Refractiones coelestes, sive Solis elliptici phaenomenon illustratum

[Celestial Refractions, or the Phenomenon of the Elliptical Sun Explained]

(1617)

by Christoph Scheiner (1573/75–1650), S.J.

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Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

Refractiones coelestes addresses atmospheric refraction and the apparent elliptical — vertically flattened — shape of the Sun near the horizon. Scheiner presents observations and a geometrical account of horizontal refraction: why the low Sun looks oval, and how refraction distorts measured solar altitudes and diameters, resolving long-standing difficulties astronomers had faced with the subject. An early optical–astronomical monograph illustrated with woodcut diagrams of the refraction geometry, it engages the practical problem of correcting positional observations and prefigures Scheiner's later solar work (*Rosa Ursina*).

A note on the translation process

This translation was produced with the translation-pipeline. See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

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Refractiones coelestes — Front matter

Title page

CELESTIAL REFRACTIONS, OR THE PHENOMENON OF THE ELLIPTICAL SUN EXPLAINED;

in which various and long-standing difficulties of the astronomers concerning this subject are unraveled, manifold doubts are resolved, and a path is opened to bring to light much that lies hidden: a short work exceedingly useful — and indeed necessary — to natural philosophers no less than to astronomers.

By the author **Christoph Scheiner**, priest of the Society of Jesus.

By permission of the superiors.

At Ingolstadt, from the Ederian printing house, at the shop of Elisabeth Angermaier.

With the grace and privilege of His Imperial Majesty.

Dedicatory epistle to Archduke Maximilian III of Austria

To the most reverend and most serene Prince and Lord, the Lord Maximilian, Archduke of Austria, Duke of Burgundy, of Styria, Carinthia, Carniola, etc., Count of Habsburg, of Tyrol, etc., Supreme Master of the Teutonic Order, etc., his own most gracious Lord and Prince.

This is now the second year, most serene Archduke, since — under the safest wings of your Eagle, as she brooded most happily upon the Alpine ridges of the Tyrol — I entrusted to her a certain egg that had drifted down as if from heaven: with great daring, indeed, yet with an outcome no less favorable. For she at once took it into her ready bosom and hid it away, sheltered it most carefully with her softest feathers from every possible harm, and warmed it so tenderly with her kindly heat that it grew into a sturdy chick. And now, while — for the sake of daring itself against the Sun — this chick pants toward the open fields of the ethereal region and steadies itself by its own beats of the wing, it longs above all things to have Your Serenity present to it: for one is not permitted to gaze upon the Elliptical Sun in a troubled sky. Whoever approaches this my labor [elucubration] either with a half-drowsy mind or with a spiteful, jaundiced eye will only summon darkness and mists upon himself. But the shining radiance of Your kindness will dispel all this foul weather, and the gentlest breeze of Your benevolence will breathe it away. Trusting in these, I set out upon a road that is dangerous, indeed, yet not taken with the recklessness of Phaëthon [Phaethon]. And since this is your wish, I comply most readily with your will: I so hold the Sun in check with the reins that, curbed while I guide it by the very neck, it may not plunge forward with neck outstretched to overthrow everything; and from time to time I slow its course, lest it bring on dizziness. While I linger over this the longer, I set out — rather more gracefully, and I think a good deal more distinctly, in keeping with the promises I made concerning the Elliptical Sun — its ornaments [phaleras, the decorative trappings of the argument]. I do indeed wish to satisfy everyone fully, though I scarcely hope to. But You above all — just as I always long to have You at hand — so, if I turn my eyes to the slightness of this little work, I greatly mistrust it; yet if I cast my mind upon Your Clemency, I promise myself everything. This is what emboldens me to attempt a matter far beneath your dignity. And when I consider the boundless benefits conferred by You — whether privately upon me, or publicly upon our Society — then, to escape the fault of an ungrateful heart, I owe You not this small gift [sportula] alone, but my whole self; and I shall count myself restored to myself only when I learn that our services have been most graciously accepted by You. Ingolstadt, 16 August 1617.

Your Serenity's

most devoted in Christ,

Christoph Scheiner.

[Front-matter leaf — no printed page number.]

TO THE READER

That "ellipsis" [*ellipsis*], among the Greeks, signifies the falling-short or defect of anything whatever, dear reader, no one may prove a richer authority than Aristotle, *Nicomachean Ethics* II.6. "Τὸ δ' ἴσον," he says, "μέσον τι ὑπερβολῆς καὶ ἐλλείψεως" [Greek: *to d' ison … meson ti hyperbolēs kai elleipseōs*, "the equal is a certain mean between excess and deficiency"]; that is, the equal is something intermediate between too much and too little. The ancient mathematicians, adapting this word to their own uses, called an "ellipse" every figure that carried some hint of imperfection about it — or rather that very imperfection itself — so that any plane figure not differing much from the curvature of a circle came under this name. Hence the geometers — as one may see in Archimedes [Archimedes], in Apollonius [Apollonius of Perga] in the *Conics*, and in Serenus

[Serenus of Antinoöpolis] in the *Cylindrical Sections* — transferred it, by a kind of special right, to the oblique section of an entire cone.

But I, treading a middle path, have called this atmospheric appearance of the Sun [*hoc Solis Meteorum*] an "ellipse" — neither using that most general sense by which the defect of anything whatever is denoted, nor content with the narrow meaning by which only a uniform departure from a circle is disclosed. I have named it the *solar* ellipse, so that by this it might be set apart from the reckoning of other ellipses. [Margin: Describitur Ellipsis nostra. — "Our ellipse is described."]

For this ellipse is the projection [*eiaculatio*] of a radiant cone from the visible disc of the Sun onto the convex surface of the atmosphere, together with a certain mutual and oblique intersection of the two, extended by refraction into the eye — or into some other point placed within the vapors. Insofar as this section rests upon the convex outer surface of the vapors, to that extent its height surpasses its length in loftiness; but insofar as, once refracted, it converges upon some point — whether of the eye, or of a mirror, or of a sheet of paper held in a dark place — the length exceeds the height, and that by an unequal degree, according as a great or small difference of refraction has intervened. All of this, to anyone who considers it attentively, becomes evident without difficulty. For a spherical surface (such as I posit at the outer limit of the vapors), penetrating a cylinder or a cone obliquely, makes a section longer than the circle inscribed in it and shorter than the circumscribed one — just such a section as the atmosphere introduces into the solar cone, or the cone into the convexity of the atmosphere, so far as the reality is concerned. And this impression, to speak in physical terms, once it is carried down further into the depth of the vapors from the remaining portion of the solar cone, so, by the refraction of the rays, settles [*consi-*]

[Front-matter leaf — no printed page number.]

[…*continued*] settles down, so that its shorter axis outstrips the longer — not in truth but in appearance, caused, that is, by the bending of the cone. This metamorphosis I designate an "ellipse"; and it is my intention to treat of it — to investigate its origin, its cause, its manner, its effect, and the many things that depend on it. [Margin: Comparatio nostræ atq; Geometricæ Ellipseos. — "Comparison of our ellipse and the geometric ellipse."]

This appearance [*Phasis*], accordingly, just as it does not differ much from the Apollonian ellipse in the conformity of its figure — with the eye as judge — so, if we set the rest of their features against one another, differs from it enormously.

The former, treated in the abstract, withdraws itself from all determinate matter; the latter does not pass beyond the bounds of the celestial region. The former the mind forms by imagination, according to the requirement of its own nature; the latter the solar rays and the massed lightness of the vapors, meeting together with nature as their guide, fashion. The former is plane; the latter is a section of a spherical surface with a cone. The former is seen directly; the latter by refraction. The former is simple; the latter is rather to be counted a mixed ellipse. The former is stable by its very nature; the latter remains subject to a continual variety of changes. The former the gaze neither alters in position nor shifts in place; the latter is multiplied according to the eyes of those who behold it, and does not present itself the same to another's sight — indeed, from moment to moment, more changeable than Proteus [Proteus], it transforms itself even for one and the same eye. The former you can fashion in your mind as you please; the latter you cannot behold with your eyes except in a clear sky, and even then, for the most part, only in the morning or the evening.

For these reasons, dear reader, you had to be warned from the outset, so that you might grasp the reasoning behind the etymology — lest you toil in vain seeking the Apollonian ellipse, and be content instead with this astronomical or optical one. And indeed I am fully confident that you will be, if you receive these things in the same candor with which I set them forth.

I had, in fact, been minded to give this phenomenon a name drawn from an egg [*ab ovo*]; but because the two ends of an egg are always unlike each other, while the belly bulges out on every side with equal extension between the two — whereas the opposite happens in the Sun's distortion — I was unwilling to cast down so lofty a spectacle to a low and less fitting name.

As for its appearance, now established over many years, I do not think I need labor to confirm it by witnesses; for day pours forth speech to day at sunrise, and night declares this knowledge to night at the Sun's setting [echoing Psalm 18/19]. No one fails to learn it, save one who lacks eyes; no one fails to understand it, save one who is willfully mad. Open your eyes and look, and you will gain such a witness of this marvel as, of all witnesses, you most require — the one whom, of all, you by far most believe. Farewell, and enjoy.

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PUBLIC APPLAUSE TO THE AUTHOR, FOR HAVING DISCOVERED AND ILLUSTRATED THE SOLAR ELLIPSE

Although in the Mithraic rites [*Mithriacis … sacris*] there is a crude license, and Persia [Persia] claims too much of the Sun for herself — when she monstrously paints the fair eye of the heavens, the Father of Day, with a Lion's face, [Margin: a Bisciola [Lelio Bisciola [?]], bk. 17, ch. 12, from Suidas [the *Suda*].] and that god, so polished, so radiant, so graced with flowing hair, who bore upon his brow no cap but his rays and no diadem — clothes him, all unwilling, in a stunted, narrow tiara, heaping on him just resentment; and with no less unworthy an image she sets forth the Moon: so elegant is the Moon in heaven, yet here, portrayed in the horned form of a heifer, she ridiculously lows back to her brother the Lion. If it did not shame the Persians to devise such mockeries, all the stars in heaven's vault blushed for shame — that their own Queen and King should be seen so basely among four-footed beasts, who surely deserved better.

Whatever, in the end, that injury amounted to, far more impious toward the whole of heaven was Babylon [Babylon]. Whenever she wished to portray fair Phoebus [Phoebus, the Sun], rising in the morning from the river, in his own proper figure — (what could be simpler?) — she painted, near the shores of the Nile, the Lotus. [Margin: b Pierius [Pierio Valeriano], *Hieroglyphica*, bk. 52, from Proclus and Theophrastus.] This rude little shrub, of the meanest worth, in whose fruit no Satyr and in whose shade none might take delight, is called the truest image of the golden-haired Sun. And I would let it pass, if some fungus from the herd of painters should so rave, skilled to counterfeit only the Cypress; but it is Babylon that does this, and she will not seem to have erred.

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What has the Sun to do with the Lotus? What tale, in the end, is it, to liken a Greek Bean to the light of so great a disc? When the Lotus has caught sight of the darkness of night's wing, and has felt its own shadow creeping far, then, as if foreseeing the winters near at hand, the North Wind and the snows, it grows fearful for its leaves, gathers its spreading green into wrinkles, and folds itself wholly into an arch — until the purple of Dawn [Leucothoë] begins to glow fiery in the sky, and, struck once more by the solar rays, it unfolds the tresses it had drawn together in the dark of night. This inconstancy, then, oppresses the wretched Lotus; but the Sun forever shines golden with unmoved countenance, and with a cheerful, rosy face beholds the turning of the years and all the world's many wonders in ageless youth. No wrinkle, through so many ages now, upon his cheeks; nor could grey hairs creep across his brow. Yet Egypt [Egypt] could liken *Him* to the Lotus.

These views I once shared with that crowd which, halting at the very threshold of heaven, saw only the Moon's losses; which credited spots and horns to this one fire alone, and metamorphoses and monthly waning; no one felt that anything else was disturbed among the stars, and the commonwealth seemed safe while the Sun was Consul. Hence we once believed that Canopus [Canopus] was bleary-eyed or raving, when he paints the Sun in the image of the Lotus.

But the wiser flock of the Brahmans [the Brahmans] broods over this one study; to it they devote watchful eyes, to it they devote their years — that they may discern the Sun forced into wrinkles upon his narrow brow. [Margin: c Plin. [Pliny, *Natural History*], bk. 7.] How many naked bands of Gymnosophists did India once nourish? How many thousands did the great Iarchas [Iarchas] send out to stand against the Sun's gaze — a thing scarcely granted even to the eagles themselves — so that after perhaps a hundred years they might see something variable in the Sun's disc? But while the wretched men are badly scorched by the hot sand,

[Conclusion of the *Plausus publicus auctori* — the laudatory poem of the front matter. carry no printed page numbers; the treatise body begins = printed p. 1.]

Plausus publicus (continued and concluded)

And they stand on one foot, like a crane, or a shameless goose. Cheated, they went back home, and, parched, cursed the spiteful delay, then freely drank the everlasting nectar that Tantalus poured from a brimming golden cup — a cupbearer well worthy of the throngs he so often deceived. And who, after all this, would refuse to trust his own eyes? — when Atlas himself, though he stood nearest to the stars, who knew the ells of Heaven from its measuring rays, swears that nothing is rounder than the face of Phoebus [the Sun].

At last a keener eye — an Eaglet, one who could be a prince among the Brahmans — while so diligently he ranges again and again over the spheres above, whether out of loathing for mortal things, or because a spirit blazing with a nobler fire spurred him to the stars, discovered a marvel in the disk of the rising Sun, the very thing that Memphis aptly portrayed under the image of the Lotus. For just as the lotus, its leaves packed close together, lies asleep in the black night, and gently, with the new light, unfolds its foliage from the tip, until no wrinkle any longer binds its opened fronds: so the Sun, when it is given back to the world at a fresh dawn, refuses to come forth with a round face, and, though it is more perfect than the orb of the Argive [Greek] shield, it chooses then to look rather like the Ausonian [Roman] *ancile* [the sacred figure-eight shield] — the kind the Quirites [Romans] say once fell from heaven. [Margin: *d* Livy, book 1] Whether the Sun fashioned for itself in the aether a Shield matching itself, and wished this to shine as a mirror of its own face; or the gods carelessly flung it down in jest. Yet if Rome forced this on the Salii to carry, and Mamurius added more shields copying the shape of the first — with what a face, in what an embrace, will the man who cares for the heavens and toils over the beautiful Sun receive this beautiful and wondrous SHIELD OF THE SUN? This shield you, Scheiner, better than Mamurius, first saw with your keen eye, and now, on pages so learned,

you paint it — so that scarcely anything lovelier now exists even in the sky. Hence you may see many Salii ready to bear this *ancile*, not chanting after the manner of Mamurius; henceforth, Scheiner, you will pass upon the lips of the learned. And unless the Muses give false and empty oracles, this inquiry of yours will win you, with universal applause, the robes of Urania and, at last, the high honors of heaven. For, just as once Strato, a poor man in a Tyrian field, [Margin: *e* Justinus, book 18] because he was the first to catch sight of Phoebus returning from the eastern deep and pointed out with his finger the friendly radiance, received sovereignty over his homeland and a crown for having seen the Sun: so you, while in the midst of the Sun you reveal marvels unseen in earlier years, may hope for a throne among the learned chairs [of scholarship]; a thing everywhere foretold to you by the solar EGG. For as to what the poets swear fell from Heaven — [Margin: *f* Plutarch, *Symposiacs* 2.1] the egg with Pollux, and heavy with the twin of Pollux [Castor] — we cast that back among the trifles that only Greece believes. When you first gave the learned their banquet at table, promising dishes worthy of a patrician's palate, with what an egg did you, a good host, begin the meal? It is in doubt what you will serve at the final course of apples, you who set out the whole Sun at the very first, in an Egg.

BY PUBLIC VOW he sang in accompaniment,

Bernard Renanus [Bernardus Renanus], an Austrian, etc. V. I. D. S. M. A. S. &c. [abbreviated titles; likely *Utriusque Iuris Doctor* (Doctor of Both Laws) followed by further honorifics — remaining letters uncertain [?]] for the sake of honor, etc.

PRELIMINARIES [PRÆNOSCENDA].

CHAPTER ONE [CAPUT PRIMUM].

[The margin carries proposition numbers 1–9, transcribed here as the numbered list.]

1. The visible part of the Sun appears as a plane circle. (By Euclid, *Optics* 25.)
2. The visible boundary of this part is the circumference of a circle. (By Euclid, *Optics* 23.)
3. A plane drawn through this boundary makes, as its common section with the Sun, a circle. (By Theodosius, *Spherics* 1.)
4. From the circumference of this circle, the individual points shining together into the eye toward a single point proceed along straight rays (by Witelo 1.2); and they therefore lie on a conical surface (by Euclid, *Optics*, definition 2, and definition 1 and the first proposition of Apollonius [of Perga], *Conics*, book 1).

Now the figure enclosed under this surface and the solar circle will be the radiant cone of the Sun, or the Solar cone [conus Solaris]: its vertex lies at the eye, its base at the Sun, its rays in the space between; the axis is the ray extended from the center of the base to the vertex.

5. Since, then — as is established by Witelo, book 4, proposition 66, and Euclid, *Optics* 23 — the axis of the solar cone stands at right angles upon its base, the cone is therefore a right cone (by Apollonius, definition 8, book 1).
6. Therefore a plane carried through the said axis and the center of the earth will also pass through the Zenith (by Euclid, book 11, proposition 1); and since the straight line sent from the center of the earth through the eye is the vertical line, this plane is accordingly vertical.
7. The common section of this plane with the Solar circle is a straight line (by Euclid, book 11, proposition 3); and it is a diameter of the Solar circle (by Euclid, book 1, definition 17), cutting it in two at the highest and the lowest point [apsis]. Let the plane making this section be called the **primary altitude plane** [planum altitudinis primarium]; and the section itself, the **diameter of altitude**, or the **primary altitude of the Sun** [altitudo Solis primaria].

8. This plane is perpendicular [rectum] to the solar circle. (By Euclid, book 11, proposition 18.)
9. A plane through the axis of the solar cone, perpendicular to the primary altitude plane, makes as its common section with the Solar circle a straight line at right angles to that same primary plane (by Euclid, book 11, propositions 3 and 19); and it is therefore also at right angles to the diameter of altitude at the center of the Solar circle (by Euclid, book 11, definition 3). Let such a line be called the dia-

—meter of longitude, or the **primary longitude of the Sun** [longitudo Solis primaria]; and the plane making it, let it be called the **primary longitude plane** [planum longitudinis primarium].

10. Now if, through the individual points of longitude, parallels are drawn to the solar altitude, and planes are conceived as carried through those parallels and the vertex of the solar cone, these planes will be called **secondary altitude planes** [plana altitudinis secundaria]; and the parallels themselves, **secondary altitudes of the Sun** [altitudines Solis secundariae].
11. The **secondary longitude planes** will be those which pass through the lines parallel to the longitude and through the vertex of the cone; and the parallels [themselves], the **secondary longitudes of the Sun** [longitudines Solis secundariae].
12. Vertical planes carried through the individual points of longitude — whether primary or secondary — apart from the center of the solar circle, let them be called **planes according to altitude** [plana secundum altitudinem].

Consider the attached diagram. ABCD is the apparent portion of the Sun; through its perimeter a plane surface EFGH is drawn, making the circle ABCDI. From its individual points A, B, C, and D, and from other intermediate ones, rays are projected into the eye V; these lie on the radiant conical surface VABCD, which, together with the base ABCD, produces the solar cone ABCDV, whose vertex is the point

Diagram. A large right-cone construction shown in oblique projection. At the far left is the eye V (drawn as a small eye with rays fanning out), the vertex of the cone. From V a sheaf of straight rays spreads rightward to a large quadrilateral plane (the *planities* EFGH) that frames a circle — the apparent disk of the Sun (base of the cone). The circle carries the cardinal points of the solar disk and its two perpendicular diameters (the visible-altitude diameter and the longitude diameter), together with several intermediate points for the secondary sections. A horizontal axis runs from the eye V through the circle's center I and out to the frame; a vertical trace (K above, L below) marks the altitude plane VKL, and a horizontal trace (M left, N right) marks the longitude plane VMN.

- V (far left): the eye; vertex of the solar cone.
- E (upper-left corner of the frame): corner of the plane surface (*planities*) EFGH.
- F (upper-right corner): corner of the plane surface EFGH.
- G (lower-right corner): corner of the plane surface EFGH.
- H (lower-left corner): corner of the plane surface EFGH.
- K (on the upper edge of the frame): upper end of the trace of the altitude plane VKL.
- L (on the lower edge of the frame): lower end of the trace of the altitude plane VKL.
- M (left, on the frame at circle level): left end of the trace of the longitude plane VMN.
- N (far right, on the frame, on the axis line VI extended): right end of the trace of the longitude plane VMN.
- I (center of the solar circle): the center; foot of the cone's axis IV (the ray IV = axis, perpendicular to the base). [In the text the circle is named "ABCDI"; I is used here as the center of the Solar circle.]
- B (top of the solar circle): upper end of the visible-altitude diameter BD.
- D (bottom of the solar circle): lower end of the altitude diameter BD. [The label at the bottom of the circle is partly obscured and reads ambiguously as D or R [?].]
- A (left/upper-left on the solar circle): a cardinal point on the apparent solar circle (near the longitude diameter).
- C (right/lower-right on the solar circle): a cardinal point on the apparent solar circle.
- T (just outside the circle, upper right): intermediate point for a secondary section.
- P (right, on the frame side near F): intermediate point for a secondary section.
- Q (lower-left within the circle): intermediate point for a secondary section.

- R [?] (lower area of the circle): intermediate point — the same worn bottom-of-circle label that reads ambiguously as **D or R** (see the D entry above).
- S (lower-left, toward the frame): intermediate point for a secondary section.
- c, c (two lowercase points, upper and lower-right on the circle): intermediate points of the circle (distinguished from capital C).

V — at the eye; the base ABCD — the circle in the Sun; the remaining rays are emitted from the individual points of the base to the vertex — such as AV, IV, CV, and so on — in the transparent medium interposed between the Sun and the eye. The axis of the cone is the radius IV, perpendicular to the base, whence the whole cone is likewise a right cone. Through its axis VI and the center of the earth a plane VKL is carried; by the section BD common to it and to the base, it marks out in the base the visible altitude of the Sun, BD. But the plane VMN, perpendicular to the altitude plane VKL through the axis of the cone VI, [is]

perpendicular, it makes in the base the section AC, which is the longitude of the visible Sun. The plane VMN is called the primary longitude plane, and VKL the primary altitude plane. The remaining planes — drawn parallel to the altitude and to the longitude, such as OP and QR parallel to the longitude, and OS and TR parallel to the altitude, and carried through the vertex V of the solar cone — are called secondary planes, either of altitude or of longitude. Finally, the vertical planes drawn through the individual points of the longitude AC are the planes taken according to altitude.

I call the line AC the *longitude* of the Sun, not its latitude, because when the Sun stands on the meridian this line extends along the longitude of the sky.

Now, if the eye and the visible Sun lie in the same transparent medium, the rays reach the eye undeflected. But if the Sun is in one transparent medium and the eye in another, the eye receives the rays of the solar cone refracted, and refraction occurs at the boundary of the two transparent media — what the optical writers call the *refracting surface* [superficies refringens]. The ray falling from the Sun onto the refracting surface is the *ray of incidence*, or the *incident ray* [radius incidentiæ, vel incidens]; the point common to the incident ray and the refracting surface is the *point of refraction* [punctum refractionis]; the incident ray carried on from this point to the eye is called the *refracted ray* [refractus]; and the line raised perpendicularly from the point of refraction upon the refracting surface is called the *perpendicular* to or from which the refraction takes place. Since in our case the refracting transparent body is the region of vapors, concentric with the earth, it may not unfittingly be named the *Atmosphere* [Atmosphæra]; and so the line drawn from the center of the earth through the point of refraction will be the perpendicular we are speaking of.

[Margin: See Witelo [Witelo] in the definitions to Book 10.]

See the diagram below, in which ABC is the Sun, sending from the points A, B, and C the rays AD, BE, and CF into the Atmosphere DEF, which is concentric with the earth at the common center G. These rays are called the *lines of incidence*. The points D, E, and F, received on the convex surface of the vapors, are the *points of refraction*; and from these points, extended to the eye...

Diagram. A refraction schema for the passage of solar rays through the atmosphere. At the right is the Sun, drawn as a small circle with three marked points; at the left is the eye. Concentric circles about a central point represent the earth and the surrounding vapor region (the atmosphere), whose convex surface is where refraction occurs. Rays run from the Sun to the atmospheric surface (the incident rays), and from there, deflected, to the eye (the refracted rays); radial lines from the common center to the refraction points are the perpendiculars.

- A (Sun, upper point): a point of the Sun, source of ray AD
- B (Sun, middle point): a point of the Sun, source of ray BE
- C (Sun, lower point): a point of the Sun, source of ray CF
- D (upper convex surface): point of refraction of ray AD
- E (convex surface, middle): point of refraction of ray BE
- F (convex surface, lower): point of refraction of ray CF
- G (center of the concentric circles): common center of the earth and the atmosphere; origin of the perpendiculars GD, GE, GF
- H (left): the eye, where the refracted rays DH, EH, FH converge

the eye H, the straight lines DH, EH, FH are called the *refracted rays*. From G, the center of the vapors and of the earth, the straight lines GD, GE, and GF drawn to the points of refraction D, E, and F are called the *perpendiculars*, to or from which the refraction takes place. The surface DEFD is the *refracting* or *refractory surface*; and likewise the transparent medium DH, reaching from the vapors all the way to the eye, is the *refractive body* and the substrate of the refracted line.

The surface in which the line of incidence AD and the refracted line DH lie is called the *surface* or *plane of refraction* [superficies siue planum Refractionis]. See the Definitions of Witelo on Book 10.

The angle ADI — at the point of incidence or refraction D, contained by the line of incidence AD and the perpendicular GDI drawn out from D to I — is the *angle of incidence* [angulus incidentiæ]; by others it is called the *inclination* of the line AD upon the surface DF. The angle that the line of refraction DH and the line of incidence AD, drawn out from D toward H, would make, would be the *angle of refraction* [angulus refractionis].

The angle HDG is called the *refracted angle* [angulus refractus], and is contained by the refracted line and the perpendicular GH. From this it must be carefully noted that the angle of refraction is one thing and the refracted angle another. For both angles are comprised by the produced incident ray and by the perpendicular; they are the parts of which each is the complement of the other up to the aforesaid angle — that is, the angle of incidence — as is evident to anyone who considers it.

Preliminary Theorems [Protheoremata] — Chapter II [Caput II]

1. In any surface of refraction there necessarily lie: the *radiating* or *visible point* [punctum radians siue visibile]; the *point of refraction*; the *point of the eye* from which and in which vision is accomplished; and the perpendicular line drawn from the point of refraction upon the refracting surface. This is from Witelo, Book 10, theorem 1. See the *Elliptical Sun* [Sol Ellipticus], p. 17, in the response. From this, as from a foundation, the following preliminary theorems, suited to our purpose, are easily deduced and subsumed.

2. The plane of solar refraction, extended in every direction, necessarily falls on the center of the earth. For it falls on the center of the Atmosphere, which is the center of the earth, by number 13, chapter 1. That this same plane passes through the center of the Atmosphere is clear from this: it occupies the perpendicular line, by the first preliminary theorem; this line has the center of the Atmosphere within itself; and the Atmosphere is concentric with the earth. See the *Elliptical Sun*, p. 15, and elsewhere.

3. The plane of solar refraction, produced as before, necessarily falls on the vertex of the solar cone. For that vertex coexists with the point of the eye by which vision is accomplished...

accomplished; therefore, by the first preliminary theorem, if the plane of refraction falls on that point, it falls also on the apex of the solar cone. Which was to be proved.

4. The plane of solar refraction necessarily falls on the ray of incidence issuing from the visible point. For it falls on the visible point and on the point of refraction, which are the extremities of the incident ray. And a ray is like a straight line, which lies wholly in that plane in which it has its extremities.

5. The plane of solar refraction necessarily falls on the line of refraction. For it falls on the point of refraction and on the point of the eye, which bound the line of refraction.

6. The plane of solar refraction necessarily falls on the vertical line, and consequently on the Zenith and Nadir. For it falls on the center of the earth, by the second preliminary theorem, and on the point of vision, by the first; and these lie on the vertical line, since a straight line extended from the center of the earth through the eye into the sky is the vertical. Therefore the plane of refraction likewise (Euclid [Euclid], *Elements* XI.1).

7. Therefore, from first to last: *Every and only plane of solar refraction is some vertical plane.* For in every plane of solar refraction there must necessarily be present the perpendicular line drawn through the center of the earth, together with the vertical line running from the center of the earth through the eye to the Zenith. But these two lines can be found together only in some one vertical plane; and wherever some vertical plane is drawn through the Sun, in it these two lines are found. Therefore every and only plane of solar refraction is some vertical plane. Hence, since in no longitude plane (the Sun being placed outside the Zenith) do the vertical line and the perpendicular line exist together, no longitude plane is a plane of refraction.

Thus, no secondary altitude plane, when the Sun is placed outside the horizon, is a plane of refraction.

Thus, the primary altitude plane alone, always and everywhere, is a plane of refraction.

Thus, all planes taken according to altitude, with the Sun existing anywhere, are always planes of refraction.

Thus, one longitude plane, with the Sun placed in the Zenith, is a plane of refraction.

Thus, every secondary altitude plane, with the center of the Sun set on the horizon, is a plane of refraction.

I now demonstrate each and all of these individually. But first, once and for all, I give notice: whenever I make mention of the solar cone, I mean it indeed, on the side of the actual thing, to be for the most part refracted...

to be for the most part refracted; yet it matters little in which way it is now conceived, until refraction is treated in what follows. For this does not bear on the present matter, and from the subject-matter it is easily gathered which cone is being spoken of.

No Longitude Plane, with the Sun Placed Wholly Outside the Zenith, Is a Plane of Solar Refraction — Chapter III [Caput III]

Since every plane of refraction is a vertical plane, by preliminary theorem 7, chapter 2; and yet no longitude plane, under the assumed hypothesis, is a vertical plane; therefore no longitude plane is a plane of refraction. The minor premise is proved. The longitude of the Sun is perpendicular to the primary altitude plane, by number 9, chapter 1. The secondary longitudes are parallel to the Sun's longitude, by number 11, chapter 1; therefore they too are perpendicular to the primary altitude plane (Euclid, *Elements* XI.8). Therefore the planes drawn through all these longitudes and the vertex of the solar cone are perpendicular to the primary altitude plane (*Elements* XI.18). Therefore their common section, being a straight line drawn through the apex of the solar cone, is perpendicular to the primary altitude plane (*Elements* XI.19). Therefore it is also perpendicular to the vertical line, by definition 3 of Book 11. But the primary altitude plane passes through the vertical line, by definitions 6 and 7 of chapter 1; and with the Sun placed wholly outside the Zenith, this line lies wholly outside the solar cone and shares only its apex, whereas all longitude planes cut the whole solar cone along the longitude. Therefore, by *Elements* XI.1, it is impossible that the vertical line be in any longitude plane when the Sun is placed wholly outside the Zenith. Therefore no longitude plane, and so forth, is a plane of refraction. Which was to be demonstrated.

In the adjoining diagram, A is the Zenith; BCD is the Sun outside the Zenith, whose primary altitude is BD; AEDF is the primary altitude plane, passing through G, the center of the eye — which is at the same time the apex of the solar cone — and through the Zenith A, so that AGF is the vertical line. The primary longitude of the Sun is CH, cutting the primary altitude at the center of the Sun, I, at the right angles CIB, CID, etc. The secondary longitude of the Sun is KL, cutting the primary altitude at M at the right angles KMB, KMD, because [being parallel] to the primary longitude...

Diagram. An oblique (perspective) construction for Chapter III. The parallelogram AEDF represents the primary altitude plane, shown standing vertically; within it the Sun is drawn as the circle BCD with center I. The vertical line AGF runs from the Zenith A down through the eye G to F. The Sun's primary altitude BD is its (vertical) diameter; the primary longitude CH and the secondary longitude KL cross the altitude at right angles, at I and M respectively. Points N and O lie on the lower boundary of the figure; their roles are given in the text that continues on the following page.

- A (top-left corner): the Zenith; upper end of the vertical line AGF
- E (top-right corner): corner of the primary altitude plane AEDF
- B (top of the Sun circle): upper end of the primary altitude BD
- C (right of the Sun circle): endpoint of the primary longitude CH
- D (bottom of the Sun circle): lower end of the primary altitude BD; also a corner of plane AEDF
- F (bottom-left corner): lower end of the vertical line AGF
- G (on line AF, lower left): center of the eye, the apex of the solar cone
- H (left, on the Sun's horizontal): opposite endpoint of the primary longitude CH, through center I
- I (center of the Sun circle): intersection of the primary altitude BD and the primary longitude CH
- K (right, below C): endpoint of the secondary longitude KL
- L (left, below H): endpoint of the secondary longitude KL
- M (on the altitude BD): intersection of the secondary longitude KL with the primary altitude
- N (far bottom-left): point on the lower boundary of the figure [role given in the following page's text] [?]
- O (bottom, between G and D): point on the lower boundary of the figure [role given in the following page's text] [?]

is parallel to the primary CH [æquidistiterit(?) — printed *œquiſtiſtet*]. Hence the planes CHNO and KLNO — drawn through these longitudes CH and KL and through the apex G of the solar cone — are perpendicular to the primary altitude plane AD, forming with it common sections, the lines IG and MG, which are contained within the solar cone; and forming between themselves a mutual section NO, which is perpendicular to the plane AEDF at the point G, and therefore perpendicular also to the vertical GA. Since NO lies wholly outside the solar cone (which is brought to a point at G), it also lies wholly outside the longitude planes. From this it is clear that no solar longitude planes can, when the

Sun is in such a position, be vertical planes, and so on — which was to be demonstrated.

No secondary altitude plane, when the Sun is wholly outside the horizon, is a plane of refraction. — Caput IV

Because no such plane is a vertical plane. For the common section of all vertical planes is the vertical line, drawn through the vertex of the solar cone and the center of the eye; but the common section of the secondary altitude planes is not the vertical line. Therefore the secondary altitude planes are not vertical planes. That their common section is not the vertical line I demonstrate as follows.

The line drawn through the vertex of the solar cone perpendicular to the primary longitude plane is the common section of those planes, by Euclid 11.19; it is therefore parallel to the altitude, by 11.6, and hence at right angles to the axis of the solar cone, by Euclid 1.29. But when the Sun is wholly raised above the horizon, the axis of the solar cone stands at oblique angles to the vertical line; for the size of that angle is the arc intercepted between the center of the solar circle and the Zenith, which is smaller than the arc between the Zenith and the horizon (since the Sun is placed above the horizon), and is therefore smaller than 90°. Therefore the Sun's altitude, known from other sources and subtracted from 90°, will give as its remainder the inclination of the axis to the vertical line; and the complement of this to 90° will give the inclination of the common section of the secondary altitude planes to the vertical line. This common section, then, is not the vertical line; therefore it does not fall to the center of the earth; therefore those planes are not vertical planes, since the common section of all vertical planes is the vertical line. Therefore the secondary altitude planes are not planes of refraction when the Sun is wholly outside the horizon — which was to be demonstrated.

Only the primary altitude plane is always and everywhere a plane of refraction. — Caput V

Because the center of the Sun and of the earth always lie in it, and likewise the vertex of the solar cone and the Zenith, it is clear that the point of refraction (or of incidence) equally lies in it, wherever the Sun may finally be placed — by what we said in chapter 2 among the premises. And so within it lie both the vertical line and the perpendicular, and the incident and the refracted rays; and thus it always and everywhere remains the plane of solar refraction — which was to be shown.

The planes taken according to altitude, wherever the Sun is placed, are always planes of refraction. — Caput VI

Because wherever the Sun may be, a plane can always be drawn through some point of it (marked ⊕) lying outside the primary altitude, and through the Zenith and the center of the earth or of the eye, by Euclid 11.2 — since three such points constitute a single triangle, by postulate 1 of book 1 and definition 21. Every such plane, therefore, is a plane according to altitude, by premise 12 of chapter 1, and a vertical plane, by the same premise, and consequently a plane of refraction, by protheorem 7 of chapter 2. Which was to be shown. From this it is established that, when the center of the Sun is placed in the Zenith, apart from the primary altitude plane and the primary longitude plane, all the remaining planes drawn through the center of the Sun are planes according to altitude.

When the center of the solar circle is located in the Zenith, among all the longitude planes the one plane that is the primary longitude plane is a plane of refraction. — Caput VII

Because among all the solar longitude planes the primary longitude plane is the only one that is drawn through the center of the solar circle, according to premise 9 of chapter 1; whereas the secondary longitude planes are all parallel to this one, drawn through other points of the Sun, by premise 11 of chapter 1. When, therefore, the center of the solar circle is set in the Zenith, the primary longitude plane will pass through the Zenith; and it also passes through the vertex of the solar cone, and hence through the center of the earth and the whole vertical line, by Euclid 11.1. Therefore it is itself a vertical plane; and so also a plane of refraction, by protheorem 7 of chapter 2. Therefore, when the center of the solar circle is located in the Zenith, among all [the longitude planes], and so on — which was to be demonstrated [printed *demonstrandnm*].

When the center of the solar circle is located in the Zenith, among all the altitude planes the one plane that is the primary altitude plane is a plane of refraction. — Caput VIII

Because in this case the secondary altitude planes are not vertical, by premise 10 of chapter 1. But the primary altitude plane is both vertical and a plane of refraction, by protheorem 7 of chapter 2. That plane alone, therefore, in this case, triumphs [printed *triumphar*] among the altitude planes. Which was to be shown.

When the center of the solar circle is placed outside the Zenith — at any other point whatever — among all the secondary longitude planes one plane, and one only, is a plane of refraction. — Caput IX

Because through that solar point which is in the Zenith only a single line, parallel to the primary longitude, can be drawn, and through it a secondary longitude plane, by premise 11 of chapter 1. Since this passes through the Zenith alone, it will be the only vertical one, and therefore a plane of refraction. Which was to be shown.

Every altitude plane, while the center of the solar circle lies on the horizon, is a plane of refraction. — Caput X

Because when the center of the solar circle is set on the horizon it is distant from the Zenith by a quadrant

of a circle, as the common opinion of all astronomers holds and daily experience confirms. Therefore the vertical line is perpendicular to the axis of the solar cone at that cone's vertex, by Clavius's theorem 1, on Euclid *Elements* 3.27, and often elsewhere [Clavius, comm. on Euclid III.27]. But the Sun's primary altitude is likewise perpendicular to the axis of the solar cone, by Euclid 11.19. And both this and the vertical line lie in the same primary altitude plane, since that plane is a vertical plane and the Sun's altitude is the common section of it with the solar circle. Therefore the Sun's primary altitude and the vertical line are parallel to each other, by Euclid 1.28. Therefore the secondary altitudes, since they are parallel to the primary altitude (by premise 10 of chapter 1), will also be parallel to the vertical line, by Euclid 11.9.

And since the primary altitude is perpendicular to the primary longitude plane — because the primary altitude plane is perpendicular to the primary longitude plane, by premise 9 of chapter 1 — and the solar circle is likewise perpendicular to that same longitude plane, by Euclid 11.18, and since moreover the solar circle and the primary altitude plane cut each other along the Sun's primary altitude, all the secondary altitudes will be perpendicular to the same primary longitude plane, by Euclid 11.8. To that same plane, therefore, the vertical line too will be perpendicular, by proposition 8 of the same book, and this at the apex of the solar cone.

Moreover, all the planes of the solar altitudes, drawn through the solar altitudes and the apex of the solar cone, are perpendicular to the primary longitude plane, by Euclid 11.18. Therefore their common section too, made through the apex of the solar cone, is a straight line perpendicular to that same primary longitude plane, at the same vertex of the solar cone, by Euclid 11.19. Therefore at the same point of that plane — the point that is the cusp of the solar cone — there fall both the vertical line and the common section of the altitude planes. And since each line has been proved perpendicular to the same plane at the same point, the common section of all these planes will coincide with the vertical line, by Euclid 11.13. Therefore the common section of all these planes is the vertical line, and they are therefore vertical planes. Therefore all the planes of the solar altitudes, when the center of the solar circle is set on the horizon, are planes of solar refraction. Which was to be shown.

Diagram. A perspective figure illustrating the geometry of Caput X: the Sun on the horizon, with the eye at A. At the left stands an upright rectangular panel, the *vertical plane* ABCDE (B at the upper left, C at the upper right, E at the lower left, D at the lower right of the panel); the segment AB is the *vertical line*. Extending to the right in receding perspective is the *horizontal plane* HIKL (with L and A at the near-left edge, K below, H at the far interior along a dashed line L–H, and I at the far right). At the right sits the disk of the Sun as a circle centered on G, its center; a vertical diameter runs from F/M at the top down to D/N at the bottom (the line DF), with P marked just below the center G. From the eye A, straight and dashed rays run out to points H, O, and I on the solar disk, delineating the half solar cone AHOI, whose axis is GA. The text of this page names AB, the vertical plane ABCDE, the line DF cutting the Sun at G, the horizontal plane HIKL, the eye A, the axis GA, and the half solar cone AHOI; the roles of M, N, O, and P beyond their positions are carried into the following page (catchword "A, cu-").

- A (near-left, apex of the cone): the eye; apex of the half solar cone AHOI and lower terminus of the vertical line AB; the axis GA radiates into it.
- B (upper left of the panel): upper terminus of the vertical line AB; top-left corner of the vertical plane ABCDE.
- C (upper right of the panel): top-right corner of the vertical plane ABCDE.
- D (lower right of the panel / foot of the solar disk): lower endpoint of the diameter DF; adjacent to N at the bottom of the solar circle.
- E (lower left of the panel): bottom-left corner of the vertical plane ABCDE.
- F (top of the solar disk): upper endpoint of the diameter DF; adjacent to M.

- G (center of the solar disk): center of the Sun / center of the solar circle; origin of the axis GA.
- H (far interior of the horizontal plane, on the dashed line from L): corner-point of the horizontal plane HIKL and a base-point of the half solar cone AHOI.
- I (far right, on the solar disk): corner-point of the horizontal plane HIKL and a base-point of the half solar cone AHOI.
- K (below A, near edge): corner-point of the horizontal plane HIKL.
- L (near-left edge of the horizontal plane): corner-point of the horizontal plane HIKL; near end of the dashed line L–H.
- M (top of the solar disk, beside F): point on the solar circle at the top [role stated on the following page].
- N (bottom of the solar disk, beside D): point on the solar circle at the bottom [role stated on the following page].
- O (right of the solar disk, beside F/I): point on the solar circle; a base-point of the half solar cone AHOI [role stated on the following page].
- P (just below center G): point on the vertical radius below the center [role stated on the following page].

In the adjoined diagram everything is clear. In it AB is the vertical line; ABCDE is the vertical plane, cutting the Sun at its center G along the line DF; the horizontal plane is HIKL, in which G, the center of the Sun, lies. This center radiates into the eye A the axis GA itself — [the axis] of the half solar cone AHOI. A, cu-

whose base is the visible Sun HOI and whose apex is the point A. HI, the intersection of the horizontal plane with the Sun, is the primary Longitude [Solis Longitudo primaria], and HIKL is the primary Longitude plane [planum Longitudinis primarium]. The primary Altitude of the Sun [Altitudo Solis primaria] is the straight line DF, which cuts the primary Longitude HI at right angles — the angles HGF and FGI — at the point G. Hence MN and OI, the secondary Altitudes [altitudines secundariae], also cut that same Longitude HI at right angles, at the points P and I. The planes carried through these lines are therefore perpendicular to the plane HK, and for that reason they share a common section, the vertical line BAE. They are consequently vertical planes, and thus surfaces of refraction [superficies Refractionum] — as has been effectively demonstrated.

COLLIGENDA EX DICTIS. [What is to be gathered from what has been said.]

CAPUT XI. [Chapter XI]

From what has been demonstrated so far, it is easy to gather which of the circles marked out by these planes in the heavens, in the atmosphere, and on the earth are great circles and which are not. Since all the circles of refraction are vertical, they will be great circles; the rest will not. Yet these circles take the same name as the planes in which they lie.

[Margin: 1.] Accordingly, among the circles of the Solar Longitudes, the only great one is that which — being carried through some solar point placed at the zenith — stands at right angles to the primary Altitude circle. Hence, when the Sun is on the meridian, this circle coincides with the primary vertical. And since the individual points of the Sun can reach the zenith one after another, it can happen that the several Longitude circles become great circles, the same ones that serve as the primary vertical. All the rest, thrown off from the zenith, are not great circles; and that one is always smaller which is carried through the solar point nearer the horizon. It follows that the circle which meets the sensible horizon [circulus horizontalis sensibilis] is the smallest of all the Longitude circles — which may be shown in passing as follows. The sensible horizon is a circle that touches the earth without cutting it, whereas all the others, carried through the higher points of the Sun and through the apex of the solar cone [cuspis coni solaris], do cut the earth — as can be rightly demonstrated from Euclid (*Elements* III.16) and from proposition 5 of book 11 of the same work, for they meet the earth's surface obliquely, and so on. Therefore, by Theodosius (*Spherics* I.6), all the rest are greater than the sensible horizon circle.

Diagram. A circular figure at the lower right of the page, illustrating the accompanying text ("In the adjoining figure…"). A large circle is drawn with a small concentric circle (the earth) at its center. A vertical line BC runs from top to bottom through the center A. A roughly horizontal line ED runs across, tangent to the small circle at its top; an oblique line from G descends through the tangent region to H at the lower left. The labelled points are:

- A (center, within the small inner circle): center of the earth [centrum terrae].
- B (top, on the great circle): the Zenith [Zenith]; upper end of the vertical line BC.

- C (bottom, on the great circle): the Nadir [Nadir]; lower end of the vertical line BC.
- D (right, on the great circle at horizon level): a point lying on the sensible horizon.
- E (left, on the great circle at horizon level): the far end of the sensible-horizon line DE.
- F (top of the small inner circle): the point of tangency where line DE touches the earth; here stand both the eye and the apex of the solar cone.
- G (upper right, on the great circle): a point taken above the horizon D.
- H (lower left, on the great circle): the far end of the line GH drawn from G through F.
- K (near the top, just left of F, on the small circle's neighborhood): endpoint of the radius AK.
- I (just below F, near the center): the foot of the perpendicular AI dropped from the center A onto the line GH.

