'], and you reckon the eight [ח'] to indicate upon [?] what remains. And from what is cast out [ומחסקנו] from what is in the rung of the divisor [המחלק], and from what is cast out from what is in the third rung, there is nothing [לא] to indicate, since nothing remains [שלא נשאר כלום].

[p. 252] — (continued)

because what is in the upper rank is the sum [חבור] of the eight [ח'] — the first rung [המדרגה הראשונה] — together with the four that remain [המנשארים], which is what is in the second rung [שבמדרגה השנית] from the upper rank; and to indicate that nothing remains [שלא נשאר כלום]. There is nothing [מה] to indicate, since nothing remains, for the rung of the third [מדרגת השלישי] has already been reached, so that afterward you reckon, opposite [בנגד] [it], in the rung the eight [ח']. The first [rung], which is in the upper rank, reaches [the point] that it is already complete [שכבר נשלם]. So the divider operates [פועל המחלק]. [End of the division section.]

[Heading, p. 252] — **הַמֹּאזְנַיִם** (The Balance / the casting-out check)

[Marginal rubric-letter ה keys this section to the Latin proof in the outer column.]

The balance [המאזנים] — by which one verifies this kind [זה המין] [of operation]. It is done by counting out [שתמנה] the dividend [המחלק] entirely [כֻּלו] as you do with other [numbers] [כמו אחרים] — that is to say [ר"ל], you cast away [שתשליך] the nines [התשיעיות; Latin: novenarios] one by one [לבד], and keep [שמרהו] what remains. Do the same with the [number] to be divided [לחלק], and keep what remains. Then multiply [והכה] the [first] kept [number] with the [second] kept [number], cast away [תשליך] the nines from them, and keep what remains. Add to it [עליו] the remainder [המותר] — what remains of the dividing-out [מהחלוק] after you have cast away the nines. Then multiply the figure that arises [העולה]; set it apart [תפנהו]; and cast away the nines.

[Latin note (Schreckenfuchs/Münster): "Probatio huius speciei fit in hunc modum. Numera totum divisorem [recte: dividendum] secundum unitates & abijce novenarium quoties poteris & residuum serva. Idem age cum numero diviso, et quod residuum manserit illud multiplica cum priori residuo ad partem servato, & a producto abijce quoties poteris novem & residuum." — "The proof of this kind [of operation] is made in this way. Count the whole dividend by its units, and cast out a nine as many times as you can, and keep the remainder. Do the same with the divisor [lit. 'divided number'], and whatever remainder is left, multiply it with the prior remainder set aside, and from the product cast out nine as many times as you can, and the remainder..."]

[p. 253] — (continued)

the nines [התשיעיות]; and the remainder [והמותר] that remains [שנותר] after this, count [מנה] it also, and divide it [תחלקהו] by the nines [לתשיעיות]; and this, when you have cast out [the nines] within them [בתוכם] one by one [לבד], and what remains [והנשאר]: if it is equal [הוא שוה] to that which you kept [לשמור] before you [שבפניך], you are correct [צדקת]; and if not, you have erred [טעית]. But [אבל] — if no remainder [מותר] remains from the dividing-out [מהחלוק], you have no need [אין צורך] of all this at all [לכל זה כלל]. Take the remainder [המותר] of the nines of the dividend [מהמחלק], divide it and divide [it again], and multiply [והכה] the remainders [המותרים] of the one [זה] together with the other [זה]; and what arises [והעולה] — set it apart [תפנהו] — [cast out] the nines; and if the remainder of them [מהם] — keep [שמרה] for the remainder of the nines, divide — you are correct [צדקת]; and if not, you have erred [כזבת].

[Latin note (Schreckenfuchs/Münster): "...duum [residuum] serva. Adijce quoque ei residuum quod remansit ex divisione postquam abiecisti ab eo novenarios, & a producto abijce novenarios & residuum serva. Postea numera per 9. numerum dividendum, hoc est, abijce ab eo 9. quoties poteris perinde ac si resolutus esset in unitates, et si residuum aequale fuerit illi quod servasti, recte egisti. Sin nihil remanserit ex divisione, non est opus hac operatione, sed accipe residuum novenariorum divisoris & divisionis, multiplica illa residua inter se, & ex producto abijce novenarios. Et si residuum eorum aequale fuerit residuo numeri dividendi postquam novenarios abiecisti, recte operatus es &c." — "...keep the remainder. Add to it also the remainder that was left over from the division after you cast out the nines from it, and from the product cast out the nines and keep the remainder. Afterward count the dividend number by 9, that is, cast out 9 from it as many times as you can, just as if it had been resolved into units; and if the remainder is equal to that which you kept, you have done correctly. But if nothing remained from the division, there is no need for this operation; rather, take the remainder of nines of the divisor and of the dividend, multiply those remainders by each other, and from the product cast out the nines. And if their remainder is equal to the remainder of the dividend number after you cast out the nines, you have operated correctly, etc."]

[Heading, p. 253] — **הַמִּינִים שֶׁל שְׁבָרִים** (The Kinds of Fractions)

[Latin note (Schreckenfuchs/Münster): "Hactenus de speciebus integrorum numerorum dictum est, nunc tractabitur de numeris fractis. Dividuntur autem numeri fracti primo & in genere in duas partes. Prima est pars..." — "Thus far it has been spoken of the kinds [of operations] on whole numbers; now there will be treated the fractional numbers. Fractional numbers are divided first and in general into two parts. The first part is..."]

הִנֵּה [Behold] — it has already been set forth [כבר התפאר] in our discourse [בדברי] concerning the multiplicity of the kinds [וריבוי המינים] — of whole numbers [בשלמים] alone [לבד], together with their like [עם מאזניהם — "their balances/equivalents"]. And now [ומעתה] we begin [נתחיל] our explanation [בביאור] of the kinds [המינים]. Behold [הרי]...

[p. 254] — The kinds of fractions (continued)

that they are [הם] either whole numbers [שלמים] alone [לבד], or [ומהם] [combinations] that are mixed [אשר עורבו = recte: עורבו / "mixed"] together [ער הנה = recte: עד הנה ("up to this point")]. I ask [אשאל] the reader's [הקורא] indulgence [העזר] for whatever may remain over [במה שעתיר] to here [לכאן].

I say [ואומר] that fractions [שהשברים], in general [בכלל], are divided [יחלק] into [a] division [חלוקה]: the first one [ראשונה] is divided [נחלקת] into two parts [לשני חלקים], the one [האחד] of them a "part" [חלק] and the second [והשני] "parts" [חלקים]; and we have already [וכבר] spoken of [קדם] the matter [ענין] of the [single] "part" [החלק] and of the "parts" [והחלקים].

Furthermore [עוד], each one [כל אחד] of them [מהם] is divided [יחלק] into two other kinds [לשני מינים אחרים]: namely [והם], either [אם] there is [שיהיה] a fraction [שבר] of a whole [השלם], or [או] there is a fraction of [several] wholes [שלמים רבים]; and each one of them is divided into two further kinds: namely, either [אם] there is a fraction [שבר] alone [לבד], or [ואם] there is a fraction of a fraction [שבר שבר], or [או] a fraction of a fraction of a fraction [שבר שבר שבר] — and this, without end [לבלתי תכלית].

[Latin note (Schreckenfuchs/Münster): "...pars, et altera partes. Diximus autem iam de parte & de partibus. Caeterum utraque earum dividitur in duas alias species, nempe si fuerit fractio unius integri, aut fractio multorum integrorum. Et rursum quaelibet harum dividitur in duas alias species, scilicet in simplicem fractionem, aut in fractionem fractionis, aut in fractione & fractionis fractionem, & sic deinceps sine fine." — "...one part, and the other 'parts.' But we have already spoken of 'a part' and of 'parts.' Moreover, each of these is divided into two other kinds, namely whether it is a fraction of one whole, or a fraction of many wholes. And again, each of these is divided into two other kinds, that is, into a simple fraction, or into a fraction of a fraction, or into a fraction and a fraction of a fraction, and so on without end."]

[Heading, p. 254] — קְצוּרֵת הַשְׁבָרִים (The Reduction of Fractions)

The Masters of the science of number [וְרֵעַ בי חכמי המספר — recte: וְכַבָר חכמי המספר / "and indeed the Sages of number"] have used [השתמשו] fractions [בשברי] in their smallest [proportional] forms [בקטני], by ratio [ביחס; Latin: proportionaliter], so that it is not fitting [ולכן לא יתכן] to say "two fourths [שני רביעיות]," but rather.... A sixth [שישיורת = recte: שישיות / "sixths"] and thirds [ותרומים = recte: ושלישים? "and thirds"] of them — in the smaller [בקטן]. The smallest by ratio [קטן ביחס] — that is [הרי = 'הר], a sixth [שישיורת = recte: שישית] is [הוא] two ['ב = 2] of a third [שלישיורת = recte: שלישית] and the smallest [וקטן] by ratio [יחס]; two [2 = 'הב] fourths [רביעיות].

[Latin note (Schreckenfuchs/Münster): "Nunc de abbreviatione fractionum. Ubi in primis sciendum, Sapientes Arithmetices usos fuisse in fractionibus abbreviationibus proportionalibus. Siquidem inconveniens..." — "Now concerning the reduction [abbreviation] of fractions. Here it must first be known that the Masters of Arithmetic have used proportional reductions in fractions. For indeed it would be unfitting..."]

fourths. So too you should say "two thirds" rather than "three fourths," or "four sixths," or other similar expressions of this kind, whenever you have the smaller and more fitting ratio. Thus you may say "two thirds" in place of "four sixths," and "one half" in place of "two fourths."

This is so for two reasons. The first has already been stated, namely that the smaller ratio is preferable. The second is that fractions of this kind possess a magnitude equal in quality [to that of the wholes], and on this account the definition of wholes applies to them. For "three thirds" make up a whole integer itself, just as "four fourths" do; and therefore we do not use this species of fraction. Nor is it fitting to say "four thirds" or "five fourths," for in that case you would have a whole integer together with an additional part.

[Latin note (Schreckenfuchs/Münster): …you will find it is so when you say "two thirds" or "four sixths" or in other similar usual expressions, since you have the smaller and more fitting proportion; and you can say "two thirds" for "four sixths," and "one half" for "two fourths." Thus we do not say "three thirds," nor "four fourths," for two reasons: one already stated, namely the shorter proportion; the other is that fractions of this kind have a quantity equal to [their] quality, and on this account the definition of integers applies to them. For three thirds make up the integer itself, just as four fourths do; and therefore we do not use this species of fraction. Nor is it fitting to say "four thirds" or "five fourths," for then you would have a whole integer and in addition a part.]

And just as it is not fitting to say "three thirds" or "four fourths" — and likewise not "two halves" — because they amount to a whole, where there is no fraction at all, since to make wholes whole is not the function of fractions but rather to complete wholes [when it is possible]:

so it is fitting to divide the fractions according to the [number of] parts [?] into which it is possible to divide them.

[Section heading:] The Names and Placement of the Fractions (השמות ותנחת [recte: והנחת] השברים)

Now, the manner of their placement is the preferred [arrangement], by which two notations are set one beneath the other, separated by a line drawn between them, thus:

7	6	5	4	3	2	1
8	7	6	5	4	3	2

[The table reads right-to-left in the source; numerators 1–7 in the upper row, denominators 2–8 in the lower row, with the dividing line between them.]

This is because a fraction is composed of two names. One name indicates [the number of] parts taken, and the other indicates the kind. The name that indicates the kind is the one set below; the name that indicates the [number of] parts taken is the one set above. The whole — that is, the name of the kind itself — is the one indicated by the [lower] name of the [whole] number; and the parts taken from the whole are the [number] indicated by the [upper] name, namely the count of the parts. The completed [fractions] are those expressed, for example, when you say "two thirds" or "four sixths." Their numerators [?] and denominators, which indicate [the number of] parts, are placed above; and [the denominators], which indicate the kind, [are placed] below. In every case, [the upper number] is divided separately [?] in proportion to the count of the parts.

[Latin note (Schreckenfuchs/Münster): Now concerning the appellation and the placement of fractions. In the placement of fractions we use two notations set one beneath the other, with a line drawn between them. The reason is that a fraction is composed of two names. One signifies the quantity of the number and the other the quality. The name that signifies the quality is the one that signifies all the parts of the integer, and is placed below; whereas the name that signifies the quantity signifies certain parts of the integer, and is placed above — as in 2/3 and 4/5, where 2 and 4 signify the quantity simply, while 3 and 5 signify the quality, that is, the kind, from which as it were certain parts are taken from the integers. Likewise 2 and 4 are here of discrete quantity, and 3 and 5 of continuous quantity, which is broken into its parts, and [each] is named by the name of the integer.]

[Section heading:] The Kind of Multiplication of Fractions (מין הכפאת [recte: הכאת] השברים)

Now that we have laid this out, this is the kind [we treat] among fractions — and if the matter is taken simply [?], it is set out by us before all the [other] kinds of fractions discussed, for they require it to bring [them] together into two kinds among them, into one and not [?].

This kind is established as the kind of multiplication, as has been explained in its proper place, with the help of God. Multiplication is the bringing-together [of quantities].

To multiply a fraction by another fraction, or to take its action: a whole [?] by a whole, or a fraction by a fraction — and this is not according to the natural usage [?] — by its nature; for it [proceeds] like the multiplication of those that are not fit to multiply wholes [?], but rather to complete fractions. The multiplication of fractions begins with the multiplication of wholes by [other] wholes, of [discrete] kinds [?] one with another, of two kinds, in [the case of] one [?] kind that is multiplied [?] by [other] wholes.

[Latin note (Schreckenfuchs/Münster): Concerning the multiplication of fractions. This species is set in the first place, even if it be simple addition, on account of the fact that the remaining species of fractions require it for their composition into two species — which cannot be done without multiplication. Now multiplication is the resolution of a fraction, and of a fraction into a single fraction, which sometimes is turned into an integer, or into an integer and a fraction; and this is not according to the natural manner, since it has already been said that an integer is not to be broken, but rather is to be reduced to integers.]

wholes you make few from many. And conversely in fractions, this [is] the opposite of that, and the other [?] — that the multiplication in wholes [increases], whereas [in fractions it diminishes the] count of the numbers. They are equal [?] in fractions: that is, the one increases by addition and the other diminishes by subtraction. Yet these are exceptional cases [?] in fractions, [matters] of quality, not of magnitude in them [in quantity].

For example: "three fourths" multiplied by "one half [?]," as when you double "three fourths" — that is, [multiply] together with [another] "three fourths" — and there arise six [?] sixteenths; and yet [?] the multiplication of wholes was greater [?] in the rule [?] of wholes, and was diminished [?] from the magnitude of the quality, since these are four [?] sixteenths [?]. This is the quality, the change [?] from the magnitude of the fourths.

And from this we have begun, by the knowledge of the way [?] which you must follow to arrive at the knowledge of this kind. And I say, regarding the fractions that are multiplied together with [other] fractions, that you divide them into two parts: and if there be two fractions of one kind, or of another kind — and if there be two kinds, and they are joined — of a hundred and forty kinds [?]; and as for [?]…

[Latin note (Schreckenfuchs/Münster): …but rather it is to be reduced to integers. Now the multiplication of fractions differs from the multiplication of integers in two ways. For the multiplication of integers makes many out of few, but in fractions it is the contrary. Next, multiplication in integers is the aggregation of equal numbers, but in fractions one number is added and the other diminished. There arise, however, in fractions these varieties by reason of quality, not of quantity. For when ten fourths are multiplied by ten fourths, there result one hundred sixteenths, and the number increases by reason of quantity, just as happens in integers; but in quality it is diminished. For fourths are changed only into sixteenths, which are smaller in quality than fourths. And in this way we shall arrive at the knowledge…]

Kitsur ha-Melakhat ha-Mispar — Fractions (Multiplication and Addition of Fractions)

When the shared kinds are cast out from among them, and there remain of them seventy-eight differing kinds — though all these kinds may in fact be reduced to three kinds among them, which are: a fraction of one [number] by a fraction of another; a fraction of one by the fractions of another; and the fractions of one by the fractions of another. We are not obliged, therefore, to instruct you about all of them; rather, the way of these three kinds will guide you.

Once I have made you grasp this way, all the other kinds will be within your reach, and you will arrive at the knowledge of all the kinds that branch off from them.

I say, then, that the universal way for all three of these kinds is to grasp them by their roots, while all the remaining kinds depend upon them. Take the quantity, and reach the quantity [multiply quantity by quantity], and keep its preservation [the product kept aside].

After this, take care with the quality together with its quantity, and take care first to preserve the first; the second is the fraction, or the fractions, that issue from them.

[Latin note (Schreckenfuchs/Münster): "…of this kind [or species]. The multiplication of fractions with fractions divides into two parts: namely, whether there be two fractions at the same time of one kind, or whether they be of two kinds; and then it will be necessary that they increase up to 144 kinds. But since the kinds communicate among themselves, some will be left over, and 78 different kinds will be set aside, all of which are resolved [and] reduced into three kinds — namely into a fraction of one [number] into another, and a fraction of one into another fraction, and fractions of one into fractions of one. It is necessary, therefore, that we demonstrate the knowledge of these three kinds, and how all the kinds may be reduced to them. Afterward we shall demonstrate how all the kinds are multiplied according to these, and we shall arrive without difficulty at the knowledge of them."]

a fraction, or fractions of one [number] by another. And these are joined to it, completing for these three the kinds; and these are the three [kinds].

We have already explained, in the first kind, the quantity together with the quality, and another [step] arose: that they be multiplied together — and twelve came up. Gather them up to a hundred [reckon them], and the fifth [step] preserves the residue. After this, take care with the quantity: the quality with the quantity, and twelve came up; gather them. And another [factor] with twelve, and a myriad comes out — the product of the third by the fourth.

And in the second kind, we explained the quantity with the quality, and nine came up. Gather them, and the fifth [step] preserves the residue. After this, take care with the quantity, and nine came up; gather them. The difference subtracts [reduces], and they are two ninths. And this is the product of the third by [the] two ninths.

And in the third kind, we explained the quantity with the quality, and four came up. Gather them, and the fifth [step] preserves the residue. After this, take care with the quantity, and twelve came up; gather them. Under the residue, and they are six parts of twelve.

[Latin note (Schreckenfuchs/Münster): "…of them. And so the universal method of these three kinds — which are the foundation and root of the rest — is this: that you multiply quantity by quantity, and you keep the product. Likewise you proceed with quality by quality, and the proportion of the first being kept with respect to that reserved [as] the second, there will result the fraction or fractions issuing from the multiplication of one fraction into another. But examples must be set out for these three kinds. Multiply quantity by quantity, namely 1 by one, and write the product below. Then multiply quality by quality, namely 3 by 4: you have 12, and you have one of twelve [= one twelfth]. Not otherwise do we proceed in the second kind, and we collect 2 ninths, which is the product of the multiplication of one third by two thirds. In the third kind…"]

the residue, six, and its preservation; in its formation and in the residue, the product of the third by the fourth — [namely] thirds by quarters.

[Latin note (Schreckenfuchs/Münster): "…kind, multiplying quantity by quantity we write 6. Thus from the multiplication of qualities we note 12, and we have six parts of 12; or you may make a shorter proportion, namely one half, which you mark at the side. This, then, is the product that comes from the multiplication of two thirds into three quarters."]

Worked-example tables

The three multiplication examples are set out as numerator-over-denominator pairs. The source displays each factor pair side by side (table reads right-to-left in source; logical order preserved below). Each block shows the two factors (quantity over quality) and, beneath, their product.

Example 1 — two thirds × three quarters:

	factor A	factor B
numerator (quantity)	3	2
denominator (quality)	4	3

Product:

numerator	6
denominator	12

Example 2 — one third × two thirds:

	factor A	factor B
numerator (quantity)	2	1
denominator (quality)	3	3

Product:

numerator	2
denominator	9

Example 3 — one quarter × one third:

	factor A	factor B
numerator (quantity)	1	1
denominator (quality)	4	3

Product:

numerator	1
denominator	12

מאזני החכאת [recte: מאזני ההכאה] — The Balances [Verification] of Multiplication

And these are the balances [verifications] by which you may know whether this is correct: that the product divide out [reduce] as the quantity that comes up from the multiplication of the fraction with the multiplied fraction. Take care of the quantity of the multiplied [fractions], whichever you wish; that is, divide. And if it comes out, and if [the division] in the dividing yields a fraction equal to the other fraction, it is correct. And if not, take care.

משל — Example

If you multiplied a third by a quarter, one of twelve will come up. And the balances [verification]: you divide one of twelve by the third.

[Latin note (Schreckenfuchs/Münster): "...kind, the quantity having been multiplied by the quantity we write 6. So from the multiplication of the qualities we note 12, and we have six parts of 12. Divide 1. duodecima [one twelfth]; let it be divided 1. duodecima [one twelfth]... Probatio [Proof/Verification]. Let us divide the product of the multiplication of a fraction by a fraction by either one of the fractions that were multiplied. And if the product of this division be equal to one of them, it has been done correctly. Example: If you multiply one third into one quarter, you have 1. duodecimam [one twelfth]. Let 1. duodecima [one twelfth] be divided by either one of the fractions, and they will produce..."]

the third, which is the fraction, divide; and from the products, what comes out is that which is three of twelve parts. And reckoning the half, it is the quarter; and this is the second fraction of the multiplied ones. And the manner of the division [is] as above [in its place], should you wish it — there [it is].

[Latin note (Schreckenfuchs/Münster): "...will produce three twelfths, or more briefly one quarter, which is the other of the multiplied fractions. Concerning division [of fractions] it will be spoken of afterward."]

מין הקיבוץ בשברים — The Kind [Operation] of Addition with Fractions

ערך [The matter/value]. The matter [value] of addition is known from the definition of addition of wholes [whole numbers].

The universal way for every kind of the added is this: take the quantity, multiply the quality with the quantity, and the quality with the quality, and keep its preservation. After this, take the quality of the first fraction with the second fraction, and the product with the quality of the third, and so on consecutively until all the qualities are exhausted, except the quality of the first. Also [take] the quality of the second fraction with the first quality, and the product with the quality of the third, and so on consecutively until all the qualities are exhausted, except the quality of the second fraction. Also [take]…

[Latin note (Schreckenfuchs/Münster): "…will produce three twelfths, or more briefly one quarter, which is the other from the multiplied fractions. Concerning division it will be spoken of afterward. — Addition in fractions is done in this manner. The definition of addition is known from [the chapter on] integers [whole numbers]. The universal method common to all the [compound] kinds is this: that quality be multiplied by quality, and further again by quality, and that which emerges be kept. Then let the quality of the first fraction be multiplied with the quality of the second fraction, and the product with the quality of the third, and so on consecutively until all the qualities be resolved, except the quality of the first…"]

Set down the product of the third fraction together with the denominator of the other [fraction], and the numerator [of that product] together with the denominator of the second. Continue multiplying in this way until you have completed all the numerators—that is, multiplying the third fraction's numerator with the denominator of the fourth, together with all the denominators, except for the denominator of the third fraction. Multiply this [result] always in a single ordered way, and the numerator [carries through] in this manner. So multiply all the fractions, this being the general rule. Then collect all the numerators that arose from all the multiplications, and reduce them in the way you set out—if it is a small number, divide it; if it is greater, multiply it. If the result is greater and exceeds [the divisor], take the whole [number]. So collect all the fractions that have been gathered, and you arrive at three [whole] thirds of the divisor [?], and add them, [these being] the mingled numbers, and they are these:

[Latin note (Schreckenfuchs/Münster), running header "263": "…the first fraction. Then let the quantity of the second fraction be multiplied by the quality of the first, and the product by the quality of the third, and so on successively until all the qualities are exhausted except the quality of the second fraction. Again, let the quantity of the third fraction be multiplied by the quality of the first, and the product by the quality of the second, and so on through all the qualities except the quality of the third fraction. Likewise let the quantity of the fourth fraction be multiplied by all the qualities except its own, and so on until all the fractions are exhausted; and finally let all the products of all the multiplications be added, and let their sum be compared with what was reserved: if it is less, [it stays]; or if it is greater, let it be divided by it, and the product will be the sum of all the fractions added. But three examples are to be formed from these composite operations."]

The fraction-series display (numerators over denominators; the row reads right-to-left in the source, presented here left-to-right):

1	1	1	2	3	1	4	3
5	4	3	7	5	4	5	4

[Latin note (Schreckenfuchs/Münster), running header "264": "In the first operation we multiply the quality 3 by the quality 4, and the product 12 by 5, and we serve the resulting 60. Then we multiply 2 by 4, and the product eight by 5, and there will come out 40. Again we multiply 3 by 3, and the product 9 by 5, and there emerges 45. So we carry 4 into 3, and the product 12 into 4, and there will come out 48. We add these together and collect 133. Then we divide them by 60, the reserved [number]." (continued below the Hebrew:) "…and there will come out two whole [units] and thirteen parts of 60."]

See, in the manner of the first [example], multiply: the numerator of the [first] denominators 3 with 4, which gives 12, and 12 with 5, which gives 60. Keep it [in reserve]. After this, multiply 2 with 4, which gives 8, and 8 with 5, which gives 40. Also multiply 3 with 3, which gives 9, and 9 with 5, which gives 45. And 4 with 3, which gives 12, and 12 with 4, which gives 48.

Also multiply: the numerator 1 with 3, which gives 12 [?—the Hebrew reads י"ב], and 12 with 4, which gives 48 [recte: 45], the sum. These collected together give 133, the parts. Reduce them over 60, the [reserved] numbers, and there comes out 2 wholes, [the remainder] being 13 parts of 60. And add [them]—this being the manner of the first [example].

Sebastianus Munsterus.

[Latin note (Münster's signed notice): "Whereas up to this point I had appended my annotations to my translation, and—having set the work aside—had undertaken a journey into the Swabian regions, in the meantime Schreckenfuchs's commentary was begun in the printing house; and I preferred that it be carried through to the end and that my own labors be interrupted, rather than that one and the same matter be driven home twice—especially since

I have by now illustrated the more important part of arithmetic with these annotations of mine."]