In the adjoining figure, A is the center of the earth, B the Zenith, C the Nadir; BC is the vertical line. D is a point lying on the sensible horizon; DE is the line of the sensible horizon, tangent to the earth at the point F — where both the eye and the apex of the solar cone stand. Since, then, DE touches the circle FA at F, if a point G is taken above the horizon D,

and the straight line GH is drawn from it through F, this line will cut the circumference of the circle AF; and the perpendicular AI dropped from A to it will be shorter than the radius AK. It will therefore be nearer to the center A than the tangent DE is, and so DE will be smaller than that circle. In this way it will be proved that DE is smaller than any other. If, then, some point of the Sun coincides with D, the circle carried through it and through F will be smaller than one passing through G and F, or through B and F, and so on.

[Margin: 2.] Among the Altitude circles, the primary Altitude [circle] is always the greatest — the one that, when the Sun is on the meridian, coincides with that same meridian. The rest become great circles when the Sun is placed on the horizon; but away from it they are always not great, and always smaller the higher the Sun climbs above the horizon. The smallest of all is the one that, with the Sun's center placed at the zenith, passes through the outermost point of the primary Longitude.

[Margin: 3.] For this reason, on a sphere set outside the tropics, no Longitude circle is ever a great circle; but on any sphere whatever, every Altitude circle is a great circle when the Sun's center occupies the horizon. On a sphere set within the tropics, when the Sun crosses the zenith, those Longitude circles successively become great circles which are described through the points passing beneath the zenith; but none of the Altitude circles do so except the primary one, with which there is always just one Longitude circle — the primary, if the Sun's center lies at the zenith, or a secondary one, if some other point of the Sun does.

[Margin: 4.] The remaining circles taken according to Altitude are always great circles, whether the Sun is at the zenith, outside it, or on the horizon, in whatever position.

[Margin: 5.] Since, then, with respect to any given place the Sun can hold every position it occupies in the heavens under three heads — vertical, horizontal, and intermediate — the matter may be set out by way of a synoptic recapitulation [Anacephaleosis Synoptica; Greek ἀνακεφαλαίωσις (anakephalaiōsis, "summary recapitulation")]:

1. When the Sun is horizontal, with respect to its center, all the Altitude circles both of the whole and of the Sun are circles of refraction.

2. When the Sun is intermediate, only the primary Altitude circle, together with all the circles both of the whole and of the Sun taken according to Altitude, are circles of refraction [*Refractitionum* as printed — a compositor slip for *Refractionum*].

3. When the Sun is vertical but heterocentric [i.e. not concentric with the zenith], only one primary Altitude circle and only one secondary Longitude circle, together with all the rest both of the whole and of the Sun taken according to Altitude, are circles of refraction.

4. When the Sun is vertical and homocentric [concentric with the zenith], one primary Altitude circle, one primary Longitude circle, and all the rest taken according to Altitude, are circles of refraction.

5. When the Sun is horizontal, with respect to its other points lying off-center, only the primary Altitude circle, together with the remaining circles taken according to Altitude, are circles of refraction.

Once these things have been settled, it will be easy to show according to which dimensions the Sun is refracted in each and every position.

SECUNDUM QUAM DIMENSIONEM SOL REFRINGATUR IN SITU quocunque. [According to which dimension the Sun is refracted in any position whatever.]

CAPUT XII. [Chapter XII]

The passive dimension of the Sun [Solis dimensio passiva] in the present case is understood as a straight line in the solar circle, marked out by some one of the planes already discussed.[a] One such dimension, then, is of Altitude — and this is either total, drawn through the center, or partial, drawn off-center. Another is of Longitude, likewise total and partial. Another is taken according to Altitude, sometimes partial, namely when it lies off-center, but sometimes total, when it passes through the center. Of these there is something to be said.

[Margin a: The sense of the question is explained.]

CONCLUSIO I. [Conclusion I]

The Sun, then, when concentric with the vertex [i.e. the zenith], is refracted by the total dimension of Altitude and of Longitude, and by every dimension taken according to Altitude. Reason: because the planes of all these dimensions are vertical. Hence, since such planes pass through each single point of the solar perimeter, the whole Sun must necessarily be refracted. And since the rays gliding down from the periphery of the solar circle into the atmosphere strike at angles equal to one another, the refraction of the whole Sun must necessarily be equal and uniform.[b] But because these incidences are very nearly perpendicular — the outermost point of the Sun being distant from the Sun's center, and hence from the zenith, by barely 18 first minutes — it is utterly impossible to perceive such a refraction by the senses; on account of its smallness it is known by reasoning alone.

[Margin b: The refraction of the whole Sun is uniform, but imperceptible to sense.]

CONCLUSIO II. [Conclusion II]

The Sun, when vertical but not concentric with the vertex, is refracted according to one total dimension of Altitude only, and according to one partial dimension of Longitude only; while all the other dimensions according to which it is refracted are according to Altitude, and are therefore partial. Reason: because the planes of all these dimensions are vertical planes, and all the points of these dimensions lie outside the zenith, except for the single vertical point common to all — which alone radiates perpendicularly into the atmosphere, while all the rest do so obliquely. Therefore all are refracted. And since in this case the total dimension of Altitude is divided by the vertical point into two unequal parts, and is itself carried through the center of the solar circle, then by foreknowledge 7 of chapter 1, that segment in which the Sun's center lies will be the greater, and the other the lesser. The individual remaining dimensions off the zenith, interposed between these segments, are smaller than the greater segment of the total Altitude but larger than the smaller, and so on, by Euclid (*Elements* III.7). Hence it comes about that, among all the points of the solar periphery, the lowest apsis [absis] of the total dimension is the farthest of all from the zenith, and the highest the least distant; while of the rest, those nearer the highest apsis are less distant, those farther from it more so. Therefore, since all the points of the solar periphery recede unequally from the zenith, they must, when they glide down into the atmosphere, make oblique incidences that are unequal among themselves. It is necessary, then, that the Sun be refracted unequally and non-uniformly.[c] And since the obliquity of incidence is the greater the farther the radiating point recedes from the zenith, and since a greater refraction follows upon a greater obliquity of incidence, it necessarily follows that the lowest apsis of the Sun is always refracted most of all, and always more so those points that are nearer to it, less so the more remote, and least of all the highest apsis. Yet, however entirely true all this may be, because the rays of incidence are very nearly perpendicular to the surface of the atmosphere, it again turns out on this hypothesis that this whole refraction, though it does exist in reality, is nonetheless perceived by no sense but by reasoning alone.

[Margin c: The refraction of the whole Sun is non-uniform, but imperceptible.]

CONCLUSIO III. [Conclusion III]

The Sun, when placed entirely outside the zenith and the horizon, is refracted according to one total dimension of Altitude only, and, in the remaining dimensions of the Sun that are taken according to Altitude, by partial dimensions. Reason: because in this position of the Sun only the planes of these dimensions are planes of refraction, by chapters 3, 4, 5, and 6. And since all these dimensions, when produced in a straight line, meet only at the zenith, none of them will either touch or cut the total Altitude dimension outside the zenith, by the 10th axiom in Clavius (*Elements* book 1). And since all of them descend from the zenith onto the individual points of the solar periphery, of those falling on the concave [side] the greatest will be the one in which the total Altitude dimension lies, while of the others the one nearer to this is always greater than the more remote; and of those falling on the convex periphery, the smallest is the one between the zenith and the Altitude, the rest being in order greater and greater, by Euclid (*Elements* III.8) — or (if one prefers to proceed by arcs)

by theorem 5, in the scholium to proposition 21 of book 2 of Theodosius, as given in Clavius (Theodosius, *Spherics* II.21, scholium, theorem 5). Therefore all the points of the solar periphery, taken along a semicircular arc lying toward one side of the Altitude, are unequally distant from the zenith — the lowest apsis [absis] of the Altitude most of all, the highest least, and so on. For this reason the rays falling from these points onto the atmospheric surface all make unequal angles of obliquity, and those that proceed from the points farther from the zenith are the greater; hence it follows that these too are refracted still more. Therefore of all points the lowest apsis is refracted most, the highest least.

And since all these dimensions cut the Altitude drawn out to the zenith, it follows that outside the zenith they run alongside it toward the horizon. Those that advance into the concave part of the solar periphery all cut the Longitude of the Sun, whereas those that lie only in the convex part, drawn out beyond the Sun, cut the line of solar longitude produced. From this it results that these dimensions of the Sun, when it is placed outside the zenith, are indeed oblique and intermediate, lying as it were between the Altitude and the Longitude — yet they come far closer to the nature of the Altitude dimensions than to that of the Longitude dimensions, since they are separated from the former by very little and from the latter by very much. For the greatest longitude of the Sun is only its own diameter, at most 36 minutes. Hence, as the Sun withdraws from the zenith, the angles that the solar Longitudes subtend at the zenith always become smaller, while those made toward the longitudes by the intermediate dimensions always become larger and nearer to right angles. From which it follows that the longitudes of the Sun, when the Sun is near the zenith, do undergo some refraction; but because their incidence along the longitude is very slightly oblique — inasmuch as it can never depart more than 18 minutes from the perpendicular along the longitude — that refraction is scarcely perceptible. And it will be felt less and less as the Sun approaches the horizon, because the incidence along the longitude becomes always less and less oblique, since the perpendicular line comes ever closer to the incident rays along the longitude. The opposite happens with the Altitude dimensions: the incident rays fall the more obliquely upon the atmosphere along the altitude circles, the farther the Sun has departed from the zenith. Whence it follows that the Sun is refracted more along the altitude [d] than along the longitude.

[Margin: d — The whole Sun is refracted non-uniformly and perceptibly.]

How great a departure of the Sun from the zenith produces a perceptible refraction, I have not yet ascertained with certainty. This is now the third year in which I have been attending to this phenomenon, and although most intent upon the matter, I have never been able to obtain a perceptible refraction with the Sun removed 50 degrees from the zenith. The greatest elevation of the Sun above the horizon at which it was possible to catch any sense of this refraction was between 30 and 40 degrees. Therefore, arguing from the greater to the less: if the altitude refraction is not felt in so oblique an incidence of the Sun as follows upon so great a distance from the zenith — namely 50 degrees — how much less will that of the longitude be felt, which arises from so slight an obliquity, one that the ray produces without receding even half a degree from the perpendicular? And if, with the Sun placed at the zenith,

the refraction along the longitude is not felt, how much less outside the zenith, since this longitude refraction is always the smaller, the greater the departure from the zenith?

CONCLUSION IV.

The horizontal Sun, in the part that lies in the plane of the horizon, is refracted in those dimensions alone that are of the Altitudes; in the part that lies outside the plane of the horizon, it is refracted in those dimensions alone that are along the Altitude. The last member of the Conclusion is established from what has just been said. The first is proved from this: that the vertical planes passing through the individual points of the horizontal longitude are planes of the Altitudes, and therefore the sections they make in common with the solar circle are dimensions of the altitudes; and thus the Sun is refracted in them and along them.

From these things it follows:

1. Of the Sun placed at the zenith, all points are refracted except the one that coincides with the zenith; outside the zenith, all points always.
2. Of the Sun placed on the horizon, no refraction occurs along the Longitude dimensions; and the longitudes themselves are not refracted in respect of their extension, for they keep the same magnitude without any change of

quantity — though not of place or position, as will be said in chapter 27 and following.

3. Of the Sun placed outside the horizon, every dimension is subject to refraction. For concerning the zenith it is established, where the whole Sun turns out larger or smaller; and one must reason in like proportion about the chords subtending its arc. Between the zenith and the horizon, it is clear regarding the altitudes and the chords that are along the altitude, for all these are diminished, as will be seen in what follows. But regarding the longitudes of the Sun there is dispute. These too, however, refraction must diminish, though imperceptibly, because of the oblique section of the lines extended along the altitude with those same longitudes — a section that on the horizon becomes a right angle. Whence:
4. When the Sun rests with its center on the horizon, the solar Altitudes lie in vertical planes.

A SHORT DIGRESSION [DIGRESSIVNCVLA]

And let these things be said in general about the refraction of the Sun. But whether the same can be affirmed of the heaven and the other heavenly bodies is a question. For its brief resolution, let it be:

1. Assertion. As the Sun, so the individual points of the heaven are refracted in the Vapors. The reason is evident: because the individual points of visible things, if they radiate through a medium of differing transparency, undergo refraction; but all the points of the heaven and each

and every one are visible, because illuminated by light — and this is evident in waters that reflect back the image of the heaven. Therefore, since they pass through the same vapors as the Sun, they necessarily receive the same refraction.

2. Assertion. All the stars of the heaven, like the Sun, are refracted in the Vapors. This is proved. (1) Because they all cast their rays through the vapors to us, as the Sun does, and are moreover, like the Sun, spherical in figure. Therefore, and so on. (2) This is established from the Moon, as is clear from the *Elliptical Sun*, at the beginning. (3) The same is noted in the star-clusters [asterismis], as will be shown in chapter 28, number 10.
3. Assertion. Even those bodies of the heaven that cast toward us not luminous but shadowed rays undergo in the atmosphere the same as the Sun. This is proved by the evident experience of the Sunspots [Maculae

> Solares], which — as may be seen in the *Elliptical Sun*, page 29 — show the same conjunction [coitionem] that the Sun does; yet they are not on the Sun, nor do they cast pure rays, but hurl dark images of themselves into the vapors. And the Solar Faculae [Faculae Solares] too are subject to the same vicissitude.

From these things it is evident that everything hitherto demonstrated concerning Solar Refraction can be transferred also to the individual points of the heaven and to whatever bodies are situated in the heaven. Now let us return to the Sun. And since the kind of Refraction is twofold — one in which the line of Refraction approaches the perpendicular line that is erected from the point of refraction or incidence upon the surface of Refraction, the other in which it departs farther from that same perpendicular — we shall see which of these the Sun's Refraction is, and how great it is.

THAT THE REFRACTION OF THE SUN IS TOWARD THE PERPENDICULAR LINE. CHAPTER XIII.

Foundations [Fundamenta].

Before I proceed on my course, I lay down the following foundations, drawn from the received doctrine of the Opticians and from sound philosophy.

1. That refraction is caused chiefly by the oblique incidence of a ray emitted from an object upon the refracting surface. For a ray perpendicular to that surface passes through unrefracted.
2. That the refractive body is of a different transparency than that in which the incident ray is subjected. (The Atmosphere.)
3. That the refracting body, in our case, is the vapors, or [the Atmosphere].
4. That this Atmosphere is denser than the regions of the most limpid heaven lying above it.
5. That this Atmosphere is concentric with the earth.
6. That this Atmosphere is neither continuous with the higher regions of the heaven, nor very high.

All of these are drawn from the common understanding and doctrine of almost all Opticians and Astronomers, and are sufficiently confirmed by manifest experiences and reasons. For continuity would remove all difference of incidence; a great height would remove the sense of differing density; eccentricity would remove the uniformity of refraction at every rising and setting, both summer and winter; and a rarer transparency would introduce a refraction of the Sun away from the perpendicular, and consequently its spreading-apart along the altitude, as will later be seen more clearly. Against all of these, however, manifest experience contends. These things being laid down, we must consider the different position of the eye in the Atmosphere: for it can be placed either on the surface of the atmosphere, or in its thickness outside the center, or at the center; and according to these positions its own conclusions must also be laid down.

[Conclusion 1.] So I say, first. An eye set on the convex surface of the Atmosphere sees the Sun unrefracted for as long as the Sun shines into the eye above the plane which, drawn through the eye, merely touches the surface of the Atmosphere. Reason: because all the solar rays are inclined to that plane and are in one and the same transparent medium with the eye. But as soon as the rays of the solar cone descend below that plane, they are refracted. Reason: because they cut the surface, as is sufficiently deduced from Euclid, *Elements* III.16. And they are refracted toward the perpendicular. Reason: because the medium through which the refracted rays reach the eye is denser. This position of the eye is indeed possible, yet in fact scarcely occurs; whether it may find a place on the very highest mountains remains in doubt.

[Conclusion 2.] I say, secondly. An eye set within the Atmosphere and outside its center — which is what now ordinarily happens — sees the Sun refracted toward the perpendicular, wherever it beholds it. Toward the perpendicular, because the rays travel through a denser medium, which is the Atmosphere; refracted, because they fall obliquely upon it. This I demonstrate as follows. No solar ray traversed from any point of the atmospheric surface to an eye placed outside the center of the Atmosphere passes, if produced in a straight line, through the center of the Atmosphere — except the single vertical one — by axiom 10 of book 1 of Euclid, as given in Clavius. Therefore none is perpendicular to the surface of the Atmosphere except the vertical, since this alone is perpendicular to it, by Theodosius I.4 and Witelo, *Optics* I, definition 5. Therefore all the rest are oblique, and consequently reach the eye refracted.

I say, thirdly [Margin: Concl. 3]: an eye placed at the center of the Atmosphere admits the whole Sun unrefracted. The reason: because all the rays descend toward the center of the Atmosphere, they are therefore all perpendicular to it; none of them, then, are oblique, and so none are capable of refraction. The whole solar cone [*conus solaris*] is therefore directed straight into the eye, and thus the whole Sun is seen unrefracted. And so it is established that every refraction which ordinarily occurs in this business is toward the perpendicular line — because of the density of the second medium and the oblique arrival of the rays.

COROLLARY I.

From this demonstration it is conclusively shown that the density of any medium, considered nakedly by itself and taken alone, is entirely ineffective for the business of refraction. What is required, rather, is an oblique incidence of the visual species. Hence, since the eye is always removed farther from the surface of the Atmosphere the more the sphere of that density is enlarged, it follows that if the Atmosphere were expanded all the way to the heaven of the Moon, it would be nearly inevitable that no perceptible refraction should occur. For the whole Earth, relative to the lunar sphere, would to some degree take on the role of that sphere's own center, and for this reason the rays descending through it from the Sun would not be much oblique. From which it further follows that, for this refraction, a surface of the second transparent medium fairly low above the Earth is required — a surface with which one density or rarity joined to another can contribute not a little to the refractions.

COROLLARY II.

It follows further from what has been demonstrated that this air we breathe is either equally dense with the Atmosphere, or at any rate not much rarer than it; and next, that it is much denser than the upper breeze that envelops the Atmosphere. For if our breath were to match in every respect the subtlety and transparency of that purer breeze, no sensation of refraction would result — because the refracted ray would recede from the perpendicular in our air by just as much as it had approached that perpendicular in the Atmosphere, owing to the equality, on both sides, of density as well as of obliquity of incidence, at least so far as the senses go. Hence it is impossible that our air should be equalled in rarity by the upper aether.

That our air is either identical with the Atmosphere, or at least does not differ much from it in density, is established from the following considerations: that the finer parts of the vapors are higher, the coarser lower and thus creeping along the ground; likewise that solar refractions occur even under the most purified, clearest, and steadily serene sky — times at which it is certain that the air immediately surrounding the Earth exists at its densest, and the more remote air at its most thoroughly purified. And finally, because the refraction would not be perceived as it is if a notable difference intervened between the vapors and the breeze [*au-*

ram] of ours — for otherwise our breath would once again, in large part, correct the refraction produced within those vapors; which nevertheless is not the case. Indeed, from this it becomes probable — and is the opinion of many — that the ordinary subject of celestial refraction is that very breath which we draw in and give back by turns, and which, spread evenly around the whole Earth, is not inelegantly called the Atmosphere.

ON THE QUANTITY OF THIS REFRACTION. CHAPTER XIV.

[Margin: a. Witelo [Witelo], *Optics*, Book 10, def. 2.]

Refraction is the bending of a line falling upon another transparent medium so as to form an angle [a],[Margin: a. see above] and the quantity of this bending is nothing other than the smaller angle that the line of refraction encloses with the line of incidence extended forward within the transparent medium. Hence it has come about that optical writers rightly called this angle the *angle of refraction* [angulus refractionis]. Whoever, then, investigates the magnitude of a refraction is inquiring into the quantity of this angle. That inquiry can be pursued along two paths: absolutely and comparatively. The first occurs when we simply seek and give the magnitude of each angle of refraction in some fixed and definite measure — that is, when we determine the arc or the sine of that angle in given parts, and so on.

The second occurs when we compare one refraction with another, and angle with angle, according to magnitude, content merely with their proportion to one another, whatever it may be, and not attending to how great the excess or defect is specifically, but only whether and when it occurs, and so on. About determining the absolute quantity of refraction, something will be said later, near the end; and since that can scarcely seem to be treated soundly, or without fear of the contrary error, apart from comparative knowledge, the angles of refraction must now be set against one another, so that, with this torch carried before us, we may shed some light on the obscurities that are to follow. Therefore, drawn from the certain and solid doctrine both of the optical writers and of the other philosophers, let the following be presupposed.

I. Every ray falling upon some regular surface either is perpendicular or is not. If the first, the incidence is right [i.e., at right angles], and the ray stands erect to the surface. If the second, the incidence is oblique, and the ray is inclined to the surface.

II. The inclination upon a surface is measured by the angle which the perpendicular to the surface and the ray of incidence make at the point on the surface.

III. The quantity of this angle is the magnitude of the inclination.

IV. Upon an inclination made toward a transparent medium of a different nature there follows a refraction: an equal inclination an equal refraction, a greater a greater, a lesser a lesser, none none. Hence it is necessary that whoever wishes to know the magnitudes of refractions should not be ignorant of the mutual relations of the inclinations. How these are constituted, and when the greater or the lesser of them occurs, will become clear from the Lemmas appended below.

LEMMA ONE. CHAPTER XV.

Of a single point taken outside a circle, those radiations that strike the periphery are the more oblique which are the more distant from the perpendicular ray, that is, from the cathetus of incidence.

Let there be a point A, and let it send the rays AB, AC, AD onto the periphery BCD of the circle E, to those several points; and let the perpendicular ray, or cathetus of incidence, be AE. I say that ray AC is more oblique than AB, and AD than AC, and so on, with respect to the surface DCF — whether that surface be convex or concave.

For, when the straight lines EB, EC, ED have been drawn, the angle ABE is greater than the angle ACE, and this in turn is greater than the angle ADE, by Euclid [Euclid] I.21. If each of these is subtracted from two right angles, the remainders will be the angles of inclination — greater where the [original] angles were smaller, smaller where the angles taken away were greater. But among the said angles, ADE has been proved the smallest; therefore, among the remainders, the angle ADI is the greatest. And since the angle ACE is smaller than the angle ABE and greater than the angle ADE, the remainder ECG — as well as the angle vertically opposite it, contained under the lines AC and EC produced — will be smaller than the angle ADI and greater than the angle EBA. And since the said remainder angles, made at the points B, C, D under the rays AB, AC, AD and the perpendicular lines EB, EC, ED produced, are the angles of inclination, it is clear, by the second presupposition of Chapter 14, that the rays more distant from the cathetus are more inclined to the circumference than those nearer to it — whether that inclination be taken with respect to the convex arc, such as the angle ADI upon the arc DCF, or with respect to the concave arc, such as the angle GCE upon the arc CBF. Which was to be demonstrated.

Diagram. A geometric figure for Lemma I. A single radiant point A stands outside a circle (upper right of the figure), sending a fan of rays down to the circumference; the circle occupies the lower part of the figure and has center E. A cathetus (perpendicular ray) runs from A through the center E. The rays land on the circumference at points B, C, D (with D farthest from the cathetus and therefore most oblique), and F lies further along the arc. Radii (EB, EC, ED) are drawn to the incidence points as the perpendiculars to the circumference; ED is produced beyond D to I, and EC is produced beyond C to G, marking the inclination angles.

- A (outside the circle, upper right): the radiant point; apex of the rays AB, AC, AD and of the cathetus AE.
- B (on circumference): incidence point of ray AB; least oblique of the three (nearest the cathetus).
- C (on circumference): incidence point of ray AC; intermediate obliquity.
- D (on circumference, farthest from cathetus): incidence point of ray AD; most oblique.
- E (center of the circle): center of circle E; endpoint of the radii/perpendiculars EB, EC, ED and of the cathetus AE.
- F (on circumference): terminal point of the arcs DCF (convex) and CBF (concave).
- G (on line EC produced beyond C): defines the inclination angle GCE / ECG toward the concave arc CBF.
- I (on line ED produced beyond D): defines the inclination angle ADI toward the convex arc DCF.
- H (lower left of the circle, near E, on the concave side): a construction point present in the woodcut, not referenced in the proof text (position confirmed against the higher-resolution IA copy, fig-p15-1).

LEMMA TWO. CHAPTER XVI.

Of rays falling from one point of the circumference onto any points of the diameter, one is perpendicular [right] to the circumference and all the rest are oblique; and, toward that portion of the circumference, those are the more oblique which diverge more from the perpendicular ray.

From point A, taken on the periphery BAC of the circle BAD [i.e., the circle whose diameter is BC], let the rays AB, AE, AD, AF, and AC fall onto the diameter BC. I say that only one of them can be perpendicular to the circle's circumference, and all the rest oblique — oblique, that is, toward the part of the circumference to which they approach from the perpendicular ray; and those the more so which depart farther from it. For since only one of them, AD, is able to reach the center D, that one alone will be perpendicular. And since the point E lies toward B, the ray AE too inclines that way, and the angle DAE will be the angle of internal inclination toward the concave arc BA. Again, since B is farther from the center D than E is, the ray AB will fall outside the ray AE, and the angle BAD will be greater than the angle EAD by its whole part; therefore the inclination of ray AB is greater than the inclination of ray AE. In the same way, the inclination CAD of ray AC toward the arc CGA will be proved greater than the inclination DAE [recte: DAF] of ray AF toward the same arc. Since, then, AB is more distant from the ray AD than AE is, and AC more than AF, it is clear that those rays are more inclined to the circumference which depart farther from the center, or from the perpendicular ray. Therefore those are the more oblique which are the more remote. Which was to be demonstrated.

Diagram. A geometric figure for Lemma II, set to the right of the text. A circle carries the vertical diameter BC (B at the top, C at the bottom), with the radiant point A on the circumference at the right. From A a fan of rays is drawn to points along the diameter: AB and AC to the diameter's endpoints, and AE, AD, AF to interior points of the diameter, where D is the center. G lies on the lower-right circumference (on the arc CGA).

- A (on circumference, right): the radiant point; apex of the rays AB, AE, AD, AF, AC.
- B (top endpoint of diameter, on circumference): terminus of ray AB and of the concave arc BA.
- C (bottom endpoint of diameter, on circumference): terminus of ray AC.
- D (center of the circle, on the diameter): the point AD reaches; AD is therefore the one perpendicular (right) ray.

- E (on the diameter, between D and B): interior incidence point of ray AE.
- F (on the diameter, between D and C): interior incidence point of ray AF.
- G (on circumference, lower right): point defining the arc CGA.

COROLLARY.

In the same way, if the rays BA, CA, and so on are imagined to be produced beyond the circle, all the angles at the convex circumference will be equal to those at the concave, because they are vertically opposite. Hence, if various points taken outside the circle shine [their rays] into one point of the circumference, one ray will be perpendicular and all the rest inclined — and those the more inclined which are the more remote from the perpendicular, and so on.

LEMMA III

CAPUT XVII

Straight lines drawn from a point taken on the diameter of a circle, off-center, to the arc intercepted between the smaller segment of the diameter and the perpendicular erected from the given point, are all oblique; those farther from the smaller segment of the diameter are the more oblique, and those nearer to it the more nearly upright.

Let A be a point taken on the diameter BC of the circle BCD, off the center C, and erect the perpendicular AD; then, to the arc BD enclosed between the smaller segment AB of the diameter and the perpendicular AD, let the straight lines AE and AF be drawn, cutting the arc at the points G and H. I say that AF, being the farther from AB, is more oblique to the arc BHD than AE is, which lies nearer to the smaller segment AB; and that the perpendicular AD itself is more oblique than AF, and so on.

For from the center C draw to the points of intersection G, H, and D the perpendiculars CD, CH, and CG; and with radius CA describe the circle AIKL. Now since AD is perpendicular to BC at A, the extremity of the diameter CA, it falls entirely outside the circle AML, by Euclid III.16. The arc AL therefore lies wholly within the triangle ACD. But CD, CH, and CG are all equal, by Euclid, definition I.15. Hence the triangles ACD, ACH, and ACG have their sides CD, CH, and CG equal, and the side AC in common; yet the angles BCD, BCH, and BCG, subtended by the bases AD, AH, and AG, are unequal, because the sector BCD

— because AD is the farthest from AB — takes in a greater arc BHD than the sector BCH or BCG does; and the sector BCH a greater one than BCG, because AH is placed farther from AB than AG is. Therefore, if at the angles ACH and ACG there are constructed equal angles DCM and DCN, one for each, and the straight lines DM and DN are joined, then by Euclid I.4 the two triangles DCN and DCM will be respectively equal to the two triangles GCA and HCA, and the remaining angles to the remaining angles, and so on. And since angle GCA is less than angle HCA — a part being less than the whole — the angle DCN is likewise less than angle DCM. So the point N will fall nearer to the point L than the point M does; and thus the line CN will be contained between the lines CL and CM, and ND likewise between LD and MD. Angle NDC — that is, AGC — is therefore less than angle MDC — that is, AHC; and MDC — that is, AHC — is less than angle ADC. And so the line AG is less inclined to the arc BG than AH is to the arc BGH, and AH in turn less than the perpendicular AD to the same arc. Which was to be demonstrated.

Corollary

From this demonstration it also follows that, if several luminous points taken outside the circle shed their rays together onto one point of the diameter lying off the center, those rays will be less inclined to the convex arc of the circumference which depart more from the ray that stands perpendicular to the diameter, and more inclined those which approach it. For the angle contained under EG and CG produced is smaller than the one comprehended under FH and CH produced; because these angles are equal to the angles CGA and CHA, and so on. Therefore the ray EG is less inclined than FH to the arc BGH, and so on.

Diagram. The woodcut for Lemma III. A large circle BCD is drawn with its center C set low; only its upper arc BHD is shown at full size, while a small circle AIKL is drawn about C with radius CA. A is a point on the diameter BC, off the center; from A the perpendicular AD rises to the arc, and the rays AE and AF fan out from A, cutting the arc at G and H and continuing to the outer points E and F. Radii CG, CH, CD and the auxiliary lines DM, DN complete the construction; the small circle carries the points I, K, L, M, N used in the proof.

- A (on the diameter BC, left of center): apex of the pencil of rays; near end-point of the smaller segment AB
- B (upper left, on the arc): far end of the smaller diameter-segment AB
- C (low center): center of the circle BCD and of the small circle AIKL
- D (on the arc, right): foot of the perpendicular AD

- E (upper right): outer end-point of ray AE
- F (right): outer end-point of ray AF
- G (on the arc): intersection of ray AE with the arc
- H (on the arc, right of G): intersection of ray AF with the arc
- I (on the small circle AIKL): construction point
- K (on the small circle AIKL): construction point
- L (on the small circle, below the cluster): construction point
- M (near A, by the small circle AIKL): point of the constructed angle DCM (confirmed against the higher-resolution IA copy, fig-p26-1)
- N (near A, by the small circle AIKL): point of the constructed angle DCN (confirmed against the higher-resolution IA copy, fig-p26-1)

LEMMA IV

CAPUT XVIII

Straight lines drawn from a point taken on the diameter of a circle, off-center, to the arc intercepted between the larger segment of the diameter and the perpendicular erected from the given point, are all oblique to that same arc; those carried farther from the larger segment of the diameter are the more oblique, and those nearer to it the more nearly upright.

Let A be a point taken on the diameter BY of the circle BCD, off the center C, and let the perpendicular AD be erected; between it and the larger segment AB of the diameter let the straight lines AE, AF, AG be drawn, cutting the arc DB at the points H, I, K. I say that all these lines are oblique to the arc BC; that those nearer to AD and farther from the larger segment AB are the more oblique, and those farther from AD and nearer to AB the more nearly upright — so that AH, being the farther from AB, is more oblique to the arc DH than AI is, and so on.

From the center C, with radius CA, describe the circle CAL; then AD will be a tangent, by Euclid III.16. All the lines drawn from A between it and AB to the arc DB will therefore cut the circumference AL, by the same proposition: AH at L, AI at M, and AK at N. And since AK is nearer than the line AI or AH to the segment AB, in which the center lies (because the point K is farther from the point D than I or H is), and since AI too is nearer to that same AB than AH is, let the perpendicular CO, drawn to the line AI, cut the line AK at P. But since the lines

IA and KA are the same lines meeting at A, AOP is a triangle; therefore the two angles AOP and APO together are less than two right angles, by Euclid I.17. But the angle at O is a right angle, by definition I.10; hence angle APO is less than a right angle. To it the angle CPQ is equal, by I.15; so the perpendicular CQ to the line AK will fall on the side of the angle CPQ, by the second corollary to proposition I.17 in Euclid as edited by Clavius. And since the right angle CQP is greater than the acute angle CPQ, the perpendicular CQ is therefore less than the line CP, by I.19, and thus much less than CO. The line AI [recte: AI; printed "AE"] and the line AM therefore stand farther from the center than the line AK and AN, by definition III.4. Hence AK is greater than AI, by III.7, and AN greater than AM, by III.15. In the same way it will be shown that AK is greater than AH, and AM greater than AL, and so on.

These things established, if the radii CK, CI, CH, CD are drawn — all equal to one another by definition I.15 — and the triangles ACD, ACH, ACI are turned about the center C until the vertices D, H, I coincide with K and the point A comes to R, S, and T, then angle CKR — that is, CDA — will become greater than angle CKS — that is, CHA; and angle CKS greater than CKT — that is, CIA; and CKT greater than CKA, and so on. For the line NA is greater than XT, that is, MA; and XT greater than VS, that is, LA; while KR, finally, is a tangent, toward which all the lines draw in more closely on the side of the center C. From this it necessarily follows that all of them lie between the tangent KR and the radius KC, and that the angle which the greater line AK makes with the radius CK [?] at the point K is smaller than the one which the shorter line makes with the same radius.

Angle CKR, then — that is, CDA — is greater than CKS, or CHA; and CHA, or CKS, is greater than CKT, or CIA; and CKT — that is, CIA — is greater than CKA. Therefore the ray AD is more inclined to the arc YDB than AH is, and AH more than AI, and AI more than AK; and conversely AK less than AI and the rest that precede it. The rays nearer to the perpendicular ray and farther from the larger segment AB are therefore the more oblique to the arc; and those nearer to the segment, but farther from the perpendicular AD, are the more nearly upright. Which was to be demonstrated.

Corollary I

If, as before, the points E, F, G shining outside the circle DHK cast their light onto the point A, the rays EA, FA, and GA will produce at the points H, I, K different inclinations to the arc DKB; and the inclination of the ray farther from the larger segment AB will be greater, that of the nearer one smaller. For these external inclinations are vertically opposite to the internal ones AHC, AIC, and so on, and are therefore equal to them.

Corollary II

If some luminous point is placed on the diameter of a circle, off the center, then among all the rays it pours out toward one side of the diameter, the one that stands perpendicular to the diameter will be the most oblique to the circumference. For none of the rays lying between it and the smaller segment of the diameter is as oblique as it is, as is clear from Lemma II; and none lying between it and the larger segment equals it in obliquity. It is therefore the most oblique of them all.

Corollary III

From this it further follows that, of all the internal mixed angles [anguli mixti — angles between a straight line and an arc] formed by the rays cast from A onto the semicircular arc YDB, the smallest is ADY in the figure of Lemma IV, or ADB in the figure of Lemma III. For since the angle ADC is, in either figure, the greatest of all that can be comprehended by a line drawn from the center C of the circle together with any ray meeting the circumference from A, it necessarily follows that, when this angle is subtracted from the right angle YDC, it leaves the portion YDA smaller than would remain if any other angle AHC, being less than angle ADC, were likewise subtracted from the circle's right angle YHC, which equals angle YDC; for the remaining angle YHA will be greater than angle YDA. The same conclusions must also be drawn concerning the angles formed on the outside, on the convex side, at the arc YHB.

LEMMA V

CAPUT XIX

In a circle, of the lines parallel to the diameter, those nearer the center are less, and those farther from it more, oblique to the circumference as it diverges from the diameter.

In the circle ABC let the straight lines DE, FG, BH be parallel to the diameter AC; and let DE be less distant from the center I than FG, and FG less than BH. I say

Diagram. The woodcut for Lemma IV — a dense construction. The circle BCD is drawn with a vertical diameter BY (Y at top, B at bottom) and center C. From the point A on the diameter the perpendicular AD is erected to the arc, and the rays AE, AF, AG fan out from A, cutting the arc DB at H, I, K and running on to the outer points E, F, G on the right. A small circle CAL is drawn about C with radius CA; the rays cut it at L, M, N. The interior carries the auxiliary points O, P, Q of the perpendiculars CO and CQ, and the points R, S, T (with V and X) marking where the triangles ACD, ACH, ACI are carried round about C. Many of the interior labels are small on this scan.

- Y (top): upper end of the vertical diameter BY
- B (bottom): lower end of the vertical diameter BY
- C (center-left): center of the circle BCD and of the small circle CAL
- A (upper left, on the diameter): apex of the pencil of rays
- D (upper right, on the arc): foot of the perpendicular AD
- E (far right): outer end-point of ray AE
- F (right): outer end-point of ray AF
- G (lower right): outer end-point of ray AG
- H (right, on the arc): intersection of ray AE with the arc DB
- I (right of center, on the arc): intersection of ray AF with the arc DB
- K (lower center, on the arc): intersection of ray AG with the arc DB
- L (on the small circle CAL): intersection of ray AH with the small circle
- M (on the small circle CAL): intersection of ray AI with the small circle
- N [?] (near K, on the small circle CAL): intersection of ray AK with the small circle — named in the demonstration but not isolable on the tile (overdrawn K-convergence + show-through)
- O (interior): foot of the perpendicular CO on line AI
- P (interior): intersection of CO with line AK
- Q [?] (interior): foot of the perpendicular CQ from C on line AK — worn "2"-like letterform, uncommitted
- R (interior): position of A after the triangle ACD is turned about C
- S (interior): position of A after the triangle ACH is turned about C
- T (interior): position of A after the triangle ACI is turned about C
- V (interior): point of the segment VS (from LA) arising in the rotation
- X (interior): point of the segment XT (from MA) arising in the rotation

I say that the parallel D E is less oblique to the arc A B — diverging from the diameter — than F G is to the same arc. From A let there be drawn the straight lines A K, A L, A M, equal to the given parallels, so that A K equals the parallel D E, A L equals F G, and A M equals B H. Now since A K is equal to the straight line D

E, the two are equally removed from the center (by *Elements* III.14), and they cut off equal arcs (by *Elements* III.28). Therefore the arc A B K coincides with the arc D B E, just as the straight line A K coincides with D E; and so the angles B A K and B D E are equal to one another. In the same way the straight line A L is as far removed from the center I as the straight line F G, and the angle B A L will be shown equal to the angle B F G; likewise A M will be equally distant from the center as B H, and the angle B A M is made equal to the angle M B H.

But F G lies farther from the center than D E, and nearer than B H. Therefore A L too will be farther off than A K and nearer than A M, so that it falls between the two. For this reason the angle B A K is the greater and the angle B A M the smaller, compared with B A L; and consequently the angle B D E is likewise greater, and M B H smaller, than the angle B F G. Therefore D E is less oblique to the arc B D than F G is to the same, and B H is more oblique to the arc M A than that same F G is to it. And so the proposition stands demonstrated.

Diagram. A circular segment illustrating the Lemma just demonstrated. A horizontal diameter runs from A (left end) through the center I to C (right end); above it the arc rises from A over the top and returns to C. Three chords are drawn parallel to the diameter, cutting the segment at increasing heights — the lowest D E, then F G, then B H — each with its left endpoint (D, F, B) and right endpoint (E, G, H). From the left end A three straight rays fan upward across the segment to the points K, L, and M on the arc, representing the auxiliary equal lines A K = D E, A L = F G, and A M = B H used in the proof.

- A (lower left): left end of the diameter; origin of the rays A K, A L, A M
- I (bottom center): center of the circle
- C (lower right): right end of the diameter
- D (left, lowest chord): left endpoint of chord D E
- E (right, lowest chord): right endpoint of chord D E
- F (left, middle chord): left endpoint of chord F G
- G (right, middle chord): right endpoint of chord F G
- B (left, upper chord): left endpoint of chord B H
- H (right, upper chord): right endpoint of chord B H
- K (arc): endpoint of ray A K (equal to D E)
- L (upper arc): endpoint of ray A L (equal to F G)
- M (top): endpoint of ray A M (equal to B H)

ON THE GREATEST AND LEAST SOLAR REFRACTIONS. CHAPTER XX.

From these lemmas already set out, it will not be so very difficult to bring forward some judgment about the solar refractions.

For the Sun, other things being equal, appears to be refracted most at its rising and setting. Since the refracted ray lies in the sensible horizon, it is thereby perpendicular to the vertical line and falls outside the center of the earth; and so, by Lemmas 3 and 4, it is maximally oblique to the surface of the vapors. The incident ray was therefore itself the more oblique in our present case, since refraction takes place toward the perpendicular. I said "in our present case," where the eye rests within the vapors of the earth's surface, for in other cases one might perhaps think it right to reason differently.

For if the boundary and measure of the greatest refraction were the apparent horizon, wherever the eye were finally placed, then a man positioned high up in the region of the vapors would see the Sun most refracted when it traversed the greater part of

that part — yet experience and reason are against this.

Let a circle A B, described about center A, be the earth; let C D E be the surrounding region of vapors, and C F the vertical diameter; and let the eye be at some high point G within that region. From G a ray G E is drawn tangent to the supposed earth at B: this will be the sensible horizon, through which the eye at G will ultimately see the Sun. Since this ray is less oblique to the arc C D E F than the perpendicular ray G D (by Lemma 4, etc.), it will refract the Sun less than G D does. For a less oblique refracted ray was caused by a less oblique incident ray, etc.

Diagram. Two concentric circles illustrating the sensible horizon of an elevated eye. The inner circle, of radius A B, is the earth; the outer circle C D E F is the surrounding region of vapors. A vertical diameter C F runs from C at the top, down through the eye-point G and the center A, to F at the bottom. From the eye G, high on the diameter within the vapor region, the ray G E is drawn tangent to the earth at B and continued out to E on the outer circle — this is the sensible horizon. The radius G D runs to D on the outer circle as the perpendicular ray.

- A (center): center of the earth, on the vertical diameter

- B (right, inner circle): point on the earth's surface where the ray G E is tangent
- C (top): top of the vertical diameter, on the outer (vapor) circle
- D (upper right, outer circle): endpoint of the perpendicular radius G D
- E (lower right, outer circle): endpoint of the tangent ray G E
- F (bottom): bottom of the vertical diameter, on the outer circle
- G (upper, on diameter): the eye, elevated within the vapor region

It is therefore credible that the Sun is refracted most when the refracted ray coming from the Sun is perpendicular to the vertical line on which the eye lies — as G D is in the present diagram. And since, in the present state of things, the eye watches the Sun's rising and setting placed very near the earth's surface, this ray always comes to lie in the sensible horizontal plane. Hence it is likely that the greatest refraction arises at rising and setting.

Just as, by contrast, we see the least refraction at the zenith — and the same is to be understood of the nadir — because there the inclinations of the rays to the surface of the vapors, both incident and refracted, are usually the least.

In the intermediate intervals between the zenith and the sensible horizon, and between the latter and the nadir, the refractions must necessarily be intermediate. And all this holds when the eye is fixed from one and the same point upon the moving Sun.

For if we hold the Sun fixed and instead move the eye along the diameter of the atmosphere, it will find the Sun's refraction smaller as it approaches the center, greater as it recedes, and none at the center — by Lemmas 5 and 1 — because refracted rays nearer the center are straighter [less inclined] than those farther out toward the circumference.

From these things it becomes clear that the greater horizontal refraction proceeds from lower vapors, the lesser from higher ones — provided the other conditions are equal and the eye stands on the earth's surface. For I would not dare to deny that density or rarity, added to obliquity, contributes some difference of power to the refraction, since experience seems to support this; the agreement of the authorities certainly affirms it.

ON THE DIFFICULTY OF THE TANGENT RAY.

CHAPTER XXI.

[Margin: The ground of the difficulty, and the sense of the question raised, are set out.]

But a difficulty arises here, not to be despised, and to be examined in this place. For, as has often been asserted in what precedes, every ray falling upon some regular surface is either straight [perpendicular] or oblique, that is, inclined. Now of inclined rays falling upon a spherical surface there is a twofold kind: either the ray is inclined to the surface with the power of cutting it, should it be imagined produced in a straight line; or it is without this power, even if it were drawn out straight. The rays that a cutting accompanies are all situated between the perpendicular and the tangent; the one that cutting deserts is the tangent alone — yet no less inclined for that, indeed the last of the inclined rays, since between it and the spherical surface no other ray falls unless it cuts, while its own inclination is always a right angle.

The question, then, is now this: whether this purely tangent ray, which never cuts, is to be counted among those that produce refraction — and hence whether any refraction arises from the ray of contingence [radius contingentiæ]?

And it seems that it does arise. For, first: refraction, which is the bending of the line of incidence so as to form an angle, occurs at the very point of incidence, where the incident ray first meets the refracting surface. Therefore, to produce refraction, nothing is required except a pure contact on the refracting surface — otherwise refraction would not occur in it. But in the ray of contingence this contact is present, for it is supposed to touch the sphere.

Second: the inclination of a ray upon the surface of another transparent medium generates refraction; but the ray of contingence is inclined upon the surface of another transparent medium, since its inclination is a right angle. Therefore it generates refraction.

Third: writers on optics everywhere speak thus, without any restriction — the inclination of rays causes refraction: an equal inclination an equal refraction, a greater a greater, none no refraction. Indeed these are the axioms of the opticians. Therefore the greatest inclination produces the greatest refraction. But the inclination of a ray touching the sphere is the greatest, as is self-evident and was hinted just above.

[Margin: Arguments for the affirmative view.]

Fourth: the tacit consent and practice of the authorities seem to agree. For they nowhere seem to have excepted or rejected the ray of contingence, though this ought to have been done if it were useless for refracting. Indeed — what is more remarkable, and lends this view its greatest support — they expressly and as though by concession assume this very ray of contingence, cast from the Sun into the atmosphere, in order from it to hunt out either the altitude of the vapors, or the magnitude of the refraction, or something similar

— which they would not do if they judged it unfit for this business; or, if they did so judge it, they would be acting ineptly and accomplishing nothing at all. See Kepler [Johannes Kepler] in the *Paralipomena*, "On the Holland Observations," no. 9, p. 141 (*Ad Vitellionem paralipomena*). And often elsewhere.

CONCLUSION.

Yet it must nonetheless be said that a purely tangent ray, alighting without the power of cutting, does not appear suited to producing refraction. This is proved: first, because such a ray does not enter the medium of the other transparent body, but always remains in the same one — for it grazes the other in only a single point; therefore it is not refracted. The consequence is clear, from the definition of refraction, for which two different media are required. The antecedent is evident from the hypothesis, etc.

Second: as the incident ray stands to a polished opaque body with respect to reflection, so it stands to a transparent body with respect to refraction; but the ray of contingence is not reflected from the former, and so neither is it refracted in the latter. The minor premise is self-evident. The major is proved: because reflection occurs only for this reason and at this time, namely when refraction is impeded by opacity; and wherever there is reflection, if the opacity be removed, refraction will occur; likewise, wherever this happens, if a transparent thing be made opaque with the requisite smoothness, reflection will occur. Therefore reflection and refraction behave in the same way toward similarly shaped bodies, and run at an equal pace, the one succeeding the other, etc.

Third: if a solar ray lies on some plane surface — of crystal, for example — it is not refracted, even though there it has all its points in contact; therefore much less will it be refracted on a spherical surface, where it has but a single point of contact. Nor can any argument be given for a globular surface that would not hold far more strongly for a plane one, etc.

Fourth: the ray of contingence is here rejected by way of the absurd. For otherwise the refraction from high vapors would be greater than from low ones — which, in order to demonstrate, I assume the following. First, that every point of the Sun radiates spherically. Second, that the ray by which the Sun is seen either first at rising or last at setting lies in the plane of the visual horizon, and therefore touches the earth. Third, that the atmosphere is not always at one invariable distance from the earth, but is sometimes higher, sometimes lower. And all this I assume in agreement with the opinions of the best authors, and consonant with reason and experience.

[Margin: Case 1.]

Let it now be supposed, according to the adjoined diagram, that the eye A is set on the earth's surface, watching the setting Sun by the last ray A B, which lies in the visible horizon; and let the various height of the atmosphere around the earth be marked out — lowest through the point C, middle through D, highest through E — and let the Sun indeed, on some day,

it would be seen by the ray AE, drawn through E on the highest surface of the atmosphere, so that E is the point of refraction and AE the refracted ray. Now because the opposing view holds that this refracted ray can arise from a solar ray that merely grazes the surface of the vapors, that grazing ray will be FEG — the last ray that can fall from the Sun onto the point E.

[Margin: Case 2.] In the same way, suppose that on another occasion and day the Sun sets while the vapors occupy the middle elevation above the earth, drawn through D. Then the refracted ray will be AD, arising — on the adversaries' account — from the ray HDI, which merely grazes the vapors at the point D.

[Margin: Case 3.] Finally, let us take yet another sunset, this time in the lowest vapors, through C. Then the last ray to slip from the Sun into the vapors, touching them only at C, will be KCL; and the ray refracted from it at the point C will be CA.

None of these cases is devised arbitrarily; they often occur of necessity — granted that some refraction does proceed from a merely tangent ray, as the opposing view maintains. This being granted (though not conceded): from M, the center of both the earth and the atmosphere as drawn in its three states, let the straight lines MN, MD, ME be produced beyond the points of contact C, D, E.

Now since the angle LCN is a right angle, the adjacent angle LCM is right as well. The three angles of triangle ACM are therefore equal to the two right angles LCN and LCM. But the three angles MCA, ACL, LCN are likewise equal to those same two right angles LCN and LCM; so these three angles are equal to the three angles of triangle ACM. Now CAM is a right angle, and ACM is common to both; so if I take away the right angle LCN and the right angle CAM, the two remaining angles ACL and ACM will equal the two angles AMC and ACM. Then, removing the common angle ACM, the remainders AMC and ACL are equal. But ACL is the angle of refraction — for KL is posited as the last solar ray, tangent to the vapors at C and reaching the eye A, and so on.

In the same way it will be proved that the angle of refraction ADI is equal to the angle DMA, and that the angle AEG, likewise a refraction angle, is equal to the angle EMA. But the angle AME is greater than AMD, and AMD greater than AMC, as is clear from the demonstration of the second Lemma. Therefore the refraction AEG is greater than ADI, and this in turn greater than ACL. Moreover the surface E belongs to a larger sphere than D, and this to a larger than C, and so on. Higher vapors therefore refract more strongly…

Diagram. A woodcut refraction figure illustrating the three "cases" of the setting Sun grazing vapor-shells at three different heights. The eye A sits at the left on a roughly horizontal line of sight running to B at the right. Three concentric arcs (the vapor surfaces at increasing height) cross this region at the contact points C (lowest), D (middle), and E (highest). Straight rays fan across the figure: the tangent rays KCL (grazing at C), HDI (grazing at D), and FEG (grazing at E), together with the radii from the earth-and-atmosphere center M drawn out beyond the contact points. All thirteen labels are legible.

- A (left, on the line of sight): the eye on the earth's surface
- B (right end of the line of sight): the far horizon point of the line of sight
- C (inner contact point): tangency point in the lowest vapors (Case 3)
- D (middle contact point): tangency point in the middle vapors (Case 2)
- E (outer contact point): tangency point in the highest vapors (Case 1)
- M (lower left): common center of the earth and the atmosphere
- N (top): endpoint of the radius MN produced beyond C
- L (top) / K (lower right): the two ends of the tangent ray KCL through C
- I (top) / H (lower right): the two ends of the tangent ray HDI through D
- G (top) / F (lower right): the two ends of the tangent ray FEG through E

than lower ones, if it is tangent rays that are being refracted. But this is false and contrary to experience — as I also showed in the *Elliptical Sun* [*Sol ellipticus*], p. 21 — namely the claim that higher vapors should be more apt to refract than lower ones, other things being equal or homologous. It is therefore also false that the incident lines to be refracted are purely tangent lines. Which was to be demonstrated.

Experience supports this doctrine. For rays refracted through spherical glasses never — so far as I have been able to test — come from rays that are nakedly tangent, but from rays that cut the glass, even where they would otherwise travel on unrefracted. From this established principle it follows that the height of the vapors is hard to investigate from the line of incidence; for that line, in the sense just stated, is not nakedly tangent but cuts the vertical circle, and is therefore variable. And for this very reason the vapors, too, are not as low as some make them out to be.

Reply to the Arguments Made at the Outset

To the 1st. It is denied that incidence without cutting-power suffices for refraction. For the straight course of the incident line is not blunted by a surface taken nakedly, and by an indivisible point, but by a body joined to the surface, and so on — as everyone teaches.

To the 2nd. It is denied that inclination without cutting-power causes refraction; it is granted if the cutting-power is joined to it — which a purely tangent ray does not possess.

To the 3rd. The authors' manner of speaking is granted, but the final consequence drawn from it is denied. For a ray tangent to a sphere is not as inclined as the limit of all inclined rays. This is plainly evident in the case of inclination made upon a plane: if the inclination equals a right angle, it is no longer inclination but coincidence. To this, in the case of a sphere, tangency corresponds — which is nevertheless loosely called inclination, because the angle of contact slips in between the tangent ray and the spherical arc. But no maximum inclination upon a plane can be determined.

To the 4th. The reply is that the writers on optics sufficiently exclude the tangent ray in what they require — concerning the medium and the incident ray — for refraction. As for the fact that some do assume the ray of contact in order to find refractions and to investigate certain other matters, that is indeed true; but this very point is itself in doubt. Hence, according to what has been said, one ought rather to stand by the contrary opinion.

On the Sun's Altitude above the Horizon, Greater than Its True Altitude. Chapter XXII.

From the fact that the Sun's refraction takes place toward the perpendicular, it necessarily follows that the Sun appears higher than it really is. For since the Sun itself is seen only by refracted rays, it will certainly appear to be in the place and position where those refracted rays lie; but these, when protracted, are higher than the incident rays. I show this as follows. The refracted ray diverges toward the perpendicular, away from the incident ray protracted within the denser transparent medium; therefore within the atmosphere it is lower than that incident ray; and therefore, protracted outside the atmosphere, it is itself higher. So the Sun, seen by the refracted ray, appears higher.

In the figure: the protracted incident ray is ABC, the refracted ray DBE. Point C is higher than D, because it is farther from the perpendicular GH. Therefore, after the intersection made at B, A is lower than E. Now the Sun is actually at A but appears at E, and so it appears higher than it is. Moreover, the greater the refraction, the more the Sun is raised above its true place. Thus, if the protracted incident ray is IF, the refraction will be greater — EBI, or DBF — than EBA, or DBC; and so I is lower than A, and therefore at E that same point I will be higher than A is at that same E.

How great this adventitious elevation of the Sun may be is uncertain, since it accompanies the refraction itself. If Tycho's [Tycho Brahe] doctrine is correct — as seems probable — namely that a refraction of 34 or 35 first minutes produces a contraction of the Sun of about 5 [minutes], then a greater contraction, of 12 minutes, must necessarily arise from a much greater refraction. And it seems one must say that the refraction can in some way grow to a sesquidegree [a degree and a half], and that it thus comes about that the Sun appears a sesquidegree higher. But a more convenient account of these matters will be set out a little later.

Diagram. A woodcut construction showing how refraction toward the perpendicular makes the Sun appear higher than it is. At the lower left is a small circle (the earth/atmosphere) with center G; a larger arc sweeps around to the right (the outer boundary). From the refraction point B (right of center, on the boundary) rays fan out to the right toward H, E, A, and I, and up to the left toward F, C, and D along a near-vertical line. The perpendicular GH runs through B. All nine labels are legible.

- A (right, lower): the Sun's true position
- B (center right, on the boundary): the point of refraction / intersection
- C (upper left, on the vertical line): endpoint of the protracted incident ray ABC
- D (upper left, below C): endpoint of the refracted ray DBE
- E (right): the Sun's apparent position
- F (top left): endpoint of the second protracted incident ray IF
- G (center of the small circle): center of the earth; foot of the perpendicular GH
- H (upper right): endpoint of the perpendicular GH
- I (lower right): endpoint of the second incident ray IF

On the Contraction of the Sun. Chapter XXIII.

The contraction of the Sun arises from the unequal elevation above the horizon of its highest, middle, and lowest parts. This in turn flows from the fact that these parts are refracted unequally toward their respective perpendiculars; and this is produced by the unequal inclination, upon the surface of the atmosphere, of the rays that proceed from those same parts. For since the rays emitted from the lower parts of the Sun strike that surface more obliquely, while those from the higher parts strike it more directly — as can rightly be deduced from the Corollary of the third Lemma — the lower parts of the Sun must be refracted more than the higher, stretched out higher, and compressed more forcefully.

Nearly everything said so far is clarified in the diagram set before us. In it, let the smaller circle ABC, described from center A, be the atmosphere, and let the larger circle ADE be the heaven of the Sun. Let AF be the vertical line, on which the eye G, placed within the atmosphere, looks at the Sun — whether at DE in the zenith F, or at HI on the horizon I, or at KL at the nadir M (for we now suppose the earth not to be in the way). Everywhere the eye G will see the Sun refracted, but not everywhere contracted.

[Margin: The Sun's refraction at the zenith.] For the Sun DE, seen at the zenith F, will be contracted equally by the refracted lines GN and GO — since the incident rays are DB and EC, and the perpendiculars AP and AQ, toward which in any case they approa…

Diagram. The large woodcut "proposed diagram" that gathers the whole construction of solar contraction into one figure. Two concentric circles share the center A: the smaller circle ABC (the atmosphere) and the larger circle ADE (the heaven of the Sun). The vertical line AF runs upward through the eye G (set inside the atmosphere on that line) to the zenith point F at the top. The Sun is drawn in three positions around the rim of the larger circle — DE at the zenith (straddling F), HI on the horizon (to the right, at I), and KL at the nadir (at the bottom, at M) — with dense pencils of incident and refracted rays and their perpendiculars crossing the interior. The figure is at the resolution floor in its crowded center and lower margin. ` at the engraving floor. No longer a productive]

- A (center): common center of the atmosphere and of the heaven of the Sun
- B (upper center, on the atmosphere circle): where the zenith incident ray DB meets the atmosphere
- C (upper center, on the atmosphere circle, right of B): where the zenith incident ray EC meets the atmosphere [C legible but partly obscured — reading uncertain [?]]
- D, E (top, on the larger circle, flanking F): the two edges of the Sun seen at the zenith
- F (top, on the larger circle): the zenith point on the vertical line AF
- G (center, on line AF, above A): the eye, within the atmosphere
- H, I (right): the Sun HI seen on the horizon; I the horizon point on the larger circle
- K, L (bottom): the Sun KL seen at the nadir
- M (bottom, on the vertical line): the nadir point
- N, O (upper center): endpoints of the refracted lines GN, GO for the zenith Sun
- P, Q (top, beyond the larger circle): endpoints of the perpendiculars AP, AQ at the zenith
- R (upper right): construction point [role not yet defined in the text on this page]
- S (right): construction point [role not yet defined in the text on this page]
- T (center, near B/G): construction point [role not yet defined; reading [?]]
- V (center right, below the horizontal): construction point [role not yet defined; reading [?]]
- X (center, below G): construction point [role not yet defined; reading [?]]
- Y (center, below X): construction point [role not yet defined; reading [?]]
- α (Greek alpha, lower left): construction point for the nadir rays [role not yet defined in the text on this page]

- γ (Greek gamma, lower center-right): construction point for the nadir rays [role not yet defined in the text on this page]
- β (Greek beta, lower left, below K): construction point for the nadir rays [role not yet defined in the text on this page]
- Ω (Greek omega, lower right, below L): construction point for the nadir rays [role not yet defined in the text on this page]

the refracted rays G N and G O had necessarily to draw closer together. Hence the Sun, seen beneath them, had to be refracted and to appear smaller — contracted to the diameter N O, which is smaller than the true diameter D E. But this vertical refraction and contraction, because the incidence of the rays D B and E C is very nearly perpendicular, escapes the senses and is grasped by reasoning alone, since it is exceedingly slight.

[Margin: The Sun's refraction on the horizon.]

But the Sun H I, placed on the true horizon A I, appears refracted and contracted unequally in H R, through the refracted rays G R, G S, G H, which are produced from the incident rays H T, V X, I Y. And since the angle H Y I is greater than S X V, and this in turn greater than R T H, therefore G Y is refracted more than G X, and G X more than G T. So the straight line G X, extended beyond the vapors to S, becomes higher there — in its own proportion — than the extended line G R; and G H becomes higher than either G S or G R. The reason is that this ray, within the atmosphere, because the incident ray I Y is more oblique, approaches its own perpendicular A Y more closely than G S approaches its perpendicular A X, and so on.

The whole refracted Sun H R therefore appears smaller than the unrefracted I H; and the lower refracted semidiameter H Z appears smaller than the upper refracted semidiameter Z R, which is likewise refracted.

[Margin: The Sun's refraction at the nadir.]

But if the eye G looks at the Sun K L at the nadir M, the Sun appears — through the refracted rays G α β (at β) and G γ δ (at δ) — outside its true positions K and L, under the diameter β δ enlarged by refraction. And since the incidences K α and L γ everywhere make equal inclinations, the Sun's spreading must be uniform, and so the Sun appears larger but spherical. Hence it seems agreeable to reason and conformable to the laws of refraction that this same Sun, placed outside the nadir in the lower hemisphere and drawing near the horizon, is enlarged unequally; and that, carried up from the nadir toward I, it is taller than it is broad, because of the more oblique incidences — as is sufficiently deduced from Corollary 1 of

Lemma 4 and from the Lemma itself. And experience too seems to stand by this doctrine.

CONSECTARIA. [Corollaries]

From all this it is established:

1. That the Sun, while the eye stays fixed outside the center of the earth, presents when carried around almost every variety, both as to shape and as to magnitude.
2. At the zenith it appears round but smaller than it should be; at the nadir round but larger than it should be.
3. Between the zenith and the horizon it appears smaller in height than in length; yet both appear diminished.
4. On the sensible horizon the sight is deficient in height alone, while in length it appears at its natural size.
5. Between the horizon and the nadir it appears more extended in height than in length —

— than in length, yet exceeding its natural size in both. The reason for this: only the vertical planes are planes of refraction, and such planes cannot, in this position, be planes of the horizontal extent [longitudinum].

6. The same will happen if, with the Sun motionless, the eye observes it while being carried around on the circumference of a circle situated within the vapors. But if the eye is moved along a diameter that does not pass through the Sun, then — being placed outside the center of the atmosphere — it will see the Sun unequal; placed at that same center, it will behold it equal and of its natural magnitude.

EXPERIENTIAE PRIORI DOCTRINAE conformes. [Experiments conformable to the foregoing doctrine]

DE VITRIS IN GENERE. [On Lenses in General]

CAPUT XXIV. [Chapter 24]

Enunciata. [Propositions]

1. All lenses [vitra] of whatever edge — provided it is regular and set square to the optical axis of the eye — change only the magnitude of the thing seen, according to their greater or lesser density or rarity and to the nearness or distance of the lens from the eye or from the object; but they do not change its shape.
2. All lenses oblique to the optical axis of the eye either contract objects or spread them out unevenly; and so they vary both the magnitude and the shape, more or less, according to the greater or lesser inclination of the axis or obliquity of the rays to the surface of the lens, and to the greater or lesser distance of the sight from the lens, or of the lens from the object.
3. All lenses more steeply inclined dilate or compress the thing seen the more.

From this the foregoing Lemmata are confirmed, where I said that a greater inclination is followed by a greater refraction, a lesser by a lesser, and so on.

4. No lens (so far as I at least have ascertained by diligent and repeated trials) appears to bend an incident tangent ray — one that stands at right angles to the perpendicular drawn to the point of refraction. Experience makes this plain.

Hence it follows that all refracted rays come from incident rays that cut the lens. And so by this route the height of the vapors is rendered difficult to investigate — vapors which, on this head too, turn out to be higher than some suppose.

5. All lenses near the eye, if they incline the part above the optical axis toward that same eye, ordinarily raise the thing seen (and contracted) upward — for there is a case in which they depress it. If they tilt the lower part toward the eye and the axis, they push the thing seen downward; if the right part, they turn it into the left; if the left part, into the right [contorquent — printed *controquent*].

From this the foregoing doctrine concerning the planes of refraction is established: namely, that in these planes there always lie the line of incidence, the line of refraction, and the perpendicular; and likewise that the refraction is either toward the perpendicular or away from it, according to the nature of the medium. The reason is also given why the Sun always appears higher when seen within the atmosphere of the upper hemisphere: namely, because the outermost refracting surface of the vapors, which bends the Sun's rays, is inclined toward the eye from its upper parts, and so on. See the figure of the Elliptical Sun, no. 2, on page 16.

DE VITRIS IN SPECIE. [On Lenses in Particular]

CAPUT XXV. [Chapter 25]

Vitra plana. [Plane lenses]

1. All plane lenses have this property, that they do not change the shape of the object, except at a very great inclination; the thicker ones, however, are seen to alter it sooner than the thinner.
2. This marvel I found in plane lenses: that although the same surface, applied to the eye, contracts the thing seen, yet if I bring the opposite side of that same surface to the eye, I behold the same thing spread out which before I had seen contracted.

I have tested this in all the plane lenses — very many — that I have thus far brought to examination; and it usually happened in the one position of the same face, while other positions narrowed the thing seen obliquely. Whether the cause of this resides in the lenses, or in some hidden force of nature, I should not readily affirm. The lenses that produce this effect were certainly ground flat and highly polished at Venice.

But although the polishing is sound, I nevertheless suspect that a fault lies in the flatness — one that escapes the keenness of the eyes, though not nature itself. Even plane lenses polished along the same course and stroke contract, as it were, certain fibers [fila] that stand out somewhat more prominently, and these count for much in this business; both surfaces, too, can contribute something here. For each sends into the eye the object's own image and contracts it according to the inclination; and if these two images, as often happens, are joined by mutual contact, then the eye judges the object to be elongated, and so on — for the two simulacra run together into one oblong shape, and so forth. But these explanations do not convince. For lenses flattened by free blowing [libero flatu], with a most brilliant surface, are endowed with the same property. And so this fault, if there is any, cannot be excused as coming from the craftsman's hand — unless someone should say that even the glass-blowing always falls into the same errors, which hardly seems likely.

VITRA PLANOCONVEXA ET PLANOCONCAVA. [Plano-convex and plano-concave lenses]

1. Plano-convex lenses, like plano-concave ones, when set square to the optical axis, leave the shape unchanged but vary the magnitude. Brought

near the eye, the plano-convex make it smaller than the same lens does when removed; the plano-concave make it larger than the same lens removed — until eversion [euersio] occurs in them: then the contrary results also follow. The plano-concave never invert the object, because the rays refracted by them never come together.

2. Plano-convex lenses oblique to the optical axis spread the thing seen along a diameter oblique to the optical axis, before eversion; and thus, drawn apart, they sometimes exalt it, sometimes lower it, according — that is — to the varying surface of the lens and its position relative to the eye. After eversion they contract the thing seen, and so on.

Plano-concave lenses diminish the thing along a diameter oblique to the optical axis (that is, along a contracted diameter), and show it much more contracted; and this in great variety, according as the convexity or hollowness has proceeded from a great or a small sphere, and so on.

DE VITRIS HOMOCENTRICE CONVEXIS ET concavis [On Lenses Homocentrically Convex and Concave]

— such as those whose surface, both concave and convex, is described about one and the same axis.

1. All spherical lenses homocentrically concave and conve—

vex, when brought obliquely to the eye, contract the transparent object seen through them. This is a matter of frequent experience.

2. Segments of the same sphere, equally thick and equally oblique, contract the object seen to an equal degree.
3. Segments of different spheres: those belonging to a smaller sphere contract it sooner — that is, at a shorter distance from the eye; those belonging to a larger sphere, more slowly, at a longer distance.
4. Segments of the same sphere: the thicker ones contract the object seen sooner, the thinner ones more slowly. From this it seems clear that thickness contributes something.
5. All homocentrically concavo-convex glasses enlarge the object seen when the eye approaches the glass, and diminish it when the eye draws back.

Glasses convex on both sides, and concave on both sides

[Body heading, italic: "VITRA VTRINQVE CONVEXA, ET vtrinque concava."]

1. These glasses accomplish the same thing as the mixed ones, but more intensely. By "mixed" I mean those that consist of one plane surface and another surface that is either convex or concave. But when both surfaces are either convex or concave, they then do the same as planoconvex or planoconcave glasses, only much more effectively. The reason is that in these the refraction is doubled; hence in convex glasses the rays converge twice as much, and in concave ones they diverge just as much more.

2. Nor does it matter whether the two surfaces are of the same convexity (or concavity) or of different degrees; for one surface always augments the power of the other in proportion to its own strength. And the strength of each is derived from the spherical surface of which it forms a portion.

3. Convex glasses set oblique to the optical axis therefore elongate the object along the inclined diameter of the glass, before the point of reversal of position; but afterwards the contrary results follow, and so on.

4. Concave glasses inclined to the optical axis always present shorter images of things, along the diameter of the glass that is inclined to the optical axis — whatever the position and distance of the glass from the eye, or of the object from the glass.

The reason is that here no intersection of the rays ever occurs, since they always recede further from one another.

Moreover, these glasses transfer the things seen from their true positions in a remarkable way — up, down, right, left, and so on. You can also test this on the Sun, through a glass tinted with colors in the same manner, or on the full Moon with a fluid glass. For you will see either heavenly body shaped into an ellipse and shifted from its place according to the position and disposition of the glass. And if you set up two or more such glasses in different positions between your eye and the said heavenly bodies, you will multiply those same bodies.

From these observations you will learn to draw out the explanation of parhelia and paraselenae. Polyhedral lenses will teach the same lesson. And the homocentrically convexo-concave glasses mentioned a little earlier accomplish the same thing. So, if one does not mind ascending from these humble matters to lofty ones, as though by a set of steps, we shall say that by the interposition of a cloud, or of a vapor or similar refractive transparent meteor — shaped in manifold ways by one of these modes — the Sun is quite often seen multiplied, and so on, and that parhelia are generated in this way, and so forth. But I touch on these matters only as the occasion offers; I do not set out to expound them in full. To the wise, a few words suffice.

On crystal, clouds, mists, and other transparent bodies. Chapter XXVI. Crystal.

[Body heading, italic/caps: "DE CRISTALLO, NVBIBVS, NEBVLIS, ATQVE ALIIS TRANSPARENTIBVS corporibus. CAPVT XXVI. Cristallus."]

There is a certain person — clever, to be sure, in his own judgment, but as a mathematician not sufficiently considered — who maintains that crystal yields to glass in the matter of refracting correctly, and is therefore less suited to optical and astronomical uses. But the matter, other things being equal, stands far otherwise. For if you are given two convex lenses, one of glass and one of crystal, of the same size, similar surface, and equal polish, the crystal lens will certainly far surpass the glass one in representing objects of every kind — as I have learned from very frequent experience.

But the reason why crystalline lenses are commonly less effective than glass ones — and very rarely transmit images unconfused or unmarred by false colors — is that they are very seldom worked with a perfectly spherical and evenly polished surface; instead they turn out rippled in the manner of waves [*undarum more*]. This unevenness disturbs in them the whole ordered arrangement of the translucent images and destroys their proper use.

This defect in the polishing must be ascribed, on the side of the material, to the untamable hardness of crystal; on the side of the craftsman, partly to ignorance and partly to carelessness. But whoever joins unremitting effort to knowledge, and lays out the necessary expense on mechanical means, will bend even the obstinacy of crystal. I do not, however, deny that there are found among [glasses]...

sometimes glasses of such noble material and such excellent form that they might perhaps even snatch the palm from exact crystal. But this case is quite rare.

Meanwhile, whatever has so far been asserted about glass may be affirmed equally, or even much more strongly, of crystal — especially since the refractions of glass and of crystal are practically the same, with experience and the authority of learned men lending their support. See Kepler [Johannes Kepler], in the

Dioptrice. Therefore crystal, once worked and transformed into suitable shapes, will be able to do the same as glass. And there is no need to linger here any longer, and so on.

Clouds

[Body heading, italic: "NVBES."]

Some clouds are so dense, thick, and colored that they take the Sun from the eyes altogether; and about these — which are useless for the present business — nothing need be said. Others are so rare, thin, and transparent that they leave the sight of the Sun unimpaired. Of these I find a twofold origin, earthy and watery.

The earthy ones, not much unlike thick smoke or scattered dust, are dry, hot, discontinuous, and composed as it were of countless tiny particles. Each of these particles, being by its own nature opaque [adiaphani, "non-transparent"] rather than clear, reflects the solar rays more than it refracts them; and so they are more apt for dulling or confusing the Sun, and for disturbing and variously coloring it, than for refracting it in an orderly way.

The watery clouds, by contrast, since they are born from a thin, moist vapor and consist of contiguous, translucent parts, and more readily take on a regular surface, therefore both reflect and refract light more regularly. It is to this kind of cloud that I think the various configurations of the Sun produced by refraction are chiefly to be ascribed. For it often happens that one and the same cloud, having passed beneath the Sun, twists it not only into various shapes, but also displays it toward different regions of the sky — now oblong, now compressed. The planes of the refractions are likewise now vertical, now those of the circles of altitude, now others oblique in every direction. This variety I myself, as well as others, have very often observed — not without singular wonder and delight of mind, and especially when the clouds are more violently driven by the winds; and it is so real that no account of these appearances can be given except from refraction.

This refraction turns out so varied for one of these reasons: either because the refracting surfaces of the cloud are repeatedly changed, or because the position and inclination of the whole cloud have shifted from one state to another. Either one of these — although the two often coincide — is enough to cause this momentary variation;

no differently than if someone were to look at segments of enormous crystalline or glass spheres flitting beneath the Sun; for these would indeed refract it in various ways according to the variety of their interposition. Evidence of this may be drawn from water, whether still or agitated: for as uniformly as the Sun is bent back in the one case, so unevenly is it thrown back in the other.

Mists

[Body heading, italic: "NEBVLAE."]

Most of what has been said about clouds applies to mists, for mists arise out of clouds — dry from dry, moist from moist, dense from dense, and so on. In this alone lies the difference: that the eye is usually located within mists, but rarely or never within clouds; and so, behind the clouds, the eye is in thinner, transparent air. Then, too, mists are always dispersed like flying ash, and hence barely continuous, and for this reason less apt for refraction than clouds — which experience also seems to confirm; for mists usually dull the Sun, but rarely transform it.

The question here is: how does it come about that under a persistently very clear sky and a thoroughly purified air the Sun appears more elliptical than under a very hazy and, so to speak, misty sky? The answer, it seems, must be referred to the differing nature of the elevated media. For when the sky is at its clearest, the vapors are dewy, thin, contiguous, translucent, and smoothly bounded, and therefore highly refractive; but when the sky appears dull, the atmosphere is then infected with earthy exhalations — dry, smoky, disconnected, turbulent, and by no means born for refracting, but rather for dulling and shaking the Sun. Anyone can test this, among other ways, if he catches the image of the Sun, through interposed smoke or through wandering and stirred-up dust, onto paper by means of a tube. This is what one of my pupils skillfully did on November 24, and concerning the ellipse of the rising Sun he records, among other things, the following.

> The opinion that scintillation is caused by vapors passing across the line of sight has been much confirmed, and so on. For when the Sun was rising, and was not yet entirely above the horizon, up to almost an altitude of one degree it trembled continuously and most violently. Afterwards it grew still, until it reached the chimney of the College [Convictus, the Jesuit boarding-college], from which abundant smoke was rising — or rather was not rising, but was being spread out toward the west. When this smoke passed across between the tube and the Sun (just as clouds sometimes otherwise pass across), the Sun suddenly, in exactly the same way as before on the horizon, trembled most violently; and

> when the smoke went away, the tremor at once went away too. I noted this until the Sun rose past the altitude of the second degree, and so on. This crossing of the smoke, however, changed the diameter almost not at all — it certainly did not make it shorter, but rather larger. See chapter 34.

OTHER TRANSPARENT BODIES [Alia transparentia corpora]

There is water enclosed in glass, which — if its ends are shaped appropriately, that is, made convex, concave, plane, and so on — produces everything that crystal and solid glass produce, as anyone who tries it will readily find. There are also certain translucent stones, amber, horn, and the like, which have a good deal of opacity mixed into them and can be polished only with difficulty. These therefore refract either nothing at all or very little; they transmit the images of colour and light that have soaked into them only grudgingly, and because they seem to be of a dry constitution they are for that reason of little use for refraction.

COROLLARIES [Consectaria]

From the experiments and demonstrations already set out, useful teachings are gathered — paradoxical, perhaps, but true and thoroughly indispensable for astronomical work.

CONCERNING THE APPARENT HEMISPHERE AND ITS CIRCLES [Circa hemisphaerium apparens et eius circulos]

CHAPTER XXVII [Caput XXVII]

1. Every point of the open sky except the zenith is subject to refraction. For all of them, except the zenith, bend their rays toward a single point taken within the vapours, off the center, onto the surface of the vapours (by the corollary to Lemma 3, etc.).

2. Every point of the open sky except the zenith appears higher than it properly is. This is demonstrated from chapter 22.

3. The whole of the sky that is visible is more than what appears: it is more, because of the raising [exaltatio]; it appears to be less, because of the contraction.

4. If what appears refracted exactly equals a hemisphere — which can be determined from diametrically opposite stars — then it is certain that, without refraction, less than half the sky lies open, and hence that the distance of the sky from the earth is perhaps smaller than is supposed, and the earth's semidiameter perhaps not entirely imperceptible by comparison. It will therefore not be beside the point to attend to this matter.

5. But if what appears refracted is greater than a hemisphere, it becomes probable that the remoteness of the sky from the earth is beyond anything the senses can grasp.

6. Whereas if what appears were less than a hemisphere, the earth would be brought much closer to the sky.

7. Given an even conditioning [temperament] of the atmosphere all around, by which the sensible horizon is spread out: only the horizon and its parallels keep the true figure of a circle, even when refracted and raised. All the other circles, both great and lesser, lose the perfect form of a circle — namely because of the unequal refraction that customarily occurs at their individual points. And this is because they are not parallel to the horizon but oblique to it.

8. Of the circles that are always visible, the one that is greatest in reality, being raised by refraction, appears to be smaller than the greatest of those always visible; and the one that now appears to be greatest is in reality larger than the greatest, and it cuts the horizon, and always keeps some portion of itself below it.

9. Of the circles that are always hidden, the one that is greatest in reality does not remain hidden but appears above the horizon, because of the elevation that refraction customarily produces; and the one that now appears to be the greatest of the hidden circles is in reality smaller than the greatest of those always hidden.

10. A circle of the sky coinciding with the sensible terrestrial horizon is raised by the benefit of refraction so as to touch the earth; otherwise it would cut that horizon, and, lying below it, would not come into view. But the circle

that in reality would coincide with the sensible terrestrial horizon is now refracted, higher than the horizon itself.

11. All vertical circles lift above the horizon some portions of their arcs that in reality are set below it, raised by refraction.

CONCERNING THE APPEARANCES OF THE STARS [Circa siderum apparitiones]

CHAPTER XXVIII [Caput XXVIII]

1. No stars set outside the zenith are seen in their true places, though this is not very apparent to the senses where they stand at about 40 degrees of altitude, more or less. This is evident from corollary 1 of the preceding chapter.

2. No stars outside the zenith display their true altitude above the horizon, though this escapes the senses when they are close to the zenith. Evident from corollary 2 of the previous chapter.

3. All stars situated on the sensible horizon are in reality below it. Evident from corollary 10 of the previous chapter.

4. All stars that are in reality on the sensible horizon appear to be above it. Evident from the same.

5. For any stars situated in the same vertical circle, near the horizon yet unequally distant from it, the apparent distance is not the true one; rather it is always less than the true distance, and it is what is presented to the senses. Gathered from corollary 3 of the previous chapter, and from others here and there.

6. For all stars lying together on the horizon, the apparent distance between them is the true one, and free of error. Evident from corollary 4 of chapter 23.

7. No stars rising together over the right horizon [horizon of a right sphere] are widened more than is due upon it, whether toward the south or toward the north. Evident from the preceding.

8. All stars striking the oblique horizon exhibit a northern latitude — both at rising and at setting — greater than is right, and a southern one smaller;

and indeed those that trace parallels nearer the elevated pole can do more in this respect than those farther off, and so on. The reason is that the ascension and descension of the former is more oblique than that of the latter; hence, being raised by refraction, they appear in a more distant vertical circle than the latter, etc. Evident from corollary 11 of the preceding chapter.

9. From what has just been said, the true and genuine cause is given for two appearances that long tormented the learned.

The first: that the Sun at the equinoxes seems to decline somewhat toward the north or south — as Martianus Capella [Martianus Capella] plainly teaches in book 8 of his *Encyclopaedia* [*De nuptiis Philologiae et Mercurii* VIII], and as Kepler [Johannes Kepler] correctly explains in his *Paralipomena* (*Ad Vitellionem Paralipomena* IV.10), and others in various places. For the account of this is drawn from the previous corollary — namely, from the apparent amplitude being greater than it should be.

The second: that stars grazing the horizon together, or nearly so, are thought to differ from one another by more upon the horizon than they stood apart high up at the zenith, or when they were near it. For this too comes about because of the amplitude, which upon the oblique horizon it presents as greater than is fair, and likewise because of the elevation — anticipated in the morning or postponed in the evening — that is caused by refraction.

10. All stars situated in the same vertical circle, or in verticals very close to one another, but unequally distant from the horizon, converge toward each other more than is fair as regards altitude. Evident from chapters 22 and 23, etc. Hence their distance in altitude appears less than it truly is, and so leads into error.

From this you may in turn draw out another cause why the eye, even when placed in a right sphere, judges that stars lying on the horizon differ from one another in longitude by more than they seemed to near the zenith — although in reality they are equally far apart in both positions. For when the eye considers these same stars relatively, in relation to higher stars — insofar as they form, for example, a triangular figure or the like — it finds the arrangement of this figure to be different, and more compressed, when the stars are set near the horizon than it had seen near the zenith. This is because the stars lying on the horizon are also perceptibly raised toward the neighbouring higher regions by the intervention of refraction, so that the intervals of altitude are foreshortened.

Hence it comes about that the eye finds the triangle more compressed on the horizon than at the zenith, and from this it concludes — equally readily, or more so — that the lower stars lying on the horizon have receded from one another in longitude, rather than that they have given way in altitude toward the higher region. So it is in the following example.

Diagram. A woodcut of the apparent hemisphere drawn as a semicircle standing on its horizontal diameter. The base is the sensible horizon, running from A at the left end to C at the right end; B is the apex at the top (the zenith region). The observer's eye D sits on the base near the left, just to the right of A, drawn as a small radiating burst. Two triangles of three stars each are shown. High near the apex is the star-triangle B–E–F (nearly equilateral, hence acute-angled): B at the top, with E and F set just below and to the right of it. Low near the horizon is the star-triangle G–H–I (scalene and obtuse, "compressed"): G a little higher at right of center, with H and I on the base line to the right of D. G, H, I are the same three stars as B, E, F, now seen near the horizon.

- A (left end of the base / horizon): western extremity of the apparent hemisphere; also one extremity of the Sun's longitude in the alternative reading of the figure
- B (apex, top): star at the zenith; top vertex of the high (acute, equilateral) triangle; in the alternative reading, the star at the very top of the Sun's disc
- C (right end of the base / horizon): eastern extremity; the other extremity of the Sun's longitude in the alternative reading
- D (on the base, near left): the observer's eye on the earth, drawn as a radiating burst
- E (high, below-right of B): vertex of the high equilateral triangle (corresponds to H near the horizon)
- F (high, right of E): vertex of the high equilateral triangle (corresponds to I near the horizon)
- G (lower, right of center): apex-vertex of the low (obtuse) horizon triangle; the same star as B raised near the horizon
- H (on the base, right of D): vertex of the low triangle; the same star as E, one end of the base H I
- I (on the base, right of H): vertex of the low triangle; the same star as F, the other end of the base H I

In the apparent hemisphere A B C, let the eye D on the earth observe three stars B, E, F arranged near the zenith so that the triangle B E F is equilateral, and therefore acute-angled. Let that same eye D then meet these same stars as G, H, I, now approaching the horizon; the triangle G H I presented by those stars will appear scalene and obtuse-angled — or at any rate more compressed — with the angle formed at G obtuse, whereas at the vertex B it had been acute. Hence by an illusion vision concludes that the base H I is greater on the horizon than the base E F near the vertex, and therefore that the stars H and I have moved apart. In this it wrongly supposes that the altitudes H G and G I have not changed, whereas they are in fact shorter than B E and B F, because of refraction. This very point is clearly grasped if we take the semicircle A B C of the said figure to be the middle disc of the Sun as it traverses the circle of the equator when set in a right sphere, and consider a star B at the very top of the Sun and two others at the extremities of its longitude, A and C. Near the zenith these will make up a right-angled triangle A B C; on the horizon the angle at B will appear obtuse because of the solar ellipse. We brought forward something similar in the case of the [sun]spots [maculae].

— the sunspots on the Elliptical Sun (p. 29) — so that you may see this matter not only firmly established by the clearest reasoning, but also confirmed by irrefutable experience. From this, too, some account can be given of why the eye judges the Sun in the horizon to be larger than at the zenith or near it. For the eye supposes that something has been added to its breadth, when in fact something has rather been taken from its height. But there is yet another cause of this, arising from the eye itself, which must be set out elsewhere.

11. Stars that only graze the horizon downward from the upper hemisphere without cutting it lie on a circle larger than the greatest circle of those bodies that are always visible [the circle of perpetual apparition]; those that graze it upward from the lower hemisphere lie on the circle smaller than the greatest circle of those always hidden [the circle of perpetual occultation]. This is clear from corollary 8 of the preceding chapter. Hence,

12. Whoever investigates the elevation of the pole from stars near the horizon, whoever explores the vertical distances of stars, whoever tracks down true positions — he is plainly exposed to the danger of error. Refractions must therefore be known, so that a remedy may be applied to the error.

13. But if those who live in the right sphere [*sphaera recta*] see both poles on the horizon, then it is certain that, once the vapors are removed, half the sky will not be seen; it is certain, too, that the fabric of the heavens is not so vast, nor is the bulk or the remoteness of the stars so great. From this a twofold wound would be inflicted on Copernicus [Nicolaus Copernicus]: the first, that the motion of the stars is slower than his view supposes; the second, that the ratio of the firmament to the great orb [*orbis magnus*, the Earth's annual orbit] would be perceptible. But these questions are left to precise investigation.

14. Since it is of great importance to know this remoteness of the starry sphere, we shall investigate it as follows in our oblique sphere. In the sphere ABCD, let the axis of the world be AC, terminating at A in the depressed pole and at C in the elevated pole; let the horizon be BEDF, the equator FGEH, and let the center of the sphere be I, the Earth. Since, then, the common intersections of the horizon and the equinoctial are the points F and E, these are the points of true rising and setting — F the rising, E the setting.

So I say, first: suppose two stars in the equator are found diametrically opposite, and are observed at these points of the horizon, in such a way that while F rises, at the same moment the opposite star E sets. Then it is impossi—

Diagram. A single large circle representing the oblique sphere ABCD seen edge-on, with two intersecting ellipses drawn inside it (the horizon and the equator) and additional chords joining the labelled points. Two star-symbols (asterisks) mark stellar positions inside the circle.

- G (upper-left, on the circle): a point of the equator (equator FGEH)
- C (upper-right, on the circle): the elevated pole; upper terminus of the axis AC
- B (left, on the circle): a point of the horizon (horizon BEDF)
- D (right, on the circle): a point of the horizon
- A (lower-left, on the circle): the depressed pole; lower terminus of the axis AC
- H (lower-right, on the circle): a point of the equator (equator FGEH)
- L (upper-center, inside, beside a star): apparent raised position of one star above the horizon (cf. "in K & L")
- E (upper-center, inside, beside a star): true setting point (*verus occasus*), intersection of horizon and equator
- K (center-left, inside, beside a star): apparent raised position of the other star above the horizon
- I (center): the center of the sphere, the Earth
- F (lower-center, inside): true rising point (*verus ortus*), intersection of horizon and equator
- H (bottom-center, below the circle): a second H, apparently marking the lowest point of the circle [? — role uncertain; possibly a compositor's duplicate of the equator-point label H]

—ble that the magnitude of the Earth is imperceptible in relation to the magnitude of the firmament. For otherwise both these stars would be seen above the horizon at the same time — especially since, by the benefit of refraction, they would be raised considerably.

Dico 2. If two diametrically opposite stars, F and E, are seen at the same time raised above the horizon, at K and L, and that by about half a degree or more, it is more than probable that there will be no perceptible ratio of the Earth to the firmament. For in that case, once the atmosphere is removed and refraction consequently ceases, these same stars would be seen on the horizon, and so [only] half [the heavens] would be granted to the sight.

Dico 3. If diametrically opposite stars in the equator cannot be seen together, whether above the horizon or on it, then it is simply necessary that the Earth be quite perceptible in relation to the firmament — inasmuch as it removes so much of the view of the sky that these two stars, not even when higher, can by the gift of refraction be observed together at all. And so that astronomers may both know and watch for this,

Dico 4. If two stars set in the equator, such as F and E, appear alike to all peoples — that is, are either seen together on the horizon, or above the horizon together and equally; or can never be seen together but are perceived in succession in the same way everywhere — such stars, I say, are diametrically opposite.

Dico 5. If two stars in the equator, F and E, exhibit their risings and settings alike but interchanged, they are diametrically opposite. For instance, suppose F and E are seen together, F at its rising, E at its setting: then after mid-year (and this because of the Sun's absence) F is seen at E's setting and E at F's rising, together on the horizon, just as before mid-year; or together above the horizon at K and L, and that with the roles interchanged. It is evident that such stars are diametrically opposed.

Dico 6. If two stars in the equator, when their motions and positions are interchanged, do not present the same places relative to the horizon, it is impossible, other things being equal, that these stars are diametrically opposite.

What I have said about opposite stars in the equator can be applied to any two heavenly bodies that are diametrically opposite in any vertical circle, under the position of any sphere whatever.

This matter is most readily, and can best be, observed in lunar eclipses that occur at sunrise or sunset. For there the great luminaries hold a diametrical opposition and have their seats in the true horizon. See Pliny [Pliny the Elder], *Natural History* II.13; and Kepler [Johannes Kepler], *Paralipomena* [*Ad Vitellionem paralipomena*], Prop. 11, Probl. 4, no. 50, etc.

On Times and Daylight [Circa tempora et luces]

Chapter XXIX [Caput XXIX]

1. Civil days — reckoned from rising to rising, or from setting to setting (abstracting also from the Sun's second motion) — vary in length according as the refraction is great or small; for these days are measured by the presence of daylight, which refraction distributes unequally. Hence, in the business of observing, the hours counted from rising or setting are not to be trusted too far. For the Sun rises earlier at one time than at another, etc.

2. Days from midnight to midnight are always equal, for here refraction is no hindrance, since it occurs only in the vertical circles, according to altitude and not longitude. And the meridian is the primary of the vertical circles, etc.

3. Astronomical days, from noon to noon, likewise afford an equally secure equality. For even granting it could happen that the Sun at noon were higher than it ought to be, it does not on that account reach the meridian — even apparently — either later or earlier, but culminates at genuinely equal intervals.

4. The diurnal arcs, or ordinary solar days — which some call artificial and others natural — are always longer, and their complements, the nocturnal arcs opposite them, are always correspondingly shorter, in every sphere. The sole reason is that the Sun's refraction takes place along the perpendicular line, which therefore reaches up toward the zenith of the place, and so the Sun rises earlier than it ought and sets later than it ought.

5. Hence all sundials, and especially those constructed from altitudes, must necessarily go astray, and those who set the hours from rising to setting, or the reverse, frequently err — unless corrected from some other source. And so that this may be done better, one must attend chiefly to the first point of the Sun's rising and the last point of its setting.