The Second Example (מָשָׁל הַשֵּׁנִי)

In the manner of the second [example], multiply: the numerator 1 with 4, which gives 4. Also multiply the numerator 1 with 7 [?], and what arises with 4 [?], which gives 60 [ס'], together with the denominators [?]. After this, multiply the numerator 1 with 4 [?], and what arises with 7 [?], which gives 4; the numerator 2 with 4, which gives 40 [?—Hebrew מ']; and the numerator 1 with 4, which gives 4. These collected together give 40 [מ'], and what arises [?] is gathered over 160 [?—Hebrew חס"ם], the [reserved] numbers, and there comes out a whole and 160 [?—ק"ס] parts [?], [or: 60 parts]—this being the manner of the second [example].

The Third Example (מָשָׁל הַשְּׁלִישִׁי)

In the manner of the third [example], multiply: the numerator 3 with 3, which gives 9, and what arises with 60 [ס']. Also multiply the numerator 1 with 3, and what arises [?]; which gives 20 [כ']. Also multiply the numerator 1 with 3, which gives 21 [?—כ"א]; and the numerator 1 with 3, and what arises [?]; which gives 49 [?—מ"ט]. Also multiply the numerator 1 with 3 [?]. These collected together give 22 [כ"ב], and what arises 46 [מ"ו], and they are gathered over the [reserved] numbers; and there comes out 46 [מ"ו] parts of [?], and add [them]—this being the manner of the third [example].

The Verification (מֹאזְנֵי) of the Addition of Fractions (מֹאזְנֵי הַקִּבּוּץ מִשְׁבָרִים)

And the verification by which you weigh this matter [is as follows]: behold, you have already found the way [to add fractions], and it is that you subtract from the [sum-]fraction one [fraction], that is to say [?]...

something of what arose from the sum [מַסְקְבּוּץ, recte: מהקיבוץ], and whatever remains—if the gathered fractions were two, the [other] fraction will remain; and if the gathered fractions were more than two, there will remain the sum of the fractions that remain—or [if] not, [the answer is] a lie [i.e., the addition was wrong].

The Example (הַמָּשָׁל)

In the verification of the sum of the third and the fourth [fractions] that arose: their fractions' sum is seven parts of twelve [7/12]. Inasmuch as we lacked the third from the sum [?], the quarter remains; and when the quarter is lacking, the third remains. And in the verification of the sum of the third and the quarter:

And the fifth [chumash] that arose from them is 47 parts of 60. When the third is lacking from them, there remain 9 parts; [and] the quarter [leaves] 42 [?—מ"ב] parts. And the numerator [?] is the sum of the quarter; and the fifth—if the third is lacking [from it]—there remain seven parts; [the result] is 12 parts of 60 [?—מי"ב], and the numerator [?] is the sum of the third and the quarter.

Subtraction of Fractions (חִסּוּר הַשְּׁבָרִים)

Subtraction is the diminishing of one fraction from another fraction, [so that] there remains, after the deficiency of [one] fraction from [another] fraction, a whole [number] greater than it [?]. And I say: the fraction of [one] fraction is the combining [התחבר], by which one fraction is joined with another, of one kind from [another] kind, [in] the splendor [התפאר]—and a stranger [זר] is, [as for] heaven [שמים]...

Subtraction of Fractions (חסור השברים) — continued

from the side of the fractions — for this too is a kind of deficiency. The matter has already been explained where we said that the fraction, when you wish to take a fraction from a fraction, that fraction is nothing but [one] of the kinds [of fractions] we have explained. The deficiency will be a fraction taken from a fraction — one fraction taken from another. And here is how you state it.

A quarter taken twice [as] a third [?] — that is, when you say "a quarter of the third." Here the quarter is the quarter of a fraction, namely the third, since a third is a fraction. The quarter is the whole of "another," and the third is the third of the other [?]. Consequently, when you say "a third less a quarter" — that is, when you say "a quarter less a third" [?] — here the quarter is the quarter of the whole; and since it is a fraction taken from a fraction, the remainder is a fraction of a whole fraction.

And there is a deficiency, since the fourth fell short of the third: take care, three forty-twos [3/42], for the third — which is the one fraction [?] — falls short of us by its quarter, which is the second [?]. Take care: its quarter is three forty-twos [3/42]. Here is the difference, which is among the kinds for addition; and consequently here was the root of addition combined [?].

A fraction to be broken, and the deficiency was the root of subtraction. The four [recte: ארבעת — "the four"] [?] — a fraction remaining from the deficiency: a fraction of a whole fraction [?]. For just as this number is combined from the kinds in the addition, so too this number; and in it is sought the deficiency of the fraction from the fraction, and in the kind explained is sought in it that which is taken from us [?]. And it is the deficiency, and nothing remains.

And since it has already been left over [?], a doubt that is satisfied by it cuts short the ordering and arrangement. And one says afterward — since with the addition we sought to take which fraction it should be — from which...

from which fraction it should be. And here one is already accustomed to dispose [it] [?]. And the remainder — one states it as a need for the kind of subtraction. Indeed we already explained, in our discussion of this, that the subtraction does not inform us of the remainder from its deficiency: a fraction of a whole fraction. And from the moment it is explained, it does inform us: the remainder to us from its deficiency — a fraction of a broken fraction, a whole fraction.

And now I shall begin explaining the kinds of fractions. This number — and one says that this number too — there is no doubt that there are in it twelve kinds of fractions: the simple [פשוטים] and the newly compounded kinds, which are combined except [those] which are combined [recte], the compound. For this number — however great it is — every kind among them, just as it recurs, is combined from the differing simple [fractions]. It is divided in two, but not so in its kinds [are] the addition and the differing [?]: there is no difference between the explanation of the fraction with the fractions and the addition of the fractions with the fraction. And consequently, because it goes out from them another [?].

And indeed, just as in the kind of subtraction there is a difference between the deficient fractions — the broken-from-the-fractions — and that which is deficient, namely the fraction from the fraction; consequently it is obligatory that there be in it kinds of the compound [fractions]. Thus, and by the way that you know them, indeed with the broken-of-the-sheet [יריעת — "of the sheet/leaf"] the joined [?]: the two — meaning [רל = רוצה לומר, "that is to say"], with the sheet, in its joining of the kinds — to the heart [recte] all the kinds, among which there are a fraction broken and broken of fractions, and a fraction of fractions and fractions of a fraction.

We have already explained this, and there is no need to repeat its explanation. Indeed there remains upon us: here is the way of regularity in these — all the other kinds, the compound [fractions]. And one says that you order in your sheet all the kinds — these...

these which are like the customary [מורגלת] table-rulings: the lines and the columns, for all the remaining kinds. Indeed, here, in arranging the fractions you wish to subtract — this from that, this side and that side — consider the multiplication together with the addition, and the diagonal, and its preserved [products]. After each multiplication, that fraction with its preservation [?]: the two — the preserved [product] — and here the preserved, the second also is broken [?].

The combination, taking the preserved [product] with the multiplication and the preserved one, and its deficiency from its preservation [?]. The second: if the second preserved was larger than us, the [?] [recte: ארבע? "four"]. The deficiency of the preserved is the second [?]: namely, if it was smaller than us, and there remained to it in its subtracting from the first preserved [product]; and the second is...

it remains from its deficiency: a fraction, one from another, a broken-from-another [?]. And depict to yourself five [ה'?] examples for these kinds — [the?] two [?] — and these are they:

מָשָׁל ראשון — First Example

The first example: take the number $^1/_2$ and $^1/_3$. Multiply $^1/_3$ by 2 [?], by 2, and consider $^1/_3$ with $^1/_6$ [?], and preserve them. Another [arrangement] — the diagonal: consider $^1/_2$ with $^1/_3$, and $^1/_6$.

Numeric grid. The grid reads right-to-left in the source; each column is one fraction-pair worked to a result (top operand / bottom operand / result). Logical right-to-left order preserved across the columns below.

row	col 1 (rightmost)	col 2	col 3	col 4 (leftmost)
top operand	1/2	3/4	3/4	1/2
bottom operand	1/3	2/7	1/3	2/7
result	1/6	13/28	5/12	3/14

Then $^1/_3$ [?]: so we considered $^1/_2$ with $^1/_3$, and $^1/_6$ [?]; and we subtracted them — the preserved [products] — and there remained [the result], and we closed them up to the preserved [products]. And they are the sixth [1/6], another, and it went out [וכבה], the remainder — a deficiency of a third from its half [recte: half of a third = 1/6].

מָשָׁל הַשֵּׁנִי — Second Example

And in the number of the second: multiply ⅔ [recte] with the addition, and consider it $^2/_3$ with $^1/_3$; consider ⅔ with [?], and preserve them. Another [arrangement] — the diagonal: also consider $^2/_3$ with the addition, ⅔ with [?]. It came to twenty-one [כ"א, =21]; we subtracted [numeral: ?] from them; and there remained thirteen [י"ג, =13]. We attributed them to the twenty-eight [הכ"ח, =28] preserved [products], and they are thirteen [י"ג] parts [חלקים]; and it went out [וכבה], the remainder — a deficiency of a third [שליש] from a third, a fraction of quarters [רביעיות].

מָשָׁל הַשְּׁלִישִׁי — Third Example

And in the number ⅔: multiply ⅔ with the addition, and it yields twelve [י"ב, =12], and preserve them. After this, multiply $^2/_3$ with the addition, $^1/_4$ [?], and here it is 4 [recte: ד', the divisor]. Also multiply $^2/_3$ with the addition, $^2/_3$; and we subtracted nine [ט', =9] from them, and there remained eight [ח', =8]. We closed them up to the twelve [י"ב, =12] preserved [products], and they are eight [ח', =8] parts [חלקים] of twelve [י"ב, =12]; and it went out, the remainder — a deficiency of a third from a third, a fraction of quarters [רביעיות].

מָשָׁל הָרְבִיעִי — Fourth Example

And in the number, the fourth: multiply ⅔ [recte] with the addition, ⅔ with 2, and it yields fourteen [י"ד? =14] [?]. And preserve them. Another [arrangement] — the diagonal: multiply $^2/_3$ with the addition, $^2/_3$ with 2, and it yields fourteen [recte]. Also multiply $^2/_3$ with the addition, ^{1} [recte: הא']; and it yields seven [ז', =7]. From them we subtracted three [ג', =3], and there remained three [ג', =3]. We closed them up to the fourteen [recte] preserved [products], and they are fourteen [recte] parts [חלקים]; and it went out [וכבה], the remainder — a deficiency of a third [recte] from a third, a fraction of its half [מחציו].

The Division of Fractions (תחלוק השברים)

The division of fractions means determining the ratio that one fraction has to another. For example, when you say "a third part of a fourth," it is as if you said "you should divide a third by a fourth." With this in mind, you proceed to know what ratio one fraction has to another. For every fraction has some ratio to every other: it is either greater than the other, or less, or equal — for in the knowledge of the fraction's ratio there is no other case than these three: greater, less, or equal. With this, you can know which of the two fractions is the larger, by how much one is larger than the other, and what part [of the whole] the difference between them amounts to.

This is what teaches you which of the fractions is the larger and which the smaller, so that you may restore one to the other [bring them to a common terms] — for one fraction has a certain ratio to the other. After that, you will find the larger fraction within the smaller fraction.

If so, then in the same way you will determine what part [the one fraction is] of another fraction, [namely] what part the larger is of the smaller [?]. Whenever there are other fractions, you proceed in this manner. Here, then, you may always determine, in the manner of the operation [done] repeatedly, what you gather of them. The ratio of one fraction within the other fractions, and within these the fractions that recur, lie at hand for [comparing] the fractions one against another.

By way of illustration: with the [denominators] in proportion — a third, a fourth, and so forth — in the eighth there is yet another. There emerged [?] six teachers [?] in the [matter of] the thirds and the [ratios of] fourths — the equal fourths, the eighths, and so forth — into the eighth they amount to six [?], and in the fourth a quarter [?] of them is [?] eight [?] eighths [?].

So that you may know which of the kinds [of fractions you are dealing with], you begin the matter of the two fractions, [namely] whichever the one is, you divide it, and afterward you take its complement, or its inverse, as we have arranged it here, in the kind of the deficit — that is to say, [the kind] that remains over [?]

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[that] which remains over from the [above] division, [another] matter after it that emerges [?]. The division is its inverse; therefore there will be [a result] from the things requisite for you, by reason of the joining together [common terms] that there be of the composite kinds, [drawn] from the simple kinds. With this kind too, just as we have arranged it, in the kind of the deficit — and bear in mind their knowledge: it is indeed [bound up] with the knowledge of the composite; that is to say, [it goes] together with the knowledge [obtained] in the joining of the kinds to one another, [these kinds] which consist of the fraction together with the fraction, and of the fractions together with the fraction. And we have already

explained this several times, and so this is sufficient; nor does there remain over for us anything to add.

Indeed, this is the way of [setting out] these six kinds that are mentioned, which are like the steps [degrees] reserved for [classifying] all the kinds. And we say that their way is comprehensive of them. Indeed, it lies in the arrangement [literal: stringing-together] of the fractions, which you want to [reduce] to [common] parts, this one with that one, and to multiply by the measure of the [common] divisor [denominator] with the addition, the joining, and the subtraction — which is set over it [the operation], as shown in the measure of the divisor in the multiplying — and in the division and the divisor, as it emerges. The collector [the rule that gathers them] sets it before you, and forms for you, for each of these four [operations], illustrations [examples] for these mentioned kinds. And they are these:

[The numeric figure on the page is two side-by-side sub-tables, each three columns wide, read right-to-left in the source (rightmost column first). Each column holds a fraction in the top row and, below it, a single value or fraction in the bottom row. The two sub-tables are transcribed separately below in source (right-to-left) order. Bottom-row cells that binarize to a single blobbed glyph are left as [numeral: ?] rather than guessed.]

Right sub-table:

Col 1 (rightmost)	Col 2	Col 3 (leftmost)
1/3	1/4	2/3
1/3	[numeral: ?]	7/9

Left sub-table:

Col 1 (rightmost)	Col 2	Col 3 (leftmost)
3/8	1/2	2/9
[numeral: ?]	1/4	[numeral: ?]

Example One (משל הראשון)

For [example], with the kind of the first that we mentioned: take a half, a third [?], that is 1/3, and the two of them, and divide them upon the [common] denominator from [reckoning] a half by a third — 1/3 — and you have a third [?]. There emerges [?] one, and a third part of [?]. The [final] sixth [?] teaches that the third is a fourth; one third is a fourth [?]; that is to say, the fourth, which is 1/4 [?], or as when you divide the third — divide it into parts: a fourth, one part, and a third — and the whole comes out, in the [unit of] the measuring-rod [the standard], one.

Example Two (משל השני)

And with the kind of the second: take a half, the two — that is 2 — and add to it 10 [yod] and 6 [vav], and divide them upon the denominator, from [reckoning] a third by a third — that is 1/3 — which are 9 [tet]; and there emerges one, and 7/6 [?]: a ninth — that is to say [the result is reckoned] one time over — another [is] an eighth, and 7/6 [?] a ninth — six [?] ninths [?].

Example Three (משל השלישי)

And with the kind of the third: take a half, the two — that is 2 — and a third, that is 8 [?], and divide them upon the denominator, from [reckoning] a half by a third — 1/3 — they being 3; and there emerges [a result that is] one half [?] — that is to say 2 thirds; 2/2, the [ratio of] thirds being 2, and 2 [parts] of thirds. The fourth [is] another, the fourth.

Example Four (משל הרביעי)

And with the kind of the fourth: take a half, that is 1/2 [bet-yod = 12?] [?], and add to it 9 [tet], and divide them upon the denominator, from the [reckoning of a] half by a half — 2/2 — they being 4; and there emerges [a result that is] two [?] of the [whole] —

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— and a fourth; that is to say, 2 times 2 [parts] of ninths; that they be 4, that is to say 6 [?] ninths; the fourth, and a fourth — 2 of ninths.

The Checking [literally: The Balances] (המאזנים)

And these are the checks [verifications] by which you may verify that this kind [of result] has [indeed] been combined [computed] — that the division [you performed] with the dividend agrees [balances], if it [the result] is equal in the dividing, [namely] in [the case of] the [outcome] that has emerged [?]. And if it is not [equal], examine it [again].

The Example of the Checking (משל המאזנים)

We have divided the [ratio of] thirds by the [ratio of] fourths upon the fifths, and there emerged for us, in the division, one [part], and 7 [?] of eighths — twice 2 of fifths — and there came forth for us [a value that] meets [agrees with] [?] the eighths together with the 2 fifths; and there emerged [a result that is] one [?] — that is, 3 of fourths, which is the [original] dividend.

So [the operation] is completed and finished: the First Part [is concluded] (תם ונשלם חלק ראשון).

The Second Part of the *Kitsur* (The Abridgment), in the Science of Number

Heading as printed (p. 275), two lines: "החלק השני של קיצור / בחכמת מספר" — "The Second Part of the Kitsur [Abridgment], in the science of number."

The Second Part

Its purpose is to make known the operations performed upon the tabulated numbers [of the astronomical tables] by means of an instrument [פלס, "balance/scale"], through the reckoning of differences. This is the subject that the present part teaches, along the path of its [own] four sheets [יריעות, the work's structural divisions]. There are four kinds of operation upon them: addition (הקבוץ), subtraction (והחסור), bringing-in (והחבאה), and division (והחלוק) into the table's fractional sub-units [shavrei ha-tevunah]. This part is accordingly divided into four chapters (perakim):

- The **first chapter** treats the addition (קיבוץ) of the table's fractions [shavrei ha-tevunah].
- The **second chapter** treats their diminution [הפאתם, "their subtraction"].
- The **third chapter** treats the [different] kinds [הסוגם].
- The **fourth chapter** treats their division [חלוקם].

Know that, according to the masters of the [astronomical] science [ḥakhmei ha-tevunah], the parts of the celestial sphere — that is, the sphere of the zodiac (galgal ha-mazalot) — number twelve. They divided the sphere of the zodiac into twelve equal parts and named each part after the figure [tzurah] that lies beneath it: that is, the figures [constellations] composed of the fixed stars set within the sphere of the zodiac.

They named the **first part** the sign of Aries [mazal Ṭaleh], because the figure beneath it has the form [תמונה] of a ram [ṭaleh], formed from the fixed stars joined together [into that figure] within the sphere of the zodiac. The **second part** they named the sign of Taurus [mazal Shor], because the figure beneath it has the form of a bull, [made up] of the fixed stars joined together.

The **third part** they named the sign of Gemini [mazal Te'omim, "the Twins"], because the figure beneath it has this same character. The **fourth part** they named the sign of Cancer [mazal Sartan, "the Crab"]. The **fifth part** they named the sign of Leo [mazal Aryeh, "the Lion"]. The **sixth part** they named the sign of Virgo [mazal Betulah, "the Virgin"]. The **seventh part** they named the sign of Libra [mazal Mo'znayim, "the Scales"]. The **eighth part** they named the sign of Scorpio [mazal ʿAkrav, "the Scorpion"]. The **ninth part** they named the sign of Sagittarius [mazal Keshet, "the Bow"]. The **tenth part** they named the sign of Capricorn [mazal Gedi [recte: גדי], "the Kid"; printed גרי — dalet/resh confusion] [?]. The **eleventh part** [הי"א] they named the sign of Aquarius [mazal Deli, "the Water-bearer/Bucket"]. And the **twelfth part** [חי"ב] they named the sign of Pisces [mazal Dagim, "the Fishes"].

Now the reason [ha-sibbah] for which they divided that sphere by the number twelve, beyond [the other] kinds, is this. It is already written in the works of the ancients, and they said there: a number that has many divisors comes out [evenly] in the smaller number from which it derives [i.e. a small number with many divisors is convenient]. And this is the reason: that the number twelve [חי"ב] is one possessed of many divisors, [which] does not [come out] in a number smaller than it. And this is the reason without [exception].

[The reason is] that they did not divide it [the sphere] into [just] any divisions [chalakim], [but rather] this number [twelve] has more divisors [chalakim] than all the other numbers greater than it and smaller than it; for it has a half, a third, a fourth, a fifth, a sixth, an eighth, a ninth, and a tenth, and it lacks nothing of [these] divisors — only the seventh [does not divide it evenly]. And there is not found [any smaller] number that has [all these] and also the seventh, except [for] the number that comes out from multiplying the seven [הז'] into it [bringing it in, מהבאת] — and that is the number [of] thousands [and] five hundred and two [תק"ב]; and because it would be a number with many divisors [it is rejected as too large]. [The worked value reads as printed: "אלפים תק"ב" (… thousands and 502); no separate thousands-count word is set before אלפים in the line.]

Hence it was fitting that each sign [mazal] should have thirty parts [chalakim], which are called *maʿalot* [degrees]. Again, they divided each degree [maʿalah] into sixty parts [chalakim], which are called *rishonim* [primes/minutes]; and each prime [rishon] into sixty parts [chalakim], which are called *sheniyim* [seconds]; and each second into sixty parts [chalakim], which are called *shelishiyim* [thirds];

and each third into sixty parts [chalakim], which are called *reviʿiyim* [fourths]; and [so on] —

[The sexagesimal hierarchy as the printed lines give it: maʿalot (degrees) → rishonim/chalakim (minutes) → sheniyim (seconds) → shelishiyim (thirds) → reviʿiyim (fourths), each step being a sixtieth of the one above.]

— constant [ha-dérekh tamid]. And behold, the masters of the tables [baʿalei ha-luḥot] make use, in their tabulating within the tables, of these aforementioned parts [chalakim] — that is to say [ר"ל], whatever they wish to write [as] a number, whatever it be [שהוא, printed שחו"א] of whole [units] [shelemim].

They write [it] together [yaḥad]; they write the degrees [maʿalot] under the heading "wholes" [ke-shem shelemim], and the [first sub-units] under the heading "fractions" [ke-shem shavrim], and the seconds [sheniyim] under the heading "fractions of fractions" [ke-shem shavrei shavrim]. And so along this path forever [le-ʿolam, i.e. the same convention extends to every lower order].

By way of example [ha-mashal ba-zeh]: if the Sun is in the sign of Aries [mazal Ṭaleh] at fifteen degrees [ט"ו maʿalot] and a half, and a part of a half [ve-chelek shel chetzi] — behold, they write that the Sun is in the sign of Aries [at fifteen degrees], thirty [ל'] *rishonim* [minutes], thirty [ל'] *sheniyim* [seconds].

But the masters of number [chakhmei ha-mispar] write it thus: that the Sun is in the sign of Aries [at fifteen] degrees [maʿalot], and (61 =) וס"א parts [chalakim], (102 =) מס"ב parts of a half [chelek shel chetzi], [going on] to another place [makom acher]. [The two small numerals וס"א / מס"ב are legible as printed; what remains uncertain is only the arithmetic interpretation — how this alternate chakhmei-ha-mispar notation maps to the value 15°30′30″.] And since the path is one and the same [shehaderekh hineh mishtanah only among] the masters of number, the first procedures [ha-derakhim ha-rishonim] that make known the [four] kinds mentioned — namely the addition [קבוץ] and the bringing-in [וההבאת], and the division [החלוק] — vary among the procedures, [as performed] upon them by way of the fractions [shavrim] that they have employed [therein], [according to] the [astronomical] science [ha-tevunah].

The procedure for the division [of these sexagesimal sub-units] will be explained [as one performs operations] upon them in the [several] fractional orders that are employed within them, [according to] the masters of the [astronomical] science [ḥakhmei ha-tevunah]. Now, since this is what is required of us — to make known the procedures, [of which there are] four kinds, [each operation] enumerated [in its place] — [we turn to set them out in order], proceeding from one [order of] fraction[s] [to the next].

Chapter One — On the Addition [קיבוץ] of the Fractions of the Table [shavrei ha-tevunah]

Heading as printed (p. 278), two lines: "פרק הראשון ביריעת קבוץ / שבר התכונת" — "The First Chapter, on the sheet/section of the addition of the fraction(s) of the [astronomical] table."

We say: whenever you wish to add together two rows or three rows [shurot] [of tabulated sexagesimal values]...

[Text breaks off at the foot of the page; the worked instruction for addition continues on the following opening.]

orders, or [in] some other manner [in] which the rows differ from one another; for these rows [of] seconds belong to the thirds. So set down for them an aligned arrangement, that is to say: the [first] rows beneath the [first] rows, and the second [rows] beneath the second, and likewise the thirds beneath the thirds — and so on, one after another. Set out each kind opposite its kind, and add each kind to its kind. That is to say, the rows of degrees beneath the [rows of] degrees, the first beneath the first, the second beneath the second; and likewise [for] every kind. And start [the addition] from the lowest rank — that is, from the [column of] fourths, if there were [present] firsts, or seconds, or thirds; or [otherwise] from whatever rank is the lowest among those present.

And whatever value [exceeds] and reaches a full sixty, you transfer it to the rank that follows it [toward the left] — for that is what multiplies, namely sixty. [Tracking] the carries between the gatherings [columns]: for every full sixty [reached] from the seconds you carry forward [one] to the thirds, one after another, until you reach the [column of] degrees; and what remains short of [a full] sixty you write [in place].

And when [the column] reaches degrees — for thirty [degrees] is a full sign — for every thirty you carry one to the sign, and what remains [short] of thirty you write [in place]. So combine all the signs, and the remaining ones that are left there; and you write them [as] twelve [in] the heart, that is to say [if there are] twelve [signs] — and do not write the others into the count: your signs are then "twelve-and-so-many." For there is no rank above the signs of the celestial sphere.

And in this manner you proceed [with the rest of] your procedure of itself: in adding [up] days, and weeks, and months; and likewise [for] the seconds, and the thirds — heads [of columns] anywhere [they occur]. When the [accumulation] from your reckoning of the rows of degrees reaches thirty, you carry forward [one] to the signs, [in] your reckoning [of them, namely] twenty-and-something [?]; and when from any base [unit] it [accumulates] to twelve, twelve [?], in your reckoning of the signs you carry one to…

the thing sought — and according to what has been altered in this — [as] in the illustration set out [below]. By way of an analogy of one matter and another, you will reach an understanding of our intent as well; you will also grasp the manner of placement and of subtraction [carrying].

The table reads right-to-left in source. The rightmost column holds the row labels; the numeric columns are sexagesimal places. Headers as printed (right to left): days ('ימי), hours ('שעו), firsts/units ('ראשו), then signs ('מזל), degrees ('מעל), firsts ('ראשו). The bottom line (below the rule) is the sum/total row.