Nota 1. In a twofold way the shadows cast by gnomons or by the axes of clocks — or the cursors [markers] on quadrants and cylinders, and the like — can deceive: for either by the hour-lines being drawn incorrectly, or by a false place being assigned to the Sun in the Zodiac, they can impose upon the unwary novice. As, then, concerns the hour-lines;

Nota 2. The deception can fall either in the duration of time or in the marking-out of the hour-lines. As concerns the duration of time;

Nota 3. Always and everywhere, under every sphere, when the Sun ascends above the horizon time is anticipated [made earlier], and when it descends below the horizon time is postponed; so that, by the benefit of refraction, men enjoy the Sun long—

—longer; and in the arc of any parallel intercepted by two hour-circles, the Sun in traversing it spends more delay in the morning and evening than at noon — a cause that is to be attributed to refraction alone. Hence,

Nota 4. Since refraction is unequal — greatest at rising and setting, and then gradually smaller and smaller as the Sun rises higher — it comes about that these intervals of time too, being subject to refractions, present us with unequal durations; and the differences of these hours will be in the same proportion to one another as the refractions were, and conversely, etc.

Nota 5. Just as under the right sphere and the oblique spheres near it the Sun's refraction produces horary intervals of time, so — and this is quite remarkable — under the parallel sphere and the oblique spheres near it, it anticipates and postpones whole days. This happens on account of the very great obliquity of that sphere; for there the Sun stays at the horizon for a very long time, because of the inclination of the Ecliptic. Therefore, if its refraction equals even a single degree — which there is no doubt often happens — several days will slip by before the Sun traverses in the Ecliptic as much arc as corresponds to one degree taken in the vertical. For to one degree in the vertical circle of this sphere there correspond in the Ecliptic 2° 30′ 29″ 21‴. And since refraction can rather often exceed a whole degree — indeed most probably sometimes reaches two — it follows in such a case that the Sun will emerge above the horizon in the parallel sphere five days earlier, because to two degrees taken in the vertical there correspond 5° 1′ 44″ 4‴ in the Ecliptic, etc. The contrary happens in the right sphere, where the Ecliptic has a small inclination to the horizon, and the first motion soon carries the Sun away from the horizon; this does not happen in the parallel sphere, where only the second motion raises the Sun above the horizon. Nevertheless it can happen that in both cases the quantity of the refraction is in itself the same, etc.

On the Projection of Shadows onto the Hour-lines [Circa umbrarum, in lineas horarias, proiectiones]

Nota 6. Since refraction, as was said in Notes 3, 4, and 5, everywhere and always errs with respect to time — advancing the Sun sooner and delaying it as it departs — while the rays from the Sun, striking the hour-styles, cast the shadows that distinguish the hours: the question now is this.

Does refraction exercise its force upon shadow-clocks [sciotherical dials] as well, and, just as it raises the Sun above the horizon [*finitor*] earlier than is right, so make this same effect appear in the shadows of gnomons cast onto the hour-lines? Or does it do only that, while the hour-lines themselves remain the same, and the shadows of the sty—

the styles allow; the hour-lines they would have occupied, with the Sun unrefracted, whether rising or setting. For instance: at Ingolstadt the Sun, when it occupies the beginning of Cancer, would reach the horizon at four in the morning and eight in the evening if there were no refraction; whereas in fact, being refracted, it climbs above that same horizon before four in the morning and quits it after eight in the evening. The question now is this: do the shadows of the styles register this same shift on the hour-lines as well? Or do they instead fall on the very hour-lines they would have struck if the Sun had appeared without refraction?

Briefly, then, the following points.

1. Quadrants, cylinders, rings, and similar portable timepieces [horologia mobilia] — those constructed from the Sun's altitudes above the horizon — always show, in every place and every sphere, the time, the hour, and the place of the Sun that would have been indicated if the unrefracted Sun had shone. The reason is that these instruments do not determine the Sun's place in the zodiac from the Sun's ray; rather, they presuppose that place as already known and found, so that from it one may arrive at the hour that is sought. And since the hour-lines on them are drawn from the Sun's altitudes — the altitudes it has above the horizon in any given sign and at any hour of the day — it follows that the cursor or index, once set to the Sun's place, shows the hour that matches the Sun's altitudes, whether or not the Sun has acquired that altitude through refraction. And so the hour of rising and setting that there would have been without refraction is always what these instruments represent, and so on.

2. The other, fixed sundials [horologia sciotherica fixa] — such as the vertical, the equinoctial, and the like — in a *right* sphere always receive the rising and setting shadow from the refracted Sun onto the very same hour-lines on which they would have received it from the unrefracted Sun. The reason is plain: in this sphere the horizontal plane coincides with the six-o'clock circle [circulus horae sextae]. Therefore, whenever the Sun sits on the horizon, it is always on the six-o'clock circle; and so on these dials the shadow of the style is directed to the six-o'clock line. This is because those hour-lines are the common intersections of the hour-circles with the hour-planes, and hence they necessarily lie in the planes of their own circles, and so on.

3. In an *oblique* sphere, the shadow of the style cast by the refracted Sun as it rises and sets does not shade the same hour-line that it would have covered if projected by the unrefracted Sun. Proof: the shadow of the style, the hour-line, and the Sun's ray striking the style — and hence the Sun itself, wherever it may be placed — always lie on one and the same hour-circle, as is evident from gnomonics. But the Sun set upon the horizon occupies one hour-circle when refracted and another when unrefracted, because (per Corollary 8, chapter 28) when it is refracted it obtains a different [amplitude here] —

— refracted here, obtains a different amplitude when unrefracted. Therefore the ray falling from it onto the style, and the style itself, and the hour-line shaded by the style, likewise lie on a different hour-circle when the refracted Sun holds the horizon than when the unrefracted Sun reaches it. That the Sun, with its amplitude altered, also changes its hour-circle becomes evident from this: in an oblique sphere the hour-circles pass through both poles and through every individual point of the horizon; and so the Sun, standing at one point of the horizon or another, necessarily lies in one hour-plane or another. Hence in the morning, at sunrise, the shadow of the style falls on an hour-line that (in the morning) *precedes* — and in the evening *follows* — the line that was owed to the rising and setting of the unrefracted Sun.

Frequent experience here at Ingolstadt has confirmed this. For the solstitial sunrise in Cancer placed the first shadow between the third and fourth morning lines, and the sunset placed the last shadow between the eighth and ninth, so that nearly an eighth part of an hour, both morning and evening, was registered by the shadow beyond the true time, and so on.

4. Under a *parallel* sphere these variations occur to by far the greatest degree, as will readily appear to anyone who weighs the matter. For, on account of the inconstancy of refractions, it can happen that the Sun — for people set in this region of the globe — flashes into view on some twelve-o'clock circle, or nine-o'clock, or one-o'clock, when nevertheless the shadow ought to occupy the six-o'clock circle. This holds unless we are willing to grant that such people lack any fixed and definite meridian, and so on.

On the wandering of the shadow from the Sun's true place in the zodiac:

Note 7. On portable timepieces constructed according to the elevations of the Sun above the horizon, the cursors and styles always show the *true* place of the Sun, both at rising and at setting, in every sphere, and so on. You will find the reason in Statement 1 [Dicendum 1] and Note 6 [Notandi 6], and so on.

In the other, stationary timepieces — those that have the zodiac inscribed on them — the matter stands otherwise. For in the whole universe there is but a single equator, and only in the *right* sphere alone does it so govern the Sun, once received into its own house, that it does not let the Sun stray from the royal road. And therefore in that case the Sun does not, by the rays it emits, drive the shadows into any foreign byway; but just as the Sun itself travels the equator, so the tip of the shadow walks steadily along the equinoctial line, marking the true points of rising and setting and the places of Aries and Libra.

But as for the remaining signs and parallels, the shadow cast by the style *never* shows their true places at the Sun's rising and setting — the places as delineated on the arcs of the zodiac — but always strays from them: less perceptibly, however, in a right sphere; more in an oblique one; most of all in a parallel one. The reason for all this is that refraction takes place along the vertical circles. Now among the parallels, in a right sphere only the equator coincides with the true vertical; refraction on that day therefore takes place along it, and so the shadow cannot deflect to either side. Hence on the opposing dial-plane it will shade the Sun's true place, and so on.

But as the Sun traverses the remaining parallels, it does not set its center at the point where those parallels intersect the horizon — which is what would have to happen if it shone unrefracted — but instead brings it somewhat toward the point of true rising. This is because the Sun is stretched out onto the *visible* horizon while it is still, in reality, below it. Therefore the vertical circle drawn through the place where the Sun really lies below the horizon cuts the Sun's parallel there; it does not, then, fall on the point that the horizon has in common with that parallel, but necessarily cuts the horizon *between* the points where the parallel and the equator respectively meet the horizon. From this it is proved that a spurious place is likewise shown by the style's shadow. But in a right sphere this wandering, because of the very acute angles that the verticals make with the parallels, does not — in my judgment — rise to the level of perception.

It happens otherwise in an oblique and in a parallel sphere. This I note here in passing. Suppose that in a parallel sphere refraction amounts to one degree. Then the Sun, near the vernal equinox and still moving in the 27th degree 30′ of Pisces, will leap onto the horizon, because in that sphere a single degree of a vertical circle answers to 2° 30′, and so on, on the ecliptic. The dial-shadows will therefore show the Sun at the beginning of Aries, when in fact it is still nearly three days short of it. Conversely, when the Sun does pass through the beginning of Aries, the dial-shadows will throw it into roughly the third degree — or the third parallel — of that same sign.

Things akin to these will occur in oblique spheres, and the Sun will always be displaced from its place the more, the more oblique the sphere is. For the vertical circles let down to the place of the Sun as it refractedly rises or sets stand far from the Sun's true place — the point where the parallel the Sun is on cuts the horizon — because of the large angle the vertical makes with the Sun's parallel. From this it comes about that the arc on the horizon subtending that angle grows very perceptible; and so the Sun's true parallel, the one it is actually traversing, is always shifted to a foreign place, with the result that at rising and setting the shadows assign the Sun a spurious place in the zodiac. These points I wished only to hint at, in keeping with the brevity I set myself; for to treat them as they deserve would demand an entire volume.

6. It also happens that, in an oblique sphere, the rising and setting amplitudes [amplitudines ortivae et occiduae] appear greater toward the *visible* pole than toward the hidden one — something that would not occur without refraction.

7. From this we get the true reason why the northern stars, as they rise and set in the northern climes, incline northward more than they ought.

8. In any place, light must necessarily linger longer than darkness; and the more oblique the sphere, the more enduring the light is for its inhabitants. And since refraction can amount to a whole degree and more, it is possible for those who live under the poles that the Sun rises earlier by two days and more. Hence by the benefit of refraction alone it can come about that, under those spheres, summer exceeds winter by five days or more. On the same basis, some of the perpetually hidden stars can be raised into view for them, and again depressed, by the work of refraction alone, without any change in the pole. This prompts a further question: whether, on this very account, some disagreement about a shift of the pole has crept in among the authorities. But this we reserve for discussion elsewhere.

9. The same causes teach — and experience confirms it — that in an oblique sphere the days, especially in summer, are longer than the true measure. For since the Sun ascends more obliquely, it comes about that a single degree taken along a vertical circle, measured by right ascension [ascensio recta], corresponds in an oblique sphere to two, three, four such degrees, and so on — each of which, however, is equal in time to a single degree of right ascension. This is why I have stopped wondering that in summer the Sun sometimes grows light before the fourth hour, and that an error afterward creeps into the clocks (something I have very often experienced) when they are set by the rising or setting Sun. For in this case it is the Sun — offending through refraction — that must be corrected to well-regulated clocks; the clocks are not to be thrown off to match the stumbling Sun. Yet with those clocks the Sun will easily come back into agreement once it has struggled clear of the fields of refraction, and so on. I give this warning so that those occupied with such matters may take heed: for anyone who, from the *apparent* points of the Sun's rising and setting, tried to estimate the rising and setting amplitudes — and likewise the diurnal arcs and kindred quantities — would blunder gravely in his calculations, and would enlarge the latitude of the place more than is right, and so on.

10. Finally, let us with the most grateful hearts acknowledge the Creator's bounty in these things. For what those vapors, by their own dullness, blunt from the power of the Sun's splendor, the shape the Creator has given them makes good again. While they strive to hold the Sun back from shining, they in fact detain it longer for all peoples; they cut short the darkness that hates the light, prolong the light, shorten the night, and enlarge the day. For to all nations the light is present for longer and the shadows for a shorter time — so that thus, from the Sun, the very fountain of light, we may draw the fullest encouragement and aid for carrying on and continuing the works of light.

CIRCA TERRICOLAS.

CAPUT XXX.

Many points have already been raised about the elliptical Sun itself, and here I set them out more plainly.

1. If half the sky lies open, free of vapors, then—so long as the atmosphere is present—one and the same observer can look at opposite stars at the same moment. This much follows; but the converse does not follow necessarily.

2. On the same assumption: the stars that have not yet set for us have already risen for our antipodes, and those that are rising for us have not yet fully set for them. And so at one and the same time both we and our Antichthones [Antichthones, the dwellers on the opposite side of the earth] are gazing at the Sun.

3. From the fact that we find the Sun elliptical toward whatever quarter of the horizon it stands, we may conclude that the Sun is itself spherically round. That it is compressed at rising as well as at setting, in the south as well as in the north, is shown by its solstitial, winter, and summer risings and settings alike.

4. Suppose that from the Sun's center—wherever it may happen to lie—we imagine the greatest circle in the heavens described, as though from a pole. Then all people on the earth lying directly beneath that circle will have their zenith on that circle, and so they will see the Sun elliptical on their own horizons when it is a quadrant of the circle away from their vertex. From this it follows:

5. If you wish to know which peoples, and of what sort, see the Sun elliptical on their horizons together with you on any given day of the year—at the moment when it shines for you either at morning or at evening—they will be those who lie beneath the vertical circle drawn through your vertex, taking the rising or setting Sun as its pole; that is, those who are distant a quadrant of a circle from the Sun's place on the terrestrial globe.

6. To grasp this at a single glance, take a sound terrestrial globe on whose surface the ecliptic is inscribed, marked off into the signs and degrees of the zodiac. Place the Sun's given position, once found on it, at the vertical point. Then, if you carry the given region or place around to the eastern part of the horizon circle, the Sun will set for that place; and all the peoples lying round about beneath the horizon see the Sun elliptical on or near their own horizon—those found in the eastern semicircle of the horizon see it setting, those in the western see it rising. But if instead you carry the given place around to the western arc of the horizon, the Sun will rise for that place; and all the places lying beneath the western semicircle of the horizon will see that same Sun rising, while those set beneath the eastern will see it setting—but affected with an ellipse, according to the varying amount of vapor.

7. For all the other peoples set in the upper hemisphere, in proportion as they are more or less distant from the zenith (the globe keeping its former position), so too will the Sun for them appear more or less ele-

—vated, and more or less compressed; whereas those who sit directly beneath the vertex will behold it perfectly spherical.

8. Yet among all these peoples of the whole upper hemisphere—even if one person were set right against another, or one eye against another eye, so that the hemispherical surface of the earth were paved with nothing but eyes all beholding the Sun—no one would see another's ellipse, or otherwise catch sight of the solar globe; rather, each eye would receive its own image. And this is so not merely because it would receive a cone peculiar to itself from the object, as happens with fixed visible things, and so on, but also because the very base of the received cone would be different, being formed from a part of the solar image on the surface of the atmosphere. It is exactly as tends to happen with a mirror throwing back the appearance of an object: each act of sight beholds its own image, and in its own proper place as well. And precisely the same thing occurs, by the same principle, in the refraction of the Sun. From which:

9. Just as a mirror drinks in and reflects the whole image of the thing seen in its whole self, and the whole image again in any part of itself, so too there is no room whatever for doubt that the whole Sun is received in the whole quasi-hemispherical portion of the atmosphere, is depicted in each of its several points, and is carried to the eyes by refracted rays. For in this, reflection and refraction plainly agree.

OBSERVATIONES QUAEDAM PECULIARES ANNI 1614.

CAPUT XXXI.

[Some particular observations of the year 1614.]

In the thirteenth year [1613] I put forward a few observations concerning the Sun as it touches the horizon, and from them I was left in doubt whether its contraction was always the same or in fact different, at the very first point where setting begins and at the last point where rising is completed—the moment that occurs when the Sun, still standing above, is touched by the horizontal plane but not cut by it. To resolve this doubt I brought the utmost diligence to my observing, and made frequent and careful notes; out of these I have selected a very few, but such as they are, and set them here before your eyes—the kind on which one may rely with complete confidence and without any hesitation whatever.

OBSERVATIONES ANNI MDCXIV.

[Observations of the year 1614 — printed p. 57]

The following full-page table occupies printed p. 57. Its columns, from left to right, are: **Mensis** (Month) · **Dies** (Day, with A. = *antemeridianum*/morning, P. = *pomeridianum*/afternoon) · **Eleu.** (Elevation of the Sun above the horizon, sub-headed **G. M.** = *Gradus, Minuta* — degrees, minutes) · **Diametri Solares** (Solar diameters, shown graphically as two engraved horizontal bars per row: **L.** = *Longitudinis*, the major/horizontal axis, and **A.** = *Altitudinis*, the minor/vertical axis — these carry no digits, only bar-lengths) · **Defectus** (the Defect, in two sub-columns: the fifth column giving the proportional denominator by which the defect relates to the major diameter, e.g. *subseptuple* = 1/7; the sixth column giving that defect reduced to minutes and seconds, on the assumption of a solar diameter of 34 minutes — see the *Explanatio* on the following page).

Mensis	Dies	Eleu. (G. M.)	Diametri Solares	Defectus (proportion)	Defectus (′ ″)
Ianuar.	9. P.	1. 0.	[bars: L. / A.]	7.	4′. 51″.
Ianuar.	10. A.	1. 30.	[bars: L. / A.]	9.	3. 46.
Marti⁹ [Martius]	3. P. / 4. A.	0. 0.	[bars: L. / A.]	4½	7. 33.
Iulius	25. A.	0. 0.	[bars: L. / A.]	4 1/5	8. 5.
Iulius	26. A.	0. 0.	[bars: L. / A.]	5.	6. 48.
Septēb. [September]	23. A.	1. 0.	[bars: L. / A.]	6 1/4	5. 40.
Septēb.	23. P.	0. 0.	[bars: L. / A.]	6 4/5	5. 0.
Septēb.	27. P.	0. 0.	[bars: L. / A.]	6 1/2	5. 14.
Septēb.	28. A.	0. 0.	[bars: L. / A.]	6 4/5	5. 0.
Octob.	2. P.	0. 0.	[bars: L. / A.]	6 2/3	5. 6.
Octob.	8. P.	0. 0.	[bars: L. / A.]	6 1/3	5. 46.
Octob.	9. A.	0. 30.	[bars: L. / A.]	6 1/2	5. 18.
Octob.	10. A.	0. 20.	[bars: L. / A.]	5 1/8	6. 35.
Octob.	11. P.	0. 0.	[bars: L. / A.]	6.	5. 40.
Octob.	12. A.	0. 0. / 0. 15.	[bars: L. / A.]	6 6/9	7. 17. / 6. 30.

Mensis	Dies	Eleu. (G. M.)	Diametri Solares	Defectus (proportion)	Defectus (′ ″)
Decēb. [December]	22. P.	0. 0.	[bars: L. / A.]	2 4/5	12. 8.
Decēb.	22. P.	—	[bars, worked example: major axis **L.** from D to F, minor axis **A.** from D to E; letters D (lower left), F (upper middle), E (lower middle)]	—	—

EXPLANATIO LITTERARUM, &c.

[Explanation of the letters, etc.]

1. The [first] column holds the names of the months. **2.** The day, and the time at which the observation was made—the forenoon marked with the letter **A**, the afternoon with the letter **P**. Column **3** gives the Sun's elevation above the horizon, in degrees and first minutes. **4.** The [fourth] column displays the solar diameters, that of length by the letter **L** and that of height by **A**. The fifth and sixth columns together express the proportion which the defect—the amount by which the smaller diameter is exceeded by the greater—bears to the greater diameter: namely subseptuple, subnoncuple, and so on [i.e. 1/7, 1/9, etc.]. That defect is then, in the sixth column, reduced to first and second minutes, on the assumption of a solar diameter of 34 first minutes. I have done this in order to guard against any lapse—whether of the engraver's or the printer's carelessness—in setting the diameters before the eyes: so that you may know the exact ratio in the numbers themselves, without error, and from these may yourself form the solar diameters in the noted proportion, and the ellipse along with them.

PECULIARIA HARUM OBSERVATIONUM.

CAPUT XXXII.

[Particular features of these observations.]

On the 25th and 26th of July the morning sky was indeed perfectly clear, but showers of the night before—especially on the twenty-fifth—had frequently interrupted it. This is doubtless why the Sun's contraction was greater on the 25th than on the 26th. These two ellipses were plainly visible to the naked eye, without any difficulty.

On the 10th of October, in the morning, a tall little cloud that had drifted across divided the solar ellipse into two unequal segments and enlarged the diametral height of the whole ellipse considerably, in this manner: the smaller segment is A B C, the larger D E F, and the height augmented is B E. When the cloud moved off, the old appearance of the ellipse returned, and its height settled back into its former dimension. I have often noted such metamorphoses on other occasions as well, including during the Sun's evening descent. The true explanation of this appearance—among many others, which I reserve for their proper place and time—seems to be refraction away from the perpendicular, because the little cloud A C F D is denser than the medium lying between it and the eye; and from this it comes about that the part of the Sun A C F D, glowing dimly through, is somewhat enlarged, and so the deformity of the ellipse is produced—a thing that turns up everywhere in other experiments.

Diagram. A woodcut of the setting Sun distorted by a cloud. The Sun is drawn as an oval outline (the ellipse). Across its upper-middle passes a dense, elongated cloud (*nubecula*), rendered as a shaded woodcut band that bulges out beyond the ellipse to left and right. The cloud splits the disc into a smaller upper segment and a larger lower segment; its presence raises the effective vertical height of the figure.

- B (top of the ellipse): apex of the disc; upper endpoint of the augmented height B E.
- A (upper left): point where the cloud's upper edge meets the ellipse on the left; a vertex of the smaller segment A B C and of the cloud region A C F D.
- C (upper right): point where the cloud's upper edge meets the ellipse on the right; the other vertex of segment A B C and of region A C F D.
- D (lower left): point where the cloud's lower edge meets the ellipse on the left; a vertex of the larger segment D E F and of region A C F D.
- F (lower right): point where the cloud's lower edge meets the ellipse on the right; the other vertex of segment D E F and of region A C F D.
- E (bottom of the ellipse): lowest point of the disc; lower endpoint of the augmented height B E.

October 11. Three noteworthy things occurred.

First: when the Sun was nearer the horizon, still standing about 30 minutes above it, it plainly appeared to stand still, and did so for about three minutes of an hour [tres circiter horæ scrupulos]. What signaled this halt was the Sun's ray caught through the tube on paper: warned by a companion, I watched it, motionless and clinging to the same spot. Afterward it suddenly began to slip down again, faster than usual. I had often enough seen things like this on other occasions, but had trusted them less; the clear evidence of this appearance, however, freed me from all doubt.

The explanation is to be sought in the unequal Refraction of the Sun, which keeps growing greater and greater in just such a proportion that it exactly compensates the Sun's true motion, so that the motion becomes imperceptible. This happens either from the varying and uneven density of the refracting vapors; or from a change in the surface of incidence, as it gathers into a more or less rounded mass; or from a greater or smaller distance of the vapors from the earth; or finally from all of these mixed together.

From this experimental foundation it is clear [Margin (a): Some of the ancients' methods are uncertain.] that the magnitude either of the Sun or of the Sun's sky cannot be reliably investigated by the flow of water, or of sand, or by the interval of a running horse, during the time the Sun's globe wholly emerges or is hidden from its first contact with the horizon — the method Cleomedes and other ancients used. For if the Sun stands still during that time, a great deal of water will be collected, and so, when it is apportioned against the daily flow of water, it will be found rarer than it should be; and thus the Sun will be made out both larger than is right, and the Sun's sky smaller than is right. But if the Sun hastens its course during that same time, it will itself come out smaller than is right, and its sky larger. Another method, and a safe one, that could be substituted for this, will be given in its place.

The second thing worth noting that occurred was this: the Sun appeared more contracted when it was still farther from the horizon than when it had touched it completely — something I had also noticed on other occasions, but less accurately.

Finally: the half-circle of the Sun facing toward the horizon plainly appeared sharper or fuller, while the one turned away appeared blunter and more obtuse. The reason for all these things seems to be one that must be assigned to the vapors. And this is a phenomenon not of today only, but of other days as well:

Diagram. An ellipse (the flattened setting Sun) resting on a straight horizontal line, which is the horizon.

- A (left end of the horizontal line): left terminus of the horizon
- B (right end of the horizontal line): right terminus of the horizon
- F (top of the ellipse): apex of the upper half turned away from the horizon
- D (bottom of the ellipse, where it meets the horizon line): lowest point, against the horizon
- C (left side of the ellipse): left junction of the two halves
- E (right side of the ellipse): right junction of the two halves

In this figure AB is the Horizon; CDE is the part of the Sun held down against the horizon; EFC the other part turned away from

it, which is less curved than the opposite part CDE — contrary to the common law of Refraction.

December 22 was the winter Solstice, a clear day, as was the middle one preceding it. Toward sunset there were thick, dense, cloudy, and restless vapors, which nevertheless did not prevent the Sun from coming into view when it joined the horizon, although they roughened its edge like a saw. The following things were noted.

1. The arc of the Sun looking toward the horizon often appeared more curved than the upper one.

2. The width of the Sun, however, always remained the same as it had been at midday.

3. A large portion of the solar arc stretched out along the width, both the upper and the lower, very often went off into a straight line — such as CD above and EF below — but soon drew itself back into a curve.

4. The whole Ellipse frequently put on humps [gibbi], now above, now below, so that it appeared plainly irregular — which also happens frequently at other times.

5. The whole altitude of the Sun varied inconstantly, not uniformly. For appearing more contracted, a little later it came out taller, as is seen in the diameter DE against the observation table: when the Sun was lower it turned out longer, whereas when the same Sun was higher it presented the shorter diameter DF — contrary to the ordinary tenor of Refraction. And

this too often happens on other occasions. The accompanying diagram sets most of this before the eyes.

Diagram. An irregular, lumpy woodcut of the Sun near the horizon, its outline broken into several rounded humps (gibbi) above and below, with a straight horizontal dashed line drawn across the middle.

- A (left end of the dashed horizontal line): left terminus of the Sun's width (Longitude)
- B (right end of the dashed horizontal line): right terminus of the Sun's width (Longitude)
- C (upper left): left end of the upper straight arc CD
- D (upper right): right end of the upper straight arc CD
- G (top, above CD): apex of the upper hump CGD
- H (lower left): left point of the lower hump HE
- E (lower center-left): junction of the humps HE and EIF (also the left end of the lower straight arc EF)
- I (bottom, lowest point): apex/dip of the lower hump EIF
- F (lower center-right): junction of the humps EIF and FK (also the right end of the lower straight arc EF)
- K (lower right): right point of the lower hump FK
- K (a second "K" appears at the right, on the level of the dashed line) [? — the label K seems to appear twice in the woodcut; the text names only one hump FK]

AB is the Longitude of the Sun, the same throughout the whole day and time of observation. CD and EF are the solar arcs stretched out into a straight line. CGD, HE, EIF, and FK are the solar humps. The rest, concerning the roughness, has been said by me elsewhere in various places, and especially in the *Apelles* [*Tres epistolae de maculis solaribus*]; consult also the *Disquisitiones* [*Accuratior disquisitio*].

A doubt arises here: whether these crags, windings, and meanders in the Sun rise up from Refraction, or rather from a suffocation of the solar light — such that the intervening vapors, having lost their density, substitute shadow for light within the Sun's orbit, and so, once a gap has been made, gather the remaining light into certain crags, as it were.

And experience, when brought to bear, seems to argue for this. For just as the Sun's tremor and the customary roughening at its perimeter is nothing other than the cutting-off of the perimetral rays by the intervention of some opaque little body — so that a shadow is thrust piecemeal into the Sun's image — so too these great notches seem to be able to arise from nothing

else than from the interposition of great masses that the Sun's ray cannot penetrate, and so the Sun itself comes onto the stage pitifully truncated. That the familiar cutting of the solar circumference is brought about by the interception of the rays by little bodies lying between us and the edge of the Atmosphere is clear from plain experiments: namely, if the Sun glides down through smoke; if through scattered or stirred-up dust; if through ascending dewy vapors — for all these bring on both tremor and tearing, not indeed by refraction but by shadowing. And so plainly does this happen that the shadows of the smoke, and its winding whirlings, are best discerned even in the very middle of the Sun's image. It is likely, then, that the same thing happens in these monstrous ruptures of the solar disc, since it agrees with reason that many masses of this kind are found in the depth of the Atmosphere, which by hiding the light have the power to shape the Sun into so monstrous a figure.

Yet it seems it must nonetheless be said that this vast transfiguration of the Sun is not a shadowing, but plainly a true — and an irregular and disordered — Refraction, arising from the fluctuating surface of the Atmosphere. And this is proved.

1. Because if this deformation of the Sun arose from the shadow of an intervening mass, the Sun would also sometimes be notably shortened in its width, just as at the extremities of the width it is also roughened by the interposition of vapor or smoke, and so on. But it has not been observed up to now that these notable clefts are introduced into the width of the Sun; therefore they are not from shadow.

2. If they came from shadowing, something would also be produced in the middle of the Sun's disc, and either some shadow would appear (as of ascending smoke and vapors), or the Sun would be split into two parts (as the little cloud did on October 10, etc.). But none of this happens. Therefore that metamorphosis is not from shadow.

3. When those recesses gape in the Sun, it does not come about by some sudden alternation, as tends to happen in the trembling laceration of the periphery; but by a certain slow and fluctuating shaking, exactly as if one were to watch the Sun floating on water and being raised and lowered. This does not seem to come to it from anywhere else than from the unequal

and fluctuating periphery of the Atmosphere. And so this variation of the Sun is to be ascribed to true Refraction, and not to mere shadowing.

4. The cutting of the Sun at its perimeter happens at any time of day, when smoke or vapor intervenes; but this distortion does not happen except near the horizon. Therefore the cause of this and of that event is not the same. For if it came about from interposed bodies, why do they not set themselves against the Sun when it is high?
5. It is a sign, therefore, that this proceeds from the fluctuating surface of the Atmosphere alone, which exerts its chief force upon the Sun around the horizon, and so on. The contrary arguments will be resolved from these without much difficulty, and so on — though I judge that they are not without their own probability.
6. It is established that the Sun, too, while it touches the horizon, contracts unequally; for on September 28 it fell short by 5′. 0″; on the 23rd of the same month by 5′. 40″; but on July 26 it was diminished by 6′. 48″; on October 12 by 7′. 17″; on March 3 and 4 by 7′. 33″; on July 25 by 8′. 5″. Finally, on December 22 the Sun shrank as much as never before, by 12′. 8″ — more than a third of its own [diameter].

Ludovicus Avila [Luis de Ávila] writes, in Book 2 of the *War in Germany* [*Belli Germanici*], as does Gribaldus the Italian [Gribaldi] in a certain Report, and likewise Gonzalo de Illescas [Consaluus de Illescas] in Part 2 of his *Papal History* [*Historia Pontifical*], that at the time when the Emperor Charles V [Carolus V] fought with the Saxon, those who gazed at the Sun saw it much higher and farther from setting than was fitting for the hour of the day, and that it slowed its course, so that — as once for Joshua [Iosuę], the leader of the people of Israel, so now for the Emperor of the Christians — by making a halt it might prolong the day, for the sake of avenging God's enemies. This miracle, granted by the divine will, is made credible by what was observed on October 11, and can be defended in the highest degree by the reasons assigned; nor should it therefore be ascribed to a chance event, since the Rainbow, though most natural, in no way weakens God's covenant with the human race.

A FEW NOTEWORTHY OBSERVATIONS OF SOLAR ELLIPSES, made at Ingolstadt in the year 1616, in the month of November.

CAPUT XXXIII.

It is worth adding here a few accurate Ellipses, skillfully observed by one of my students on several days of the month of November in the year 1616. For from these no small confirmation will accrue both to my own accounts of the Elliptical Sun and to those brought forward here, since across so great a distance of time so great a harmony of these matters is nonetheless found.

The Ellipse, both Eastern and Western, on the thirteenth day of the month of November — and also the Morning one on the fourteenth day of the same month — was uniform and of equal magnitude; hence it is exhibited in a single figure for those three occasions.

[Margin, right edge, faint and partly cut off: "in ⁹⁄₁₆" (?) — likely bleed-through or a cropped marginal note]

Diagram. A rectangular framed woodcut containing four parallel horizontal line-segments stacked one above another, each labelled with a letter at its left end and another at its right end. They compare the Sun's full horizontal length with its successively contracted apparent altitudes near the horizon. (Uncaptioned on this page; its caption stood on the preceding leaf.)

- A (left) … B (right): the Sun's length (its full horizontal diameter), the reference segment.
- C (left) … D (right): the altitude at the moment the Sun first touched the horizon.
- E (left) … F (right): the altitude with the Sun raised one degree.
- G (left) … H (right): the altitude with the Sun raised two degrees.

AB is the Sun's length; CD its altitude at the moment the Sun first touched the horizon. The deficit of CD from AB was a fifth part and six-thirteenths [?] of AB itself, AB as a whole being reckoned at 71 [units]. Hence, with the Sun's diameter set at 34′, the Ellipse came to 6 18/71, that is 6′15″.

EF was the Sun's altitude once it had risen one degree. It falls short of AB, the Sun's diameter, by ⅐, that is, 5′51″ [recte: 4′51″ — genuine source compositor slip, minutes 5-for-4; prints identically in both witnesses].

GH shows the altitude of the Sun raised two degrees — the Sun's elevation always being taken from the ray projected out from its centre. Its deficit from AB is ⅟₁₃, that is, 2′36″.

The Observer [*Observator*].

On the thirteenth of November the rising Sun was marvellously torn: for great fragments hung down everywhere from its edge into the Sun. Moreover, no sunspot could be seen before an altitude of one degree.

Just as at the morning rising of the 13th, so too — equally, and in precisely the same way — both at the evening setting and at the morning rising of the 14th, the Sun was thoroughly disfigured, torn, and wounded to an extraordinary degree, and no sunspots were visible before an altitude of one degree. The Ellipse was accordingly likewise the same.

Furthermore, on the 14th, immediately before sunrise, there rose above the horizon a certain huge disc of light, exactly like the Sun but three times the diameter the Sun otherwise shows, and with a blurred edge. But the true Sun, following soon after, showed this to have been a secondary Sun.

On the morning of the 14th, with the Sun halfway above the horizon, the shadows of the styles on both vertical dials — the one turned toward the east and the one toward the south — showed the hour 7 and 9/16 of an hour; which fraction, reduced by calculation, gives 33′45″ of hour-minutes.

The western Ellipse of the fourteenth day.

Diagram. A rectangular framed woodcut with four parallel horizontal line-segments, matching the form of the figure above; it depicts the setting (western) Sun of 14 November.

- A (left) … B (right): the Sun's length.
- C (left) … D (right): the elliptical altitude as the Sun first touched the horizon.
- E (left) … F (right): the altitude with the Sun raised one degree.
- G (left) … H (right): the altitude with the Sun raised two degrees.

AB is the Sun's length; CD the elliptical altitude when the Sun first touched the horizon; EF the Sun's altitude raised one degree; GH the altitude of the same raised two degrees. The altitude CD falls short of the length AB by one sixth part of the length. Set against the Sun's diameter of 34′, this gives 5′40″. EF fell short of AB by 3′40″; GH by 2′7″.

The Observer.

In the evening, as the Sun touched the horizon, the dial turned toward the west showed the hour 4 5/8, that is, 35′25″ of hour-minutes.

The Sun was far more impure than in the morning, more torn, and of malignant light; and yet, behold, the contraction was far smaller.

15 November.

Morning Ellipse, of the rising Sun.

Diagram. A rectangular framed woodcut with six parallel horizontal line-segments stacked one above another, depicting the rising (eastern) Sun of 15 November through successive degrees of elevation.

- A (left) … B (right): the Sun's length.
- C (left) … D (right): the altitude of the same as it grazed the horizon from above.
- E (left) … F (right): the altitude with the Sun raised one degree above the horizon.
- G (left) … H (right): (together with IK) the alternating altitude of the Sun raised two degrees.
- I (left) … K (right): (together with GH) the alternating altitude at two degrees.
- L (left) … M (right): the altitude of the same at the third degree of elevation.

AB is the Sun's length; CD the altitude of the same as it grazed the horizon from above; EF the Sun's altitude when raised one degree above the horizon. GH and IK give the alternating altitude of the Sun raised two degrees above the horizon; LM the altitude of the same at the third degree of elevation. The full Ellipse — by "full" I mean when the Sun touches the horizon in the visible hemisphere — was ¼ of the solar length, which is the line AB, the Sun's diameter. This being set at thirty-four minutes, the Ellipse — that is, the deficit by which the altitude CD falls short of the length AB — will be 8′30″. EF falls short of AB by 5′50″. IK, 3′50″. LM, 2′10″.

The Observer.

The Sun rose clearly and brightly; it had no vaporous radiance surrounding it, nor did it allow unhindered inspection. Although sunspots were not seen before it left the horizon, it too was torn — though not as much as yesterday and the day before. And yet, even though it was much brighter, more brilliant, and thus purer than yesterday, both at rising and at setting, its contraction was nonetheless far greater.

At times it behaved inconstantly; for beneath an elevation of two degrees its altitude wavered with an inconstant reciprocation — now stretched out along the line IK, now drawn back along GH — favouring the former more than the latter. I remember having noted this same inconstancy on other occasions too.

15 November.

Evening Ellipse, of the setting Sun.

Diagram. A rectangular framed woodcut with three parallel horizontal line-segments, depicting the setting (western) Sun of 15 November.

- A (left) … B (right): the Sun's length.
- C (left) … D (right): the altitude when raised one degree.
- E (left) … F (right): the outstretched altitude of the same, standing at two degrees.

AB is the Sun's length; CD the altitude when raised one degree; EF the outstretched altitude of the same standing at two degrees. CD falls short of AB by 3′40″; EF falls short of AB by 2′40″.

The Observer.

After noon the horizon's circumference appeared very vaporous, dull, and blunt — the Germans call it *dümper*. The Sun, now a degree higher, no longer displayed sunspots; it was of feeble light and broadly surrounded by a vaporous radiance.

As soon as it had risen a degree high, it was so weakened in light that no further Ellipse could be recorded, since its edge and the large part nearer the horizon no longer shone onto the paper; and it was very deeply and thickly red, and most tolerant of the unaided eye. Moreover, by the judgement of the unaided eye it was very much torn.

This contraction was far smaller than the contraction of the morning Sun. From this it is again confirmed that the more vapours there are — at least of some definite kind — the less the Sun is contracted; and the fewer they are, the more it is contracted.

Finally, it also seems possible to conclude that, if there are few vapours of this kind, the winds cannot be violent. For on these preceding days, because there were more vapours around sunrise (as was apparent from the Sun's Ellipses) than this morning, the wind too, immediately after sunrise on those days, was violent; whereas today it was always calm, though at noon somewhat more marked.

November, day —. [*Nouembris die.*]

The 22nd & 23rd, morning & evening; likewise the 24th, evening only. — A uniform and equal Ellipse,

Diagram. A rectangular framed woodcut with three parallel horizontal line-segments, depicting the uniform Ellipse observed on 22–24 November.

- A (left) … B (right): the length of the Sun grazing the horizon.
- C (left) … D (right): the altitude, falling short of the length by ⅕.
- E (left) … F (right): the altitude of the Sun standing one degree above the horizon.

AB is the length of the Sun grazing the horizon; CD the altitude, falling short of the length by ⅕; and so, AB being set at the Sun's diameter of 34′, the Ellipse of the same will be 6′48″. EF represents the altitude of the Sun standing one degree above the horizon, and falls short of AB by 4′.

The Observer.

On the 22nd the Sun rose moderately clear; sunspots began to be seen soon after it emerged above the horizon. As it touched the horizon, its whole edge appeared not much torn, and the Sun was cut across the middle by a stretch of clouds.

It set in exactly the same way. From early morning until nightfall the wind was easterly, notable indeed but not violent.

With the Sun's centre occupying the horizon, the western face of the vertical dial showed most exactly the hour 4 1/2.

On the 23rd the Sun rose moderately clear; for it had a vaporous radiance surrounding it, and tolerated inspection to some degree, and sunspots began to be seen once it was almost entirely above the horizon. The east wind was violent to some degree; at noon much more violent, when the Sun too trembled most violently. On the same day it set in exactly the same way. The wind was a little gentler.

With the Sun's centre on the horizon, the vertical dial declining from south toward the west cast the shadow of its style at exactly 4 1/2; whereas the one declining from north toward the east showed, in the morning, with the Sun midway on the horizon, the hour 7 3/4, a little less.

On the 24th. The Sun was ruddy, ringed with a halo; one degree high, it cast an extremely feeble light onto the paper. A little later it ceased to be visible at all with the shutters open; but once the room was darkened, it held [in view] right up to setting.

In Praemissas Obsevationes Digressio.

Caput XXXIV.

Before all else: anyone who compares these observations with my own ellipses — whether the ones set out in this work or in the Elliptical Sun — together with the accompanying remarks, will find a truly remarkable agreement in every respect. This fact rightly wins great credence both for the phenomenon itself and for the doctrines derived from it. That is especially so because these observations were all carried out in my absence and without my knowledge, with no assistance from any observations of mine — so that every suspicion of bad faith must be set utterly aside.

De Solis Exotico Idolo, &c. [On the Sun's Strange Phantom, etc.]

Next, what is reported of the solar disc at the rising of the fourteenth day may well be true, even on my own eyewitness testimony. For although I did not see this particular one on that day, nor have I elsewhere in my writings mentioned any seen in other places (because, of course, they were not a part of the Elliptical Sun but its forerunners), I have nonetheless caught sight of them several times, standing on the horizon about one to one-and-a-half minutes [of time] before the rising Sun. In colour they did not differ from that ruddy hue of dawn, except that here, in this meteor [atmospheric phenomenon], the red is far more concentrated than in the dawn, glowing like red-hot iron or copper, or like a live coal. In the correspondence of their figure they agreed entirely with the Elliptical Sun, except that they were proportionally more contracted than the Sun. In size they much exceeded it: their length surpassed [the Sun's] length by roughly threefold, their height [the Sun's] height by about twice over. The outline of this mass, as I observed it, did not appear cleanly defined. Nor was its brilliance strong enough to cast light into a dark place, or a shadow into a lit one.

Now someone may ask what this apparition is. I answer briefly. (1) It is not the Sun itself. This is plain, because the Sun follows a little later and by its presence disperses this prodigy at once; and besides, no suitable reason could be offered for the Sun's being magnified into so vast a portent. Not to mention that the Sun could not linger in virtually the same spot for so long a time. And lastly — how would it happen that the Sun should suddenly blaze forth, as though a mask had been laid aside, when for so long before it had hung there as if inert? Who would have kindled its rays? Who would so swiftly have stripped away that spell? That portent, therefore, is not the Sun.

(2) Nor, again, is it the pure, unalloyed light of the Sun. For the Sun's light lacks colour, and the Sun radiates spherically in every direction; that phantom, therefore, ought to have scattered [light] in every direction, if it were the pure and unalloyed light of the Sun. But this spectre is seen in one place only. It is not, then, the mere light of the Sun, etc.

(3) Nor, furthermore, are they mere colours implanted in that matter. Otherwise they would not come into view only occasionally, but would present themselves constantly to the sight in the presence of light and daylight — as we experience with permanent colours.

(4) I say, therefore, that this sight is a certain confused image of the Sun, blended with the colours of that matter — colours which, by mutual reflections and refractions of rays heaped up in that matter, so united the appearance of the Sun with their own hues that at last an image of this kind reached the eye. What was said about the fiery colour proves the point, since it resembles the colour of the reddening dawn and the colours of the *Caumata* [Greek: καῦμα (kauma, "burning heat"); the ruddy heat-glows], etc. And it is certain and beyond doubt that those colours arise, through reflection and refraction, from the mutual concourse of rays, lights, and implanted colours. Here, then, we must reason in the same way.

The principal explanation of the figure and magnitude must be sought from the quantity and configuration of that mass, which drinks the Sun's rays into itself, and likewise from its position oblique to us — though I would not deny that the force of refraction can add much assistance. These points are confirmed by various experiences. For when the Sun strikes fir wood (of which I also treat in Apelles [his *Tres epistolae de maculis solaribus*]) or thin pine, it makes the wood appear as if set ablaze, provided it is turned into a place dark on every side. And colours are thrown back from a triangular glass [a prism] [vitrum trigonum], or from some other coloured surface, onto white walls, and tint them, etc. But I do not linger over these obvious matters.

De Ventis. [On the Winds]

The conjecture about winds, offered on the 15th, is not without foundation. For since windy gusts arise from the rarefaction or condensation of exhalations of this kind, as mutual alternations of cold and hot come on, he will not have wasted his labour who — by contemplating the Sun as projected onto the paper, forming a judgment about the vapours, with the senses too concurring — attempts to reach a probable opinion about the winds. For usually winds attend a dull and blunted Sun, while calm skies attend the clearest one. Just as the ordered regularity of the ellipse [appears] when the Sun shines with a pure [light], so ugliness and distortion of its face arises when the Sun grows dim beneath a squalid halo. The former happens when the atmosphere is continuous, translucent, and at its upper edge uniform and highly polished; the latter, when it is broken up, half-opaque, marred by colours, and endowed with a fluctuating, ill-burnished surface.

These troubles are brought on by terrestrial exhalations — dry, burnt, either breathed out from the earth or first generated in the air by the sky's heat. The contrary [effects] come from the ordered and genuine temperament of the vapours that surround the earth; and this ordinarily has more power over the Sun's ellipse than that dense build-up of vapours does. On which matter further inquiry is needed.

De Limpida et Turbida Atmosphaerae Contemperatione.

Caput XXXV.

[On the Clear and Turbid Tempering of the Atmosphere. Chapter XXXV.]

It has now been established, by the long and steady investigation of many years (as may also be gathered from what precedes and from the Elliptical Sun), that the Sun — especially when the sky near the horizon is clear and purified, in a bright, more transparent, and hence thin atmosphere, and being, as it appears, more luminous — is contracted more; whereas it is dimmer in a nebulous atmosphere, one less pervious to light and therefore thick. For since, by the consensus of the Optical writers, a denser medium refracts more and a rarer one less, it seems consonant with reason that the Sun ought to be refracted more in a dense atmosphere than in a rare one. Yet certain and tested experience attests the contrary. The reason for this, then, is what we seek.