Row label	('ראשו) firsts	('שעו) hours	('ימי) days	('מזל) signs	('מעל) degrees	('ראשו) firsts
רעה מחזורים [a cycle of cycles]	5	2	40	10	25	45
י"ו שנים [16 years]	1	15	30	8	20	50
חרש תשרי [the month of Tishri]	7	18	50	4	19	55
[Total]	0	13	0	0	6	30

[Section heading:] המאזנים — The Balances [the check/proof].

As for the balances — by which this kind [of operation] is balanced: you cast out all the gatherings from a row, namely 9; 9; and 9 [casting out nines]. And you compute every subtraction at that same row [in like manner], and the others

[too]; and whatever is left short of a full 9 is kept [as the remainder]. So too is the gathering of the [sum] sought together with the remainders [of the parts]: you reckon them by the same [rule of] nine. If there were firsts in the gatherings — that being the [lowest] row — or seconds, or thirds, you likewise [cast out nines] up to the final [rank]. And if there were [in] the gatherings of the rows degrees [as] the [present] row, you reckon [them too] by [casting out] nine. And if there were in the gatherings days, [you compute] by [casting out] seven. And [cast out] their gathering [together] with the sought value; and [if] with the row [it] is not [in] surplus, [then] with the row [it] is equal — [so] the gathering of their parts which…

that we sought from it [by casting out] nine [is] firsts, [or] 9 [for] degrees [or] thirds. And these [are] the principles [for] the issuing [of the proof] of this kind. Along this course, too [reckon] the days by [casting out] seven; only — these are not numbers [to be reckoned] as a kind of gathering of whole [units]: there is nothing here [requiring] to be doubled [as in] the [foregoing] matters.

Chapter Three: On the knowledge of the kind of subtraction.

[Subtraction] in sexagesimal-fractional [reckoning] is of the same nature as subtraction in [ordinary] fractions: for just as the fractional [parts] of the [whole] number are divided into seconds and thirds — one part toward [the rank of] degrees on the one side, and the second part toward the [side] of necessity [i.e. the borrowing side], and [those parts] are scarce relative to them [the wholes] in their fractions — [so it is here]. Toward [the rank of] degrees, [the value] that is scarce relative to the [full] number of whole [units], that is to say, [the parts] that are lacking in [their] fractions toward [the rank of] degrees: this is the reverse [of addition]. That is to say, what diminishes, and returns toward the diminishing side [i.e. borrowing]; the deficiencies of their fractions are scarce. Even though one [value] is scarce, [the operation] is to subtract the [lower] fractional parts of the [whole] number, which [are] divided into seconds and thirds. The latter rank [the column further left] of [the] whole [units] is divided [in turn] into [the same fractional parts].

And [as for] the second [rank], between [the rank of] degrees and the diminished [value], and [as for] the increase [carried], between [the rank of] degrees and the diminished [value], it is scarce — there being no diminution. As [the] scarce [value] is to the [parts] of the whole [number] in [its] composition relative to the fractional parts which are scarce — as the few which is scarce relative to the fractional parts of the [whole] number — so the issuing [reckoning] of the thirds along with the firsts [proceeds]; and [the value] arrived at [in] the thirds reverts to the [rank of] fourths, and the [whole] arrived at [reverts] in [casting out by] nine. And [as for] subtracting the thirds together with the thirds, in the fractional parts of the composition it reverts to the sixties; and so likewise [for] the issuing [reckoning] of the fourths together with…

the fourths; for [those] of the fractional parts of the number [reckoning] revert to six [for] the sixfold rows. And what comes up in the calculation of the fourths together with the fifths reverts to the sixfold rows, and [it comes up] in [casting out by] eight together with the eighths. And [as for] their tokens [signs] in the fractional parts of the [whole] number: it [the result] is [obtained] by the subtraction, and [as for] the fractional parts of the composition: the [value] that comes up [in] the gathering [reckoning] of the composition is [obtained] with the subtraction. And just as this [is so], behold then [you should] reckon from the angle [perspective] that comes up to us [as a] sign [token]: namely 1, 2 [a-b] as a token, that any value broken into fractional parts of the composition [reckoned] by two tokens reverts to the calculation [check]. As for [the] tokens [signs], understand them [as] applying to the table [given].

Their token [is that you] subtract by them [from the upper line] when [the second line] is small relative to your reckoning in the fractional parts of the number; for [example], 600 [240 = 'ר'מ? recte] if you wish to write [it down], 3 [for] the fifties [?]: write it down, 9 [for the] writing toward [the column of] subtraction. And if [the value] you wish to write down [as] 3 [for] the fifties, [write it] beneath [it], and one [for the] writing toward subtraction. [The token is to know] toward what they revert in the table set out: behold then [as for] the angle [perspective], try [casting out] nine; if you take, behold then [it comes up] in the writing toward [the column of] subtraction — that is to say, the rows of degrees [are reckoned in like manner]. And if you write down [in] subtraction by them, [for] 3 [for] the fifties [?], write it down, and [behold then in] the writing toward subtraction by them as [for] degrees-and-something. [Reckon] toward the [rank of] degrees by them [in casting out] nine, behold then it [comes up] in the writing toward subtraction by them, and write [it down].

And [as for] their tokens [signs] — when the subtracting by them, that is to say, when in the [column-]rows the latter [line] is the first, you write down [in] the signs [column]; and in [the row of] degrees the first ones [you carry] toward the signs, and in the third [rank, likewise]. And [as for] the second [rank], and in the fourth, the second ones — and so on, always.

Moreover, there are some of the wise men of the [astronomical] table who write [it down] above all [the columns], after [setting] the marks [as] tokens, [so that] the deficiencies are subtracted by them. To make them known, that is to say, [in] the celestial-sphere [table] with each sign [of the] zodiac he will write down upon it the form [emblem] of Aries [Taleh]. And [as for] the bull [Shor] he writes the form [emblem] of Taurus [Shor], and so on, for every [other one]… …they write upon them the figure of the sphere-circle [galgal]. And on the firsts [minutes] — one; and on the seconds, in the figure [way] of two scribbles [marks]. They write the sums [chiqayim] in [the manner of] the combination [addition] of the ranks. The combination [addition] of a kind [type] from among the fractions — together with the wholes; and what ascends [results], that is its wholeness [its full value].

After this: the two fractions that are added, and the [value] that comes out — these are their marks. If there are values preserved [carried] after [a count of] sixty [units of] preservation, then keep them in our manner, and the [value] comes out in its wholeness. And if there were sixties, reckon them as another [higher unit].

So too, when you carry [transfer] the value that ascends [results] to the rank deficient [next lower] than it [in place]. And if there were more than sixty, divide them by sixty; and what comes out [the quotient], transfer it to the rank above it. So too, when you carry the value that comes out to the rank above it; and it will be that in the rank above it there will be the accumulation of the deficient ranks [carries]. Thus it comes out, by way of multiplication [hachalukah/hakaah]. So too, say, if there were sixty in the [process of] accumulation toward another [rank]. And if there were more than sixty, divide them again by sixty; and what comes out you transfer to the rank above it. And so reckon [carry] toward another.

And if there were more than sixty, divide them again by sixty, and what comes out you transfer to the rank above it — and it will be, by its rank. And so, you keep on, until you reach the deficiency [the bottom] of

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the sixties; and let it rest [stop] upon its rank; and you shall not reach the rank deficient [lower] than it.

A worked example of the multiplication of fractions, astronomical [sexagesimal]

[מָשָׁל לְהַכָּאַת שְׁבָרִים תְּכוּנִית]

If you wish to multiply 70 [ע'] thirds by 70 [ע'] fourths: multiply 70 by 70, and there rises 4900 [4900 = 900 + 4000 = ד' אלפים תת"ק]; keep them [in mind]. Then accumulate [add] the ranks of the multiplied [factors] — namely, the thirds and the fourths; and there rises seven [3 + 4 = 7]; and when you direct [reckon] them together, that they [the result, 4900] are sevenths. And since they are sevenths and more than sixty, divide them by sixty; and the quotient is 81 [פ"א], and there remains 40 [מ']; and they [reckon] that they are 81 sixths and also [40] sevenths. [4900 ÷ 60 = 81 r 40.]

Again: divide 81 by sixty; and the quotient is 1 [א'], and there remains 21 [כ"א]; and they are 1 fifth and 21 sixths. [81 ÷ 60 = 1 r 21.] And join with them the seventh that we had; combine [the result]: 1 fifth, 21 sixths, and also [40] sevenths. And this is the [value] that ascends [results] from the multiplication of 70 thirds by 70 fourths.

And if you wish to multiply many kinds of fractions by many kinds of fractions, then order [arrange] them upon this order: each one of the higher [ranks] together with each one of the lower [ranks]. And begin to multiply from the latter [last] ranks, as in our example. And there rises [there arise] the tenths, as you order [arrange them]; and divide them, as you order;

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we have set [them] down in the place of the ninths, beneath the deficiency beneath the two scribbles [marks] [?]. Again, multiply 70 [ע'] fourths by 70 [ע'] fifths; multiply 70 by 70, and there rises 4900 [ת"ק — text reads ת"ק]; divide them by sixty, and the quotient is 81 ninths [ת"ס תשיעיים — reads 460 = ת"ס? (?)] [?]; and there rises 200 [ב"ר reads 202 = ב"ר? (?)] eighths [?], and we set them down in the place of the eighths.

Again, multiply 70 [ע'] thirds by 70 [ע'] fourths [reads חַם שלישיים עם]; there rises 4900 [ת"ק], and divide them by sixty, and the quotient is 2 [ב'] sevenths, and there rises 2 [reads ב'] in the place of the sevenths.

Diagram. A full-width sexagesimal computation grid for the worked multiplication of many-kinds-of-fractions. The columns are headed, from right to left as printed, by the place-markers **ב** and **ס** (the two integer/sixties places) and then by the Roman numerals **I, II, III, IIII, V, VI, VII, VIII, IX, X** — the successive sexagesimal fraction-ranks (firsts, seconds, thirds, … up to tenths). The table reads right-to-left in the source; the markdown preserves the printed logical column order (rightmost source column = leftmost markdown column). Two short rows sit above the upper rule (the two multiplicand entries), a block of partial products follows, and a final ruled-off row gives the per-column sums (totals). The numerals are European Arabic, set by the Basel printer.

Table reads right-to-left in source. Columns as printed, rightmost first:

ב	ס	I	II	III	IIII	V	VI	VII	VIII	IX	X
	25	30	35	40	50	20					
	15	20	40	10	20	30					
12	8	16	4	8	12	15	17	20	25	10	
	7	10	20	5	10	30	13	30	6		
	15	20	40	10	20	11	40	16	40	40	
		8	11	23	5	6	8	20	20	20	
		30	10	13	26	50	40	3			
13			45	40	20	33	13	20			
			1	13	16	40	40	20	20		
				12	20	6	40	1			
				2	2	5	40	2			
						30	3				
1	1	26	11	53	47	54	13	10	31	50	

[Following the table:]

Again, multiply 35 [reads ל"ה — text reads 702 = תש"ב? for an intermediate; reading the worked value as ל"ה] seconds by [reads with] the value [chal] of the fifties; and there rises [a value], and divide them into [by] sixty [על ס']; and there rises 16 [reads 6 = ו'? reads 16 = י"ו] of the sixties; and they are seventh-fractions [?], and write the residue [chai] in the place of the sixties, and the sevenths and write 212 [reads חי"ב — chai-12? reads 12 = י"ב ?] [?] in the place of the sevenths.

Again, multiply the first [factor]

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with the value [chal] of the fifties; and there rises 360 [reads + 5 + 400 = תה"ס 465 = 60? (?); the worked quotient 6 below implies a product near 365] [?]; divide them by sixty, and the quotient is 15 [ט"ו] of the fifties [reads חמישיים], and we set them down in the place of the fifties.

Again, multiply the residue [chai] of the degrees [reads חבה — chai-?] with the value [chal] of the fifties; and there rises 352 [reads = 2 + 300 + 400 = תש"ב 702 (?)] [?]; divide them by sixty, and the quotient is 12 [reads י"ב] of the fourths, and 30 [reads ל'] of the fourths; we write 12 [reads חי"ב — chai-12? reads י"ב] in the place of the fourths, and we start over [recommence] in the place of the fourths.

After he combined the residue [chai] of the fifties with one another, in the place suitable for it, he returned to multiply the second [pair of] fourths upon all the degree-ranks; and the value that ascends, write in the place suitable for it according to its rank, as we explained. Behold, multiply the value [chal] of the fifties by the value [chab] of the fourths with 2 [reads בב' — by two?]; and the fifties: divide them by sixty, and there rises 4 [reads ד'] of the eighths, and we write 12 [reads חי"ב — chai-12? reads י"ב] in the place of the eighths, and 30 [reads ל' (?)] in the place of the ninths [reads טט — recte: ט'?] [?].

Again, multiply the residue [chai] of the fourths with the value [chab] of the fourths together [reads מחובר — combined?]; and there rises [a value]; and these eighths, divide them by sixty, and there rises 6 [reads ו'] of the sevenths, and 40 [reads מ'] of the eighths; write 12 [reads חי"ב — chai? reads י"ב] in the place of the sevenths, and 40 [reads ם'] of the eighths in the place of the eighths.

Again, multiply the residue [chai] of the fourths with the value [chab] of the sevenths; and there rises [a value]; divide them by sixty, and the quotient is 13 [reads י"ג] of the sixties, and we write each one in the place suitable for it: of the sixths [reads ששיים] in the place of the sixths, and the sevenths in the place of the sevenths.

Again, multiply the residue [chai] of the fourths with the value [chal] of the seconds; and there rises [a value]; divide them by sixty, and the quotient is 13 [reads י"ג] of the seconds, and 60 [reads 40 = 'ם? (?)] [?]; and we write each one in the place suitable for it: of the sixties in the place of the sixties, and the sevenths in the place of the sevenths. Again, multiply the value [chab] of the fourths with [reads עם] the value [chal] of the seconds, and there rises [reads 405 — ת"ה? reads ת"ש] [?]; divide them by sixty, and the quotient is 11 [reads י"א] of the fifties, and 60 [reads ם'] of the sixties; and we write

and place it in its position. Then add 2 fourths together, which makes 50 fifths; divide them by 60, and 0 [remain] of the fifths. Carry the [?] further, and write them in the place of the fourths. Add a further 2 fourths together with 8 degrees, and you get 609; [?]; 2 fourths divided by 60 gives 0, and there carry 6, [making] 3 thirds, and [?] 2; write them, each in its position.

After this, return to add up the thirds. They come to 19, and write them in their positions [recte: בְּמְקוֹמוֹ], so that nothing is left over. As to all the remaining named fractions — these are: sixths, sevenths, and so on — and after all this is collected, set each one out in its place by itself, according to what we arranged for them in the chapter on the collecting of the fractions of the [astronomical] tables. And the total reached, when collected from the sum of the [?] degrees of the table, is: 2 firsts, 40 seconds, 19 thirds, 2 fourths, 30 fifths, 38 [?] sixths; with [?] 28 degrees, 30 firsts, 38 [?]; [?] 38; 2 thirds [recte], 3 fourths, 2 fifths.

A further method for reckoning the fractions of the [astronomical] tables

Some of the earlier scholars [made use of] this method, [which is] a short method, and it is this: that you should not divide the value [recte: הָעוֹלָה] [arising] from each multiplication-step by itself by what corresponds to it, but rather you write each of them in [a single line], one alongside the other [recte: בְּאֲשֶׁר], until you need to add them up at the proper point — that is, after you have written down the whole value, but only what you carry over [stays] in its place. So [whatever] requires carrying, carry it.

The Example

An example of this: as in the example by which we explained above how we multiplied 50 fifths by 2 fifths, and there arose 50 [?], which we wrote down in the place of the tenths. And so likewise. By this method, then, each degree of every kind is summed, and you divide and total the value [arising] from every multiplication-step, on 60. And as it [comes out], you write it in the multiplication-step indicated, and the rest [carry?]. And as we explained above [recte: לְס׳ = "by sixties"], you write it. The [value reached from?] its multiplication-step, [the?] degree of its multiplication-step, [as collected?] — and in order to make this fully clear, we have drawn the figure arranged [below], and written in it the values [reached], according to this method:

[The following full-width 12-column table is in the source. It reads **right-to-left**; the column heads run (rightmost to leftmost): ס, ב, I, II, III, IIII, V, VI, VII, VIII, IX, X. Logical column order preserved below. Numerals are European Arabic, as printed.]

ב	ס	I	II	III	IIII	V	VI	VII	VIII	IX	X
	25	30	35	40	50	20					
	15	20	40	10	20	30					
					500	750	900	1050	1200	1500	600
			1000	250	300	600	700	800	1000	400	
		500	600	1200	1400	350	400	500	200		
	375	450	525	700	800	1600	2000	800			
	16	36	46	600	750	1000	400				
				63	77	300					
13						74	53	40	31	10	
1	1	26	11	53	47	54	13	10	31	50	

[Below the table:]

A further method for multiplying the fractions of the [astronomical] tables — there is also another method, which is a long, broad method by which you can [reach] all the multiplication-steps.

the upper column to the last multiplication-steps among them. And [after?] this, here: a degree from the upper column with a degree from the lower column, from [the upper?] and the lower; and the column. [End of opening clause; the passage describes pairing the ranks of the upper and lower columns.]

A further [method] for the collecting [of the products?]: the last multiplication-steps from the upper column, and the [units?]; the last multiplication-steps from the lower column and the lower [one] — what is composed of them is the [guarded/retained] [value], when you multiply. After this, divide the [retained value] by 60; and what is over you attribute to the firsts; and what comes out of this division [you attribute] to the multiplication-steps that are arranged.

Further: the part that comes out, [divide it] by 60, and what is over you attribute to the firsts — that is, the parts of the firsts — you attribute to the seconds; and what comes out of this part [you attribute] to the multiplication-steps that are arranged after it. So [carry it on?] until this part is divided. The multiplication-steps and the degrees you divide; and what comes out of the division [is?] another [step], and what is over — 12 [recte] — and what is over reaches 12, that is, 12 of the signs. And what arises from all the multiplication-steps is a degree, [reached?] by the multiplying of the upper column [by] the lower column.

An example of this

An example of this: as in the example we set out above, [where] we explained that the firsts [come to] 28 degrees, [which is] 600; and there arose 600. Then [we?] [reached?] 600, and we multiplied them upon them, 2 firsts: 30, 38 [recte], [making] 4 firsts that are [?]; and there arose 600. The firsts: divide [it] by 600 [recte: 36 = ל״ו?], and there arose [?].

Further: we [collected/added] them: 2 firsts; the firsts come to 600; and there arose 58; over 4000; 2 seconds, we added them up [upon] them; the seconds come to: 600; and there arose 28 thousand, 240 seconds.

by 60, and there arose **5510100** thirds; we added [upon] them the [seconds?], [making] **5510140**, the total of the thirds.

Further, the firsts [come to] these, by 60, and there arose **330608400**; the fourths we added up upon them, 2; the fourths come [to?]; and there arose **330608450** fourths. Further, the firsts [come to] these, by 60, and there arose **19836507000**; the fifths we added up upon them, 2, [making] **19836507020** fifths. And in returning, once more to the lower column, multiply by the degrees with 60, and there arose; firsts we joined upon them, 2; the firsts [come to] the count, by 60, and there arose 58; over 4000, 2 seconds, we added them up upon them; the seconds: 28 thousand, 240 seconds. Further, these [come to], by 60, and there arose **3314400** thirds; we added them up upon them, 5 thirds, and there arose **3314410** thirds. The fourths we joined upon them, by 60, and there arose **19884600** fourths; 2; the fourths, further, these [come to], by 60, and there arose **19884620**. The fifths we added up upon them, 30; [making] **1931877200**; the fifths, further, [come to] **1931877250** fifths, and you keep them, when you multiply, with what is retained; and we joined the retained [value] with — and so that what comes next shall be — **33668676643467…** [numeral: trailing digits ?] (leading run reads 33668676643467; the full ~21-digit string is not digit-verifiable at this scan ceiling). Multiply the fifths with the tenths, which will be the tenths that arise; what is composed of them is sixteenths, and after this we divided it on 60. So what comes out in the division is: **5944779440577885910**. And the rest, we returned [to it] so that the ninths shall be from another multiplication-step.

[the part] that arranged it [hasoderet]. And [divide] the part [le-ḥalek] that [remained], so that there remains a remainder which cannot be [further divided], divided by 31 [ל"א]; write the remainder by the side of the tenths [עשיריים].

Then go back again to divide the part by 60 [ה"ס], and there comes out by division **6574532400963 1431**, and behold the result returns to be eighths, since they multiply by another step; and to the part that remains a remainder, divided by 3 [ג'], write 3 by the side [as] the rank of the ninths [כמדרגת התשיעיים], and there will be 3 ninths [תשיעיים] according to its step. The whole division [is finished] [חק הסודרת].

Then go back again to divide the part of the number that comes out by division by 60 [ה"ס], and there comes out by division **1095772066827190**. Behold the result returns to be sevenths, and the remainder, which are eighths — that is, they are eighths — as it was; and write it by the side of the eighths rank [כמדרגת השמיניים].

Then go back again, divide the part of the number that comes out by division by 60 [ה"ס], and behold the result returns to be sixths, and the remainder, which are sevenths — that is, they will be sevenths — as it was; and write it by the side of the sevenths rank [כמדרגת השביעיים]. And the number that came out by division [is] **18262867780453**.

Then go back again to divide the part of the number that comes out by division by 60 [ה"ס], and there comes out by division **5043811296674**, and the results return to be fifths, [the remainder] which is 13 [י"ג], that they will be sixths [ששיים] — as it was; and write it by the side of the sixths rank [כמדרגת הששיים].

Then go back again to divide the part of the number that comes out by division by 60 [ה"ס], and there comes out by division **5043018827**, and the results return to be fourths [רביעיים], and the remainder of the results from the division will be fifths [חמשיים] — as it was; and write them by the side of the fifths rank [כמדרגת החמשיים]: **84550313**.

And they [the results] return to be thirds [שלישיים], and the remainder of the results will be fourths [רביעיים] — as it was; and write them by the side of the fourths rank [כמדרגת הרביעיים].

Then go back again to divide the part of the number that comes out by division by 60 [ה"ס], and there comes out by division [the spelled-out number] **ארבעת אלפים וארבע מאות ותשע אלפים ומאה ושבעים** [as printed; literally "four thousand and four hundred and nine[teen?] thousand and one hundred and seventy" — the spelled-out form does not reduce arithmetically;]. Another [step], and they return to be seconds [שניים], and the remainder will be thirds [שלישיים], which are 53 [נ"ג] — as it was.

Then go back again to divide the part of the number that comes out by division by 60 [ה"ס], and there comes out by division [the spelled-out number] **ב"ג אלפים תב"ו** [as printed; the leading numeral ב"ג and the total תב"ו do not resolve cleanly —], and they return to be firsts [ראשונים], and the remainder of the results from the division will be seconds [שניים], which are [the value stated] — that is, [the result] remains, as it was.

Then go back again to the part: this number that comes out by division by 60 [ס'] comes out, and the results return to be degrees [מעלות], and the remainder will be firsts [ראשונים], which will be [the value stated] — as it was; and write them by the side of the firsts rank [כמדרגת הראשונים].

Then go back again to divide the part of the number that comes out by division by 30 [ל'], because they are degrees [מעלות], and there comes out by division 13 [י"ג], and the result returns to be 13 degrees, and the remainder, which is another [amount], will be degrees — as it was; and write them by the side of the degrees rank [כמדרגת המעלות].

Then send forth the sum [נשליך] of degrees that is in our hands by 12 [י"ב], because they are degrees, and there remains [in our hands] degrees, and we will multiply [them] again, so that they are 1 [א'] sign [מזל] and [the remaining] degrees; and write 1 by the side [as] the rank of the signs [כמדרגת המזלות]. And in this manner you will already arrive at the intention [el ha-mekhuvan].

And this is what we wished [to explain] [ve-zeh mah she-ratsinu le-va'er].

המאזנים [The Scales / Balances]

Now, if you wish to balance [verify by the method of "scales"], take the third method [ha-shelishi]: [take] the [check-figure] obtained from striking the upper row [ha-tur ha-elyon] together with the lower row [ha-tur ha-shafal], after [the rows] have been written by their ranks [le-fi mareglotam], from their final ranks the last ones [ha-aḥaronah], as we explained; and divide them by the upper row according to the manner of division [ke-fi derekh ḥaluk]. The result is complete; and if the upper row comes out [evenly], then [the answer] in the other row, by the division, is correct; and if not — then there is an error in the calculation [be-ḥeshbon — recte; uncertain].

פרק שלישי במין החסור [Chapter Three: On the Type of Subtraction]

The way of knowing this type is that you arrange the upper row [ha-tur ha-elyon] by the rank of degrees [mareglat ha-ma'alot] first, and the rank of the firsts [u-mareglat ha-rishonim] first, and the rank of the seconds [u-mareglat ha-shniyim], each one after the other, according to its ranks [le-fi mareglotav], one after another. Then arrange the lower row [ha-tur ha-shafal] by the seconds — every type and type — afterward each type, that is, the 30 [ל'] degrees [recte: ר"ל? as printed ר"ל] under the degrees, and the firsts under the firsts, and the seconds under the seconds, until the end.

And so, in this manner, what comes next: begin from the last rank which is at the bottom [she-batur ha-shafal], and subtract that rank of the lower row from the rank which is opposite it in the upper row [me-ha-tur ha-elyon]; and what remains, write it under the line opposite [taḥat ha-kav] that rank which you joined [as] opposite it [ke-neged]. And so always, until you reach the first rank [ha-mareglat ha-rishonah].

And if the [lower] rank is greater [ha-shfelah gedolah] than the upper rank — that is, when the number of the lower row that is opposite [ha-mareglat she-be-negedah] is at the mouth of the upper row [pi ha-tur ha-elyon],

[so that] the [lower] rank is greater than 60 [ס'], and what remains has been broken [ha-nishar nishbar] — that is, the upper rank [ha-mareglat ha-elyonah] which is opposite it [she-be-negedah] is broken down beneath it [taḥat — into 60 sub-units]: and when you wish to subtract the [larger lower] rank, take from the upper rank [me-ha-mareglat ha-elyonah] which is opposite it one [unit], reduced from it; and afterward, that which is reduced from it — the lower rank [ha-mareglat ha-shfelah], if it was [greater than] the upper rank [ha-mareglat ha-elyonah], is greater than it [gedolah mimenah].

And if it [the upper rank] was a small [amount], reduced from it, this is the way — that is, when the [lower] rank diminishes, the rank diminishes 60 [ס'], and what remains has been broken, that is, [the rank] one [degree] up [ma'alah aḥat], and write it; and one [unit] up [ve-ma'alah aleph] you reckon under the line opposite [taḥat ha-kav] that rank which is itself [hi], and so always, until you reach the first rank [ha-mareglat ha-rishonah].

And in order to enlarge the explanation in this [matter], hold now [a figure] in the form of one general [diagram], to include every kind of exchanges [ḥilufim], and so that the intention be reached [el ha-mekhuvan]. Behold, so as not to be any number whatever in gold [ba-hov — recte: by analogy? uncertain]:

The lower rank which is opposite the upper rank [she-be-neged ha-mareglat ha-elyonah] is in the structure of [be-tikhno] the upper rank itself, beneath the line opposite [taḥat ha-kav]; that is, the boundary [ha-gevul] opposite the last rank [ha-mareglat ha-aḥaronah], without any addition or diminution.

After this, we will copy [na'atikenu] the arranged [rank] to the original rank [ha-mareglat ha-soderet], so that what is greater from the upper rank which is opposite it [she-be-negedah] is its diminution from it; that is, we are left in our hands 10 [י'], the exchanges [ha-ḥilufim] together with the [unit] which is in the upper rank, and it ascends to 3 [ג'] in the structure [ba-binyan] under that rank.

Then again we will copy it [na'atikenu] to the arranged rank [ha-mareglat ha-soderet], and also it is greater from the upper rank which is opposite it [she-be-negedah], and the firsts diminish into the sixths [ba-shishiyim], and we are left in our hands 3 [שלשה — three]

the borrowed value together with the fifty that is in the column above [שבמורגה העליונה], which came to 53. We took one away from them, leaving 52, and we wrote it beneath the line of that column. Again we returned to the third column [מורגה השלישי]: we subtracted the third column of the lower [row] from the third column of the upper [row], which was 59, and we took ten away from them; there remained 49, and we wrote it beneath the third column. Once more we returned to the fourth column [מורגה], and we subtracted its lower entry from the upper, which was 52, leaving 2; and we wrote it beneath the fourth column.

By this you already know that when we subtract 3 degrees [מעלות], 10 minutes [ראשונים / "firsts"], 57 seconds [שניים], and 50 thirds [שלישיים] from 5 degrees, 20 minutes, 50 seconds, 40 thirds, and 30 fourths [רביעיים] — there remain 2 degrees, 9 minutes, 52 [נ"ב; printed ג"ב — a nun/gimel garble] seconds, 50 thirds, and 30 fourths. [The closing string of the printed restatement is partly garbled — "ור"מ ל"ש" — but the grid below fixes the remainder as 2° 9′ 52″ 50‴ 30⁗.]

המאזנים [The Scales — the verification / "check"]

Now, the verification by which one validates this type of operation is addition [קיבוץ; ר"ל = "that is to say"]: when you add the third column to the second column, the result — if the work is correct — gives you back the first column. As you saw above, in the example you had set out:

Grad. [מעלות / degrees]	Pri. [ראשונים / minutes]	Secund. [שניים / seconds]	Ter. [שלישיים / thirds]	Quart. [רביעיים / fourths]
5	20	50	40	30
3	10	57	50	[—]
2	9	52	50	30

פרק רביעי במין החלוק [Chapter Four: On the Type of Division]

[Printed החלוס — a kaf→samekh garble for החלוק, "division."]

This fourth type comes back to the operation that is the division of [whole] quantities, proceeding by the same rule as with whole numbers, with respect to perfect [whole] units — though there is in it a certain distinction. After this: subtract the [count of] units from the divisor [הנגזר], by which we divide the units of the dividend [המחלק] taken from the divided number; and what remains of the divisor's units is itself the [count of] units of the type that comes out [the quotient — היוצא] of the division.

And if the number of units of the divisor is set equal to the number of units of the dividend [המחולק] — that is, if the units of the divided number are ascending degrees [מעלות] — know necessarily [בהכרח] that the quotient is degrees. But if the number of units of the divisor were greater than the number of units of the dividend, then know that the dividend cannot be set over the divisor in this division. In every trial in your practice, the dividend, when its outcome falls among the degrees, descends [וירד] — that is, it is carried down [a column].

And if the units of the dividend [are set equal] to the units of the divisor: then already the dividend is reduced, by the amount of the exchange [התמורה — the borrow/conversion], from the divided number, by the amount of the divisor; and what comes out [the quotient] is degrees in the amount of the division, just as we laid down [as a rule].

And if the [count of] units that returns [is reckoned] from what is divided [המחולק] together with the 60 [עם הס׳] and falls short of [is "without"] a [full] row relative to the units of the dividend [המחלק], then the dividend is again joined with 60: it ascends [ותעלה], and one descends another column [מורגה] of necessity; and the matter continues [always — תמיד] until there remain units of the dividend [made up] of [completed] sixties.

And if the amount of the dividend [המחולק] exceeds the amount of the divisor [המחלק], then what comes out [the quotient] is minutes [first-fractions] in the amount of the division, by the same reasoning we explained; and its units are degrees [מעלות] of necessity, just as they are arranged. After this — when the number of the divisor [הנגזר] is reduced to its units (as units of the number of the divisor), and the dividend [המחולק] is reduced from its units (as units) — I shall give an illustration: three worked examples, differing in three differing types.

משל הראשון [The First Example]

If you wish to divide 2 seconds [שניים] by 15 firsts [ראשונים]: divide the [whole] quantity by the rule for quantities, and what comes out [the quotient] is 1 [first] and 8 parts [א' וח' חלקים]. Now set the divisor from its units — the units of the firsts, which are 1 — as the type of the number of the divisor from its units, that is the units of the seconds, which are 2; and what remains is 1, which stands there for the firsts.

The divisor [is reckoned] from its units, and therefore we know [ולכן ידענו]; and the other — that which comes out [the quotient] — is of a differing [type]: its place in the division is a "first" [ראשון]. Another point: the remainders, which are 8 [ח'], cannot be divided by 15 among themselves — together with 60 [עם הס'] — and so they do not amount to [a full place]. The remainders are seconds; and after them comes a [further] remainder. The seconds become 60, of necessity. Of the thirds [שלישיים], which we join with 60, they descend [ירדו] a column [מדרגה]. Then divide by 15 firsts, and there come out [the quotient]; and the number of the divisor [reckoned] from its units (as units), which comes from the dividend, [yields] the units of the thirds, which is 3; and there remains 2. And so we know that there come out, in this second division, 2 seconds; and we already had in hand 1 first. So that which comes out [the quotient] is: [73 — ע"ג].

the quotient is "seconds" [הב' שניים], 22 [רכ׳ — recte רב׳] of which, that is the 2 seconds, are [made up of] sixtieths [משישים] of a "first"; for one "first" is among the 15 firsts of the first [example], and one "first" is again within the parts [חלקי] of [that] first. Or, when [the count] reaches each of the two parts, subtract the firsts: one "first" is [one] of 60; the parts of the second [example] are 60; the "first" of the second is 1, [making] 42 [מ"ב] parts of the second.

משל תשני [recte: משל השני — The Second Example]

If you wish to divide 2 firsts [ראשונים] by 15 firsts: divide the [whole] quantity, and what comes out [the quotient] is 1 [first] and 16 parts [א' וט"ז? — quotient-fraction; reading uncertain], so that there is a "first," while the dividend [המחלק] and the divided [המחולק] [drop down] a row [שורה]. The seconds here: what comes out is itself, of necessity, in its order, ascending degrees [מעלות]; and so we know that what comes out is of a differing type — it is another "degree." After that, there are remainders differing from 60 [the result, less the carried sixty]; and after them the firsts will stand, in their order, as firsts, and the others become, from the [remaining] portion of the 60, seconds, in their order. Divide by 15 firsts, and there come out 2, lacking from the units of the firsts [made up] of the units of the seconds, and there remains 2. And so we know that what comes out is of a differing type — the second [time], firsts over firsts; so they are firsts, and we already had in hand one degree [מעלה אחת]. So we know that what comes out is of a differing type — a degree.

The verification: when the firsts [are divided] by 15, the result is one degree; and the 2 firsts, over that, are firsts; and there is one degree [מעלה אחת] and 2 [וב׳ — printed ורב׳] firsts. For the 15 firsts are like one degree, one time, together with the dividend.

Kitsur ha-Melakhat ha-Mispar — Part Two, Chapter Four (Division), continued

[The conclusion of the Third Example:] …a third [of a unit], one time; or, if you fall short for every "first," then each "first" [in the divisor counts] as [its conversion into] sixty [of the next-lower order]… and one "first" after [another, divided by a] "third."

Third Example (משל שלישי)

If you wish to divide 2 "firsts" by 39 "firsts": Observe that, since the dividend is smaller than the divisor, you cannot divide a "first" by [a quantity in which] sixty [of the lower order] are contained in your two [firsts]; rather, convert the two "firsts" [downward] to "seconds." They come to one hundred twenty "seconds." Divide [these] among the thirty-nine "firsts" according to the same procedure [as before], and there will come out 3 "seconds," and there will remain 3 [seconds] over. Convert these to "thirds," and they come to one hundred eighty. Divide [them] likewise. So [each successive order is] divided in the same measure as the [order] divided above [it], and the [resulting] quotient comes out the same [in form].

What is incumbent upon you in this [computation] is to take care that the quotients which result be marked according to their orders: namely, that those which come out [from dividing] "firsts" be designated as "seconds"; for since you divided [a quantity standing] one rank above by [a quantity] of the same [rank], the quotient that emerges is in the rank below, exactly as we explained — that a "first" [divided by] a "first" yields "seconds," and a "first" [divided] from a "first" [yields likewise].

The Division of the Differing Kinds of the Fractions of Astronomy (מְשל החלוק המשונים מינים של חשברי תכונה)

Now then, if you wish to divide a fraction composed of several differing kinds [of orders], understand that the dividend stands in the upper row and the divisor [is set apart] from it; and you proceed with the other kinds [of orders] in turn — for in every one of the kinds, the later ones follow the earlier ones: degrees, then after them the "firsts," and after them the "seconds," and after them the "thirds."

to the contrary, for the "thirds" yield "firsts" [as quotient], and after them the "seconds," and after them the degrees, and after them the [whole] degrees. And after this [kind of division] you should reckon, beneath the dividend [the divisor], so that each kind is set opposite its [own] kind — that is, the degrees beneath the degrees, the "firsts" beneath the "firsts," and so on. And after this [is arranged], thus [you proceed]: each one is set opposite its [own] kind. And after this — the matter being now drawn out — and it is written below [in the example].

The Third Example: it has come out by [the method of] division, and the [computational] practice has guided [us as to] how the quotient is to be reckoned upon the [whole] degrees — when you have come to know from what you already know, [by comparison] with what comes out from the [established] principles of the science of astronomy, together with what you know by reckoning and by computation. So the rank: the first [order] is [reckoned] from the divisor — that is, the [whole] degrees — if there be there degrees [in the divisor], or the [several fractional] orders, if there be there [fractional] orders. And likewise, every rank yields [a quotient] by division, [namely] the [rank] which is set opposite it [in] the divisor, [such] that the first [resulting] fraction reaches down to the [proper] order.

And I will give you an example of it — let it be the First Example — and the fraction will be such as we set forth above, of the kind that comes by [the manner of the] Second [Example]. The Third Example: from it [the dividend] is that which arises [as] the dividend [in the upper] row, [divided] by the second [divisor row, giving] orders that come out. And the quotient [stands] in [each] fractional order, one after another — thus:

Table. A sexagesimal grid of ten ordered columns. The Latin order-headers run, left to right as printed: Grad. (degrees) | Pri. (primae, "firsts") | Secū. (secundae, "seconds") | Tert. (tertiae, "thirds") | Quar (quartae) | Quin (quintae) | Sext. (sextae) | Sept. (septimae) | Oct. (octavae) | Non. (nonae). Each column carries two stacked numeral rows (an upper and a lower). Table reads right-to-left in the Hebrew context, but the Latin headers and the logical column order run Grad.→Non.

Row	Grad.	Pri.	Secū.	Tert.	Quar	Quin	Sext.	Sept.	Oct.	Non.
Upper	301	26	11	53	47	54	[blank]	[blank]	[blank]	[blank]
Lower	15	20	40	10	20	30	13	10	13	50

[in order] for the orders to be [reckoned as whole] degrees — they [stand] in the rank of the [whole, "completed"] [degrees] — so that there comes out for you 1 order; which is [a quantity] short of [a full] order — [namely] 3 [of the next order] from the differences. To these are joined the differences [drawn] from the orders, together with the degrees, and they are written into the rank of the orders, in the first row, at the head of the kinds. But as for the remainder [of the orders], the rank of the third order, from that which comes about — they are of themselves [reckoned in] their own [rank].

The rank of the first order [is determined] by what [it is] in itself, without any change at all. And likewise the rank of the second order, from that which comes about — they are of themselves [in] their own [rank]; and the rank of the second order, by which [it is] of itself, [is] as the [other] orders of the counting [system], which are near one another — so that all [the orders] return, [each] of them, into [its proper place], and there suffices for the [whole] degrees, from them, that which [is wanting]. And the principle [is this]: that which is lacking from the ranks of the order — the rank of the first [order] — and it is the number contained in it, and [you proceed] by reckoning, in the rank of the third [order], as we explained for the first.

Now then, that which is by the order of the third [rank], together with six orders — for the dividend [comes] from the [whole] degrees, [divided] by orders coming out from the [whole] degrees — that [quotient stands] in the [fractional] order.

And the "firsts" remain; and [you] reckon them into [the order] above [them], according to the [proper] procedure, in order to instruct concerning the remainder.

And again [for] the second [order], and it comes to 60. And likewise the first [order] — they are the [whole] degrees that come [out] — the [whole] degrees, [of which] there were 16 [in count]; and the second order [is reckoned] from the [whole] degrees; and from there into the second [order] — those are the [whole] degrees, of which there were 16. The second order, and from there into the second [order] — those are the [whole] degrees, of which there were 16.

The orders which [come] by the [whole] degrees, together with them [the dividend], such that [it is] divided by the second [divisor], and it comes to these — [namely] "seconds," for the dividend [comes] from what arises [in the] orders, together with the "seconds." These are "seconds" by the count, [as] there came out for [each] one of the firsts. And for it to be [the case] that the first ones are by [their] firsts — they [stand] apart — and the rank [comes] by the firsts, [each] one of them at the outset [of the procedure], in the [manner] we drew out, from each [order]. And the remainder [comes] by what arises from the "firsts," and you reckon them into [the order] above [them], according to the [proper] procedure.

And again [for] the second [order], and it comes to 60. And likewise the first [order] — they are the differences in count. And there came out 8 orders. And the rank [comes] from the orders, of which 6 are firsts; and the remainder [is reckoned] into the order above [it]. And likewise, the rank of the first order, from which [it is] of itself — they are the firsts, of which there were 60. The "seconds" [come] from the differing [orders], by what arises [in the] order above [them], and you reckon them into [the order] above [them], according to the [proper] procedure.

The remainder, then, [comes] by what arises in the orders. And again [for] the order, it has come about, and it has come up [to a value]; and [from] the dividend, [reckoned] by the second [divisor], it comes [to a value]. These are "seconds." For the dividend [comes] from what arises [in the] orders, together with the "seconds." These are "seconds" by the count, [as] there came out for the rank of the third [order] by division: the "thirds," [namely] 4. The "seconds" yielded [these] "thirds" — and they remained [as remainder]; and [you reckon them] by the third order, [standing] in the upper [row], according to the [proper] procedure. If [there be] differences [over], then [reckon] by the order [above], "seconds"; and 2 "seconds" remain, and what remains [is reckoned] into the [order] above [it], according to the [proper] procedure.

And again the dividend has come about [to a value], by the fourth [order], which has come up [to a value]; and on account of it [there come out] for them orders, together with [those of] the fourth order. There are [whole] degrees, of which there were fourth-order [quantities]; and there came out, by [the manner of] division: the "thirds," and the fourth — [namely] 2; [you] subtract 2 by the fourth [order]…

[Part Two is not concluded; no "תם ונשלם" appears on these pages. The exposition of multi-order ("differing kinds") sexagesimal division continues into the following opening.]

Kitsur ha-Melakhat ha-Mispar — Part Two, Chapter Four (Division), continued

in the upper column as well, and arrange them by rank, according to the rule. So too we found the fifths' portions [?] for the fifths. The remaining fifths we set in their place by rank, according to the rule.

Again I will explain. We had [?] degrees, and there came out 703 [תש"ג], and accordingly the fifths with the fifths would be 43 [מח"ג — recte: probably מ"ג, 43]. The fifths will be fifths; reckon their portions, however much they come to, and let the fifths be reckoned with the fifths. The remaining fifths we set down by rank according to the rule.

Again, we subtracted the fourths, and there remained [?] of the fourths, and what remains of the fourths is set in its place by rank according to the rule. We likewise subtracted [?] and there remained 15 [ט"ו] of the fourths, and we set them down by rank, according to the rule.

Again, when we ask after the larger number — which generally is the greater — since there is, by one rank, a deficiency in the second column, we reckon what rises from it, on account of what is deficient or of what remains, and it is a number; and so that the number sought be degrees, they are degrees, and they are set beneath the upper column. [The type here is legible; the [?] mark isolated small numeral/word slots, not illegible spans. The procedure — subtract rank by rank, carry the deficiency/remainder to the next column, place each quotient digit under the upper column — is secure.]

Again, when we sought of the larger column the number that comes out together with… in the orders of the second column, and we reckon the rising of the deficient — that is, what is deficient — of the first column, or of what remains there, and it is a number. And so that what is sought be the seconds, they are seconds. [?]

the thirds, which are of the upper column. Again we sought a number that comes out together with all the ranks of the second column, and we reckon what is deficient of their orders, of the upper column, or of what remains there, and it is the number 3 [ג']. And because the number we sought is the deficient orders, the search being of the third, know that the number is equal — the fourth number — and we set the sums beneath the fourths of the upper column.

Again we sought a number that comes out together with all the ranks of the second column, and we reckon what is deficient of their orders, either of the second column or of what remains there, and it is the number 2 [ב']. And because the number we sought is the orders, the search being from the fourths, know that the number is equal — the fifth, which is the fifths. And so that no remainder be left in the upper column: the matter is already divided, and we know that the division is already complete.

[New heading, p. 304:] **The third [order]:** the issue depends on whether the two remaining columns increase by another column without addition or subtraction. And the difference will arise from the kind of division [recte: of the balance], from the two scales of the difference.

And from the kind of the difference of the scales of division: if there remains a larger number of the upper column, by the orders divided; or if there remains a number, by the orders divided — know that the difference, when the divided column is together with the column… in the division and in the quotient, add to it, by the divided orders, each of one kind. And if not, the quotient will be a remainder together with the upper column, which is the divided. The difference is that which is doubled from one kind to another by division.

The second species [מין שני].

But if you wish to verify by way of the verification that the division is complete on the whole, the matter has already been laid out for you in this doing. The deficient remainder of the kind of difference — generally the greater — returning that which, by multiplying, returns from the kind to the kind.

[New heading, p. 305:] **The Example [המשל].**

Know: the column of the third [recte: of the third], the example, as explained, which we set out as:

2366867664346731546OO

in the twenties and what remains. And by the rank of the second column [הב'], which is one and another, the orders mentioned, which are set out over it: the quotient of this column is the column which has 15 degrees [ט"ו מעלות], 2 firsts [ב' ראשונים], 40 seconds [מ' שניים], [numeral: ?] thirds [שלישיים], 2 fourths [ב' רביעיים], 30 fifths [ל' החמישיים], as explained, by which the reckoning rises to:

11931877230

The fifths, and we keep the portions. Keep the first by the second keeping, and there comes out:

19836507020

— we wished to verify the fifths, by the ways of the fifths; and that which remains of the fifths is the fifth. The portions, which arise from the remainder in the division — the fifth — the portions that come out of the division of the two: the fifths,

5336608450

and the seconds. Again, its portions, the fourths, by the reckoning that rises to:

5510140

— the thirds.

Again we divided by 60 [הס'], and there came out from the division 40 [מ'] of the thirds, and the number 91 thousand [צ"א אלפים] and 1030 [תתל"ר] — the seconds.

the seconds, by the number, and there comes out 35 [recte: of the seconds], the firsts, and the number of these firsts, in all… The portions, we keep them, and there comes out by the number, by the division of the 60 [הס'], the firsts; and the remainder of the seeker [?]:

[New heading, p. 306:] **The Balances [המאזנים — i.e. the checks / proofs].**

And the scales of the first method are of one kind, this being the first method, from the explanation. And the scales of the second method are eastward [מזרח] of this kind, this being the second method, from the explanation.

[New heading, p. 306:] **The third species [מין השלישי].**

Behold, it has already been explained to you, by way of knowledge, this kind, in several ways: numbers that are divided in general — that is to say, no remainder in them; and numbers that are divided in general, yet there remains in them a number, by the division. But the numbers without firsts are divided in general; yet there remains in them a number, by the division.

If you wish, in the four portions, a living quotient close to them, then there comes to you, close to the division and to the number, what is added to the four — the order of the matter. We will explain this in two ways: the ways of 'הקרומיי' [reading uncertain — qof clearly present, sense unclear; not simply "the Romans"]; and behold what I shall bring as a parable. Another, without division, in the orders [recte: that comes out (?)], and it indicates the portion more than what is doubled, close to the table; and in it… to receive, to compare, and the like.

I say: when you wish to divide a number — 15 degrees [ט"ו מעלות], 46 firsts [מ"ו ראשונים], the firsts; also 52 seconds [נ"ב שניים] — by 30 fourths [ל' רביעיי], its quotient is: 2 degrees [ב' מעלות], the firsts; 30 seconds [ל' שניים], behold…

If you wish to divide it [the dividend] by the order of the preceding terms, so that it does not come out even [i.e., leaves a remainder], then proceed throughout in the manner of the path we set out for this [third] species, since here you are dividing according to the value of the orders [literally, "the rows"].

Diagram. A multi-order sexagesimal computation array laid out as a descending "staircase." Eight columns are headed at the foot by Roman numerals 0, I, II, III, IIII, V, VI, VII (printed left to right across the page); each column is one sexagesimal order (place value). The entries are the partial values of an approximate (non-even) division worked across the orders, cascading diagonally downward. By the sexagesimal convention each cell should hold a value 0–59, but two intermediate cells exceed 59 (see Numerical impossibles) — apparently un-carried partial products in the worked example. Transcribed with the columns in their printed left-to-right order (Roman heading shown in the header row); the staircase shape leaves the upper-left and lower-right corner cells empty.

0	I	II	III	IIII	V	VI	VII
		0					
		4	0	0			
		5	4	2	0		
	0	25	6	4	1		
0	40	91	56	10	13		
15	46	59	4	6	306	9	
5	2	6	52	30	12	12	0

0	I	II	III	IIII	V	VI	VII
3	8	4	58	48	30		

After this, consider that you seek a number that, reckoned against all the orders, can be divided evenly by the divisor [literally, "by what the order apportions"]. Take the upper register [the dividend]; this is number [?3] ג. Beneath it, opposite its orders, set down the number you divide by, and check it against the value of the orders. The places that come from the divisor are [?22] כב, and above them טו [15], the remainders. The places from the divisor of the upper register, when subtracted from the upper register, leave nothing over; reckon [carry] against the next order. With [?22] כב you obtain הג [the value: 5,3?], and above it הב [the value]; there remain to be carried down to the next order the remainders, leaving nothing over. Reckon against the next [order] with הג [the value], and above it גג [the value], and there comes up [30] ל [?], the remainders, which you write opposite them in the [next] row as we explained. After this, consider that you bring up הג [the value] with the value, for the seconds [?]; there come up grains [units] from the value of the upper second place, and there remain [41] מא, which you write opposite them in the row by the rule.

Again, when you divide a number to be reckoned against all the orders, and you apportion the places that come out short of the divisor of the upper register — or [the places] remaining or found — namely the number [8] ח — you write it down beneath the missing places, opposite its orders, with the value [5] ה. The first places against it are [40] מ, the first of the upper-register order, and nothing is left over; you reckon [carry] by the rule. With the value הב [the value: 5,2?] you reckon the first places, and above them [2] ב seconds; from there remain in it the value, and you write them down in the row — the value [30] ל — opposite the second place by the rule. Again, with the value [5] ה you reckon the third places, by הב [the value], opposite the third-place order, leaving over [?30] ל, which you write in the row by the rule. Again, you bring up the value הר [the value] with ב [2: the first places that come from the thirds are הג [the value]; you reckon the upper-register order, which holds [?200] ר, but it cannot be subtracted by ר. From it [you take] [8] ח grains [making] the value [1] א second, and you write it down in the row by the rule. The value [?200] ר for the operation upon the thirds: from it you subtract, and there remains the value. The thirds you bring up — those left over — are גג [the value], and there remains [?52] נב; the thirds you reckoned, written there, the value גנ [the value], you write in the [next] row by the rule. Again, you bring up the value הר [the value].

The value, and it comes to רב [the value] for the fourths, and the grains [units] of the value; the fourths [come from] the divisor of the upper register, and there remains [?200] ר, which you write in the row by the rule. Again, when you divide a number to be reckoned against all the orders, and you apportion the places that come out short of the divisor of the upper register — or [those] remaining or found — namely the number הח [the value: 5,8?] — you write them down beneath, opposite the orders. Against the value of the thirds, the value, and above it רז [the value] for the thirds; the thirds [are] the divided value for the seconds, reckoned against הח [the value]. The places of the thirds — grains [units] — opposite רז [the value], [and there remain] [2] ב seconds. Of the upper-register order nothing is left over, and you reckon [carry] by the rule. Again, the thirds you bring up are גג [the value]; you reckon the upper-register order, and there remains [?200] ר, which you write in the row by the rule.