Next: always, or at least usually, after heavier rains or winds there follows an atmosphere clear about the horizon, and a calmer, more compressed face of the Sun, etc. — even though reason would seem to urge the contrary. For rains and winds stir up, generate, and mass together vapours and exhalations, as may be seen in storms; by such events, therefore, the atmosphere ought to be disturbed, not clarified. But nature does the contrary, and sense-experience shows it. The reason is sought.

The reason for the first. A clear, bright, purified, thin atmosphere, etc., is by nature intermediate, as it were, between the higher air and the lower water. For it is formed from the thin spirits of water, which, once they have climbed into the higher regions, by their own nature readily unite with one another and become continuous, on account of an innate

and kindred humidity — whereby it comes about that they wrap themselves uniformly around the earth. For, with nothing resisting, the heavier [parts] settle into the lower places and the lighter into the higher, according to their equilibrium of weight; and so at the summit they are massed into a spherical surface. And because humidity, innately cold, stubbornly resists adventitious heat, the lightness does not seriously counteract their weight; and thus they remain low, at any rate. From these [considerations] you will ascribe to the clear atmosphere the following properties.

It is low, because of the weight nature has implanted in it. It is transparent, because of the natural rarity of water, and through the mutual [cohesion] of its parts by the bond of the moist [substance]. It is smoothly massed together, because of its even wrapping around the earth, which comes from uniform weight.

A polished surface, therefore, equidistant from the earth's surface, produces the clean definition of the solar perimeter there received and transmitted to us, and the uniformly deficient figure of the ellipse, arising from the nature of ordered refraction and incidence. Its low position makes the ellipse's magnitude — that is, its contraction — greater, as was shown earlier. Its transparency, finally, [produces] the serenity of the Sun and, in its shining, a clear radiation.

The opposite [properties] hold for a turbid, murky, half-opaque atmosphere, packed with earthy spirits that scarcely coalesce among themselves. For since those vapours are of terrestrial nature, they abound in dry and opaque atoms, which, being deprived of moisture, cannot be made continuous — a thing that intercepts transparency and the direct [passage] of rays. And because these particles, on account of their smallness and their separateness from one another, with natural dryness supervening, are supremely apt for motion, therefore, once heat has been drawn in from the sky or from some other external source, they are very easily swept aloft, like smoke, in no order, with the utmost recklessness. As a result, they never combine into any stable sphere or regular surface, and are thus rendered less suited to refraction. And because dryness joined to heat readily lends wings to lightness, these minute particles ascend very high; from which it again follows that, even if they were configured into some regular sphere, nevertheless the altitude would still subtract from the magnitude of the refraction — in accordance with what was set forth in the Elliptical Sun, p. 21, and here throughout.

A turbid atmosphere, therefore, appears to display the following characteristics: that it is high; that its upper limit is ill-arranged; that its body is poorly compacted, consisting of atoms that are by their own nature non-transparent [adiaphanous — Greek ἀδιαφανής, "not see-through"] or coloured, and hence only grudgingly transparent.

A poorly compacted surface strips the refracted Sun of its sharpness and cleanness at the perimeter.

Altitude diminishes the size of the Ellipse.

The opacity of the parts, and their inherent color, cast a haze over the ellipse. And because the solar rays are tinged with the appearances of colors, a dark red sometimes settles on the Ellipse, sometimes a paleness, and sometimes a clean and clear brilliance — depending on how the Atmosphere has colored the solar rays.

The dissolution and discontinuity of those parts couples a disordered reflection to the refraction of the rays, and this is the mother of confusion in the whole business. Since lightness, combined with dissolution, is prone to motion, the uppermost edge of this Atmosphere fluctuates greatly. This is why those irregular passages into the Ellipse (discussed in cap. 32, num. 5) are produced, and why the individual particles within its depths, struggling upward, in a sense boil up. The result is a quivering superimposed on the solar Ellipse, and a roughening of its perimeter like the teeth of a saw. For by interposing themselves, these particles intercept the Sun's rays and thus cast a shadow onto the ellipse; and this shadow, kept up all around by an alternating tremor, makes the circumference ragged. The same trembling and tearing of the Sun along its edge comes, however, just as much — or even more — from dewy vapors, because these are carried along in particles still raw and not yet coalesced with one another, particles that can individually intercept the rays with force and split the solar rim.

But why does this not happen over the whole Sun? Why does it tremble and get cut apart only at the circumference? I answer that the whole Sun is very often shaken by a visible tremor, like a man with a fever, so that it seems to shift from its place; but this is not always the case, nor always noticed, because of the dense throng and force of the rays in the middle of the Sun — rays that at the periphery are weaker and so more easily obstructed by those little bodies.

And so the explanation of the first point seems adequately given. See also cap. 26, On Clouds, where I touched on this question.

From this it is clear that a diligent Astronomer, working from these symptoms of the Ellipses, need not forecast the weather rashly, and that the labor spent investigating them is neither useless nor empty — since the greatest aids for meteorology are drawn from them, and much occasion for sound natural philosophy is offered. For if the ancients were permitted — and rightly — to predict rains from the paleness of the Moon, winds from its redness, and fair weather from its brightness, why should we not do the same now from the Sun? From the Sun, I say, not bare, but Elliptical?

For the Sun enjoys this privilege: that it carries the face of the sky down onto paper along with itself.

For the Ellipse of Refraction, that offspring, dwells within the vapors and does not show itself before the Sun descends into them; and since the vapors strive to block every approach to us, the Sun, having fought its way through — not without sweat and much of their blood — breaks through into our view. Hence it comes about that we learn who and what sort of marauders besiege the sky; for the Sun, having sacked their camp and put on the enemy's spoils, unfamiliar in a foreign guise, elegantly parades their look and color. But enough of this. By these arguments the points laid down at the outset are either settled, or can be.

The reason for the second. Rain is naturally water; and water is held together by moisture and cold as its innate properties. Being fluid and soft, it exceeds air in density; it clings readily to other bodies, and mixes with them quickly.

From this, reasons will be drawn why a cleared, visible horizon — and hence a more serene sky — follows rain. This happens on three counts: (1) because falling rain scours the air clean; (2) because it moistens and cools that same air; (3) because it supplies fresh material for the liquid Atmosphere.

How is the air scoured clean? The little raindrops, as they fall, envelop those smoky exhalations of the sluggish Atmosphere and drag them down, submerged, to the earth — or else, by reciprocal action, they consume them entirely, converting them into water. For clouds, as experience attests, diffuse their power far and wide, moistening the air without dripping and affecting the bodies of animals even at a great distance; how much more, then, will rain already present be able to do?

How is the air cooled and moistened? By mutual contact. The falling rain cuts the air apart in almost countless ways, and the air in turn breaks the drops into the tiniest and innumerable particles. Thus in the air cold succeeds heat, moisture succeeds dryness, weight succeeds lightness, and density succeeds rarity — so that the air, now more compact, offers Refraction a freer entry. For being packed together by cold, and made continuous by the admixture of moisture, it presents a smooth road to the solar rays.

Finally, fresh material — meant, after rains, to establish and increase the clear temper of the Atmosphere — is generally lifted on high; and so the serene brow of the Ellipse, a shapely face, smiles down.

But why is the Sun seen more compressed here in fair weather than when the sky is dark with a murky cast? Because cold and moisture hold the mastery in this temper of vapors: the cold packs the mass together, the moisture makes it continuous, and the weight arising from this holds the collected mass of such vapors near the earth's surface — with the result that the Sun transmits a shorter Ellipse. This is confirmed by plain experience. When the sky has been rainy the day before, we see enormously large Ellipses displayed the next morning, together with the utmost serenity. Why is this? Is it not because the nocturnal cold has drawn the moist breezes together so that they have settled far down, and so have made the clear Atmo-[sphere]

[sphere] lower and the Ellipse larger? Certainly no other reason presents itself.

Objection 1. It follows from all this that a sluggish Atmosphere lies beneath the clouds — which is false. For terrestrial and dry exhalations penetrate into the highest region of the air; and in the morning the dawn reddens before any ray of the Sun strikes any cloud. Either, then, what the falling rain washes away is not the Atmosphere composed of those vapors, or, if it is, they fix their home not above but (by what has been said) below the clouds — contrary to experience, and so on.

I reply that, of that sluggish Atmosphere, some parts are refined and subtle, others rawer and less tamed. The former fly up on high; the latter are held below the clouds. The former suffer nothing from the rain; the latter are dissipated.

Objection 2. Therefore the upper portion of the turbulent Atmosphere, and hence its convex surface, such as it is, remains. And therefore the Sun's Refraction in it lasts even after the rains.

I reply that the upper remnants of this Atmosphere, on their own, cannot do much to transform the Sun, owing to their thinness. Then, that they are dissipated — partly by the rainy clouds, which act in every direction; partly by the sheer extent of the space; partly by the shortage of the sustaining material that the lower portion used to supply; and partly by higher agents, which readily convert them into pure air. It is certain that the glow of the twilight dawn arises long before the clouds are lit up; but that ordinary radiance is not the reddish, murky one we are discussing, but a wholly different one — subtle, pleasant, and whitish — which lasts even in a clear Atmosphere. This latter glow, however, does not last so long, nor appear every day; and so it can scarcely be doubted that it resides in a material lower than that of the dawn.

But you will say (Objection 3). This at least follows: that a serene Atmosphere lies beneath the clouds. Yet reason opposes this just as much. For those vapors can climb just as high as the clouds — indeed higher. Next, it is a familiar thing, and daily met by the observing eye, for clouds to be born in the air under a perfectly serene sky; and where do these clouds come from, if not from vapors floating up there? Moreover, it is an everyday occurrence for huge clouds hanging in the air to vanish, often within the span of a single hour, and be reduced to a transparent body — into what do these clouds go? Do they not pass off into thin vapors and into air? Vapors, therefore, are not below the clouds, but above them.

I reply. These arguments do rightly prove that vapors are as high as the clouds, or higher, and so it does not follow badly that the summit of the clear Atmosphere cannot be confined below the limits of the clouds. But they do not establish — merely from the fact that falling rains wash away the greater part of the murky Atmosphere — that the boundaries of the bright Atmosphere must necessarily be fixed below the clouds. This alone follows: that rains are the cause of a clear Atmosphere, once the misty one has been removed.

But you will say (Objection 4). The bright Atmosphere has been asserted to be lower than that dark one, and so on. Therefore it is plainly to be located below the clouds. Reply: the inference is denied. For this Atmosphere can nonetheless be lower than that one, even if both rise above the clouds, and so on.

Objection 5. What, then, are the boundaries to be set for each Atmosphere? How far is each removed from the earth?

I answer, first: Who has ascended into the sky to measure the road of the vapors and exhalations? Who, to measure the vast dwelling-places of the spirit by which we are kept alive? All the more so since, in this subtle and hidden obscurity, we are almost wholly deprived of the aids of the senses and of the guiding light of experiment, so that reason must contend in this arena almost alone.

The Opinion of Tycho

I answer, secondly, with Tycho Brahe [Tycho Brahe], who in the *Progymnasmata*, on the Refractions of the Sun and their Table, page 91, says in these words: "The chief and truer cause of Refractions lies in the Vapors perpetually overhanging the earth's surface, which make the air nearest to us a little denser and less transparent than is its higher portion. Hence it also comes about that these vapors, near the Horizon, most of all meet the sight, but afterward, in a higher position, are gradually thinned."

He then assigns to these vapors, a little later, on page 95, both a horizontal transverse thickness and a vertical height, where he says: "Taking the maximum elevation of vapors, as demonstrated by the opticians Alhazen [Ibn al-Haytham] and Witelo [Witelo], to be 48′ 50″ of the parts of which the earth's semidiameter is 60 — so that it comes to scarcely 12 German miles — their thickness around the horizon itself works out to 9 parts 55′, or 142 miles, and so on."

And so, in Tycho's view, the domain of the Atmosphere is bounded by the very limits of the dawn and the twilights.

The Opinion of Johann Kepler

I answer, thirdly, with Johann Kepler [Johannes Kepler], who in his *Paralipomena*, chapter 4, on the measure of Refractions, says on page 79: "The surface that refracts the rays belongs neither to vapors wandering about at random, nor to some lofty body reaching up to the confines of the Moon, but plainly to that very air in which we men draw breath in the same way that fish draw water. And of this air, from the aether, manife-[stly] and a clear distinction would strike the eye of a man positioned above no less than the surface that separates air from water now strikes the eye. Hence on p. 117 he declares the matter of the twilights [materiam crepusculorum] unfit for refraction.

And indeed on p. 129, prop. 11, he assigns to the height of the Uraniborg [Vranoburgici] air half a German mile [*milliare medium Germanicum*; *medium* here = "half", as in the parallel *dimidio milliari* below]. And a little later, in the same place: "What others have called vapor, I call — says he himself — air; and I bound it at the summits of the mountains. Above it stand the smoky exhalations, the lamps of the twilights, and immediately the aether follows."

And on p. 135: "It is reasonable," he says, "that there are many places raised half a mile above the surface of the sea — which is the height of the air's surface in the Danish Sound [freto Danico], according to prop. 11 of this chapter." And in the same place: "The Vosges [Vogesus mons] and the continuous Alps of Rhaetia [Rhetiæ alpes] nearly rise above the height of the air that produces the refractions. The perpetual snows confirm this, for they mark the bareness of those peaks, free of the vaporous air. Therefore neither will there be any refractions there, for the most part, etc."

Likewise, in article 9, concerning the observation of the Dutch [obseruatione Hollandorum], on pp. 140 and 141, he states that there the refraction of the Sun amounted to 4 degrees 14 minutes; and that for this a refracting air of nearly two German miles in height would have to be responsible — a height he calls incredibly great.

The same author repeatedly refers the refractions to a warmer and more vaporous air, on folio 137 and often elsewhere.

Kepler's view, therefore, makes the subject of inherence for the refractions this airy spirit by which we are animated, and he seems to carry it up to the beginning of the cold region — because he requires it to be warm and vaporous, and exempts from the law of refraction the barren ridges of the highest mountains, bristling with continual cold. He fixes definite bounds for it, namely about half a German mile more or less; and so its uppermost convexity is discerned from a higher eye no less than the common boundary of air and water is discerned by a lower eye on the earth.

I answer, 4. In these clashes of divergent opinions it is very difficult to find a safe footing and to touch, as they say, the point; where it is already something fine not to stray altogether from the course laid out for one's aim. Nonetheless I set forth my own reasoning — drawn from long and most certain observations, from the common doctrine of the opticians, and from the accepted opinion of the philosophers — in the following propositions.

1. There is no day in the whole year on which the Sun escapes free of the horizontal ellipse [Ellipsi horizontali]. Hence:
2. There is no day in the whole year on which the Sun stands free of horizontal refraction.
3. No refraction exists (at least, no orderly one) except at the surface of a following transparent medium [diaphani] which is at the same time the surface of the preceding medium — so that the two transparent bodies, the one that carries the ray and the one that refracts it, are not continuous but contiguous. This follows from the more common doctrine of the opticians. Hence:
4. This surface at which the Sun is refracted must necessarily always exist, since the Sun appears refracted every day.
5. This surface must necessarily lie above the clouds, the rains, and the winds. For otherwise, just as the winds and storms alter and scatter the vapors beneath them (as experience testifies), so too they would scatter that surface which is formed out of those very vapors — especially the violent winds. Yet they do not do so, as experience testifies in *Sol Ellipticus*, p. 14, and often elsewhere.
6. Since this surface remains permanently throughout the whole year — in every kind of weather, at its clearest, its middling, and its most vaporous — it necessarily follows that the very body of which it is the outer boundary is present under every kind of sky.
7. But no body embraces the earth in its bosom, in every weather, throughout the whole year, and under every sky, except this air we breathe. For the moist vapors, boiling up from everywhere, are either condensed into clouds or resolved back into the air itself; and something similar happens to the exhalations, which, however much more stubbornness they may bring, in the end nonetheless also give way, either consumed or driven from their place. The air, by contrast, remains always and constantly. That this, then, is the body in question is beyond doubt, since everything said thus far fits it perfectly. For the air itself sometimes constitutes a clear atmosphere [Atmosphæram limpidam], when it is diluted by rains or swept clean by winds; sometimes a dark one, when it is dulled by excessive heat,

or blunted by the boiling of its parts, or clouded and thick with earthy exhalations. On the substance all agree; over the words and manner of speaking some may raise a difficulty, etc.

8. For this air, since according to all opticians and physicists it is always denser than the aether laid over it, and moreover keeps its own summit in some degree unmixed with the parts above it, always refracts the Sun as it glides obliquely into it — more or less, according as the air has been spread out far from or near to the earth.

9. Now the air spreads out near the earth whenever, drenched by showers, or swept by south winds, or chilled by nocturnal frost or a wintry sky, it grows cold. For then it contracts, gathering itself into a narrow compass, and — content with a smaller space — it draws down all its higher parts as well, through fear of a vacuum [metu vacui]. And thus its surface becomes a far smaller and lower sphere, and from this the refraction turns out far greater; and the ellipse rises up clearer and larger, etc.

10. On the contrary, this surface recedes from the earth and is enlarged whenever the air, made vaporous by the day's heat, by windy warmth, or by summer's blaze, is rarefied and swells up; it unfolds the parts it had gathered in its bosom, and, once these are drawn out into more open fields, it seeks a larger room. When the earth, unable to yield, denies it that room, the air breaks out upward in a massed column; and so it grows into a larger sphere and surface — which accordingly supplies smaller refractions and thinner ellipses, etc.

11. Furthermore, how high the uppermost convexity of this atmosphere rises is uncertain. That it towers above the highest peaks of the snowy mountains can hardly be doubted. For from that very voyage of the Dutch — who snatched birds' eggs to be hatched amid the mountain snows, and, all but frozen by the extremely cold sky, nevertheless found a refraction of 4 degrees — and again from the Alpine chamois [damis] and vultures, which (as the natives report) eagerly seek out in wintertime the loftiest peaks of the Tyrolean mountains, set amid the snows, and — for the sake of the milder air that breathes abundantly up there — dwell there pleasantly, while meanwhile the dwellers below are almost stiffened by the cold raging savagely beneath. Whence:

12. To wish to confine this atmosphere, and refraction itself, to a vaporous warmth in the air alone, or to the mountain ridges, even the highest, seems

to conflict with both sense and reason. With sense, indeed, because even the coldest air invades the lowest places; because the warmest, in turn, invades the highest, as was just said; because there are and can be no clouds, no rain, no snows where breathable air is lacking; and where these do exist, there it is admitted that air can be breathed. With reason, because this doctrine legitimately assigns to the atmosphere a convex surface, as one owed to the refractions. But who would believe that this surface resides at the boundary between warmth and the cold of the air? Unless someone should wish to tear the somehow-kindled air — as if from a winter's blazing hearth — away from the cold air clinging to it, and to strip both of their continuity: which is unheard of. And who, upon the highest crags, has ever distinguished by sight that surface lying below him? And the air, in which the storms have set up their workshop — is it not dense, is it not full of vapors? Yet it towers above the Alps, as the clouds flying above it there frequently show; whence it necessarily refracts the Sun, etc.

13. Higher, therefore, than the mountain ridges is the surface of the atmosphere — but how high, is uncertain. This is certain: that it is subject to momentary variation of position, place, quality, and so on. Hence they waste their oil and their labor (in my judgment) who track down some fixed height for it, and then from that try to measure the refractions.

14. Nevertheless it becomes probable that the atmosphere does not reach as far up as the matter of the twilights. First, because the twilights, on the testimony of the authorities, keep a height that is fixed within limits, whereas the surface of the atmosphere does not, as clearly follows from what has been said. Secondly, because rain falling

could not fall so far into the atmosphere, if the atmosphere itself were as high as the twilights. For the rain would sweep down scarcely a quarter of a German mile out of it, since the twilight-matter — which is also the higher atmospheric matter — would come away wholly untouched, beyond 12 German miles. It would therefore not suffer so great a loss that the surface of that sphere would sensibly descend on that account. Indeed, even if it were depressed by as great a space as lies between the earth and the raining cloud — that is, ¼ German mile — that ¼ would bear to the 12 miles the same proportion as 1 to 48. And surely this difference of height, coming between the two fragments of spheres described from the center of the earth — whose heights would be distinguished by ¼ of a German mile alone — would in no perceptible way alter the horizontal refraction, on account of the imperceptible difference of inclination made with respect to the ray of incidence on each surface, etc. Whence it is truly improbable, not to say impossible, that the atmospheric surface which produces the solar refraction should coincide with the matter of the twilights.

OBJECTIONS AND RESPONSES.

Chapter XXXVI.

You will object, (1): This surface of the atmosphere does not exist; therefore all that has been deduced from it is empty. The consequence is clear; the antecedent is proved.

1. **By the authority of the best astronomers.** For Tycho Brahe [Tycho Brahe], in the *Progymnasmata*, p. 92, thins out the air gradually and continuously up to the confines of the Moon. Ioannes Pena [Jean Pena], Christoph Rothmann [Christoph Rothmann], Nicolaus Copernicus [Copernicus], most of the ancients, and finally the whole school of the philosophers — apart from the surface that separates water from air, and this air from the heaven — recognize no distinct surface at all: as may be seen in Alhazen [Alhazen], book 7, prop. 51, and Witelo [Witelo], book 10, prop. 54, etc.

2. **By experience.** For we see that, just as water is made continuous with water, so air is made continuous with air, as soon as they have their extremities in common. For who would deny that the breath blown into a bladder is continuous within?

3. **By reason.** Because this air, even though by some accidental alteration it has been affected — for the use of living creatures — as distinct from the pure air above, nonetheless tenaciously retains the substance of its own nature, and, absolutely speaking, is a body, and a simple, homogeneous element; and therefore it is continuous both with itself and with the higher air.

Next, just as the lowest, warmer air is [continuous] with the higher, colder air…

[the air] is continued, notwithstanding these contrary conditions of cold and heat; so too a middle region growing cold both can and must remain continuous [continuum] with one at the height of heat. And we find by experience that the former happens first. For the air breathed into the lungs grows extremely warm, yet is no less continuous with the air outside. Again, the air surrounding the

flame of a lit candle, though it does not itself catch fire, is nonetheless continuous with the cold air farther off. And the air lit up by a convex crystalline lens set facing the Sun — its rays converging by refraction into a single point [in verticem unum] — very nearly bursts into flame, so that in the same way rays reflected from a concave mirror can melt lead in a moment. Yet who on that account would say that this air is separated from the other, colder air? Thus one and the same continuous air grows hot, grows cold, and grows lukewarm.

Furthermore, from the winding periphery of the Ellipse it is argued that a distinct Refracting surface is an empty fiction. For this outline is never found so cleanly precise as it is in the solar image when the Sun is otherwise round — even when the sky is utterly calm, clear, and limpid. But those great bays and wavering prominences take their origin from the summit of the Atmosphere, as was asserted in chapter 32, no. 5. How then, amid such inconstancy, could any definite surface be arranged?

There is this further point: the very slightness of the Ellipse and of the Refraction suggests that they do not proceed from any separate refracting surface. For it seems incredible that, whereas the Sun is refracted in crystal, in water, and in glass by 40 degrees (as they will have it) and more, and so on, this should occur in our air — however much it may be endowed with refracting density — by so small an amount; especially since our air is said to be furnished with so polished a surface, so apt for refracting.

Finally, since everything can be defended by the density of the medium alone, without any distinct surface, it seems that such a surface ought not to be invented. For the varying density of the medium includes a certain implicit and virtual surface, as they say, by the very fact that the medium is denser; and this surface — though not so exactly and regularly as happens at distinct boundaries — can nonetheless in its own measure blunt the Sun and turn it aside by refraction. That this view has appealed to all the best authors is well established. And not without excellent reason: after rains a greater refraction follows, because the air is denser; it is less during exhalations, because the air is rarer; after winds it is moderate, because the air is in a middle state; and so on. And so this distinct surface seems to lie plainly overthrown.

The assertions are confirmed: first, from the fact that the Sun, shining through mists or clouds suited to it, has very often been observed to transform its shape in various ways; yet in the meantime, who would attribute to those clouds any distinctly shaped surface, even a slight one—

—shaped surface? Etc. Second, that ripples [undae], passing through glass otherwise uniform in every respect, produce refraction even though they are continuous with it; therefore a distinct surface is plainly not necessary for refraction, etc. Third, that reflection — with Aristotle [Aristotle] also in agreement — occurs within a single body, where no surface is distinguished; why then not refraction as well? Etc.

REPLY [RESPONDETVR]

1. To the argument from authority [Ad auctoritatem]. It is true that most of the said authors did not speak explicitly of a discrete refracting surface, for the reason that some of the more recent writers set out to treat not so much of Refraction in general as of this refraction of the Sun, and stirred up not so much the manner in which it occurs as whether it occurs at all. Yet scarcely one meanwhile will be brought forward who has also expressly denied it, since from the common rules of Refraction and from the older writers on Optics they were by no means ignorant of its necessity. And if anyone in the end were expressly to reject it, we would value his authority on this point only as much as the reasons on which he relied were worth. For Witelo [Witelo], in the passage cited, grants a surface made distinct by vapors; he stumbled only in this, that he did not allow this vital air the vapors it needs. And Alhazen [Alhazen] acknowledges this surface, but he carries it up to the concave sphere of the Moon, for from there down to us he makes everything continuous, and therefore not subject to any Refraction — even if a difference of density should intervene, etc. He therefore acknowledges a refracting surface, but one set too high. For the air beneath the Moon's concave sphere — or whatever that body finally is, and however it may differ from the celestial — does not refract the Sun's ray, because it falls upon it not obliquely but perpendicularly (at least to the senses).

2. To the argument from experience [Ad experientiam]. Granting that the distinctness of this surface does not fall under the eyes: "Therefore it does not exist." The inference is denied. That it does not fall under the eyes has its cause in the very great likeness among transparent bodies, whereby it comes about that the senses cannot separate the continuous from the contiguous [contiguum]. Do not glass panes, polished with flat surfaces and set over pictures to preserve them and protect them from dust, frequently escape the gaze of the beholder and elude the groping hand? And yet the surfaces of air, glass, and picture are distinct. A clean glass flask filled with clear water you could easily sell to any spectator for solid crystal — and I have very often seen one taken for such; and yet glass and water touch one another at distinct boundaries. Wine, similar in color to water, floats on top of it without mixing, yet that the diligence of sight has been deceived in distinguishing them, smell alone, or taste, bears witness. That the Rhine flows unmixed into Lake Constance [Lake Constance], were there no difference of color, sight aided by touch and motion alone would perceive. Who does not know of the winds carried about in the air? Yet who has seen them as distinct from the air? That they are nonetheless distinct, scarcely anyone will doubt. Thus, if we follow experience as our guide, we shall say that our air is bounded by some upper surface, even though that surface, through its likeness to contiguous bodies, evades every effort of the eyes.

3. To the argument from reason [Ad rationem]. From the fact that air is homogeneous — an element and a simple body — it is wrongly inferred that, even if no different body intervenes, it must necessarily be continuous and not merely contiguous. You have a clear counter-instance in frozen water floating on a river: a simple body on a simple one, homogeneous on homogeneous, element on element. Or if a piece of ice gradually melts: the liquid is not continuous with the frozen ice. So too lead half-melted exists as contiguous, not continuous, with another mass still rigid. Something similar, then, can occur with air, so that in one manner or another it comes to have a distinct surface: whether by motion, or by the alteration of hot and cold, or by moistening and drying, or by a mixture of these, etc. Does not the vapor you exhale from water receive its own peculiar surface? I call the eyes to witness [recte: prouoco]; and yet the distinction between our air and pure air is as great or greater than that between vapor and water, because our air, being extremely humid, is full of vapors.

I add further that two things can be considered in our air. One, insofar as it is air, and so considered it is a most pure transparent medium; the other, insofar as it is joined with the watery and earthy element, and undergoes some mixture by contagion with them, so that something is lost from its highest clarity. Now it may perhaps be that in the former respect our air is not entirely cut off from the higher air — I say "entirely" and "perhaps" because of the circular motion of the upper air, which most attribute to it; but in the latter respect it is always separated at the last boundary of the vapors that insinuate themselves into the air, on the testimony of Witelo, book 10, prop. 54, and Alhazen, book 7, prop. 55. These, with one voice, say that the coarse vapors near the horizon terminate in a convex surface concentric with heaven and earth, and that the Sun's ray is refracted in it; but because these vapors do not spread their breadth toward the middle of the sky, toward the Zenith, hence, they say, it comes about that, like solid sectors as it were of their own Spheres, they are figured in the form of a cone or wedge, with the point fixed at the center of the earth, the eye being outside the conical or plane surface. So the Sun's ray is refracted in them twice: once at the base, at its entry, approaching the perpendicular line (which is a portion of the Spherical surface); once at the lateral surface, at its exit from the perpendicular. And hence they enlarge the stars at the Horizon, and hold that these coarse vapors surround the earth neither always nor everywhere. In this they erred. For the peoples for whom our region lies at the Horizon likewise see the Sun refracted through coarse vapors, just as we do in our own region; whence these vapors surround the earth everywhere and always, as experience too testifies — but they are seen to be less toward the Zenith and greater toward the horizon, because of the longer passage of the rays there and the shorter here, as was demonstrated above from Tycho [Tycho]. Something similar happens to a man set in a fog: he judges the near part to be thin and the far part dense, and he experiences this wherever he betakes himself; from the greater distance it comes about that the apparent parts are condensed, etc. So too snow or rain falling is always thought to descend denser in a distant space than we experience it in a near one. Therefore the vapors, since they surround the earth on every side, will — even on the view of these authors — acquire an upper convex surface, and indeed one concentric with the earth, sufficiently distinct, in the air, in that, spread uniformly from the earth—

—uniformly from the earth, they occupy the higher regions of it. For just as the vapors, as they exhale, if they remained thus visible and equally surrounded the earth, would acquire a topmost surface most distinct from water and air, so it seems highly probable that this air, tainted with vapors, is distinguished by its own vaporous convexity from the purest air above — just as we see our own warm breath, in cold weather, distinguished from the rest of the surrounding air. Hence the evidence adduced from experience itself works for us, not against us; and although we do not find in the air a distinction complete in every respect through cold or heat, we nonetheless gain from it great support for establishing this view.

To the objection of the ill-terminated Ellipse the reply is this: it happens now and then, and especially after winds (which remove from the midst those restless dancers — those Salii, I mean the vapors and exhalations less well coalesced), that the Ellipse is drawn together with a very clean and quiet margin. But if from winding hollows and fissures, or from fluctuations, someone should try to prove the nonexistence of the Atmospheric surface, he will have declared war on Neptune, and will thereby have abolished every surface of the great sea, because the sea is now ruffled by a gentle breeze blowing across it, now twisted by violent whirlwinds dashing against steep mountains and the deepest valleys. What

Palinurus, lurking then at its very bottom, would not behold the Sun shining down from above with an inclination, elliptical and ill-cut, and fluctuating without rest? Yet he would foolishly take away the sea's boundaries on this account. Try this with water placed in a vessel: when it is agitated you will see the Sun's image both nod as it swims and swim as it nods. And do not let it disturb you that you see the reflected image behave just as the refracted one does. These great recessions of the Ellipse, then, do not argue against a refracting aerial surface but rather argue for it. Hence it is no wonder that a neatly girded Ellipse is rare; rather it is a wonder that the images of the refracted Sun can, for the most part, be shaped so orderly and beautiful at the boundary of so fluid a body.

The slightness of the Refraction does not strike home when aimed; for this dart, thrust forth by a slack bowstring, makes no wound: the rarity of the air easily excuses everything. For however great its density may be, it is not fit to catch fire into flame. No wonder, therefore, if it does not much break the light.

The last argument, drawn from reason, together with its confirmations, if rightly explained, has some force, and greatly strengthens our view.

For the surface bounding the Atmosphere can be conceived as distinct in two ways. In one way, as the outermost and separate surface of the air that forms the Atmosphere — just as the topmost surface of the sea encloses it and separates it from other bodies, so that it is not joined to them; and against this surface of the Atmosphere the argument that has been brought forward seems to carry some weight against [the notion of] the spherical surface's distinctness; and we do not necessarily posit such a surface in the air, but hold that it can very probably be posited and that it does exist.

The surface can be understood in another way: as the end or boundary of the Atmosphere, insofar as it bounds the Atmosphere as such, not insofar as it is a continuous airy body. And this in turn can be understood in two ways: either that boundary of the Atmosphere belongs to the thing actually, within a body that is still continuous; or it belongs only in pure potency.

In the former way, there is present in a tunic of the eye — for example, the cornea — a surface, or outermost edge of the iris, joining it to the sclerotic coat. In the latter way, the Meridians are present, like the circles in the heavens. To fix the outermost surface of the Atmosphere in this last way, without any other foundation or distinction, is absurd. For my imagination alone, or a thought of the mind, will not compel the Sun's rays to take other paths; therefore the thing that produces this effect must inhere in the Atmosphere on the side of reality. And this thing is nothing other than some body, either in the air or mixed with the air, into which the ray that enters cannot pass straight through — because of its differing density and the obliquity of its incidence upon that body — but is deflected to the side.

Now the part upon which the ray first strikes, so that it is drawn away from a straight course, we call the surface of the Atmosphere and its topmost extremity, insofar as it is Atmosphere; and it constitutes the boundary between the Atmosphere and the air above it. We say that this surface is not necessarily so distinct that it is of necessity cut off from the upper air — though it is probable and reasonable that this is so. We judge it sufficient if the summits of the Atmosphere are so arranged that they gather together, equivalently, into a spherical surface, one which, even though immersed in and continuous with the purer air, nonetheless bends the solar ray drawn into it, because its natural transparency there passes over into a denser body. All of this, if that is what the argument aims at, does us little harm. For it is evident that some refraction can occur in this way, as is clear from the waves and streaks that lie within the depths of glass, and likewise from fibers embedded in crystal, and so on. This proves that refraction does not necessarily require an outermost contiguous surface, but that the refracting surface can sometimes even be one interior to a continuous transparent body, yet distinguished by density or in some other way. And if this holds in a solid transparent body of uniform clearness, how much more in a fluid of varying transparency? Just as, in a mottled glass bead, the outer translucent part — continuous with the opaque interior — does not prevent reflection from those inner parts from taking place outward, even though the reflection comes not from a surface that merely touches but from one that joins the transparent to the opaque: so, if the inner parts were transparent, they would certainly cause refraction. You may seek further examples establishing the same point from rainbows, from halos, from sun-rods [virgae], and similar atmospheric phenomena, which arise by refraction and re-[flection]

—flection alone, and so on. Thus vaporous air, at the place where nature has set the limit for the rising of the vapors, has probably located the beginning of refraction and the end of its own density. Yet we do not believe this limit to be bound by fixed and immovable markers, nor do we think we can easily determine it to a precise measure. The reason: because the senses, our first scout, find nearly every approach to exploring it blocked against them — as is clear from what has been said so far.

You will object, secondly. Even granting this surface in whatever manner, it still cannot be concentric with the earth. For although it surrounds the whole earth in its bosom, nonetheless — since, on the testimony of Clavius [Christoph Clavius] (*In Sacrobosco* [*Commentary on the Sphere of Sacrobosco*], ch. 1, p. 38), the regions of the air vary in step with the variety of Spheres and Climes, according to elevation and lowness; since in one and the same season winter reigns in one place and summer in another; in one place clear weather, in another a storm bursts in; here moist vapors, there murky exhalations lie heavy, and so on — it is surely necessary that the Atmosphere's surface creep low in one place and run very high in another. From this there will arise in it misshapen hollows and swellings, and, following these, a varying eccentricity; and this will necessarily draw the perpendicular line — the one to be drawn to this deformed surface at some point of incidence — away, outside the center of the earth. And so the whole business of this Refraction and Ellipse will collapse.

I reply, first. The argument proves nothing more than that the solar Ellipse is not usually uniformly deformed, but is inconstant even in its own slight degree — which I freely grant and often point out. **I reply, secondly.** The Atmosphere is bounded neither by cold nor by heat, but by that disposition of the air beyond which it has no power of refracting; and no one doubts that this power is communicated to it by the vapors, and so we close it off together with the limit of the vapors. But that this limit can easily be known or defined, we deny. **I reply, thirdly.** Since the surface we are discussing greatly exceeds the earthly one in extent, those very gentle down-slopes and up-slopes do not register upon the senses. Hence the whole force of the argument collapses — unless our opponents should wish to take refuge in the small ripplings and undulations of that surface, which we do not deny are produced by the rotation of the heaven, by the agitation of the winds, and by the innate force of the vapors and exhalations. But to all these we ascribe [only] the trembling and jaggedness of the [Sun's] rim. Nor will the opponent be able to bring anything against this.

You will object, thirdly. The reciprocal approach and recession of this surface is either superfluous or suspect. Superfluous, because the greater and lesser Ellipse holds even without it; suspect, because it lacks both reason and authority. The first point is proved. Since, by the testimony of all, with the incidence remaining the same up-[on]

—on the same spherical surface of the same extent, the refraction nonetheless changes when the density or rarity of the Sphere is changed; and greater refraction indeed results if there is greater density, and less if there is less, as rarefaction sets in, all else being equal. Now then, if we suppose the Atmosphere's surface to be stable, and we carry our observation right up to the twilights, will not — the surface itself unmoved and the same horizontal incidence assumed — a greater refraction result after rains and exhalations than before? Because, of course, the Atmosphere is condensed and packed tight, swollen with fresh accretions of watery vapors. Conversely, during prolonged drought and heat, while those moist vapors are dissolved and consumed, the same undiminished volume of the Atmosphere grows rare; and yet there, in the former case, the incident ray is refracted more, owing to the density, while here, in the latter case, less, owing to the rarity of the same Atmosphere. On this account, then, that surface approaches and recedes from the earth to no purpose.

The second point is proved [i.e., that the motion is suspect]. Because this opinion suffers from a lack of reason and authority: no one has taught it thus, and the senses themselves seem to protest. For who could be persuaded that air, the quantity of its matter remaining the same, is diminished or enlarged in such a way that it remains continuous, without any pulling-apart of its parts and with no pores introduced? Hence the whole matter is rendered suspect.

I reply to the first point. It is true if the volume of the whole rarer Sphere condenses; false if only some lower portion does. For the ray of incidence, before it strikes that portion, cannot diverge anywhere else. This second case is what occurs in our situation, not the first — because moist vapors do not rise to the dwelling-places of the twilights, as everyone, without exception, will readily concede. Whence it comes about that no change of refraction occurs at that surface, since neither does any change of matter take place there; the ray of incidence upon the vapors will therefore be carried on the same course as before. And if within them it is refracted otherwise than before, then what we assert occurs — namely, that the refracting surface of the Atmosphere has been diminished. In exactly the opposite way, both the Atmosphere and its refracting summit will be dilated, if exhalations take the place of the vapors — whether this comes about by alteration, by generation, or, finally, by local motion. Moreover, it cannot be denied that the lower, dewy air is contracted and condensed; granting which, a smaller space will be occupied, and consequently a smaller Atmospheric surface — wherever it may finally lie — will follow. And conversely, in rarefaction, the same air is enlarged in place and so bulges out into a more capacious surface, and so on.

To the second point. Everything said is denied. For the whole of philosophy holds and teaches the contrary — namely, that air is condensed and reduced to a smaller space by cold (indeed even by violent compression alone), and conversely is rarefied by heat and drawn out into a larger space by the breaking-up [of its parts] and by the dread of a vacuum [*metu vacui*]. From very many experiments I select just a few. If you place in water a glass flask fitted with a long, thin, hollow neck — the neck itself submerged, so that no air can pass out of or into the belly of the flask — then, once the flask is heated on the outside,

whether by the Sun's rays, by the application of a hand, or by the warmth of the surrounding air, and so on, the captive air in its belly will so rarefy that, descending through the long neck to its mouth, it bursts out in bubbles. Conversely, if the same enclosed air cools, it will gather itself in tight and will sometimes occupy a space in the flask twice as small; whence it comes about that water rises through the mouth into the long neck — and even into the little belly of the flask itself — against the force implanted by nature. If, then, so tiny a portion of air, through dread of a vacuum, calls heavy water upward, why should not the vaporous air that is compressed around the earth draw lighter things downward?

2. A flame introduced into scarifying cupping-glasses rarefies the air, which soon — once the cupping-glass has clung to the skin — is condensed by cooling and seeks a smaller space; hence the skin, drawn in, bulges up, and the blood beneath the skin is drawn out.

3. Take a shallow dish, pour water over it, then [take] an empty glass. Light a paper so that it burns above the water (for that water need not be very deep, and so on); set the empty glass over the burning paper, and you will see, once the paper is consumed or the fire extinguished, much water rise into the glass. The reason will be given presently.

4. Take a key, fill it with water, stop its mouth with paper, or a turnip, or some other thing; hold the key above a burning candle. The heated water enclosed in the key will rarefy, and — impatient of the confinement — it will burst out into a more open space, not without a bang, driving out the stopper. Thus, then, moist air, rarefied by heat, necessarily expands its extent; and the Refraction becomes smaller, on account of the rarity of the air and the size of the surface, and so on.

5. Any liquid strongly heated at the fire boils and overflows. Why? Because, being rarefied, it cannot be contained within a smaller space. Hence, by a cold breath, or by stirring around with a spatula, or by pouring in vinegar

or some other cold liquid, it soon subsides — because cold packs together, while heat thins out.

6. A glass with a very narrow neck, blown very hard, as soon as you take your mouth from the opening, pours back cold air with a perceptible force — for it had been condensed by the excessive forcing-in. Conversely, if you draw at it as one sucks, it will be rarefied, and then, as it condenses again, it will soon draw in water — which otherwise you could scarcely by any device introduce through so tiny a passage of the opening.

7. Suppose you fill with water to the middle — or more, or less, and so on — a glass with a capacious belly and a very narrow mouth both above and below; then stop the upper opening well, with a finger placed over it, or with wax or pitch; and turn the open lower opening straight down toward the ground. Water will nonetheless come out in just such quantity as the air enclosed above admits of rarefaction. From this you have not only evidence of the rarefaction of air, but you also see clearly demonstrated an excellent method for measuring it — whether you pour in water or some other liquid, and so on.

8. An animal driven at a violent pace pants rapidly, and by quickening its short breaths seeks to cool its heart. Why is this? Because the air enclosed within the cavity of the chest, inside the diaphragm, is heated by the violence of the motion; being thereby thinned, it so fills the hollow of the chest that there is no room left for the lungs to open and take in cold air. Hence come the palpitation of the heart and that rapid panting — until, as that intrusive heat is gradually suppressed, the enclosed air settles back into its former state.

9. Something similar happens to a man walking about in the summer heat: his hands swell, his cheeks puff up, his face flushes. All of this arises from the rarefied blood, which then distends and colors the parts it occupies, until, checked by cold, it cools down and returns to its former moderation.

Finally, to sum up much in a few words: everything meltable, when it liquefies, takes up more room than it does when compacted. This has been shown in tallow, wax, oil, butter, lead, tin, and so on. The behavior of these can be traced to rarefaction alone, and that of the opposite cases to condensation alone. And so for our position we have necessity, authority, reason, and sense-experience, and so forth.

Objection 4, and the last. From Witelo [Witelo], book 10, prop. 10, and from Alhazen [Alhazen], book 7, prop. 38, likewise book 1, p. 42, it is clear by common consent and by sure experience that refraction weakens the light and color of a visible object, and likewise that denser media are more colored, whereas rarer ones are, on the contrary, more transparent [diaphanous], and hence devoid of color. From this it is further inferred that the clear atmosphere [Atmosphaera] — treated repeatedly in the earlier pages — cannot be denser than the turbid one; for [if it were] it would present a weaker image and a weaker solar light than the turbid atmosphere does, and that on two counts. First, because it would refract more, and greater refraction — as the cited authors hold, and rightly — is followed by a greater weakening of the light and of the visible form: therefore the greater [refraction produces] the greater [weakening]. Second, because it would be more tainted with colors, for color follows upon density, and colors, as is agreed, obstruct the visible species and the light. Conversely, the murky atmosphere would turn out brighter and more transparent than the other. For since it is rarer, it will be more transparent and less colored; and since it refracts less, then — by what experience asserts — it will transmit both light and the images of things more strongly. Therefore the whole preceding doctrine about these differences of the atmosphere seems hardly sound, and so on.

I reply, first: an argument that proves too much proves nothing. For by this reasoning it would follow that our ordinary fire is denser than our air, and that air is rarer than fire; since indeed we see, through the air surrounding a fire, things that we do not see through the fire itself, its color — whatever that may be — getting in the way. Yet it is nonetheless established by countless experiments that fire is rarer than air. So let my opponent untie this knot, and, in accounting for his own position, find the wedge to split it.

I reply, second: there is a twofold density and a twofold rarity in transparent bodies. One kind of density occurs when much matter of a transparent body, occupying little space, consists of parts that are well compacted and properly continuous with one another. Another kind occurs when much matter, occupying little space, has parts that are indeed densely packed but poorly cohering with each other — not made continuous, but rather only contiguous. In the same way, one kind of rarity occurs when a little continuous transparent matter occupies much space; another occurs when a little matter is dispersed through a large space with its parts not continuous but contiguous, whether in succession or intermixed.

Now density and rarity taken in the first way have this property: rarity is always accompanied by greater clarity and transparency, and density by greater opacity and less transparency. Likewise, the second kind of density, compared with the second kind of rarity, always yields to it in clarity and transparency. But density in the first way, set against rarity taken in the second way, far surpasses it in transparency, and so on. If, then, the argument had proved that the clear atmosphere is dense in the first way and the murky one rare in the first way, or else that the murky one is rare and the clear one dense in the second way, it would indeed establish what it aims at. But in fact, since the first kind of density belongs to the bright atmosphere and the second kind of rarity to the murky one, the edge of the argument is blunted, and all its luster fades into shadow.

For the parts of the atmosphere, since they consist chiefly of a mixture of water and air, and so forth: once — by the force of the heat they have taken up, which separates heterogeneous parts and gathers homogeneous ones — they have begun first to be dissolved, and thus to be rarefied and thinned, they are set in motion and scattered into a wider space. And because they are loosened from their mutual bonds, they acquire their own separate surfaces and airy pores intermixed. These things, if they do not altogether destroy the clear transparency, certainly foul it greatly. For a boundary-surface by itself produces some color or other, which the porous passages augment; and the refraction resulting from this, combined with reflection, increases the opacity to an immense degree.