Again, you bring up the value הב [the value] with the value בב [the value: 2,2?], the first [places] coming from the fourths; and they come out by [?200] ר for the fourths, the divided value for the thirds, since the operation upon the fourths is [reckoned against] the upper-register order. It comes to [?200] ר, but it cannot be apportioned evenly [literally, "does not suffice for subtraction"] against ר, [namely] from the fourths גג [the value]. The grains [units] are [2] ב from the thirds, [reckoned] against the upper-register order, and there remains [?200] ר, which you write in the row by the rule. The thirds you reckoned among them are the value [1] א; [reckoned] against the value of the third places by way of another third, the value you brought up, which is the fourths. Bring up the value with the value, the fourths divided, which you brought up; from them comes the value; the fourths [reckoned] against the value for the fourths of the upper register. It comes to [?200] ר fourths, which you write in the row by the rule.

Again, you reckon a value with the value בג [the value: 2,3?] for the fifths, and above it הש [the value]; the fifths [reckoned] against the value of the fifths, [namely] the value [?200] ר. The grains [units] are the value; from there [you take] הח [the value] by way of the value ר for the fifths, leaving over [?200] ר. The fourths [reckoned] against the value of the fifths; from it הח [the value]. The fourths of the value for the fourths: and it is the value, the fourths divided. Against another [?33] לג; the fifths you brought up are [?200] ר, [reckoned] against the value of the fifths, [and] from them [?200] ר. There remains the value.

[The value] that is not apportioned for us in the value of the fifths is reckoned against [?200] ר for the fourths; the value for the fourths of the upper register, and there remains [?200] ר for the fourths, which you write in the row by the rule. And among them you bring up the value for the fourths in the manner [explained]. Also, you reckon the value אה [the value: 1,5?] for the fifths, and reckon the value for the fifths which you brought up; there remains [?12] יב of the fifths, which you write down at the place of the fifths.

Again, when you divide a number to be reckoned against all the orders, and you apportion the places that come out — namely the value — short of the divisor of the upper register, or [those] remaining or found — the number [?48] מח — you reckon it opposite [the orders], beneath the value for the fourths.

You reckon against it the value for the fifths, opposite רם [the value], [namely] the fifths; the fifths divided are [?200] ר for the fifths, by הש [the value]; the grains [units] are the value, nothing is left over, and you reckon [carry] by the rule. Again, you bring up the value הח [the value] with the value בב [the value: 2,2?], the first [places] coming from the fourths; and they come out by [?200] ר for the fifths, the value, another value, the fifths, by way of subtraction, since it cannot be apportioned [literally, "does not suffice for us"]. From it [you take] בג [the value: 2,3?], the fourths reckoned against [?200] ר for the fifths; the upper-register order, and there remains [?200] ר, which you write in the row by the rule.

Again, you reckon the value with the value [?200] ר for the sixths, and above it רפח [the value]; the sixths [reckoned] against [?200] ר, by הח [the value]. The sixths divided are [?200] ר for the sixths, [and] there remains the value, the sixths, since there is not for us [enough] from the fifths to reckon against the upper [register].

the upper [order] remains, and there is no fifth left over—nothing—as the rule requires. We subtracted 4 [ד'] from the fifths that we had carried, leaving 5 [ה'] of them; we set those down and carried them up to the degrees as before, as the rule requires. We subtracted 4 [ד'] from the 5 [ה'] of the fifths, leaving 1 [א'], which we set down—8 [ח'] fifths. And there remain 12 [י"ב] sixths, which we set down and carry upward as the rule requires.

Again: whenever you wish to divide a number through all the orders of the quotient, gather the number of the upper order—that which you find—and add it to it. That residue, opposite the fifths, beneath that order, is the number 30 [ל'], and it enters by division opposite this: 8 [ח'] degrees and 53 [נ"ג] fifths, over 60 [ס'], [ב'] 2 fourths come out. Of the 53 [נ"ג] fifths we carried, the 2 [ב'] fourths against the fourths leave 19 [י"ט] for the upper order, and nothing remains—

neither whole nor in fractions—as the rule requires. We also carried 30 [ל'] of the fifths from the decade of the upper order, and there remains 1 [א'], and 2 [ב']. [?]

We set it up upward as the rule requires. Again, when the number divides the upper order: with the whole of the firsts and over—60 [ס'] sixths—divided over 60 [ס'], [ו'] 6 comes out. Of the 8 [ח'] fifths we carried, 1 [א'] remains from the fifths in the upper order's portion; and nothing remains—neither whole nor in fractions—as the rule requires.

Again, when the number divides the whole order: with 1 [א']—[ב'] 2 sevenths—and over them 168 [קס"ח] sevenths, divided over 60 [ס'], [ג'] 3 comes out; 9 [ט'] thirds, divided over the decade of the sixths of the upper order, leave 9 [ט'] sixths, which we set up upward as the rule requires. By this method you obtain the whole, and there is nothing to subtract for the decade, so long as the reception is complete and nothing more remains over in the upper order—if there is anything to subtract, as may be possible there. Observe, then, whether it is impossible. It is possible here that all the remainders left over complete the bookkeeping. Examine this carefully, then: this—the first of these residues, the first remainders—two species…

The Fourth Species — מין הרביעי

Now observe: if you wish to use this together with the second: halve the divisor into the wholes [the integer part], less by a hundred, and over it there comes up 20455'550 [Latin/Hindu-Arabic digits set left-to-right in the Hebrew line; the mark between the two clusters is a separator-apostrophe, not a leading "1"] fourths, which is the divisor. [?]

Halve the divisor into the wholes, the seconds that are in it—18 [י"ח]—into thousands; reckon the residue of the divisor's portions, the divisible portion of the divisor over the wholes: it comes out by division 11 [י"א] thousands, 284 [רפ"ד], and there remains also a residue of the divisor's residue, 17 [י"ז] thousands. For they are exhausted; and since the divisor is likewise in fourths, while the divisor is in seconds, and their residue is the residue of the divisor from the residues of the divisible number, there remains the residue of the seconds. [?]

By this division, reckon the residue: the seconds, divided over 60 [ס'], come out—168 [קס"ח] firsts—and there remain 4 [ד'] seconds; and we kept one of the seconds, 1 [א']. Also the portions—200 [ר'] firsts—come out over 60 [ס'], [ג'] 3 of the degrees come out, and there remain 8 [ח'] firsts; and against the second order there had already been 4 [ד'] seconds, so there are two orders. We have in hand 3 [ג'] degrees, 8 [ח'] firsts, 4 [ד'] seconds.

We reckoned the residue of our halving, for which the divisor is in our hand—from the number 17 [י"ז] thousands; we set it right with 1 [א'], and over it there comes up, with 60 [ס'], ['א] 1—these 17 [י"ז] thousands—as firsts; and over them 60 [ס'], the fifths—and these are fourths, according to what had been firsts; and we set them together over 60 [ס'] and set them in a single decade; one more. By this opening, from a counted portion of the parts over division—the divisor, which is 18 [י"ח] thousands—we reckoned, and there comes out to us by division 54 [נ"ד] thirds, together with 9 [ט'], because the fifth order divides the fifths and the divisor is in seconds; and as their residue—4 [ד'] seconds—and the residue of the seconds is in the fifths, there shall remain…

the residue of the thirds, and we kept them. Again we reckoned for the fourth: there remains of the species of the second division, which is 210 [ר"י] thousands, 605 [תרג"ב]; and over them another residue, so that within them is the residue of the divisor, which is the fifth order—the seconds—in division with 60 [ס'], and over it there comes up 408 [ת"ח], [ע"ט] 79 thousands, 102 [ק"ב]; and over them 60 [ס'] sixths divided over the decade of the divisor, it comes out to us—1 [א'] thousand, 18 [י"ח]—as the quotient 46 [מ"ו]; and them over 60 [ס'] fourths, because the number divides the order in the sixths while the divisor is in the seconds. [?]

And when we reckon the residue of the seconds, from the sixths there shall remain the residue of the fourths, and we kept them. Again we reckoned for the fourth: there remains of the species of the third division, which is nine thousand [תשעה אלפים]; and over them another residue, so that within them is the residue of the divisor, which is 6 [ו'] thousands, in division with 60 [ס'], and over it there comes up 542 [שמ"ב] thousands, 302 [ש"ב]; and them over 60 [ס'] divided over the decade as quotient—the divisor, which is 8 [ח'] thousands—and we reckoned, and there comes out to us by division, opposite the 30 [ל'] fifths, because the number divides—which is the seventh order—while the divisor is in seconds. [?]

And when we reckon the residue of the seconds, from the sevenths there shall remain the residue of the fifths; and the residues of the fourths we also kept, for they are kept in safe-keeping. We have in hand 3 [ג'] degrees, 8 [ח'] firsts, 4 [ד'] seconds, 58 [נ"ח] thirds, 48 [מ"ח] fourths, 30 [ל'] fifths—and this is itself the result, according to the first reckoning we set out from. [So the half is 3° 8′ 4″ 58‴ 48⁗ 30^{5}.]

By this method you obtain the result of the reckoning, until you complete the whole division—if a residue should come, whether it can be approximated but no further, so near that your requirement is met in the residues; and the divisions approximate to the decade, and a residue may reach its completion.

Part Three — the Third Part of this Compendium [Kitsur], on the Discovery of Roots — חלק שלישי זה הקצור הוא מהמציאות השרשים

[Large initial] **Know** [דע] — that numbers are divided into two parts: one [reckoned] in itself, and the other whose fractions are derived [from it]. A like-number is a number from the rank of comparables—a number that, of itself, in its own right, is fit to be reckoned—such as the number 9 [ט'], which is itself a third of three [taken] in itself, or [is] a number in its own right. As for the derived numbers, they are drawn from proportional numbers, proportioned in themselves, as in our own case—such as 13 [י"ג]; for from them there proceed the numbers called "derived," which have no roots among the wholes in their own right. And I have distinguished [the matter of] the derived number.

[A derived number] is a number whose beginning is something other than a number [arising] from the product of some whole number by itself—such as the number 8 [ח']; for there is found a number, in general, that, when squared upon itself, is called a number of the ranks of the proportional numbers, whose beginnings are according to their squaring: from the first [order] are their beginnings—for example, as in our case—but it exists only as a derived number, a number from the reverse [side] of the product, 22 [כ"ב] with...

all of that, with two orders—and they are distinct; and so, when there is a reverse number, then the number divides the order. You are obliged to know the two derivations [the two orders of derivation]—a sowing [seed]—and the middle figures, spoken of among the masters of the third [ranks] above, of those who stand in relation, in order to make known the number, [its] reckoning hidden within the squarings: the two productions of the root—the number conjoined from its components, in the figure of the sides of the squares.

Now, then: if it is required of you to make this known, by way of the extraction of the roots [תשרשים] [recte: השרשים]...

the suitable [roots] for the related numbers (המספרים הנגררים) and the multiplied numbers (הנשנשים) [recte: הנשנים?], which are near to numbers without [an exact] root. [?]

Know that the first ones, when you place them in the knowledge of this kind [of operation] — and that is the way the number is ordered (שנגסרר) [recte: שנסדר] — when you require a root, take one step (דרגה) at a time. [?] When you require the root of one rank (מדרגה), one alone — then this is a sought root (שורש מבוקש). If there is an additional [rank] near to it, then when you reconsider it among the others… [?]

If there is a number of ranks (מדרגות) separated apart, then the root sought is the last [rank] joined (מחוברת) with the sums (הסכומת) that remain to it; or, if there is an additional [rank] near, when you reconsider the last rank with respect to the tens (לעשרות) and the sums (הסכומת) that remain to it among the others — if it is a number of ranks (מדרגות), a pair (זוג).

After this, the root, [if it is] of two [ranks]: if there is one, the rank rises (העולה) and is multiplied by others. Insert it (בהתבחו) [recte: בהתבכו?] under the rank sought. And if there is one rank rising, and the whole [value] is multiplied by ten under one root, and beneath one rank (תחת מדרגה) the sums are written; and if there were tens [in the] rank — that which rises is written after [the rank] beneath the rank, the sums, and the tens beneath the root multiplied. [?]

After this, request a number whose product (מהבאתו) [arises] from multiplying the rising [value] by itself, [a number] which is subtracted (שיחוסר) from the numbers of the upper row (טור העליון), from the numbers of the product. And the [number] of the ranks corresponding to the numbers of the product — and the [number] corresponding to the number that is sought, which is the desired product (המופך) [recte: המוכפל], by itself: every one [of them] which corresponds in [the] ranks of the upper row (הטור העליון), and the number that is left over (הנשאר) — examine the third rank (המדרגה השלישית) from the place of the first root, and consider (ויקרא) the second root (השורש השני).

After this, multiply the second root by [the value] examined just as [for] the first root, and in the [text]…

the rising [value] of the product — one [step] beneath the rank, the sums, for the root, the second [root], if there is one. Either beneath one [rank] or beneath the second root, if there are tens beneath the rank, the rank, [its] sums are recounted (סיפרא) [?] or others beneath the rank, the sums, from the root, and beneath the tens of the ranks if there is one — then there are second tens [that] remain, [and they] are left over. And you have shown by multiplying the first root upon (אל) the rank of the sums to it. And if it should happen that the second root [is] from the multiplied [value] beneath the rank, the sums — the second root, examine its companion (חברתו) by itself. [?]

After this, request a number which [, when] it adds up (שיספיק) [for] the rising [value] from its product (מהבאתו) — that is, all the numbers from the part of the first root, and from its second product (תשני) [recte: השני] by itself, which is subtracted from the numbers of the upper row corresponding to the ranks of the upper row, [which are] of equal magnitude (שכנגד) [to] the numbers of the doubled (הכפל) — and a number corresponding to the requested number, which is each [number] that corresponds in [the] ranks of the upper row. And the number — let it be written, [its] product (חמכופש) [recte: המכופל], beneath the rank, beneath the [third?] rank — the sums [are] placed [in] the third rank, the second root. And consider that the third root that remains by [the] same path [is] always preserved (שמיר) [recte: שמור?], until you complete [it]. As I have placed the ranks of the upper row, which is the upper row, recount its roots (חדרגש שרשו) [recte: דרוש שרשו — "seek its root"]. [?]

After this, here is everything in [the value of?] the perfect-squares (החשנשים) [recte: המרובעים?], and their order (וסדרם) in [the] row, after [the way] I placed the foot of the rank of the rising [value] in the upper row, which is the root, the number [recte: sought]. [?]

But if a number is left over (נשאר) [in] the upper row, it shall be a root [recte: with the root] approximate (השורש הקרוב) [recte: בקירוב — "approximately"]. And if it [should] happen, if you desire (תרצה) to draw the matter still nearer (להקריבו יותר), what is between the upper row… [I have related/composed?] that this is the foundation (יסוד) — for [the row] is full of [significant] figures (ספרות), upon the upper row [in] that which you draw near. And every [thing] which is full of figures…

by significant figures (בריבוי הספרות) [recte: ברבוי?] you will add to the precision (תוסיף בדקדוק) in the extraction of the root (בקרקוס השרש) [recte: בחקירת השרש], and in [that] alone — [a precision] that will exist [as] doubt and error with the path [of] those sums always, until the last root reaches the final account (אל הספרא האחרונה).

After this, take everything that is among the roots (המשרשים) [recte: המשורשים] and their order (וסדרם) in [the] row, after [the way] I have laid out its ranks. This is [the rule] like this matter. After this, your path: from the ranks of the first row, [take] the number of ranks [equal to] half [of the] figures (חצי הספרות), and that which remains of the ranks of the first row [is] in [the] number of ranks [equal to] half the figures (חצי הספירות) [?], and that which remains of the latter ranks (המדרגות האחרונות) which are preserved (שמורם) [recte: שמורים]; and them [, the] degrees (מעלות). After this, [there is] yet a product (חבח) [recte: כפל/הכאה] of the ranks of the firsts (תחנשלבות) [?] with these [, the] preserved (חס) [?]; and [for] the rising [value] (וחעילה) [recte: והעולה] — recount it (חש) [recte: דרוש?] from the ranks of the first row, [in the] number of [the] ranks [equal to] half [of the] figures, and that which remains of the latter ranks shall be preserved (שמרם). And them, [the] seconds (שניים) — and among them are the firsts (ראשונים). [?]

Yet another product of this: [take] the ranks completed (נשלבות) [recte: נשלמות] with the [value] rising (העולה), and the rising [value] in [the] number of ranks [equal to] half the figures, and that which remains of the latter ranks [is] preserved (שמרם), and them are the seconds (שניים). And among them are seconds (שניים), until that which comes to all the ranks completed (נשלבות) [recte: נשלמות] without [remaining] figures (אל ספרות) [recte: בלא ספרות].

After this, take all that is preserved (השמורים) which are among them [, the] degrees (מעלות), and the firsts (הראשונים), and the seconds (השניים), and the thirds (השלישיים), and the fourths (הרביעיים), and seek (ודרוש) a root [for] the dispersed [value] (המרושרס) [recte: המרוסס/המפוזר] from the number, without (כלכתי) [recte: בלתי] a related [root] (נגרר).

First example (משל ראשון)

An example of the related number (המספר הנגרר), [for] which you seek its roots, which is…

this. And because of the ranks — this row [is] indeed related (נגרר), you are obligated to request two roots [for] the latter ranks (המדרגות האחרונות), which are sixty (ששים), and after [that] their root is seven (7 = ז). And place its product (הביגנוחו) [recte: הכאתו — "you multiply it"] by itself, and upon [it]…

[The worked computation is set as a right-hand numeral column, ruled (a subtraction rule runs beneath the sixth entry). Read top to bottom:]

Order (top → bottom)	Value
1	2
2	16
3	13712
4	48247
5	12457612

Order (top → bottom)	Value
6	6169002849 *(ruled underneath)*
7	78543

forty-nine (מ"ט) [recte: 49], subtract from those (מהם) one (אחד) [recte: one digit], and that which remains (ונשאר) [is] twelve (י"ב) [recte: 12]. And in writing the [numbers] above (ובהתכנים) [recte: ובכתבם] for the [purpose of] subtracting (לחורות) [recte: לחסר] upon those left over (על הנשארים) in [the] halves (בפלגי) — that is, the rising [value] (העולה) [is] fourteen (14 = י"ד). And in writing them (בכתבנים) [recte: בכתבם] one (חד) beneath one (תחת חד), and this... in the ranks the sums [are placed] for the root.

[Second numeral cluster, set off to the right beneath the prose:]

Order (top → bottom)	Value
1	4608
2	1155770
3	115

After this, request a number which adds up (שיספיק) for the rising [value] from its product (מהבאתו) [together] with [its] halves (בפלגי) — the root and its product (ומהבאתו) [recte: והכאתו] by itself, that which is subtracted (שיחוסר) from the ranks of the upper row, of equal magnitude (שכנגד) to the doubled (הכפלים) which are brought (המובאים), each one of the ranks that are [of] equal magnitude (שכנגדה), and that is one (א).

And in writing it (ובהתבנוחו) [recte: ובכתבו], in the third rank (במדרגה השלישית), the sums [are] preserved (השמורות) for the root. And gather them (וקבץ אותם), [the] second root (שורש שני), [from] its product (הביגנוחו) [recte: הכאתו] by itself with that which arises (עם הבא) — [the value] that arises from the path [of] the first root, and upon it (ועלי) [numeral: ?]; subtract (חסר) from them (מהם) [those] which are doubled, and that which is left over (ונשארו) [is] written (ונכתבם) for [the] degrees (למעלות) by the rule (כמשפט).

[Then:] And its product (הביגנוחו) [recte: הכאתו] [together] with three (3 = ג'), and upon it (ועלי) thirty-two (32 = ל"ב) — those subtracted (הנחסרים) [are] doubled from the upper row (העליון), and that which remains (ונשארו) [is] fourteen (= י"ד 14), and in writing them (ובכתבם) for the degree (למעלה) by the rule (במשפט).

Again, place its product (הביגנו) [recte: הכאתו] by itself, and upon it (ועלי) twenty (20 = כ') subtracted (הנחסרים) from the numbers (ממספרי) of the upper row (טור

העליון), and that which remains (ונשאר) here (פה), and in writing them (ובכתבם) for the degree (למעלה) by the rule (במשפט).

Again, double it (כפלנו) one (חד), by itself (בעצמו), and upon it (ועליו) fourteen (= י"ד 14) in the ranks the sum (הסכומה) [by] the second (בשני), and one (וחד) that is (שהוא) the difference (תמורה)… [?]

Kitsur ha-Melakhat ha-Mispar — Part Three: Extraction of Roots (continued) …the opening that we set out, [namely] that the remainders [left over] when you square the [approximate] root [must] be placed under the ranks [מדרגות] from which they arose. Whatever is left over from them goes into [the next] rank below, since we are obliged to [carry it down].

So too, when the ranks of the first root lie under [those of] the second root, and there [the carried figure] meets with a perfect square, place it under [its rank]. Square the [remaining] value and add it to its companions [the partial products], and [continue].

Likewise, if you have left over from the first [partial], when you square it, a number which the second rank can accommodate, place it together with all the partials. With its own [square], whatever falls short [is carried] from the upper register [טור העליון] of the ranks belonging to the lower partials, which [now] meet up with the [lower] partials, [as you] continue.

Whatever is left over from the third rank when you square the third root, and the third [root] arises [paired] with the fourth root — proceed.

[Worked steps follow; the values are remainders and doublings from the running computation.]

With one [unit] left over under [its] rank, subtract from the upper register [טור העליון] and [there remains] 5; and 5 remains, [together with] 3; and the [remainders carried over] are 3 and 2 [?], and write [them] above [the rank], according to the rule [כמשפט].

Square them again [עוד הכינוחו]: with one [unit] that falls below [its] rank, and 5 remains, [under] the 2 deficits [חסרונים] that arose, [proceed]; from the upper register [טור העליון], with the ranks summed [together] [חמחושבת = חמשבת, the computed (sum)], and there remains [the figure], and write [it] above the rank, according to the rule.

Square them again [עוד הכינוחו] with their own [value]: with the deficit subtracted from the upper register, and there arise 30 deficits [?] — these [are] kept [reserved] for the value of this rank, 100 [?] — and write [them] above the rank, according to the rule.

Square them again [עוד הכינוחו] with their own [value]: there arise 28 [?] deficits, from 30 [?] of the deficits kept [reserved] for the value of this rank — 700 [= תש 700] — and write [them] above the rank, according to the rule.

according to the rule, each one to the place that is fitting for it.

Afterward, square [the divided portion]: it is the one that is under the third root, and there arise 10; and write [it] above [the rank], in the rank from which it began. Subtract; with what remains [carried] from the first root, [add] the other value [?]. Place it at the rank from which it [arose], and let it be set there. After [this], take all the remainders, return [them], and put each one in its proper place — for what meets with a perfect square, place it; square that value, [pairing] it with what [remains].

The third [root], being [taken] from the latter [register], is squared [מכפלו] [paired] with its companions, and there arise 7; and 6 [?]. We computed [further] from the lower partials [the figures]. After this, when we [asked] whether the number suffices [to complete] the rise [of the root] together with all the partials — and with their own [value], for whatever falls short [is supplied] from their ranks — [we found that the figure] arose [paired] with the upper register, which meets [it]. And so [the situation] of the fourth root, as the rule [prescribes]; and we found a remainder of 200 [ר = 200]; and we squared it, [paired] with the third rank, [the values being] computed [as ranks/figures]. The third root, and what we read [for] the fourth root — [proceed].

We squared it [הכינוחו] with the one that falls below [its] rank, and there arose 4 of the deficits [?] that were squared [shifted] into the upper register; and there remained 2; and we set it down [in its place], according to the rule. Square it again [עוד הכינוחו] with 8 of the doubling [partial], and there arose 2 [?] of [the deficits] that were squared [shifted] into the upper register; and there remained 7; and we set it down, according to the rule. Square it again [עוד הכינוחו] with 7 of the doubling [partial], and there arose [a figure] from what was squared [shifted] into the upper register; and there remained 47 [47 = מז]; and write [them], according to the rule.

Square it again [?] [עוד דלגנו, "again we skipped/passed over"]; we squared it with its own [value], and there arose 16 [16 = יו] of the deficits, [taken] from 28 [?] [מכ"ח] that were [shifted] into the upper register; and there remained 12 [= י"ב 12]; and write [them] above the rank, according to the rule.

After this, [for] the divided portion — it is the one that is the fourth root — and there arose 8 [?]; and write [them] in the rank that arises from [pairing with] the [fourth] root. Square it again [התקנו]: the [lower] partials…

[pair with] the first partials, each one [going] to the rank from which it [arose], when you square the first partials [הבפלי הראשון]. After this, when we asked whether the number suffices [to complete] the rise [of the root] together with all the partials — and with their own [value], for whatever falls short [is taken] from the ranks of the upper register, which meets [them] — [we proceed]. The fifth root, which is the third figure [in the count], and what we read [for] the [pairing of] the fourth and fifth ranks of the roots — and we [read] the fifth root.

We squared it [paired] with what arose; and there arose 3 of the deficits [taken] from the fourth root; and there remained [a figure]; and we set it down [in its place], according to the rule.

Square it again [עוד הכינוחו] with 7 [of the partial]; and there arose 0 [בא = "there came," i.e. it came out even — no deficits], for the [computation] from the upper register, with nothing remaining; and 30 [?] [ולבן = ולבל? — "and to none," i.e. nothing] remains. So we wrote above them [in] the columns [ספרא] [the symbol] for "ranks that have nothing remaining," that is, [a sign of] nothing. [עוד דלגא ספרא = "again we skipped a column," writing a zero-cipher.] When you square it [הכפל], with one [unit], and there arise 0 [?] deficits, [taken] from 12 [מי"ב = from 12] that were [shifted] into the upper register; and nothing remains; and write [it down as] ciphers, [marking] the value of the ranks that have nothing remaining — that is, [a sign of] nothing.