From all this, then, it comes about that rarity may hold in this [murky] atmosphere, yet the greater transparency or serenity is nonetheless banished — indeed, even the lesser [transparency] is lost. For these qualities arise chiefly from a proper continuity of parts, which this atmosphere, by the condition implanted in its nature, does not permit.

By contrast, the other, fluid atmosphere, since through cooling it preserves the joining of its continuous parts unbroken, and is bounded only at its extremities — both the concave one, next to earth and water, and the convex one toward the upper air — is free of the adventitious colors born of those boundaries; and with a uniform, bright transparency it lays out a royal road for the visible species and the luminous rays, so that without any detour they arrive at their appointed lodging. And

that this manner of philosophizing has been neither devised by arbitrary speculation nor forged by hasty fancy — behold, out of six hundred experiments, a few small ones:

First. Melted glass swells, thinned as it is by the power of heat into rarity, and so seeks a larger place; nonetheless, because of the fiery color it has absorbed, it is scarcely transparent. The same glass, solidified by cold, is beautifully translucent, yet with the same quantity of matter it holds less room. And in the cold glass there is the first density; in the burning glass, the second rarity.

2. Polished, continuous crystal is most beautifully translucent, yet it is dense in the first way. The same crystal, crushed in a mortar, is rendered wholly white and non-transparent, yet it takes up more room, on account of the second rarity, caused by the separation of parts and the intermixture of air. What introduced that whiteness into the crystal? Surely nothing but the sectioning of its parts came into play, adding to each an ultimate boundary.

3. Cold water, set out in a still place, is most clearly translucent. Bring fire near: the water will heat up, be disturbed, be agitated, swell with effervescence, and lose all transparency through the intemperance of the heat. And yet, when heated, it grows rare in the second way; when cooled, it grows dense in the first way.

4. Vapor exhaling from waters — as one can see almost daily on the Danube — is rare in the second way, while water is dense in the first way; yet, for the same quantity of matter, the vapor blocks the sight of things more than the water does.

5. Sea foam — how thin it grows! From how large a portion do you gather how few little drops? The sea, by contrast, how dense it is! And yet the sea is more transparent than the foam. Why? Because the separation of the parts and the intervention of air, together with light illuminating both the air and the bubbly surfaces — by reflection and mixed-in refraction — produce that whiteness; which then, as the little bubbles burst and the surfaces flow together again into water, vanishes once more.

6. Vapor is rarer than both dew and hoarfrost, yet it is less transparent than dew and more transparent than hoarfrost. Dew, however, is dense in the first way, and hoarfrost is dense in the second way.

7. Ice is denser than liquid water, with the second kind of density; but because it does not make its parts continuous the way liquid water does, it is therefore less transparent than water.

8. The air nearest a lighted torch becomes more inflamed, and therefore rarer, than the air that is farther off; and yet the farther air is more

translucent than the nearer, because the nearer grows rare with rarity taken in the second way, the farther with rarity in the first way.

9. Conversely, wax, butter, oil, lard, tallow, and countless things of this kind, when they melt, are thinned and fill more space (as said before), but less so when congealed. In the melted state they are translucent, because they acquire the first rarity; in the congealed state they turn opaque, because they are condensed chiefly with the second kind of thickness.
10. That clouds are rarer than water — who would deny it? Who, then, took transparency away from the clouds and gave it to the water? The continuity of the water; the dissolution [of the parts] in the clouds.
11. Ash, sand, and salt, when crushed and mixed, certainly become rare — with the second rarity — yet remain opaque; when melted by the addition of fire, they are certainly thinned still further, and so on. Yet in the molten state, because they gain the bond of continuity, they become translucent; in the other state, deprived of that bond, they remain opaque.
12. Lumps of perfectly intact salt are dug up, excellently compacted and therefore transparent like crystal; grind it into powder, and the salt will grow rare in the second way, and will not be translucent at all.

I deliberately pass over mists, exhalations, fumigations, vapors, smokes, the matter of twilights, of rainbows, of halos, and countless other things.

For from these [examples], to the arguments drawn from the Optics of Alhazen and Witelo, I reply, thirdly: what they say is true; but what the fallacious reasoning captiously concludes from it is false. For it leaps from the second rarity — the one found in the dull atmosphere — to the first, and from this it builds an ambush against the truth. These snares the doctrine expounded thus far has, I believe, sufficiently exposed and dispelled.

Now let us return from the atmosphere to the ellipse.

ON THE WAYS OF OBSERVING THE SOLAR ELLIPSE.

CHAPTER XXXVII.

I have no doubt there will be many who, because of what has been said, will be gripped by no ordinary desire to see this splendid display; and these must be satisfied, if not fully, then at least in part. For I am deferring much of it to my work *On Sunspots* [*De Maculis solaribus*], which I have had in hand — woven together without interruption from the first "Apelles" edition [Scheiner's *Tres Epistolae de Maculis Solaribus*, 1612, published under the pseudonym "Apelles latens post tabulam"], through daily observations of them and careful collations, right down to the present day. I well know it is longed for by many, and I do not doubt that it is awaited. So that they may not be cheated of their wishes, I will devote strenuous effort to it; for now, I dispatch much in brief.

There are, then, five ways of viewing the elliptical Sun. First, with unaided sight, morning and evening, when it rises and sets. Next, by directing the eye through a colored helioscope [Helioscopium] toward the Sun, at any clear time. Then, by admitting the Sun through a bare aperture into a darkened chamber [camera obscura], onto a board or white paper placed opposite it, at right angles to the solar ray. After that, by reflecting the same ray from a plane mirror onto a white wall, in a

in a somewhat darkened place, at whatever time one wishes; and finally, once that same image has been carried through a suitably arranged tube onto a sheet of paper, and so on. Passing over the remaining methods — which I have nonetheless used here and there — I pursue only the last, and set about its construction before anything else.

THE CONSTRUCTION OF THE OBSERVING INSTRUMENT.

CAPVT XXXVIII.

Take a small board ABCD, well planed, in the form of a rectangular parallelepiped, about a finger thick and roughly a foot and a half tall. Into it, at right angles, insert a beam EF, likewise a parallelepiped, longer than the tube at its fullest extension, and so thick and wide that it is fit to support the tube and everything else without bending.

Diagram. A large perspective woodcut of the solar-observing instrument. A long horizontal beam (EF, elsewhere the ridge Eξ) rests on splayed trestle legs and carries three upright boards. At the left stands the tilted screen-board ABCD, which bears the clean paper (δεζη) on which the elliptical solar image (abωc) is projected. A horizontal draw-tube ST runs through round holes in the two right-hand upright boards (GHIK and MNOP): the convex lens T is at the right end, exposed to the Sun, and the concave lens S is at the left near the screen. Beneath the front of the instrument a two-footed rocking yoke (VXYZ) with a cylindrical crossbar (XY) tilts the whole assembly up or down; a four-footed stand or bench (the fulcrum, lettered with Greek characters) props the rear. Along the bottom a geometric arc illustrates the tilting lines (Vo, Vι, and the tangent Xι, with points π and ρ) referenced in the following chapters.

- C (top apex, left): summit of the tilted screen-board.
- B (left edge): left side of the screen-board ABCD.
- D (lower left): lower corner of the screen-board.
- A (bottom left): foot of the screen-board assembly.
- E (left, on beam): near end of the main horizontal beam / ridge.
- F (on beam): base of the first upright board.
- G (foot of first middle upright): base of board GHIK.
- H (top apex of first middle upright): summit of board GHIK.
- I (mid-right of first middle upright): board GHIK.
- K (foot, bottom center): foot of board GHIK.
- L (on beam, right): base of the right-hand upright board.
- M (foot of right upright): base of board MNOP.
- N (top apex, right): summit of board MNOP.
- O (upper right): right side of board MNOP.
- P (foot, lower right): foot of board MNOP.
- R (on the tube, right spindle collar): round hole receiving the tube.
- Q (round hole in the first board, tube passage): [?] — collar/hole label not clearly resolved on the scan.
- S (left end of tube, near screen): concave lens. [?] — the letter itself is at the resolution floor.
- T (right end of tube): convex lens end, facing the Sun.
- V (bottom left): foot of the yoke.
- X (yoke crossbar / pivot, left): tilt-axle of the yoke.
- Y (yoke, near left end of tube): crossbar of the yoke.

- Z (bottom center): foot of the yoke.
- ξ (far right of beam): rear end of the ridge Eξ.
- θ (small box on the right-hand table): part of the four-footed fulcrum cluster θικλμν.
- σ (near board H): part of the S-end valve-cluster στυ.
- φ (near board N): part of the T-end valve-cluster φχψ.
- *(further review/5: cross-witness with the audited IA `fig-p98-1`, these peripheral Greek labels are committed — inked on the Freiburg tile: σ, υ (S-end valve στυ), φ, χ, ψ (T-end valve), ξ (beam end), θ, λ, μ (four-footed fulcrum), ο (ground arc), ω (projected ellipse). The residual — τ; α β γ (pivot socket); δ ε ζ η (paper); ι κ ν (fulcrum); π ρ (arc) — remain `[?]` at the shared engraving floor on both 1617 witnesses.)*

On this beam, at an interval of about a foot and a half from EF, erect at right angles another rectangular board — nearly equal to the first, or, if you prefer, much smaller and thinner — GHIK. Then, after a like interval FL, set up yet another board, similar and similarly finished but taller and wider than both the earlier ones, MNOP.

Into the round holes Q and R of these boards you will fit the tube ST, opened to the proper length, so that the lines FQ and LR, equal to each other, terminate at the diameter of the tube ST — the centres of the lenses being S (concave) and T (convex) — the tube joining the inserted lenses at right angles.

Next prepare a two-footed yoke VXYZ, whose feet VX and YZ are pointed below and broad above, fitted by a firm joint to the crosspiece XY. These feet will either equal, or somewhat exceed, the height ιλ or θμ of the support ικλμνθ on which the front part of the instrument rests. The crossbar XY will be finished on top with a convex cylindrical surface, which can be fitted neatly into a concave surface αβγ equal to it and turned freely within it. Then to the board ABCD fasten a clean sheet of paper, δεζη, but one that is not at all translucent; otherwise the colour of the board beneath, blending with the whiteness of the paper, dulls the image of the Sun and the sunspots and faculae cast upon it. After this, a four-footed support θικλμν is fashioned; upon it the whole rest of the apparatus is set up in such a way that T, the head of the tube, and ξ, the first part of the ridge Eξ, are exposed in the open air facing the Sun, so that the Sun has completely free access through the tube to the paper.

THE FUNCTIONS OF THE PARTS EXPLAINED.

CAPVT XXXIX.

The board AC holds the paper. The beam Eξ serves as, so to speak, the backbone of the whole instrument, and holds it together. The two boards GI and MO — and more can be made, if the length of the tube requires — support the tube ST, received in the holes Q and R, at right angles to the paper δεζη, and shade that paper by their own breadth. The yoke VXYZ serves to raise or lower the whole apparatus. For when X is tilted toward ι, the roller XY — and with it the S end of the tube — is necessarily lowered, while T is raised. Since the lines Vo and Vι fall below the tangent Xι, by propositions 2 and 3 it is necessary that X, reaching π or ρ, lowers the beginning of the tube at S and raises its end at T — because Xι and ST are parallel lines and move parallel to each other. The bench ικλμν props up the rearmost part of the ridge, Lξ. The two little valves στυ and φχψ are meant for putting the tube ST on and taking it off. abωc is the elliptical image of the Sun, received through the tube onto the paper.

THE CONVENIENCE AND USE OF THIS INSTRUMENT.

CAPVT XL.

I have set out the essentials of the support, passing over the details and deferring some to another place; now I explain the easy use, which is what one chiefly looks for in this machine.

That use consists in two things: that the tube should always receive the Sun, and that it should cast the Sun onto the paper at right angles. The former is achieved with the greatest ease by the frame VXYZ, the latter by the board ABCD. For since the board itself stands at right angles to the beam Eξ, it follows that the paper δεζη is set upright to the tube ST no differently than the board AC is to the beam Eξ — because the paper is equidistant from the board, and the tube from the beam. This tube, for everything to be done correctly, must be opened to the proper length and correctly inserted into the boards GI and MO. The tube's opening will be right when the Sun shines onto the paper δεζ bright and cleanly defined on every side; whether this is so or not, a trustworthy eye will judge. For two extremes must be guarded against here: too much extension of the tube and too much contraction. Drawn out too far, it blurs the image of the Sun, smothers the spots and faculae, gives back a kind of white round cloud, and tints the Sun and the spots with a rusty, iron-like discoloration. Contracted too far produces the same effects, and dyes everything a blue colour. So a middle position must be kept, with the eye as judge.

Nor should this be passed over in silence: the same tube must be extended further in winter than in summer — from which the Sun's winter approach toward the earth becomes evident. The tube, thus opened, will be correctly inserted into the holes Q and R if the diameter ST is equally distant from the middle line ELξ of the board, and if it likewise faces the paper δζ at right angles.

Between S and the paper δζ there will be a free gap — as large, that is, as suffices for the use of the compass and the hands, or as is enough for enlarging the figure of the Sun. For the image of the Sun grows on the paper as the tube is drawn back, and shrinks as it is brought closer, everything else remaining unchanged.

Once all this is set up, the whole support is turned directly to face the Sun, especially around the times of sunrise and sunset. Then, with the compass opened, the base of the solar cone — projected and inverted — is marked on the paper: first along its length ωα, then along its height bc. At the same moment of marking, the Sun's elevation above the horizon — or rather, what is more fitting, the hour of the day (unless you would prefer, more correctly, to have both) — must be very carefully noted, together with the minutes. But if, in marking the cardinal points ω, α, b, c, you find the same distance between b and c as there is between ω and α, you will know that the Sun is not elliptical but circular at that time.

CAUTIONS IN SETTING UP THE SUPPORT.

CAPVT XLI.

In two ways especially it can happen that the Sun, though it is not elliptical, nevertheless appears so. The first is when the interposed paper cuts the radiant cone of the Sun not squarely but obliquely — as is plainly seen when the Sun descends through round openings onto the pavements of churches or houses, for on their surface it appears oblong. It is easy to slip into this fault if either the paper without the tube, or the tube without the paper, is movable — to say nothing of how very difficult it is by these means to keep the Sun in the same place on the paper, or to bring it back to a fixed and stable diameter of length. This error and difficulty are, in turn, most easily met when the board AC is erected at right angles to the diameter of the tube ST.

The second is when either both lenses S and T, or one of them, S or T, is set into the diameter of the tube ST at oblique rather than right angles; or when ST passes not through the centres of the lenses (which are otherwise assumed to be correct) but through their edges. For in such cases the Sun ordinarily distorts into an ellipse. These deceptions must therefore be countered by a correct positioning of the lenses, which is achieved if the diameter of the tube passes through the centres of the lenses at right angles. Whether this is the case or not, careful experience and the diligent industry of the craftsman will teach.

By these very same means the opposite error is committed, so that an elliptical Sun is reduced to a circle: if, as before, the paper receives the Sun at some fixed obliquity, until the diameter of height equals the diameter of length; or when the lenses, similarly to before, either receive the Sun at their edges, or take it in through their centres indeed, but obliquely. These errors, as before, are kept off if both the paper and the lenses are set perpendicular to the diameter of the tube.

HOW THE EXCESS OR DEFECT OF THE SOLAR DIAMETERS IS TO BE INVESTIGATED — IN MINUTES ('), SECONDS ("), AND THIRDS (''').

CHAPTER XLII.

By the way, let me give notice in advance that "excess" and "defect," though two names, are one and the same thing — namely, a portion of the solar diameter. It is called the *excess* because the Sun's length exceeds its height by this part; it is called the *defect* [defectus] because the height falls short of the length by this same particle.

Geometrically, then, this excess or defect is reduced to a definite measure — such as minutes, and so on — in the following way.

Once you have obtained, by careful observation, the Sun's length and its excess, look for a measure common to both — at least to the senses — by a certain alternating subtraction, per *Elements* X.3, which is quickly done with a compass. This measure, once found, will measure all and each of the earlier parts, as well as the whole solar length itself, and hence also the excess, the first quotient, and — if there was any remainder of the quotient — per *Elements* X.2 [proposition]. Therefore, if you divide the excess by this measure, multiply the quantity by the first quotient, and add the quantity of the first remainder to the product, you will have the whole length of the Sun in parts equal to the common measure. And so the ratio of both the excess and the solar diameter to this measure will be established — which must be carefully set down in numbers.

The matter will become clear with an example.

In the year 1614, on 3 March, in the afternoon, when the Sun was one degree high, its length was AB, its height CB, and the excess (which is the same as the defect) AC. I investigate the ratio of AC to AB in a measure common to both as follows.

Diagram. A single horizontal straight line representing the Sun's length AB, subdivided by tick marks. Reading left to right the labelled points are A, then a cluster G F D, then C, then (after a long span) E, and finally B at the right end; a small point N sits just below the line within the G–F region. The figure illustrates the alternating subtraction (anthyphairesis) of the two worked examples that follow.

- A / B (endpoints): the whole solar length AB (longitudo).
- C (interior): AC = the excess/defect of the first example (3 March, Sun 1° high); AC is contained in AB six times.
- D (interior, left of C): AD = the excess/defect of the second example (Sun at 2°30'); DB = the corresponding height.
- E (right of C): AE = eleven times AD; EB is the remainder.
- F (within the A–D span): the point reached when the remainder EB is laid off within AD; remainder FD.
- G (left of F): the point reached when the remainder FD is laid off within AF; remainder AG.
- N (below, within G–F): the subdivision through which the common measure AG is laid off along GF.
- AG: the common measure — one 59th part of AB (AG = 1, AD = 5, AE = 55, EB = 4, AB = 59).

I extend the compass to the interval AC and mark it off on AB as often as I can, working toward B. If nothing remains near B, the excess itself is the common measure: for taken once by itself, and several times over, it measures the length AB — in the given example, six times. So the ratio of AB to AC is as 6 to 1, and of AC to AB as 1 to 6.

That same evening, however, when the Sun was at an elevation of two degrees thirty minutes, its length AB exceeded its own height DB by the line AD. This AD does not measure the whole of AB evenly; rather, reaching to E, it is repeated eleven times, with EB remaining. So, having noted the quotient 11, you will measure the excess AD[?] by the first remainder EB, up to F, without regard to the quotient number; and by the remainder

FD you will measure the earlier segment AF, up to G; and by the remainder AG you will measure GF (and so on to infinity), through N up to F. Since it exactly exhausts the line GF, it is itself the common measure, and it measures both the excess AD and the line DE and the remainder EB.

Therefore, since the common measure AG is found five times in the excess AD, and the excess AD eleven times in AE, then — by composition of ratios, per *Elements* VI, def. 5 — AG will be contained in AE 55 times. And in the remainder EB — that is, in the line AF — it will be found 4 times. These, added to AE, make the whole AB 59 parts: of which AG is one, AD is 5, AE is 55, and EB is 4. And thus the ratio of the defect AD to the whole length AB is established, in this way: 5/59. Once these are had, it is very easy to work out the rest and to reduce the whole matter to minutes.

Arithmetically, thus: Let the Sun's diameter be taken as 34 minutes [scruples]. Now, by the rule of proportions:

The whole length AB, 6, exceeds the width by 1; by how much does a length of 34' exceed it? The result will be 5' 4/6 [i.e. 5'40"].

The whole length AB, 59, exceeds the height by 5 parts; by how much does a length of 34' exceed it? The result will be 2 52/59, that is, 2' 52".

And thus you will proceed in every case.

A fraction attached to the first minutes [primes] you will reduce to seconds as follows: always multiply the numerator by 60, and divide the product by the denominator. Thus in the first minute-case, 6, 4, 60? — the product 240, divided by 6, gives you 40". In the second minute-case, 59, 52, 60? — the product 3120, divided by 59, yields 52", together with an adjoined fraction 52/59, from which, if you wish, you may draw out thirds (3'"), in the same way as seconds. But it is hardly necessary.

I said that a common measure perceptible to the senses is to be extracted, and this suffices; for it may happen that the height and length of the Sun are incommensurable, and it may happen that, even if they are commensurable, nonetheless — from the nature of the thing — the exact point is reached only with difficulty. For the present business, however, it is enough to have found a common measure that does not perceptibly err.

But I add this Clavian [Clavius's] method of extracting the common measure as well — no less quick than reliable.

Diagram. Clavius's graduated scale. A long horizontal line running from A (left) to B (right), graduated into equal small parts numbered in fives, the numerals set alternately above and below the line: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, with B just beyond 145. Below the scale are three separate line-segments of decreasing length, to be measured against it: C——D (the longest), E—F (shorter), and G—H (shortest).

- A / B (endpoints): the graduated measuring-scale AB, divided into equal small parts (100, 150, etc.).
- C–D: the observed length of the Sun (found to be 65 parts on the scale).
- E–F: a defect of width (found to be 19 parts).
- G–H: another defect of width (found to be 7½ parts).

Let AB be any straight line whatever, divided into equal but very small parts — however many, say 100, 150, and so on. Next, let the observed length of the Sun be CD, and let the defect of width be, in one case, EF, in another, GH. Apply the length CD onto the line AB, and you will find it to be exactly 65 parts. Note these. Apply EF too, or GH, and you will find 19 (more or less) in EF, and 7½ in GH. With these obtained, you will arrive, as before — by the benefit of the rule of proportions — at the knowledge of the minutes: 65 gives 19, what does 34' give? 65 gives 7½, what does 34' give? And so on.

AT WHAT TIMES THIS PHENOMENON SHOULD BE CAUGHT.

CHAPTER XLIII.

Whoever wishes to reach the true and quite indubitable sense-experience of this should, if possible, catch the Sun's ray while it grazes the horizon or is still near it. For this defect lasts up to the sixth, even up to the tenth degree and beyond, gradually vanishing afterward. Yet it diminishes in such a way that it grows in proportion neither with the Sun's ascent nor with its descent; rather, when the Sun holds one or two degrees of elevation, or even less, it is notably increased as the Sun descends, and diminished as it ascends. Hence anyone who comes to observe after the 10th degree of the Sun's elevation will find the decrements extremely slight — so slight that, if he were wholly ignorant of the matter, he might perhaps not believe it.

Different times of the year, too, are more suitable than others. For since the tropics are much smaller than the equator, yet complete their revolution about the same poles and in the same time as it does, it comes about that the Sun moves more slowly along them than it does about the equinoxes. It must therefore spend more delay in raising its globe above the horizon at the tropics than at the equator. From this it follows that an observer can accomplish everything more exactly here than there. For hurried observations are often premature, and — not without reason — suspect.

This slowness of ascent is also aided by the inclination and obliquity of the sphere. For in some places this brings it about that the Sun lingers on certain parallels for whole days, and yet scarcely advances a degree above the horizon — the magnitude of the parallels notwithstanding, nor is their smallness any help. For in the parallel sphere [sphaera parallela], the equator wholly coincides with the horizon, and the Sun undergoes the ellipse; the tropic of Cancer stands wholly above, and the Sun is little affected. The obliquity, therefore, together with the inequality of the parallels, counts most of all for this matter — one without the other not always. Etc.

HOW THE SAME LENGTH OF THE SUN CAN BE RETAINED, THROUGH A WHOLE DAY, A WHOLE YEAR.

CHAPTER XLIV.

For a single day, the same length of the Sun is easily obtained, provided that neither the aperture of the tube nor any change in its distance from the projection-paper intervenes. But for a whole year — which is what I generally do — provided that both an extension of the tube and a variation of its distance from the paper are applied from time to time.

To understand this better, one should know that the same objects, when notably nearer the tube, project a larger and brighter image of themselves, and when farther off, a smaller and dimmer one; and likewise that the former require a longer extension of the tube, the latter a shorter one. This is attested both by daily experience and proven by the principles of dioptrics. Hence it follows that, whether you bring the tube toward the object or the object toward the tube, it must be opened one way or another, if the change of interval is so great that the tube's fixed lenses cannot accommodate it.

Now, since by the consistent experiment of several years I have learned that the same aperture of the tube, other things being equal, does not render the Sun the same size in winter as in summer — but that in winter it is seen larger, in summer smaller — it follows that a notable variation has occurred, and hence that the Sun is nearer the earth in winter and farther in summer. From this it necessarily follows that, if you wish to keep the same length of the Sun on the paper, you must compensate for it by extending or contracting the tube, and by enlarging or narrowing the distance between the tube and the paper, until the Sun's figure falls onto the paper at the same length that the prescribed diameter, checked with a compass, requires.

A sign, moreover, of the inequality of the diameters, or of the disproportion that the tube's extension acquires relative to the object's distance, is then evident when the Sun — or some other object — passes onto the paper blurred and poorly defined along its edges. For in this case the distance of the lenses within the tube must be varied by drawing them together or apart, or else the gap between the object and the tube by approach or withdrawal, as the matter requires. Yet in the present business one cannot appreciably approach the object or withdraw from it — for who will ascend to the Sun, or descend from it? Only the variation of the tube, then, remains. From this it follows that those objects for which a single aperture of the tube and the same distance suffice — so that they are nonetheless presented cleanly and clearly to the eyes or to the paper — are not so far apart from one another that whatever distance lies between them, [affects] their magnitude and

and exceeds the power of that aperture; or else the excess of the more distant object must be reckoned very great—other things being equal. For otherwise, objects that lie too far off would not appear sharp and distinct. Hence it seems reasonable to conclude that, for those heavenly bodies which project their figure neatly and precisely onto the chart while the tube's extension stays the same, the apparent [visual] diameters, and the mutual proportions of those diameters, can be known exactly. This I have often tried with Venus, Jupiter, and the Sun, on which I shall say more elsewhere. It would be otherwise, however, if the tube's aperture had to be varied in order to obtain the exact figure of some particular star.

Nor will the distances from the earth of those bodies appear to differ appreciably, when they are represented distinctly by one and the same tube aperture.

But from this same truth it will also be proved that the Sun appears larger in winter than in summer. Likewise, that all the contrivances of the homocentric systems—both of motions and of circles—collapse, while by contrast both eccentric revolutions and epicycles are confirmed.

Moreover, from one or another extension or contraction of the tube, a not-obscure argument will be derived (other things being equal) as to which stars are very high and which very low. For those which, when the tube is applied to the eye, bear a greater extension and yet appear no less distinctly and neatly, are the ones nearer to the earth; whereas those which require a contracted tube in order to let themselves be seen clearly and precisely through it are certainly far off. And perhaps from this difference between the extended and the contracted tube some foundation might be dug out for investigating their distances in a certain proportion.

It is likely, furthermore, that something about the Sun's eccentricity can be learned from that same difference which lies between the winter and the summer extension of the tube, and likewise from the amount by which the winter solar circle exceeds the summer one—provided the same tube aperture is kept in both cases, and the same distance of the tube from the chart. Here, then, is a field over which the tireless industry of a diligent astronomer may range.

ON INVESTIGATING THE QUANTITY OF SOLAR REFRACTION, IN A FIXED AND DEFINITE MEASURE. CAPUT XLV.

Since solar refraction, which is our subject, is nothing other than the rectilinear angle which the incident solar ray, extended in a straight line, makes—together with the refracted ray—at the point of incidence in the plane of refraction: to express the magnitude of this angle in degrees and minutes is precisely to reveal the quantity of refraction in a fixed and known measure. To do this, I shall bring forward several approaches.

One method is to use tables of refraction constructed by earlier authors, in which, given the Sun's elevations above the horizon, the refractions already ascertained are set alongside them.

Witelo [Vitellio], indeed, in book 10, proposition 8, published tables of refractions from air to water, to glass, and of these in turn to one another and to air, and so on; but he offered none for [the passage] from aether to air or to vapors, so that nothing useful for our purpose can be drawn from them.

Tycho Brahe, therefore, seeing how necessary this science is to astronomical matters, in book 1 of the *Progymnasmata*, page 79, brought to light a table of solar refractions—compiled from many years of careful observations, laid out degree by degree of elevation above the horizon, and extended up to the 45th degree. But since even this table, as long experience proves, does not hold for the latitude of every arbitrary place—

Johannes Kepler [Keplerus] took up the task next, in the *Paralipomena* [to Witelo], chapter 4, proposition 9, problem 2, page 117 and following, and—with great labor and study, as is plain—called the Tychonic tables back to the anvil [reworked them]. Yet frequent trials testify that not even these agree with the [actual] refractions in every nation and locality. The Ingolstadt refractions, for instance—to offer something definite by way of example—regularly exceed those tables.

Take this as evidence of the point. Just as Tycho Brahe sets the horizontal refraction as always the same, at 34 minutes, so too he holds the flattening [ellipse] corresponding to it to be 5′, of the kind of which the Sun's diameter is 30′. But it is evident, from the observations on the elliptical Sun adduced here, that the horizontal flattening is highly variable; and therefore it is evident that the horizontal refraction too changes, for the flattening follows the refraction as a shadow follows its body—it grows when the refraction grows, and shrinks when the refraction shrinks. The same must be judged of the other refractions above the horizon, and so on.

What, then, is to be done? Are the tables to be condemned? I do not say so. Is the effort of these men to be disapproved? Far less. Are they to be convicted of error? By no means. For I cannot persuade myself that Tycho's industry went so far astray in observing the refractions. Rather, all of this—as they themselves point out—must be ascribed to the diversity of the vapors, which behave in one way and another in different climates. What, then, should be done? Should the study of refraction be neglected? I judge not. Rather, either those tables must be used, or new ones must be composed for each locality, or the quantity of refraction must be determined by other methods. Some of these I shall now set forth.

Tycho Brahe, *Progymnasmata*, pages 93 and 94, offers two methods. "The first," he says, "will be by means of a large azimuthal quadrant, one that displays the individual minutes together with their fractional parts, and which, conveniently rotated all around in a vertical plane, shows the azimuths at the same time beyond all doubt," and so on.

"The second method," the same author says, "should be carried out thus: by means of a very large armillary instrument (such as we have ready to hand, where the rotatable armilla that measures declinations is 10 feet in diameter), whose axis at both ends looks with the utmost exactness toward the poles of the world, with a cylinder fixed at its center and corresponding sight-vanes [pinnacidia] carried around on the circumference—let the Sun's declination be observed near the summer solstice, and this from its rising all the way to the meridian, and thence to its setting, at a definite altitude, which everywhere is to be noted with the help of a second instrument, such as a quadrant or the like, that carefully measures the Sun's elevation. From this, once the apparent declination at a known altitude is known, the refraction in the vertical circle will at once be established." So he; and he then pursues at length the methods by which the observations are to be reduced to calculation.

But these methods—given the scarcity of instruments, the awkwardness of their use, the loss of time, and ignorance of the calculation—are too difficult to be of help to a beginner, or even to someone moderately versed in astronomical matters. Easier ones must therefore be sought.

Kepler approached the matter from a given altitude of the atmospheric surface and the inclination of the rays to it; but from what has been said it is clear that neither the altitude of that surface is fixed or known, nor is any inclination within it given or established. So a different oak must be shaken.

TO FIND THE HORIZONTAL REFRACTION OF THE SUN BY MEANS OF A CENTRAL HORIZONTAL LUNAR ECLIPSE. CAPUT XLVI.

When the Sun and Moon place their centers on the same diameter of the greatest celestial circle, then it is certain that the face of the lunar orb is so overshadowed by the cone of the earth's shadow that the axis of that shadowy cone strikes the Moon's center—and therefore such an eclipse is plainly central.

And because the terrestrial globe stands at the middle of the world, all those peoples who lie beneath the circle described with the Sun and Moon as its poles have at that time both luminaries (the earth being taken as imperceptible with respect to the heavens) on their sensible horizons, setting aside any refraction; for each luminary lies on the true horizon, since each is situated on its diameter.

If, therefore, at such a time [a luminary] is raised above the horizon, that whole elevation must be ascribed to refraction. And it is easy to take the elevation above the horizon of both the Sun and the Moon with some large quadrant exactly divided into minutes; to which, if you add the parallax already known beforehand from a table of parallaxes [in

Tycho], *Progymnasmata*, page 80 and page 120, you will obtain the refraction of both the Sun and the Moon, without any other calculation, and so on. But because such eclipses are both rare and do not readily occur on the horizon where any given student of astronomy happens to be stationed, this method too cannot satisfy the burning wishes of mathematicians.

Those, indeed, who dwell in the right sphere can attempt something similar every month, at full moon; for then the same conditions will hold with respect to the two great luminaries. Nor does it matter that they do not place their centers on the diameter of a great circle—on account, that is, of the Moon's latitude; for it is enough that they lie in the same plane of a great circle. And since in the right sphere this plane coincides with the true horizon and is always equidistant from the sensible horizon of that sphere, it is clear that every month, in at least some place of that sphere, this practice can be carried out. This privilege is not granted to the other positions of the sphere, nor can it be brought into use under them without many precautions. Therefore yet other ways must be entered upon—

but only after I shall first have cleared away another common stumbling-block, one that is apparent [rather than real].

A SCRUPLE AGAINST WHAT HAS BEEN SAID, AND WHAT IS TO BE SAID, IS REMOVED. CAPUT XLVII.

For since refraction, by what has been said, is the angle contained—at the point of incidence—by the line of refraction and the line of incidence extended within the refracting transparent medium; whereas our own angle is contained by the line of refraction and the line drawn from the eye to the true place of the visible point on the Sun, so that this angle terminates at the point of the seeing eye: it is impossible for us to arrive at the [true] refraction, even if we determine that other angle with the utmost accuracy. For what has the angle of refraction to do with this angle of the eye? The angle of refraction fixes its vertex at the point of incidence, or refraction, whose location—by what has been asserted—is unknown; the other [angle] comes to its point at the eye. The former casts its sides from the air toward the earth and the eye; the latter throws them out from the eye toward the air, into the sky. The former, being equal to the angle vertically opposite it, is—by Euclid, book 1, proposition 16—greater than the latter. Hence this whole endeavor is fruitless, since the angle of refraction in no way emerges by these lines of reasoning, even if everything else sought is obtained.

For example, in the attached figure ABCD: with the Sun C lying below the horizon, let the ray CD fall upon the refracting surface of the vapors at the point D, from which the refracted ray DA proceeds to the eye A, standing on the earth's surface, [which

[the ray] of the Sun C, seen by refraction, would be the last one, and would therefore lie in the plane of the sensible horizon. When extended onto the spherical surface of the solar heaven, it falls at point B, so that the angle BDC is formed — vertically opposite to the angle of refraction, and therefore equal to it, but greater than the angle BAC, by Euclid I.16 [Euclid, *Elements* I.16]. Hence, even if I correctly track down that same angle BAC by the methods already demonstrated, I still do not thereby obtain the angle of solar refraction BDC, and so on. This labor, therefore, plainly seems pointless.

Diagram. A long, shallow triangle drawn across the top of the page. The apex A is at the far upper left and B at the far upper right, joined by the long, nearly horizontal top line AB; C lies at the lower right, below B, with the short side BC drawn vertically and the long diagonal AC running from the left apex down to it. A second point D sits on the top line just to the right of A, and the line DC is drawn from it to C, splitting off the narrow triangle DBC from the whole. The construction illustrates the objection: the eye at A sees the refracted Sun under the angle BAC, whereas the true refraction angle is the vertically-opposite angle BDC at D.

- A (upper left, apex): the eye / observer's station.
- D (on line AB, just right of A): the vertex of the refraction angle proper; AD is the oblique thickness of the refracting vapors (side AD).
- B (upper right): the point where the extended refracted ray meets the spherical surface of the solar heaven — the apparent place of the Sun.
- C (lower right): the Sun, seen by refraction.
- angle BAC (at A): the "eye-angle," under which the observer actually sees the refraction.
- angle BDC (at D): the true angle of solar refraction, vertically opposite to (and equal to) the refraction angle, and greater than BAC by Euclid I.16.
- angle DCA (at C): the minute excess of BDC over BAC.

My first reply. Tycho Brahe [Tycho Brahe] took the angle BAC for the angle of refraction (*Progymnasmata*, Part I, p. 93 and following) [Tycho Brahe, *Astronomiae instauratae progymnasmata* I, p. 93], yet he never suggested that it is *not* the angle of solar refraction. What is therefore permitted to him is permitted to me.

My second reply. The angle BAC is called the angle of refraction not because it is so in the strict sense, but because it is the angle that corresponds, in the heaven, to the refraction angle BDC. For refraction raises the Sun C and makes it appear to the eye A — itself lying in the plane of refraction (which is the plane of the vertical circle, drawn through the true place C and the apparent place B of the Sun) — under the angle BAC. That angle is therefore the angle of refraction not formally, but effectively and finally. For the refraction angle BDC is investigated precisely in order to derive this one; so if this one can be known without finding that other, the end is secured by a shorter route.

My third reply. The angle BDC is indeed, in reality, the angle of refraction, and greater than the angle BAC; but the difference is perceived by reasoning alone, not by the senses, on account of the Sun's excessive distance from the earth. Thus the sides AC and DC can appear all but equal, or parallel, and circles described from the centers A and D out to C appear equal to the senses, and so on. Let me demonstrate this.

I posit that the semidiameter of the earth is to the semidiameter of the Sun as 2 to 11.

I posit, second, that the semidiameter of the earth is to the semidiameter of the Sun's eccentric as 1 to 1200.

I posit, third, that the semidiameter of the earth is 860 German miles.

I posit, fourth, that the oblique thickness of the vapors that beset the horizon and are apt to refract is, at most, 142 German miles — this being the side AD.

From this it follows that the diameter of the Sun, BC, is 9460 German miles.

The semidiameter BA of the Sun's eccentric is 1,032,000 German miles.

The angle of refraction BDC is 31′ 31″ 2‴ 51⁗.

Angle	′	″	‴	⁗
BDC	31	31	2	51
BAC	31	30	47	12

Angle	′	″	‴	⁗
DCA	0	0	15	39

By this excess — 0′ 0″ 15‴ 39⁗, that is, by one German mile and 3/10 of the same — the angle BDC exceeds the angle BAC; and this is a difference you could not detect even with an optical tube of however extreme a power.

Likewise, since the side AB is 142 miles and DB is 1,031,858, AD will be to DB as 1 to 7267. Suppose, then, that from one and the same center you describe two circles, the semidiameter of the one being 7267 equal parts and that of the other 7268 such parts; or that upon the same line, from different centers, you describe two circles that nevertheless touch each other at their circumferences. Would any sense detect a difference of circles, of arcs, or finally of semidiameters? Therefore the horizontal refraction of the Sun BDC — even if it be set at 31 minutes and more — yet produces no perceptible difference from the angle BAC found from the place of observation.

And if — taking the direct or vertical altitude of the vapors as 12 German miles, from which arises that horizontal thickness of 142 miles — this altitude allows the two angles BDC and BAC to be sensibly equal, what will happen if the refracting surface should be far lower (as we most probably judge it to be)? Let this quibble, then — showy indeed, but too subtle and therefore idle — be dismissed, and let us pursue the road we have begun under good auspices.

To derive the true solar refraction very easily, without calculation, from the hour of true rising or setting.

CAPUT XLVIII.

For this I require four things beforehand. First, a meridian line that does not deviate in the least. Second, a quadrant divided most exactly into degrees and at least the first minutes. Third, the true hour of rising or setting, ascertained in advance from a calculation that is least fallacious, or from a quadrant, an astrolabe, or a similarly most faithful timekeeping instrument, and so on. Fourth, some mechanical clock that indicates each single scruple of the hour precisely, and runs through the hours so accurately that, reset with the Sun to the meridian in one natural day, it returns perfectly, not disagreeing by even a single scruple of an hour from the Sun's daily motion. Exacting requirements indeed, but not impossible ones—

—that I require; seeing that here at Ingolstadt we supply them with no excessive labor. For the small wheel set beneath the pointer has each single hour-space divided into 60 equal parts, and so the pointer easily shows the scruples of the hour. The weights, too, are to be increased or decreased until one revolution of the pointer is made equal to the Sun's course. Once these have been reliably secured:

When the Sun occupies the meridian — which you will recognize without difficulty from the shadow cast along the meridian line — let the pointer be set to the twelfth hour "usual" (German) or the 24th "astronomical." Or, if you have some sundial clock, a very reliable one, marked out even to the scruples, you will adjust that other mechanical clock in accordance with it, for as long as the Sun stands outside any perceptible refractions. Then, when the pointer of the mechanical clock has reached precisely the point of the Sun's rising or setting, you will at the same time take, most accurately, the Sun's altitude above the horizon with the aforesaid astronomical quadrant; for that altitude itself will be the refraction sought for that moment. This I show as follows.

[Margin: The quantity of solar refraction, found without calculation.]

The arc of the vertical circle intercepted between the true and the apparent place of the Sun is the solar refraction. But the elevation of the Sun, at the moment when the mechanical pointer marks the true point of the Sun's rising and setting, taken by the quadrant, is the arc of the vertical circle intercepted between the true and the apparent place of the Sun. Therefore it is the solar refraction. The major premise is evident from the preceding chapter; the minor is proved.

For the elevation of the Sun above the horizon is nothing else than the distance of the seen or apparent Sun from the horizon, taken along the vertical circle. Now the place of the seen Sun is the same as that of the point of incidence or refraction, and the true place the same as that of the visible or radiating point; and these two points are always necessarily found in one and the same plane, as was shown in Chapters 1 and 2, and elsewhere throughout. It is therefore necessary that, while the Sun's true place is set on the horizon, the apparent place lie in the same vertical circle that is drawn through the true place, and conversely — because each lies in the plane of refraction, and this alone is the vertical plane, and so on. Hence it necessarily follows that the arc which, in the vertical circle, reaches from the apparent place of the Sun raised above the horizon down to the horizon, is at once both the elevation of the Sun above the horizon and enclosed between the true and the apparent place of the Sun — because at the time of that elevation the Sun is, in reality, supposed to stand on the horizon. The whole arc, then, by which the Sun is seen above the horizon while in reality it occupies—

—the horizon itself, is to be assigned not to reality but to appearance, and so is to be transferred entirely to refraction. Which was to be demonstrated.

And do not object that this practical procedure smacks of something "merely mechanical." For whoever would have mechanics excluded from practical matters builds cities and citadels in the air. Where would Archimedes [Archimedes] have been, had mechanics not first raised him to such a height — as he himself admits? Where the outstanding inventions of Hero [Hero of Alexandria]? Where those of others?

Therefore mechanical things are not foreign to mathematics, provided they are certain — indeed they are so necessary that without them nothing solid is accomplished. And distinguished mathematicians always either carry over into speculation something excellent drawn from mechanics, or, from a theory laid down in advance, bring to light mechanical marvels, not without the astonishment of mortals, and so on.

To find the horizontal or full solar refraction, under the apparent hour of rising or setting, most easily, without calculation.

CAPUT XLIX.

I require beforehand, as before, a mechanical clock, the true hour of rising or setting known in advance, the meridian line, a quadrant for clocks most accurately worked out even to the scruples of the hour, on which the hour-lines are extended somewhat beyond the horizontal line as well, and whose limb likewise renders the degrees together with the minutes exactly. Once these are secured:

Let the mechanical clock, previously set to the time of the rising or setting of the unrefracted Sun, be turned by its own rotation up to this time; from this time on, count carefully those hour-scruples that the pointer of the clock runs through while the Sun completes the remaining space up to the apparent setting. For the arc in the limb of the quadrant corresponding to these time-scruples is the full solar refraction. To obtain it exactly: with the cursor set on the thread to the Sun's place in the zodiac, you will advance it beyond the horizontal line of the quadrant, up until it has exactly completed the moments of time intercepted between the true and the apparent setting; and at the end of this advance, the arc of the limb intercepted between the horizon and the thread will show the solar refraction sought.

This is evident from the following. That arc, found on the quadrant, corresponds to the arc of the vertical circle intercepted between the true and the apparent place of the Sun; which is clear from this, that it is equal to that arc which the horizon and the vertical circle drawn through the apparent place of the setting Sun cut off from the parallel then described by the Sun — from the fact that that arc corresponds to the time-minutes elapsed from the true setting of the Sun to the apparent one. And thus we find the full refraction of the apparently setting Sun, which was to be done. And if you should further wish to elicit from these very data the refraction by astronomical calculation, you will accomplish it without difficulty, thus:

The hour-scruples elapsed from the time of the true solar setting to the apparent one — reduce them to minutes of the equator, and seek in the vertical circle the arc corresponding to them,

the arc, which will be the refraction. I would gladly bring forward practical examples drawn from observations made with notable diligence. But in a matter of such moment one ought not to publish anything unless it has been thoroughly sifted by long and repeated examination and confirmed by consistent practice. On the one hand, it must rest on the exact construction of the instruments and on the correct declination of the sundials [horologia sciotherica] — a thing that in such delicate work must be got exactly right [ad unguem], and is therefore laborious — as well as on the proper setting of the gnomon [stilus]. On the other hand, our refractions display against the Tychonic ones [à Tychonicis] a discrepancy in magnitude — I cannot say quite how great — a quite noticeable and indeed regular one. For these reasons I did not want, as yet, to be thought premature in producing examples or rash in compiling tables. I judged it better to set out the theory first, to be reinforced and clarified in due time by exercises derived from its own application. Meanwhile others too can make their own private trial of what I am saying. For now there is no harm in explaining the matter by an example, whether real or invented.

Suppose, then, that while the Sun passes from its true to its apparent place on the horizon, 4 horary scruples [scrupula horaria] have elapsed. I convert these into arcs of the Equator as follows. One hour contains 60 scruples; and on the Equator the time corresponding to an hour is 15 degrees, which multiplied by 60 yield 900 minutes. So by the rule of proportion I find how many minutes of the Equator correspond to 4 horary scruples, in this way: if 60 scruples of one hour give 900 minutes on the Equator, what do 4 scruples of an hour give? The result is 60 minutes, that is, 1°. And you will do likewise for the rest.

Now for this arc of the Equator one will easily find, by tables of Ascensions, the corresponding co-ascending or co-descending arc on the Ecliptic. When the Sun has moved appreciably away from the equinoxes, this arc must be brought in so that through it we may arrive at knowledge of the refraction; near the equinoxes the investigated portion of the Equator itself suffices. In an oblique sphere this portion is the side of a right-angled spherical triangle subtending the right angle — the side contained by the arc of the horizon and the vertical circle passing through the Sun's apparent place. The sought quantity of that side will therefore emerge once one has first found the angle that the arc of the Equator or of the Ecliptic makes with the horizon: the arc of the Equator when the Sun is at or near the equinoxes, that of the Ecliptic when it is far from them. For then three things will be given: the arc of the Equator or Ecliptic, and the two angles at the circumference of the horizon.

In the adjoining planispheric diagram, orthographically projected, let the horizon be represented by the straight line A B, and the vertical circle by C D. Let the true place of the setting Sun, lying on the vertical C D, be E, and the apparent place of the refracted Sun above it be F. Gradually, as the Sun passes from E below the horizon by the primary motion into the vertical G H, toward the true place K, meanwhile the apparent place of the refracted Sun reaches the horizon at I. And there arises the arc I K, enclosed between the apparent place I and the true place K of the Sun, which is the sought solar refraction.

Diagram. An orthographically projected planispheric construction for horizontal solar refraction. Two straight lines cross at right angles at the centre E: the horizontal line A B (the horizon) and the vertical line C D (the vertical circle through the true setting place). A second vertical line G H stands parallel to C D, to the right, cutting the horizon at I. An oblique straight line runs from upper left (L) down to lower right (M), representing the Sun's diurnal parallel projected as a straight line, with the true places E and K on it. A second, nearly parallel oblique line F O I represents the apparent (refracted) path. Labels read:

- A (left end of the horizontal line): western end of the horizon A B.
- B (right end of the horizontal line): eastern end of the horizon A B.
- C (top of the left vertical): upper end of the vertical circle C D.
- D (foot of the left vertical): lower end of the vertical circle C D.

- E (centre, intersection of A B and C D): true place of the setting Sun.
- F (on the vertical C D just above E): apparent (refracted) place of the Sun corresponding to E.
- G (top of the right vertical): upper end of the second vertical G H.
- H (foot of the right vertical): lower end of the second vertical G H.
- I (on the horizon, where G H meets A B, right of E): apparent place of the refracted Sun when the true Sun has descended to K.
- K (on the right vertical, below I): true place of the Sun after descending.
- L (upper-left end of the oblique line): upper extremity of the projected diurnal parallel.
- M (lower-right end of the oblique line): lower extremity of the projected diurnal parallel, below K.
- O (on the apparent path between F and I): intermediate point on the apparent (refracted) line F O I.

A short dotted arc is drawn near F, marking the refraction between the true and apparent place.

becomes known through the triangle I E K, of which E I is the arc of the horizon intercepted between the true and apparent place of the setting Sun, E K is the portion of the diurnal parallel then described by the Sun, and K I is the vertical arc intercepted between the Sun's true place K and its apparent place I. Now since the Sun descends from its true place E to its true place K in the same time in which it moves from the apparent place F to the apparent place I — along the arc or straight line F O I — it follows that if I measure the time F O I with a mechanical clock, I also have the time of the arc E K. If this is a portion of the Equator or of a neighbouring parallel, the procedure is straightforward: for the angle E I H is given as a right angle, the angle I E K as the complement of the pole's altitude, and the side E K is also given, from the known time. Therefore the refraction arc I K cannot remain hidden. It is found thus: the whole sine of the right angle E I K gives the sine of the arc E K; what will the sine of the complement of the polar altitude — that is, the sine of the angle I E K — give? The side I K will result. From the given time of four horary scruples this comes to 0° 39′ 37″ 30‴.

But when the Sun has traversed some parallel appreciably distant from the Equator, then the time-arc E K found on the Equator cannot be taken for an exact calculation. For although it is equal in number of minutes to the arc of the parallel traversed, it nevertheless exceeds it in the size of its parts, since the parallels are not great circles; and so the refraction arc I K would be found larger than it should be, if the arc of the Equator were taken in place of the arc of the parallel circle. You will meet this difficulty, then, if for the arc of the Equator E K (found by the earlier method) you substitute the co-ascending or co-descending arc on the Ecliptic, which you will find from the ascensions of the true and apparent solar place situated on the horizon. And once you know that arc of the Ecliptic — let it be taken again in the earlier diagram as E K — you will easily determine the angle I E K, which the Ecliptic makes with the horizon, from the known angle of greatest declination and from the arcs comprehending it, both of the Equator and of the Ecliptic extended to the horizon, and this by means of the ascensions and descensions. So, once this angle I E K is had, there will be, as before, the known right angle at I, the arc of the Ecliptic E K, and the angle I E K. Therefore, by carrying out the operation as before, the refraction I K will result — and a full one [plenaria], one than which no greater could result for that day and for the state of the atmospheric surface, as may be gathered from the lemmas already set out. But if you choose to employ the Ecliptic in the way described, apart from the very days of the equinoxes, you will free the calculation entirely from any suspicion of error.

For the rest, as I hinted before, the true hour of rising or setting can be known exactly from a quadrant worked out into horary minutes, so that you may dispense with the tedious labour of calculation and save much time, once the effort of constructing one such quadrant has been carefully spent. I would affirm the same of some large cylinder, an astrolabe, or a similar instrument.

I would also have this noted: that the apparent place of the Sun, on account of refraction of whatever sort, does not describe a line that is either similar or proportional to the one the true place marks out. For the line F O I, even if it is straight, is nevertheless never parallel to the straight line E K, but is always a side of the trapezium I F E K, whose sides F E and I K are unequal, because the refractions are unequal. And if that same line turns out to be curved — as can happen — it will be far less similar to the line E K, which, orthographically projected, is always straight.

It should further be understood that in the Sun's elevation above the horizon a threefold difference can arise. For since the Sun has a perceptible magnitude, and its diameter measures at least 30 minutes, its altitude above the horizon can be reckoned either to the Sun's centre, or to its highest point, or to its lowest point — the latter being the common section of the arc of the vertical circle with the circumference of the solar disk. If it is reckoned to the Sun's centre, then the true place of the solar centre must be on the horizon, and the time counted from that moment, until the apparent place of the centre reaches the horizon, belongs to the solar centre, as does the refraction found. If to the highest point, then its true place on the horizon must be known in advance, and from there the time must be measured, until the apparent place of that same point is brought to the horizon — which happens when the Sun is last seen and its uppermost part just touches, as it were, the sensible horizon; for this time, and the refraction drawn from it, pertains to the top of the Sun. If to the lowest point, nearest the horizon, its true place on the horizon must likewise be foreknown, and from that moment the horary scruples must be recorded, until the Sun apparently begins to set — which occurs at its first contact with the horizon; for this time, elapsed from the true setting of this lowest point to its apparent setting, together with the refraction drawn from it, belongs to the lowest part of the Sun.

Hence it is necessary to know in advance the true hour of rising and setting of each of these points, if indeed you wish to examine the refraction of any of them without error. For if, when the Sun's centre is truly setting, you observe the Sun's central altitude above the horizon, you will indeed have the refraction due to the centre, but not thereby the one due to the Sun's lowest edge. To obtain the latter at that time, the Sun's elevation must be taken from its lower marginal ray, not the central one; and to this elevation must further be added the true depression of that same limb below the horizon, which, when the Sun's centre is placed on the horizon, is the Sun's semidiameter, and therefore at least 15 minutes. And this whole sum will be the refraction of the lower limb. But it is better to hunt it down while the point truly occupies the horizon.

Again, if from the time of the true setting of the solar centre you note the scruples elapsed up to the true setting of the lower limb, and derive the refraction from them, it will come out much smaller than it should be, because this time is shortened more than it should be. For when the Sun's centre truly lies on the horizon, the lowest edge of that same Sun has already set, and its true place is depressed 15 minutes below the horizon — minutes to which the corresponding arc on the Equator amounts to about 23′, equivalent to 1′ 32″ of horary scruples. Hence it is safer to have noted the true hour of the setting of each point, and to observe the horary scruples slipping by up to the apparent setting, than to seek compensations from elsewhere. Yet I do not think any perceptible error is admitted if one applies the time intervening between the true and apparent setting of the solar centre also to the true and apparent descent of both the highest and the lowest point on the Sun. For it is not credible that in so tiny an interval, in which these points succeed one another, either the surface of the atmosphere or anything else has changed appreciably. So it is more than probable that the refraction of these three points is the same when they truly lie on the horizon, and likewise the same when they are set equally above or below it — since the incidences upon the same surface and the same atmosphere are the same; and this, indeed, for the most part. For if it happens otherwise, the ellipse itself will show it.

The refraction of any point on the Sun is therefore full [plenaria] when the point is apparently joined to the horizon; not full, when it is truly joined. The latter is had if, at the time of the true conjunction, the apparent elevation of that same point above the horizon is taken; the former, if the time between the true and apparent conjunction is exactly noted and converted into an arc of the Equator, and the arc corresponding to it on the Ecliptic is sought from the ascensions or descensions. Then at last the portion on the vertical circle due to the refraction is disclosed.

HOW THE TRUE AS WELL AS THE APPARENT CONTACT OF THE HIGHEST, MIDDLE, AND LOWEST POINT OF THE SOLAR ORB WITH THE HORIZON IS TO BE KNOWN

CAPUT L.

The true contact of any point is elicited either arithmetically, by calculation — commonly to be found among astronomers, and especially the gnomonicists, for a given pole altitude, from the size of the diurnal arcs, and so on, which we therefore forbear to bring in here — or geometrically, by mechanical instruments, of which…

by which — in the manner just described — we select a particular horologe-quadrant [Quadrantem horologicum], constructed for the given elevation of the pole in degrees and horary scruples and with its limb likewise divided into minutes. With its aid we shall find the true places of the said points on the horizon, by this device:

1. Draw the runner (or bead) hanging on the thread to the place of the Sun as marked in the Zodiac, and then bring the thread back over the horizontal line. The runner will show the true place and time of the Sun's center at rising and setting.

2. Keeping the runner at the place of the Sun taken from the Zodiac, draw the same thread back from the horizontal line toward the meridian side of the quadrant, until there lie on the limb, between the horizon and the thread,

as many minutes as the visual semidiameter of the Sun comprises. The runner will then show the time at which the lowest point of the Sun brings its own true place into union with the horizon.

3. Move the thread outward beyond the horizontal line by as great an arc of the limb as the visual semidiameter of the Sun subtends in the sky, and the runner will show the hour at which the highest point of the Sun truly rests upon the horizon.

The apparent place of any solar point whatever, wherever it may lie, can be known by no calculation, because it is Refraction that produces it — and Refraction becomes known through careful observation alone.

The lowest point of the Sun, then, touches the horizon at the moment when nothing lies between the Sun and it. This is observed, first, with the naked eye assisted by lenses [specularibus]; second, with an optical tube [tubo optico] or a colored Helioscope [Helioscopio colorato]; and third — most accurately of all — by admitting the Sun's image [intromissione Solis], of which we spoke in the earlier chapters on capturing the Ellipse. For there the contact of the Sun and the horizon falls upon the sight so clearly and so distinctly that you could not wish for anything better.

The center of the Sun is noticed with the naked eye only with considerable difficulty, because the Sun, in the eye, always strongly assumes a circular figure — as do all bright objects, etc. — the cause of which we assign elsewhere. But a colored Helioscope, or blue or green lenses, are very well suited to this task; and above all, the admission of the image, of which we spoke a little earlier, is best here. But how shall we distinguish the location of the center? For the Sun, unequally contracted, does not place its center at the middle of its altitude, but keeps it outside. Here above all a careful and discerning observer's eye is required; and next, the Sunspots [Maculæ Solares] — with which the Sun rarely rises — will afford no little help. If they are central, they will remove every difficulty entirely; if below or above, they will supply the sight with strong guidance, so that you will not perceptibly stray from the middle. It will also help to take the lowest altitude of the Sun at 15′ above the horizon, and from there to measure the time up to contact; for in its descent the middle part of the Sun sinks below the horizon by just as small an interval as, by 15′, it takes in approaching that same horizon, etc.

The highest point of the Sun is treated in the same way as the lowest — with the naked eye, through the Helioscope, through admission [immissionem], etc.

HOW THE APPARENT MAGNITUDE OF THE SUN, AND ITS ELEVATION ABOVE THE HORIZON, MAY BE OBSERVED WITH RESPECT TO THESE GIVEN POINTS. CHAPTER LI.

There are various methods — which I shall treat elsewhere as planned — but for now let us be content with the quadrant in the manner just described. Let two sight-vanes [pinnacidia] be made, and let both be pierced with the tiniest holes, like points, and fixed to the side of the quadrant at an equal distance, so that a solar ray transmitted through both runs parallel to the side of the quadrant. Then the radiant cone of the Sun, when it reaches the hole of the vane nearer to it, will fix its apex there because of that hole's minuteness; and all the rays, cut there in mutual decussation [crossing], after passing through, will spread out into opposite regions. The reversed cone will be in every respect proportional [to the direct one], and onto the flat surface of the other vane it will project a bright base, which is a circle — especially if the surface of that vane is clean and white, or covered with paper — in every respect proportional to the disk of the Sun. Therefore, if you take it near noon, where the Sun appears least elliptical, you will find the Sun's altitude least truncated, in this way. The topmost extremity of the base in the reversed cone represents the lowest point [absidem imam] of the Sun in the sky; the lowest extremity, the highest point. So raise the side of the quadrant, with the vanes, until the top of the base coincides with the center — that is, with the hole of the vane — and then mark the position of the plumb-line [perpendiculi]. Next lower the vanes, with the side of the quadrant, so that the bottom of the base falls on the hole, and mark the position of the plumb-line. The arc intercepted between the two positions of the plumb-line is the diameter of the Sun. You will obtain the same result if you apply your eye to the vanes, look at the edges of the Sun, and mark the sections of the plumb-line. Thus, with a bare quadrant and no tube, you can precisely hunt out the magnitude of the Ellipse — for the altitude will not appear as great as the [horizontal] length of the Sun, but shortened, in proportion to the Refraction.

In the same way you will obtain the elevation of these points above the horizon. For the top of the base of the reversed cone, brought to the hole, will yield — in the intersection of the plumb-line with the limb of the Quadrant — the elevation of the lowest point of the Sun's limb; the bottom, that of the highest; the center, that of the center; etc. And you will obtain these same values by looking at the Sun directly, etc.

But if the reversed solar cone falls too weakly through the front hole onto the opposite wall of the rear vane, or the elliptical image of the Sun is depicted faintly and hazily on the paper, then darken the whole chamber [cameram], so that only the solar pyramid [pyramis Solaris] has access, and you will see everything most brilliantly.

TO ELICIT MOST EASILY ANY REFRACTION OF THE SUN WHATEVER, FROM SOME ONE ELLIPSE TOGETHER WITH ITS REFRACTION KNOWN FOR CERTAIN IN ADVANCE. CHAPTER LII.

The chief reason I have set forth so many ways of investigating Refraction was this: that at last some one Refraction, most exact, might be obtained together with its Ellipse. For it holds that, as any Ellipse is to any other Ellipse, so is the Refraction that causes the [one] Ellipse to the Refraction that produces the latter Ellipse. (The Ellipse is the difference between the two extremities of the Sun, the highest and the lowest, as they are refracted; and once this difference is removed, then just as there will be no differing Refractions, so too there will be no Ellipses, etc.) Therefore, if even once we have obtained one most accurate Refraction together with its Ellipse, we shall be able to use it as a root [radice] from which to investigate all the Sun's Refractions — at any elevation of the Sun above the horizon, at any time, in any latitude of place — provided we also have the observed Ellipse whose Refraction we seek.

For in this way the three terms of the proportion will be known: the radical or conjugate Ellipse [Ellipsis radicalis seu coniugata] as the first; the Refraction corresponding to it as the second; and the Ellipse observed in isolation, or "orphaned" [orba], as the third — whose parent, accordingly, we wish to raise up, and which will readily come forth if the three aforesaid terms are at hand. I call an Ellipse "joined" or "conjugate" [coniugata] when it has been observed together with its Refraction at the same time, and "widowed" or "orphaned" [orba] when it alone, without its Refraction, is known. By way of an example:

In the year 1616, on 14 November, the evening horizontal Ellipse was ⅙ — that is, 5′40″ — of the units in which the solar diameter is reckoned at 34′. The true hour of setting was 4h 31′; the apparent hour, 4h 35′; the difference, 4′. Its Refraction was 39′37″. If this were so correct that no error at all had been committed — neither in the sundial [horologio sciotherico], nor in noting the Ellipse, nor in taking the time — we should indeed have obtained a conjugate Root, from which we might draw out the refractions of the other Ellipses by the Rule of Proportion, in this way:

5′40″ — that is, 340″ — yield the Refraction 39′ (for I now waive the seconds). What, then, do 4′40″ [recte: 4′7″] — that is, 247″ — yield? There will result 28′20″, for the Refraction of the plenary Ellipse [Ellipseos plenariæ] observed on 17 September, at the apparent hour when the Sun touches the horizon, 6h 12′30″; the true hour, 6h 10′.

But in such tiny quantities [minutiis] the clocks easily go astray — especially declining ones [declinantia], on account of the difficulty of finding the declination, of erecting the style [stili] correctly, and likewise of obtaining the meridian line exactly, etc. — by a whole horary scruple or a scruple and a half [uno aut sesquiscrupulo horario]; or the observer slips by so tiny a lapse of time in discriminating [the contact]; or errs in marking the elliptical limit. For these reasons, what I have brought forward thus far, for the sake of example and of teach…

and of teaching, I have said — not so that anyone should use fixed roots as already established, but so that from them he may himself learn to construct established ones. These, up to now, amid my occupations, I have not been able to work out — for all my heart's desire and for the public good of letters. In the meantime, however, if we believe Tycho, we do have the Ellipse known from his own Refractions, as I said in [the section on] the Elliptical Sun. For from the horizontal Refraction there are lost to it (as Kepler also testifies, in his *Paralipomena*, p. 131) 5′ from the Sun's diameter, of the units in which that diameter itself is 30′. If we use this as our root, we shall find, for the difference of the Ellipses, differing Refractions as well.

And in this way a certain universal table of Ellipses could be constructed for the whole world, from the greatest Ellipse down to the smallest, and that by a twofold route. The first route is from some one radical Ellipse, by the rule of proportions in the manner just handed down: as the Ellipse 5′ gives the Refraction 34′, what does the Ellipse 1″ give? 7″ will come out. What does 2″ give? 13″. 3″? 20″. 4″? 27″. 5″? 34″. 6″? 40″. 7″? 47″. 8″? 54″. 9″? 1′1″. And so on in succession. For these I touch on only by way of example, drawing on others' [data], intending to derive them from my own once I have obtained leisure and certain roots.

The other route would proceed from practical observations, by which one would hunt out at once the Ellipse and at once the corresponding Refraction, from the smallest Ellipse to the greatest. Though this is more laborious, it would in my judgment be by no means a little more fruitful and more reliable. For here it would be plainly discerned whether the Ellipses are proportional to the refractions, and these to those, etc.

You will say: why so much sweat over such minute matters? Oil and labor seem to be wasted. I answer: from what has been brought forward it can be sufficiently established that this science is in every way necessary. For this matter has not only set the more recent Mathematicians vehemently at odds among themselves — as is everywhere evident from Tycho — but also, neglected by the ancients, has introduced a great many errors into Astronomy, in the eliminating of which even posterity will sweat. Among its other, uncommon uses, you will gain this one: if you wish to gird yourself for observing the heavenly bodies, etc., you should first consult the solar Ellipse, so as to learn from it the Refraction in a constructed table; or else, from a root already at hand, derive it by the Rule of Proportion as I have just taught. And so, for that day, you may enjoy the knowledge of it in your observations. Otherwise you will surely fall into many slips, which by this method you will in any case avoid.

You will object, first, that the proportion between the Ellipses and the Refractions does not appear stable — because it can happen that the upper part of the Sun is compressed more than the lower, as experience has confirmed; next, that the lower part of the Sun is sometimes refracted equally with the upper, and yet the Altitude of the Sun is nonetheless not equal to its Length [Longitudini], but falls far short of it; and moreover, that refraction can exist without any Ellipse at all — as is plain when the Sun is placed in the Zenith, for then the Sun is refracted equally on every side and therefore lacks an Ellipse, since its altitude is equal to its length.

Finally, that argument drawn from the mutual increase and decrease of Refraction and of the Ellipse is not conclusive. For by the very same reasoning one could prove that the angles of Inclination are proportional to the angles of Refraction, since the same manner and force of argument can be produced there too. A greater Refraction arises from a greater Inclination, a smaller from a smaller, none from none; therefore, speaking absolutely, as one Inclination is to another, so is one Refraction to another. But if nature refutes this inference, how will the Astronomer admit the other? A greater Ellipse arises from a greater Refraction, a smaller from a smaller, none from none; therefore, as Ellipse is to Ellipse, so is Refraction to Refraction. This, I say, does not follow: for it can nonetheless happen that Refraction stands in a different relation to Refraction than Ellipse does to Ellipse, even though a greater follows upon a greater, and so on — just as happens in the case of Inclination. And many other similar examples could be adduced from motion, from the intensification of qualities, from the increase and decrease of light, and so on.

Reply 1. Most of these arguments attack not only the proportion of the Refractions to the Ellipses, but also the very order of the Refractions themselves and the Tables established by Tycho [Tycho Brahe]. For since the order of the tables is always uniform and stable, whereas the Refractions (by what has been said) are irregular and unstable, it is impossible that a proportion should exist between the Refractions and the Tables. Therefore, since this argument must be resolved on every view:

Reply 2. It does indeed sometimes happen that the upper edge of the Sun is refracted more than the lower; but this is neither ordinary nor lasting, and must be attributed to the sinuous folds in the volume of the Atmospheric surface. Even so, in this very case the proportion of the Ellipse to the Refraction still holds, and if the Ellipse is increased or diminished, the same happens to the Refraction as well — the only discrepancy being that both the Refracting surface and the perpendicular line are altered. For when the upper part of the Sun is refracted more than the lower, then the Refracting surface is not concentric with the earth [homocentrica terræ], but in its lower concavity is nearer the eye than in its upper. Yet this takes nothing whatever away from the proportion of Ellipse and Refraction — as is evident from the experiments with lenses cited above.

Reply 3. If an equal refraction of the upper and lower solar limb occurs, and the Ellipse nonetheless remains (which I do not recall having observed for my own part, and whether it can happen I am left in doubt), this comes about for the cause stated earlier — namely that it happens neither often nor for long. And in this case the Refracting surface makes an equal inclination above and below to the axis of the eye, and has the form of a surface that is, as it were, semicylindrical, and so forth — which accordingly also alters the Refraction in its proportion. And in this case the difference of Refraction can be taken either from the center of the Sun, or from its longi-

-tude, and so on. But as I said, these cases are rare, especially as regards the equality of the Ellipse, and so on.

Reply 4. Refraction at the Zenith is not perceptible. And then, even if it were, the defect [defectus] would nonetheless fall uniformly over the whole Sun, whose diameter would accordingly turn out smaller than that of the unrefracted Sun; and this defect would not be an Ellipse in figure but in effect, for the Sun is indeed diminished, but into a smaller circle, and so on. And thus it can well be brought back to an Ellipse in a more general sense, and so on.

Reply 5. The increases and decreases of the Ellipses do not proceed by some fixed rule, as the angles of inclination do, but are precipitated about the horizon in exactly the same manner as the angles of Refraction — from which it is evident, on further grounds passed over in silence, that they imitate the nature of the Refractions, not of the Inclinations.

You will object, secondly: even granting this proportion, tables still cannot be struck from it, on account of the Sun's eccentricity, which makes its visual diameter now greater, now smaller — whereby it comes about that the Ellipse, though produced in the same matter and from the same refraction, is nevertheless of a different size at the Sun's Apogee and Perigee. This is shown as follows. If the lowest point of the Sun, apparently resting on the horizon, finds one and the same temper and height of the Atmosphere both at Perigee and at Apogee (which scarcely anyone would dare deny to be possible and usual), it is clear that its Refraction will be the same; yet the Ellipse will not turn out the same in both cases, because the Sun at Apogee occupies a smaller arc of the greatest circle, at Perigee a greater one. Consequently the Sun's uppermost apex is nearer the horizon at Apogee than at Perigee, all else being equal; and so the difference of the Refractions arising from the lowest and highest point of the Sun will also be smaller at Apogee than at Perigee. Therefore the Ellipse too will be smaller. The proportion, then, does not hold, and the construction of the Tables wavers, and so on.

Reply 1. This argument fights equally against any Refraction tables whatever hitherto struck by anyone at all. For those authors who composed Tables of Refractions disregarded the Sun's eccentricity entirely; so if it does no perceptible damage to their Tables, neither will it be able to inflict any harm on ours — or if on us, then on them too. On every view, therefore, it must be resolved.

Reply 2. From this fastidious dispute over Apogee and Perigee the truth shines forth all the more strongly, and our proportion is most solidly established. For just as at Perigee the Sun, appearing larger, obtains a larger Ellipse, and at Apogee, appearing smaller, obtains a smaller one — both from one and the same

Refraction — so the proportion of the larger Perigeal Ellipse to the larger visual diameter of the Sun at Perigee is the same as that of the smal-

-ler Apogeal Ellipse to the smaller diameter of the Sun at Apogee, provided that the Refraction is posited as the same, all else being equal. And so it must plainly be — nor can it be otherwise — that, from the given proportion of Refractions and Ellipses, Ellipses of this kind arise. And thus it is clear how beautifully, and how everywhere and always, the true agrees with the true.

Caput LIII — Hypotyposis dictorum hactenus [Chapter 53 — A Diagram Summarizing What Has Been Said Thus Far]

It seems good to enclose the Iliad in a nutshell, and to represent most of what has been disputed up to now in a single Diagram. That these things may be the better understood, the following points are to be carefully noted beforehand and kept in mind. In the present projection of the Sphere: the straight line **A B** is the Horizon; **C D** the primary vertical circle; **E F** the axis of the world, elevated 48 degrees 40 minutes; **E** the elevated pole, **F** the depressed; **G H** the equator; **I K** the Ecliptic; **I L** the Tropic of Cancer; **K M** the Tropic of Capricorn; **A**, the North point [Septentrio]; **B**, the South [Meridies]; **C**, the Zenith; **D**, the Nadir; **N**, the center and pole of the Meridian circle A C B, which moreover represents the point of true rising and setting — in short, the center of the whole Sphere and of all the greatest circles in it; the arcs **C P D**, **C Q D**, and **C R D** are secondary vertical circles; **A S** is the greatest of the ever-apparent [circles], **B T** the greatest of the ever-hidden. These things being premised:

If there were no refraction in the nature of things, then the Sun would always appear in that place of the horizon which it occupies in reality. Now the Sun's true place on the horizon is the one produced by the common section of the horizon with the parallel in which the Sun is then moving. Thus, if the Sun is moving in the first degree of Capricorn, the parallel will be K M, and its section with the horizon V; but if the Sun is at the beginning of Cancer, the true place on the horizon is X; and so at the beginning of Aries, the Sun truly occupies the place N, which is the common section of the equator and of the vertical C D with the Horizon.

[Margin: The Sun's true place on the horizon.]

But now, refraction being granted in the manner we have described, it is necessary that the Sun — while still wandering below the horizon in its vicinity — be seen above it, upon those vertical circles whose common sections with the horizon the Refracted Sun rests upon (or onto which it falls), and in which at that time the Sun really exists. And thus the Sun's apparent place on the Horizon is the section, common to the vertical circle and the horizon, on which the Sun is seen, and so on. For example, suppose the straight line **Y Z** parallel to the horizon A B; if the Sun reaches it, then, being refracted, it is raised onto the horizon. Therefore, in Capricorn, if it reaches α [?], it will shine forth at the point R; from β in Cancer, at Q; from γ

[Margin: The Sun's apparent place.]

of Aries, at P. And so in Capricorn it will obtain both its rising and its setting amplitude, R N, smaller than the true, by the defect R V; so, dwelling in Cancer [♋], it will have the same amplitude N Q larger than the true, by the excess Q X; while in Aries,

Diagram. A large full-page stereographic projection of the celestial sphere onto the plane of the meridian, summarizing the whole refraction argument. The bounding circle is the meridian circle A C B D: the North point **A** at the left end and the South point **B** at the right end of the horizontal diameter (the horizon A B); the Zenith **C** at the top and the Nadir **D** at the bottom of the vertical diameter (the primary vertical C D). Within it are drawn the inclined axis of the world E F (elevated pole E to the upper left, depressed pole F to the lower right), the equator G H, the ecliptic I K, and the two tropics — Tropic of Cancer I L and Tropic of Capricorn K M — together with a dense pencil of secondary vertical circles (azimuth arcs) through C and D, and, near the center, a small circle (*circellus*) around the point N. A line Y Z is drawn parallel to and below the horizon. Numerous small Greek minuscules mark the Sun's sub-horizon positions along Y Z and near the horizon; at this scan's engraving floor they are individually illegible and are flagged below.

Labels I can positively read on the plate, with the role assigned in the text legend above:

- **C** (top): Zenith.
- **D** (bottom): Nadir.
- **A** (left end of horizon): North point (Septentrio).
- **B** (right end of horizon): South point (Meridies).
- **E** (upper left): elevated pole of the axis E F.
- **F** (lower right): depressed pole.
- **G** (upper right) / **H** (lower left): ends of the equator G H.
- **I** (upper right): northern end of ecliptic I K / Tropic of Cancer I L.

- **K** (lower left): southern end of ecliptic I K / Tropic of Capricorn K M.
- **L** (left, below A): far end of the Tropic of Cancer I L.
- **M** (right, below B): far end of the Tropic of Capricorn K M.
- **N** (center, inside the small circle): center and pole of the meridian; true equinoctial rising/setting point.
- **P** (on the horizon, just left of N): apparent place for Aries.
- **R** (on the horizon, right of N) and **V** (further right of R): apparent place (R) and true place (V) for Capricorn.
- **X** (on the horizon, left of N): true place for Cancer.
- **Q** (lower left): apparent place for Cancer (foot of arc C Q D).
- **T** (bottom, just left of D): foot of arc B T (the ever-hidden maximum).
- **O** (left flank, mid-lower): point on the perimeter [role not stated in legend].
- **Y** (left, below A) / **Z** (right, below B): ends of the sub-horizon line Y Z.
- **S** — confirmed **not lettered on the plate at all**: the legend names it ("A S, the greatest of the ever-apparent"), but the great circle A S simply reaches the rim unlabelled. The earlier "not locatable" is corrected to **absent** — do not place S.
- Greek minuscules along the horizon and line Y Z (the Sun's sub-horizon positions): now committed by cross-witness reading (IA-verified + Freiburg-tile-consistent): **σ, λ, ρ, ω, Φ, ε, ζ, θ, α** (the α also resolves the Capricorn sub-horizon point-letter, l.37). About ten further interior minuscules remain `[?]` at the **shared engraving floor** (stroke-overlap on both 1617 witnesses — a physical, not scan, limit).
- [Correction, further] The earlier claim that "zodiac symbols are engraved on the perimeter (Cancer/Leo by C, Capricorn/Virgo by D, Aries lower-right)" was a **misread of the Greek minuscules — there are NO zodiac glyphs on the figure.** The zodiac signs (♋, ♑, ♈, ♌) occur only in the surrounding prose. The apparently-duplicated capitals **M×2 / H×2 / Y×3 are confirmed real repeated occurrences**, not misreads.

**No longer a (cross-witness reconciled; residual `[?]` confirmed at the shared engraving floor).

being truly established there, where it ought to be deprived of all amplitude whatsoever, it acquires the same amplitude, N P. But if the Sun truly is at the equinoctial rising — which in the given diagram is referred to the point N (as is clear from what was premised at the beginning of the chapter) — it will be seen by us at [?] above the horizon A B, and, at the same moment and all else being equal, by our Antipodes at e below the same; and, universally, by all the earth's inhabitants who dwell under the same meridian A C B D — whether Periœci [Periæci], such as are under

C and ζ; or the Antoeci, such as those under C and H; or finally the Antipodes, such as those under C and D. At one and the same moment of time, if the Sun in reality stands at N, the true point of rising and setting, and so occupies the true point of rising and setting, these peoples see the Sun itself outside that point, above their own horizon, along an arc drawn through their zenith and the Sun's center. Such lines of sight are represented by the straight lines CN and DN, and could equally be represented by ζN and LN, and by countless others dropped from the individual points of the meridian to the center N — points that lie a whole quadrant distant from N, the true point of rising and setting. There, accordingly, they can behold the Sun as it stands at that point.

That the true rising and setting can rightly be represented by the single point N — which here is in fact the center of the circle ACBD — is clear from this. An eye placed at either pole of the meridian, if it looks toward the opposite pole, projects both poles together with the axis into a single point, such as N in the present case. That point at once represents the poles and the axis of the meridian, and at the same time the diameter of the prime vertical [verticalis primarius], of the Equator, and of the Horizon, and so on. Once these things are rightly understood, it follows that:

1. If the same disposition of vapors is assumed everywhere, the Sun — truly occupying N, the equinoctial point — will be seen all around at once, within a certain small circle, or rather within a portion δ [?] P ε of a spherical surface, by those who lie beneath the meridian ACBD; and yet it has its true place at the single point N alone, and only an apparent place at the remaining points δ, P, ε, and so on.

2. It follows, second, that the Sun, while it stands at the point N, simultaneously fills the whole vaporous hemisphere δPε by refraction. For all those peoples who dwell in either region cut off from the meridian ACBD behold that same Sun — and indeed each at a different elevation above his own horizon, according to a different disposition of the refracting vapors. It is just as a mirror receives the whole Sun everywhere, yet returns it to each eye at one definite spot.

3. It follows, third, that among all the peoples who observe the Sun in this way, it is seen in its true place only by those in whose zenith it actually stands — although, so far as the senses can tell, the Sun does not depart

from its true place even for others, from whose zenith it lies 30 degrees or less away.

4. It follows, fourth, that the true equinoctial point is obtained without any error, of sense or of fact alike, only by those for whom the Sun's center occupies both the vertex and the equinoctial parallel at once; less perfectly by those the farther that point stands from their vertex; and least of all by those in whose horizons the Sun then lies.

5. It follows, fifth, that even though in reality the Sun's annual motion from equinox to equinox always remains uniform, an inequality of the annual motion may nevertheless be detected in the same place, because of the variation of refraction — which

is caused by the variation of the Atmosphere in different places, owing to the difference in the obliquity of the Sphere. Hence it is no wonder if the first practitioners of these matters disagreed, whether among themselves or with others, in their observations; for it cannot be otherwise, given the unstable magnitude of refraction. And so, unless that magnitude is ascertained through this very phenomenon or from some other source, it makes it impossible to investigate reliably the equinox, which customarily occurs near the horizon.

6. It follows, sixth, that it may well be — and is quite probable — that the quantity of the solar year is entirely constant and unvarying, and that every inequality in that matter is to be attributed to this phenomenon.

7. It follows, seventh, that for those positions of the Sun which lie on the Equator, the ortive and occiduous amplitudes are determined, with less liability to error, by means of the rising and setting stars; whereas for the other positions, as the Sun recedes to either side, these same amplitudes spread apart. From which,

8. It comes about, as a consequence, eighth, that the Sun, standing at the same point of true rising and setting, is drawn simultaneously toward the arctic pole, toward the antarctic, toward the zenith, and toward the nadir — more or less — according as the obliquity of the Sphere is greater or smaller toward the South or the North. Johannes Kepler writes, in the *Paralipomena* [*Ad Vitellionem Paralipomena*] chapter 4, no. 8, page 137, and no. 9, page 139, that the Sun's refraction was sometimes seen in Germany to be greater by two degrees; and from a Batavian [Dutch] observation made in the north on 24 January 1597, at a pole-altitude of 76 degrees, he concludes that the same refraction reached a truly monstrous 4 degrees

and somewhat more, since for 14 days they caught sight of the Sun at least earlier than was due. From which,

9. It follows, ninth, that the ortive amplitude can be greater than proper by 13 degrees; from which you gather that the more oblique the Sphere is, the greater the anticipation of time that occurs. From which,
10. It can happen — not only that the Sun rises on the same day earlier than is due (which is seen on the hour-circle EγF, when the Sun describes the equinoctial parallel; for then the Sun, set at γ, would appear at the horizon-point P, and so indicates the sixth hour by its presence, though it is not yet placed on ENF, the six-o'clock hour-circle) — but even that it anticipates by whole days. For as the Sun describes the parallel δι, the armilla will show the equinox when the Sun has reached ε; but εN is smaller than that portion of the Ecliptic which is intercepted between N and κ, at the Sun's place. So if we set εN at one, two, or three degrees, then the true equinox will be anticipated by the apparent one by one, two, or three days, and so forth. AS is the greatest of the always-apparent circles, but it is itself wholly raised by refraction — most at A, least at S, and at the points situated toward S less, toward A more; and thus all the oblique circles behave. From which it is clear how they lose the perfect form

of a circle. In the place of this circle AS, γλ is stretched out, cutting the horizon, refracted, and only touching it at the point Y. Similar things happen around BT, which is always the greatest of the hidden ones; for it ascends by refraction and cuts the horizon, and in its place there always succeeds the hidden Zμ. Thus the whole portion YCZY, greater than a hemisphere, is raised right up to the horizon, and at each single point — except the vertical point C — it is refracted, whereby it sinks down somewhat, into a shape in some fashion spheroidal.

The rest, whether said here and there or left unsaid, will readily be applied to this figure by anyone who studies these matters, or will be drawn from it easily enough — such as could be the following.

Practical Corollaries. Chapter LIV.

Admonition

I would have you know, kind Reader, that in this place I do not intend — given the brevity I aim at, since I am presenting Corollaries — to teach systematically either the construction or the use of the Analemma. Both of these I rather presuppose, as things to be found in Clavius [Christoph Clavius] here and there, and in others. Nor do I intend that you should take this diagram of mine, here set down, and use it in practice for finding the things that follow; for a diagram of that sort must be exceedingly elaborate and large. Rather, I have set it down only to teach, and adduced it as a kind of example, so that you may learn by what method and skill, and from what givens and prior knowledge, one must arrive at the knowledge of those things you do not yet know in this business. Hence, just as in examples exactness is not required, so neither have I adapted this figure to any other purpose; and for that reason I have not distinguished the horizontal line AB, nor reckoned it for the true horizon only, but taken it now for the true (or rational) horizon, now for the sensible one — as the case at hand and the subject matter require.

The reasons for this are as follows. First, that those two horizons differ imperceptibly from each other with respect to the firmament — namely by 9 seconds — but with respect to the Sun and the solar heaven, by about three first-minutes [arc-minutes]; so that if the Sun sits with its center on the rational horizon, it always projects twelve minutes above the sensible one, and thus to the senses always occupies both. Second, that astronomers, otherwise most exacting, in practical matters and mechanical instruments entirely neglect that difference between the sensible and the natural horizon — as may be seen in Clavius on the *Sacrobosco* (*In Sphaeram Ioannis de Sacro Bosco*, chapter 1, page 162) and in others everywhere. Finally, that in this small diagram I could not have marked the sensible horizon above the true horizon AB without a confusion of lines and letters; so it was fitting that this single AB should serve the office of both. These things understood, then, let us proceed to the Corollaries.

Corollary I

To assign the Sun's true place in the Zodiac from a given refraction and an observed ortive or occiduous amplitude, by means of the given diagram constructed after the manner of an Analemma.

Let the refraction be given, for example, as two degrees, and let the ortive amplitude found on the horizon AB be Nν. Through the terminus ν, and the Zenith C, and the Nadir D, draw the vertical arc CνD, which cuts the parallel YZ to the horizon — that parallel being depressed two degrees below the horizon according to the given refraction — at the point ξ. Then the arc νξ will be the refraction of two degrees, and the point ξ the Sun's true place at the time of rising or setting. Therefore the circle ξϖπ, drawn parallel to the Equator, will cut the Ecliptic KI at the point ϱ, which will show the sign, together with the degree in which the Sun stands — provided, however, that the parallels GH of the equator have been inscribed on either side within the Zodiac ILKM in the usual way, or at least marked off point by point upon the Ecliptic IK.

Corollary II

To investigate the ortive or occiduous amplitude from a given refraction and the Sun's true place, known in the Zodiac.

Let the full refraction be given as two degrees. In the resumed diagram, measure from N, the point of true rising, toward D, along the line ND of the true Vertical, as far as the sine of the two given degrees of refraction, and draw YZ parallel to the horizon AB. Let the Sun's place on the Ecliptic be known as ϱ; through it the equatorial parallel οϖ, drawn, will cut the parallel YZ of the horizon at ξζ. Now the vertical arc drawn through ξ will cut the horizon at ν, and the straight line Nν will be the ortive amplitude — or rather the arc of which Nν is the sine. That arc σC, or KD, you will have if through ν you draw the straight line Kσ parallel to the prime vertical CD; or [if you draw] the perpendicular from the point ν to the horizon AB.

Corollary III

To elicit the full refraction from an observed ortive or occiduous amplitude, together with the Sun's true place known in the Zodiac.

Let the observed amplitude Nν be, for example, 25 degrees. Through the point ν describe the vertical arc CνD; let the Sun's true place in the Zodiac be known as ϱ; through it drop the straight line πϱο parallel to the equinoctial HG. This line will be the parallel of the Sun described, or to be described, by it on that day, and it will cut the vertical arc CνD at the point ξ. Therefore νξ is the quantity of refraction, which the arc AY, or BZ — cut off from the line YZ (parallel to the horizon) drawn through ξ — will determine for you in numbers.

Of these three quantities, any two can be known in advance: the place of the Sun, from ephemerides, an astrolabe, a quadrant, and the like; the observed ortive or occidental amplitude, from armillaries; and the refraction, from the ellipse or defect of the Sun — not to mention the other methods already hinted at here and there above.

COROLLARIUM IV.

To hunt out the refraction from the observed or apparent ortive amplitude and the anticipated time, when these are known.

Taking up again what has gone before: let the apparent ortive amplitude be Nv. Through the point v and both poles E and F draw the arc of the hour-circle EvF, which will cut the equinoctial HG at τ. From this point, toward H, count off the anticipated time — already known from elsewhere — either in degrees and their minutes, or in parts of hours and their sixtieth scruples, and so on: so that the time represented on the equator by τv is, in the present case, two degrees, that is, eight horary scruples, which give one-seventh of an hour plus one twenty-seventh [?: the stacked fractions sit at the resolution floor]. Through v and the poles E and F let the hour-circle EvF be drawn, which will cut the vertical circle CvD — carried through the terminus v of the apparent ortive amplitude — at ξ; and ξv, that is the arc AY, will be the refraction sought.

COROLLARIUM V.

From the place of the Sun in the Zodiac and the anticipated time being known, to find the true and apparent ortive amplitude, their difference, and also the true place of the Sun in its parallel, together with the refraction.

The anticipated time here is nothing other than the interval by which the Sun flashes above the horizon earlier in the morning, or sinks below it later in the evening, than it ought to. This delay [mora] is found as follows. From calculation, or from some suitable instrument, one knows at what hour the Sun truly crosses the horizon; and the same is established from the diurnal parallels, by their intersections with the horizon. But because, on account of refraction, this happens daily earlier than it should, and at unaccustomed points, one must therefore find this contact on an hour-circle nearer the 6th hour and farther from the 12th than it really is. This can be known in the following way. Take the true place of the Sun in the Zodiac — let it be, for example, ρ — and through it draw the parallel to the equator, προ; then, through the apparent point of the rising Sun v,

draw the vertical circle CvD, which will cut πο at ξ. Therefore the two hour-circles EvF and EξF, drawn through v and ξ, will intercept on the parallel πο the anticipation of time ξφ. Since, then, it is shown from the shadows of sundials to which hour the circle EvF belongs, one also knows — through the arc ξφ, that is vτ, or χψ (which is obtained by means of the parallels φψ and vχ) — once this is recognized and subtracted, what EξF is.

Or thus: by an accurate mechanical clock, or a water-clock [clepsydra], most exactly fitted to the diurnal motion of the unrefracted Sun, I know that, at the time when the Sun rises at v, it is still in fact situated at ξ. The mechanical clock indicates and names for me the hour-circle EξF on which the Sun truly stands, just as the shadow of the dial's gnomon shows me the hour-circle EvF, drawn through v, the apparent place of the Sun, and so on.

So let the true place of the Sun be ρ; through it draw the diurnal parallel, which cuts the hour-circle of the apparent rising at the point φ. From here count the anticipated time up to ξ — which is the true place of the Sun at the time of apparent rising, recognized by the hour indicated by the mechanical clock as the Sun moves along. This will give the apparent ortive amplitude Nv; while the parallel πο, where it cuts the horizon at ω, gives the true amplitude Nω; the horizon [gives] the difference of the two in the segment vω; and the refraction will be ξv.

COROLLARIUM VI.

From a given refraction and a given point of apparent rising, to find the anticipated time.

Let the given refraction be vξ, and the point of apparent rising v; through it draw the hour-circle EvF, and likewise through ξ draw EξF, and the parallel οξω, whose portion ξφ is the anticipated time by which the Sun leaps forth from its true place ξ into the apparent place v earlier than it should. If to this you add the time φω — the interval lying between the hour-circle EvφF, owed to the apparent rising, and EωF, appointed to the true rising — you will have the whole time ξω, enclosed between the two true places of the Sun ξ and ω: ξ indeed, when the Sun apparently occupies the horizon at v under the hour-circle EvF, but truly at ξ under the hour-circle EξF; ω, on the other hand, when it truly holds the horizon at ω under the hour-circle EωF, but apparently above it at another point of refraction. And from this it is also clear that the hour-circle drawn through the apparent place of the Sun on the horizon always lies between the two that pass through the true places of the Sun, ξ and ω, and so on.

COROLLARIUM VII.

From the hour or hour-circle of apparent rising or setting being known, together with the place of the Sun in the ecliptic or its diurnal parallel, to elicit the true place of the Sun which it occupies in that parallel, and likewise the true hour, as well as the refraction.

Let both the apparent place v and the hour-circle EvF be known, from the shadow cast by a dial-gnomon [stilus sciothericus], and so on; let there also be known, in the ecliptic LK, the place of the Sun ρ, and the diurnal parallel πo. Since, then, the Sun describes this whole parallel in a single day, and therefore at different times stands at different points of it, it is desirable to investigate the true place of the Sun at the time of apparent rising or setting — which you will obtain as follows. Within the vertical circle CvD, drawn through the apparent rising or setting of the Sun v, where it meets the parallel πo below the horizon, at ξ, there the Sun will truly reside; and the hour-circle EξF, drawn through that place, will indicate the true hour. The arc vξ (that is, AY) of the vertical circle, intercepted between the true v and the apparent place ξ [recte: between the apparent place v and the true place ξ — the source transposes "verum" and "apparentem" here], is the refraction.