Square it again [עוד הכינוחו] with their own [value], and there arise 9 [9 = ט] deficits, [the residue] from what was [shifted] into the upper register there [שב]; [it] arose [paired] with the root; and there remains nothing; and to none [ולבן =

nothing] [remains] when you write [it] down. [It] arose [as] a column [ספרא], according to the rule, [to mark] ranks that have nothing remaining upon them — [a sign of] nothing.

And so, since [the residues] have already reached the rank of the first roots [המדרגה הראשונת], and since [their] value has been completed in our hands [i.e. the computation is exhausted], and since there remains no further excess [carried over] from the upper register — we know that [the figure] which arose is the number we sought, [namely] the root.

After this, to total all our [tabulated] numbers [מספרי] and ranks [שרשי], and the additions [וסכרגום = the accumulated sums], arrange each one according to its [proper] rank.

A Second Method [דרך שני]

Now, I have already explained one method above; [here is] another method, more concise. [In it] you arrange the [tabulated] roots, [working] in the manner of [the procedure for the] number that is broken up [נסתבר = transmitted/laid out] — that is, the procedure of multiplication [הכאה]. [You begin] with the [highest] register, in the manner of [setting out] the other [tabulated] ranks; [you proceed] with the ranks of the remainders [מרגות = מדרגות], the rest [ר"ל = "that is to say"], the latter [ranks]; and [with] the tenths [התעשרות = the sexagesimal sub-ranks], in their ranks, each one [placed] according to its place.

If the number of ranks [in the radicand] is even, begin from the latter [last] rank. If it is odd [זוג, "even" — but here read as a pair/odd], begin from the [next] latter rank, joined [מחוברת] to the [adjacent] one — [namely] the last rank — [pairing them] when you proceed. The last rank [is taken together] with the tens [העשרות], and the remainders [are arranged] for the others, as we explained [כמו שכבר].

Now, you ask for the roots [שרשי], that is, the latter [ranks], or you sought [whether] the rank can accommodate [a root-digit] — and you read [off] the first root. After [reading] this, ask [whether] the number suffices [to complete] the rise [of the root], [taking it] together with the root, twice over [פעמים], and with its own [value]. After this, whatever falls short [is supplied] from the rank of the first root; and what is left over from [it goes to] the rank of the second root; you read [off] the second root, and write [it] in the rank from which it [arose], [paired] with the first root.

After [reading] this, ask [whether] the number suffices [to complete] the rise [of the root] from the [highest] register, [taking it] together with the first root and the second, twice over [פעמים], and with its own [value] one time [further]. After this, whatever falls short [is supplied] from the rank of the roots, [taking] the second and third together; and we [shift] into the upper register [for] the deficits [מסורמת = computed?] — [you read off] the third root, and write [it] in the rank from which it [arose], [paired] with the second root.

After [reading] this, ask [whether] the number suffices [to complete] the rise…

[The passage breaks off mid-sentence at the foot of p. 322.]

Kitsur ha-Melakhat ha-Mispar — Part Three: Extraction of Roots (continued)

The Second Method of root-extraction (continued)

[?] its product with the first root, and with the second, and with the third — two times each — and with itself one time, another rank. Therefore you write the products on the rank below the third root [למדרגת השורש]. And from the product of the third [root], the products written [המסורגת] are called the [third] root; and below it you write the products of the roots, and so on until you reach what is below [the lowest rank]. Continue to verify until you reach the opening rank; and then, when the products array (המסורגת) is exhausted [המבוסרת/הנגסרת] [?] at the first rank, the digits [המספרי, recte המספרים] written beneath the ranks will be the root itself. ✻

The Worked Example (המשל)

Suppose you want to find [recte לדעת] the root of "five times a thousand-thousand, and 909[?] thousand, and 28" [ה' פעמים אלף אלפים ותתט"ט(?) אלף וכ"ח — the printed number reads roughly 5,909,028; but cf. the tableau on p. 326, whose radicand is 5,499,02⅖ and whose root is 2345; the middle figure is degraded — flag the radicand]. And note, since the number whose root [is sought] [שמספר המסורגת זה] is the last [highest] rank, this number is to be analyzed [נפרך] starting from the last products array [מהמסורגת האחרונה]. We seek its near [whole] root [שרשו הקרוב], and consider [תבונן] [ב'] 2 with itself, one time; its root being near, there remain 4 [ד/(?)'ר'] from the products of the last array, which we placed [as] 8 [ח'], and we wrote it below the rank in the proper order [כמשפט]. And this rank — that is, what the first root [is] in its arrangement [בתכבנותו, recte בתכונתו] beneath the last rank.

The rank below the other [array]: here, from a number that suffices [שיכפיס, recte שיספיק] to bring up [להעלות] its product with the second root, one time, and with itself one time, since it is to be diminished [שיחוסר] from the products of the rank set out [המסורגת] for the rank of the root; and that [residue] for the first root which is 3 [הראשון, שהם ג'(?)] — that they are 3. And from the products [ומהמסורגת] set out for it, which is 49 [שהם מ"ט], with its product [וחו"א, recte והוא] with itself 3 [ג'] beneath it [בתכבנותו, recte בתכונתו] — that [it is set] beneath the last rank, which is the rank of the products set out for the first root; and it is called [ויקרא] the second root. The second [root] with its product with itself, and upon it [the number of] deficits…

what its products are, and there remains 2 [ב'], and you write them in the proper place [על מק', recte על מקומו]. ✻

Again, consider 3 [הג'] with itself, and there remain 9 [ט'] deficits [חסרונים] from [the array] — that is the products array [המסורגת] of the rank set out for the second root; and there does not remain [ולא נשאר] anything, so you write upon it a cipher [סיפרא — the zero-mark]. After this, from a number that suffices [שיכפיס, recte שיספיק] to bring up [מהבאתו] its product with the first and second roots, two times each, and with itself one time, [that is] what is diminished [שיחוסר] from the products array set out [המסורגת] for the second root. And from the products [ומהמסורגת] set out for it, which is the number 4 [recte והוא מספר ד'], you write them in its arrangement [ובתכבנותו, recte ובתכונתו] in the rank of the products set out for the second root, and it is called [ויקרא] the third root. [Consider] its arrangement [בתכבנותו, recte בתכונתו] with its product with itself, which is 2 [שהם חב', recte ב'] — that it is [set] beneath the rank set out [for the root]; and to be [ולהיות] [י"ו] 16, that there remain [שנשארו] two [ב'] in value [בערך] from the array, [namely] the products array set out [המסורגת], [there are] 2 [ב'] deficits from them, and there remains 4 [ד'], and you write them in its arrangement [ובתכבנותו] in the proper manner [במשפט]. ✻

Again, consider [עוד הבן] 1 [חד', recte א'] with the second root, 2 [ב'] times, and upon it 2 [ב'] deficits [חסרונים], [namely] 4 [ר'/ד'] — that is the products array, the second [השנית], set out for the rank of the products array set out [המסורגת] for the rank set out for the second root [הג', recte the second]; and one [וחד', recte וא'] that is opposite [שבנגד] the root 3 [השורש הג'] — that they are 49 [שהם מ"ט]; and there remains 2 [ב'], and you write upon it in its arrangement [ובתכבנו, recte ובתכונתו] to instruct [להורות] upon what remains. ✻

Again, consider another [עוד אחר תבונן] 1 [חד', recte א'] with itself one time, and upon it 16 [י"ו(?)] deficits [חסרונים — flag the value] that are in value [בערך] upon the products array [המסורגת] set out [המסורמת, recte המסורגת] for 3 [ה־ג'], and there remains 35 [ל"ה], and you write them in its arrangement [ובתכבנום, recte ובתכונה] in the proper manner [במשפט] to instruct [להורות] upon what remains. After this, [from] a number that suffices [שיכפיס, recte שיספיק] to bring up its product with the first, second, and third roots, one time each, since it is to be diminished [שיחוסר] from the products array set out [המסורגת] for the root…

for the first root; and from the third products array [ומהשלש מסורגת, recte ומהשלישי] set out for the root [recte והוא], it is the number 8 [ח'], and you write them in its arrangement [ובתכבנותו, recte ובתכונתו] in the rank of the products array set out for the third root, and it is called [ויקרא] the third root. ✻ Consider it [recte תבונן] with the first root, 2 [ב'] times, and upon it deficits [חסרונים], [the residue] upon the root 3 [שהם על הג', recte שעל], [(?)ד'] 4. [Consider] the root 1 [חד', recte א'] again [עוד] with 2 [חב', recte ב'] — that [is] the products array [שבמרורגת, recte שבמסורגת] that is drawn [הנמשבת, recte הנמשכת] for [the rank], that they remain [שהם] [כ"ג] 23, and there remains 3 [ג'], and you write them above [למעלה] in the proper manner [במשפט]. ✻

Again, consider [it] with the second root, 2 [ב'] times, and upon it 30 [ל'] and grains [וגרעינים — residual units/"seeds"], [namely the residue] of what is in the products array [שבמרורגת, recte שבמסורגת] set out for the rank set out [למורגת, recte למסורגת] for the [second] root 1 [חד', recte א'] with 2 [עזר חג', recte ב'] that is in the products array [שבמרורגת] that is drawn [הנמשבת, recte הנמשכת] for [the rank], that they remain 35 [ל"ה], and there remains 3 [ג'(?)], and you write them in the proper manner [במשפט]. ✻

Again, consider it [עוד הבינוהו] with the [third] root 3 [הג'], [ב'] 2 times, and upon it grains [גרעינים] [the residue, recte מהן] that there is in the products array [המרורגת, recte המסורגת] 3 [חג', recte ג'] set out [המסורמת, recte המסורגת] for the rank of the root 1 [חר', recte א'], with the residue [העזר] 1 [חר', recte א'] that they are in the products array [שבמרורגת] that is drawn [הנמשבת] for it, that they remain 42 [מ"ב], and there remains 2 [חב', recte ב'], and you write them above in the proper manner [למעלה במשפט]. ✻

Again, consider it [עוד הבינוהו] with itself one time, [taking it] alone [לבר, recte לבד], and upon it deficits [חסרונים] [the residue, recte בה] that there is the products array set out [המסורמת, recte המסורגת] for the rank of the products array [למורגת, recte למסורגת] set out for the [first] root 1 [חר', recte א'] with the residue 2 [עזר חב', recte ב'] that there is in the products array [שבמרורגת] that is drawn [הנמשבת] for it, that they remain [?], and there does not remain anything [ולא נשאר כלום]; and since [ולהיות שכבר] the residues [הגרעינים] are now insufficient to diminish [להגענו לחסר, recte הגענו לחסר] from the first products array [מהמסורגת הראשונה], for that reason [על כן] we knew [ידענו, recte ירענו] that the root [השורש] is already complete [שכבר גשלם, recte שלם]. ✻

And so that there should not remain any residual number at all [ולהיות שלא נשאר מספר בכלל] [in/of] the demanded number [על המספר הדרוש], we know that it is a square number [מספר מרובע], and its root [ושרשו] is the number written above [הכתוב למטרה, recte למעלה], which is "2 thousand[s]" [שהם ב' אלפים] there [שמה]. [The prose names the root as "2 thousand[s]" — i.e. the leading 2-thousands rank of the full root 2345 set out in the tableau on p. 326; transcribed as printed.]

Worked-computation layout. A four-line arithmetic tableau set at the head of the page in European (Hindu-Arabic) numerals, laying out the root-extraction just completed in prose. Read top to bottom; the two lower rows are each set under a horizontal rule. Verified under magnification.

Row	Value
Line 1 (top)	2 3
Line 2	1 2 4 5 4
Line 3 (rule above)	5 4 9 9 0 2 ⅖
Line 4 (rule above)	2 3 4 5

The Method for [extracting the root of] a number without a [whole-number] root (הדרך המספר בלת נגדר)

[Body heading p. 326: "הדרך המספר בלת נגדר" — "The Method of [a] number lacking a [perfect] root"]

You should know that if, after extracting the root of a number, there remains some number that has no [whole] root [בלתי נגדר] — first divide [תחלק] by [some divisor, recte בע…], because the number whose root [we seek] has no perfect root [בלתי נגדר]; and the way [והדרך] to know its root [שרשו] approximately [קרוב, "near"] is this: you add [שתוסיף] to the number whose root [we seek] [some] ciphers [סיפרות — appended zeros], so that you increase it [שתרצה] and conduct [ותנהיג] it in this manner [בזה] until you reach the [point of] deficit [לחסר] from the products array [מהמסורגת] again. After this you bring out [תוציא] the result [היוצא] by subtraction [בחס', recte בחסור], and continue forever [ולעולם] until you reach the cipher [סיפרא] at the head of [בראש] the upper register [הטור העליון] — [casting out] half [חצי] the appended ciphers [הסיפרות הנוספות]. And the value that comes up [ותעלה] will be like the number of times [כמספר הפעמים] reckoned

[שחושבה] — that is to say [ר"ל]: if you appended one time [פעם אחת], these are firsts [ראשונים]; and if two — they are seconds [שניים]; and if 3 — they are thirds [שלישיים]; and if 4 — they are fourths [רביעיים]. ✻

Afterwards, you cast out [תשלך, recte תשליך] from the number that comes up [העולה] the ciphers [הסיפרות], and the remaining ones [והנשארים] are sexagesimal parts [חלקים] — [obtained] upon the subtraction [על חס', recte על חסור] — and there comes out one rank [ותיצא מ(ע)לה אחת, recte מעלה — "one degree"] from the products array [מהמסורגת], divided [הנחלקים]; that is to say [ר"ל], if the parts were fourths [רביעיים], the result of the division [היוצא בחלוק] would be thirds [שלישיים]; and the remaining ones, the [parts] not divided [הבלתי נחלקים], will be fourths [רביעיים] — and keep them [ושמרם, recte ושומרם]. ✻ Again, [if] a part [חלק] of those that came out [היוצאים] is divisible [מתחלק], so that they are thirds [שלישיים] over 60 [על ס'], then those coming out of the division [והיוצאים מחלוק] will be...

seconds [שניים]

seconds, and I divided them by sixty; the result is thirds, and you keep them. The remaining part of those extracted are the two-thirds over 60; and what comes out in the division will be firsts, and I divided them by sixty, and the result is seconds, and you keep them. Again, the part of those extracted over 60 — and what comes out in the division will be firsts — and you keep them. After all of this is gathered, the irrational root [תשורש הקרוב] of a number lacking a perfect root [למספר בלתי נגדר] comes out.

The Worked Example [המשל]

If you wish to extract the root of the number, proceed by the descending chain of values below. The right-hand column gives the running computation; the Hebrew narration runs in the left-hand column. The figures read top-to-bottom as the successive products of the root-extraction, then the sexagesimal reduction by repeated multiplication by 60.

Upper tableau (the root-extraction figures), reading top to bottom:

Hebrew line label (left column)	Value (right column)
[the trial digit]	5
the root is 2 thousands [שורש ב' אלפים], here are [these]	383
many thousands, according to	114272

Hebrew line label (left column)	Value (right column)
[כב' אלפים לפי]	
the numbers, and we added upon it [המספרים ונוסית עליו]	2323136
eight digits of multiplication [שמנה ספרות פרה]	2689582573
we have led them along the row of sources [הנהגנו הדרך מקורם]	48441266941
and this number came out [ויצא זה המספר]	266000000000
	447213
	60

Lower part — the sexagesimal reduction. The running residue 447213 is repeatedly multiplied by 60, each product divided out into the next sexagesimal rank:

Operation (Hebrew, left column)	Working value (right column)
...447213, with [the residue] [הביגום עם 447213]	—
and this number came up [וחס' ועלה זה המספר]	26832780
...26832780, and divide [ותחם]	—
firsts [ראשונים]. We divided [the result] with... [הביגו עם]	1609966800
...and this number came up [ס' ועלה זה המספר]	96598008000
...1609966800, and divide [ותחם]	—
seconds [שניים]. We divided [הביגום עם]	5795830480000
...and this number came up [ס' ועלה זה המספר] 96598008000, and divide [ותחם]	[the result continues on p. 328]
thirds [שלישיים]	

thirds [שלישיים], divided out with 60, and the number comes up: **5795830480000**. And divide them, and from them come fourths [רביעיים]; from them four [ד'] digits [ספרות], for what came out of them is four. And to be a number of the same order, there are in it four digits, and at the head of the column [בראש הטור] there is half a digit [חצי ספרה] in addition; the divided number comes out: **96594400**. From them, thirds [שלישיים], for they ascend by one rank [מדרגה אחת], and what remains is **48** — the fourths [רביעיים] — and we have kept them.

Again, the divided-out [residue] over 60: the divided number comes out **1609966**. From them, two [ב'] seconds [שניים], for they ascend by one rank, and what is left over is **2** thirds [שלישיים], and we have kept them.

Again we divided out what came out, divided over 60: the divided number comes out **3683**. From them, **2** firsts [ראשונים], for they ascend by one rank, and what is left over is **16** seconds [י"ו שניים], and we have kept them.

Again we divided out what came out, divided over 60: the divided number came out **44** degrees [מ"ד מעלות], for they ascend by one rank, and what is left over is **43** firsts [מ"ג ראשונים], and we have kept them.

So we write down all that we have kept — the degrees and the rest: the degrees are **44**, the firsts **43**, the seconds **16**, the thirds **2**, the fourths **48** [מ"ג ראשונים י"ו שניים ב' שלישיים מ"ח רביעיים]; and this is the irrational root [השורש הקרוב] of **2 thousands** [לב' אלפים] — that is, of 2000.

The Common Fractions [המאזנים]

The common fractions [המאזנים] — that is, that this kind is like a "Schnabel" [כשנאב"ר] [?] — its root is in itself; and if the question is posed for the upper figure to be doubled the value over [the lower], [you proceed] as with whole numbers [שלמים]; and if not, then with appended fractions [במספרים הגרורים]...

the appended [fractions] [הגרורים]. And likewise with whole numbers I extracted the appended [fractions]. We said that the root is taken by the result of the source [מתצאת השורש] in itself, with the remainder [המותר]; and if you ascend [ותעולה — *recte* ותעלה] to the number kept, the upper figure squared — fine [צדקת]; and if not, then check again [בדקת].

The Extraction of the Root in Fractional Numbers [המציאות השורש במספרים השבורים]

After we explained the way of extracting the roots of whole numbers, as with appended [fractional] numbers — and if, with appended fractional numbers, our division of fractions [is] appended — what is required of us is to make known the way in two halves [בשני חלקים], the remaining thirds of the number. Now, if there is from this what is required of us, it is to make known the way of extracting the roots of whole things [שרשי השלמים]. And the extraction of things [ומציאות] —

the root that is appended [שבחר] together with the whole numbers [השלמים], and we say that the extraction of the things that are whole is a way in itself [בעצמו] — that is to say [ר"ל], the point from which their meaning is complete. And likewise, the divisor [מצר] of their multiplication [איבותם] is appended for an operation other [than that]; and that which is found — that the root of the number which is some [other] thing is appended and is derived from it and goes out [נגזר ממנו ויוצא] to a root — there, appended from it, is their multiplication [איבותם].

The First Worked Example [המשל הראשון]

If you wish to extract the root of **60** [חק'] — and the fourths [רביעיים] are found there — we add to it the source numbers [המקורים] in themselves; the root of **60** [חק'] is **8** [ח'], and [the operation continues]:

After this we seek there a root, **60** [תר'], which there in the fourths [הרביעיים] is derived [נגזר] from its multiplication [ממכפלו]; and we found that its root is **2** [שנים], for its root is derived from the [number] **2**, that is, half [of it], and we knew that the kept ones are halves [חציים]; and by the result we knew that they are fourths [רביעיים]. After this we knew that **5** [ה'] is derived from a root, behold, there in the **fourths** they are **2** halves [ב' חצאים].

After this, if you wish to bring them back [להשיבם] to whole numbers [השלמים], a half-part over the number of the seconds [השנים] which is their multiplication, derived [איבותם נגזר] from its root, and the result [והעולה] is **5** [ה']; and by this we knew that the root of **60** [in the fourths] is **5** wholes [ה' שלמים].

The Second Worked Example [משל השני]

If you wish to extract the root of **60** [חק'], [ששה עשורי?] **66** [?] — its accounting [חשבן], we found with the compounds [המרכבים] their forms; that its root, **60** [ששורש חק'], is **6** [ו'] of them, and that the remainder [ושהמותר] is derived. After [there is] a root, behold, taken to a number, **6** [?] there, from which the **60** [degrees] are derived; and we ask its root, and it comes out **four** [ארבעת].

And by this we knew that the kept ones [השמורים] are these fourths [רביעיים], after they are derived from a root once; and by this we knew that the root that came out is **6** [חק'] of them, and **66** there are **4** fourths [ו' רביעיים]. And if you wished to bring them back [להשיבם] to whole numbers [השלמים] — and they are **4** degrees [מ"ר מעלות] which are doubled there, derived from them, and the result [והעולה] is **2** halves [ב' חצי], and **66** [וחצי] [?]; and by this we knew that the root that is derived is **6** [חק'], **66** there are **20** wholes [עשרים שנים] — that is, **22** wholes [נחצי].

These are the two ways [האפנים] that teach us [המודיעים] the extraction of the roots of things [שרשי השברים — *recte* שרשי השלמים?] derived in things whose multiplications are appended; just as their nature is some root, like the fourths [רביעיים], so their nature is a number derived, and their multiplication is some [other thing]; for the truth of the number of the four [הארבעת] is a number derived [נגזר].

Kitsur ha-Melakhat ha-Mispar — Part Three: Extraction of Roots

Roots of mixed (compound) numbers

and likewise the fractions, which when you multiply them together and obtain their product, will be one of two cases: either numbers each of which is a perfect square in itself, or [numbers] such that their product alone [is square] but not each by itself. Thus the squaring required for the task — which is [the product of] the square together with [the square of] each by itself — is itself the [number] whose root we seek, and its root is the root we are after. [?: this clause is densely paratactic; reading approximate]

And we have already explained, regarding mixed numbers, that they too have a root just as the integers and fractions do, when the squaring of their product yields a square. So take heed: look first whether their product is a perfect square in itself. Then consider their squaring further — if it was the true [product] of each by itself, seek the root of their product; but if it was the product alone, then when the [resulting] number is mixed and its root irrational, what comes out [of the extraction] is approximate. After that, we seek the root of their product, and its root will be the root we are after. Take heed.

Example of the first kind [מין הראשון]

The example of the **first kind** [min ha-rishon], which is when [the number] is 4 fifths [ד' חמשיים — i.e. 4 fifth-rank (sexagesimal) places], and we multiply them together with their products, [these] being not irrational; here is the path: by extraction of their root. This is [the case] where one squares [she-nabbach] the number of the five [ha-chamishah] together [?]... by itself, where the fifths [ha-chamishiyim] are extracted [nigrar] from them by themselves. Again one squares [nabbach] a number [?] which... and it comes to **28** [כ"ח]. We took fractions [namit chavarim?] in the five [ba-chamishah] which are [?] their products, and they are [?]... not integers [שלמים]: [ל"ז] **37** firsts [first-rank places], **12** [י"ב] seconds, and these are kept [u-shemarnum]. We take the root [lakchu shoresh] of הב"מ [reading of this composite numeral uncertain] [?], reduced [shachut] [by] five, and we squared it [ve-nirbaʿenu]... ר"ד [reading uncertain] [?]...

that are kept, which are among us [she-be-yadeinu], are the fiftieths [chamishiyim — the sexagesimal places]; and there is no [whole] number, only the fraction of the [base] one remains [raki ha-chomesh] from [the count of] another. And [there are] **12** [י"ב] [places] which... upon **5** [ה'], and there comes out **30** [ל'] wholes [shelemim]... **49** [מ"ט] parts of the whole [chelkei ha-shalem], and another part such that it makes up the [missing] thirds [chelkei] of a whole. [?: a worn sexagesimal accounting of root-remainder ranks; the structure is recovered, several individual values flagged]

Example of the second kind [מין השני]

The example of the **second kind** [min ha-sheni], which is when [the number] is one of two mixed numbers whose product [we have multiplied] alone, and whose root [we have] taken. Right away [tekhef], in this case you will not need [to add] the [cross-term] for any [particular] number; rather, it is enough to take the product of each in itself to be its root. Likewise you will not need to add the half-times [the cross-terms]; you only seek the root nearest to the count, and this number is 4 [ד'], whose root is whole [shalem], the firsts [first-rank places] being 2 [ב'] for the seconds [shanayim]. Take from it the [number], where the fourths [reviʿiyim] are decreed from it, and [there are] 2 [ב'] of them. And we will see that our root [extracts]... whole [shalem], decreed as a half [chatzi]. And if [there are] in it [a value] of 4 [ד'], the [products] come out — and [also] the firsts, and the half of each one; and as for the [count] of two of the seconds, the half [chatzi] being one after another, the parts [chalakim] being upon 2 [ב'], there comes out **7** [ז'] of wholes [shelemim] and a part [chelek], and another part [chelek] of the **49** [מ"ט]

thirds [reading of this run uncertain] of a whole [she-mash chelkei ha-shalem]. [?: heavily worn sexagesimal accounting; the recovered structure is given, individual values flagged]

[New section] — On extracting the roots of [perfect] squares of computation [hatevunah]

[Heading: דֶּרֶךְ הַמְּצִיאַת שָׁרְשֵׁי שְׁבָרִים שֶׁל הַתְּבוּנָה — "The way of extracting the roots of fractions of the computation"]

By this path, then, you shall know how to extract the roots of perfect squares of computed [or: many — תבים] [numbers] in fractional squares [be-shivrei ha-tevunah], or in fractions and integers together likewise.

the path is that when you have multiplied [or: completed], cast away from the number what remains over and above them [the squares], and make use of [the computation] in their irrational way [be-darko ha-sorim — "of those that fall short"].

The example [המשל]

If we wished to know the root of a number — [say] degrees, [with]... firsts [?], **20** [כ'] seconds, **30** [ל'] thirds — then squaring [nafach] this [number] gives: it comes to **a thousand** [אלף], [ר'] **200** firsts; and it comes [further] to a thousand and... together with **60** [ס'], and it comes to **73** [ע"ג]; [now] with 2 [ב'] seconds, and it comes to **73 thousand** [ע"ג אלף]. The squaring [nafach] then, [taken] with [the next rank], comes to **200** [ר'] times a thousand thousand [200 thousand-thousands], **and 400 thousand and 7,000 and 600** [וארבע מאות אלף ושבע אלפים ושש מאות], which are of the thirds [shelishiyim]. We add them together with the [running total of] thirds, and it comes [again] to **200 times a thousand thousand, and 400 thousand, and 7,000, and 600**, of the thirds of [the] third. [?: numeral-dense; the place-magnitudes are now recovered from the tiles — the chain is 200·(1000·1000) + 400,000 + 7,000 + 600]

After that — for the [rank] which is the third of thirds — see, it is the separate [odd] rank [ha-margah ha-nifredet]. The squaring [nafach] gives, with [this rank], and it comes to [a value], and it is a pair [zug], and there is **רס"ר** [reading of this composite uncertain — the page prints both רס"ר and רס"ב on this page] times a thousand [?]; and there are a thousand **16** [י"ו] fourths [reviʿiyim]... thousands and **רס"ב** [reading of the final letter uncertain]. We took half of the four [ranks] in which there are fourths [reviʿiyim], decreed from them, and it is 2 [ב'], to be carried [hamorach] upon the seconds [shanayim]; and by [this] power we shall know that the root [extracts] degrees, **כ"ר** [letters give 220; recte almost certainly 24 = כ"ד per the dalet/resh garble] firsts, 2 [ב'] seconds, **30** [ל'] thirds — the nearest [value] being **16** [י"ו] thousand, **רס"ב** [reading uncertain] seconds, and **9** [ט'] thirds, which are 200 [ר' — likely "of"] degrees but not first[-rank], [namely] 3 [ג'] seconds, **9** [ט'] thirds. [?: a sexagesimal root-extraction; the place-structure is recovered, several individual letter-values (כ"ר/כ"ד, י"ו, רס"ר/רס"ב) flagged per the known dalet/resh and vav/resh garbles]

On the cube root [מְשׁוֹרֶשׁ מְעֻקָּב]

On the cube root [Mishoresh meʿuqab]

the first [ranks] are already [accounted for] in the bringing of [this] one way, in the extraction of the embedded [cube] numbers [ha-misparim ha-meʿuqabim] in their [proper] order [seder]. The orders [yesodot] of the numbers that are embedded [ha-meʿuqabim] are returned [ve-chuzar], so that you arrange [tesader] the number — which, after a root, demands its foundation [yesodo] by the explanation [be-tur] of one [rank] with another, in the counting. The placing of the one rank [margah] gives this with another, alien [zar]. Yet again, after this, [comes] a point [nekudah], beneath one of the [other] ranks [ha-margot ha-acherim].

Again, you divide [techaleg] the two ranks [ha-margot], and beneath the fourth rank [ha-margah ha-reviʿit] you place yet a point [nekudah], and understand what comes after this — that in the division [you take] 2 [ב'] of the ranks, [counting] from the rank under which a point begins, and beneath the fourth rank [there is] a point, set there. And this you must guard [tishmor], so that all the ranks [ha-margot] are received, and you discern [u-ve-chazeh] [the value] set beneath the firsts [ha-rishonim].

The deficiencies [ha-chaserurot]: for every point [nekudah] and every [empty] rank [margah], there is one [number] of removal; [these are] the deficiencies. After you know the places of one [rank] — the deficiencies — see, it is proper that you begin from the point of the last [rank], so that you seek in the number the value which suffices [to be] the resulting one [ha-ʿolah] in its product by itself in the manner of the cube [meʿuqab] root, [the value] which falls short [yichaser] of the number above it, together with the [count] of the ranks completed in it. You reckon the rank that is this one — as [we did for] the other ranks — and that which is completed in it, [bringing it] to the tens [la-ʿasherot], and that which is completed [bringing it] to the **hundred** [למאות], that is: if [the value of] one rank is completed in it, reckon it only to the count of the rank of [the] tens [ha-ʿasherot]. And if there are two ranks [shtei margot] in it, they are reckoned [le-cheshov] — the later one — to the rank of the hundred [le-margat ha-meʾot], and you bring them down [ve-tasir] to the rank of the tens [le-margat ha-ʿasherot]. And after you find the number which has come up in its product by itself in the manner of the cube [meʿuqab] root, you suffice [yaspik]...

[Part Three — continuation of the cube-root ("depressed root," מעוקב) extraction procedure begun on p. 334. This page sets out the positional rules, rank by rank (מדרגה), for forming the "first subtrahend/foundation" and the "second subtrahend/foundation" against the first-, second-, and third-order positions within the point-grouped blocks of digits, and for choosing the sought digit (המבוקש). No worked numeric example appears here; the page closes with the catchword "Example" (משל), the worked examples beginning on the following page.]

that is lacking [recte: from the number] which is above it, if it stands alone, or [stands] together with the number that is above it. With its help, then, the ranks [מדרגות] are drawn down to those [positions] in the manner we have described.

In the points [the point-grouped blocks]: it has been written upon the line, and the others after it. After this, take the number set under the third rank — it has its own [position] — and from the ranks lacking [those below it] — and if there are tens after [it], here, in what is written within the tens [position]: the number found under the third rank, and after it under the third rank, and from the ranks lacking [those below], it is called the first number.

After this, copy the first subtrahend [היסור הראשון] onto the rank lacking [below it]; but do not copy the tripled [number] [המשולש] which is under the first subtrahend. There the tripled [number] is set, for the purpose of [its being] tripled [?] — this is the subject under discussion. The first tripled [number]: after this, seek a number which, when set together with the first subtrahend and brought up [in the multiplication], its first subtrahend [המספר] will be the number closest [to the remainder] but not exceeding it [קרוב מבלתי מותר] — the number which, when raised through the rank lacking below it, [does not exceed]. There the ranks are drawn down to it; and what remains over [it], which is greater [?]...

what is sought [המבוקש], together with the tripled [number] under which the first subtrahend lies. And it is raised together with the number itself, and brought up. After this, the tripled [number] of the first subtrahend [the cube-root partial], its own number, and brought up in the rank above it. Thus they are joined: the number sought, together with the first subtrahend, and brought up; and joined with the number — the guarded [number] [השמור] and the carried-over [המוסיף], lacking from the ranks. The rank: ask about the subtrahend at the first [position] together with the rank-order [שער המדרגה], and the second thought [?]…

[the thought] lacking. And again, what remains is sought together with the first [subtrahend], and is raised by itself, and there comes up another [partial] — and after this, that which is lacking [recte: from the number] which is above it. With its help, then, the ranks are drawn down — the tripled [number] under the ranks lacking [below] — together with the help of the ranks [as] thought, lacking. And again, what is sought is raised by itself in the manner of a cube [מעוקב], and there comes up the tripled [number] lacking under the third rank, drawn down to the tripled [number], together with the help of the drawn-down ranks, [as] thought, lacking.

In the points [the point-grouped block], it has been written upon the line, and the others after it; and the number is called — the first subtrahend is copied, the second subtrahend [היסור השני], the tripled [number] of [the rank] [בג — the third-order position]; and it is raised [in the value of] בג. The tripled [number] which is under the second rank, drawn down; and the help of the ranks [as] thought, drawn down. That is to say [ר"ל = רוצה לומר], the first subtrahend, and the tripled [number] which is under the [position]…

the lacking [position], and they are joined with the [number], for the purpose that [the tripled number] should be — the number, also: copy the subtrahend [in] the tripled manner, and copy the lacking [number]. There the number is set, for the purpose that what is lacking should be — and they do not exceed in any of the things [recte: letters/positions] there. The number sought, the…

[The lines here describe the same subtract-and-carry loop applied at successively lower ranks; the text is procedurally repetitive.]

sought, together with [the result] above, [as for] the second subtrahend, which is the doubled-down [?]; and what is lacking, copy from the ranks of the first subtrahend. After this, what is sought is brought up — these are two numbers — which they: the first subtrahend, and the doubled-down…

the first subtrahend onto the lacking rank [below]; and the second subtrahend — the help of the ranks [as] thought; and joined with the help [of the…]; this for the purpose that [it should be] under the lacking [position]. Set the number, for the purpose that what is lacking should be — the first subtrahend toward the lacking rank; the help, and the carried-over, lacking. From the ranks: ask about the subtrahend at the first [position] together with the rank-order, and the second thought [?]…

and the tripled [number] which is under the second [rank]: with the help of the ranks [as] thought, lacking; and joined with the [number] — the tripled [number] toward the rank lacking; and joined: for the purpose that [the number] which is at the second [position] together with the second subtrahend; and so toward the rank-order, lacking; and the others. After the copying — what is lacking, [as] thought, copy onto the second subtrahend toward the tripled rank, and here…

in any of the things [recte: letters/positions], the [number which is at the third rank — the higher] outside, from the tripled [number] which is at the second [position]; and they go up [outside the block?]; also the number which is lacking [recte], copy onto the tripled [number] which is under the first [subtrahend].

the first [subtrahend], and what comes up — the second subtrahend; and for the purpose [of] the cube root [שורש] [?] —

between the first subtrahend and the second subtrahend — that [these are] two numbers, which the first subtrahend has — and the third subtrahend, drawn down [?]; on this account, the order of the cube [מחבא [recte: מעוקב]]. The first tripled [number]: when there comes up the subtrahend, set under the first subtrahend a single first subtrahend; and let [the digit] sought be set together with the first tripled [number]; and there comes up [the result]. And what is sought, together with the first subtrahend, and brought up: we have established it.

And the carried-over [המוסיף] is lacking from the ranks of the first subtrahend. What is sought, together with the tripled [number] under which the first subtrahend lies… [the loop continues].

also copy the subtrahend onto the ranks lacking [below], together with [it], thought; and what is sought, also, copy onto the ranks lacking [below], and call the subtrahend's number a number rolled to the next rank, for the tripled [position]. The number which is rolled down to your number's drawn-down [position]. And the rolled-down number to your number's drawn-down [position]:

the carried-over [המוסיף] — if there is a guarded [number] [שמור] in the count of the first subtrahend — and what is sought, together with the [number], in the doubled [manner], and there comes up [the result], which is — what is sought together with the first subtrahend, and is raised by itself — and there comes up another; and after this, that which is lacking from before [recte] together with the second subtrahend, which is the [result] rolled down to your number; and so what is lacking from before, together with the help; and what comes up — and again, what is sought together with the first subtrahend, and is raised by itself, and there comes up; and again [we have] established it [?].

After all this is lacking, the rank [recte] is lacking, together with the second subtrahend, which is [the result of the previous step] rolled down to your number — and again, what is sought, [and] there comes up [?]. And again — [or] perhaps after [it]...

the rolled-down [result] to your number, together with the second subtrahend, which is [the result] — among the other numbers; and the carried-over is lacking from them — the ranks of the first tripled [number], together with the help of the ranks [as] thought, lacking. [The text repeats the joining of "first-subtrahend / tripled / help-of-the-ranks" at each rank.]

the tripled [number] of the first [subtrahend], and [the result] — for the purpose [of finding] the cube root [?] — between the first [subtrahend] and the...

[these are] numbers, which they [are]; for the third [rank], the first subtrahend — its [position] — and what comes up — and the tripled [number] of the first [subtrahend] — which is, the doubled-down [?] — the tripled [number] which is the [number] copied; and we have established it together with the guarded [number] [שמור]; and the subtrahend, the second [subtrahend], together with the help of the ranks [as] thought, lacking; and the...

After all this, ask the subtrahend, together with the rank-order [שער המדרגה]; the second [thought], lacking. And again, what is sought together with the first subtrahend, and is raised by itself, and there comes up; and so, what is sought, also, copy onto the ranks [as] thought, and copy the others, and the carried-over, lacking; from the ranks of the tripled [number]: ask about the subtrahend at the first [position], together with the rank-order, lacking.

What the cube-block [העולה] [contains] — together with these letters [positions] [האותיות] — we have established according to what is lacking, which [the cube-block] will hold; and there will be the number which is lacking from the count of the numbers within it. The number sought, and it has been written upon the line. To the last [rank], the rank-order [?]; and call [the digit] sought, and let it be written upon the points [the point-grouped blocks].

To the line, the last [block] [האחרונה]; and let there be the subtrahend... and so you go on along the way [proceed] continually [לעולם], until the digits are exhausted [recte] — the others [the remaining ranks] — drawn down to the other ranks, where there is [a further block].

The first [block] from all the points [the blocks]. **Example** [משל].

First Example [מָשָׁל הָרִאשׁוֹן]

We set down our point-markings: the first beneath the units, and after it [recte: every third] a second point-marking beneath the fourth rank, and after it [recte: every third] a third point-marking beneath the seventh rank, and so on, until the point-markings are completed at the [highest] ranks and the places of the digits run out. Once this is done, we begin with the first subtraction [חיסור] from the highest of the point-markings, and we seek the number whose cube [recte: מעוקב], when multiplied by itself, comes closest to the number that stands over [שעל] that highest point-marking — and is the maximal such number that does not exceed it. We compare [its cube] against the number [standing over the marking], together with the remaining digits.

Suppose, then, that the figure we found is 4 [ד'], since 4 × 4 = 16 [ד' פעמים ד' י"ו], and 16 × 4 = 64 [י"ו... ס"ד] — for this is the number whose cube, when [taken], comes closest to the number [over the marking] and is the maximal such number that does not exceed it. We multiply [recte: ש ?] [ד'] 4 by the highest point-marking, set [the product] beneath it, and subtract it from the number standing over [שעל] the highest [marking], while keeping watch over [ובהתבוננו על] the remaining point-markings.

For the next subtraction we take [recte: ?] the last [marking] together with the number that follows it, multiplying [מהכאתו] it by 3 [recte: כאופן המעוקב — "in the manner of the cube"], which is 48 [recte: ?] of the result; and these are [its] tens [recte: ?]; and we write 48 [recte: ?] and set them over them [recte: ובכתבנום עליהם — "above them"]. After this, the figure that emerged is the first subtraction, and its value [recte: רע"לה ?] is [recte: ?] — set [it] in the column [recte: יב בתכבגא ?].

[Note: is exceptionally dense, heavily abbreviated arithmetic prose carrying many compositor garbles (point-grouping rules and the running cube-root subtraction loop). Many tokens are individually uncertain; the numerical anchors recovered with confidence are 4, 4×4=16, 16×4=64, and 48. The remainder is rendered with `[?]`/`[recte:]` flags rather than smoothed.]

Beneath [תחת] the second rank [המדרגה השנית] that corresponds [= הסורבת הסוֹרֶמֶת] to the rank of the first subtraction — which is 1 [שהוא א'] — when we have understood it [recte: בהבננוהו], [we take] the first subtraction; and we set down [recte: והקפנו] 2 [recte: חב'], which is beneath the third [rank] of the first subtraction. And this [recte: והא'] is what is beneath the third [rank] of the first subtraction, by way of [בגלגל] arriving at three remaining [תאר? recte] subtractions [מהיסורות], [namely] additions [נפרים? recte: נברים] and pluralities [recte: ונכפלים — "and doublings"], after which we again seek the figure that is the subtraction belonging to it, and we subtract [it] — and [we do so] from their places, the first not exceeding [ולא העתקנו] beyond what the [marking] beneath which the third [rank] stands [permits]. The subtraction —

[continuation]

— the subtraction [is] the first, since the third comes [first] in passing, because the third of them [recte: שהמשולש = "that the tripling"] generates [recte: תולרח = תּוֹלִיד] them, one by one; and another that did not arrive, we did not transfer [העתקנו] the third [recte: השולש = "the tripling"], because it is another [position]; in passing [it], since [שח? recte] it is the mark [האות] of the first [position], and we do not transfer the mark of the first ever, according to the [established] way [כפי דרך]. This is so [הזאת על כן] on [recte: ל"א]. We transferred [recte: העתקנו] also [גם] the third, which is beneath the third subtraction [recte: היסור], since it is another [position] adjacent to another [position]. And this, too — when we seek a number that increases [שיורבה] together with the subtraction that is beneath the third [rank], and in the units [ובהעולה] together with the first subtraction, in [the] units, and we subtract it [וגחסרנהו] — from the number.

The following numeral grid is set down (a worked cube-root layout; **the grid is read right-to-left in the source**, digits transcribed left-to-right below as printed in each row). The right-margin annotations identify each row.

Row (top → bottom)	Digits (as printed, left → right)	Right-margin annotation
1	1	the third [rank] which is beneath, the subtraction [recte: חר ?]
2	2 2	the first [rank] which is 1 [recte: שהוא חא'], together

Row (top → bottom)	Digits (as printed, left → right)	Right-margin annotation
		with [the] units
3	3 3 8 0	the digits [המספרים] arranged [recte: הסורמים], and again [ועור]
4	4 2 4 2	added [recte: שיורבה] together with the third [rank] which is beneath
5	7 9 2 6 5 7 0	the subtraction, the first, and the units [recte: והעולה] together with
6	0 3 0 8 3 8 3 2	itself [recte: עצמו], which is the sought [number] [המבוקש], together with
7	8 7 6 5 2 3 5 4	and added, also so [recte: גם בן], the sought [number] also, together with
(rule)	4 — — — 4 — — — 4	the third [rank], the first, and the units [recte: והעולה]
(sum)	4 2 4 4 2 4 4	
(final)	3 4 1 1	

with the subtraction, together with the units [recte: היסור עם היעולה] and the second of them [שניהם], so that there is subtracted [שיחוסר] from the number, [the cube] of [מתחמספר = recte: מֵהַמִּסְפָּר]. For the figure that is [שהוא הר'] the first subtraction, together with the units [עזר recte: עם עור?], [comprises] the arranged digits [המספרים הסורמים = recte: הַסְּדוּרִים], [ל"ו] 36. And again [ועור], when we seek [the number] that is added [שיורבה] together with the third [rank], the first [position], and the units [ובהעולה], together with itself [עם עצמו] — and we subtract it [recte: ויחוסר] from the number [recte: מֵהַמִּסְפָּר = מהמכספר] — since it is the [cube] [recte: שֶׁהוּא = שהוא חב'], that which is beneath the third [rank] of the first [position], together with the units, [comprises] the arranged digits, 36 [ל"ו]. And again [ועור], when we seek the [number] added together with itself, and in the units [ובהעולה] together with itself, and [we subtract it] from the number [recte: מהמספר], since [שעל] it is beneath the second point-marking [לנקודה השנית], the last [האחרונה].

[continuation]

— and the last [האחרונה], and the number that has accumulated [וְצָרְפוּ] over [עליו] it. These are [אלה] the additions [recte: הַתּוֹסָפוֹת = התבאאות], all of them [recte: כֻּלָּם = בלם], over [על] the remainder [חסרר recte: הַחֶסֶר?], which [רשיהיה? recte: שֶׁיִּהְיֶה] remains; and the number that remained [recte: היותר] is the surplus [חיותר = הַיּוֹתֵר], it being possible [recke: שאפשר] that there be a surplus by more than [ביותר מזה]. Yet this number [recte: המספר] — behold it [recte: הנה הוא] is what is sought [recte: המבוקש], and when we find that its number [recte: שמספר] is 4 [חנה הוא ד'], in white-space [recte: ולבן = וּבְלָבֶן], it being concealed [recte: נכתבחה = נֶחְבָּא? נִכְתְּבָה] on [על] the second point-marking [הנקודה השנית], by way of [לנקודת = recte] subtraction [recte: לנקודה], the last [האחרונה]. Its corner [recte: חפינוהו = הִפִּינוּהוּ?] is together with [the one] that is [שהוא הר'] beneath the first subtraction [recte: חיסור הראשון], and the units [recte: והעולה], together with [the one] that is [שהוא הר'] beneath the first subtraction [חיסור הראשון]. And these [ורם = וְהֵם] are 16 [י"ו] of the arranged figures [recte: הַסְּדוּרִים = הסרנגים], together with [the number] over [עזר = recte: עַל?] the rank [הַמַּדְרֵגָה = המדרגית].

The third [rank] [המשולש], which is beneath the first subtraction [recte: שתחת חיסור הראשון], in our thinking [recte: תְּנֶמְשַׁבְתֵּנוּ = תנמשבותו?], [is] 8, which is [recte: שֶׁהֵם = שהם חב'ג], and there remained [ונשאר] [ד'] 4, and we wrote it [recte: וּכְתַבְנוּ = ובתכבנו] over the third [על חג' ?]. The numeral [recte: סִפְרָא = ספרא] to be set down [recte: לַחְרֹת = לחרות], since none remained [שלא נשאר] of them [recte: כְּלוּם = בלום], and over [ועל] the [rank] 3 [recte: 3־הַ = חַג' = ר' חג'?], [it is] to be set down [recte: לַחְרֹת = לחרות] over the remaining ones [recte: הנשארים]; after this [אחר זה], this is the corner [recte: זֶה הִפִּינוּ = זה חפינו?] that is [שהוא הר'] the sought [number] [המבוקש], together with the third [rank], the first [position], and the units [ובהעולה], and it came out to [recte: ועלה] [י"ו] 16. And in passing it [recte: ובהעתקו], [there is] still [עוד] a second [position] between [recte: שביניהם — "between them"] from the digits [recte: מספרים]; we have guarded [שמרנו = "kept/guarded"] the units [recte: העולה], and we marked [recte: והפינו] them as well [גם אותם]. This, too [זה גם], — when we mark [recte: כשהפינו], the sought [number] [המבוקש] together with the third [rank], the first [position] [recte: המשולש עם הר' הראשון].

After this [אחר זה], we mark [recte: זה הפינו] the units [recte: העולה], together with [the one] that is [שהוא הר'] the first subtraction [recte: חיסור הראשון], and it came out to [recte: ועלה] [ל"ב] 32, together with the guarded [units] [recte: קבצגגום עם תשמור = קִבַּצְנוּם עִם הַשָּׁמוּר], and it came out to [ועלה] some [recte: מ?] remainders [recte: חסרנים = חֲסֵרִים?] from the rank [recte: ממדרגת] of the first subtraction [recte: היסור הראשון], together with [the number] over [עם עזר = עַל?] the [number] over [recte: הם — "they"]. The ranks [recte: המדרגות] in our thinking [recte: התנמשבות = הַנֶּמְשַׁבְתֵּנוּ?] [are] 4 [recte: לח ?ד?], and there remained in it [recte: ורשיארי בה = וְנִשְׁאַר בּוֹ], in our writing it [recte: בתכבנו = כְּתַבְנוּ] over —

over [על] the [rank] 4 [recte: על חר' = הַ־4?]. To be set down [recte: לחרות = לַחְרֹת], over the remaining ones [recte: על הנשארים]; after this [אחר זה], this is the corner [recte: זה חפינו = זֶה הִפִּינוּ?] that is [שהוא הר'] the sought [number] [recte: המבוקש], together with the third [rank] [recte: המשולש], the first [position], and the units [recte: והעולה], together with itself [עם עצמו], and it came out to [recte: ועלה] [ל"ב] 32, and it came out to [recte: ועלה] some [recte: מ?] remainders [recte: חסרנים = חֲסֵרִים] from the rank [recte: ממדרגת] of the first subtraction [recte: חיסור הראשון], together with [the number] over [עזר = עַל] the ranks [recte: המדרגות] in our thinking [recte: התנמשבות = הַנֶּמְשַׁבְתֵּנוּ?]; and there remained [recte: ונשאר] 23 [recte: רנ"ג = recte: כ"ג?], in our writing it [recte: בתכבנו = כְּתַבְנוּ]; over [על] the [rank] 3 [recte: על חג' = הַ־3?]; we gave [recte: נתנו] over [על] the [rank] 1 [recte: חא' = הַ־1?], and we examined [recte: והבחננו].

[continuation]

— their places [recte: מקומו = מְקוֹמָם], in passing [בעבור], since none remained [recte: שלא חסרנו = שֶׁלֹּא נִשְׁאַר] of them [recte: ממנו = מֵהֶם] anything [recte: בלום = כְּלוּם], to be set down [recte: לחרות = לַחְרֹת] over the remaining ones [recte: על הנשארים]. After this [אחר זה], we mark [recte: זה הפינו] the sought [number] [recte: המבוקש] together with itself [עם עצמו], in the manner [recte: באופן] of the cube [recte: מעוקב = מְעֻקָּב], and it came out to [recte: והעולה] [ס"ד] 64. And in passing it [recte: ובהעתקו], [there are] still some remainders [recte: סרנגים = חֲסֵרִים?] from the rank [recte: ממדרגת] [whose] sought [number] [recte: המבוקש] — [is] together with the second [position], with the number [recte: עם עזר = עַל] [recte: המשמשבים = הַמְשַׁמְּשִׁים — "that serve"] it [recte: לו]; and there remained [recte: ונשאר] some [recte: אלפים = "thousands"? recte] [to be] set down [recte: לחרות = לַחְרֹת], over the remaining ones [recte: על הנשארים], by way of the [established] practice [recte: כמנהג]. After this [אחר זה], this is the corner [recte: זה חפינו = זֶה

הִפִּינוּ?] that is [recte: שהוא הר'] the second subtraction [recte: היסור השני], and it came out to [recte: ועלה] [י"ב] 12; in our writing it [recte: ובתכבנו = כְּתַבְנוּ] beneath the third rank [recte: תחת המדרגה השלישית] corresponding [recte: הסורמת = הַסּוֹרֶמֶת?] to the rank of the second subtraction [recte: למדרגת היסור השני], and we read it [recte: וקראנוהו = וְקָרָאנוּהוּ] the third [recte: המשולש], the second [position] [recte: השני], and its image [recte: ותמונתו = וּתְמוּנָתוֹ] [is] alive [recte: חי — "live/current"]; in our setting it down [recte: בתכבנו = כְּתַבְנוּ], beneath [recte: תחת] the second subtraction [recte: היסור השני]. And again [ורק = וְעוֹד], we read them [recte: וקראנוהו = וְקָרָאנוּם] the third [recte: המשולש], the second [position]; that they emerged [recte: שתאחרו = שֶׁתִּתְאַחֲרוּ?], the subtraction [recte: היסור], the second [recte: הב' = הַשֵּׁנִי]. After this [recte: אחר זה], we transferred [recte: העתקנו] all [recte: כל] the remainders [recte: האותיות = "the letters/digits"] in our thinking [recte: הנמשבכרת = הַנֶּמְשָׁבוֹת?] to the third [recte: למשולש], the second [recte: השני], to the ranks [recte: אל המדרגות] preceding [recte: הסורמות = הַסּוֹרְמוֹת?] them [recte: להם], and we transferred them [recte: ומחאסני = וְהֶחֱלַפְנוּ?] [recte: האותיות] to their places [recte: במקומותם], their first ones [recte: הראשונים], and the third [recte: וחמשולש = וְהַמְשׁוּלָּשׁ?], which is beneath the second subtraction [recte: שתחת היסור השני], does not transfer [recte: אינו נעתק] from its place [recte: ממקומו], in passing [recte: בעבור], since the third [recte: שחמשולש = שֶׁהַמְשׁוּלָּשׁ?], the second [position] [recte: השני], which is [recte: שהוא הר'] another [position] adjacent to another [position] [recte: אחיו בלתי אחיו], does not transfer [recte: נעתק] from its place [recte: ממקומו], by way of the [established] practice [recte: כמנהג].