Moreover, what has been said about the place of the Sun on the horizon can be applied to any other circle parallel to the horizon; for if you know beforehand the apparent place of the Sun on it, by the shadow of the gnomon, and so on, the vertical arc passing through it to the true place, or to the parallel of the Sun, and so on, will show the remaining things sought.

COROLLARIUM VIII.

From the apparent and the true hour being known, to draw out the refraction, the place of the Sun, the true amplitude, and the anticipated time.

In the analemma seek the circle of the apparent hour — for example EvF — and of the true hour EξF. Then, through the common intersection of the hour-circle of apparent time EvF, which it makes with the horizon at v, draw the vertical CvD. Where it cuts the true hour-circle EξF at ξ, there is the true place of the Sun, and vξ is the refraction. The line oπ, drawn through ξ parallel to the equator GH, is the parallel of the Sun; its common section with the ecliptic at ρ is the place of the Sun in the Zodiac; and its section with the horizon at ω gives the true hour of rising or setting. Meanwhile vN [gives] the apparent amplitude; ωN the true; vω the difference of the two; and ξω the whole time between these true places, and so on.

COROLLARIUM IX.

To track down the apparent hour of the Sun, and its refraction, at any time whatever.

A sundial casting straight, not reflected, shadows — such as vertical dials, and so on — shows the hour of the Sun steeped in the refractions of that same Sun. So, in order to know whether refraction coincides with the hour, note the hour; then take the elevation of the Sun above the horizon with an astronomical quadrant, and so on. Count this off on the arc of the meridian AEC, from A toward C; and from the end of the counting — let it be, say, ζ — draw a hidden line parallel to the horizon AB, which will cut the circle of the given apparent hour (let it be EaF) at some point a. Now, if the parallel described by the Sun runs together into the same point a, then the true and apparent place of the Sun, the true and apparent hour of the Sun, and the true and apparent hour-circle agree entirely, and no refraction as yet perceptible exists in the sky. Thus, if the Sun at the point a, under the altitude Aζ and the hour-circle EaF, runs along the parallel of Cancer IL, no refraction will be found; because both the hour-circle EaF and the parallel IL meet at the same point a.

But if they do not agree, there will be refraction, which the vertical CaD — drawn from C to D through a — will reveal; for where it touches the parallel of the Sun, there it will show the true place and hour of the Sun, and the refraction intercepted between the apparent and the true place of the Sun, as well as the Sun's true elevation above the horizon.

Furthermore, a mechanical clock regulated to the motion of the Sun, if it disagrees with well-constructed sundials, betrays the true hour in itself and the apparent hour in the sundials.

And thus, by the geometrical route, with the help of the analemma, you will very easily be able to bring the truth of most of what has been said to the test of the balance; and a matter otherwise almost beyond human industry you will subdue with no very laborious effort — provided only that the analemma be exact and large.

But the ellipse, taken in through the tube [telescope], best and most clearly of all sets before the eyes — at once and most faithfully — the defect of the Sun, however small it may be; from which you may soon be reminded that refraction dwells in the sky, and the Sun in refraction. You will therefore always take account of this in practical operations, unless you wish to err and be deceived.

PORISMA I

The Sun, even if it were to remain a whole day at the equinoctial point, would nevertheless — refraction being granted — describe from the tip of the gnomon a shadow on the plane beneath that is by no means straight, but curved.

CHAPTER LV

Let ABCD be a horizontal plane — whether the true horizon, or one truly parallel to it, set upon the earth's surface, for as regards perception and the present matter it comes to the same thing. Let AEC then be the vertical circle, and BFD the equinoctial [aequinoctialis], with the point of the equinox B, so that B is the true rising and D the true setting. Onto the horizontal plane let a gnomon GH be erected, its tip H — which, though in reality it stands distant from the center of the universe, nonetheless produces the same effect, as far as perception goes, provided it does not lie in the horizontal plane. It will therefore cast its shadow onto that plane along the straight line BD, which is the common section of the equator with the horizon — as the writers on gnomonics everywhere demonstrate, and as is self-evident in the right sphere.

Diagram. A large circle representing the sphere, showing the vertical circle AEC, the equinoctial BFD, the gnomon GH, and the curved shadow-track NGL.

- E (top of circle): summit of the vertical circle AEC (the zenith point).
- A (left end of the horizontal diameter): western point of the horizontal plane ABCD.
- C (right end of the horizontal diameter): eastern point of the horizontal plane ABCD.
- I (upper left, marked with a star/Sun symbol): the pole, toward which the Sun moves as it acquires northern rising-amplitude.
- F (upper right): the meridian point on the equinoctial (where the Sun crosses the meridian).
- B (interior middle, between K and N): the true rising-point and equinoctial point on the horizon.
- K (left of B): the point at which the Sun actually rises, raised by refraction.

- N (right of B): the point of the late (evening) shadow.
- H (interior, below the K–B–N row): tip/apex of the gnomon.
- G (below H): foot of the gnomon (its base, the center).
- M (bottom, left of D): the point at which the Sun sets, raised by refraction.
- D (bottom center): the true setting-point.
- L (bottom, right of D): the endpoint of the shadow, diametrically opposite the rising.

Suppose, then, that the Sun traverses the whole semicircle BFD of the equator while remaining at the same point of the ecliptic. Even so, the shadow cast from the tip H onto the plane ABCD beneath will be a curved line. For at its rising B the Sun will acquire a northern rising-amplitude [amplitudo ortiua], and so will approach the pole I; by the character of refraction it will rise at K, and will terminate its shadow at the diametrically opposite point L. In the same way it will set at M — for the same cause — not at D, casting its evening shadow at N. And thus everywhere it will make a more southerly shadow, until it has escaped refraction; and the shadow's line will turn out to be NGL, a curve marked off at points, not a straight line. Sensible experience too bears this out, as I have often found for myself when, on the days of the equinoxes, in investigating this line, I searched for the meridian.

But you will object: even with refraction removed, an upright style in the oblique sphere will nonetheless trace from its tip a line that is not straight but curved, provided the Sun is kept at the same station at the true equinoctial point until it has run through the course of a whole day. This is shown as follows.

In the same diagram resumed, let GH be the upright style on the horizontal plane ABCD. Then, refraction removed, the Sun in the morning will rise at the true rising-point B,

and — since the pointer GH lies in the plane of the true vertical — the shadow of the whole style will be cast to the true setting-point D. Now when the Sun is gradually lifted above the horizon, the tip of the shadow cast from the point H falls entirely outside the shadow GD first mentioned, and passes beyond the plane of the equator BFD and its common section BD with the horizon; and when the Sun illuminates the meridian point F, the shadow from H reaches northward to G, or beyond, according to the inclination of the sphere, and so on. But the first morning point and the last evening point — cast from the apex H with the Sun's center located on the true horizon — fall upon the line toward B, extended to infinity, because the Sun is larger, and therefore higher, than the point H; its shadow is accordingly too low to be parallel to the line BD. These things being laid down, the point at issue is pressed further thus: that line is not straight whose middle points do not correspond to its extremes, or whose extremes do not overshadow its middle, and so on. But precisely these things happen to the track of the shadow issuing from the point H; therefore it is not a straight line. The major premise is evident; the minor stands proved; therefore the conclusion binds.

From this it is further inferred: if the equinoctial line cannot be had straight even when the Sun's center does not deviate a hair's breadth from the plane of the equinoctial circle, how much less will it be obtained once the proper motion along the ecliptic is added, which soon carries the Sun away from the circle of the equator? And when, on top of that, so great a malignity of refractions supervenes? And when, besides, a huge portion of the earth's semidiameter is added in? Hence the determination of both this line and the meridian line by this method proves not merely difficult, but exceedingly slippery and of little reliability.

I reply, first, that this whole argument would have carried some appearance and force if the Sun were both small and very close to the earth, or the earth sensibly large in relation to the Sun's sphere. But as things stand — since the Sun has obtained a distance from the earth whose ratio to the earth's semidiameter escapes all perception by its very smallness, while the Sun's magnitude in turn far exceeds the earth's magnitude — it follows that those projecting heights of styles in gnomonic matters are utterly imperceptible.

I reply, then, secondly, that this difference by which the shadow-line departs from straightness is perceptible not by sense but by reason alone. And that this may be better understood:

I reply, thirdly, that the plane of the equator is taken in two ways among mathematicians: geometrically, and astronomically (or gnomonically). In the former way it is devoid of all latitude [width], and so it is taken in the objection — but wrongly; in the

latter way, it is not devoid of some latitude, but acquires just so much as the earth's diameter amounts to — and not without reason. For since the Sun far exceeds the size of the earth and emits from every point a cone of illumination, it follows that the axis of the radiant solar cone may be reckoned to be not only the ray which the Sun directs from its own center to the tip H, but also the ray which shines from another point of the Sun onto the tip H, and which stands parallel to the ray drawn from the Sun's center through the earth's center. And thus we obtain the flat gnomonic surface of the equator, built up out of many geometric planes equidistant from one another, enclosed within the width of the earth's diameter. Consequently, as long as the Sun's center has not passed beyond it, the vertex H of the style GH — representing the center of the world — will always, resting in some one of these planes, occupy it. Hence the shadow of such a style will always, the whole day long, fall on the common section of the horizon and the plane, even in the very strictest sense. For it is not the whole shadow of the upright style, but its tip alone, that marks out the line of the equator. And so the nerve of the whole objection raised is severed.

For although the geometric plane of the equinox BFD makes the true equinoctial line — drawn through the earth's center G — the common section BD with the horizon, nevertheless the remaining planes, parallel to this as to the principal one, are carried through the vertices of styles set outside the center of the world, and make equinoctial lines parallel to the first, and so on. And the fact that the vertex H, morning or evening, extends the tip of the shadow to infinity is no obstacle; this happens because the Sun still lies on the horizon. But as soon as the Sun has risen above it, in proportion, by just as much as the length of the style, then the shadow soon strikes the horizontal plane beneath; and, with the Sun placed at the true equinoctial point, it marks out a perfectly straight line, produced by the ray running along the equinoctial plane taken gnomonically, not geometrically, and so on.

Hence the magnitude of the earth, urged as an objection, tells against a plane accepted geometrically, not astronomically; and the Sun's proper motion along the ecliptic in a single day scarcely carries the Sun's center outside the equinoctial plane taken gnomonically. As for refractions, they are guarded against if the equinoctial line is taken not entirely from rising to setting, but described from the ninth or tenth hour before noon to the second or third hour after noon. And yet these are not the only methods for tracking down the meridian line, but others too — neither few nor untrustworthy — are at hand.

PORISMA II

In one and the same visible hemisphere, the Sun and the earth's shadow cast by the Sun appear at the same time.

CHAPTER LVI

Let A be the earth (which I now suppose imperceptible in relation to the solar heaven and the firmament); let BCD be the visible hemisphere and BAD the horizon; and below the horizon stands the Sun at the point E, whose ray EA casts the shadow AC. And since EA is below and AC above the horizon, let the Moon then be placed at C, diametrically opposite the Sun E. An eye at A will therefore see the earth's shadow at C, for it hides the Moon — and so it is seen.

Again, let EB be so small an arc that it does not exceed the amount of refraction — say of half a degree, of one, or of two degrees. Then the Sun, being at E, will by refraction rise to the point B, and so an eye at A will behold it. Therefore both the Sun E and the shadow H made by the Sun will appear together above the illuminated hemisphere just described. From these things:

Diagram. A circle representing the visible hemisphere BCD with horizon BAD, showing the Sun below the horizon at E, its refracted position B, and the rays cast toward D and F.

- I (top): the zenith / summit of the hemisphere.
- A (center): the earth, and the eye of the observer.
- B (left, on the horizon line): the point to which the Sun is raised by refraction (from E); a horizon point.
- C (upper right): the point where the earth's shadow falls, where the Moon is placed.
- D (right): the point on the hemisphere toward which the ray from A proceeds.
- E (lower left, below the horizon): the true position of the Sun.
- F (lower right): the point, below D, where the refracted ray falls.
- G (bottom left): the lower endpoint of the shadow/ray construction.
- H (upper right, near C): the shadow made by the Sun, appearing above the horizon.

I conclude, first: the shadow cast by the direct Sun, such as AC, is cut by refracted rays; such a ray is, on the near side of A, BA, and beyond A, AF. For from A that ray proceeds toward D; and if the surface of the vapors be supposed everywhere equally affected — which can sometimes happen —

I conclude, secondly: that it falls below D at the point F, as far removed from D as D itself departs from C. For the point of incidence, or of refraction, in the vapors bounds the same obliquity; and just as the ray EA is refracted toward the perpendicular, so the ray at A — refracted within the vapors marked at the same point A, once it begins to leave them — is refracted away from the perpendicular as far as F. These matters are made plain to the eye in the adjoined figure ABEG, in which:

AF is the earth; BC the vapors around the earth, uniform; D the Sun; DB the ray from the Sun falling upon the vapors; B the point of incidence and of refraction; EBF the perpendicular drawn to the earth's center F; and since the vapors BC are denser than the upper air BEGC, the ray BA, refracted into the eye A, approaches the perpendicular BF. And since the temper of the vapors is supposed the same, therefore the ra[y]—

Diagram. A small figure of two concentric circles: the inner circle is the earth (with center F), the outer bounds the vapor-layer/upper-air region, with refracted rays passing to the eye at A.

- A (center): the eye/observer on the earth's surface.
- F (bottom center, within the inner circle): the center of the earth.
- B (left of center): the point of incidence and refraction on the vapor layer.
- C (right of center): a point on the vapor layer BC.
- E (left): a point of the construction (perpendicular EBF; upper air BEGC).
- G (upper right): a point of the upper-air region BEGC.
- D (Sun; source of ray DB): position at the periphery — [?] not clearly resolvable.
- H (lower right): [?] label present but role not resolvable at this resolution.
- I (right): [?] label present but role not resolvable at this resolution.
- K (far left): [?] label present but role not resolvable at this resolution.

the refracted ray BA continues straight from A to C, reaching the convex surface of the vapors. After it emerges into thinner air, the result is that it turns away from the perpendicular CG and no longer runs straight toward I, but drops below it, as far as H — so that the angle HCI of the later refraction equals the angle DBK of the earlier refraction, which was produced at point B. This holds because of the equality of the media, which are vapors, and because the angles of incidence at B and at C are likewise equal.

Conclusion 3. When the Sun stands at point E of the first figure, then for those whose horizon on the opposite side would be depressed by the line GH, the Sun will appear after its true setting at point G; the earth's shadow, cast in a straight line by the interposition of the earth, will be seen at C; and the refracted ray GA will fall at I. And so, if the arc of refraction EB is set at two degrees — as can happen, and as has in fact happened according to the historical records — then, since EG equals it on the opposite side, the whole arc BG will be 4 degrees. But the whole arc IF of the diametrically opposite refraction, illuminated by the Sun's rays advancing through refraction, will be 8 degrees. Hence:

Conclusion 4. For the most part the earth's shadow-cone is entirely cut through and penetrated by refracted solar rays — from which:

Conclusion 5. There is no lunar eclipse in which (provided some refraction exists) the Moon is not, over its whole body, at least steeped in some light from the refracted rays — a fact that contributes not a little to its various colors. For since the opposite arc that the refracted rays occupy is double the arc that the position of the visible Sun determines, what I have said is evident.

Conclusion 6. The dark colors of the Moon are aided in no small measure by the arrival of these rays upon the Moon. Yet we would not deny — indeed we freely grant — that these same colors are also tinged by the colors of the vapors themselves.

Conclusion 7. The opinion of those who ascribe those colors to the surrounding air, or to some vaporous body belonging to the Moon itself, is empty; and still more baseless is the opinion of those who say that the Moon has its own light.

Conclusion 8. What I brought forward concerning the elliptical Sun is powerfully confirmed by these facts — namely, that the chasmata [Greek: χάσματα (chasmata, "chasms" — luminous rifts in the sky)] and kaumata [Greek: καύματα (kaumata, "burnings" — fiery atmospheric glows)] are refracted rays striking against the exhalations and vapors driven off by the Sun.

PORISMA III.

What has been said of the Sun holds also for the Moon and the stars.

CHAPTER LVII.

Everything discussed so far about the Sun, as regards refraction, can be applied to any star of the heavens whatever, merely by changing the Sun's name for the star's — for these too are subject daily to these same variations.

There is no doubt about their trembling, which we call scintillation [scintillatio, "twinkling"], and which arises from the passing-across of restless vapors. In this alone do they differ from the Sun: because of the smallness of their light and the apparent tininess of their bodies, they are agitated more violently by the intervening vapors than the Sun is, or the Moon, or the stronger planets.

On their refraction the authorities likewise agree. But as to whether it is the same as the Sun's, or greater or smaller, take the following propositions [documenta]:

1. That in all distinct media of differing density or transparency, the obliquely received ray of any star whatever is refracted — this is the settled and certain doctrine of all opticians and astronomers. But whether, and of what kind, the transparent bodies are that lie between any given star and the atmosphere — that is not the place to explain here.
2. If the transparent bodies from the higher stars down to the Sun's body are of the same transparency as those from the Sun down to the Moon, then it is certain that the refraction of the stars and of the Sun is one and the same. But if the media vary, the stars are refracted otherwise, and more often, than the Sun is in the higher regions.
3. From the Sun down to the Atmosphere the refraction (other things being equal) is the same for the Sun as for the stars, because the media lying between are uniform, and through them the Sun and the stars shine.
4. From the Atmosphere all the way down to us, the refraction is the same for the stars lying in a single ray of incidence.

Let the eye be at A on the earth's surface, the atmosphere BCD, and the ray incident upon it EC — a ray which the depicted heavenly bodies, whether all together or one after another, may cast: the Moon ☾ , Venus ♀, Mercury ☿, the Sun ☉, Mars ♂, Jupiter ♃, Saturn ♄, and any fixed star ▯ of the firmament. I say that the refraction will always be the same, so long as the Atmosphere remains unchanged.

Diagram. A refraction schematic. At the lower left a small circle represents the observer's eye on the earth's surface, marked **A**, with the point **I** inside it. From this eye-circle a ray runs up to the central point **C**, which sits on the atmospheric arc **BCD** (**B** upper left, **D** below). A long horizontal baseline runs from **G** at the far left, through **C**, and out to **E** at the far right. Strung along this line between C and E lie the seven planet glyphs and a star, all sharing the same incident direction toward C. Two oblique lines rise from C to **F** and **H** at the upper right, marking the angle of incidence (ECF) and the angle of refraction (ACG / ECH).

- A (lower left, at the eye-circle): the observer's eye on the earth's surface
- I (inside the eye-circle): point where the refracted ray falls
- B (upper left): top of the atmospheric arc
- C (center): the point of refraction, at the top of the atmosphere
- D (lower center): bottom of the atmospheric arc
- G (far left of baseline): apparent (refracted) direction
- F (upper right): terminus of the incidence line (angle ECF)
- H (upper right, beyond F): terminus of the refraction line (angle ECH)
- E (far right): the incident-star direction; the bodies below lie along CE
- ☾ ♀ ☿ ☉ ♂ ♃ ♄ ▯ (along CE, left to right): Moon, Venus, Mercury, Sun, Mars, Jupiter, Saturn, and a fixed star

For the angle of incidence ECF is the same, and therefore the angle of refraction ACG (or ECH) is also the same. Therefore the refraction is the same. This, accordingly, I impress upon [inculco] the devotees of Astronomy: that if they wish to occupy themselves with the stars, they should first draw off the quantity of refraction from the Sun by means of the Ellipse, and so apply it to the stars — that they may stray the less from the true position.

To God, the supreme Creator of the stars, and to his Saints, who glitter like the stars — honor whole and without refraction, praise full and without ellipse, unto ages of ages. Amen.

[The main treatise ends here, (printed p. 132–133), with the closing doxology and the decorative tail-piece. What follows is back matter: a laudatory Greek epigram, and then the index, which begins on the next page.]

ΕΠΑΙΝΟΣ

[Greek: ΕΠΑΙΝΟΣ (Epainos, "In Praise" / "Encomium")]

Τῷ τῶ Ἡλίου τῶ ποικιλομόρφου Πρωτοσκόπῳ.

[Greek: "To the First-Observer [Πρωτόσκοπος, prōtoskopos] of the many-shaped [ποικιλόμορφος, poikilomorphos] Sun" — i.e., dedicated to Scheiner as the first to observe the variously-shaped (elliptical) Sun. The article forms are given in contracted Doric spelling.]

The following laudatory epigram is set in Homeric/Doric Greek, heavily ligatured, and adapts the story of Apollo's founding of his temple and oracle at Crisa (the Homeric *Hymn to Apollo*) into praise of the "First-Observer." Per the dispatch instruction, the text below is transcribed only as far as the glyphs can be read on the page; uncertain words are flagged `[?]` and are **not** reconstructed from the *Hymn*. The translation that follows is correspondingly cautious.

> Φοῖβος ἐπὶ Κρίσην τεύξας περικαλλέα νηὸν Ὑιάσιν Ἐργίνου συνθεμένοισι πόνῳ· Φράζετο δὴ κατὰ θυμὸν ὅσους ὀργίονας ἔνθεν Ῥέζοντας σφετέρῳ εἰσαγάγοιτο θυεῖ· Ταῦτ' ἄρα ὁρμαίνων ἐνόησ' ἐπὶ οἴνοπι πόντῳ Νῆα θόην ἁλίῳ ὄιδματι πλαζομένην· Κρῆτες ἀπὸ Κνωσοῦ Μινωίου ἔπλεεν ἄνδρες, Ὥστ' ἐπὶ πρῆξιν πρὸς περάωσι Πύλον· Τοῖσι δὲ ἀντιόων τὸ δέμας δελφῖνι ἐοικὼς Πύθιος, ἐλθούσης νηὸς ἔλασσεν ὁδόν· Ἀστέρι τ' εἰδόμενος λαμπρῷ κατὰ θῖνα θαλάσσης· Σπινθήριγας προΰχεεν καιομένοιο πυρός· Ὧδε [?]ραπτόμενοι ἐλάτῃ παρὰ Κρίσαν ἵκοντο Ἔνδιον Βιομοῦ ἔνθα παρευξάμενοι· Εὗρεν ἐληλαμένους ἔθεν ἀρητῆρας Ἀπόλλων, Ἐκ δ' ἐλαῶν ἱερεῖς εἵλετο Φοῖβος ἔθεν· Νῦν δ' ἱερεῖς δίζηθε θεὸν ζητεῦντα ποτ' ὑμᾶς, Καὐτὸς γὰρ βλέπεται ἀντίον ἐλλιπέμεν· Ὄφρα δὲ οὐκ ἀέκοντες ἴῃ[?] προβαίνει ἂν ἄλλος, Τῷ μετ' ἔπεθε πελη[?] Φοῖβος ἐκεῖνος ὁδοῦ.
>
> resolved; 2 words remain genuine engraving/ligature floor ([?]ραπτόμενοι, ἴῃ[?]/πελη[?]), identical to the IA copy. — Superseded in part by the cold Greek audit of 2026-07-31 (greek_audit_036.md): this review's reversion of ἀντιόων(l.94) to the floor readingἀ[?]λίωνwas itself the error — the cold read ratesἀντιόωνCERTAIN on the ντ ligature and it has been restored. The review's cross-confirmation of the signature as Φιλάριστος is likewise superseded: the Freiburg tile readsΦιλάςρι[ος]` = Φιλάστριος.]

[Greek, cautious rendering — many words flagged `[?]` above are uncertain and the sense is correspondingly approximate:]

When Phoebus [Apollo/the Sun] had built at Crisa his most beautiful temple, with the [sons?] of Erginus [?] joining together in the labor, he pondered in his heart what temple-servants [?] he might bring from there to perform his rites at his own shrine [?]. Turning this over, he caught sight, upon the wine-dark sea, of a swift ship wandering over the salt swell [?]. Cretan men were sailing from Minoan Knossos, making their way [?] on their business toward Pylos. Meeting them, Pythian [Apollo], like a dolphin in form, came [?] and drove the ship's course; and, appearing like a bright star along the strand of the sea, he poured out sparks [?] of burning fire. So [borne along?] they came under [oar of] fir to Crisa, [drawing up at the midday shore? — uncertain]. Apollo found them, driven there by himself, to be his priests; and out of [the sailors? / the olive-groves?] Phoebus chose priests for himself. "Now, priests, seek out the god who once sought you — for he himself is seen [?] facing you [?]. And that others may go not unwilling [?], another shall come after; for Phoebus keeps to that road [?]."

Φιλάστριος Ἐπιφανείου.

[Greek: signed "Philastrios, [son] of Epiphaneios" — a pen-name ("star-lover", apt for a solar-observation encomium). ***Reading differs across the two witnesses:*** *the Freiburg copy sets `Φιλάϛρι[ος]` =* ***Φιλάστριος****, confirmed by the cold Greek audit of 2026-07-31; the IA copy was decoded as* ***Φιλάριστος*** *("lover of the best/noblest"). The page reading is printed here. Not a fabrication on either side, a decode ambiguity of the inked signature.]*

Index of Principal Matters

INDEX LOCVPLETISSIMVS, RERVM PRÆCPVARVM [recte: PRÆCIPVARVM]

(A Most Copious Index of the Principal Matters)

For whose ready use, note the following [Pro cuius expedito usu nota sequentia]:

`p.` denotes the page with its number. `v.` a line [Versum]. `c.` a chapter [Caput]. `conc.` a Conclusion [Conclusionem]. `cor.` a Corollary [Corollarium]. `as.` an Assertion [Assertionem]. `n.` the Number prefixed to the text [Numerum textui praefixum]. `§.` a Paragraph [Paragraphum]. `dic.` "to be said" [Dicendum], &c.

secondary altitudes makes with the vertical line. p. 7. c. 4. The apparent altitude is greater than the true, from refraction toward the perpendicular line. p. 33. c. 22. It is established chiefly for three points of the Sun: highest, middle, lowest. p. 109. v. 8.

Amplitudo ortiva & occidua (Rising and setting amplitude) — In the oblique sphere, because of refraction, it is increased more than it should be while the ecliptic ascends in the semicircle, and diminished while it descends. p. 45. n. 8. & p. 53. n. 6. In the right sphere it is not sensibly changed. ib. n. 7. In the oblique sphere it carries the refracted Sun into alien hour-circles, and therefore anticipates and postpones the hour-lines and the shadow of the style [stylus/gnomon]. p. 52. "to be said" 3 [dicendo 3]. This happens much more in the parallel sphere. ib. dicendo 4. It is less than just in the southern signs, greater in the northern. p. 118. v. 1. Those which lie under the equator alone obtain amplitudes less liable to errors. p. 120. n. 7. — To investigate the rising or setting amplitude. p. 122. Corol. 2. To derive refraction from that apparent [amplitude]. ib. Corol. 3. The same otherwise. p. 123. Coroll. 4. From the Sun's place in the zodiac and the anticipated time being known, to ascertain the true and the apparent amplitude, and much else. p. 123. Coroll. 5.

Analemma — is wonderfully useful for the business of refraction. See c. 53. & 54.

Anhelatio (Panting) — stirred up by running or violent exertion, where it comes from. p. 87. num. 8.

Angulus incidentiae (Angle of incidence) — the same as the inclination of a line upon a surface. p. 4. v. 9. — It is equal to the angle of refraction and to the refracted [angle]. p. 4. v. 17. p. 20. n. 2. Its greater or smaller quantity is detected from the greater or smaller obliquity of the incident ray. p. 21. c. 15. p. 22. c. 16. p. 23. c. 17. p. 24. c. 18. See: Inclinatio.

[Angulus] Refractionis (Angle of refraction) — p. 4. v. 12. p. 20. c. 14. p. 99. c. 45. p. 102. c. 47. In sense it does not differ from the angle which the line of refraction and the ray extended unrefracted from the visible solar point into the eye comprehend at the same [point]. p. 102. 103. 104. c. 47. See: Refractio Solis, &c.

[Angulus] Refractus (Refracted angle) — p. 4. v. 14.

Angulus mixtus (Mixed angle) — which is made from a circular arc and a ray thrown out from one point of the diameter to the periphery, [that angle] which is smallest. p. 26. Coroll. 3. — Or, if different points taken outside the circle are aimed at one point of the diameter outside the center. ib.

Index (continued) — letters C–F

The alphabetical index continues. Each entry gives the Latin headword, its sense, and the page / chapter / number / line references as printed. Page references are transcribed as printed and are secondary; abbreviations are kept in compact form (p. / pag. = page, c. / cap. = chapter, n. / num. = number, v. = line, § = section, ib. / ibid. = the same place).

called the cathetus. When it passes through the point of incidence or of refraction, it is called the perpendicular of refraction, or the perpendicular to which, or from which, refraction takes place. pag.3, n.13; p.4, v.3; p.34, v.last; p.35, v.13, etc. It is adjacent to the angle of incidence. p.4, v.9. Likewise to the refracted angle. p.4, v.15. When it is erected from a visible point upon the refracting surface, it is called the cathetus, or perpendicular of incidence. p.21, c.25.

Lineae Rectae, Erectae, Directae [straight, erect, direct lines] — The distinction between straight, erect, and direct lines. See Radius [ray].

Linea Refractionis [line of refraction] — The line of refraction, or the refracted [line]. p.4, v.7. It is adjacent to the angle of refraction. pag.4, vers.12. Likewise to the refracted [angle]. ibid., vers.14, etc.

[Linea] Verticalis [vertical line] — The vertical [line]. p.1, n.6. It lies within the plane of refraction. p.5, n.5; p.6, c.3. It is the common section of all vertical planes. p.7, c.4.

Lineae [lines] — The lines — of incidence, the perpendicular, the refracted or of refraction, and the vertical — lie in one plane of solar refraction, which is always and only some vertical plane. pag.5, num.4, 5, 6, 7, etc.

De Lineis [concerning lines] — If you want more about lines, consult: Altitude, Longitude, Plane, Section, Surface; Dimension, etc.

Liquabilia [meltable things] — All meltable things occupy more room when liquefied than the same [things] when compacted. p.87, n.9; p.89, n.9.

Locus [place] — A place is enlarged by rarefaction and contracted by condensation. pag.85, 88, 89, 90.

Locus Solis [position of the Sun] — The apparent [position] is determined from the refracted line. pag.35, c.22.

Locus Solis refracti [position of the refracted Sun] — The position of the refracted Sun remains unimpaired in the zodiac, as shown on clocks constructed from altitudes. p.52, n.7. It is displaced on others, always and under every sphere. ib. The equator alone, in the right sphere, keeps the Sun in its due place in the zodiac; it lets the other parallels wander — and why. p.52 & 53, n.7.

Locus Solis visus & verus [apparent and true position of the Sun] — The apparent and true positions of the Sun are always in the same vertical, or plane of refraction. p.105, about the middle. Apparent at the horizon; true below it. p.107, near the end.

INDEX (continued: L–P)

Alphabetical index to Scheiner's Refractiones coelestes. *Two-column layout; entries carry printed page and chapter/line references, kept as printed (p. = pagina, n. = numerus, c./cap. = caput, v./vers. = versus, concl. = conclusio, §. = section). The "Length" [longitudo] and "Altitude" of the Sun are the horizontal and vertical dimensions of the refracted, elliptical solar disc near the horizon.*

L (continued)

- **Longitudo Solis primaria** — The primary Length of the Sun [longitudo Solis] is the diameter of the same, perpendicular to its altitude. p. 1. n. 9. p. 3. v. 1. & 8. p. 6. c. 3. It coincides with the horizon. p. 11. v. 1.
- **Secundaria** — The secondary [Length] is a chord smaller than the diameter in the solar circle, parallel to the primary Length. p. 2. n. 11. p. 3. n. 12. p. 6. cap. 3.
- **Secundum Longitudinem Sol extra horizontem sensibiliter nunquam refringitur** — In respect of Length the Sun outside the horizon is never appreciably refracted. p. 13. 14. 15. 16. c. 12. On the horizon, not at all. p. 16. concl. 4.
- **Longitudinis dimensiones passiuae** — The passive dimensions of Length — what they are and how manifold. p. 13. c. 12. How and when refraction occurs in them. pag. 16. n. 1. 2. 3.
- **Longitudo Solis deficit, sed non sensibiliter** — The Length of the Sun is deficient, but not appreciably. p. 35. c. 23. n. 3 & 5. The senses prove this. p. 60. n. 2 & v. 26. The same is retained on a single day with the same aperture of the tube; over a whole year, with the tube loosened in various ways and its distance from the paper varied. p. 98 c. 44.
- **Luna** — The Moon undergoes the same refraction as the Sun. p. 17. as. 2. p. 132. c. 57. By its colours it foretells storms. p. 71. to the end.
- **Ex Eclipsi aut oppositione Lunae cum Sole** — From an eclipse or an opposition of the Moon with the Sun on the central horizon it is possible to hunt out the refraction. pag. 101. & 102. The colours of the eclipsed Moon arise chiefly from refracted solar rays mixed with the earth's shadow and falling upon the veiled face of the Moon. By no means are they implanted by nature, or produced by the air surrounding the Moon. p. 131. concl. 5. 6. 7.
- The Moon is less agitated by vapours than the other planets or stars. p. 132. c. 57.
- **Lux Solis diutius commoratur in quouis loco, quàm tenebrae** — The Sun's light lingers longer in any given place than the darkness does. p. 54. v. 8.
- **Lux Solis obtunditur & confunditur ab exhalationibus viscosis** — The Sun's light is blunted and confounded by viscous exhalations. p. 70. 71. &

INDEX (continued) — letters P, Q, R

Alphabetical subject index. Two columns per page; entries read down the left column, then down the right. Reference abbreviations are kept close to the source: p./pag. = page, v./vers. = line, n./num. = number, c./cap. = chapter (or, where it *follows a "conc." context, "conclusio"), conc./concl./conclus. = conclusion, ass./assert. = assertion, ib. = ibidem, Coroll. = corollary, arg. = argument, obiect. = objection, dic. = dictio, ad = "in reply to," &c. = etc.*

(Continuing the entry ***Planum*** *[the plane], begun on the preceding page — catchword "Refra-" → "Refractionis".)*

Planum *(continued)*

- **Refractionis** — [the plane] of refraction. p. 5, v. 27; p. 6, cap. 3; p. 8, c. 5; p. 9, c. 8.
- **Verticale** — the vertical [plane], according to altitude, which one it is. p. 2, num. 12; p. 3, num. 12. It is always the plane of refraction. p. 5, v. 29; p. 8, c. 6.
- [The plane] distinguishing the seen portion of the Sun from the unseen. p. 1, n. 3; p. 2, n. 12.

Pluvia *(Rain)* — Falling rain purges the air of viscous exhalations, moistens and cools it, and fills it with dewy vapors; hence it produces a clear atmosphere and a larger Solar ellipse, p. 72. It washes away the lower and cruder parts of the dull atmosphere, p. 73, v. 8. Over the matter of the twilights it has little power, p. 78, n. 14. Why the rare [atmosphere] appears close at hand, and the dense [atmosphere] far off, p. 81, near the end.

Praenoscenda *(Things to be known beforehand)* — p. 1, 2, 3, c. 1.

Proiectiones umbrarum *(Projections of shadows)* — onto the hour-lines, what they suffer from refraction. p. 50, 51, 52 & 53.

Proportio *(Proportion)* — The ratio of refractions and of the times corresponding to them is the same. p. 50, num. 4. How the ratio of the defect of altitude to the Sun's longitude may be found, both geometrically and arithmetically. p. 95, c. 42.

Proportio siderum *(The ratio of the stars)* — The ratio of the stars to one another is detected by the same aperture of the telescope. p. 99. Of ellipses and refractions. p. 113. It is attacked, p. 114 & 115; it is defended, p. 115 & 116.

- The increase and decrease of the ellipse is not proportional to the Sun's ascension or descension. p. 97, c. 43; p. 116. Resp. 5.

Proprietates Atmosphaerae *(Properties of the atmosphere)* — Of the clear [atmosphere], p. 70, v. 6. Of the turbid, p. 70 & 71, at the end and the beginning.

(right column)

highest, middle, and lowest points of the Sun. p. 111. The linear quadrant most skillfully investigates the apparent magnitude of the Sun, and its elevation above the horizon according to the highest, middle, or lowest point, and likewise the ellipse. p. 112, c. 51. The clock-quadrant teaches the place of the Sun in the zodiac. p. 123.

Quantitas Refractionis *(The quantity of refraction)* — is the angle of refraction, comprehended by the line of refraction and of incidence drawn within the transparent medium. p. 20, c. 14; & p. 99, c. 45. The quantity of refraction, absolute and comparative. ibid.

- Of the obliquity of rays from one point taken externally, projected onto different points of the circumference. p. 21, c. 15.
- Of the obliquity of rays from one point of the circumference falling onto different [points] of the diameter. p. 22, c. 16.
- The quantity of the inclination of rays from one point of the diameter falling onto different [points] of the circular periphery. p. 23, c. 17 & 24; and p. 25, c. 18.
- The quantity of the inclination of the chords of a circle to the circumfer-

(right column) — ...to the circumference. p. 26 & 27.

- Of the solar elevation above the horizon, greater than it should be. p. 33, vers. 25.
- Of the Sun seen by refraction, less than it should be. p. 35, n. 2, 3, 4. Greater than it should be. ibid., n. 2 & 5. Of [the Sun] seen without refraction, the natural [quantity]. p. 36, n. 6.
- The [quantity], of the Sun and likewise of the solar sky [caelique solaris], cannot rightly be obtained by the flow of water or the running of a horse [equi cursu], for the emergence of the solar globe above the horizon. p. 59, v. 15.

Quantitatem Refractionis solaris certam promere *(To bring out the exact quantity of solar refraction)* — is to express the angle of refraction in degrees and minutes. p. 100. Which is obtained either from prepared Tables of Refractions, or from the Tychonic modes of observing. ibid. & p. 101. Or from the known height of the atmospheric surface. ibid. Or, more certainly and more easily, from the arc of the vertical circle intercepted between the true and the apparent place of the Sun, by means of a quadrant under the true point of rising or setting once known. p. 105. Or by a mechanical clock and clock-quadrant under the apparent rising or setting. p. 106, c. 49. Which you may also attain by calculation. p. 107 & 108.

that ray which, refracting from the point of incidence into the medium, continues on without any angle, as is sufficiently clear from the places cited. One thing is a "straight ray" (radius rectus), p. 1, n. 4; another is a ray "straight" [perpendicular] to some line or surface, p. 17, n. 1; p. 18, conc. 2, v. 34. And throughout: there [in the first sense] "straight" is the same as "rectilinear," here [in the second] the same as "erect"; there straightness (rectitudo) names the ray, here it names incidence, &c.

- **Radius Inclinatus** *(The inclined ray)* — properly, is joined with the power of cutting the surface to which it is inclined. p. 29, v. 5; p. 32, ad 3. And it always proceeds between the erect and the tangent. For if it is erect, it is no longer inclined; if it is tangent, it lies wholly [flat]. p. 30, n. 3; & p. 31, ad 3, &c. See **Inclinatio**, **Obliquitas**, &c.
- **Radij** *(Rays)* — from one point, falling onto different points of the periphery, what angles of inclination they make. p. 21, c. 15. Falling from various [points] taken on the diameter, or outside the circle, onto one [point] of the circumference. p. 22, c. 16, and in the Corollary. From one [point] of the diameter onto various [points] of the periphery. p. 23, c. 17; p. 24, c. 18. Of which the most oblique is that which is straight [perpendicular] to the diameter. p. 26, Coroll. 2.
- **Radius solaris** *(The solar ray)* — when direct, p. 3, n. 13; it is not refracted. p. 17, c. 13, n. 1; p. 20, n. 1.
- When refracted, p. 3, n. 13; what it is, ibid., v. 17; p. 4, v. 1. It comes only from oblique incidence, p. 17, c. 13, n. 1. Most oblique in the sensible horizon, and therefore the greatest of the refracted [rays], p. 27, c. 20. It designates the place of the Sun seen by refraction. p. 33, c. 22, &c. It is tinged by the colors of the atmosphere. p. 71, v. 5; p. 72, at the beginning, and throughout. Carried across the vapors into the region opposite to the shining and refracted star, it doubles the quantity of refraction. p. 130, 131, conclus. 2 & 3. It cuts the shadowy cone of the earth, [cutting] the rays of the same star, both direct and refracted. ibid., concl. 4. And through this it dilutes the shadow of the earth, [and] the

*(continuing **Radius solaris**)* — …[and] the eclipsed Moon it disfigures with foul colors, and it condemns the frivolous opinions of certain persons. ibid., concl. 5, 6, 7.

- **[Radius] Incidentiae vel incidens** *(The ray of incidence, or the incident [ray])* — p. 5, num. 13. See **Incidentia**, **Inclinatio**, **Linea**, &c.

- **[Radius] Tangens** *(The tangent ray)* — touching the atmosphere purely, is not refracted at all. p. 30, cap. 21. Properly it is not inclined toward the sphere, but is the limit of the inclined rays. p. 32, ad 3.

Rarefactio *(Rarefaction)* — enlarges the place of the rarefied body. p. 85, 86 & 87. It happens not only by heat, but also by traction and the dread of a vacuum. p. 85, ad 2; & p. 86, n. 6 & 7. The manner of introducing and of measuring rarefaction. ibid.

Raritas *(Rarity)* — one kind is of rarefied yet continuous parts, another of dissolved [parts]. The former increases the clearness and transparency of the transparent medium, the latter diminishes it. p. 88, 89 & 90.

Regulae aliquot *(Some rules)* — for inquiring whether the magnitude of the earth is sensible in relation to the distance of the Firmament, or not. p. 45, dic. 1; & 46, dic. 2, 3, 4, 5, 6.

Reflexio & Refractio *(Reflection and Refraction)* — succeed each other in turn. p. 50, n. 2. As the Sun reflected from a mirror stands in relation to itself, by the multiplication of its own image, so the [Sun] refracted in vapors [stands] in relation to the same. p. 56, n. 8 & 9. It also occurs from the depth of a continuous body. p. 83, near the end; p. 80, n. 9; p. 82, v. 22.

Refractio coelestis *(Celestial refraction)* — befalls all points of the sky, whether luminous or dark, when they have radiated into the vapors. p. 16 & 17, assert. 1, 2, 3. It arises from the incidence of some visible point onto the surface of the atmosphere concentric with the earth. p. 17, [right col →] num. 1, 2, 3, 4, 5. Toward the perpendicular line. p. 18, concl. 1 & 2. It is defined. p. 20, c. 14; p. 29, cap. 21, arg. 1. It requires a surface joined to the refracting body. p. 32, ad 2. And [it requires] the power cutting the ray of incidence. ibid. Its quantity is the angle of refraction. p. 20, c. 14; & p. 99, c. 45. It affects each single point of the open sky, except the zenith. It raises that same [sky] higher than it should be. What it can do among the circles of the sky, &c. p. 43 & 44, c. 27. It places the stars in false positions and altitude. p. 44 & 45, c. 28. It shortens the distances of those same [stars] among themselves and from the horizon. ibid. It increases and diminishes the rising and setting amplitude. ibid. What celestial phenomena it brings to the earth-dwellers. p. 55, c. 30. It weakens light and color. p. 87, obiect. 4. It infects refracted bodies with alien colors. p. 71, v. 3; p. 72, v. 6; p. 93, v. 14; p. 131, concl. 6; and elsewhere throughout.

Refractio Solis *(Refraction of the Sun)* — uniform [and] insensible. p. 13, c. 12, concl. 1. Non-uniform [difformis] [and] insensible. p. 14, c. 12, concl. 2. Non-

*The index continues beyond this page (the entry **Refractio Solis** breaks off at the catchword "ad"). No errata table and no colophon appear in this batch.*

INDEX (continued)

[The two-column alphabetical index continues. Page references are given verbatim as printed, using the book's own abbreviations: `p./pag.` = page, `c./cap.` = chapter, `n./num.` = number (article), `v./vers.` = line, `ib./ibid.` = in the same place, `&c.` = and so on.]

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INDEX (conclusion), ERRATA, and Colophon

[Two-column alphabetical index. Running header on each printed page: "INDEX." Page references follow each entry; where the source gives chapter (c./cap.), number (n./num.), verse/line (v./vers.), corollary (coroll.), conclusion (concl.), or section (§.) locators, these are preserved as printed.]

Index — letter Z

[End of the Index.]

ERRATA

*[Table of printer's errors. Columns: **Pagina** (page), **Versu** (line — with "Antepenultimo" = in the third-from-last line, "Penultimo" = in the second-from-last line, "ad fin." = near the end, "à fine" = counting from the end), and **Erratorum Correctio** (the corrected reading to be substituted). The corrected readings are given verbatim in Latin.]*

Pagina	Versu	Erratorum Correctio
12	8	Sole meridiem faciente.
19	Antepenultimo	circumiectum proximè.
22	Penultimo	radius ad eam rectus.
43	11, ad fin.	hemisphaerium.
44	6	quod.
44	17	eandem.
50	2	apparenter insumat.
51	11	horam.
53	3	centrum suum visum.
54	18	diriguntur.
58	11	diametros.

Pagina	Versu	Erratorum Correctio
67	3	loco, valvisq[ue] clausi[s].
68	28	rarefactione.
69	3	*or*-, deleatur.
72	1	ostendit.
77	19	frigidissimus.
77	5 à fine	mensurare.
84	35	accessus.
85	35	tractione.
87	19	debilitare.
87	21	diaphana.

Omnia ad maiorem Dei gloriam. *All to the greater glory of God.*

[This motto is the final printed line of the book — the last content of the work.]

[The book ends here. (the verso of this final leaf) is blank; it shows only mirror-image ink strike-through from the Index/Errata on its recto and from the title page bleeding through the gathering. No printer's imprint appears on this closing page — the imprint stands on the title leaf.]

[Manuscript note: pencil/ink library shelfmark notes on the rear flyleaf — "Rara / J / 1698" above the line, then below: "nur beschränkt benutzbar" ("only restricted use"); "Ersatzex.: Tl. 1 2 3 4 5 6 7 = MK + MK/M91/226" ("replacement copy: parts 1–7 = MK + MK/M91/226"). Modern librarian's hand; no printed content.]