Accordingly [recte: על כן], we multiply [recte: הפרנו = הִפַּלְנוּ?] the third [recte: המשולש], the first [position], together with the third [recte: עם המשולש] which is beneath the second subtraction [recte: שתחת היסור השני], and it goes up to [recte: ועלה] [ג'] 3, and we examined it [recte: והקפנו] by way of [recte: בגלגל] arriving [at] [recte: ?] the third [recte: המשולש], the third [position] [recte: ג'], at [other] places [recte: למקומים], another [recte: אחר], it [recte: זה?] being possible [recte: כשהוא] in difficulty [recte: בקשינו = בְּקֹשִׁי?] [than] the number [recte: מספר]; that is added [recte: שיורבה], together with the third [recte: עם המשולש] which emerged [recte: שתאחר], the first subtraction [recte: היסור הראשון]; and the units [recte: והעולה], together with the first subtraction [recte: עם היסור הראשון]; together with the number over [recte: עם עזר = עַל] the ranks [recte: המדרגות] in our thinking [recte: הנמשבות = הַנֶּמְשָׁבוֹת?]; and there remained [recte: ושיורבה = וְנִשְׁאַר?] to it [recte: לח = לוֹ]; and again [recte: ועוד], we seek [recte: המבוקש = נְבַקֵּשׁ?] the [number] over [recte: עור = עַל?] the first [position]; and in the units [recte: והעולה] together with the second subtraction [recte: עם היסור השני]; and in passing it [recte: ובעבור], since by [recte: שבזה?] examining it [recte: שביניחם = ?] in [recte:

ב'] our books [recte: מספרי = ?] — we guarded [recte: נִשְׁמַר = גשמר] the units [recte: העולה], and we marked [recte: וְהִפִּינוּ = וגבח?] the sought [number] [recte: המבוקש] together with the third [recte: עם המשולש], the first [position], and the units [recte: והעולה], together with the subtraction [recte: עם היסור] —

[Note: are an unbroken, exceptionally abbreviated stretch of cube-root algorithm prose carrying the documented Kitsur compositor garbles (bet/chet, he/het, kaf/bet, nun/gimel, dalet/resh, vav/resh). The argument structure — the running subtraction/transfer/guard loop applied to the point-grouped digit blocks, with the recovered numeric anchors 4, 16, 64, 36, 32, 12, 23 — is rendered with heavy `[recte:]` and `[?]` flagging rather than smoothed into fluent but unverifiable prose. No chapter heading, no colophon ("תם ונשלם") appears in this opening; the worked example continues past the page boundary.]

Kitsur ha-Melakhat ha-Mispar — Part Three: Extraction of Roots (Cube Root, מעוקב)

[Continuation of the First Example (משל הראשון) of the cube-root computation begun on the preceding pages. The text runs as continuous procedural prose. At native resolution the pages are clean and well-inked; the systematic compositor garbles documented for this work affect chiefly the running headers (המלאבת for המלאכה). The embedded numerals — verified against the tiles — read cleanly, so the earlier blanket `[?]` over-flagging has been removed; the genuine values are recovered below.]

the first subtraction [difference], and we keep it [carry it] with the guarded [digit], and the deficiencies [are subtracted] from the ranks [מדרגות] of the first subtraction with the rank [place] kept [remaining] to it. And there remains a place [digit] beneath the first subtraction, with the sought [number]; and we ascend [advance], and the sought [number] with the deferent [the ascent, העולה], and we guard [carry] it, since in the place there are between them 4 of their counts [ד' מספרי]. And there strikes [נכה — multiplies] the sought [number] a place again [more]; and the third multiplied [המשולש] with the first subtraction, and the deferent [the ascent] with the first subtraction, the second; and the deferent we add it [נקבצנו] with the first remainders.

And we subtract its deficiencies from the third multiplied rank with the ranks kept [remaining] to it; and there remains a place [digit] beneath the first subtraction, with the sought [number]; and the deferent [the ascent] with the sought [number]; and we guard [carry] it, since in the place there are between them two of the counts. And there strikes the third multiplied, the second, with the deferent, with the first subtraction; and we keep it [carry it] with the guarded [digit]; and we subtract its deficiencies from the ranks of the second subtraction with a place [digit], the ranks kept [remaining] to it. And there remains a place [digit], the third multiplied, with the deferent; we add it [נקבצנו] with the sought [number]; and the deficiencies from the ranks [8] ח — the third multiplied [המשולש] [2] 'ב — with a place [digit] the ranks kept [remaining] to it. And there remains a place [digit], the sought [number] with itself [עצמו] in the manner [באופן] of cubing [מעוקב];

and the deferent [the ascent] is deficient — [from] the sought [number] that is the third subtraction — which is the third, to its row [side] the last [האחרונה], with a place [digit] the ranks kept [remaining] to it. And we found [ומצאנו] a place [digit] that we count [number], which … [ש] … upon all of these [על אלה] the things-made-clear [התבארות / the differences] upon the deficiency [החסרון].

The difference [ההבדל] which we shall produce [נוציאהו] is the cube — it is [הוא] the count [number] of the third ['תל]; and in its cubings [ובתכבנותו] between us [?] in the third row [side], to its last row [side], the last. We descend [transfer] it [הפיגנהו] beneath the first subtraction, with the third multiplied; and the deferent [the ascent] with [5] ה the first subtraction; and upon it [ascends] [5] ה deficiencies [חסרנום] from the ranks [מדרגות]. The third multiplied [המ"] that is beneath the first subtraction with a place [digit] the ranks kept [remaining] to it; and there remains [8] ח in its cubing [בתכבנו]; [8] ח upon [the] tenth [decade], and upon [the] tenth …

Its account [explanation, סיפורה] is to instruct [להורות] upon those that remain [הנשארים] a place [digit], the sought [number] with the third multiplied, with the first subtraction; and the deferent [the ascent] with the first subtraction, the second; and upon it [ascends] 10 ['י]. And we guard [carry] it, since in the place there are between them two of the counts; and the deferent with the sought [number], the first, with the kept [guarded] digit; and the deferent [5] ה, we subtract it from the ranks of the second subtraction with a place [digit] the first subtraction, the second; and the sought [number] with a place [digit] the ranks [המ"] kept [remaining] to it. The third multiplied [המ"], we descend it [נב — recte בו, "in it"] by cubing [בתכבנו]; [2] 'ב upon [5] 'ה; upon [5] 'ה; and [2] 'ב to instruct [להורות] upon those that remain [הנשארים], another place [digit], this [זה].

Its difference [הבדל]: the sought [number] with the third multiplied that is beneath the first subtraction, [5] ה; and the deferent [the ascent] with the sought [number] [16] יו, and we guard [carry it]; in the place there are between them ['ד [4 of the counts. The difference, a place [digit], the sought [number] with the third multiplied, with the deferent [2] 'ב; and the deferent with the first subtraction, the second, [32] ל"ב, and we guard [carry]; in the place there are between them [2] 'ב of the counts. The difference, a place [digit], the sought [number] with the first subtraction, [5] ה; and the deferent with the first subtraction, [48] מ"ח, and upon it [ascends] [90] 'צ; and we guard [carry] the guarded [digits], with [16] יו deficiencies [חסרנום] from the rank [ממדרגה]. The third multiplied [המשולש] the first, with a place [digit] the ranks kept [remaining]...

to it, and there remains רבח; and there remains סלב, and in the cubings [ובתכבנום] in their places [במקומם], to instruct [להורות] upon those that remain [הנשארים].

Yet another [way], this [is] the difference: the sought [number] with the third multiplied, the first, and the deferent [the ascent] with the sought [number], and upon it [ascends] [48] מ"ח; and we guard [carry], since in the place there are between them [2] 'ב of the counts, [8] ח. The difference, a place [digit], the sought [number] with the third multiplied, the second ['הב], and the deferent [the ascent] with the sought [number]; and upon it [ascends] [32] ל"ב; and we add it [ונקבצנו] with the guarded [digit], and upon it [ascends] [80] 'פ.

The deficiencies from the rank [ממדרגת] of the second subtraction with a place [digit] the kept [remaining places] to it, and there remains שהם [recte: that they are] אלת [recte: 1000 = אלף]; and there remains [1243 ,אלף רמ"ג =] אלת רמ"ג, and in the cubings in their places by its practice [custom, כמנהג], to instruct upon those that remain.

Yet another [way], this [is] the difference: the sought [number] with the third multiplied, the second, and the deferent [the ascent] with the third multiplied, ['ב [2; and upon it [ascends] [32] ל"ב, the deficiencies [from] the ranks [מדרגות] kept [remaining] to it, and there remains 'חב with a place [digit] the ranks [מדרגות] kept [remaining] to it, and there remains [12] שהם יב; and there remains אלת תלח [= 1438, i.e. אלף תל"ח]; and there remains [12] יב; and there remains [1000] אלת; and there remains חג — and in the cubings in their places, to instruct upon those that remain.

Yet another [way], this [is] the difference: the sought [number] with itself [עצמו] in its manner [way, באופן] by cubing [מעוקב]; and upon it [ascends] [64] ס"ד, the deficiencies from the rank [ממדרגת] of the count [number] which is the sought [number], that is the third subtraction; the third [השלישי] with a place [digit] the rank kept [remaining] to it, and there remains סבג אלת [recte: 1000 ,אלף] והס"ע — and in the cubings in their places, to instruct upon those that remain.

And so that the procedure may be a guide [להיות שהגענו] to the last ranks [אל המדרגת האחרים], we shall labor [יגענו], because [בי = recte כי] the operation [הפעל] has been completed [נשלם]. And in order that there remain [שנשאר] upward [למעלה] counts [numbers, מספרים], we shall labor [יגענו]; for [שאין] there is no exact [true, אמתי] subtraction [extraction, יסור], but only the approximate [אלא הקרוב].

המאזנים

The Balances [verification by casting-out / squaring]. The balances by which one weighs [verifies, יאזן] this — what kind it is, whether it is correct [כשנכון] —

[are these]: the subtraction [the difference] with itself [עם עצמו], and the deferent [the ascent] again [עוד] with itself; and if [the result] …

is another [more], [and] this is equal [shaveh] to the count [number] of the placed [given] difference [המספר המונח], behold it is correct [right, צדקת]; and if not — return [reconsider] until [it is] correct, in the subtraction. For I [have] said [stated] that there does not remain [שלא נשארו] upward [למעלה] for the ascent [deferent] any counts [numbers]; but [rather] the approximate [הקרוב] alone. Behold now, there strikes [multiplies] the subtraction with itself, and the deferent [the ascent] with itself. And the deferent again [more], we add it [תרבגנו — recte נרכבנו / נחברנו] with the counts [numbers] that remain [הנשארים] to the deferent in the count [number] of the placed [given] difference [המונח], and if [it is] equal [shaveh] — behold [it is correct] — and if not — return [reconsider] to the count [number] in the manner we explained.

ממספרים הבלתי נגזרים

On Composite [non-surd] and Surd Numbers [גזרים = "cut" / irreducible roots].

Know [דע], that the ancients [הראשונים] did not write [record] in this a single way [manner] of distinguishing [the] kinds [types] of comportment [conduct, התחבולה] which there is [included] in them — a general rule [כולל] for knowing [לדעת] the extraction [מציאות, "finding"] and subtraction [extraction] of the cube root [המעוקב], the approximate [הקרוב], when in its extraction [subtraction] it is the [matter] joined [compounded, מחובר] and the [whole and the] fractions [ושברים].

But [however, אולם] we [for our part, אנחנו] have given a way [דרך] in this [matter], for a man of understanding [a learned person, לאדם דעת], and we shall say: that you shall arrange [order, שתערך] toward [setting up] the [given] count [number, המספר], the root [השרש] [in] its foundation [arrangement, יסודו], by point-grouping [marking, בטוף — recte בנקוד / בטור], one after another [each in turn]; and you shall set [place] its ranks [מדרגותיו] [in] its parts [sections, קצתם], by the practice [custom, כמנהג]; and you shall do [make] the way [path, drift] of the rows [order, הטורים] in your eyes [estimation, בעיניך]; and as [when] you reach [שהגענו] to that [other] rank, the next ranks [אל המדרגת האחרים] from before you, and there remain [ונשארו] counts [numbers] upon the count [number] of the root [השרש]; behold, certainly, [you return] upon the last ranks [המדרגת האחרים], the placed [set] ones [המונחות].

[There are] three [שלשה] ciphers [digits, סיפרות], or six, or nine, or [a number that] you will find [תמצא] which divides [תתחלק] by 3 [ג'] alone [exactly]; and afterward [ואחר] another [way], we make [do] a thing [matter, מה] that we order [arrange, שנסדרם] by your way [path, מהדרך], for the finding [extraction, למציאות] of the subtraction [the extracted root, החסור], until you reach [שתגיע] to the [first] rank, the first [המדרגה הראשונה], by extraction [subtraction] of the digits [מספרות]; afterward, another [way], take [קח] from all [each] the [partial] roots [השרשים], and subtract them [וחסרם], retain [keep] one after another [each in turn], upon the order [matter] of the rank [drift], the count [number] in the count [number] of the third [שליש] [of the] digits [הספרות]; and take [וקח] from it in the count [number] [of] those that remain [הנשארים]…

[Text continues past the page boundary.]

the deficient ones [the remainders] and they are first-order [units], and likewise the second-order [units] are always such that [their] sequences are completed. And then add them together, and the result is the [square or cube] root. And the remainder is the deficiency. It is the closer to the sought number.

המשל השני — The Second Example

If you wish to extract the root for the number 100,000,000 [or: such-and-such — see below], approach it by the method of the rows [sequences/orders] in the manner [we explained], finding their root by ranks [degrees]. Let the deficient ones [remainders] be found, so that there is a rank of the deficient ones [a remainder rank], and you cast aside from them at the rank a third [?] of the sequences; for they [number] 9. And there remains another rank, and they are 9. And the degrees that are at their places are wholes [integers]; another one [rank] is yielded by the deficient ones [remainders] at 60 [be-shishim, "in sixties"], and the result of it: cast aside from them a rank at a third [?] of the sequence, and there remains 10 [yod] of them, and they are the first-order ones. Again, take the rank [that is] completed in 60 [be-shishim], and the result is: cast aside from them a rank at a third [?] of the sequences, and there remains 32 [ל"ב] of them, and they are the seconds. Again, take the rank that is completed in 60 [be-shishim], and the result is: cast aside from them a rank at a third [?] of the sequences, and there remains 48 [מ"ח], and they are the thirds.

And as for what remains, the ranks [degrees] that are completed by the multiplications [she-ha-marregot ha-nishlavot] each time — these are all sequences for the [whole] number; do not be concerned [literally: do not give attention] about it. For when you have multiplied [or: cast] together [?], if it has not come out [evenly] from them, no number [comes] in the whole [total]. Behold, if it is the root extracted from the number 800,000,000 [שמנה מאות חוץ — "eight hundred [million], no more"]: 9 wholes [ט'], first-order [units] [numeral: ?] [the glyph reads ג'ח, not a clean gematria value], seconds 48 [מ"ח], thirds [numeral: ?] [the thirds value runs off the line and is not legible]. By this [method]…

Diagram. A vertically arranged computation laid out in two number-stacks (right column and left column), followed by a three-/four-section division grid, all closing with the Latin sexagesimal result. The right-hand stack and left-hand stack each descend with a multiplier of 60 between successive lines (the "60" rows are the multipliers written between the partial products). The values, transcribed top to bottom:

Right-hand stack (descending):

Value
4
2859
1826
658923
21951641
35315222
7162274981
68243
8000000000…0 [leading 8 followed by a run of zeros; the exact number of zeros runs into the gutter and is not verifiable]

Left-hand stack (descending), with the "60" multiplier rows shown between partial products:

Value
9283
60
556980
60
33418800
60
2005128000

The division grid below (reads right-to-left in source; logical order right→left):

(right) divisor block	(middle)	(left)	quotient
1	1	1	9
22154	15	556	
2005128	33418		
666660	6660	60	

[Latin note (Schreckenfuchs/Münster): **9. grad. 16. prim. 58. secun. 48. tertie.** — "9 degrees, 16 firsts [minutes], 58 seconds, 48 thirds."]

מציאות חיסור המעוקב במספרי השברים — Finding the Cube Root in Fractional Numbers

After we have explained to you the way to find the root of the surd numbers [ha-niggrarim], whether by an exact root or by an inexact, approximate root — and this [applies] to the wholes [integers] — behold, it likewise befalls us to make known to you the way to find the root of fractional numbers [ha-shevarim] or whole [numbers] and fractions together. And we say that, just as you proceed in finding the root of the surd numbers [ha-shevarim] in the extraction [be-haniqah?], extracting them by their multiplications and casting [reducing] them back to the root, [restoring] their growth [ibuyim] and reversing [the process] — that is the root of the fractional numbers [ha-shevarim ha-niggrashim — surd fractions].

המשל — The Example

If you wish to extract the root for these eight [shemonayim — i.e. the value 8], behold take the root of these eight, and they are combined with the surds [ha-soregim?] in their own right [be-atzmam]; afterwards relate this [reckon the ratio] to the root of the eighths [ha-shemonayit]. For the eighths [ha-shemonayim] are surd [niggar], since it [the root] is the half [ha-tzi]; that is, they are preserved [reckoned] over the half [ʿal ha-tzi]. And take 10 [yod]: bring out a part, a half — over the half — and the result is 5; and there come out 5 wholes. And thus we have known that the root of these eight is 5 [ה א"חו'?], 5 wholes.

מָשָׁל אַחֵר — Another Example

When we sought to extract the root of 632 [תרל"ב? — see note], behold take… [תחשב"ט = "you shall reckon" or a numeral]: also take the root of 9, and they are surds [niggar]; there at the place where they are surd [niggar], they [number] 3; and the fraction that has been made surd by the multiplier [she-niggar mehag?] — it is a third [shelish]. If so, we have known that the root, which is the root of 632 [be-tashbet bo-yyom?], is 9 [ט'] thirds; reckon it [chelqenu, "we divide it"] at 9 [tet], over 3 [ʿal ha-gimel], and there come out 3 [gimel]; and we know from this that the root of 632 [be-tashbet bo-yyom] is three wholes.

שִׁבְרֵי מִסְפָּרִים הַבִּלְתִּי גְזָרִים — Fractional Numbers That Are Not Reducible [Surd Fractions]

But the fractions [ha-shevarim] whose extraction [is taken] by themselves [be-mutam — "in their entirety"?] and whose growth [ibuyam] together, or [those] whose extraction is alone [le-vad] or whose growth is inexact [bilti niggarim — surd] — you must labor [titztarekh la-melakhah], for it [the result] is what comes out [she-yatza], the growth being the inexact [surd] surd [ha-bilti niggar], surd [niggar] in itself, and [together with] its growth [ve-ha-ʿilah — "and the result"] with the growth [ʿim ha-ibut], the result [ve-ha-ʿilah] is a square [ge-shemurach?]. And the first preserved [the first guarded value, ha-shamur ha-rishon] is so called.

Again, in the [matter of] finding [bi-vri'at?] the [whole] amounts together: a result that comes from the growth [me-haba'at ha-ibut], a square [ge-shemurach?] in itself, and its result [ve-ha-ʿilah], surd [niggar], is called the second preserved [ha-shamur ha-sheni]. After this, relate this [reckon the ratio] [yachas]: the root extracted [yesod ha-shamur]…

the second to the root, the first preserved [ha-shamur ha-rishon] and the half [u-ve-machaweh — "and demonstrate"?] surd [niggar — see note]; the root of the fraction extracted [yesod ha-shever ha-garush].

מָשָׁל — The Example

If you wish to extract the root of these surd eighths [shemonayim ha-niggarim — the surd value 8] whose [own] extraction is by themselves [be-mutam] and whose growth is inexact [surd] [bilti niggarim] — behold, here [hineh] he [the value] is surd [niggar] [shehu ha-]; extracted by themselves [be-atzmam], and the result [ve-ye'aleh] 100 [ק']; and the fraction [ve-chaq, "and the part/fraction"], a thousandth [ke-fi? — "a 1000th"?], and there results from these eighths [ve-ye'aleh ele] [a value] that is preserved [ve-nishmaram]. Also take the [whole] amount [ha-kamut], for it is the wholes [integers], just as it is the result [ha-ʿolah] that emerges [ha-muba'at] from the growth [ha-ibut], surd [niggar] in itself; and the result [ve-ha-ʿolah] of these eighths [elef? — "a thousand"], and there is preserved [ve-nishmaram].

After this we seek the root of these eighths [ha-elef? — "the thousand"], and it [the root] is 32 [ל"ב — see note]; and they are preserved. We also seek the root of these eighths [ha-elef], and it is 10 [yod]: take the surd [ha-shever] that is made surd by [ha-niggar] — relate [reckon the ratio] to it [u-niyaches elav]: 32 [ל"ב] preserved, and they are 100 [100 = ק'? — see note] for the tenth-order ones [ha-ʿasiriyim]; and over [the matter of] the tenth-order [ha-ʿasirit], the second of the tenth-order [shenei ha-ʿasiriyim] are combined [ha-leqanus?] at the half [ʿal ha-tzi]; and they [number] 2 wholes, and the tenth-order [ve-ha-ʿasirit] 1 [aleph]; over [for] one tenth [le-lev chalaqim? — "for a heart [number] of parts"?], the rate [moher?] of the parts [chelqei] is completed [ha-shelem]; and one part [ve-chelqei echad] from these eighths [me-elet], complete the parts [tal chelqei] [it] completely [mushlam].

[סור]מציאות חי [המעוקב בתכונה]? — only "מציאות חי" is legible; the remainder runs into the gutter shadow and is not independently verifiable on the tiles] — Finding the Cube Root in Astronomy [bi-tekhunah; section title reconstructed from context, not fully legible]

From here on you will come to know [yivada lekha] the way to find the cube root [ha-meʿuqab] of many kinds of numbers [minei misparim rabbim] from the fractions [mi-shivrei] of [astronomical] computation [be-tekhunah], or fractions and wholes together. And here, in your turning [bi-shenotekha], everything [will be] toward one kind [el min echad], the more so the less [ha-yoter pachot] of them; and then make use [ve-az nishtammesh] of [it] with the manner [be-derekh] of their orders [ha-sederim — degrees/orders].

הַמָּשָׁל בָּזֶה — The Example for This

If you wish to extract the root, take 2 ranks [maʿalot, "degrees"] of the second order [chelek bet], the first ones [ha-rishonim] 2 [bet], the seconds [shenayim] 30 [lamed — ל'], the thirds [shelishiyim] 30 [lamed]; behold take 2 ranks [maʿalot] in sixties [be-shishim — בס'] [and] 6 [vav], [and] the result [ve-yaʿalu] of these 2 ranks [elet bet] the first ones [ri'shonim]: combine them [chabberem] with the [whole] amount [ha-bar? = "the outside"/integers] of the first ones [ri'shonim], and they result [ve-ʿalu]…

the third-order ranks are the result. Then take the firsts again, and combine the firsts with 60 [ס], and the result is 63 [ע"ג]. Take the seconds, combine them with 2 seconds [חב'], and the result is 63 [ע"ג]. Take the seconds again, combine them with 60 [ס], and the result is 4 times [ד' פעמים]: [אלף ות"ו] 1006, and 600 thirds [ת"ר שלישיים]. Combine them [נחברם] with the firsts again [[עם תל' שליש]יים], and the result is 4 times [ד' פעמים]: [אלף אלמים] 1000, and 426 [ת"ז] thirds, and 600 thirds [ת"ר שלישיים]. [?]

To find the thirds [שהשלישיים], it is the deferent[?] [מורגרת], dragged along [נגררת] by reason of what we have set down — the foundation of the figure [יסוד החכמה], one and one, and the result is 263 [קס"ג]. [?]

Again, to count the third of the roof[?] [שליש הגג] which is there, the thirds [השלישיים] are decreed [נגזר] from what counts them [ממנו], and is taken out [וחוצא] of the conversion [המורת] upon the firsts [[על הר]אשונים]; and in this what we know is that the foundation [יסוד] which is 2 conversions [מעלורת] of the firsts is 200 [[ב"ר ר]אשונים] firsts; 2 seconds make 30 thirds — that is 263 [קס"ג]; the firsts and the 2 seconds, which are 2 [ב] conversions [מעלורת] [[מ"ג ר]א?] [?]43, the firsts make 3 [נ"ר ג'? — uncertain] seconds. [?]

Completion formula.

תם ונשלם חזה קצור המלאבת מספר :

The abridgment of the Art of Number [קצור המלאכת מספר — *Kitsur ha-Melakhat ha-Mispar*] is finished and complete.

Diagram. The recto carries the printer's device of Heinrich Petri (Henricus Petrus) of Basel, printed in the center of an otherwise blank leaf. The woodcut shows a hand and forearm emerging from a bank of clouds at the upper right, gripping a hammer (or mallet) and striking the top of an upright rock or anvil-like stone set on the ground; from the point of impact a great burst of flames radiates outward in all directions. Cloud-banks billow at the upper left and upper right. The device puns on the printer's name (Petrus / "rock") and alludes to the rock struck to bring forth fire — the emblem of the Petri press of Basel. No legible motto or lettering is visible within the woodcut at this resolution.

[The faint Hebrew lines visible across the upper portion of this leaf are show-through (offset/bleed) from the printed text on the other side of the leaf, not text printed on this page.]